Tidal effects on scalar cloud (numerical simulation)

Taishi Ikeda (CENTRA, Lisbon)
with Vitor Cardoso, Francisco Duque

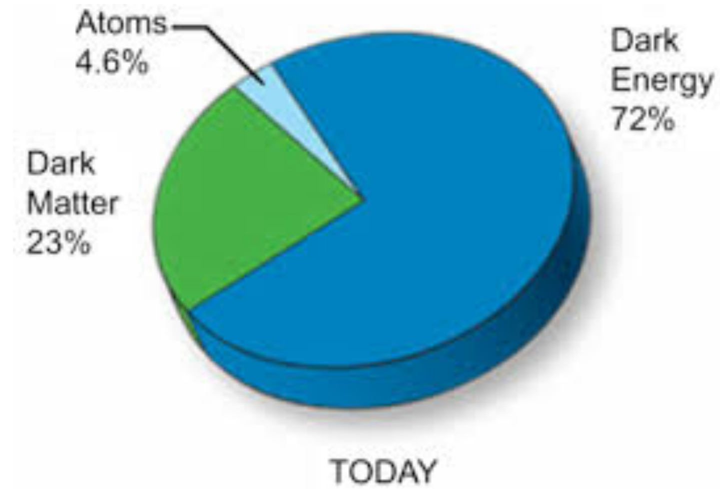
Outline

1. Introduction
2. Our work
   - How to add tidal force
   - Weak tidal
   - Strong tidal
   - Application
3. Summary

Outline

1. Introduction
2. Our work
   - How to add tidal force
   - Weak tidal
   - Strong tidal
   - Application
3. Summary

Light scalar field



Energy components

Dark Matter

- QCD axion
- string axion
- PBH et al

Dark Energy

- Cosmological constant
- Modified gravity
  - Scalar tensor theory
  - F(R) gravity
  - massive gravity et al

➡ Several models predict light scalar field.

Superradiant clouds

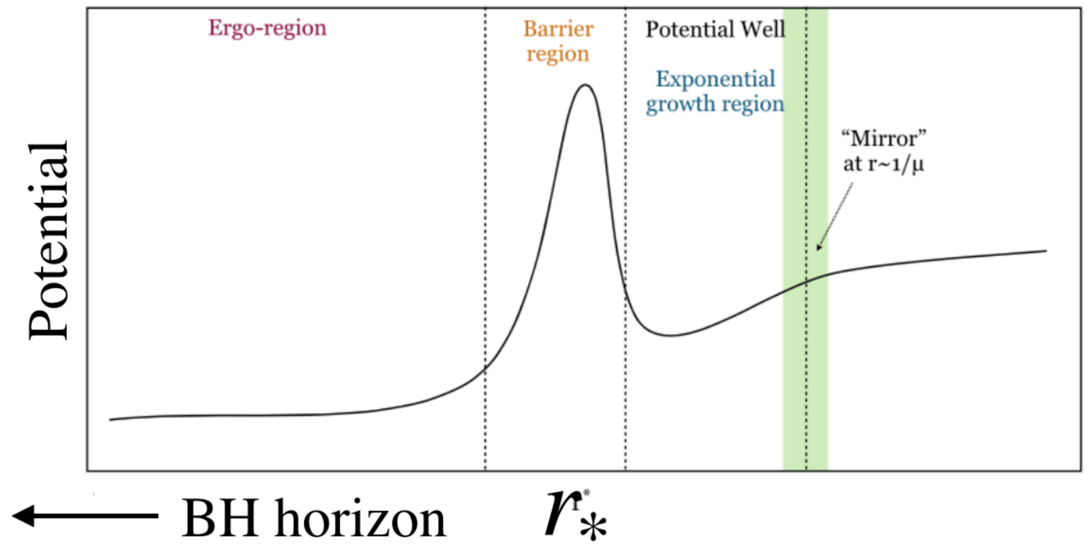
- Supperradiance

$$\Phi(x) = e^{-\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r)$$

$$\Rightarrow \quad \mathrm{Re}(\omega) < m\Omega_{\mathrm{H}} = \frac{ma}{2Mr_+}$$

$$\tau \sim 100\tilde{a} \left(\frac{10^6 M_\odot}{M} \right)^8 \left(\frac{10^{-16}\mathrm{eV}}{\mu} \right)^9 \sec$$

- Scalar cloud



Superradiant clouds

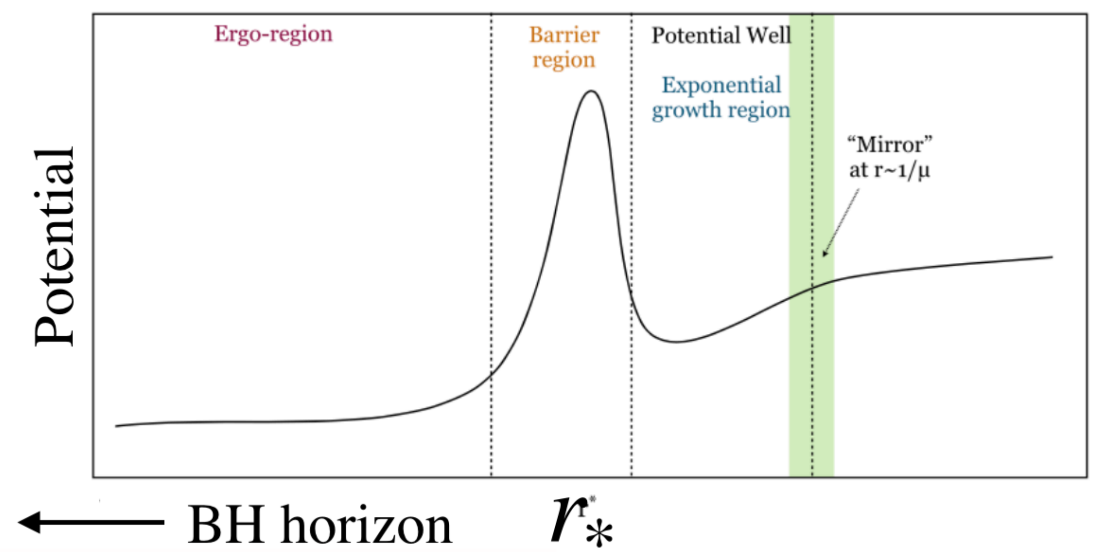
- Supperradiance

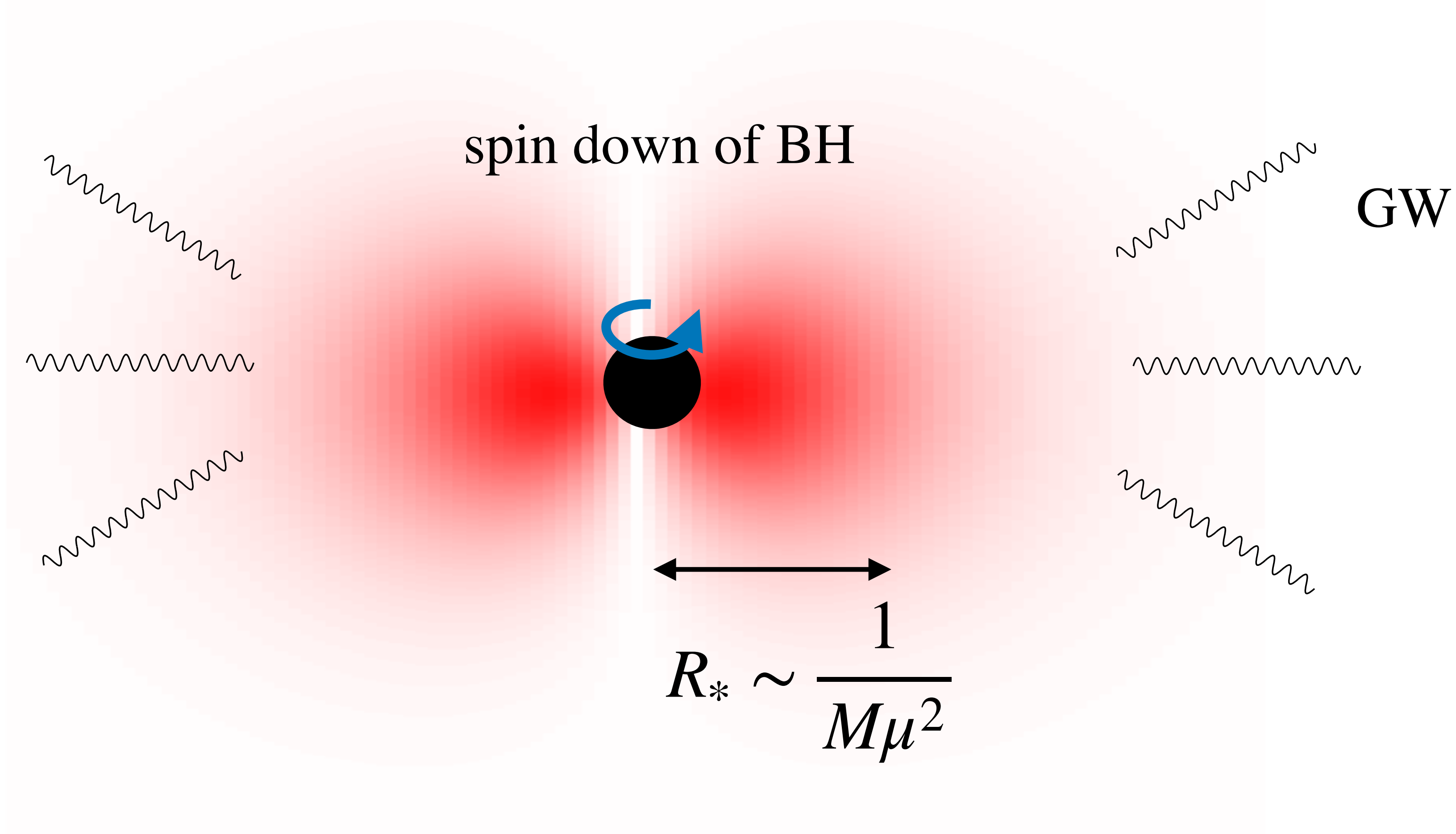
$$\Phi(x) = e^{-\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r)$$

$$\mathrm{Re}(\omega) < m\Omega_{\mathrm{H}} = \frac{ma}{2Mr_+}$$

$$\tau \sim 100\tilde{a} \left(\frac{10^6 M_\odot}{M} \right)^8 \left(\frac{10^{-16}\mathrm{eV}}{\mu} \right)^9 \mathrm{sec}$$

- Scalar cloud





Superradiant clouds

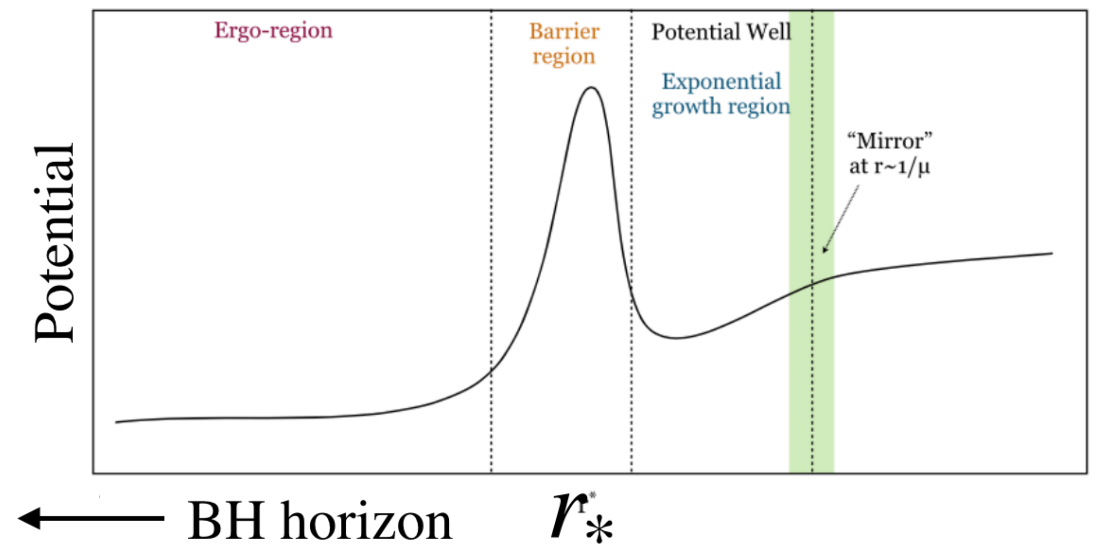
- Supperradiance

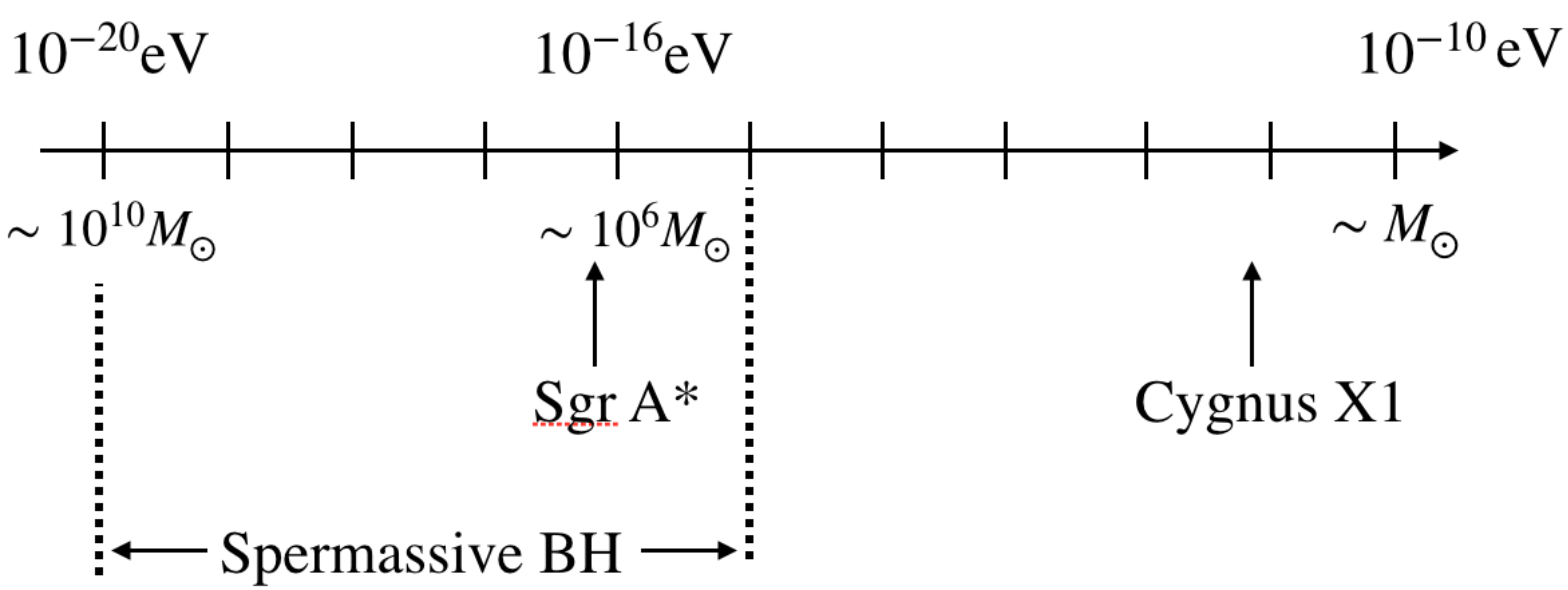
$$\Phi(x) = e^{-\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r)$$

$$\Rightarrow \mathrm{Re}(\omega) < m\Omega_{\mathrm{H}} = \frac{ma}{2Mr_+}$$

$$\tau \sim 100\tilde{a} \left(\frac{10^6 M_\odot}{M} \right)^8 \left(\frac{10^{-16}\mathrm{eV}}{\mu} \right)^9 \mathrm{sec}$$

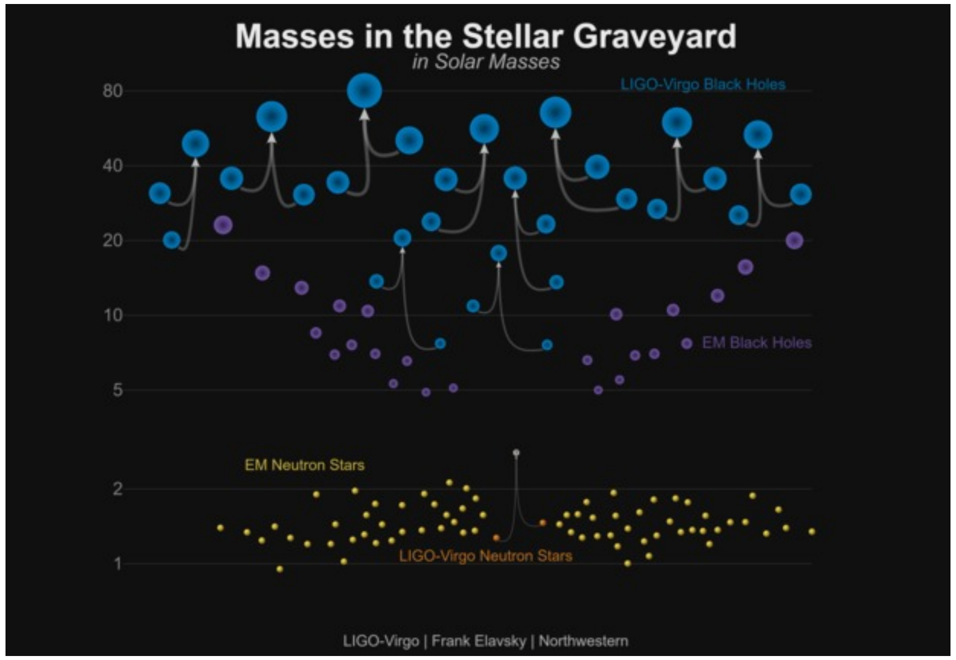
- Scalar cloud

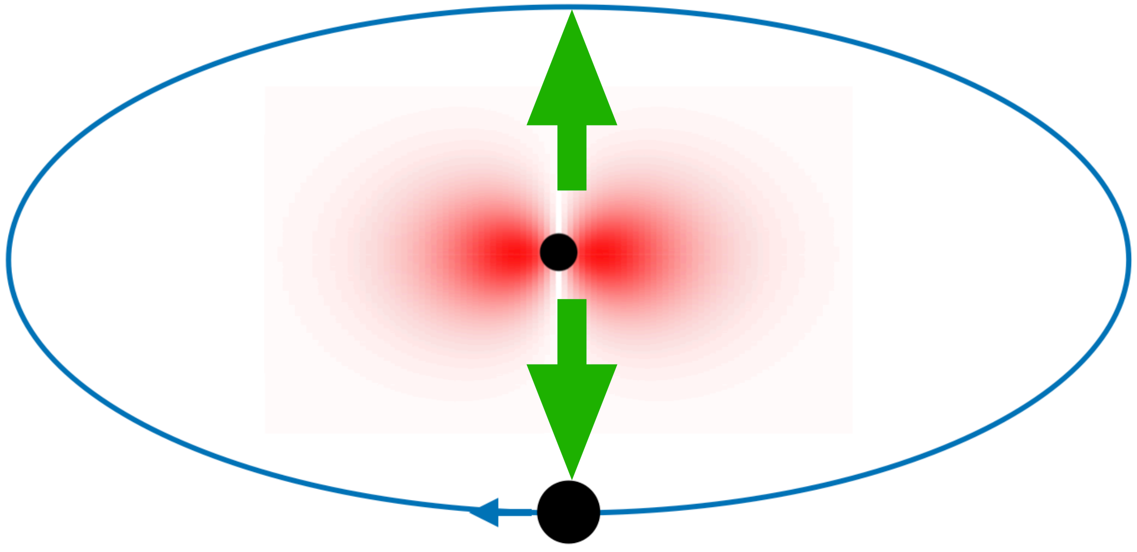




Black Hole has a companion.

- There are a lot of BH binaries in our Universe.
- Sgr A* and Cygnus X1 have companion stars.
- Does tidal force from companion star change history of the cloud ?
  - tidal disruption ?
  - GW emission from the cloud et al





Previous work

$$V(r) = \frac{\alpha}{r}$$

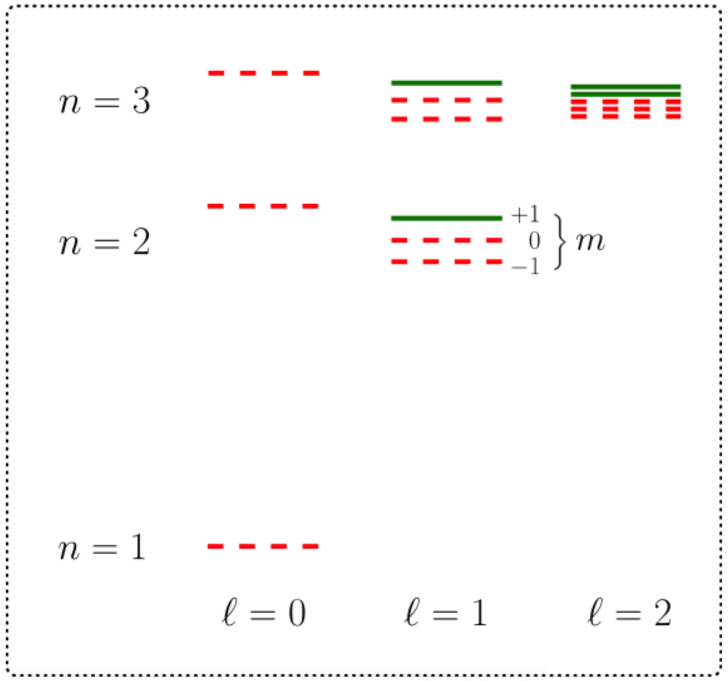
- Mode mixing (D.Baumann et al PRD99,044001, E.Berti et al PRD99,104039)
  - single BH
    - $(\Box - \mu^2)\Phi = 0 \Rightarrow i\partial_t \Psi = \left(-\frac{1}{2\mu^2}\nabla^2 + \underline{V(r)}\right)\Psi \Rightarrow \begin{cases} |n,l,m> \\ \omega_{n,l,m} \end{cases}$

      $\begin{cases} M/r \ll 1 \\ \text{non-relativistic limit} \end{cases}$

      cf : QM of Hydrogen atom
    - higher order correction

$$\Delta\omega_{nlm} = \mu\left(-\frac{\alpha^4}{8n^4} + \frac{(2l-3n+1)\alpha^4}{n^4(l+1/2)} + \frac{2\tilde{a}m\alpha^5}{n^3 l(l+1/2)(l+1)}\right)$$

    - $\mathrm{Im}\left(\omega_{\mathrm{nlm}}\right) \propto \mathrm{m}\Omega_{\mathrm{H}} - \omega$
      - decaying mode $\mathrm{Im}\left(\omega_{nlm}\right) < 0$
      - growing mode $\mathrm{Im}\left(\omega_{nlm}\right) > 0$



Previous work

$$i\partial_t \Psi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r)\right)\Psi$$

- Binary BH
  - The tidal effect deforms the potential.

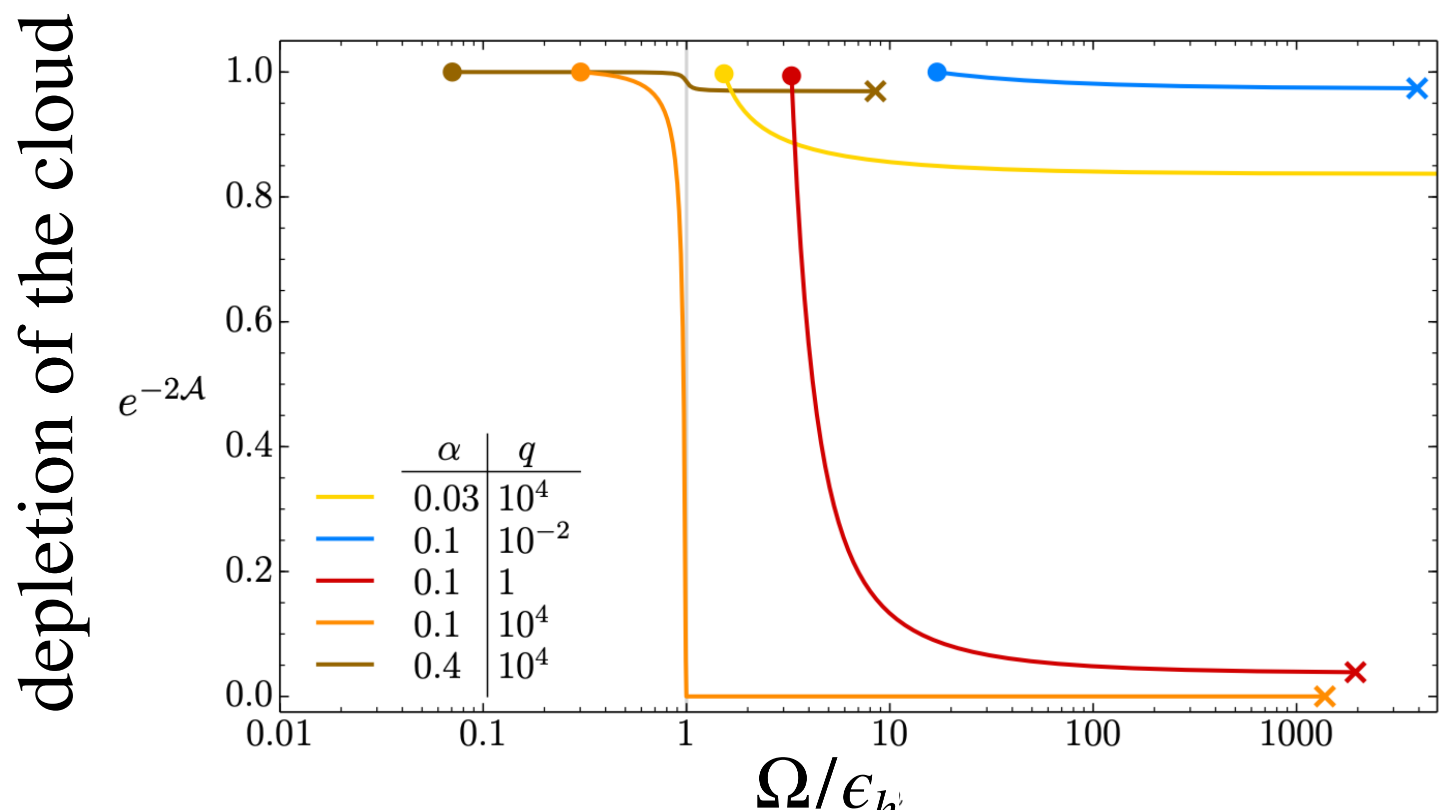
$$V(r) \to V(r) + \underline{\delta V(t, r, \theta, \phi)}$$

cf : Perturbation theory in QM

  - mode mixing

$$< n, l, m \,|\, \delta V \,|\, n', l', m' > \neq 0$$

➡ Growing mode is coupled to decaying mode.



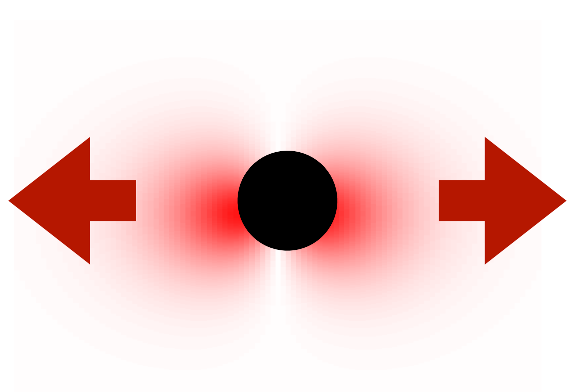
time

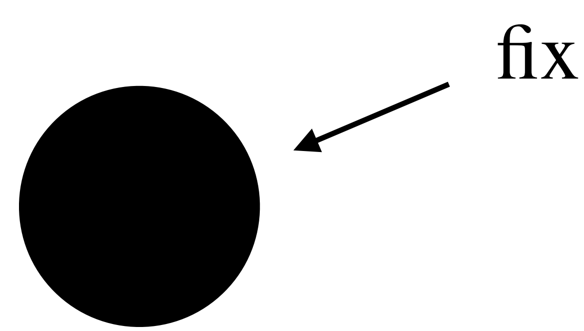
What we want to do

- Previous works : perturbation theory of QM
  - mode mixing between decaying and growing mode
  - depletion of the cloud
- Questions
  - What happens beyond perturbation theory ?
  - Is the cloud disrupted due to the strong tidal force ?

➡ Numerical simulation is good approach.

- For simplicity, we focus on static tidal field.
- Weak tidal : consistency check with perturbation theory
- Strong tidal : threshold of the tidal disruption





Outline

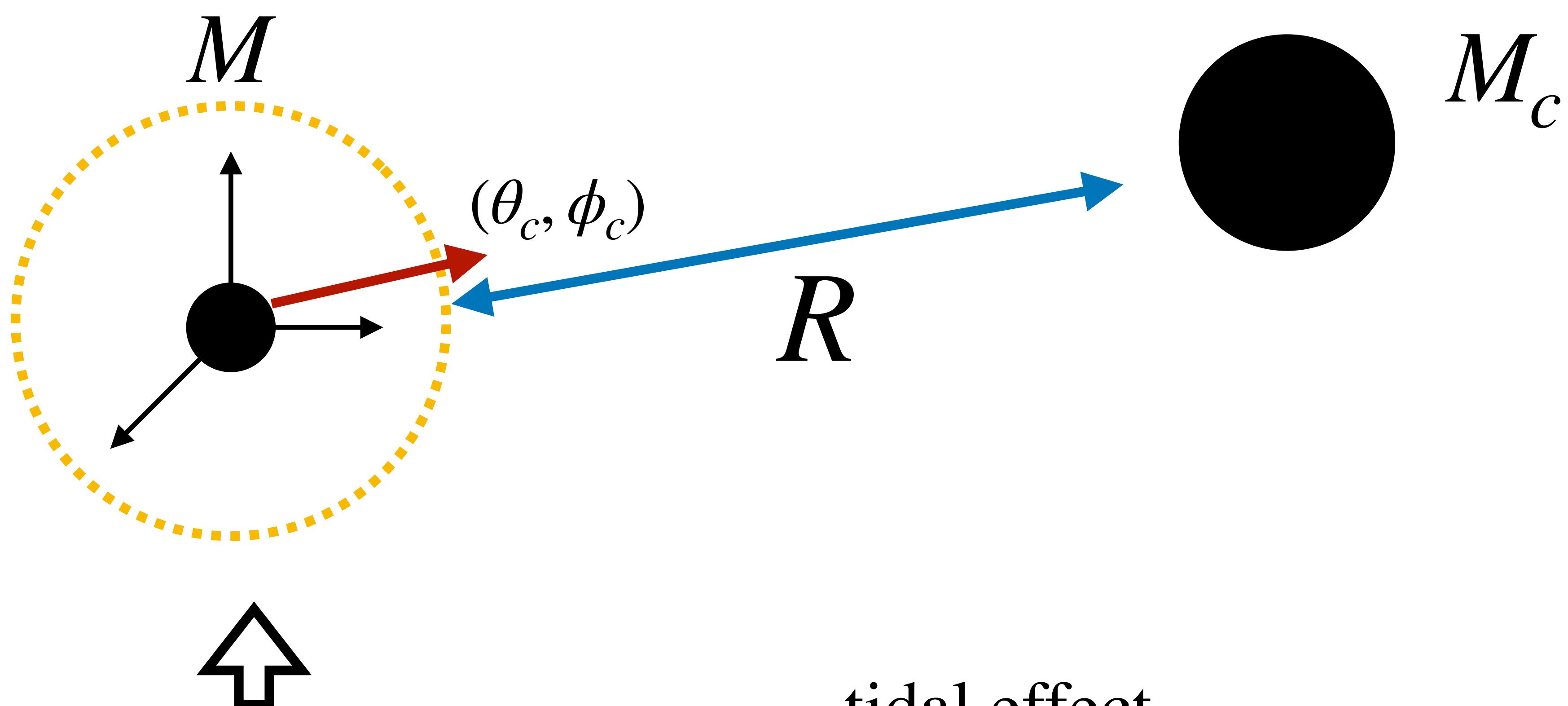
1. Introduction
2. Our work
   - How to add tidal force
   - Weak tidal
   - Strong tidal
   - Application
3. Summary

Tidally deformed BH

- How to add tidal effects ?

KG equation

$$(\Box - \mu^2)\Phi = 0$$



tidal effect

$$ds^2 = ds^2_{\rm BH} + \sum_m \left(\frac{r}{M}\right)^2 \frac{8\pi\epsilon}{5} Y^*_{2m}(\theta_c, \phi_c) Y_{2m}(\theta, \phi)\left(f^2 dt^2 + dr^2 + (r^2 - 2M^2) d^2\Omega\right) + \cdots$$

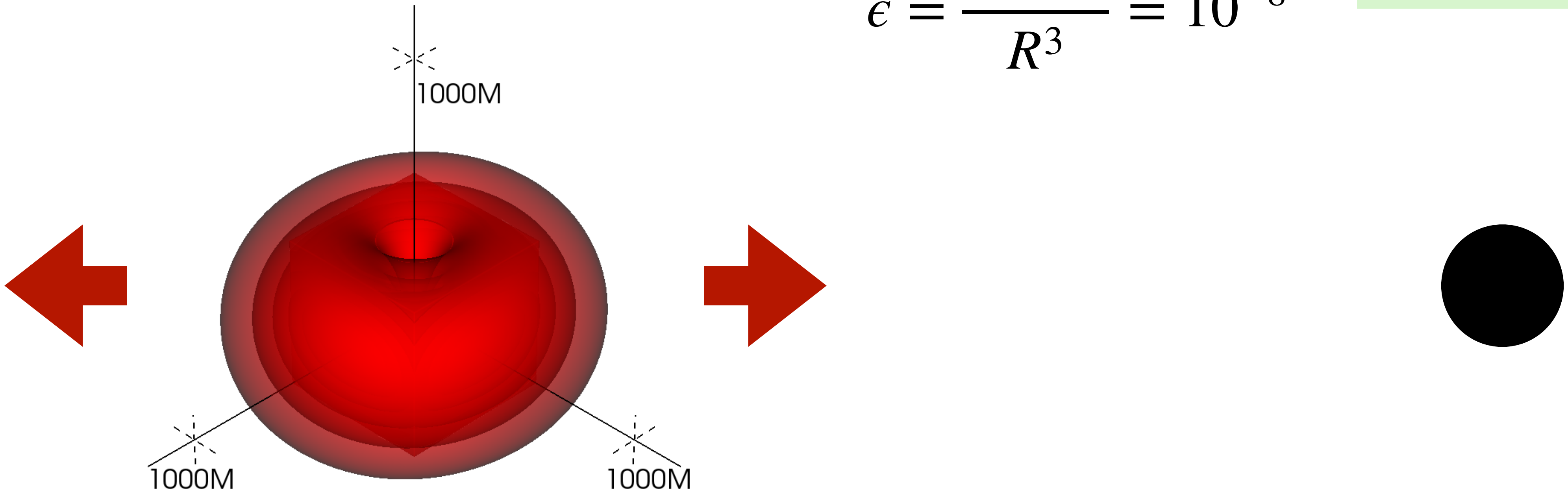
with Regge Wheeler gauge

$\epsilon = \dfrac{M_c M^2}{R^3}$: the strength of tidal force

$$f = 1 - \frac{2M}{r}$$

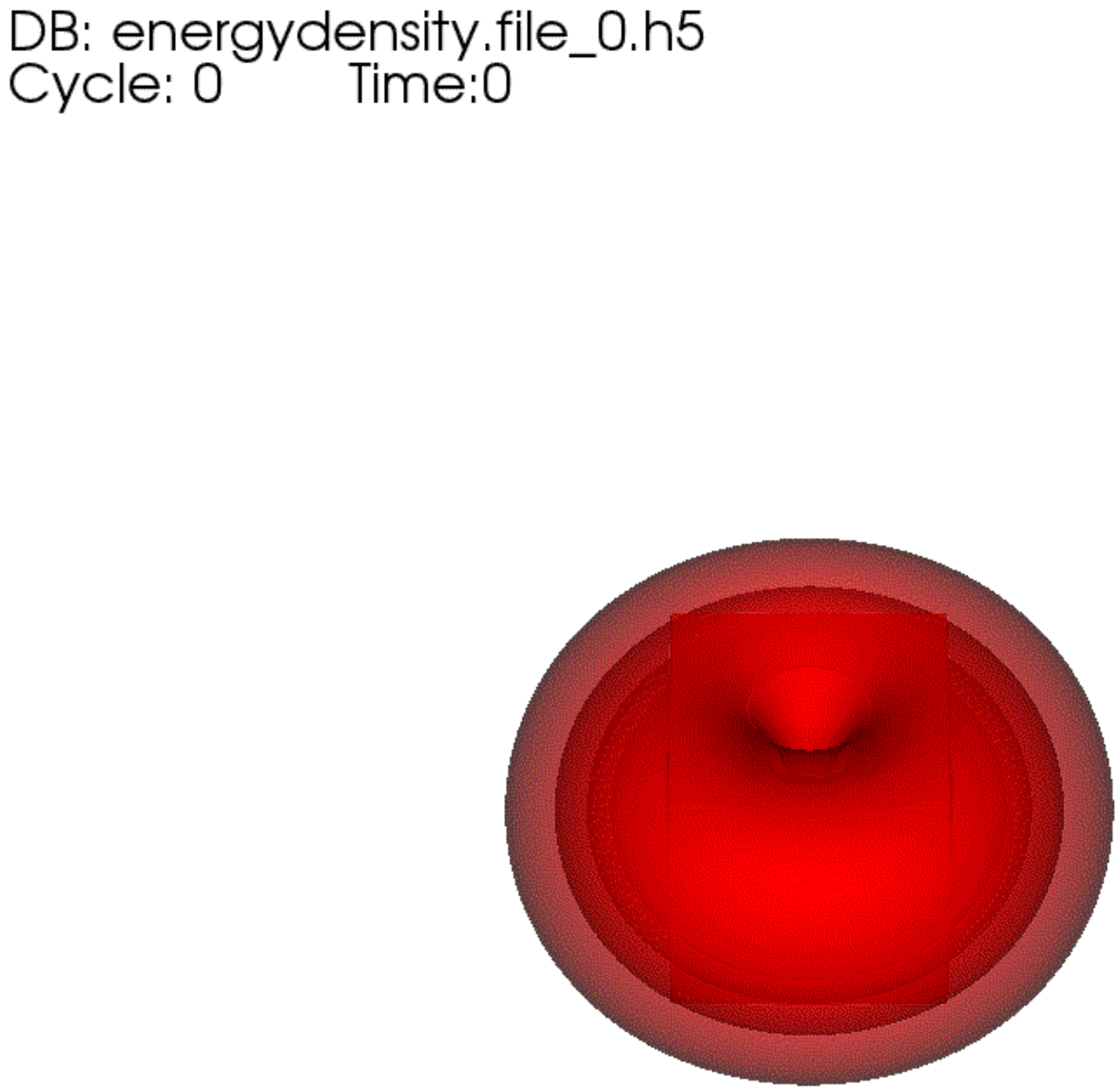
$$\text{cf: } R = 10^4 M$$
$$M_c = 10^4 M$$

$$\epsilon = \frac{M_c M^2}{R^3} = 10^{-8}$$

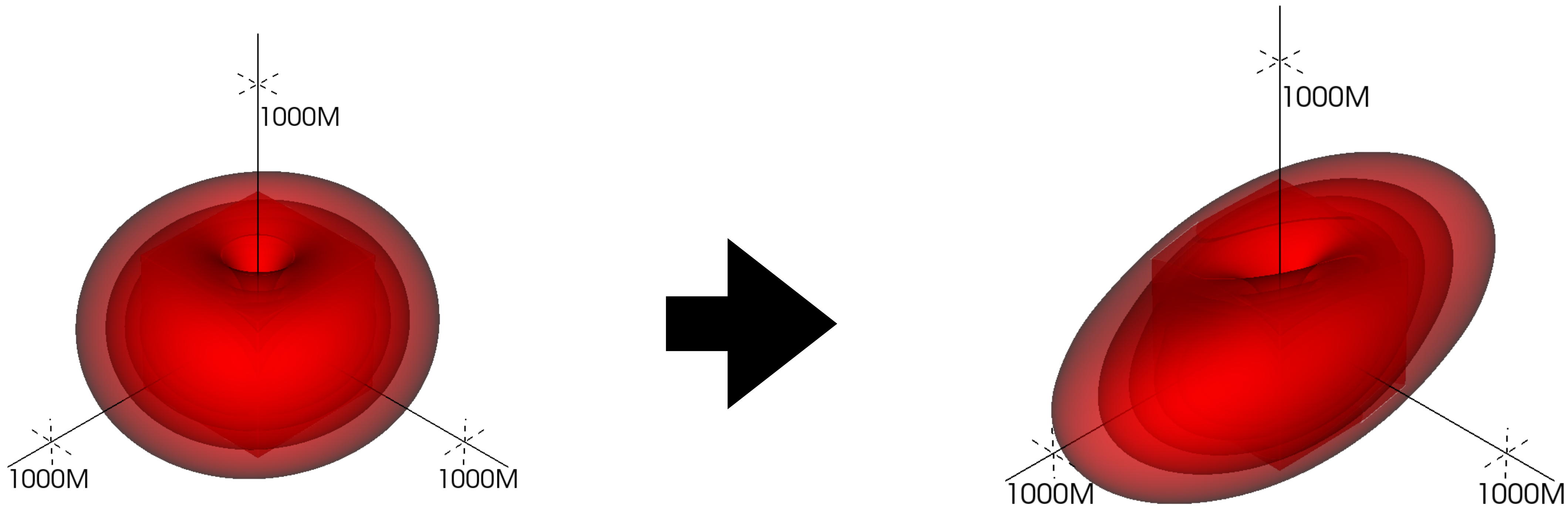


Simulation 1 : Weak tidal case

Weak tidal case

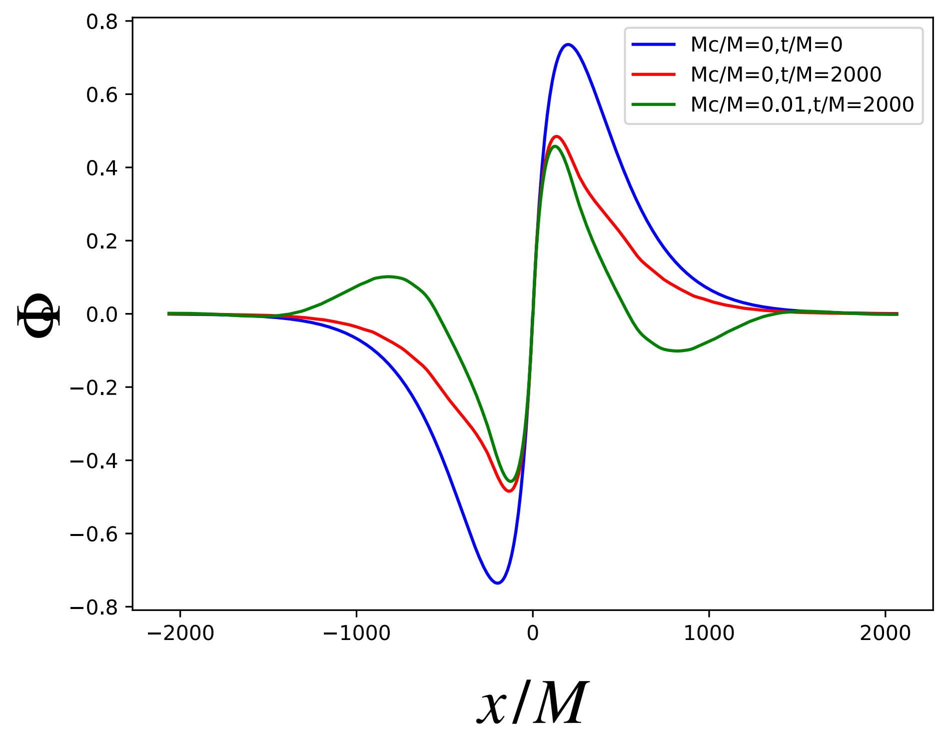


Weak tidal case

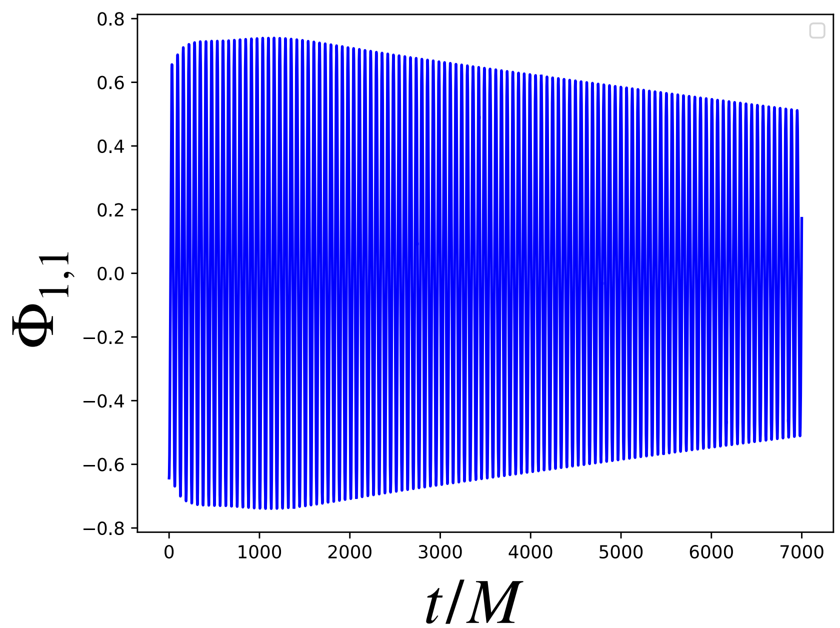


Weak tidal case

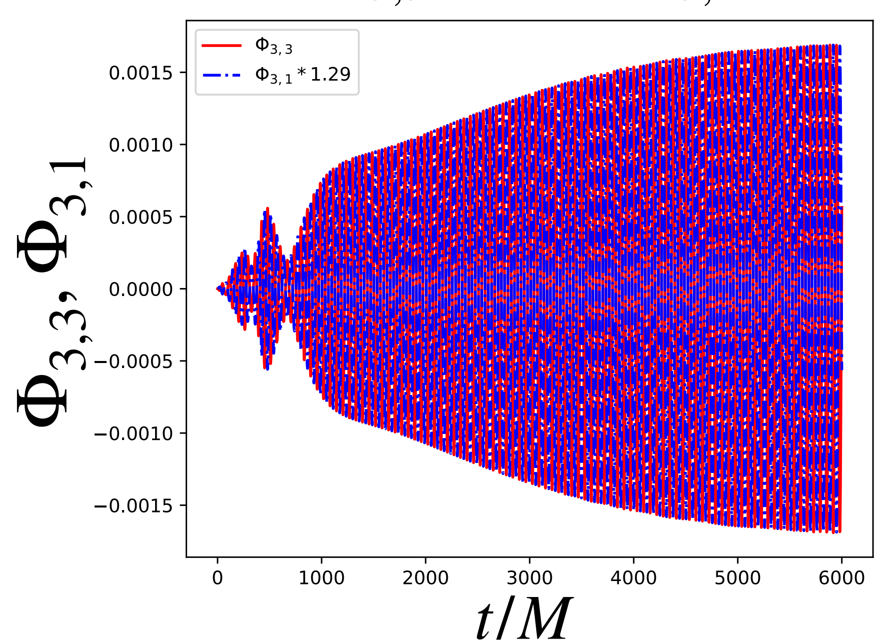
- Excitation of overtone mode.
  - $n = 3{,}4$ modes are excited.
  - consistent with perturbation theory of QM. (Up to a few factor)



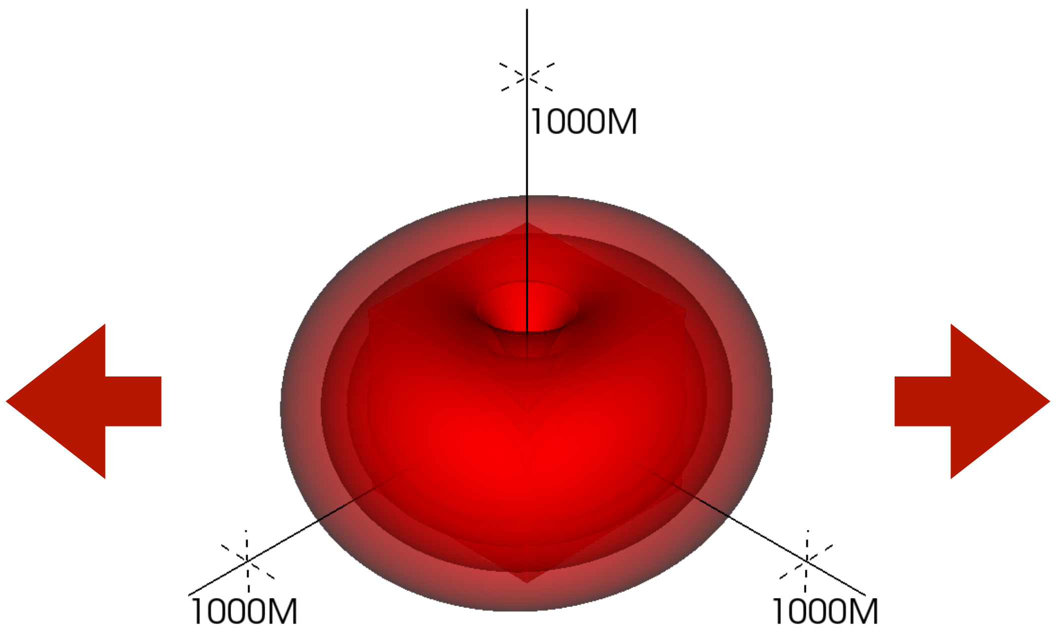
- Excitation of higher l mode.



$\Phi_{3,3} \simeq 1.29\Phi_{3,1}$

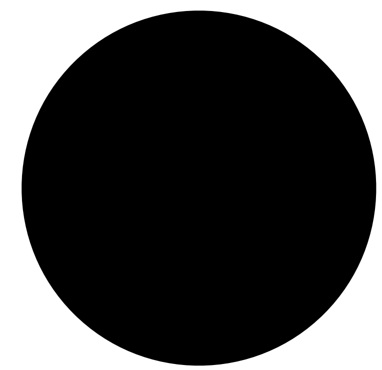


➡ Strong gravitational wave emission is expected.



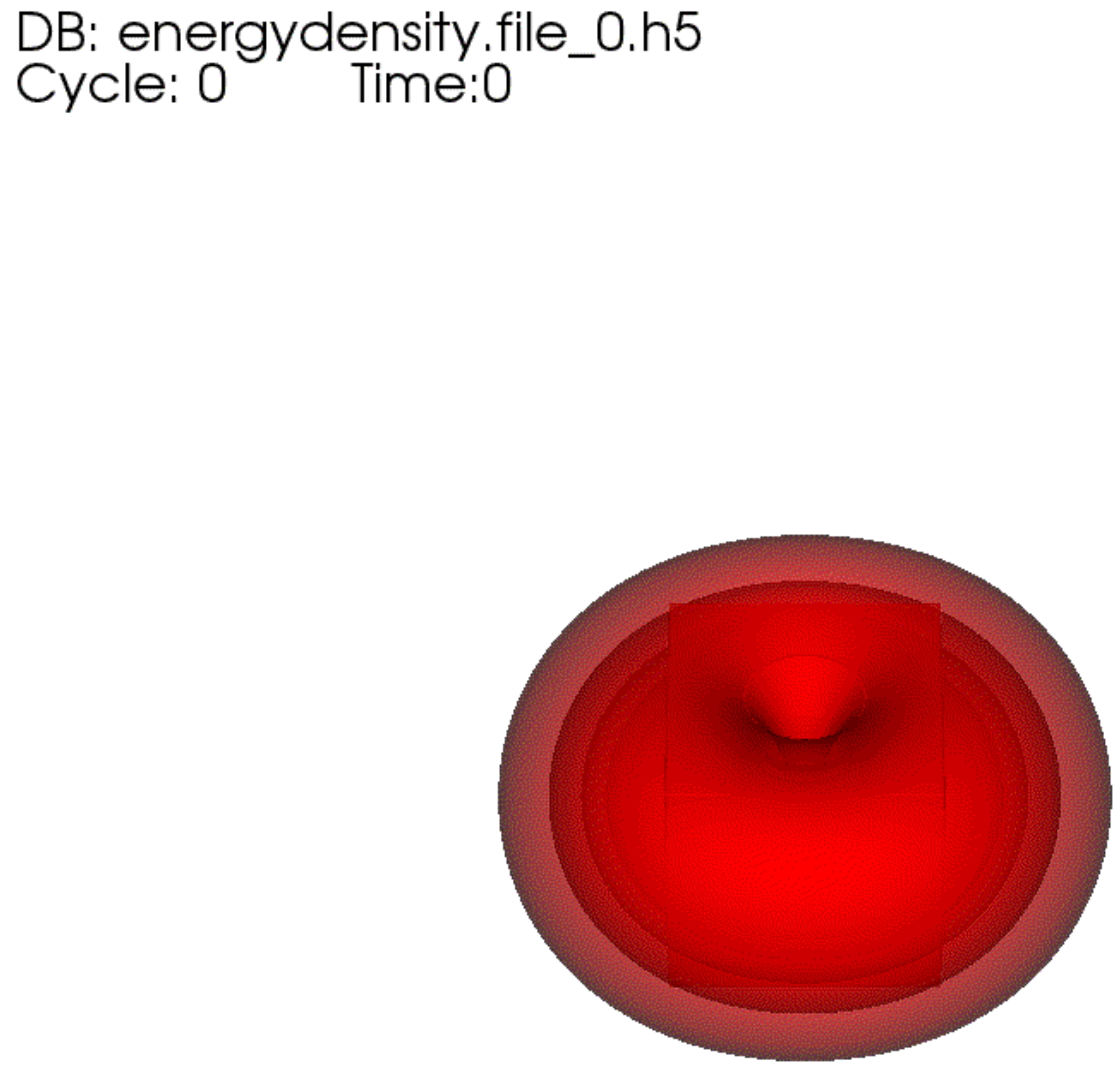
$$\epsilon = \frac{M_c M^2}{R^3} = 10^{-7}$$

cf: $R = 10^4 M$
$M_c = 10^5 M$

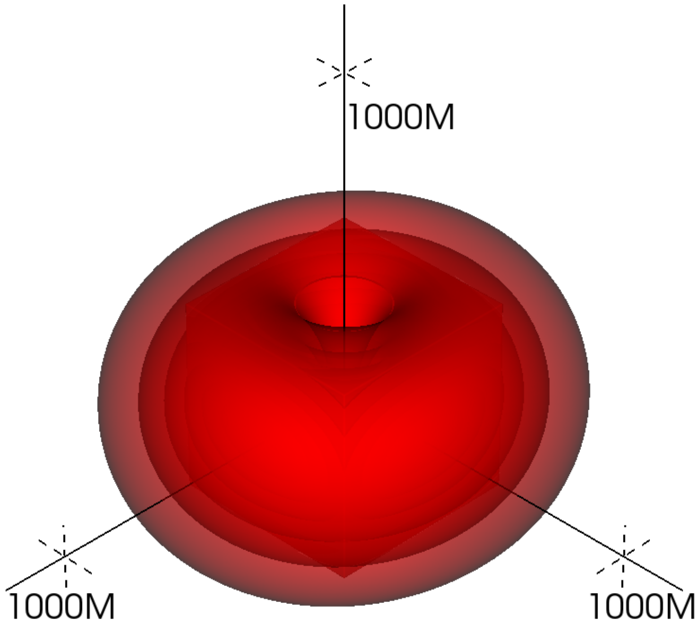


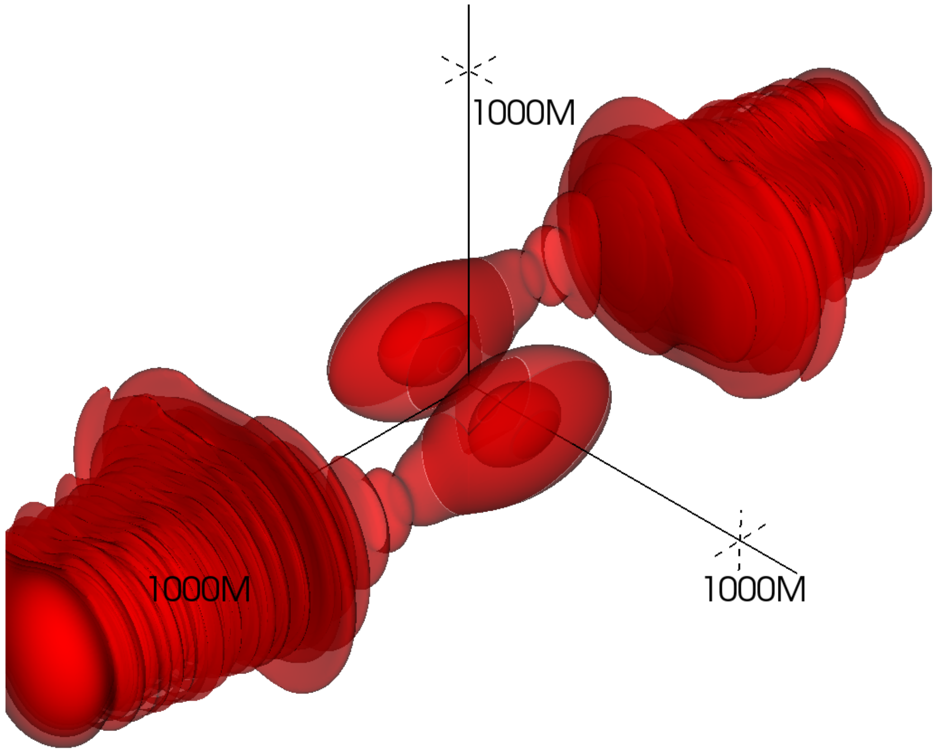
Simulation 2 : Strong tidal case

Strong tidal case



Strong tidal case



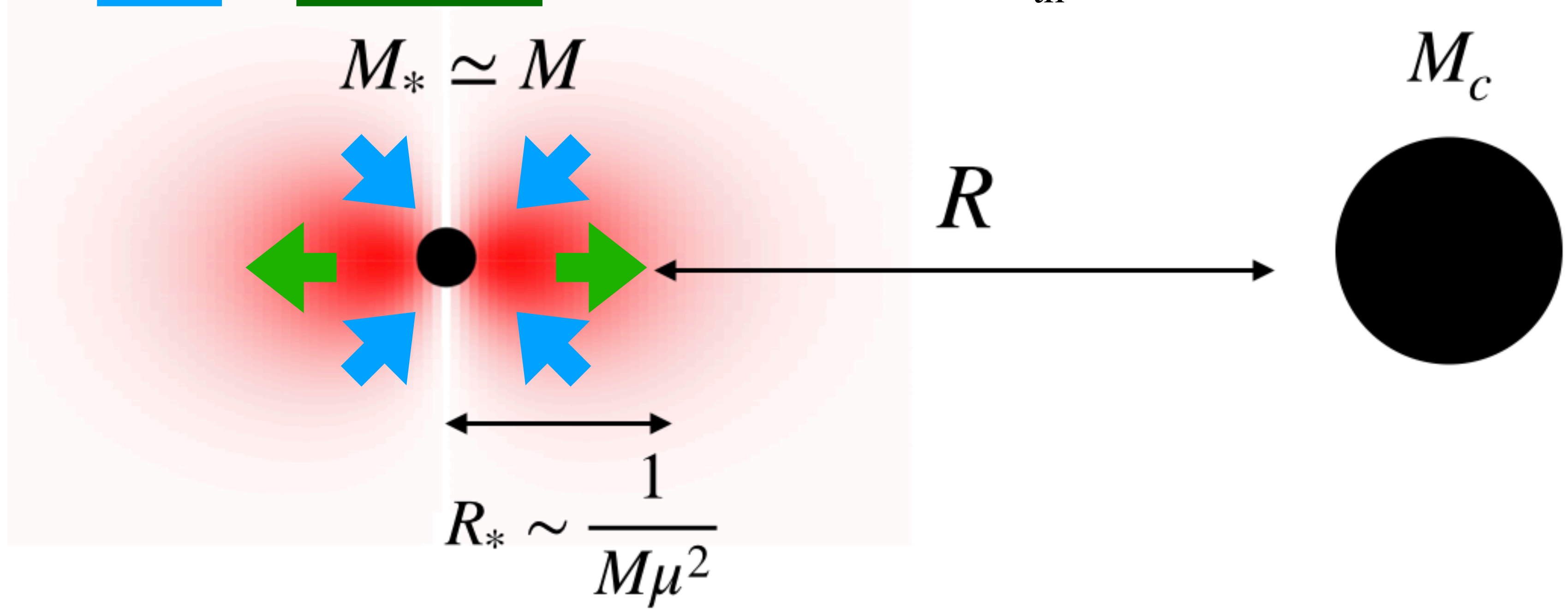


Strong tidal case

- Tidal disruption

cf: Roche limit

$$\underbrace{\frac{M_*^2}{R_*^2}} \sim \underbrace{\frac{R_*}{R}\frac{M_c M_*}{R^2}} \quad \Rightarrow \quad \left.\frac{M_c M^2}{R^3}\right|_{\rm th} \sim (M\mu)^6 \begin{cases} 10^{-6} \ (\text{ for } M\mu = 0.1 \ ) \\ 6\times 10^{-5} \ (\text{ for } M\mu = 0.2 \ ) \end{cases}$$

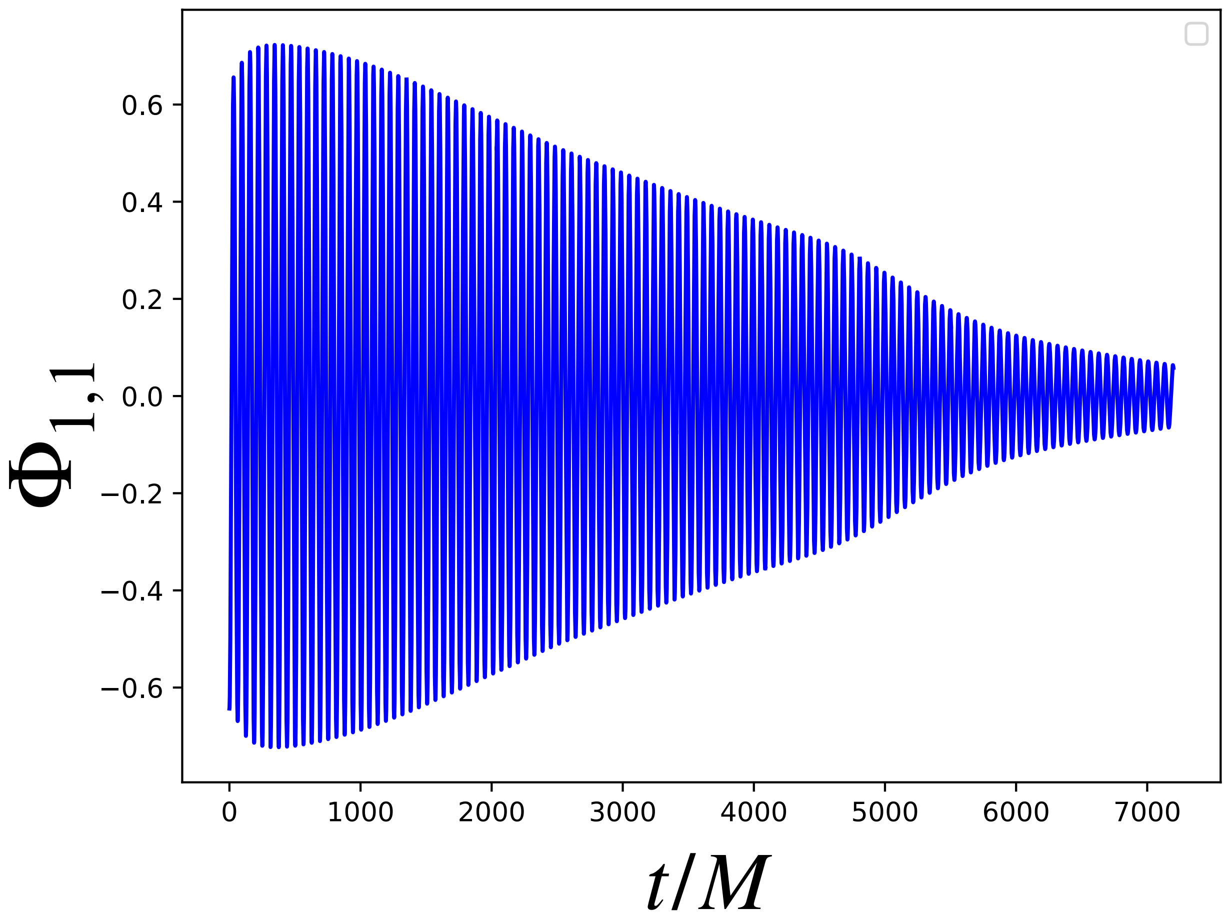


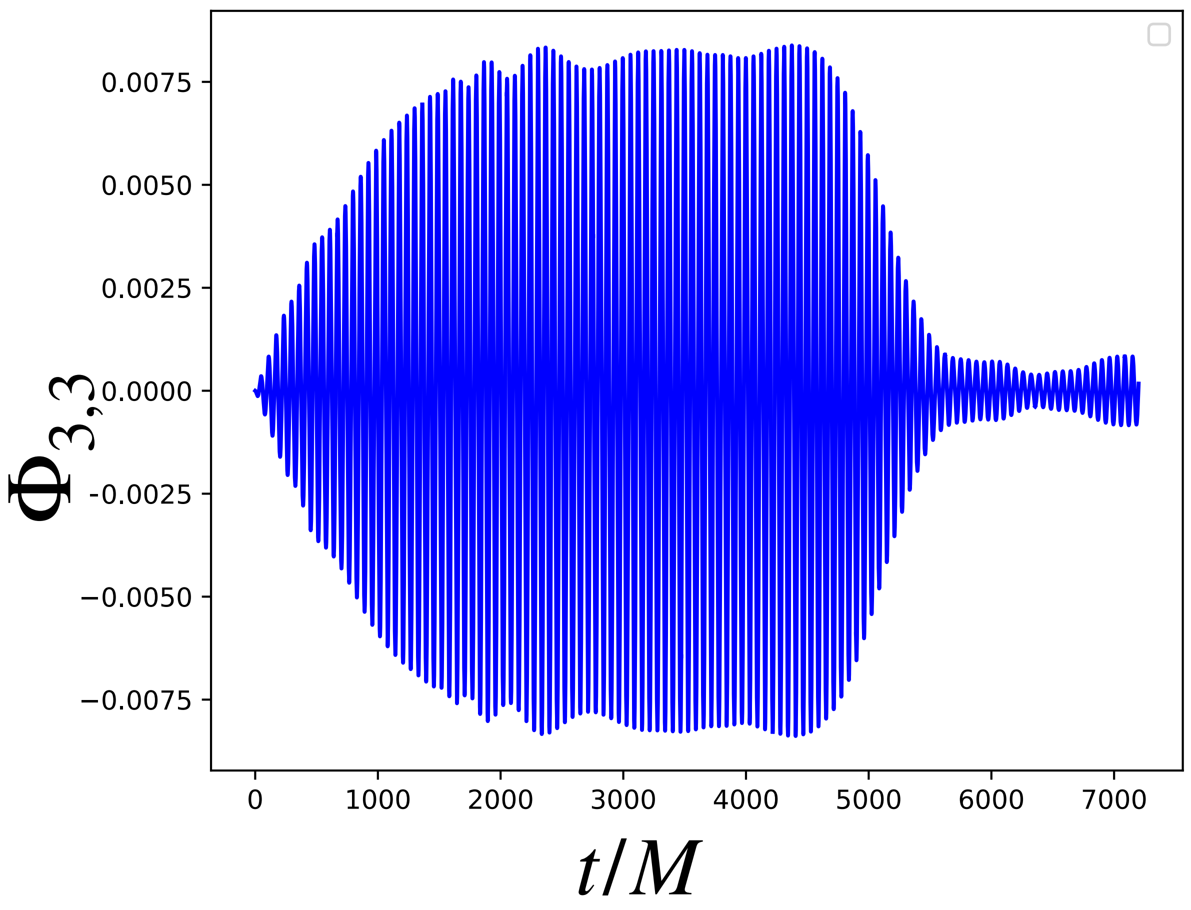
- Numerical result

$$\epsilon_{\rm th} = \left.\frac{M_c M^2}{R^3}\right|_{\rm th} \sim \begin{cases} 10^{-8} \ (\text{ for } M\mu = 0.1 \ ) \\ 2\times 10^{-7} \ (\text{ for } M\mu = 0.2 \ ) \end{cases} \sim \frac{1}{250}(M\mu)^6$$

Strong tidal case

- After higher mode is excited, the cloud is disrupted.





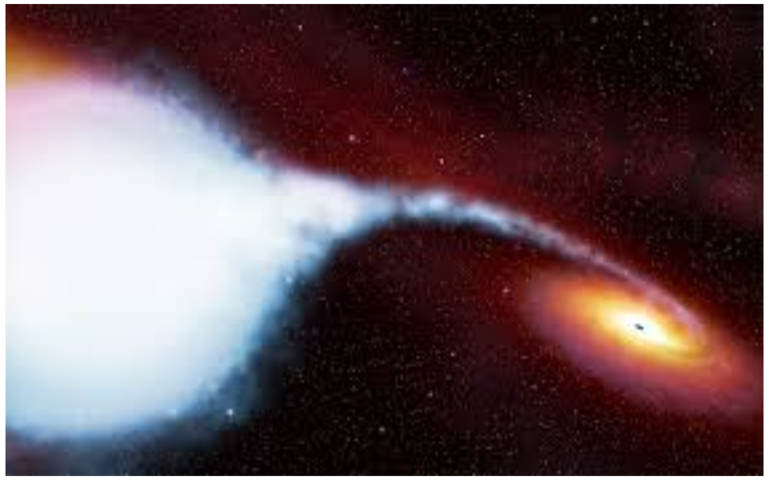
Astrophysical application (In progress)

cf: Superradiance time scale

$$t_s \sim M(M\mu)^9$$

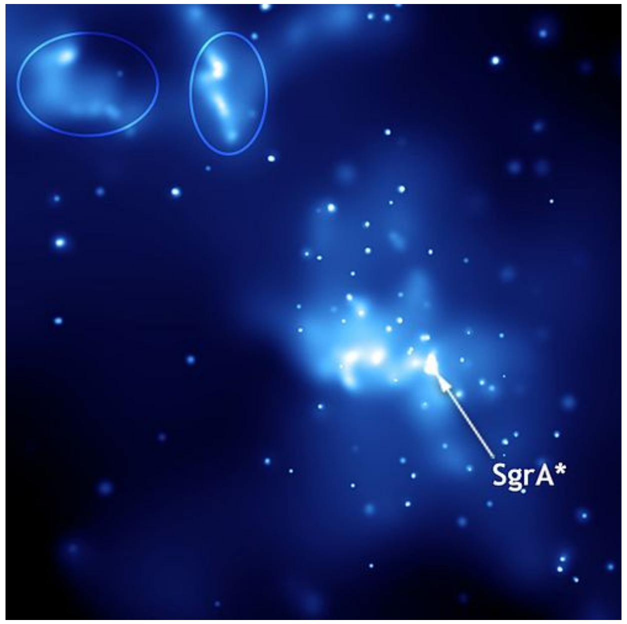
- Cygnus X-1 (J.A.Orosz et al (2011))
  - $M_{\rm BH} \sim 15 M_\odot$
  - $M_c \sim 20 M_\odot$ $\Rightarrow$ $\epsilon \simeq 5 \times 10^{-19}$
  - $R \sim 3 \times 10^{10}$ m

$\Rightarrow$ Scalar cloud with $M\mu \lesssim 2 \times 10^{-3}$ is disrupted.



- Sgr A* (S2) (R.Abuter et al (2018))
  - $M_{\rm BH} \sim 4 \times 10^6 M_\odot$
  - $M_c \sim 20 M_\odot$ $\Rightarrow$ $\epsilon \simeq 2 \times 10^{-15}$
  - $R \sim 1400 M_{\rm BH}$

$\Rightarrow$ Scalar cloud with $M\mu \lesssim 9 \times 10^{-3}$ is disrupted.



- Time scale of super-radiance is irrelevant in our Universe.
- The cloud around the threshold overlap the companion star (?)

Outline

1. Introduction
2. Our work
   - How to add tidal force
   - Weak tidal
   - Strong tidal
   - Application
3. Summary

Summary

- We considered tidal effect on scalar cloud.
- We investigate the time evolution of the cloud under tidal force.
  - Higher multipole mode is excited.
    - Strong gravitational wave emission is expected.
  - Tidal disruption

$$\epsilon_{\mathrm{th}} \sim \frac{1}{250}(M\mu)^6$$

- Astrophysical application (In progress)
  - Cygnus X1
  - Sgr A*