Tidal effects on scalar cloud (numerical simulation)

arXiv:2001.01729

Taishi Ikeda (CENTRA, Lisbon)
with Vitor Cardoso, Francisco Duque

Outline

1. Introduction
2. Perturbation theory (D.Baumann et al PRD99,044001)
   - Mode mixing
   - Resonance & cloud depletion
3. Time evolution (Our result)
   - Our strategy
   - Excitation of higher multipole mode
   - Tidal disruption
4. Summary

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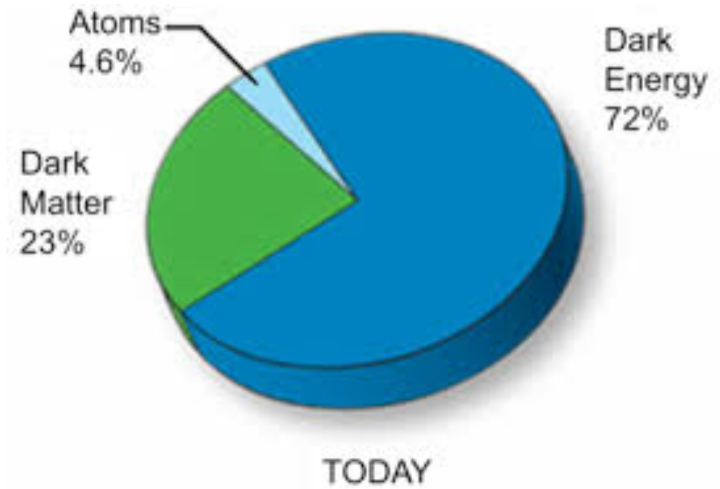
Light scalar field



Energy components

<u>Dark Matter</u>

- QCD axion
- string axion
- PBH et al

<u>Dark Energy</u>

- Cosmological constant
- Modified gravity
  - Scalar tensor theory
  - F(R) gravity
  - massive gravity   et al

➡ Several models predict light scalar field.

Superradiant clouds

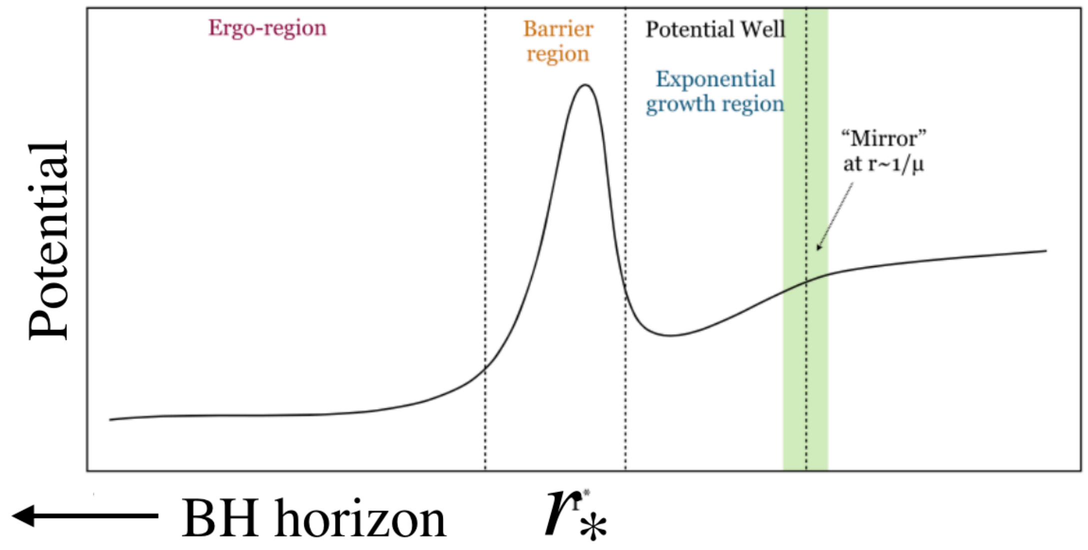
- Supperradiance

$$\Phi(x) = e^{-\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r)$$

$$\mathrm{Re}(\omega) < m\Omega_{\mathrm{H}} = \frac{ma}{2Mr_+}$$

$$\tau \sim 100\tilde{a} \left(\frac{10^6 M_\odot}{M} \right)^8 \left(\frac{10^{-16}\mathrm{eV}}{\mu} \right)^9 \mathrm{sec}$$

- Scalar cloud



Superradiant clouds

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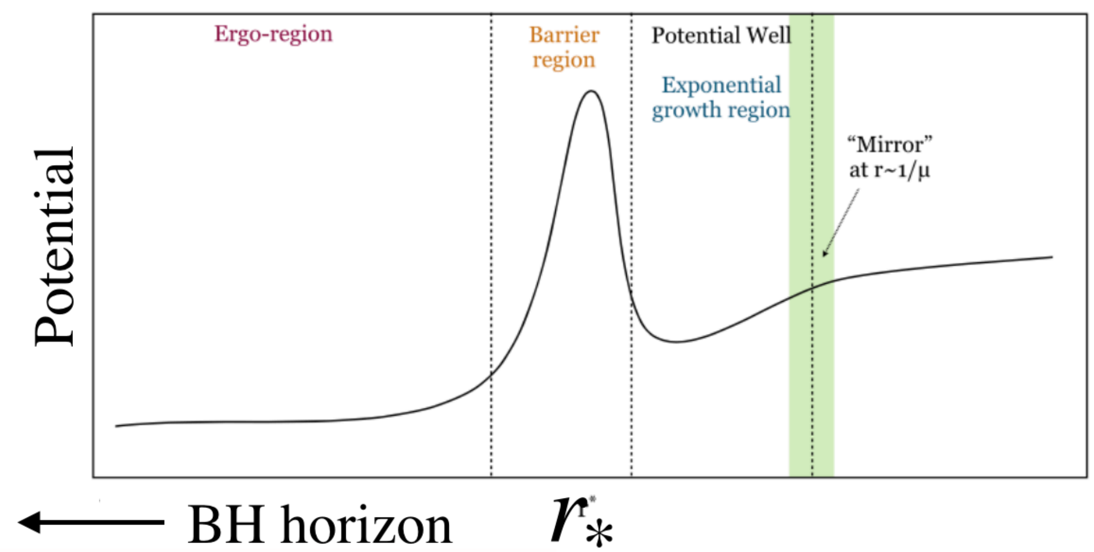
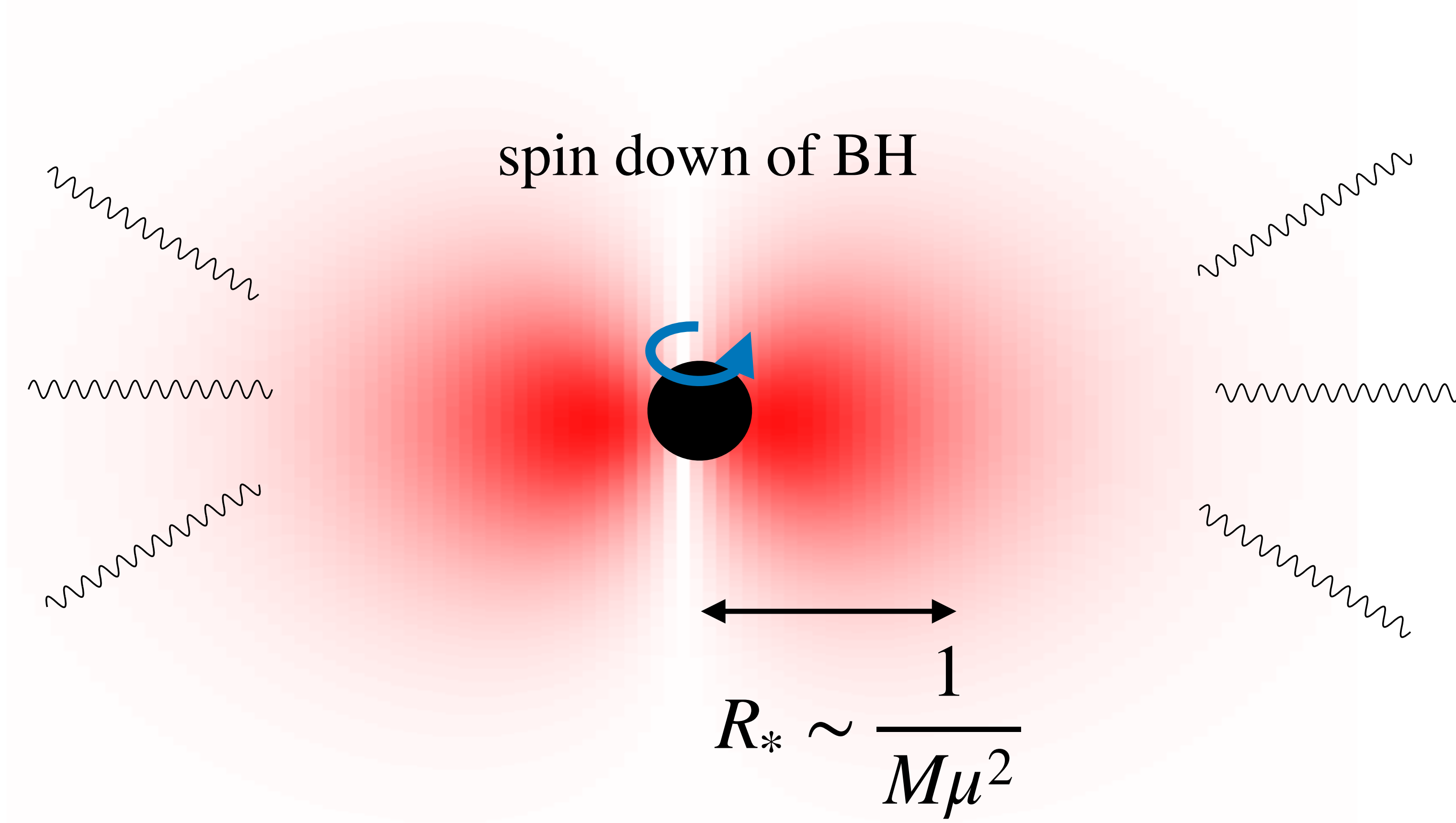
- Scalar cloud

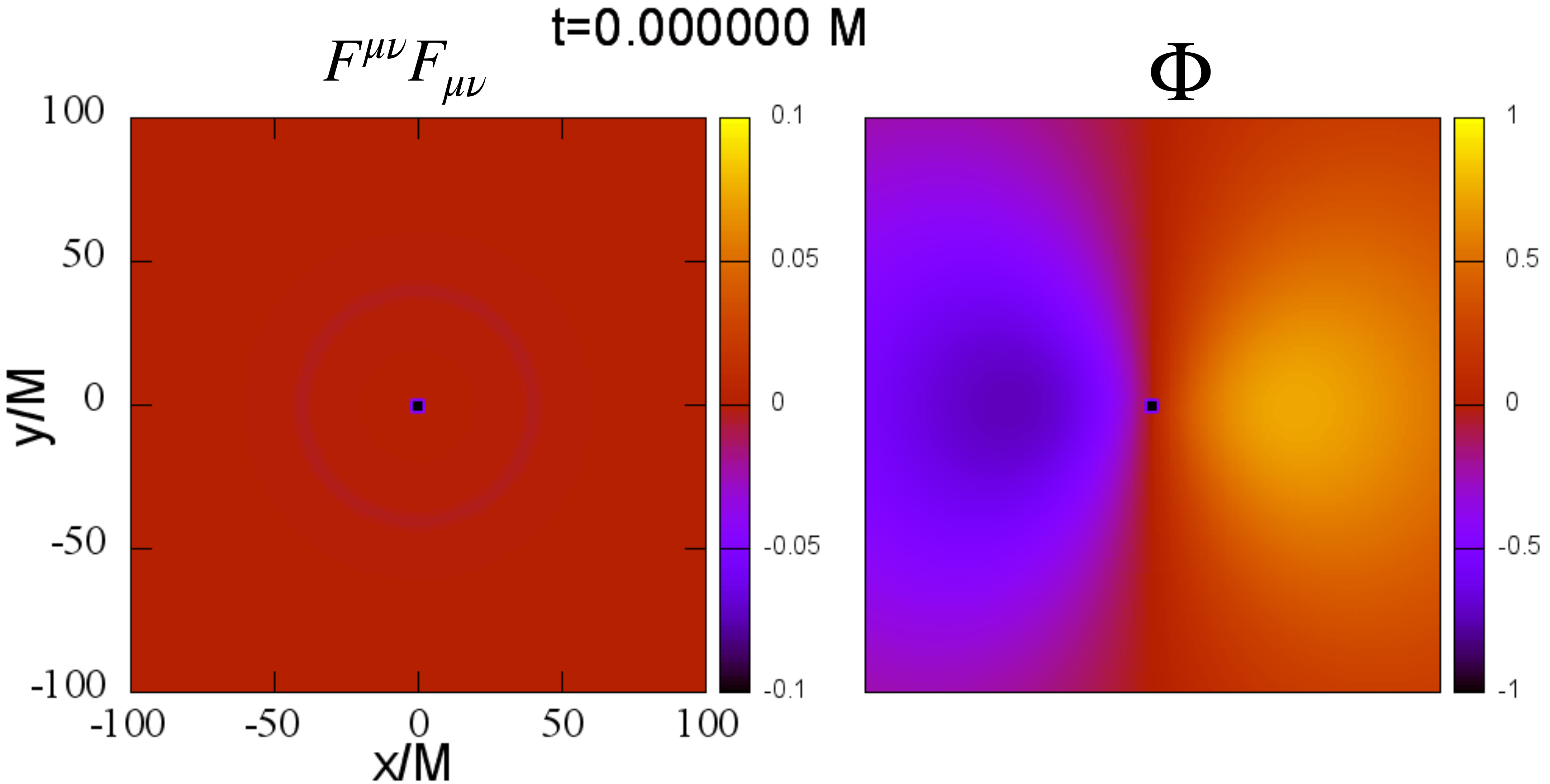


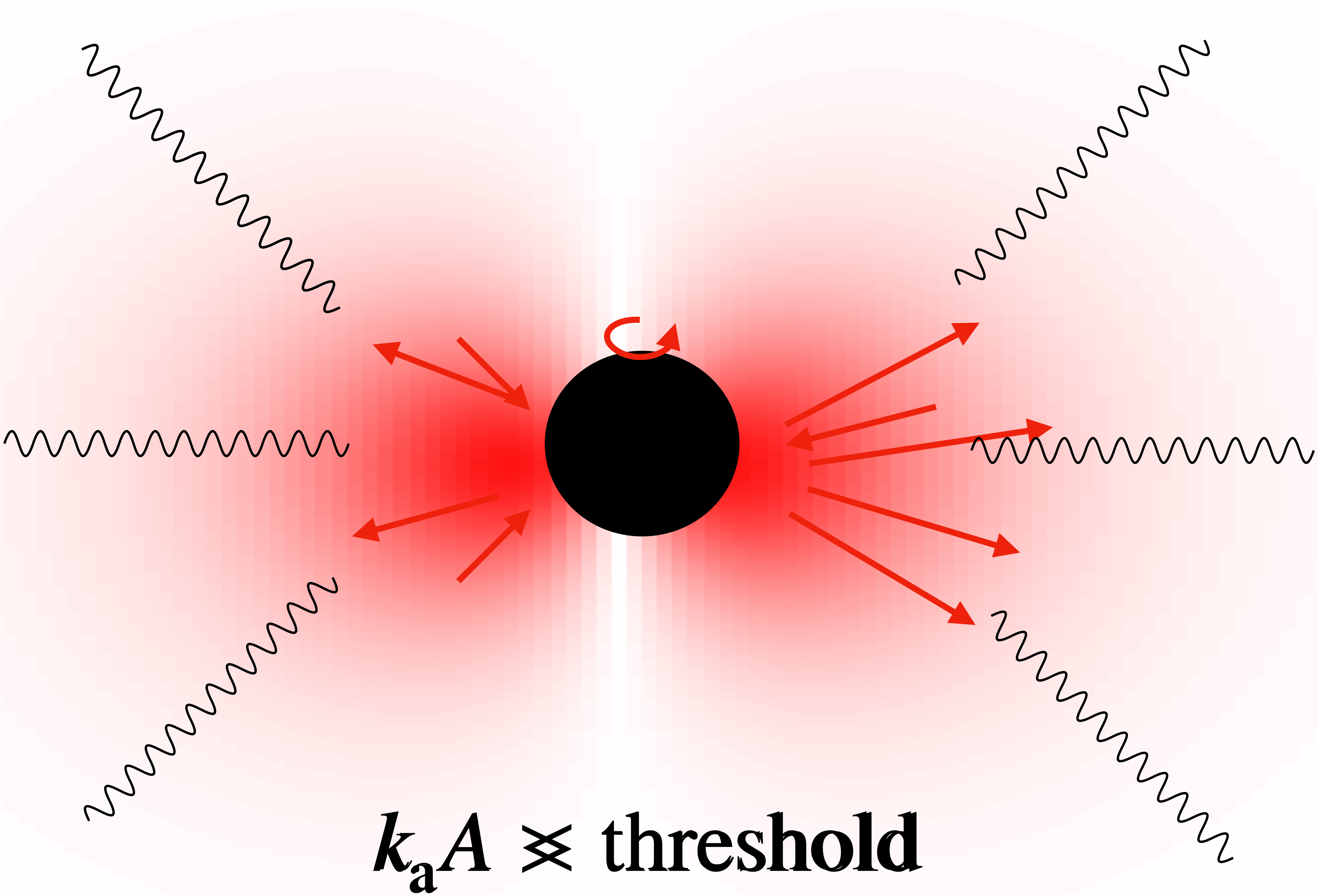


Superradiant clouds

- Coupling between the scalar field and SM particle effects the time evolution of the cloud. (Ikeda 2019)

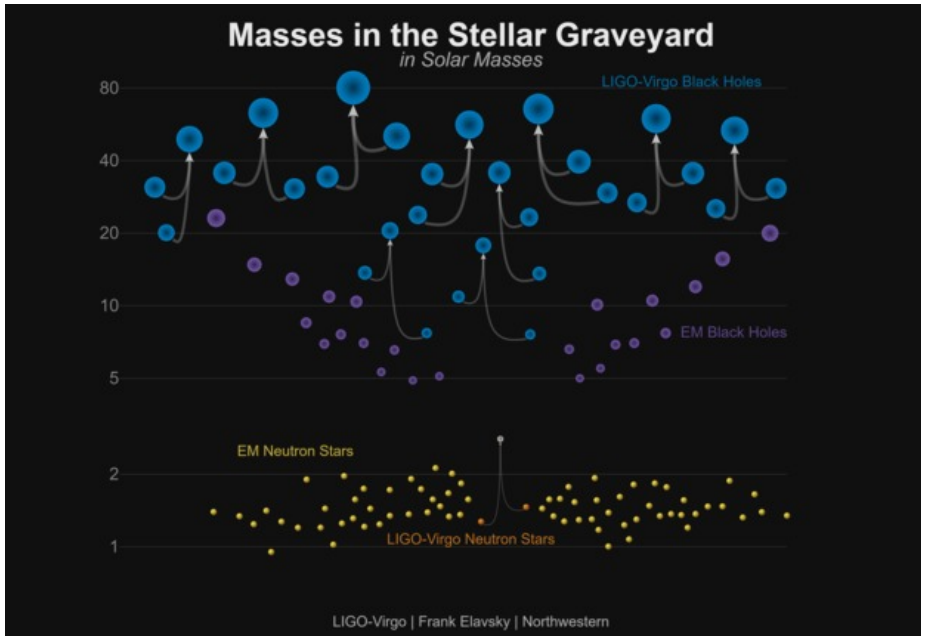
$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \dfrac{k_\mathrm{a}}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_\mathrm{a}\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$

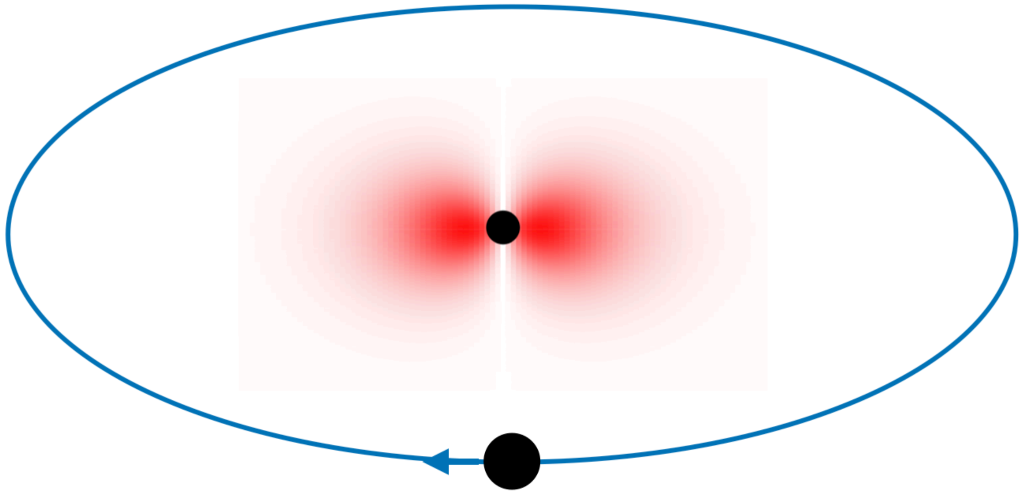



$k_a A \gtrless$ threshold

Black Hole has a companion.

- There are a lot of BH binaries in our Universe.
- Sgr A* and Cygnus X1 have companion stars.
- Scalar cloud around BH with companion star
    - Scalar cloud feels a tidal force.
    - Does tidal force change the dynamics of scalar cloud ?
    - Is scalar cloud disrupted ?





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Previous work

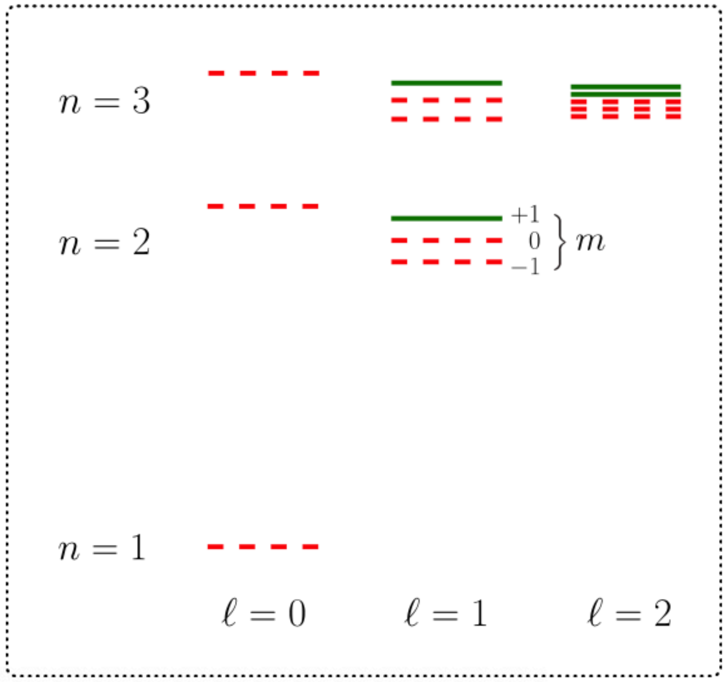
$$V(r) = \frac{\alpha}{r}$$

- Mode mixing (D.Baumann et al PRD99,044001, E.Berti et al PRD99,104039)
  - single BH
    - $(\Box - \mu^2)\Phi = 0$ ➡ $i\partial_t \Psi = \left(-\frac{1}{2\mu^2}\nabla^2 + \underline{V(r)}\right)\Psi$ ➡ $\begin{cases} |n,l,m> \\ \omega_{n,l,m} \end{cases}$

      $\begin{cases} M/r \ll 1 \\ \text{non-relativistic limit} \end{cases}$

      cf : QM of Hydrogen atom
    - higher order correction

      $$\Delta\omega_{nlm} = \mu\left(-\frac{\alpha^4}{8n^4} + \frac{(2l-3n+1)\alpha^4}{n^4(l+1/2)} + \frac{2\tilde{a}m\alpha^5}{n^3l(l+1/2)(l+1)}\right)$$
    - $\mathrm{Im}(\omega_{nlm}) \propto m\Omega_H - \omega$
      - decaying mode $\mathrm{Im}(\omega^{(d)}_{nlm}) > 0$
      - growing mode $\mathrm{Im}(\omega^{(g)}_{nlm}) < 0$



Tidal effect on the cloud

$$i\partial_t \Psi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r)\right)\Psi$$

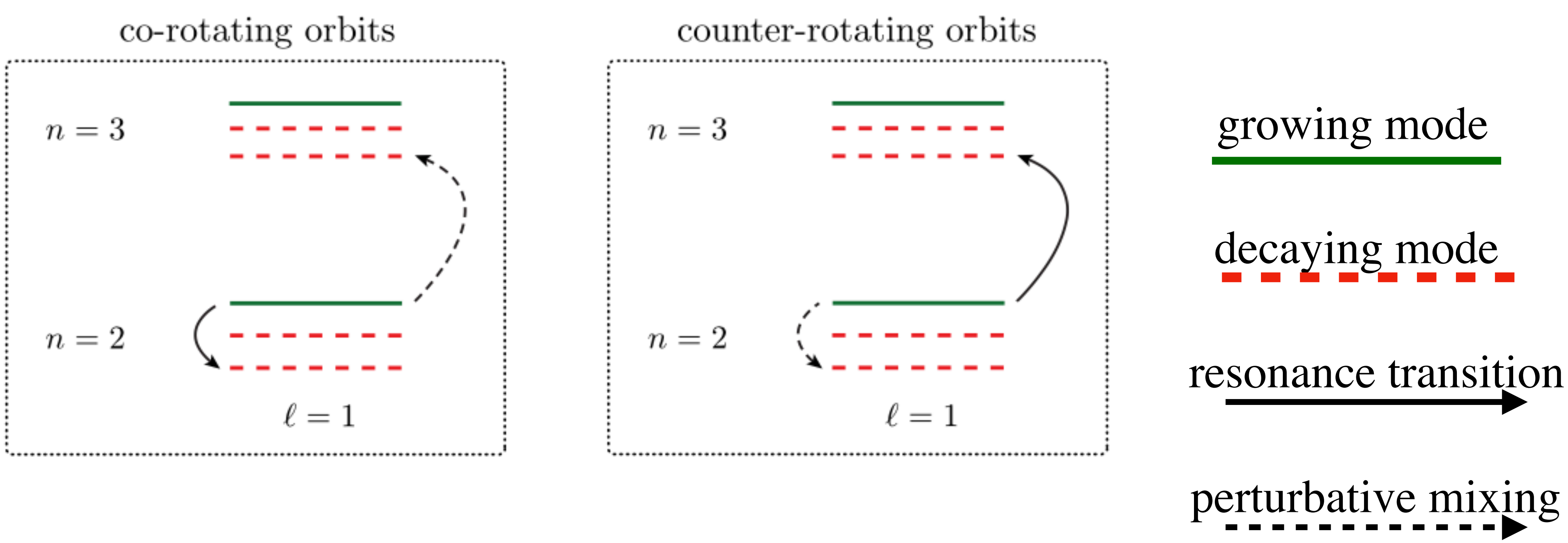
- Binary BH
  - The tidal effect deforms the potential.

$$V(r) \to V(r) + \underline{\delta V(t, r, \theta, \phi)}$$

cf : Perturbation theory in QM

  - mode mixing

$$< n, l, m \,|\, \delta V \,|\, n', l', m' > \neq 0$$



Tidal effect on the cloud

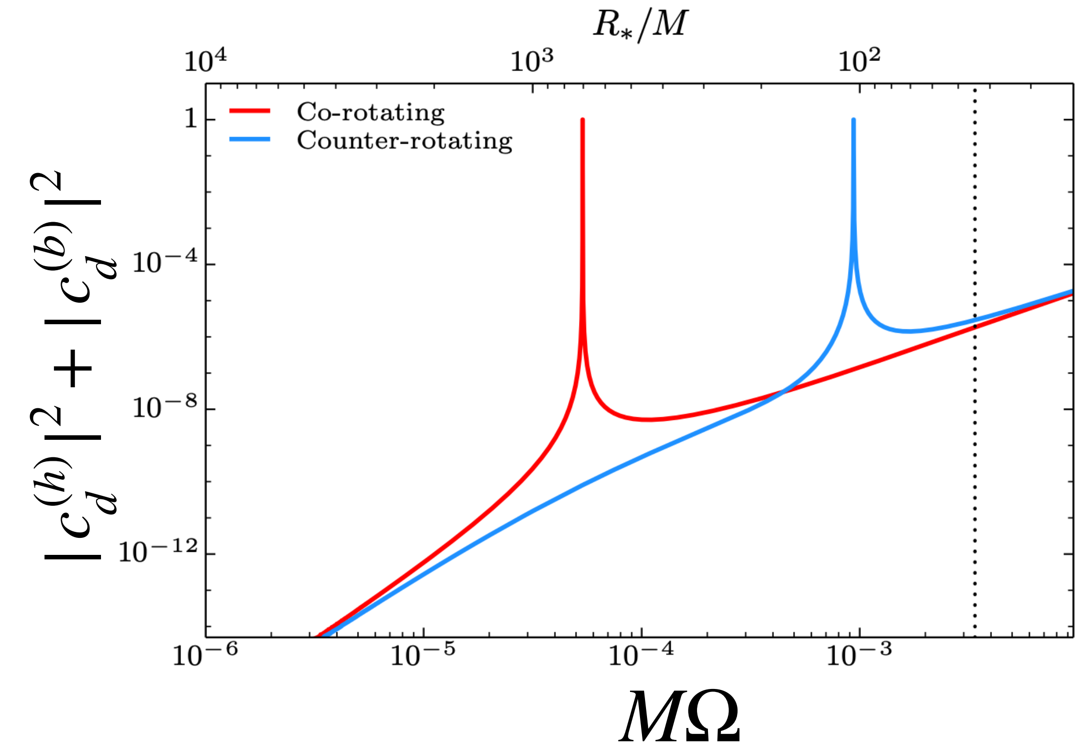
- resonance
  - $\Omega \sim \epsilon_h = \dfrac{\mu \tilde{a} \alpha^5}{12}$

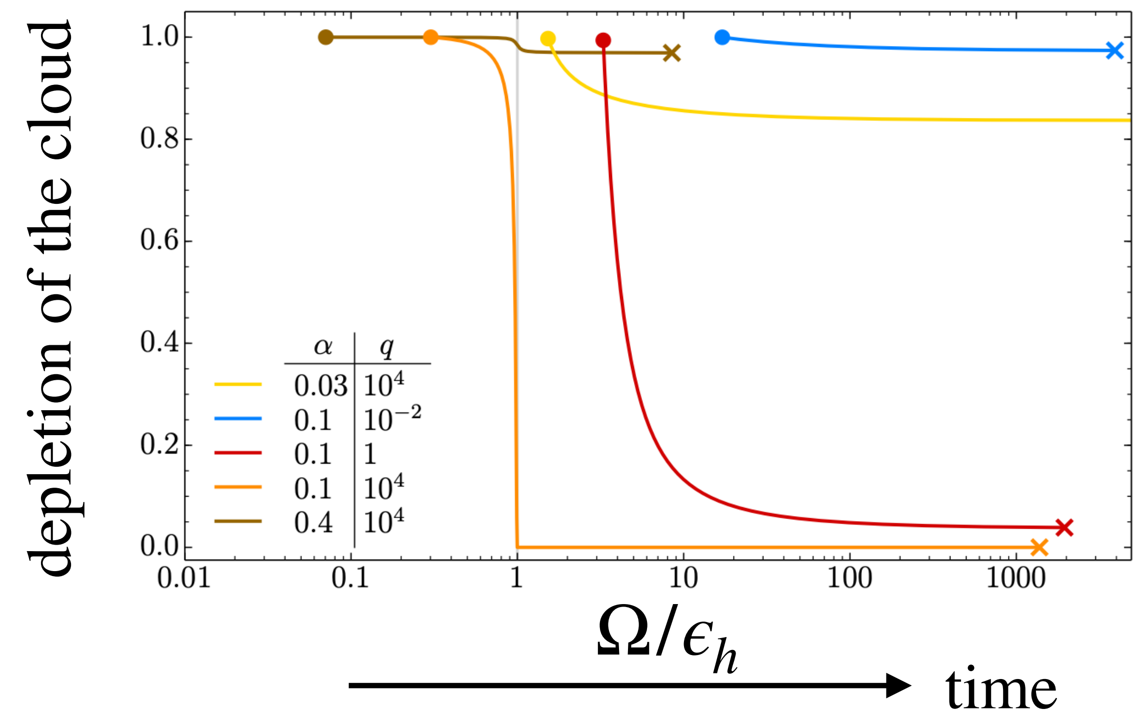
$$\Rightarrow \begin{cases} |\Psi_g> = |2,1,1> \\ |\Psi_d^{(h)}> = |2,1,-1> \end{cases}$$

  - $-\Omega \sim \epsilon_b = -\dfrac{5}{144}\mu\alpha^2$

$$\Rightarrow \begin{cases} |\Psi_g> = |2,1,1> \\ |\Psi_d^{(b)}> = |3,1,-1> \end{cases}$$

- Cloud depletion

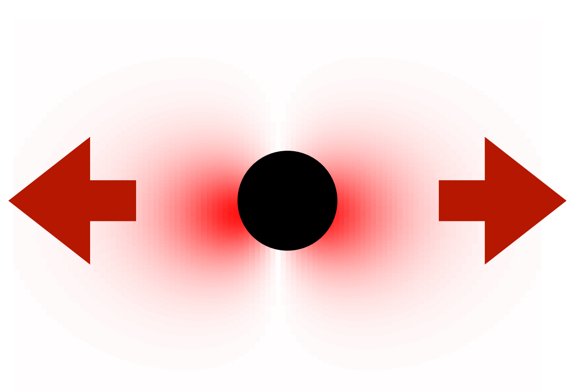


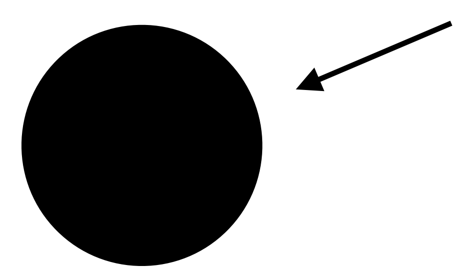


What we want to do

- Previous works : perturbation theory of QM
  - mode mixing between decaying and growing mode
  - depletion of the cloud
- Questions
  - What happens beyond perturbation theory ?
  - Is the cloud disrupted due to the strong tidal force ?

➡ Numerical simulation is good approach.
  - For simplicity, we focus on static tidal field.
  - Weak tidal : consistency check with perturbation theory
  - Strong tidal : threshold of the tidal disruption





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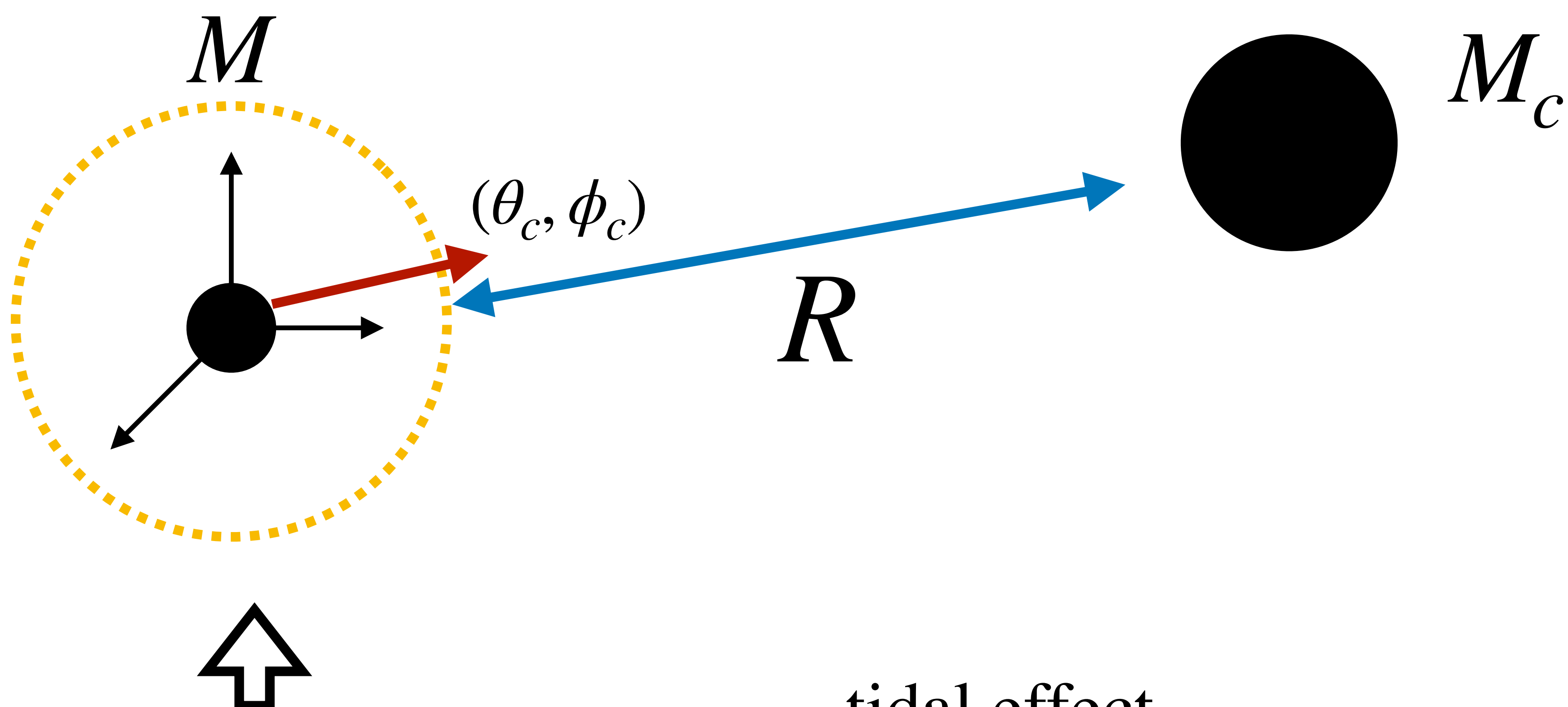
Tidally deformed BH

- How to add tidal effects ?



tidal effect

$$ds^2 = ds^2_{\rm BH} + \sum_m \left(\frac{r}{M}\right)^2 \frac{8\pi\epsilon}{5} Y^*_{2m}(\theta_c, \phi_c) Y_{2m}(\theta, \phi)\left(f^2 dt^2 + dr^2 + (r^2 - 2M^2) d^2\Omega\right) + \cdots$$

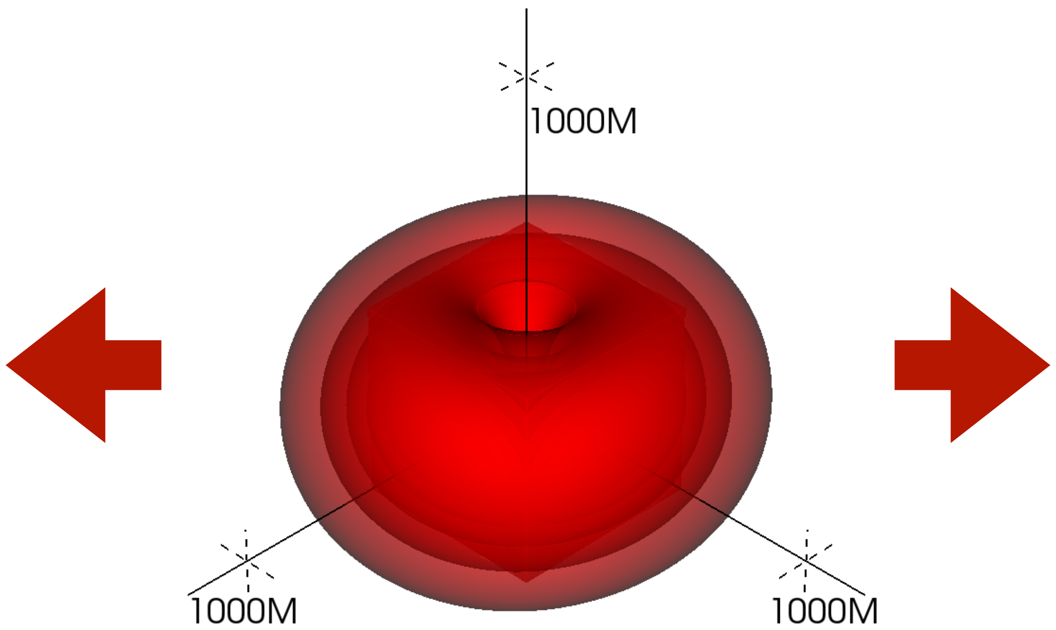
with Regge Wheeler gauge

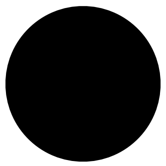
$\epsilon = \dfrac{M_c M^2}{R^3}$: the strength of tidal force

$$f = 1 - \frac{2M}{r}$$

cf: $R = 10^4 M$
$M_c = 10^4 M$

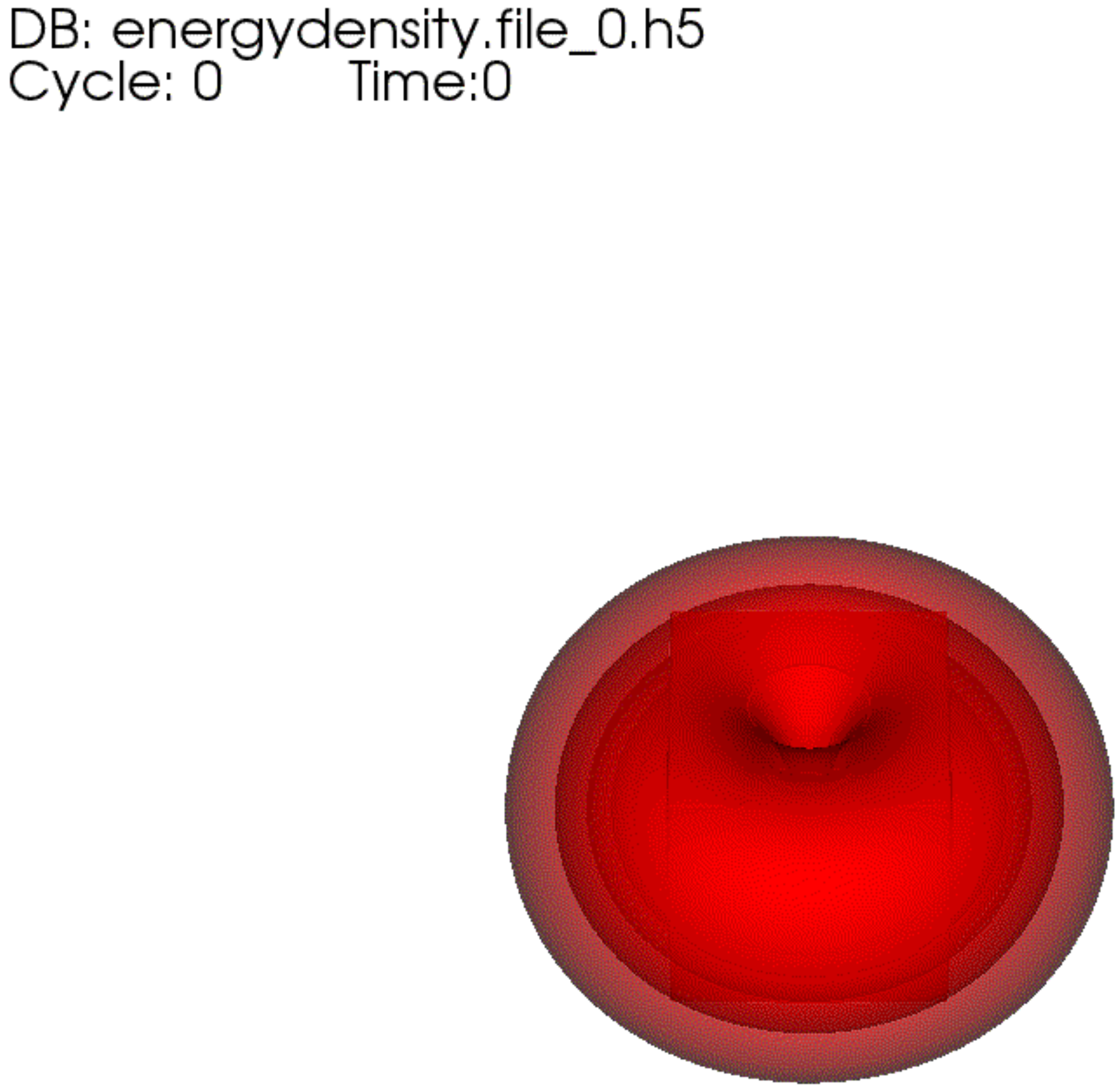
$$\epsilon = \frac{M_c M^2}{R^3} = 10^{-8}$$



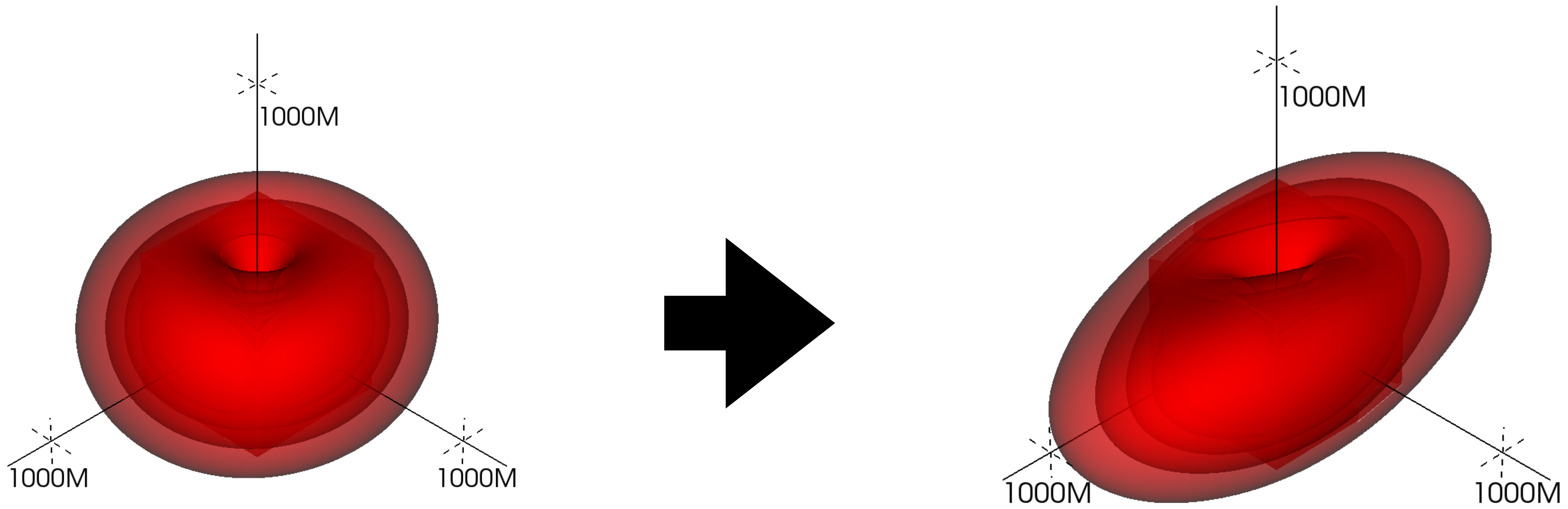


Simulation 1 : Weak tidal case

Weak tidal case

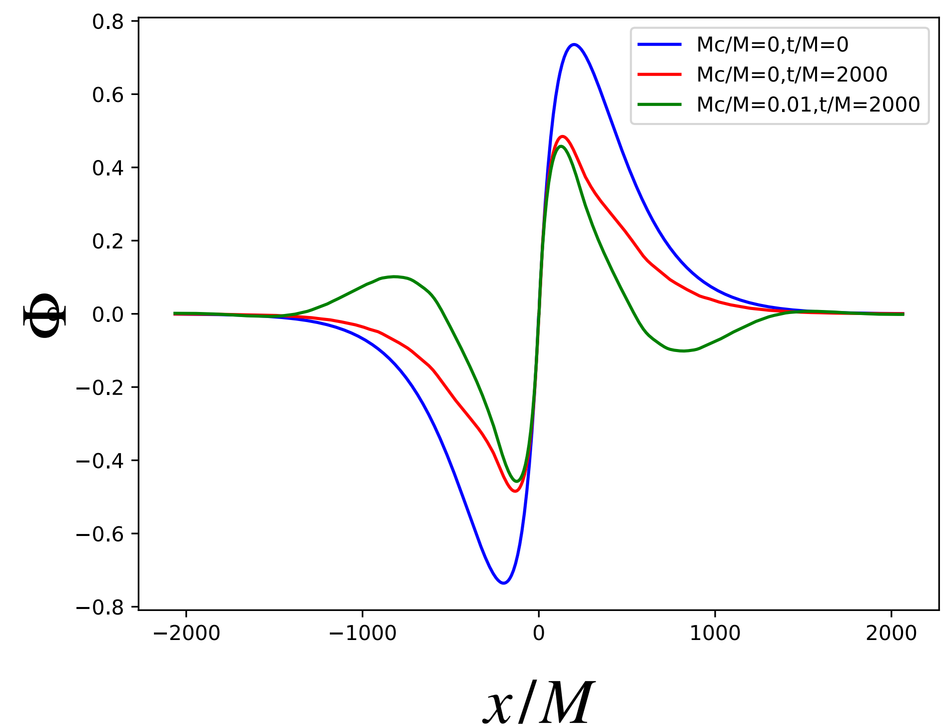


Weak tidal case

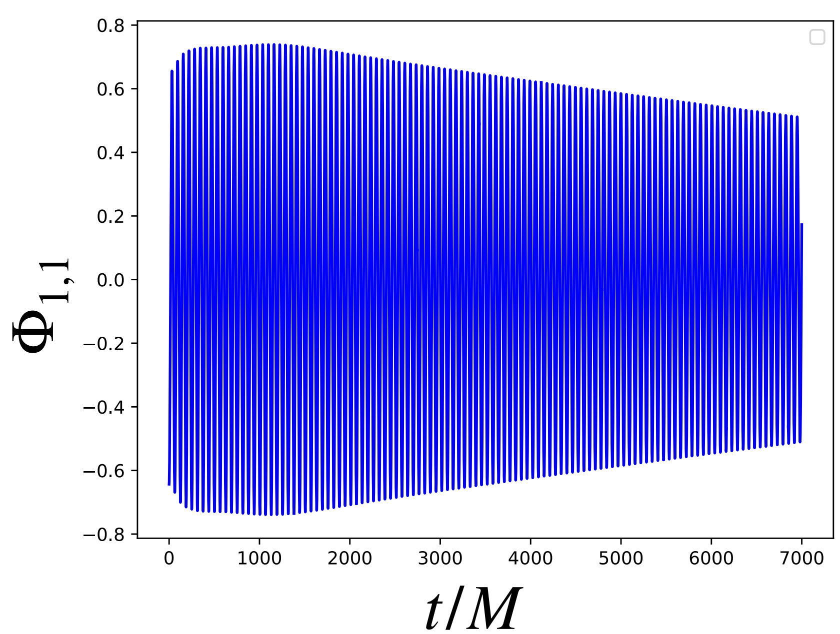


Weak tidal case

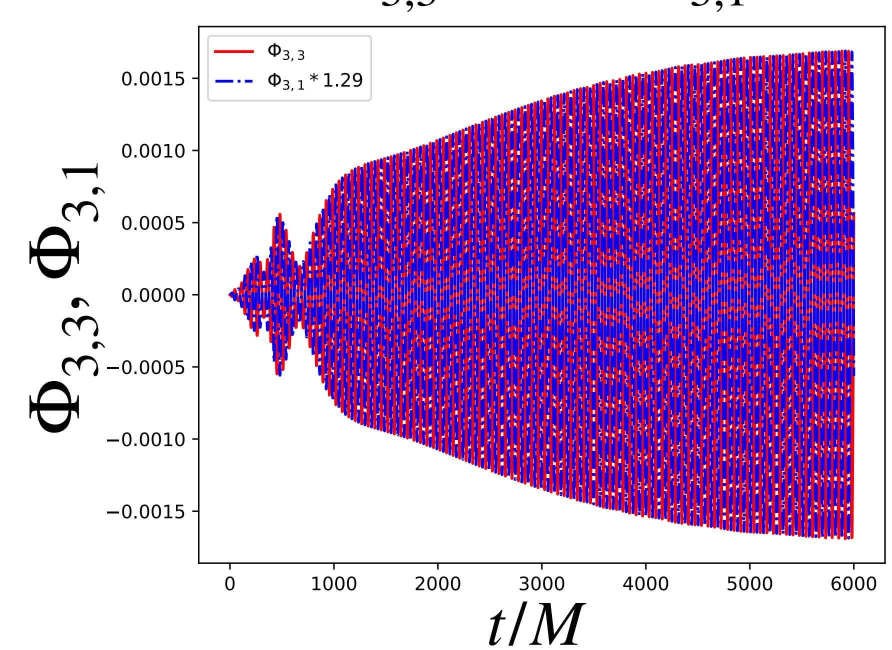
- Excitation of overtone mode.
  - $n = 3{,}4$ modes are excited.
  - consistent with perturbation theory of QM. (Up to a few factor)



- Excitation of higher l mode.



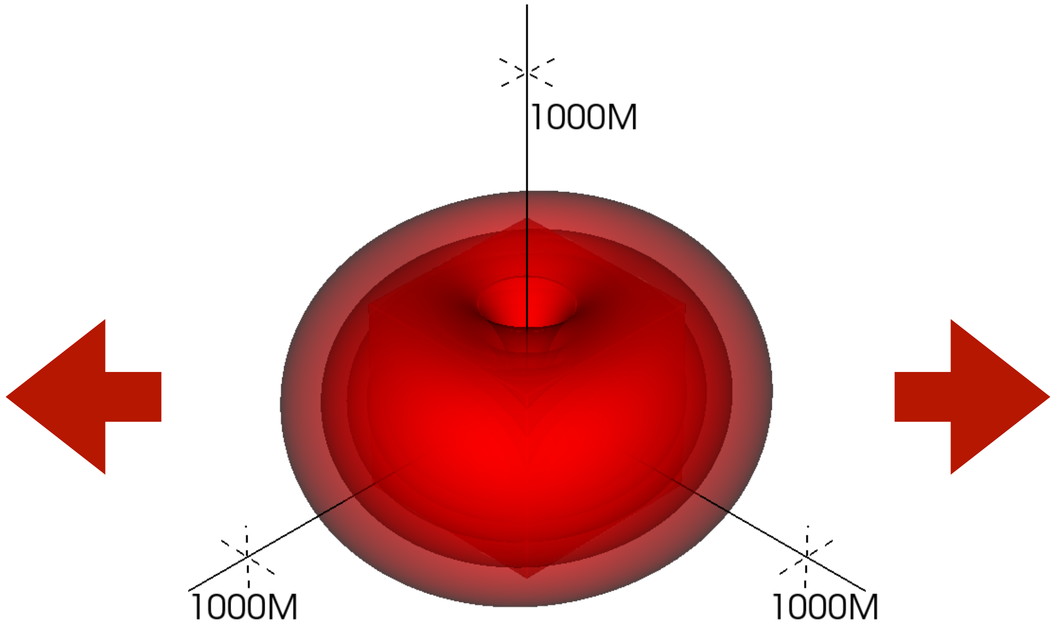
$\Phi_{3,3} \simeq 1.29\Phi_{3,1}$

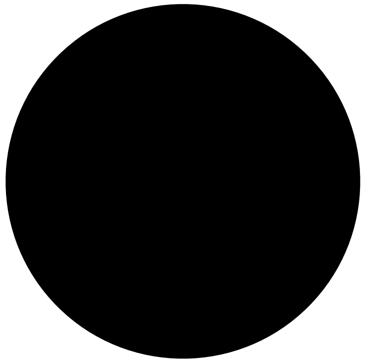


➡ Strong gravitational wave emission is expected.

cf: $R = 10^4 M$
$M_c = 10^5 M$

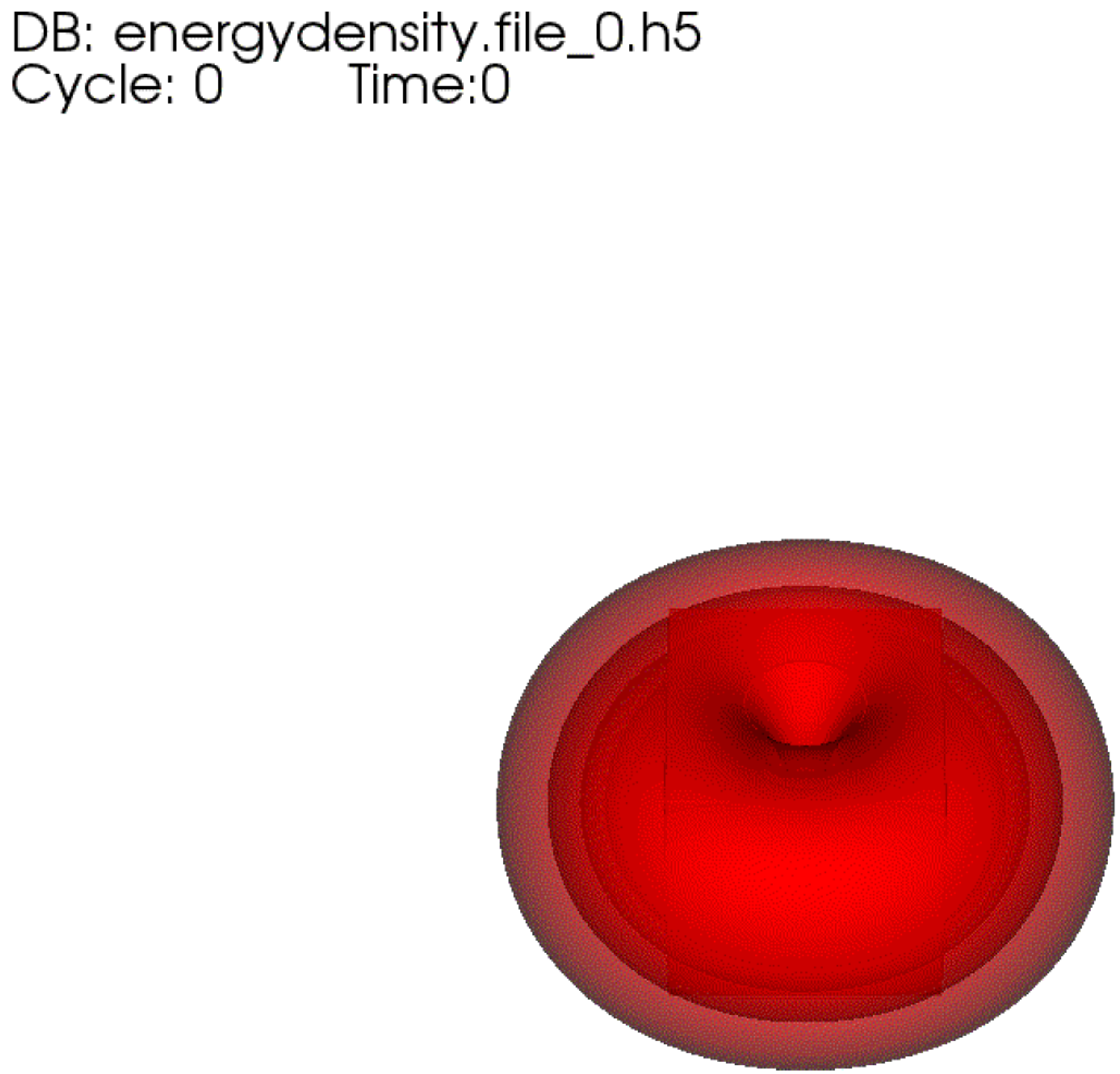
$$\epsilon = \frac{M_c M^2}{R^3} = 10^{-7}$$



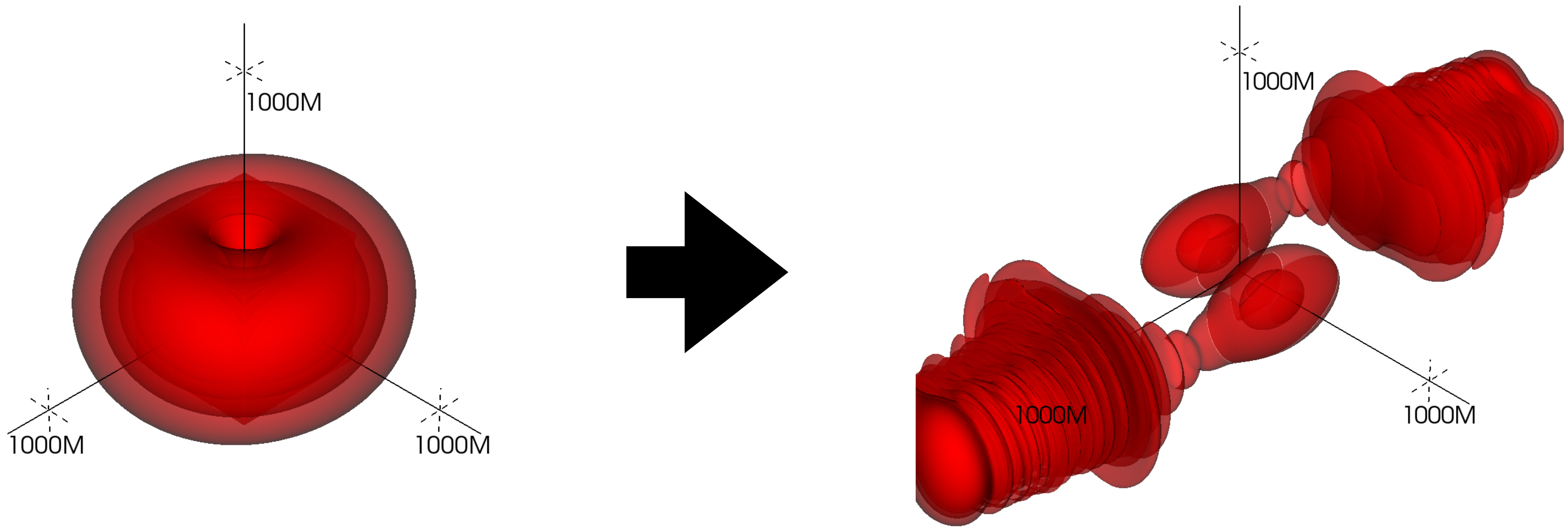


Simulation 2 : Strong tidal case

Strong tidal case



Strong tidal case

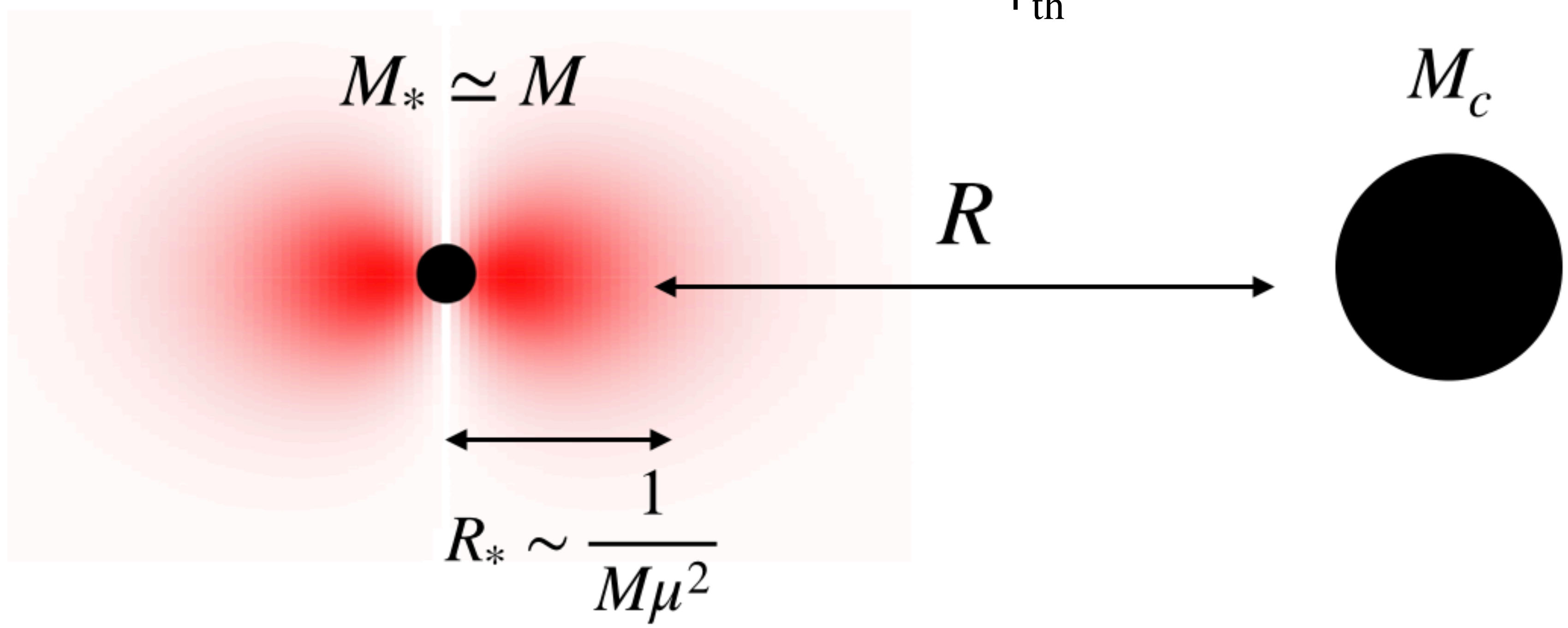


Strong tidal case

- Tidal disruption

cf: Roche limit

$$\frac{M_*^2}{R_*^2} \sim \frac{R_*}{R}\frac{M_c M_*}{R^2} \quad \Rightarrow \quad \left.\frac{M_c M^2}{R^3}\right|_{\rm th} \sim (M\mu)^6 \begin{cases} 10^{-6} \ (\text{ for } M\mu = 0.1 \ ) \\ 6\times 10^{-5} \ (\text{ for } M\mu = 0.2 \ ) \end{cases}$$

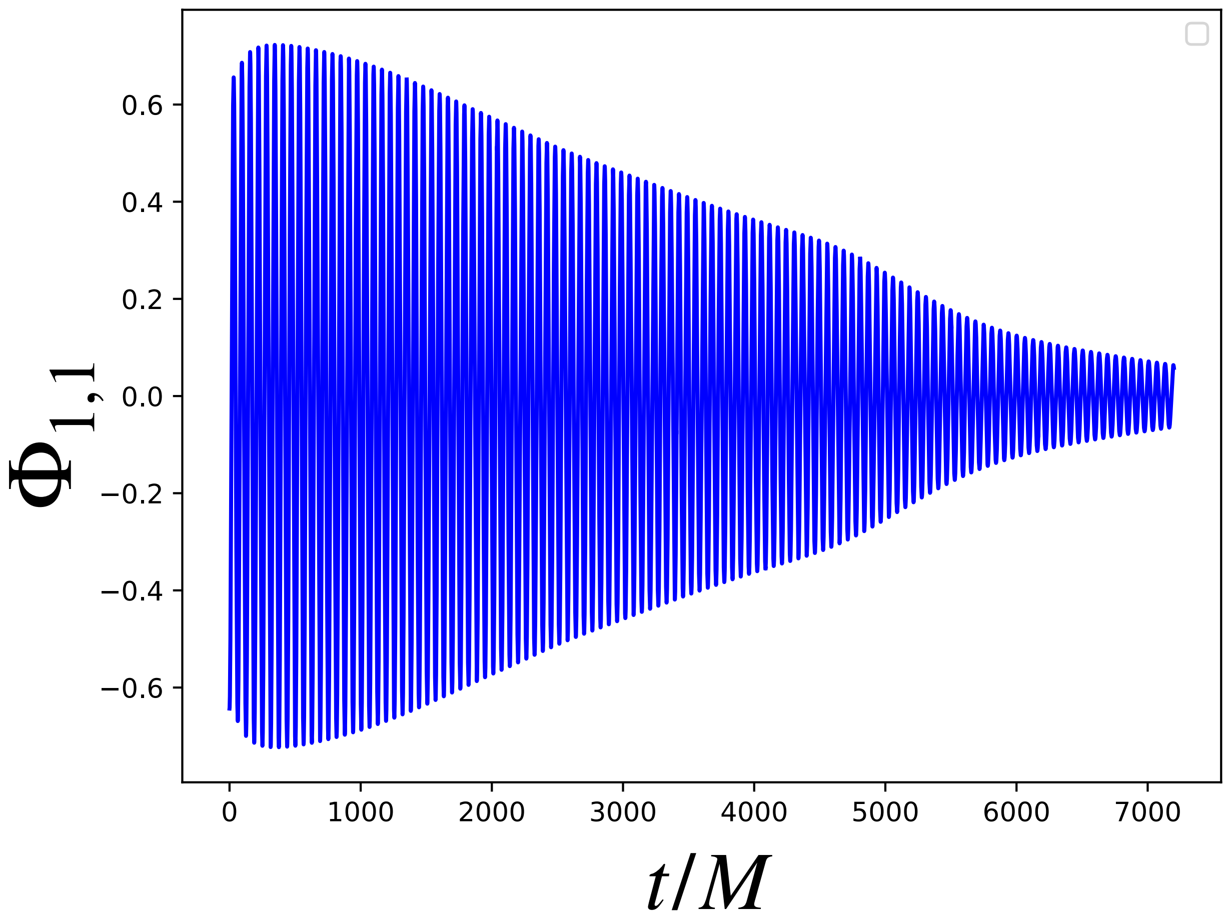


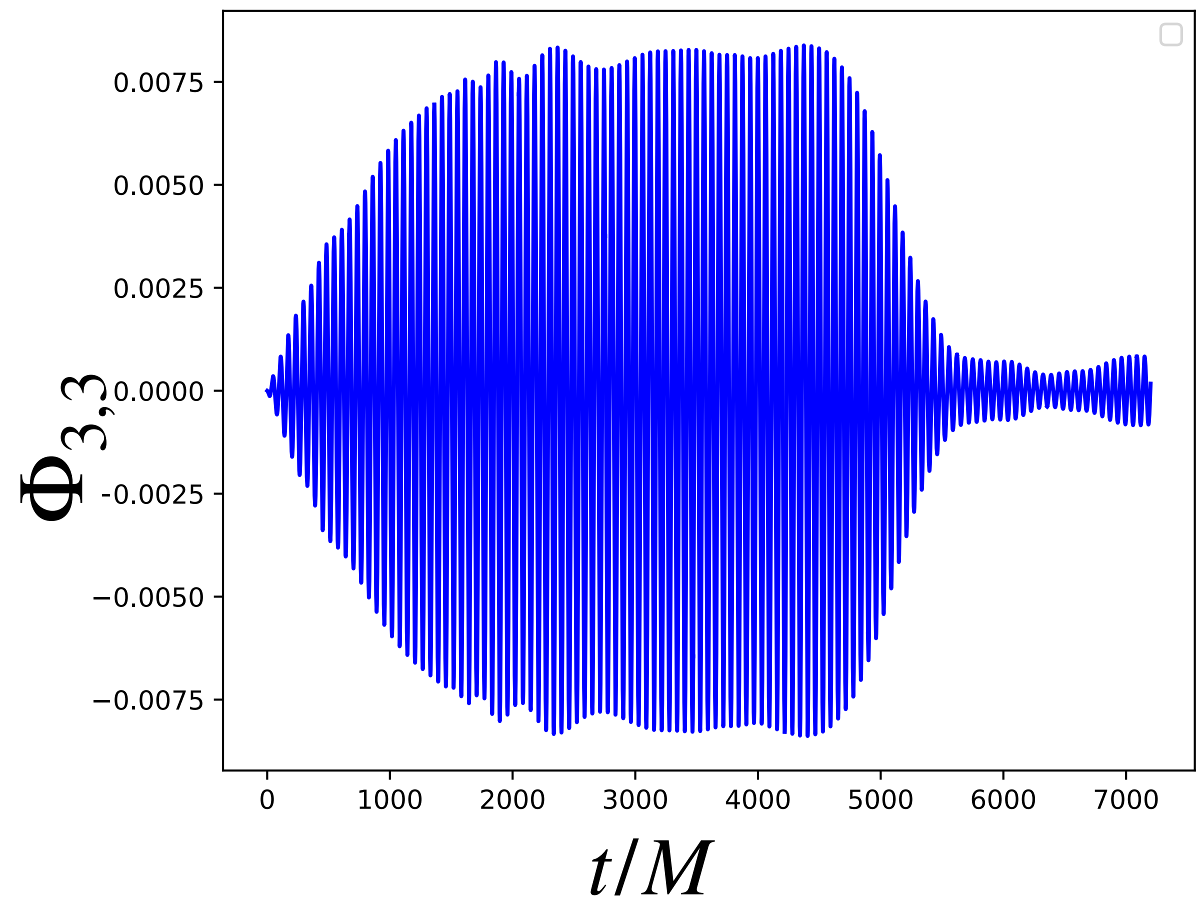
- Numerical result

$$\epsilon_{\rm th} = \left.\frac{M_c M^2}{R^3}\right|_{\rm th} \sim \begin{cases} 10^{-8} \ (\text{ for } M\mu = 0.1 \ ) \\ 2\times 10^{-7} \ (\text{ for } M\mu = 0.2 \ ) \end{cases} \sim \frac{1}{250}(M\mu)^6$$

Strong tidal case

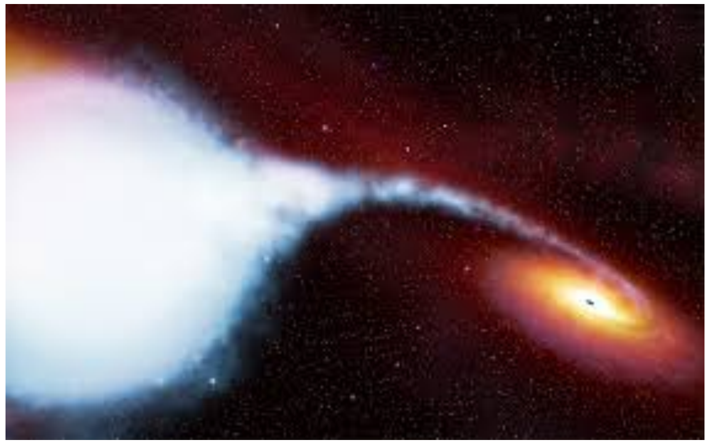
- After higher mode is excited, the cloud is disrupted.





Astrophysical application (In progress)

cf: $t_s \sim M(M\mu)^{-9}$

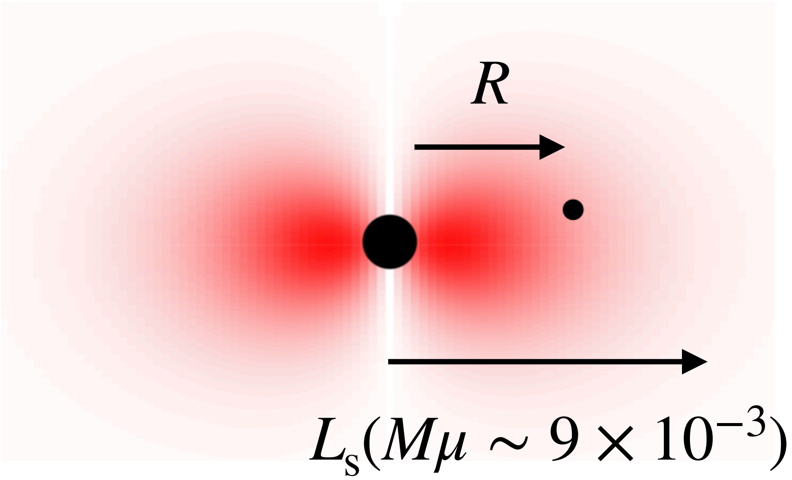


- Cygnus X-1 (J.A.Orosz et al (2011))
  - $M_{\mathrm{BH}} \sim 15 M_\odot$
  - $M_c \sim 20 M_\odot$ ➡ $\epsilon \simeq 5\times 10^{-19}$
  - $R \sim 3\times 10^{10}$ m

➡ Scalar cloud with $M\mu \lesssim 2\times 10^{-3}$ is disrupted.

- Sgr A* (S2) (R.Abuter et al (2018))
  - $M_{\mathrm{BH}} \sim 4\times 10^{6} M_\odot$
  - $M_c \sim 20 M_\odot$ ➡ $\epsilon \simeq 2\times 10^{-15}$
  - $R \sim 1400 M_{\mathrm{BH}}$

➡ Corresponding mass scale : $M\mu \lesssim 9\times 10^{-3}$



* This is beyond our approximation.

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Summary

- We considered tidal effect on scalar cloud.
- We investigate the time evolution of the cloud under tidal force.
  - Higher multipole mode is excited.
    - Strong gravitational wave emission is expected.
  - Tidal disruption

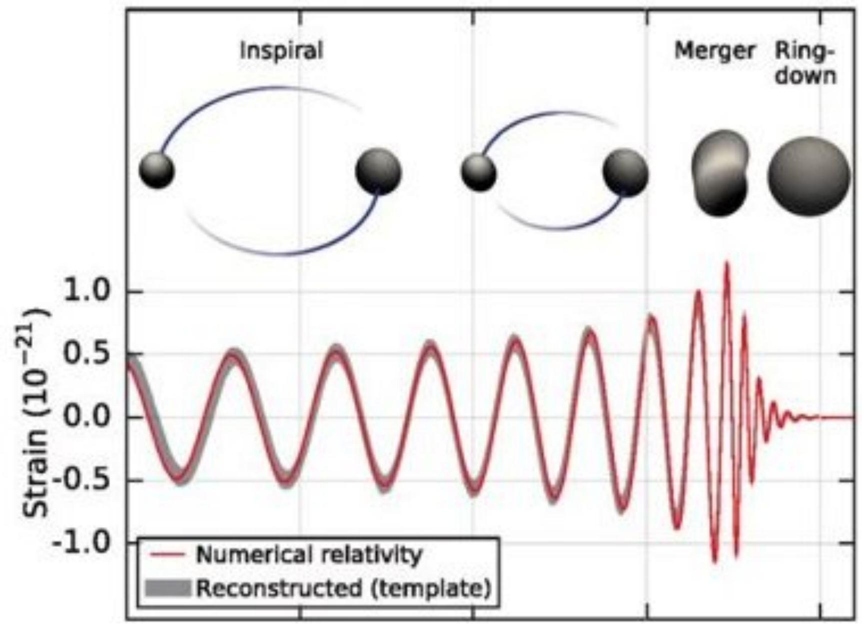
$$\epsilon_{\mathrm{th}} \sim \frac{1}{250}(M\mu)^6$$

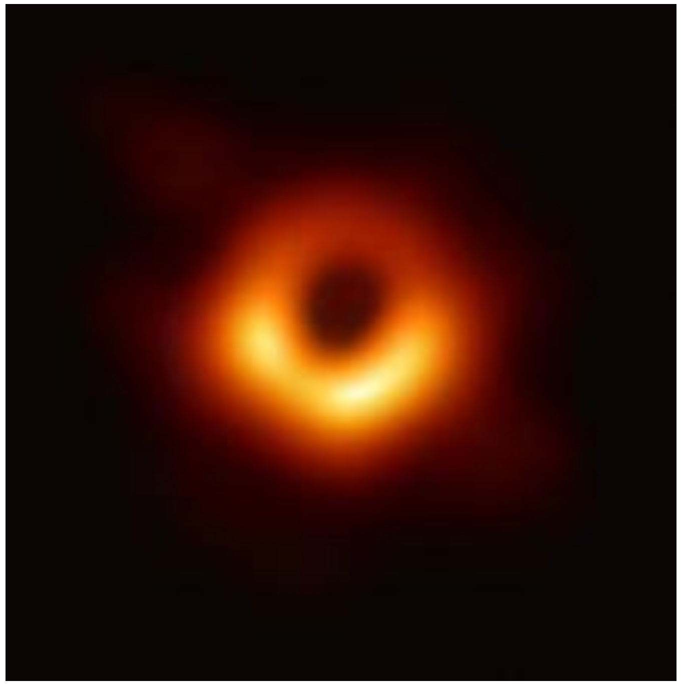
- Future work
  - Time dependent tidal force
  - Gravitational wave from deformed cloud

Fin.

Black Hole Physics

- Gravitational wave
- BH shadow
- Test beds of Gravity theory
  - No-hair theorem
- Prove of early Universe
  - Primordial BH
  - Inflation
- Theoretical features
  - Hawking area law
  - BH entropy
- "Particle detector"



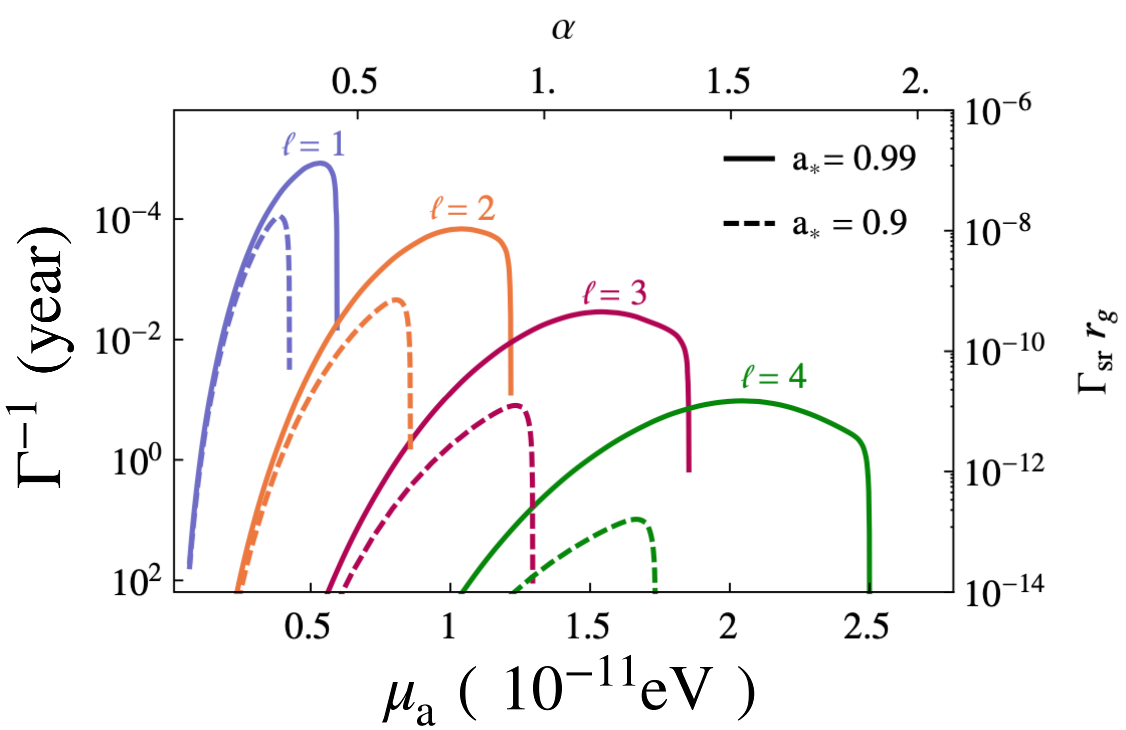


Black Hole as a particle detector

- Supper-radiance
  - $\Phi(x) = e^{-\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r)$
  - condition

$$\mathrm{Re}(\omega) < m\Omega_{\mathrm{H}} = \frac{ma}{2Mr_+}$$

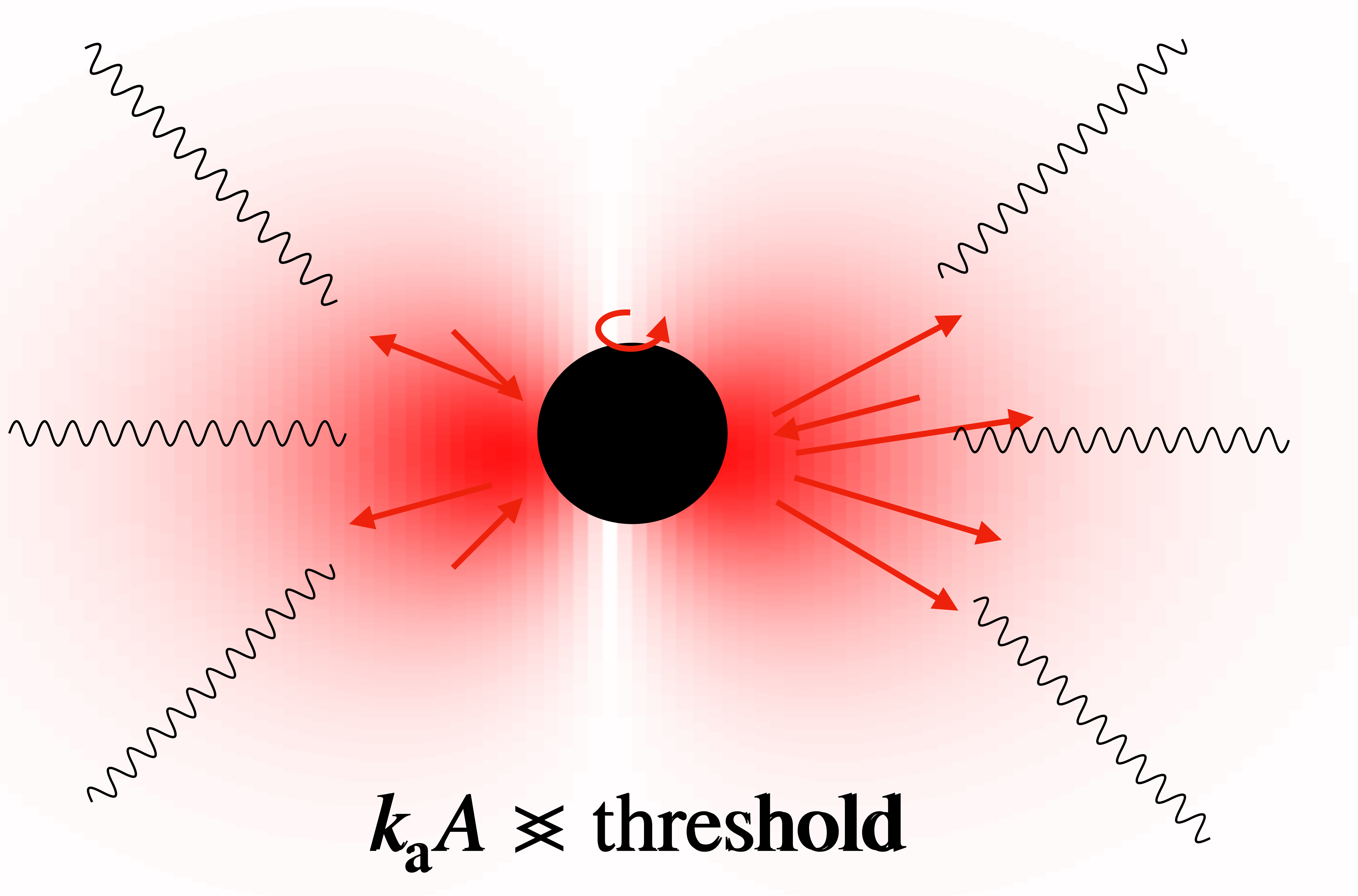
$$\Rightarrow \mathrm{Im}(\omega) > 0$$



Arvanitaki et.al(2015)

- The energy of axion cloud can leak to other fields.
  - Gravitational wave
  - Photon emission (Ikeda et al. (2019))

$$\mathscr{L}_{\Phi\gamma\gamma} = -\frac{1}{2} k_{\mathrm{a}} \tilde{F}_{\mu\nu} F^{\mu\nu} \Phi = -2k_{\mathrm{a}} \vec{E} \cdot \vec{B} \Phi \qquad k_{\mathrm{a}} = \frac{\alpha C}{2\pi F_{\mathrm{a}}}$$

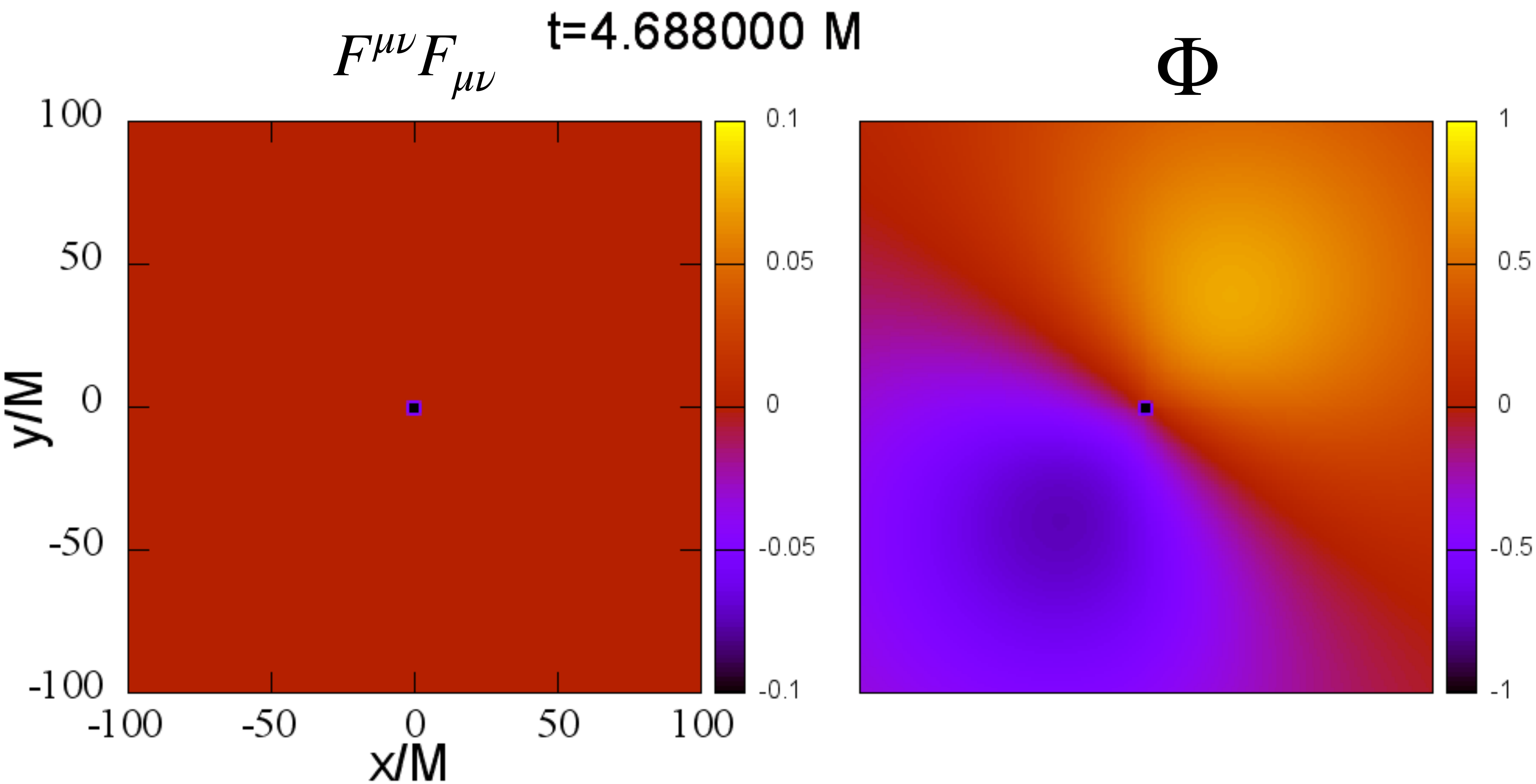

$k_a A \gtrless$ threshold

Black Hole as a particle detector

- Burst case (Localized initial profile)

$$\mu M = 0.2, \quad k_{\mathrm{a}} A_0 = 0.4$$

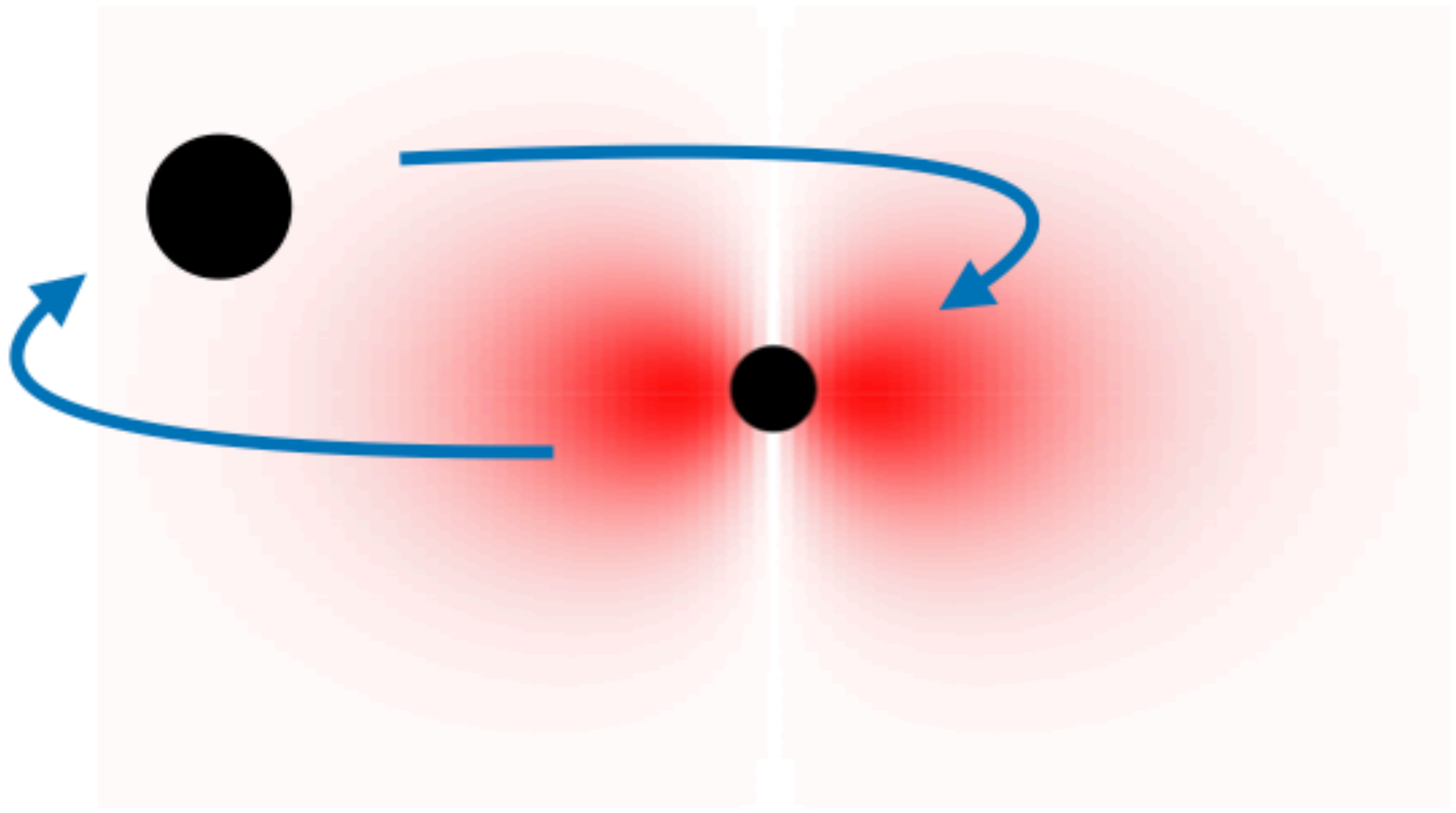
$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \dfrac{k_{\mathrm{a}}}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_{\mathrm{a}}\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$



Our Questions

- In these story, the dynamic of axion cloud is important.
- There are a lot of binary BH in our Universe.
- Does history of axion cloud change around binary BH ?

→ We investigate the tidal effects on the cloud.



What we want to do

- We will solve the tine evolution of axion cloud under the tidal force.
- And, compare with perturbation theory.

Perturbation theory $\Longleftrightarrow$ Time evolution

(D.Baumann et al PRD99,044001)

$$(\Box - \mu^2)\Phi = 0$$

$$i\partial_t \Psi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r) \right) \Psi$$

$$V(r) \to V(r) + \delta V(t, r, \theta, \phi)$$

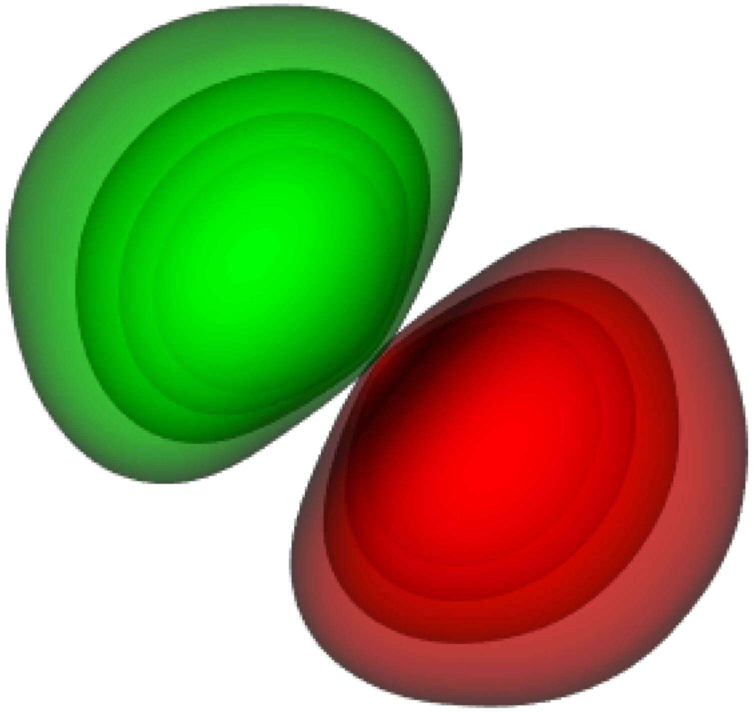
Numerical simulation

- Initial condition : Axion cloud

$$\Phi = A_0 r M\mu^2 e^{-rM\mu^2/2}\cos(\phi - \omega_{\rm R} t)\sin\theta \qquad \omega_R \sim \mu$$

$n = 2, l = 1, m = \pm 1$

- We focus on static tidal for simplicity.
- Parameters
  - axion mass : $\mu M = 0.1,\ 0.2$
  - tidal force : $\dfrac{M_c M^2}{R^3}$
- Numerical code
  - 4th order RK
  - MPI-Open MP
  - fixed mesh refinement et. al.



Thank you.