Falsification of Cyber-Physical Systems with Robustness-Guided Black-Box Checking

Masaki Waga*

National Institute of Informatics and the Graduate University for Advanced Studies Chiyoda, Tokyo, Japan mwaga@nii.ac.jp

ABSTRACT

For exhaustive formal verification, industrial-scale cyberphysical systems (CPSs) are often too large and complex, and lightweight alternatives (e.g., monitoring and testing) have attracted the attention of both industrial practitioners and academic researchers. Falsification is one popular testing method of CPSs utilizing stochastic optimization. In stateof-the-art falsification methods, the result of the previous falsification trials is discarded, and we always try to falsify without any prior knowledge. To concisely memorize such prior information on the CPS model and exploit it, we employ Black-box checking (BBC), which is a combination of automata learning and model checking. Moreover, we enhance BBC using the robust semantics of STL formulas, which is the essential gadget in falsification. Our experiment results suggest that our robustness-guided BBC outperforms a state-of-the-art falsification tool.

CCS CONCEPTS

- Computer systems organization \rightarrow Embedded and cyber-physical systems; Real-time system specification;
- Software and its engineering \rightarrow Formal software verification; Search-based software engineering.

KEYWORDS

cyber-physical systems, falsification, black-box checking, automata learning, model checking, signal temporal logic, robust semantics

*JSPS Research Fellow

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1 INTRODUCTION

Falsification of cyber-physical systems. Due to their safety-critical nature, safety assurance of cyber-physical systems (CPSs) is a vital problem. For exhaustive formal verification, e.g., reachability analysis, industrial-scale cyber-physical systems (CPSs) are often too large and complex. Therefore non-exhaustive but lightweight alternatives (e.g., monitoring and black-box testing) have attracted the attention of both industrial practitioners and academic researchers. Optimization-based falsification is one of the search-based testing methods to find bugs in CPSs, and many algorithms [15, 18, 43, 49–51] have been studied. The problem is formulated as follows.

The falsification problem:

INPUT: a CPS model \mathcal{M} (given an input signal σ , it returns an output signal $\mathcal{M}(\sigma)$) and a specification φ of the CPS model \mathcal{M} .

Problem: Find a violating input signal σ such that the corresponding output signal $\mathcal{M}(\sigma)$ violates the specification φ i.e., $\mathcal{M}(\sigma) \not\models \varphi$

The technical essence of optimization-based falsification is to reduce CPS safety assurance to the *simulation-based optimization* problem through the *robust semantics* [21] of *signal temporal logic (STL)* formulas [37]. The robust semantics of an STL formula shows a *quantitative* satisfaction degree: if the robust semantics of an STL formula φ is negative, φ is violated. Thus, the falsification problem can be solved by minimizing the robust semantics of the given STL formula φ using an optimization technique, e.g., covariance matrix adaptation evolution strategy (CMA-ES) [6], through simulations. The analysis of differential equations tends to be expensive, and falsification often finds a bug more efficiently than formal verification of CPSs such as reachability analysis of hybrid automata.

Thanks to the robust semantics of STL, optimization-based falsification often falsifies an STL formula effectively even if

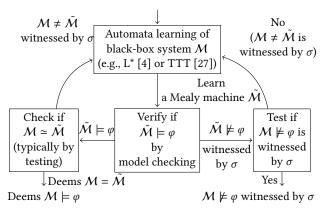


Figure 1: A workflow of black-box checking [40, 44].

it is hard for a random testing. Falsification usually requires many simulations to find a violating input signal. This can be a problem due to the simulation cost of CPSs. A simulator of a self-driving car—involving the obstacles (e.g., pedestrians and other cars) and road conditions as well as the ego car—typically runs in a speed that is more or less real-time. A single simulation of it thus would take several seconds, at least. Thus, we want to reduce the number of the simulations.

Black-box checking. Black-box checking (BBC) [44] or learningbased testing (LBT) [40] is another testing method of blackbox systems. The speciality of BBC is the combination of automata learning [9] and model checking [7]. As described in [44], an outline of BBC is shown in Fig. 1. Here, a black-box system $\mathcal{M} \colon \Sigma^* \to (\mathcal{P}(AP))^*$ is a function from a discrete input sequence $\iota \in \Sigma^*$ to a sequence $\mathcal{M}(\iota) \in (\mathcal{P}(AP))^*$ of the sets of atomic propositions satisfied at each time. By automata learning, a Mealy machine $\tilde{\mathcal{M}}$ is constructed from the previous simulation results (the top box of Fig. 1). The learned Mealy machine \mathcal{M} is used to approximate the blackbox system \mathcal{M} . By model checking, one checks if the learned Mealy machine $\tilde{\mathcal{M}}$ satisfies the given property φ (bottom center of Fig. 1). Since the behavior of the black-box system \mathcal{M} and the learned Mealy machine $\tilde{\mathcal{M}}$ can be different, their consistency is confirmed through additional simulations of \mathcal{M} (bottom left and right of Fig. 1). We note that the learned Mealy machine $\tilde{\mathcal{M}}$ is independent of the property φ , and we can use the learned Mealy machine \mathcal{M} for model checking of properties other than φ .

Thanks to the soundness of *conformance testing* used as *equivalence testing* (left of Fig. 1), BBC can guarantee that the given black-box system certainly satisfies the given property [44] although the soundness relies on additional assumptions on the black-box system (e.g., the upper bound of the number of the states). A recent survey [25] reports that at the early stage of the automata learning, it is beneficial for the equivalence testing to try to find a counterexample

 $\iota \in \Sigma^*$ satisfying $\mathcal{M}(\iota) \neq \tilde{\mathcal{M}}(\iota)$ instead of trying to prove the equivalence by conformance testing, e.g., W-method [11] and Wp-method [23]. *Random testing* is one typical choice of the equivalence testing other than conformance testing. Random testing usually samples the inputs uniformly, and it is good at covering various inputs. But, due to its uniform nature, random testing is not good at finding *rare* counterexamples existing only in a small area of the input space.

Robustness-guided black-box checking. Our contribution is to combine optimization-based falsification and BBC aiming at the improvement of both of them. We enhance BBC by the robust semantics of STL, which is the essential gadget in optimization-based falsification. We utilize BBC to solve the falsification problem.

As an improvement of BBC, we employ the robust semantics of STL to enhance the search of a counterexample exploiting the following observation. If the CPS \mathcal{M} violates the given STL formula φ , but the learned Mealy machine $\tilde{\mathcal{M}}$ satisfies φ , there exists a discrete input $\iota \in \Sigma^*$ such that the output $\mathcal{M}(\iota)$ of \mathcal{M} violates the specification φ , and we have $\mathcal{M}(\iota) \neq \tilde{\mathcal{M}}(\iota)$. Minimizing the robust semantics of φ , our equivalence testing of \mathcal{M} and $\tilde{\mathcal{M}}$ focuses on a subspace of the input space where a counterexample more likely exists. To minimize the robust semantics, we use, e.g., hill climbing or genetic algorithms [34].

As an improvement of optimization-based falsification, we aim at reducing the number of the simulations when we try to falsify a CPS over multiple STL formulas. Multiple STL formulas are used in falsification, e.g., because for one abstract requirement in engineers' minds, many STL formulas realize it, and we want to try some STL formulas out of them. Through the automata learning in BBC, we reuse the knowledge on the CPS obtained when falsifying other STL formulas, and reduce the number of the simulations. See Section 6 for related works on model learning for falsification.

Another big problem of optimization-based falsification is that we can obtain very small information when we failed to falsify it. Since BBC generates a learned Mealy machine $\tilde{\mathcal{M}}$ even if the given specifications are not falsified, we can potentially use it to explain why the BBC failed.

We note that the existing robust semantics, e.g., [2, 17, 21], are incompatible with the finite semantics of LTL in [12], which is implemented in LTSMin [29]. Although the novelty is limited, we define and employ a suitable robust semantics of STL with a soundness and correctness theorem.

We implemented a prototypical tool FalCAuN for robustness-guided BBC and compared its performance with: *i*) Breach, which is one of the state-of-the-art falsification tools; and *ii*) a baseline BBC using random search for the equivalence testing. Our experimental result suggests that

- (1) on average, robustness-guided BBC using genetic algorithm falsifies more properties Breach and the baseline BBC method; and
- (2) robustness-guided BBC is much more scalable than Breach with respect to the number of the properties we try to falsify.

Contributions. Our contributions are summarized as follows.

- · By combining optimization-based falsification and blackbox checking (BBC), we proposed robustness-guided BBC to improve both of them.
- guided BBC.
- Our experimental results show that our robustnessguided BBC outperforms baseline BBC and one of the state-of-the-art falsification algorithms.

Organization. After reviewing some preliminaries in Section 2, we show the robust semantics of STL in a discrete-time setting in Section 3. This semantics is compatible with the finite semantics of LTL in [12]. In Section 4, we show how to enhance BBC by the robust semantics of STL, which is the main contribution of this paper. We show our experimental evaluation in Section 5. We review some related works in Section 6. We conclude and show future works in Section 7.

PRELIMINARIES 2

Notations. For a set X, we denote its powerset by $\mathcal{P}(X)$. We denote the empty sequence by ε . For a set X, an infinite sequence $\overline{x} = x_0, x_1, \dots \in X^{\omega}$ of X, and $i, j \in N$ satisfying $i \le j$, we denote the subsequence $x_i, x_{i+1}, \dots, x_j \in X^*$ by $\overline{x}[i, j]$. For a set X, a finite sequence $\overline{x} \in X^*$ of X, and an infinite sequence $\overline{x'} \in X^{\omega}$ of X, we denote their concatenation by $\overline{x} \cdot \overline{x'}$. For a set X and its subsets $X', X'' \subseteq$ X, we denote the symmetric difference of X' and X'' by $X' \triangle X'' = \{x \in X \mid x \in X', x \notin X''\} \cup \{x \in X \mid x \in X'\}$ $x \notin X', x \in X''$. For a function $f: X \to Y$ and a finite sequence $\overline{x} = x_1, x_2, \dots, x_n \in X^*$, we let $\overline{f}: X^* \to Y^*$ as $\overline{f}(\overline{x}) = f(x_1), f(x_2), \dots, f(x_n)$. For closed intervals $I_1 =$ $[a_1, b_1], I_2 = [a_2, b_2] \text{ over } \mathbb{R} \cup \{\pm \infty\}, \text{ we let } -I_1 = [-b_1, -a_1]$ and $\max(I_1, I_2) = [\max(a_1, a_2), \max(b_1, b_2)]$ and $\min(I_1, I_2) =$ $[\min(a_1, a_2), \min(b_1, b_2)].$

LTL model checking

Linear temporal logic (LTL) [45] is a commonly used formalism to describe temporal behaviors of an infinite or finite sequence $\pi \in (\mathcal{P}(AP))^{\infty}$ of a set $\pi_i \subseteq AP$ of atomic propositions representing valuations of atomic propositions.

Definition 2.1 (linear temporal logic). For the set AP of the atomic propositions, the syntax of *linear temporal logic (LTL)* is defined as follows, where $p \in AP$ and $i, j \in \mathbb{N} \cup \{+\infty\}$ satisfying $i \leq j$.¹

$$\psi, \psi' ::= \top \mid p \mid \neg \psi \mid \psi \lor \psi' \mid \psi \; \mathcal{U}_{[i,j)} \; \psi' \mid \mathcal{X} \psi$$

For an LTL formula ψ , an *infinite* sequence $\pi = \pi_0, \pi_1, \dots \in$ $(\mathcal{P}(\mathbf{AP}))^{\omega}$ of subsets of atomic propositions, and $k \in \mathbb{N}$, we define the satisfaction relation $(\pi, k) \models \psi$ as follows.

• By combining optimization-based falsification and black-
box checking (BBC), we proposed robustness-guided
$$(\pi,k) \models \neg \psi \iff (\pi,k) \not\models \psi$$

• BBC to improve both of them. $(\pi,k) \models \psi \lor \psi' \iff (\pi,k) \models \psi \lor (\pi,k) \models \psi'$
• We implemented a prototypical tool FalCAuN for robustness-
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We denote $\pi \models \psi$ if we have $(\pi, 0) \models \psi$. An LTL formula ψ is safety if for any infinite sequence $\pi \in (\mathcal{P}(AP))^{\omega}$ satisfying $\pi \not\models \psi$, there exists $i \in \mathbb{N}$, such that for any j > i and for any $\pi' \in (\mathcal{P}(AP))^{\omega}$, we have $\pi[0,j] \cdot \pi' \not\models \psi$. For a safety LTL formula ψ , the violation of ψ can be monitored by a finite prefix $\pi[0,i] \in (\mathcal{P}(AP))^*$ of $\pi \in (\mathcal{P}(AP))^{\omega}$. In [12], the finite semantics $\llbracket \psi \rrbracket$ of LTL ψ is defined by the set of finite prefixes potentially satisfying the property ψ . We note that this semantics is also utilized in the latest version of LTSMin.

Definition 2.2 (finite semantics of LTL [12]). For an LTL formula ψ , $\llbracket \psi \rrbracket \subseteq (\mathcal{P}(AP))^*$ is the following set of *finite* sequences of subsets of atomic propositions

$$\llbracket \psi \rrbracket = \{ \pi \in (\mathcal{P}(\mathbf{AP}))^* \mid \exists \pi' \in (\mathcal{P}(\mathbf{AP}))^\omega . \ \pi \cdot \pi' \models \psi \}$$

Definition 2.3 (Mealy machine). For the input and output alphabet Σ and Γ , a *Mealy machine* is a tuple $\mathcal{M} = (L, l_0, \Delta)$, where L is the finite set of locations, $l_0 \in L$ is the initial location, and $\Delta \colon (L \times \Sigma) \to (\Gamma \times L)$ is the transition function.

For a Mealy machine $\mathcal{M} = (L, l_0, \Delta)$ over Σ and Γ , the *lan*- $\exists l_1, l_2, \dots, \forall i \in \mathbb{N}. \Delta(l_i, a_i) = (b_i, l_{i+1}) \}$. For an infinite signal $\sigma = (a_0, b_0), (a_1, b_1), \dots \in (\Sigma \times \Gamma)^{\omega}$, we let $\mathbf{pr}_1(\sigma) =$ $a_0, a_1, \dots \in \Sigma^{\omega}$ and $\mathbf{pr}_2(\sigma) = b_0, b_1, \dots \in \Gamma^{\omega}$. For a Mealy machine \mathcal{M} , the *input language* $\mathcal{L}_{in}(\mathcal{M}) \subseteq \Sigma^{\omega}$ and the *out*put language $\mathcal{L}_{out}(\mathcal{M}) \subseteq \Gamma^{\omega}$ are $\mathcal{L}_{in}(\mathcal{M}) = \{\mathbf{pr}_1(\sigma) \mid \exists \sigma \in \mathcal{L}_{in}(\mathcal{M}) \in \mathcal{L}_{in}(\mathcal{M}) \mid \exists \sigma \in \mathcal{L}_{in}(\mathcal{M}) \in \mathcal{L}_{in}(\mathcal{M}) \}$ $\mathcal{L}(\mathcal{M})$ and $\mathcal{L}_{out}(\mathcal{M}) = \{ \mathbf{pr}_2(\sigma) \mid \exists \sigma \in \mathcal{L}(\mathcal{M}) \}.$ We employ a Mealy machine over Σ and $\mathcal{P}(\mathbf{AP})$ to model a system.

Definition 2.4 (LTL model checking). Let Σ be the input alphabet and let AP be the set of the atomic propositions. Given an LTL formula ψ over **AP** and a Mealy machine \mathcal{M} over Σ and $\mathcal{P}(AP)$, LTL model checking decides if we have

¹In the standard definition of LTL, the interval [i,j) in $\mathcal{U}_{[i,j)}$ is always $[0, \infty)$ and it is omitted. We employ the current syntax to emphasize the similarity to STL. We note that this does not change the expressive power.

 $\forall \pi \in \mathcal{L}_{out}(\mathcal{M}). \pi \models \psi.$ Moreover, it answers $\sigma \in \mathcal{L}(\mathcal{M})$ satisfying $\mathbf{pr}_1(\sigma) \not\models \psi$ if such σ exists. We denote $\forall \pi \in \mathcal{L}_{out}(\mathcal{M}).\pi \models \psi$ by $\mathcal{M} \models \psi$.

In the rest of this paper, we only consider safety LTL formulas [35]. For any safety LTL formula ψ , if we have $\mathcal{M} \not\models \psi$, there is a finite counterexample $\sigma \in (\Sigma \times \mathcal{P}(\mathbf{AP}))^*$ such that $\mathbf{pr}_2(\sigma) \in (\mathcal{P}(\mathbf{AP}))^* \setminus \llbracket \psi \rrbracket$ and there exists $\sigma' \in (\Sigma \times \mathcal{P}(\mathbf{AP}))^\omega$ satisfying $\sigma \cdot \sigma' \in \mathcal{L}(\mathcal{M})$. Thus, we use such a *finite* counterexample $\sigma \in (\Sigma \times \mathcal{P}(\mathbf{AP}))^*$ as a witness of $\mathcal{M} \not\models \psi$. We let $\mathcal{L}^{\mathrm{fin}}(\mathcal{M}) = \left\{ \sigma \in (\Sigma \times \mathcal{P}(\mathbf{AP}))^* \mid \exists \sigma' \in (\Sigma \times \mathcal{P}(\mathbf{AP}))^\omega . \sigma \cdot \sigma' \in \mathcal{L}(\mathcal{M}) \right\}$.

2.2 Active automata learning

Active automata learning is an automata learning method pioneered by L* algorithm [4], which learns the minimal DFA $\mathcal{A}_{\mathcal{L}}$ over Σ recognizing the target language $\mathcal{L} \subseteq \Sigma^*$. L* algorithm learns a DFA through the queries to membership and equivalence oracles. Given a word $w \in \Sigma^*$, the membership oracle answers if w belongs to the target language \mathcal{L} i.e., $w \in \mathcal{L}$. Given a candidate DFA \mathcal{A} , the equivalence oracle answers if \mathcal{A} recognizes the target language \mathcal{L} i.e., $\mathcal{L}(\mathcal{A}) = \mathcal{L}$, where $\mathcal{L}(\mathcal{A})$ is the language of the candidate DFA \mathcal{A} . When \mathcal{A} does not recognize \mathcal{L} , the equivalence oracle also answers a counterexample $w \in \Sigma^*$ such that $w \in \mathcal{L}(\mathcal{A}) \triangle \mathcal{L}$. We note that a Mealy machine \mathcal{M} can also be learned similarly. See e.g., [47].

Equivalence testing. In practice, the target language \mathcal{L} is usually given as a black-box system, and a sound and complete equivalence oracle is often unimplementable while the given black-box system itself can be a membership oracle. Therefore, we need an approximate strategy for equivalence testing. For example, LearnLib [28] implements deterministic exploration (e.g., complete, depth-bounded exploration), random exploration (e.g., random words testing), and tonformance tests (e.g., W-method [11] and Wp-method [23]).

Alphabet abstraction. Another practical issue is that the input and output alphabet can be huge or even infinite, and the automata learning algorithm does not perform effectively or does not terminate. For instance, the input and output of a CPS model is usually real-valued signals, which are infinitely many. To overcome this issue, alphabet abstraction is employed to reduce the alphabet size. For example, a variant of Mealy machines is used to map the concrete and large alphabet to the abstract and small alphabet in [1].

2.3 Black-box checking

Black-box checking (BBC) [44], is a black-box testing method² combining model checking and active automata learning.

Given a black-box and potentially infinite locations Mealy machine \mathcal{M} over Σ and $\mathcal{P}(\mathbf{AP})$, and a safety LTL formula ψ , BBC deems $\mathcal{M} \models \psi$ or returns a counterexample $\sigma \in (\Sigma \times \mathcal{P}(\mathbf{AP}))^*$ such that we have $\mathbf{pr}_2(\sigma) \in (\mathcal{P}(\mathbf{AP}))^* \setminus \llbracket \psi \rrbracket$ and there exists $\sigma' \in (\Sigma \times \mathcal{P}(\mathbf{AP}))^\omega$ satisfying $\sigma \cdot \sigma' \in \mathcal{L}(\mathcal{M})$. In contrast to the usual testing methods, BBC also constructs a Mealy machine $\tilde{\mathcal{M}}$ through automata learning. Thus, we can reuse some part of the previous testing results through the extracted Mealy machine $\tilde{\mathcal{M}}$.

Fig. 1 shows a workflow of BBC. First, we learn a Mealy machine \mathcal{M} from the black-box system \mathcal{M} by an automata learning algorithm e.g., L* [4] or TTT algorithm [27]. We note that the learned Mealy machine $\tilde{\mathcal{M}}$ may behave differently from the original black-box system \mathcal{M} because our equivalence testing is an approximation, or even the equivalence testing might be omitted at this point. Then, we check if we have $\tilde{\mathcal{M}} \models \psi$ by model checking. If we have $\tilde{\mathcal{M}} \not\models \psi$, we also obtain a counterexample $\sigma \in (\Sigma \times \mathcal{P}(AP))^*$. We feed the counterexample σ to the original system \mathcal{M} and check if σ is a witness of $\mathcal{M} \not\models \psi$, too. If σ is a witness of $\mathcal{M} \not\models \psi$, we conclude $\mathcal{M} \not\models \psi$ and return the counterexample σ . Otherwise, we have $\sigma \in \mathcal{L}^{fin}(\tilde{\mathcal{M}})$ but $\sigma \notin \mathcal{L}^{fin}(\mathcal{M})$, and we use σ to refine our learning of \mathcal{M} . If we have $\mathcal{M} \models \psi$, we check if the behavior of \mathcal{M} and $\tilde{\mathcal{M}}$ are similar enough by equivalence testing. If we find a counterexample $\sigma \in \mathcal{L}^{fin}(\mathcal{M}) \triangle \mathcal{L}^{fin}(\tilde{\mathcal{M}})$, we conclude that the learned Mealy machine \mathcal{M} is not similar enough to the original system \mathcal{M} , and we use σ to refine our learning of M. If we could not find such σ , we deem \mathcal{M} to be equivalent to \mathcal{M} and return $\mathcal{M} \models \psi$, which is not always correct.

3 DISCRETE-TIME SIGNAL TEMPORAL LOGIC AND ROBUSTNESS

Signal temporal logic (STL) [37] is a formalism to represent behavior of continuous-time, real-valued signals with quantitative satisfaction degree called robust semantics [17]. Due to the discrete nature of BBC, we need to represent discrete-time, real-valued signals. In this section, we introduce discrete-time STL, which is a variant of LTL for real-valued signals. We define the robust semantics for both infinite and finite signals.

Definition 3.1 (signal). For a finite set Y of variables, a signal $\sigma \in (\mathbb{R}^Y)^{\infty}$ is a (finite or infinite) sequence of valuations $u_i \colon Y \to \mathbb{R}$. For a finite signal $\sigma \in (\mathbb{R}^Y)^*$, we denote the length n of $\sigma = u_0, u_1, \dots, u_{n-1}$ by $|\sigma|$.

Definition 3.2 (signal temporal logic). For a finite set Y of variables, the syntax of signal temporal logic (STL) is defined as follows, where $y \in Y$, $\bowtie \in \{>, <\}$, $c \in \mathbb{R}$, and $i, j \in \{>, <\}$

does not hold in most of the CPS application, and we use BBC just as a testing method. See also Section 6.

²Under some assumption, BBC is sound i.e., BBC proves the correctness of the black-box system. See e.g., [38]. However, the soundness assumption

 $\mathbb{N} \cup \{+\infty\}$ satisfying i < j.

$$\varphi, \varphi' ::= \top \mid y \bowtie c \mid \neg \varphi \mid \varphi \lor \varphi' \mid \varphi \mathcal{U}_{[i,j)} \varphi' \mid \chi \varphi$$

We use the following standard notation: $\bot \equiv \neg \top$; $y \ge c \equiv \neg (y < c)$; $y \le c \equiv \neg (y > c)$; $\varphi \land \varphi' \equiv \neg ((\neg \varphi) \lor (\neg \varphi'))$; $\varphi \Rightarrow \varphi' \equiv (\neg \varphi) \lor \varphi'$; $\top \mathcal{U} \varphi \equiv \top \mathcal{U}_{[0,\infty)} \varphi$; $\diamondsuit_{[i,j)} \varphi \equiv \top \mathcal{U}_{[i,j)} \varphi$; and $\Box_{[i,j)} \varphi \equiv \neg (\diamondsuit_{[i,j)} \neg \varphi)$.

For an STL formula φ over Y, an *infinite* signal $\sigma = u_0, u_1, \dots \in (\mathbb{R}^Y)^\omega$ over Y, and $k \in \mathbb{N}$, the satisfaction relation $(\sigma, k) \models \varphi$ is inductively defined as follows.

$$(\sigma, k) \models \top \qquad (\sigma, k) \models y > c \iff u_k(y) > c$$

$$(\sigma, k) \models y < c \iff u_k(y) < c$$

$$(\sigma, k) \models \neg \varphi \iff (\sigma, k) \not\models \varphi$$

$$(\sigma, k) \models \varphi \lor \varphi' \iff (\sigma, k) \models \varphi \lor (\sigma, k) \models \varphi'$$

$$(\sigma, k) \models X\varphi \iff (\sigma, k + 1) \models \varphi$$

$$(\sigma, k) \models \varphi U_{[i,j)} \varphi' \iff \exists l \in [k + i, k + j). (\sigma, l) \models \varphi'$$

$$\land \forall m \in \{k, k + 1, \dots, l\}. (\sigma, m) \models \varphi$$

The satisfaction relation $(\sigma, k) \models \varphi$ gives a *qualitative* verdict of the satisfaction of the STL formula φ by the signal σ . The *robust semantics* $\rho(\varphi, \sigma, k)$ gives a *quantitative* satisfaction degree of the STL formula φ by the signal σ .

Definition 3.3 (robust semantics). For an STL formula φ over Y, an infinite signal $\sigma = u_0, u_1, \dots \in (\mathbb{R}^Y)^\omega$ over Y, and $k \in \mathbb{N}$, the robust semantics $\rho(\varphi, \sigma, k) \in \mathbb{R} \cup \{\pm \infty\}$ of the STL formula φ and the signal σ at k is defined as follows.

$$\begin{split} \rho(\top, \sigma, k) &= + \infty \qquad \rho(X\varphi, \sigma, k) = \rho(\varphi, \sigma, k + 1) \\ \rho(y > c, \sigma, k) &= u_k(y) - c \quad \rho(y < c, \sigma, k) = -u_k(y) + c \\ \rho(\neg \varphi, \sigma, k) &= -\rho(\varphi, \sigma, k) \\ \rho(\varphi \lor \varphi', \sigma, k) &= \max(\rho(\varphi, \sigma, k), \rho(\varphi', \sigma, k)) \\ \rho(\varphi \ \mathcal{U}_{[i,j)} \ \varphi', \sigma, k) &= \\ \sup_{l \in [k+i,k+j)} \min(\rho(\varphi', \sigma, l), \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi, \sigma, m)) \end{split}$$

Theorem 3.4 (soundness and completeness). For an STL formula φ over Y, an infinite signal $\sigma = u_0, u_1, \dots \in (\mathbb{R}^Y)^\omega$ over Y, and $k \in \mathbb{N}$ we have the following.

$$\rho(\varphi, \sigma, k) > 0 \Rightarrow (\sigma, k) \models \varphi \quad (\sigma, k) \models \varphi \Rightarrow \rho(\varphi, \sigma, k) \ge 0$$

If we have $(\sigma,0) \models \varphi$, we denote $\sigma \models \varphi$. The *safety* fragment of STL is defined similarly to that of LTL. For an STL formula φ , we define two finite semantics: the *supremum* finite semantics $[\![\varphi]\!]_{\Diamond} \subseteq (\mathbb{R}^Y)^*$ and the *infimum* finite semantics $[\![\varphi]\!]_{\Diamond}$ is the set of prefixes *potentially* satisfying the property φ , and corresponding to the finite semantics of LTL in [12]. The infimum finite semantics $[\![\varphi]\!]_{\Box}$ is the set of prefixes *surely* satisfying the property φ .

Definition 3.5 ($\llbracket \varphi \rrbracket_{\diamondsuit}$, $\llbracket \varphi \rrbracket_{\square}$). For an STL formula φ , the supremum finite semantics $\llbracket \varphi \rrbracket_{\diamondsuit} \subseteq (\mathbb{R}^Y)^*$ and the infimum finite semantics $\llbracket \varphi \rrbracket_{\square} \subseteq (\mathbb{R}^Y)^*$ are defined as follows.

$$\llbracket \varphi \rrbracket_{\diamondsuit} = \{ \sigma \in (\mathbb{R}^Y)^* \mid \exists \sigma' \in (\mathbb{R}^Y)^{\omega}. \ \sigma \cdot \sigma' \models \varphi \}$$
$$\llbracket \varphi \rrbracket_{\square} = \{ \sigma \in (\mathbb{R}^Y)^* \mid \forall \sigma' \in (\mathbb{R}^Y)^{\omega}. \ \sigma \cdot \sigma' \models \varphi \}$$

As the robust semantics for the finite signals, we employ *robust satisfaction interval (RoSI)* [13].

Definition 3.6 (robust satisfaction interval). For an STL formula φ over Y, a finite signal $\sigma \in (\mathbb{R}^Y)^*$ over Y, and $k \in \mathbb{N}$, the robust satisfaction interval $\text{RoSI}(\varphi, \sigma, k)$ is the following closed interval over $\mathbb{R} \cup \{\pm \infty\}$.

$$\operatorname{RoSI}(\varphi, \sigma, k) = \left[\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi, \sigma \cdot \sigma', k), \sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi, \sigma \cdot \sigma', k) \right]$$

Theorem 3.7 (soundness and completeness). For an STL formula φ over Y, a finite signal $\sigma = u_0, u_1, \dots, u_{n-1} \in (\mathbb{R}^Y)^*$ over Y, and $k \in \mathbb{N}$, we have the following.

$$\begin{split} \sup & (\operatorname{RoSI}(\varphi,\sigma,k)) > 0 \Rightarrow \sigma[k,|\sigma|-1] \in [\![\varphi]\!]_{\diamondsuit} \\ & \sigma[k,|\sigma|-1] \in [\![\varphi]\!]_{\diamondsuit} \Rightarrow \sup (\operatorname{RoSI}(\varphi,\sigma,k)) \geq 0 \\ & \inf (\operatorname{RoSI}(\varphi,\sigma,k)) > 0 \Rightarrow \sigma[k,|\sigma|-1] \in [\![\varphi]\!]_{\square} \\ & \sigma[k,|\sigma|-1] \in [\![\varphi]\!]_{\square} \Rightarrow \inf (\operatorname{RoSI}(\varphi,\sigma,k)) \geq 0 \end{split}$$

One computational issue on the robust satisfaction interval $RoSI(\varphi, \sigma, k)$ is that its definition is not inductive and it is unclear if it is effectively computable. Instead, we use the following inductive overapproximation $[\rho](\varphi, \sigma, k)$ of $RoSI(\varphi, \sigma, k)$ as a quantitative satisfaction degree in our method.

Definition 3.8 ($[\rho](\sigma, \varphi, k)$). For an STL formula φ over Y, a finite signal $\sigma = u_0, u_1, \dots, u_{n-1} \in (\mathbb{R}^Y)^*$ over Y, and $k \in \mathbb{N}$, $[\rho](\varphi, \sigma, k)$ is the closed interval over $\mathbb{R} \cup \{\pm \infty\}$ inductively defined as follows.

$$\begin{split} [\rho](\top,\sigma,k) &= [+\infty,+\infty] \\ [\rho](y>c,\sigma,k) &= \begin{cases} [u_k(y)-c,u_k(y)-c] & \text{if } k<|\sigma| \\ [-\infty,+\infty] & \text{if } k\geq|\sigma| \end{cases} \\ [\rho](y$$

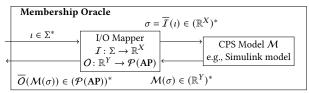


Figure 2: Membership oracle for a CPS model using alphabet abstraction: I and O are applied to each element. See Section 2 for the notation \overline{I} and \overline{O} .

THEOREM 3.9. For any STL formula φ over Y, a finite signal $\sigma = u_0, u_1, \dots, u_{n-1} \in (\mathbb{R}^Y)^*$ over Y, and $k \in \mathbb{N}$, we have $RoSI(\varphi, \sigma, k) \subseteq [\rho](\varphi, \sigma, k)$.

Corollary 3.10 justifies the use of $[\rho](\varphi, \sigma, 0)$ as a quantitative satisfaction degree of $\sigma \models \varphi$.

COROLLARY 3.10. Let φ be an STL formula over Y, let $\sigma = u_0, u_1, \dots, u_{n-1} \in (\mathbb{R}^Y)^*$ be a finite signal over Y, and $k \in \mathbb{N}$. If we have $\sup([\rho](\varphi, \sigma, 0)) < 0$, for any $\sigma' \in (\mathbb{R}^Y)^\omega$, we have $\sigma \cdot \sigma' \not\models \varphi$. If we have $\inf([\rho](\varphi, \sigma, 0)) < 0$, there exists $\sigma' \in (\mathbb{R}^Y)^\omega$ satisfying $\sigma \cdot \sigma' \not\models \varphi$.

4 BLACK-BOX CHECKING OF CYBER-PHYSICAL SYSTEMS

In this section, we show how to solve the falsification problem by BBC. Moreover, we enhance the membership testing by the robustness in STL. This is our main contribution. Let X and Y be the finite sets of the input and output variables, respectively. We define $CPS \ model \ M$ over (X,Y) as a function $M \colon (\mathbb{R}^X)^* \to (\mathbb{R}^Y)^*$ satisfying $|\sigma| = |M(\sigma)|$. The input signal $\sigma \in (\mathbb{R}^X)^*$ shows the inputs (e.g., the angle of the brake pedal) at each time step, and the output signal $M(\sigma) \in (\mathbb{R}^Y)^*$ shows the states (e.g., the speed of the car) at each time step.

We solve the falsification problem using BBC, where the given black-box system is a CPS model $\mathcal{M}\colon (\mathbb{R}^X)^* \to (\mathbb{R}^Y)^*$. Although the input and the output domain of the CPS model \mathcal{M} is *continuous*, we construct a Mealy machine $\tilde{\mathcal{M}}$ with *finite* input and output. In what follows, we present how to implement the membership and equivalence oracles for CPSs, and how we employ BBC to falsify multiple STL formulas. We note that we can still use LTL model checking because discrete-time STL can be interpreted as LTL and we assume that there is a constant sampling rate for the CPS trajectories.

4.1 Membership oracle with alphabet abstraction

Same as the usual BBC and automata learning for software testing, we use the CPS model \mathcal{M} as the membership oracle. As we discussed in Section 2.2, we have to abstract the alphabet due to the real-valued input and output of \mathcal{M} . As the abstract input and output alphabets, we use a finite set Σ and

Algorithm 1: Search-based equivalence testing

```
Input: CPS model \mathcal{M} \colon \mathbb{R}^{X} \to \mathbb{R}^{Y}, input mapper I \colon \Sigma \to \mathbb{R}^{X}, output mapper O \colon \mathbb{R}^{Y} \to \mathcal{P}(AP), STL formula \varphi, and Mealy machine \tilde{\mathcal{M}} \colon \Sigma^{*} \to (\mathcal{P}(AP))^{*}

Output: Returns \iota \in \Sigma^{*} satisfying \overline{O}(\mathcal{M}(\overline{I}(\iota))) \neq \tilde{\mathcal{M}}(\iota), or \bot when no such \iota was found /* sample the initial population */

1 I \leftarrow \text{genPopul}()

2 until isTimeout() do

3 | if \exists \iota \in I. \overline{O}(\mathcal{M}(\overline{I}(\iota))) \neq \tilde{\mathcal{M}}(\iota) then

4 | return \iota

/* Generate the next population e.g., by random sampling or robustness-guided optimization */

5 | I \leftarrow \text{genNextPopulation}(I, \mathcal{M}, \varphi)

6 return \bot
```

the power set $\mathcal{P}(\mathbf{AP})$ of atomic propositions, respectively. For simplicity, we employ a stateless mapper. Namely, for the input alphabet Σ , we define the *input mapper* $I:\Sigma\to\mathbb{R}^X$, which assigns one input signal valuation to each $a\in\Sigma$, and for the output alphabet $\mathcal{P}(\mathbf{AP})$, we define the *output mapper* $O\colon\mathbb{R}^Y\to\mathcal{P}(\mathbf{AP})$, which returns the set of the atomic propositions satisfied for the given output signal valuation. We apply I and O to each element of the sequences. See Fig. 2 for an illustration. We note that the construction of I and O as well as the choice of the input alphabet Σ are done by a user.

4.2 Robustness-guided equivalence testing

As we discussed in Sections 1 and 2.2, we need an equivalence testing method to find a counterexample even if it is too rare for random search. Algorithm 1 shows a general outline of search-based equivalence testing (including random search) of a CPS model \mathcal{M} and a Mealy machine $\tilde{\mathcal{M}}$.

In random search, after randomly sampling the initial inputs $I \subseteq \Sigma^*$ (line 1), we test the equivalence of \mathcal{M} and $\tilde{\mathcal{M}}$ for each input $\iota \in I$ (line 3). If we find no counterexample, we again randomly sample the next inputs $I \subseteq \Sigma^*$ (line 5) and test the equivalence again. We repeat such a sampling (line 5) and testing (line 3) until we find a counterexample ι or we reach the timeout.

The main observation in robustness-guided equivalence testing is as follows. If we have $\overline{O} \circ \mathcal{M} \circ \overline{I} \not\models \varphi$ and $\tilde{\mathcal{M}} \models \varphi$, by a discrete input $\iota \in \Sigma^*$ witnessing $\overline{O} \circ \mathcal{M} \circ \overline{I} \not\models \varphi$, we can also witness $\overline{O} \circ \mathcal{M} \circ \overline{I} \not\models \tilde{\mathcal{M}}$, where \circ is the function composition. Thus, by minimizing the robustness of the CPS model \mathcal{M} , we can guide the search to the inputs witnessing the difference between $\overline{O} \circ \mathcal{M} \circ \overline{I}$ and $\tilde{\mathcal{M}}$. Specifically, in line 5 of Algorithm 1, we use optimization to sample such inputs I that makes the robustness of the CPS model \mathcal{M} low.

Algorithm 2: BBC for multiple specifications

```
Input: CPS model \mathcal{M}: \mathbb{R}^X \to \mathbb{R}^Y, input mapper
               I: \Sigma \to \mathbb{R}^X, output mapper O: \mathbb{R}^Y \to \mathcal{P}(AP), and
               STL formulas \varphi_1, \varphi_2, \ldots, \varphi_n.
    Output: A set result of pairs (\iota_i, \varphi_i), where i \in \{1, 2, ..., n\}
                  and \iota_i \in \Sigma^* is a witness of \mathcal{M} \not\models \varphi_i.
 1 result \leftarrow \emptyset; notFalsified \leftarrow \{1, 2, ..., n\}
    /* Extract a Mealy machine from the actual CPS model
         (above of Fig. 1)
_{2} \mathcal{M} \leftarrow learnMealy(\mathcal{M})
3 repeat
           cex \leftarrow \bot
4
          for i \in \text{unfalsified do}
5
                 /* Model checking (center of Fig. 1)
                                                                                             */
                 if \mathcal{M} \not\models \varphi_i then
                        \iota_i \leftarrow \text{the witness of } \mathcal{M} \not\models \varphi_i
                       /* Feed \iota_i to \mathcal M (right of Fig. 1)
                       if \mathcal{M} \not\models \varphi_i is witnessed by \iota_i then
 8
                              push (\iota_i, \varphi_i) to result
                              remove i from notFalsified
10
                       else cex \leftarrow \iota_i; break
11
          if cex = \bot then
12
                 for i \in \text{unfalsified } \mathbf{do}
13
                       /* Search-based equivalence testing in
                             Section 4.2 (left of Fig. 1)
                        cex \leftarrow searchEquivTest(\mathcal{M}, \mathcal{I}, \mathcal{O}, \varphi_i, \tilde{\mathcal{M}})
                       if cex \neq \bot then
15
                              break
16
           if cex \neq \bot then
17
                \tilde{\mathcal{M}} \leftarrow \text{learnMealy}(\mathcal{M}, \tilde{\mathcal{M}}, cex)
18
19 until cex ≠ ⊥
20 return result
```

For example, we can use *local search* e.g., *hill climbing* and *genetic algorithm* [34], where the objective is to minimize $\sup([\rho](\varphi, \sigma, 0))$. We can continue this optimization along different equivalence testing calls by taking over the inputs in line 1 instead of generating randomly.

4.3 BBC for multiple specifications

Algorithm 2 shows how we employ BBC to falsify multiple STL formulas. In line 2, we extract a Mealy machine $\tilde{\mathcal{M}}$ from the CPS model \mathcal{M} . Then, in line 6, for each STL formula φ_i which is not falsified yet, we check if $\tilde{\mathcal{M}} \not\models \varphi_i$ holds by LTL model checking. When $\tilde{\mathcal{M}} \not\models \varphi_i$ holds, we obtain a witness $\iota_i \in \Sigma^*$ of $\tilde{\mathcal{M}} \not\models \varphi_i$. In line 8, we check if ι_i also witnesses $\mathcal{M} \not\models \varphi_i$ by checking if we have $\overline{O}(\mathcal{M}(\overline{I}(\iota_i))) \not\models \varphi_i$. When ι_i also witnesses $\mathcal{M} \not\models \varphi_i$, we store ι_i in *result* as a witness of $\mathcal{M} \not\models \varphi_i$. Otherwise, we have $\overline{O}(\mathcal{M}(\overline{I}(\iota_i))) \not\models \tilde{\mathcal{M}}(\iota_i)$, and we use ι_i to refine the learned Mealy machine $\tilde{\mathcal{M}}$ (in line 18). When $\tilde{\mathcal{M}} \models \varphi_i$ holds, in line 14, we use the search-based equivalence testing (Algorithm 1) to find $cex \in \Sigma^*$

satisfying $\overline{O}(\mathcal{M}(\overline{I}(cex))) \neq \tilde{\mathcal{M}}(cex)$. When we find such cex, we use it to refine the learned Mealy machine $\tilde{\mathcal{M}}$ (in line 18). Otherwise, we deem $\overline{O} \circ \mathcal{M} \circ \overline{I} = \tilde{\mathcal{M}}$ and return result as the final result: the set of the falsified specifications φ_i with inputs ι_i witnessing $\mathcal{M} \not\models \varphi_i$.

5 EXPERIMENTAL EVALUATION

We implemented a prototypical tool FalCAuN for robustness-guided BBC of CPSs in Java using LearnLib [28], jMetal [19], and LTSMin [29]. As the optimization method in the robustness-guided equivalence testing (i.e., line 5 of Algorithm 1), we employ a hill climbing (HC) and the genetic algorithm [34] (GA). In HC, for each discrete input sequence $\iota \in \Sigma^*$ in the current population set, we generate "children" input sequences by a random mutation. Then, we construct the next population set by taking the children with the smallest robust semantics. HC is one of the simplest algorithm to exploit the robust semantics, but we may get stuck in local optima. In GA, we avoid local optima by using larger population size and combining mutation, crossover, and selection. Our implementation is in https://github.com/MasWag/FalCAuN.

We conducted experiments to answer the following research questions.

RQ1 Does BBC falsify as many specifications as one of the state-of-the-art falsification tools?

RQ2 For which equivalence testing, BBC performs the best? **RQ3** Does BBC falsify multiple specifications effectively?

Benchmarks. As the CPS model \mathcal{M} , we used the Simulink model of an automatic transmission system [26], which is one of the standard models in the literature on falsification. Given a 2-dimensional signal of throttle and brake, the automatic transmission model M returns a 3-dimensional signal of *velocity* v, *rotation* ω , and *gear* q. The range of throttle and brake are [0, 100] and [0, 325], respectively. The domains of velocity v and rotation ω are reals, and the domain of gear q is $\{1, 2, 3, 4\}$. As the specifications, we used the sets of the STL formulas in Table 1. Each benchmark consists of multiple and similar STL formulas. For example, φ_1 consists of 6 STL formulas and all of them are instances of the parametric STL formula $\Box (v < p)$. This setting reflects our motivating example illustrated in Section 1: we do not know the exact threshold in the specification and we want to test the CPS model over various specification instances. The benchmarks φ_1 – φ_5 are taken from [50] and the benchmarks φ_6 and φ_7 are our original.

Experiment. We compared the robustness-guided BBC methods HC and GA with a baseline BBC method RANDOM and one of the state-of-the-art falsification tools Breach.

In HC, for each discrete input sequence *i* in the current population, we generate 60 "children" discrete input sequences

Table 1: List of the STL formulas sets in our benchmarks. φ_1 , φ_2 , φ_3 , φ_4 , and φ_5 are taken from [50]. The other benchmarks are original. The STL formulas in $\varphi_{6,\text{tiny}}$, $\varphi_{6,\text{small}}$, $\varphi_{6,\text{medium}}$, $\varphi_{6,\text{large}}$, $\varphi_{6,\text{huge}}$, and $\varphi_{6,\text{gigantic}}$ have the same structure. These benchmarks are mainly used to compare the scalability with respect to the size of the benchmark.

	STL template	parameter valuations	size
φ_1	$\Box(v < p)$	$p \in \{100, 102.5, 105, 107.5, 110, 112.5, 115, 117.5, 120\}$	9
$arphi_2$	$\Box(g=3\Rightarrow v>p)$	$p \in \{20, 22.5, 25, 27.5, 30\}$	5
φ_3	$\diamondsuit_{\lceil p_1, p_2 \rceil}(v < p_3 \lor v > p_4)$	$(p_1, p_2) \in \{(5, 20), (5, 25), (15, 30), (10, 30)\}, (p_3, p_4) \in \{(50, 60), (53, 57)\}$	8
$arphi_4$	$\square_{[0,26]}(v < p_1) \vee \square_{[28,28]}(v > p_2)$	$p_1 \in \{90, 100, 110\}, p_2 \in \{55, 65, 75\}$	9
$arphi_5$	$\square(\omega < p_1 \vee \mathcal{X}(\omega > p_2))$	$p_1 \in \{4000, 4700\}, p_2 \in \{600, 1000, 1500\}$	6
$\varphi_{6, ext{tiny}}$	$\Box(v < p_1 \Rightarrow \Box_{[0,p_2]}(v < p_3))$	$p_1 \in \{30, 40\}, p_2 = 8, p_3 = 80$	2
$\varphi_{6,\mathrm{small}}$	$\Box(v < p_1 \Rightarrow \Box_{[0,p_2]}(v < p_3))$	$p_1 \in \{30, 40\}, p_2 = 8, p_3 \in \{70, 80\}$	4
$\varphi_{6,\mathrm{medium}}$	$\Box(v < p_1 \Rightarrow \Box_{[0,p_2]}(v < p_3))$	$p_1 \in \{30, 40\}, p_2 \in \{8, 10\}, p_3 \in \{70, 80\}$	8
$\varphi_{6,\mathrm{large}}$	$\Box(v < p_1 \Rightarrow \Box_{[0,p_2]}(v < p_3))$	$p_1 \in \{30, 40, 50\}, p_2 \in \{8, 10\}, p_3 \in \{70, 80\}$	12
$\varphi_{6,\mathrm{huge}}$	$\Box(v < p_1 \Rightarrow \Box_{[0,p_2]}(v < p_3))$	$p_1 \in \{30, 40, 50\}, p_2 \in \{8, 10\}, p_3 \in \{60, 70, 80\}$	18
$\varphi_{6, m gigantic}$	$\Box(v < p_1 \Rightarrow \Box_{[0,p_2]}(v < p_3))$	$p_1 \in \{30, 40, 50\}, p_2 \in \{6, 8, 10\}, p_3 \in \{60, 70, 80, 90\}$	36
$arphi_7$	$\square(((g \neq p_1) \land \mathcal{X}(g = p_1)) \Rightarrow \square_{[0,p_2]}(g = p_1))$	$p_1 \in \{1, 2, 3, 4\}, p_2 \in \{1, 2, 3\}$	12

by random swap: given a discrete input sequence $\iota = a_1, a_2, \ldots, a_n$ random swap returns $a_1, a_2, \ldots, a_{i-1}, a, a_{i+1}, \ldots, a_n$, where $a \in \Sigma$ and $i \in \{1, 2, \ldots, n\}$ are randomly chosen. Among the "children" discrete input sequences, 5 input sequences realizing the smallest robust semantics are chosen to the next population.

In GA, we used uniform mutation, uniform crossover, and tournament selection. The population size, mutation probability, and crossover probability in GA are 150, 0.01, and 0.5, respectively.

In RANDOM, we used a random equivalence testing.

We used TTT algorithm [27] for active automata learning in BBC. For the experiments on BBC, the timeout is 4 hours *in total*. In BBC, the input length is fixed to 30. The abstract alphabet Σ is $|\Sigma| = 4$ such that the throttle is either 0 or 100 and the brake is either 0 or 325. The atomic propositions **AP** is the coarsest partitions of the output space (i.e., the valuations of v,ω , and g) compatible with the inequalities in STL formulas in each benchmark.

We used Breach [16] version 1.5.2 as a baseline. Breach provides several optimization algorithms including *covariance matrix adaptation evolution strategy (CMA-ES)* [6], *global Nelder-Mead (GNM)*[36], and *simulated annealing (SA)* [33]. Among them, we only used CMA-ES because it is reported to outperform the other optimization methods in [51]. For the experiment on Breach, the timeout is 15 minutes *for each* specification. In Breach we generated piecewise constant signals with 30 control points. We note that the signals generated by Breach take floating-point values while the discrete input sequences generated by FalCAuN take 4 values. Thus, the search space of Breach is larger but there can be specifications falsifiable only by Breach.

Since the optimization algorithm in GA, HC, Breach as well as the random sampling in Random are stochastic, we executed each benchmark and algorithm for 10 times. For

each execution, we measured the number of the falsified specifications and the time to falsify all the falsified specifications. For Breach, we used the sum of the time to falsify all the falsified specifications. Table 2 shows the summary of the experiment result. We also show the result of a pure random sampling process (Purerandom) to confirm the hardness of the benchmarks. We also note that φ_1 and φ_7 contain AT1 and a variant of AT5 specifications in [20]. Both of the specifications are falsified by GA 10 times out of 10 trials. We conducted the experiments on an Amazon EC2 c4.large instance (2 vCPUs and 3.75 GiB RAM).

5.1 RQ1: Comparison with Breach

In Table 2, we observe that on average, GA falsified as many properties as Breach does for any benchmark φ_i . HC also falsified as many properties as Breach does for any benchmark φ_i except for φ_1 . Even for φ_1 , the number of the falsified properties of HC is comparable to that of Breach. We also observe that Random falsified as many properties as Breach except for φ_1 , $\varphi_{6,\text{medium}}$, and $\varphi_{6,\text{large}}$.

One reason of the good performance of GA and HC is that the equivalence testing in these methods utilizes a *discrete* optimization and tends to work well even if the different part of the input sequence contributes to the robust semantics differently. For example, in order to falsify φ_4 , we have to find an input that makes the velocity high in the beginning and suddenly decreases the velocity at 28 time units. Such an optimization is not easy for *continuous* optimization methods e.g., CMA-ES.

Another reason is that CMA-ES does not work well when the fitness function has very small slope. For example, for the benchmark φ_2 , when the gear is not 3, the change of the robustness is almost discrete and the slope can be 0. This is a difficult situation for many continuous optimization methods based on the slope. Especially when the slope is too small,

Table 2: Summary of the experiment result. The numbers N/T in each cell are the number N of the falsified specifications and the time T [min.] to falsify all the falsifiable specification. For each experiment setting, the average and the standard deviation are shown. For each benchmark φ_i , the best cell in terms of the following order is highlighted: N/T is better than N'/T' if and only if we have N>N' or we have both N=N' and T<T'. For each benchmark, the largest average of the number of the falsified properties is shown in bold blue font.

	PureRandom	Rani	ANDOM HC		GA		Breach		
	aver. # of spec.	average	std. dev.	average	std. dev.	average	std. dev.	average	std. dev.
φ_1	5.70	8.80/11.10	0.60/2.73	8.90/28.53	0.30/49.81	9.00 /68.96	0.00/64.64	9.00 /12.05	0.00/0.19
φ_2	0.00	4.90 /75.99	0.30/45.98	4.80/82.56	0.40/61.28	4.90 /74.12	0.30/77.88	2.00/0.20	0.00/0.00
φ_3	0.00	8.00 /9.34	0.00/2.88	8.00 /12.68	0.00/4.45	8.00 /12.87	0.00/4.96	8.00 /22.43	0.00/0.58
φ_4	0.60	6.10/100.83	0.70/76.80	5.90/124.88	0.70/73.43	6.90 /163.03	0.30/24.56	2.60/22.37	0.80/7.41
φ_5	2.40	6.00 /139.72	0.00/132.73	3.30/72.99	2.49/124.15	6.00 /133.66	0.00/140.54	3.00/5.78	0.00/0.45
$\varphi_{6,\mathrm{tiny}}$	2.00	2.00 /2.24	0.00/1.14	2.00 /2.44	0.00/1.11	2.00 /3.54	0.00/1.47	2.00 /3.12	0.00/0.09
$\varphi_{6,\mathrm{small}}$	4.00	4.00 /2.98	0.00/1.38	4.00 /2.58	0.00/1.44	4.00 /3.20	0.00/1.03	4.00 /4.41	0.00/0.18
$\varphi_{6,\mathrm{medium}}$	6.10	7.20/141.83	2.40/416.15	8.00 /2.50	0.00/1.31	8.00 /4.07	0.00/2.52	8.00 /7.74	0.00/0.04
$\varphi_{6,\mathrm{large}}$	9.00	10.80/288.46	3.60/566.25	12.00 /3.00	0.00/2.02	12.00 /3.47	0.00/1.46	12.00 /9.99	0.00/0.04
$\varphi_{6,\mathrm{huge}}$	12.00	18.00 /2.36	0.00/1.21	18.00 /2.00	0.00/0.74	18.00 /3.21	0.00/0.78	18.00 /12.45	0.00/0.06
$\varphi_{6, \mathrm{gigantic}}$	30.00	31.00 /5.59	0.00/1.93	31.00 /12.15	0.00/10.05	31.00 /7.95	0.00/3.93	31.00 /36.30	0.00/0.45
φ_7	0.00	12.00 /1.35	0.00/0.76	12.00 /1.25	0.00/0.72	12.00 /1.84	0.00/0.50	9.00/0.38	0.00/0.01

CMA-ES stops deeming there is no better inputs. On the other hand, the behavior of the robustness-guided equivalence checking methods is much like the random search and it successfully falsified the specifications.

5.2 RQ2: Best equivalence testing method

In Table 2, we observe that on average, the number of the falsified properties of GA is greater than or equal to that of Random and HC. Moreover, GA has smaller standard deviation of the number of the properties than Random and HC. This is because GA has a good balance of exploitation of exploration and the equivalence testing tends have a good performance constantly while Random and HC occasionally fails to find a counterexample in the equivalence testing.

On the other hand, we also observe that GA tends not to be the fastest among the BBC methods. This makes the number of the highlighted cells of GA smaller than that of HC and equal to that of RANDOM although GA falsified the largest number of properties. This is because the genetic algorithm in GA is more complicated than the hill climbing in HC and the random search in RANDOM while these simple optimization is enough for easy benchmarks. However, even though GA is not the fastest BBC method, the additional time caused by GA is only a few minutes and it is acceptable for many practical usages. Therefore, we conclude that GA performed the best among the three BBC methods.

5.3 RQ3: Effectiveness to falsify multiple specifications

Fig. 3 shows the average of the number of the falsified properties and the time to falsify these properties for $\varphi_{6,\text{tiny}}$, $\varphi_{6,\text{small}}$,

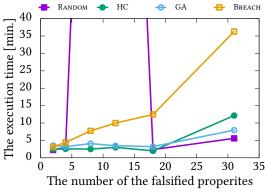


Figure 3: The average of the number of the falsified properties and the time to falsify them [min.] for $\varphi_{6,\text{tiny}}$, $\varphi_{6,\text{small}}$, $\varphi_{6,\text{medium}}$, $\varphi_{6,\text{large}}$, $\varphi_{6,\text{huge}}$, and $\varphi_{6,\text{gigantic}}$.

 $φ_{6,\text{medium}}$, $φ_{6,\text{large}}$, $φ_{6,\text{huge}}$, and $φ_{6,\text{gigantic}}$. We observe that except for $φ_{6,\text{medium}}$ and $φ_{6,\text{large}}$ of HC, the execution time of the BBC algorithms tends to be shorter than that of Breach. Especially, for $φ_{6,\text{tiny}}$, $φ_{6,\text{small}}$, $φ_{6,\text{medium}}$, $φ_{6,\text{large}}$, and $φ_{6,\text{huge}}$, we observe that the execution time of HC and GA is more or less constant while the execution time of Breach increases linearly. This is because in BBC, once we learn a sufficiently accurate Mealy machine $\tilde{\mathcal{M}}$, we often find counterexamples for several specifications immediately. On the other hand, in Breach, each falsification trial is independent and the execution time increases linearly. We note that the huge execution time of Random for $φ_{6,\text{small}}$ and $φ_{6,\text{medium}}$ is due to the outliers as the large standard deviations suggest.

Table 3: Result of falsification only using the extracted Mealy machine. The second column shows the number of the counterexamples found by model checking of the Mealy machines $\tilde{\mathcal{M}}$ extracted during the BBC. The third column shows the number of the actual counterexamples confirmed through a simulation of the CPS model \mathcal{M} . The fourth and the fifth columns show the average and the standard deviation of the robustness, respectively.

STL formula φ	# of $\varphi \not\models \mathcal{M}$	# of $\varphi \not\models \mathcal{M}$	Average of $\llbracket \varphi \rrbracket$	std. dev. of $\llbracket \varphi \rrbracket$
$\Box(v < 90)$	10	5	1.10	1.94
$\square_{[0,26]}(v < 90) \vee \square_{[28,28]}(v > 40)$	4	0	4.19	0.00
$\Box_{[0,26]}(v < 90) \lor \Box_{[28,28]}(v > 50)$	10	0	3.80	0.60
$\square_{[0,26]}(v < 90) \vee \square_{[28,28]}(v > 60)$	10	0	3.24	0.76

Table 4: Average of the number of the states of the extracted Mealy machine

	RANDOM	HC	GA
φ_1	181.90	270.90	441.50
φ_2	612.60	661.60	610.00
φ_3	154.20	200.20	198.30
φ_4	1372.70	1194.70	1353.30
φ_5	948.60	1442.14	888.60
$\varphi_{6, ext{tiny}}$	26.60	32.10	35.20
$\varphi_{6,\mathrm{small}}$	45.30	40.60	39.50
$\varphi_{6,\mathrm{medium}}$	41.44	37.80	47.70
$\varphi_{6,\mathrm{large}}$	32.89	48.70	44.80
$\varphi_{6,\mathrm{huge}}$	41.20	36.70	44.80
$\varphi_{6, \mathrm{gigantic}}$	1912.00	1714.40	1891.10
φ_7	24.00	21.10	20.00

5.4 Discussion on the extracted Mealy machines

One natural question on BBC is whether the extracted Mealy machine $\tilde{\mathcal{M}}$ is a good approximation of the original system \mathcal{M} . Especially, since the robustness-guided equivalence testing focuses on the inputs realizing low robustness, it is unclear if the extracted Mealy machine $\tilde{\mathcal{M}}$ behaves similarly to the original system \mathcal{M} even for the inputs not realizing low robustness. We note that as shown in Table 4, the extracted Mealy machines tend to be huge and a manual inspection is unrealistic.

In order to obtain insights on the aforementioned question, we conducted the following additional experiments.

- (1) For a Mealy machine $\tilde{\mathcal{M}}$ generated through BBC and an STL formula φ not used when $\tilde{\mathcal{M}}$ is learned, we conducted model checking to obtain a witness $\iota \in \Sigma^*$ of $\tilde{\mathcal{M}} \not\models \varphi$. We note that if we have $\tilde{\mathcal{M}} \models \varphi$, we cannot obtain such ι .
- (2) By feeding the generated witness $\iota \in \Sigma^*$ to the original system \mathcal{M} , we checked if ι also witnesses $\mathcal{M} \models \varphi$. Precisely, we checked if we have $\overline{O}(\mathcal{M}(\overline{I}(\iota))) \models \varphi$ by running a simulation.

As the Mealy machines, we used the 10 Mealy machines generated by GA with the benchmark $\varphi_{6,\text{gigantic}}$. As the STL formulas, we used variants of the STL formulas in φ_1 and φ_4 .

Table 3 shows the experiment result. In the second column of Table 3, we observe that we tend to be able to falsify the STL formula φ with respect to the extracted Mealy machine M. On the other hand, in the third column of Table 3, we observe that the witness $\iota \in \Sigma^*$ of $\varphi \not\models \mathcal{M}$ is usually not a witness of $\varphi \not\models \mathcal{M}$. This suggests that if we directly reuse a Mealy machine generated through BBC of different STL formulas, falsification does not perform well. However, in the fourth column, we observe that the robustness is much smaller than the threshold in the STL formulas, and the witness ι of $\varphi \not\models \mathcal{M}$ actually witnesses "near violation" of $\varphi \models \mathcal{M}$. We note that this is not due to outliers as we observe the small standard deviation in the fifth column, Therefore, it seems that the extracted Mealy machine \mathcal{M} is not a very precise abstraction of the original system \mathcal{M} , but we can potentially use $\tilde{\mathcal{M}}$ as a rough approximation of \mathcal{M} .

6 RELATED WORKS

Black-box checking (BBC) [44] (or learning-based testing (LBT) [40]) is initially presented as a *sound* black-box testing method utilizing Vasilevskii and Chow (VC) algorithm [11, 48] as the equivalence oracle. The correctness of the VC algorithm relies on the upper bound of the size of the state space of the black-box system. In [38], Büchi acceptance condition in the state space of the black-box system is used for the sound equivalence checking.

A great effort has been devoted to a more practical direction of BBC, including the testing of automotive systems. For example, case studies on testing of automotive software systems are shown in [32] and an application to the CPSs with continuous dynamics is presented in [31, 39]. However, up to our knowledge, there is no work exploiting the quantitative satisfaction degree of the requirements in addition to Boolean satisfaction. For BBC, as far as we are aware of, two tools have been presented: LBTest [41] and an implementation [38] in LearnLib [28]. Our prototypical tool FalCAuN relies on the implementation [38] in LearnLib.

Falsification is one of the well-known quality assurance methods of CPSs with two well-matured tools: Breach [16] and S-Taliro [5]. Moreover, a friendly competition [22] has been held every year since 2017.

Among many algorithms for falsification, only a few algorithms utilize model learning. For example, in [14], for a CPS model $\mathcal M$ and an STL formula φ , a probabilistic model is constructed to approximate the function from an input signal σ to the robust semantics of φ over the output signal $\mathcal M(\sigma)$, and Bayesian optimization [10] is used to make falsification efficient. In [3], deep reinforcement learning [42] is used for a similar optimization. One drawback of these algorithms is that the learned model depends on the STL formula φ , and it is (at least) not straightforward to apply for the falsification of multiple STL formulas.

In [30], reinforcement learning is used to falsify one specification for multiple but similar systems effectively. We note that our BBC approach is also applicable for falsification of multiple but similar systems by *adaptive model checking* [24].

7 CONCLUSIONS AND FUTURE WORK

Combining optimization-based falsification and black-box checking (BBC), we presented robustness-guided BBC, which is a method to falsify multiple specifications efficiently. Our main technical contribution is to use the robust semantics of STL to enhance the equivalence testing in active automata learning. Our experiment results suggest that robustness-guided BBC by genetic algorithm (GA) tends to outperform baseline algorithms of both optimization-based falsification and BBC. Namely, we compared with BREACH, which is one of the state-of-the-art falsification tools, and RANDOM, which is a BBC method with random equivalence testing.

One future direction is to reuse the extracted Mealy machine $\tilde{\mathcal{M}}$ for BBC over the STL formulas φ other than the formulas φ' examined when $\tilde{\mathcal{M}}$ is extracted. As we observed in Section 5.4, $\tilde{\mathcal{M}}$ may not be a good approximation of \mathcal{M} for falsification of φ , but it seems $\tilde{\mathcal{M}}$ roughly captures the behavior of \mathcal{M} . Thus, we need to (hopefully only slightly) refine $\tilde{\mathcal{M}}$ to obtain a witness of $\mathcal{M} \not\models \varphi$. When $\tilde{\mathcal{M}} \models \varphi$ holds, we have to find an input to refine $\tilde{\mathcal{M}}$ by robustness-guided equivalence testing. It is an interesting future work to make this robustness-guided equivalence testing efficient utilizing $\tilde{\mathcal{M}}$. We note that when we have $\tilde{\mathcal{M}} \not\models \varphi$, we can use the counterexample obtained by the model checking to $\tilde{\mathcal{M}}$. It is also a future work to use $\tilde{\mathcal{M}}$ to explain why the BBC failed.

Another future direction is an efficient falsification method over a family of similar systems using *adaptive model checking* [24].

It is also a future work to conduct further detailed experimental evaluation to compare with more tools by using the ARCH-COMP benchmark [20], or to optimize some parameters. For example, for the alphabet size, there should exist a trade-off between the computation cost and covering a larger class of signals. For the input mapper, we used a very

simple input mapper in the explained as explained in Section 5. Investigation of a good method to give an appropriate alphabets or an input mapper is future work. It is also an interesting future work to use an optimization-based conformance testing of CPSs [46] instead of our robustness-guided equivalence testing.

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A OMITTED PROOFS

A.1 Proof of Theorem 3.4

Theorem 3.4. We prove by induction on the structure of φ .

When $\varphi = \top$, we have $\rho(\top, \sigma, k) = +\infty > 0$ and $(\sigma, k) \models \top$.

When $\varphi = y > c$, we have $\rho(y > c, \sigma, k) = u_k(y) - c$. If we have $\rho(y > c, \sigma, k) > 0$, we have $u_k(y) - c > 0$, and $(\sigma, k) \models y > c$ holds. If we have $(\sigma, k) \models y > c$, we have $u_k(y) - c > 0$, and $\rho(y > c, \sigma, k) \ge 0$ holds.

When $\varphi = y < c$, we have $\rho(y < c, \sigma, k) = -u_k(y) + c$. If we have $\rho(y < c, \sigma, k) > 0$, we have $-u_k(y) + c > 0$, and $(\sigma, k) \models y < c$ holds. If we have $(\sigma, k) \models y < c$, we have $-u_k(y) + c > 0$, and $\rho(y < c, \sigma, k) \ge 0$ holds.

When $\varphi = \neg \varphi'$, we have $\rho(\neg \varphi', \sigma, k) = -\rho(\varphi', \sigma, k)$. If we have $\rho(\neg \varphi', \sigma, k) > 0$, we have $\rho(\varphi', \sigma, k) \leq 0$. Therefore, we have $(\sigma, k) \not\models \varphi'$ and we have $(\sigma, k) \models \neg \varphi'$. If we have $(\sigma, k) \models \neg \varphi'$, we have $(\sigma, k) \not\models \varphi'$. By induction hypothesis, we have $(\sigma, k) \models \varphi' < 0$ and we have $(\sigma, k) \models \neg \varphi' \geq 0$.

When $\varphi = \varphi' \vee \varphi''$, we have $\rho(\varphi' \vee \varphi'', \sigma, k) = \max\{\rho(\varphi', \sigma, k), \rho(\varphi'', \sigma, k)\}$. If we have $\rho(\varphi' \vee \varphi', \sigma, k) > 0$, we have $\rho(\varphi', \sigma, k) > 0$ or $\rho(\varphi'', \sigma, k) > 0$. By induction hypothesis, we have $(\sigma, k) \models \varphi'$ or $(\sigma, k) \models \varphi''$, and therefore, we have $(\sigma, k) \models \varphi' \vee \varphi''$.

When $\varphi = X\varphi'$, we have $\rho(X\varphi', \sigma, k) = \rho(\varphi', \sigma, k+1)$. If we have $\rho(X\varphi', \sigma, k) > 0$, we have $\rho(\varphi', \sigma, k+1) > 0$. By induction hypothesis, we have $(\sigma, k+1) \models \varphi'$ and therefore, we have $(\sigma, k) \models X\varphi'$. If we have $(\sigma, k) \models X\varphi'$, we have $(\sigma, k+1) \models \varphi'$. By induction hypothesis, we have $\rho(\varphi', \sigma, k+1) \geq 0$, and therefore, we have $\rho(X\varphi', \sigma, k) \geq 0$.

When $\varphi = \varphi' \mathcal{U}_{[i,j)} \varphi''$, we have $\rho(\varphi \mathcal{U}_{[i,j)} \varphi', \sigma, k) = \sup_{l \in [k+i,k+j)} \min(\rho(\varphi',\sigma,l), \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi,\sigma,m))$. If we have $\rho(\varphi \mathcal{U}_{[i,j)} \varphi', \sigma, k) > 0$, there exists $l \in [k+i,k+j)$ such that we have $\rho(\varphi',\sigma,l) > 0$ and for any $m \in \{k,k+1,\dots,l\}$, we have $\rho(\varphi,\sigma,m) > 0$. By induction hypothesis, there exists $l \in [k+i,k+j)$ such that we have $(\sigma,l) \models \varphi'$ and for any $m \in \{k,k+1,\dots,l\}$, we have $(\sigma,m) \models \varphi$. Therefore, we have $(\sigma,k) \models \varphi \mathcal{U}_{[i,j)} \varphi'$. If we have $(\sigma,k) \models \varphi \mathcal{U}_{[i,j)} \varphi'$, there exists $l \in [k+i,k+j)$ such that we have $(\sigma,l) \models \varphi'$ and for any $m \in \{k,k+1,\dots,l\}$, we have $(\sigma,m) \models \varphi$. By induction hypothesis, there exists $l \in [k+i,k+j)$ such that we have $\rho(\varphi',\sigma,l) \geq 0$ and for any $m \in \{k,k+1,\dots,l\}$, we have $\rho(\varphi,\sigma,m) \geq 0$. Therefore, we have $\rho(\varphi \mathcal{U}_{[i,j)} \varphi',\sigma,k) \geq 0$,

A.2 Proof of Theorem 3.7

First, we prove the following lemma.

LEMMA A.1. For an STL formula φ over Y, a finite signal $\sigma = u_0, u_1, \dots, u_{n-1} \in (\mathbb{R}^Y)^*$ over Y, and $k \in \mathbb{N}$ we have the following.

$$\sigma \notin \llbracket \varphi \rrbracket_{\square} \iff \sigma \in \llbracket \neg \varphi \rrbracket_{\diamondsuit}
\sigma \notin \llbracket \varphi \rrbracket_{\diamondsuit} \iff \sigma \in \llbracket \neg \varphi \rrbracket_{\square}$$

PROOF. The first part is proved as follows.

$$\sigma \notin \llbracket \varphi \rrbracket_{\square}$$

$$\iff \neg (\forall \sigma' \in (\mathbb{R}^Y)^{\omega}. \, \sigma \cdot \sigma' \models \varphi)$$

$$\iff \exists \sigma' \in (\mathbb{R}^Y)^{\omega}. \, \sigma \cdot \sigma' \not\models \varphi$$

$$\iff \exists \sigma' \in (\mathbb{R}^Y)^{\omega}. \, \sigma \cdot \sigma' \models \neg \varphi$$

$$\iff \sigma \in \llbracket \neg \varphi \rrbracket_{\diamondsuit}$$

The second part is proved as follows.

$$\sigma \notin \llbracket \varphi \rrbracket_{\diamondsuit}$$

$$\iff \neg (\exists \sigma' \in (\mathbb{R}^{Y})^{\omega}. \, \sigma \cdot \sigma' \models \varphi)$$

$$\iff \forall \sigma' \in (\mathbb{R}^{Y})^{\omega}. \, \sigma \cdot \sigma' \not\models \varphi$$

$$\iff \forall \sigma' \in (\mathbb{R}^{Y})^{\omega}. \, \sigma \cdot \sigma' \models \neg \varphi$$

$$\iff \sigma \in \llbracket \neg \varphi \rrbracket_{\square}$$

Theorem 3.7 is proved as follows.

Theorem 3.7. We prove by induction on the structure of φ .

When $\varphi = \top$, we have $\sup(\text{RoSI}(\top, \sigma, k)) = \inf(\text{RoSI}(\top, \sigma, k)) = +\infty > 0$ and $\sigma[k, |\sigma| - 1] \in (\mathbb{R}^Y)^* = [\![\top]\!]_{\Diamond} = [\![\top]\!]_{\Box}$. When $\varphi = y > c$, we have the following.

$$\sup(\operatorname{RoSI}(y>c,\sigma,k)) = \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(y>c,\sigma \cdot \sigma',k) = \begin{cases} u_k(y) - c & \text{if } |\sigma| > k \\ +\infty & \text{if } |\sigma| \leq k \end{cases}$$

$$\inf(\operatorname{RoSI}(y>c,\sigma,k)) = \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(y>c,\sigma \cdot \sigma',k) = \begin{cases} u_k(y) - c & \text{if } |\sigma| > k \\ -\infty & \text{if } |\sigma| \leq k \end{cases}$$

If we have $\sup(\operatorname{RoSI}(y>c,\sigma,k))>0$, we have $u_k(y)>c$ or $|\sigma|\leq k$, and we have $\sigma[k,|\sigma|]\in [\![y>c]\!]_{\diamondsuit}$. If we have $\sigma[k,|\sigma|]\in [\![y>c]\!]_{\diamondsuit}$, we have $u_k(y)>c$ or $|\sigma|\leq k$, and thus, we have $\sup(\operatorname{RoSI}(y>c,\sigma,k))\geq 0$. If we have $\inf(\operatorname{RoSI}(y>c,\sigma,k))>0$, we have $|\sigma|>k$ and $u_k(y)>c$, and we have $\sigma[k,|\sigma|]\in [\![y>c]\!]_{\square}$. If we have $\sigma[k,|\sigma|]\in [\![y>c]\!]_{\square}$, we have $|\sigma|>k$ and $u_k(y)>c$, and thus, we have $\inf(\operatorname{RoSI}(y>c,\sigma,k))\geq 0$. When $\varphi=y< c$, we have the following.

$$\sup(\operatorname{RoSI}(y < c, \sigma, k)) = \sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(y < c, \sigma \cdot \sigma', k) = \begin{cases} -u_k(y) + c & \text{if } |\sigma| > k \\ +\infty & \text{if } |\sigma| \le k \end{cases}$$

$$\inf(\operatorname{RoSI}(y < c, \sigma, k)) = \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(y < c, \sigma \cdot \sigma', k) = \begin{cases} -u_k(y) + c & \text{if } |\sigma| > k \\ -\infty & \text{if } |\sigma| \le k \end{cases}$$

If we have $\sup(\operatorname{RoSI}(y < c, \sigma, k)) > 0$, we have $u_k(y) < c$ or $|\sigma| \le k$, and we have $\sigma[k, |\sigma|] \in \llbracket y < c \rrbracket_{\diamondsuit}$. If we have $\sigma[k, |\sigma|] \in \llbracket y < c \rrbracket_{\diamondsuit}$, we have $u_k(y) < c$ or $|\sigma| \le k$, and thus, we have $\sup(\operatorname{RoSI}(y < c, \sigma, k)) \ge 0$. If we have $\inf(\operatorname{RoSI}(y < c, \sigma, k)) > 0$, we have $|\sigma| > k$ and $u_k(y) < c$, and we have $\sigma[k, |\sigma|] \in \llbracket y < c \rrbracket_{\square}$. If we have $\sigma[k, |\sigma|] \in \llbracket y < c \rrbracket_{\square}$, we have $|\sigma| > k$ and $u_k(y) < c$, and thus, we have $\inf(\operatorname{RoSI}(y < c, \sigma, k)) \ge 0$. When $\varphi = \neg \varphi'$, we have the following.

$$\begin{split} \sup(\operatorname{RoSI}(\neg\varphi',\sigma,k)) &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\neg\varphi',\sigma \cdot \sigma,k) = \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} -\rho(\varphi',\sigma \cdot \sigma,k) \\ &= -\inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi',\sigma \cdot \sigma,k) \\ \inf(\operatorname{RoSI}(\neg\varphi',\sigma,k)) &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\neg\varphi',\sigma \cdot \sigma,k) = \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} -\rho(\varphi',\sigma \cdot \sigma,k) \\ &= -\sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi',\sigma \cdot \sigma,k) \end{split}$$

If we have $\sup(\text{RoSI}(\neg \varphi', \sigma, k)) > 0$, we have $\inf(\text{RoSI}(\varphi', \sigma, k)) < 0$. By induction hypothesis, we have $\sigma[k, |\sigma|] \notin \llbracket \varphi' \rrbracket_{\square}$. By Lemma A.1, we have $\sigma[k, |\sigma|] \in \llbracket \neg \varphi' \rrbracket_{\diamondsuit}$.

If we have $\sigma[k, |\sigma|] \in [\![\neg \varphi']\!]_{\diamondsuit}$, by Lemma A.1, we have $\sigma[k, |\sigma|] \notin [\![\varphi']\!]_{\square}$. By induction hypothesis, we have $\inf(\text{RoSI}(\varphi', \sigma, k)) < 0$, and thus, we have $\sup(\text{RoSI}(\neg \varphi', \sigma, k)) \ge 0$.

If we have $\inf(\text{RoSI}(\neg \varphi', \sigma, k)) > 0$, we have $\sup(\text{RoSI}(\varphi', \sigma, k)) < 0$. By induction hypothesis, we have $\sigma[k, |\sigma|] \notin \llbracket \varphi' \rrbracket_{\Diamond}$ and by Lemma A.1. By Lemma A.1, we have $\sigma[k, |\sigma|] \in \llbracket \neg \varphi' \rrbracket_{\Box}$.

If we have $\sigma[k, |\sigma|] \in [\![\neg \varphi']\!]_{\square}$, by Lemma A.1, we have $\sigma[k, |\sigma|] \notin [\![\varphi']\!]_{\diamondsuit}$. By induction hypothesis, we have $\sup(\text{RoSI}(\varphi', \sigma, k)) < 0$, and thus, we have $\inf(\text{RoSI}(\neg \varphi', \sigma, k)) \ge 0$.

When $\varphi = \varphi' \vee \varphi''$, we have the following.

$$\begin{split} \sup(\operatorname{RoSI}(\varphi' \vee \varphi'', \sigma, k)) &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi' \vee \varphi'', \sigma \cdot \sigma', k) \\ &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \max \bigl\{ \rho(\varphi', \sigma \cdot \sigma', k), \rho(\varphi'', \sigma \cdot \sigma', k) \bigr\} \\ \inf(\operatorname{RoSI}(\varphi' \vee \varphi'', \sigma, k)) &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi' \vee \varphi'', \sigma \cdot \sigma', k) \\ &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \max \bigl\{ \rho(\varphi', \sigma \cdot \sigma', k), \rho(\varphi'', \sigma \cdot \sigma', k) \bigr\} \end{split}$$

If we have $\sup(\operatorname{RoSI}(\varphi' \vee \varphi'', \sigma, k)) > 0$, there exists $\sigma' \in (\mathbb{R}^Y)^\omega$ satisfying $\rho(\varphi', \sigma \cdot \sigma', k) > 0$ or $\rho(\varphi'', \sigma \cdot \sigma', k) > 0$. By Theorem 3.4 there exists $\sigma' \in (\mathbb{R}^Y)^\omega$ satisfying $(\sigma \cdot \sigma', k) \models \varphi'$ or $(\sigma \cdot \sigma', k) \models \varphi''$, and thus, we have $\sigma[k, |\sigma|] \in [\![\varphi' \vee \varphi'']\!]_{\diamondsuit}$. If we have $\sigma[k, |\sigma|] \in [\![\varphi' \vee \varphi'']\!]_{\diamondsuit}$, there exists $\sigma' \in (\mathbb{R}^Y)^\omega$ satisfying $(\sigma \cdot \sigma', k) \models \varphi' \vee \varphi''$. By Theorem 3.4 there exists $\sigma' \in (\mathbb{R}^Y)^\omega$ satisfying $\rho(\varphi', \sigma \cdot \sigma', k) \geq 0$ or $\rho(\varphi'', \sigma \cdot \sigma', k) \geq 0$, and thus, we have $\sup(\operatorname{RoSI}(\varphi' \vee \varphi'', \sigma, k)) \geq 0$.

If we have inf $(\operatorname{RoSI}(\varphi' \vee \varphi'', \sigma, k)) > 0$, for any $\sigma' \in (\mathbb{R}^Y)^\omega$, we have $\rho(\varphi', \sigma \cdot \sigma', k) > 0$ or $\rho(\varphi'', \sigma \cdot \sigma', k) > 0$. By Theorem 3.4 for any $\sigma' \in (\mathbb{R}^Y)^\omega$, we have $(\sigma \cdot \sigma', k) \models \varphi'$ or $(\sigma \cdot \sigma', k) \models \varphi''$. Therefore, we have $\sigma[k, |\sigma|] \in [\![\varphi' \vee \varphi'']\!]_{\square}$.

If we have $\sigma[k, |\sigma|] \in [\![\varphi' \vee \varphi'']\!]_{\square}$, for any $\sigma' \in (\mathbb{R}^Y)^{\omega}$, we have $(\sigma \cdot \sigma', k) \models \varphi' \vee \varphi''$. By Theorem 3.4 for any $\sigma' \in (\mathbb{R}^Y)^{\omega}$, we have $\rho(\varphi', \sigma \cdot \sigma', k) \geq 0$ or $\rho(\varphi'', \sigma \cdot \sigma', k) \geq 0$. Therefore, we have $\inf(\text{RoSI}(\varphi' \vee \varphi'', \sigma, k)) \geq 0$.

When $\varphi = X\varphi'$, we have the following.

$$\begin{split} \sup(\operatorname{RoSI}(X\varphi',\sigma,k)) &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(X\varphi',\sigma \cdot \sigma,k) = \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi',\sigma \cdot \sigma,k+1) \\ &= \sup(\operatorname{RoSI}(\varphi',\sigma,k+1)) \\ \inf(\operatorname{RoSI}(X\varphi',\sigma,k)) &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(X\varphi',\sigma \cdot \sigma,k) = \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi',\sigma \cdot \sigma,k+1) \\ &= \inf(\operatorname{RoSI}(\varphi',\sigma,k+1)) \end{split}$$

If we have $\sup(\text{RoSI}(X\varphi', \sigma, k)) = \sup(\text{RoSI}(\varphi', \sigma, k + 1)) > 0$, we have $\sigma[k + 1, |\sigma|] \in [\![\varphi']\!]_{\diamondsuit}$ and $\sigma[k, |\sigma|] \in [\![X\varphi']\!]_{\diamondsuit}$. If we have $\sigma[k, |\sigma|] \in [\![X\varphi']\!]_{\diamondsuit}$, we have $\sigma[k + 1, |\sigma|] \in [\![\varphi']\!]_{\diamondsuit}$, therefore, we have $\sup(\text{RoSI}(X\varphi', \sigma, k)) = \sup(\text{RoSI}(\varphi', \sigma, k + 1)) > 0$ and

If we have $\inf(\operatorname{RoSI}(X\varphi', \sigma, k)) = \inf(\operatorname{RoSI}(\varphi', \sigma, k + 1)) > 0$, we have $\sigma[k + 1, |\sigma|] \in [\![\varphi']\!]_{\square}$ and $\sigma[k, |\sigma|] \in [\![X\varphi']\!]_{\square}$. If we have $\sigma[k, |\sigma|] \in [\![X\varphi']\!]_{\square}$, we have $\sigma[k + 1, |\sigma|] \in [\![\varphi']\!]_{\square}$, therefore, we have $\inf(\operatorname{RoSI}(X\varphi', \sigma, k)) = \inf(\operatorname{RoSI}(\varphi', \sigma, k + 1)) \ge 0$.

When $\varphi = \varphi' \mathcal{U}_{[i,j)} \varphi''$, we have the following.

$$\begin{split} &\sup(\operatorname{RoSI}(\varphi' \ \mathcal{U}_{[i,j)} \ \varphi'', \sigma, k)) \\ &= \sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \sup_{l \in [k+i,k+j)} \min(\rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &\inf(\operatorname{RoSI}(\varphi' \ \mathcal{U}_{[i,j)} \ \varphi'', \sigma, k)) \\ &= \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \sup_{l \in [k+i,k+j)} \min(\rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi', \sigma \cdot \sigma', m)) \end{split}$$

If we have $\sup(\operatorname{RoSI}(\varphi'\mathcal{U}_{[i,j)}\varphi'',\sigma,k)) > 0$, there exist $\sigma' \in (\mathbb{R}^Y)^\omega$ and $l \in [k+i,k+j)$ such that we have $\rho(\varphi'',\sigma\cdot\sigma',l) > 0$ and for any $m \in \{k,k+1,\ldots,l\}$, we have $\rho(\varphi',\sigma\cdot\sigma',m) > 0$. By Theorem 3.4, there exist $\sigma' \in (\mathbb{R}^Y)^\omega$ and $l \in [k+i,k+j)$ such that we have $(\sigma \cdot \sigma',l) \models \varphi''$ and for any $m \in \{k,k+1,\ldots,l\}$, we have $(\sigma \cdot \sigma',m) \models \varphi'$. Therefore, there exist $\sigma' \in (\mathbb{R}^Y)^\omega$ and $l \in [k+i,k+j)$ satisfying $(\sigma \cdot \sigma',k) \models \varphi' \mathcal{U}_{[i,j)} \varphi''$ and we have $\sigma[k,|\sigma|] \in [\varphi' \mathcal{U}_{[i,j)} \varphi'']_{\diamond}$

and $l \in [k+i,k+j)$ satisfying $(\sigma \cdot \sigma',k) \models \varphi' \mathcal{U}_{[i,j)} \varphi''$ and we have $\sigma[k,|\sigma|] \in [\varphi' \mathcal{U}_{[i,j)} \varphi'']_{\diamondsuit}$. If we have $\sigma[k,|\sigma|] \in [\varphi' \mathcal{U}_{[i,j)} \varphi'']_{\diamondsuit}$, there exist $\sigma' \in (\mathbb{R}^Y)^{\omega}$ and $l \in [k+i,k+j)$ satisfying $(\sigma \cdot \sigma',k) \models \varphi' \mathcal{U}_{[i,j)} \varphi''$ and we have $\sigma[k,|\sigma|] \in [\varphi' \mathcal{U}_{[i,j)} \varphi'']_{\diamondsuit}$. Therefore, there exist $\sigma' \in (\mathbb{R}^Y)^{\omega}$ and $l \in [k+i,k+j)$ such that we have $(\sigma \cdot \sigma',l) \models \varphi''$ and for any $m \in \{k,k+1,\ldots,l\}$, we have $(\sigma \cdot \sigma',m) \models \varphi'$. By Theorem 3.4, there exist $\sigma' \in (\mathbb{R}^Y)^{\omega}$ and $l \in [k+i,k+j)$ such that we have $\rho(\varphi'',\sigma \cdot \sigma',l) \geq 0$ and for any $m \in \{k,k+1,\ldots,l\}$, we have $\rho(\varphi'',\sigma \cdot \sigma',m) \geq 0$, and thus, we have $\rho(\varphi'',\sigma \cdot \sigma',k) \geq 0$.

If we have $\inf(\operatorname{RoSI}(\varphi'\mathcal{U}_{[i,j)}\varphi'',\sigma,k))>0$, for any $\sigma'\in(\mathbb{R}^Y)^\omega$, there exists $l\in[k+i,k+j)$ such that we have $\rho(\varphi'',\sigma\cdot\sigma',l)>0$ and for any $m\in\{k,k+1,\ldots,l\}$, we have $\rho(\varphi',\sigma\cdot\sigma',m)>0$. By Theorem 3.4, for any $\sigma'\in(\mathbb{R}^Y)^\omega$, there exists $l\in[k+i,k+j)$ such that we have $(\sigma\cdot\sigma',l)\models\varphi''$ and for any $m\in\{k,k+1,\ldots,l\}$, we have $(\sigma\cdot\sigma',m)\models\varphi'$. Therefore, for any $\sigma'\in(\mathbb{R}^Y)^\omega$, there exists $l\in[k+i,k+j)$ satisfying $(\sigma\cdot\sigma',k)\models\varphi''\mathcal{U}_{[i,j)}\varphi''$ and we have $\sigma[k,|\sigma|]\in[\varphi'\mathcal{U}_{[i,j)}\varphi'']_{\square}$

If we have $\sigma[k, |\sigma|] \in [\varphi' \mathcal{U}_{[i,j)} \varphi'']_{\square}$, for any $\sigma' \in (\mathbb{R}^Y)^{\omega}$, there exists $l \in [k+i, k+j)$ satisfying $(\sigma \cdot \sigma', k) \models \varphi' \mathcal{U}_{[i,j)} \varphi''$. Therefore, for any $\sigma' \in (\mathbb{R}^Y)^{\omega}$, there exists $l \in [k+i, k+j)$ such that we have $(\sigma \cdot \sigma', l) \models \varphi''$ and for any $m \in \{k, k+1, \ldots, l\}$, we have $(\sigma \cdot \sigma', m) \models \varphi'$. By Theorem 3.4, for any $\sigma' \in (\mathbb{R}^Y)^{\omega}$, there exists $l \in [k+i, k+j)$ such that we have $\rho(\varphi'', \sigma \cdot \sigma', l) \geq 0$ and for any $m \in \{k, k+1, \ldots, l\}$, we have $\rho(\varphi', \sigma \cdot \sigma', m) \geq 0$. Thus, we have $\rho(\varphi', \sigma \cdot \sigma', m) \geq 0$.

A.3 Proof of Theorem 3.9

Theorem 3.9. Since both $[\rho](\varphi, \sigma, k)$ and $RoSI(\varphi, \sigma, k)$ are nonempty closed intervals, we have $RoSI(\varphi, \sigma, k) \subseteq [\rho](\varphi, \sigma, k)$ if and only if we have $\inf([\rho](\varphi, \sigma, k)) \leq \inf(RoSI(\varphi, \sigma, k)) \leq \sup([\rho](\varphi, \sigma, k))$. We prove the theorem by induction on the structure of φ .

When $\varphi = \top$, we have $\sup(\text{RoSI}(\top, \sigma, k)) = \inf(\text{RoSI}(\top, \sigma, k)) = +\infty$ and $[\rho](\varphi, \sigma, k) = [+\infty, +\infty]$. Therefore, we have $[\rho](\top, \sigma, k) = \text{RoSI}(\top, \sigma, k)$.

When $\varphi = y > c$, we have the following.

$$\begin{split} \sup(\operatorname{RoSI}(y>c,\sigma,k)) &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(y>c,\sigma \cdot \sigma',k) = \begin{cases} u_k(y) - c & \text{if } |\sigma| > k \\ +\infty & \text{if } |\sigma| \leq k \end{cases} \\ \inf(\operatorname{RoSI}(y>c,\sigma,k)) &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(y>c,\sigma \cdot \sigma',k) = \begin{cases} u_k(y) - c & \text{if } |\sigma| > k \\ -\infty & \text{if } |\sigma| \leq k \end{cases} \end{split}$$

Therefore, we have $[\rho](y > c, \sigma, k) = \text{RoSI}(y > c, \sigma, k)$.

When $\varphi = y < c$, we have the following.

$$\begin{split} \sup(\operatorname{RoSI}(y < c, \sigma, k)) &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(y < c, \sigma \cdot \sigma', k) = \begin{cases} -u_k(y) + c & \text{if } |\sigma| > k \\ +\infty & \text{if } |\sigma| \le k \end{cases} \\ \inf(\operatorname{RoSI}(y < c, \sigma, k)) &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(y < c, \sigma \cdot \sigma', k) = \begin{cases} -u_k(y) + c & \text{if } |\sigma| > k \\ -\infty & \text{if } |\sigma| \le k \end{cases} \end{split}$$

Therefore, we have $[\rho](y < c, \sigma, k) = RoSI(y < c, \sigma, k)$.

When $\varphi = \neg \varphi'$, we have the following.

$$\begin{split} \sup(\operatorname{RoSI}(\neg\varphi',\sigma,k)) &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\neg\varphi',\sigma \cdot \sigma,k) = \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} -\rho(\varphi',\sigma \cdot \sigma,k) \\ &= -\inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi',\sigma \cdot \sigma,k) \\ &= -\inf(\operatorname{RoSI}(\neg\varphi',\sigma,k)) \\ \inf(\operatorname{RoSI}(\neg\varphi',\sigma,k)) &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\neg\varphi',\sigma \cdot \sigma,k) = \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} -\rho(\varphi',\sigma \cdot \sigma,k) \\ &= -\sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi',\sigma,k)) \end{split}$$

Therefore, we have the following.

$$RoSI(\neg \varphi', \sigma, k) = -RoSI(\varphi', \sigma, k) \subseteq -[\rho](\varphi', \sigma, k) = [\rho](\neg \varphi', \sigma, k)$$

When $\varphi = \varphi' \vee \varphi''$, we have the following.

$$\begin{split} \sup(\operatorname{RoSI}(\varphi' \vee \varphi'', \sigma, k)) &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi' \vee \varphi'', \sigma \cdot \sigma', k) \\ &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \max \bigl\{ \rho(\varphi', \sigma \cdot \sigma', k), \rho(\varphi'', \sigma \cdot \sigma', k) \bigr\} \\ &= \max \bigl\{ \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi', \sigma \cdot \sigma', k), \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi'', \sigma \cdot \sigma', k) \bigr\} \\ &= \max \bigl\{ \sup(\operatorname{RoSI}(\varphi', \sigma, k)), \sup(\operatorname{RoSI}(\varphi'', \sigma, k)) \bigr\} \\ &\leq \max \bigl\{ \sup([\rho](\varphi', \sigma, k)), \sup([\rho](\varphi'', \sigma, k)) \bigr\} \\ &= \sup([\rho](\varphi' \vee \varphi'', \sigma, k)) \\ &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi' \vee \varphi'', \sigma \cdot \sigma', k) \\ &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi', \sigma \cdot \sigma', k), \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi'', \sigma \cdot \sigma', k) \bigr\} \\ &\geq \max \bigl\{ \inf(\operatorname{RoSI}(\varphi', \sigma, k)), \inf(\operatorname{RoSI}(\varphi'', \sigma, k)) \bigr\} \\ &= \max \bigl\{ \inf([\rho](\varphi', \sigma, k)), \inf([\rho](\varphi'', \sigma, k)) \bigr\} \\ &= \inf([\rho](\varphi', \sigma, k)), \inf([\rho](\varphi'', \sigma, k)) \bigr\} \\ &= \inf([\rho](\varphi', \sigma, k)), \inf([\rho](\varphi'', \sigma, k)) \bigr\} \\ &= \inf([\rho](\varphi', \varphi'', \sigma, k)), \inf([\rho](\varphi'', \varphi, k)) \bigr\} \\ &= \inf([\rho](\varphi', \varphi'', \varphi, k)), \inf([\rho](\varphi'', \varphi, k)) \bigr\} \\ &= \inf([\rho](\varphi', \varphi'', \varphi, k)), \inf([\rho](\varphi'', \varphi, k)) \bigr\} \\ &= \inf([\rho](\varphi', \varphi'', \varphi, k)), \inf([\rho](\varphi'', \varphi, k)) \bigr\} \\ &= \inf([\rho](\varphi', \varphi'', \varphi, k)), \inf([\rho](\varphi'', \varphi, k)) \bigr\} \\ &= \inf([\rho](\varphi', \varphi'', \varphi, k)), \inf([\rho](\varphi'', \varphi, k)) \bigr\} \\ &= \inf([\rho](\varphi', \varphi'', \varphi, k)), \inf([\rho](\varphi'', \varphi, k)) \bigr\} \\ &= \inf([\rho](\varphi', \varphi'', \varphi, k)), \inf([\varphi](\varphi'', \varphi, k)) \bigr\}$$

Therefore, we have $RoSI(\varphi' \lor \varphi'', \sigma, k) \subseteq [\rho](\varphi' \lor \varphi'', \sigma, k)$. When $\varphi = \mathcal{X}\varphi'$, we have the following.

$$\begin{split} \sup(\operatorname{RoSI}(X\varphi',\sigma,k)) &= \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(X\varphi',\sigma \cdot \sigma,k) = \sup_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi',\sigma \cdot \sigma,k+1) \\ &= \sup(\operatorname{RoSI}(\varphi',\sigma,k+1)) \\ \inf(\operatorname{RoSI}(X\varphi',\sigma,k)) &= \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(X\varphi',\sigma \cdot \sigma,k) = \inf_{\sigma' \in (\mathbb{R}^Y)^\omega} \rho(\varphi',\sigma \cdot \sigma,k+1) \\ &= \inf(\operatorname{RoSI}(\varphi',\sigma,k+1)) \end{split}$$

Therefore, we have the following.

$$RoSI(X\varphi', \sigma, k) = RoSI(\varphi', \sigma, k+1) \subseteq [\rho](\varphi', \sigma, k+1) = [\rho](X\varphi', \sigma, k)$$

Table 5: The ratio of the time to falsify as many properties as Breach. There is no entry for φ_4 because the number of falsified properties by Breach was not constant. The cells with N/A show that the method could not falsify as many properties as Breach.

	Breach/Random	Breach/HC	Breach/GA
φ_1	N/A	N/A	0.174747
$arphi_2$	0.0657534	0.0468933	0.0426136
φ_3	2.40107	1.76962	1.74259
$arphi_5$	3.32821	3.43027	2.70304
$\varphi_{6, ext{tiny}}$	1.39079	1.27869	0.880527
$arphi_{6, ext{small}}$	1.48069	1.7071	1.38028
$\varphi_{6,\mathrm{medium}}$	N/A	3.10013	1.90094
$\varphi_{6,\mathrm{large}}$	N/A	3.33556	2.88035
$\varphi_{6,\mathrm{huge}}$	5.2717	6.225	3.87649
$\varphi_{6, ext{gigantic}}$	6.4918	2.98888	4.56795
$arphi_7$	0.363057	0.36248	0.24333

When $\varphi = \varphi' \mathcal{U}_{[i,j)} \varphi''$, we have the following.

$$\begin{split} &\sup(\operatorname{RoSI}(\varphi' \ \mathcal{U}_{[i,j)} \ \varphi'', \sigma, k)) \\ &= \sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \sup_{l \in [k+i,k+j)} \min(\rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \min(\rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &\leq \sup_{l \in [k+i,k+j)} \min(\sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &\leq \sup_{l \in [k+i,k+j)} \min(\sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} (p)(\varphi'', \sigma, l)), \min_{m \in \{k,k+1,\dots,l\}} \sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\sup_{\sigma' \in (\mathbb{R}^Y)^{\omega}} (p)(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &\geq \sup_{l \in [k+i,k+j)} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \min(\rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \min_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', m)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', l)) \\ &= \sup_{l \in [k+i,k+j)} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{m \in \{k,k+1,\dots,l\}} \inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi', \sigma \cdot \sigma', l)) \\ &= \sup_{l \in [k+i,k+j]} \min(\inf_{\sigma' \in (\mathbb{R}^Y)^{\omega}} \rho(\varphi'', \sigma \cdot \sigma', l), \min_{\sigma'$$

Therefore, we have $RoSI(\varphi' \mathcal{U}_{[i,j)} \varphi'', \sigma, k) \subseteq [\rho](\varphi' \mathcal{U}_{[i,j)} \varphi'', \sigma, k)$.

B OMITTED EXPERIMENT RESULT

Table 5 shows the ratio of the time to falsify as many properties as Breach.