ブラックホール周りのアクシオン雲からの電磁放射

(Electromagnetic radiation from axion cloud around Kerr BHs)

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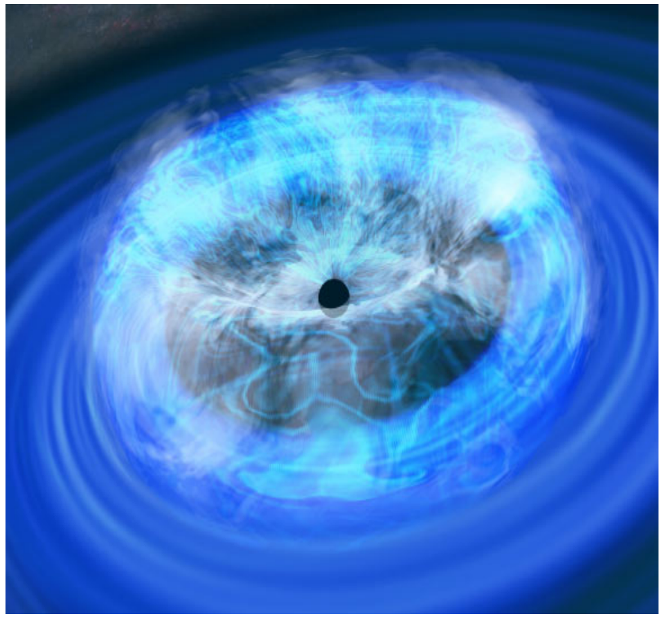
arXiv: 1811.04950, 1811.04945

Black hole physics

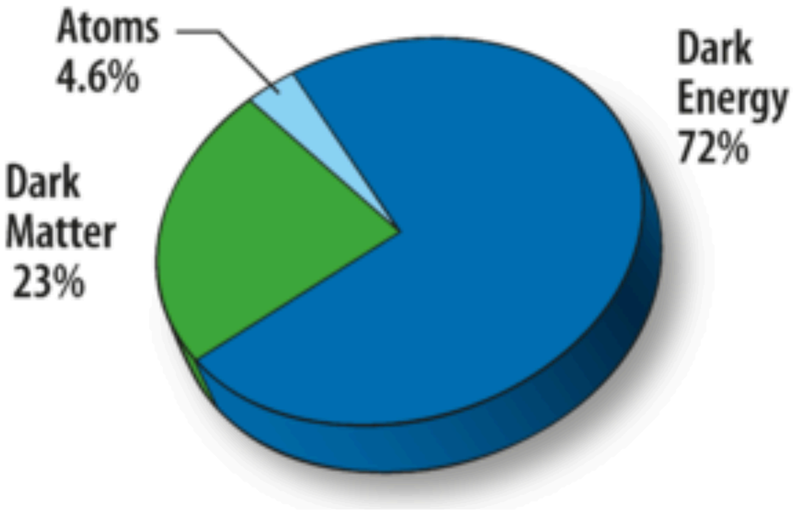
- Physics around BH
  - No Hair or Hairy BH
  - BH formation
  - Gravitational wave
  - Test of gravitational theory
  - Fundamental field around BH

et al



- BH as “particle detector”
  - By using these physics, BH can become “particle detector”.
  - We focus on the scalar field.

ex) axion



Scalar field around Kerr BH

- Super-radiance

$$\omega < m\Omega_H$$

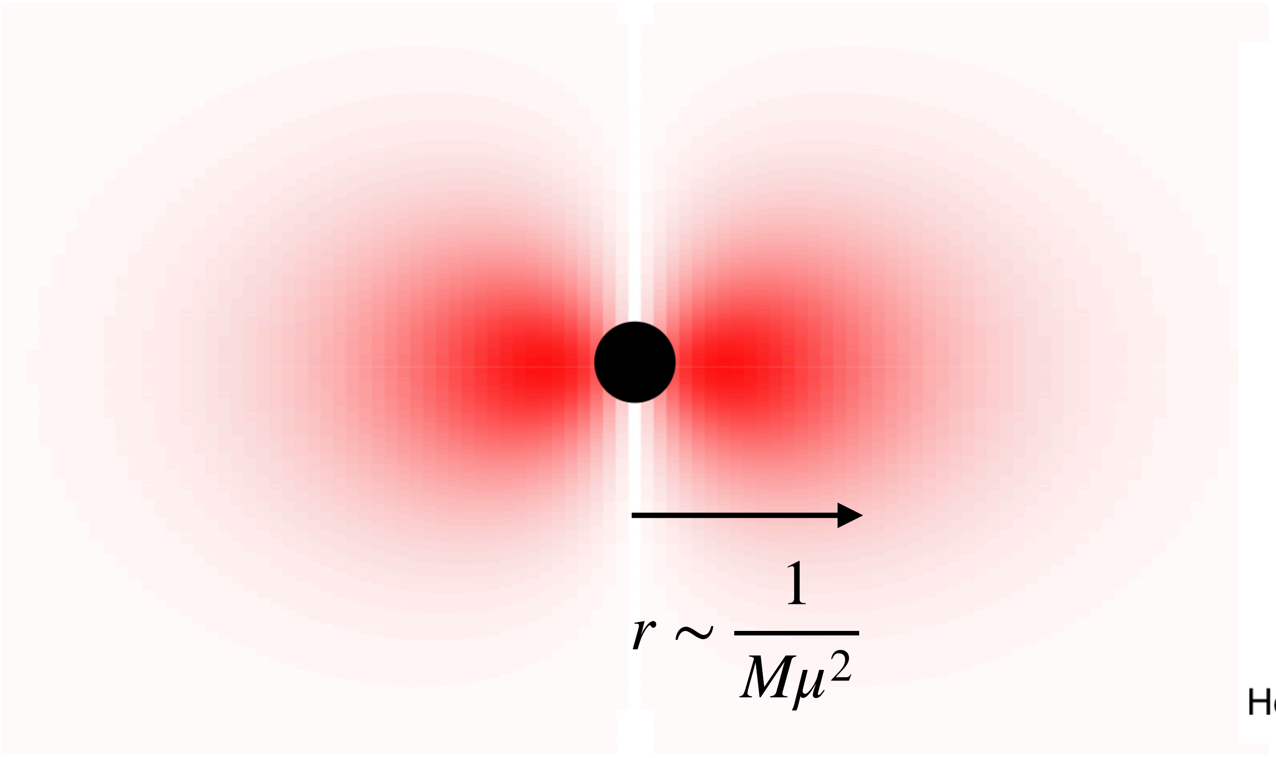
$$\tau = 2 \times 10^4 a \left(\frac{\mu}{10^{-5}\mathrm{eV}}\right)^{-1} \left(\frac{\mu M}{0.03}\right)^{-8} \mathrm{s}$$

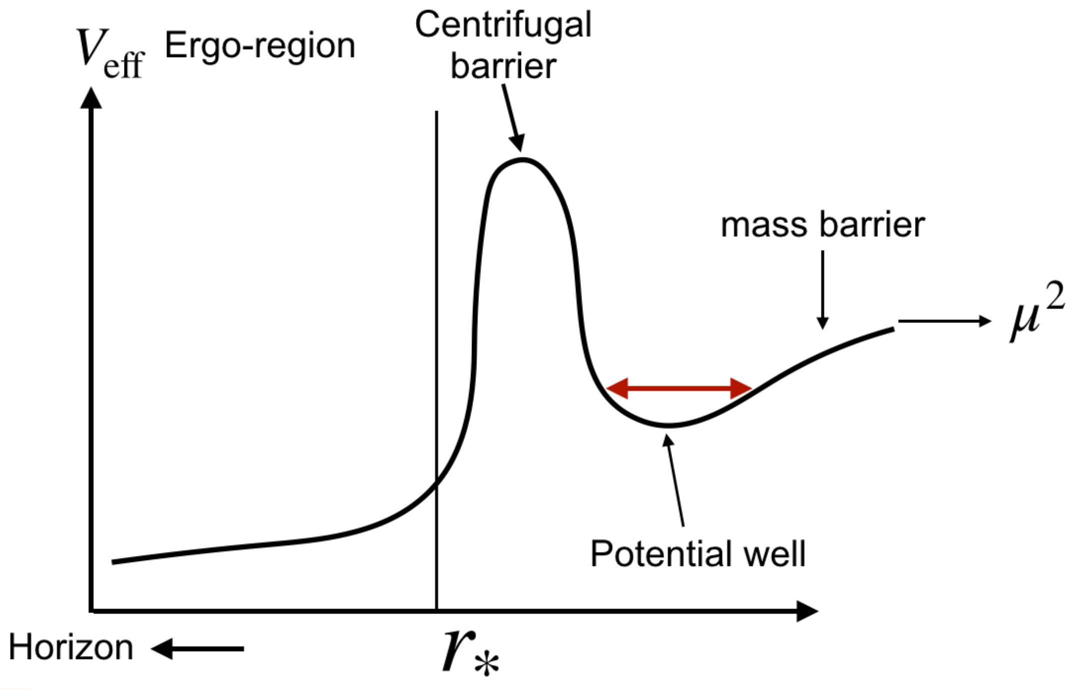
Ω_H : angular velocity

$\Phi(x) = e^{-i\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r)$

- Scalar field can be localized as cloud around BH

$$r_{\mathrm{cloud}} \sim \frac{(l+n+1)^2}{(M\mu)^2} M$$





Set up

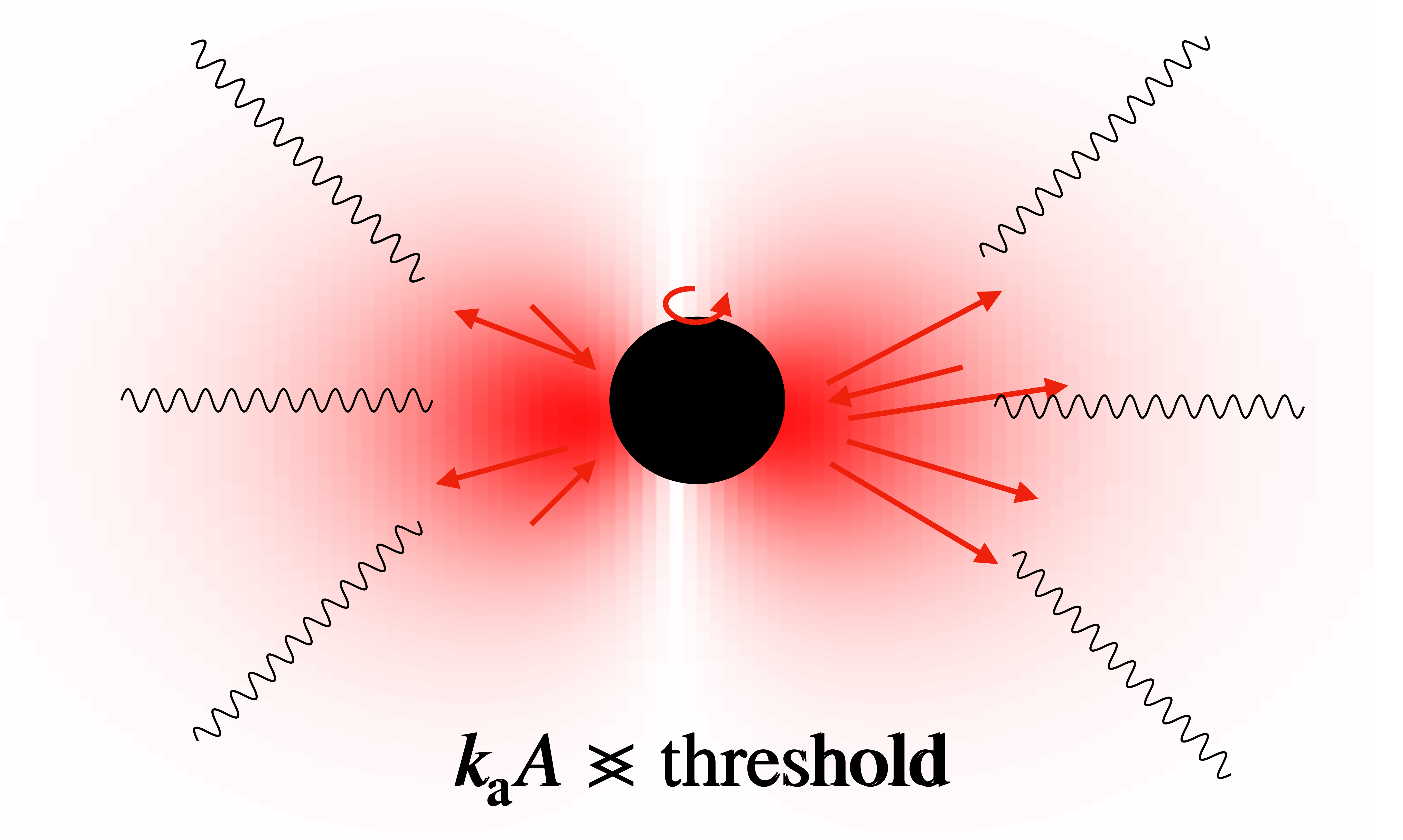
- Matter contents (Kerr BH background)
  - Massive scalar field : Φ
  - Electro-magnetic field : A_μ
- Interaction
  - axion type interaction

$$\mathscr{L}_{\mathrm{a}} = -\frac{k_{\mathrm{a}}}{2}\Phi^* F^{\mu\nu}F_{\mu\nu} = -2k_{\mathrm{a}}\Phi\vec{B}\cdot\vec{E}$$

cf: Axion decay to photon.
Axion convert into photon under magnetic field.

  - scalar type interaction

$$\mathscr{L}_{\mathrm{s}} = -\frac{(k_{\mathrm{s}}\Phi)^p}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{(k_{\mathrm{s}}\Phi)^p}{2}\left(\vec{B}^2 - \vec{E}^2\right)$$


$k_a A \gtrless$ threshold

Outline

1.Introduction

2.Known fact

- Simple toy model (Sen, (2018))
- BLAST of light from axion cloud (J.G.Rosa et al(2018))

3.Our work

- Formulation & Initial data
- Flat space
- Around Kerr BH
- Supper-radiance effect

4.Summary

Simple toy model

- EM field grows exponentially under spatially uniform coherent oscillating axion field. (Sen(2018))
  - Maxwell equation with uniform coherent oscillating scalar field

$$\nabla_\mu F^{\mu\nu} = 2k_{\mathrm{a}}\tilde{F}_{\nu\mu}\nabla^\mu \underline{\Phi} \qquad \Phi = \Phi_0 e^{-i\mu t} + \Phi_0^* e^{i\mu t}$$

μ : mass of scalar field

  - We use following ansatz

$$A_\mu(\vec{x}, t) = \frac{1}{2\sqrt{V}}\sum_{\vec{k}}\left(\underline{\alpha_\mu(\vec{k}, t)e}^{i(\vec{k}\cdot\vec{x}-\omega_{\vec{k}}t)} + \alpha_\mu^*(\underline{\vec{k}, t)e}^{-i(\vec{k}\cdot\vec{x}-\omega_{\vec{k}}t)}\right)$$

  - Coupled ordinary differential eqs. for transverse modes

$$\begin{cases} -\ddot{\tilde{\alpha}}_{(1)}(\vec{k}, t) + i2\omega_{\vec{k}}\dot{\tilde{\alpha}}_{(1)}(\vec{k}, t) + m_{\mathrm{a}}k_{\mathrm{a}}|\vec{k}|\tilde{\alpha}^*_{(2)}(-\vec{k}, t)\left(\Phi_0 e^{i(2\omega_{\vec{k}}-m_{\mathrm{a}})t} - \Phi_0^* e^{i(2\omega_{\vec{k}}+m_{\mathrm{a}})t}\right) = 0 \\ -\ddot{\tilde{\alpha}}_{(2)}(-\vec{k}, t) + i2\omega_{\vec{k}}\dot{\tilde{\alpha}}_{(2)}(-\vec{k}, t) + m_{\mathrm{a}}k_{\mathrm{a}}|\vec{k}|\tilde{\alpha}^*_{(1)}(\vec{k}, t)\left(\Phi_0 e^{i(2\omega_{\vec{k}}-m_{\mathrm{a}})t} - \Phi_0^* e^{i(2\omega_{\vec{k}}+m_{\mathrm{a}})t}\right) = 0 \end{cases}$$

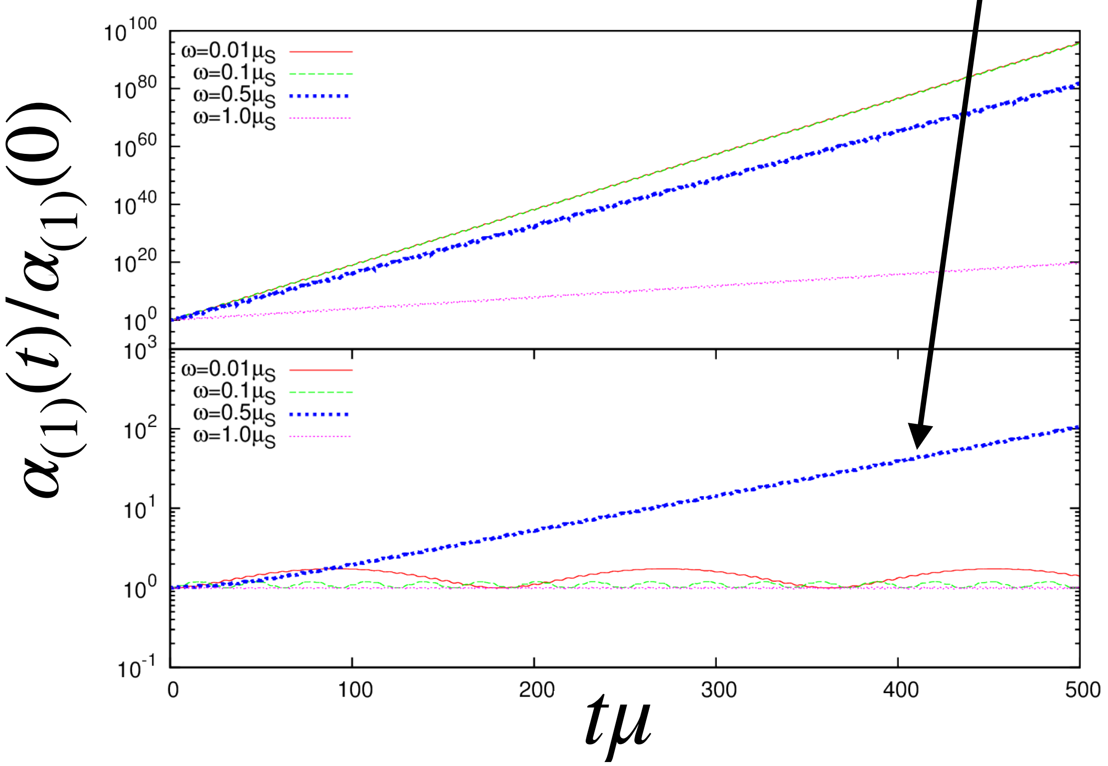
Simple toy model

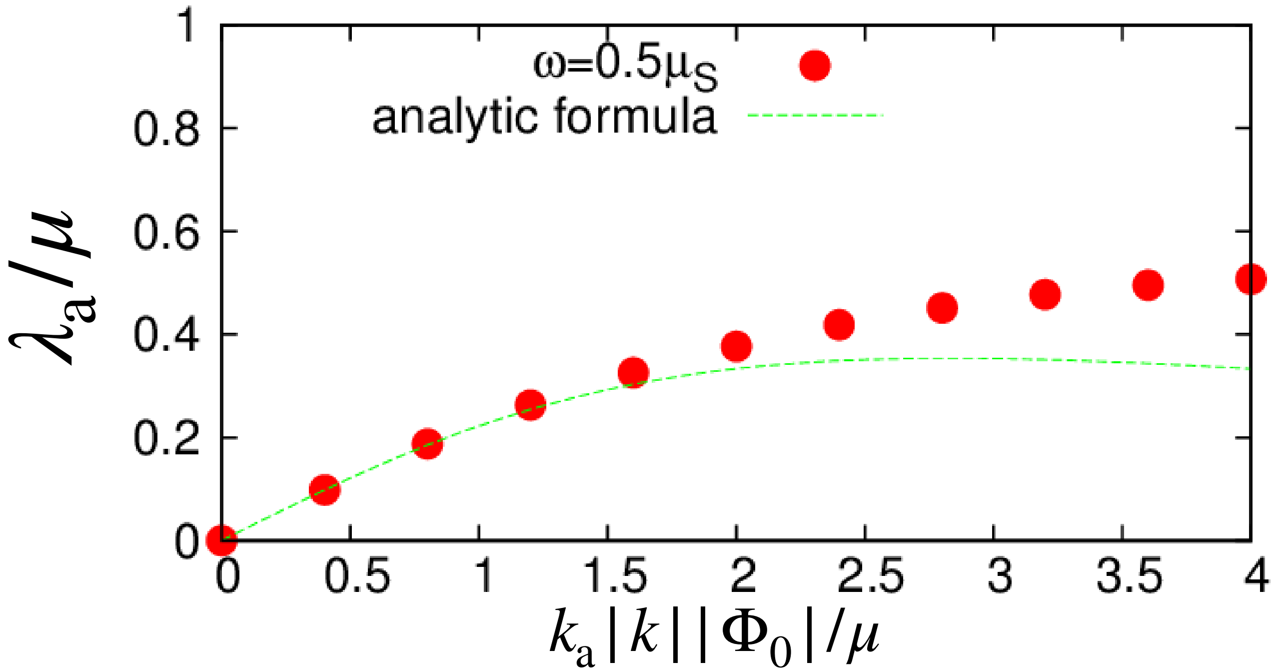
- We can show that
  - the fastest growing mode : $\omega = 0.5\mu$
  - $\tilde{\alpha}_{(I)}(t, \omega = 0.5\mu) \sim e^{\lambda_a t}$ for $k_a|k||\Phi_0|/\mu \ll 1$
  - growth rate

$$\lambda_a = \frac{\mu\epsilon}{1 + \frac{1}{2}\epsilon^2} \qquad \epsilon = k_a|k||\Phi_0| \qquad \text{(arXiv:1811.04945)}$$

- Numerical result

$\omega = 0.5\mu$





BLAST from axion cloud

- BLAST(**B**lack hole **L**asers powered by **A**xion **S**uper-radian**T** instabilities) J.G.Rosa et al(2018)
  - From Boltzmann equation (for 2p state)

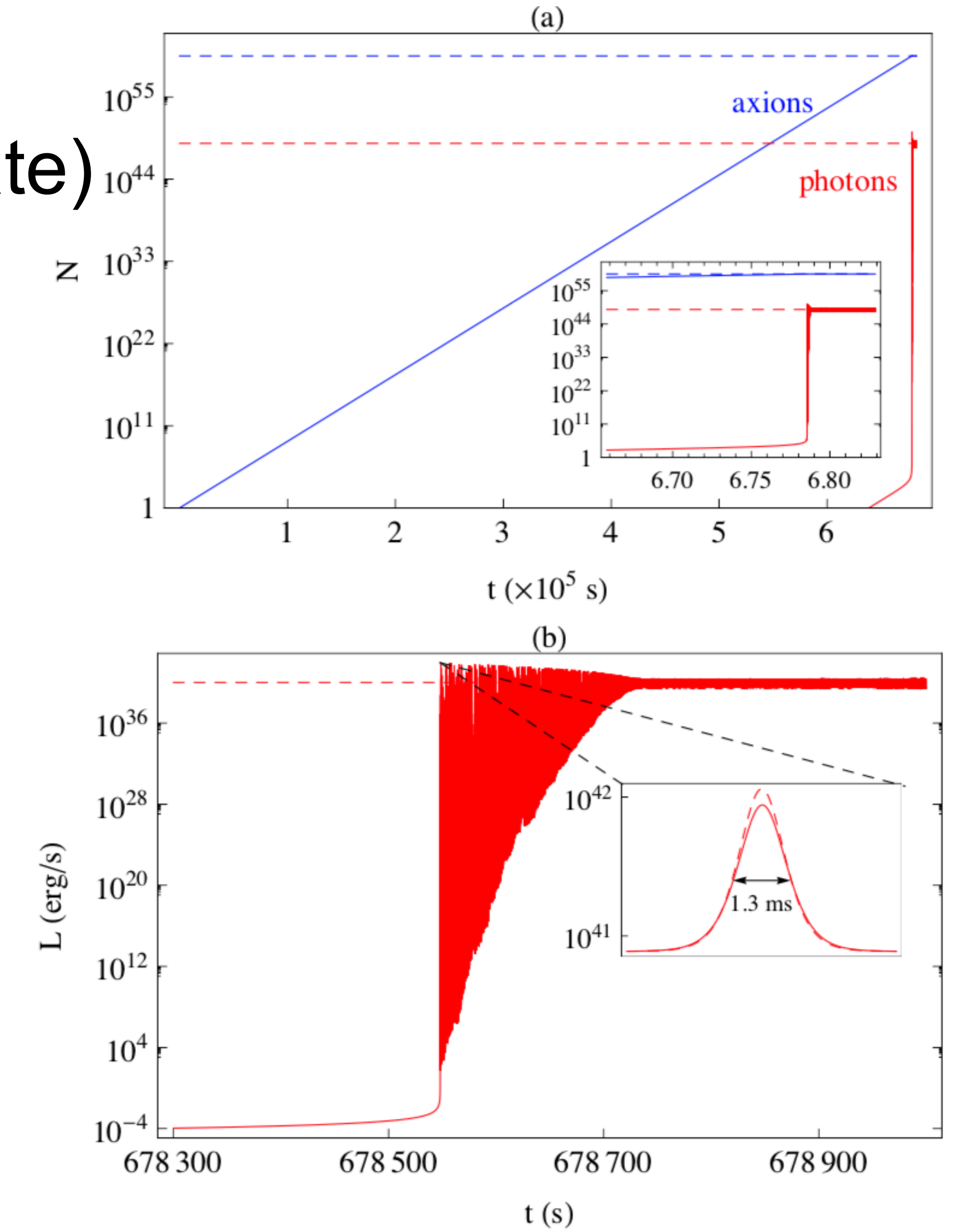
N_ϕ : number of axions

N_γ : number of photons

Γ_ϕ : spontenious axion decay width

$$\frac{dN_\phi}{dt} = \underbrace{\Gamma_s N_\phi}_{\text{super-radiance effect}} \underbrace{- \Gamma_\phi \left(N_\phi (1 + AN_\gamma) - B_1 N_\gamma^2 \right)}_{\text{from interaction}}$$

$$\frac{dN_\gamma}{dt} = \underbrace{- \Gamma_e N_\gamma}_{\text{escape from cloud}} \underbrace{+ 2\Gamma_\phi \left(N_\phi (1 + AN_\gamma) - BN_\gamma^2 \right)}_{\text{from interaction}}$$



  - They predicted bright laser from axion cloud around PBH.

What we want to do

- Summary of known fact
  - Spatially uniform coherent oscillating axion field induces the exponential growth of EM field in flat space (Sen, 2018)
  - The laser like emission of EM field from axion cloud is predicted by solving Boltzmann eq. (J.G.Rosa et al 2018)

- What we want to do is
  - solving Klein-Gordon equation and Maxwell equation with interaction around Kerr background.

$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \dfrac{k_a}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_a \tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$

  - discussing the burst of EM field from axion cloud.

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3. Our work
    - Formulation & Initial data
    - Flat space
    - Around Kerr BH
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4. Summary

Formulation

- We ignore dynamics of gravity sector.
- Equations

$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \dfrac{k_\mathrm{a}}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_\mathrm{a}\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$

metric (Kerr-Schild form)

$$ds^2 = (\eta_{\mu\nu} + 2Hl_\mu l_\nu)dx^\mu dx^\nu$$

$$H = \frac{r^3 M}{r^4 + a^2 z^2}$$

$$l_\mu = \left(1, \frac{rx + ay}{r^2 + a^2}, \frac{-ax + ry}{r^2 + a^2}, \frac{z}{r}\right)$$

- Formulation
  - 3+1 formulation (with Z term)

$$\partial_t \Pi = \alpha(-D^2\Phi + \mu_\mathrm{s}^2\Phi + K\Pi - 2k_\mathrm{a}E^iB_i) - D^i\alpha D_i\Phi + \mathscr{L}_\beta\Pi$$

$$\partial_t\Phi = -\alpha\Pi + \mathscr{L}_\beta\Phi$$

$$\partial_t\mathscr{A}_i = -\alpha(E_i + D_i\mathscr{A}_\phi) - A_\phi D_i\alpha + \mathscr{L}_\beta\mathscr{A}_i$$

$$\partial_t E^i = \alpha(KE^i + D^iZ - (D^2\mathscr{A}^i - D_kD^i\mathscr{A}^k)) + 2\alpha k_\mathrm{a}(+\epsilon^{ijk}E_kD_j\Phi + B^i\Pi) + \epsilon^{ijk}D_k\alpha B_j + \mathscr{L}_\beta E^i$$

$$\partial_t A_\phi = \alpha(KA_\phi - D_i\mathscr{A}^i - Z) - \mathscr{A}_jD^j\alpha + \mathscr{L}_\beta A_\phi$$

$$D_iE^i + 2k_\mathrm{a}B_iD^i\Phi = 0$$

$$\partial_t Z = \alpha(D_iE^i - \kappa Z) + 2k_\mathrm{a}\alpha B_iD^i\Phi + \mathscr{L}_\beta Z$$

Initial data

- Scalar field : axion cloud

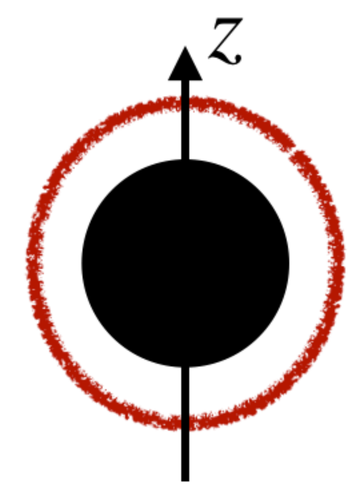
$$\Phi = A_0 g(\tilde{r})\cos(\varphi - \mu t)\sin\theta$$

$k_a A_0$: effective coupling for EM field

$$\begin{cases} g(\tilde{r}) = \tilde{r}e^{-\tilde{r}/2} \\ \tilde{r} = rM\mu^2 \end{cases}$$

- EM field
  - 1. extended profile

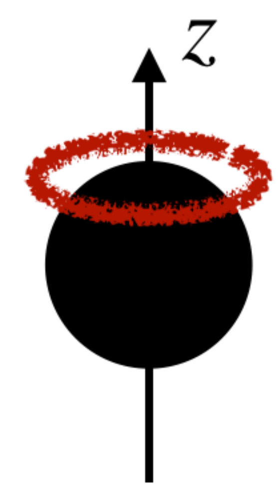
$$E^\varphi = E_0 e^{-(\frac{r-r_0}{w})^2} \quad E^r = E^\theta = B^i = 0$$



  - 2. localized profile

$$E^\varphi = E_0 e^{-(\frac{r-r_0}{w})^2}\Theta(\theta) \qquad E^r = E^\theta = B^i = 0$$

$$\Theta(\theta) = \begin{cases} \sin^4(4\theta) & (0 \leq \theta < \frac{\pi}{4}) \\ 0 & (\frac{\pi}{4} \leq \theta < \pi) \end{cases}$$



initial parameter : E_0, r_0, w
(These profile satisfy Gauss's law.)

  - The result qualitatively does not change.

Instability in flat space

$$(FF)_{00} = \int d\Omega F^{\mu\nu} F_{\nu\nu} Y_{00}$$

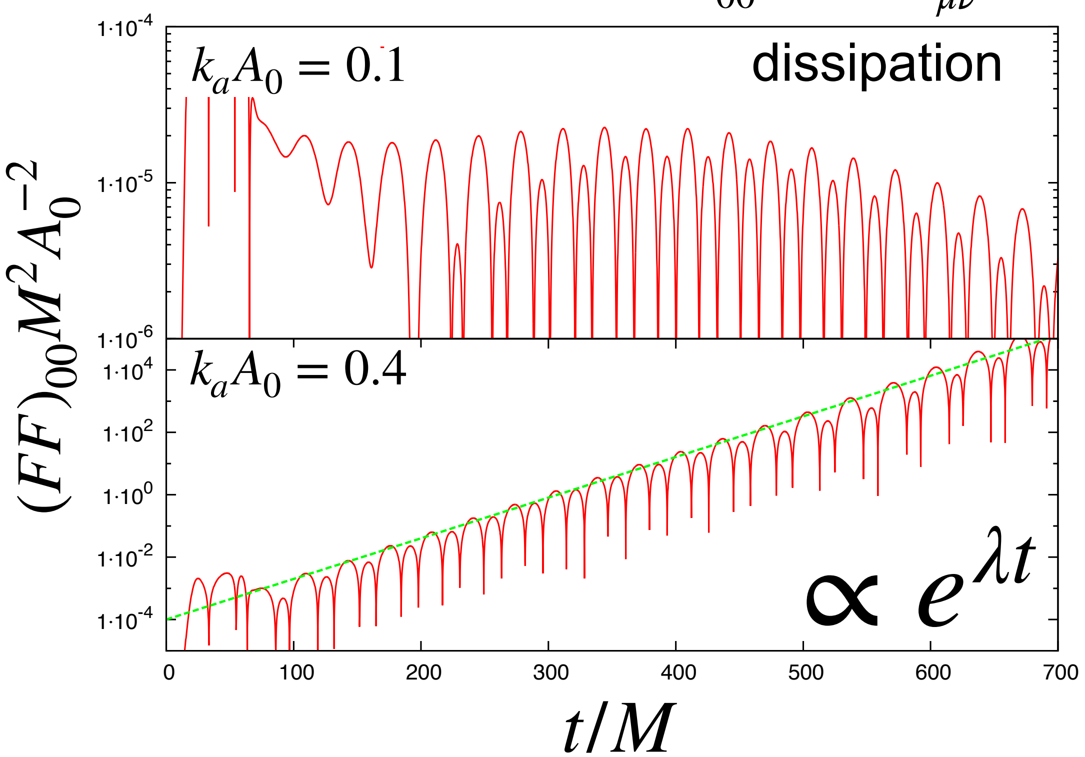
- EM field under the fixed axion cloud in flat space.

$$\nabla_\mu F^{\mu\nu} = 2k_{\mathrm{a}} \tilde{F}_{\nu\mu} \nabla^\mu \underline{\Phi}$$

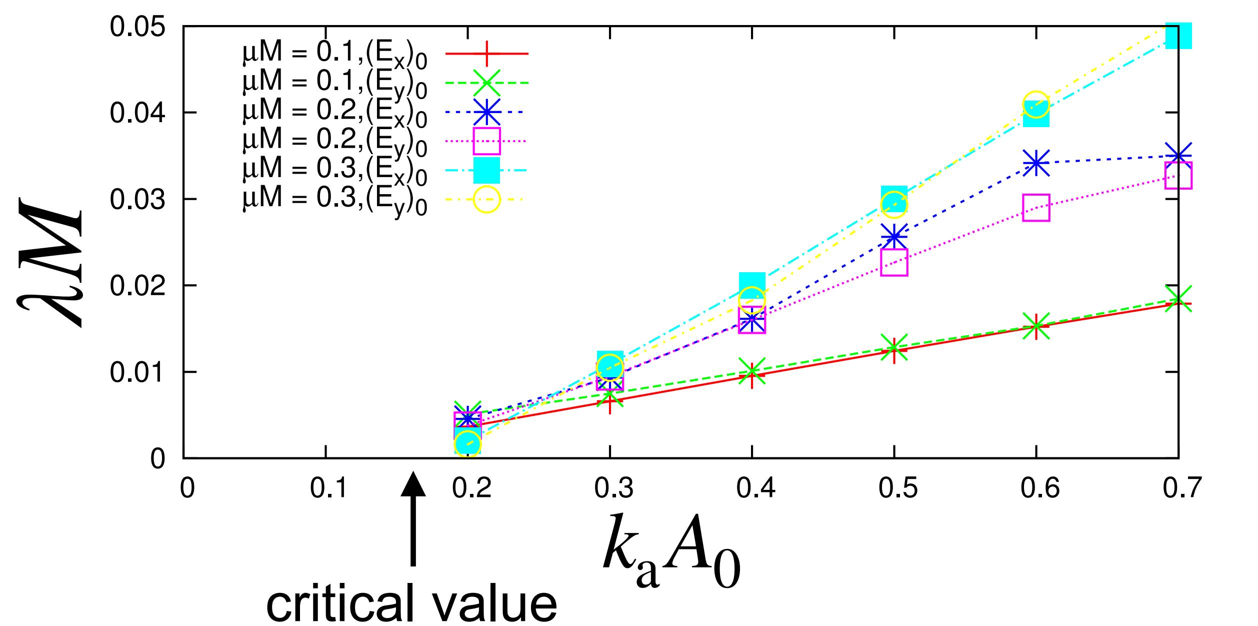
axion cloud : $\Phi = A_0 g(\tilde{r}) \cos(\varphi - \mu t) \sin\theta$

- When the coupling is larger than a certain value, the EM field grows exponentially.

Time evolution of Y_{00} of $F^{\mu\nu}F_{\mu\nu}$



Relation between λM and $k_{\mathrm{a}} A_0$ for each mass μM



Around Kerr BH

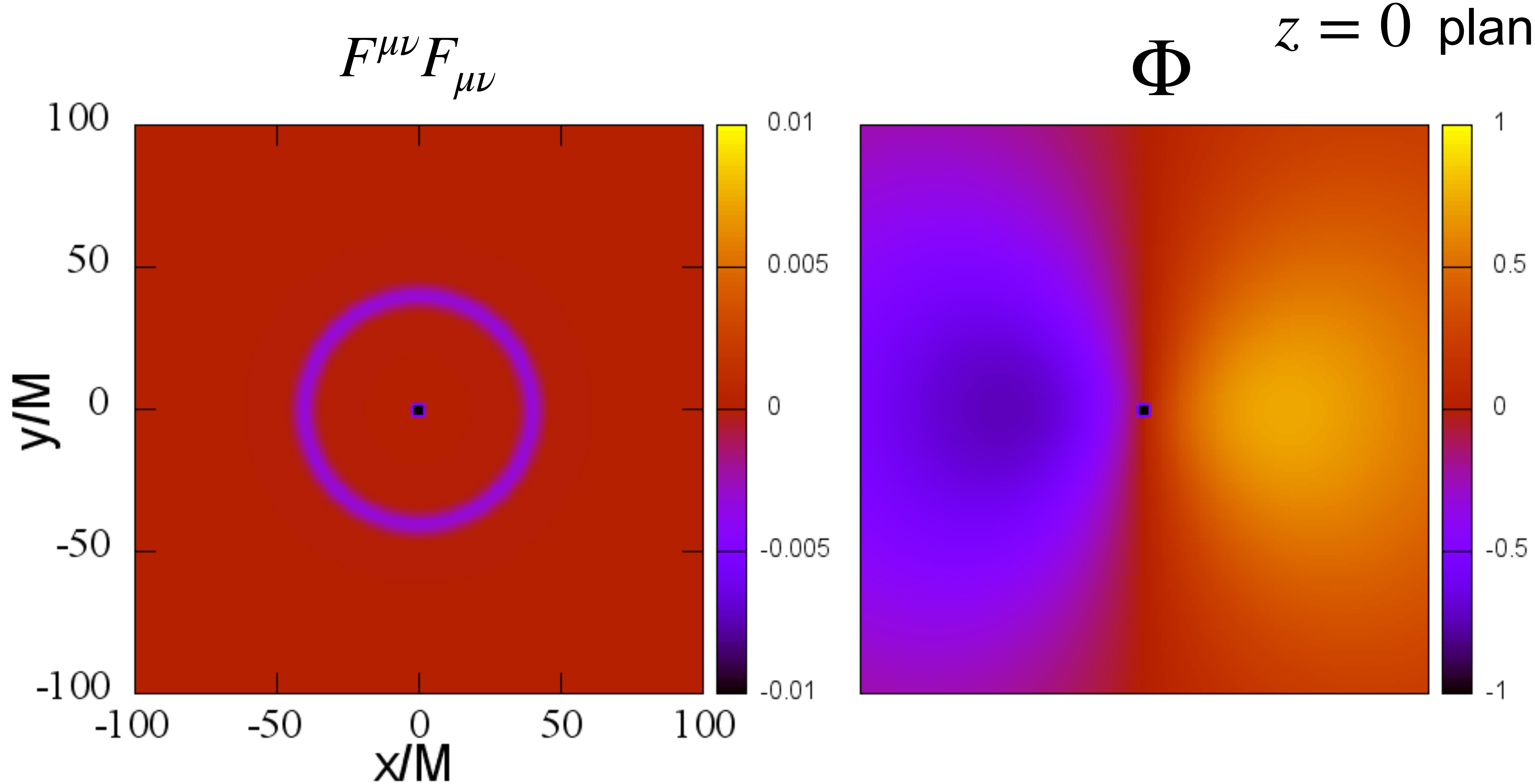
- Dissipation case (Extended initial profile)

$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \dfrac{k_\mathrm{a}}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_\mathrm{a}\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$

$$\mu M = 0.2, \quad k_\mathrm{a}A_0 = 0.1$$

t=0.000000 M

$z = 0$ plane

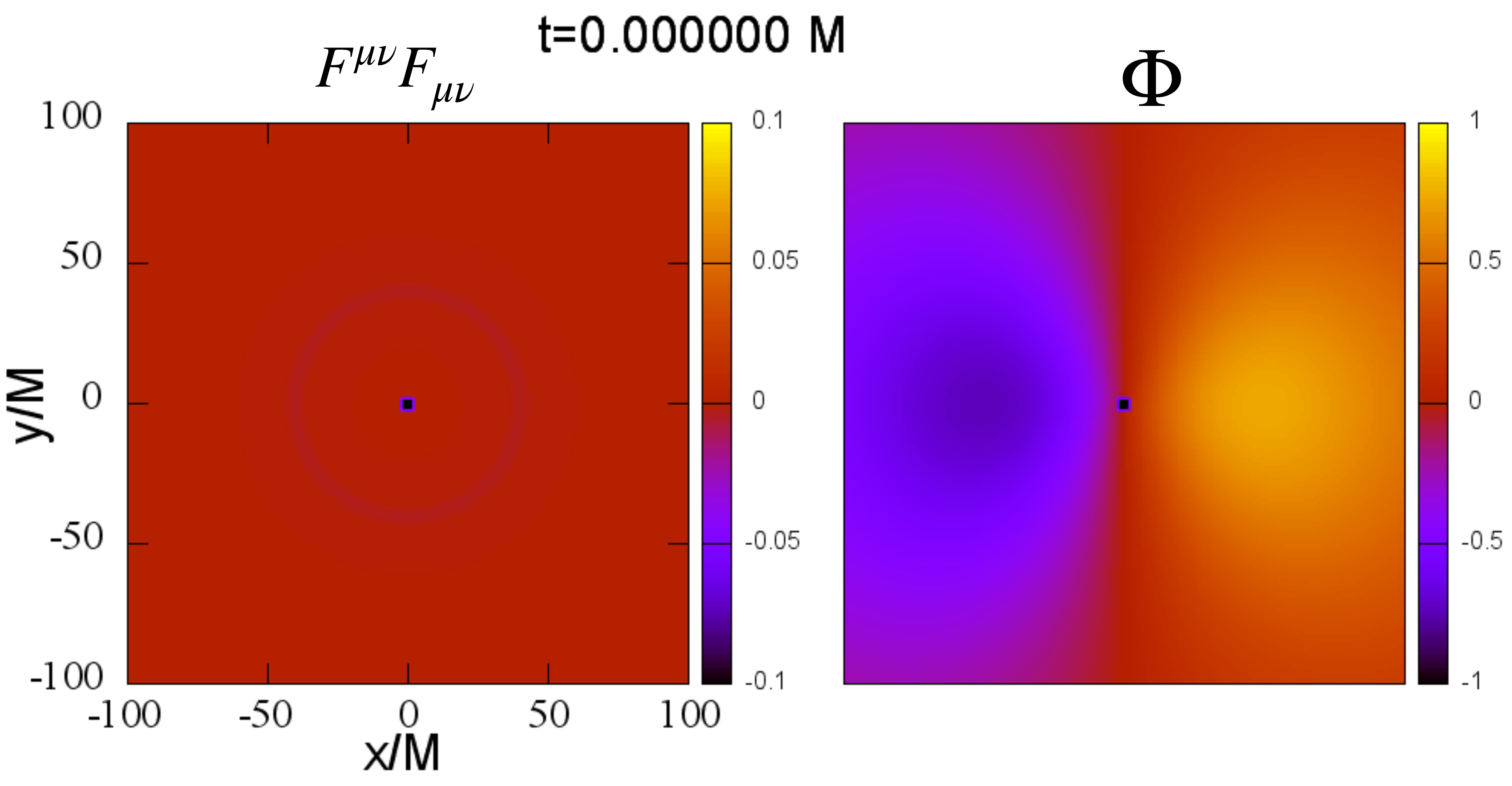


Around Kerr BH

- Burst case (Extended initial profile)

$$\mu M = 0.2, \quad k_{\rm a} A_0 = 0.3$$

$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \dfrac{k_{\rm a}}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_{\rm a}\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$

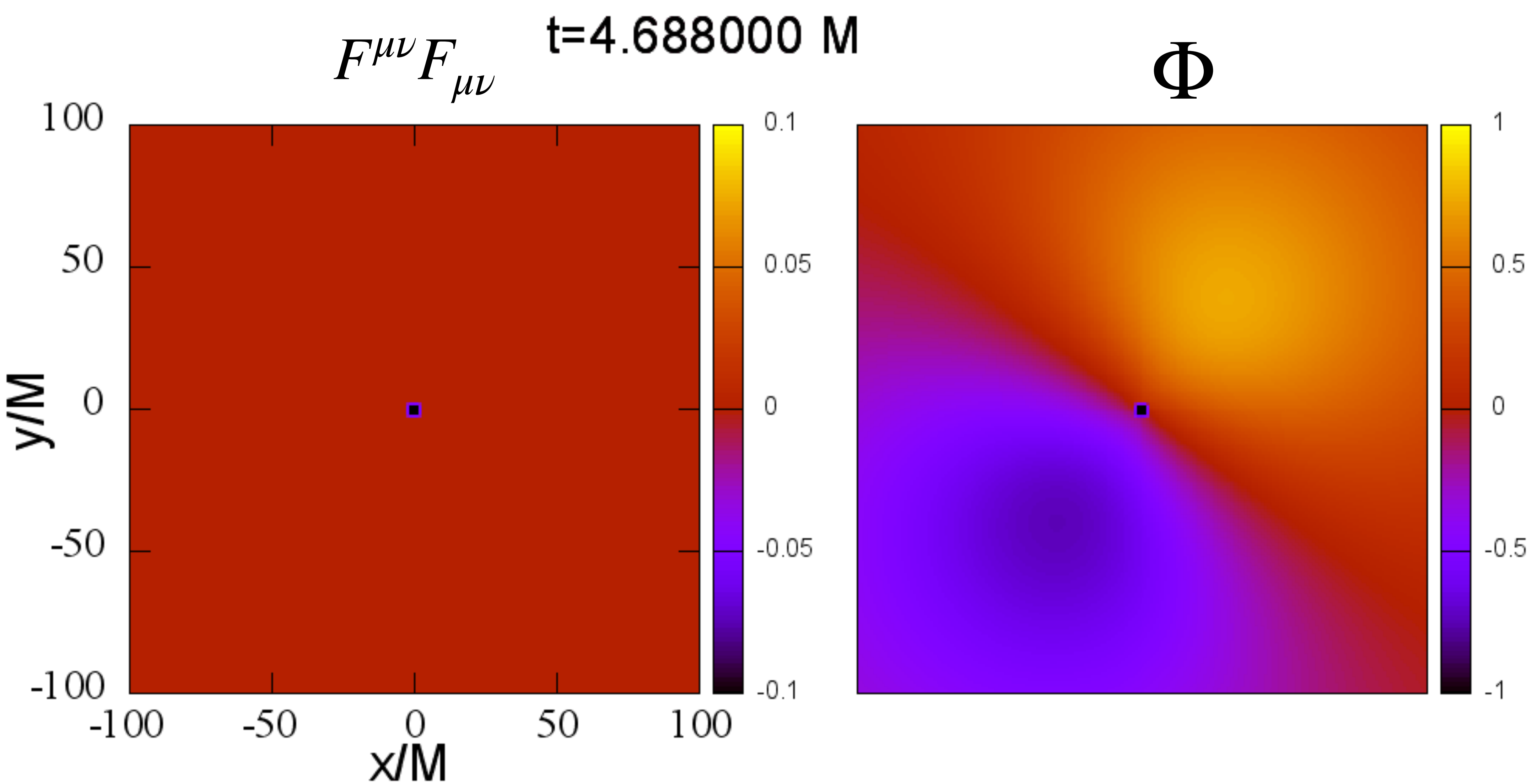


Around Kerr BH

- Burst case (Localized initial profile)

$$\mu M = 0.2, \quad k_a A_0 = 0.4$$

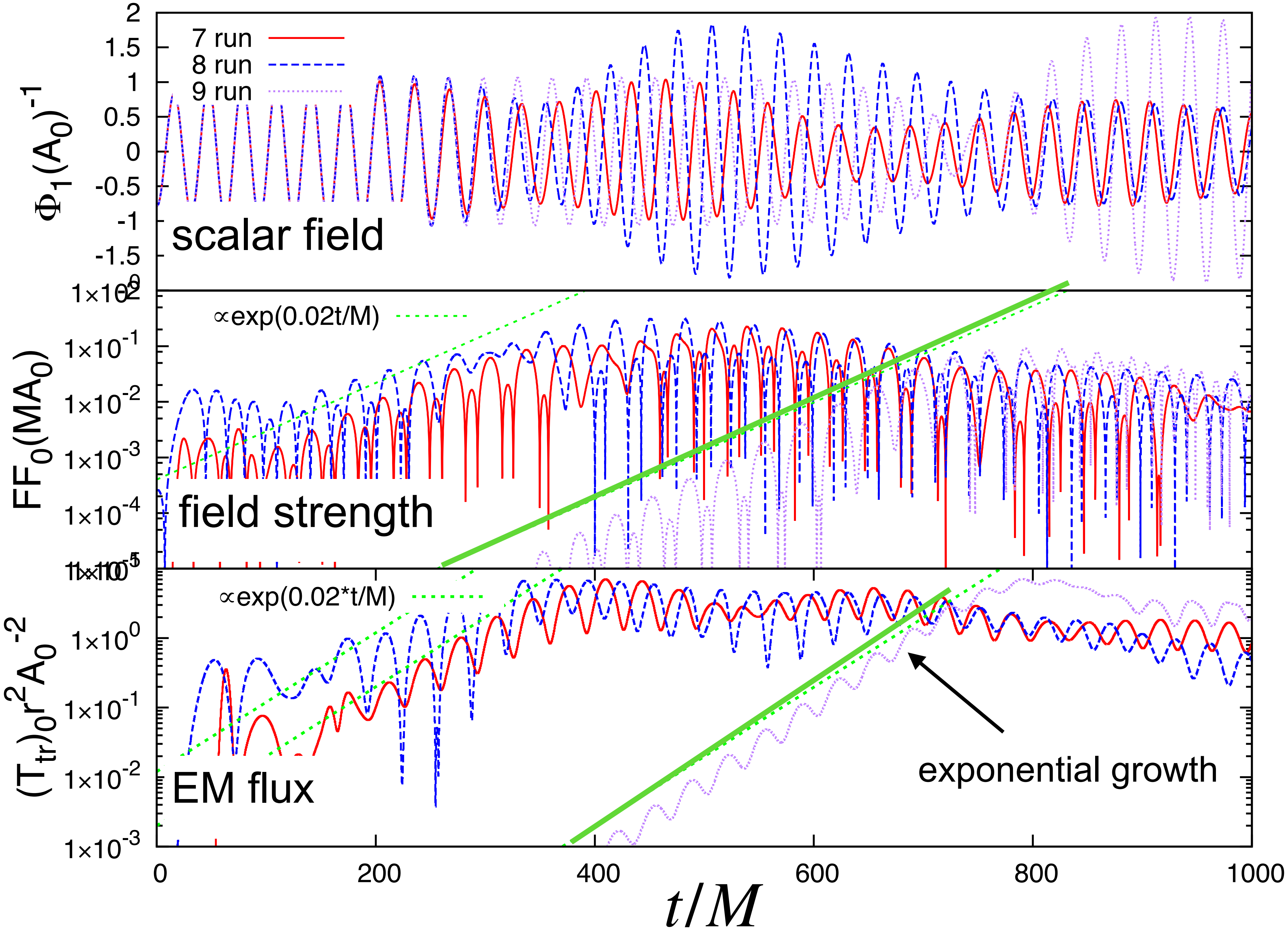
$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \dfrac{k_a}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_a \tilde{F}_{\nu\mu}\nabla^\mu \Phi \end{cases}$$



Around Kerr BH

cf: $\Phi_1 = \int d\Omega \Phi \frac{1}{2}(Y_{1,1} + Y_{1,-1})$

- Typical time evolution of the burst



Around Kerr BH

- We could checked
  - the exponential growth for several initial data for $k_{\mathrm{a}} A_0 \geq \text{threshold}$
  - typical frequency of EM field : $\omega = \mu/2$
- Scenario
  - 1.Initial EM pulse dissipates.
  - 2.EM field grows exponentially.
  - 3.Energy of EM field propagates as radiations.
  - 4.Energy of scalar field decrease, and new coupling is below the threshold.
- We get
  - luminosity formula: $\dfrac{dE}{dt} = 5.0 \times 10^{-6} \left(\dfrac{M_{\mathrm{S}}}{M} \right) \dfrac{c^5}{G}$
  - threshold for the burst : $\dfrac{\sqrt{\hbar}}{k_{\mathrm{a}}} < 6 \times 10^{18} \left(\dfrac{M_{\mathrm{S}}}{M} \right)^{1/2} (\mu M)^2 \ \mathrm{GeV}$

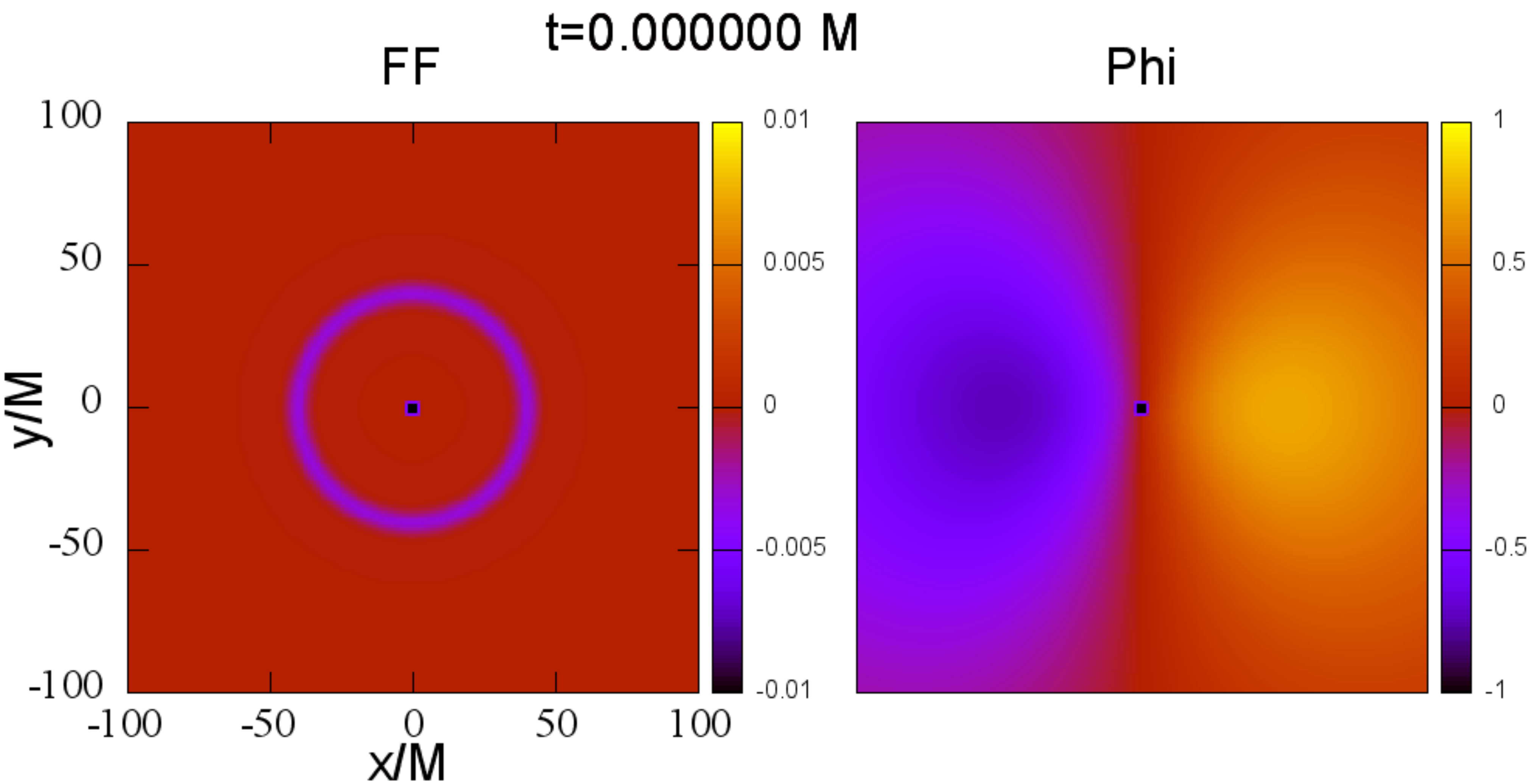
$\begin{cases} M & \text{: BH mass} \\ M_{\mathrm{S}} & \text{: total mass of axion cloud} \end{cases}$

Supper-radiance effect

- The burst is induced by super-radiance effect.
  - We add term to scalar field eq. which induces “super-radiant” like effect.

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\Phi + \underline{C\frac{\partial\Phi}{\partial t}} = \mu^2\Phi + \frac{k_a}{2}\tilde{F}_{\mu\nu}F^{\mu\nu}$$

“super-radiance” time scale $\sim 1/C$



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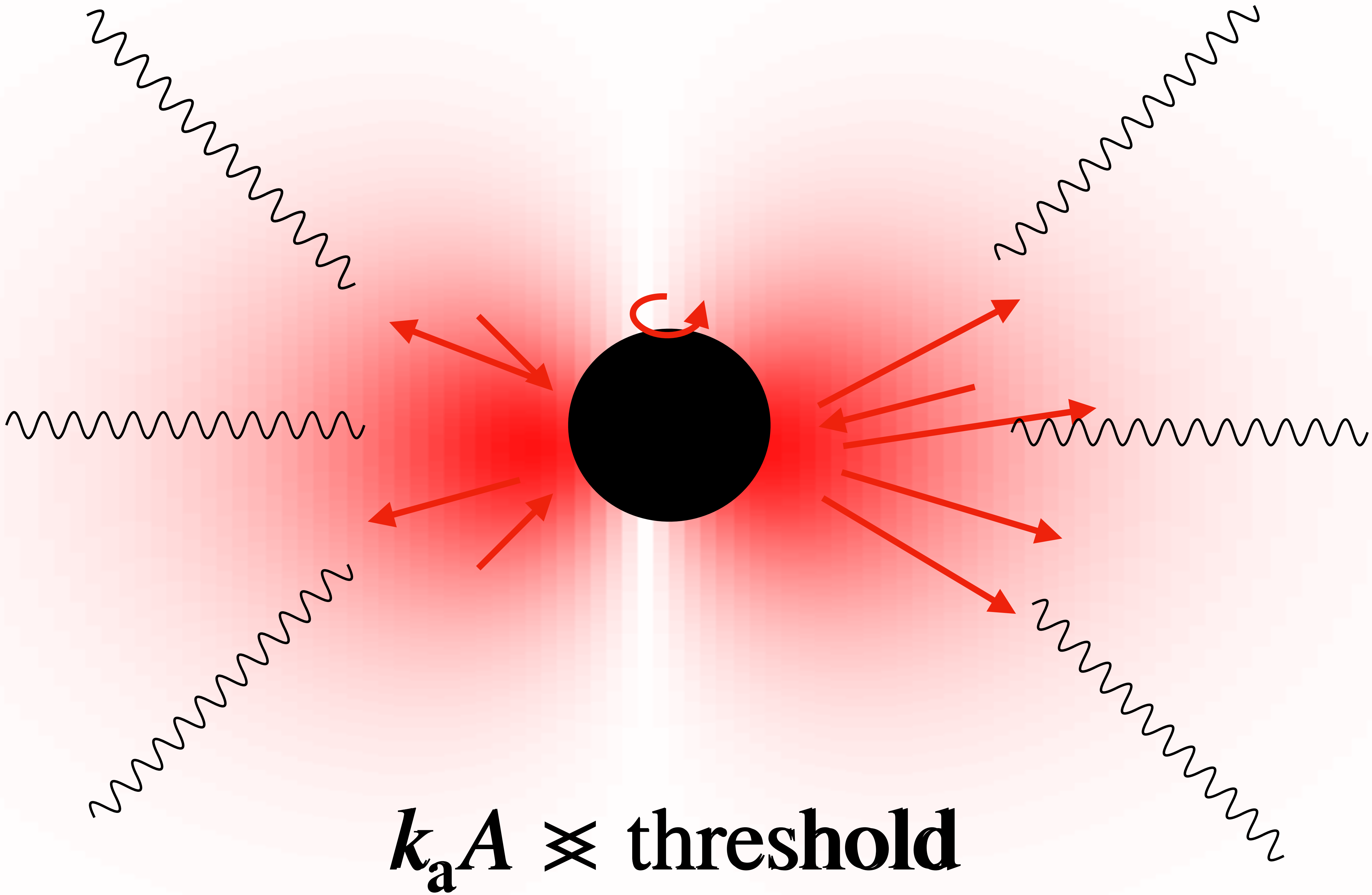
$k_a A \gtrless$ threshold

Summary

- Result
  - Energy of axion cloud transfers to EM field
  - EM field grows exponentially, and burst of EM field occurs.
  - Luminosity of burst

$$\left\{\begin{array}{ll} M & \text{: BH mass} \\ M_{\mathrm{S}} & \text{: total mass of axion cloud} \end{array}\right.$$

$$\frac{dE}{dt} = 5.0 \times 10^{-6} \left(\frac{M_{\mathrm{S}}}{M}\right) \frac{c^5}{G}$$

  - Threshold for axion couplings

$$\frac{\sqrt{\hbar}}{k_{\mathrm{a}}} < 6 \times 10^{18} \left(\frac{M_{\mathrm{S}}}{M}\right)^{1/2} (\mu M)^2 \ \mathrm{GeV}$$

  - The burst is induced by “super-radiance” instability.
  - We also get similar result in the case of scalar type coupling.

cf : $\mathscr{L}_{\mathrm{s}} = -\frac{(k_{\mathrm{s}}\Phi)^p}{4} F^{\mu\nu}F_{\mu\nu} = -\frac{(k_{\mathrm{s}}\Phi)^p}{2}\left(\vec{B}^2 - \vec{E}^2\right)$

Thank you.