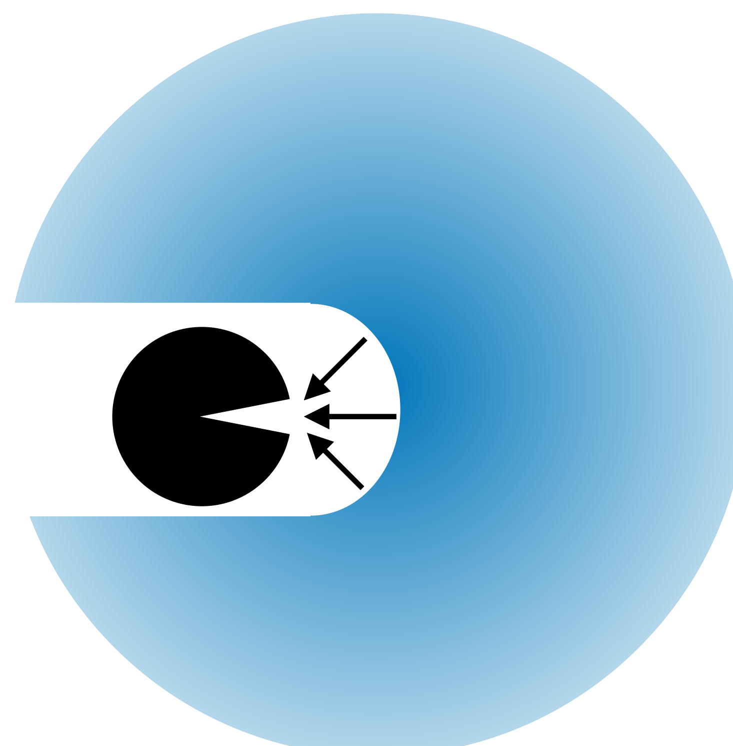
Black hole eating boson star



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(on going work)

Outline

- Introduction
  - Motivation
  - Set up
- Gravitational atom
- Numerical relativity simulations
- Summary

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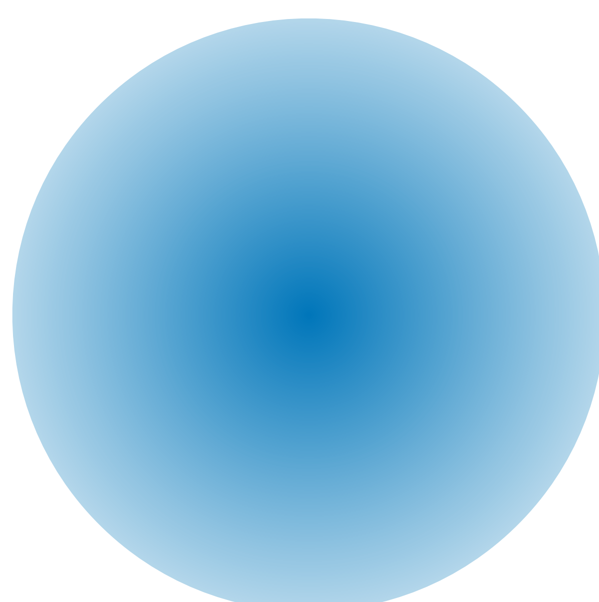
Light scalar field in our Universe

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi} - g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi^* - \mu^2|\psi|^2\right)$$

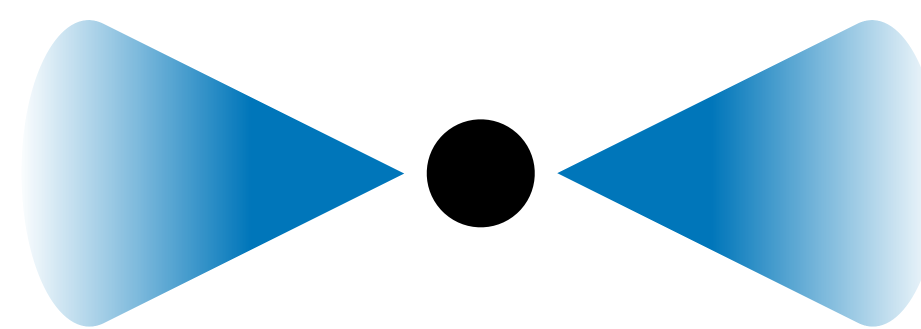
ψ : complex scalar field

Boson star
= self-gravitating object



- compact object
- dark matter halo

Gravitatioanl atom
= test field configuration
around BH



- source of GW
- superradiant instability

Boson star as DM halo

- Large boson star may be dark matter.

$$\frac{M_{BS}}{M_{\odot}} = 9 \times 10^{9} \frac{100\text{pc}}{R_{BS}} \left(\frac{10^{-22}\ \text{eV}}{\mu} \right)^{2}$$

Rotation curve

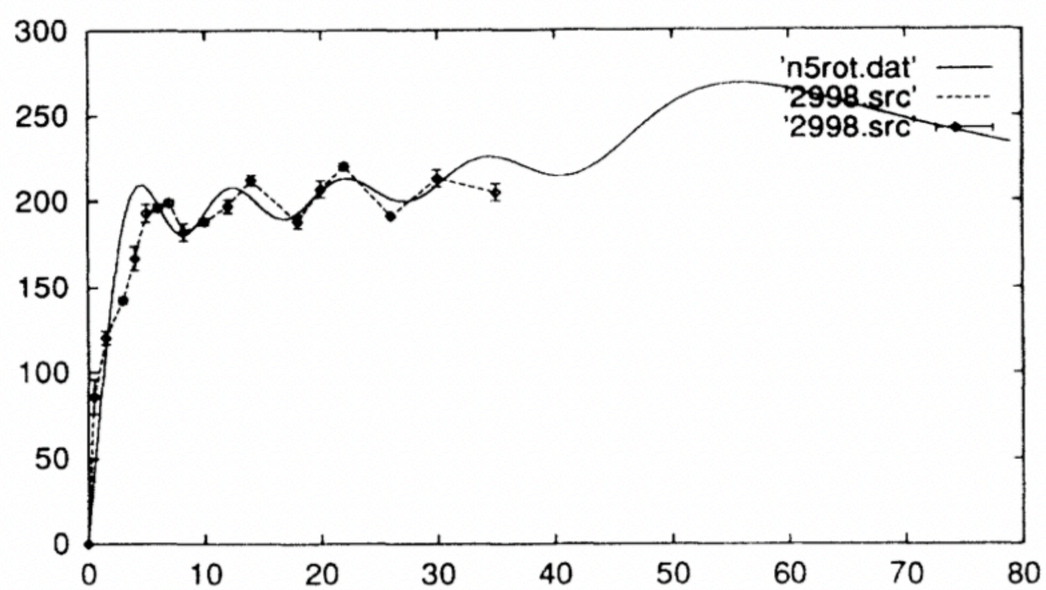


FIG. 3. Comparing theory and observation for NGC2998; velocity (km/sec) vs distance from the center (kpc). $n = 5$. The solid line is the theoretical curve and the dotted line with the error bar is the observed data.

Sang-Jin(1994)

Cosmic structure

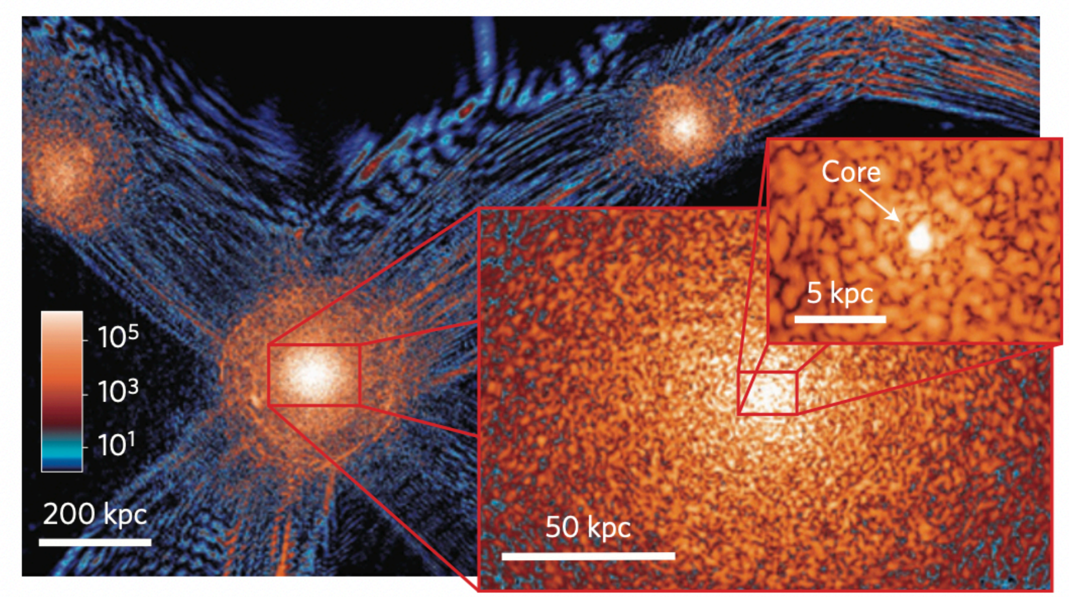
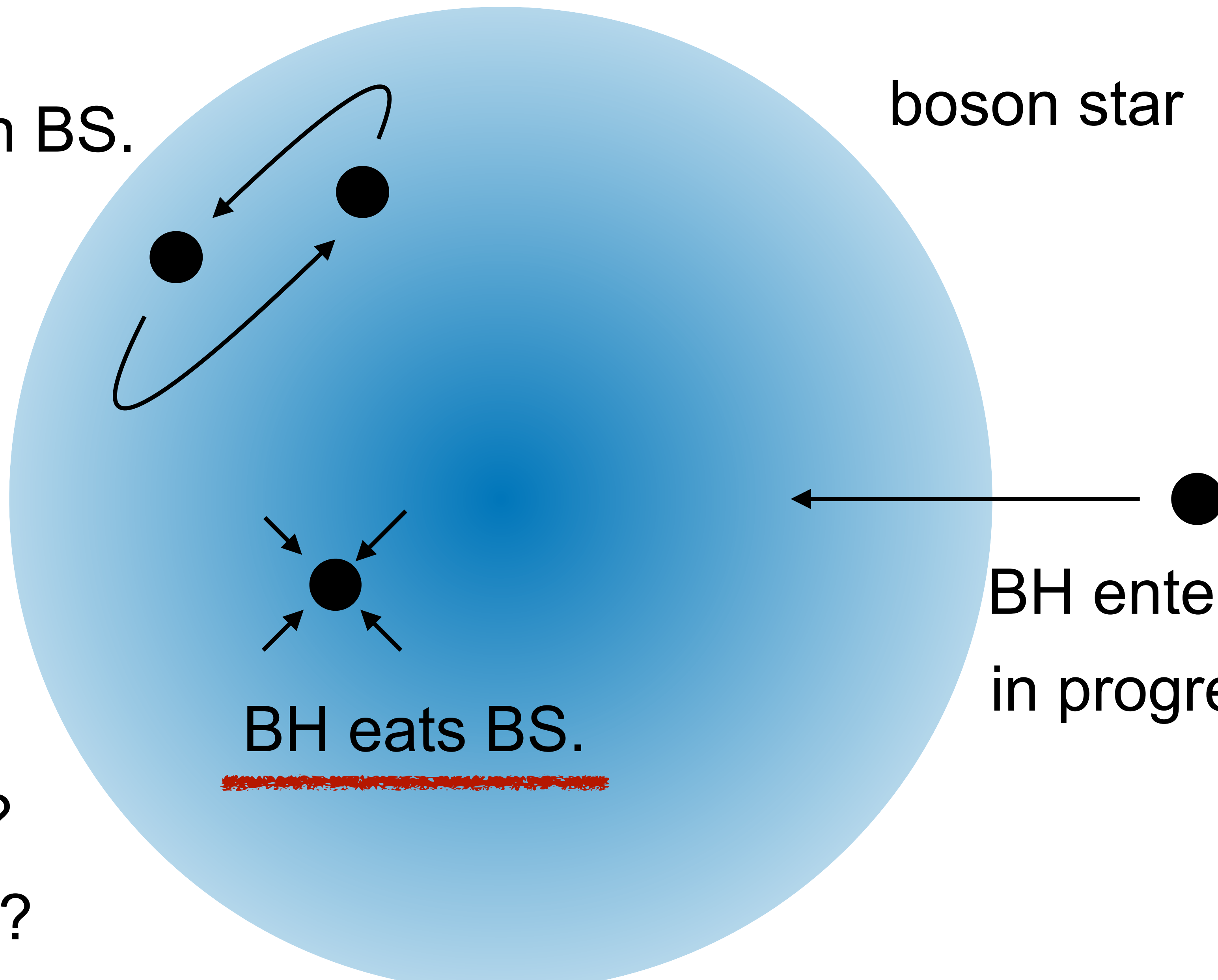


Figure 2 | A slice of the density field of the ψDM simulation on various scales at $z = 0.1$. This scaled sequence (each of thickness 60 pc) shows how quantum interference patterns can be clearly seen everywhere from

Schive et.at.(2014))

Possible interactions with BHs

- Dark matter halo interacts with BHs



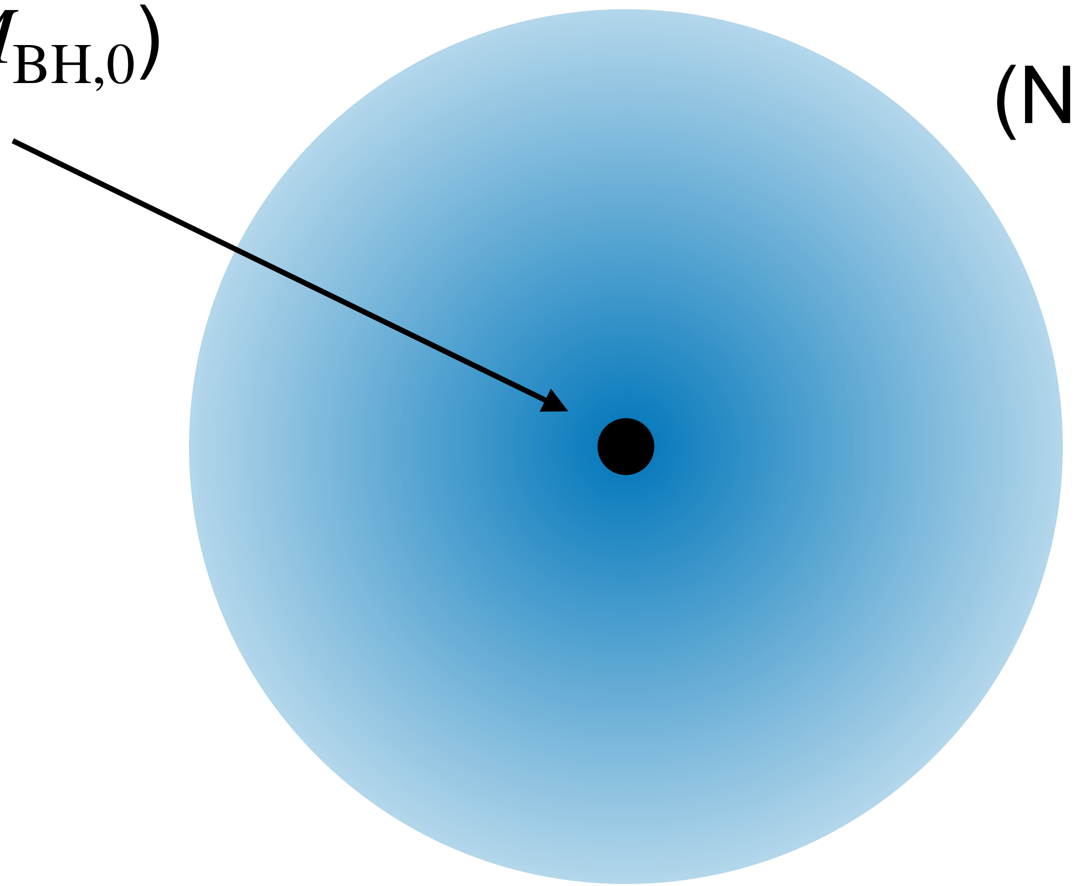
- Time scale ?
- Observable ?
- Typical dynamics ?

Set up

- Set up : BS-BH system
  - Spherical symmetry (for simplicity) : non-spinning BH, BS
  - Initial profile is boson star profile with BH
  - We consider the evolution of metric and the complex scalar field.

Black hole ($M_{\mathrm{BH},0}$)

(Nodeless) boson star



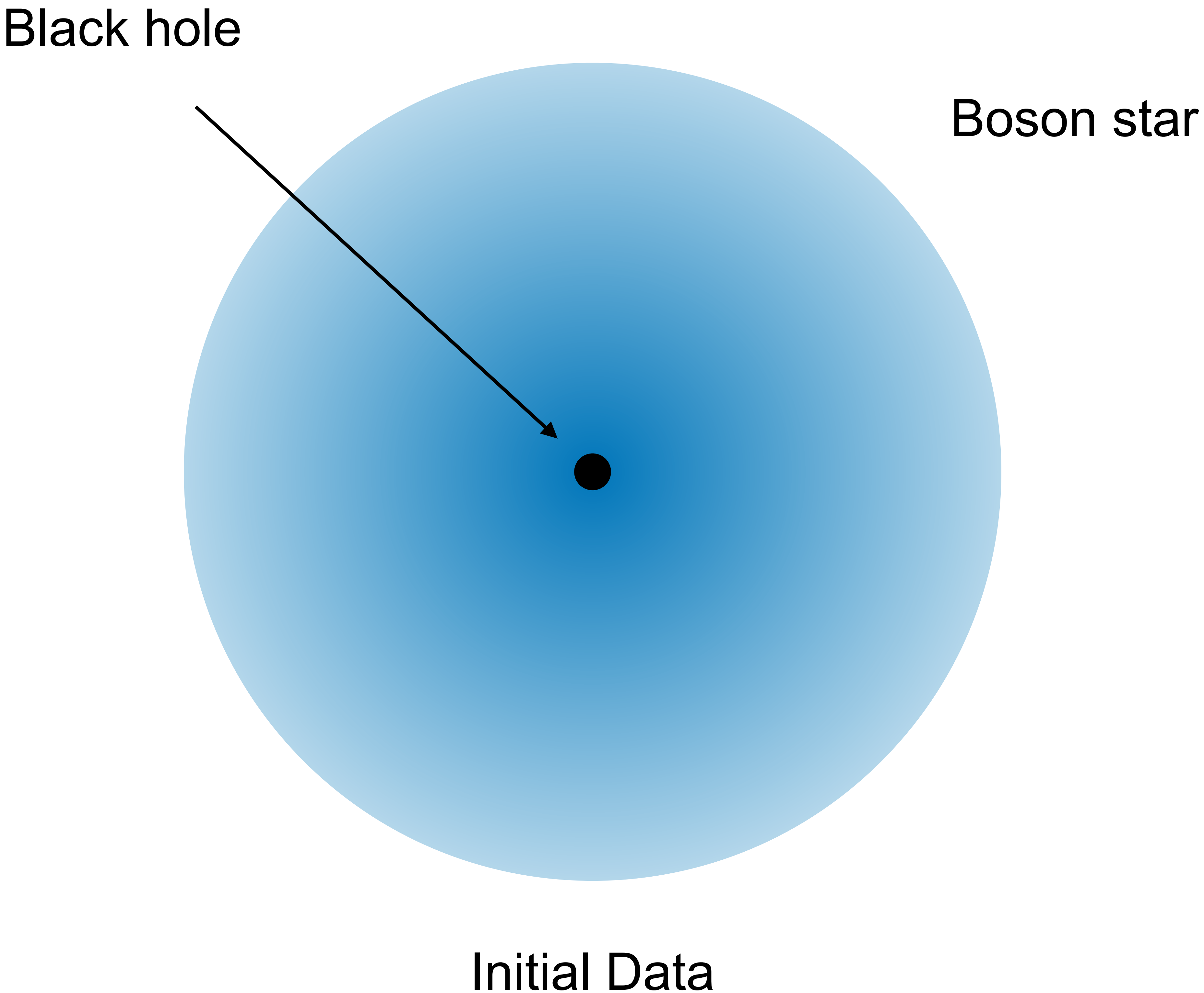
Initial Data

Parameters

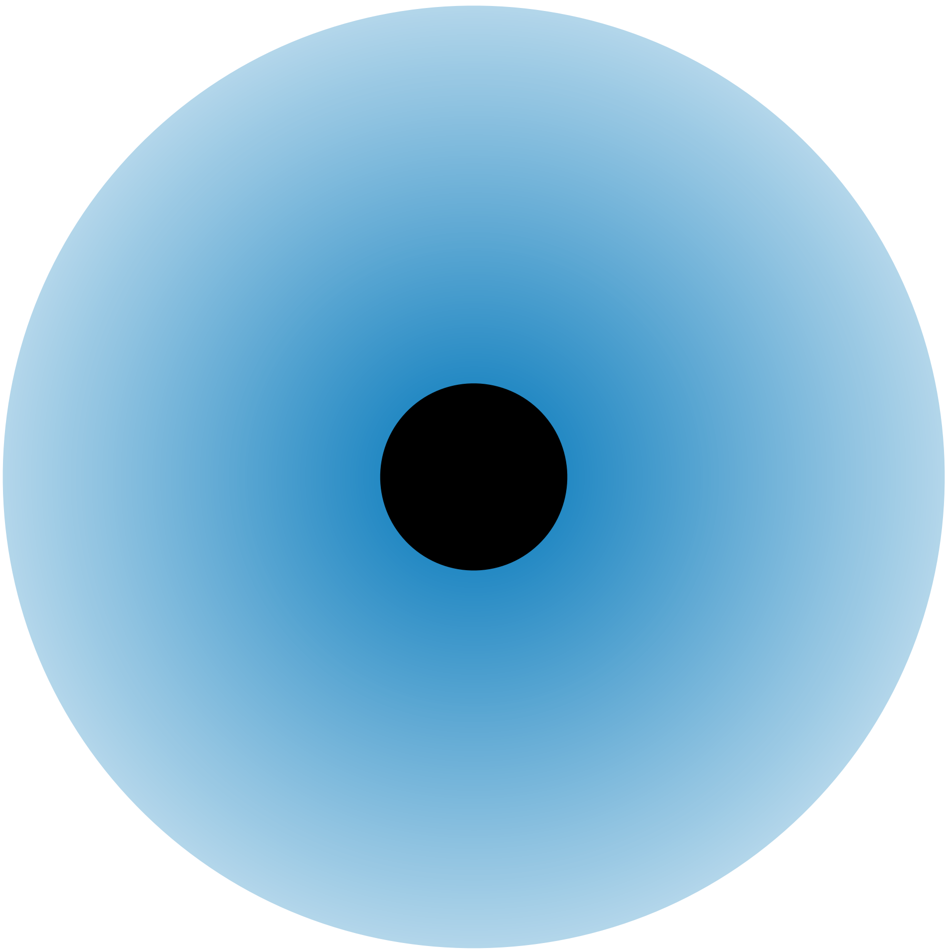
$\mu M_{\mathrm{BH},0}, \Psi_c$

$\Rightarrow M_{\mathrm{BS}}, R_{\mathrm{BS}}$

Possible Scenario

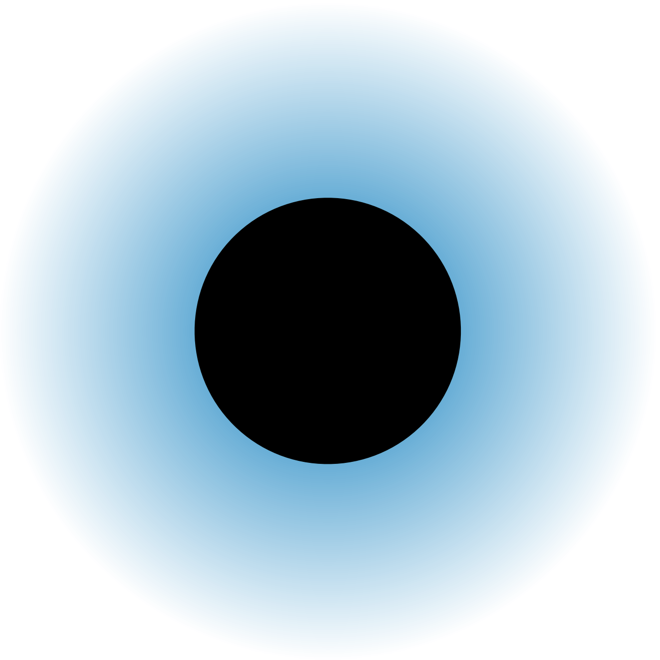


Possible Scenario



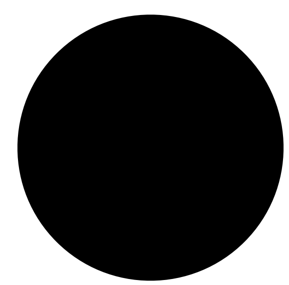
BH eats boson star.

Possible Scenario



Formation of gravitational atom ?

Possible Scenario



Final state

Our methods

- Gravitational atom in late time
  - Test field configuration ($E_\Psi \ll M_{\rm BH}$)

$$\Box_{\rm BH}\Psi - \mu^2\Psi = 0 \quad \longrightarrow \quad \omega = \omega_{\rm Re} + i\omega_{\rm Im}$$

- Newtonian limit
  - Newtonian approximation

$$\begin{cases} ds^2 = -(1+2\Phi_N)dt^2 + \delta_{ij}dx^i dx^j \\ \Psi = \mu^{-1/2}\bar{\Psi}e^{-i\mu t} \end{cases} \quad \longrightarrow \quad \begin{cases} i\dfrac{\partial\bar{\psi}}{\partial t} = -\dfrac{1}{2\mu}\Delta\bar{\psi} + \mu\left(-\dfrac{M_{\rm BH}}{r} + \delta\Phi_N\right)\bar{\psi} \\ \Delta\delta\Phi_N = 8\pi\mu\bar{\psi}_0^2 \end{cases}$$

  - BH horizon effect is not included.

skip the detail in this talk

- Numerical relativity simulation
  - All effects are included.
  - Simulations with $\mu M_{\rm BH} \ll 1$ or $\Psi_c \ll 0.01$ are difficult.

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Gravitational atom

$$f(r) = 1 - \frac{2M}{r}$$

- Test scalar field around BH

$$\Box_{\mathrm{Sch.BH}}\psi - \mu^2\psi = 0 \qquad \psi = \sum_{lm}\frac{\sigma_{lm}(r)}{r}Y_{lm}(\theta,\phi)e^{-i\omega t}$$

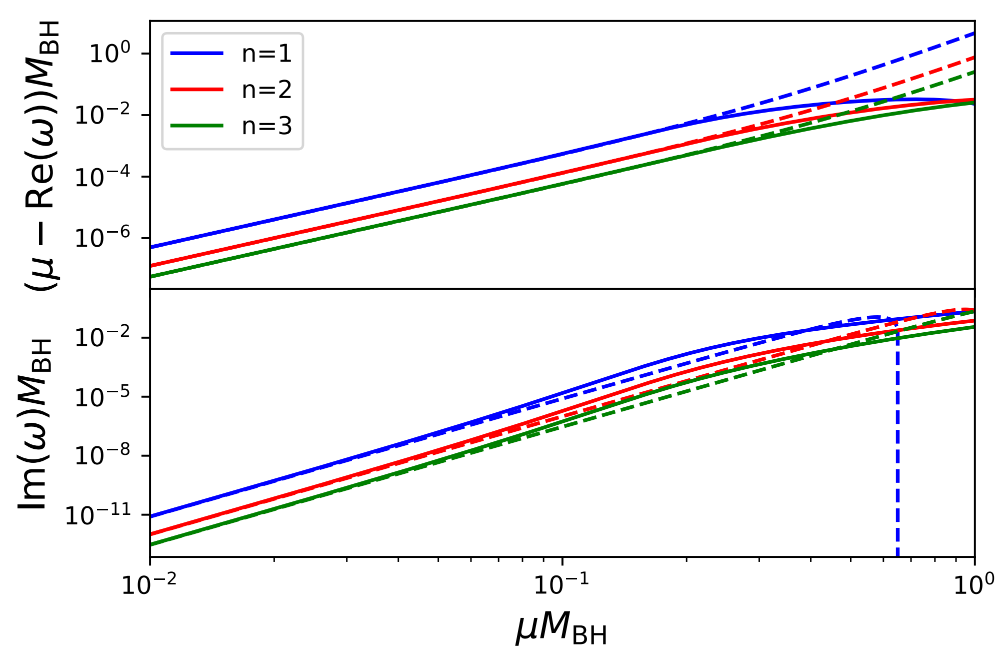
$$\Rightarrow \quad f^2\sigma''_{lm} + ff'\sigma'_{lm} + \left(\omega^2 - f\left(\frac{l(l+1)}{r^2} + \frac{f'}{r} + \mu^2\right)\right)\sigma_{lm} = 0$$

$$\omega = \omega_{\mathrm{Re}} + i\omega_{\mathrm{Im}}$$

- Gravitational atom $\omega < \mu$

$$\begin{cases} \sigma_{lm}(r \to 2M_{\mathrm{BH}}) \propto \left(\frac{r}{2M_{\mathrm{BH}}} - 1\right)^{i2M_{\mathrm{BH}}\omega} e^{i\omega r}, & \text{(Ingoing into horizon)} \\ \sigma_{lm}(r \to \infty) \propto e^{-\sqrt{\mu^2 - \omega^2}\,r}. & \text{(Decaying at infinity)} \end{cases}$$

- Spectrum of gravitational atom
  - Direct Integration
  - Leaver method
  - Detweiler approximation et.al.



$\Rightarrow$ Life time of the gravitational atom

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Numerical formulation

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- We use (generalized-)BSSN formulation.
  - conformal decomposition

$$\begin{cases} \gamma_{ij} = e^{4\phi}\tilde{\gamma}_{ij} \\ K_{ij} = e^{4\phi}\tilde{A}_{ij} + \frac{1}{3}\gamma_{ij}K \end{cases}$$

  - auxiliary field

$$\tilde{\Lambda}^k = \tilde{\gamma}^{ij}(\tilde{\Gamma}^k_{ij} - \bar{\Gamma}^k_{ij})$$

$\bar{\gamma}_{ij}$: reference metric

  - spherical symmetry : (t, r, θ, ϕ)

$$\begin{cases} \tilde{\gamma}_{ij} = \mathrm{diag}(\tilde{a}, \tilde{b}r^2, \tilde{b}r^2\sin^2\theta) \\ \tilde{A}_{ij} = \mathrm{diag}(A, Br^2, Br^2\sin^2\theta) \\ \tilde{\Lambda}^k = (\tilde{\Lambda}, 0, 0) \end{cases}$$

  - constraint eq.

$$\mathscr{H} \equiv \left(\frac{\phi''}{a} + \frac{\phi'^2}{a} - \left(\frac{a'}{2a^2} - \frac{b'}{ab} - \frac{2}{ar}\right)\phi'\right)e^{\phi} - \frac{e^{\phi}}{8}\tilde{R} + \frac{e^{5\phi}}{8}\left(\frac{A^2}{a^2} + 2\frac{B^2}{b^2}\right) - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}E$$

$$\mathscr{M} \equiv 6\phi'\frac{A}{a} + \frac{A'}{a} - \frac{a'A}{a^2} + \frac{b'}{b}\left(\frac{A}{a} - \frac{B}{b}\right) + \frac{2}{r}\left(\frac{A}{a} - \frac{B}{b}\right) - \frac{2}{3}K' - 8\pi p = 0$$

Numerical formulation

- evolution eq.

$$
\begin{cases}
\partial_t \phi = \beta \phi' - \frac{1}{6}\alpha K + \sigma \frac{1}{6}\mathscr{B} \\
\partial_t a = \beta a' + 2a\beta' - 2\alpha A - \sigma \frac{2}{3} a \mathscr{B} \\
\partial_t b = \beta b' + 2\beta \frac{b}{r} - 2\alpha B - \sigma \frac{2}{3} b \mathscr{B} \\
\partial_t K = \beta K' - \mathscr{D} + \alpha(\frac{1}{3}K^2 + \frac{A^2}{a^2} + 2\frac{B^2}{b^2}) + 4\pi\alpha(E+S) \\
\partial_t A = \beta A' + 2A\beta' + e^{-4\phi}(-\mathscr{D}^{TF}_{rr} + \alpha(R^{TF}_{rr} - 8\pi S^{TF}_{rr})) + \alpha(KA - 2\frac{A^2}{a}) - \sigma\frac{2}{3}A\mathscr{B} \\
\partial_t B = \beta B' + \frac{e^{-4\phi}}{r^2}(-\mathscr{D}^{TF}_{\theta\theta} + \alpha(R^{TF}_{\theta\theta} - 8\pi S^{TF}_{\theta\theta})) + \alpha(KB - 2\frac{B^2}{b}) + 2\frac{\beta}{r}B - \sigma\frac{2}{3}B\mathscr{B} \\
\partial_t \tilde{\Lambda} = \beta\tilde{\Lambda}' - \beta'\tilde{\Lambda} + \frac{2\alpha}{a}(\frac{6A\phi'}{a} - \frac{2}{3}K' - 8\pi p) + \frac{\alpha}{a}(\frac{a'A}{a^2} - \frac{2b'B}{b^2} + 4B\frac{a-b}{rb^2}) + \sigma(\frac{2}{3}\tilde{\Lambda}\mathscr{B} + \frac{\mathscr{B}'}{3a}) + \frac{2}{rb}(\beta' - \frac{\beta}{r}) - 2\frac{\alpha' A}{a^2} + \frac{1}{a}\beta''
\end{cases}
$$

skip the details…..

- Our numerical code
  - Time integration : 4th order Runge-Kutta method
  - Radial derivative: 4th order accurate centered finite difference
  - Open MP, KO dissipation, excision procedure ….

Construction of ID

- We construct BS-BH initial data by solving constraint equation.
  - assumptions for initial data
    - Parameters: ψ_c , $\mu M_{0,\mathrm{BH}}$
    - momentarily static : $K = A = B = 0 \quad \Longrightarrow \quad \mathscr{M} = 0$
    - conformally flat : $a = b = 1$
    - Profile of the scalar field is same as boson star profile.
    - precollapse lapse, zero shift : $\alpha(0,r) = e^{-4\phi(0,r)}$, $\beta(0,r) = 0$
  - 1. Construct the BS star profile in isotropic coordinate.

$$\begin{cases} ds^2 = -\alpha_{\mathrm{BS}}^2(r)dt^2 + \Phi_{\mathrm{BS}}(r)^4(dr^2 + r^2 d^2\Omega) \\ \psi(t,r) = \psi_{0,\mathrm{BS}}(r)e^{i\omega t} \end{cases}$$

Construction of ID

- 2.Sum of conformal factor

$$\Phi = e^{\phi}$$

$$\Phi = \Phi_{\mathrm{BS}} + \Phi_{\mathrm{BH}} - 1 + \delta\Phi$$

$$\Phi_{\mathrm{BH}} = 1 + \frac{M_{0,\mathrm{BH}}}{2r}$$

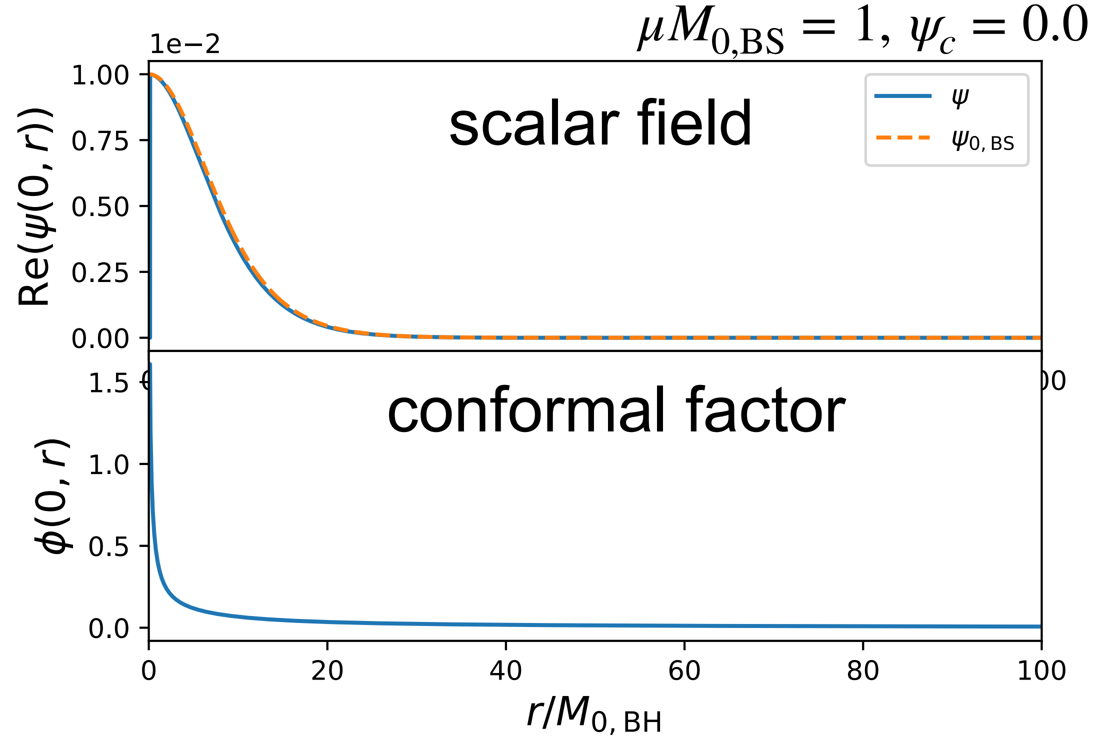
- 3.Solve Hamiltonian constraint for $\delta\Phi$

$$\mathscr{H} = \Phi'' + \frac{2}{r}\Phi' + 2\pi\Phi^5 E^{\mathrm{W}}_{\mathrm{BS}} = 0$$

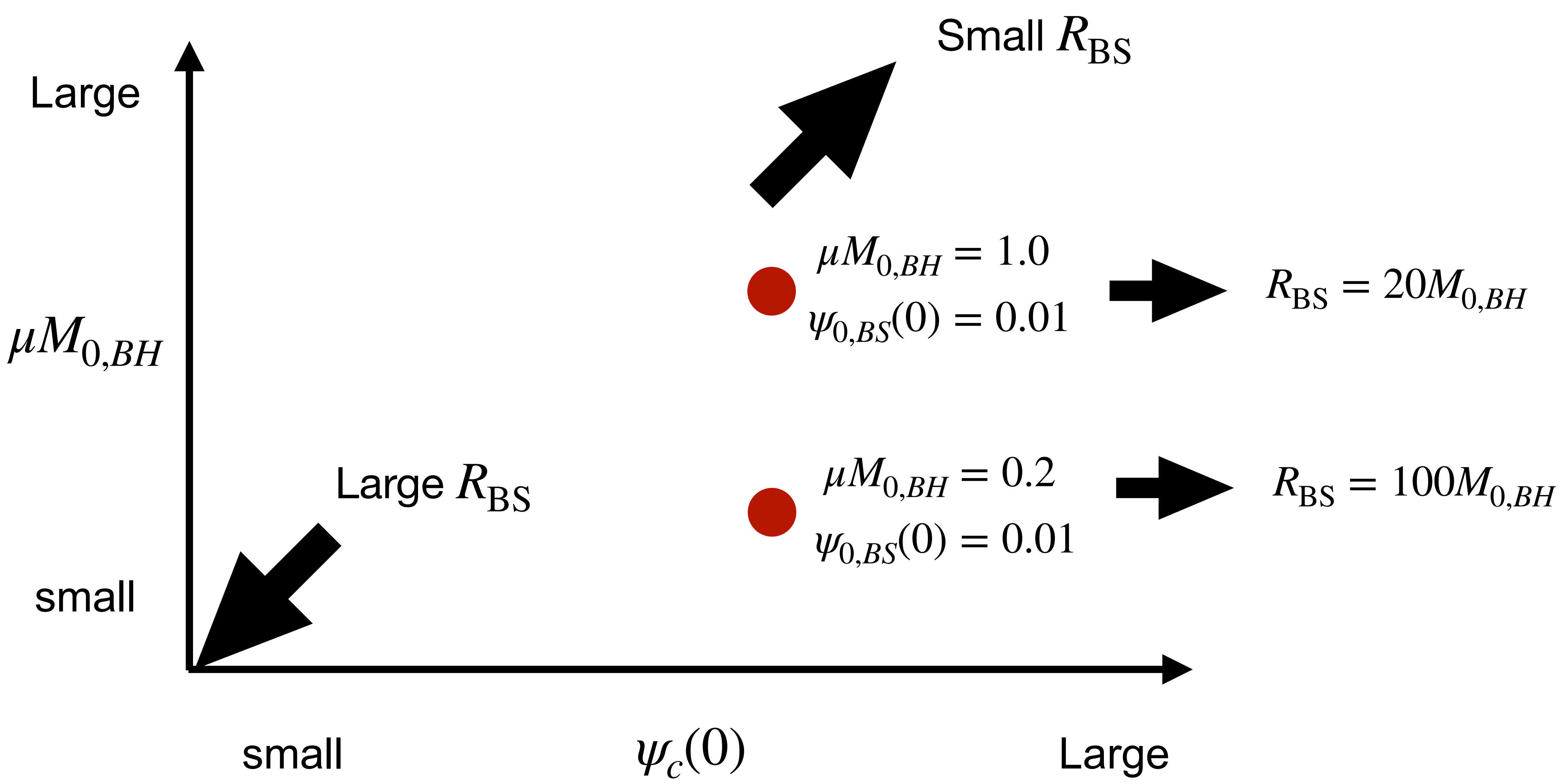
$$\Rightarrow \delta\Phi'' + \frac{2}{r}\delta\Phi' + 2\pi\left(\Phi^5 E^{\mathrm{W}}_{\mathrm{BS}} - \Phi^5_{\mathrm{BS}} E_{\mathrm{BS}}\right) = 0$$

Integrate from infinity with $\delta\Phi(\infty) = \partial_r \delta\Phi(\infty) = 0$

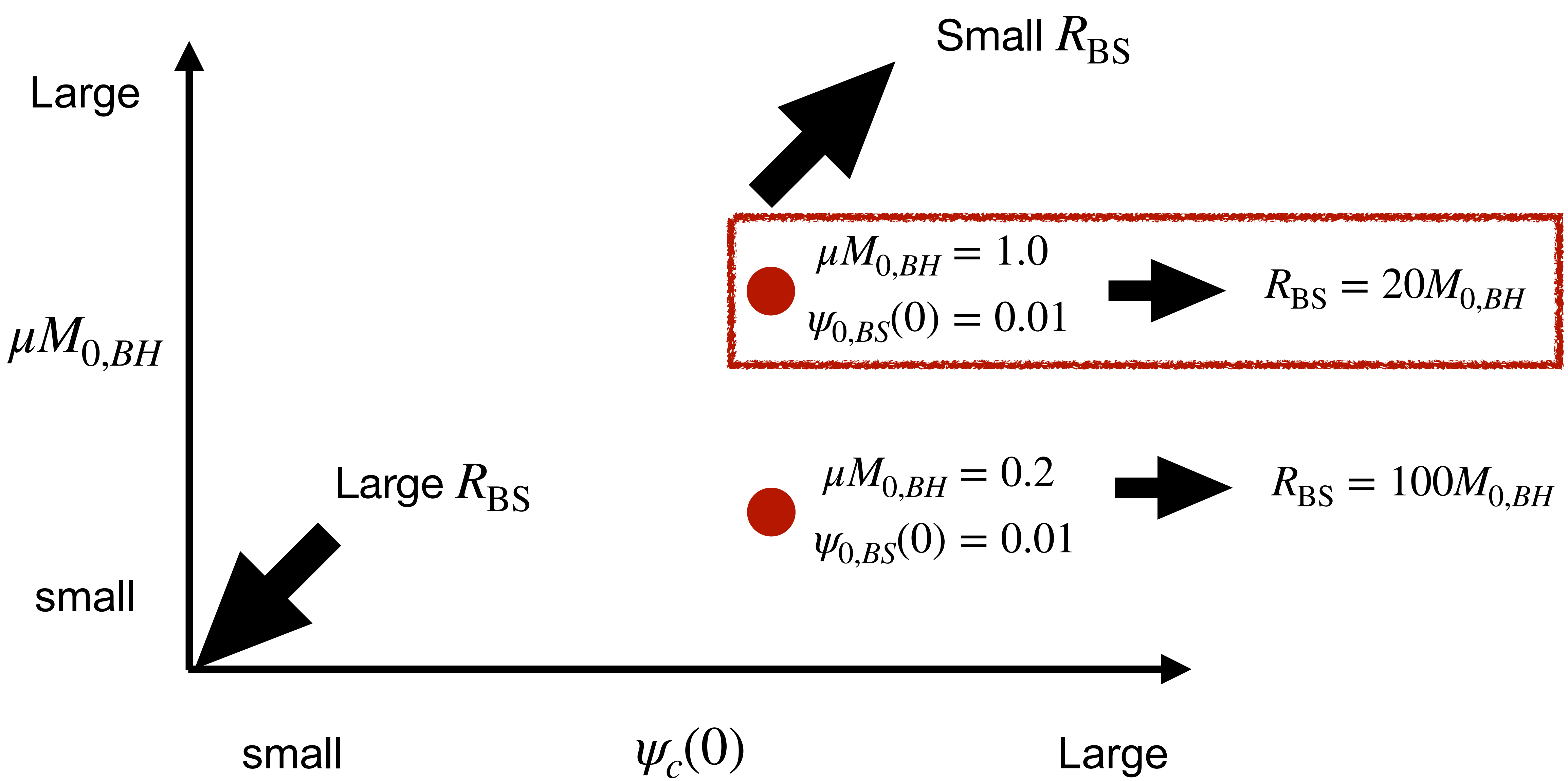
$\mu M_{0,\mathrm{BS}} = 1,\ \psi_c = 0.01$



Parameter space



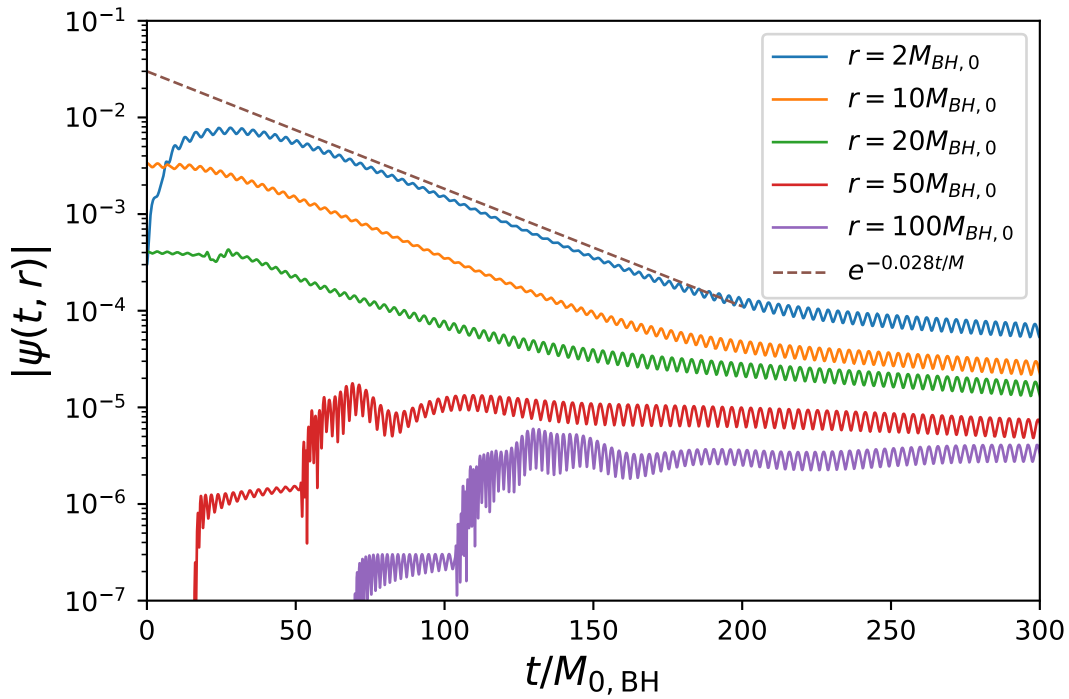
Parameter space



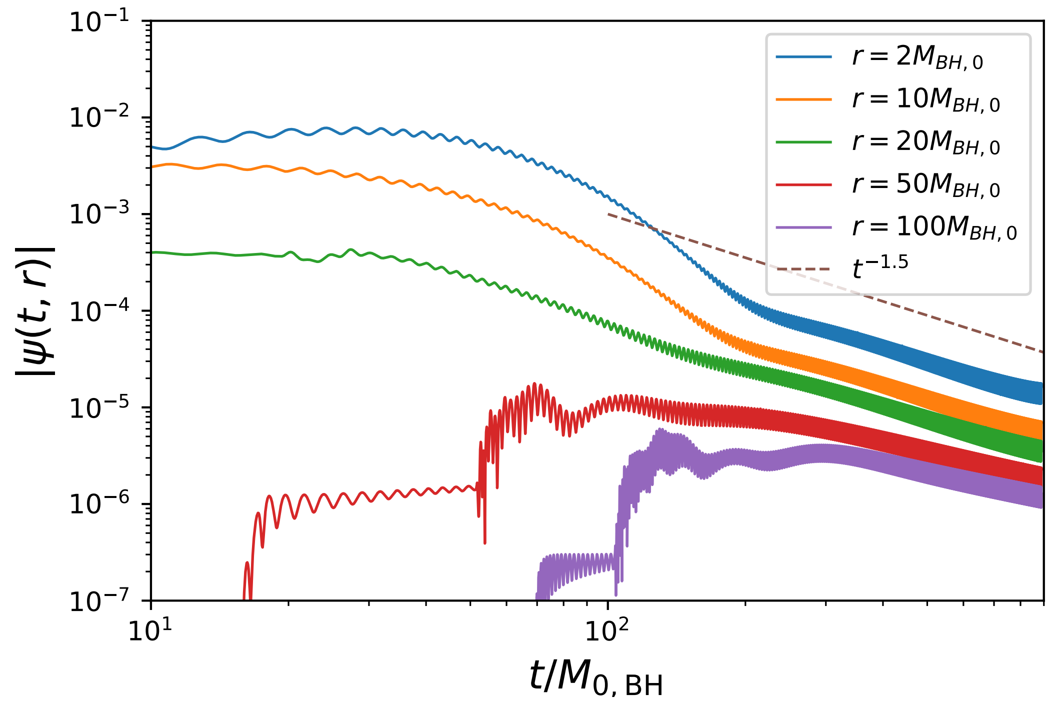
Preliminary results

$$\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01$$

exponential decay
in early phase.



power-law tail in late time



In general, we can expect power low tail

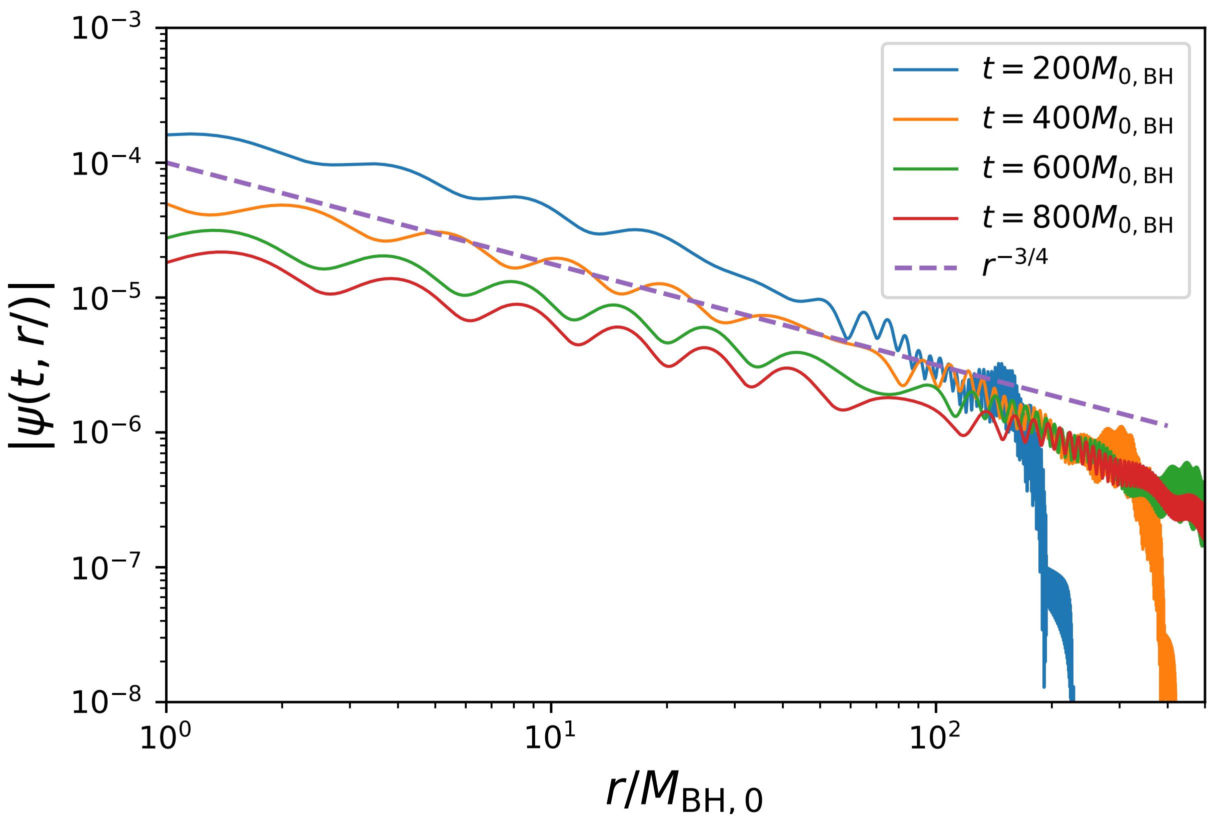
$$\psi \sim t^p \sin(\mu t)$$

$$\begin{cases} p = -(l + 3/2) & \text{at late time} \\ p = -5/6 & \text{at very late time} \end{cases}$$

Preliminary results

$$\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01$$

Late time radial profile of scalar field is $\sim r^{-3/4}$

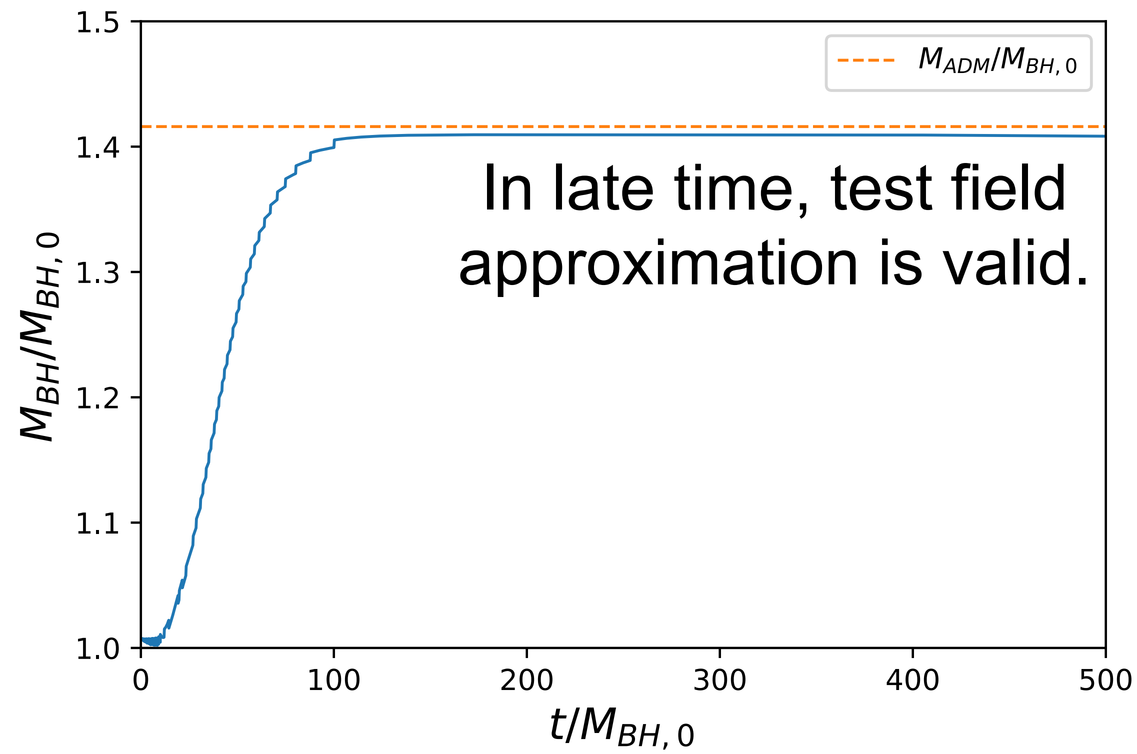


cf: Clough et.al.(2019), Hui et.al (2019)

$$\psi(t, r) \propto r^{-3/4} \cos(\cdots)$$

for $\mu M \gtrsim 1$

BH eat almost boson star energy in early phase.



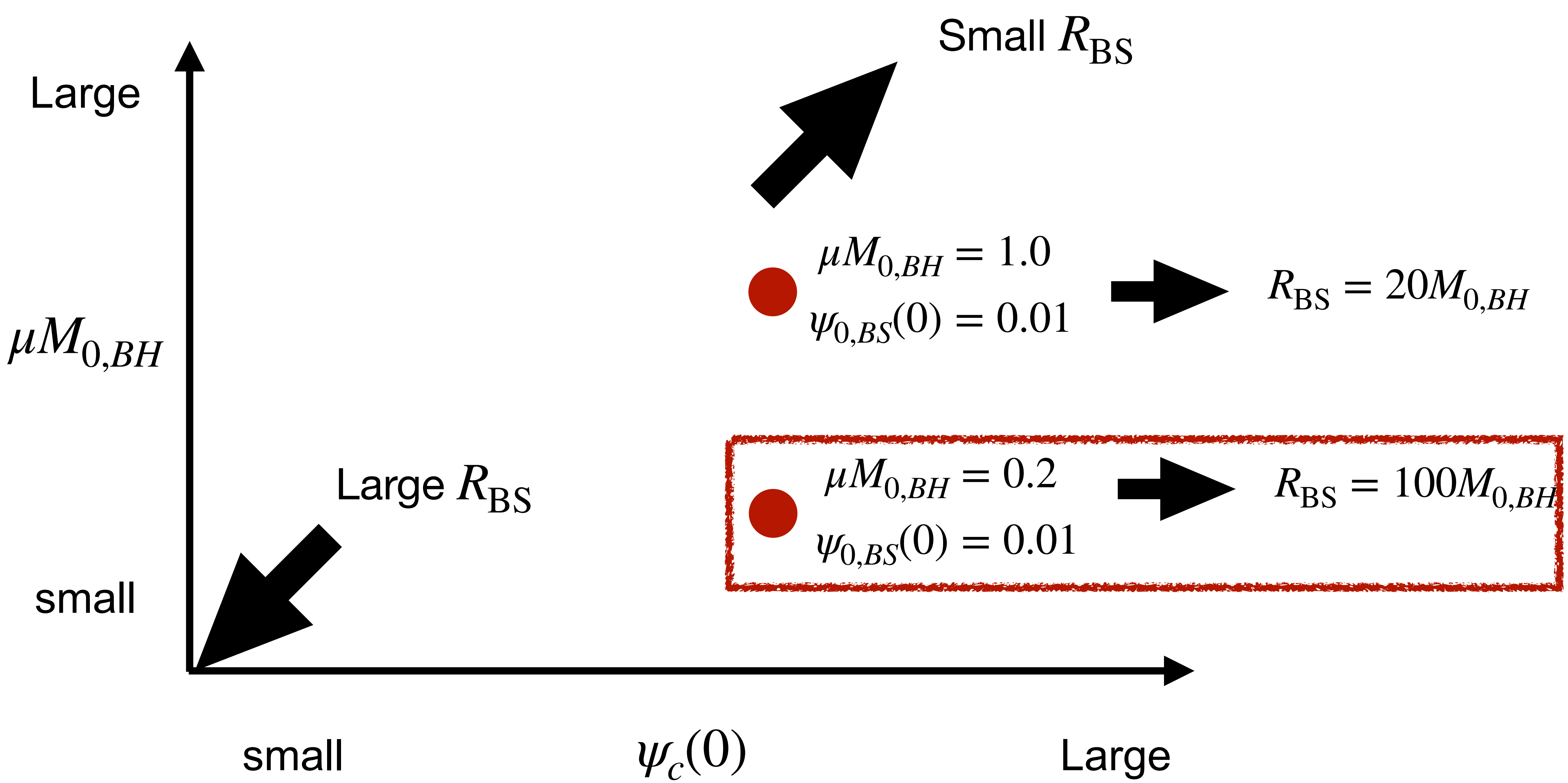
In late time, test field approximation is valid.

In late time, $\mu M_{BH} \simeq 1.4$

The life time of the corresponding gravitational atom is very short.

→ power low behavior dominates.

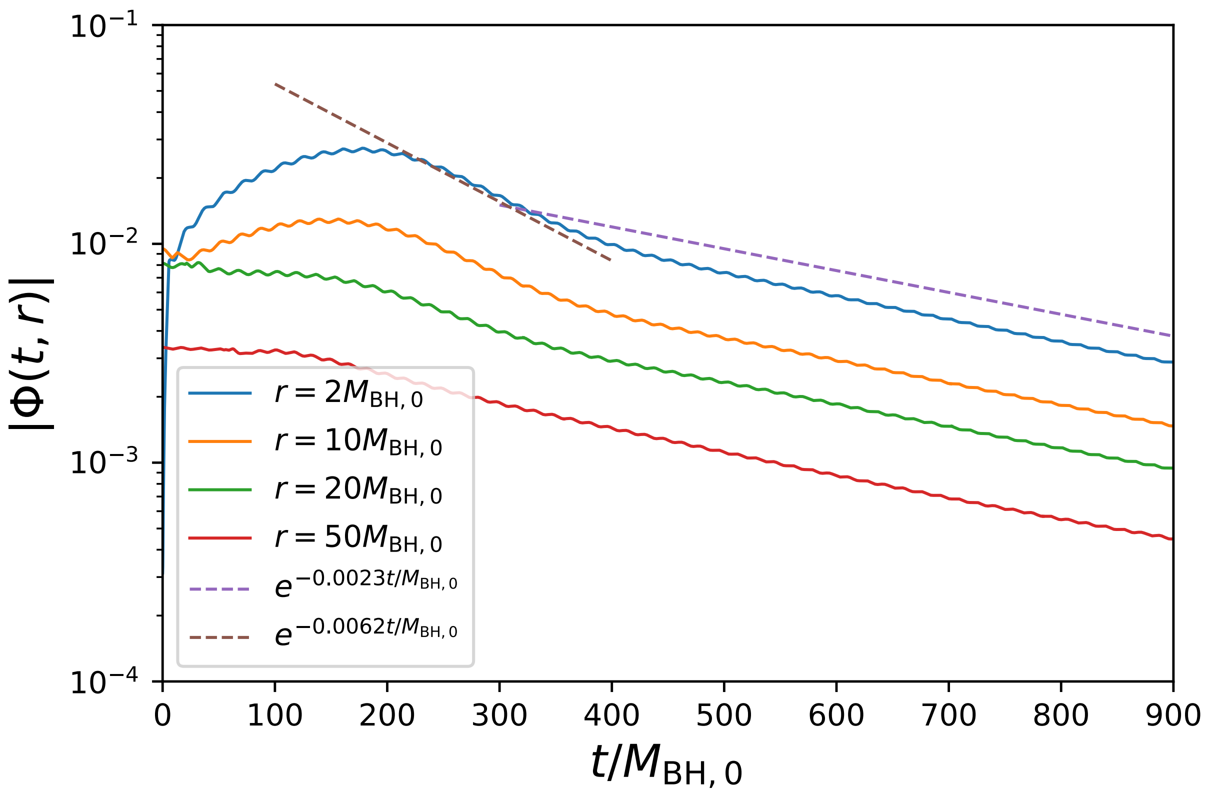
Parameter space



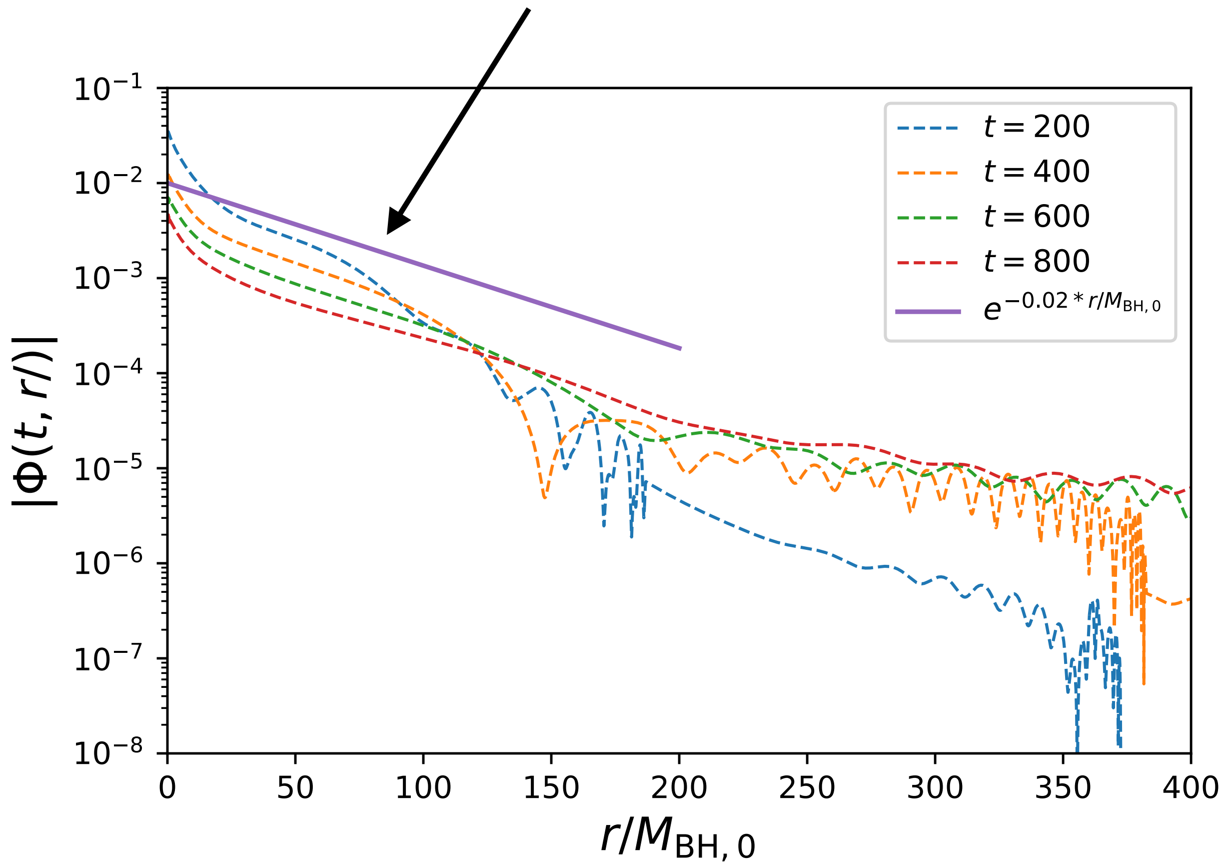
Preliminary results

$$\mu M_{0,\mathrm{BH}} = 0.2, \psi_c = 0.01$$

Exponential decay



Consistent with Newtonian limit



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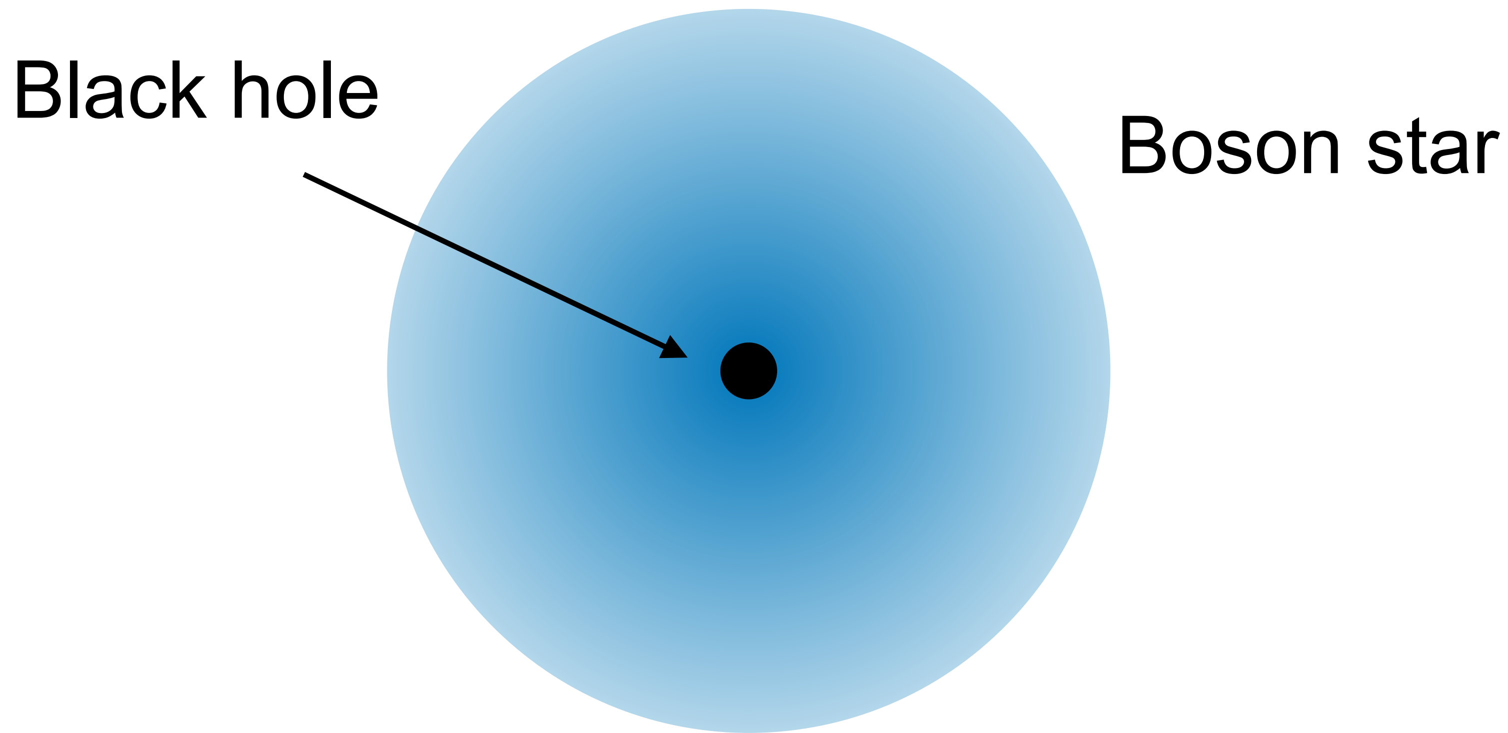
- We discussed the accretion process of boson star into black hole.



- Gravitatioanl atom : spectrum
- Schrodeinger-Poisson eq : configuration in Newtonian limit
- Numerical relativity simulations
  - power low profile $r^{-3/4}$ $(\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01)$
  - Newtonian profile $(\mu M_{0,\mathrm{BH}} = 0.2, \psi_c = 0.01)$
- We need further simulations….

Finish