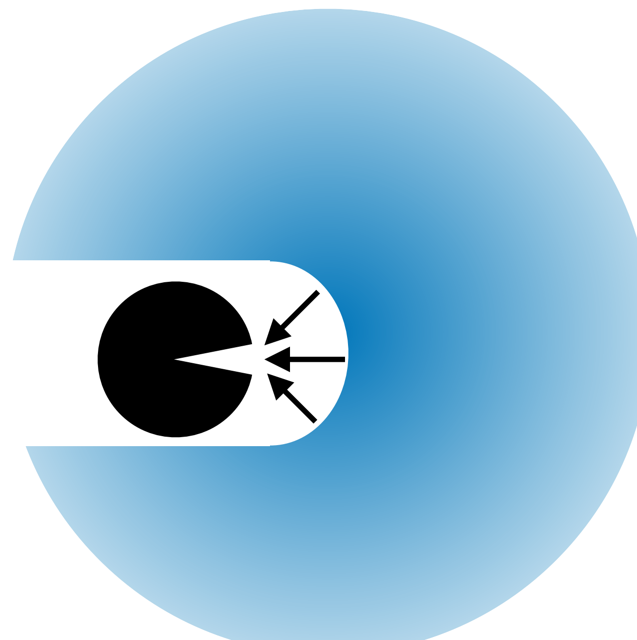
P21

Black hole eating boson star



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Outline

- Introduction
  - Motivation
  - Set up
- Numerical formulation
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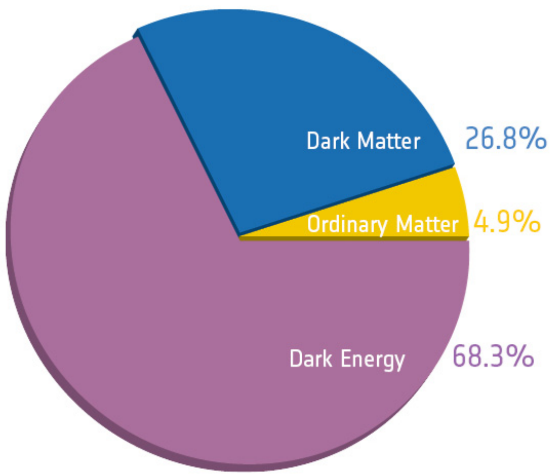
Scalar field beyond GR and SM

- Mystery in our Universe
  - Dark matter ??
  - Dark energy ??
  - Quantum theory of gravity ??



- Light scalar fields are smoking guns for new physics beyond SM.
- Boson stars of the complex scalar field may be dark matter.

Sang-Jin (1994)

  - Boson stars interact with other astrophysical objects.
- The simplest model of boson stars

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi} - g^{\mu\nu} \nabla_\mu \psi \nabla_\nu \psi^* - \mu^2 |\psi|^2 \right)$$

ψ : complex scalar field

Boson star

- Non-rotating Boson star profile

$$\begin{cases} ds^2 = -\alpha_{\rm BS}^2 dt^2 + a_{\rm BS}^2 dr^2 + r^2 d^2\Omega \\ \psi_{\rm BS}(t,r) = e^{i\omega t}\psi_{0,\rm BS}(r) \end{cases}$$

ω : boson star frequency

  - Einstein eq. and KG eq.

$$\begin{cases} \partial_r \alpha_{\rm BS} = \cdots \\ \partial_r a_{\rm BS} = \cdots \\ \partial_r^2 \psi_{0,\rm BS} = \cdots \end{cases}$$

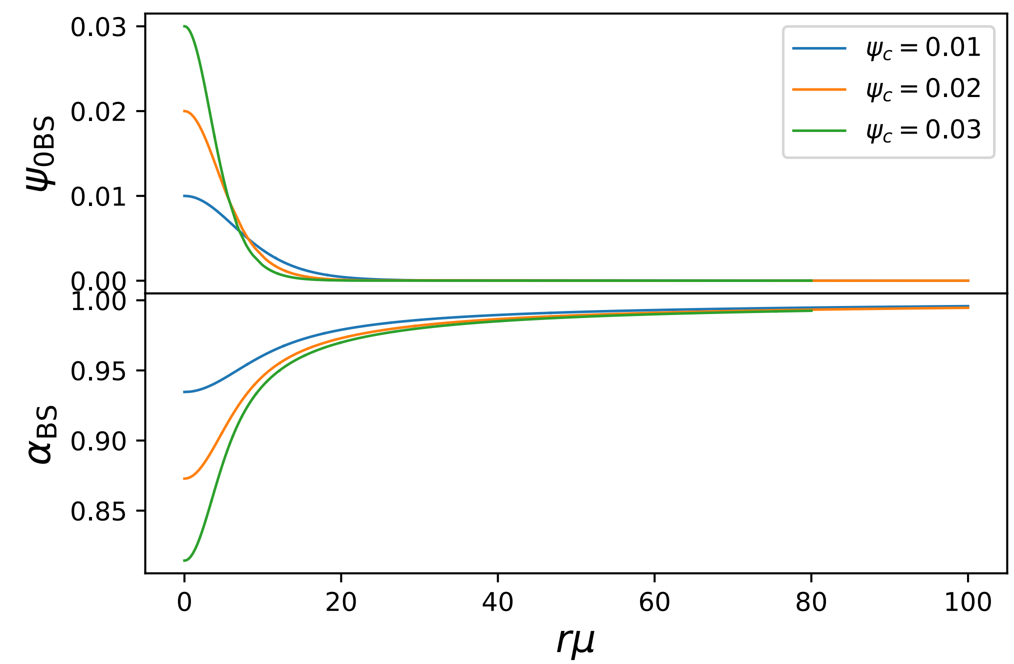
  - Boundary conditions

$$\begin{cases} a_{\rm BS}(\infty) = \alpha_{\rm BS}(\infty)^{-1} \\ \psi_{0,\rm BS}(\infty) = 0 \end{cases}$$

  - Eigenfrequency : $\omega_{\rm BS}(\psi_c, n)$

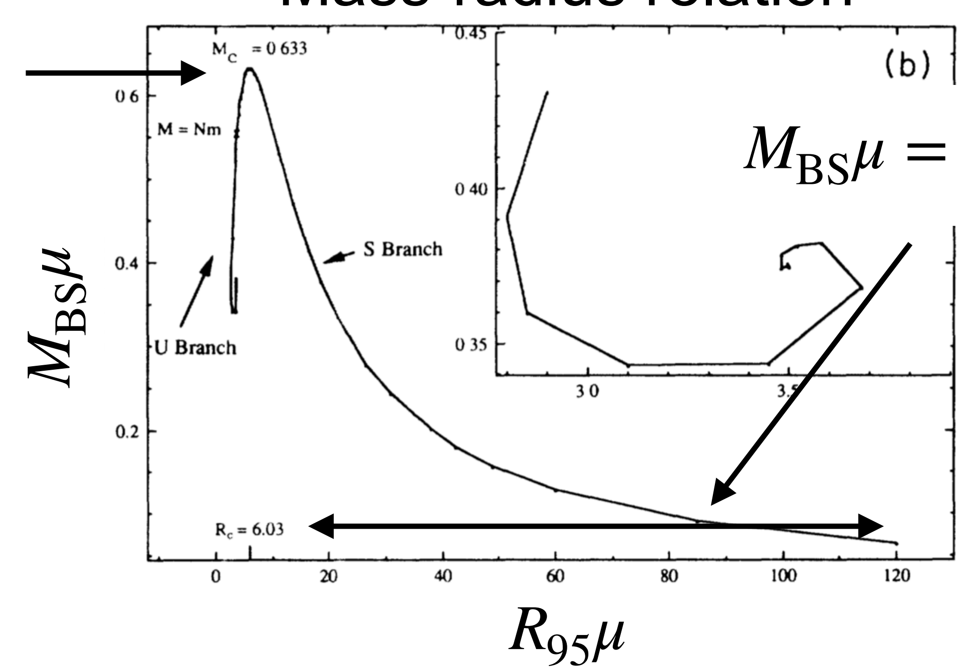
$\psi_{0,\rm BS}(0) = \psi_c$ $\quad$ n : # node

solutions with $n = 0$



Mass-radius relation

$$M_{\rm max} \simeq 0.6 \frac{M_{\rm p}^2}{\mu}$$



$$M_{\rm BS}\mu = \frac{9.1}{R_{\rm BS}\mu}$$

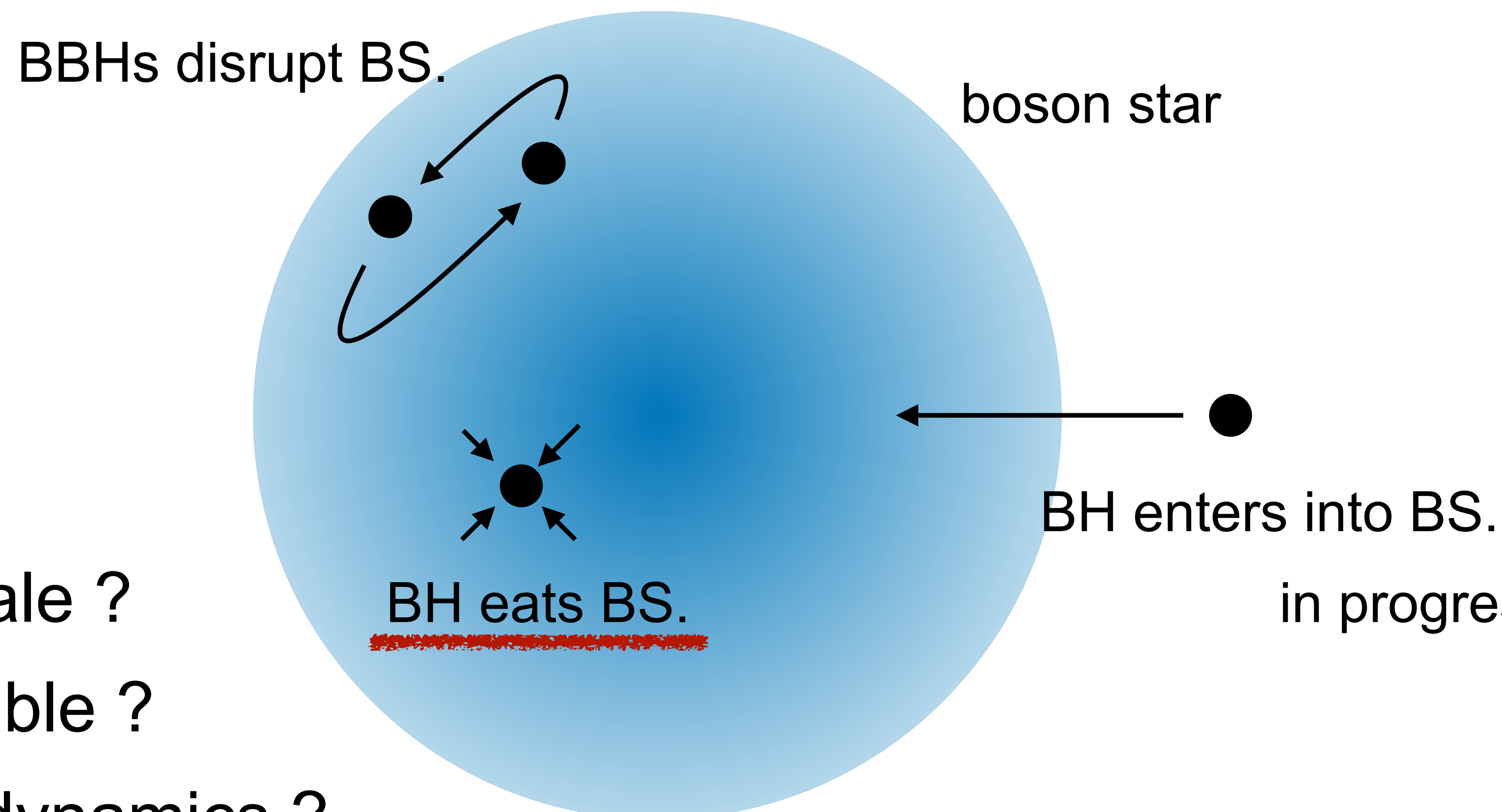
Seidel et.al 1990

Possible interactions with BHs

- Boson stars with light fields

$$\frac{M_{\rm BS}}{M_\odot} = 9\times 10^9 \frac{100{\rm pc}}{R_{\rm BS}} \left(\frac{10^{-22}\ {\rm eV}}{\mu} \right)^2$$

- Boson stars can interact with BHs.

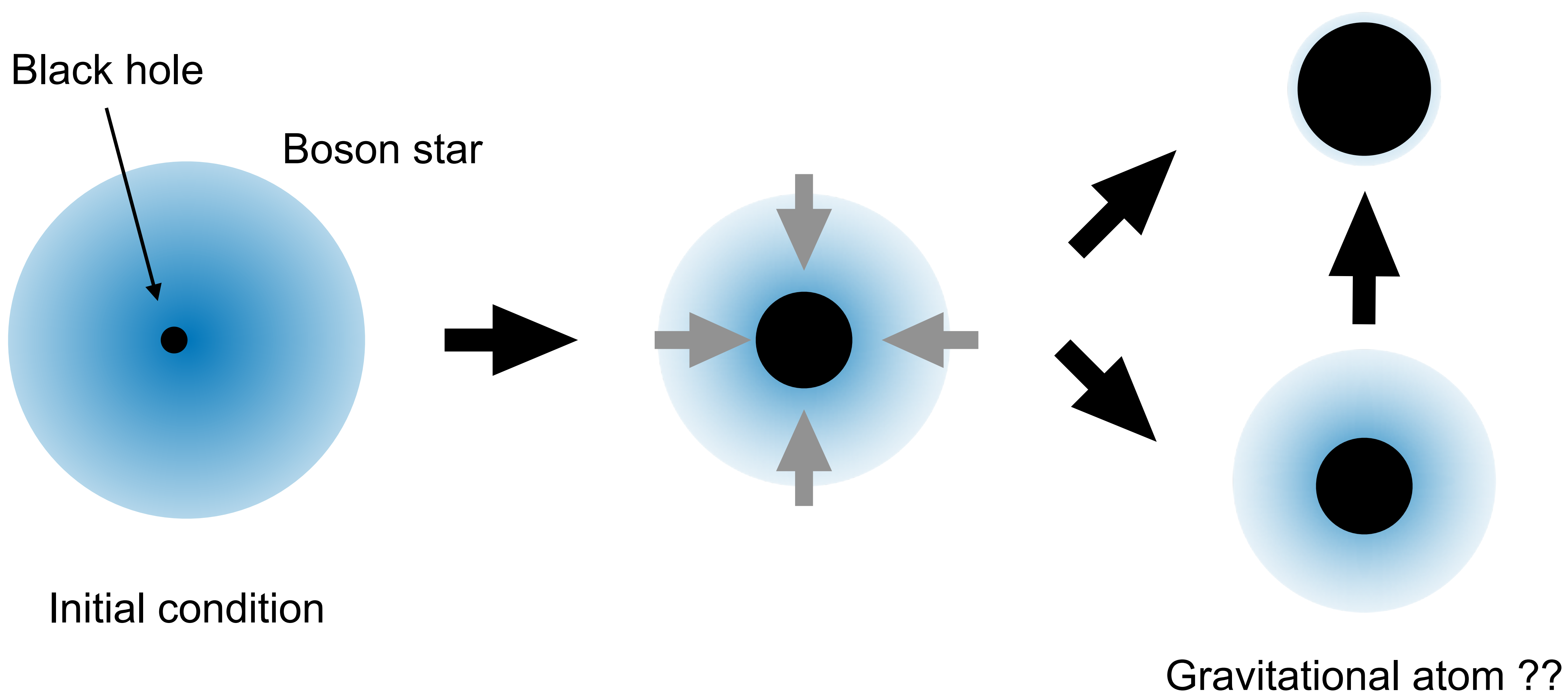


in progress with Zhen

- Time scale ?
- Observable ?
- Typical dynamics ?

Set up

- Set up : BS-BH system
  - Spherical symmetry (for simplicity)
  - Initial profile of the scalar field is same as boson star profile
  - We solve the evolution of metric and the complex scalar field.



Gravitational atom

- Gravitational atom is long-lived state of the scalar field around BH.

$$\left(\Box_{\text{Kerr BH}} - \mu^2\right)\psi = 0 \qquad \psi = e^{-i\omega t + im\phi}R(r)S(\theta)$$

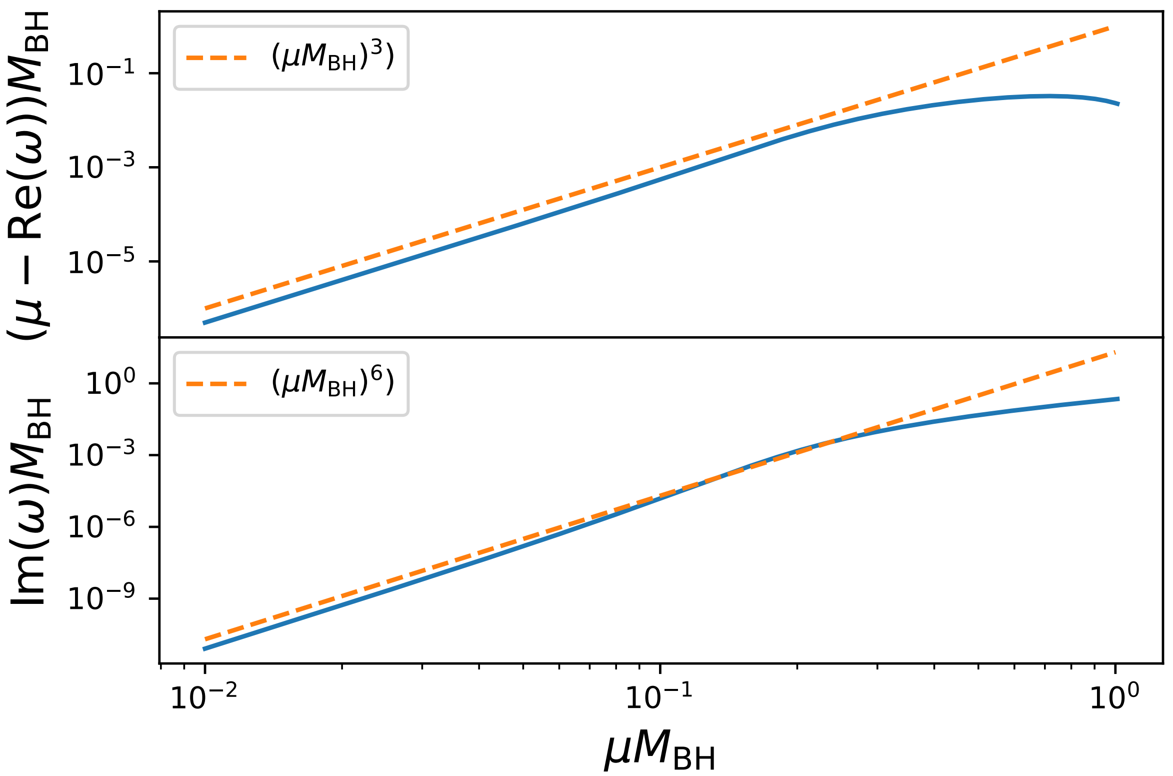
$$\Delta\frac{d}{dr}\left(\Delta\frac{dR}{dr}\right) + \left(\omega^2(r^2+a^2)^2 - 4aMrm\omega + a^2m^2 - \Delta(\mu^2r^2 + a^2\omega^2 + \Lambda)\right)R = 0$$

- Boundary condition
  - Ingoing on BH horizon
  - decaying at infinity
- Leaver method

→ $\omega = \omega_R + i\omega_I$

  - We can estimate typical life time of the gravitational atom.

$a = 0, l = 0$, nodeless



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Numerical formulation

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- We use (generalized-)BSSN formulation.
  - conformal decomposition

$$\begin{cases} \gamma_{ij} = e^{4\phi}\tilde{\gamma}_{ij} \\ K_{ij} = e^{4\phi}\tilde{A}_{ij} + \frac{1}{3}\gamma_{ij}K \end{cases}$$

  - auxiliary field

$$\tilde{\Lambda}^k = \tilde{\gamma}^{ij}(\tilde{\Gamma}^k_{ij} - \bar{\Gamma}^k_{ij})$$

$\bar{\gamma}_{ij}$: reference metric

  - spherical symmetry : (t, r, θ, ϕ)

$$\begin{cases} \tilde{\gamma}_{ij} = \mathrm{diag}(\tilde{a}, \tilde{b}r^2, \tilde{b}r^2\sin^2\theta) \\ \tilde{A}_{ij} = \mathrm{diag}(A, Br^2, Br^2\sin^2\theta) \\ \tilde{\Lambda}^k = (\tilde{\Lambda}, 0, 0) \end{cases}$$

  - constraint eq.

$$\mathscr{H} \equiv \left(\frac{\phi''}{a} + \frac{\phi'^2}{a} - \left(\frac{a'}{2a^2} - \frac{b'}{ab} - \frac{2}{ar}\right)\phi'\right)e^{\phi} - \frac{e^{\phi}}{8}\tilde{R} + \frac{e^{5\phi}}{8}\left(\frac{A^2}{a^2} + 2\frac{B^2}{b^2}\right) - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}E$$

$$\mathscr{M} \equiv 6\phi'\frac{A}{a} + \frac{A'}{a} - \frac{a'A}{a^2} + \frac{b'}{b}\left(\frac{A}{a} - \frac{B}{b}\right) + \frac{2}{r}\left(\frac{A}{a} - \frac{B}{b}\right) - \frac{2}{3}K' - 8\pi p = 0$$

Numerical formulation

- evolution eq.

$$
\begin{cases}
\partial_t \phi = \beta \phi' - \frac{1}{6}\alpha K + \sigma \frac{1}{6}\mathscr{B} \\
\partial_t a = \beta a' + 2a\beta' - 2\alpha A - \sigma \frac{2}{3} a \mathscr{B} \\
\partial_t b = \beta b' + 2\beta \frac{b}{r} - 2\alpha B - \sigma \frac{2}{3} b \mathscr{B} \\
\partial_t K = \beta K' - \mathscr{D} + \alpha(\frac{1}{3}K^2 + \frac{A^2}{a^2} + 2\frac{B^2}{b^2}) + 4\pi\alpha(E+S) \\
\partial_t A = \beta A' + 2A\beta' + e^{-4\phi}(-\mathscr{D}_{rr}^{TF} + \alpha(R_{rr}^{TF} - 8\pi S_{rr}^{TF})) + \alpha(KA - 2\frac{A^2}{a}) - \sigma \frac{2}{3} A \mathscr{B} \\
\partial_t B = \beta B' + \frac{e^{-4\phi}}{r^2}(-\mathscr{D}_{\theta\theta}^{TF} + \alpha(R_{\theta\theta}^{TF} - 8\pi S_{\theta\theta}^{TF})) + \alpha(KB - 2\frac{B^2}{b}) + 2\frac{\beta}{r}B - \sigma \frac{2}{3} B \mathscr{B} \\
\partial_t \tilde{\Lambda} = \beta \tilde{\Lambda}' - \beta' \tilde{\Lambda} + \frac{2\alpha}{a}(\frac{6A\phi'}{a} - \frac{2}{3}K' - 8\pi p) + \frac{\alpha}{a}(\frac{a'A}{a^2} - \frac{2b'B}{b^2} + 4B\frac{a-b}{rb^2}) + \sigma(\frac{2}{3}\tilde{\Lambda}\mathscr{B} + \frac{\mathscr{B}'}{3a}) + \frac{2}{rb}(\beta' - \frac{\beta}{r}) - 2\frac{\alpha' A}{a^2} + \frac{1}{a}\beta''
\end{cases}
$$

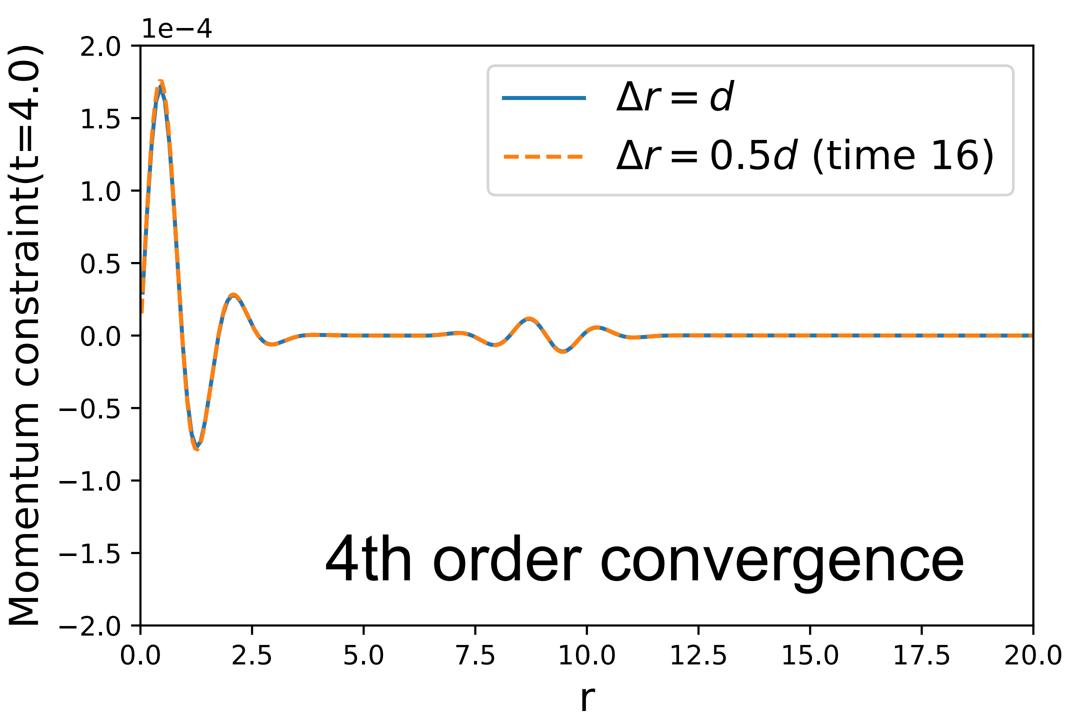
skip the details…..

- Our numerical code
  - Time integration : 4th order Runge-Kutta method
  - Radial derivative: 4th order accurate centered finite difference
  - Open MP, KO dissipation, excision procedure ….

Test simulations

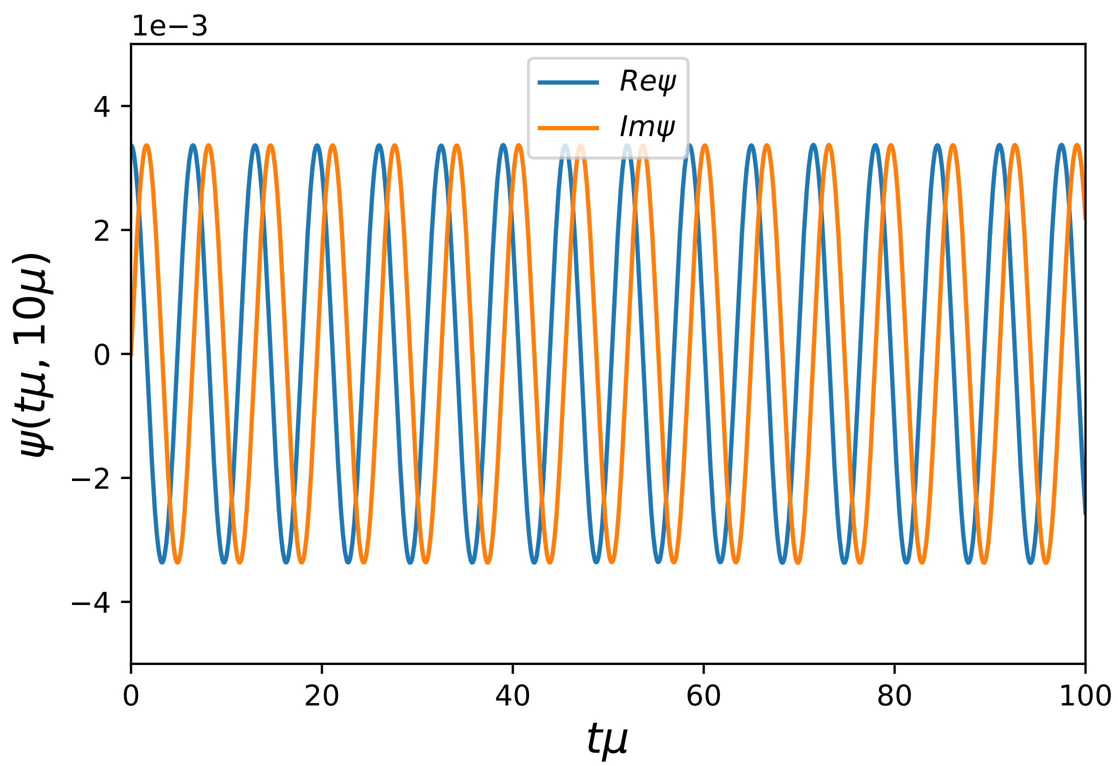
- Numerical convergence, and boson star evolution
  - Pure gauge evolution

$$\alpha(r,0) = 1 + \frac{\alpha_0 r^2}{1+r^2}\left(e^{-(r-r_0)^2} + e^{-(r+r_0)^2}\right)$$



  - Boson star evolution

$$\begin{cases} \phi(0,r) = \phi_{\mathrm{BS}}(r) \\ \psi(0,r) = \psi_{\mathrm{BS}}(r) \\ \alpha(0,r) = e^{-4\phi_{\mathrm{BS}}(r)} \end{cases} \qquad \psi_c = 0.01$$



Evolution is consistent with stable boson star configuration.

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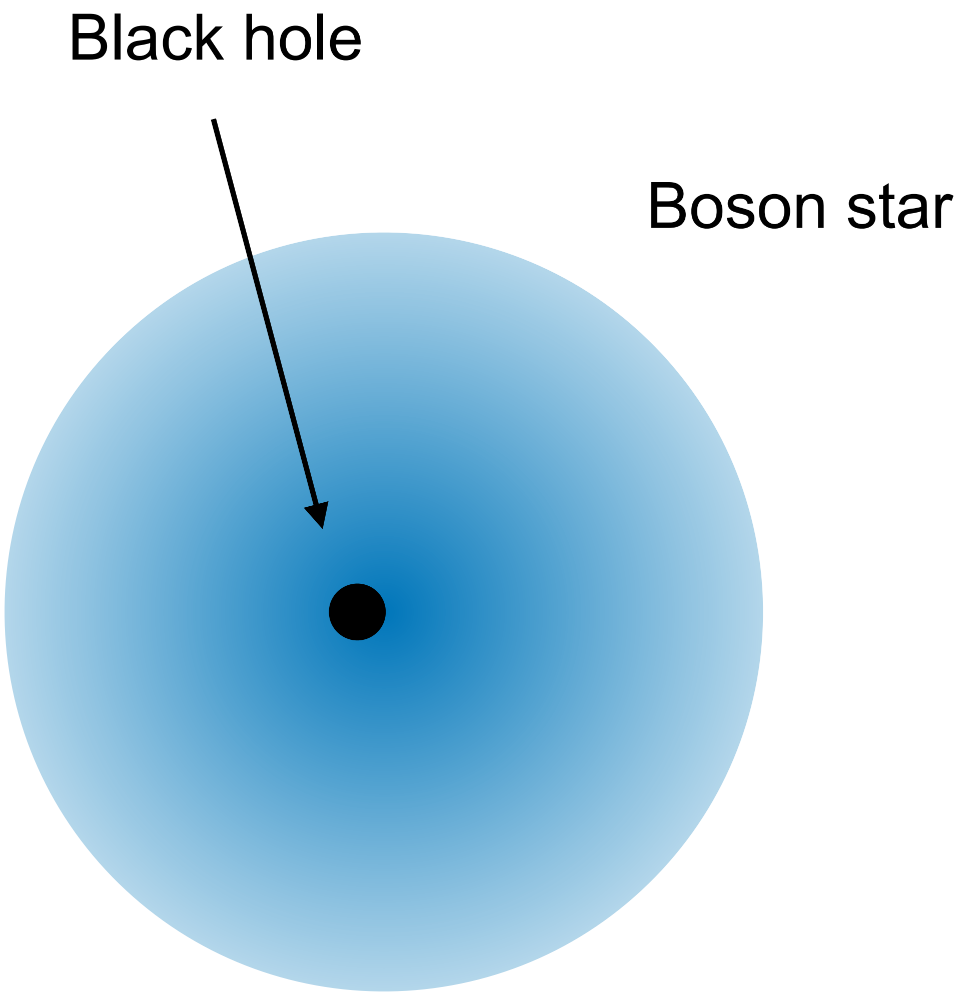
Construction of ID



Initial condition

Construction of ID

- We construct BS-BH initial data by solving constraint equation.
  - assumptions for initial data
    - momentarily static : $K = A = B = 0$ ➡ $\mathscr{M} = 0$
    - conformally flat : $a = b = 1$
    - Profile of the scalar field is same as boson star profile.
    - precollapse lapse, zero shift : $\alpha(0,r) = e^{-4\phi(0,r)}$, $\beta(0,r) = 0$
  - Parameters: ψ_c , $M_{0,\mathrm{BH}}$
  - 1. Construct the BS star profile in isotropic coordinate.

$$\begin{cases} ds^2 = -\alpha_{\mathrm{BS}}^2(r)dt^2 + \Phi_{\mathrm{BS}}(r)^4(dr^2 + r^2 d^2\Omega) \\ \psi(t,r) = \psi_{0,\mathrm{BS}}(r)e^{i\omega t} \end{cases}$$

  - 2. Apply the window function to scalar field

$$\psi_{\mathrm{BS}}^{\mathrm{W}}(r) = W(r)\psi_{0,\mathrm{BS}}(r)$$

$$W(r) = \begin{cases} 0 & (r < M_{0,\mathrm{BH}} + \epsilon_1) \\ f(r) & (M_{0,\mathrm{BH}} + \epsilon_1 < M_{0,\mathrm{BH}} + \epsilon_1 + \epsilon_2) \\ 1 & (M_{0,\mathrm{BH}} + \epsilon_1 + \epsilon_2 < r) \end{cases}$$

Construction of ID

- 3.Sum of conformal factor

$$\Phi = e^{\phi}$$

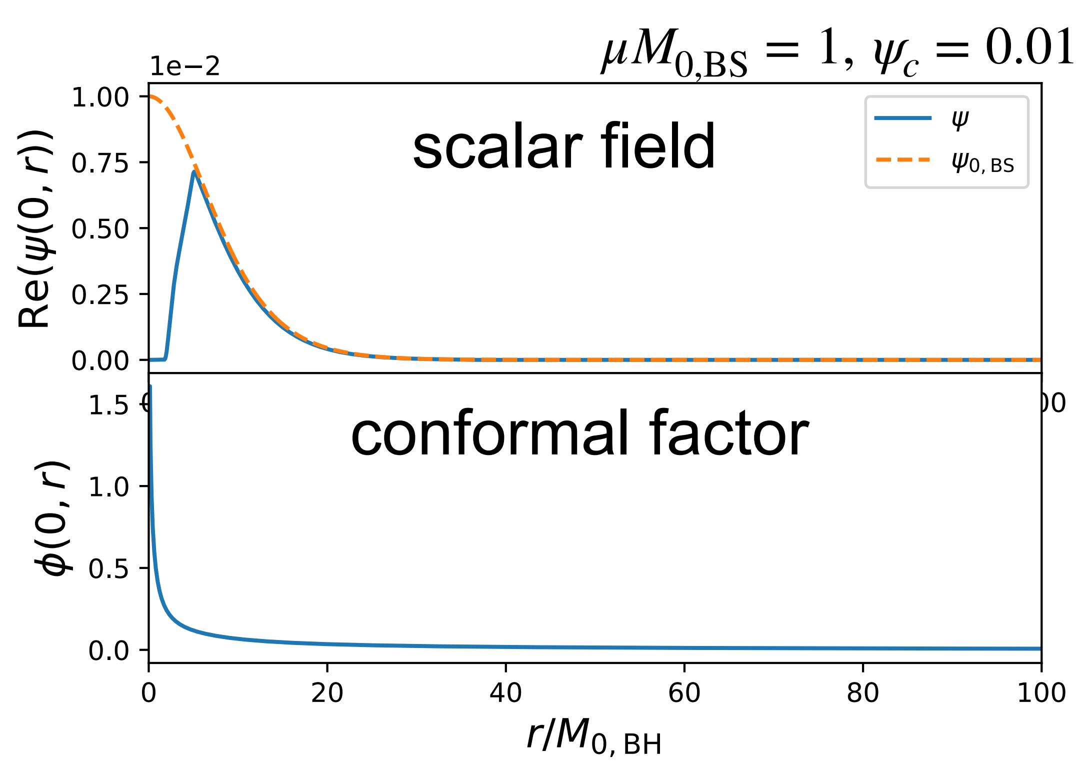
$$\Phi = \Phi_{\mathrm{BS}} + \Phi_{\mathrm{BH}} - 1 + \delta\Phi \qquad \Phi_{\mathrm{BH}} = 1 + \frac{M_{0,\mathrm{BH}}}{2r}$$

- 4.Solve Hamiltonian constraint for $\delta\Phi$

$$\mathscr{H} = \Phi'' + \frac{2}{r}\Phi' + 2\pi\Phi^5 E^{\mathrm{W}}_{\mathrm{BS}} = 0$$

$$\Rightarrow \quad \delta\Phi'' + \frac{2}{r}\delta\Phi' + 2\pi\left(\Phi^5 E^{\mathrm{W}}_{\mathrm{BS}} - \Phi^5_{\mathrm{BS}} E_{\mathrm{BS}}\right) = 0$$

Integrate from infinity with $\delta\Phi(\infty) = \partial_r \delta\Phi(\infty) = 0$



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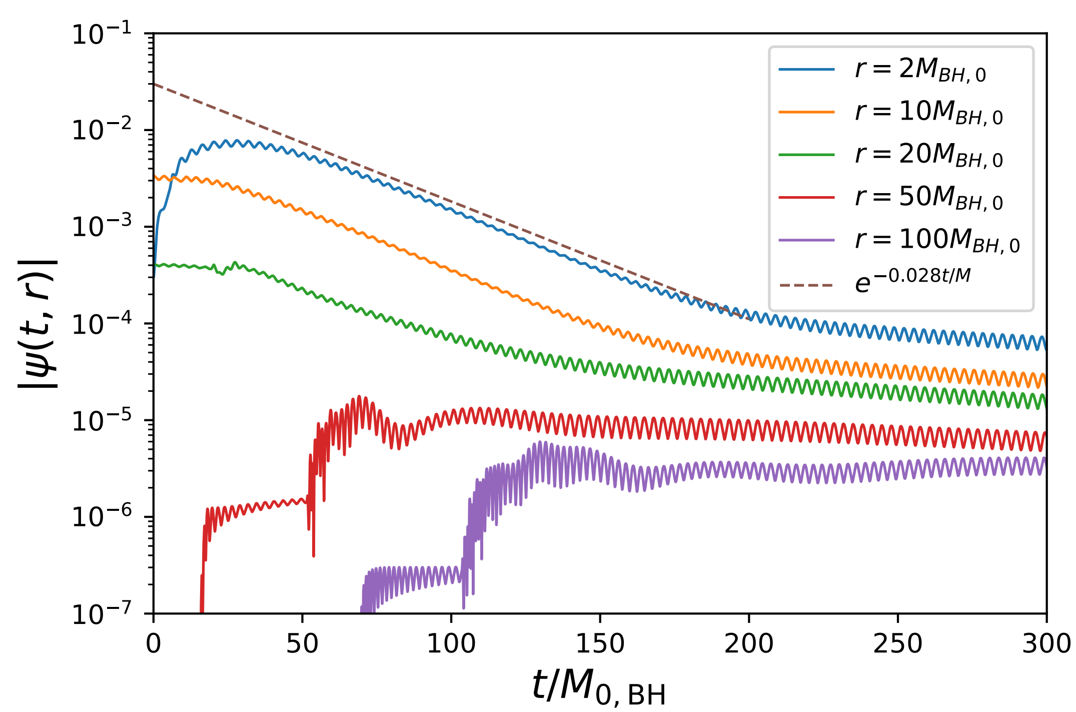
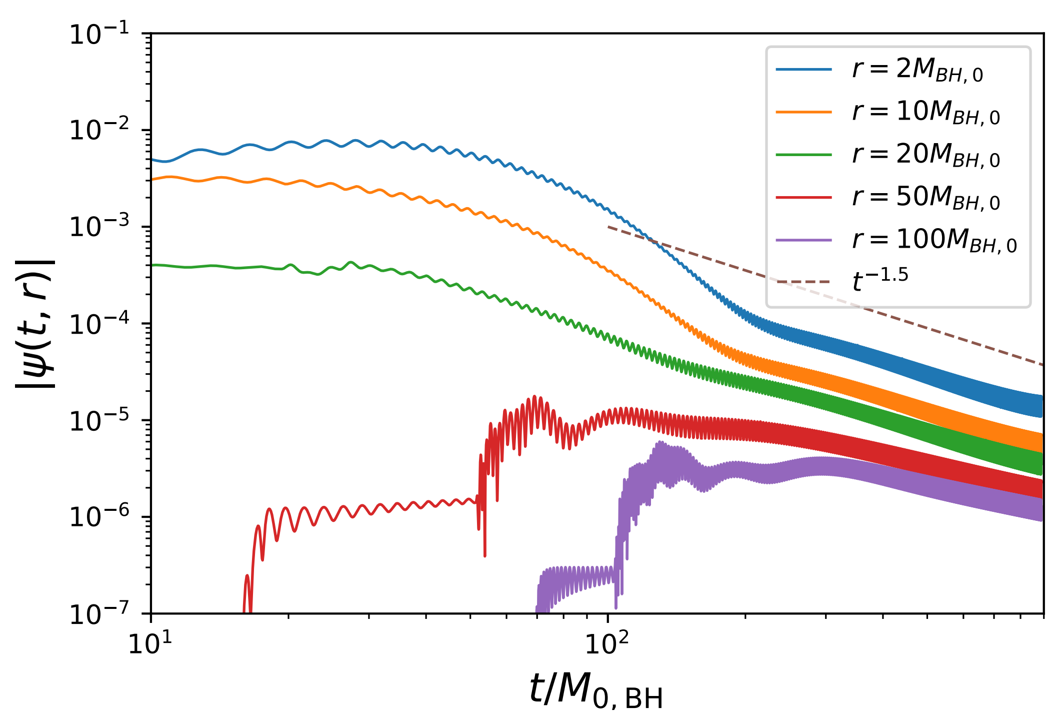
Preliminary results

- Here, we show one preliminary result. $\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01$

The scalar field decays exponentially around BH in early phase.



In late time, we observe power-law tail.



In general, we can expect power low tail

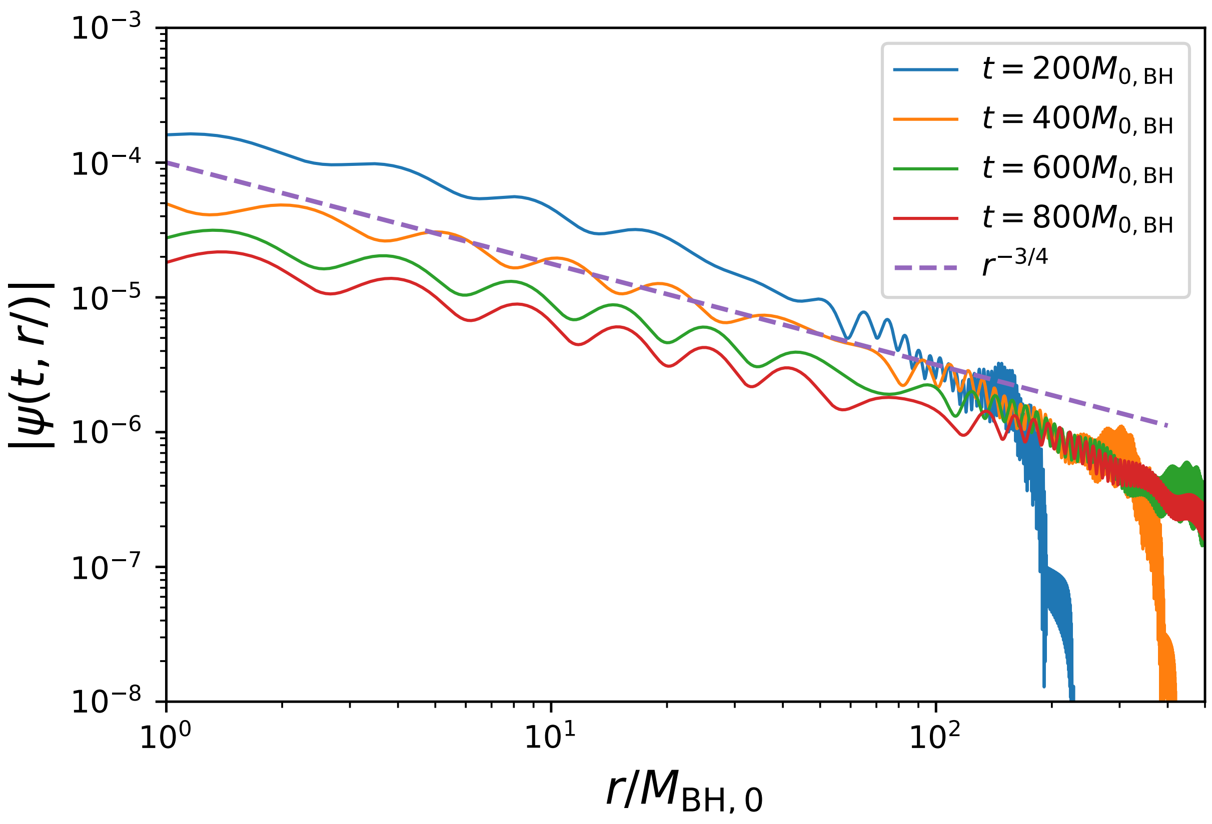
$$\psi \sim t^p \sin(\mu t)$$

$$\begin{cases} p = -(l + 3/2) & \text{at late time} \\ p = -5/6 & \text{at very late time} \end{cases}$$

Preliminary results

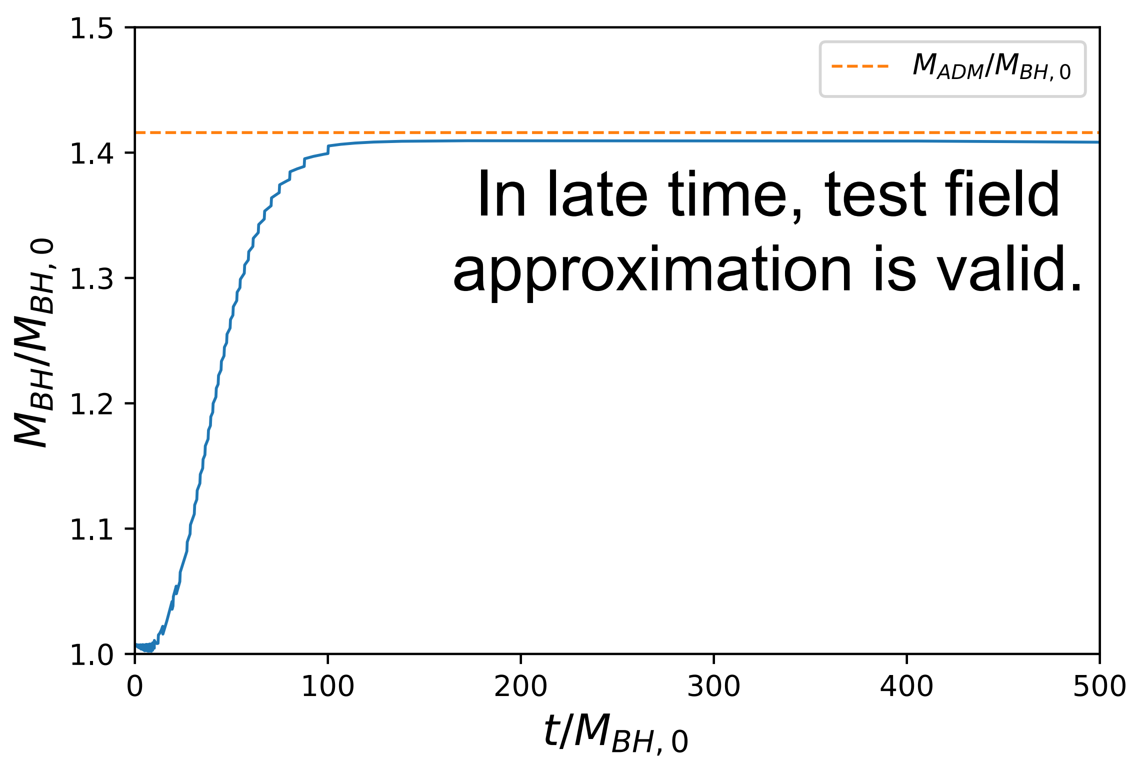
$$\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01$$

Late time radial profile of scalar field is $\sim r^{-3/4}$



cf: Lam (2019)

BH eat almost boson star energy in early phase.



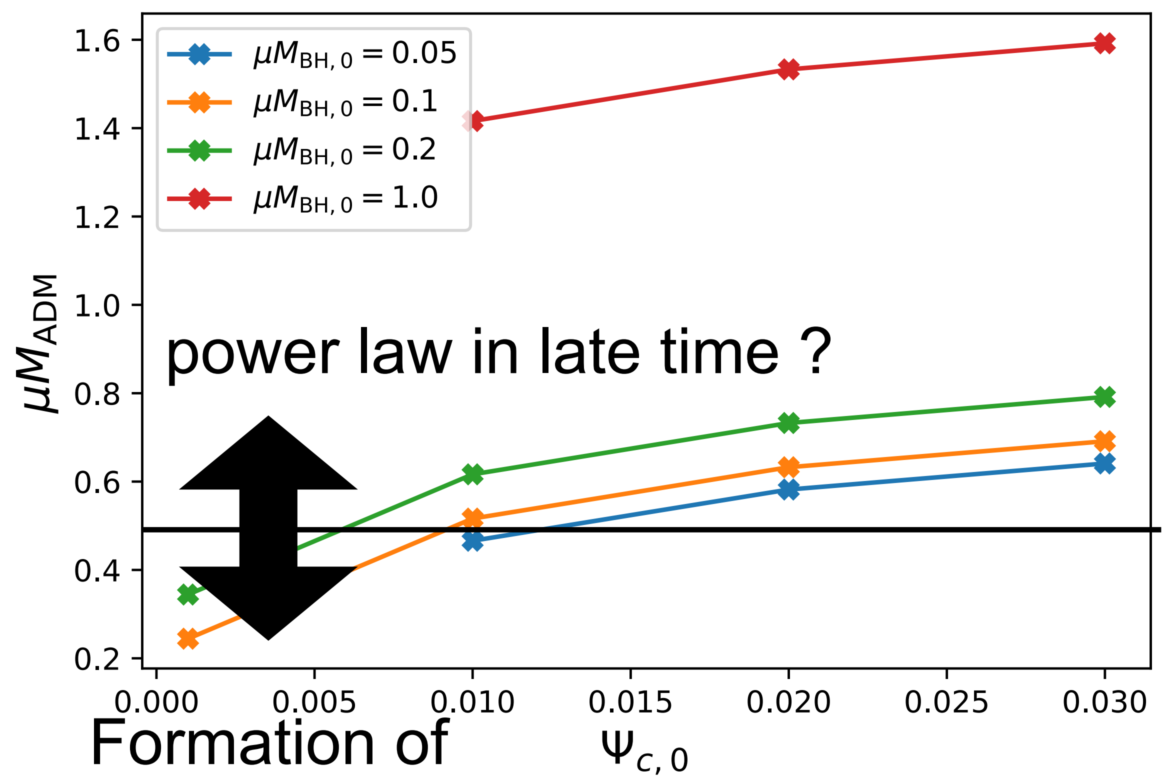
In late time, $\mu M_{BH} \simeq 1.4$

The life time of the corresponding gravitational atom is very short.

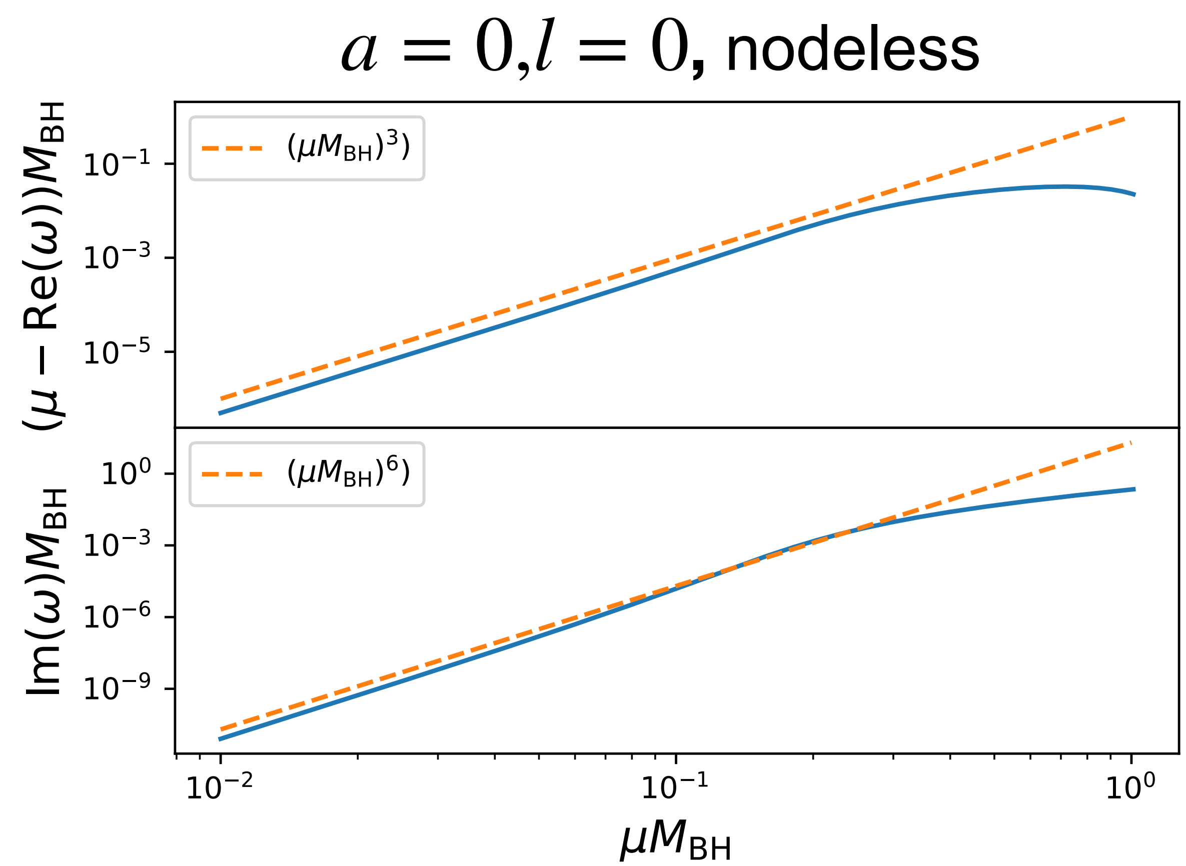
→ power low behavior dominates.

Preliminary results

- We guess late time behavior from ADM mass and mass of the scalar field.



Formation of gravitational atom ??



We need further simulations to check the expectations.

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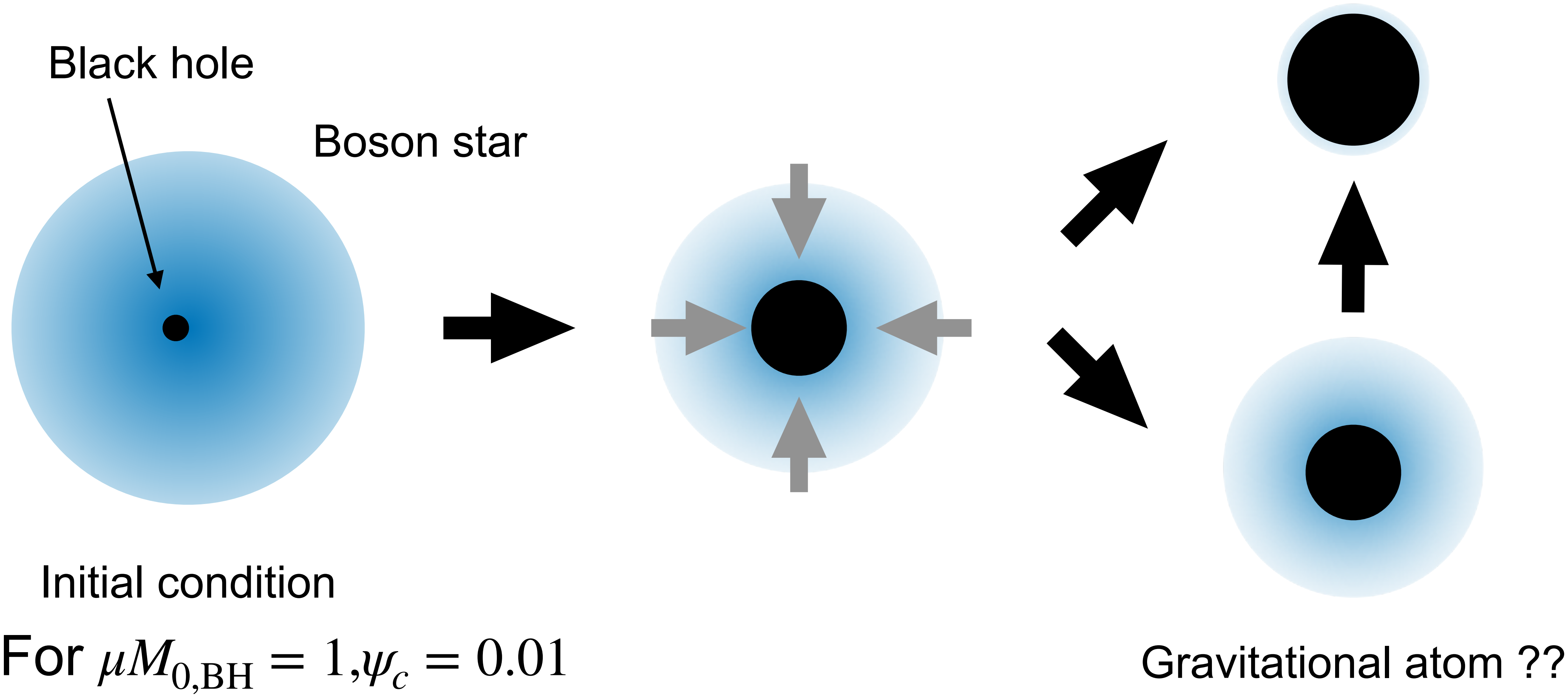
Summary

- We considered the accretion process of boson star into black hole.



  - For $\mu M_{0,\mathrm{BH}} = 1, \psi_c = 0.01$
    we observed late time power low decay, and power low profile.
  - We can guess late time behavior from ADM mass and mass of the scalar field.
- We need further simulations….

Finish