Instabilities of scalar fields around oscillating stars

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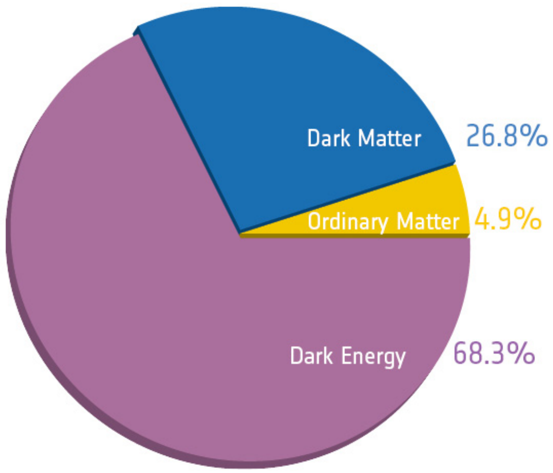
DarkGRA

Scalar field beyond GR and SM

- Mystery of our Universe
  - Dark matter ?? Dark energy ??
  - Quantum theory of gravity ??



- Scalar-tensor theory is natural extension of general relativity.

$$\mathscr{L} = \frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} \mu_0^2 \Phi^2 + \cdots$$

- The scalar field may have nonminimal coupling to matter.

$$\mathscr{L} \supset \mathscr{L}_{\mathrm{matter}}(\Psi_{\mathrm{m}}, A(\Phi)^2 g_{\mu\nu})$$

  - Chameleon screening mechanism
  - Spontaneous scalarization in neutron star   et al….

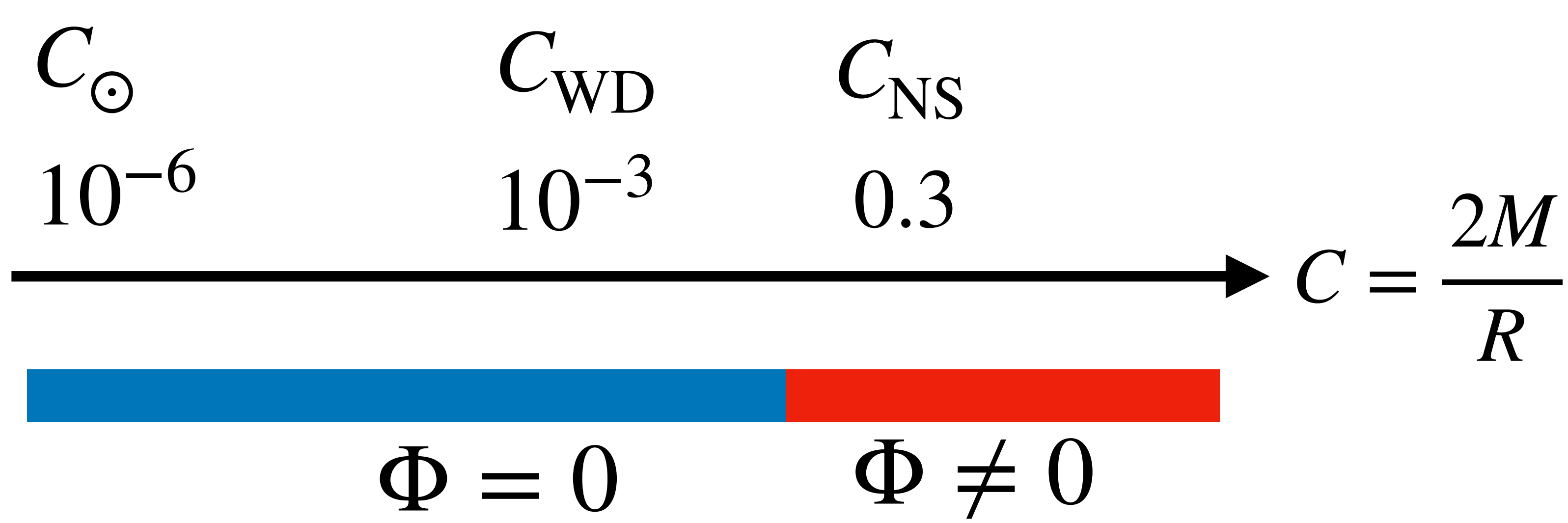
Scalar field beyond GR and SM

$$\Box\,\Phi = V'_{\rm eff}(\Phi,\rho) \qquad V_{\rm eff}(\Phi,\rho) = \frac{1}{2}\left(\mu_0^2 - |\beta|\rho\right)\Phi^2 \qquad \left(A(\Phi) = e^{\frac{\beta}{2}\Phi^2}\right)$$

$$\beta < 0$$

threshold for scalarizarion : $|\beta_c^*| \sim \left(1 + \left(\mu_0 R_*\right)^2\right) C_*^{-1}$

- Spontaneous scalarization in neutron star *(Damour et al (1993))*
  - around Sun or WD : $\Phi = 0$ $\qquad |\beta_c^{\rm NS}| < |\beta| < |\beta_c^{\rm WD}|$
  - around NS : $\Phi \neq 0$ ➡ scalar hair



Scalar field beyond GR and SM

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threshold for scalarizarion : $|\beta_c^*| \sim \left(1 + \left(\mu_0 R_*\right)^2\right) C_*^{-1}$

- No Spontaneous scalarization in all normal stars
  - around all normal stars : $\Phi = 0 \qquad |\beta| < |\beta_c^{\mathrm{NS}}|$

→ How to constraint the parameter region ??

$C_\odot$ 10^{-6} $\quad$ C_{WD} 10^{-3} $\quad$ C_{NS} 0.3

$C = \dfrac{2M}{R}$

$\Phi = 0 \qquad \Phi \neq 0$

When does the field become relevant ??

- Our model and parameter region

$$|\beta| < |\beta_c^{\mathrm{NS}}|$$

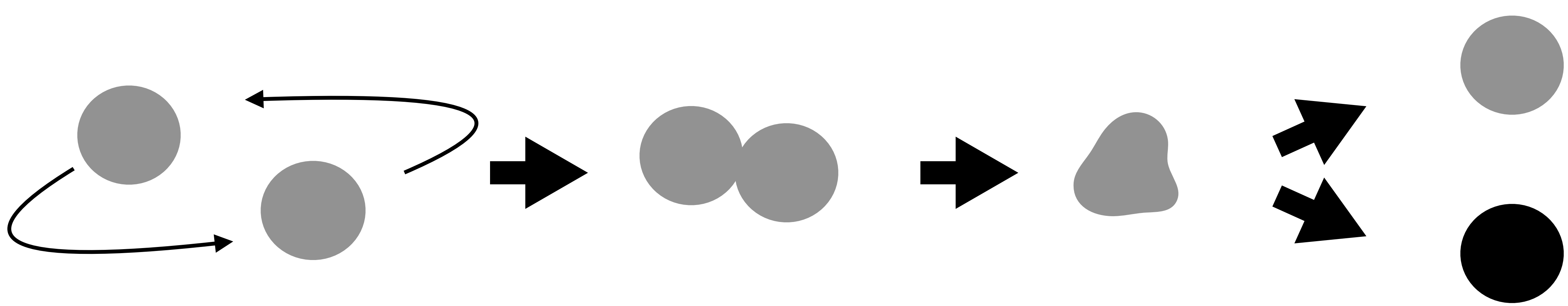
$$\mathcal{L} = \frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - V(\Phi) + \mathcal{L}(A(\Phi)^2 g, \Psi_m) \qquad A(\Phi) = e^{\frac{\beta}{2}\Phi^2}$$

  - The stationary profile around all normal stars is same as GR.

$$\Phi = 0$$

  - The dynamical situations ??

eg) Neutron star merger remnants et al ….



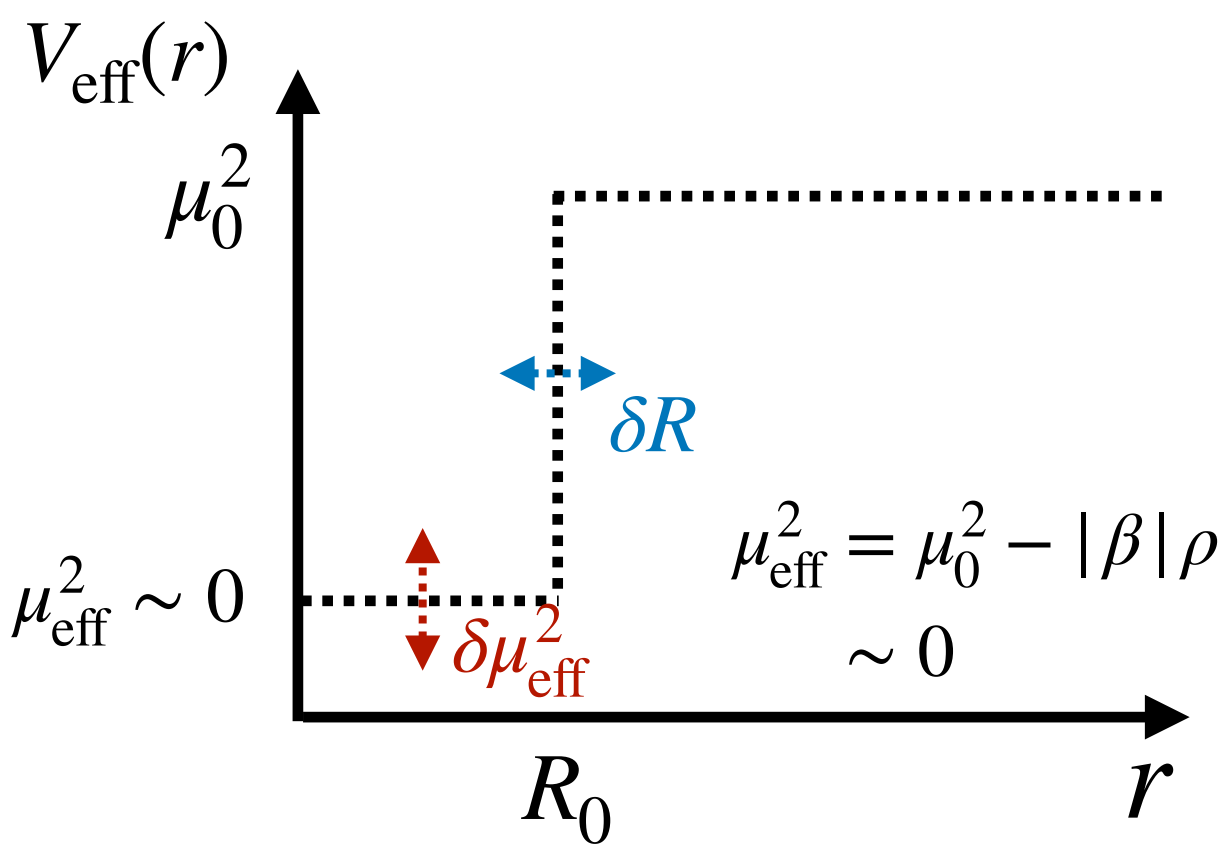
Effective potential

$$\Phi(t,r,\theta,\phi) = \sum_{l,m} \frac{\psi_{lm}(t,r)}{r} Y_{lm}(\theta,\phi)$$

- Effective potential of scalar field around relativistic star.

$$-\partial_t^2 \psi - \frac{\alpha^2}{a^2}\partial_r \left(\log \frac{a}{\alpha}\right) \partial_r \psi + \frac{\alpha^2}{a^2}\partial_r^2 \psi - V_{\rm eff}(r)\psi = 0$$

$$V_{\rm eff}(r) = \cdots + \alpha^2 \left\{ \mu_0^2 - |\beta| \left(\tilde{\rho} - 3\tilde{p}\right) \right\}$$



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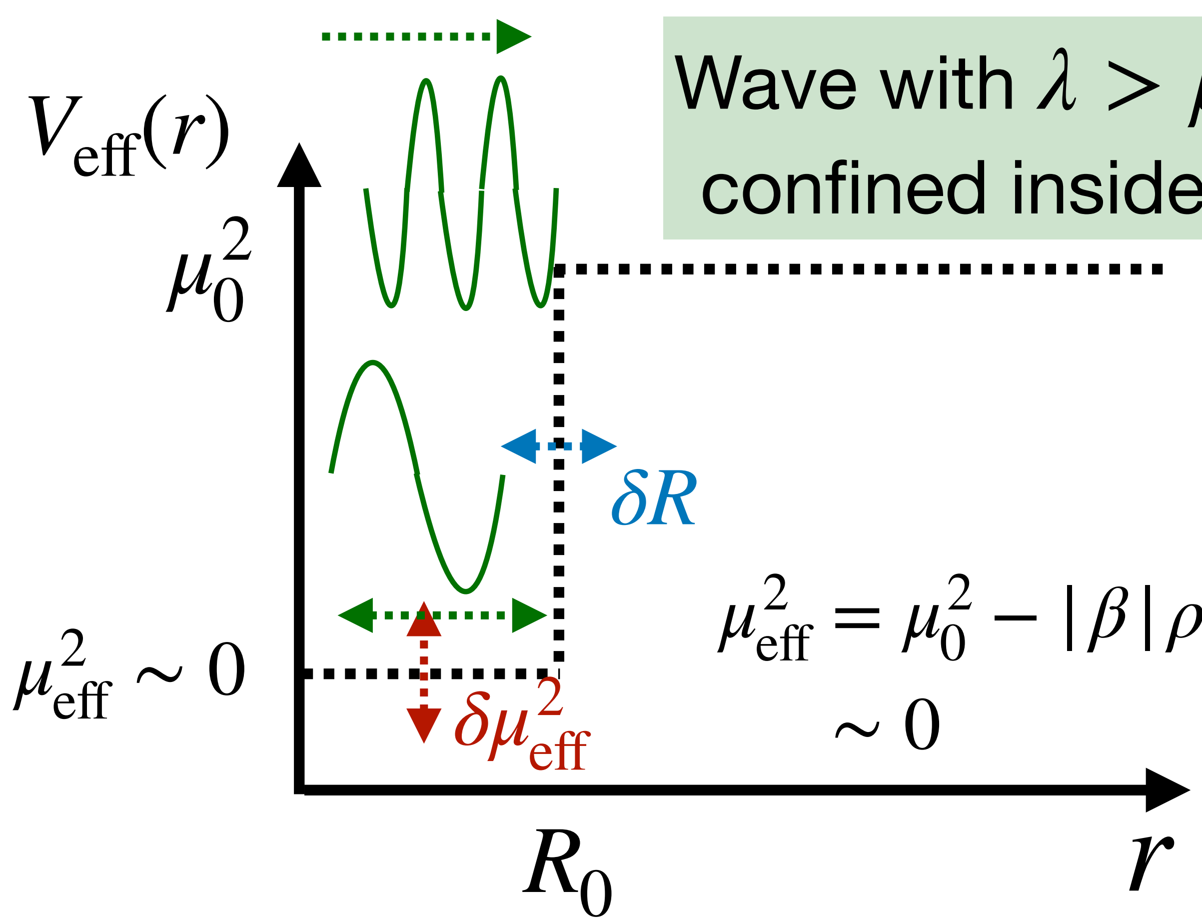


Wave with $\lambda>\mu_0^{-1}$ is confined inside star.

$\mu_{\rm eff}^2=\mu_0^2-|\beta|\rho$
~ 0

parametric instability

blueshift instability

Effect of δR

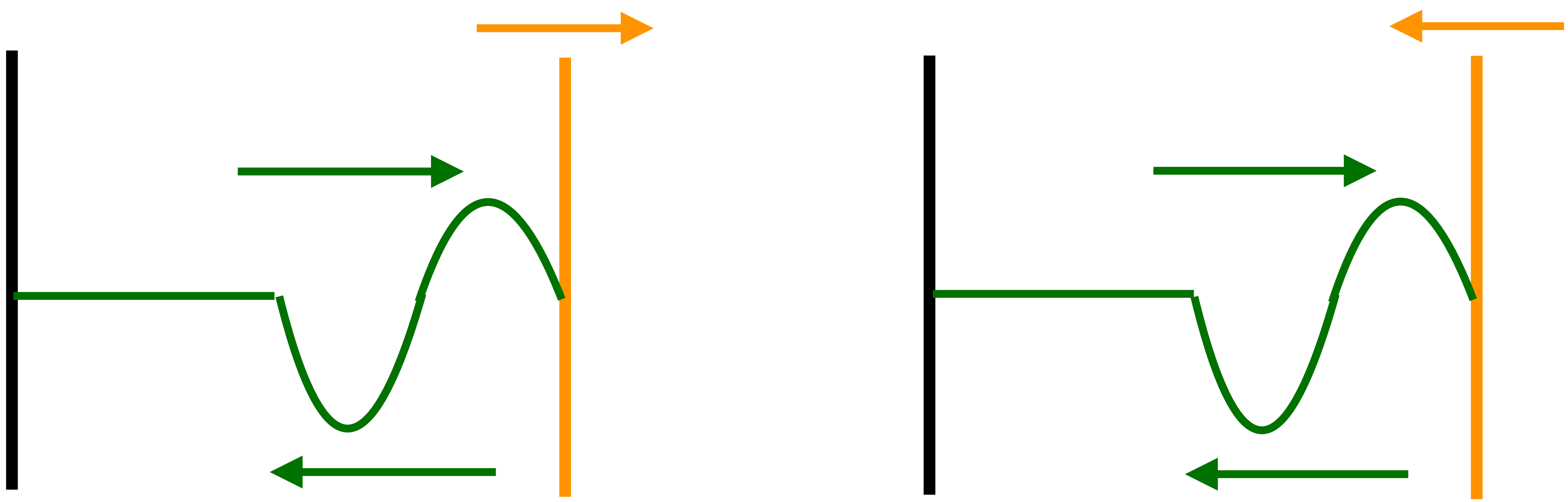
Blueshift instability (1+1 dim)

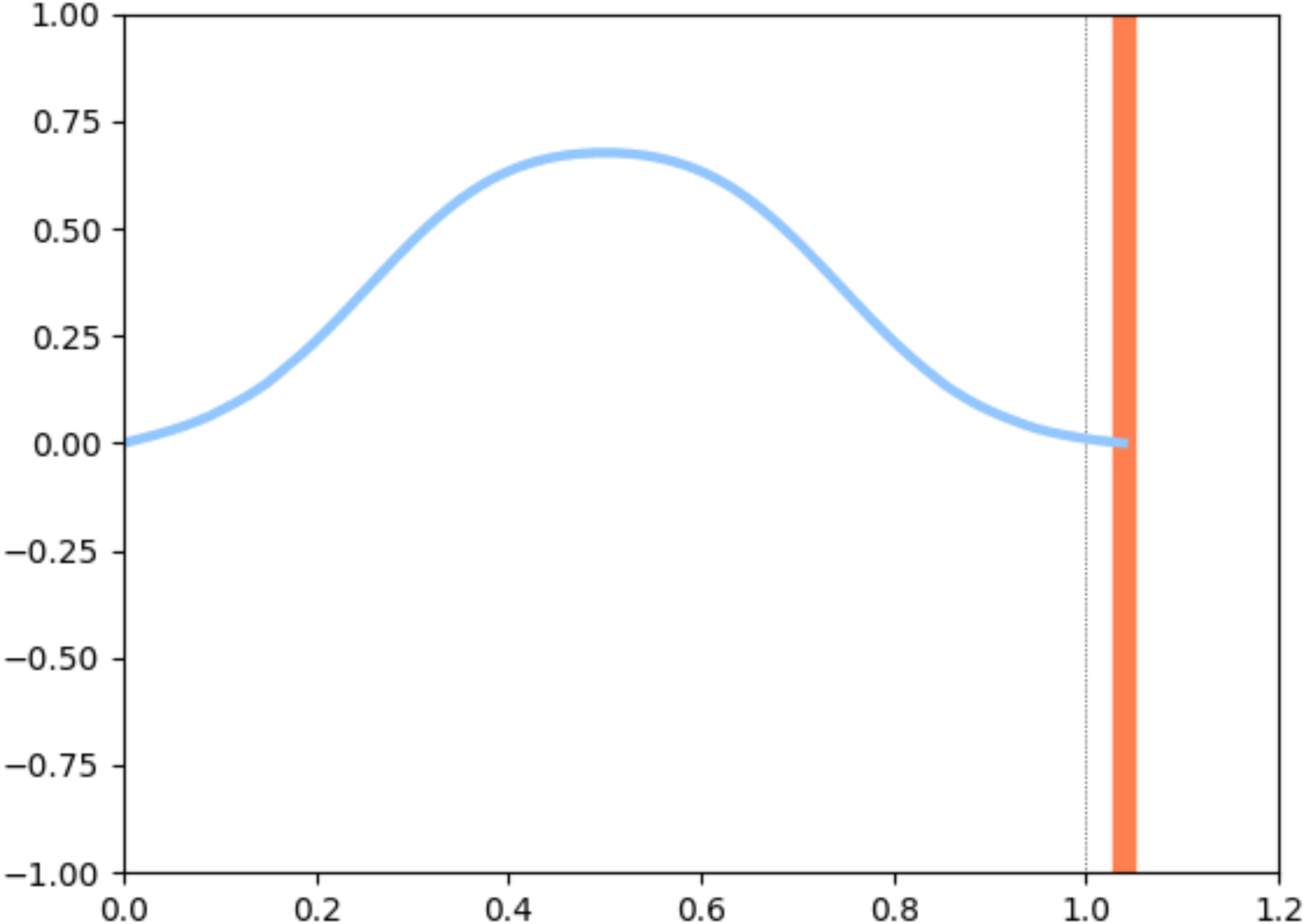
- 1+1 dim. wave eq. with oscillating boundary *(Dittrich et al 1994)*

$$-\partial_t^2\Phi + \partial_x^2\Phi = 0 \quad \text{with} \quad \begin{cases} \Phi(t,0) = 0 \\ \Phi(t, L(t)) = 0 \end{cases}$$

$$L(t) = L_0 + \delta L \sin(\omega t)$$

Redshift   Blueshift



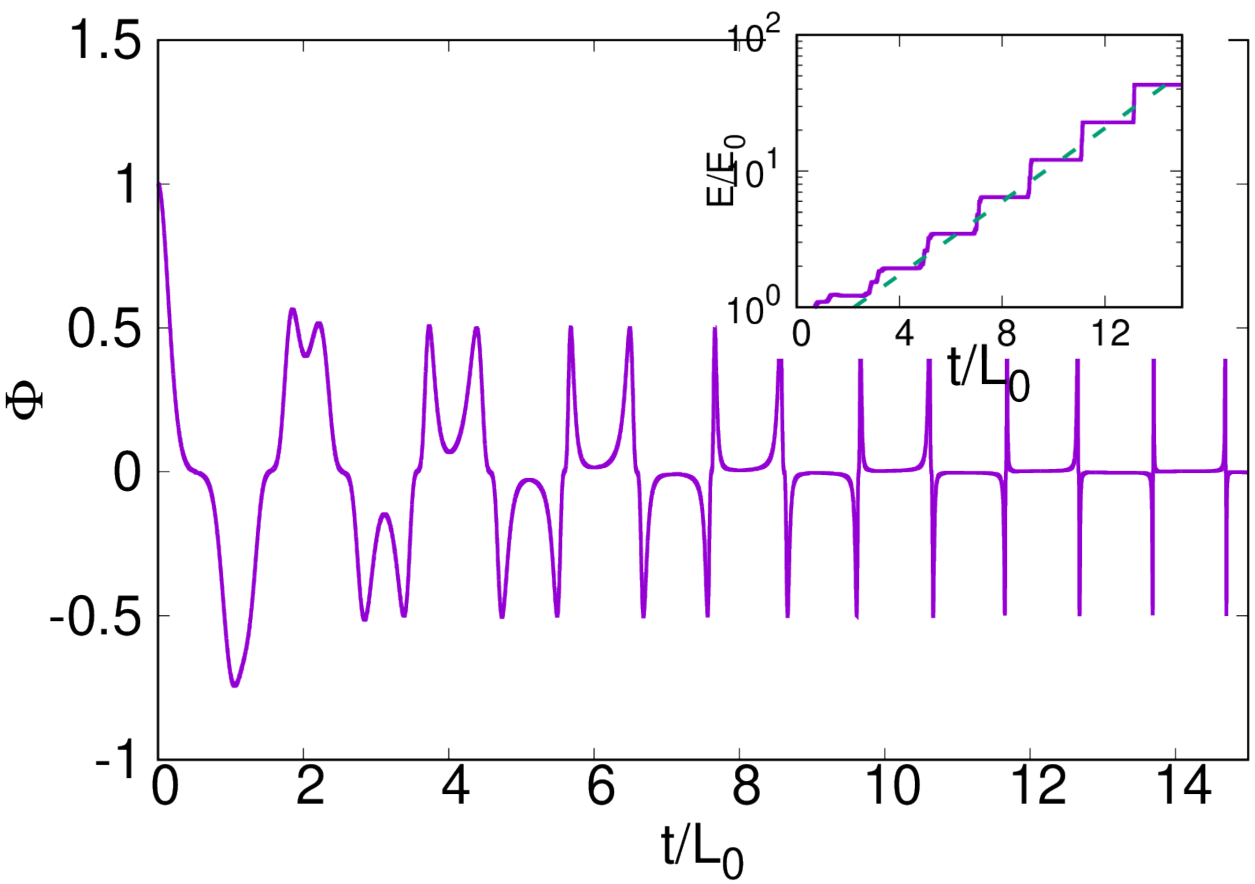


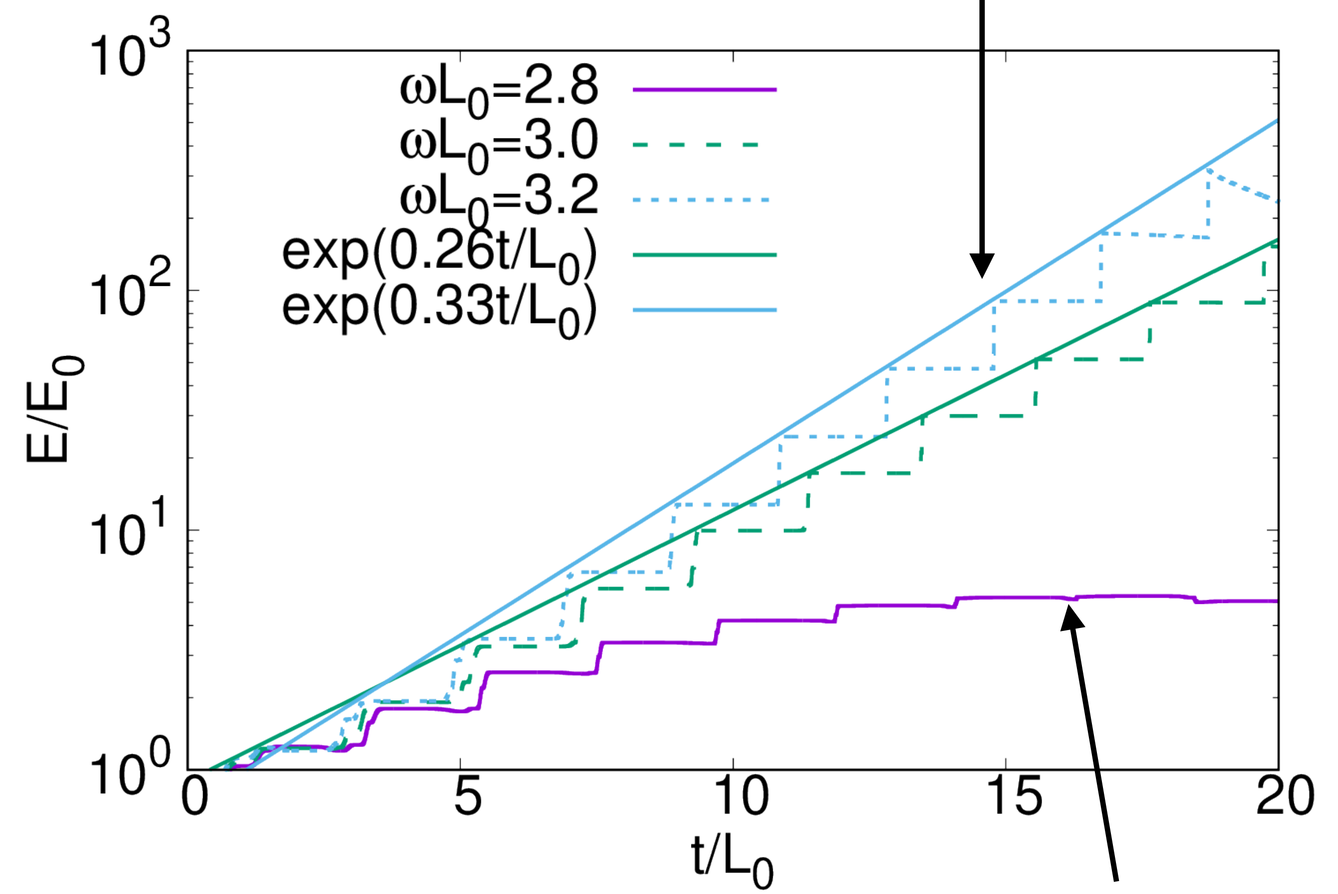
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0
1.2

Blueshift instability (1+1 dim)

- Condition for the blueshift instability

$$\frac{\pi}{L_0+\delta L} < \omega < \frac{\pi}{L_0-\delta L}$$ ➡ The blueshift accumulates. $E \propto e^{\lambda_B(\omega,\delta L)t}$





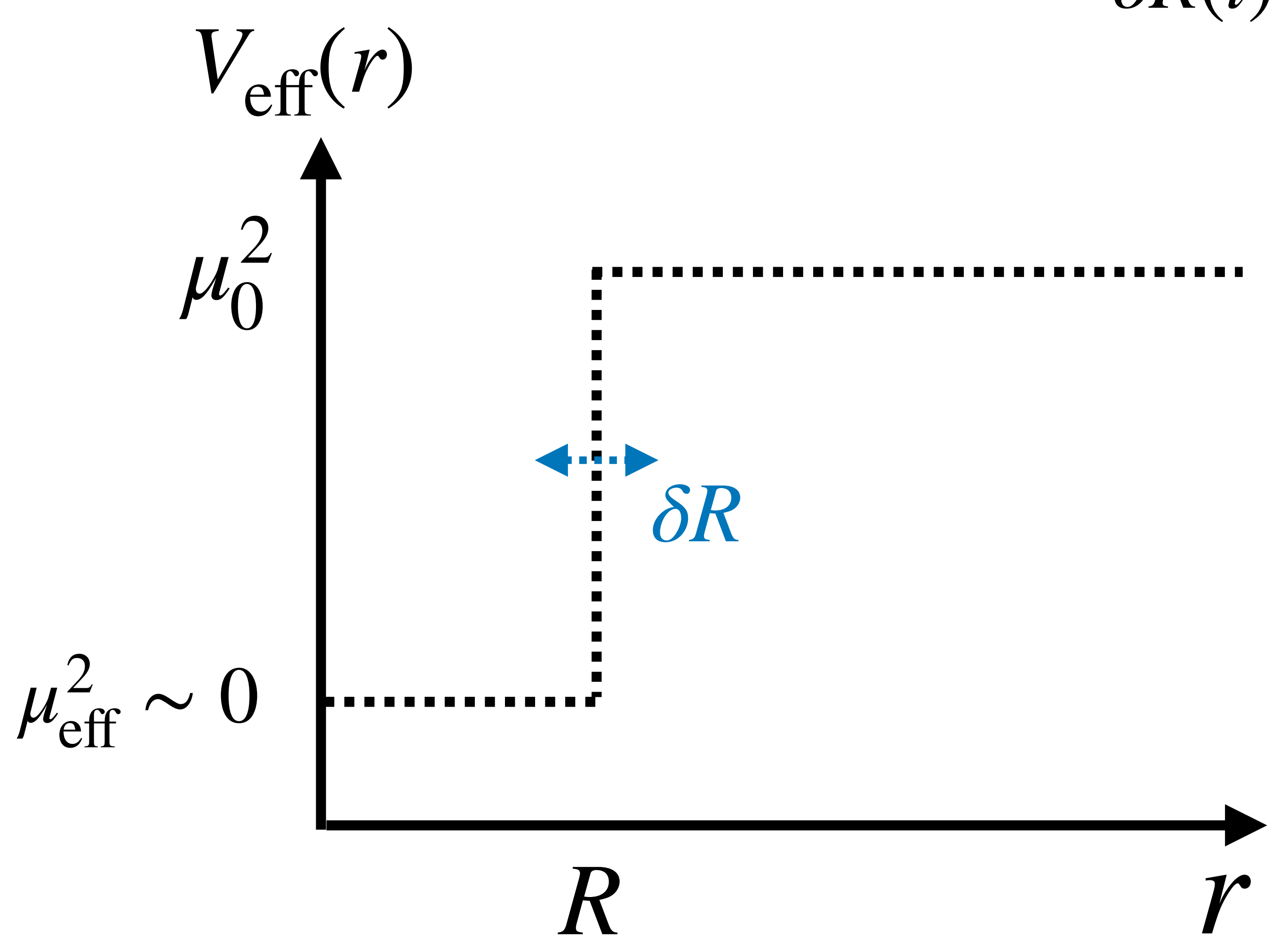
not satisfy

Blueshift instability (3+1 dim)

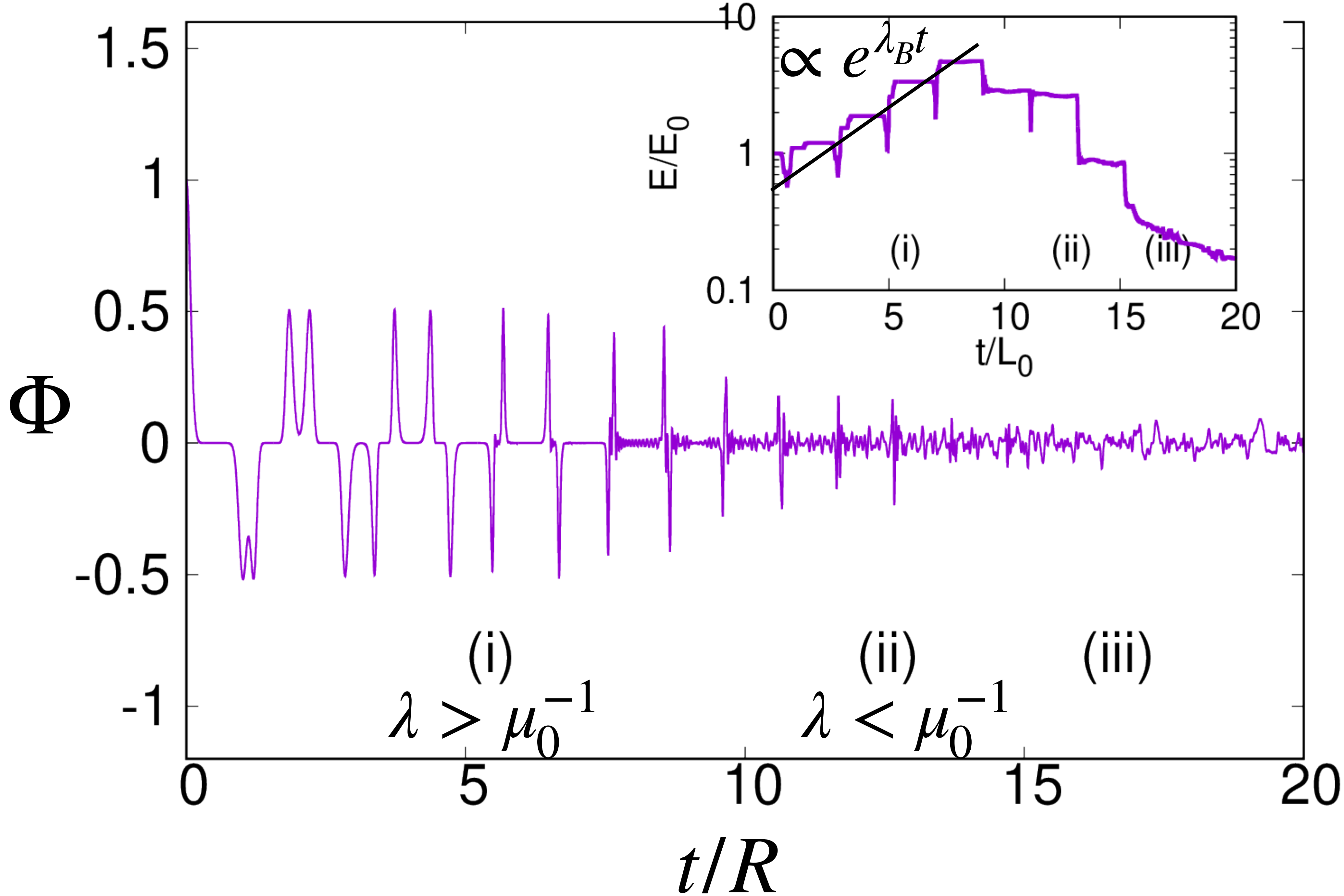
- 3+1 evolution in flat spacetime

$$\Box_{\text{flat}}\Phi = V_{\text{eff}}(r)\Phi \qquad V_{\text{eff}}(r) = \begin{cases} \mu_0^2 & (R+\delta R(t) < r) \\ \mu_{\text{eff}}^2 & (r < R+\delta R(t)) \end{cases}$$

$$\delta R(t) \sim \sin(\omega t)$$



Blueshift instability (3+1 dim)



$$t_B \sim 3 \times 10^{-2}\mathrm{s} \left(\frac{10^{-3}}{\lambda_B(\omega) L_0} \right) \left(\frac{L_0}{10\mathrm{km}} \right) \ln \left| 10 \frac{\mu}{10^{-10}\mathrm{eV}} \frac{\sigma}{10\mathrm{km}} \right|$$

Effect of $\delta\mu^2$

Parametric instability (1+1 dim)

- 1+1 dim. wave eq. with oscillating effective mass

$$-\partial_t^2\Phi + \partial_x^2\Phi - \mu^2(t)\Phi = 0$$

$$\mu^2(t) = \bar{\mu}^2 + \delta\mu^2 \sin(\omega t)$$

- This equation can be reduced to Mathieu equation.

$$\partial_\tau^2\phi_k + \left(\delta + 2\,\epsilon \sin(\tau)\right)\phi_k = 0$$

$$\tau = \omega t$$

$$\delta = \frac{k^2 + \bar{\mu}^2}{\omega^2}$$

- Instability condition

$$\delta \sim \frac{n^2}{4}$$

Parametric instability (3+1 dim)

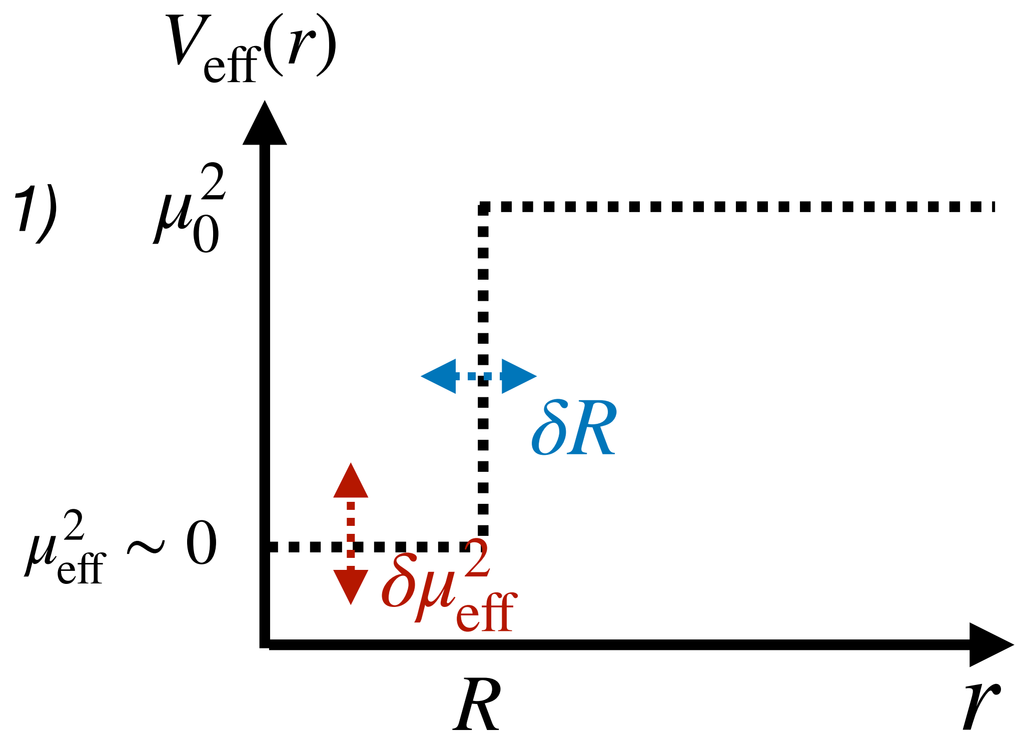
- 3+1 evolution in flat spacetime

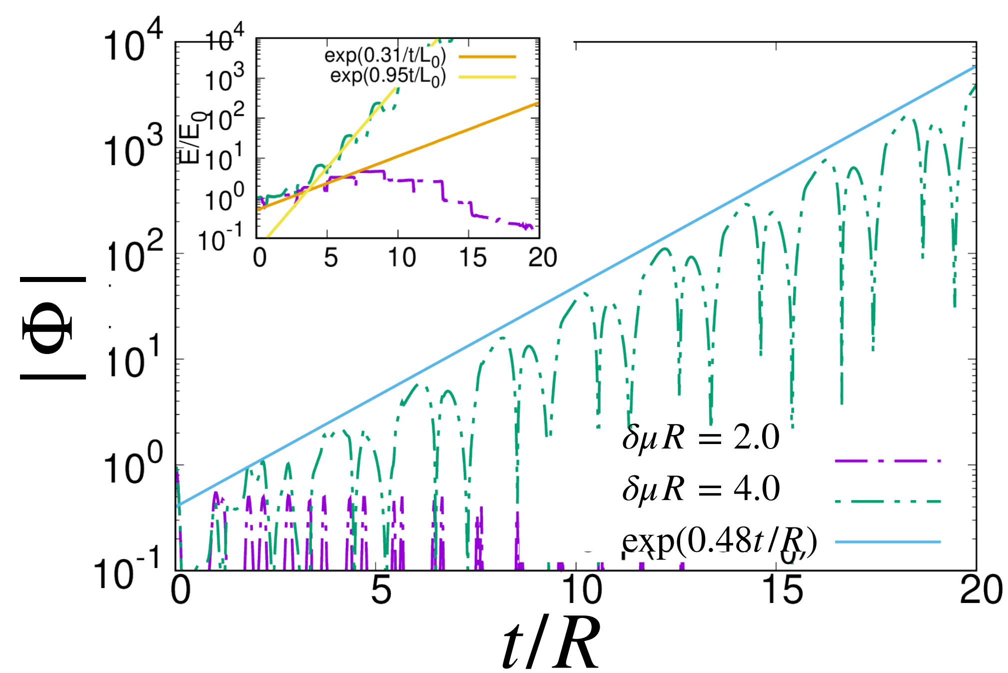
$$\Box_{\text{flat}}\,\Phi = V_{\text{eff}}(r)\Phi$$

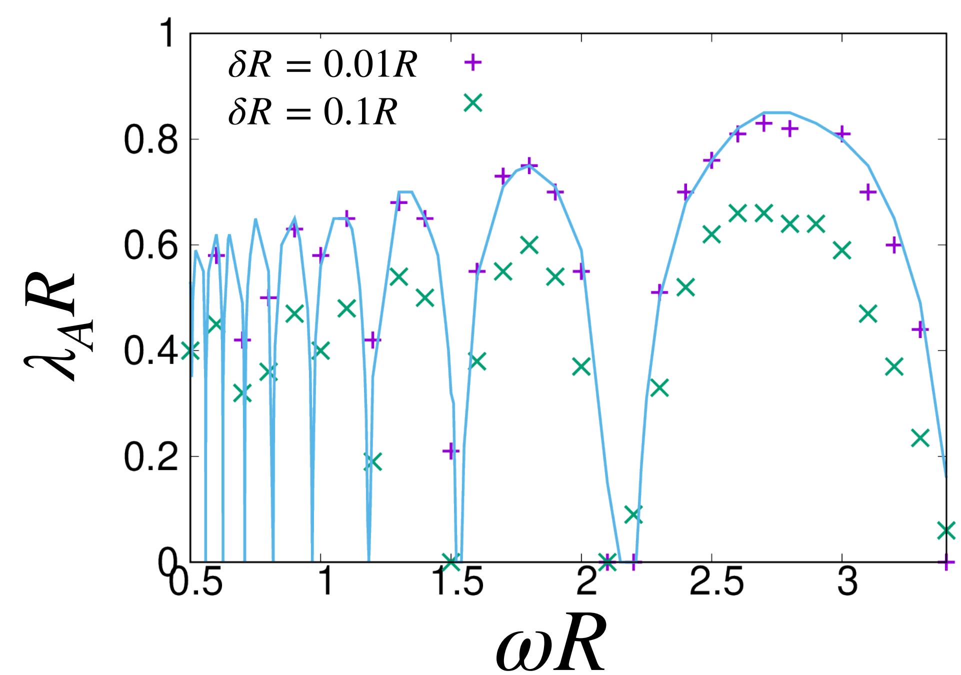
(Wang et al (2011))

$$V_{\text{eff}}(r) = \begin{cases} \mu_0^2 & (R+\delta R < r) \\ \mu_{\text{eff}}^2 + \delta\mu_{\text{eff}}^2 \sin(\omega t) & (r < R+\delta R) \end{cases}$$

$$\delta R(t) \sim \sin(\omega t)$$



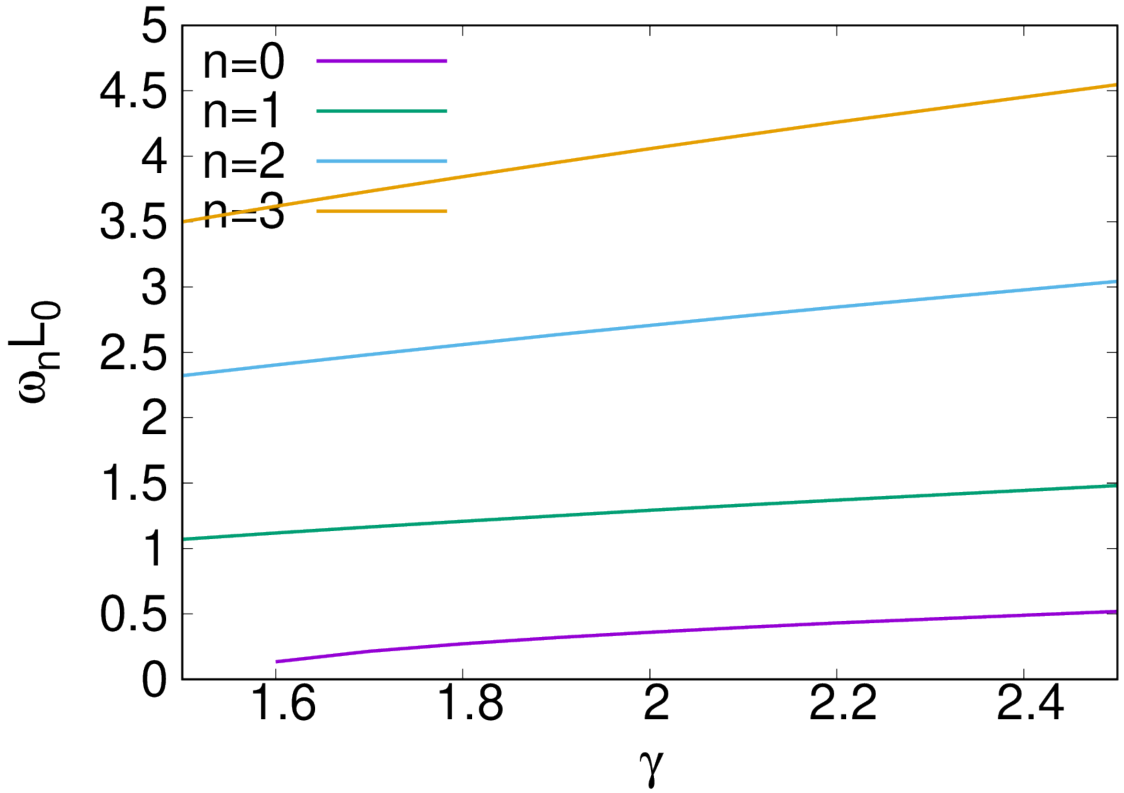


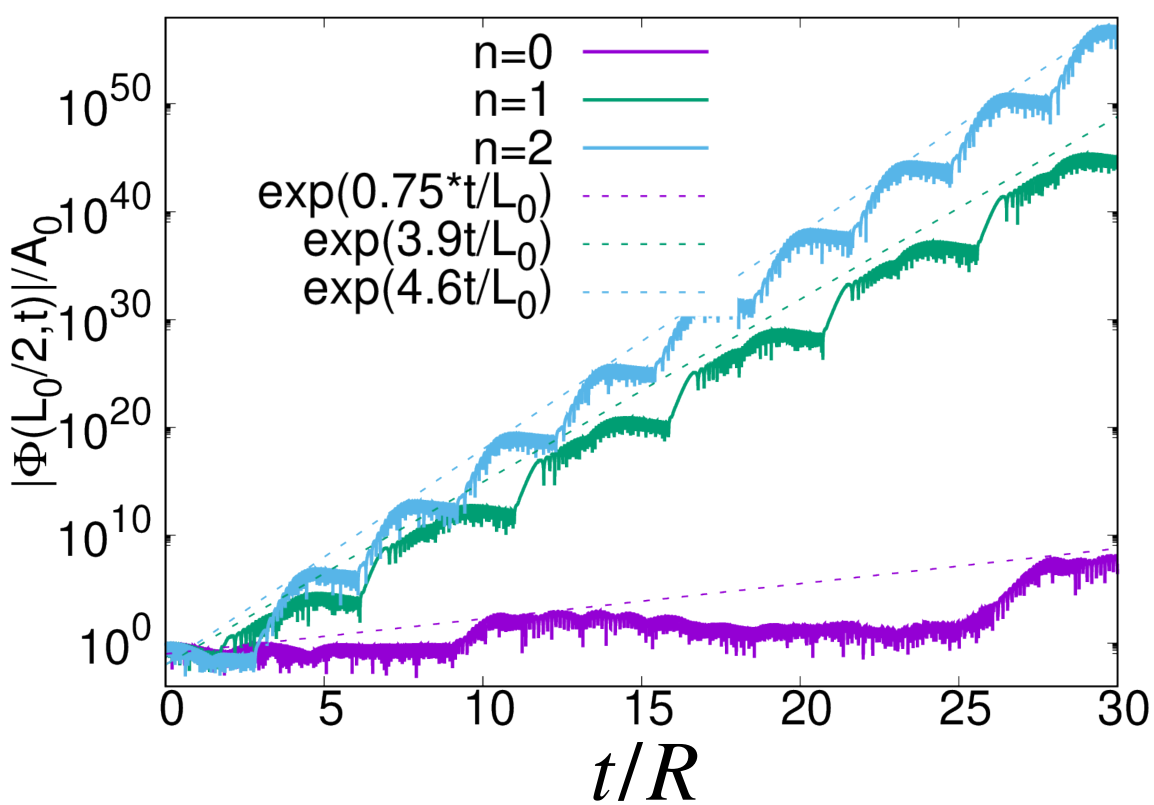


$$t_A \sim 2\text{s}\left(\frac{10\text{km}}{L_0}\right)\left(\frac{\omega L_0}{4\pi}\right)\left(\frac{10^{-2}\rho_{17}}{\delta\tilde{\rho}}\right)\left(\frac{10}{|\beta|}\right)$$

Parametric instability in relativistic stars

- We checked the parametric instability in relativistic constant density star.
    - solve the normal mode of the star
    - solve the scalar field around star with normal mode perturbations.





Summary

$$\mathscr{L} = \frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - V(\Phi) + \mathscr{L}(A(\Phi)^2 g, \Psi_m) \qquad A(\Phi) = e^{\frac{\beta}{2}\Phi^2}$$

$$|\beta| < |\beta_c^{\mathrm{NS}}|$$

- Two instabilities of the scalar field around oscillating star.
  - blueshift instability

$$t_B \sim 3 \times 10^{-2}\mathrm{s} \left(\frac{10^{-3}}{\lambda_B(\omega) L_0} \right) \left(\frac{L_0}{10\mathrm{km}} \right) \ln \left| 10 \frac{\mu}{10^{-10}\mathrm{eV}} \frac{\sigma}{10\mathrm{km}} \right|$$

  - parametric instability

$$t_A \sim 2\mathrm{s} \left(\frac{10\mathrm{km}}{L_0} \right) \left(\frac{\omega L_0}{4\pi} \right) \left(\frac{10^{-2} \rho_{17}}{\delta \tilde{\rho}} \right) \left(\frac{10}{|\beta|} \right)$$

- We can expect the similar effect in other models.

$$\mathscr{L} \supset f(\Phi) \mathscr{G}_{\mathrm{GB}}$$

Thank you !!