Overview

- Theory and Background (Andrea, 15m)
- **Properties and Taxonomy (Thomas, 12m)**
  - Skeletonization Properties
  - Taxonomy of Skeletons
  - Question (5m)
- Skeletonization Methods (Andrea, 12m)
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- Applications (Thomas, 10m)
- Conclusions (Andrea, 10m)
  - Questions (10m)

Properties of skeletonizations

Property	[CSM07]	[SYJT13]	[SJT14]	[SBdB15]
Homotopy	✓	✓	✓	✓
Invariance	✓	✓		
Thinness	✓	✓	✓	
Centeredness	✓	✓	✓	
Smoothness	✓	✓	✓	
Details	✓	✓		
Regularization	✓	✓	✓	✓
Reconstructibility	✓			
Scalability			✓	✓

skeleton(ization) properties and their coverage in previous surveys

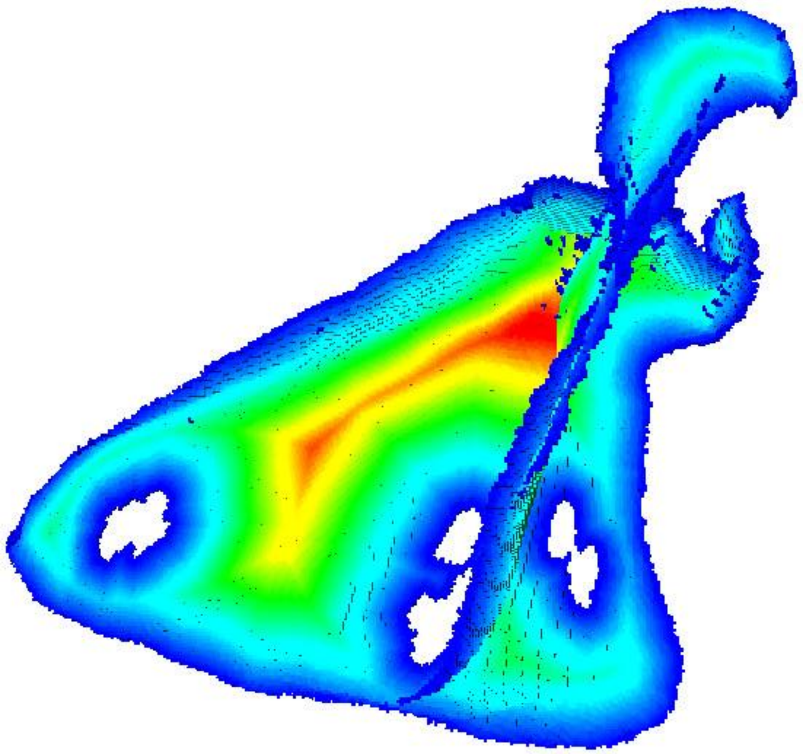
[Cornea et al. TVCG'13] Curve-Skeleton Properties, Applications and Algorithms
[Sobiecki et al. ISMM'13] A survey on voxel-based skeletonization algorithms and their applications
[Sobiecki et al. PRL'14] Comparison of curve and surface skeletonization methods for voxel shapes
[Saha PRL'15] A survey on skeletonization algorithms and their applications

Homotopy

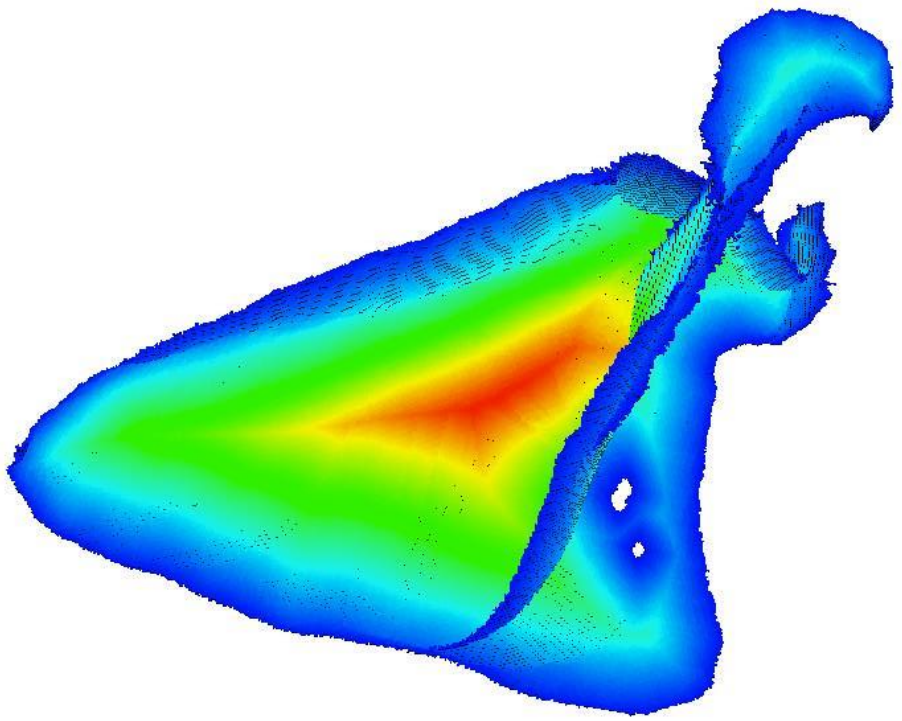
Practical skeletons should maintain the homotopy of their formal def.

- **disconnected parts** when regularization is too aggressive
- **tunnels** (dis)appear for low resolution input models [SYJT13,SJT13]

➞ defects affect topology-based analyses [SSGD03]



from lower resolution input shape



from higher resolution input shape

Invariance

For T an isometric transform, $MAT\big(T(O)\big) = T(MAT(O))$

- analytic methods (in $\mathbb{R}^3$) are **invariant**
- voxel-based methods **cannot be fully invariant**
    - especially true for chamfer distances
    - better for exact Euclidean distance transforms [MQR03,HR08]

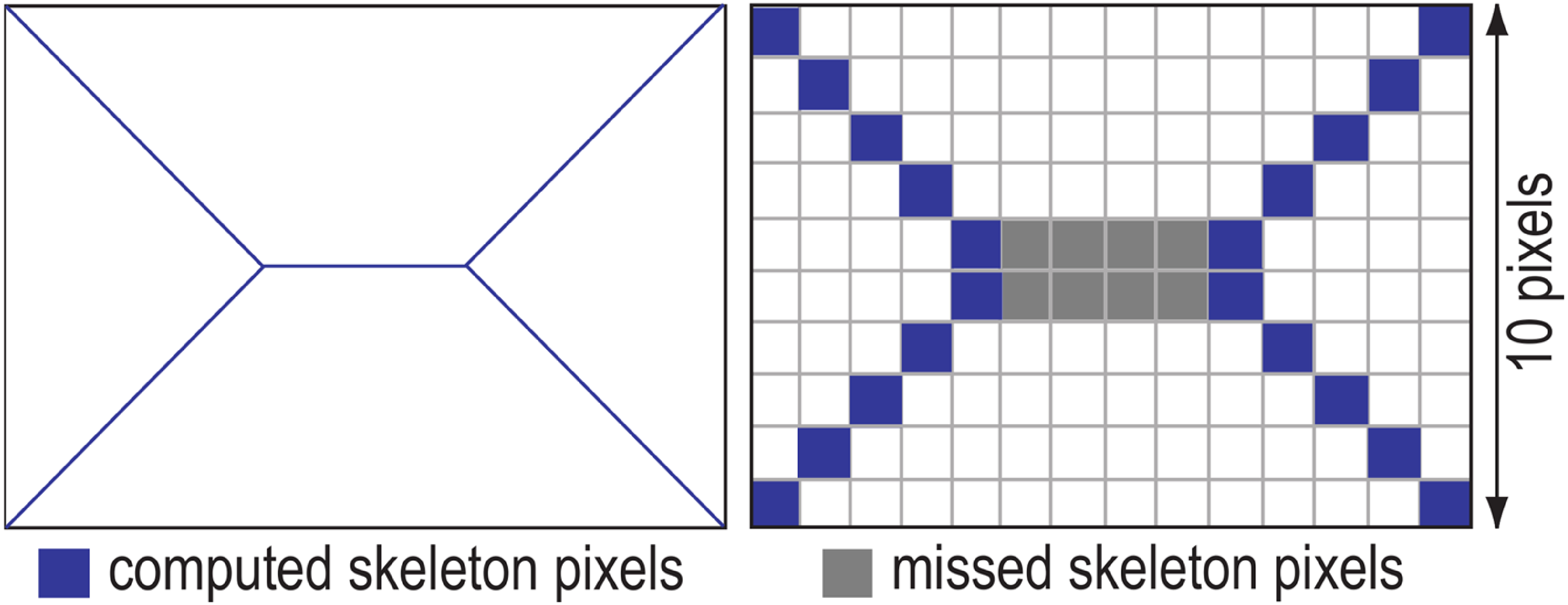
→ without invariance one needs to be careful about shape orientation

Thinness

Practical skeletons should be as thin as allowed by the space sampling

- mesh-based skeletons achieve zero-thickness
- issues for voxel-based skeletons
  - lower bounded by fixed grid resolution
  - conflicts with centeredness

➡ cannot use exact distance comparisons in Maxwell set definition

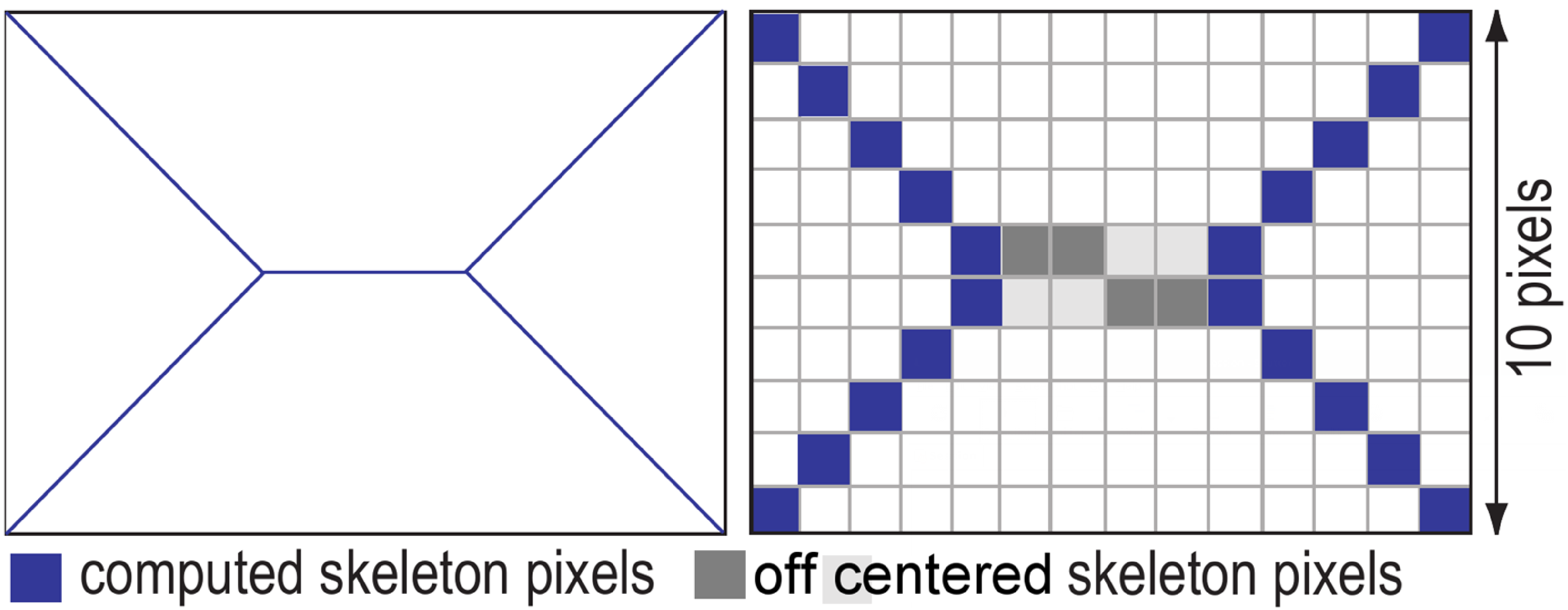


Centeredness

Skeleton points should be at equal distance from $n > 2$ surface points

- voxel-based skeletons **cannot be perfectly centered**
- no universally accepted definition for curve skeletons

➡ critical for shape reconstruction [ASS11] and metrology [JKT13]



Smoothness

Practical skeletons should be piecewise-smooth (C^2)

- **how to assess** when skeletons are smooth enough?
- limited by space sampling in $\mathbb{Z}^3$
- depends on the local surface point density in $\mathbb{R}^3$
- improved by filtering [ATC*08,HF09,JT12]

➔ unconstrained smoothness adversely affect centeredness

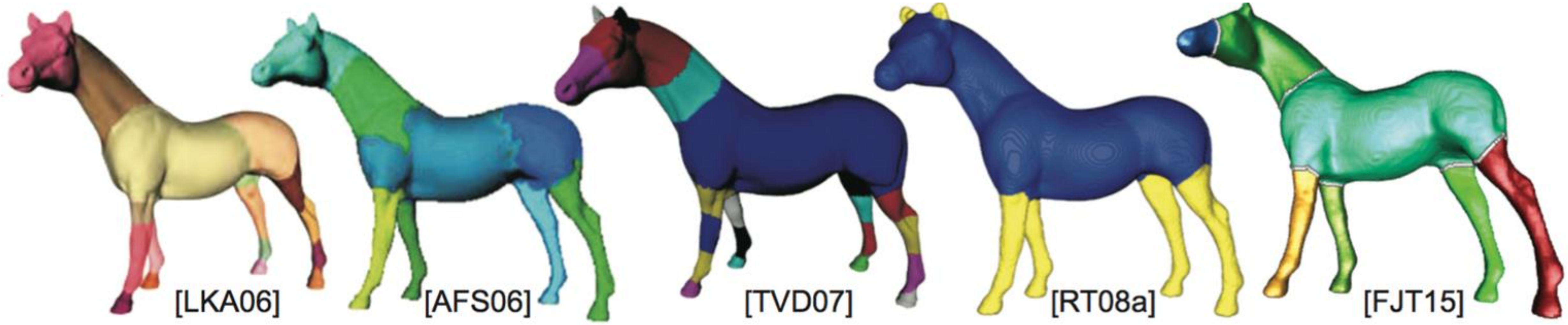
smoothness improved surface skeletons [JT12]

Detail Preservation

Practical skeletons should capture all shape topology and geometry

- detect junction, perform component-wise differentiation of input shape
- conflicts with semi-continuity/instability of the MAT
- distinction **shape details vs noise**?

➡ important for global shape matching, retrieval & reconstruction [CSM07,RvWT08a]



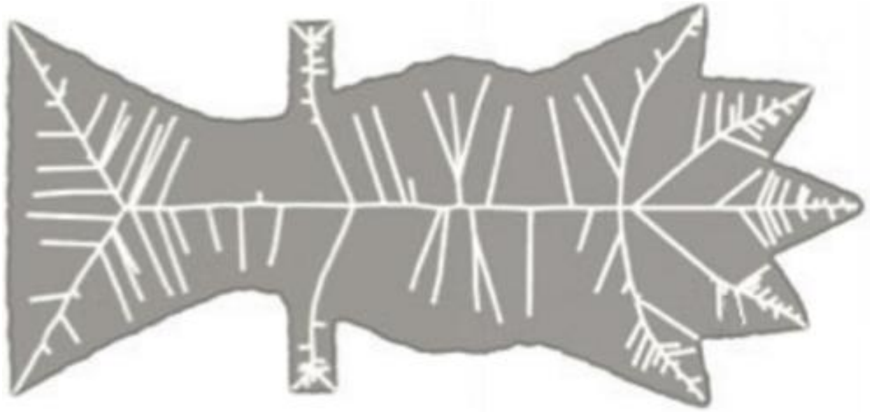
part-based shape segmentation using skeletons

Regularization

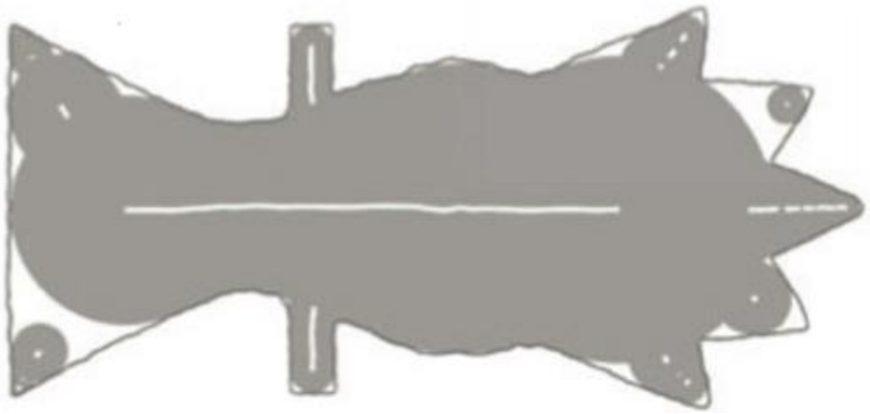
Key MAT’s challenge: sensitivity to small shape changes / noise

➡ regularization: removal of instability to make the MAT robust to noise

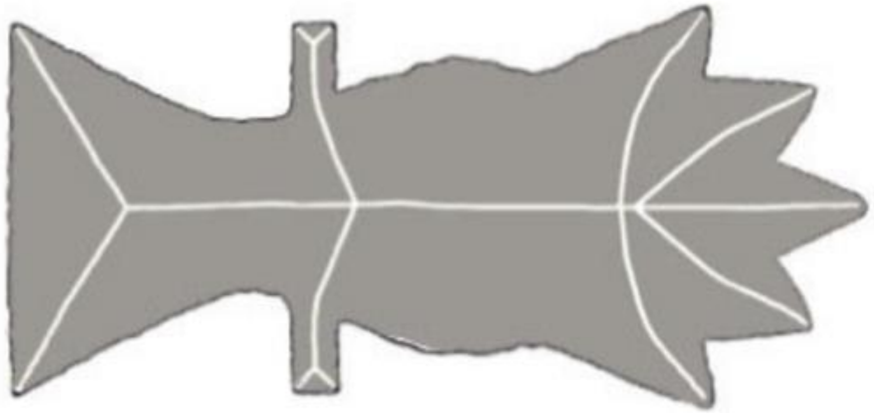
- local criteria [ACK01,HR08,FLM03,CL05a]
  - no way to separate locally identical, yet globally different, contexts
  - simple to compute, can disconnect skeletons
- global criteria [BGP10,DS06,RvWT08a]
  - measures monotonically increase from skeleton boundary inwards
  - thresholding measures preserves homotopy
- conflicts with detail preservation



MAT of noisy input



local regularization [CL05a] (λ-Axis)



global regularization [BGP10] (Scale Axis)

Reconstruction

In theory, we can exactly reconstruct a shape from its MAT, but:

- representation & computation approximations
- sampling limits
- regularization & smoothing filters

→ exact reconstruction is rarely possible

Input Shape

Reconstruction

Scalability & Speed

Need for **interactive & scalable** 3D skeletonizations [CSM07]

- Voronoi-based $O(n \cdot \log(n))$ for n shape samples*
- distance-based $O(\mathrm{T} \cdot \log\|S\|)$, $\|S\|$ shape boundary length, T average shape thickness [TvW02,FSL04]
- contraction & ball-inscription $O(n \cdot s)$ for n samples and s iterations [MBC12,JKT13]

Parallelizing **practical** skeleton detection operations

- e.g. ball inscription [MBC12,JKT13], distance transform [CTMT10]
- highly increase speed
- complex implementations

* **[Attali et al., SCG'03] Complexity of the delaunay triangulation of points on surfaces: the smooth case**

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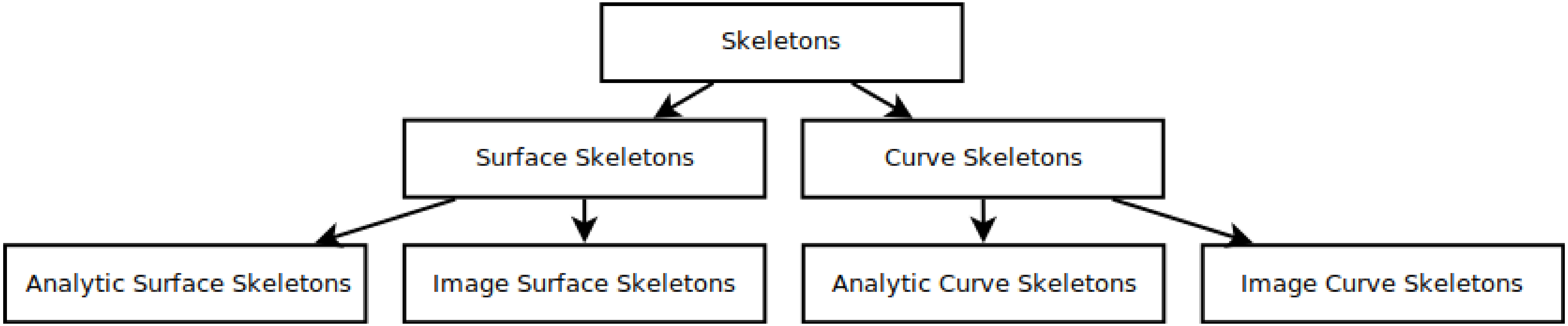
Taxonomy of Skeletons

Skeletonizations as a multidimensional attribute space

- points are skeletonization methods
- attributes describe how well a method complies with properties
- present such space via a taxonomy



Type of components

- curves only for Curve Skeleton
- surfaces as well for Surface Skeleton

Space sampling

- $\mathbb{R}^3$ sampling for Analytic Skeleton
- $\mathbb{Z}^3$ sampling for Image Skeleton