

Presentation No.12



Black hole binaries and light fields

- gravitational molecule -

Taishi Ikeda (池田大志)

Sapienza University of Rome

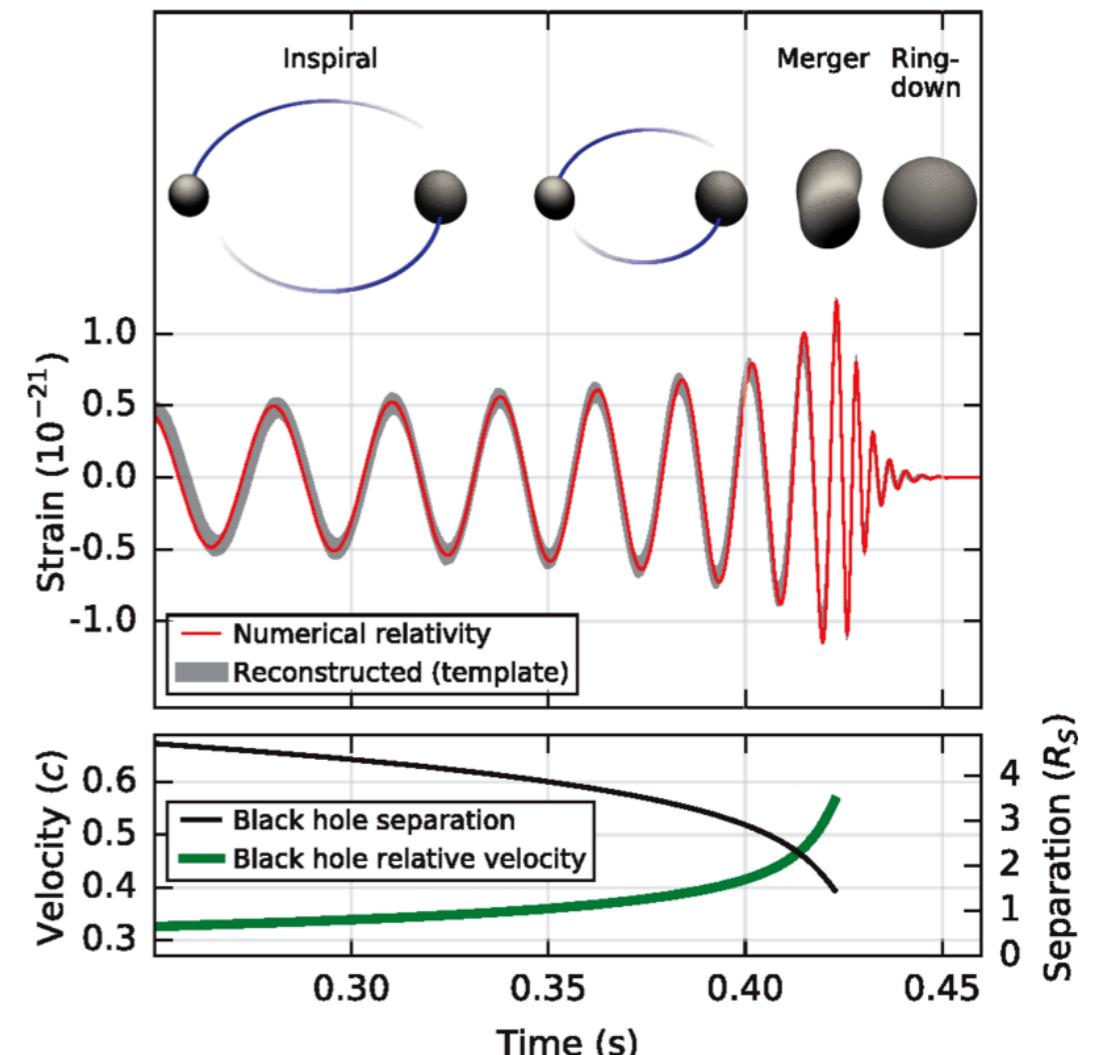
with Laura Bernard, Vitor Cardoso, Miguel Zilhão,

Phys.Rev.D 103 (2021) 2, 024020

Gravitational wave astronomy

- Binary Black hole is one of the main sources of GW.
- We can predict the wave form using....
 - ▶ PN approximation
 - ▶ BH perturbation
 - ▶ Numerical relativity et al
- However...

How deeply do we understand fundamental physics around binary BH ?



Black hole vs Black hole binary

- We don't deeply understand the physics around BH binary spacetime.

Black Hole

- Photon sphere
- Quasi normal mode
- Bound state of massive fields
(Gravitational atom)



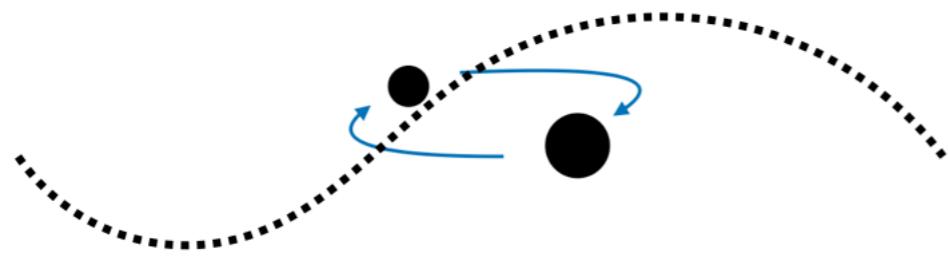
Black Hole Binary

- “Global ” photon surface?
- “Global” QNM ?
- ~~Please see PRD100(2019) 4,044002.~~
 - “Global” bound state of massive fields ?
(Gravitational molecule)

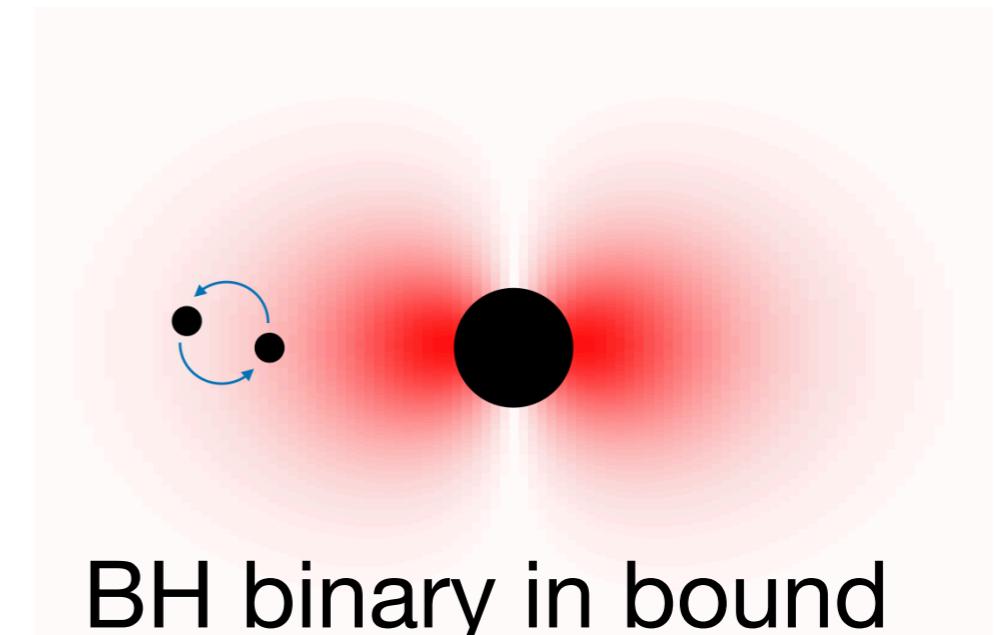


Massive field around binary

- When massive scalar fields become relevant...

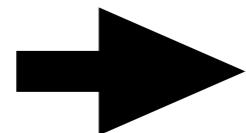


BH binary in coherent
oscillating scalar field



BH binary in bound
state around 3rd BH

What is the typical state of the massive scalar
field around BH binary ?



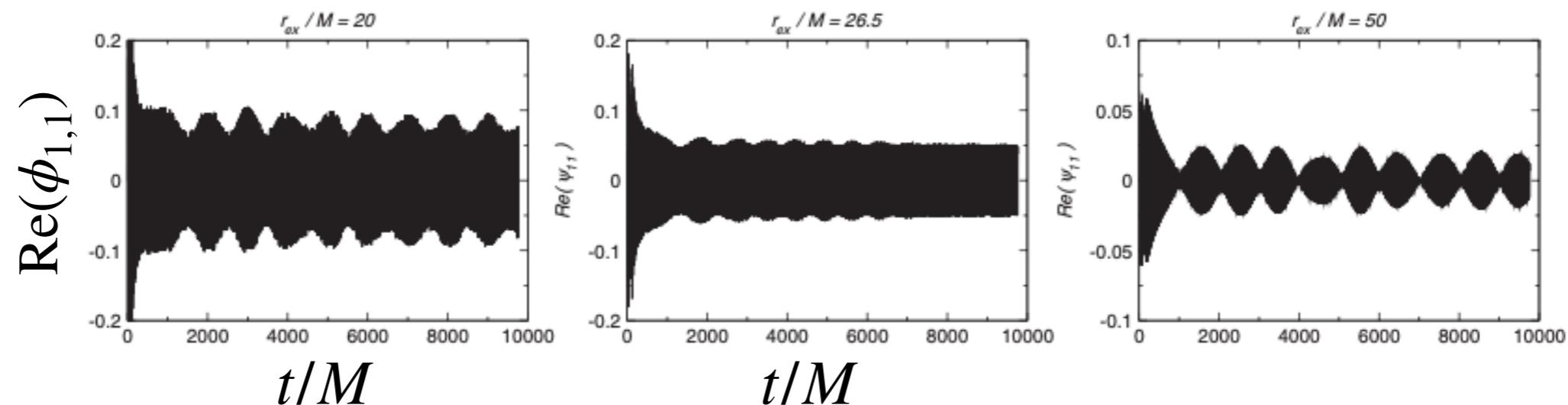
Gravitational molecule !!

Gravitational atom

- Gravitational atom = bound state of massive field around BH
 - ▶ Very long life time.

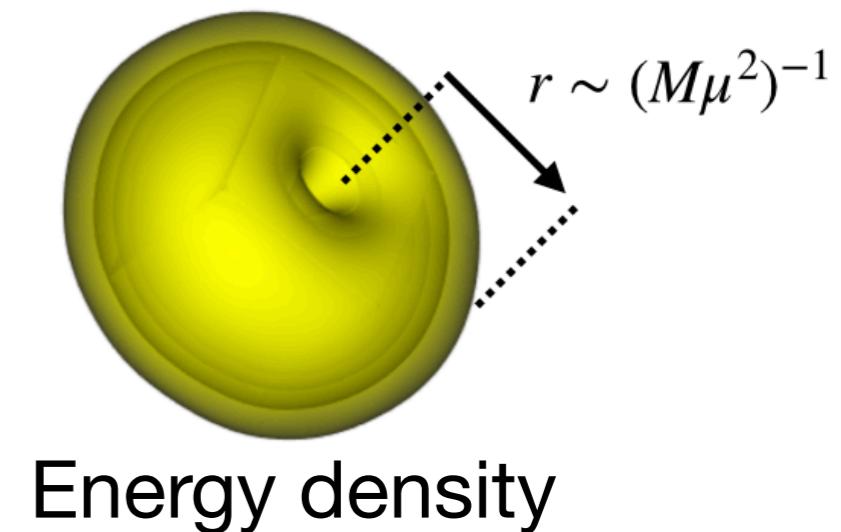
Witek et al (2013)

$$\mu M = 0.42 \quad a = 0.99M$$



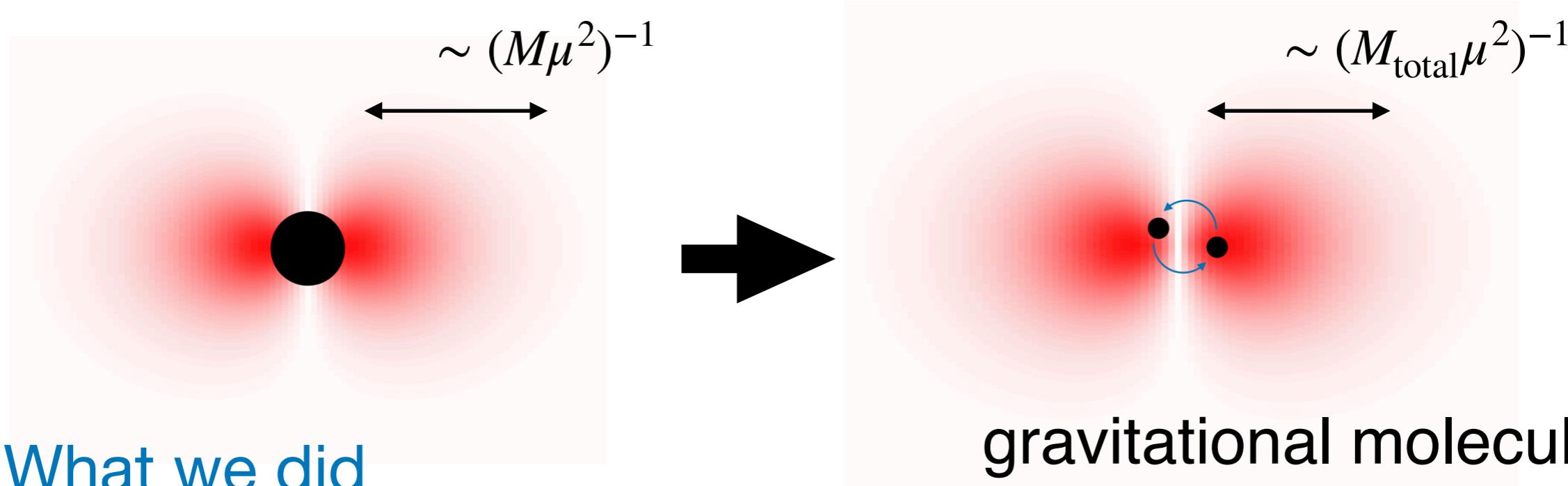
- ▶ the spectrum is similar to hydrogen atom

→ **Gravitational atom !!**



Our expectation

- Is there gravitational molecule ?
 - It seems possible to replace single BH with BH binary keeping bound state.



- What we did
 - We numerically solve the test scalar field around BH binary, and construct the gravitational molecule.

Initial data

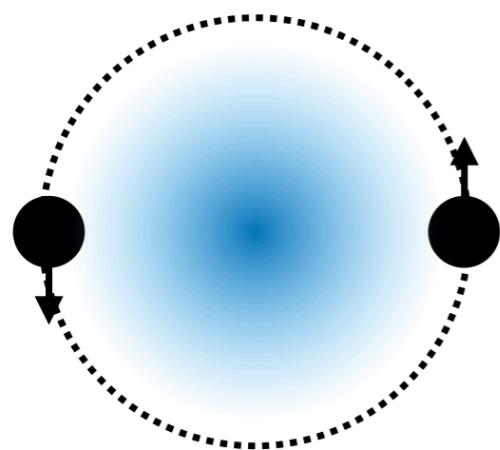
- We use three different initial data.

$$\phi = R(r)\mathcal{A}(t, \theta, \varphi)$$

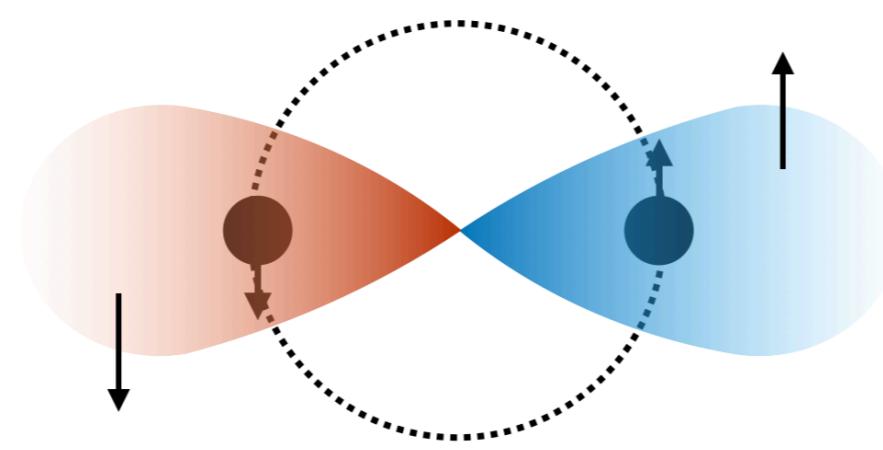
$$\begin{cases} \phi = Ae^{-\frac{1}{2}\left(\frac{r}{w}\right)^2} \\ (\partial_t - \mathcal{L}_\beta)\phi = 0 \end{cases}$$

$$\begin{cases} R = A_0 r e^{-\frac{r}{2w}} \\ \mathcal{A} = \operatorname{Re} \left(Y_{1,1}(\theta, \varphi) e^{-i\mu t} \right) \end{cases}$$

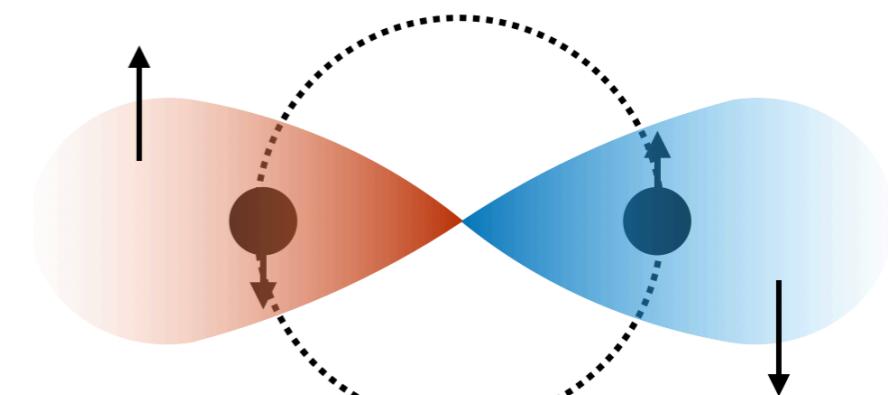
$$\begin{cases} R = A_0 r e^{-\frac{r}{2w}} \\ \mathcal{A} = \operatorname{Re} \left(Y_{1,1}(\theta, \varphi) e^{+i\mu t} \right) \end{cases}$$



Gaussian profile



Co-rotating dipole

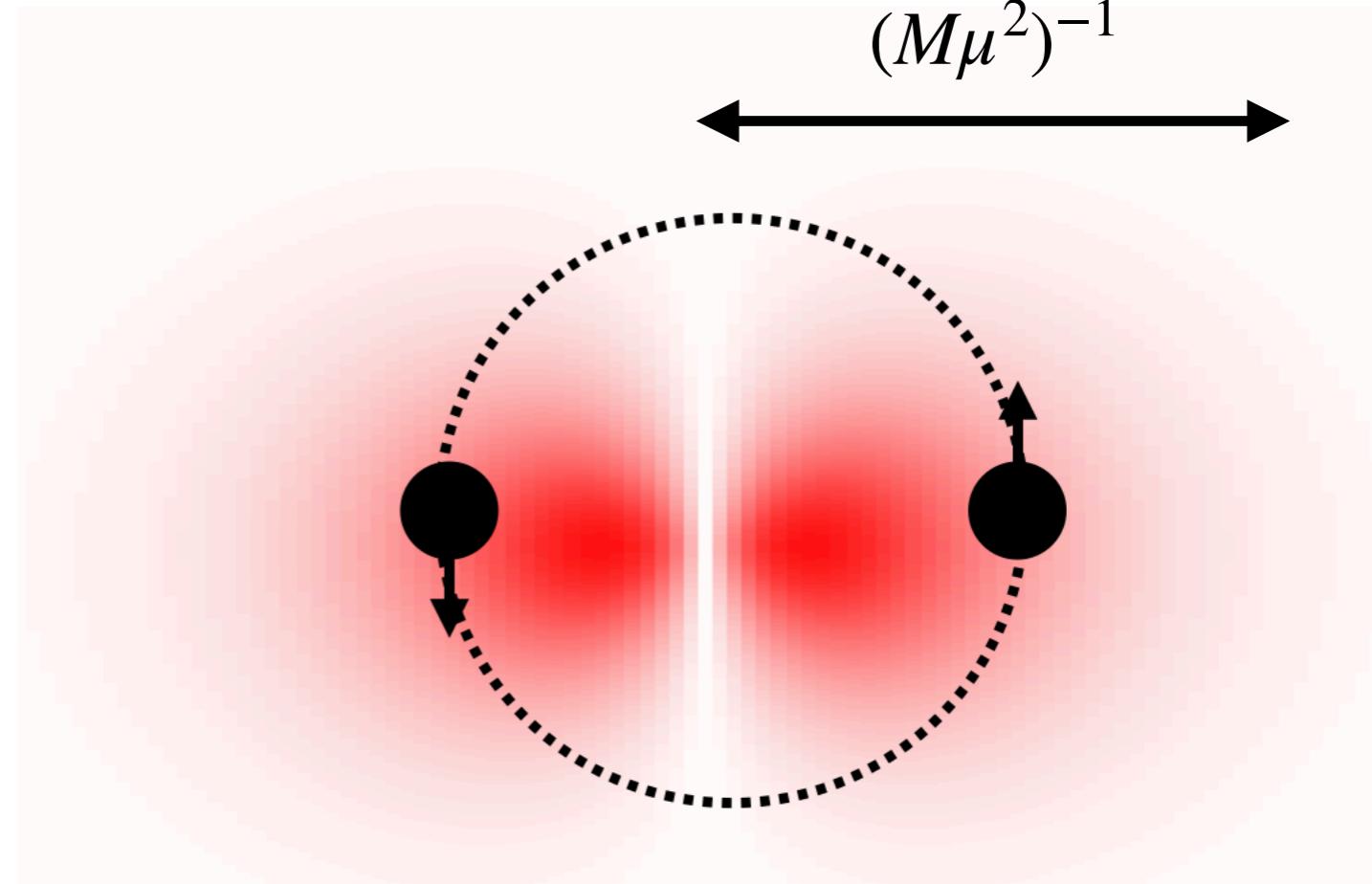
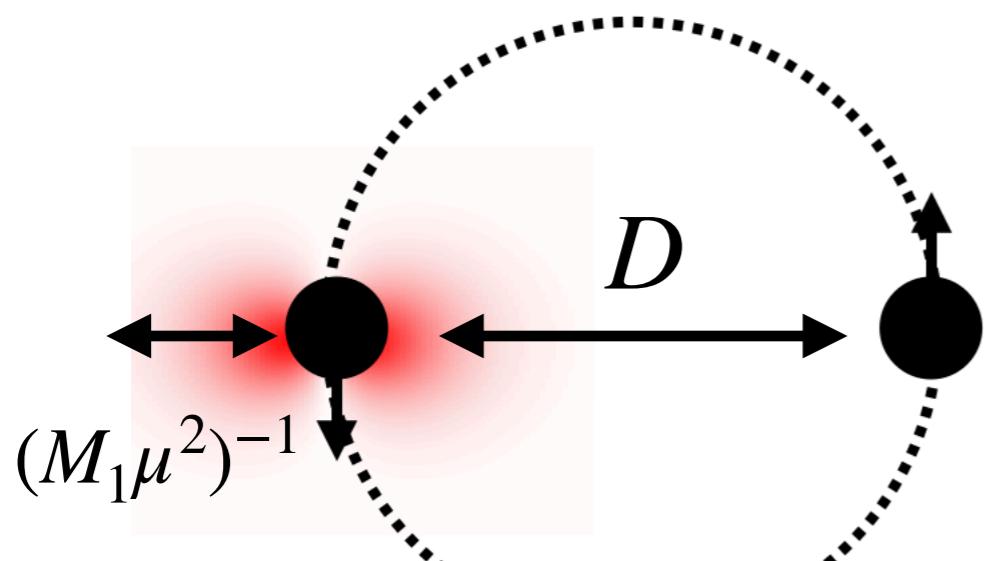


Counter-rotating dipole

Typical length scale

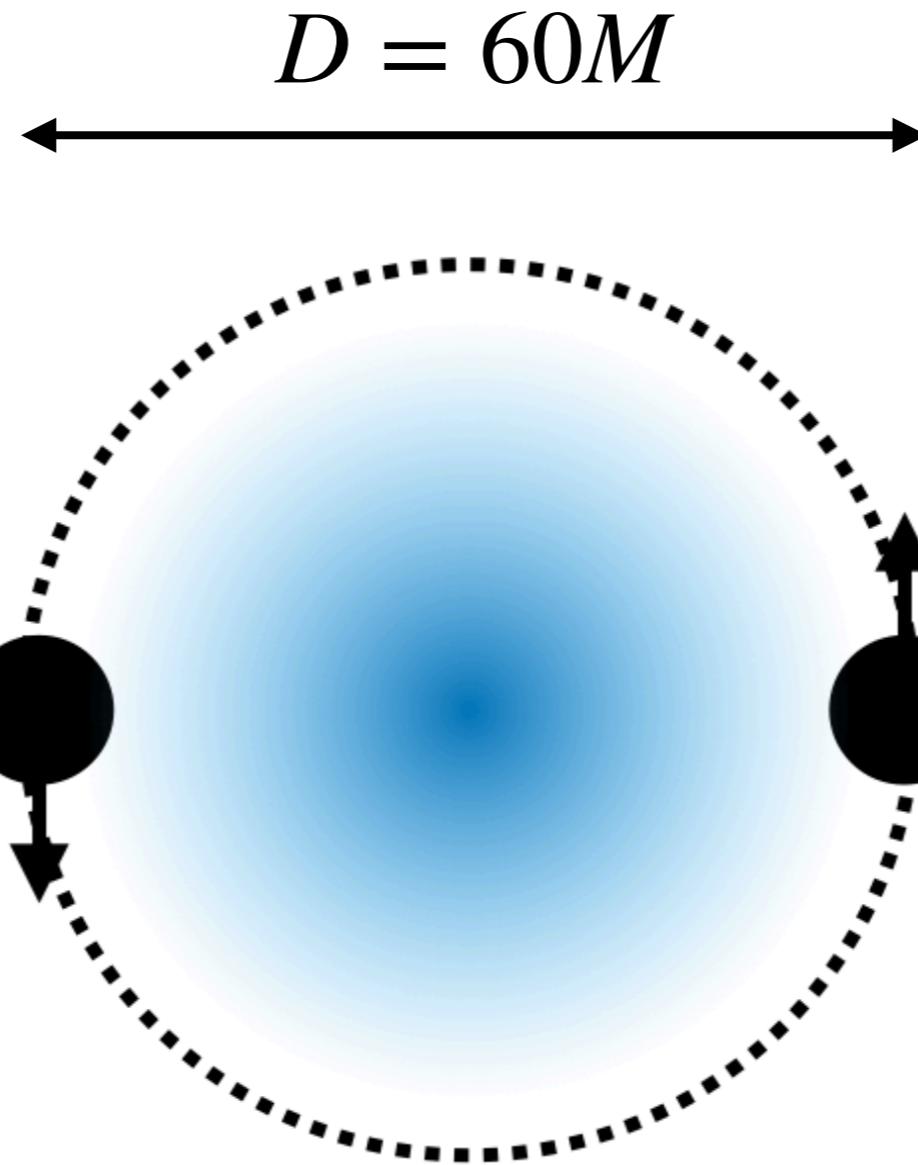
$$(\square - \mu^2) \phi = 0$$
$$M_1 = M_2 = \frac{M}{2}$$

- We have three length scales.



$\mathcal{O}((M_1\mu^2)^{-1}) \ll D$: gravitational atom around individual BH

$\mathcal{O}((M\mu^2)^{-1}) \gtrsim D$: gravitational molecule around BH binary



$$\mu M = 0.5$$

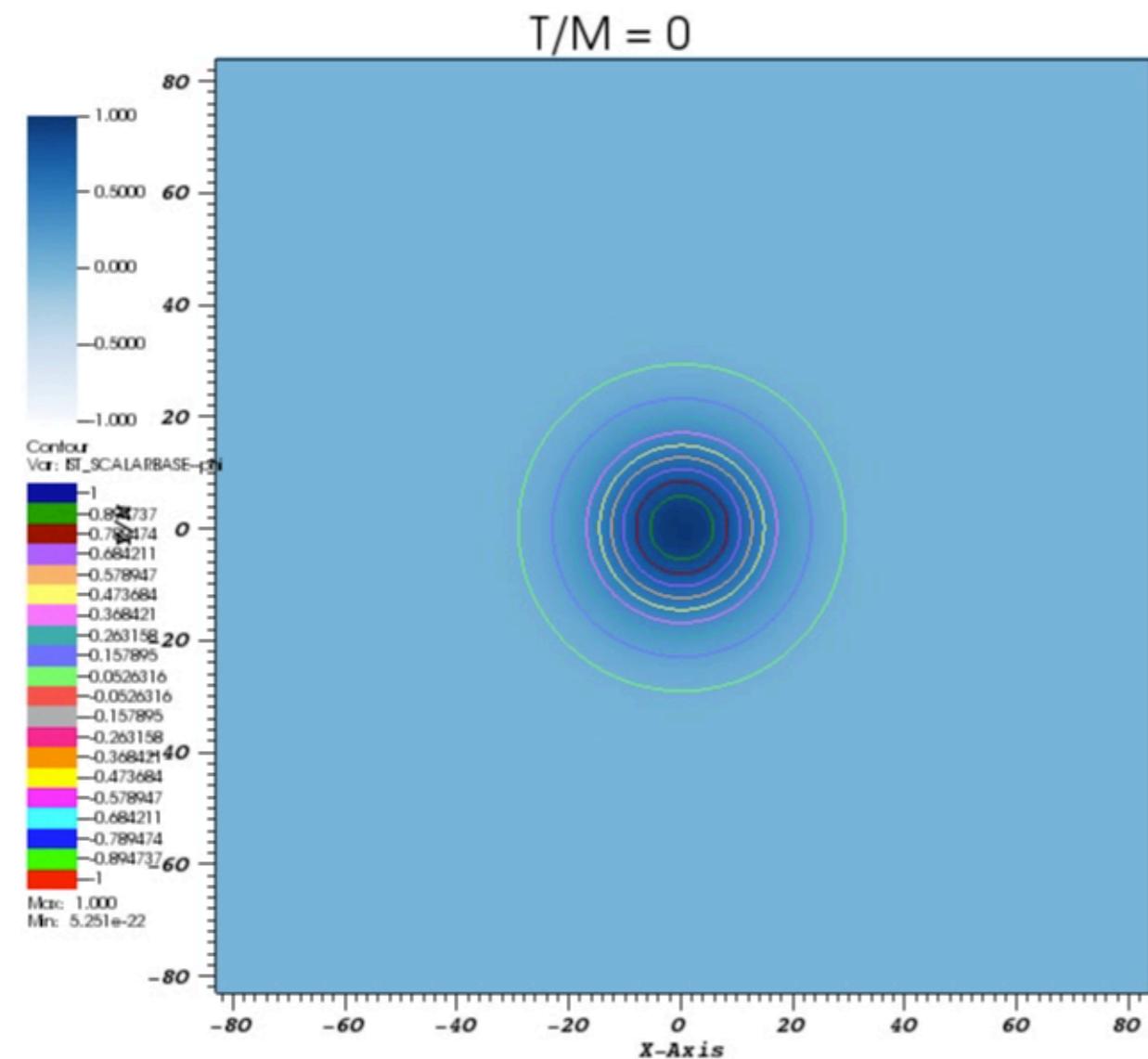
$$w = 25M$$

$$\mathcal{O}((M_1\mu^2)^{-1}) \ll D$$

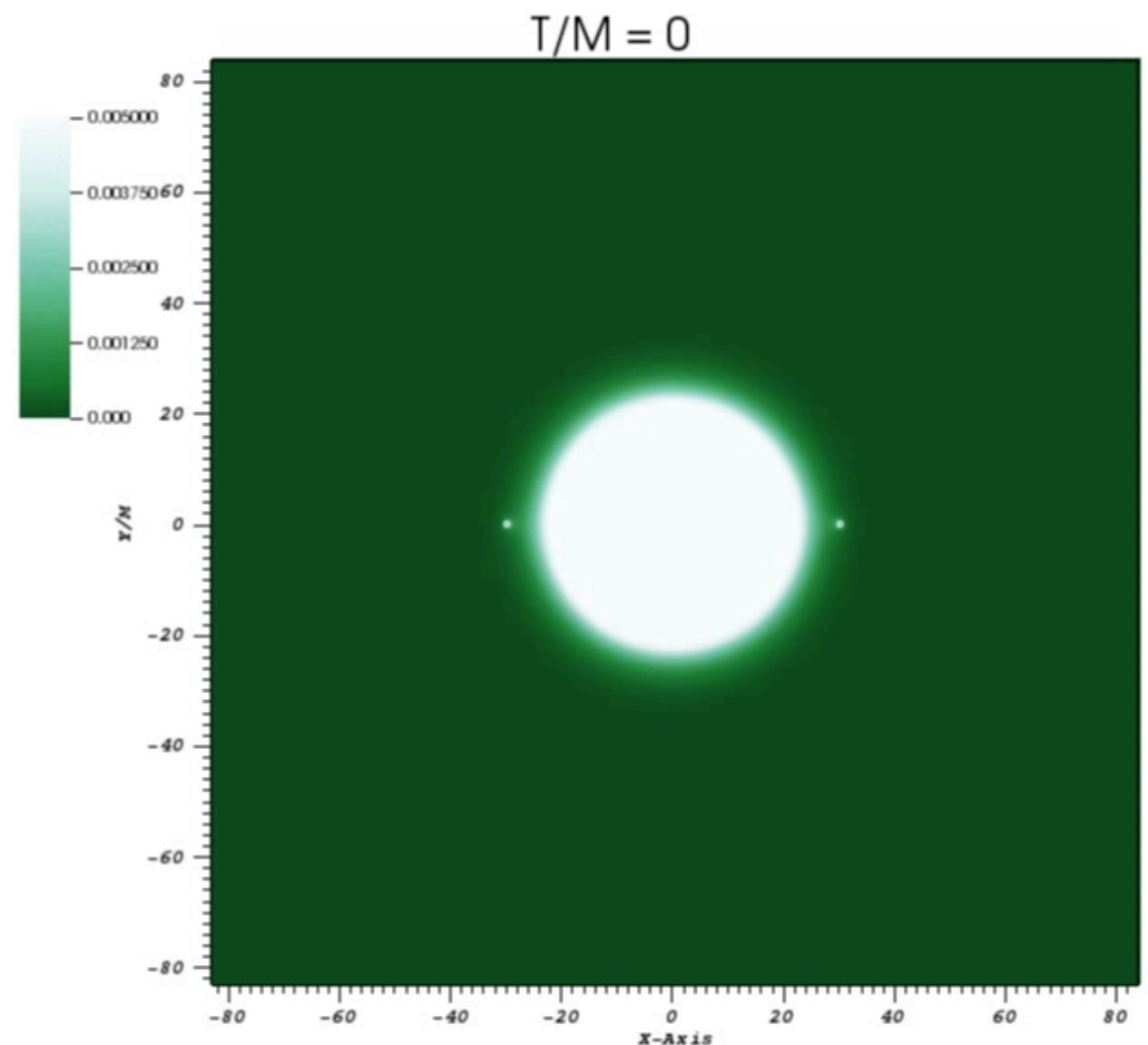
Simulation 1 : Gaussian initial data

Simulation 1

Scalar field

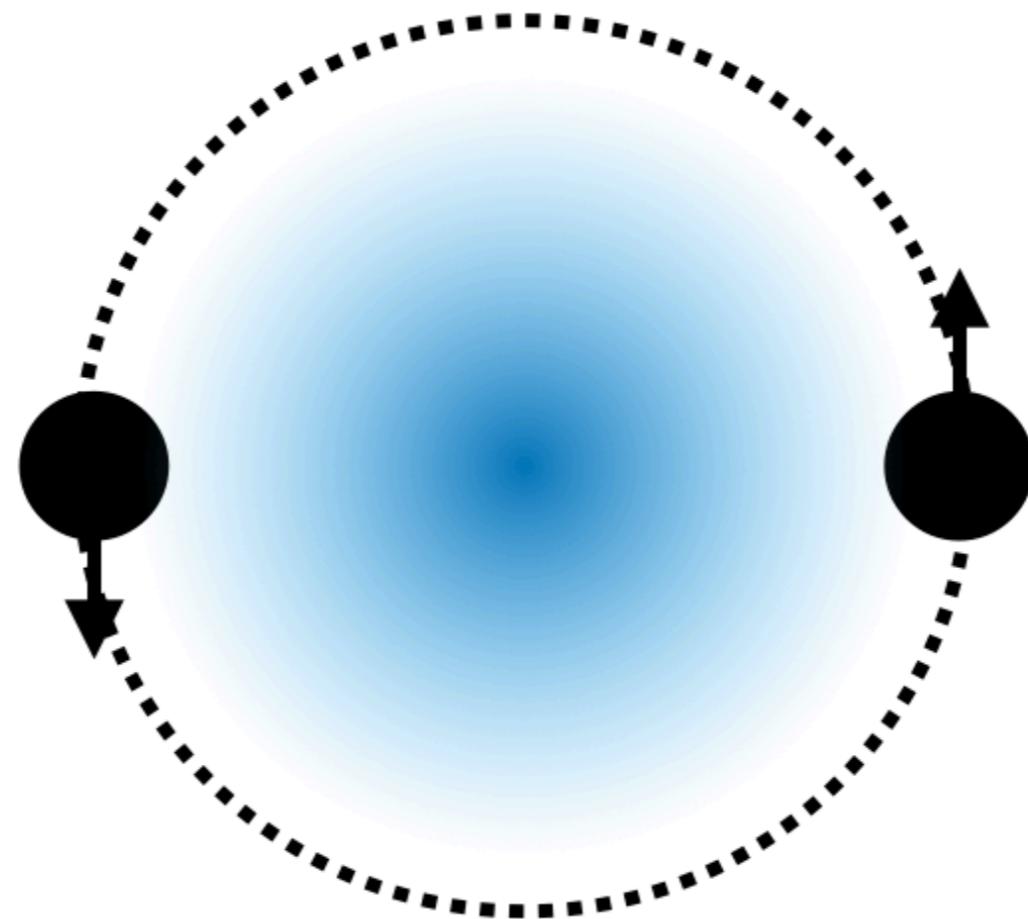


Energy density



- Gravitational atom around individual BHs.

$$D = 10M, 60M$$



$$\mu M = 0.2$$

$$w = 25M$$

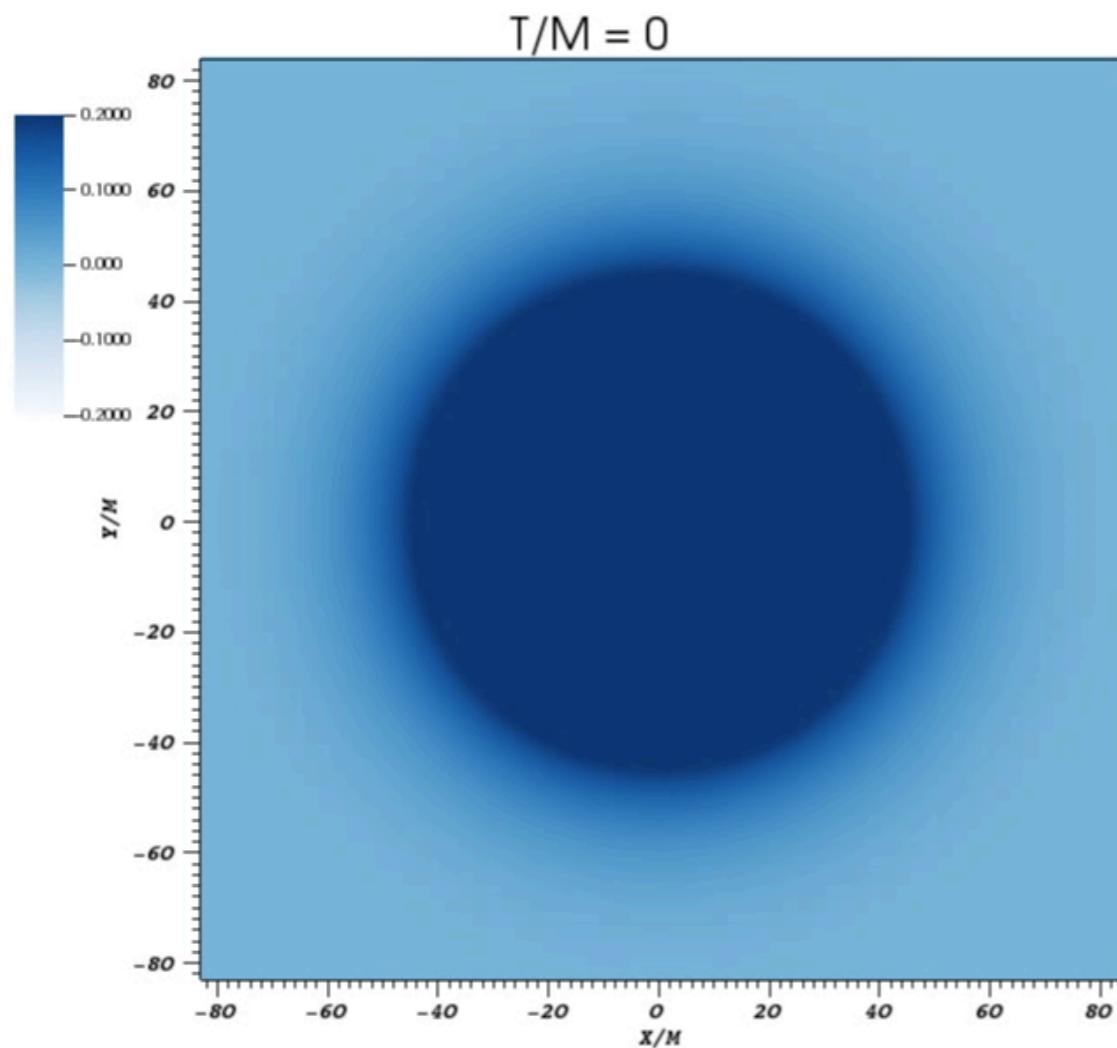
$$\mathcal{O}((M\mu^2)^{-1}) \gtrsim D$$

Simulation 2 : Gaussian initial data

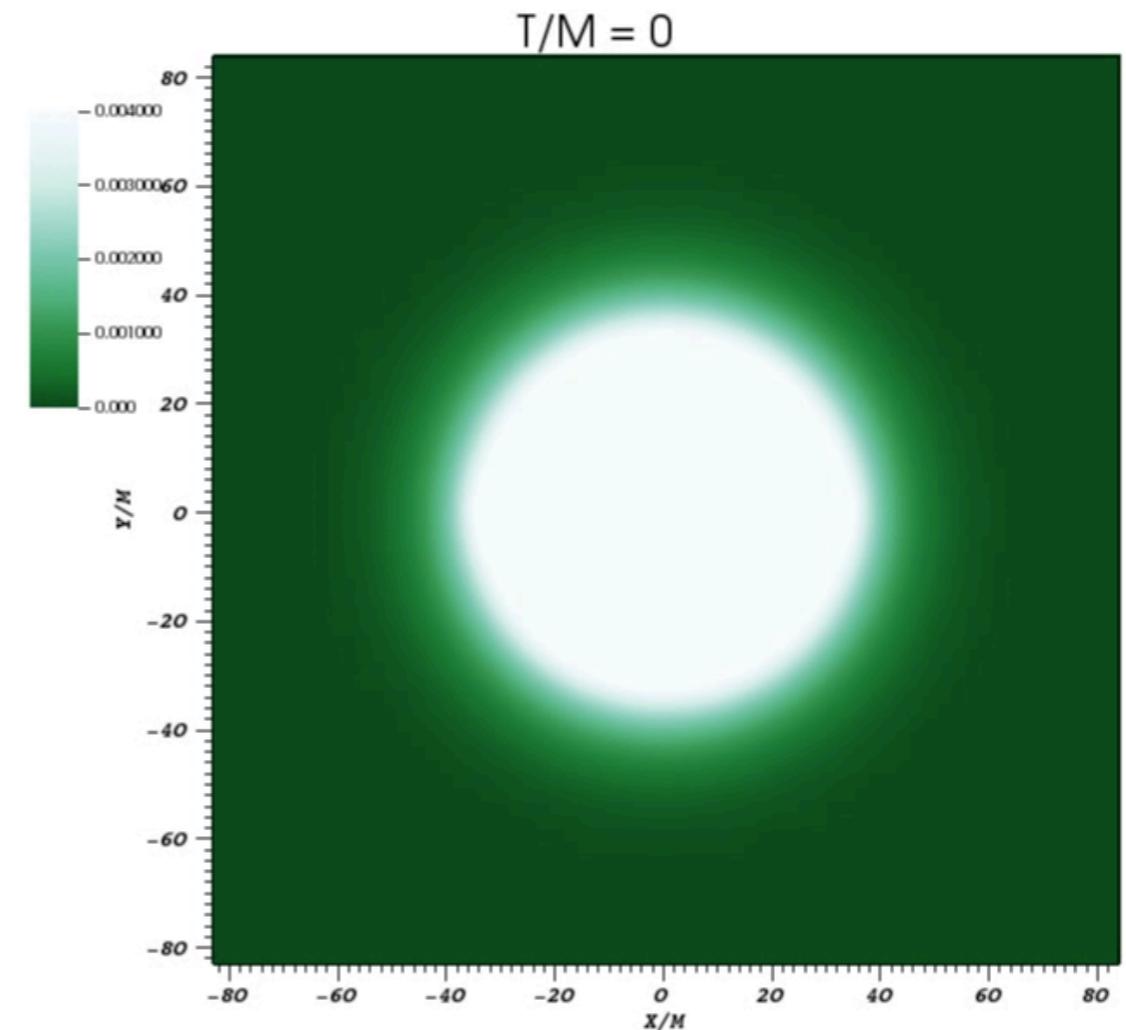
Simulation 2

$$D = 60M$$

Scalar field



Energy density

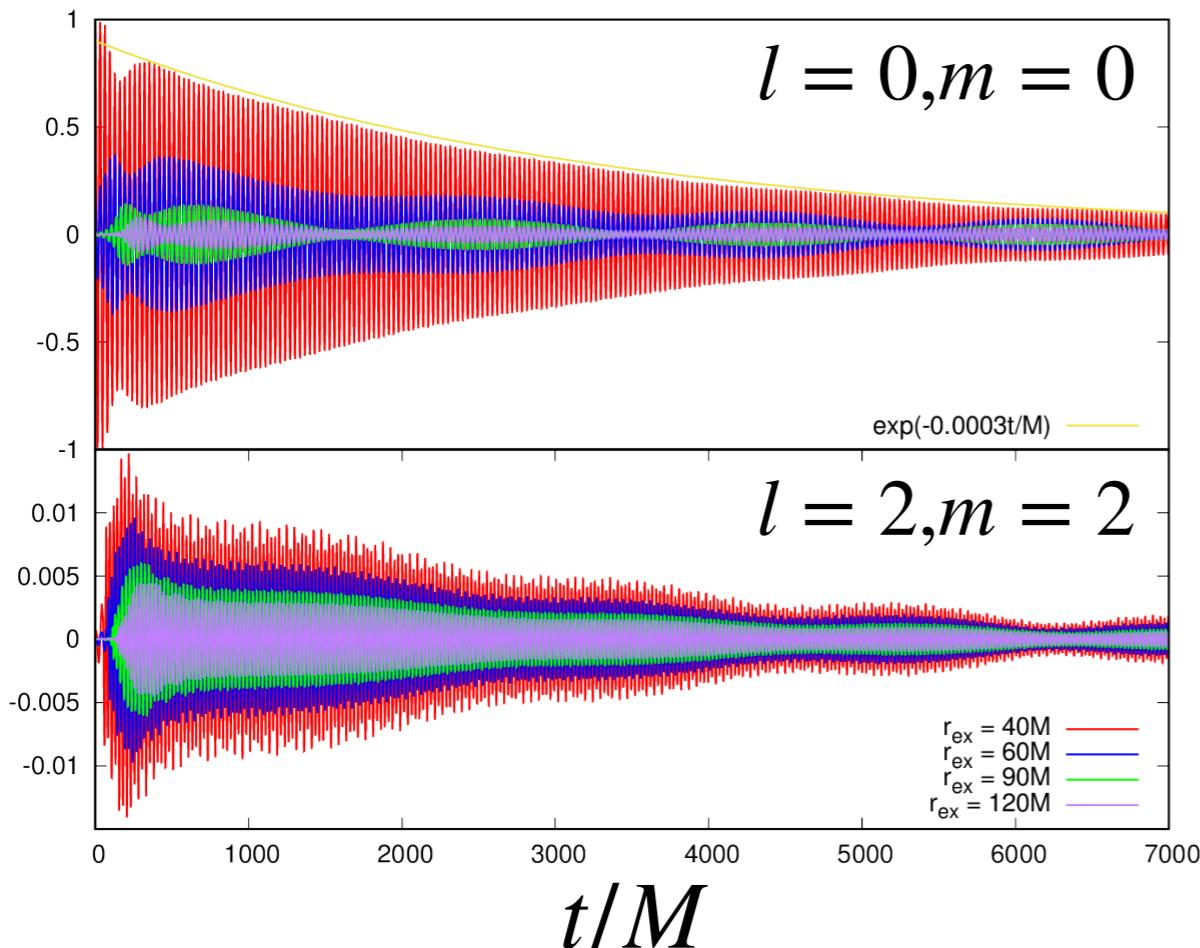


- “Monopole” gravitational molecule around BH binary.

Simulation 2

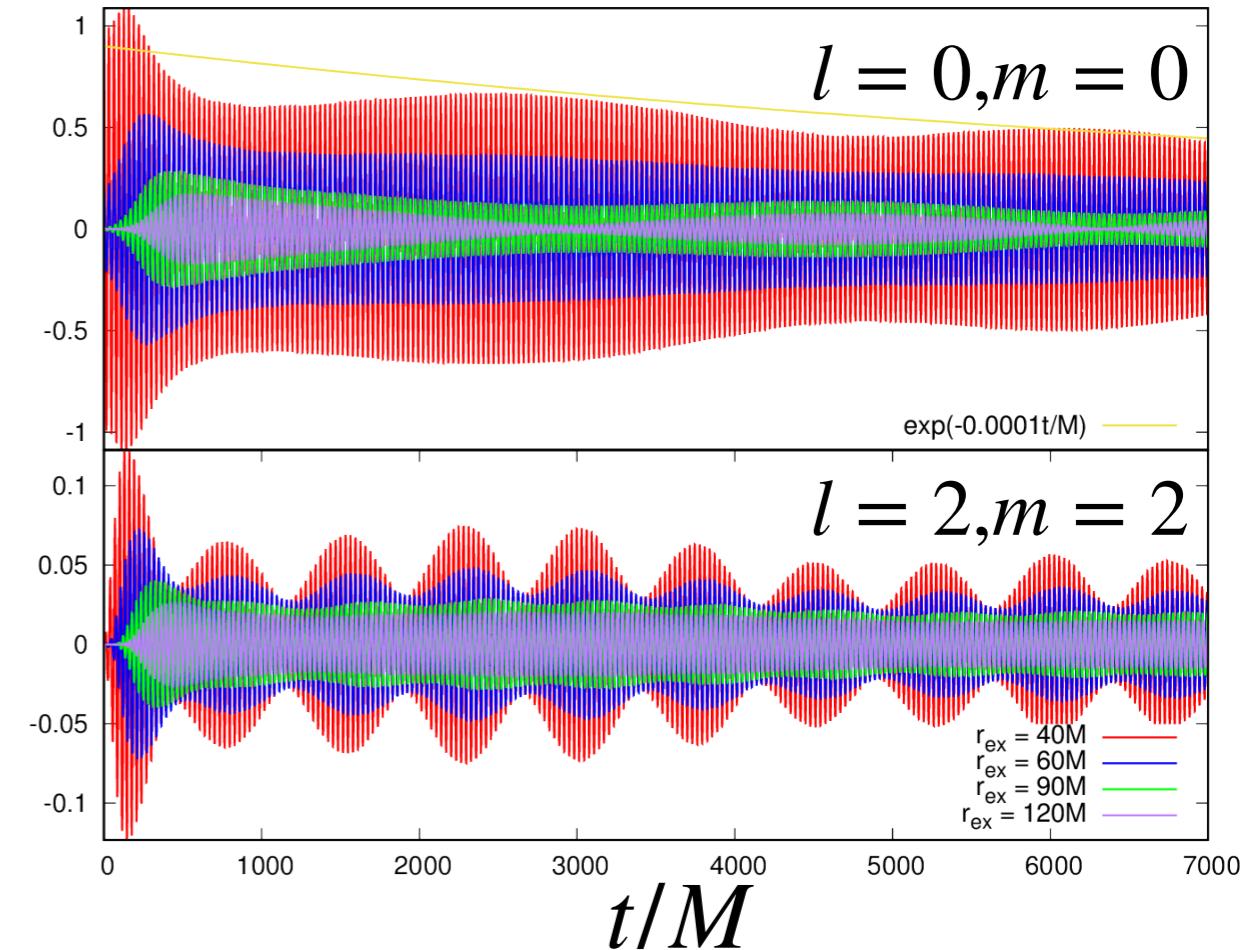
$$\text{cf: } \phi_{00} \sim e^{-\frac{t}{\tau}}$$

$D = 10M$



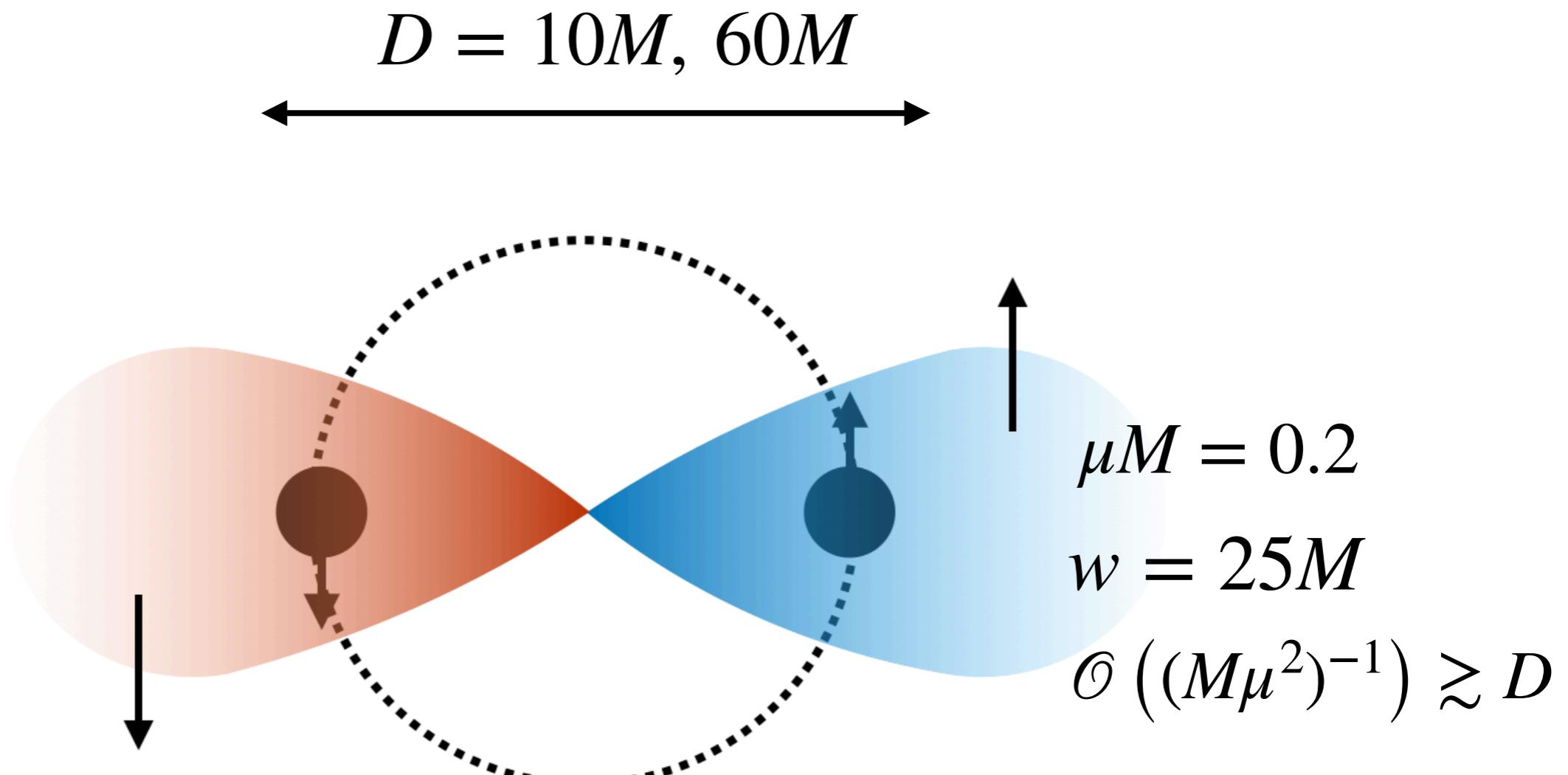
$$\tau \simeq 3 \times 10^3 M$$

$D = 60M$



$$\tau \simeq 1 \times 10^4 M$$

- The spectrum of numerical simulation is good agreement with Di-hydrogen atom.

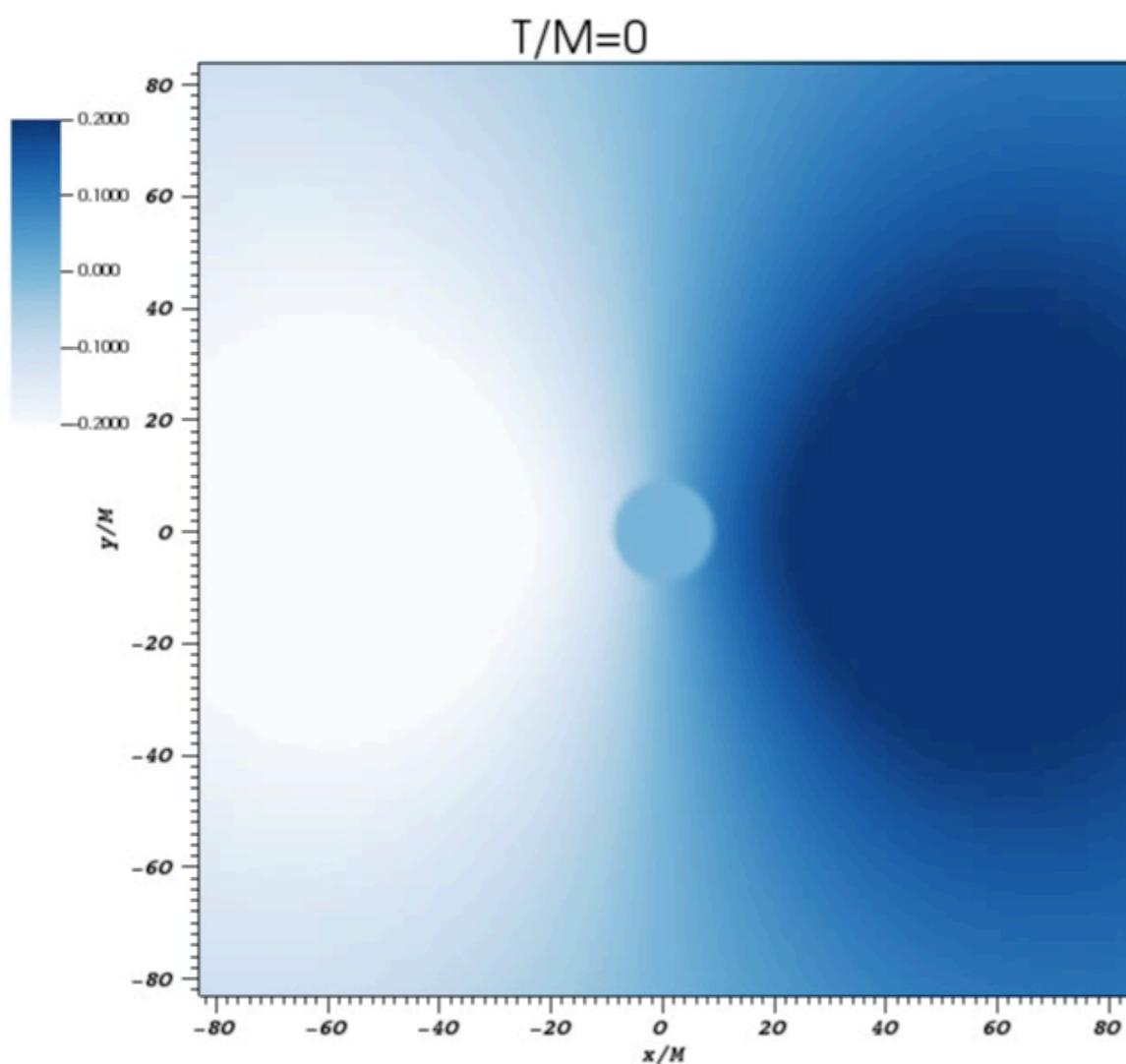


Simulation 3 : Co-rotating dipole initial data

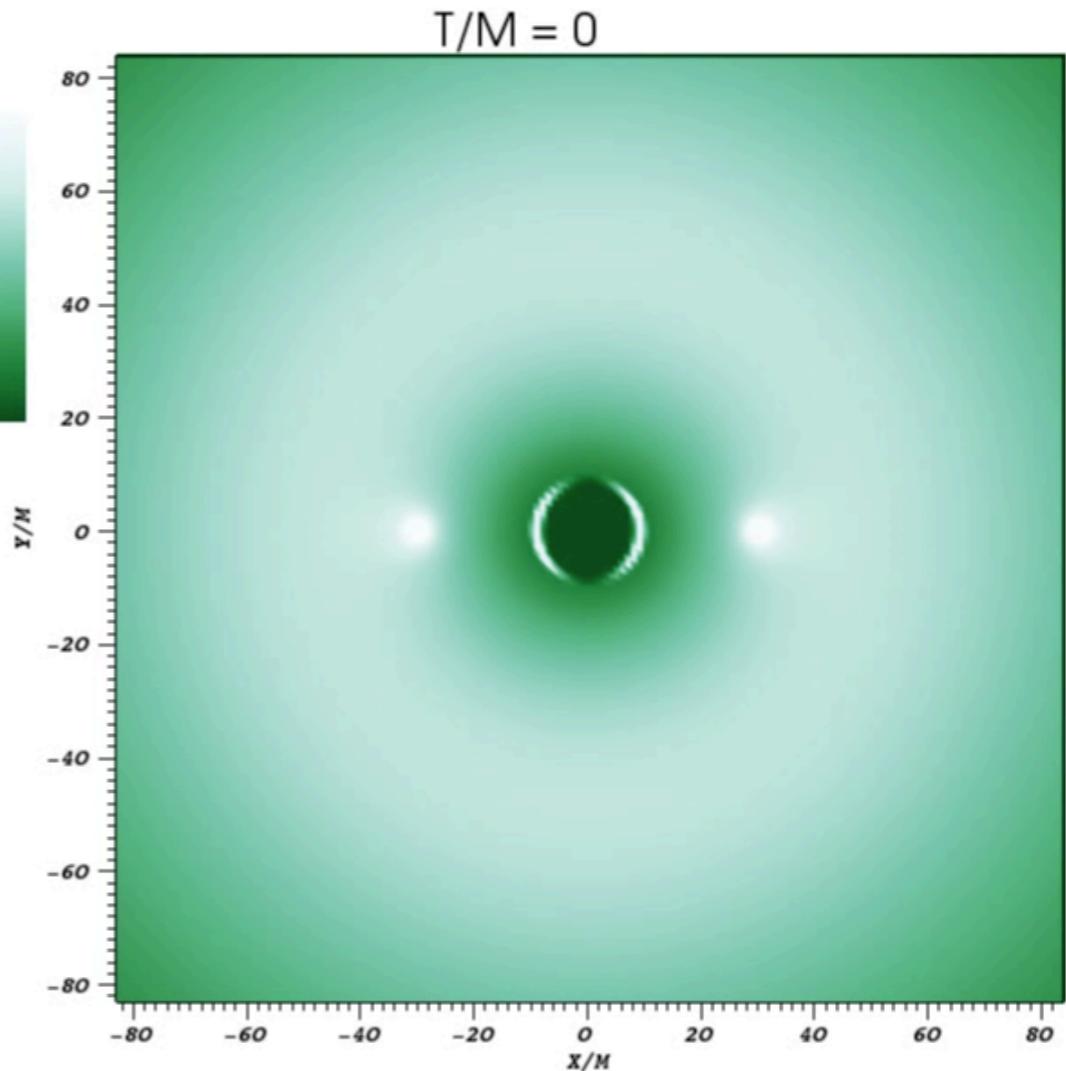
Simulation 3

$D = 60M$

Scalar field



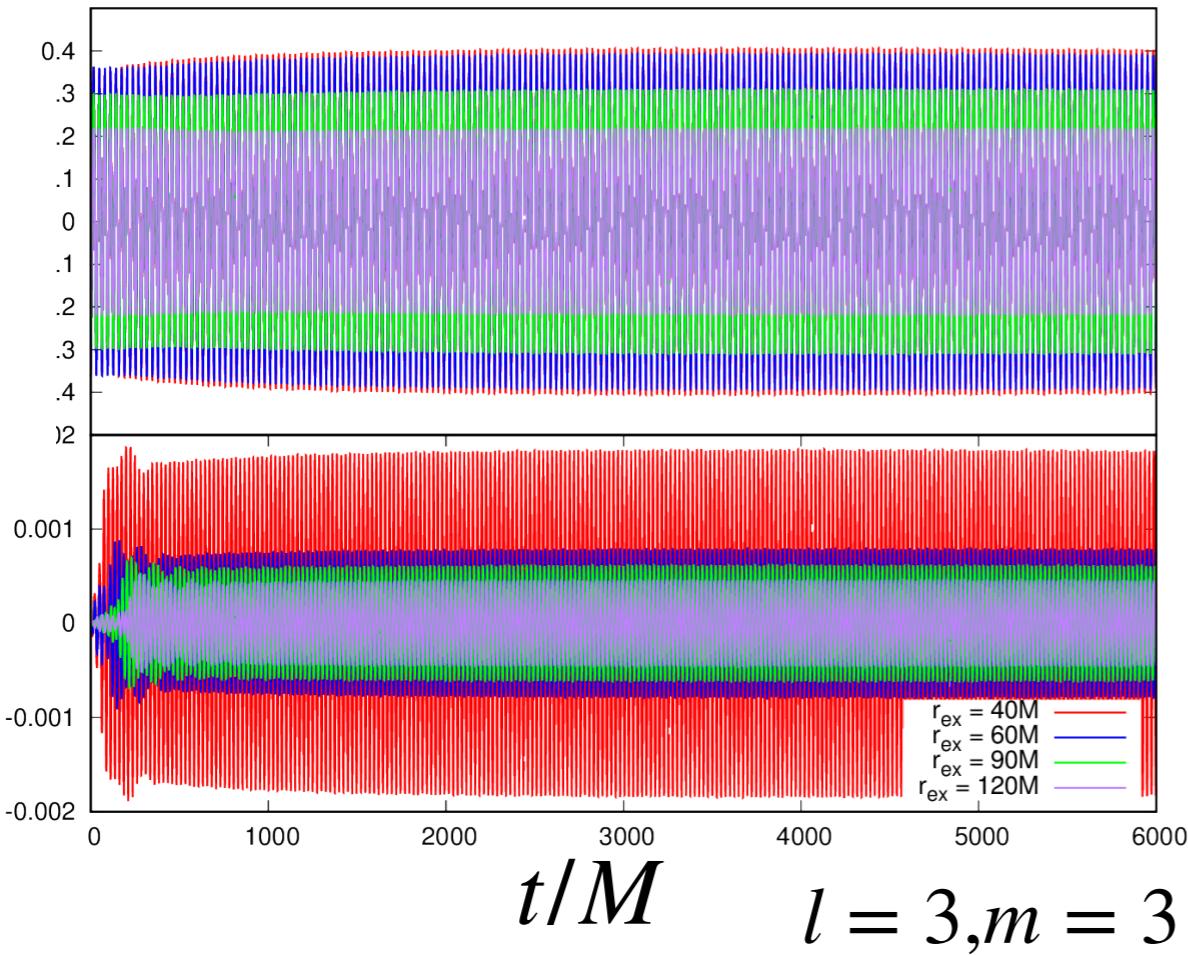
Energy density



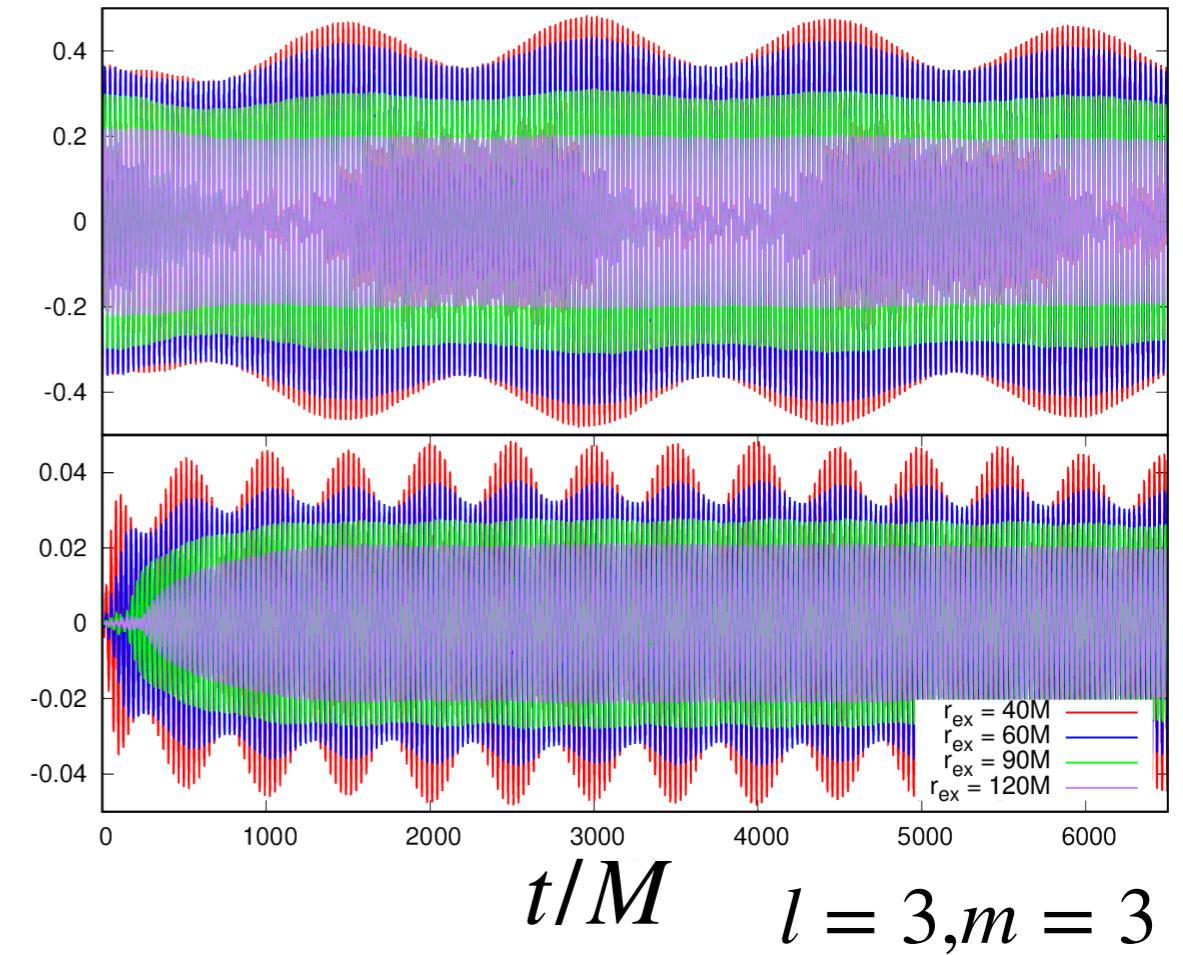
- Energy density rotates with binary.
- “Dipole” co-rotating gravitational molecule around BH binary.

Simulation 3

$$D = 10M \quad l = 1, m = 1$$

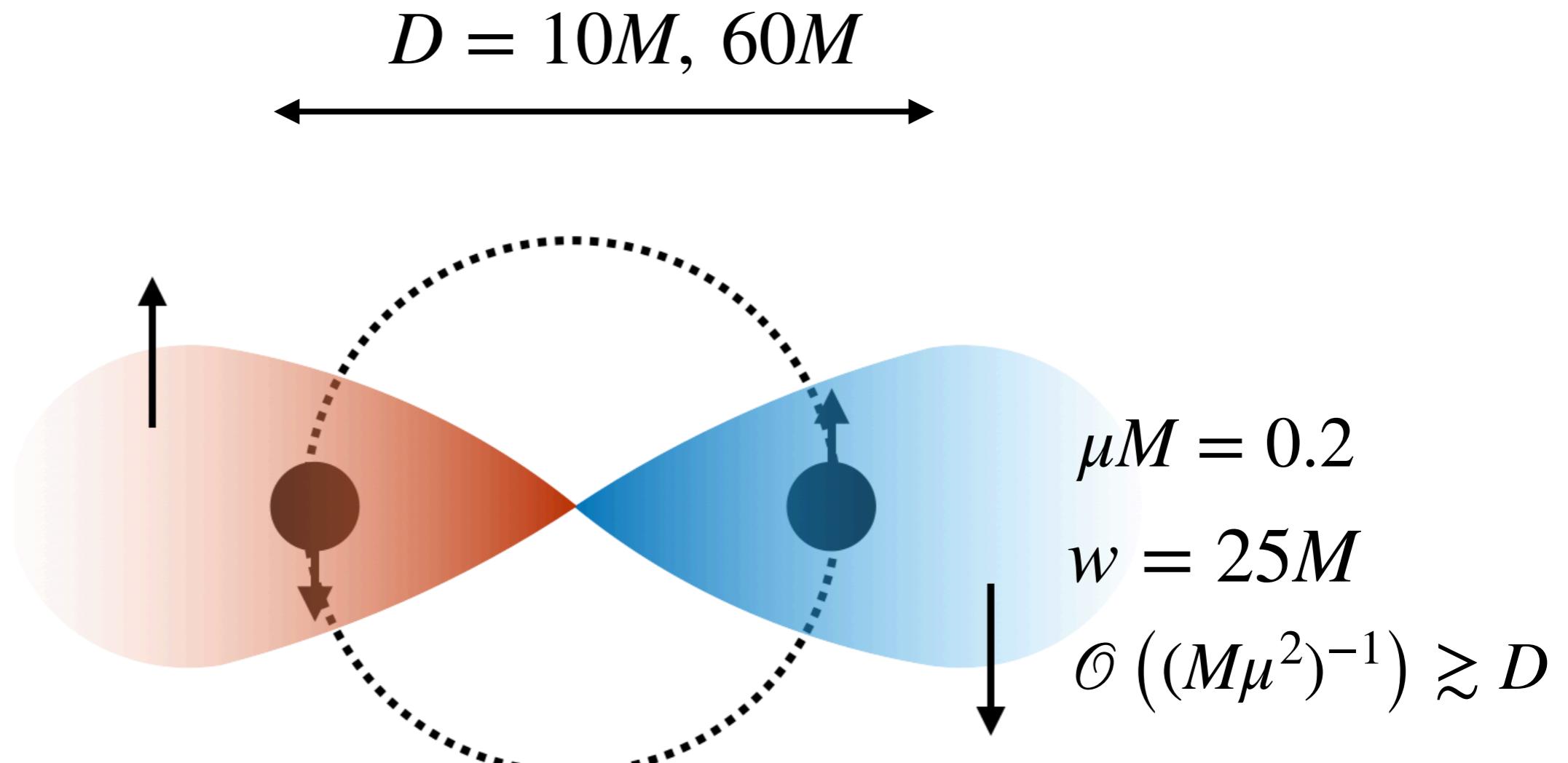


$$D = 60M \quad l = 1, m = 1$$



We could not get decaying time scale.

At least, $\tau \gg 10^4 M$

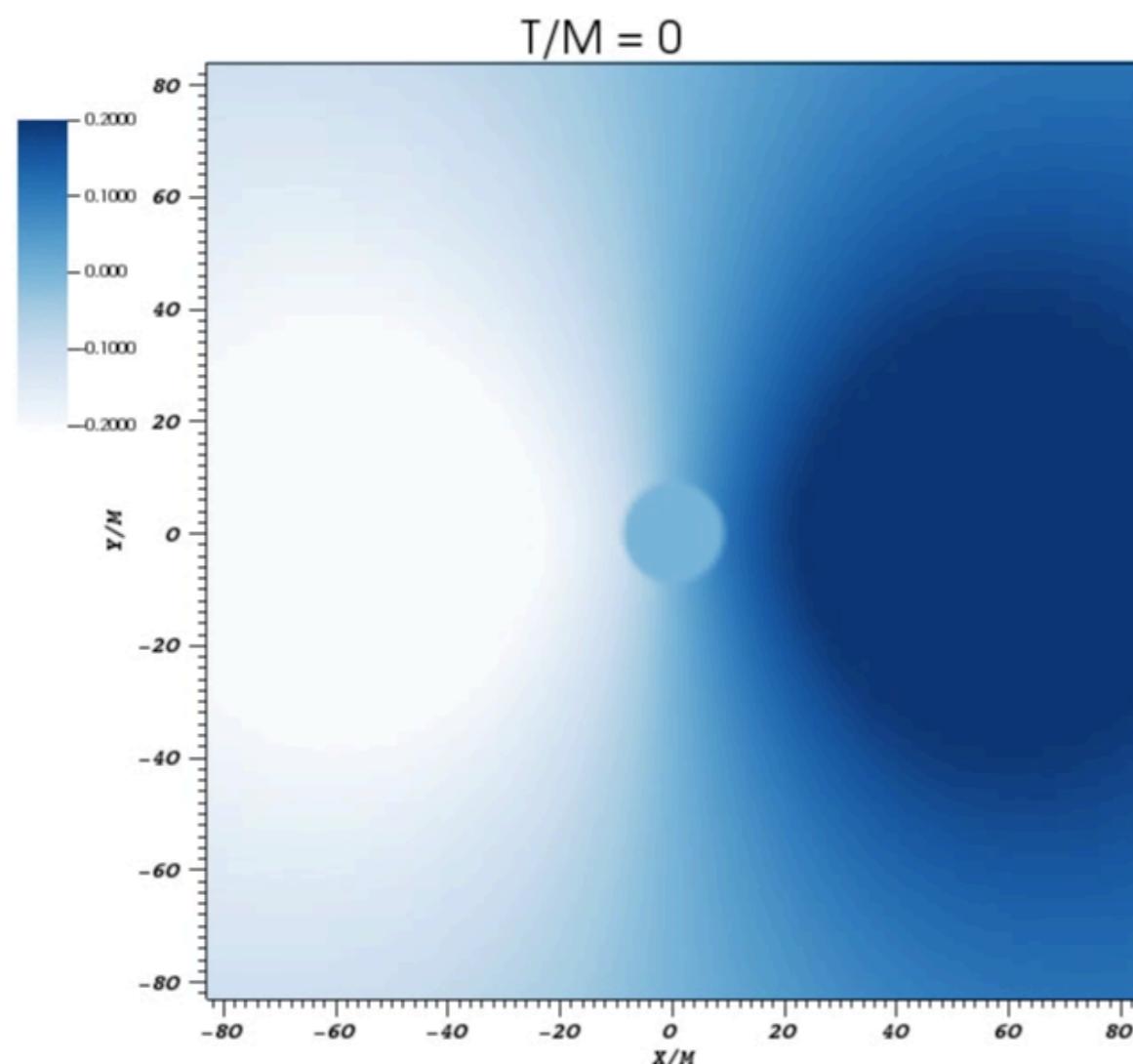


Simulation 4 : Counter-rotating dipole initial data

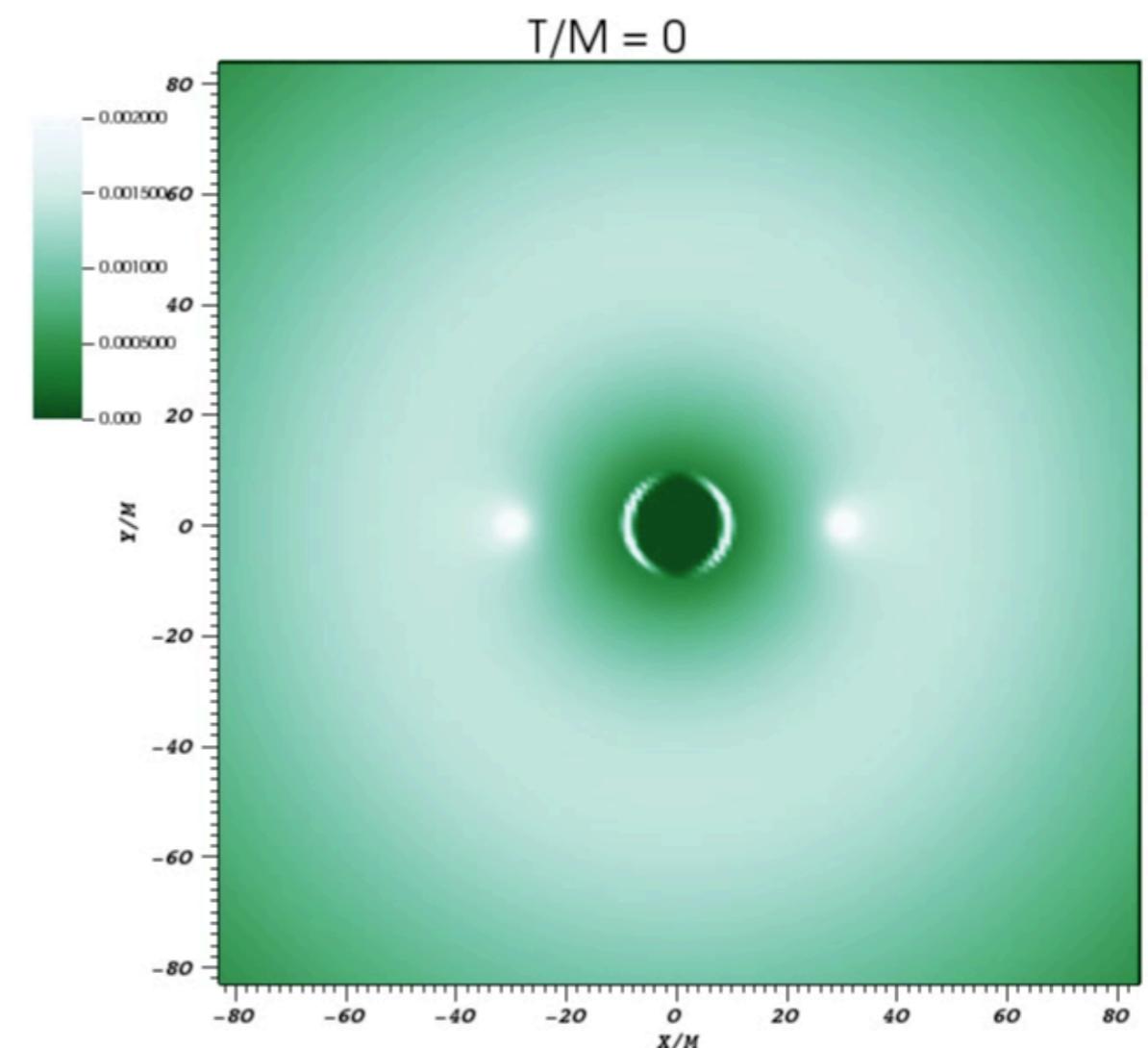
Simulation 4

$D = 60M$

Scalar field



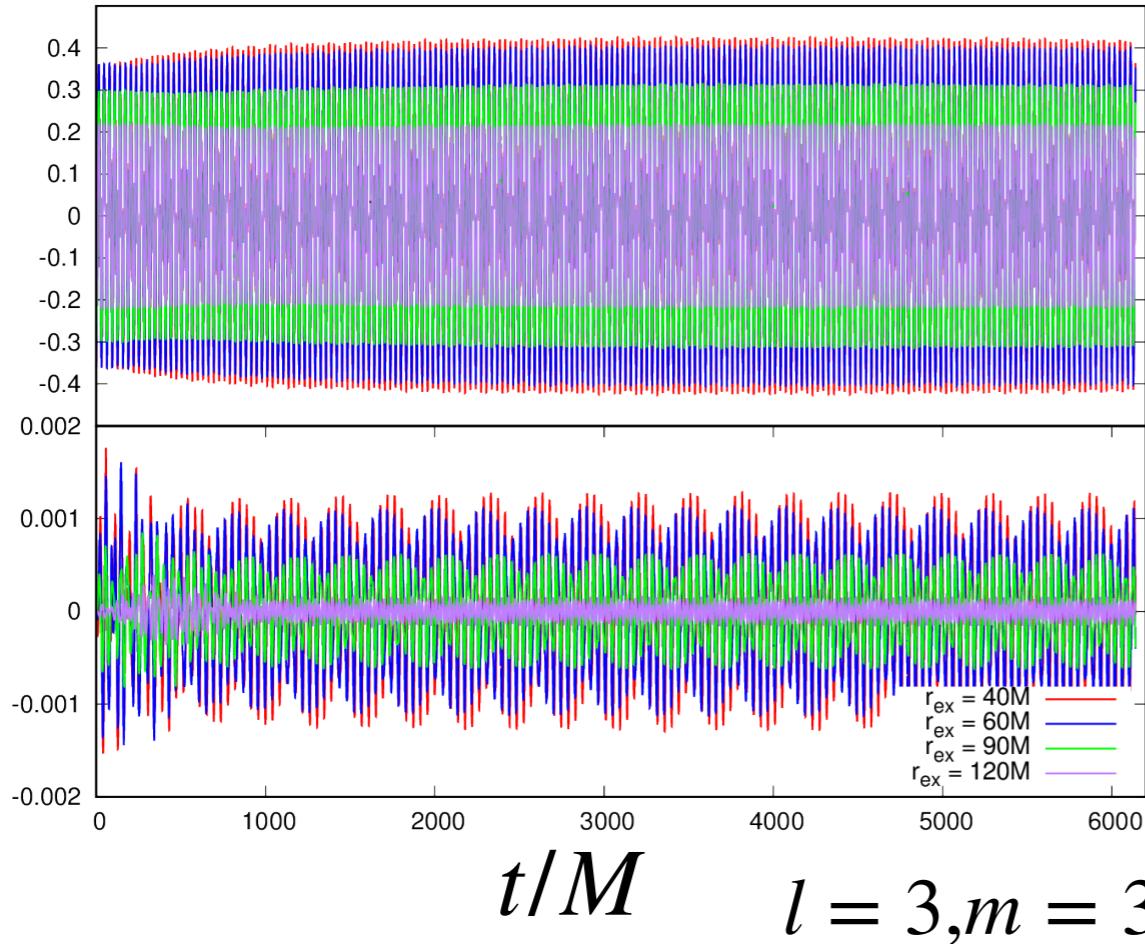
Energy density



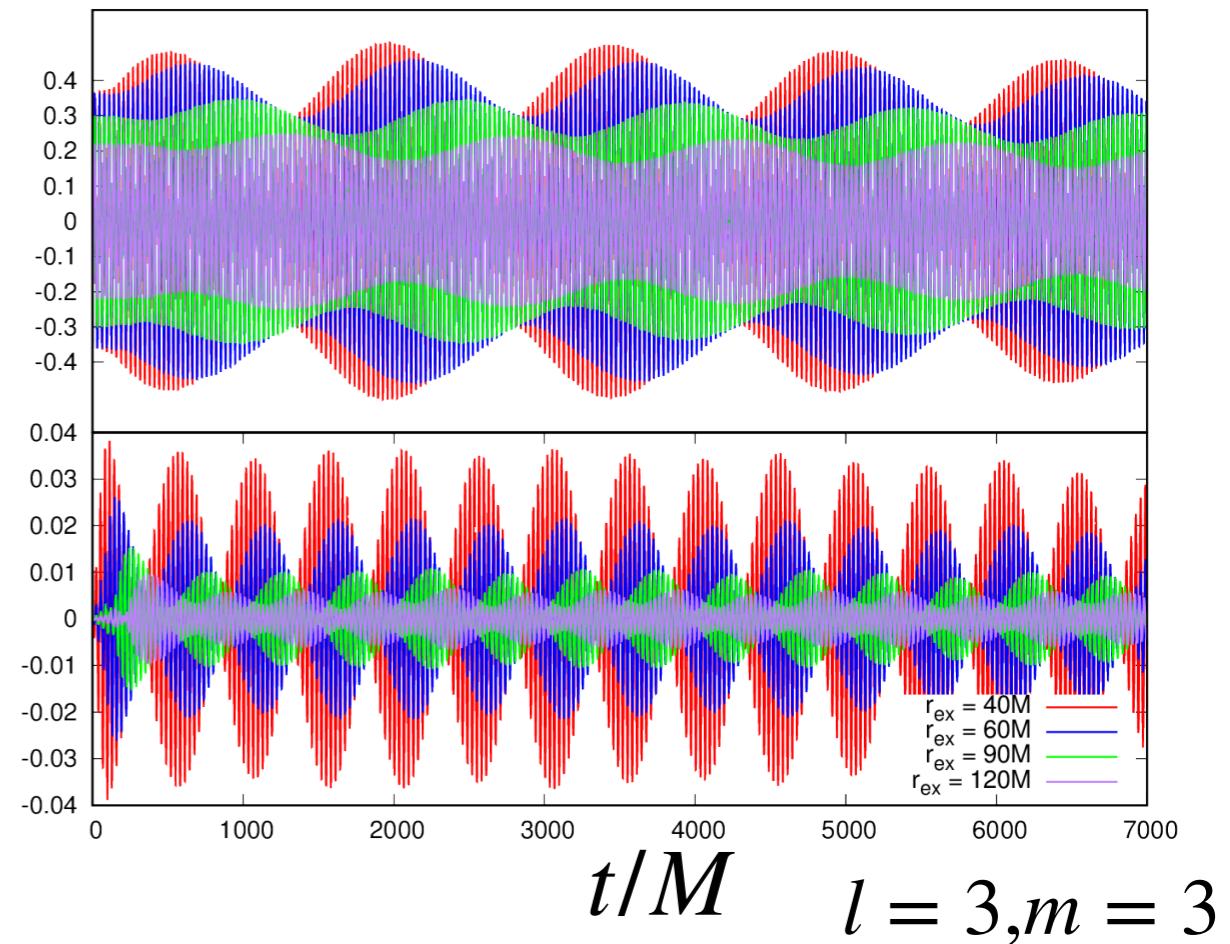
- Energy density rotates with binary.
- “Dipole” counter-rotating gravitational molecule around BH binary.

Simulation 4

$$D = 10M \quad l = 1, m = 1$$



$$D = 60M \quad l = 1, m = 1$$



We could not get decaying time scale.

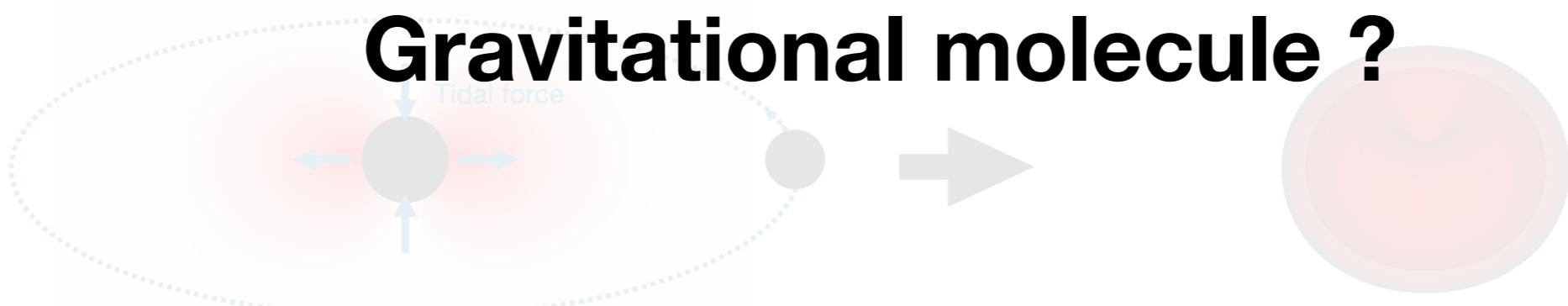
At least, $\tau \gg 10^4 M$

Summary

- Our result
 - Strong evidence for existence of global bound state (Gravitational molecule).
 - Analogy with QM of di-hydrogen atom.
- (a lot of) Future works
 - eccentricity orbit ?
 - Gravitational wave from the gravitational molecule ?
 - Force between BHs due to the molecule ?
 - Can we observe the force using GW from binary ?
 - We can apply a lot of well-known physics of QM for molecule !! et al

In these setup, massive scalar fields can
be relevant around BH binary.

We must understand typical behavior of
the fields around the binary.



Outline

- 1. Introduction**
- 2. Gravitational atom**
- 3. Analytic description**
 - **Gravitational molecule**
 - **Particle picture**
- 4. Numerical simulations**
 - **Set up**
 - **Numerical construction**
- 5. Summary**

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Gravitational atom

$$c^2 = -(\mu M)^2 a^2 (\mu^2 - \omega^2)$$

$$\Delta = r^2 - 2Mr + a^2$$

- Bound state of massive scalar field around Kerr BH

S.Detweiler (1980)

$$(\Box_{\text{BL Kerr}} - \mu^2) \Phi = 0 \quad \Phi = e^{-i\omega t + im\phi} R(r) S(\theta)$$

- Angular part (spheroidal harmonics)

$$\left(-\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) - c^2 \cos^2 \theta + \frac{m^2}{\sin^2 \theta} \right) S = \Lambda S$$

Bound state
 $EM = (\omega - \mu)M < 0$

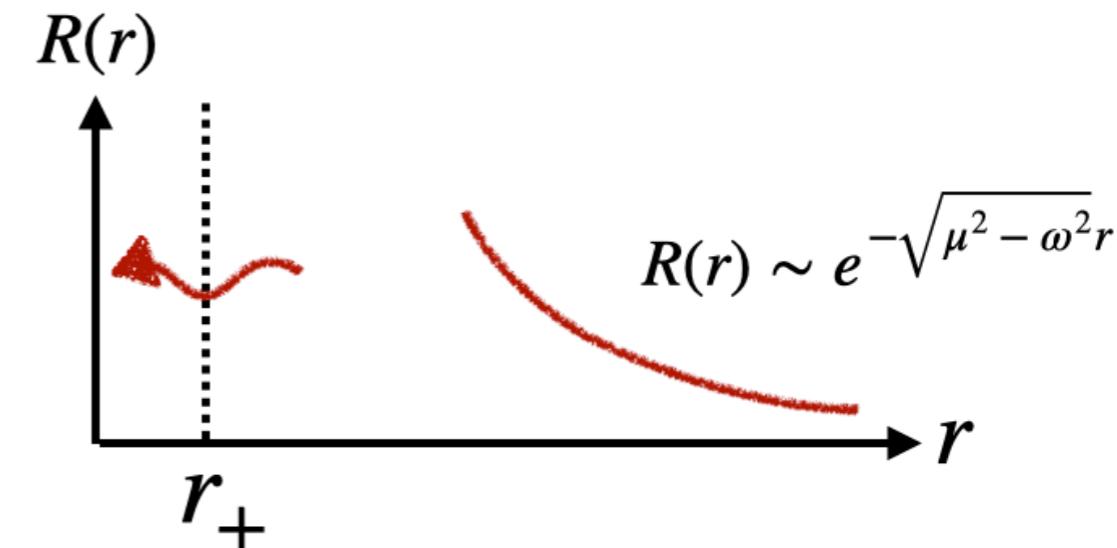
- Radial part

$$\Delta \frac{d}{dr} \left(\Delta \frac{dR}{dr} \right) + (\omega^2 (r^2 + a^2)^2 - 4aMr m\omega + a^2 m^2 - \Delta (\mu^2 r^2 + a^2 \omega^2 + \Lambda)) R = 0$$

- Boundary condition

○ vanish at infinity

○ ingoing at the horizon



Gravitational atom

- Let us assume $\mu M \ll 1$, $|EM| = |(\omega - \mu)M| \ll \mu M$
- In far region $r \gg M$,

Non-relativistic limit

$$\left\{ -\frac{1}{2\mu} \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) + \frac{l(l+1)}{r^2} \right] - \frac{\mu M}{r} \right\} R^{\text{far}}(r) = E R^{\text{far}}(r)$$

- Near horizon

This equation is same as
QM of Hydrogen atom.

$R^{\text{near}}(r) \sim \text{ingoing mode}$

- From matching condition, we have spectrum.

$$E_{nlm} = \omega_{nlm} - \mu = -\frac{\mu(\mu M)^2}{2n^2} + i\Gamma_{nlm}$$

$\Gamma_{nlm} \propto m\Omega_H - \omega_{nlm}$

This term is same as
spectrum of Hydrogen atom.

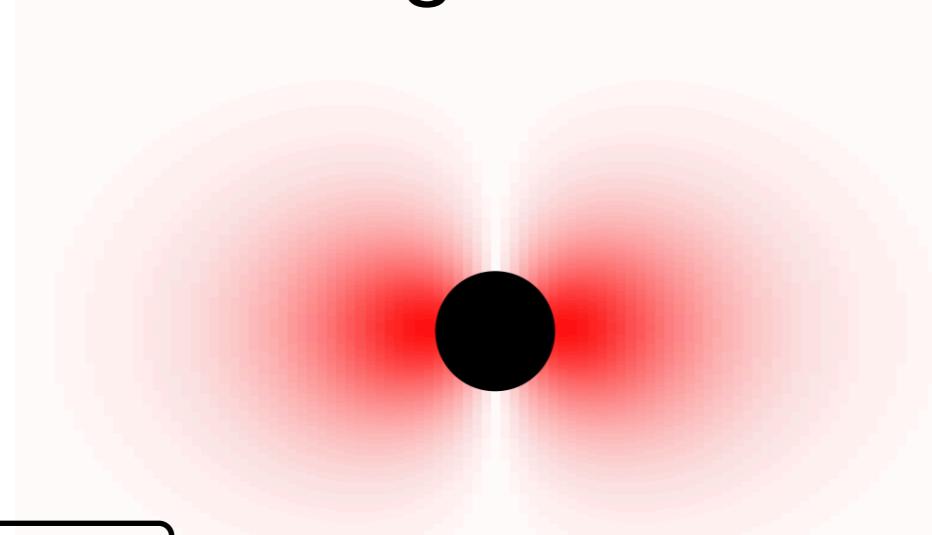
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Lessons from single BH

$$\phi = \frac{1}{\sqrt{2\mu}} (\varphi e^{-i\mu t} + \text{c.c.})$$

Single BH



Clear conditions

$$(\square_{\text{Kerr}} - \mu^2)\phi = 0$$

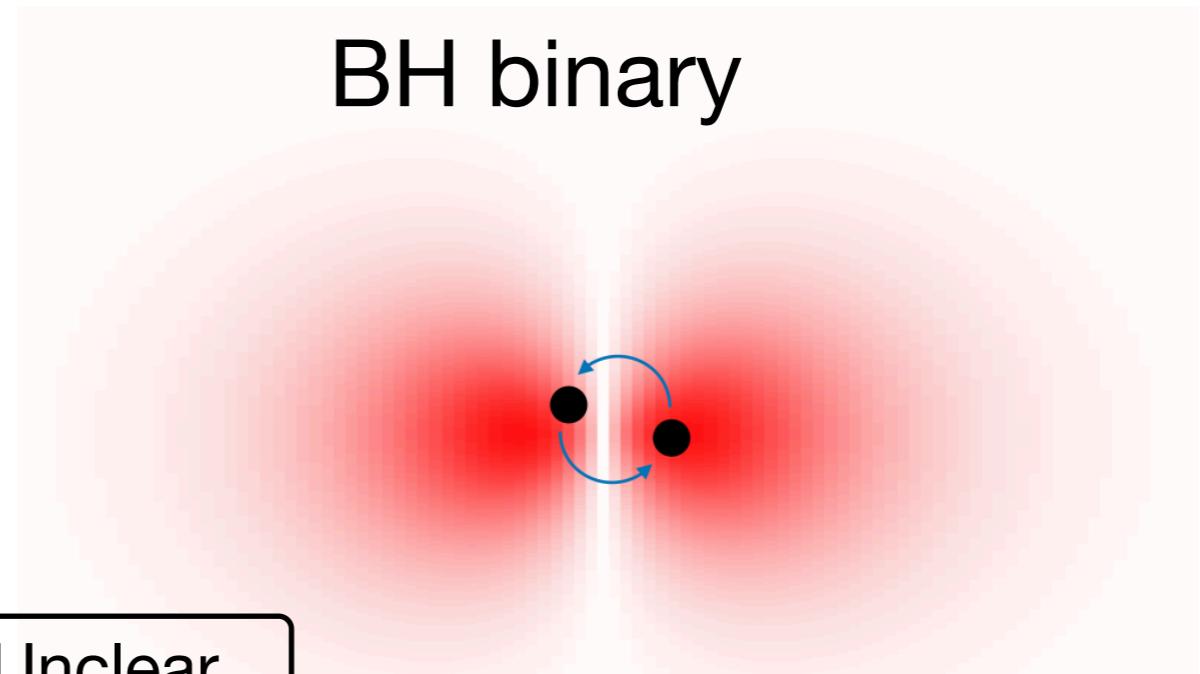
$$i\partial_t\varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r) \right) \varphi$$

$$V(r) = \frac{\mu M}{r}$$

Hydrogen atom : $|n, l, m\rangle$

Gravitational atom

BH binary



Unclear conditions

$$(\square_{?} - \mu^2)\phi = 0$$

$$i\partial_t\varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r) \right) \varphi$$

$$V(r) = \frac{\mu M_1}{|r - r_1(t)|} + \frac{\mu M_2}{|r - r_2(t)|}$$

Di-hydrogen atom

Gravitational molecule

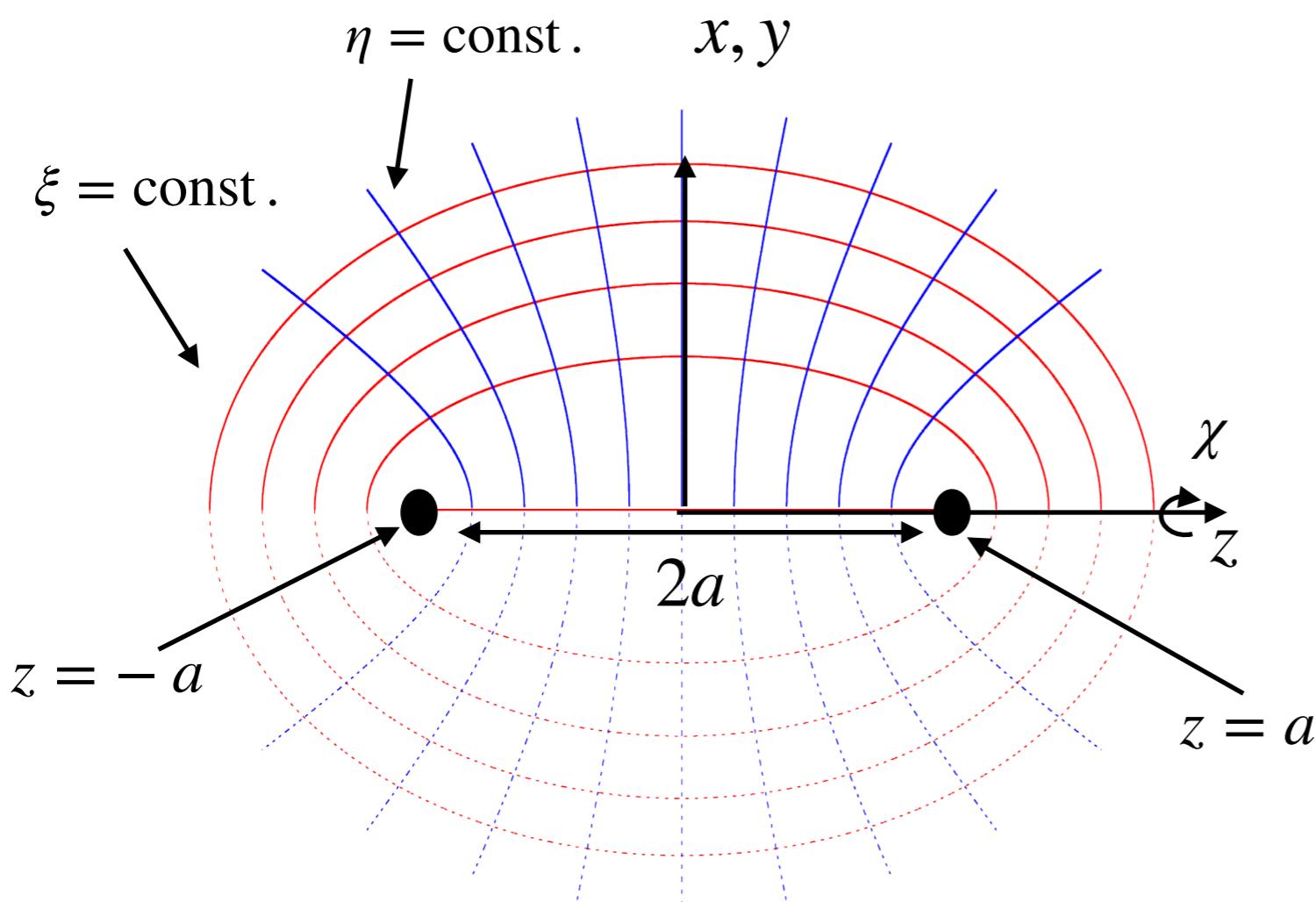
QM in di-hydrogen atom

- In co-moving frame with binary, the problem becomes QM in di-hydrogen atom with “fixed” nuclei.

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2} \nabla^2 + V(r) \right) \varphi$$

$$V(r) = \frac{\alpha_1}{|r - r_1|} + \frac{\alpha_2}{|r - r_2|}$$

$$\mu M_i = \alpha_i$$



Coordinate transformation

$$x = a\sqrt{1 - \eta^2}\sqrt{\xi^2 - 1} \cos \chi$$

$$y = a\sqrt{1 - \eta^2}\sqrt{\xi^2 - 1} \sin \chi$$

$$z = a\eta\xi$$

prolate spheroidal coord.

$$0 \leq \chi < 2\pi, -1 \leq \eta \leq 1, 1 \leq \xi$$

QM in di-hydrogen atom

- Schrödinger equation is separable.

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2} \nabla^2 + V(r) \right) \varphi \quad V(r) = \frac{\alpha_1}{|r - r_1|} + \frac{\alpha_2}{|r - r_2|}$$

→ $\varphi \propto e^{-i\bar{E}t + im_\chi \chi} R(\xi) S(\eta)$

- Angular part : spheroidal harmonics → Λ
- Radial part :

$$\partial_\xi \left((\xi^2 - 1) \partial_\xi R \right) + \left(\Lambda - \frac{m_\chi^2}{\xi^2 - 1} + 4\alpha\mu a\xi + 2\mu a^2 \bar{E} \xi^2 \right) R = 0$$

- Boundary condition

- $R \sim e^{-\sqrt{2\mu a^2 |\bar{E}|} \xi}$ ($\xi \rightarrow \infty$) → \bar{E}
- $R \sim (\xi - 1)^{\frac{m_\chi}{2}}$ ($\xi \rightarrow 1$)

QM in di-hydrogen atom

- We can solve these equations using Leaver's method and direct integration method.
- The eigenstate can be labeled by (m_ξ, m_η, m_χ) .

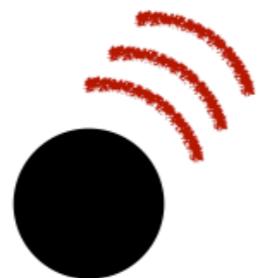
Spectrum of molecule

(m_ξ, m_η, m_χ)	$a = 5$		$a = 30$	
	Λ	$10^2 \times \bar{E}$	Λ	$10^2 \times \bar{E}$
(0,0,0)	-0.0129	-0.386	-0.342	-0.272
(1,0,0)	-0.00327	-0.0981	-0.0993	-0.0817
(2,0,0)	-0.00146	-0.0439	-0.0468	-0.0387
(0,2,0)	5.998	-0.0445	5.915	-0.0453
(1,2,0)	5.999	-0.0250	5.952	-0.0254
(2,2,0)	5.999	-0.0160	5.970	-0.0162

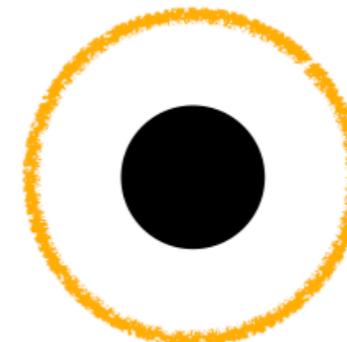
Particle description

- Particle analogies of wave equation often play an important role in several physical phenomena.

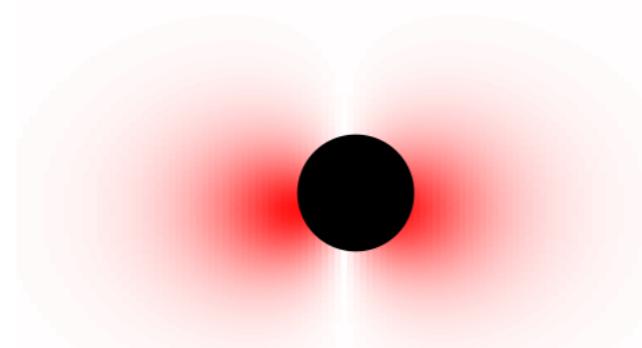
QNM



Photon sphere

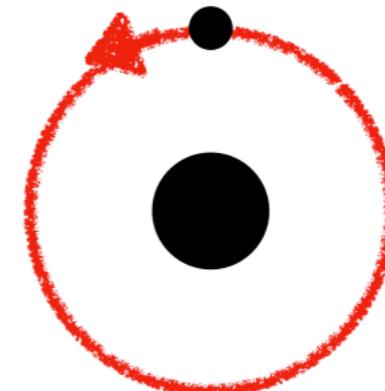
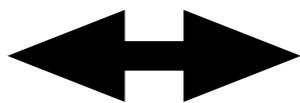


Bond state of scalar field



Bound state of massive particle

WKB app.



Schrödinger equation

Hamilton-Jacobi equation

WKB approximation

- The bound state is described by Schrödinger equation.

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2} \nabla^2 + V(r) \right) \varphi \quad V(r) = \frac{\mu M_1}{|r - r_1|} + \frac{\mu M_2}{|r - r_2|}$$

- WKB approximation

$$\varphi = e^{iS(t, \xi, \eta, \chi)} \quad S(t, \xi, \eta, \chi) = S_\xi(\xi) + S_\eta(\eta) + m_\chi \chi - Et$$

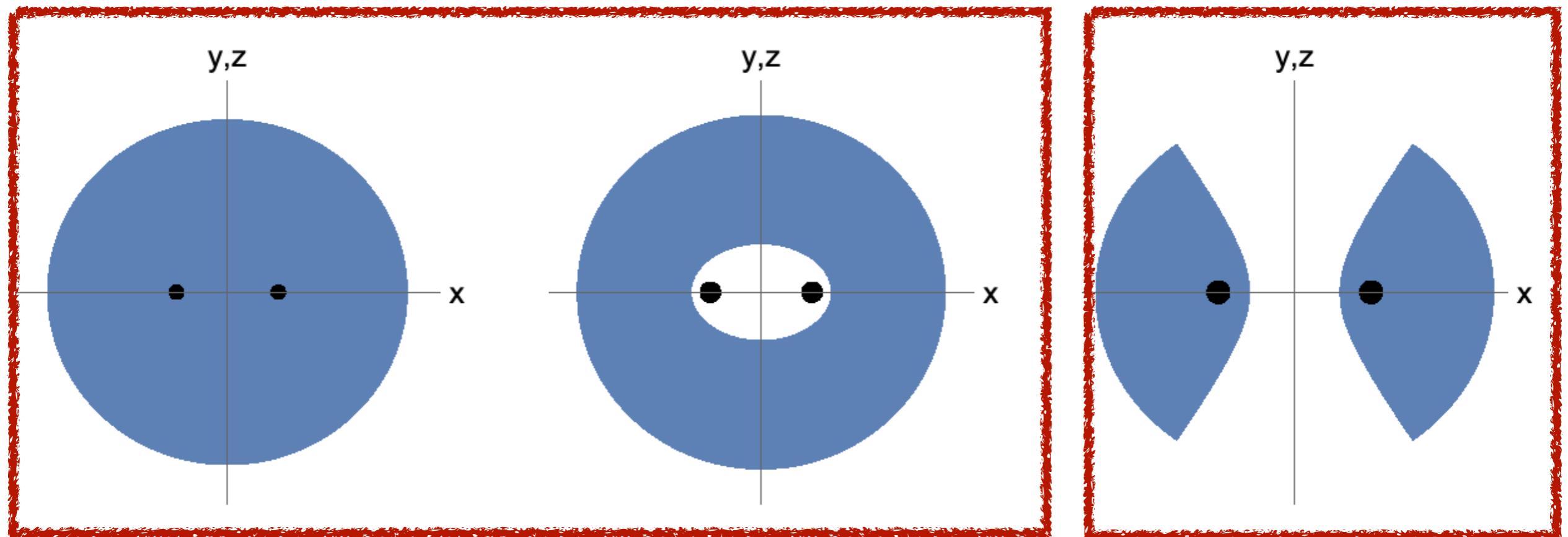
- Hamilton-Jacobi equation

$$(S'_\eta)^2 = \frac{2a^2\mu}{1-\eta^2} \left(\frac{l^2}{2a^2\mu} - \bar{E}\eta^2 + \frac{\mu\Delta M}{a}\eta - \frac{m_\chi^2}{2a^2\mu} \frac{1}{1-\eta^2} \right) > 0$$

$$(S'_\xi)^2 = \frac{2a^2\mu}{\xi^2 - 1} \left(-\frac{l^2}{2a^2\mu} + \bar{E}\xi^2 + \frac{\mu M}{a}\xi - \frac{m_\chi^2}{2a^2\mu} \frac{1}{\xi^2 - 1} \right) > 0$$

Classically allowed region

- Classically allowed region gives us hint of the topology of bound state.



Gravitational molecule

Gravitational atom
around individual BHs

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3. Analytic description

- Gravitational molecule
- Particle picture

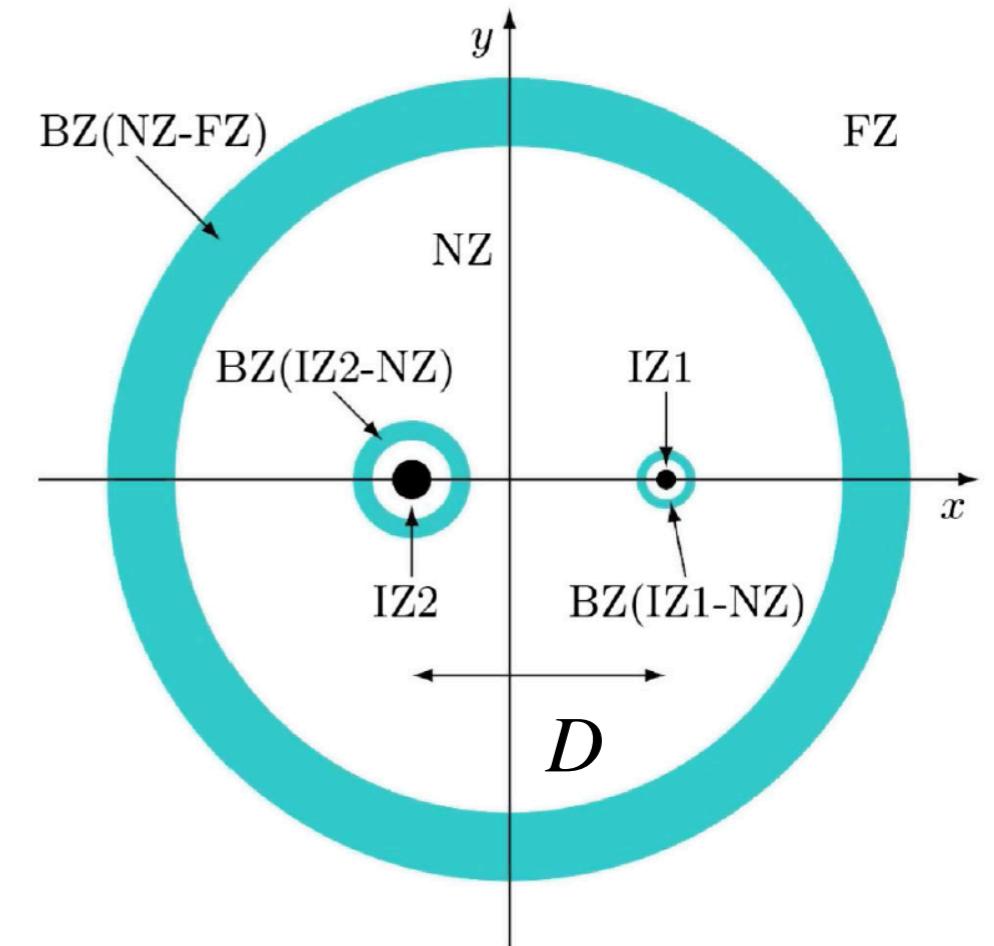
4. Numerical simulations

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5. Summary

Approximated BH spacetime

- Construction of the metric $A = 1, 2$ ref: PRD89,084008(2014)
 - ▶ Inner Zones (IZ) : $0 < r_A \ll r_{12}$
 - a perturbed Schwarzschild
 - ▶ Near Zone (NZ) : $m_A \ll r_A \ll \lambda$
 - PN approximation
 - ▶ Far Zone (FZ) : $\lambda \ll r < \infty$
 - PM approximation
 - ▶ Buffer Zone (BZ)
 - Asymptotic matching
- We assume circular orbit.
- We neglect emission of GW.
 - ▶ BH separation is constant.

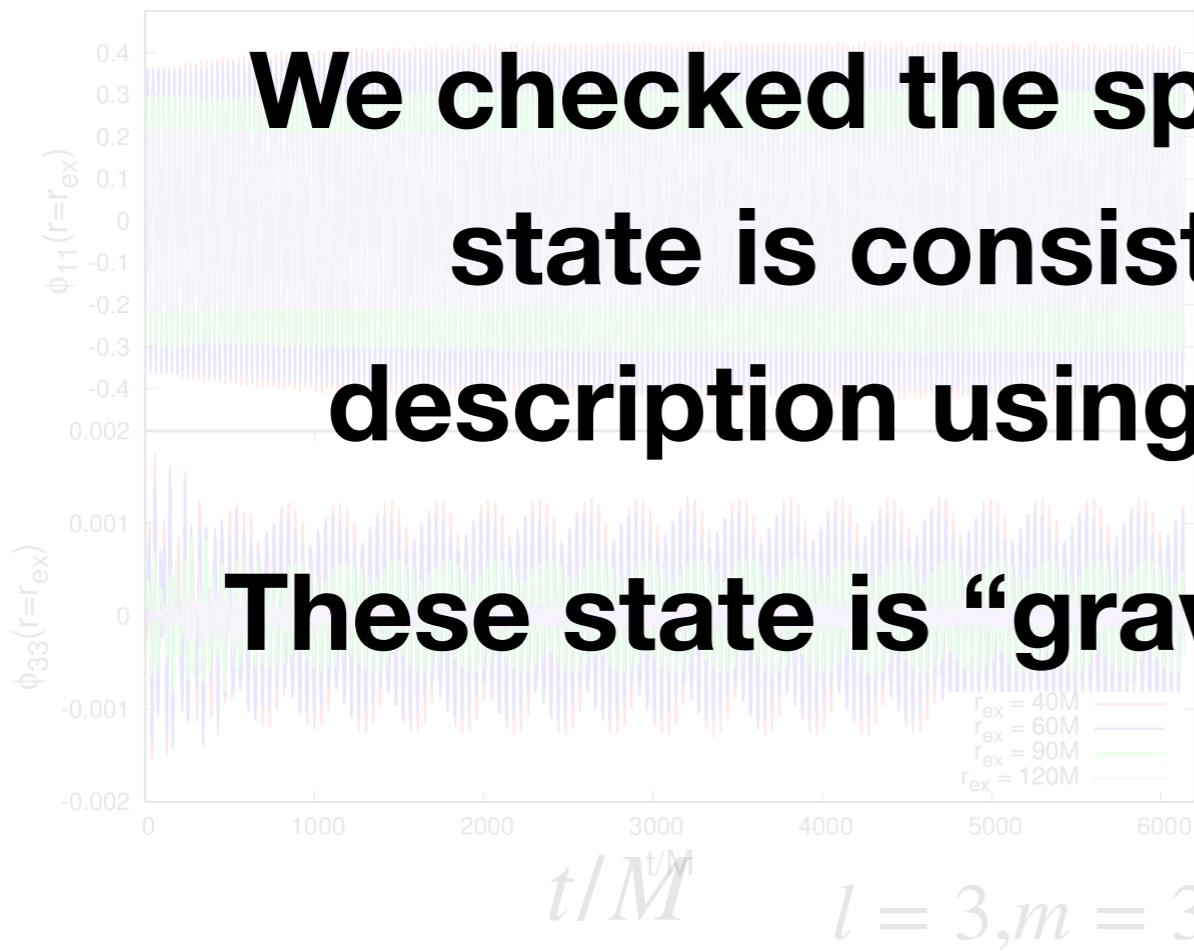


BHB spacetime (BH1, BH2)

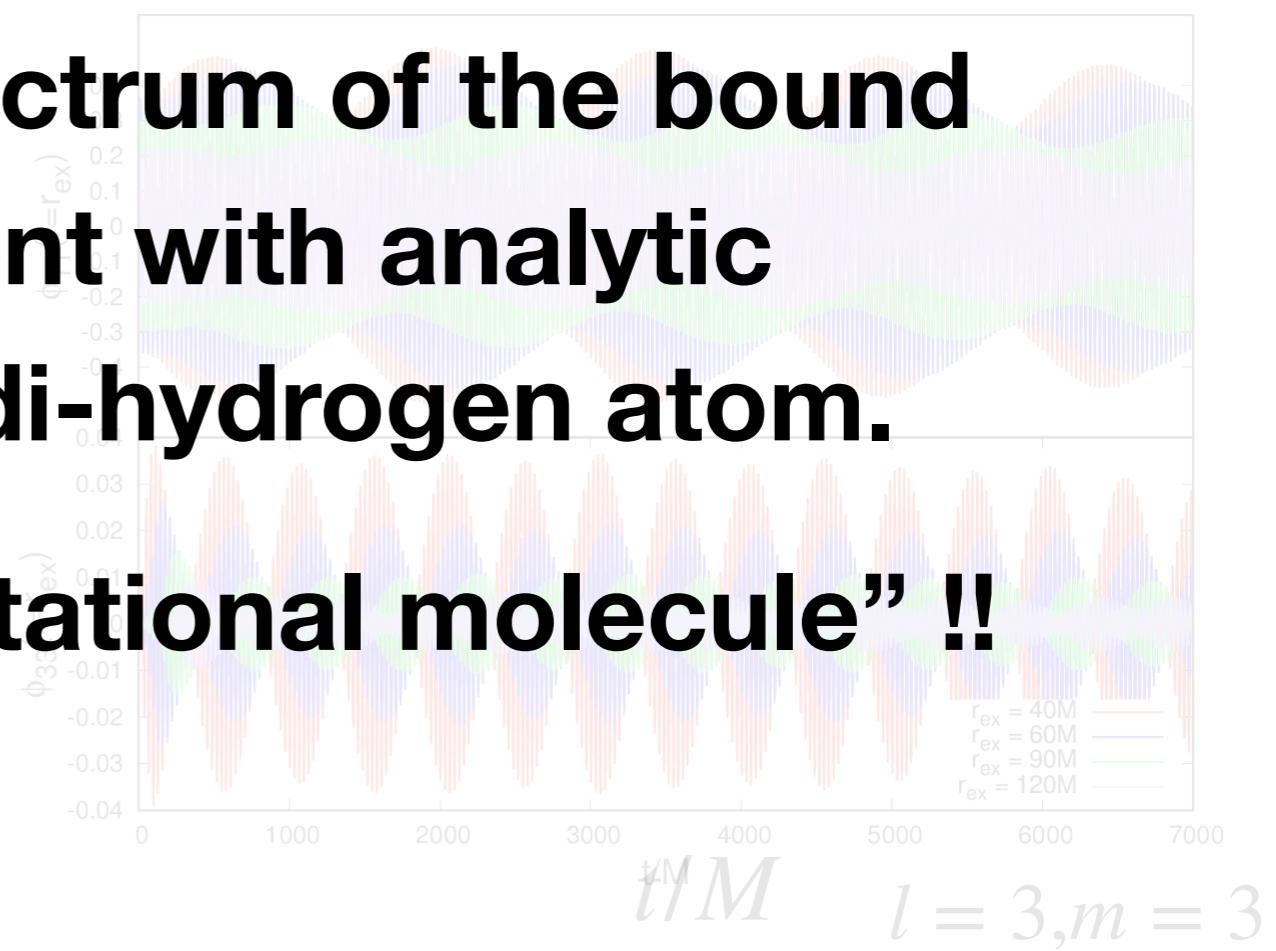
D : BH separation

$m_1 = m_2 = M/2$

$D = 10M$
 $l = 1, m = 1$



$D = 60M$
 $l = 1, m = 1$



We checked the spectrum of the bound state is consistent with analytic description using di-hydrogen atom.

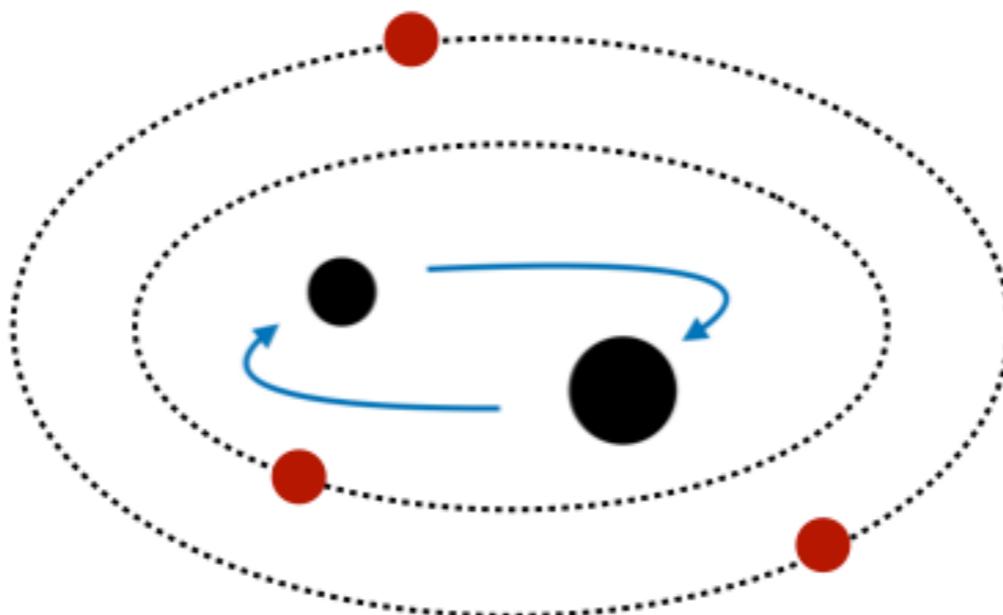
These state is “gravitational molecule” !!

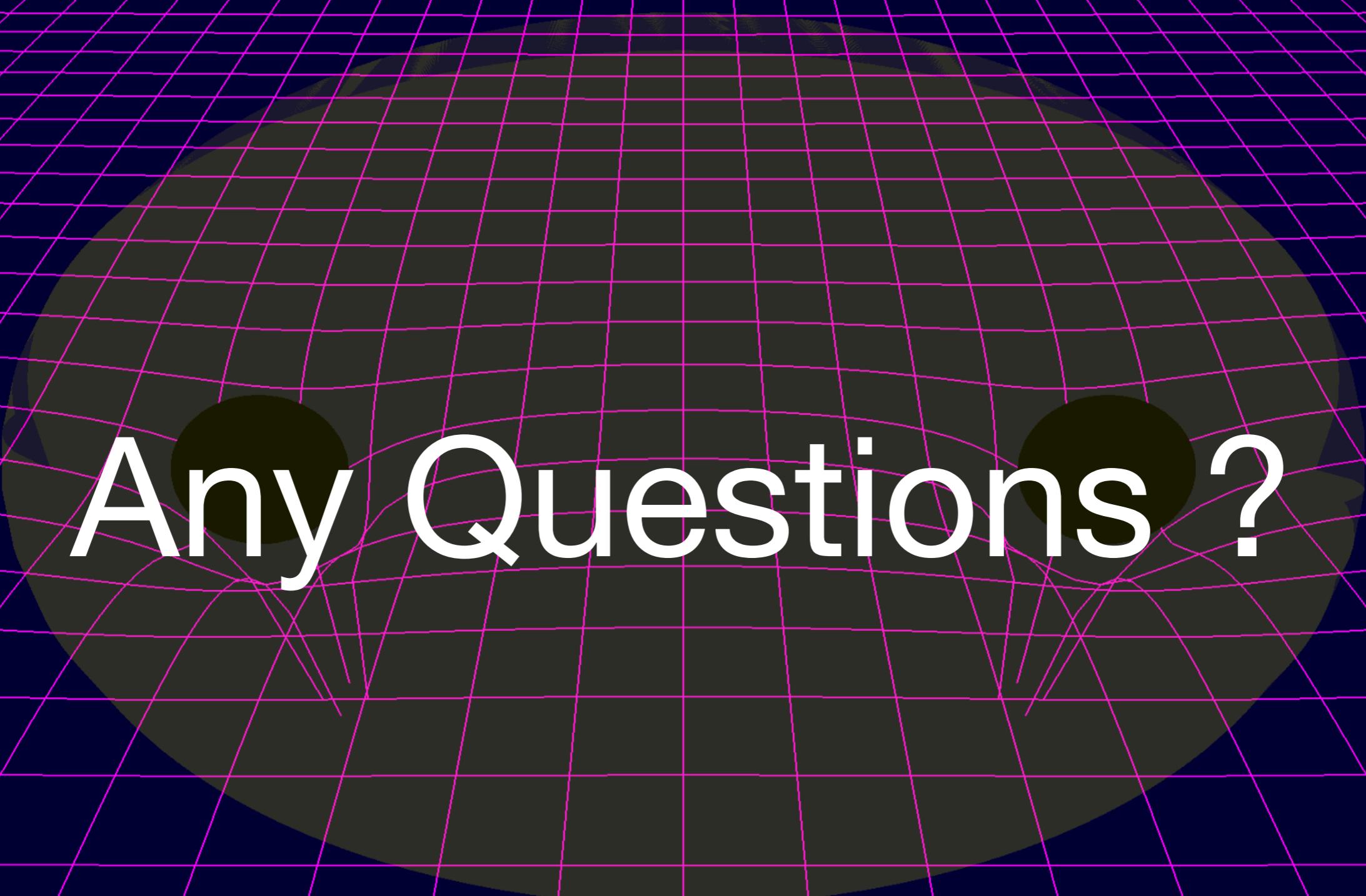
$$\tau \gg 10^4 M$$

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Thank you for your attention !!





Any Questions?