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Serie 01

06.03.18

A1.1) Gegeben: Stützstellen $(1,4,9)$ für Funktion $f(x) = \frac{1}{\sqrt{x}}$

Gesucht: Ein Polynom 2.ten Grades mit $\frac{1}{\sqrt{x}}$ bei $(1,4,9)$

Polynom $p(x) = \sum_{i=1}^{n} y_i \cdot L_i(x)$

$$L_i(x) = \frac{(x-x_1)\cdot(x-x_2)\cdot\ldots\cdot(x-x_n)}{(x_i-x_1)\cdot(x_i-x_2)\cdot\ldots\cdot(x_i-x_n)}$$

Stützpunkte: $(1,1)$, $(4, 0.5)$, $(9, 0.\bar{3})$

$$L_i(1) = \frac{(x-4)\cdot(x-9)}{(1-4)\cdot(1-9)} = \frac{x^2-13x+36}{24}$$

$$L_i(4) = \frac{(x-1)\cdot(x-9)}{(4-1)\cdot(4-9)} = \frac{x^2-10x+9}{-15}$$

$$L_i(9) = \frac{(x-1)\cdot(x-4)}{(9-1)\cdot(9-4)} = \frac{x^2-5x+4}{40}$$

$$\Rightarrow p(x) = \frac{x^2-13x+36}{24} - \frac{1}{2}\cdot\frac{x^2-10x+9}{15} + \frac{1}{5}\cdot\frac{x^2\cdot 5x+4}{40}$$

$$= \frac{1}{60}x^2 - \frac{1}{4}x + \frac{148}{120} = \underline{\underline{\frac{1}{60}x^2 - \frac{1}{4}x + \frac{37}{30}}}$$

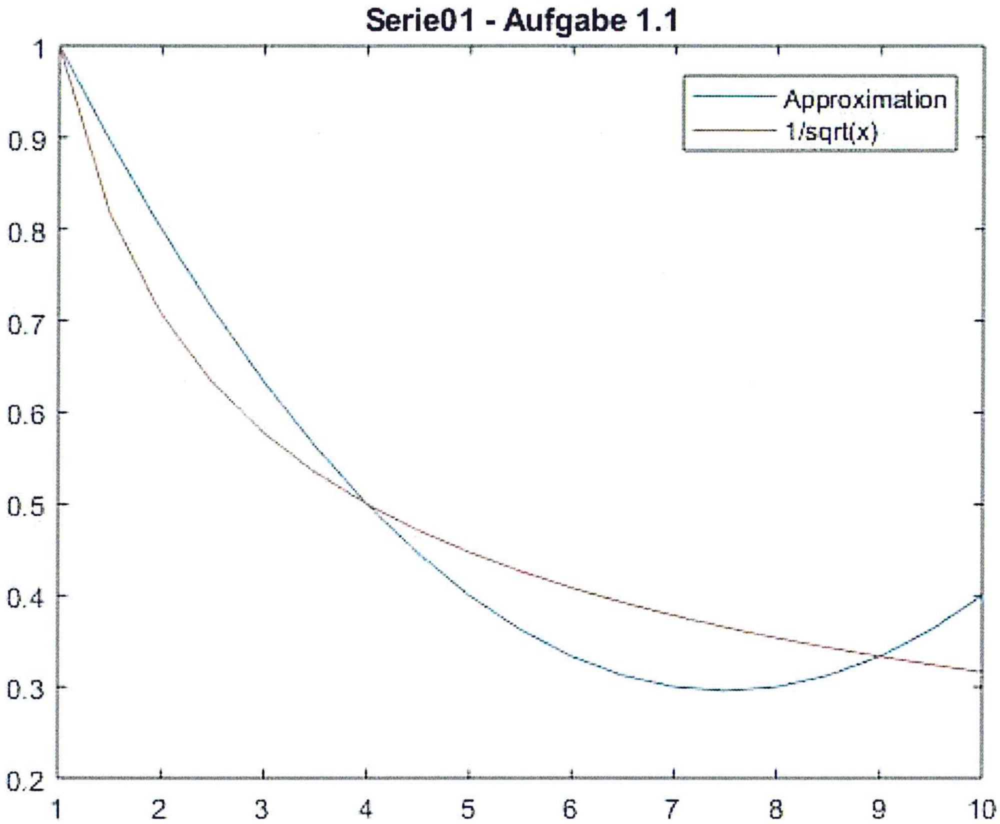
Vergleiche Approximation von $p(x=2.25)$ mit exatem Wert $2/3$.

$$p(2.25) = \frac{1}{60}\cdot 2{,}25^2 - \frac{1}{4}\cdot 2.25 + \frac{37}{30} = 0.75521$$

Error: $\left|0.75521 - \frac{2}{3}\right| = \underline{\underline{0.08854}}$

```
clear;
clc;

x = 1:0.5:10;
figure(1);
plot(x, 1/60.*x.^2-1/4.*x+37/30, x, 1./sqrt(x));
title("Serie01 - Aufgabe 1.1")
legend("Approximation", "1/sqrt(x)");
```



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A1.2) Gegeben: Stützpunkte/werte Paare $p(-3)=0,\ p(-1)=1,\ p(1)=0$

Gesucht: Polynom p. zweiten Grades, Lagrange & Newton Interpo

Lsg.: $L_i(-3) = \frac{(x+1)\cdot(x-1)}{(-3+1)\cdot(-3-1)} = \frac{x^2-1}{8}$

$$L_i(-1) = \frac{(x+3)\cdot(x-1)}{(-1+3)\cdot(-1-1)} = -\frac{x^2+2x-3}{4}$$

$$L_i(1) = \frac{(x+3)\cdot(x+1)}{(1+3)\cdot(1+1)} = \frac{x^2+4x+3}{8}$$

$$P_L(x) = \frac{x^2+2x-3}{4} = \underline{\underline{-\frac{1}{4}x^2 - \frac{1}{2}x + \frac{3}{4}}}$$

Lagransch

Newton

	x	y
0.	-3	0
1.	-1	1
2.	1	0

$c_0 = 0$

$$\delta y[x_0, x_1] = \frac{1-0}{-1+3} = \frac{1}{2} \quad (c_1)$$

$$\delta y[x_1, x_2] = \frac{0-1}{1+1} = -\frac{1}{2}$$

$$\delta y[x_0, x_1, x_2] = \frac{-\frac{1}{2}-\frac{1}{2}}{1+3} = -\frac{1}{4} \quad (c_2)$$

$$P_N(x) = 0 + \frac{1}{2}(x+3) - \frac{1}{4}\cdot(x+3)\cdot(x+1)$$

$$= \frac{1}{2}x + \frac{3}{2} - \frac{1}{4}(x^2+4x+3) = \frac{1}{2}x + \frac{3}{2} - \frac{1}{4}x^2 - x - \frac{3}{4}$$

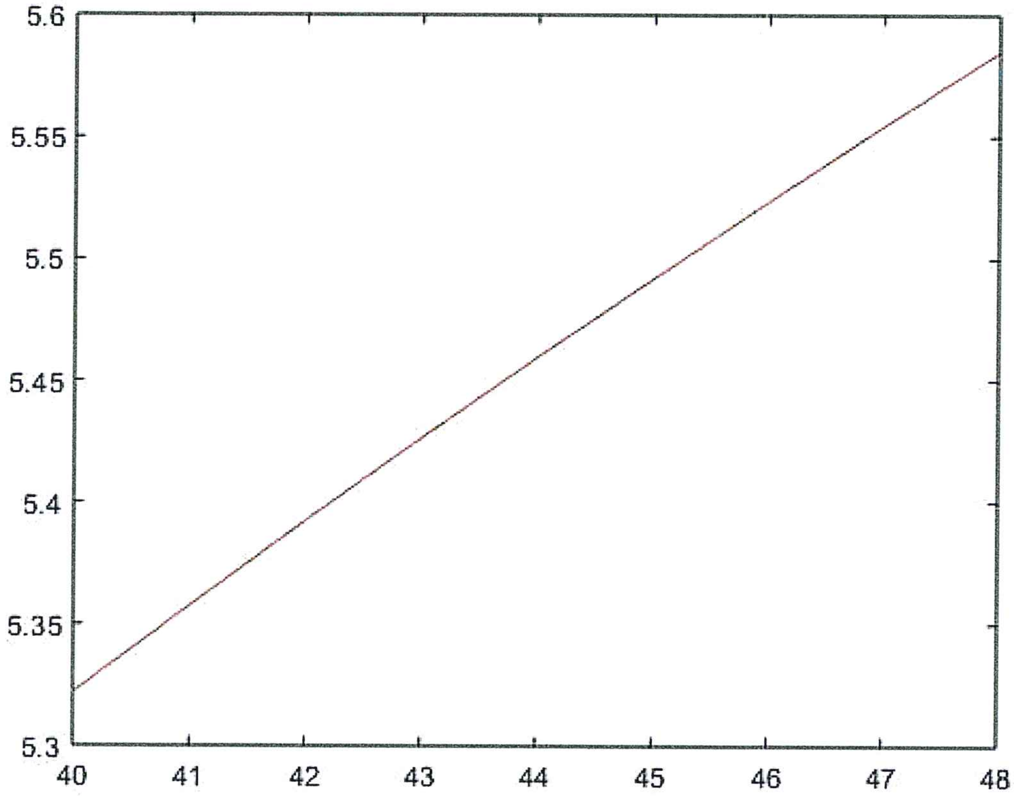
$$= \underline{\underline{-\frac{1}{4}x^2 - \frac{1}{2}x + \frac{3}{4}}}$$

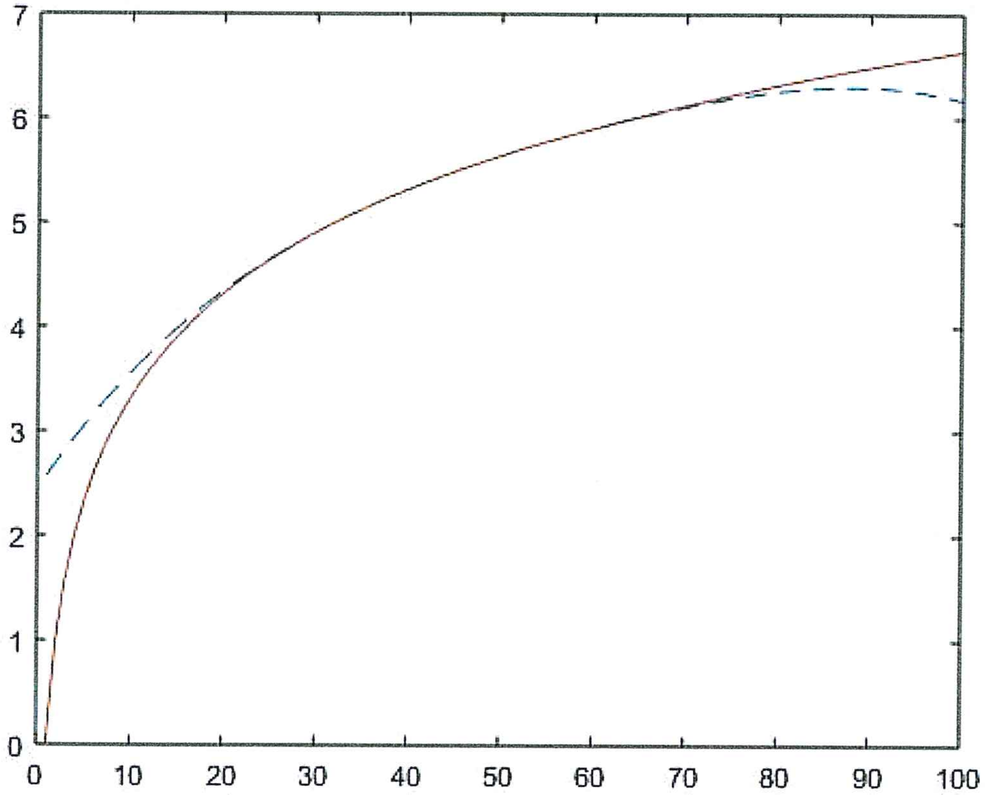
$P_L = P_N \Rightarrow$ Beide Interpolationen ergeben dasselbe Polynom!

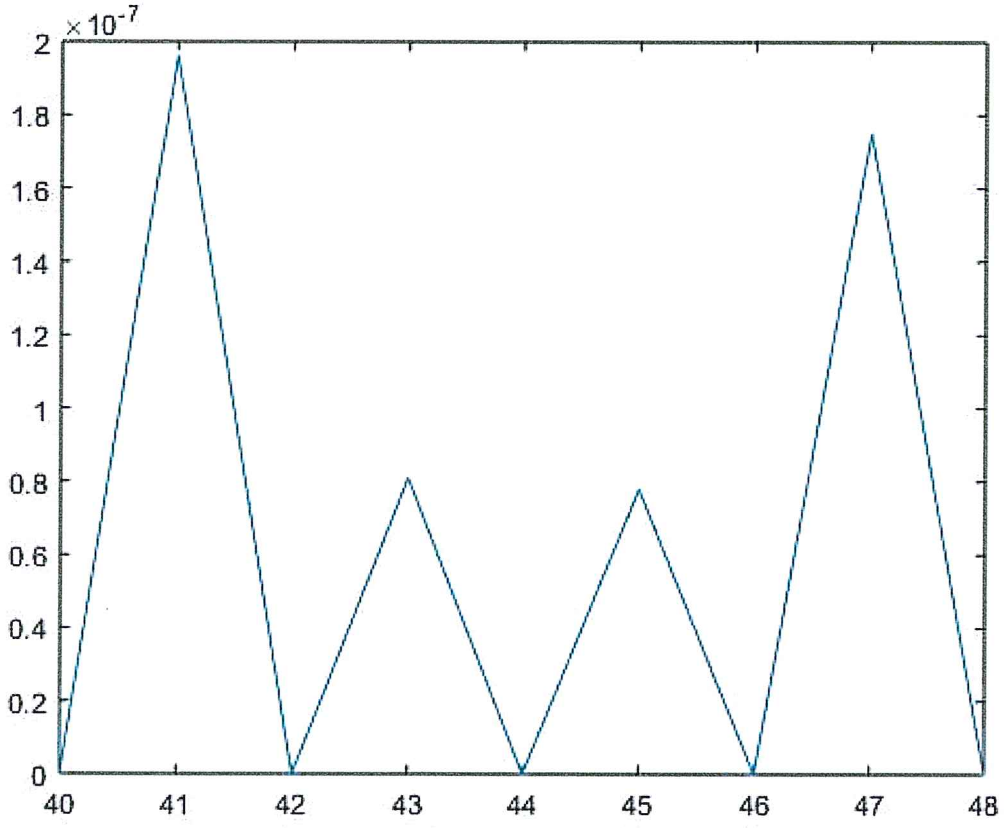
A1.3 Serie 1 Luc Kury

```
clear;
clc;

format long

x=40:1:48
figure();
title('LogInterpol(x) vs Log2 [40,48]')
xlabel('x')
ylabel('y')
plot(x,LogInterpol(x),'--',x,log2(x))

x=1:100;
figure();
title('LogInterpol(x) vs Log2 [1,100]')
xlabel('x')
ylabel('y')
plot(x,LogInterpol(x),'--',x,log2(x))

x=40:48;
figure()
title('Fehlerfunktion')
plot(x,abs(log2(x)-LogInterpol(x)))

y=LogInterpol(45.254834)
x=log2(45.254834)
```

```
x =

    40    41    42    43    44    45    46    47    48


y =

   5.499999924541300


x =

   5.500000000129461
```


5.6
5.55
5.5
5.45
5.4
5.35
5.3
40
41
42
43
44
45
46
47
48


7
6
5
4
3
2
1
0
0
10
20
30
40
50
60
70
80
90
100



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```
function [p] = LogInterpol (t)
x=[40 42 44 46 48];
y=log2(x);

n = length (x);

for i=1:n
    c(i) = y(i);
end

for k=2:n
    for i=n:-1:k
        c(i)=(c(i)-c(i-1)) ./ (x(i)-x(i-k+1));
    end
end

for k=1:length(t)
    p(k) = c(n);
    for i=n-1:-1:1
        p(k) = c(i)+(t(k)-x(i)) .* p(k);
    end
end
```

```
Not enough input arguments.

Error in LogInterpol (line 17)
for k=1:length(t)
```