

# Blast of light from axionic cloud around BHs

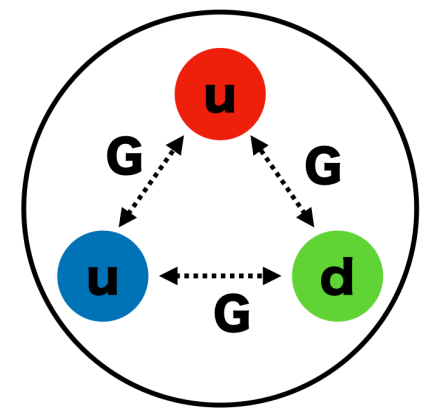
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PRL122(2019)no.8,081101



# What is Axion ?



phase of quark  
mass matrix

$$\tilde{\Theta} = \Theta + \text{Arg Det} M$$

parameter of  
correct vacuum

- QCD axion

- Strong CP problem

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{SU}(3)} + \underline{\tilde{\Theta} \frac{g^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{a\mu\nu}}$$

- Electric dipole moment of neutron

$$d_n \simeq 10^{-16} |\tilde{\Theta}| e \text{ cm} < 10^{-25} e \text{ cm} \rightarrow |\tilde{\Theta}| < 10^{-9}$$

Why this value is small ?

- Peccei Quinn Mechanism is one of the plausible solution.

- Light scalar field “**axion**” was predicted

$\Phi$  : axion field

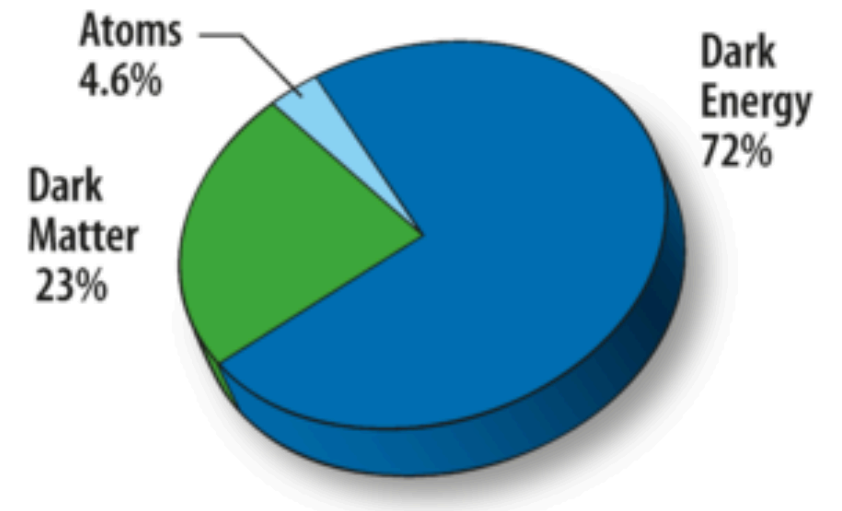
$$\mathcal{L}_{\text{axion}} = -\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2} \mu^2 \Phi^2 + C_a \frac{\Phi}{F_a} \frac{g^2}{32\pi^2} \tilde{G}^{a\mu\nu} G_{\mu\nu}^a + \dots$$

- String axion

- String theory also predicts the scalar field with very light mass.

# What is Axion ?

- Axion physics
  - CMB polarization
  - Axion cooling of star ...
- Axion is dark matter candidate.
  - Our universe may be filled with axions.
- Axion around BH
  - Axion can localized around BH, as **axion cloud**.



$$r_{\text{cloud}} \sim \frac{(l + n + 1)^2}{(M\mu)^2} M$$

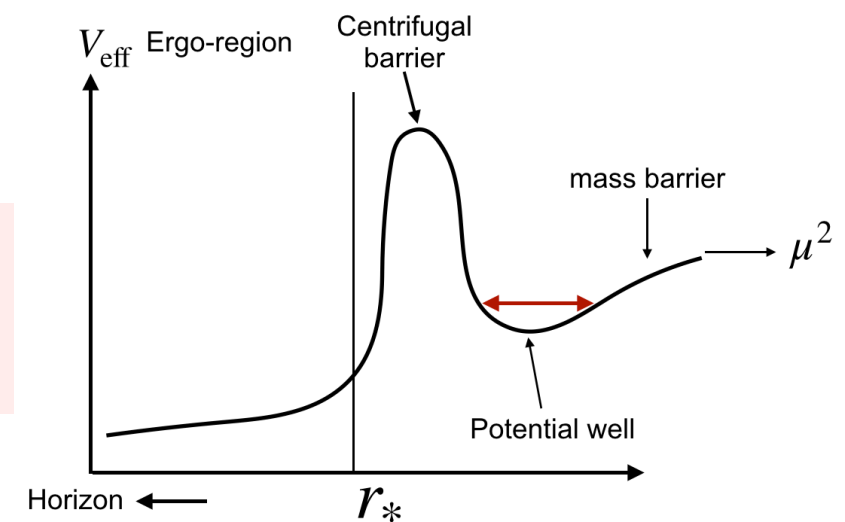
- Superradiant instability

$$\omega < m\Omega_{\text{H}}$$

$$\tau = 2 \times 10^4 a \left( \frac{\mu}{10^{-5} \text{eV}} \right)^{-1} \left( \frac{\mu M}{0.03} \right)^{-8} \text{ s}$$

$\Omega_{\text{H}}$  : angular velocity

$$\Phi(x) = e^{-i\omega t} e^{im\phi} S_{lm}(\theta) R_{lm}(r)$$



# Interaction with EM

- Interaction with photon

$$\mathcal{L}_{\Phi\gamma\gamma} = -\frac{1}{2}k_a\tilde{F}_{\mu\nu}F^{\mu\nu}\Phi = -2k_a\vec{E}\cdot\vec{B}\Phi \quad k_a = \frac{\alpha C}{2\pi F_a}$$

For QCD axion

$$F_a \sim 6 \times 10^{11} \left( \frac{10^{-5}\text{eV}}{\mu} \right) \text{GeV}$$

- Axion life time (QCD axions)

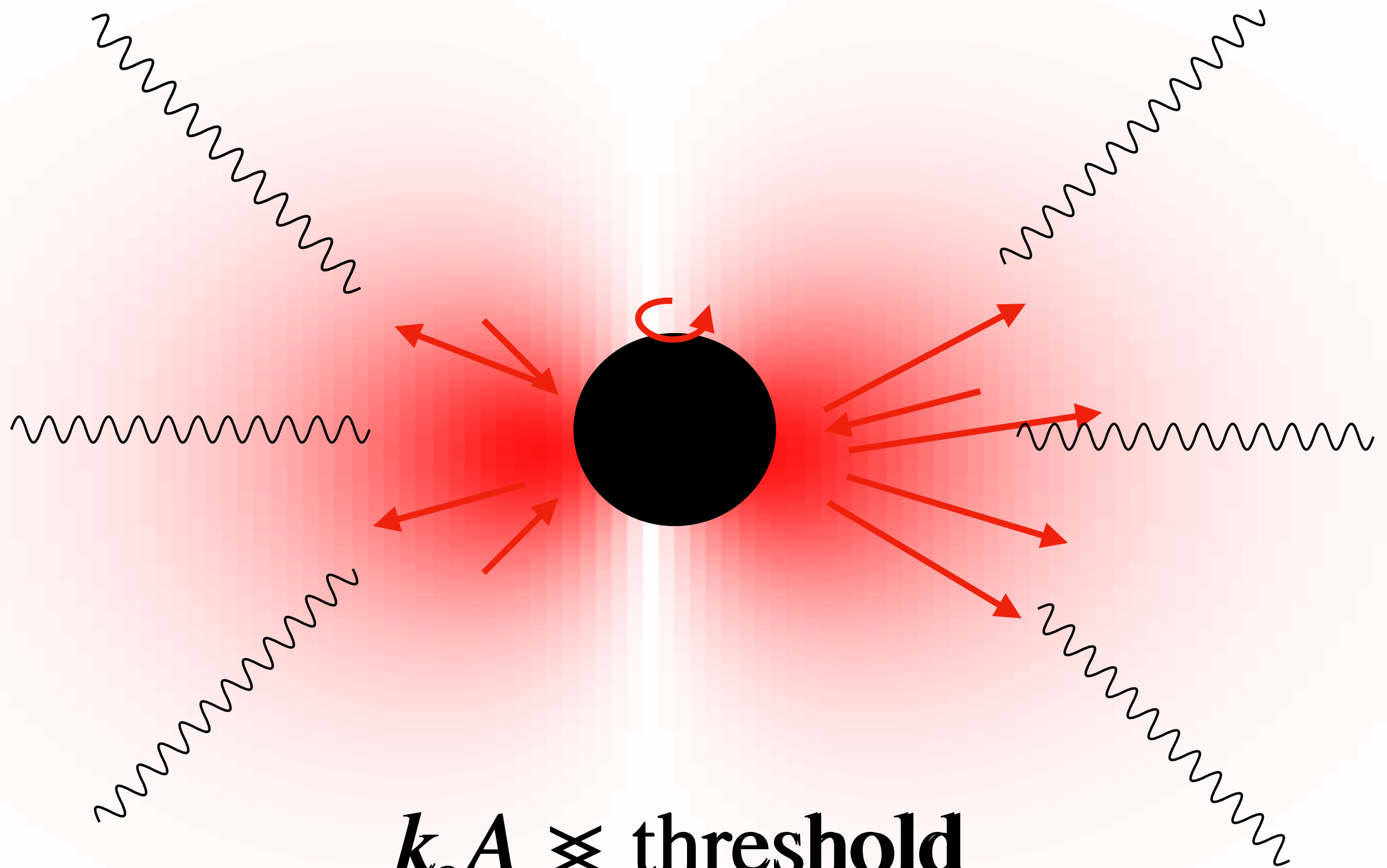
$$\tau_\phi \sim 3 \times 10^{32} \left( \frac{\mu}{10^{-5}\text{eV}} \right)^{-5} \text{Gyr}$$

- Axions convert into photon in magnetic field

$$\Gamma \sim 7 \times 10^{-11} \text{yr} \left( \frac{10^{16} \text{GeV}}{F_a} \right)^2 \left( \frac{\mu_a}{6 \times 10^{-10} \text{eV}} \right) \left( \frac{B}{4 \times 10^8 \text{G}} \right)^2$$

- Question

- If we consider the interaction around BHs, what happens ?



$k_a A \not\approx \text{threshold}$

# Outline

## 1.Introduction

## 2.Known fact

- Simple toy model (Sen, (2018))
- BLAST of light from axion cloud (J.G.Rosa et al(2018))

## 3.Our work

- Formulation & Initial data
- Flat space
- Around Kerr BH
- Supper-radiance effect

## 4.Summary

# Simple toy model

- EM field grows exponentially under spatially uniform coherent oscillating axion field in flat space. (Sen(2018))
  - Maxwell equation with uniform coherent oscillating scalar field

$$\nabla_\mu F^{\mu\nu} = 2k_a \tilde{F}_{\nu\mu} \nabla^\mu \underline{\Phi}$$

$$\Phi = \Phi_0 e^{-i\mu t} + \Phi_0^* e^{i\mu t}$$

$\mu$  : mass of scalar field

- We use following ansatz

$$A_\mu(\vec{x}, t) = \frac{1}{2\sqrt{V}} \sum_{\vec{k}} \left( \underline{\alpha_\mu(\vec{k}, t)} e^{i(\vec{k} \cdot \vec{x} - \omega_{\vec{k}} t)} + \underline{\alpha_\mu^*(\vec{k}, t)} e^{-i(\vec{k} \cdot \vec{x} - \omega_{\vec{k}} t)} \right)$$

- Coupled ordinary differential eqs. for transverse modes

$$\left\{ \begin{array}{l} -\ddot{\tilde{\alpha}}_{(1)}(\vec{k}, t) + i2\omega_{\vec{k}} \dot{\tilde{\alpha}}_{(1)}(\vec{k}, t) + m_a k_a |\vec{k}| \tilde{\alpha}_{(2)}^*(-\vec{k}, t) \left( \Phi_0 e^{i(2\omega_{\vec{k}} - m_a)t} - \Phi_0^* e^{i(2\omega_{\vec{k}} + m_a)t} \right) = 0 \\ -\ddot{\tilde{\alpha}}_{(2)}(-\vec{k}, t) + i2\omega_{\vec{k}} \dot{\tilde{\alpha}}_{(2)}(-\vec{k}, t) + m_a k_a |\vec{k}| \tilde{\alpha}_{(1)}^*(\vec{k}, t) \left( \Phi_0 e^{i(2\omega_{\vec{k}} - m_a)t} - \Phi_0^* e^{i(2\omega_{\vec{k}} + m_a)t} \right) = 0 \end{array} \right.$$

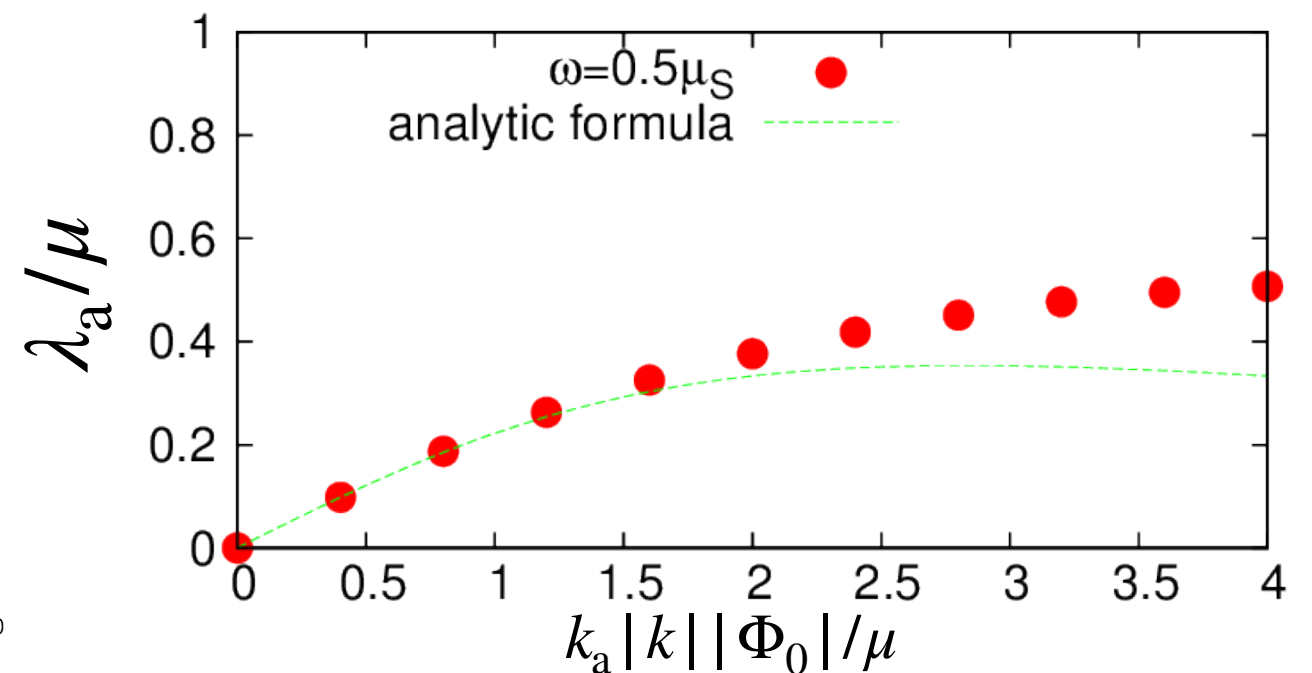
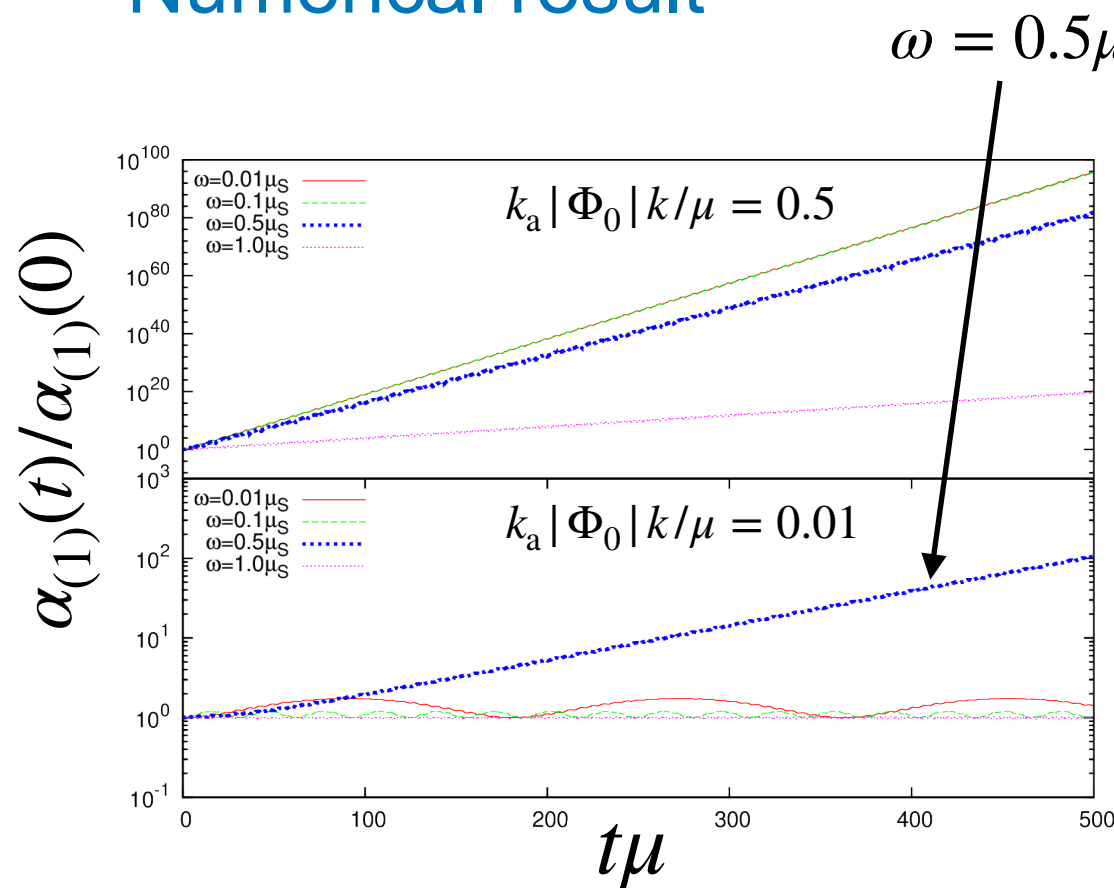
# Simple toy model

## ► We can show that

- the fastest growing mode :  $\omega = 0.5\mu$  for weak coupling
- $\tilde{\alpha}_{(I)}(t, \omega = 0.5\mu) \sim e^{\lambda_a t}$  for  $k_a |k| |\Phi_0| / \mu \ll 1$
- growth rate

$$\lambda_a = \frac{\mu\epsilon}{1 + \frac{1}{2}\epsilon^2} \quad \epsilon = k_a |k| |\Phi_0| \quad (\text{PRD99(2019)no.3,035006})$$

## ► Numerical result





# BLAST from axion cloud

- BLAST(Black hole Lasers powered by Axion Super-radianT instabilities) J.G.Rosa et al(2018)

- From Boltzmann equation (for 2p state)

$N_\phi$  : number of axions

$N_\gamma$  : number of photons

$\Gamma_\phi$  : spontanious axion decay width

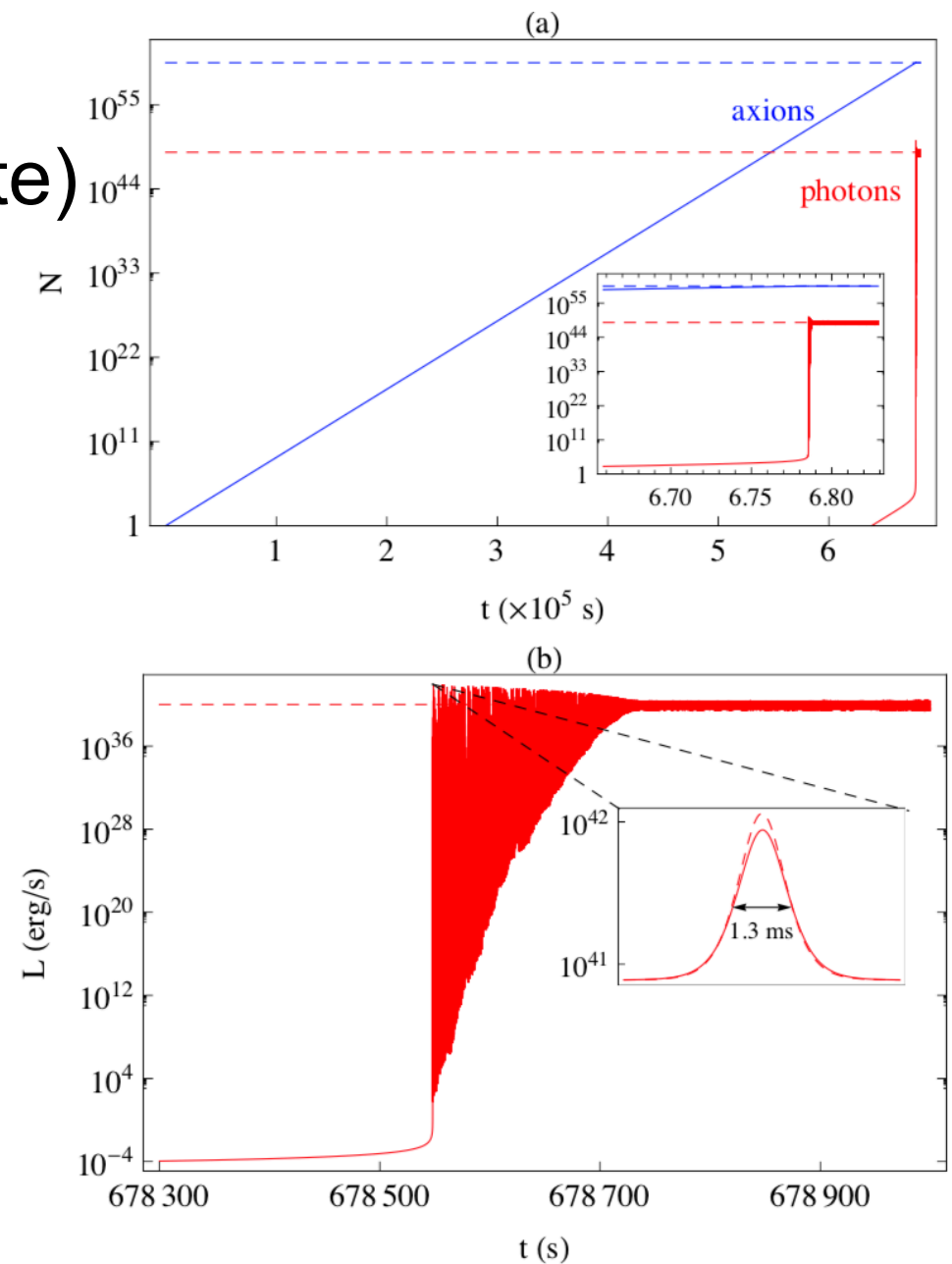
$$\frac{dN_\phi}{dt} = \Gamma_s N_\phi - \Gamma_\phi \left( N_\phi (1 + A N_\gamma) - B_1 N_\gamma^2 \right)$$

↑  
super-radiance  
effect

↑  
from interaction

$$\frac{dN_\gamma}{dt} = -\Gamma_e N_\gamma + 2\Gamma_\phi \left( N_\phi (1 + A N_\gamma) - B N_\gamma^2 \right)$$

↑  
escape  
from cloud



- They predicted bright laser of photon from axion cloud around BHs.

# What we want to do

- Summary of known fact
  - **Spatially uniform coherent** oscillating axion field induces the exponential growth of EM field in flat space (Sen, 2018)
  - The laser like emission of EM field from axion cloud is predicted by solving **Boltzmann eq.** (J.G.Rosa et al 2018)
- What we want to do is
  - **solving Klein-Gordon equation and Maxwell equation** with the interaction around Kerr background.

$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \frac{k_a}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_a\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$

- discussing the burst of EM field from axion cloud.

# Outline

## 1.Introduction

## 2.Known fact

- Simple toy model (Sen, (2018))
- BLAST of light from axion cloud (J.G.Rosa et al(2018))

## 3.Our work

- Formulation & Initial data
- Flat space
- Around Kerr BH
- Supper-radiance effect

## 4.Summary

# Formulation

- We ignore dynamics of gravity sector.
- Equations

$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \frac{k_a}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_a\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$

- Formulation

- 3+1 formulation (with Z term)

$$\partial_t\Pi = \alpha(-D^2\Phi + \mu_s^2\Phi + K\Pi - 2k_aE^iB_i) - D^i\alpha D_i\Phi + \mathcal{L}_\beta\Pi$$

$$\partial_t\Phi = -\alpha\Pi + \mathcal{L}_\beta\Phi$$

$$\partial_t\mathcal{A}_i = -\alpha(E_i + D_i\mathcal{A}_\phi) - A_\phi D_i\alpha + \mathcal{L}_\beta\mathcal{A}_i$$

$$\partial_tE^i = \alpha(KE^i + D^iZ - (D^2\mathcal{A}^i - D_kD^i\mathcal{A}^k)) + 2\alpha k_a(+\epsilon^{ijk}E_kD_j\Phi + B^i\Pi) + \epsilon^{ijk}D_k\alpha B_j + \mathcal{L}_\beta E^i$$

$$\partial_tA_\phi = \alpha(KA_\phi - D_i\mathcal{A}^i - Z) - \mathcal{A}_jD^j\alpha + \mathcal{L}_\beta A_\phi$$

$$D_iE^i + 2k_aB_iD^i\Phi = 0$$

$$\partial_tZ = \alpha(D_iE^i - \kappa Z) + 2k_a\alpha B_iD^i\Phi + \mathcal{L}_\beta Z$$

metric (Kerr-Schild form)

$$ds^2 = (\eta_{\mu\nu} + 2Hl_\mu l_\nu)dx^\mu dx^\nu$$

$$H = \frac{r^3 M}{r^4 + a^2 z^2}$$

$$l_\mu = \left(1, \frac{rx + ay}{r^2 + a^2}, \frac{-ax + ry}{r^2 + a^2}, \frac{z}{r}\right)$$

# Initial data

$k_a A_0$  : effective coupling for EM field

- Scalar field : axion cloud

$$\Phi = A_0 g(\tilde{r}) \cos(\varphi - \mu t) \sin \theta$$

$$\begin{cases} g(\tilde{r}) = \tilde{r} e^{-\tilde{r}/2} \\ \tilde{r} = r M \mu^2 \end{cases}$$

- EM field

- 1. extended profile

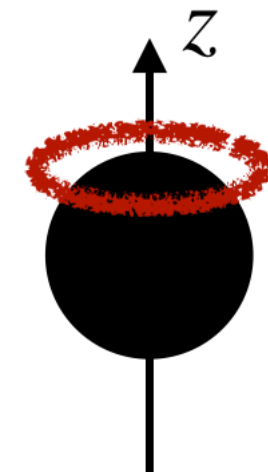
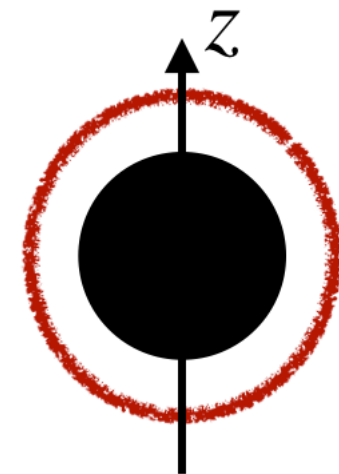
$$E^\varphi = E_0 e^{-(\frac{r-r_0}{w})^2} \quad E^r = E^\theta = B^i = 0$$

- 2. localized profile

$$E^\varphi = E_0 e^{-(\frac{r-r_0}{w})^2} \Theta(\theta) \quad E^r = E^\theta = B^i = 0$$

$$\Theta(\theta) = \begin{cases} \sin^4(4\theta) & (0 \leq \theta < \frac{\pi}{4}) \\ 0 & (\frac{\pi}{4} \leq \theta < \pi) \end{cases}$$

initial parameter :  $E_0, r_0, w$   
(These profile satisfy Gauss's law.)



- The result qualitatively does not change.

# Instability in flat space

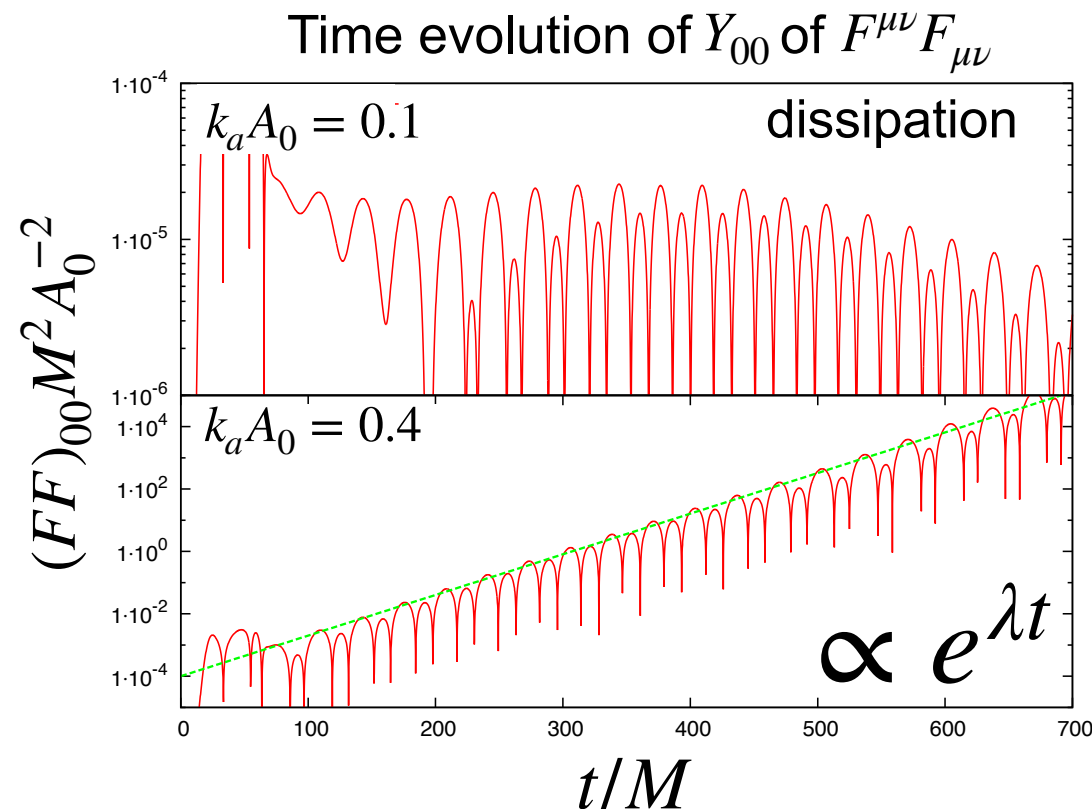
$$(FF)_{00} = \int d\Omega F^{\mu\nu} F_{\nu\mu} Y_{00}$$

- EM field under the fixed axion cloud in flat space.

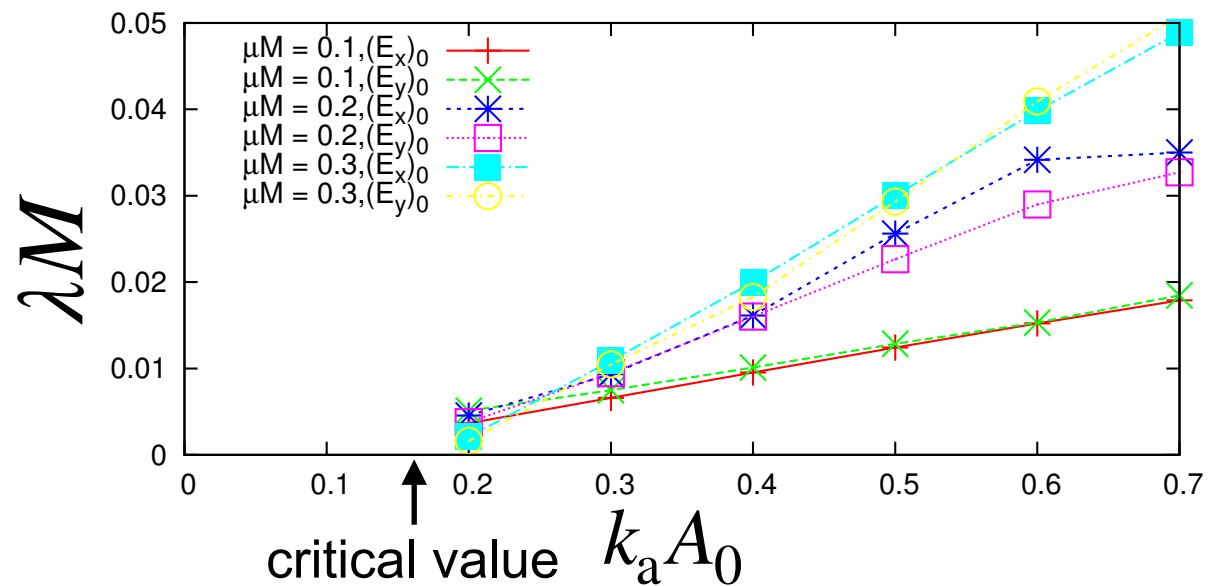
$$\nabla_\mu F^{\mu\nu} = 2k_a \tilde{F}_{\nu\mu} \nabla^\mu \Phi$$

axion cloud :  $\Phi = A_0 g(\tilde{r}) \cos(\varphi - \mu t) \sin \theta$

- When the coupling is larger than a certain value, the EM field grows exponentially.



Relation between  $\lambda M$  and  $k_a A_0$  for each mass  $\mu M$



$$\lambda \simeq \lambda_* \langle \Phi \rangle - \lambda_\gamma$$

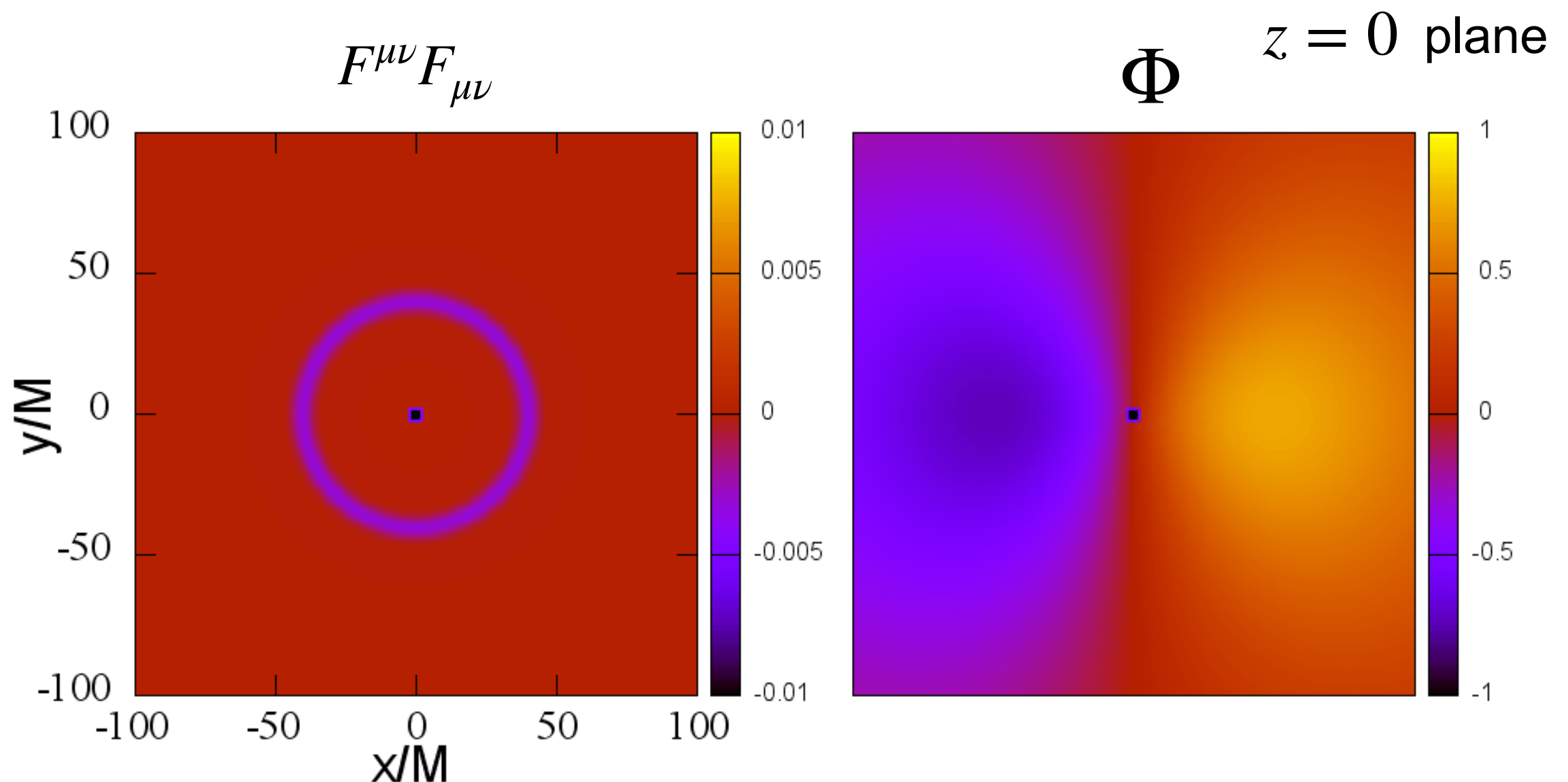
estimate from  
homogeneous  
configuration

time scale for  
photon to leave  
the cloud

# Around Kerr BH

- Dissipation case (Extended initial profile)  $\left\{ \begin{array}{l} (\nabla^2 - \mu^2)\Phi = \frac{k_a}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_a\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{array} \right.$   
 $\mu M = 0.2, \quad k_a A_0 = 0.1$

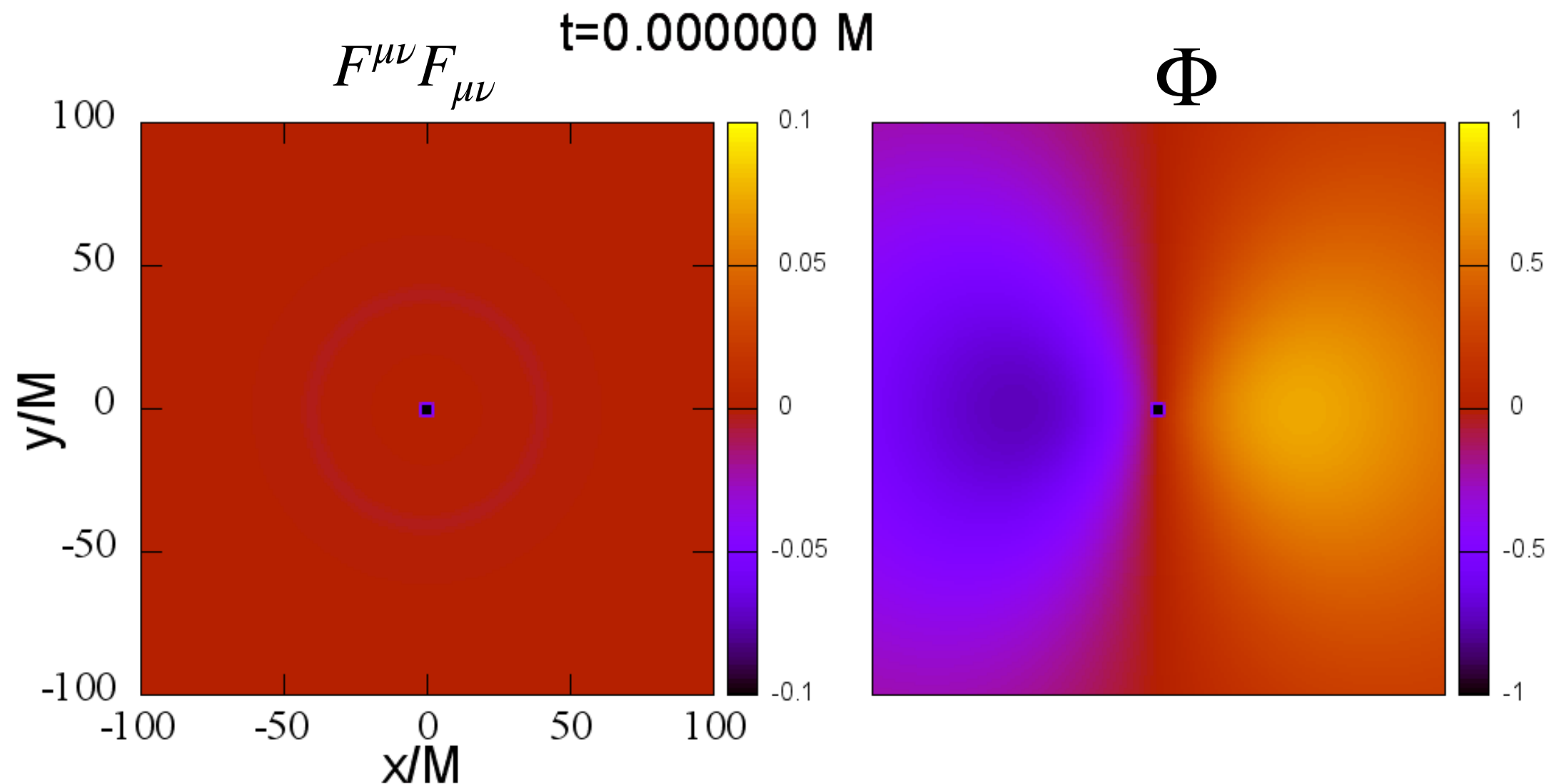
$t=0.000000 \text{ M}$



# Around Kerr BH

- Burst case (Extended initial profile)
 
$$\mu M = 0.2, \quad k_a A_0 = 0.3$$

$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \frac{k_a}{2} \tilde{F}_{\mu\nu} F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_a \tilde{F}_{\nu\mu} \nabla^\mu \Phi \end{cases}$$



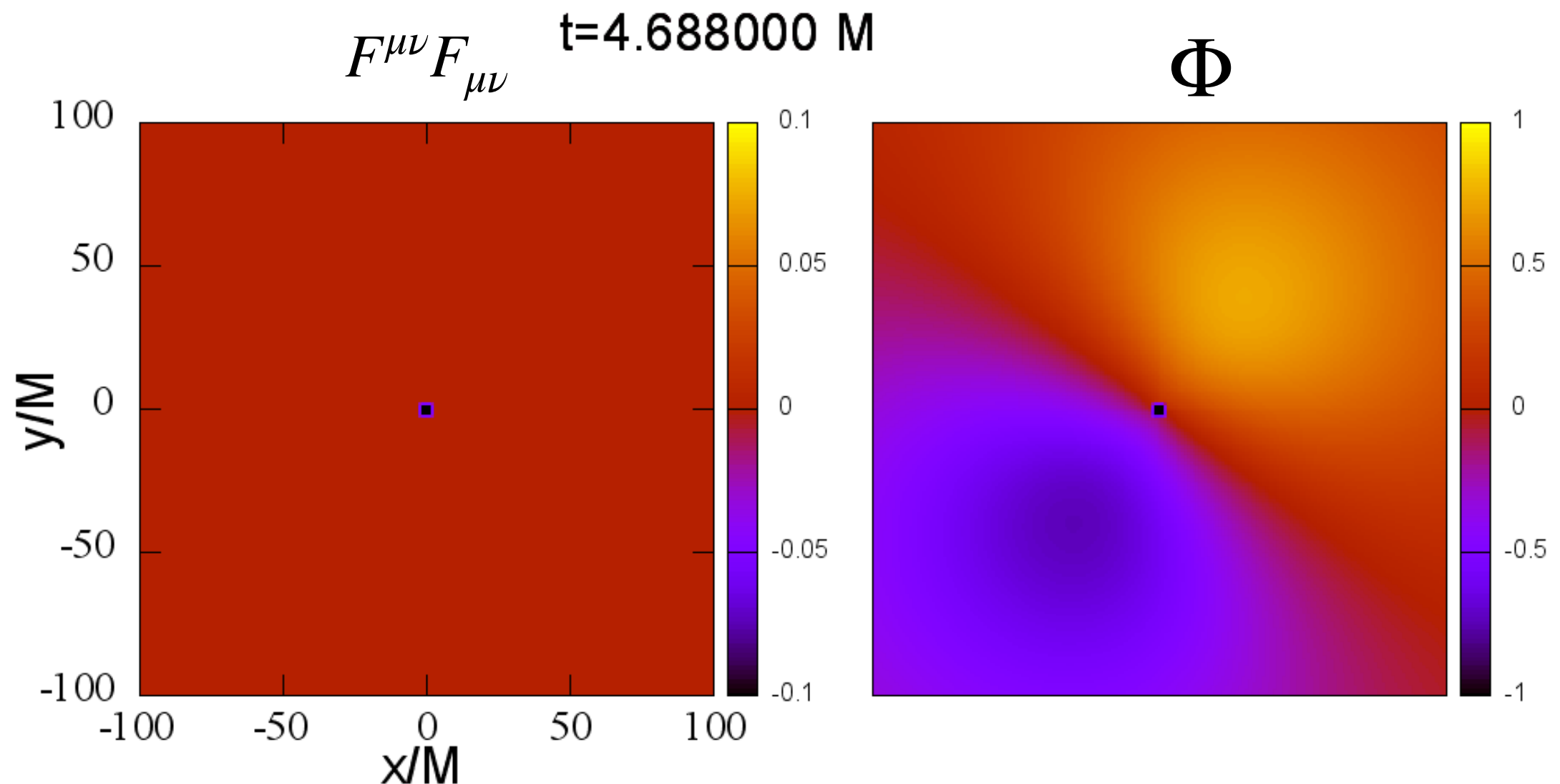


# Around Kerr BH

- Burst case (Localized initial profile)

$$\mu M = 0.2, \quad k_a A_0 = 0.4$$

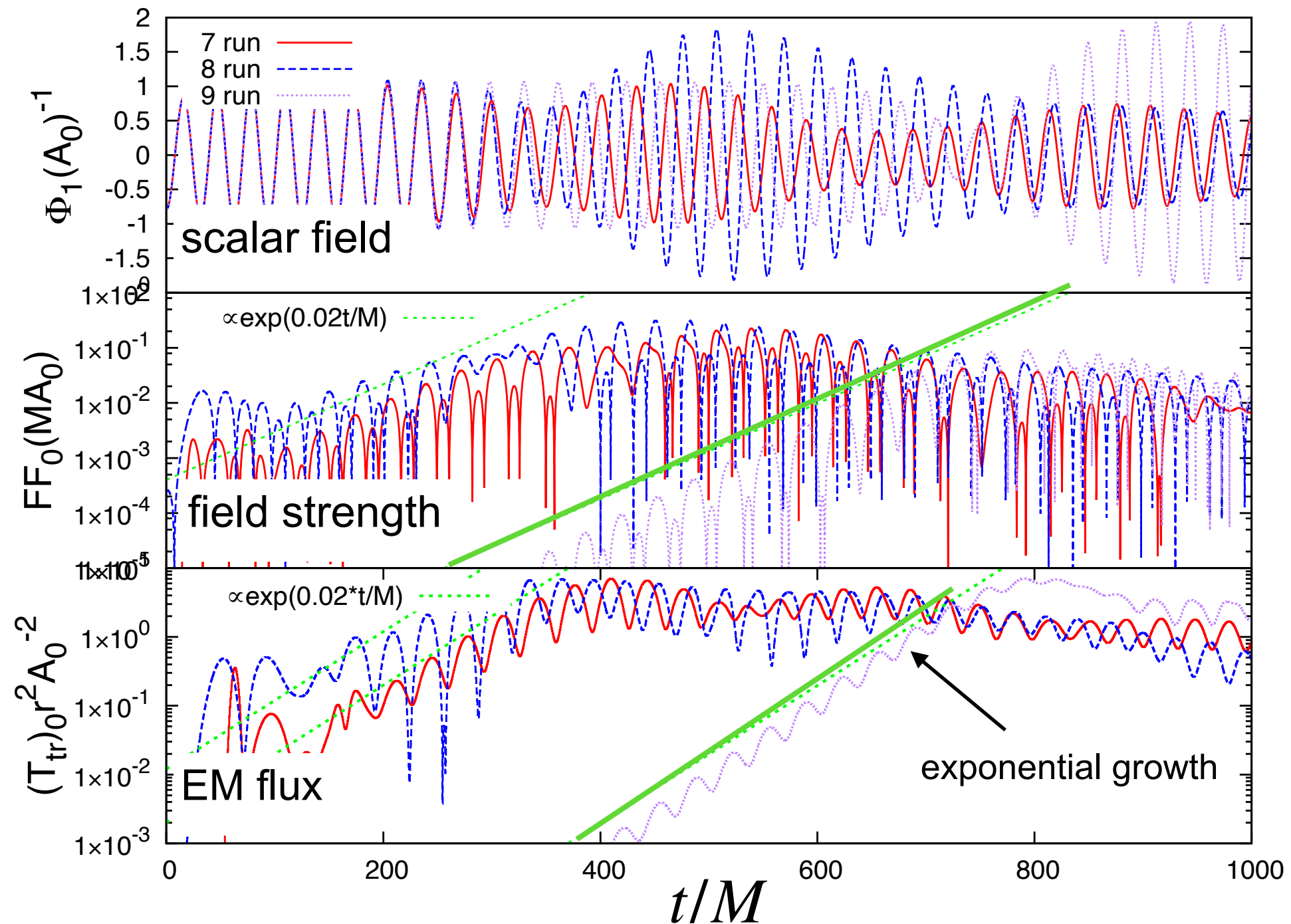
$$\begin{cases} (\nabla^2 - \mu^2)\Phi = \frac{k_a}{2}\tilde{F}_{\mu\nu}F^{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 2k_a\tilde{F}_{\nu\mu}\nabla^\mu\Phi \end{cases}$$



# Around Kerr BH

$$\text{cf: } \Phi_1 = \int d\Omega \Phi \frac{1}{2} (Y_{1,1} + Y_{1,-1})$$

- Typical time evolution of the burst



# Around Kerr BH

- We could checked

- the exponential growth for several initial data for  $k_a A_0 \geq \text{threshold}$
- typical frequency of EM field :  $\omega = \mu/2$

- Scenario

- 1.Initial EM pulse dissipates.
- 2.EM field grows exponentially.
- 3.Energy of EM field propagates as radiations.
- 4.Energy of scalar field decrease, and new coupling is below the threshold.

- We get

- luminosity formula:  $\frac{dE}{dt} = 5.0 \times 10^{-6} \left( \frac{M_S}{M} \right) \frac{c^5}{G}$

$$\begin{cases} M & : \text{BH mass} \\ M_S & : \text{total mass of axion cloud} \end{cases}$$

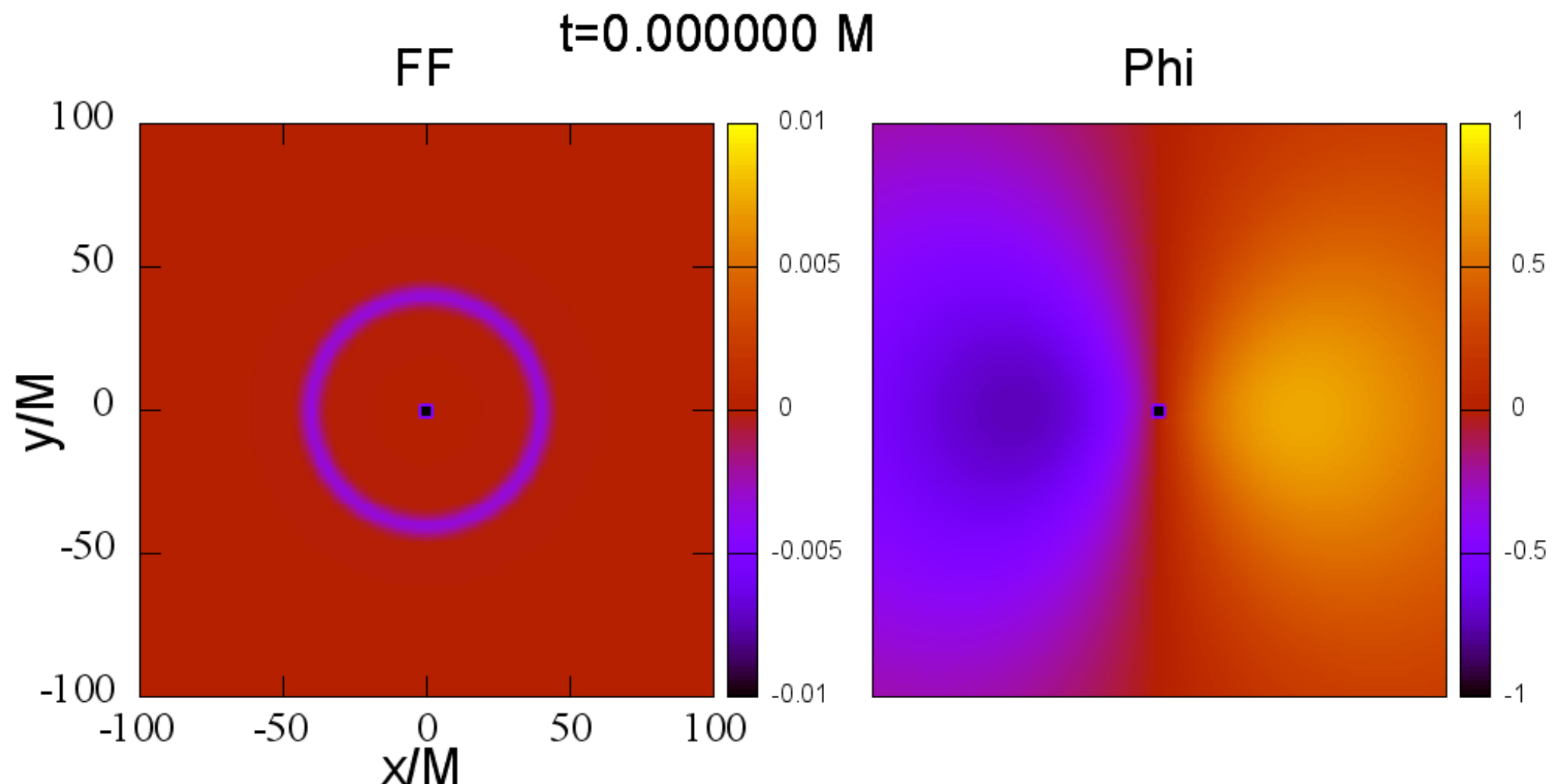
- threshold for the burst :  $\frac{\sqrt{\hbar}}{k_a} < 6 \times 10^{18} \left( \frac{M_S}{M} \right)^{1/2} (\mu M)^2 \text{ GeV}$

# Supper-radiance effect

- The burst is induced by super-radiance effect.
  - We add term to scalar field eq. which induces “super-radiant” like effect.

$$g^{\mu\nu} \nabla_\mu \nabla_\nu \Phi + \underline{C \frac{\partial \Phi}{\partial t}} = \mu^2 \Phi + \frac{k_a}{2} \tilde{F}_{\mu\nu} F^{\mu\nu}$$

“super-radiance” time scale  $\sim 1/C$



# Outline

## 1.Introduction

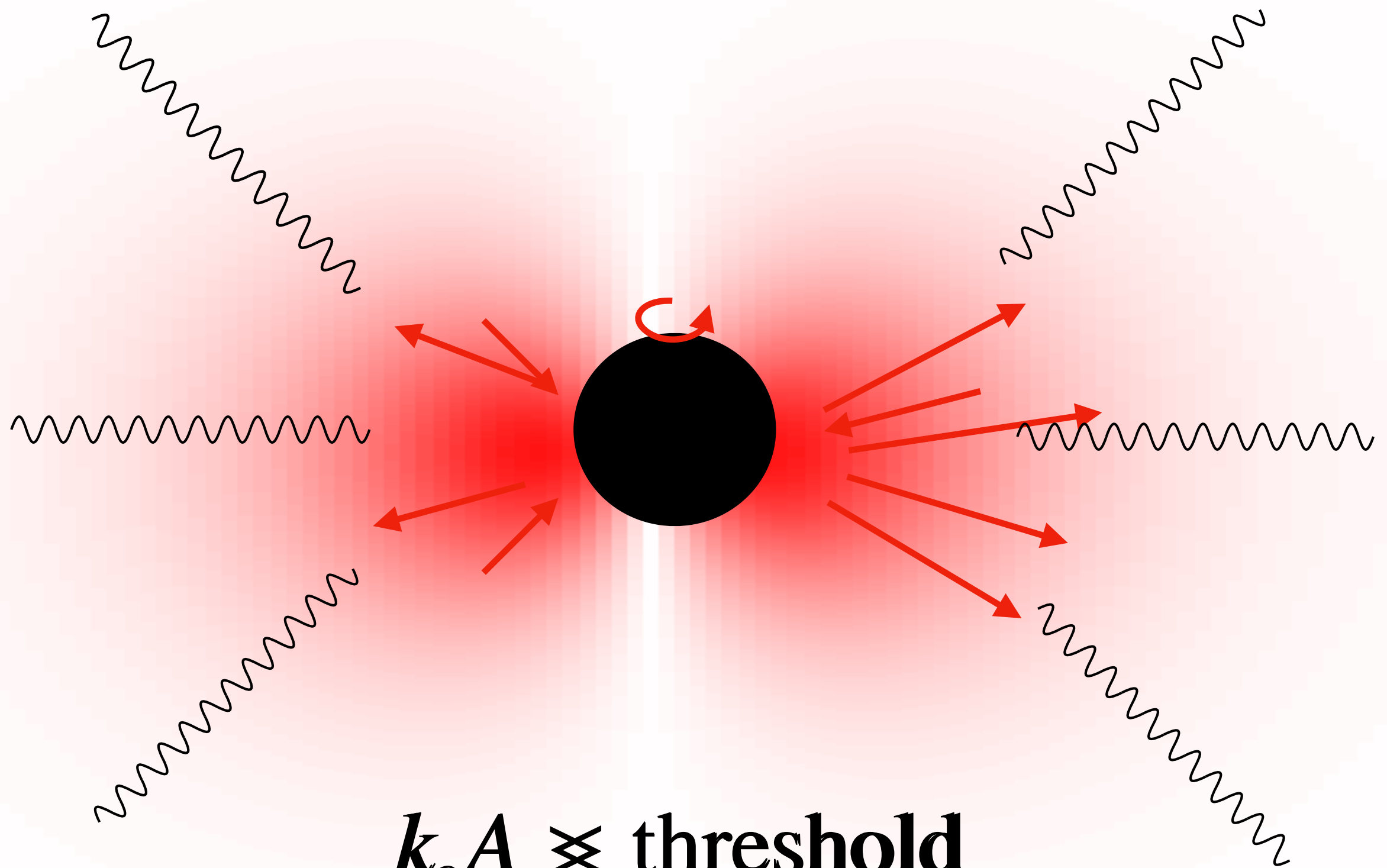
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## 4.Summary



**$k_a A \not\approx \text{threshold}$**

# Summary

- Result

- Energy of axion cloud transfers to EM field
- EM field grows exponentially, and photons emit from the axion cloud.

- Luminosity of burst

$$\frac{dE}{dt} = 5.0 \times 10^{-6} \left( \frac{M_S}{M} \right) \frac{c^5}{G}$$

$$\begin{cases} M & : \text{BH mass} \\ M_S & : \text{total mass of axion cloud} \end{cases}$$

- Threshold for axion couplings

$$\frac{\sqrt{\hbar}}{k_a} < 6 \times 10^{18} \left( \frac{M_S}{M} \right)^{1/2} (\mu M)^2 \text{ GeV} \quad \text{for } \mu M \sim 0.2$$

- The burst is induced by “super-radiance” instability.

# Summary

- We also get similar result in the case of scalar type coupling.

$$\mathcal{L}_s = -\frac{(k_s\Phi)^p}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{(k_s\Phi)^p}{2}\left(\vec{B}^2 - \vec{E}^2\right)$$

- Future work

- Light scalar field can interact with other particle ?
  - coupling with fermions

$$\mathcal{L}_{\text{int}} \supset i\partial_\mu\Phi\bar{\psi}\gamma^\mu\gamma_5\psi, \quad i\Phi\bar{\psi}\psi$$

 Can fermions be emitted from the cloud ?



The background features a large, semi-transparent red circle centered on the page. Overlaid on this circle are four thin, grey, wavy lines that resemble stylized sound waves or scribbles. These lines are positioned diagonally, with two on the left and two on the right, framing the central text.

**Thank you.**