# Tidal effects on axion cloud (numerical simulation)

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**with Vitor Cardoso, Francisco Duque**


# Outline

1. Introduction
2. Perturbation theory (D.Baumann et al PRD99,044001)
   - Mode mixing
   - Resonance & cloud depletion
3. Time evolution (Our result)
   - Our strategy
   - Excitation of higher multipole mode
   - Tidal disruption
4. Summary

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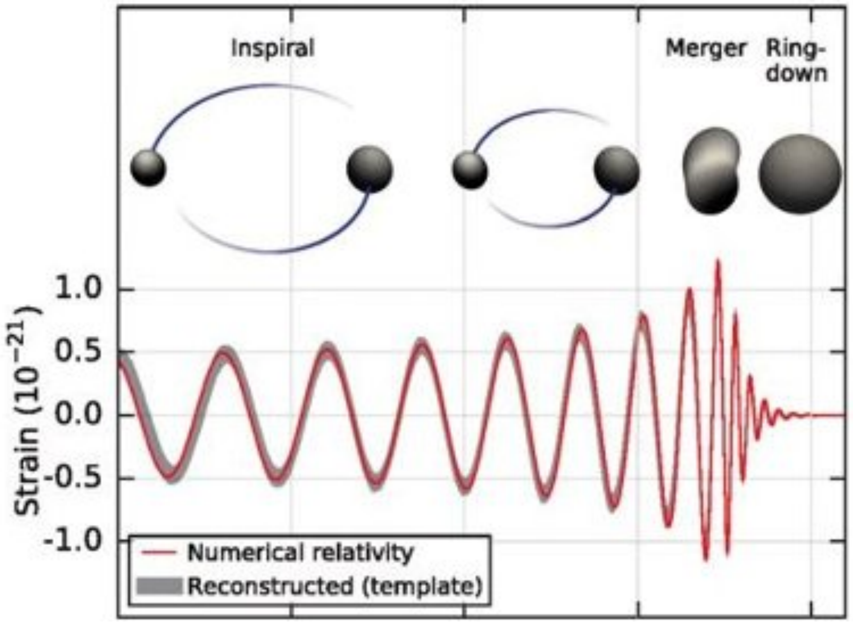
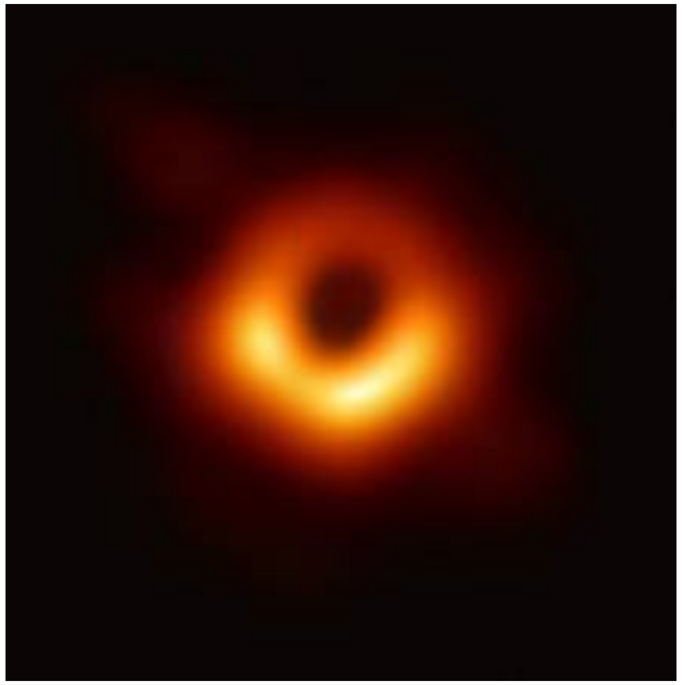
# Black Hole Physics

- Gravitational wave
- BH shadow
- Test beds of Gravity theory
  - No-hair theorem
- Prove of early Universe
  - Primordial BH
  - Inflation
- Theoretical features
  - Hawking area law
  - BH entropy
- “Particle detector”



# Black Hole as a particle detector

- There may be a lot of massive particles in our Universe.
  - QCD axion

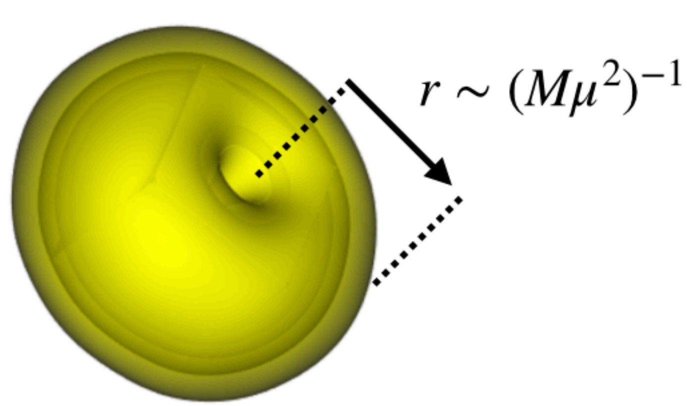
$$\mathscr{L}_{\rm QCD} = \mathscr{L}_{\rm quark} + \mathscr{L}_{\rm SU(3)} + \underline{\tilde{\Theta}\frac{g^2}{32\pi^2}G^{a\mu\nu}\tilde{G}_{a\mu\nu}}$$

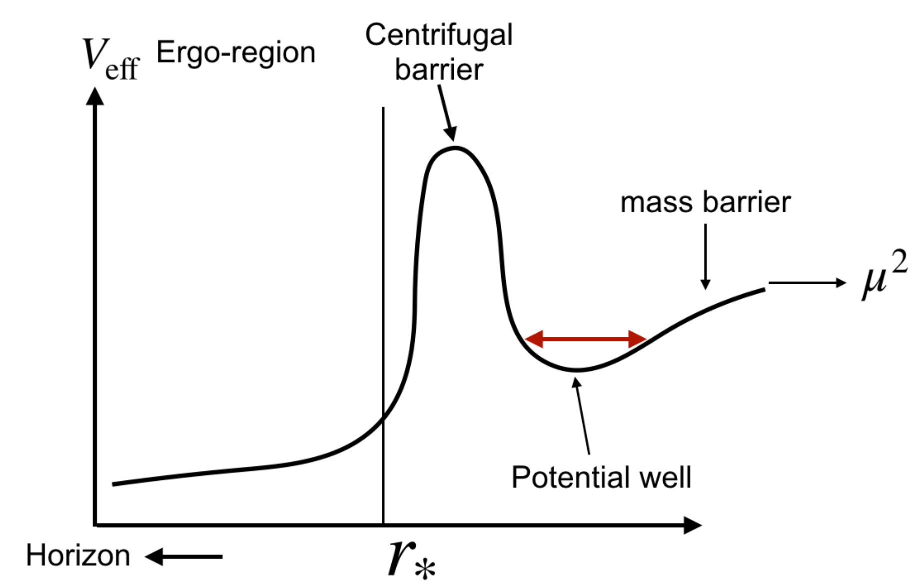
$|\tilde{\Theta}| < 10^{-9} \longrightarrow$ PQ mechanism

  - String axion
  - Modified gravity theory

cf: $\mu \sim 10^{-10}\mathrm{eV} \longleftrightarrow M \sim M_{\odot}$

- Massive field can be localized around BH



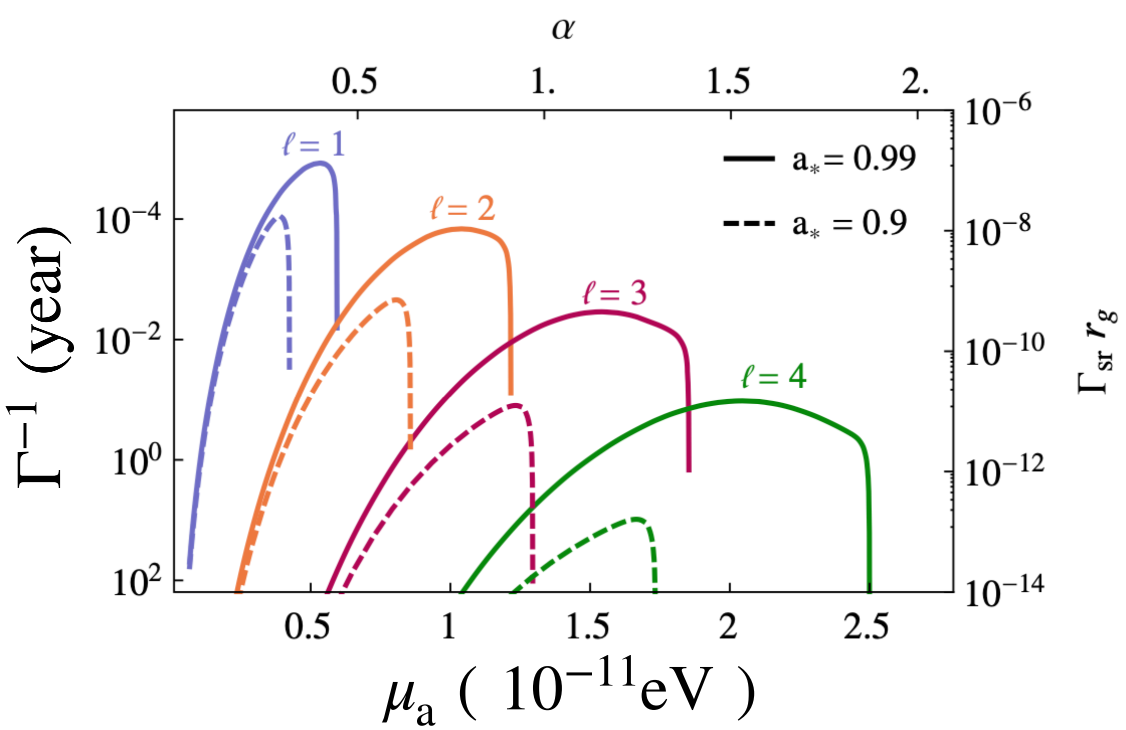



# Black Hole as a particle detector

- Supper-radiance
  - $\Phi(x) = e^{-\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r)$
  - condition

$$\mathrm{Re}(\omega) < m\Omega_{\mathrm{H}} = \frac{ma}{2Mr_+}$$

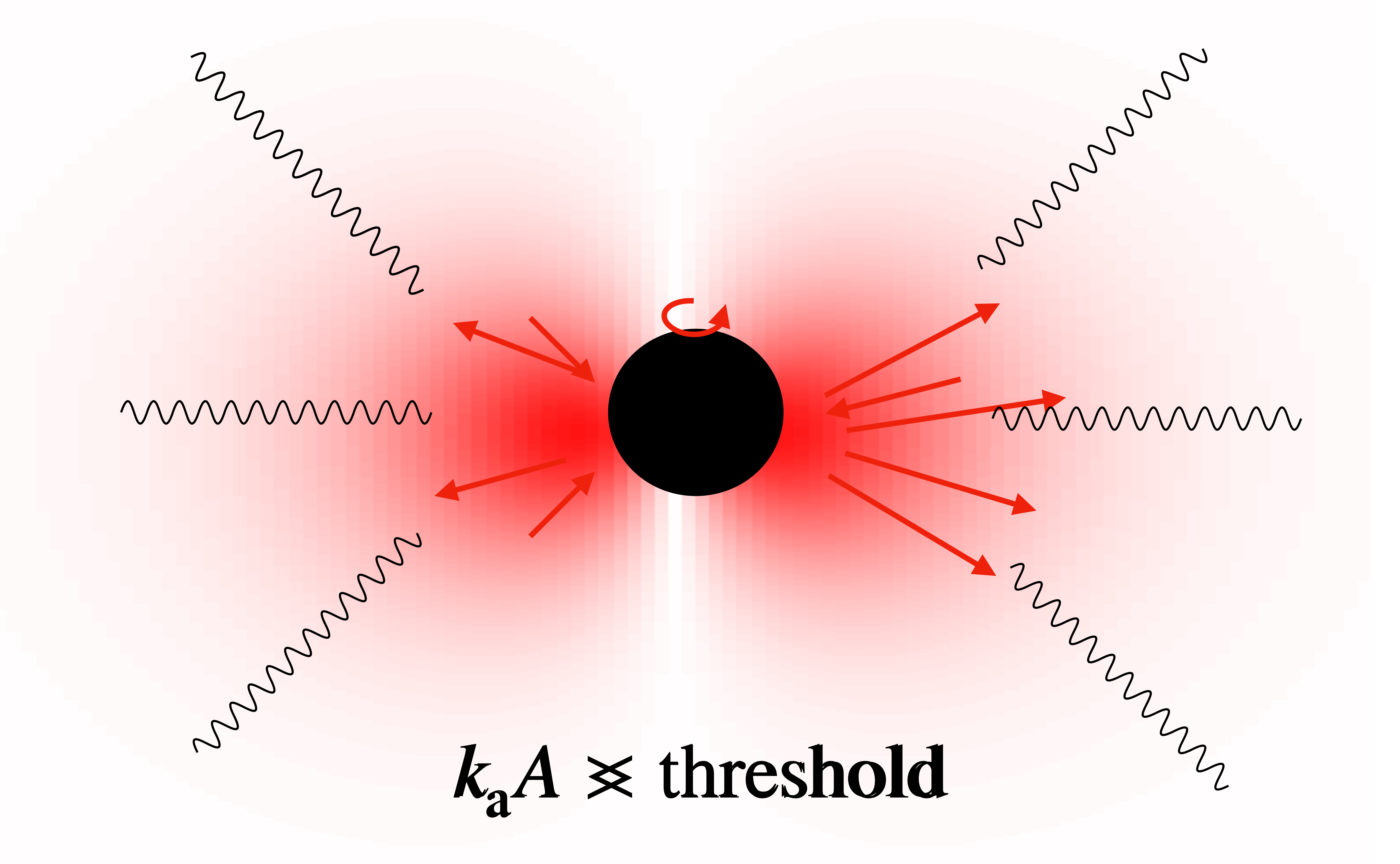
$$\Rightarrow \mathrm{Im}(\omega) > 0$$




Arvanitaki et.al(2015)

- The energy of axion cloud can leak to other fields.
  - Gravitational wave
  - Photon emission (Ikeda et al. (2019))

$$\mathscr{L}_{\Phi\gamma\gamma} = -\frac{1}{2} k_a \tilde{F}_{\mu\nu} F^{\mu\nu} \Phi = -2k_a \overrightarrow{E} \cdot \overrightarrow{B} \Phi \qquad k_a = \frac{\alpha C}{2\pi F_a}$$

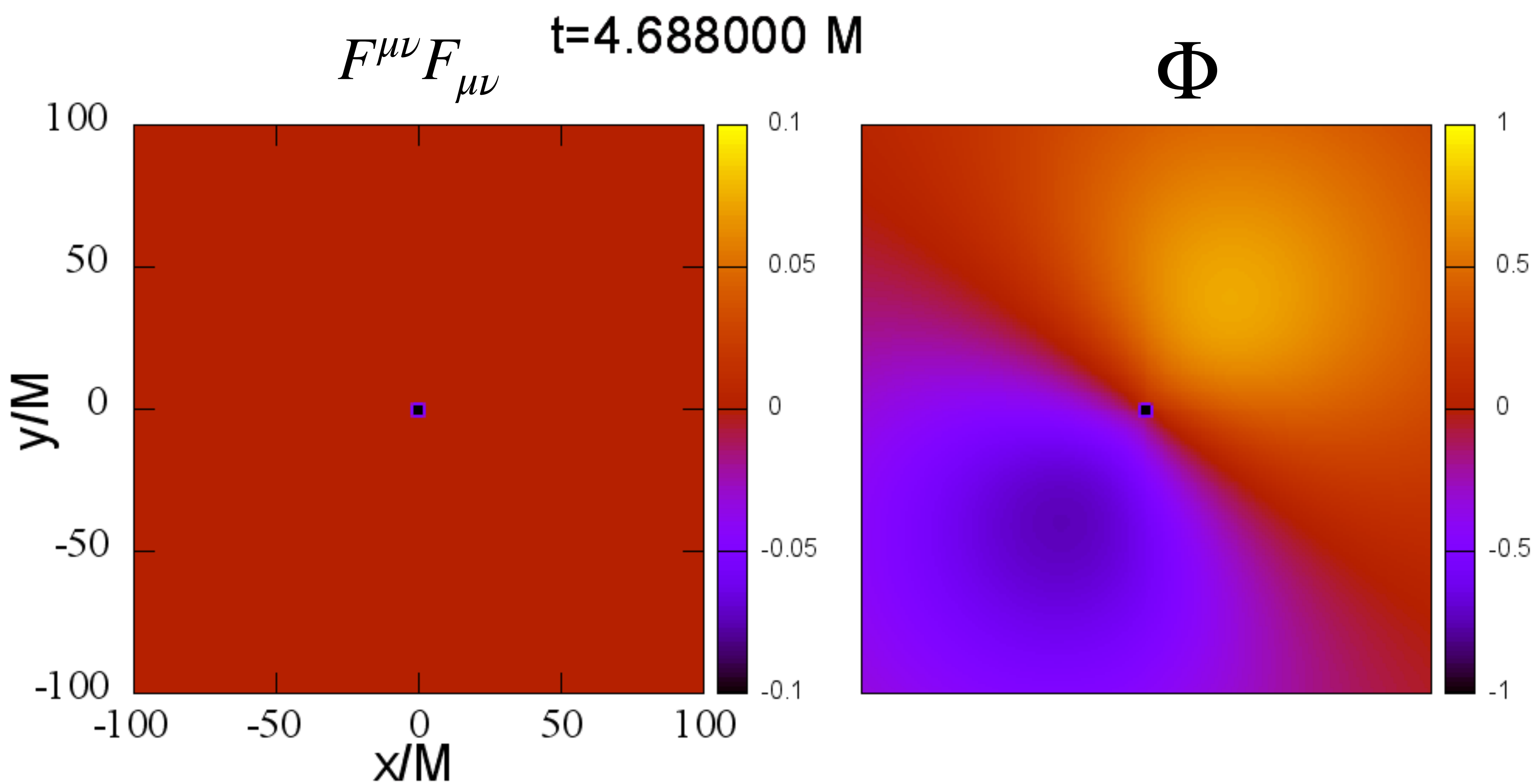

$k_a A \gtrless$ threshold

# Black Hole as a particle detector

- Burst case (Localized initial profile)

$$\mu M = 0.2, \quad k_{\mathrm{a}} A_0 = 0.4$$

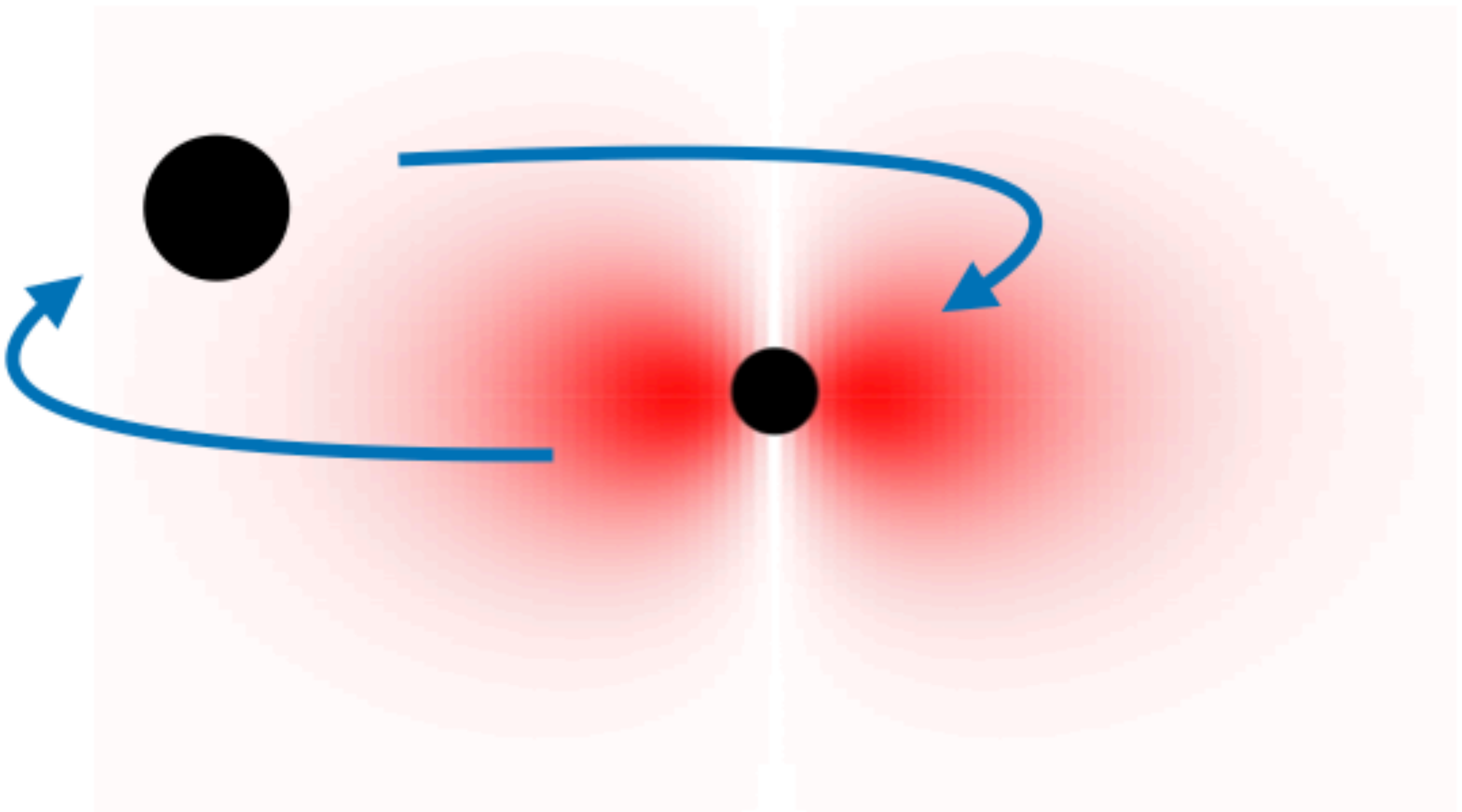
$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \dfrac{k_{\mathrm{a}}}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_{\mathrm{a}}\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$



# Our Questions

- In these story, the dynamic of axion cloud is important.
- There are a lot of binary BH in our Universe.
- Does history of axion cloud change around binary BH ?

⟶ We investigate the tidal effects on the cloud.



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# Tidal effect on the cloud

$V(r) = \frac{\alpha}{r}$

- Mode mixing (D.Baumann et al PRD99,044001)
  - single BH

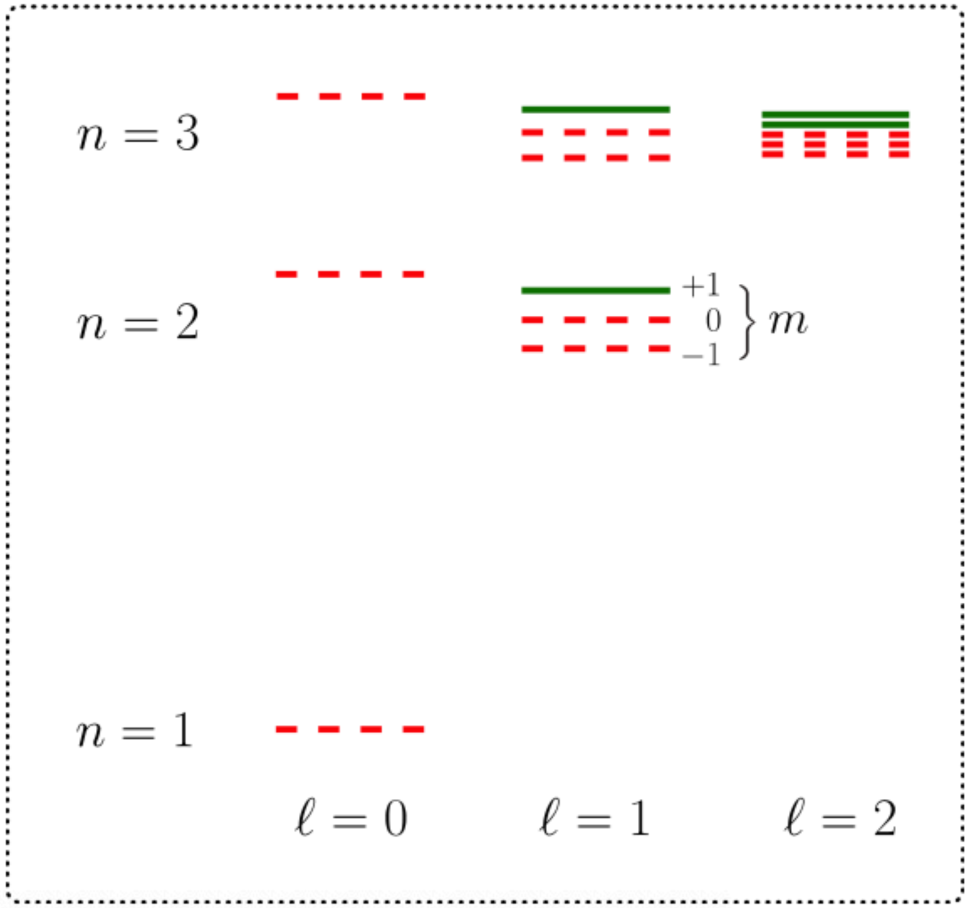
    non-relativistic limit

    - $(\Box - \mu^2)\Phi = 0$ ➡ $i\partial_t \Psi = \left(-\frac{1}{2\mu^2}\nabla^2 + \underline{V(r)}\right)\Psi$

      ➡ $|n, l, m>$ $\omega_{n,l,m}$ cf : QM of Hydrogen atom

    - higher order correction

      $$\Delta\omega_{nlm} = \mu\left(-\frac{\alpha^4}{8n^4} + \frac{(2l - 3n +}{n^4(l + 1/}\right.$$




    - decay width $\Gamma_{nlm} \propto m\Omega_H - \omega$
      - decaying mode $\Gamma^{(d)}_{nlm} > 0$
      - growing mode $\Gamma^{(g)}_{nlm} < 0$

# Tidal effect on the cloud

$$i\partial_t \Psi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r)\right)\Psi$$

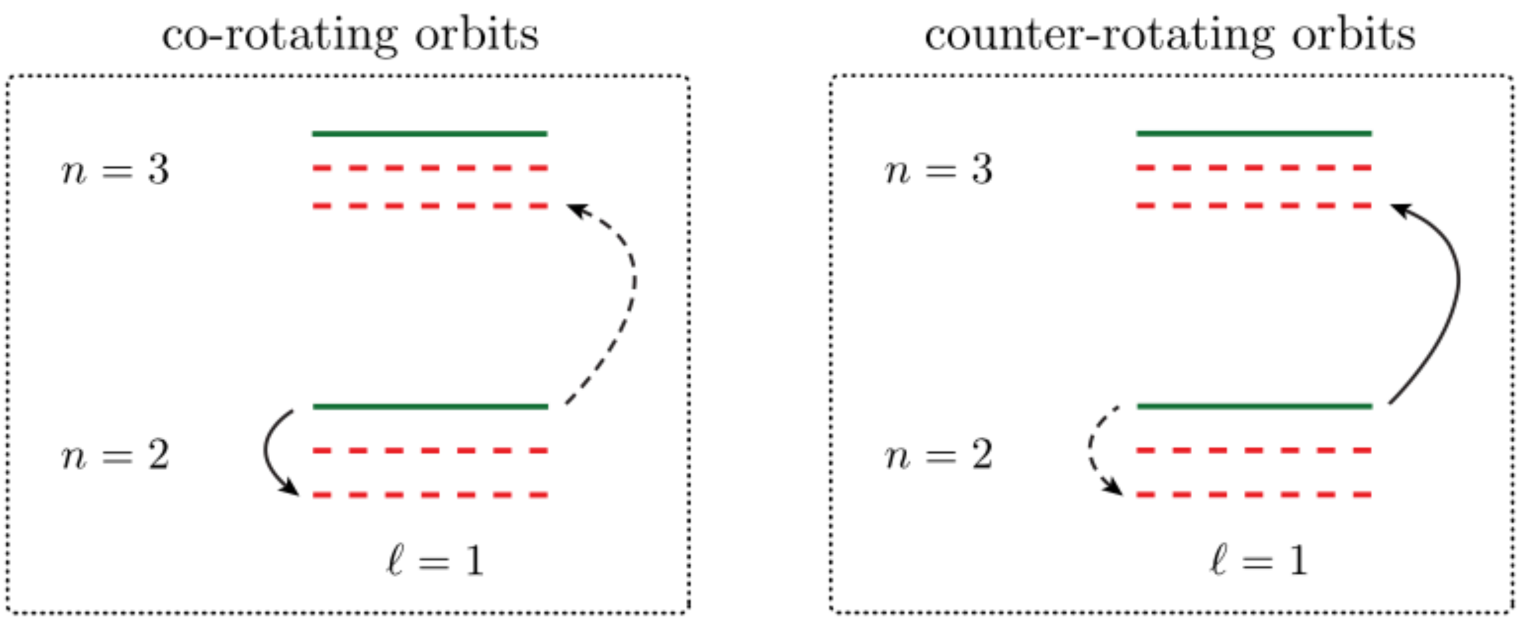
- Binary BH
  - The tidal effect deforms the potential.

$$V(r) \to V(r) + \underline{\delta V(t, r, \theta, \phi)}$$

cf : Perturbation theory in QM

  - mode mixing

$$< n, l, m \,|\, \delta V \,|\, n', l', m' > \neq 0$$




growing mode

decaying mode

# Tidal effect on the cloud

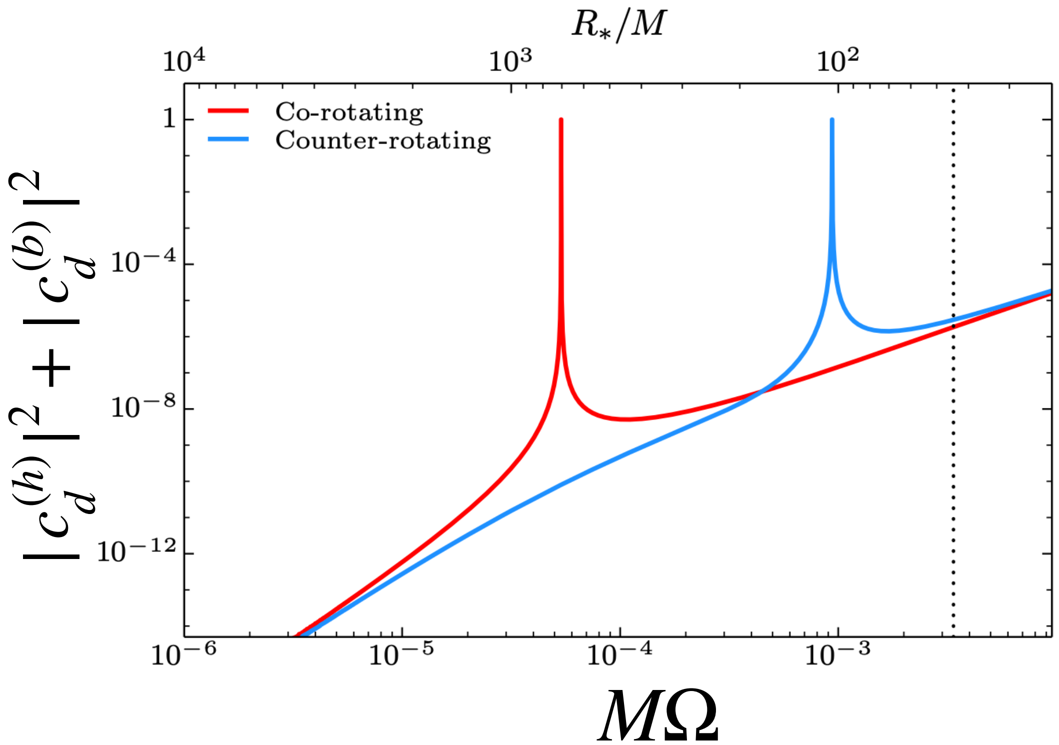
- resonance
  - $\Omega \sim \epsilon_h = \dfrac{\mu\tilde{a}\alpha^5}{12}$

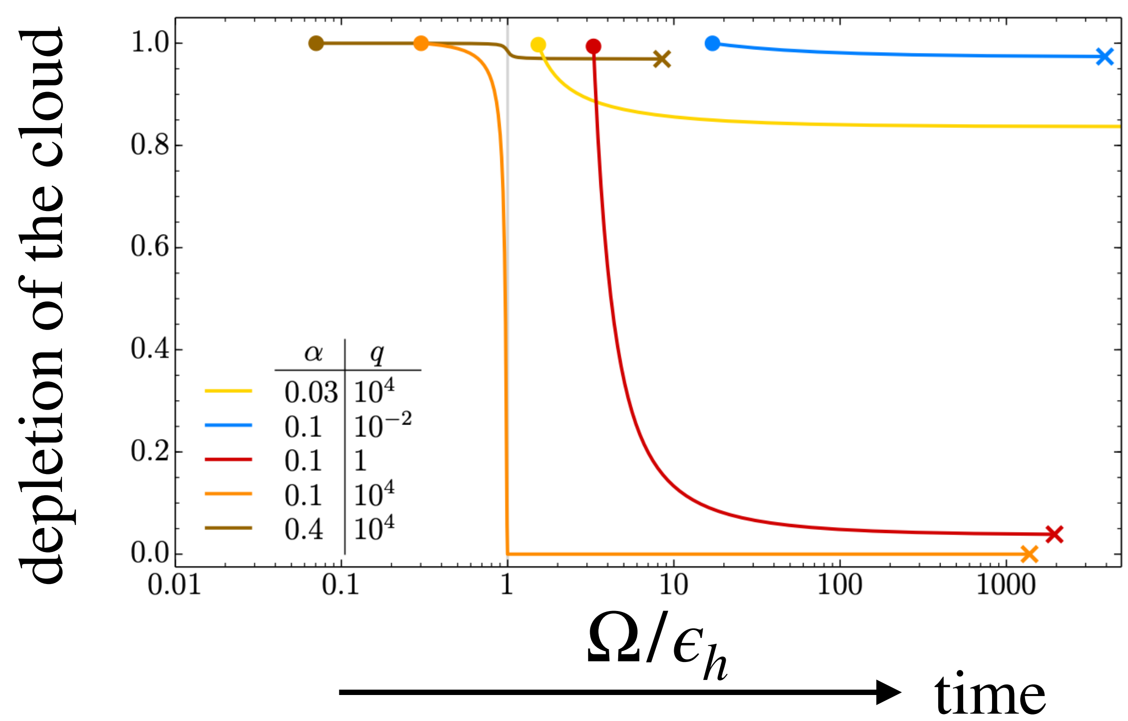
$$\Rightarrow \begin{cases} |\Psi_g> = |2,1,1> \\ |\Psi_d^{(h)}> = |2,1,-1> \end{cases}$$

  - $-\Omega \sim \epsilon_b = -\dfrac{5}{144}\mu\alpha^2$

$$\Rightarrow \begin{cases} |\Psi_g> = |2,1,1> \\ |\Psi_d^{(b)}> = |3,1,-1> \end{cases}$$

- Cloud depletion



# What we want to do

- We will solve the tine evolution of axion cloud under the tidal force.
- And, compare with perturbation theory.

Perturbation theory $\Longleftrightarrow$ Time evolution

(D.Baumann et al PRD99,044001)

$$(\Box - \mu^2)\Phi = 0$$

$$i\partial_t \Psi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r)\right)\Psi$$

$$V(r) \to V(r) + \delta V(t, r, \theta, \phi)$$

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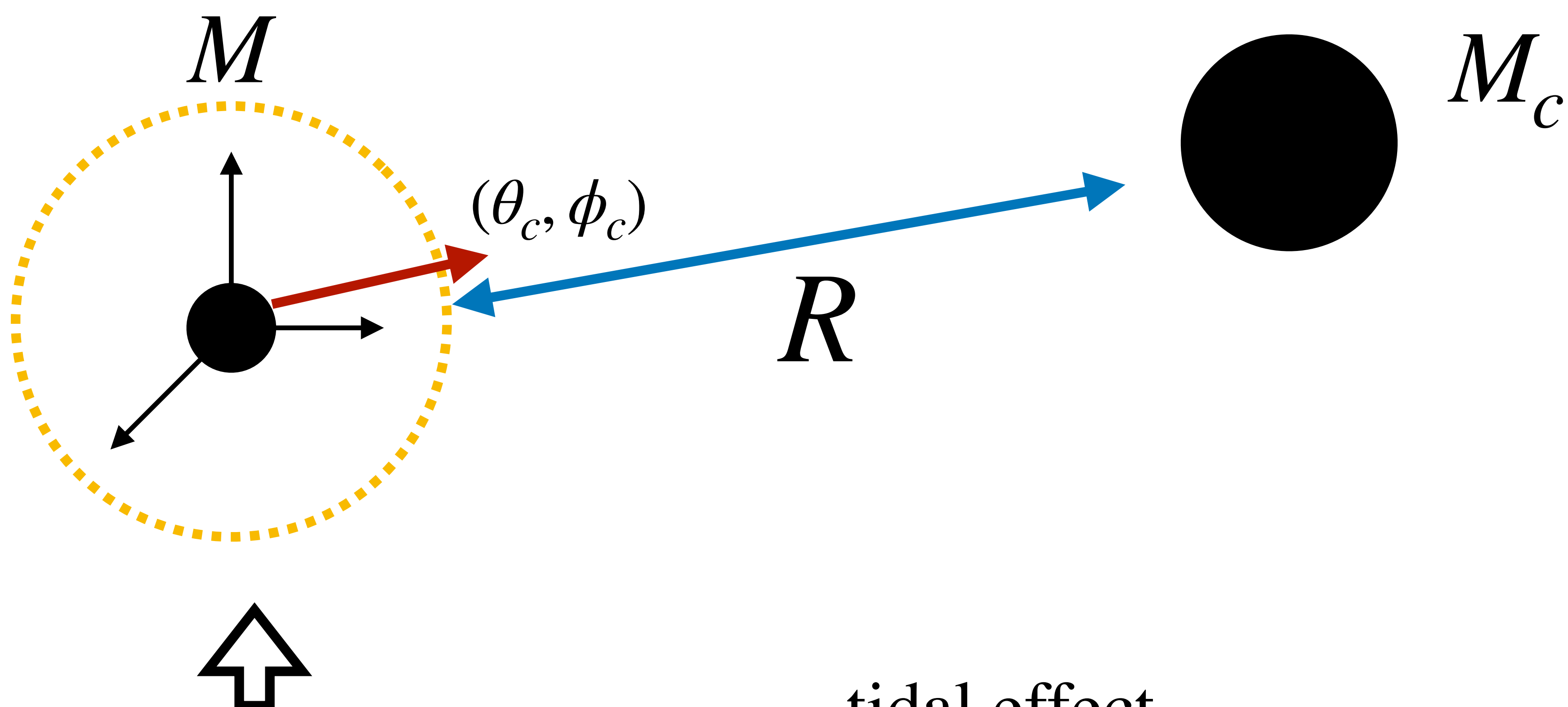
# Tidally deformed BH

- How to add tidal effects ?




tidal effect

$$ds^2 = ds^2_{\rm BH} + \sum_m \left(\frac{r}{M}\right)^2 \frac{8\pi M_c M^2}{\underline{5R^3}} Y^*_{2m}(\theta_c, \phi_c) Y_{2m}(\theta, \phi) \left(f^2 dt^2 + dr^2 + (r^2 - 2M^2) d^2\Omega\right) + \cdots$$

with Regge Wheeler gauge

$\dfrac{M_c M^2}{R^3}$ : the strength of tidal force

$$f = 1 - \frac{2M}{r}$$

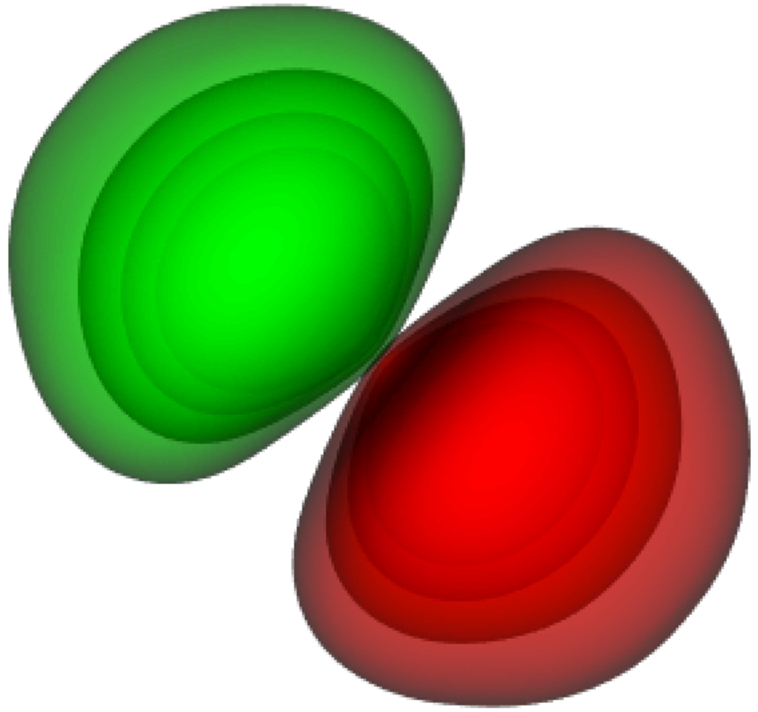
# Numerical simulation

- Initial condition : Axion cloud

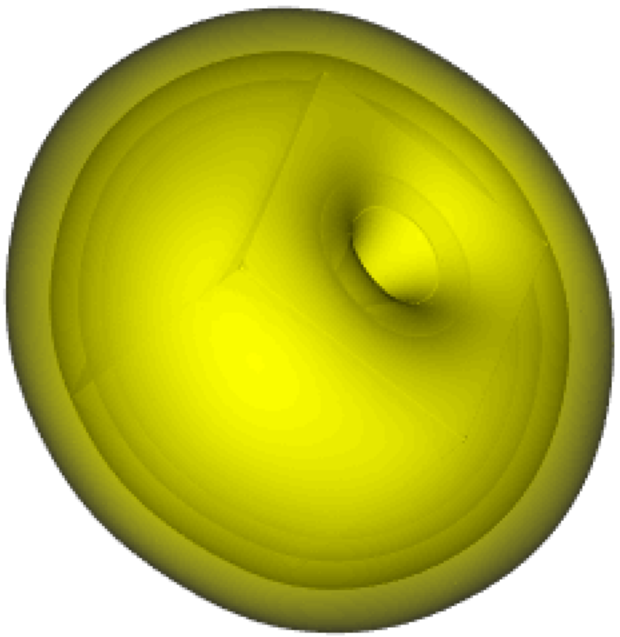
$$\Phi = A_0 r M \mu^2 e^{-rM\mu^2/2} \cos(\phi - \omega_{\mathrm{R}} t) \sin\theta \qquad \omega_R \sim \mu$$

$$n = 2, l = 1, m = \pm 1$$

- We focus on static tidal for simplicity.
- Parameters
  - axion mass : $\mu M = 0.1,\ 0.2$
  - tidal force : $\dfrac{M_c M^2}{R^3}$
- Numerical code
  - 4th order RK
  - MPI-Open MP
  - fixed mesh refinement et. al.

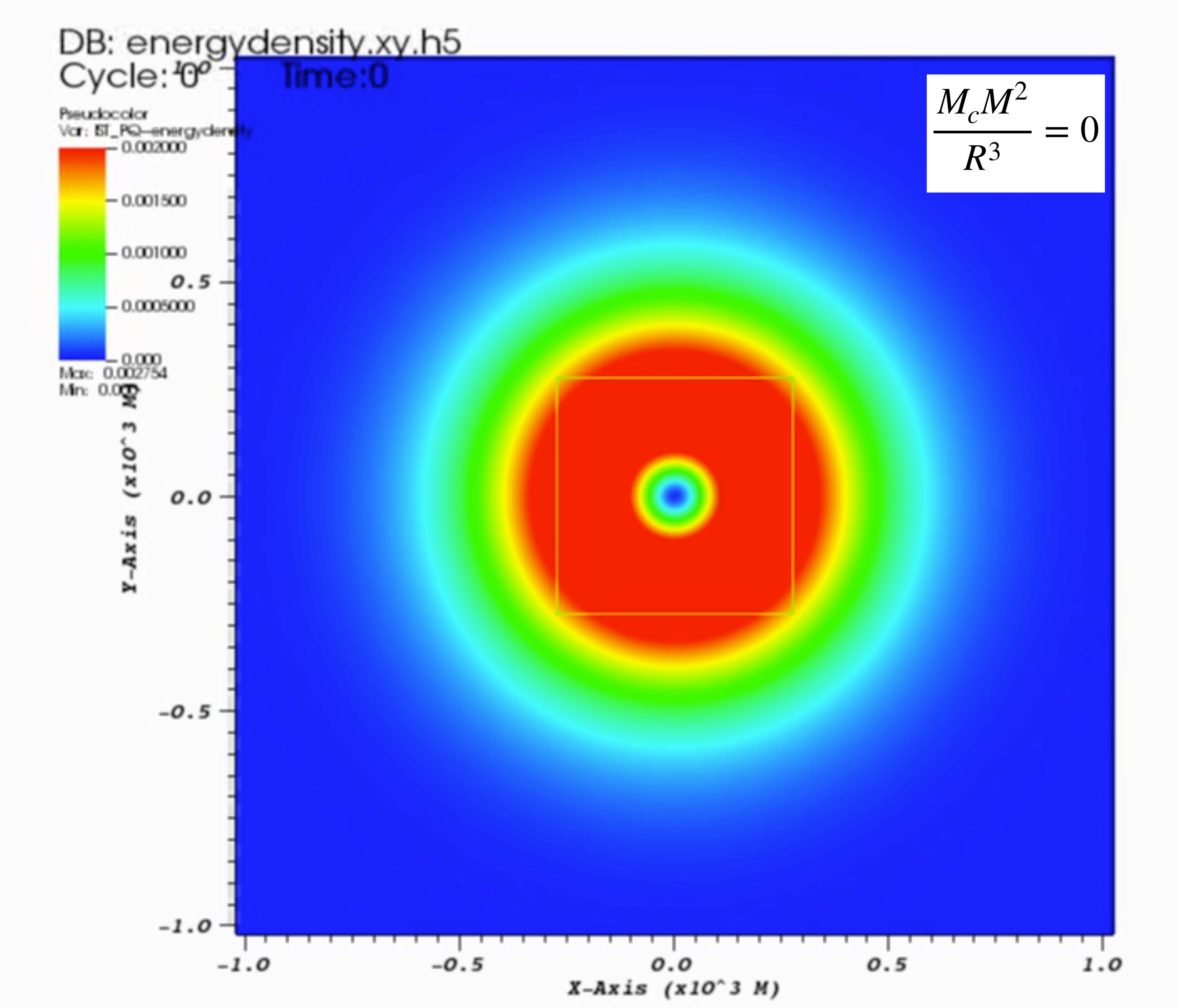


$$\frac{M_c M^2}{R^3} = 0$$



# Simulation 1 : No tidal case

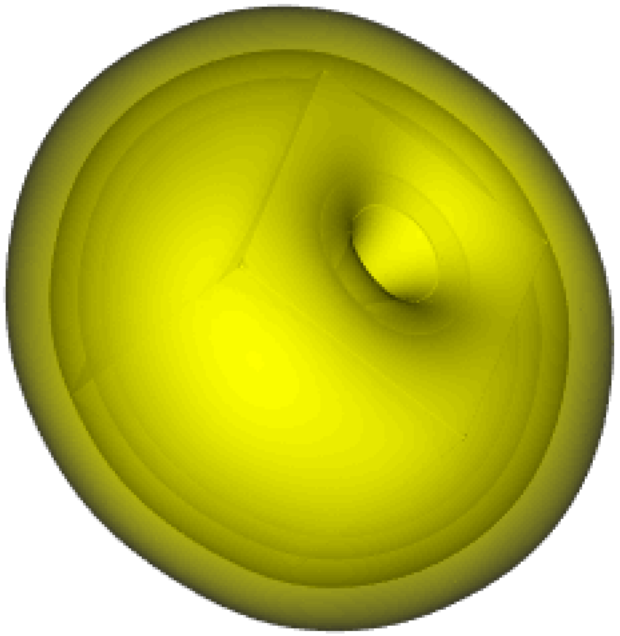
# No tidal case

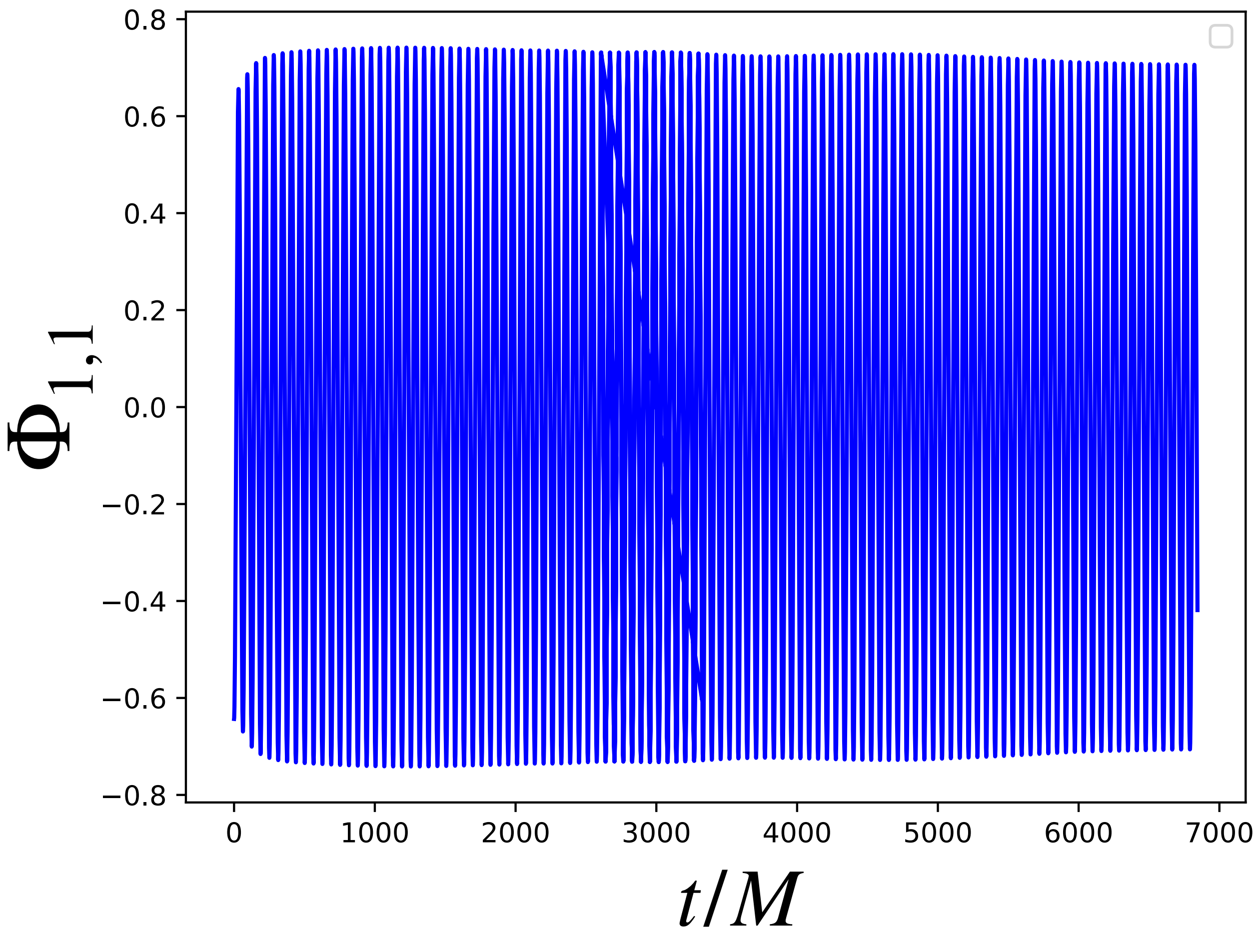



$$\frac{M_c M^2}{R^3} = 0$$

# No tidal case

- Scalar field is “almost” stationary in numerical time scale.

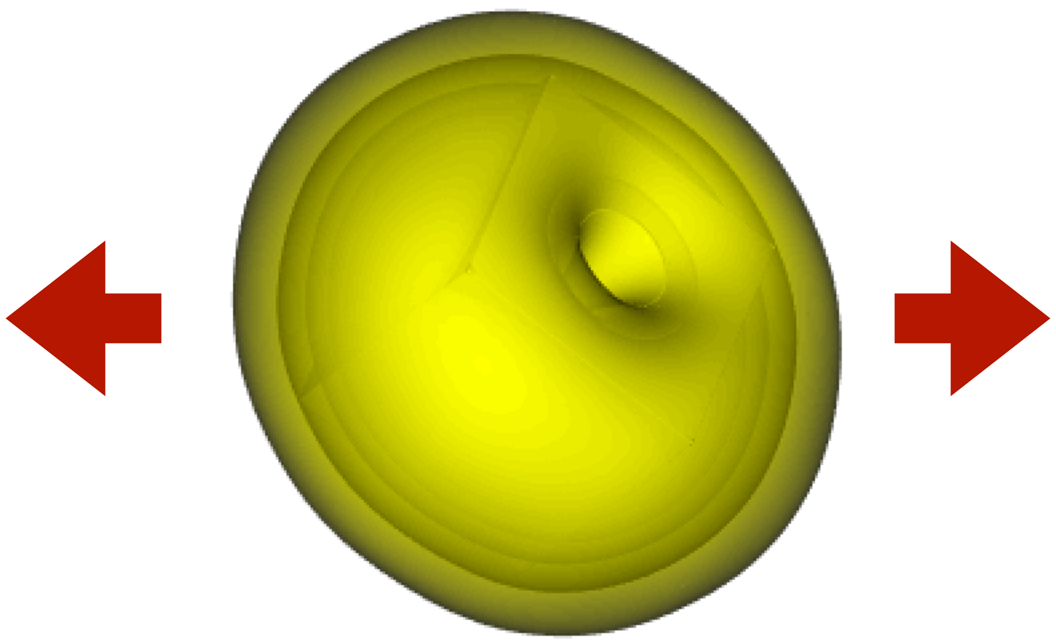


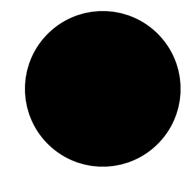


$$\frac{M_c M^2}{R^3} = 10^{-8}$$

cf: $R = 10^4 M$

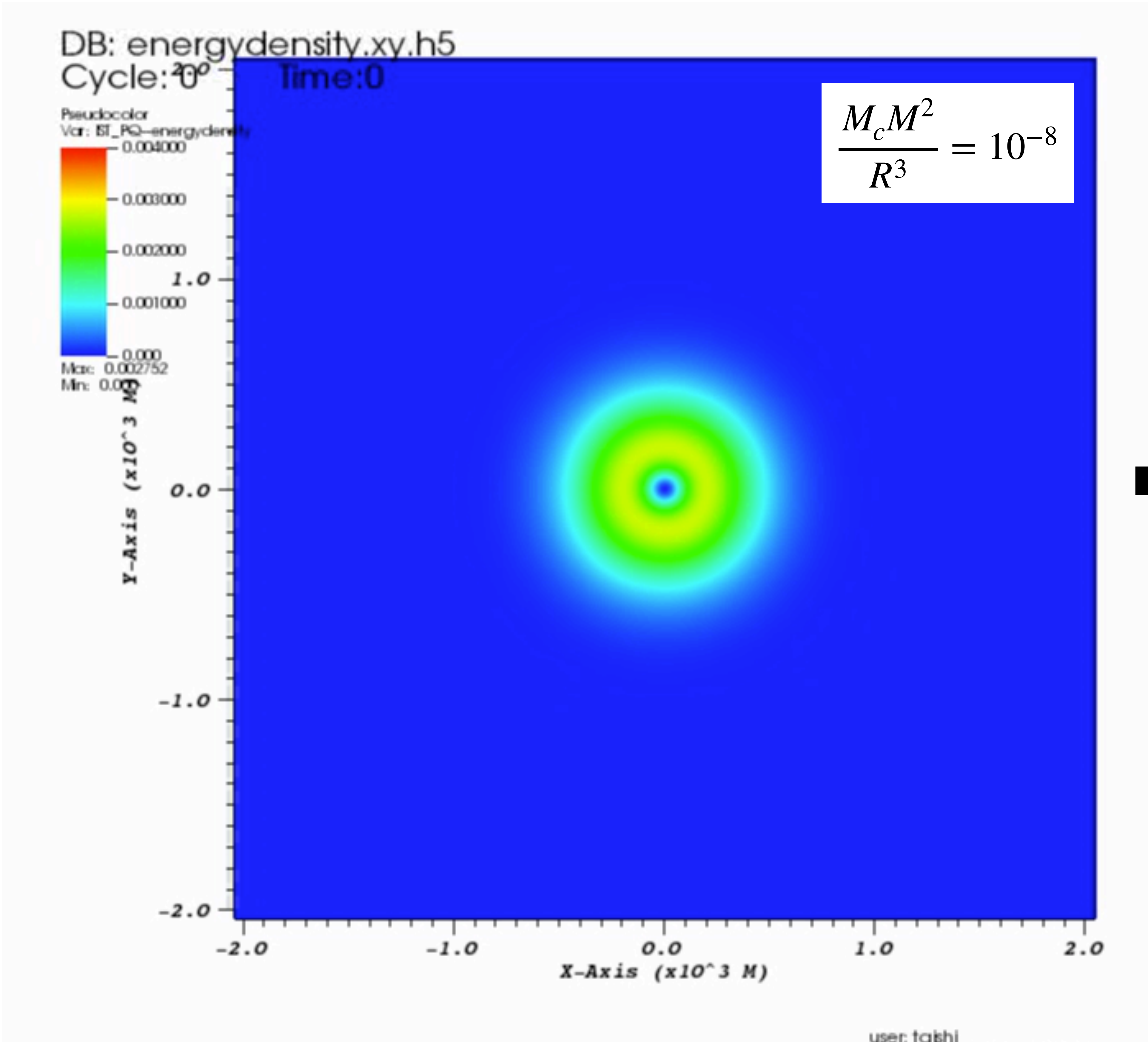
$M_c = 10^4 M$





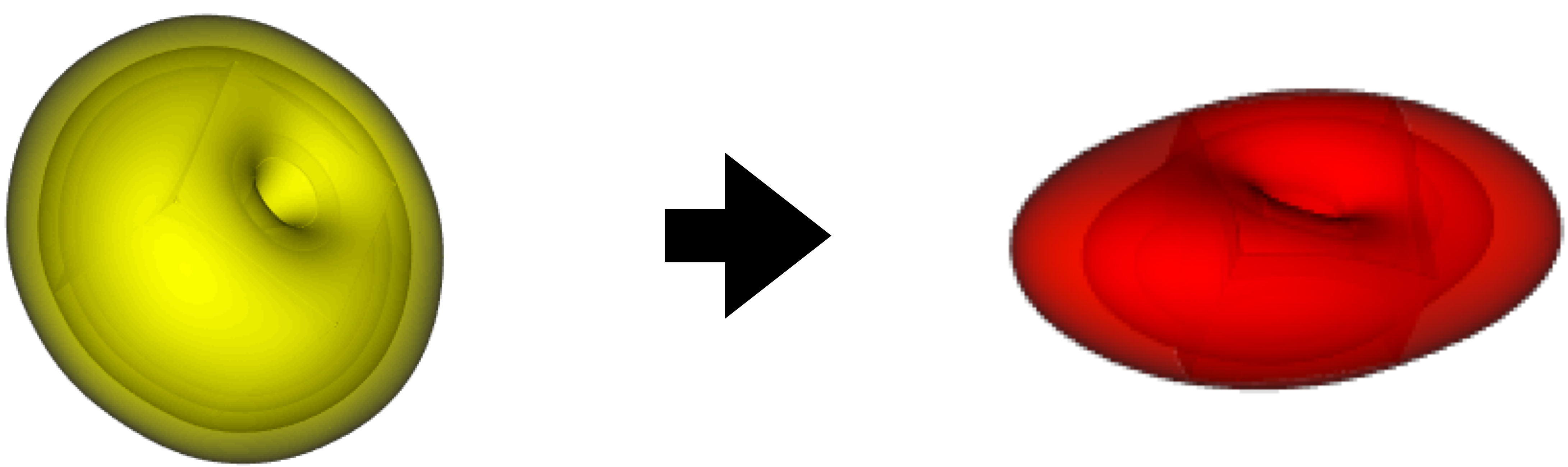
# Simulation 2 : Weak tidal case

# Weak tidal case



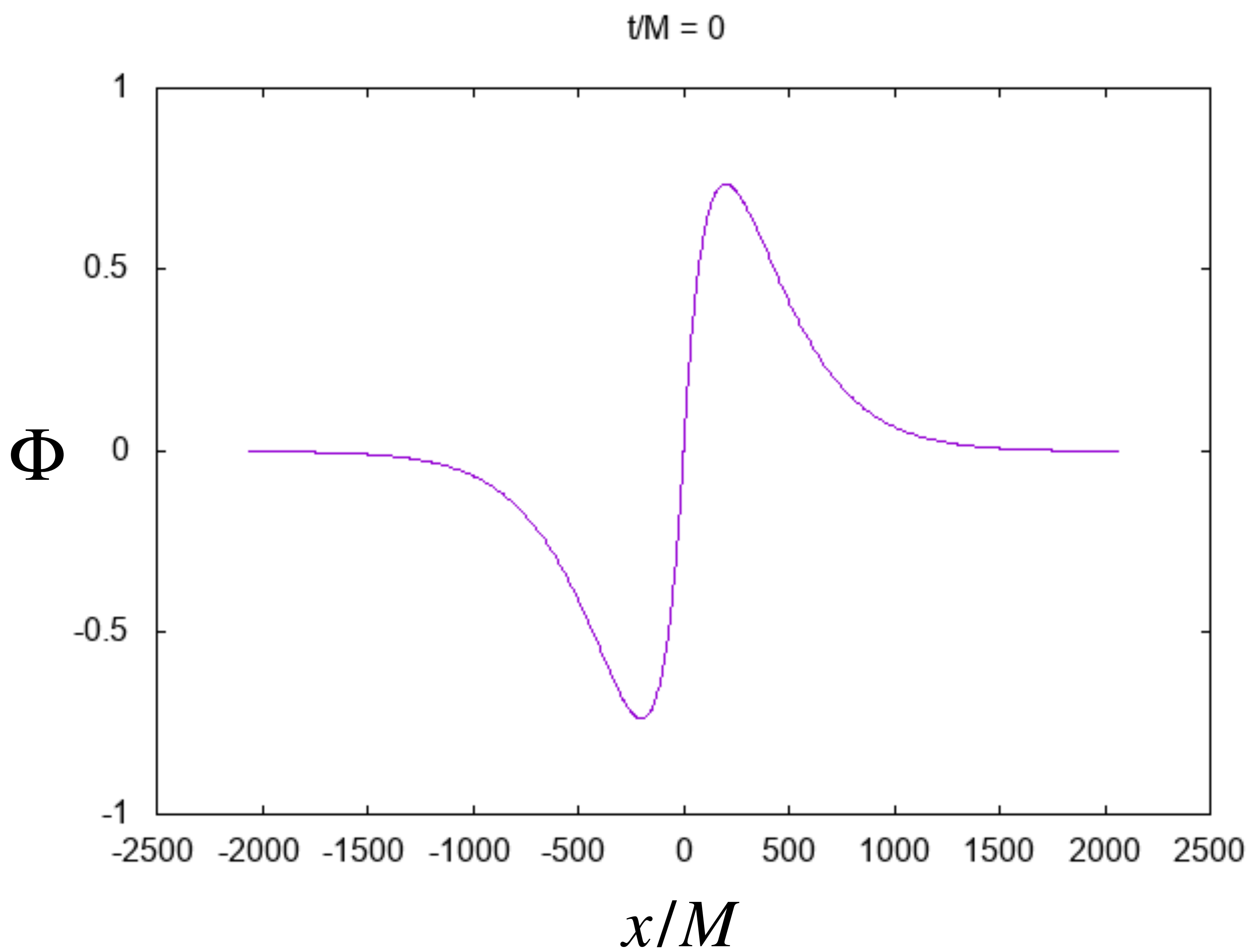

$$\frac{M_c M^2}{R^3} = 10^{-8}$$

# Weak tidal case



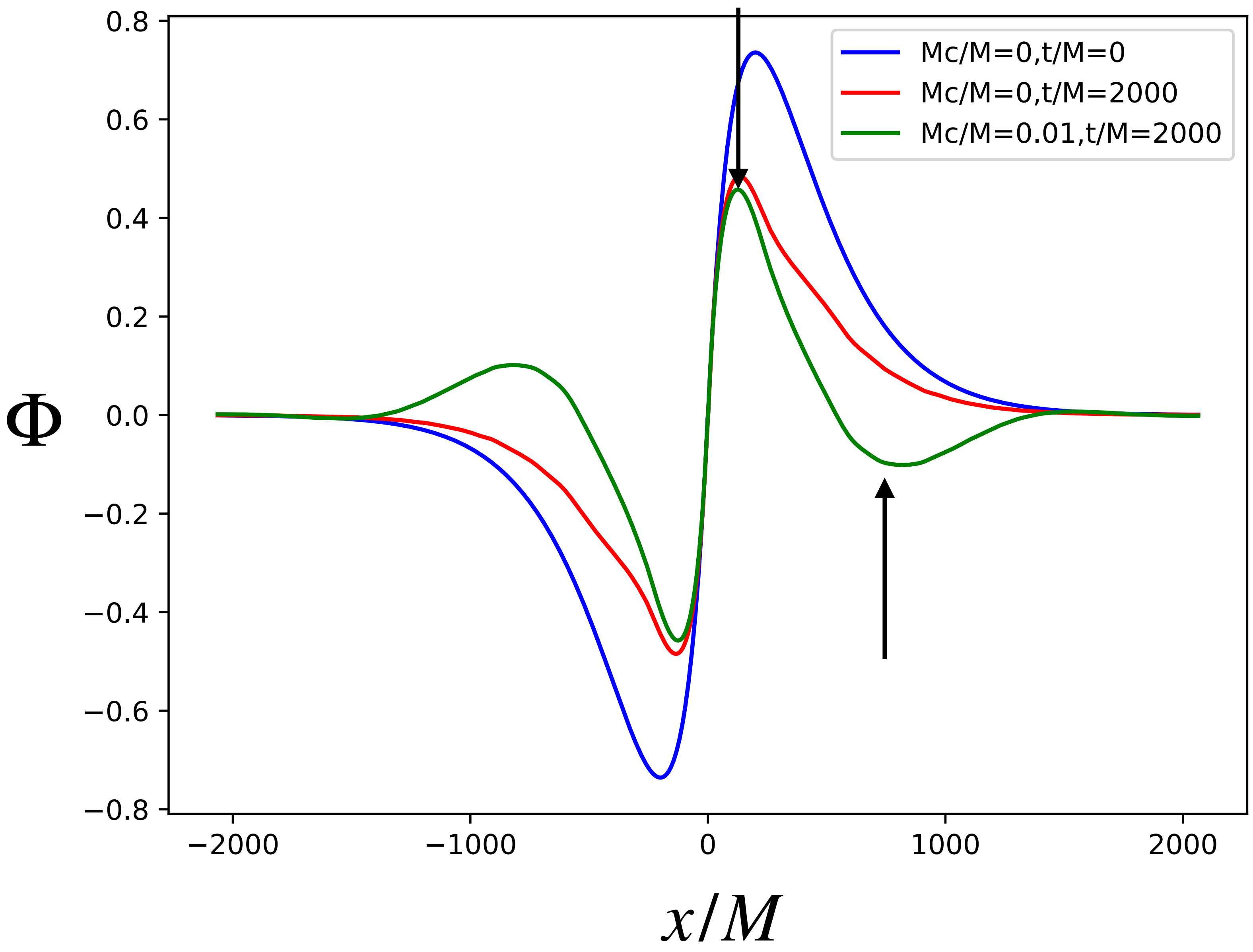
# Weak tidal case

- Excitation of overtone mode



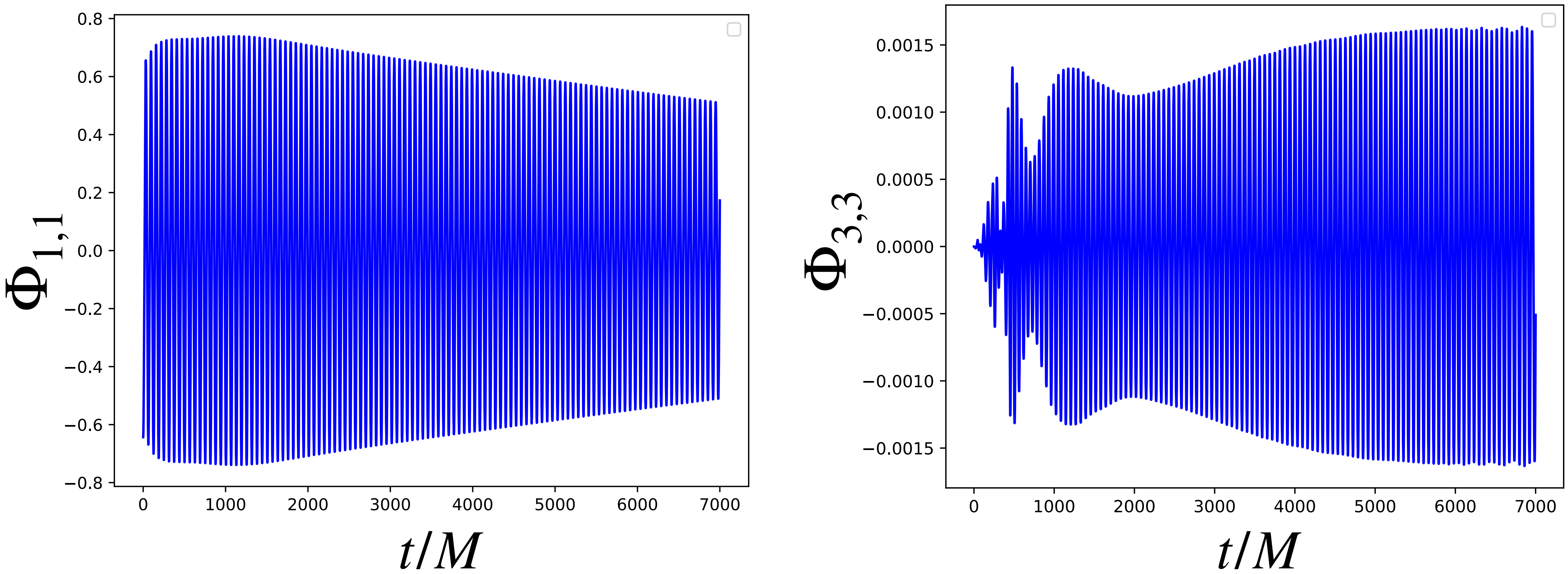
# Weak tidal case

- Excitation of overtone mode
  - $n = 4$ mode is excited. $r_{\rm ex}/M \simeq 170,\ 875,\ 2155$



# Weak tidal case

- Higher mode is excited.

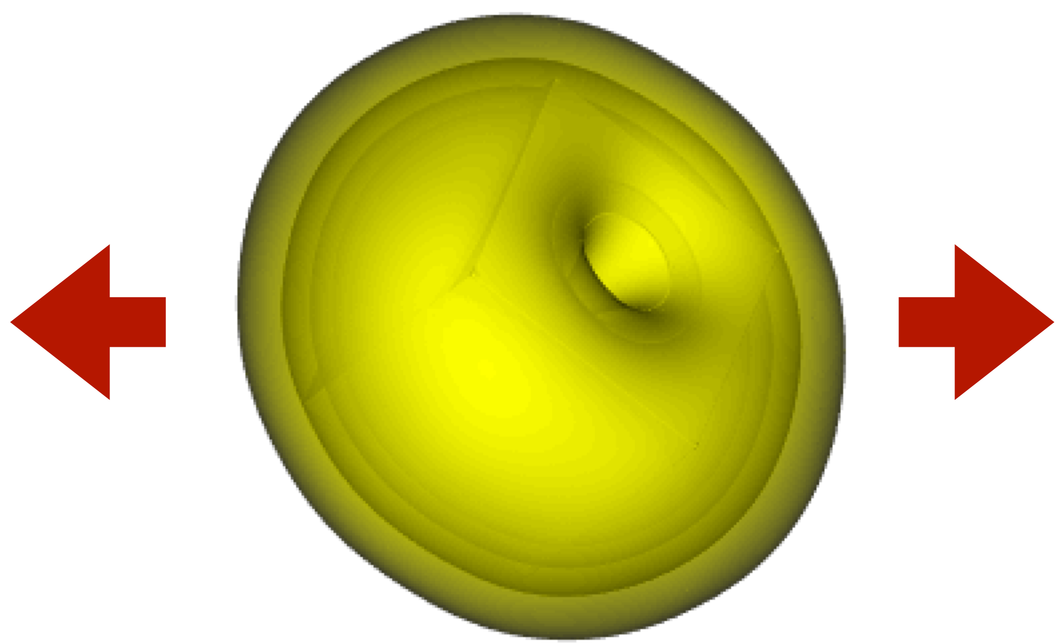


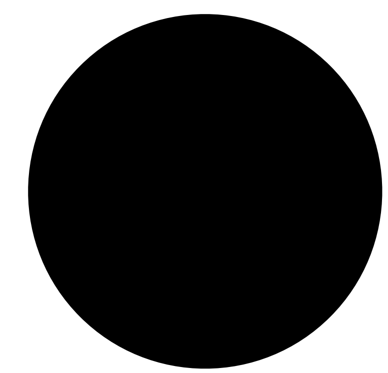

Gravitational wave may be strongly emitted.

cf: $R = 10^4 M$

$M_c = 10^5 M$

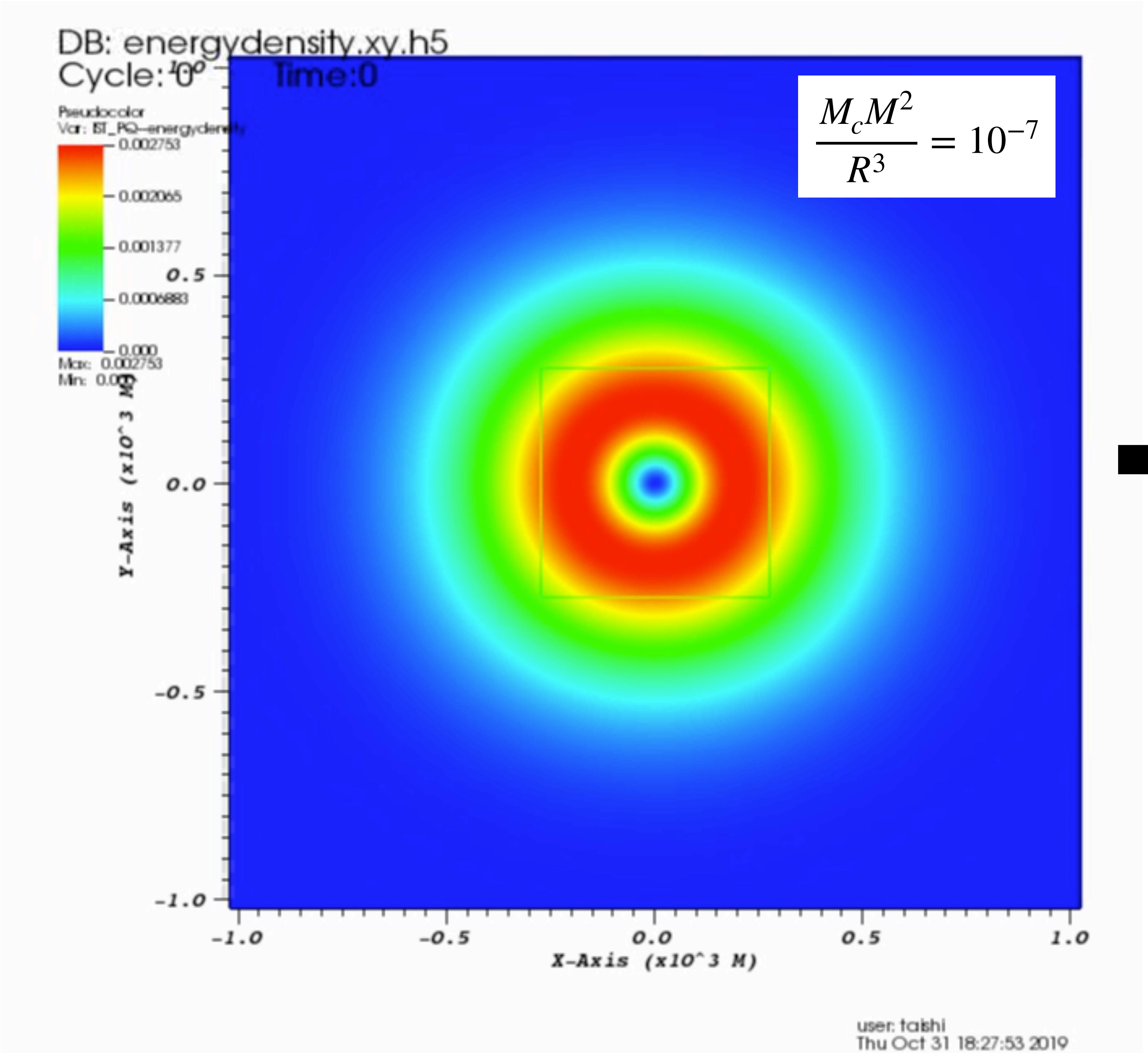
$$\frac{M_c M^2}{R^3} = 10^{-7}$$



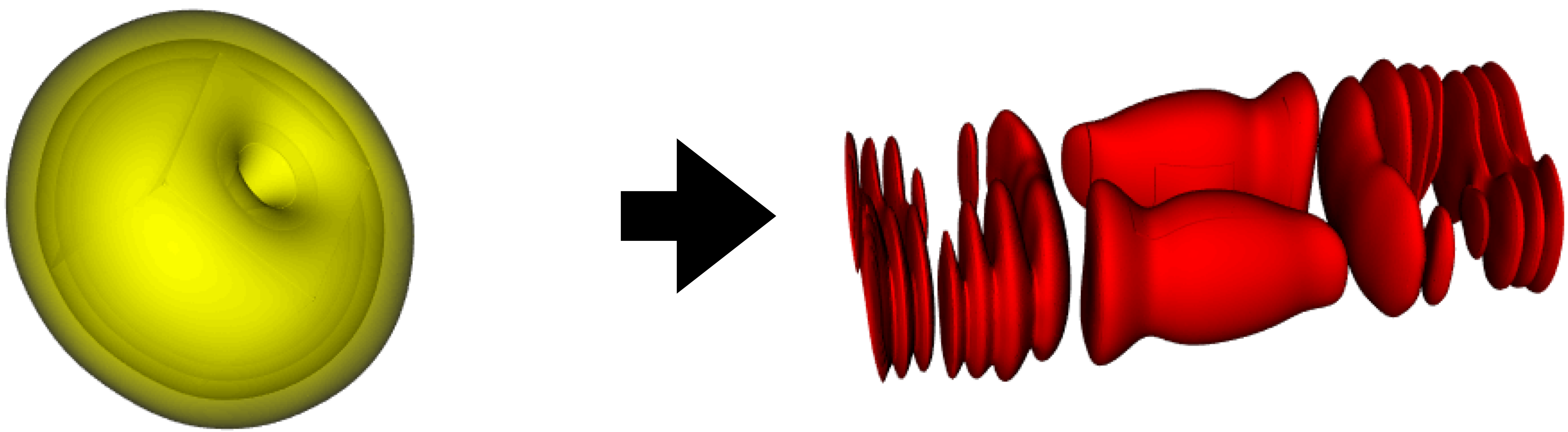


# Simulation 3 : Strong tidal case

# Strong tidal case



# Strong tidal case

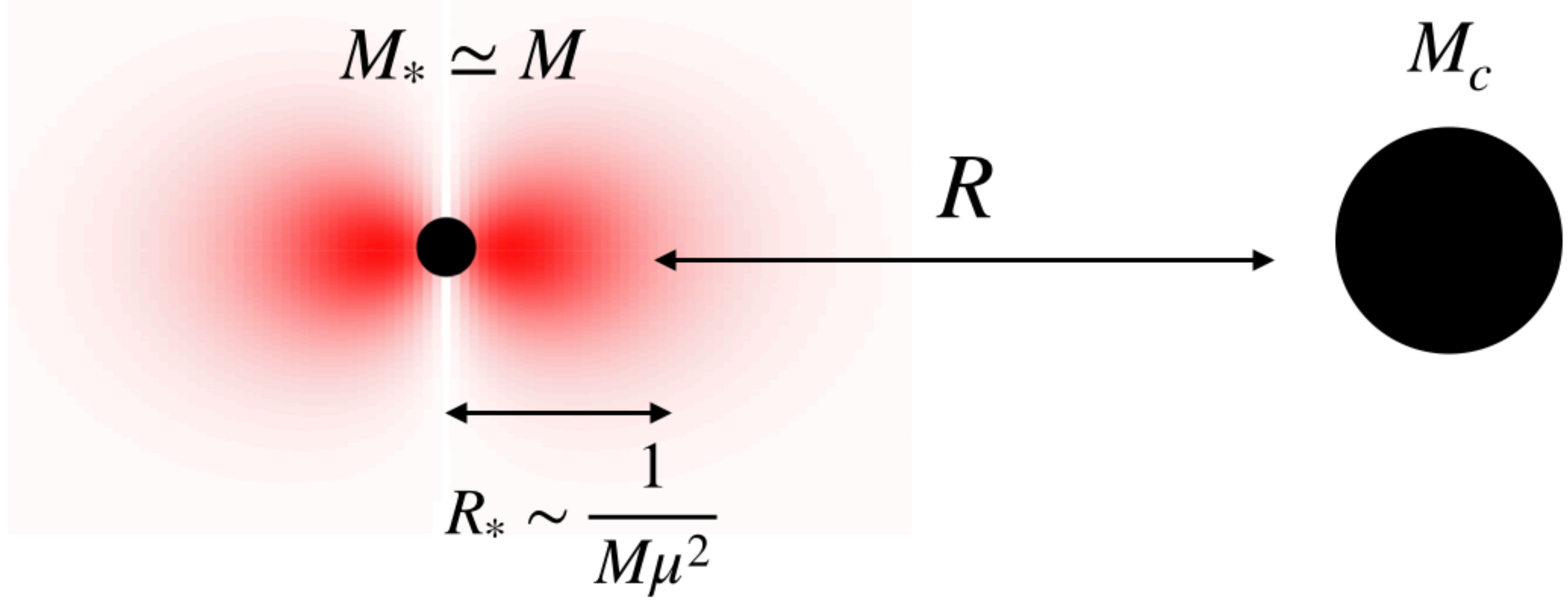


# Strong tidal case

cf: Roche limit

- Tidal disruption

$$\frac{M_*^2}{R_*^2} \sim \frac{R_*}{R}\frac{M_c M_*}{R^2} \quad \Rightarrow \quad \left.\frac{M_c M^2}{R^3}\right|_{\text{th}} \sim (M\mu)^6 \begin{cases} 10^{-6} \ (\text{ for } M\mu = 0.1 \ ) \\ 6 \times 10^{-5} \ (\text{ for } M\mu = 0.2 \ ) \end{cases}$$

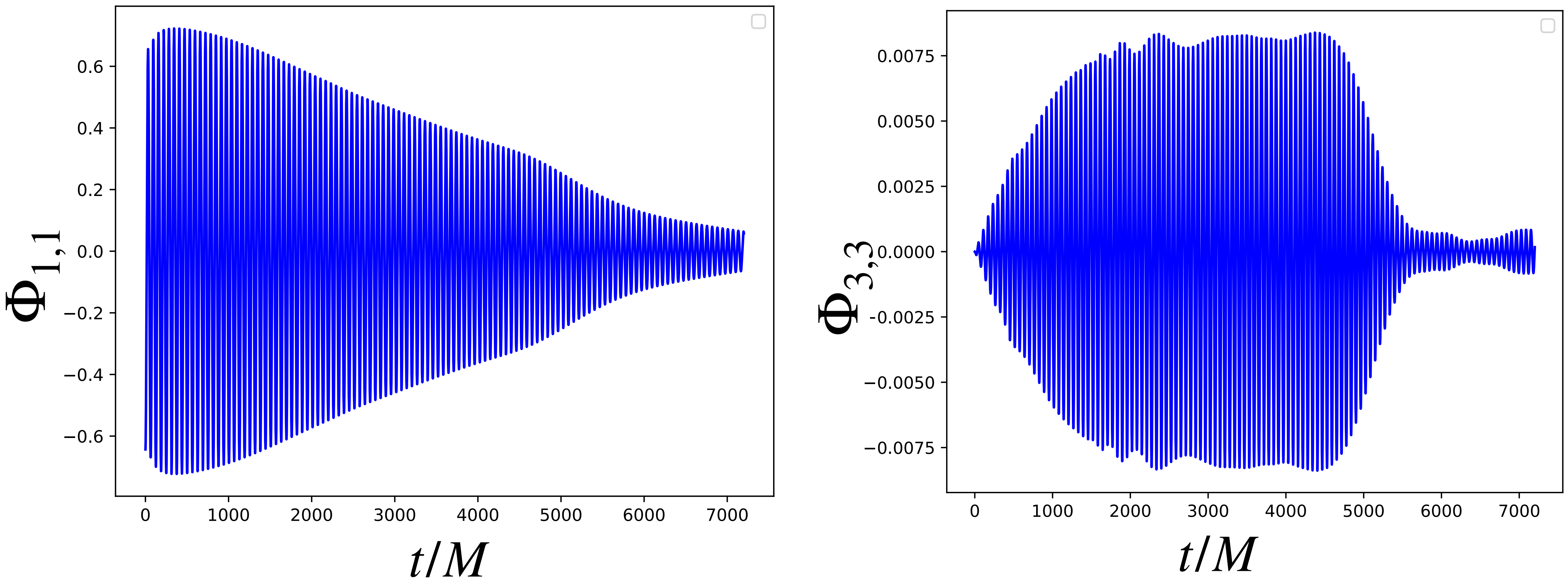



- Numerical result

$$\left.\frac{M_c M^2}{R^3}\right|_{\text{th}} \sim \begin{cases} 10^{-8} \ (\text{ for } M\mu = 0.1 \ ) \\ 5 \times 10^{-7} \ (\text{ for } M\mu = 0.2 \ ) \end{cases} \sim 10^{-2}(M\mu)^6$$

# Strong tidal case

- After higher mode is excited, the cloud is disrupted.



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- We considered tidal effect on axion cloud.
- Perturbation theory (D.Baumann et al PRD99,044001)
  - Mode mixing
  - Resonance and cloud depletion
- Time evolution - static tidal (Our result)
  - Higher multipole mode is excited.
    - Strong gravitational wave my be emitted.
  - Tidal disruption
- Future work
  - Dynamical tidal
  - Gravitational wave (?)

# Thank you.