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Black hole binaries and light fields

- gravitational molecule -

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arXiv:2010:00008

Black hole vs Black hole binary

- We don't deeply understand the physics around BH binary spacetime.

Black Hole

- Photon sphere
- Quasi normal mode
- Bound state of massive fields
(Gravitational atom)



Black Hole Binary

- “Global ” photon surface?
- “Global” QNM ?
- ~~Please see PRD100(2019) 4,044002.~~
 - “Global” bound state of massive fields ?
(Gravitational molecule)



Outline

1. Introduction

2. Gravitational atom

3. Gravitational molecule

- Equivalence with di-Hydrogen molecules
- Numerical simulations

4. Summary

Gravitational atom

$$c^2 = -(\mu M)^2 a^2 (\mu^2 - \omega^2)$$

$$\Delta = r^2 - 2Mr + a^2$$

- Bound state of massive scalar field around Kerr BH

S.Detweiler (1980)

$$(\Box_{\text{BL Kerr}} - \mu^2) \Phi = 0 \quad \Phi = e^{-i\omega t + im\phi} R(r) S(\theta)$$

- Angular part (spheroidal harmonics)

$$\left(-\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) - c^2 \cos^2 \theta + \frac{m^2}{\sin^2 \theta} \right) S = \Lambda S$$

Bound state
 $EM = (\omega - \mu)M < 0$

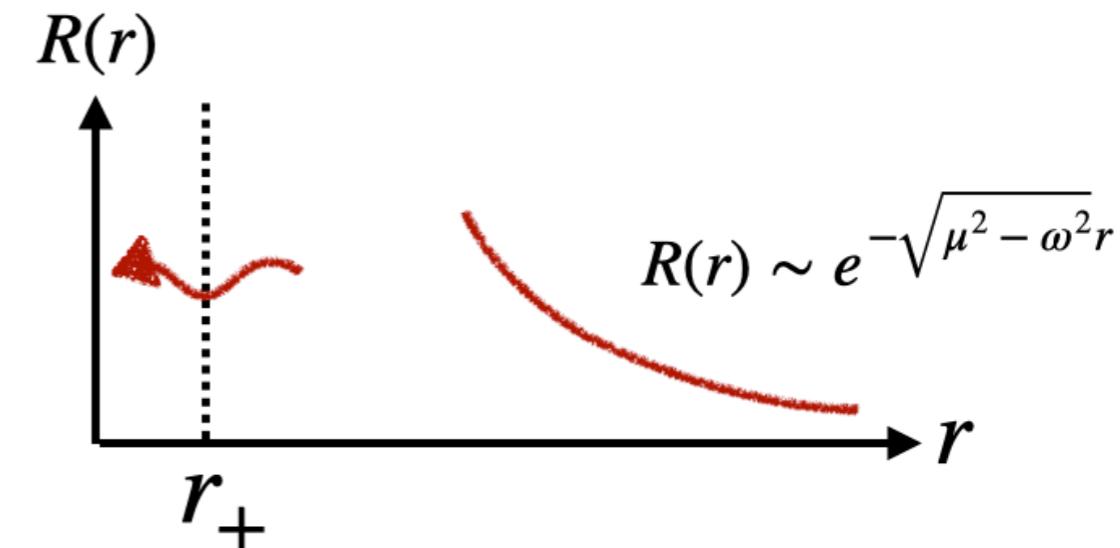
- Radial part

$$\Delta \frac{d}{dr} \left(\Delta \frac{dR}{dr} \right) + (\omega^2 (r^2 + a^2)^2 - 4aMr m\omega + a^2 m^2 - \Delta (\mu^2 r^2 + a^2 \omega^2 + \Lambda)) R = 0$$

- Boundary condition

○ vanish at infinity

○ ingoing at the horizon



Gravitational atom

- Let us assume $\mu M \ll 1$, $|EM| = |(\omega - \mu)M| \ll \mu M$
- In far region $r \gg M$,

Non-relativistic limit

$$\left\{ -\frac{1}{2\mu} \left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) + \frac{l(l+1)}{r^2} \right] - \frac{\mu M}{r} \right\} R^{\text{far}}(r) = E R^{\text{far}}(r)$$

- Near horizon

This equation is same as
QM of Hydrogen atom.

$R^{\text{near}}(r) \sim \text{ingoing mode}$

- From matching condition, we have spectrum.

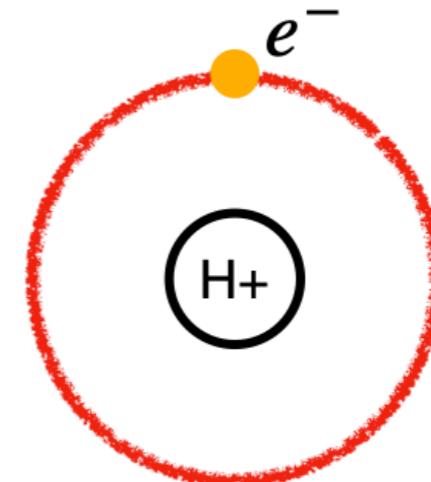
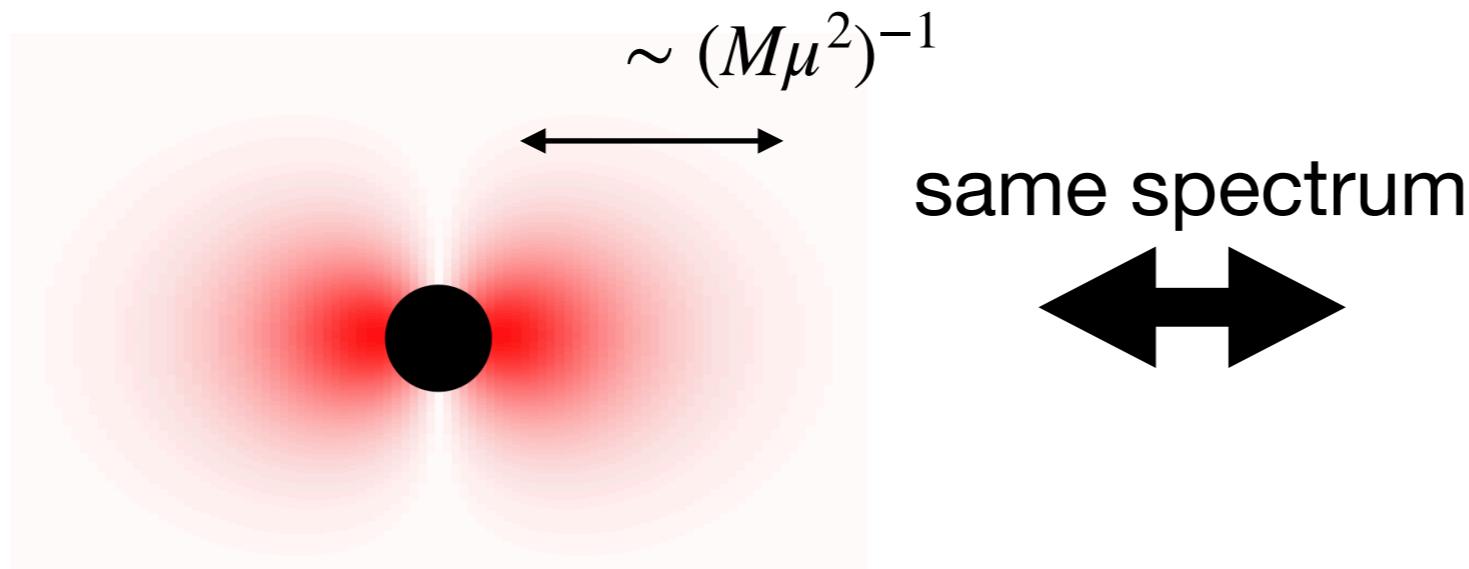
$$E_{nlm} = \omega_{nlm} - \mu = -\frac{\mu(\mu M)^2}{2n^2} + i\Gamma_{nlm}$$

$\Gamma_{nlm} \propto m\Omega_H - \omega_{nlm}$

This term is same as
spectrum of Hydrogen atom.

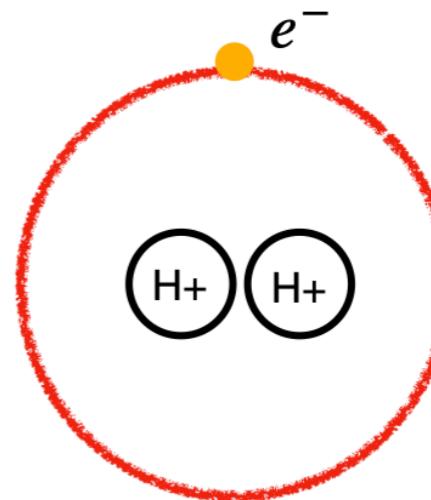
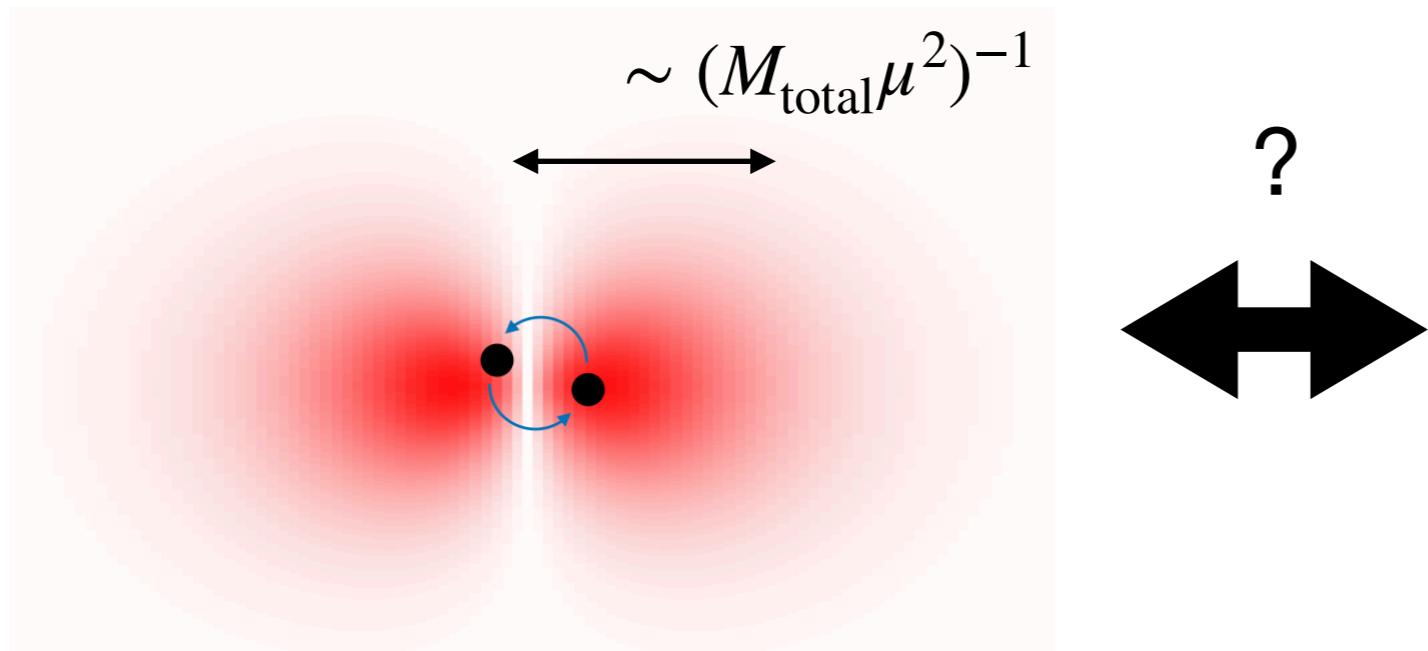
Our expectation

- Gravitational atom



Hydrogen atom

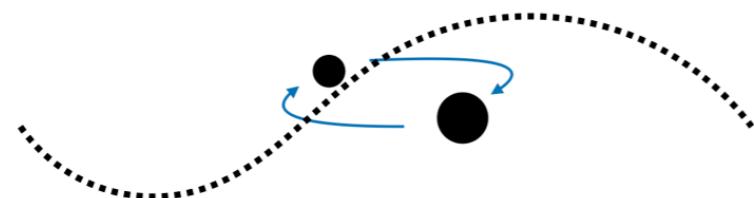
- Gravitational molecule (?)



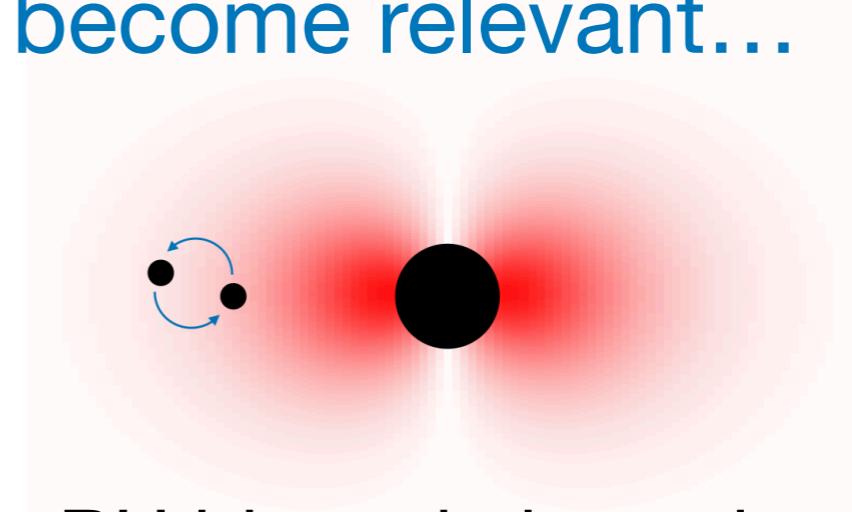
Di-Hydrogen molecule

Massive field around binary

- When massive scalar fields become relevant...



BH binary in coherent
oscillating scalar field



BH binary in bound state
around 3rd BH

- Gravitational molecule may be typical state around binary.
- What we want to do....
 - Analytic description of the gravitational molecule.
 - Numerical construction of the state

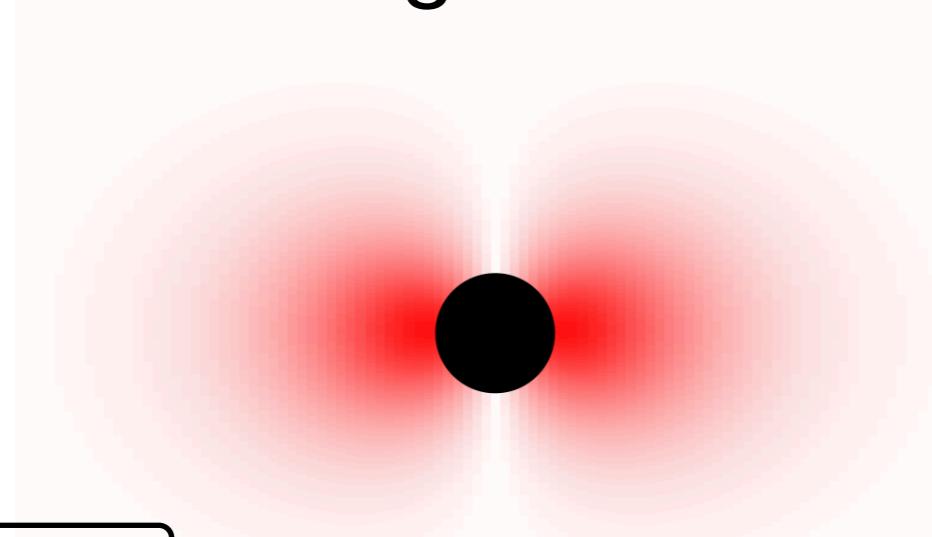
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Lessons from single BH

$$\phi = \frac{1}{\sqrt{2\mu}} (\varphi e^{-i\mu t} + \text{c.c.})$$

Single BH



Clear conditions

$$(\square_{\text{Kerr}} - \mu^2)\phi = 0$$

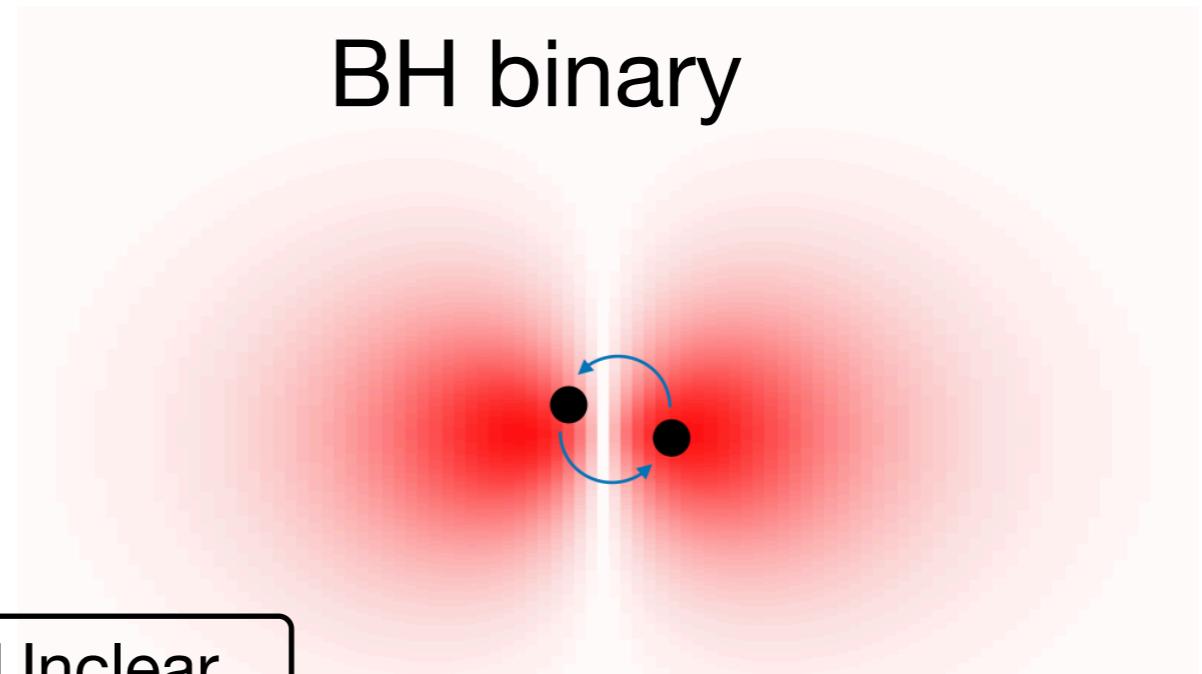
$$i\partial_t\varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r) \right) \varphi$$

$$V(r) = \frac{\mu M}{r}$$

Hydrogen atom : $|n, l, m\rangle$

Gravitational atom

BH binary



Unclear conditions

$$(\square_{?} - \mu^2)\phi = 0$$

$$i\partial_t\varphi = \left(-\frac{1}{2\mu^2}\nabla^2 + V(r) \right) \varphi$$

$$V(r) = \frac{\mu M_1}{|r - r_1(t)|} + \frac{\mu M_2}{|r - r_2(t)|}$$

Di-hydrogen molecule

Gravitational molecule

QM in di-hydrogen atom

- In co-moving frame with binary, the problem becomes QM in di-hydrogen molecule with “fixed” nuclei.

$$i\partial_t \varphi = \left(-\frac{1}{2\mu^2} \nabla^2 + V(r) \right) \varphi \quad V(r) = \frac{\alpha_1}{|r - r_1|} + \frac{\alpha_2}{|r - r_2|} \quad \mu M_i = \alpha_i$$

- This system is separable, and we can calculate the energy spectrum.

$$\bar{E} = \mu - \omega$$
 - Eigenstate can be labeled by (m_ξ, m_η, m_χ) .

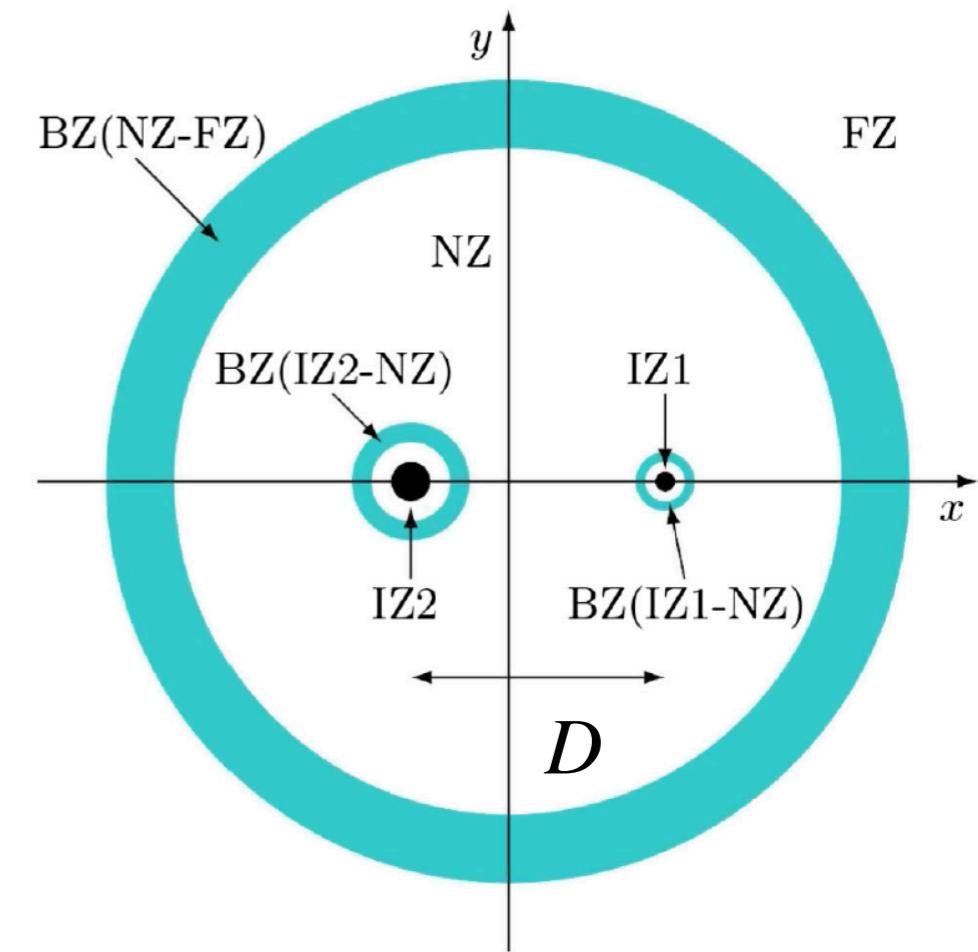
(m_ξ, m_η, m_χ)	$D = 10M$		$D = 60M$	
	A	$10^2 \times \bar{E}$	A	$10^2 \times \bar{E}$
(0,0,0)	-0.0129	-0.386	-0.342	-0.272
(1,0,0)	-0.00327	-0.0981	-0.0993	-0.0817
(2,0,0)	-0.00146	-0.0439	-0.0468	-0.0387
(0,2,0)	5.998	-0.0445	5.915	-0.0453
(1,2,0)	5.999	-0.0250	5.952	-0.0254
(2,2,0)	5.999	-0.0160	5.970	-0.0162

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Approximated BH spacetime

- We prepare the analytic “BH metric.” ref: PRD89,084008(2014)
- Construction of the metric $A = 1,2$
 - ▶ Inner Zones (IZ) : $0 < r_A \ll r_{12}$
 - a perturbed Schwarzschild BH
 - ▶ Near Zone (NZ) : $m_A \ll r_A \ll \lambda$
 - PN approximation
 - ▶ Far Zone (FZ) : $\lambda \ll r < \infty$
 - PM approximation
 - ▶ Buffer Zone (BZ)
 - Asymptotic matching
- We assume circular orbit.



BHB spacetime (BH1, BH2)

D : BH separation

$m_1 = m_2 = M/2$

Initial data

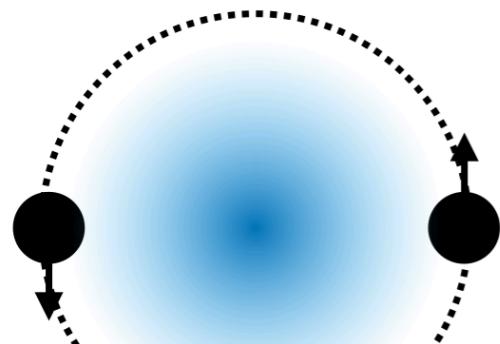
- We evolve the test massive scalar field around binary BH.
- Initial data.

$$\phi = R(r)\mathcal{A}(t, \theta, \varphi)$$

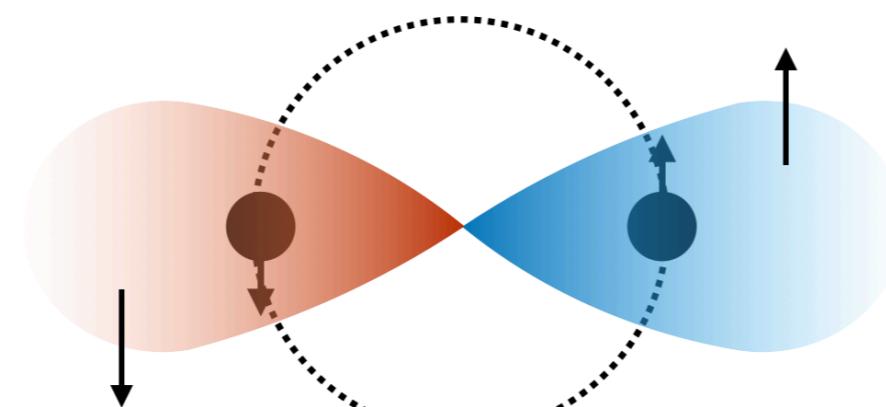
$$\begin{cases} \phi = Ae^{-\frac{1}{2}\left(\frac{r}{w}\right)^2} \\ (\partial_t - \mathcal{L}_\beta)\phi = 0 \end{cases}$$

$$\begin{cases} R = A_0 r e^{-\frac{r}{2w}} \\ \mathcal{A} = \text{Re} \left(Y_{1,1}(\theta, \varphi) e^{-i\mu t} \right) \end{cases}$$

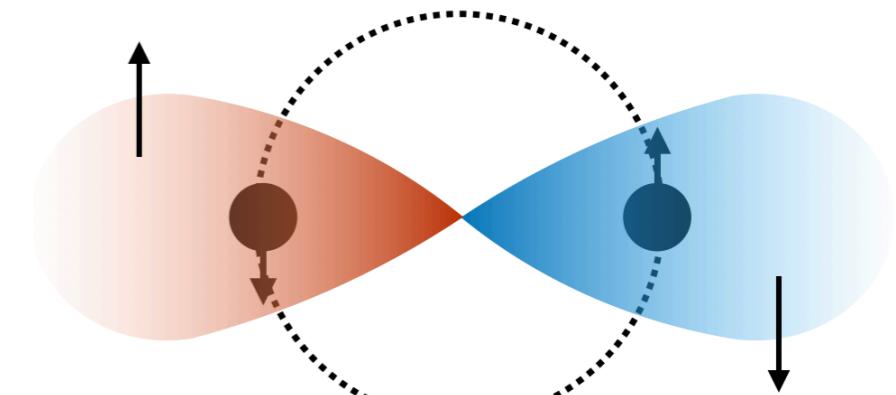
$$\begin{cases} R = A_0 r e^{-\frac{r}{2w}} \\ \mathcal{A} = \text{Re} \left(Y_{1,1}(\theta, \varphi) e^{+i\mu t} \right) \end{cases}$$



Gaussian profile



Co-rotating dipole



Counter-rotating dipole

Initial data

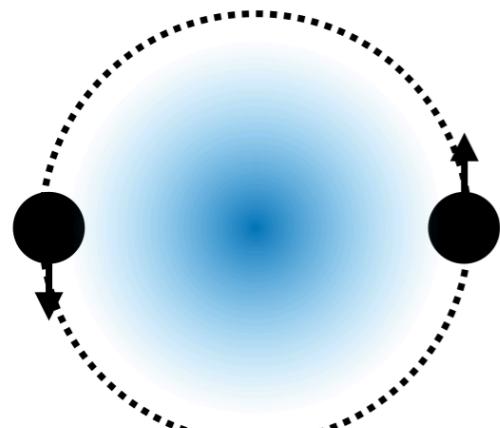
- We evolve the test massive scalar field around binary BH.
- Initial data.

$$\phi = R(r)\mathcal{A}(t, \theta, \varphi)$$

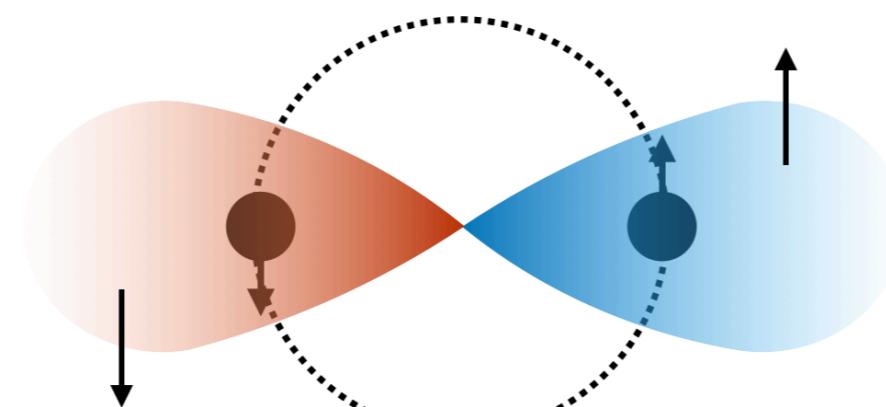
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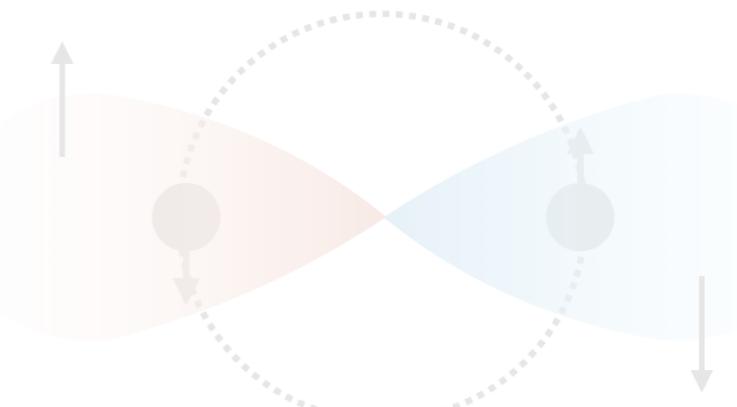
$$\begin{cases} R = A_0re^{-\frac{r}{2w}} \end{cases}$$



Gaussian profile



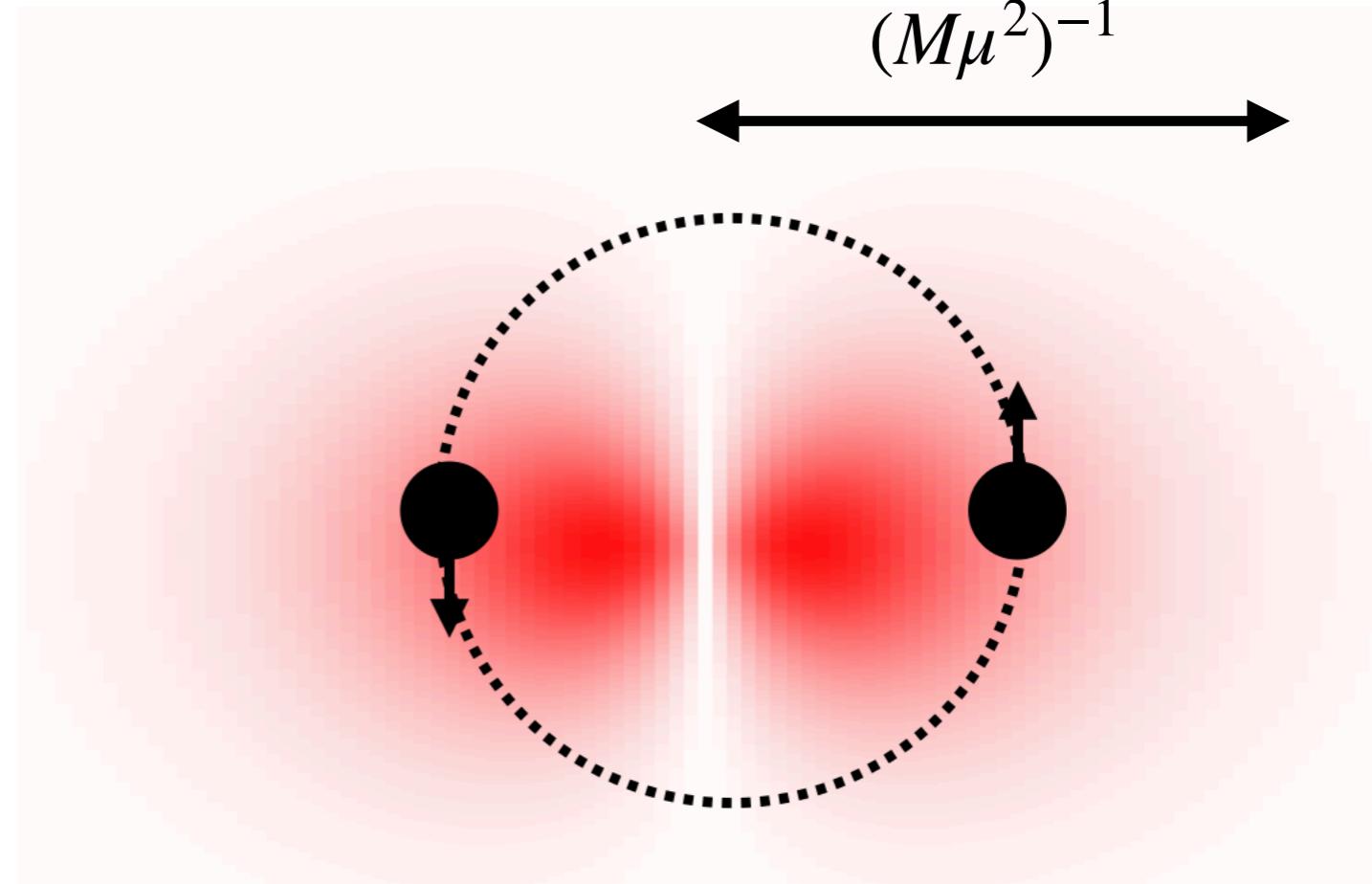
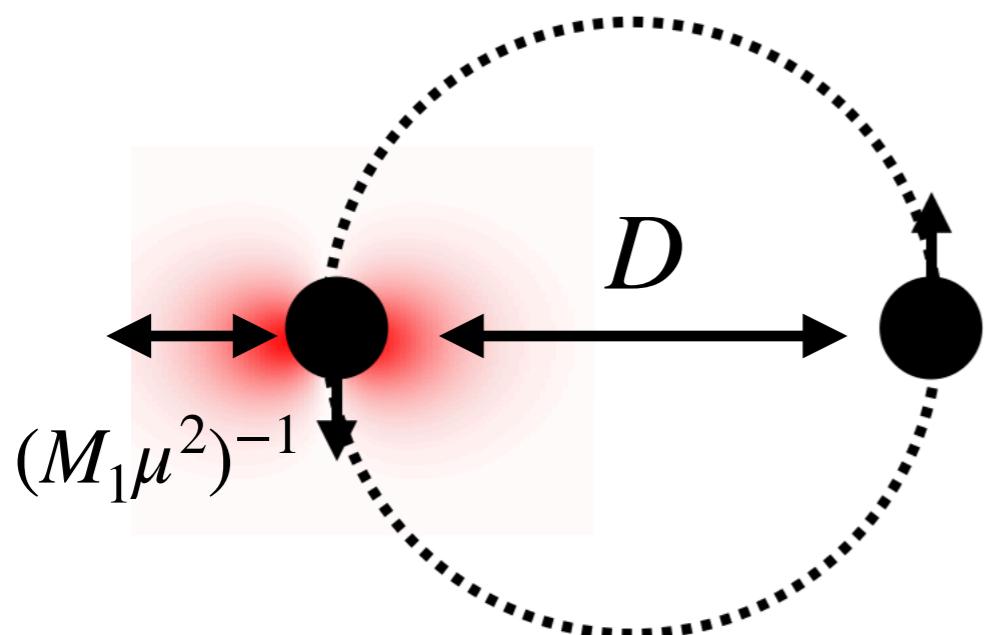
Co-rotating dipole



Typical length scale

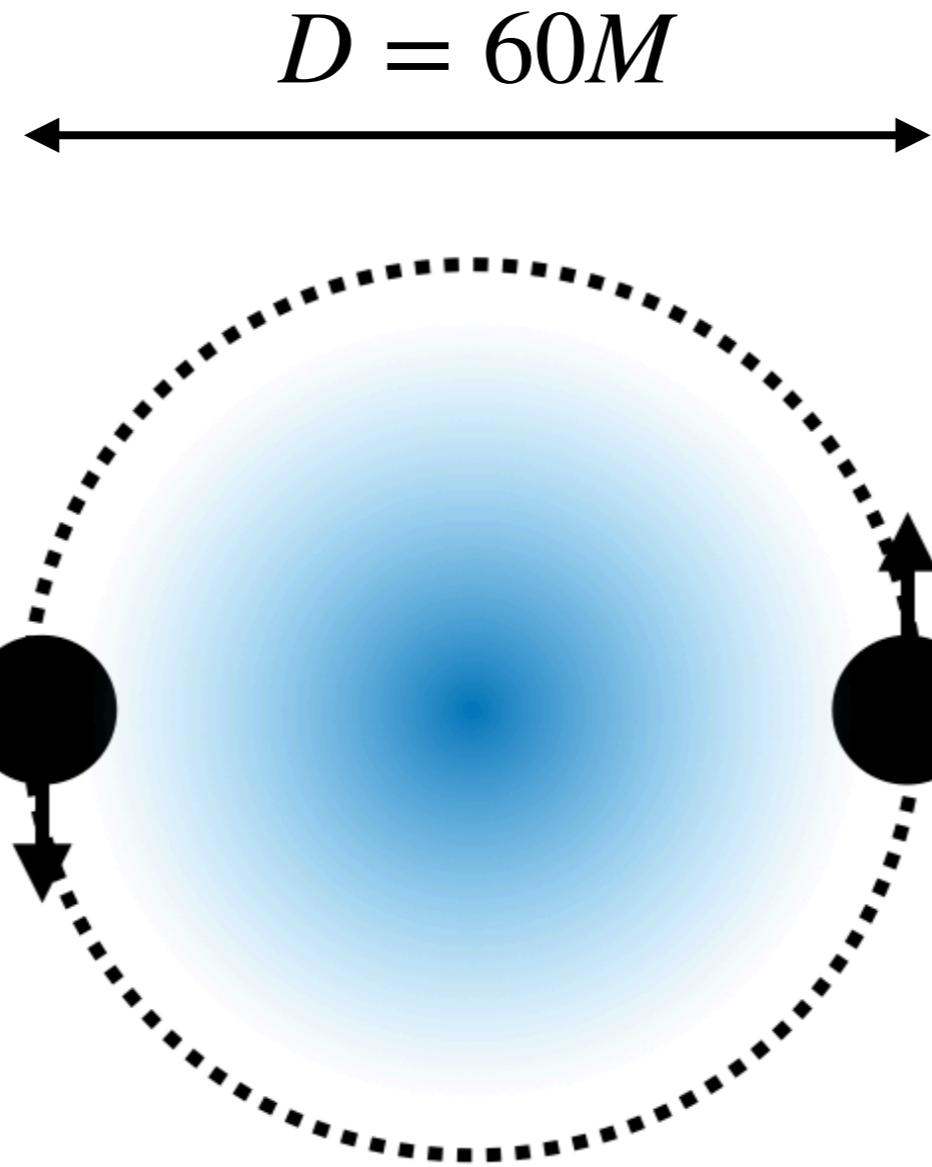
$$(\square - \mu^2) \phi = 0$$
$$M_1 = M_2 = \frac{M}{2}$$

- We have three length scales.



$\mathcal{O}((M_1\mu^2)^{-1}) \ll D$: gravitational atom around individual BH

$\mathcal{O}((M\mu^2)^{-1}) \gtrsim D$: gravitational molecule around BH binary



$$\mu M = 0.5$$

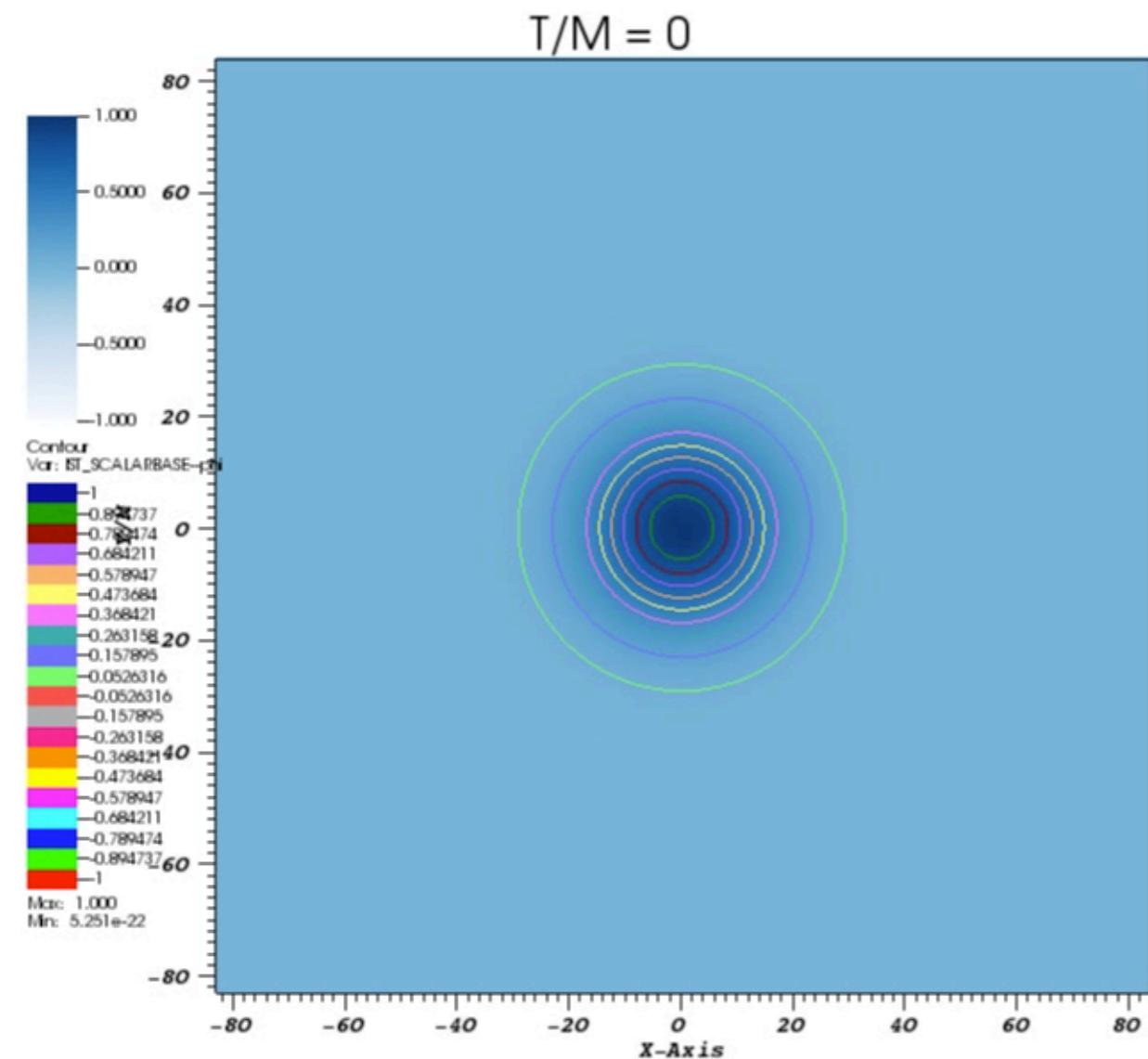
$$w = 25M$$

$$\mathcal{O}((M_1\mu^2)^{-1}) \ll D$$

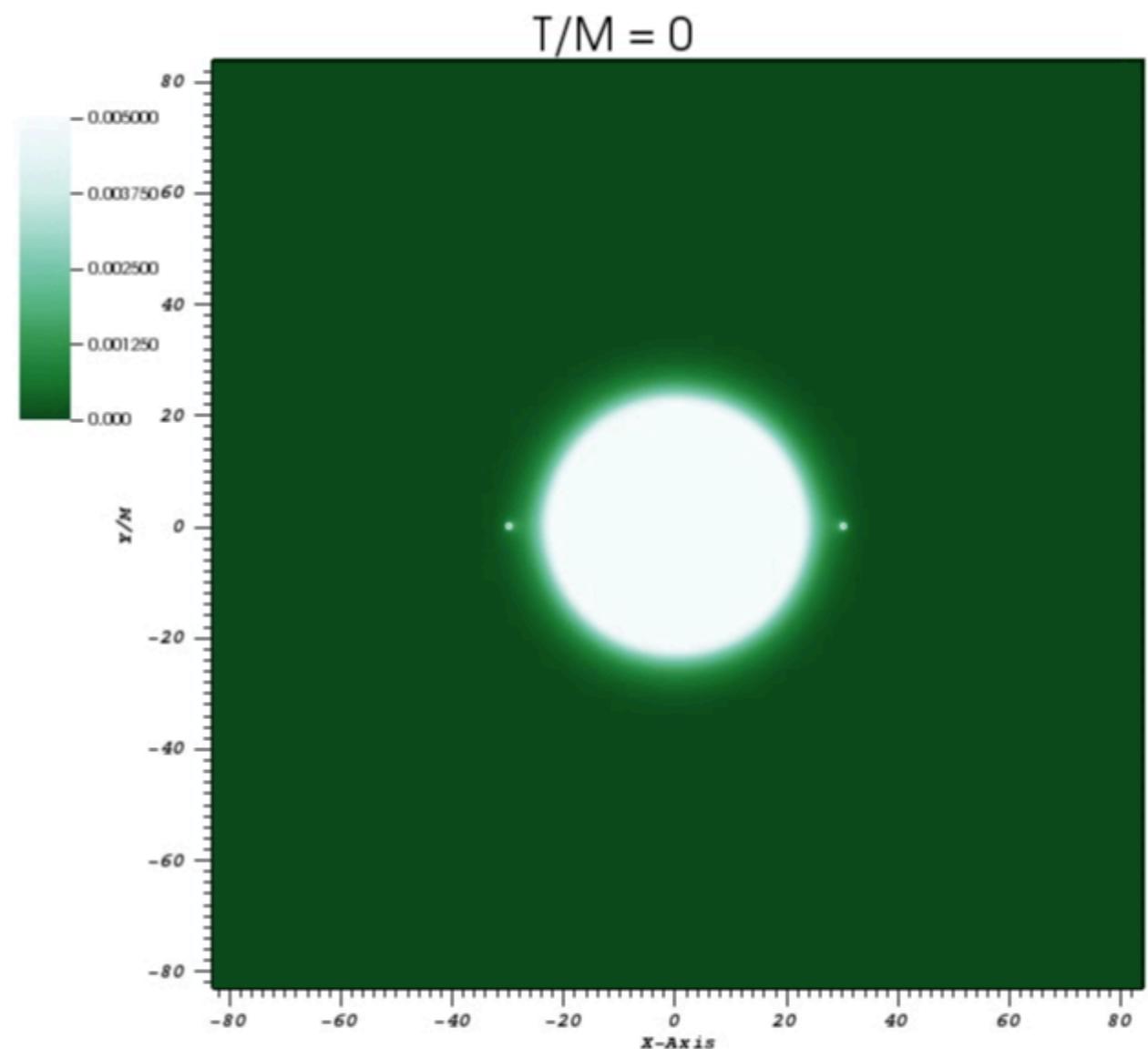
Simulation 1 : Gaussian initial data

Simulation 1

Scalar field

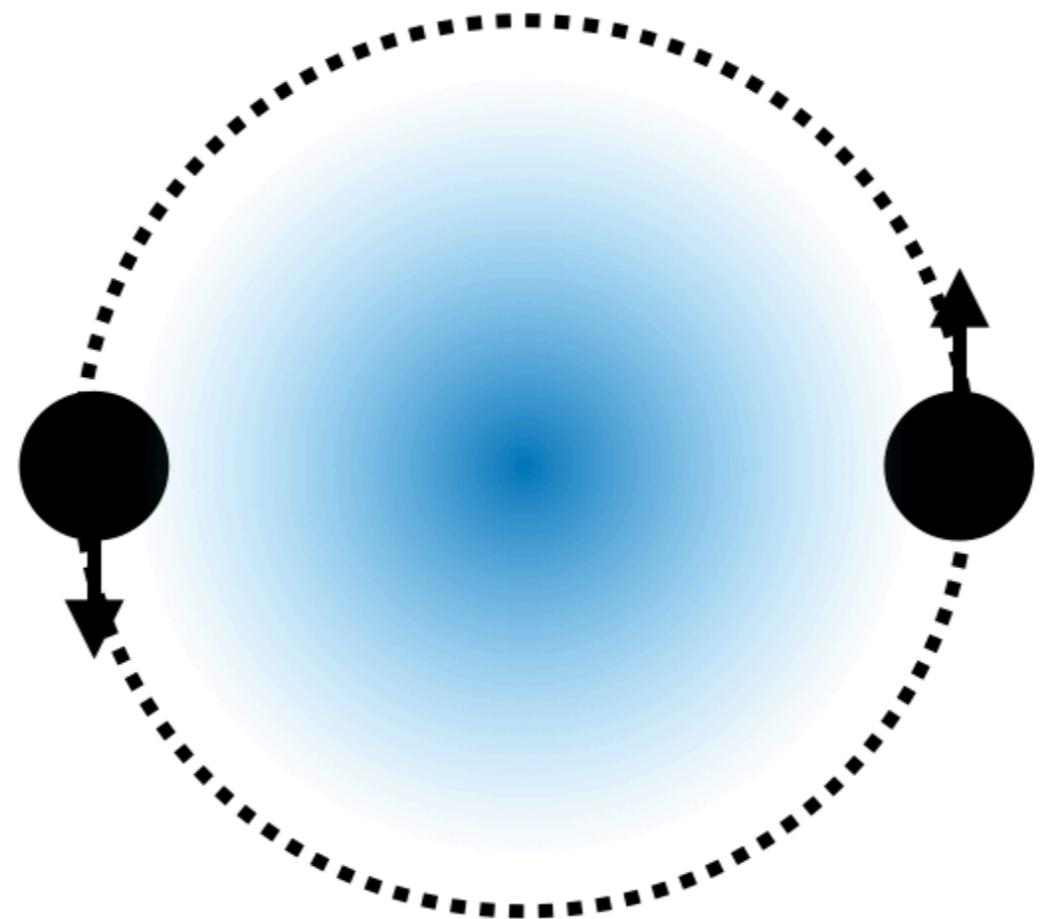


Energy density



- Gravitational atom around individual BHs.

$$D = 10M, 60M$$



$$\mu M = 0.2$$

$$w = 25M$$

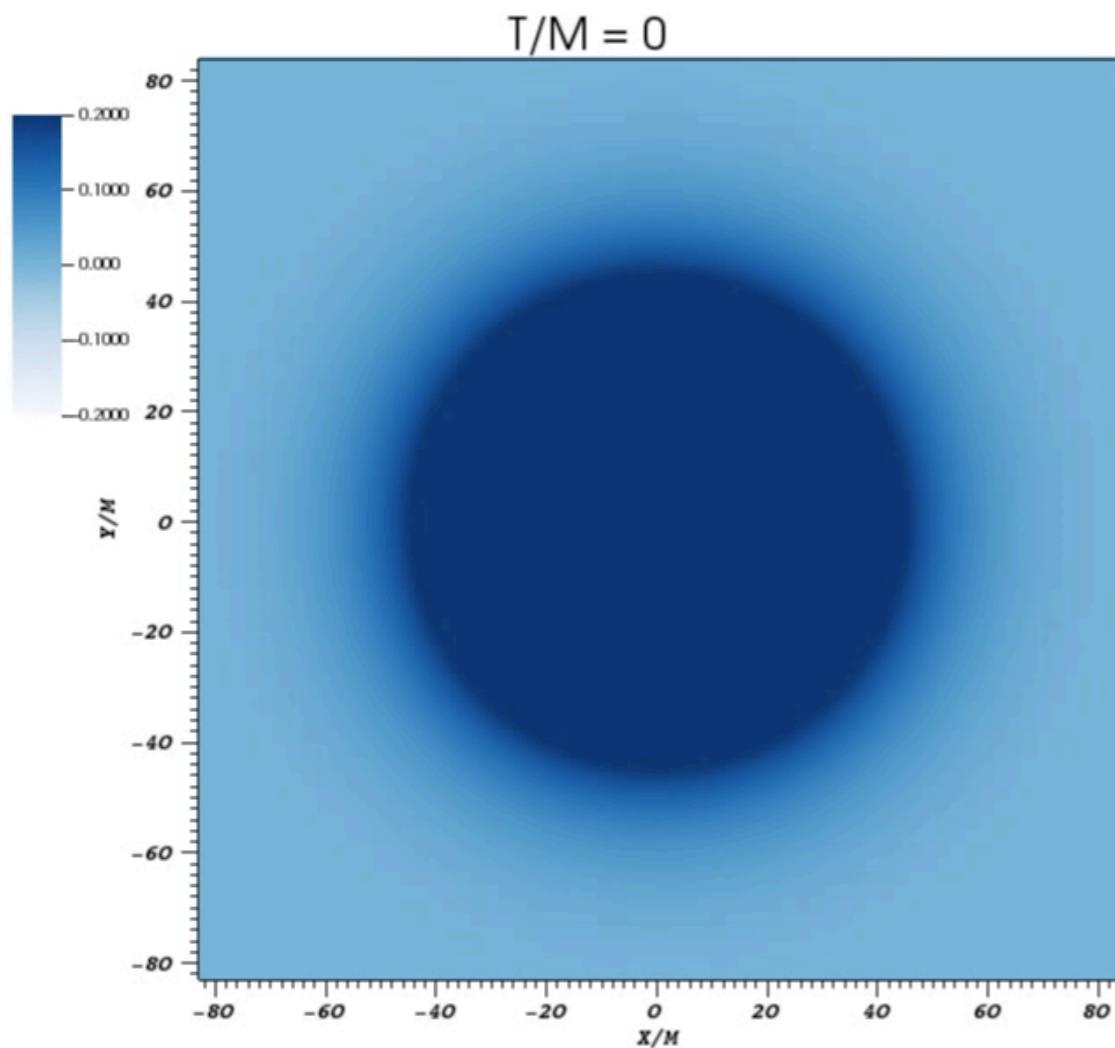
$$\mathcal{O}((M\mu^2)^{-1}) \gtrsim D$$

Simulation 2 : Gaussian initial data

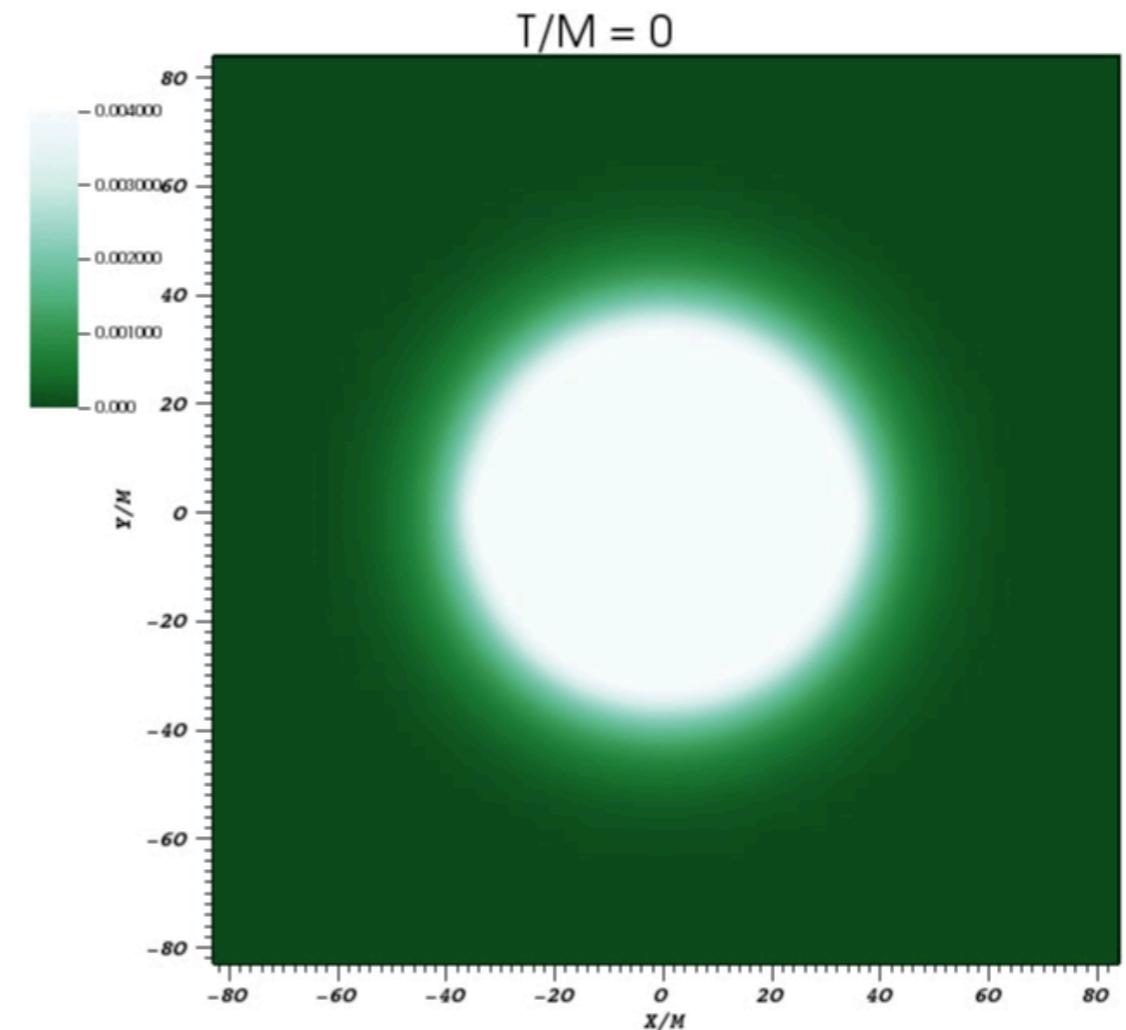
Simulation 2

$$D = 60M$$

Scalar field



Energy density

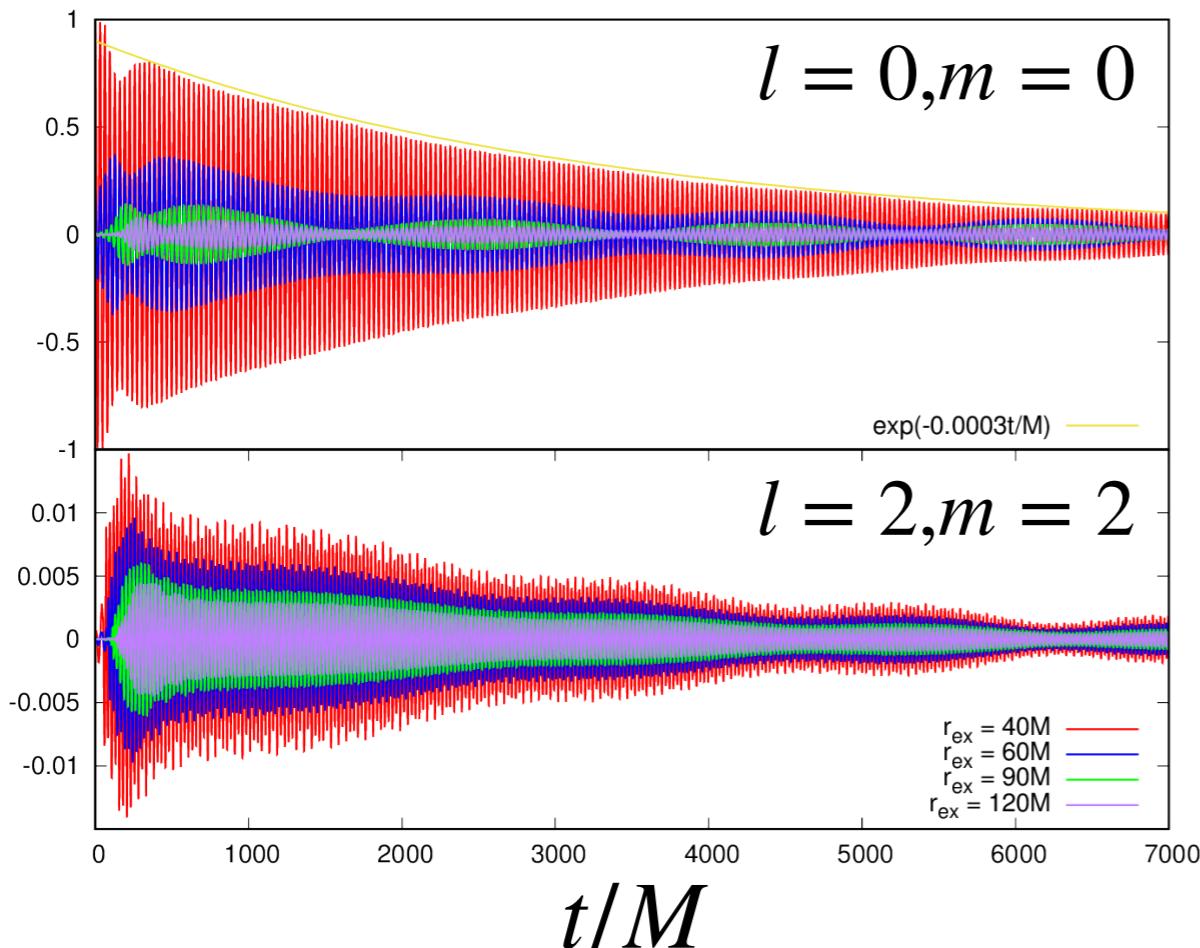


- “Monopole” gravitational molecule around BH binary.

Simulation 2

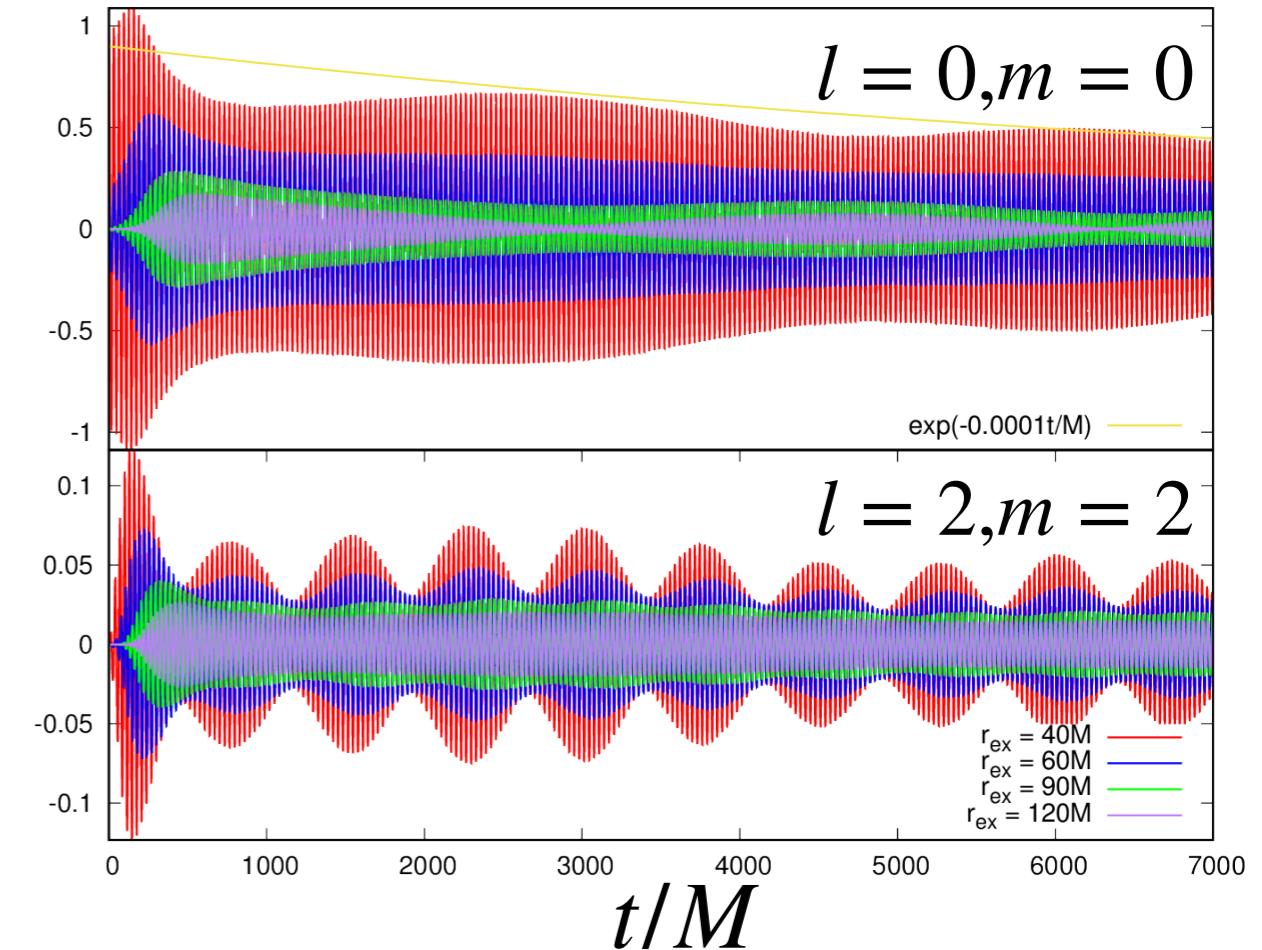
$$\text{cf: } \phi_{00} \sim e^{-\frac{t}{\tau}}$$

$D = 10M$



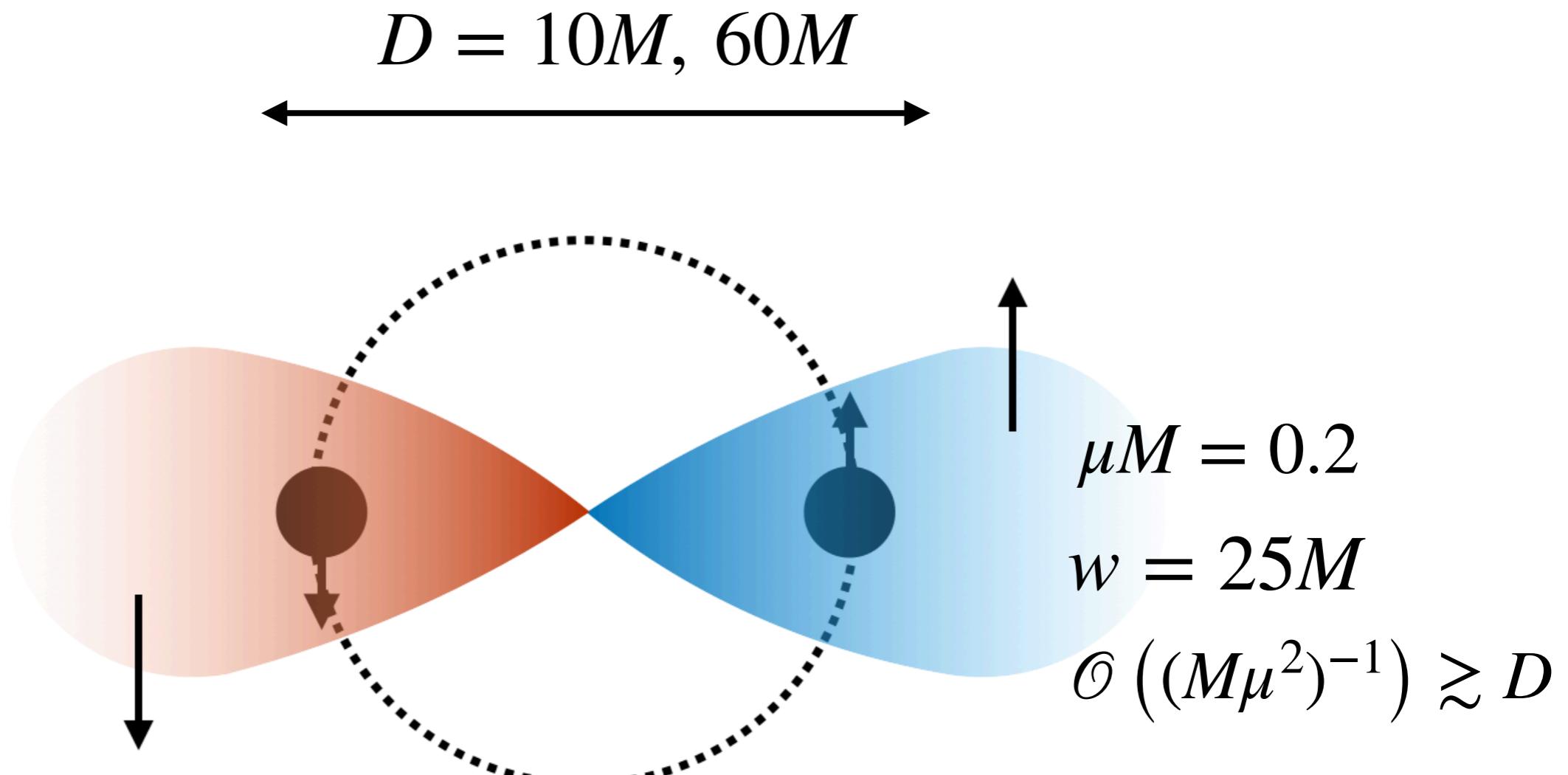
$$\tau \simeq 3 \times 10^3 M$$

$D = 60M$



$$\tau \simeq 1 \times 10^4 M$$

- The spectrum of numerical simulation is good agreement with Di-hydrogen molecule.

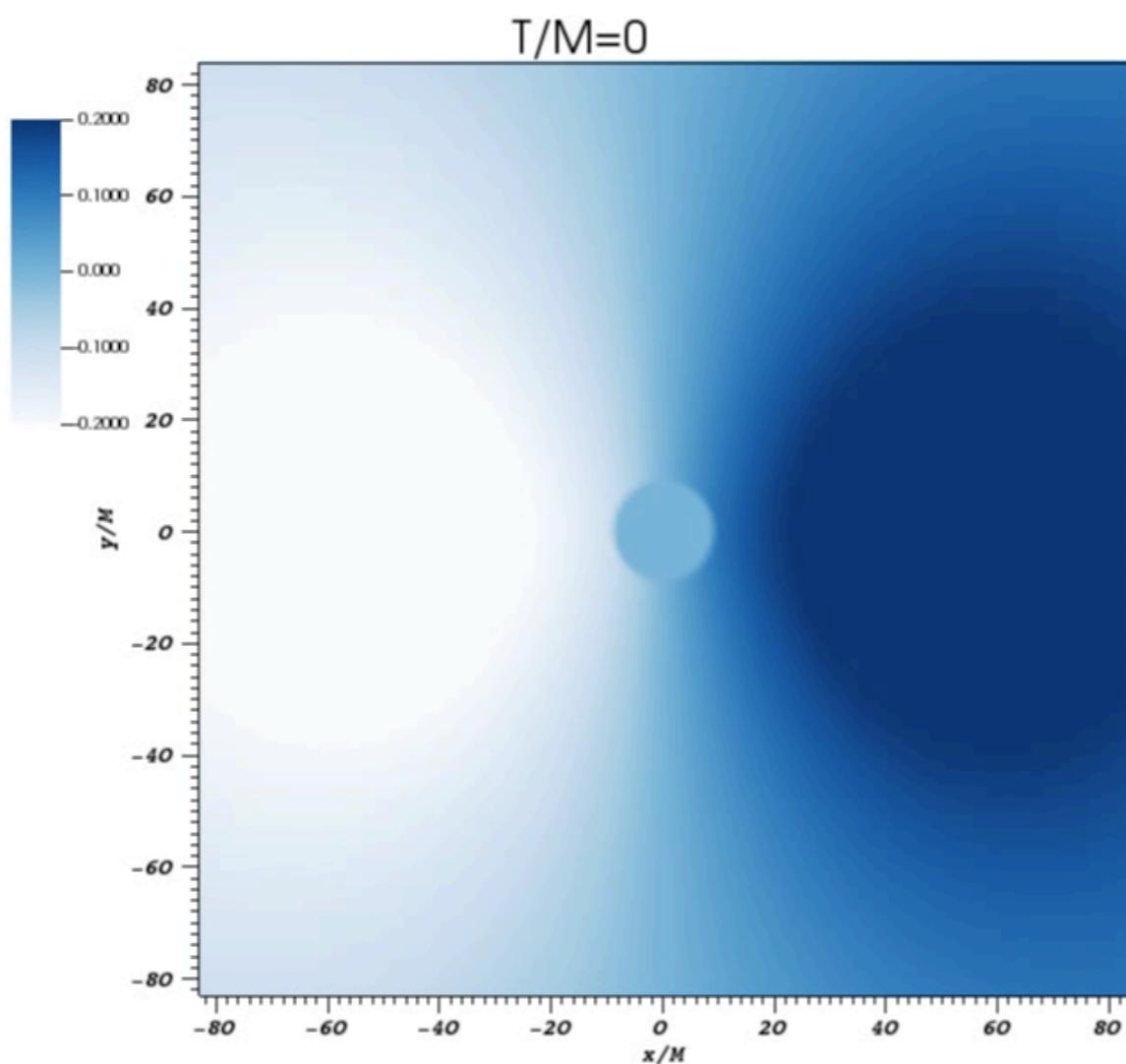


Simulation 3 : Co-rotating dipole initial data

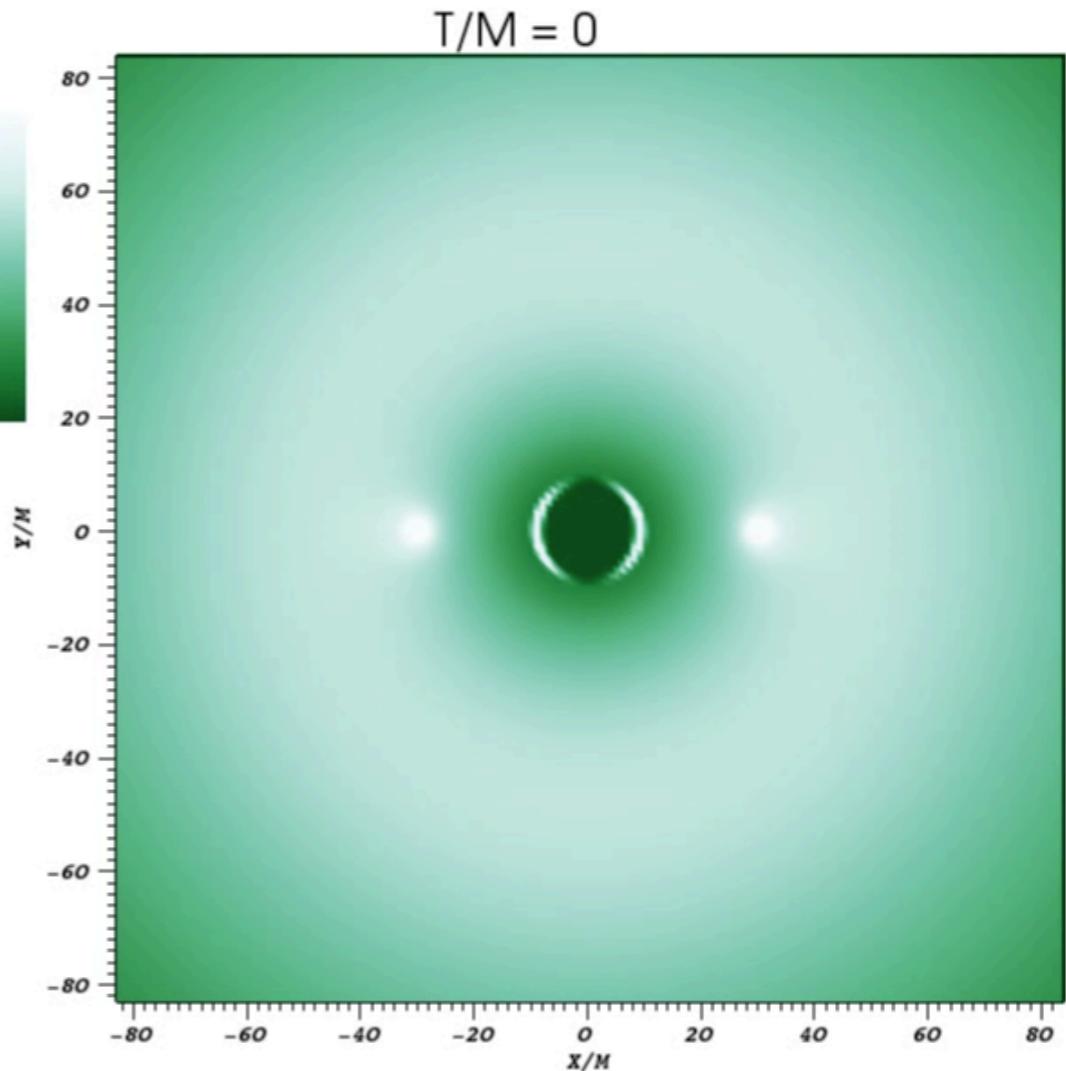
Simulation 3

$D = 60M$

Scalar field



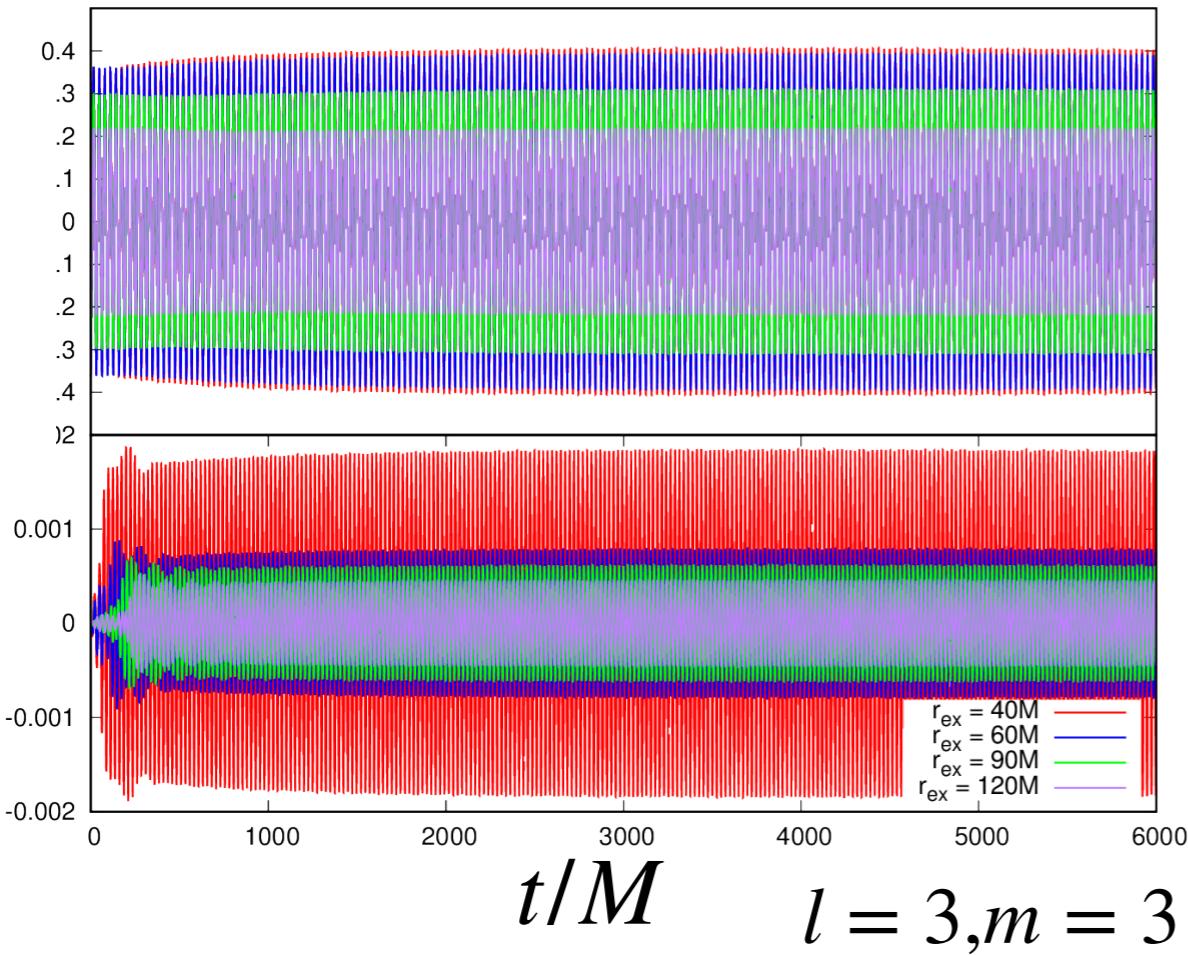
Energy density



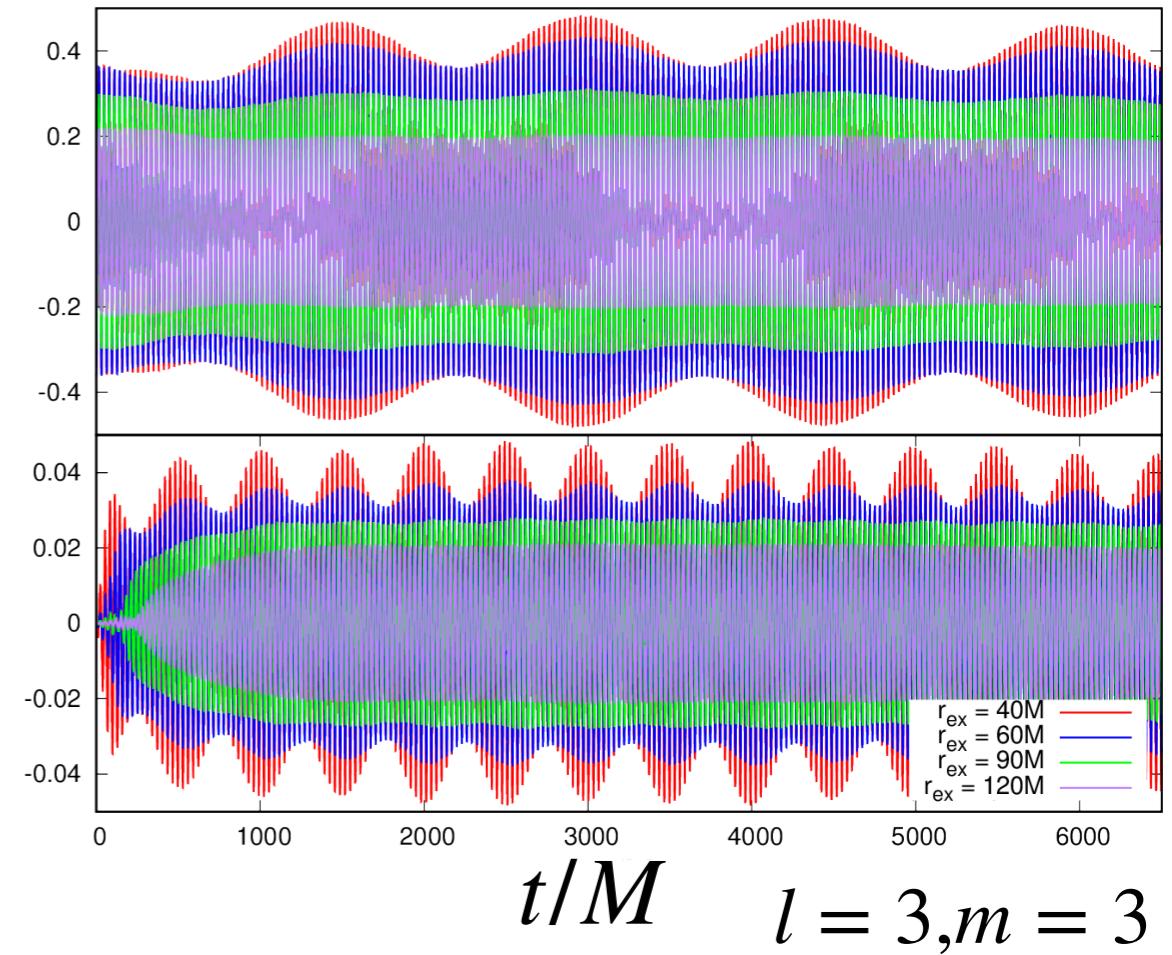
- Energy density rotates with binary.
- “Dipole” co-rotating gravitational molecule around BH binary.

Simulation 3

$$D = 10M \quad l = 1, m = 1$$



$$D = 60M \quad l = 1, m = 1$$



We could not get decaying time scale.

At least, $\tau \gg 10^4 M$. This is long-lived bound state.

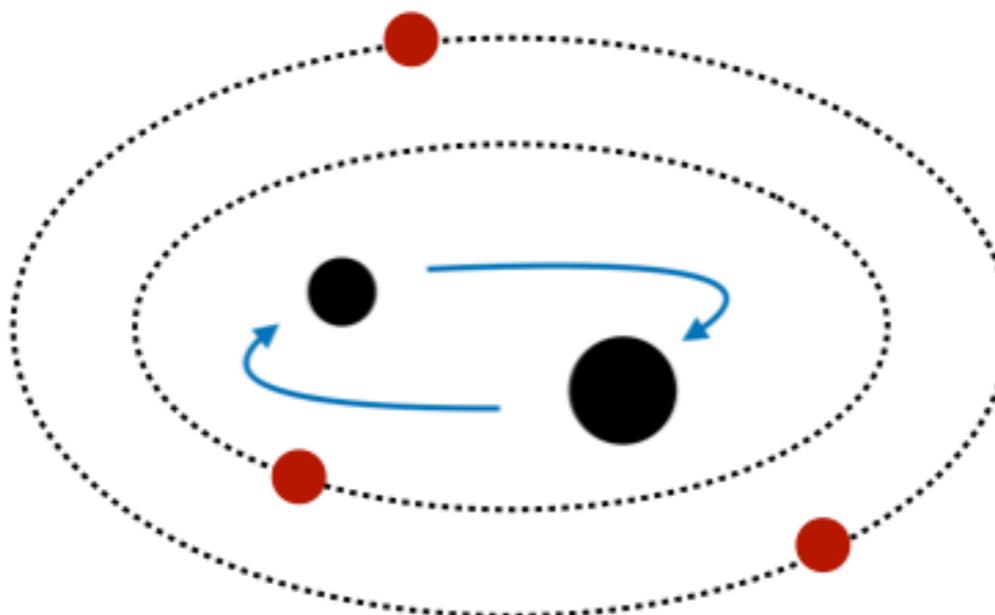
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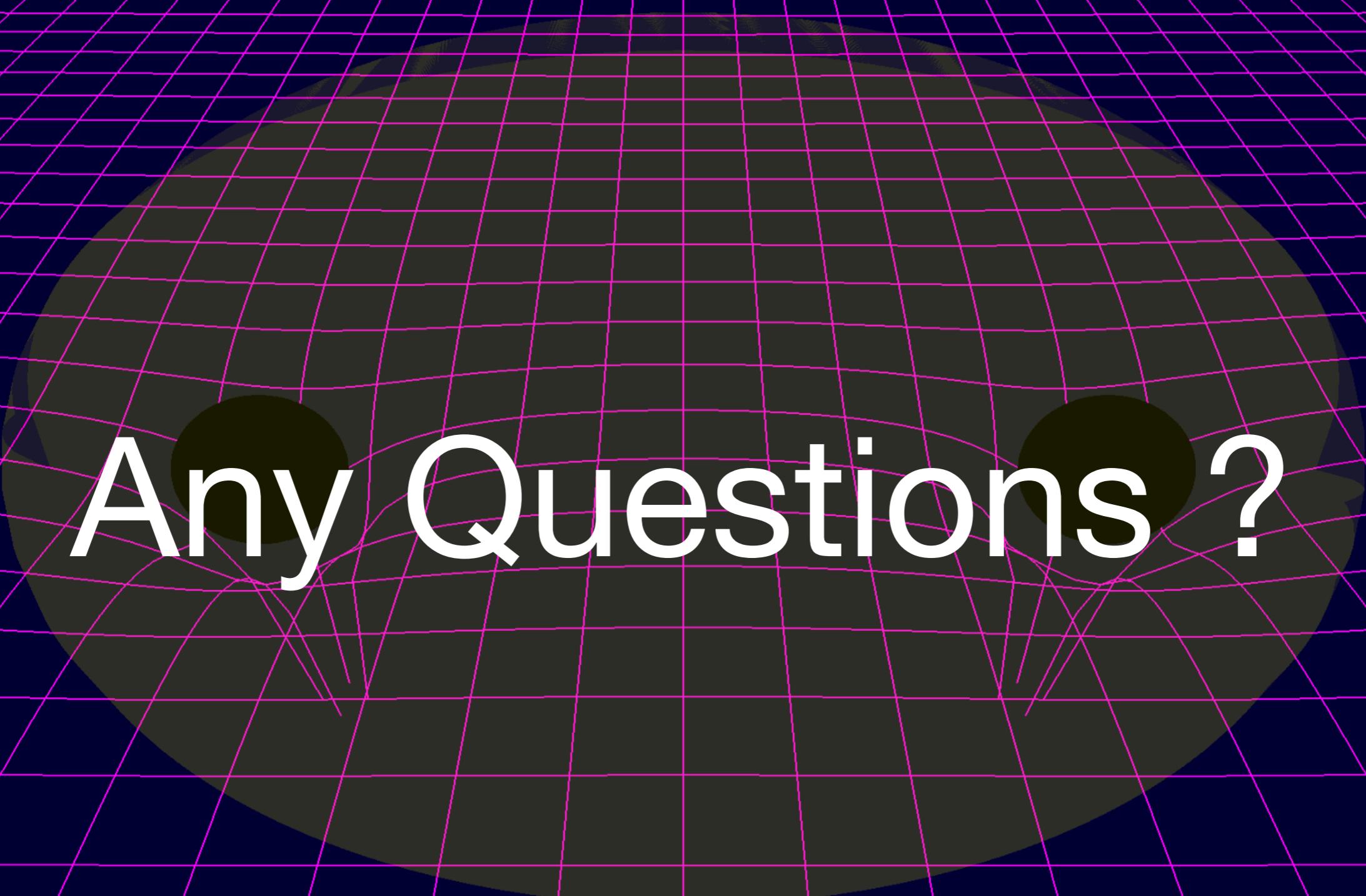
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Summary

- Our result
 - Strong evidence for existence of global bound state (Gravitational molecule).
 - Analogy with QM of di-hydrogen atom.
- Future works
 - eccentricity orbit ?
 - Gravitational wave from the gravitational molecule ?
 - Force between BHs due to the molecule ?
 - Can we observe the force using GW from binary ?
 - We can apply a lot of well-known physics of QM for molecule.

Thank you for your attention !!





Any Questions?