Black hole binaries and light fields

- gravitational molecule -

Taishi Ikeda (池田大志)

Sapienza University of Rome

arXiv:2010:00008

My collaborators

Laura Bernard

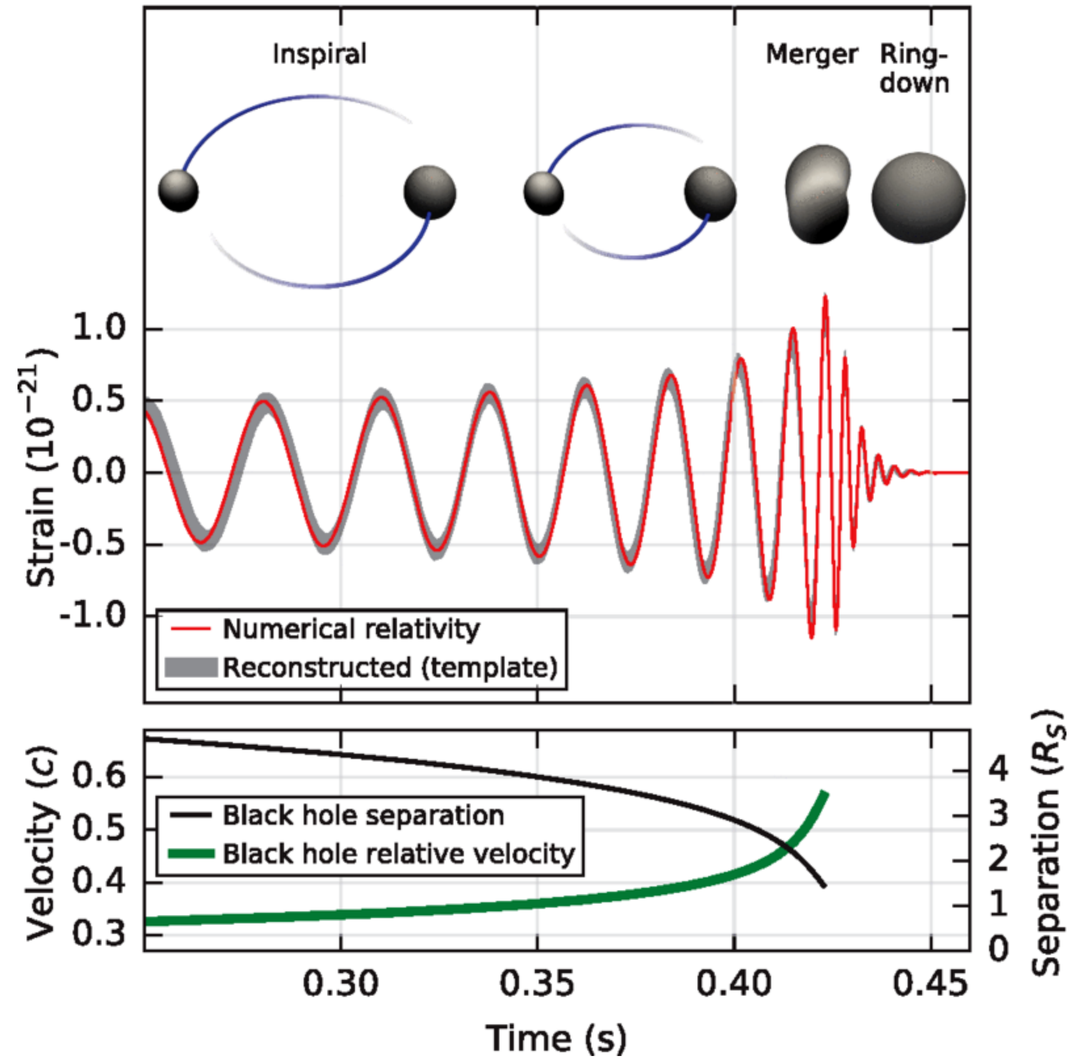
Vitor Cardoso

Miguel Zilhão

Gravitational wave astronomy

- Binary Black hole is one of the main sources of GW.
- We can predict the wave form using….
  - PN approximation
  - BH perturbation
  - Numerical relativity et al
- However…

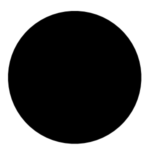
How deeply do we understand fundamental physics around binary BH ?



Black hole vs Black hole binary

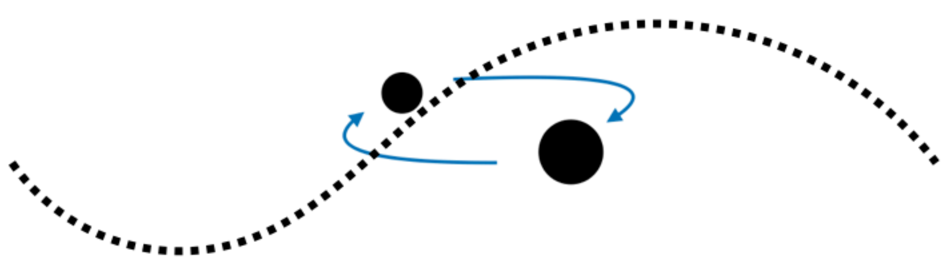
- We don't deeply understand the physics around BH binary spacetime.

Black Hole	Black Hole Binary
- Photon sphere	- "Global " photon surface?
- Quasi normal mode	- "Global" QNM ?
	Please see PRD100(2019) 4,044002.
- Bound state of massive fields (Gravitational atom)	- "Global" bound state of massive fields ? (Gravitational molecule)

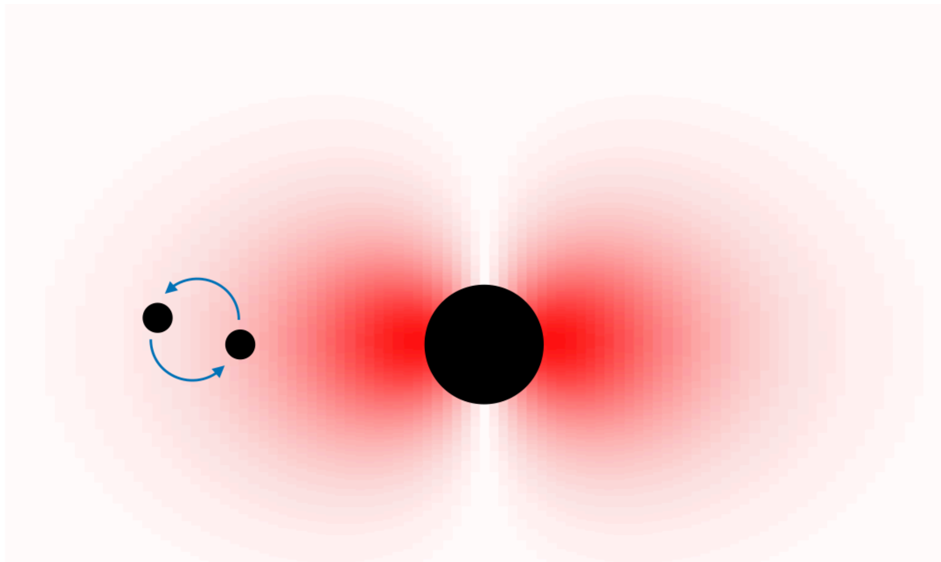


Massive field around binary

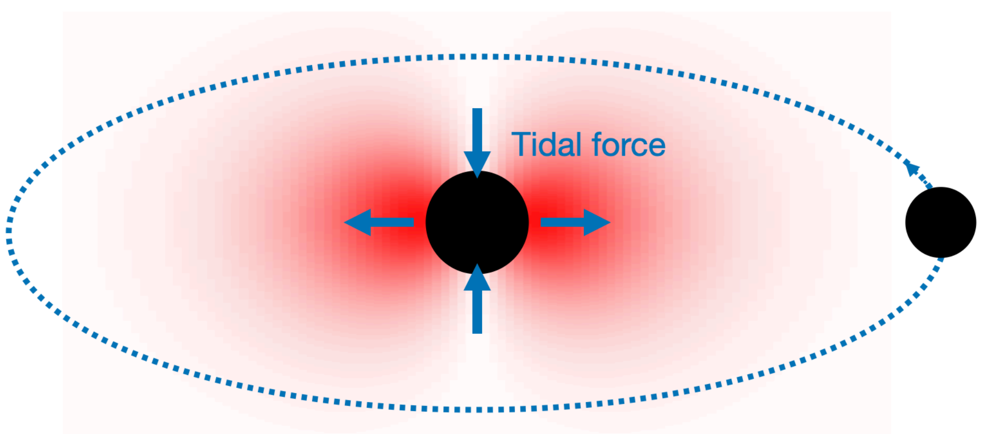
- When massive scalar fields become relevant…

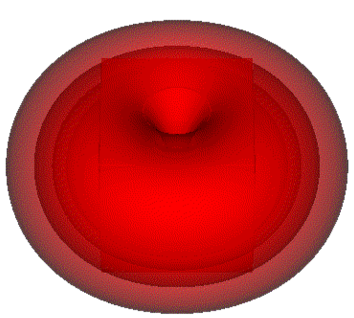


BH binary in coherent
oscillating scalar field



BH binary in bound state
around 3rd BH





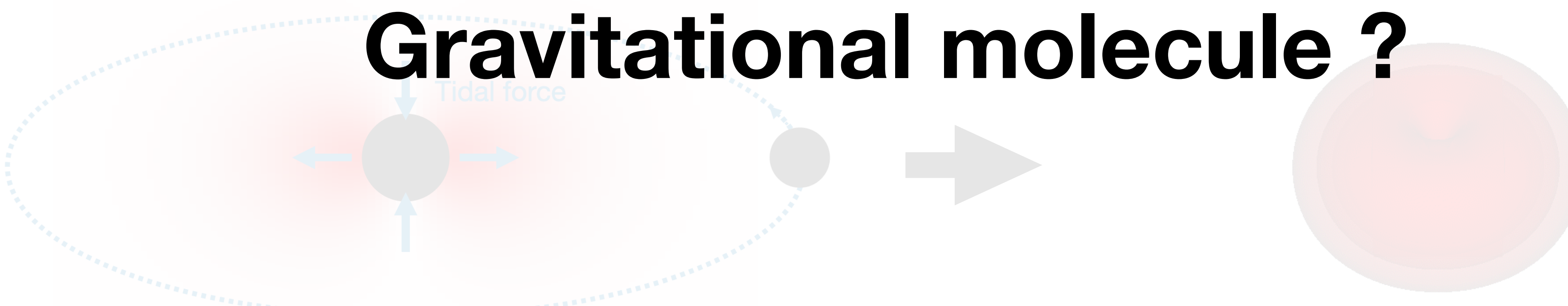
Tidal disruption of bound state of massive field by companion star

Cardoso et al. (2020)

In these setup, massive scalar fields can be relevant around BH binary.

We must understand typical behavior of the fields around the binary.

Gravitational molecule ?



Outline

1. Introduction
2. Gravitational atom
3. Analytic description
    - Gravitational molecule
    - Particle picture
4. Numerical simulations
    - Set up
    - Numerical construction
5. Summary

Outline

1. Introduction
2. Gravitational atom
3. Analytic description
   - Gravitational molecule
   - Particle picture
4. Numerical simulations
   - Set up
   - Numerical construction
5. Summary

Gravitational atom

$$c^2 = -(\mu M)^2 a^2(\mu^2 - \omega^2)$$
$$\Delta = r^2 - 2Mr + a^2$$

- Bound state of massive scalar field around Kerr BH

S.Detweiler (1980)

$$\left(\Box_{\text{BL Kerr}} - \mu^2\right)\Phi = 0 \qquad \Phi = e^{-i\omega t + im\phi}R(r)S(\theta)$$

- Angular part (spheroidal harmonics)

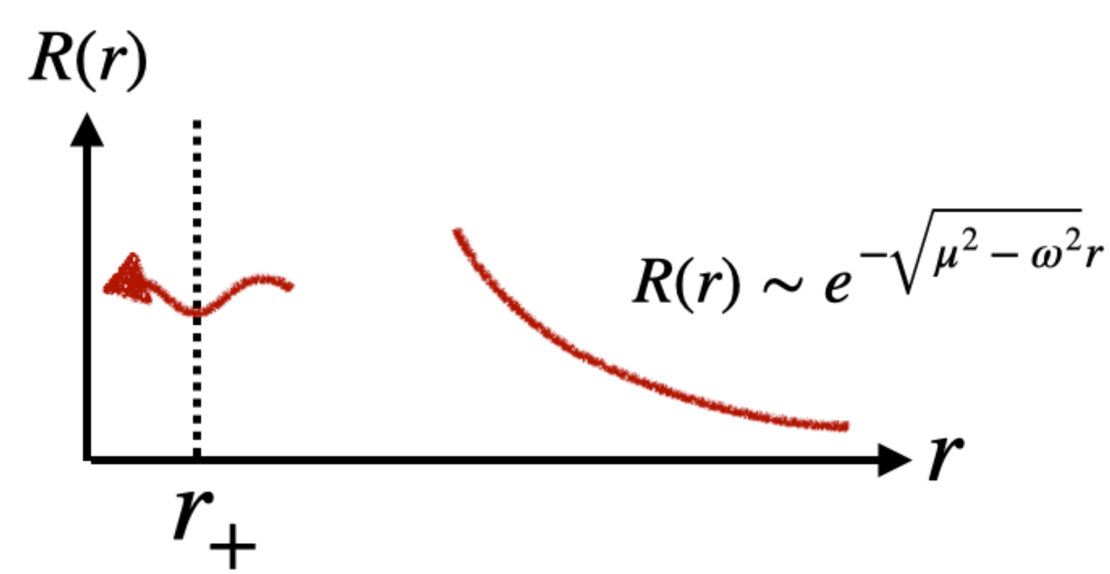
$$\left(-\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right) - c^2\cos^2\theta + \frac{m^2}{\sin^2\theta}\right)S = \Lambda S$$

Bound state
$EM = (\omega - \mu)M < 0$

- Radial part

$$\Delta\frac{d}{dr}\left(\Delta\frac{dR}{dr}\right) + \left(\omega^2(r^2+a^2)^2 - 4aMrm\omega + a^2m^2 - \Delta(\mu^2r^2 + a^2\omega^2 + \Lambda)\right)R = 0$$

- Boundary condition
  - ◉ vanish at infinity
  - ◉ ingoing at the horizon



Gravitational atom

- Let us assume $\mu M \ll 1$, $|EM| = |(\omega - \mu)M| \ll \mu M$

Non-relativistic limit

- In far region $r \gg M$,

$$\left\{ -\frac{1}{2\mu}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{l(l+1)}{r^2}\right] - \frac{\mu M}{r} \right\} R^{\text{far}}(r) = E R^{\text{far}}(r)$$

This equation is same as QM of Hydrogen atom.

- Near horizon

$$R^{\text{near}}(r) \sim \text{ingoing mode}$$

- From matching condition, we have spectrum.

$$E_{nlm} = \omega_{nlm} - \mu = -\frac{\mu(\mu M)^2}{2n^2} + i\Gamma_{nlm}$$

$$\Gamma_{nlm} \propto m\Omega_{\mathrm{H}} - \omega_{nlm}$$

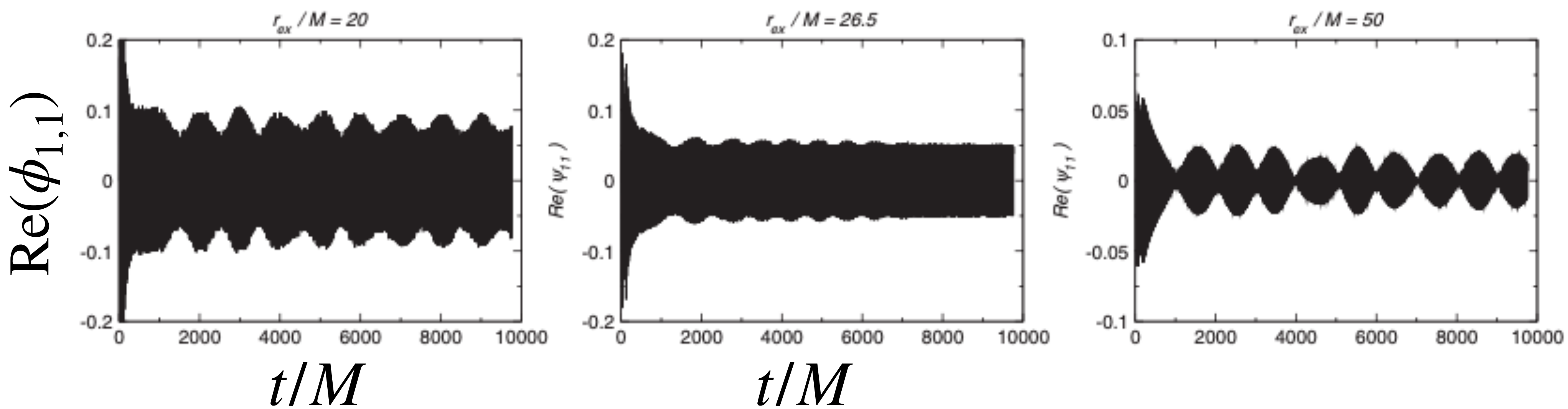
This term is same as spectrum of Hydrogen atom.

Gravitational atom

- Time evolution

Witek et al (2013)

  - Very long life time. $\mu M = 0.42$ $a = 0.99M$

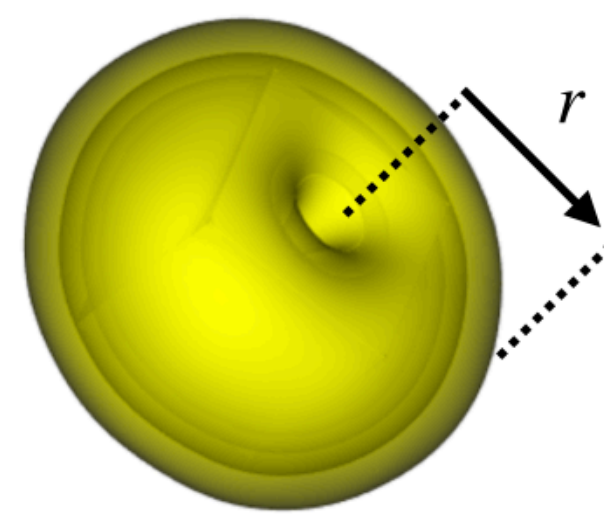


cf: $l = m = 1$

- bound state around BH

➡ QM of hydrogen atom.

Gravitational atom !!

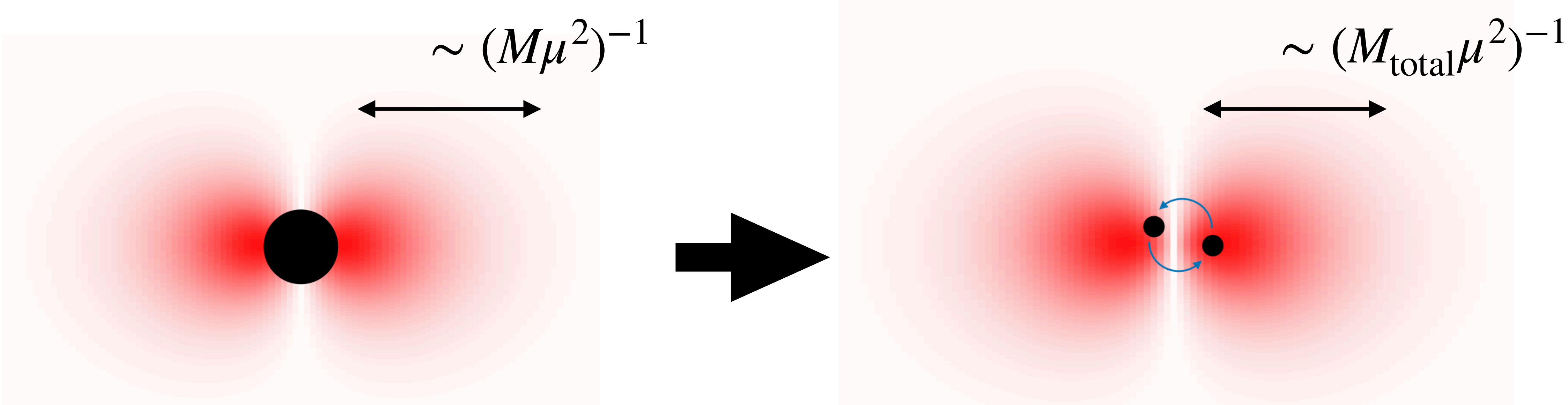


$r \sim (M\mu^2)^{-1}$

Energy density

Our expectation

- Is there gravitational molecule ?
    - It seems possible to replace single BH with BH binary keeping bound state.



gravitational molecule ?

- What we want to do is …
    - Analytic description of the gravitational molecule.
    - Numerical construction of the state

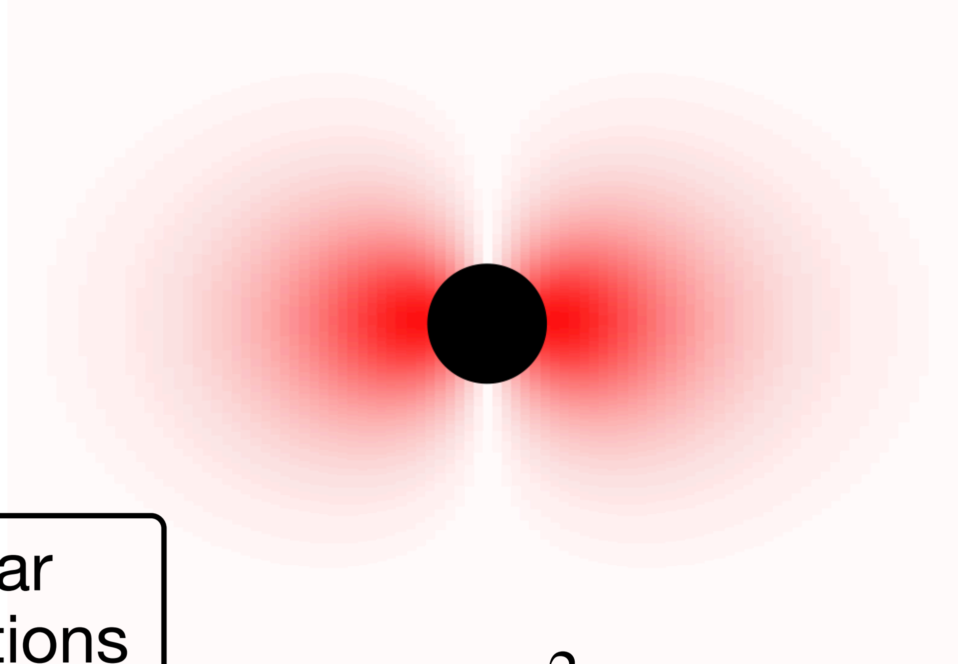
Outline

1. Introduction
2. Gravitational atom
3. Analytic description
   - Gravitational molecule
   - Particle picture
4. Numerical simulations
   - Set up
   - Numerical construction
5. Summary

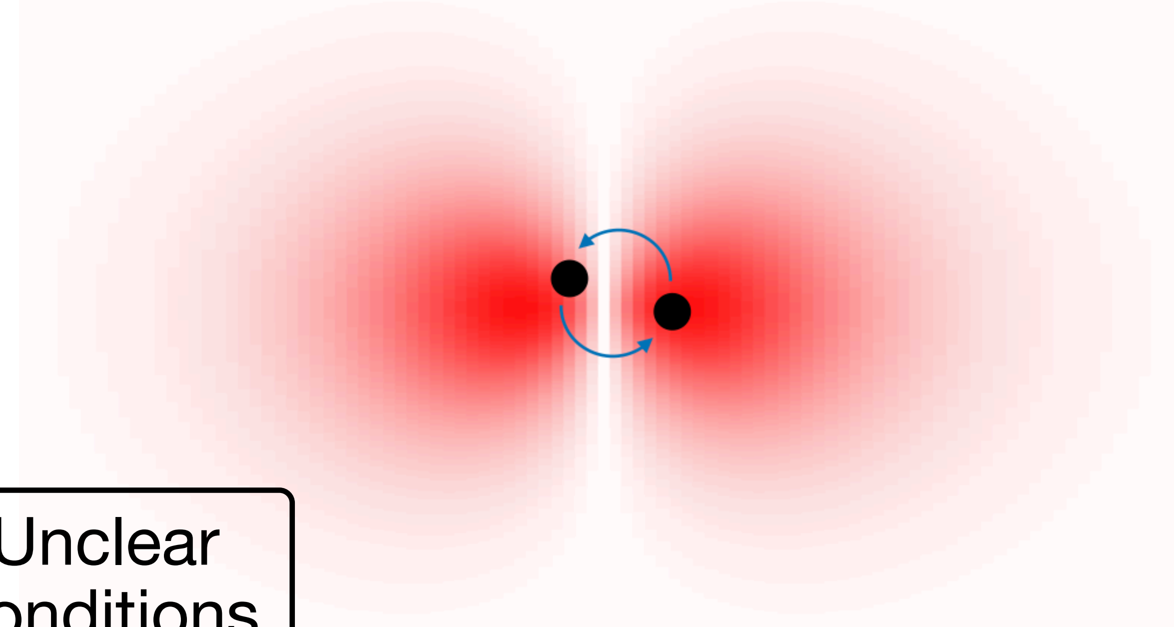
Lessons from single BH

$$\phi = \frac{1}{\sqrt{2\mu}}\left(\varphi e^{-i\mu t} + \mathrm{c.c.}\right)$$

Single BH



BH binary



Clear conditions

$$(\Box_{\mathrm{Kerr}} - \mu^2)\phi = 0$$

Unclear conditions

$$(\Box_{?} - \mu^2)\phi = 0$$

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r)\right)\varphi$$

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r)\right)\varphi$$

$$V(r) = \frac{\mu M}{r}$$

$$V(r) = \frac{\mu M_1}{|r - r_1(t)|} + \frac{\mu M_2}{|r - r_2(t)|}$$

Hydrogen atom : $|n, l, m>$

Di-hydrogen atom

Gravitational atom

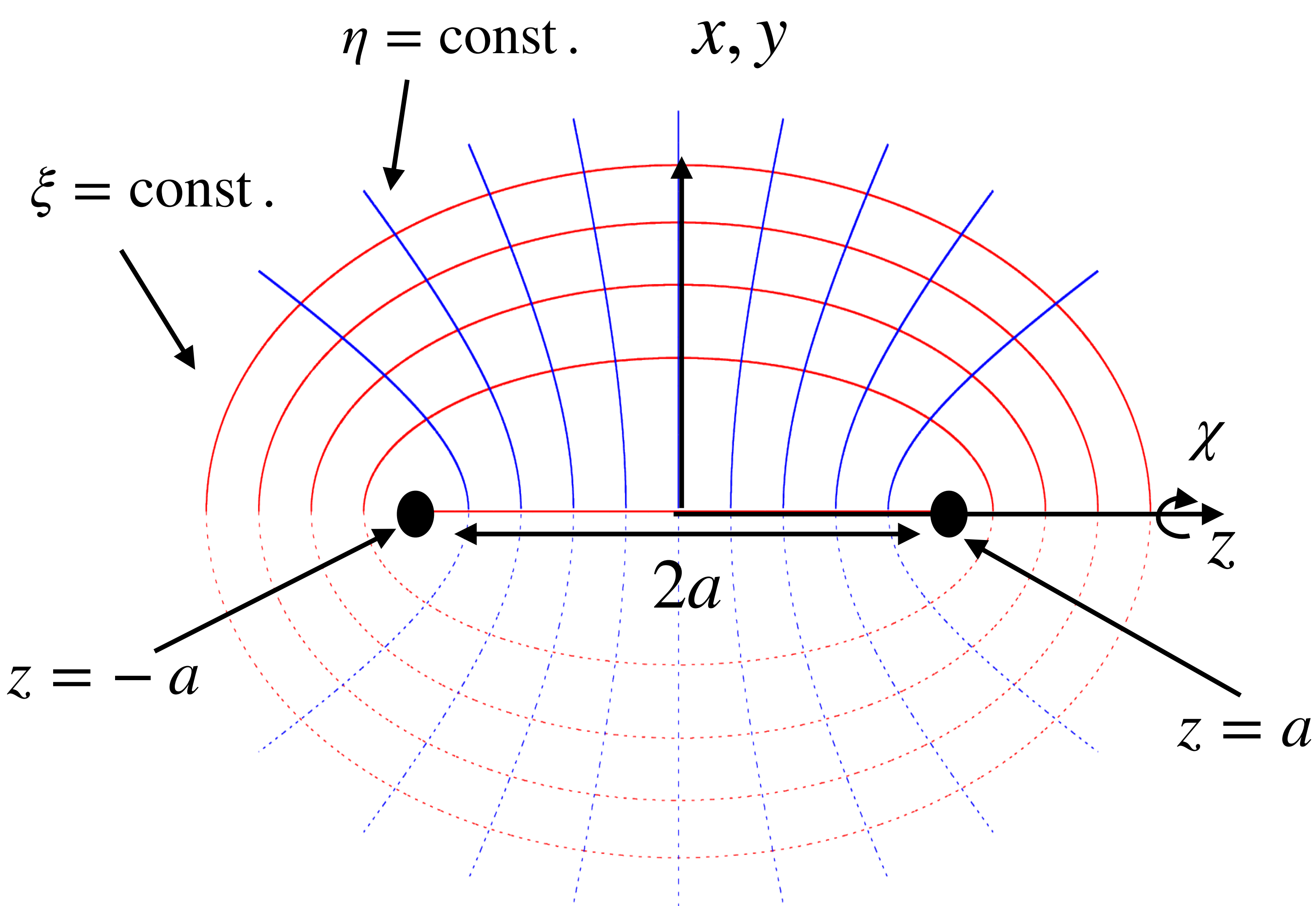
Gravitational molecule

QM in di-hydrogen atom

- In co-moving frame with binary, the problem becomes QM in di-hydrogen atom with “fixed” nuclei.

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r) \right) \varphi \qquad V(r) = \frac{\alpha_1}{|r - r_1|} + \frac{\alpha_2}{|r - r_2|}$$

$$\mu M_i = \alpha_i$$



<u>Coordinate transformation</u>

$$x = a\sqrt{1-\eta^2}\sqrt{\xi^2 - 1}\cos\chi$$

$$y = a\sqrt{1-\eta^2}\sqrt{\xi^2 - 1}\sin\chi$$

$$z = a\eta\xi$$

prolate spheroidal coord.

$$0 \le \chi < 2\pi, -1 \le \eta \le 1, 1 \le \xi$$

QM in di-hydrogen atom

- Schrödinger equation is separable.

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r)\right)\varphi \qquad V(r) = \frac{\alpha_1}{|r - r_1|} + \frac{\alpha_2}{|r - r_2|}$$

$$\Rightarrow \quad \varphi \propto e^{-i\bar{E}t + im_\chi \chi} R(\xi) S(\eta)$$

- Angular part : spheroidal harmonics $\Rightarrow \Lambda$
- Radial part :

$$\partial_\xi \left((\xi^2 - 1)\partial_\xi R\right) + \left(\Lambda - \frac{m_\chi^2}{\xi^2 - 1} + 4\alpha\mu a\xi + 2\mu a^2 \bar{E}\xi^2\right) R = 0$$

- Boundary condition
  - $R \sim e^{-\sqrt{2\mu a^2 |\bar{E}|}\,\xi} \quad (\xi \to \infty)$
  - $R \sim (\xi - 1)^{\frac{m_\chi}{2}} \quad (\xi \to 1)$

$\Rightarrow \bar{E}$

QM in di-hydrogen atom

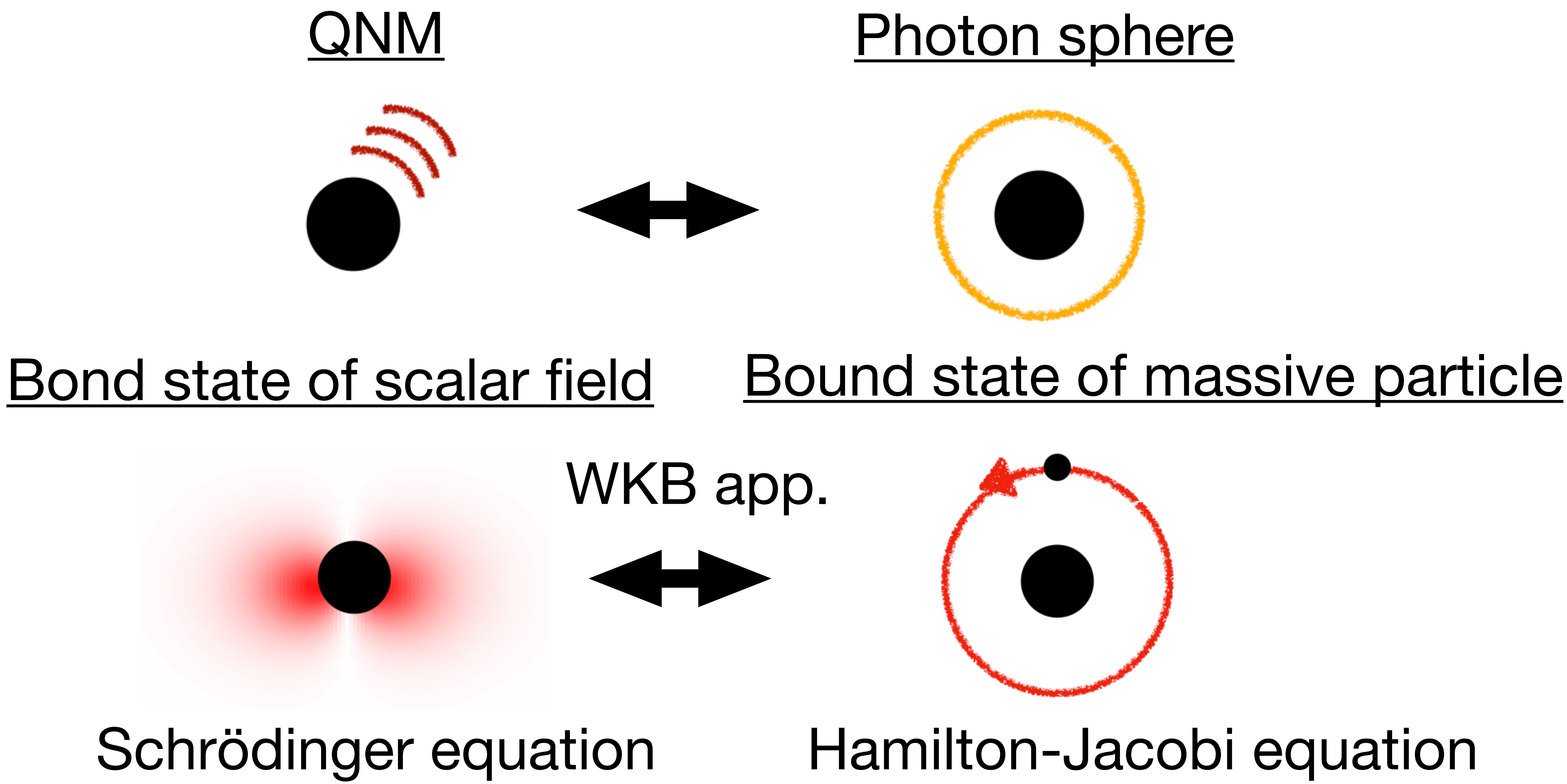
- We can solve these equations using Leaver's method and direct integration method.
- The eigenstate can be labeled by (m_ξ, m_η, m_χ).

Spectrum of molecule

(m_ξ, m_η, m_χ)	$a = 5$		$a = 30$	
	Λ	$10^2 \times \bar{E}$	Λ	$10^2 \times \bar{E}$
(0,0,0)	-0.0129	-0.386	-0.342	-0.272
(1,0,0)	-0.00327	-0.0981	-0.0993	-0.0817
(2,0,0)	-0.00146	-0.0439	-0.0468	-0.0387
(0,2,0)	5.998	-0.0445	5.915	-0.0453
(1,2,0)	5.999	-0.0250	5.952	-0.0254
(2,2,0)	5.999	-0.0160	5.970	-0.0162

Particle description

- Particle analogies of wave equation often play an important role in several physical phenomena.



WKB approximation

- The bound state is described by Schrödinger equation.

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r) \right)\varphi \qquad V(r) = \frac{\mu M_1}{|r - r_1|} + \frac{\mu M_2}{|r - r_2|}$$

- WKB approximation

$$\varphi = e^{iS(t,\xi,\eta,\chi)} \qquad S(t,\xi,\eta,\chi) = S_\xi(\xi) + S_\eta(\eta) + m_\chi \chi - Et$$

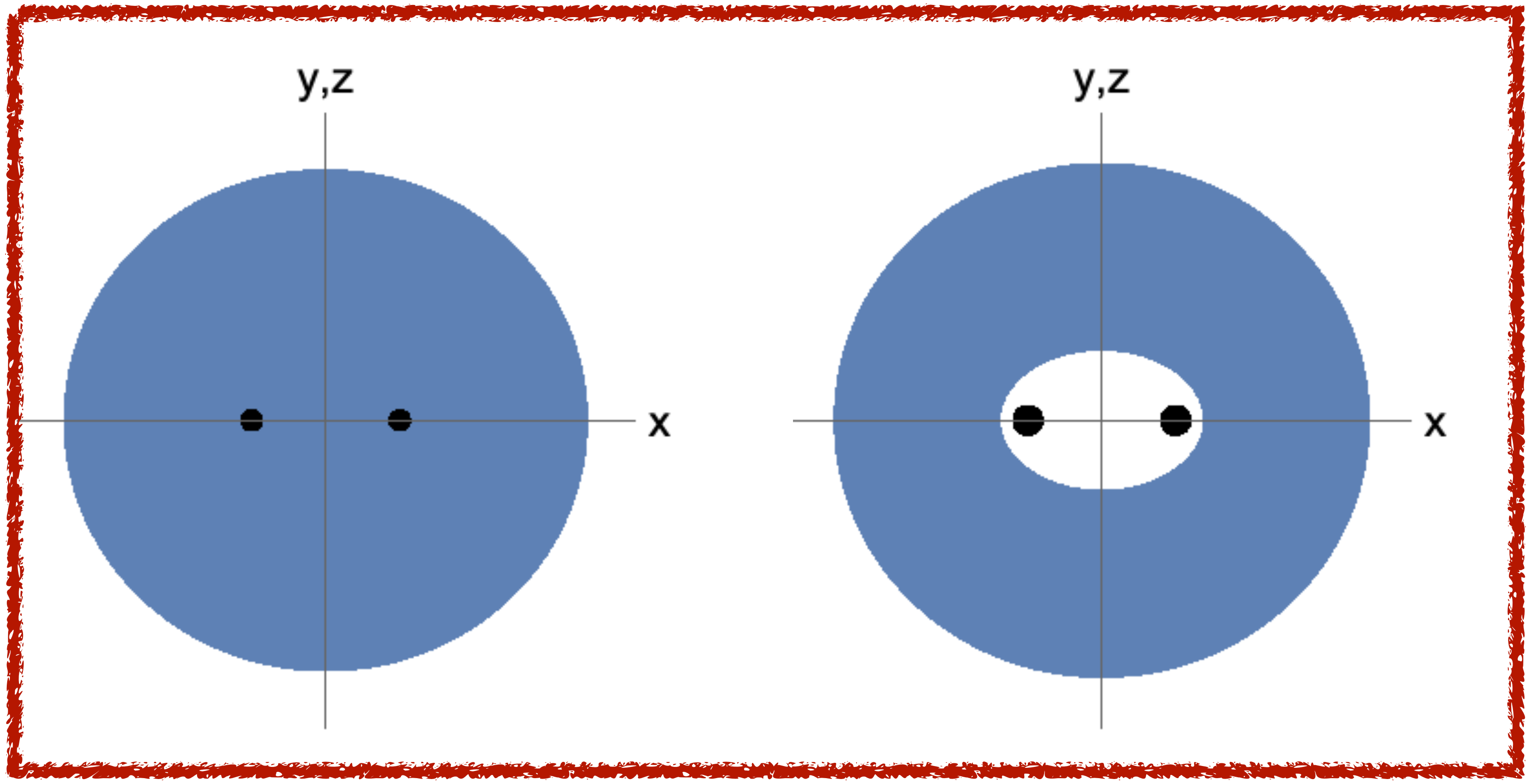
- Hamilton-Jacobi equation

$$(S'_\eta)^2 = \frac{2a^2\mu}{1-\eta^2}\left(\frac{l^2}{2a^2\mu} - \bar{E}\eta^2 + \frac{\mu \Delta M}{a}\eta - \frac{m_\chi^2}{2a^2\mu}\frac{1}{1-\eta^2} \right) > 0$$

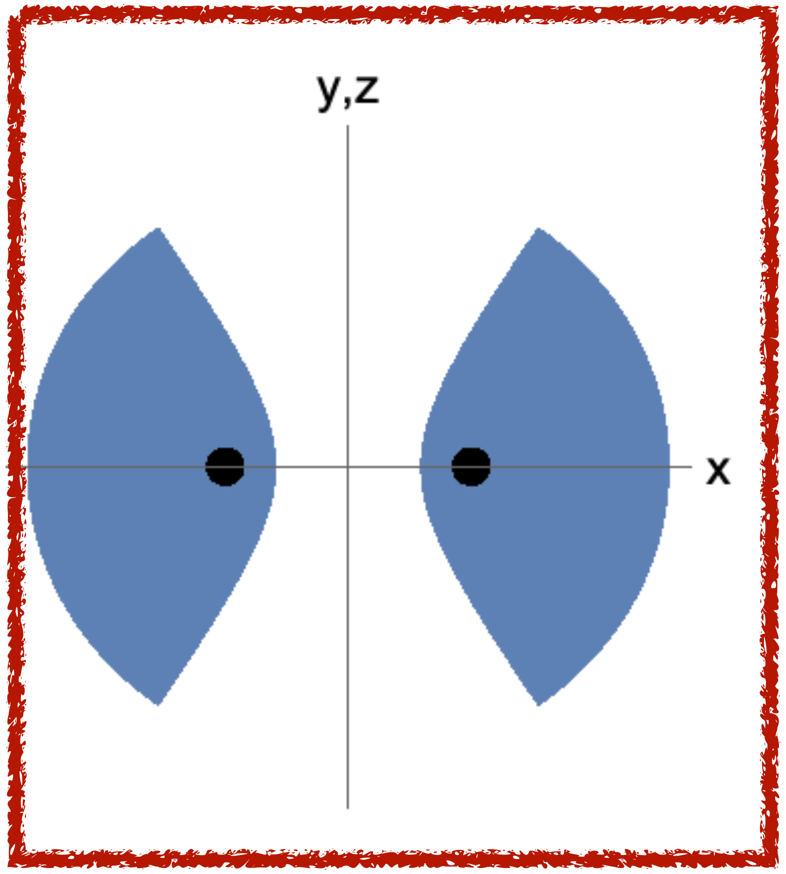
$$(S'_\xi)^2 = \frac{2a^2\mu}{\xi^2-1}\left(-\frac{l^2}{2a^2\mu} + \bar{E}\xi^2 + \frac{\mu M}{a}\xi - \frac{m_\chi^2}{2a^2\mu}\frac{1}{\xi^2-1} \right) > 0$$

Classically allowed region

- Classically allowed region gives us hint of the topology of bound state.



Gravitational molecule



Gravitational atom around individual BHs

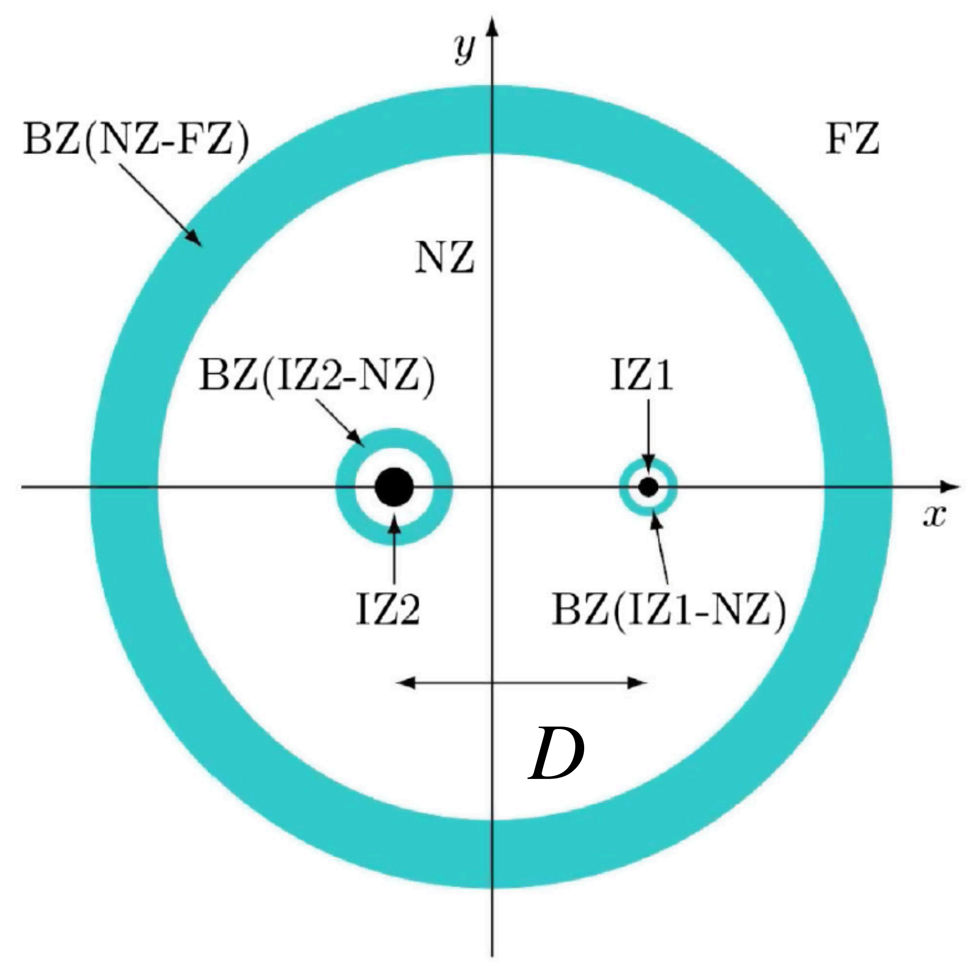
Outline

1. Introduction
2. Gravitational atom
3. Analytic description
   - Gravitational molecule
   - Particle picture
4. Numerical simulations
   - Set up
   - Numerical construction
5. Summary

Approximated BHB spacetime

- Construction of the metric $A = 1{,}2$
  - Inner Zones (IZ) : $0 < r_A \ll r_{12}$
    - a perturbed Schwarzschild B
  - Near Zone (NZ) : $m_A \ll r_A \ll \lambda$
    - PN approximation
  - Far Zone (FZ) : $\lambda \ll r < \infty$
    - PM approximation
  - Buffer Zone (BZ)
    - Asymptotic matching
- We assume circular orbit.
- We neglect emission of GW.
  - BH separation is constant.

ref: PRD89,084008(2014)



BHB spacetime (BH1, BH2)

D : BH separation

$m_1 = m_2 = M/2$

Initial data

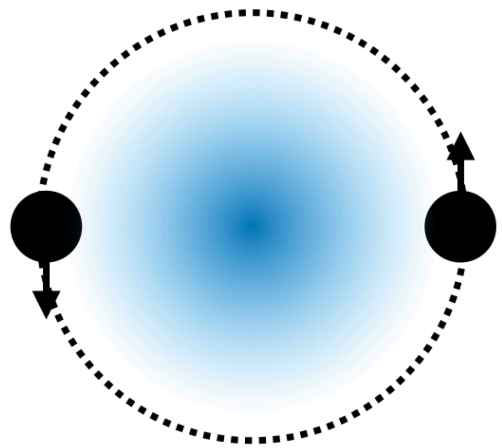
- We use three different initial data.

$$\phi = R(r)\mathscr{A}(t,\theta,\varphi)$$

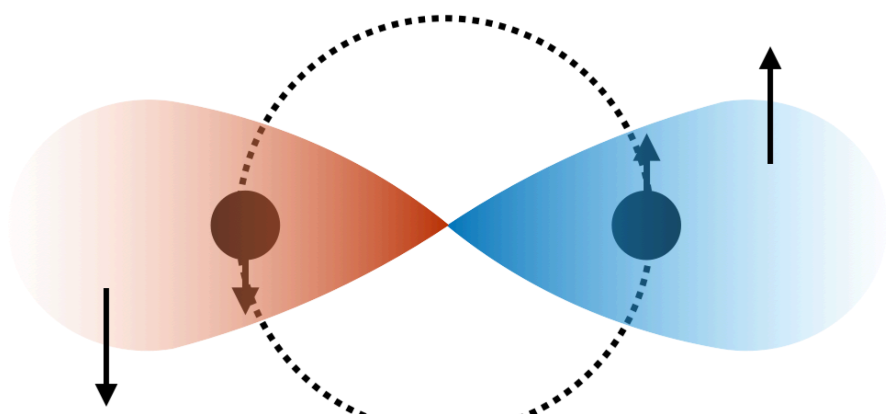
$$\begin{cases} \phi = Ae^{-\frac{1}{2}\left(\frac{r}{w}\right)^2} \\ (\partial_t - \mathscr{L}_\beta)\phi = 0 \end{cases}$$

$$\begin{cases} R = A_0 r e^{-\frac{r}{2w}} \\ \mathscr{A} = \mathrm{Re}\left(Y_{1,1}(\theta,\varphi)\ e^{-i\mu t}\right) \end{cases}$$

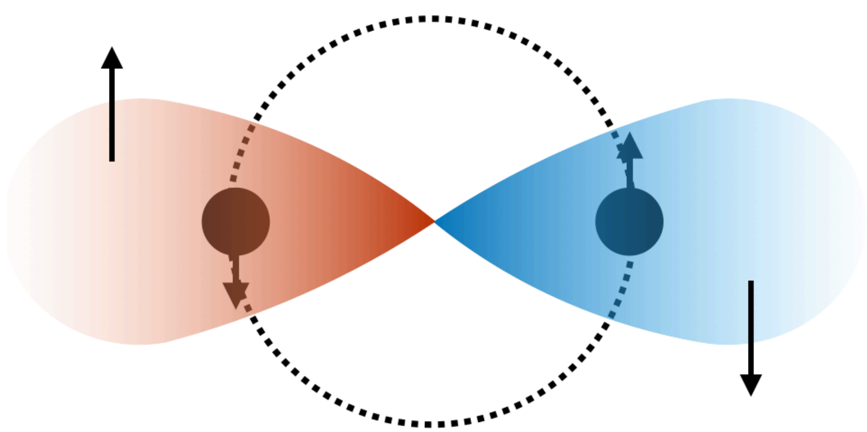
$$\begin{cases} R = A_0 r e^{-\frac{r}{2w}} \\ \mathscr{A} = \mathrm{Re}\left(Y_{1,1}(\theta,\varphi)\ e^{+i\mu t}\right) \end{cases}$$



Gaussian profile



Co-rotating dipole



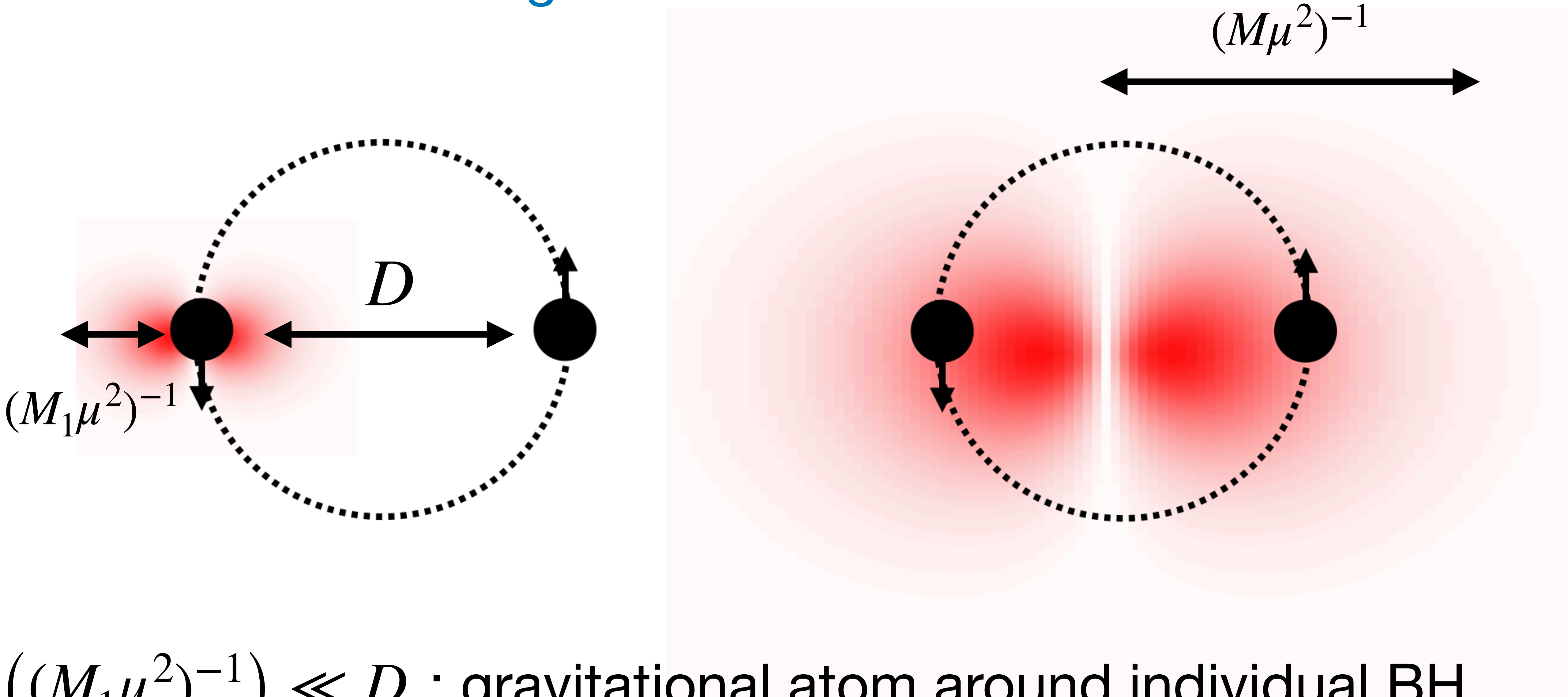
Counter-rotating dipole

Typical length scale

$$\left(\Box - \mu^2\right)\phi = 0$$

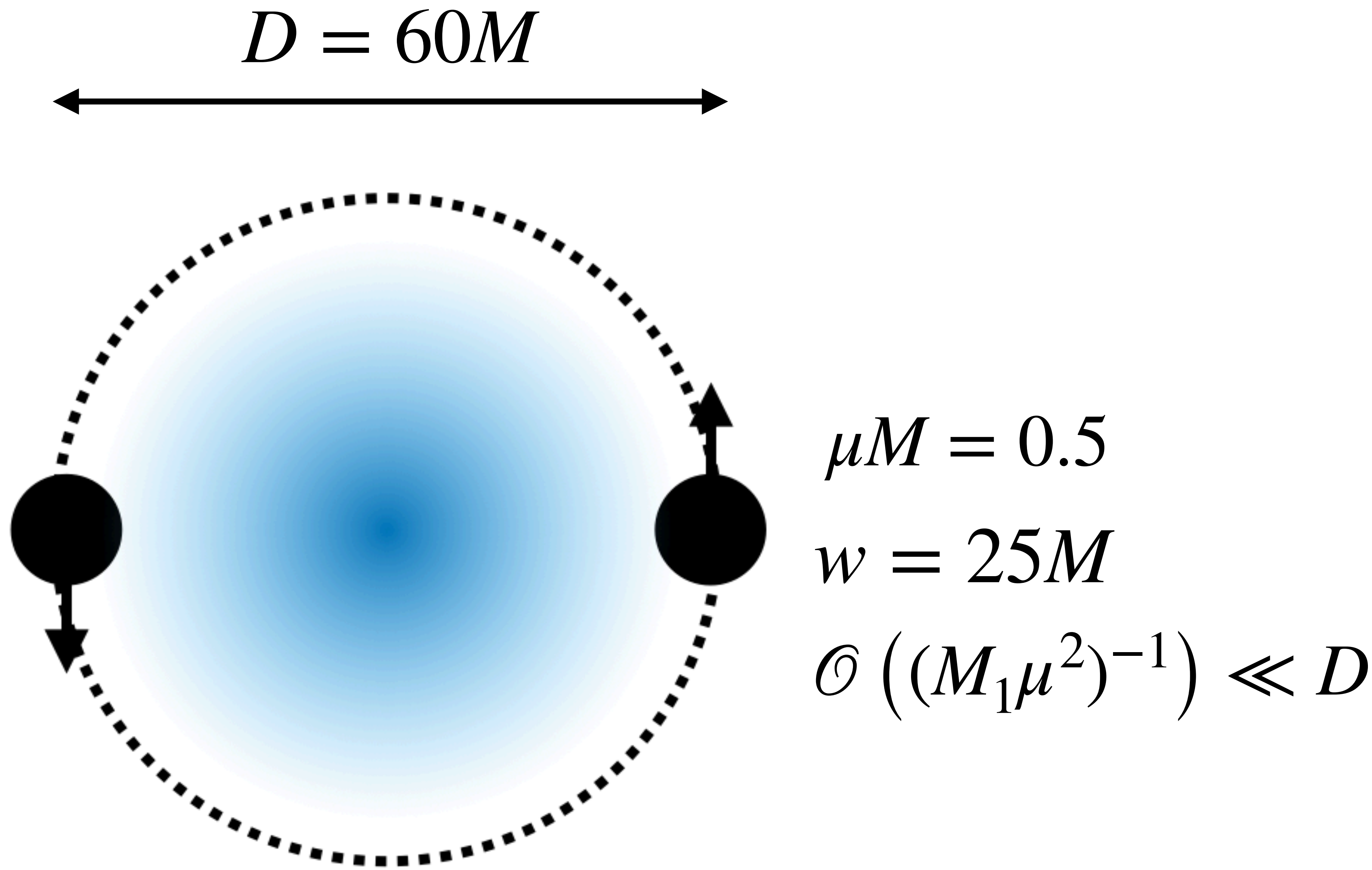
$$M_1 = M_2 = \frac{M}{2}$$

- We have three length scales.



$\mathcal{O}\left((M_1\mu^2)^{-1}\right) \ll D$: gravitational atom around individual BH

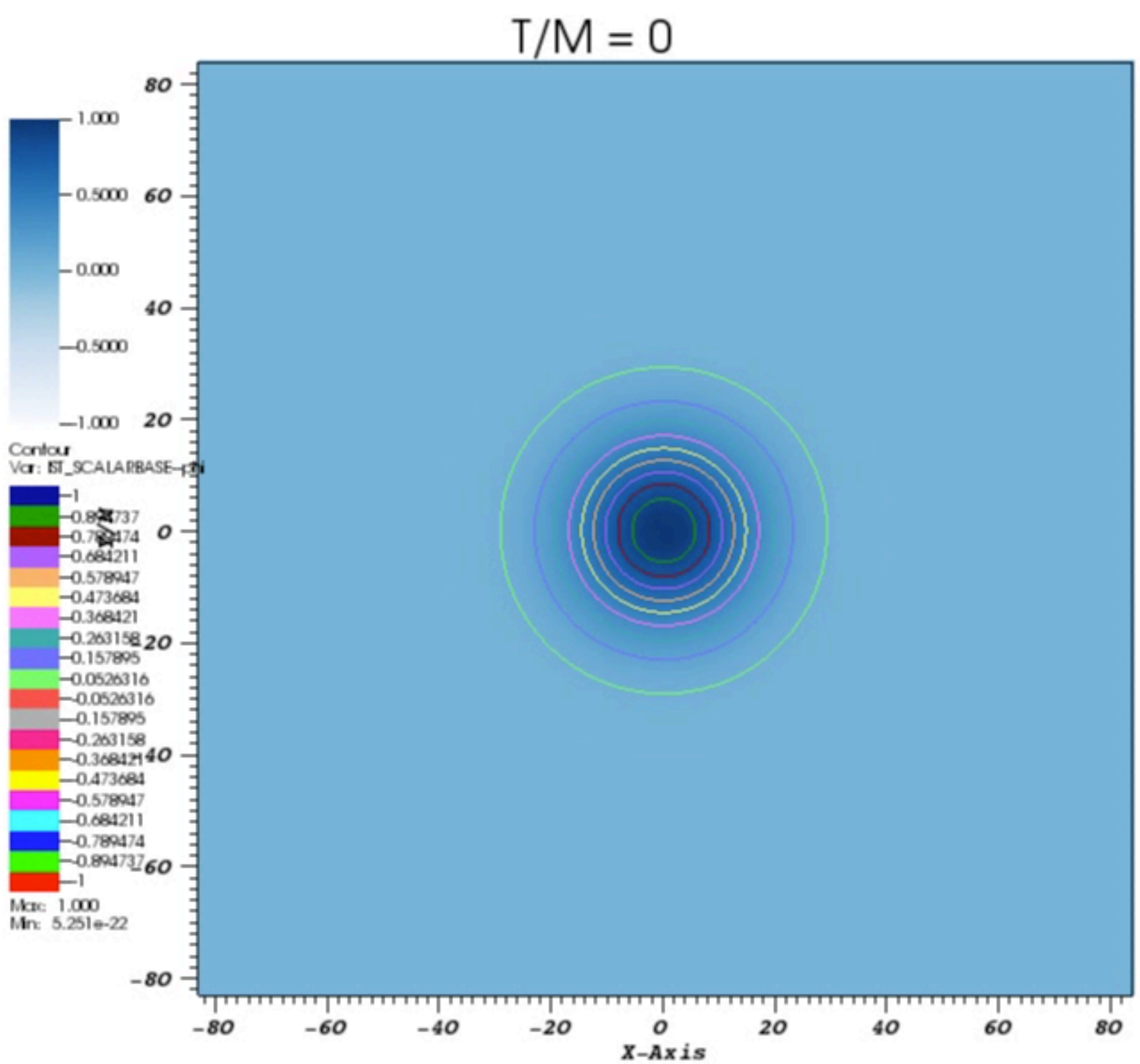
$\mathcal{O}\left((M\mu^2)^{-1}\right) \gtrsim D$: gravitational molecule around BH binary



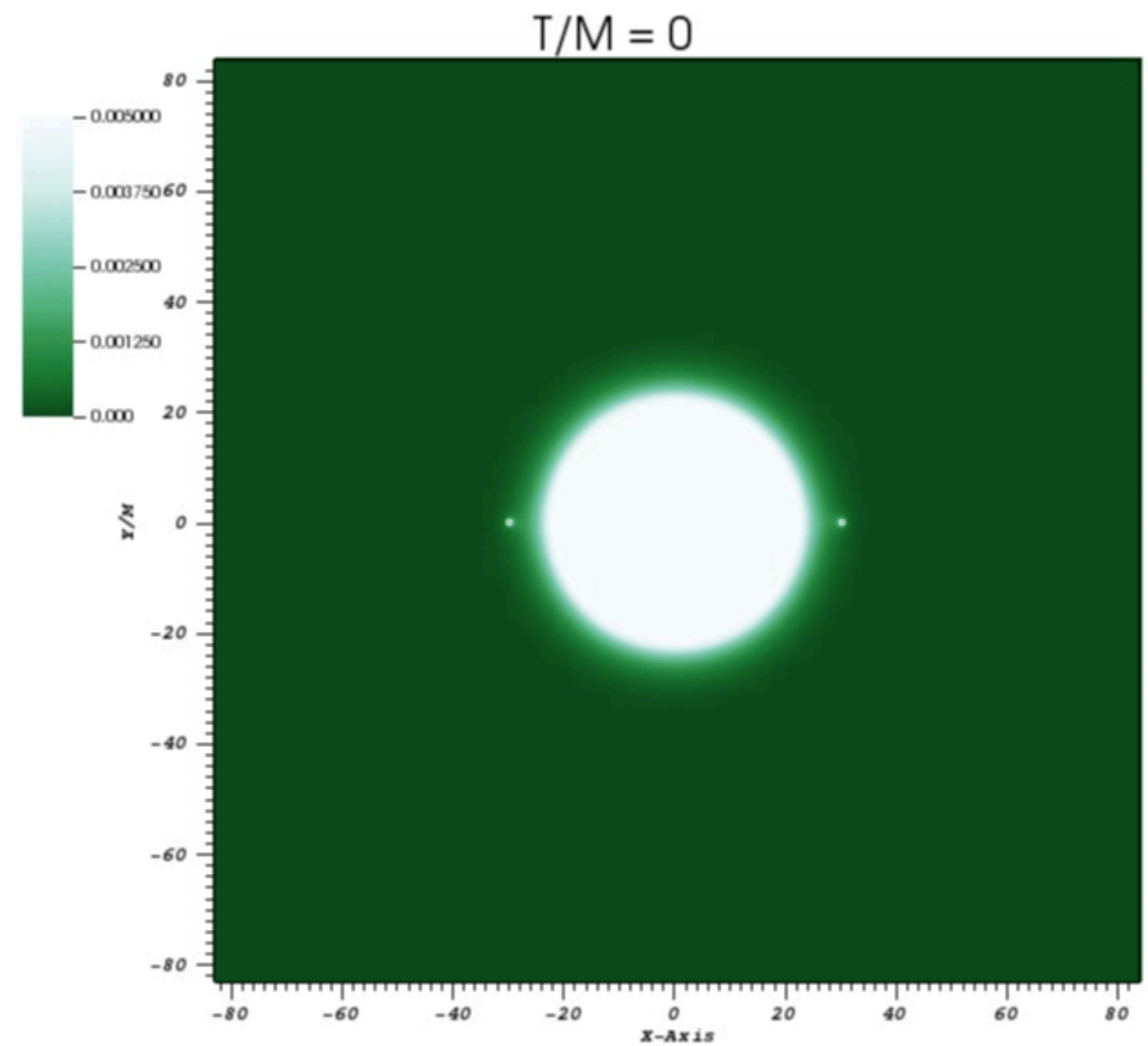
Simulation 1 : Gaussian initial data

Simulation 1

Scalar field

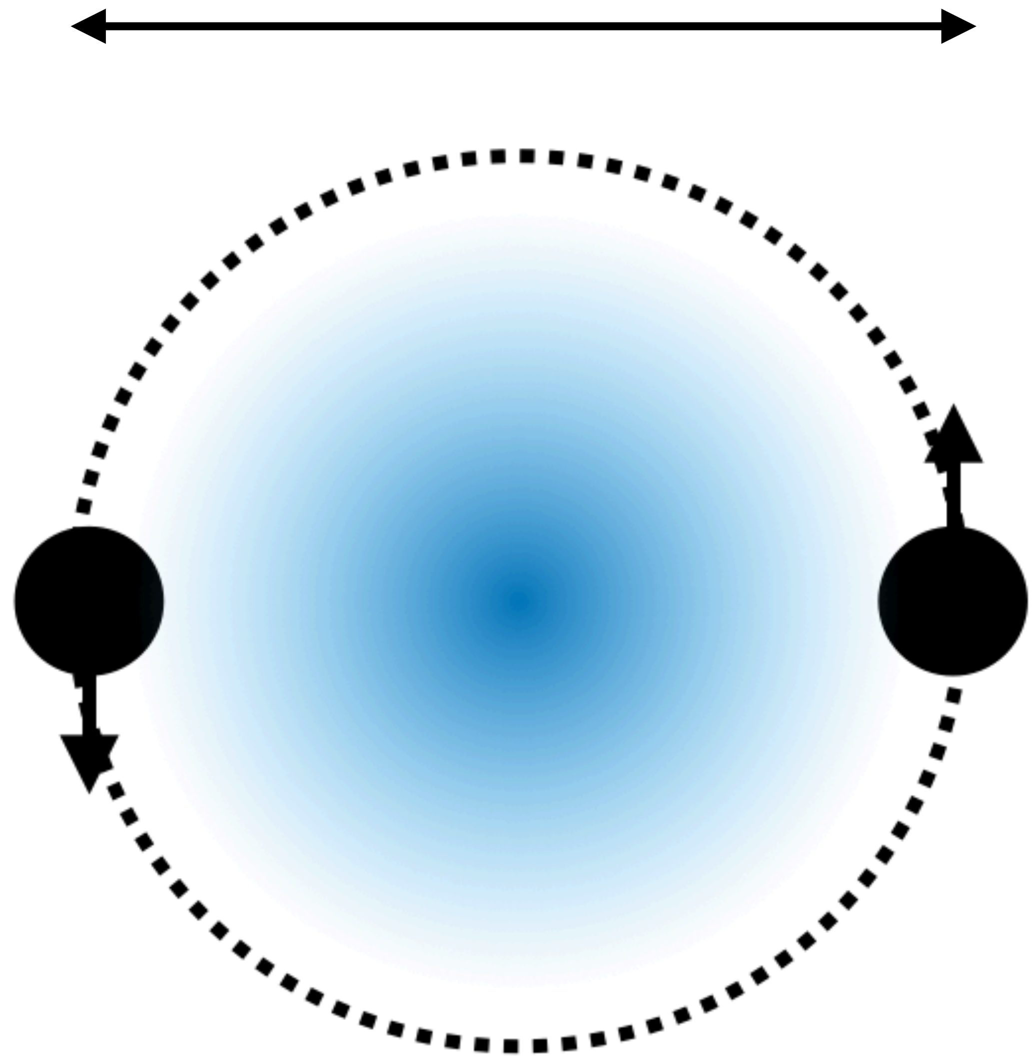


Energy density



- Gravitational atom around individual BHs.

$D = 10M,\ 60M$



$\mu M = 0.2$

$w = 25M$

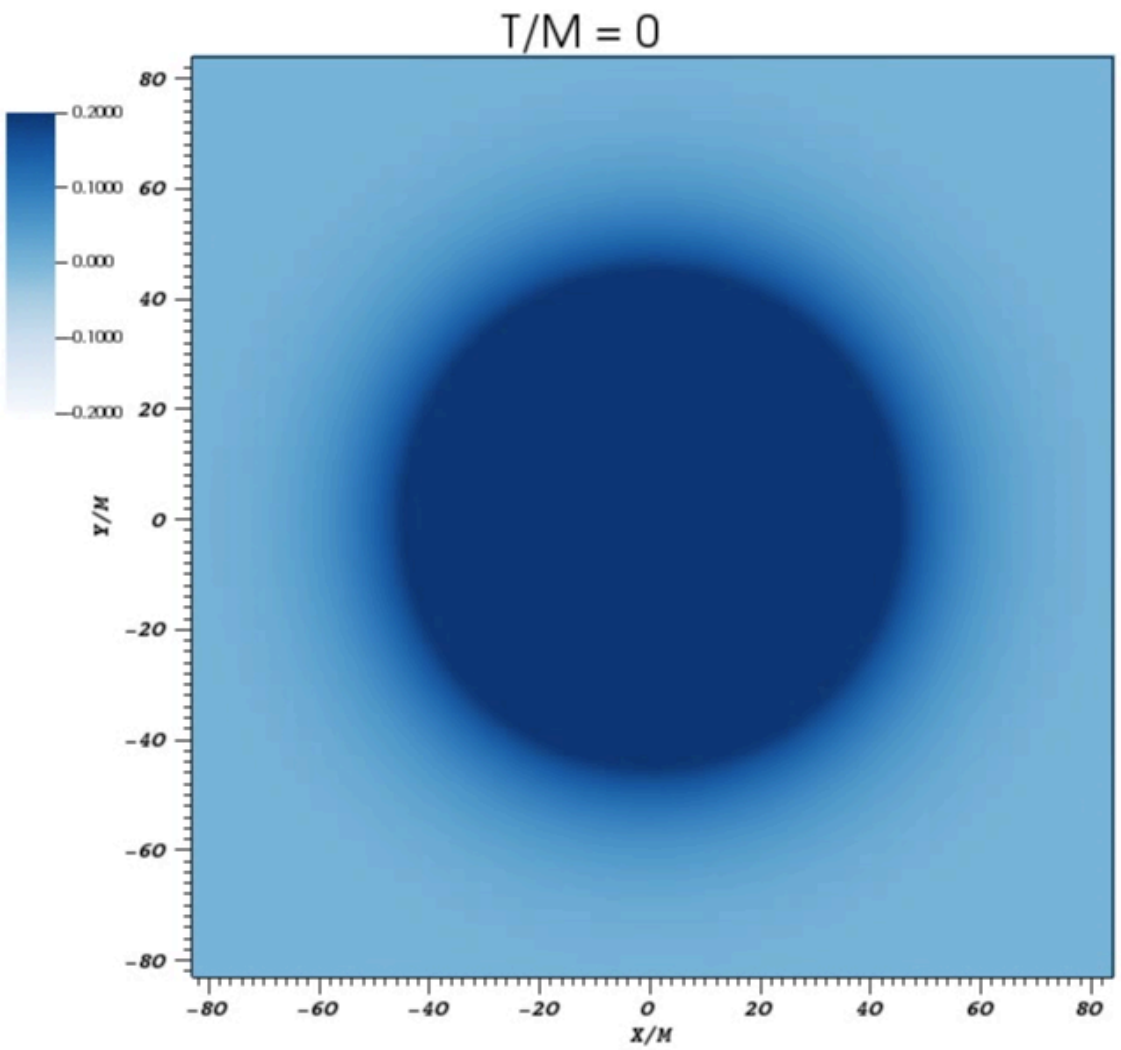
$\mathcal{O}\left((M\mu^2)^{-1}\right) \gtrsim D$

Simulation 2 : Gaussian initial data

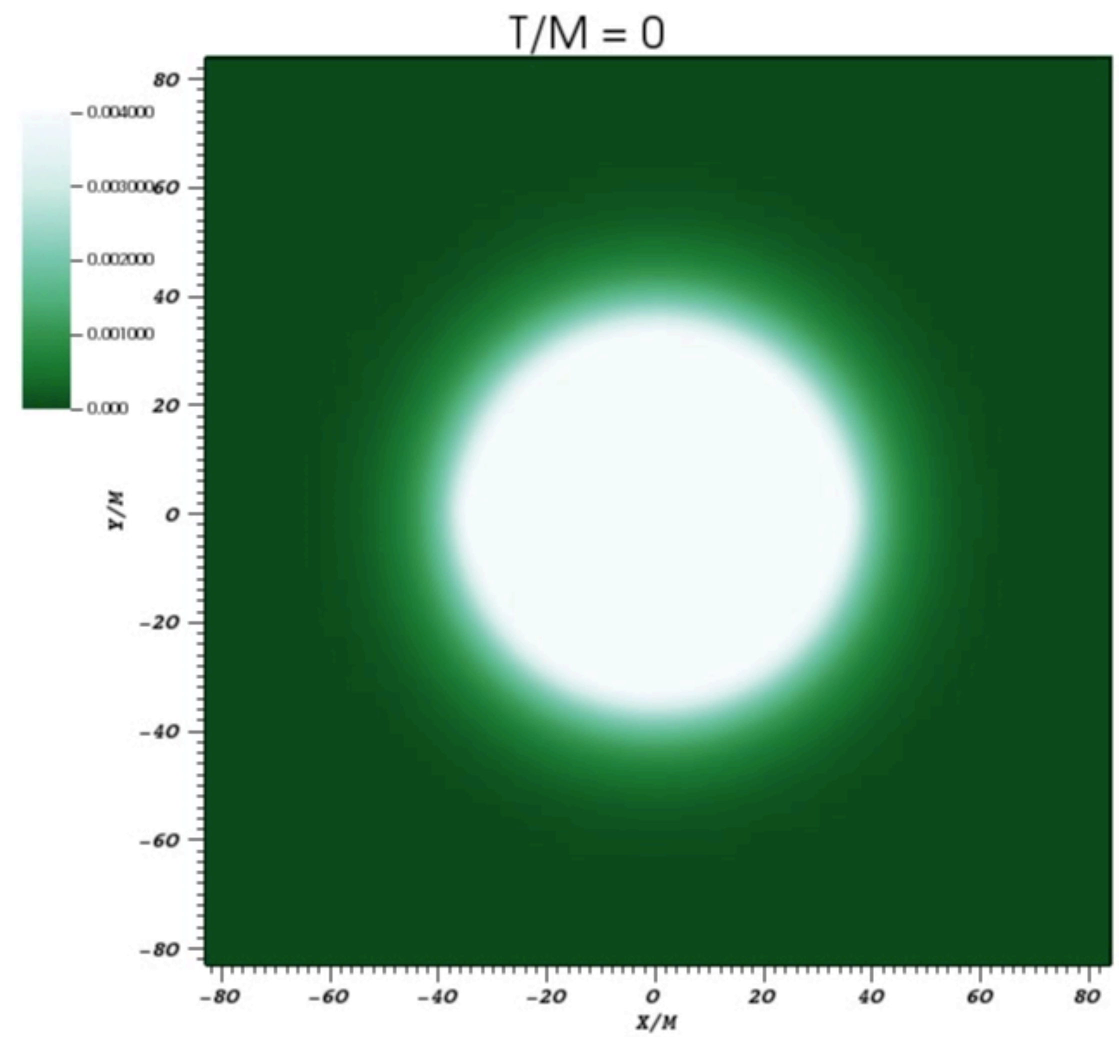
Simulation 2

$D = 60M$

Scalar field



Energy density

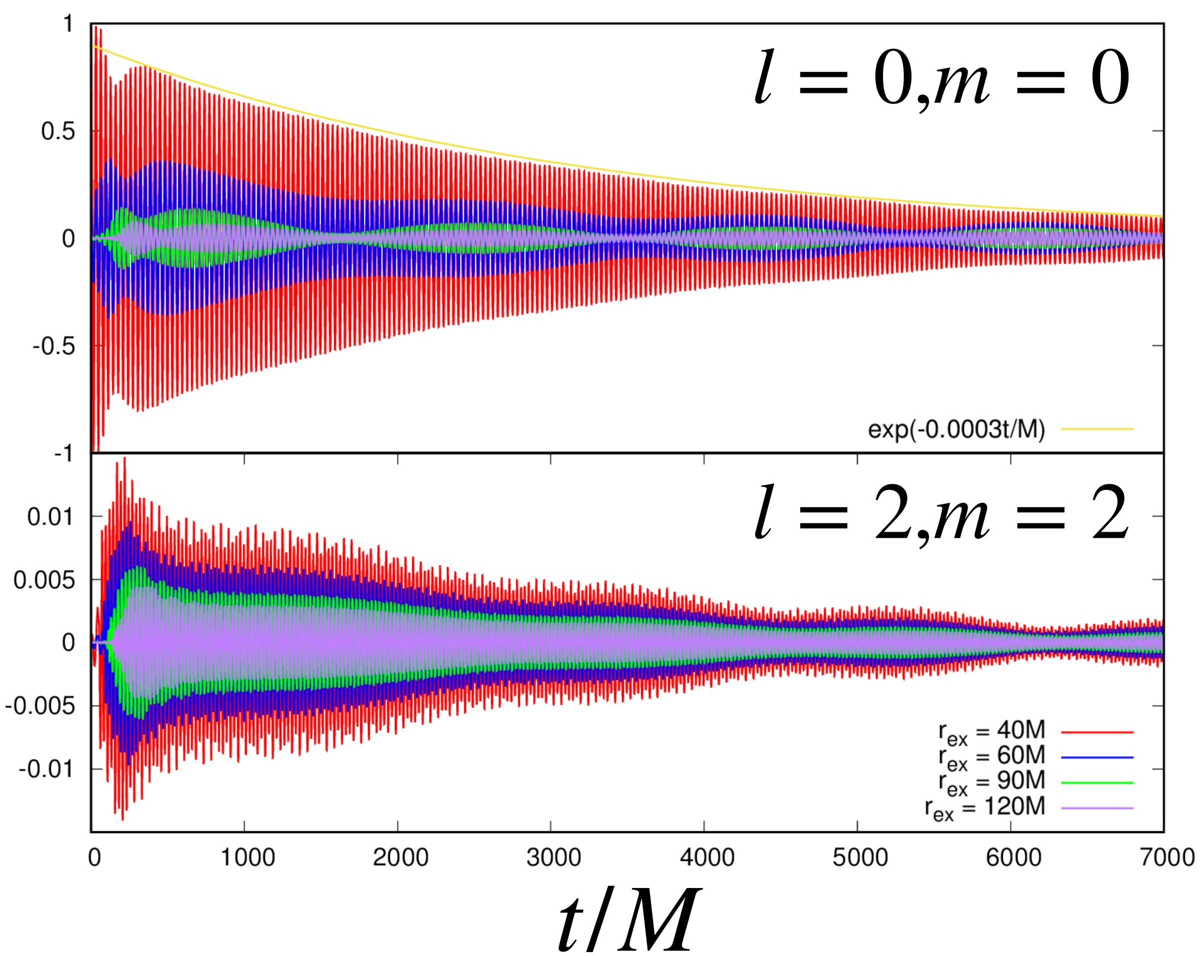


- “Monopole” gravitational molecule around BH binary.

Simulation 2

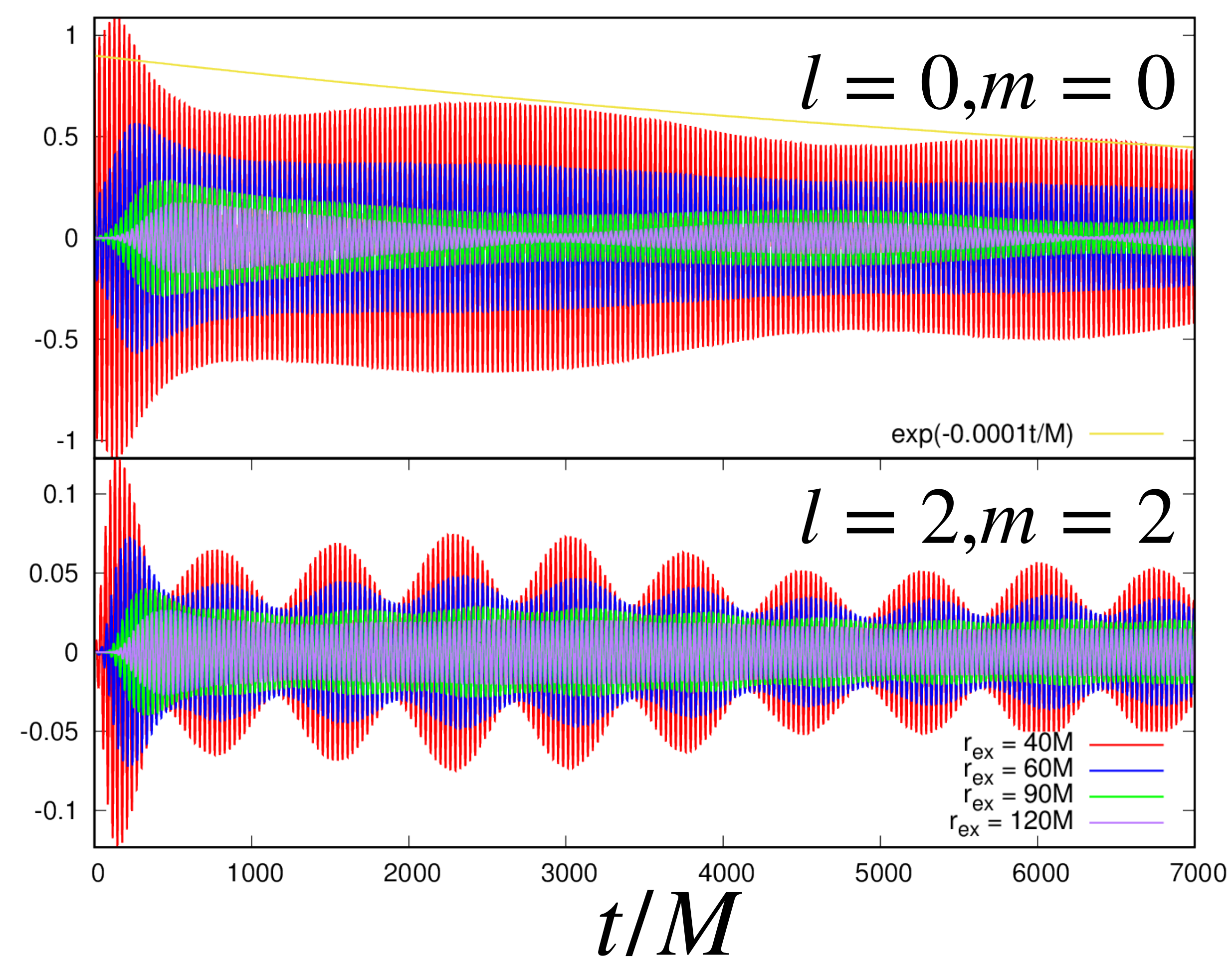
cf: $\phi_{00} \sim e^{-\frac{t}{\tau}}$

$D = 10M$



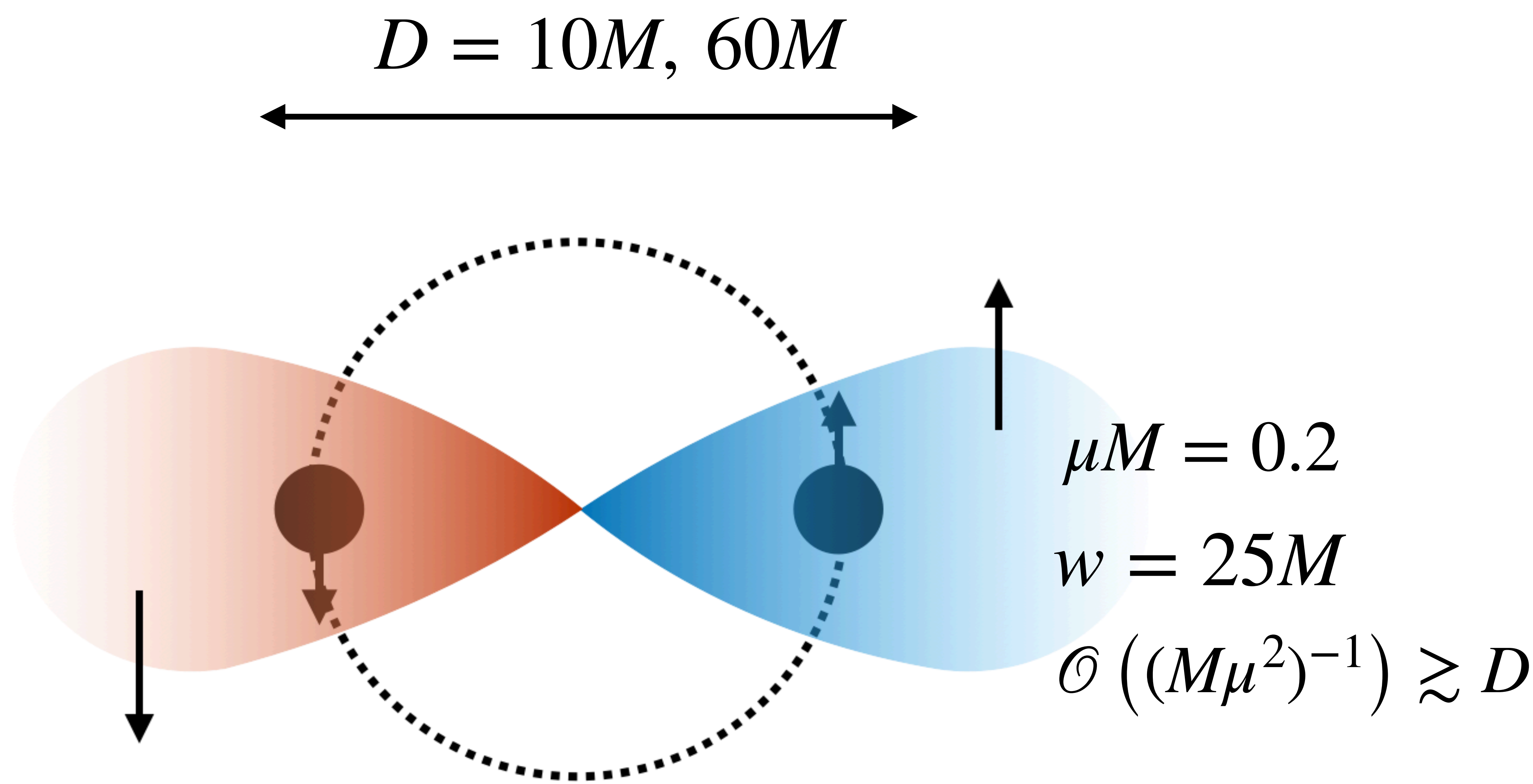
$\tau \simeq 3 \times 10^3 M$

$D = 60M$



$\tau \simeq 1 \times 10^4 M$

- The spectrum of numerical simulation is good agreement with Di-hydrogen atom.

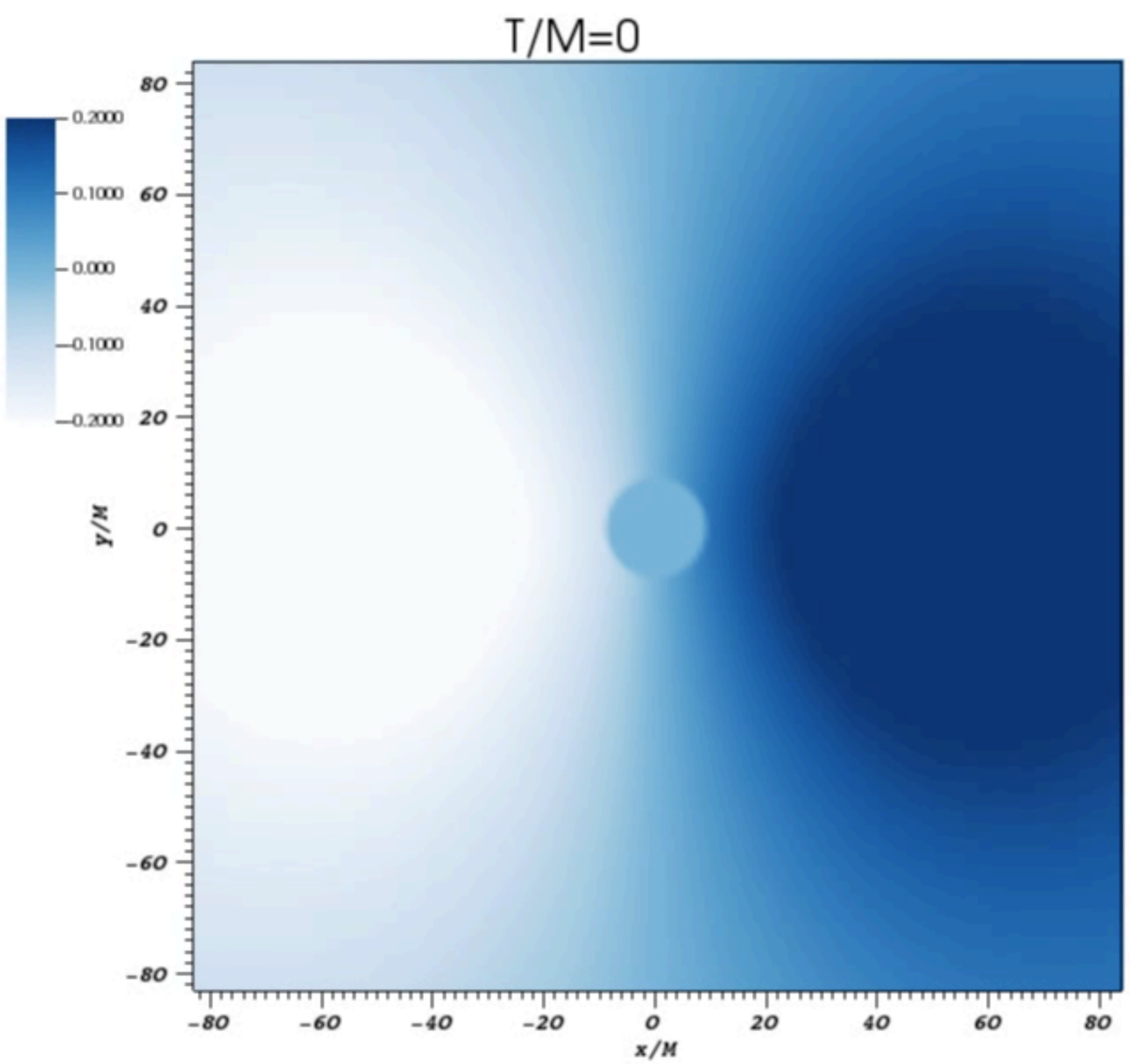


Simulation 3 :
Co-rotating dipole initial data

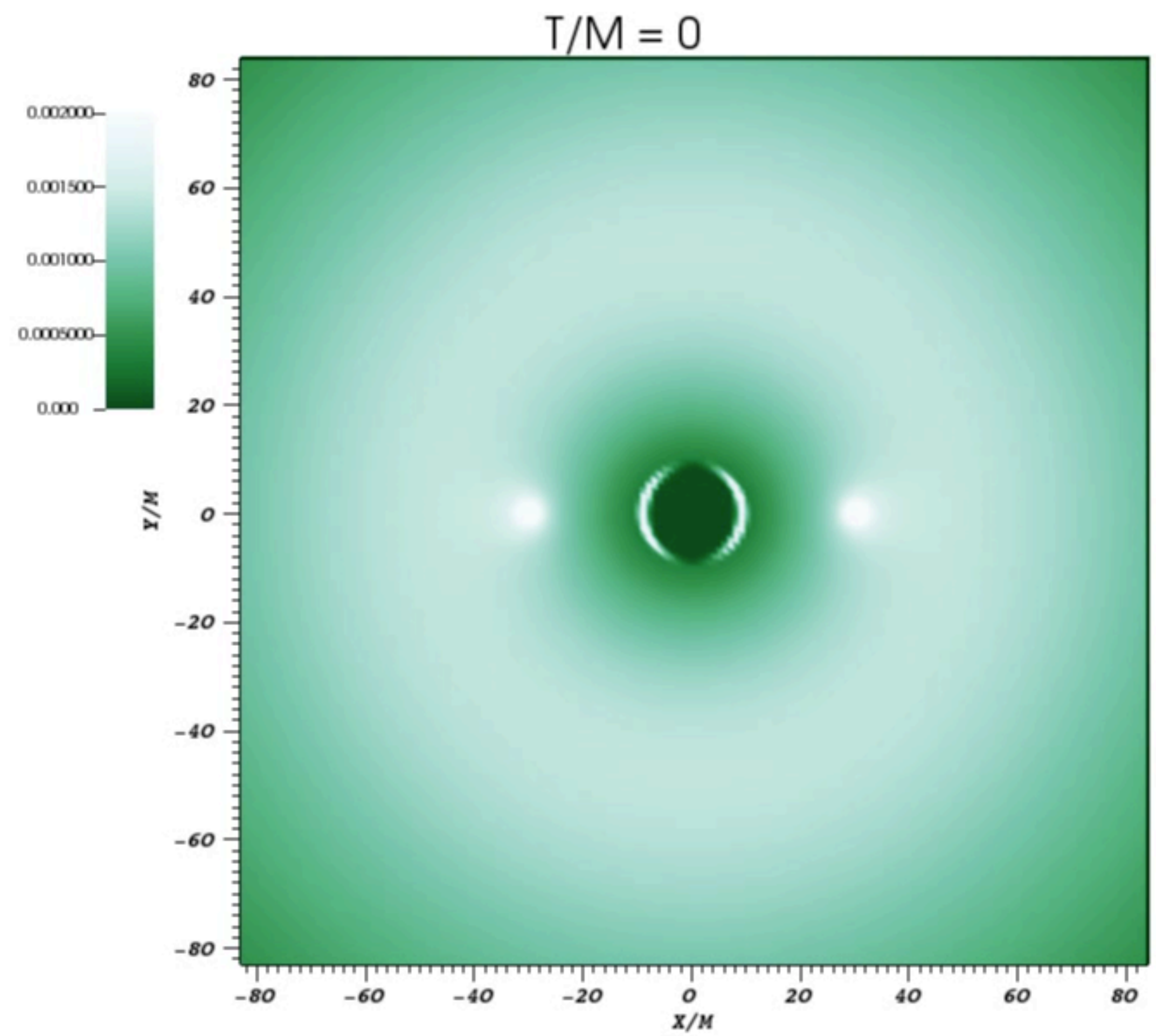
Simulation 3

$D = 60M$

Scalar field



Energy density

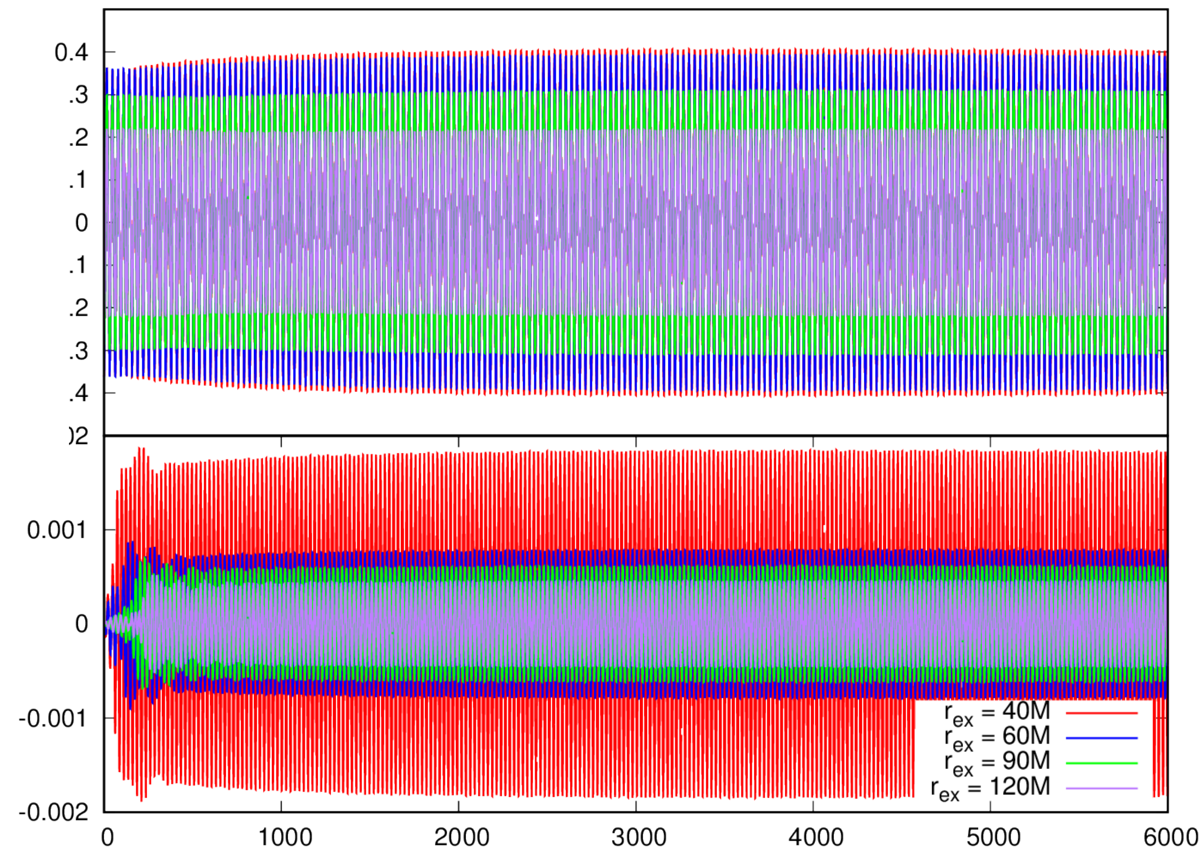


- Energy density rotates with binary.
- “Dipole” co-rotating gravitational molecule around BH binary.

Simulation 3

$D = 10M$

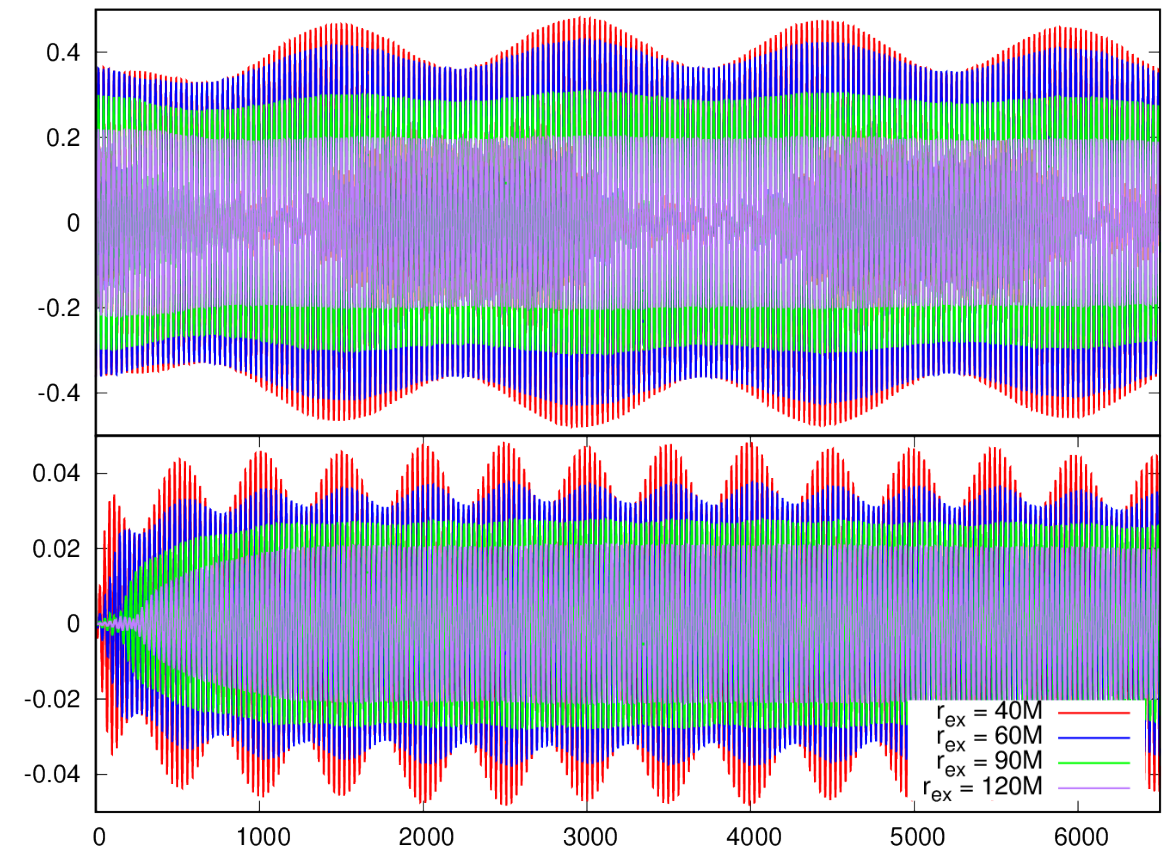
$l = 1, m = 1$



t/M $\quad l = 3, m = 3$

$D = 60M$

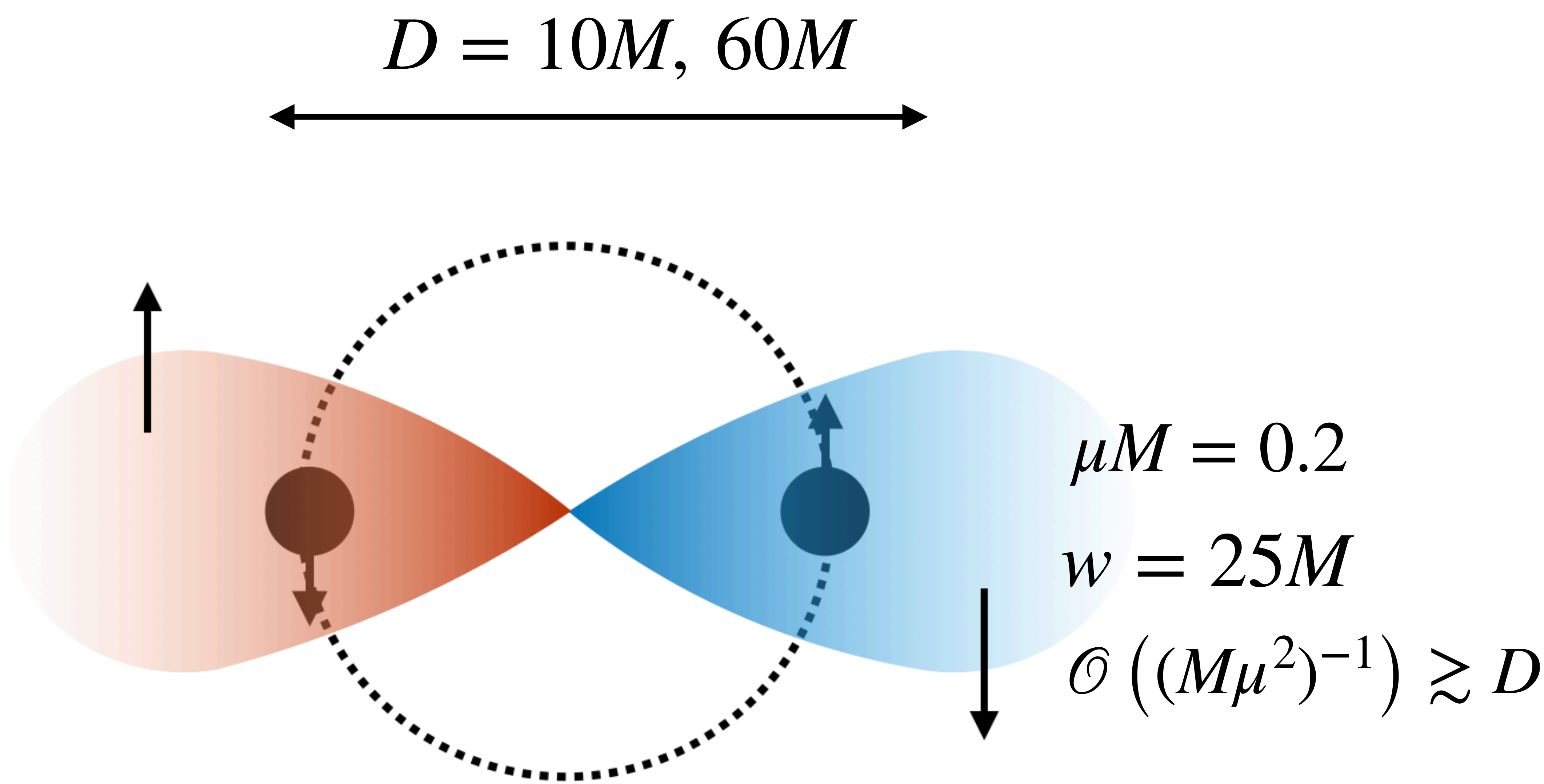
$l = 1, m = 1$



t/M $\quad l = 3, m = 3$

We could not get decaying time scale.

At least, $\tau \gg 10^4 M$



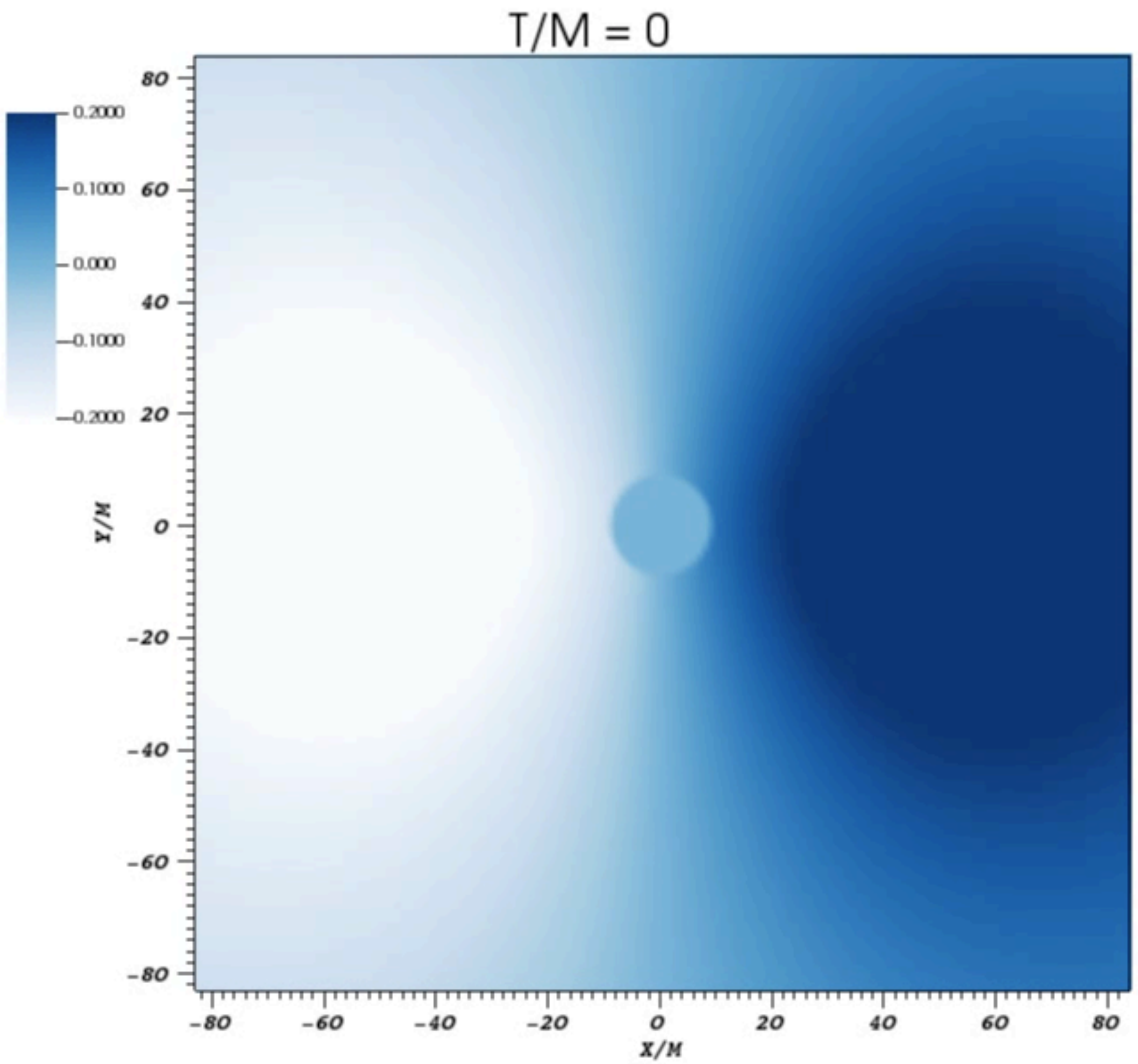
Simulation 4 :
Counter-rotating dipole initial data

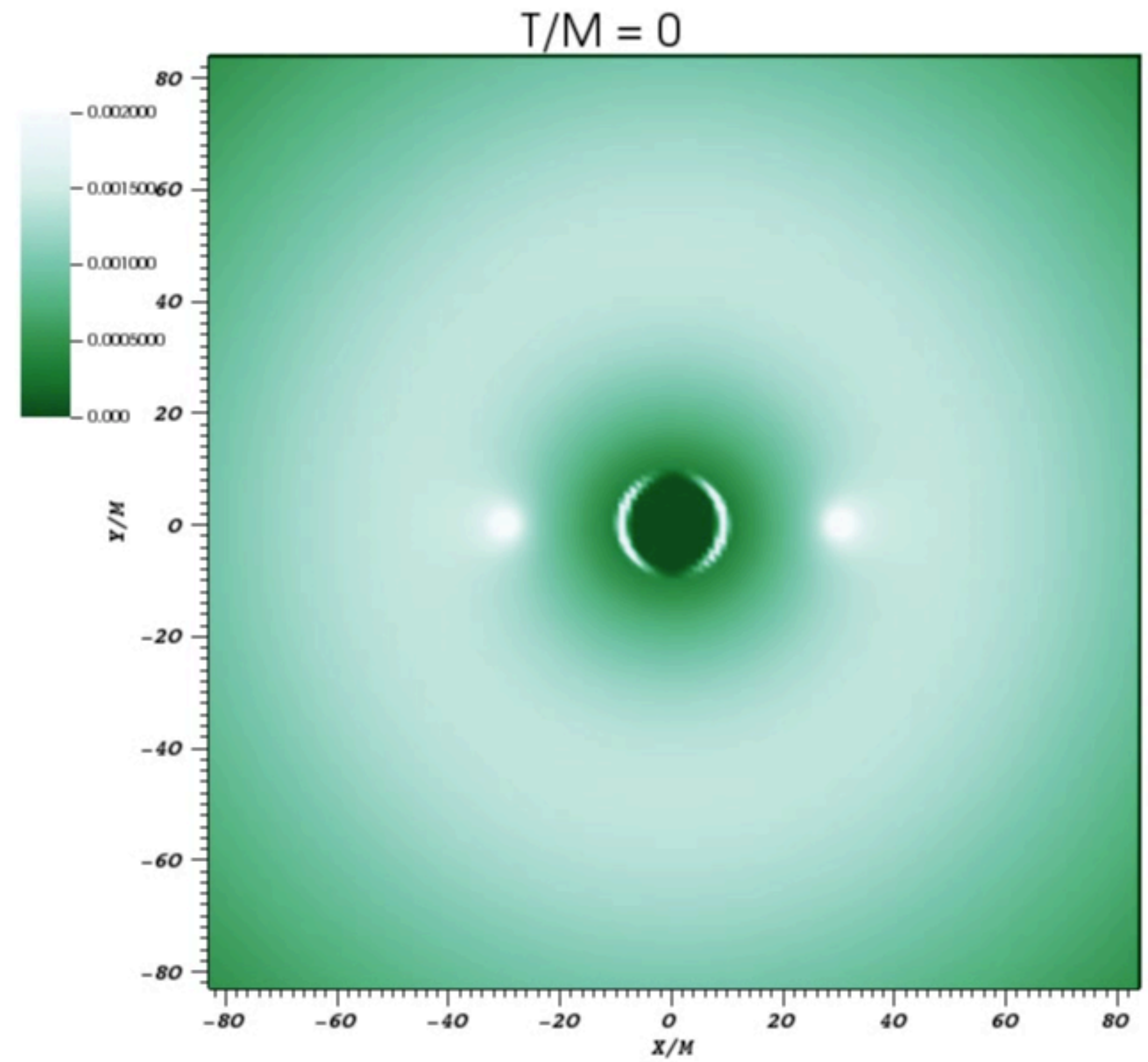
Simulation 4

$D = 60M$

Scalar field

Energy density



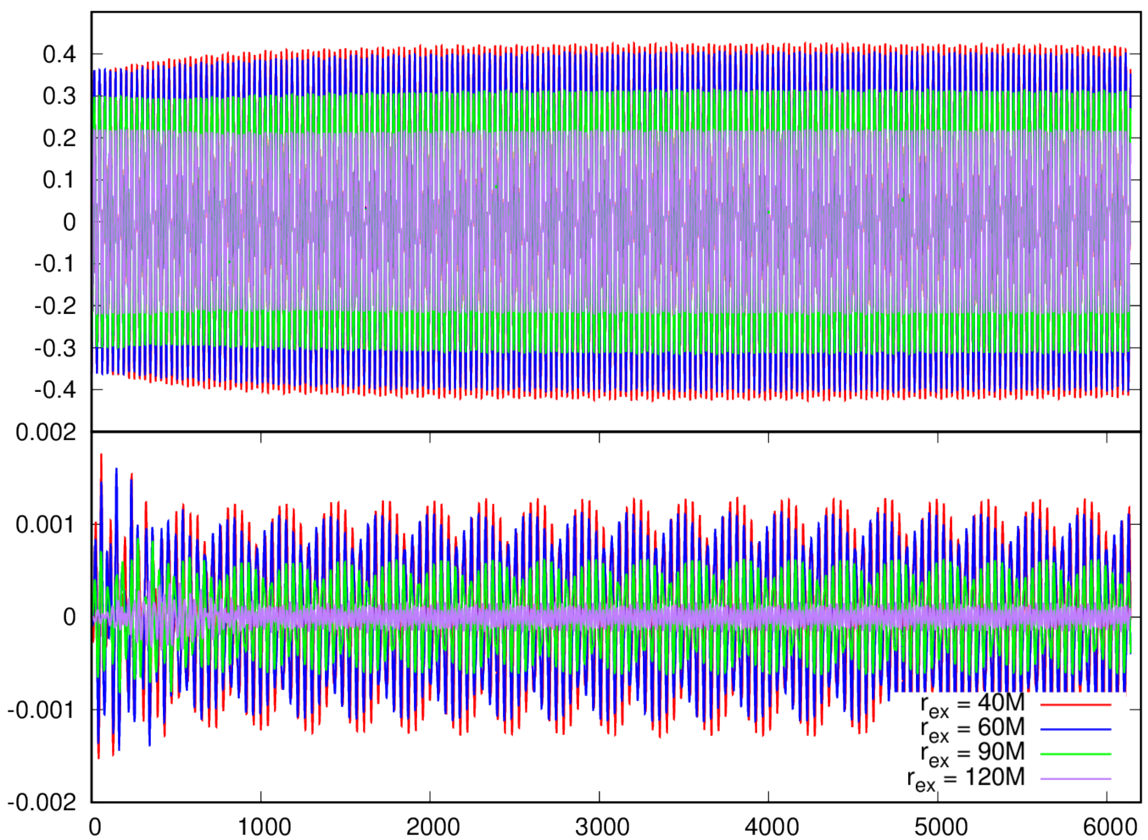


- Energy density rotates with binary.
- “Dipole” counter-rotating gravitational molecule around BH binary.

Simulation 4

$D = 10M$

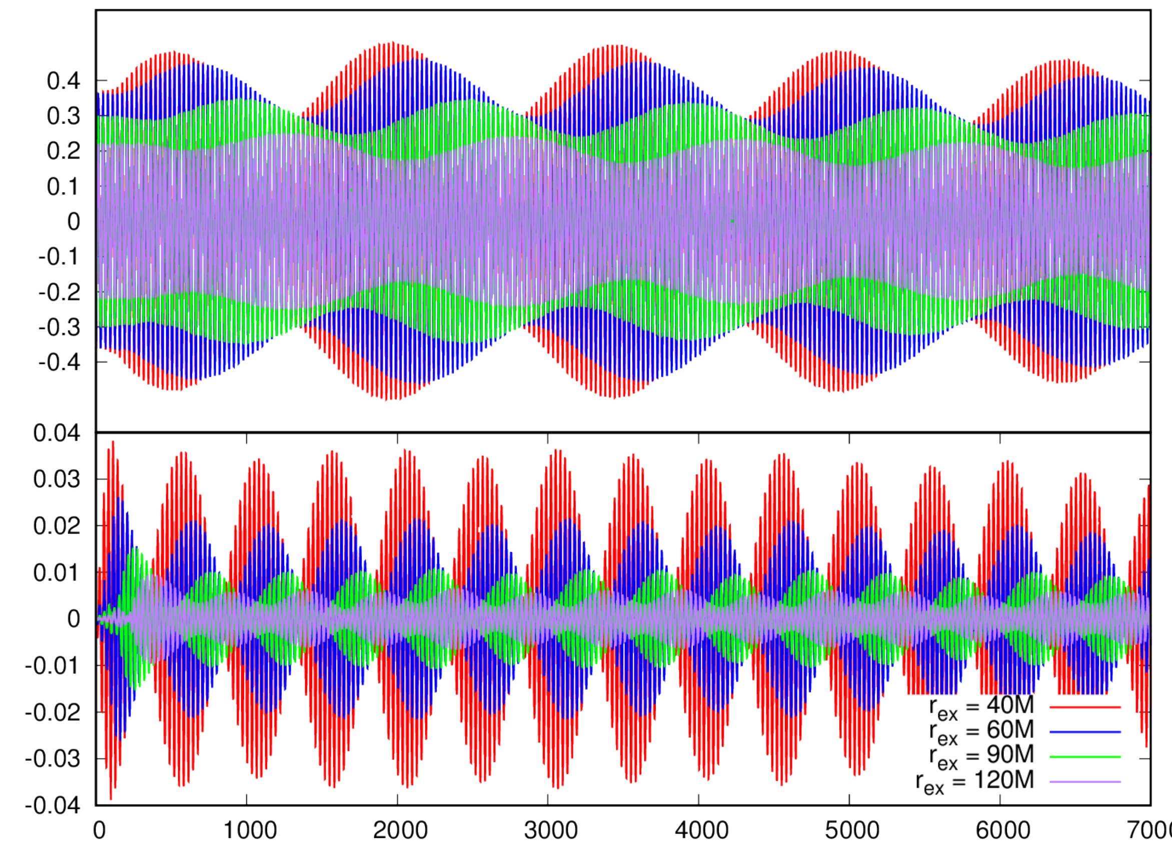
$l = 1, m = 1$



t/M $\quad l = 3, m = 3$

$D = 60M$

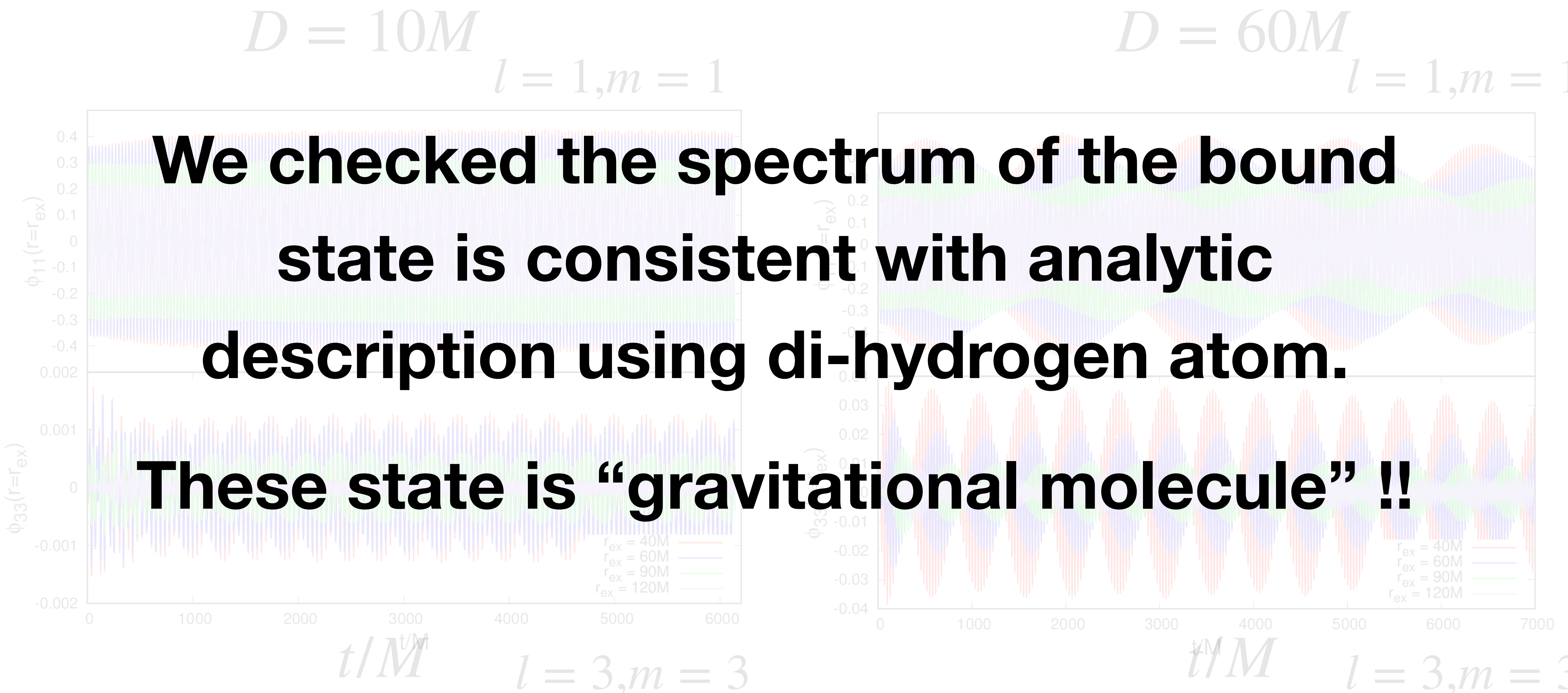
$l = 1, m = 1$



t/M $\quad l = 3, m = 3$

We could not get decaying time scale.

At least, $\tau \gg 10^4 M$



We checked the spectrum of the bound state is consistent with analytic description using di-hydrogen atom.

These state is “gravitational molecule” !!

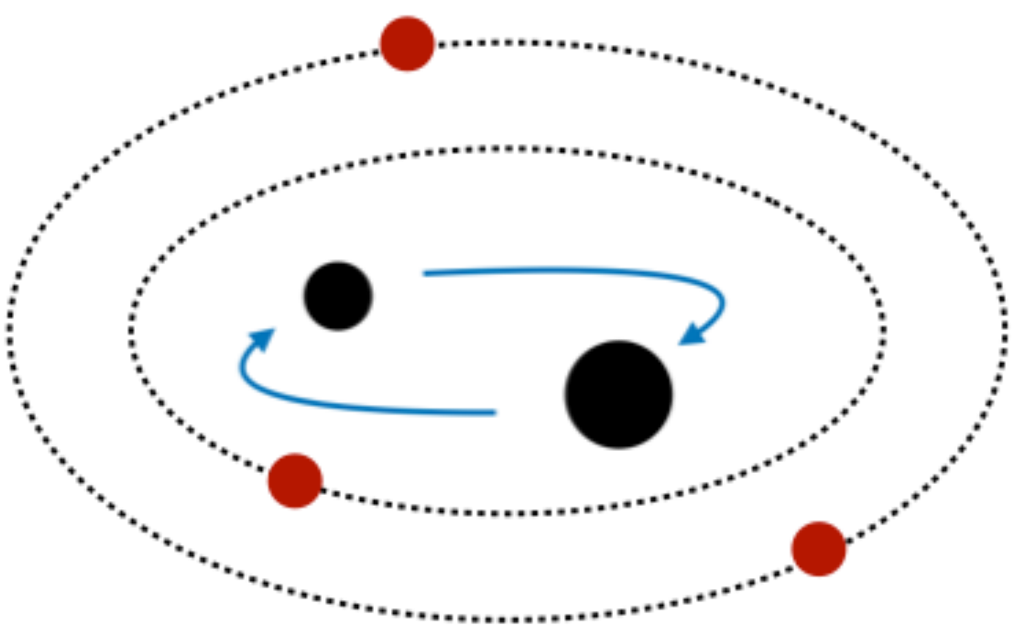
Outline

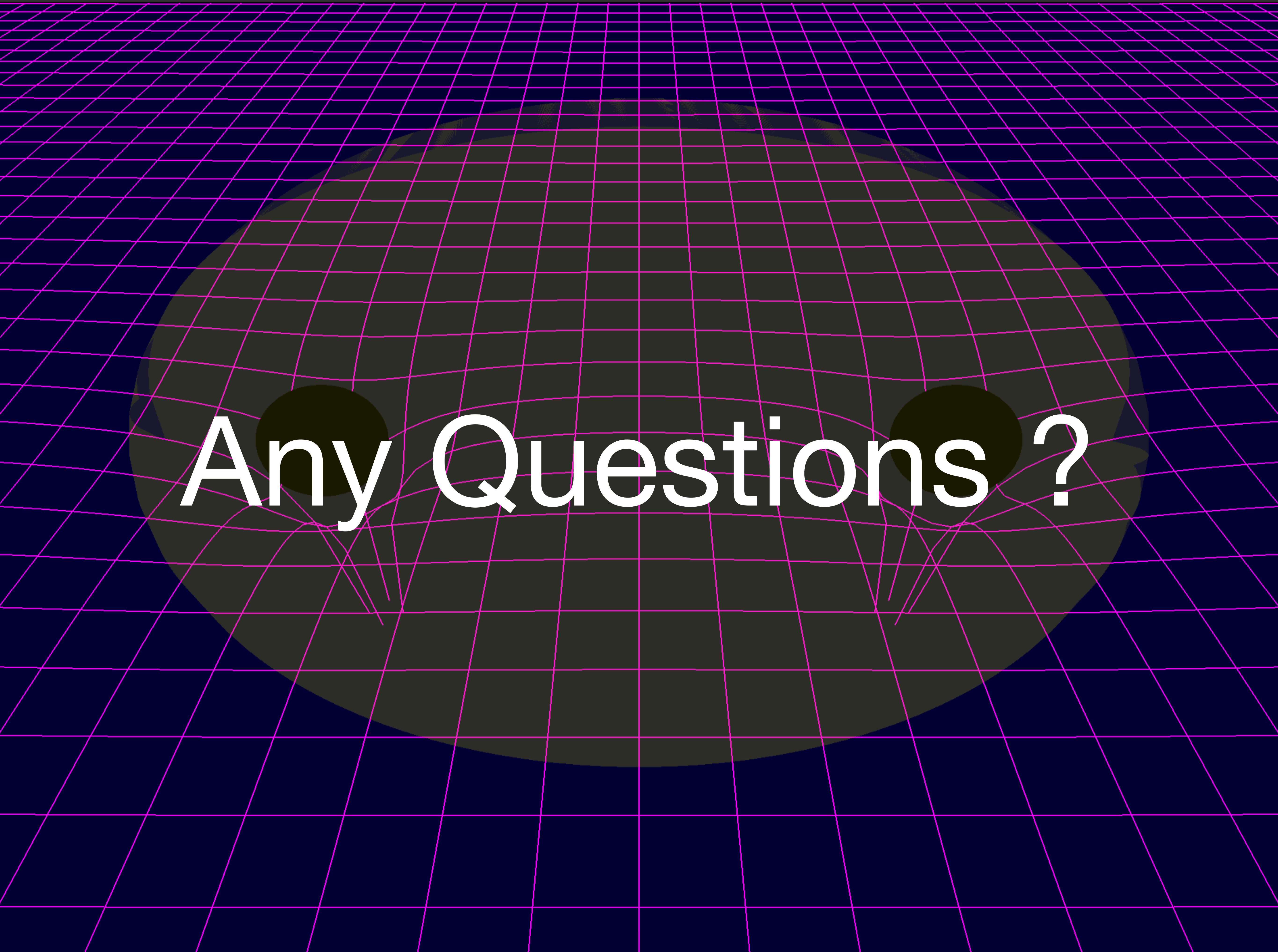
1. Introduction
2. Gravitational atom
3. Analytic description
   - Gravitational molecule
   - Particle picture
4. Numerical simulations
   - Set up
   - Numerical construction
5. Summary

Summary

- Our result
  - Strong evidence for existence of global bound state (Gravitational molecule).
  - Analogy with QM of di-hydrogen atom.
- (a lot of) Future works
  - eccentricity orbit ?
  - Gravitational wave from the gravitational molecule ?
  - Force between BHs due to the molecule ?
  - Can we observe the force using GW from binary ?
  - We can apply a lot of well-known physics of QM for molecule !!   et al

Thank you for your attention !!




Any Questions ?