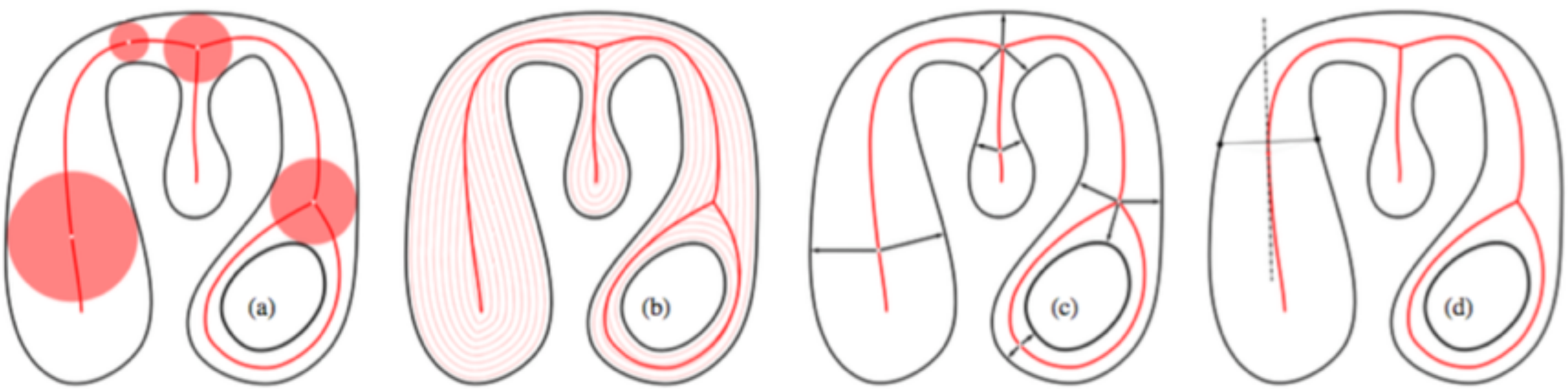
State of the Art Report

3D Skeletons



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http://gfx.uvic.ca/pubs/2016/skelstar/slides

Speakers

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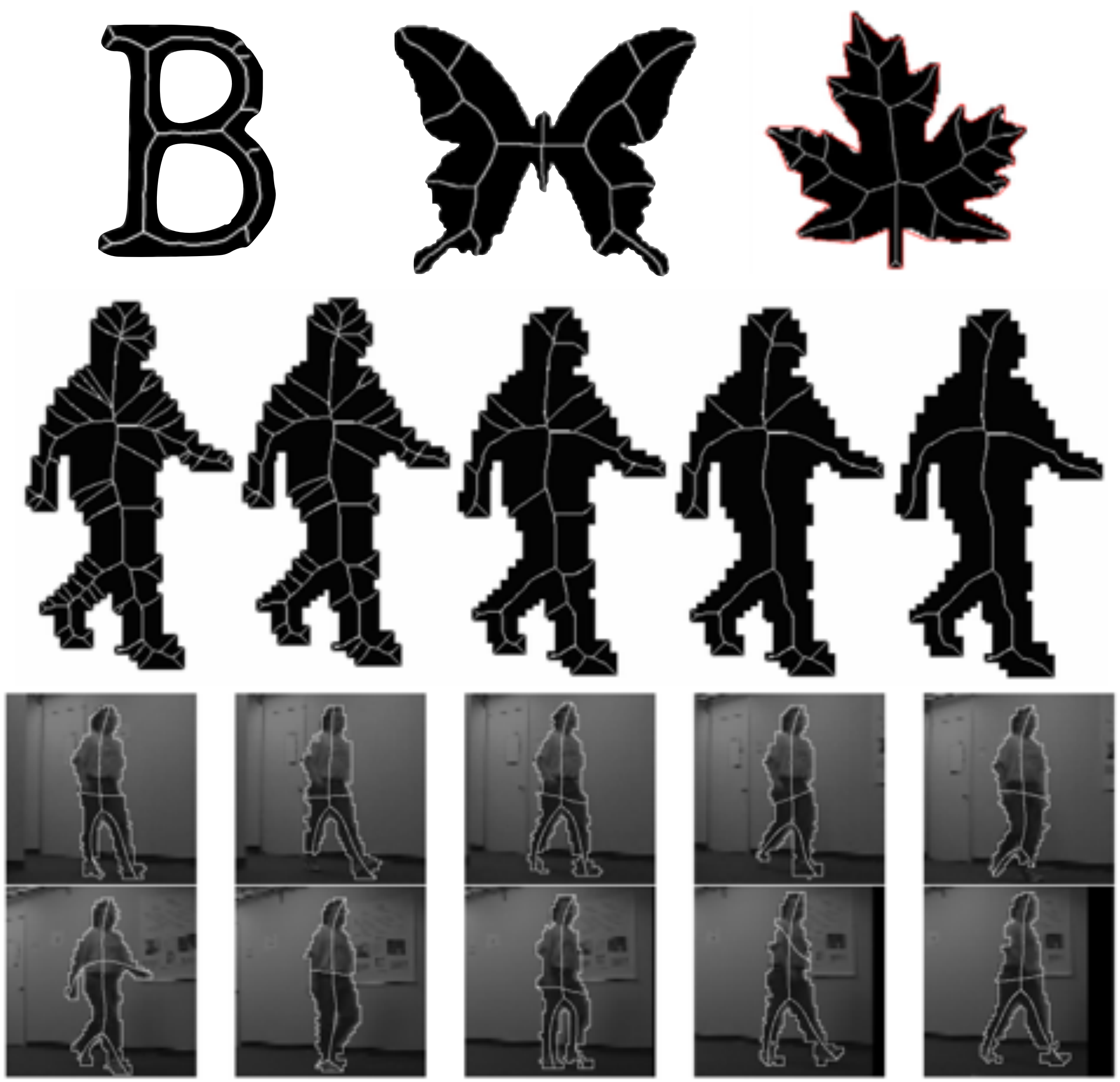
Why skeletons?

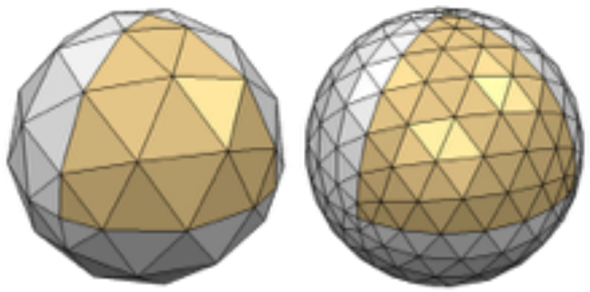
"I have approached the problem of shape by assuming the current mathematical tools were somehow missing essential elements of the problem. For despite more than two millennia of geometry, no formulation which appears natural for the biological problem has emerged"

Blum 1967

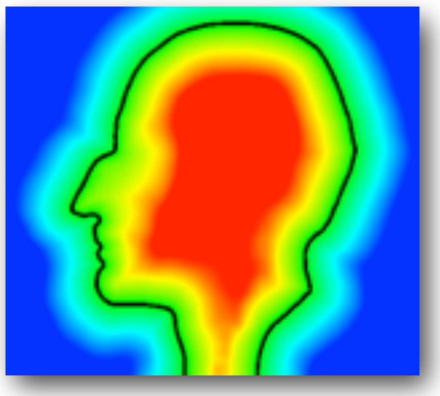


Geometry Representations

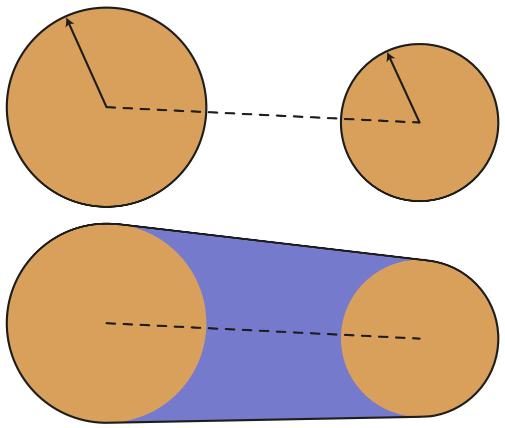
- **Explicit**
  - Encode the boundary
  - e.g. Bezier, triangular meshes



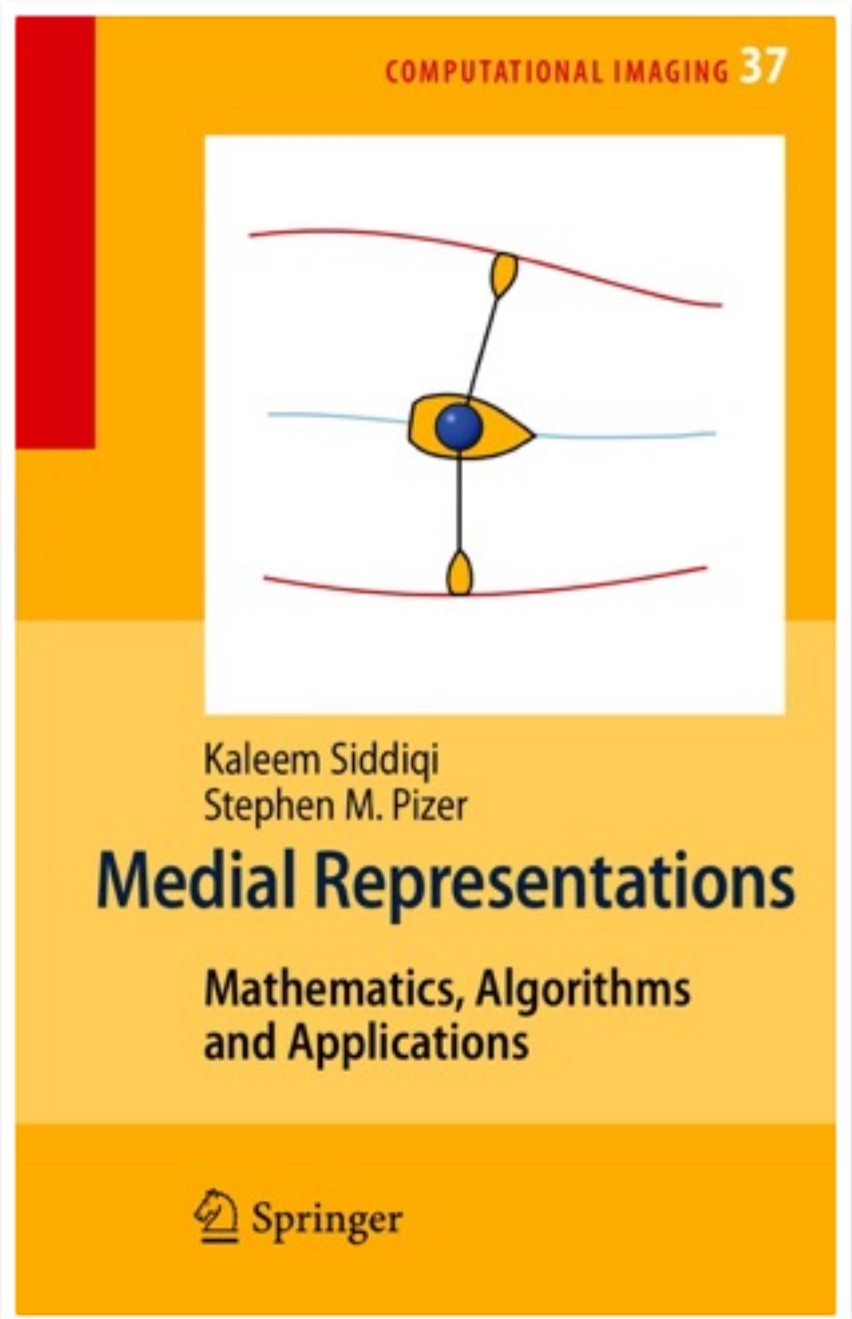
- **Implicit**
  - Encode the volume
  - e.g. signed distance, indicator

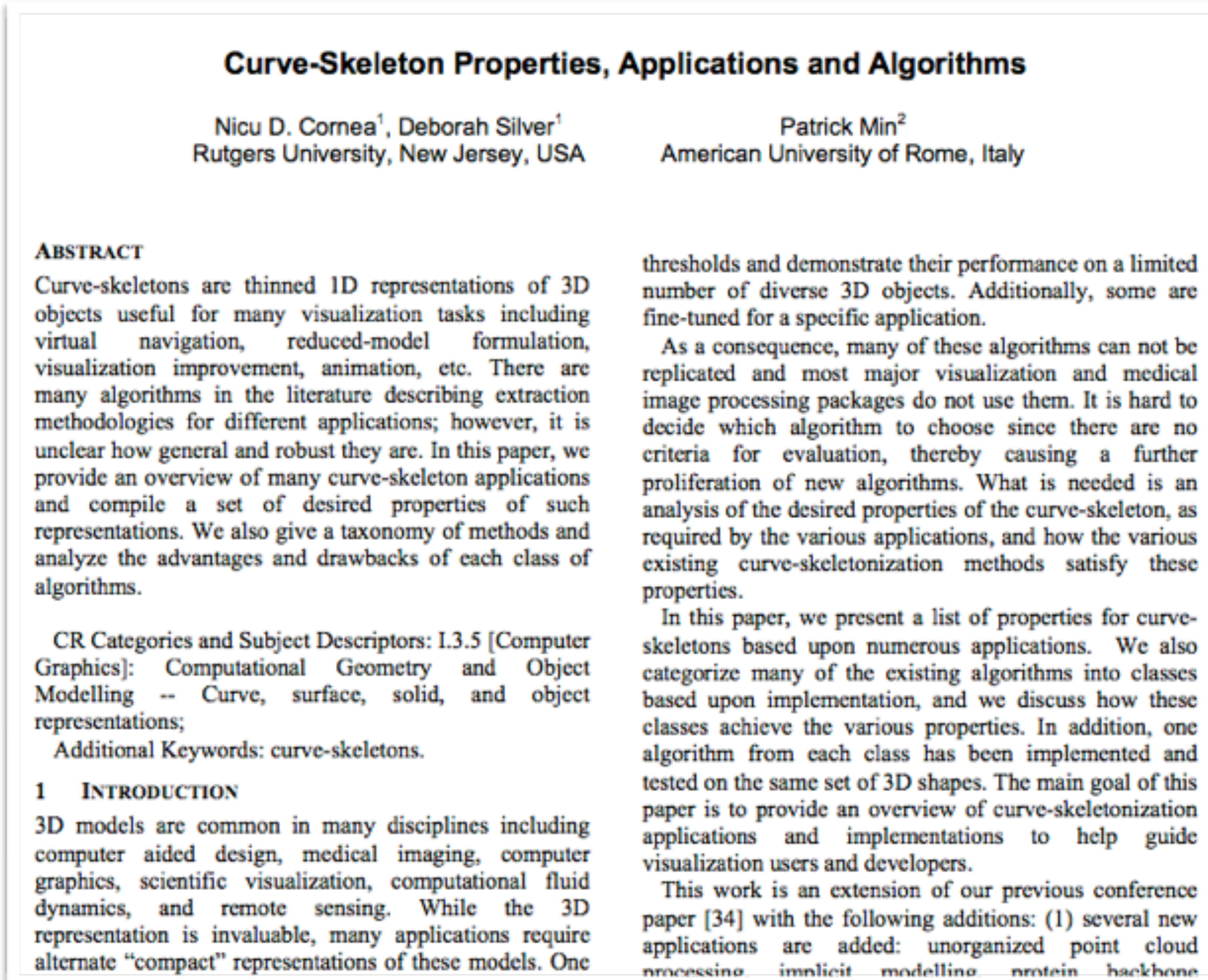


- **Hybrid**
  - mixture of implicit/explicit
  - e.g. sphere meshes, MAT



Some Existing Resources





Curve-Skeleton Properties, Applications and Algorithms

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ABSTRACT

Curve-skeletons are thinned 1D representations of 3D objects useful for many visualization tasks including virtual navigation, reduced-model formulation, visualization improvement, animation, etc. There are many algorithms in the literature describing extraction methodologies for different applications; however, it is unclear how general and robust they are. In this paper, we provide an overview of many curve-skeleton applications and compile a set of desired properties of such representations. We also give a taxonomy of methods and analyze the advantages and drawbacks of each class of algorithms.

CR Categories and Subject Descriptors: I.3.5 [Computer Graphics]: Computational Geometry and Object Modelling -- Curve, surface, solid, and object representations;

Additional Keywords: curve-skeletons.

1 INTRODUCTION

3D models are common in many disciplines including computer aided design, medical imaging, computer graphics, scientific visualization, computational fluid dynamics, and remote sensing. While the 3D representation is invaluable, many applications require alternate "compact" representations of these models. One thresholds and demonstrate their performance on a limited number of diverse 3D objects. Additionally, some are fine-tuned for a specific application.

As a consequence, many of these algorithms can not be replicated and most major visualization and medical image processing packages do not use them. It is hard to decide which algorithm to choose since there are no criteria for evaluation, thereby causing a further proliferation of new algorithms. What is needed is an analysis of the desired properties of the curve-skeleton, as required by the various applications, and how the various existing curve-skeletonization methods satisfy these properties.

In this paper, we present a list of properties for curve-skeletons based upon numerous applications. We also categorize many of the existing algorithms into classes based upon implementation, and we discuss how these classes achieve the various properties. In addition, one algorithm from each class has been implemented and tested on the same set of 3D shapes. The main goal of this paper is to provide an overview of curve-skeletonization applications and implementations to help guide visualization users and developers.

This work is an extension of our previous conference paper [34] with the following additions: (1) several new applications are added: unorganized point cloud

[Saha PRL'15] A survey on skeletonization algorithms and their applications **(only 2D)**
[Sobiecki et al. PRL'14] Comparison of curve and surface skeletonization methods for **voxel shapes**
[Sobiecki et al. ISMM'13] A survey on **[voxel-based]** skeletonization algorithms and their applications

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- Theory and Background (Andrea, 15m)
  - **Questions (5m)**
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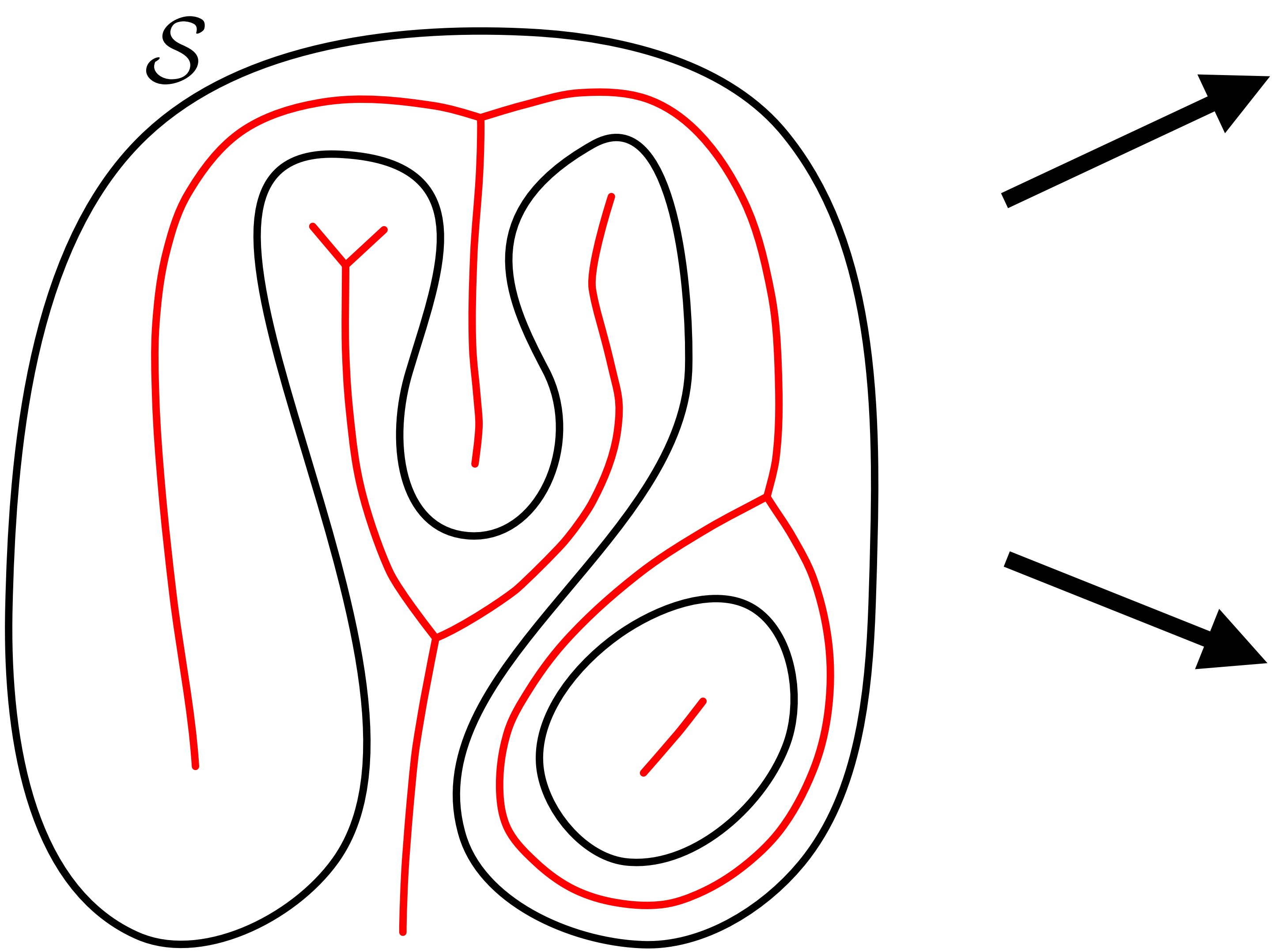
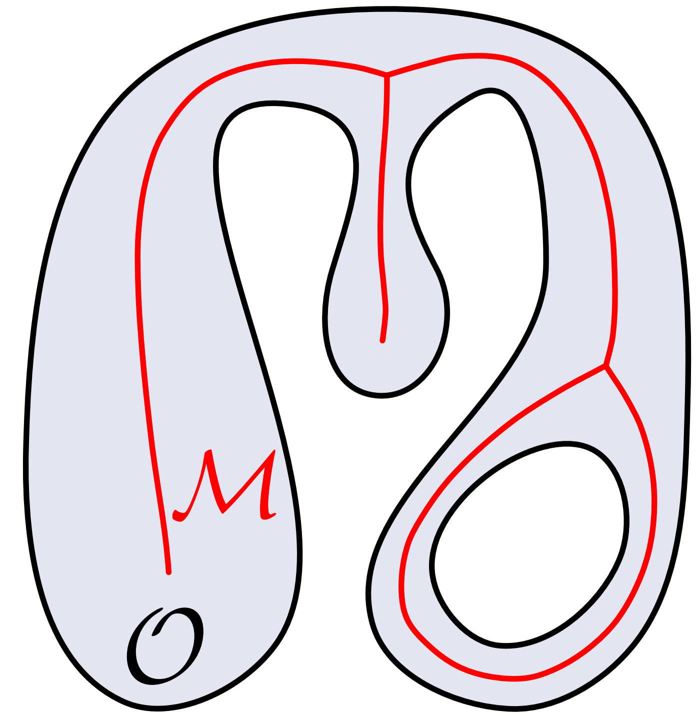
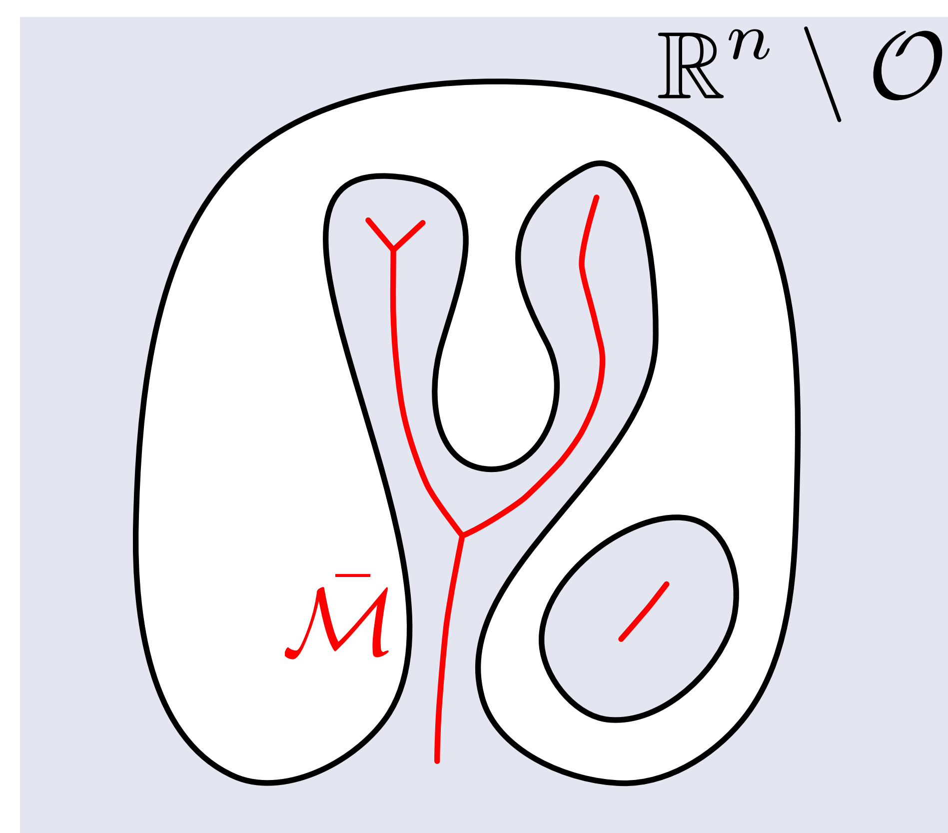
Foreground v.s. Background

- Solid object $\mathcal{O}$
- Boundary $\mathcal{S} = \partial\mathcal{O}$



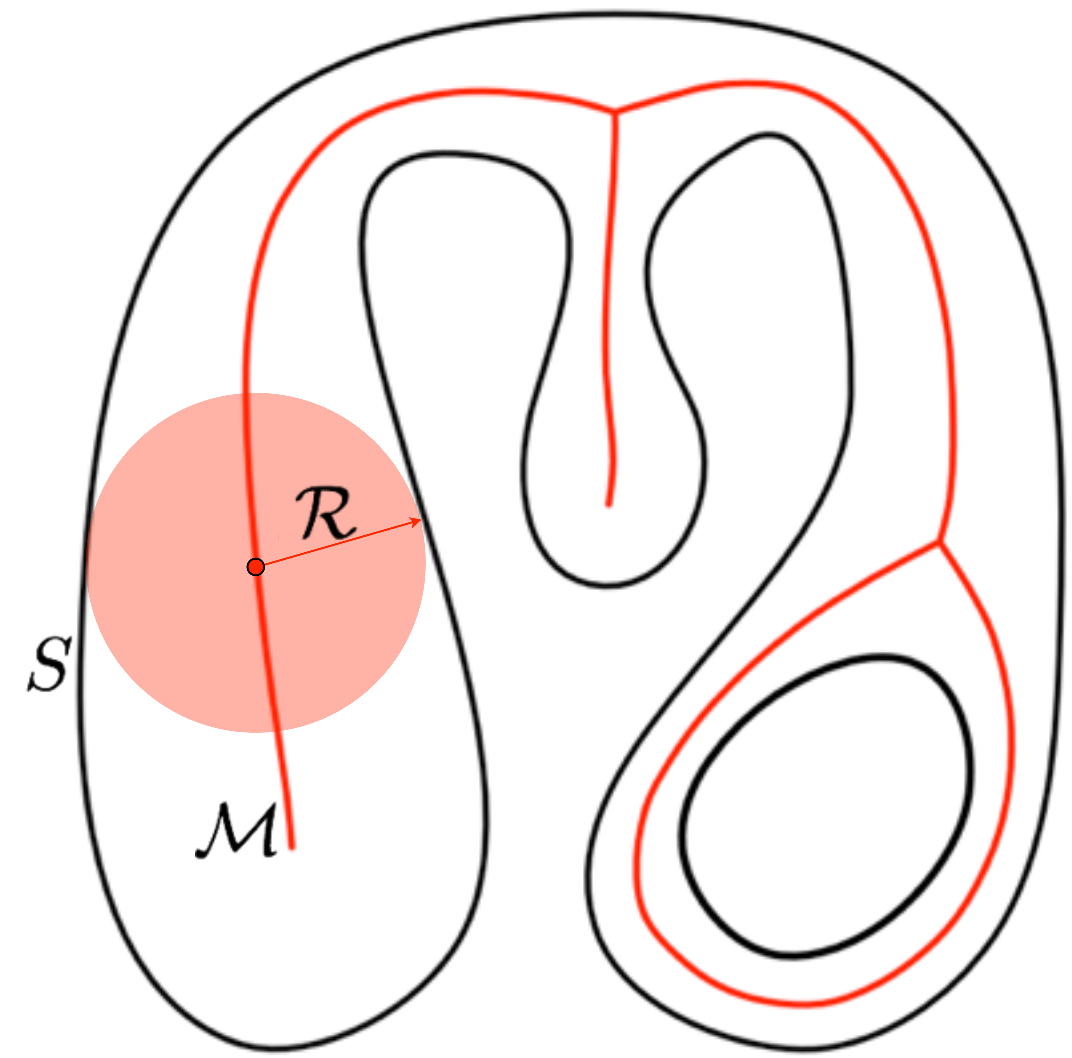




Skeletons in 2D

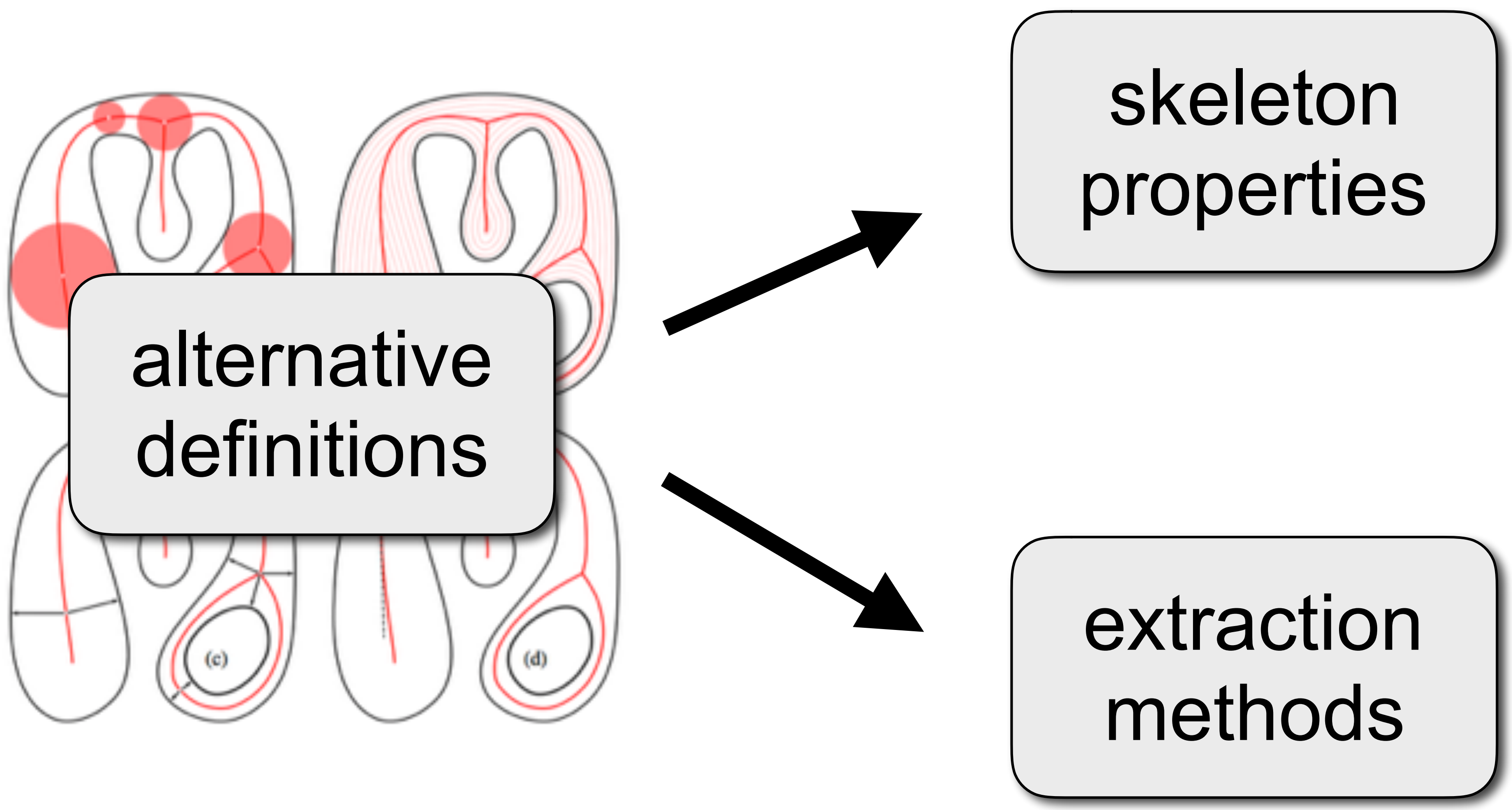
- The **M**edial **A**xis **T**ransform (MAT)
  - Medial: mid-shape
  - Axis: axial representation
  - Transform: invertible

$$\boxed{\mathcal{M}}, \boxed{\mathcal{R}} = MAT(S)$$



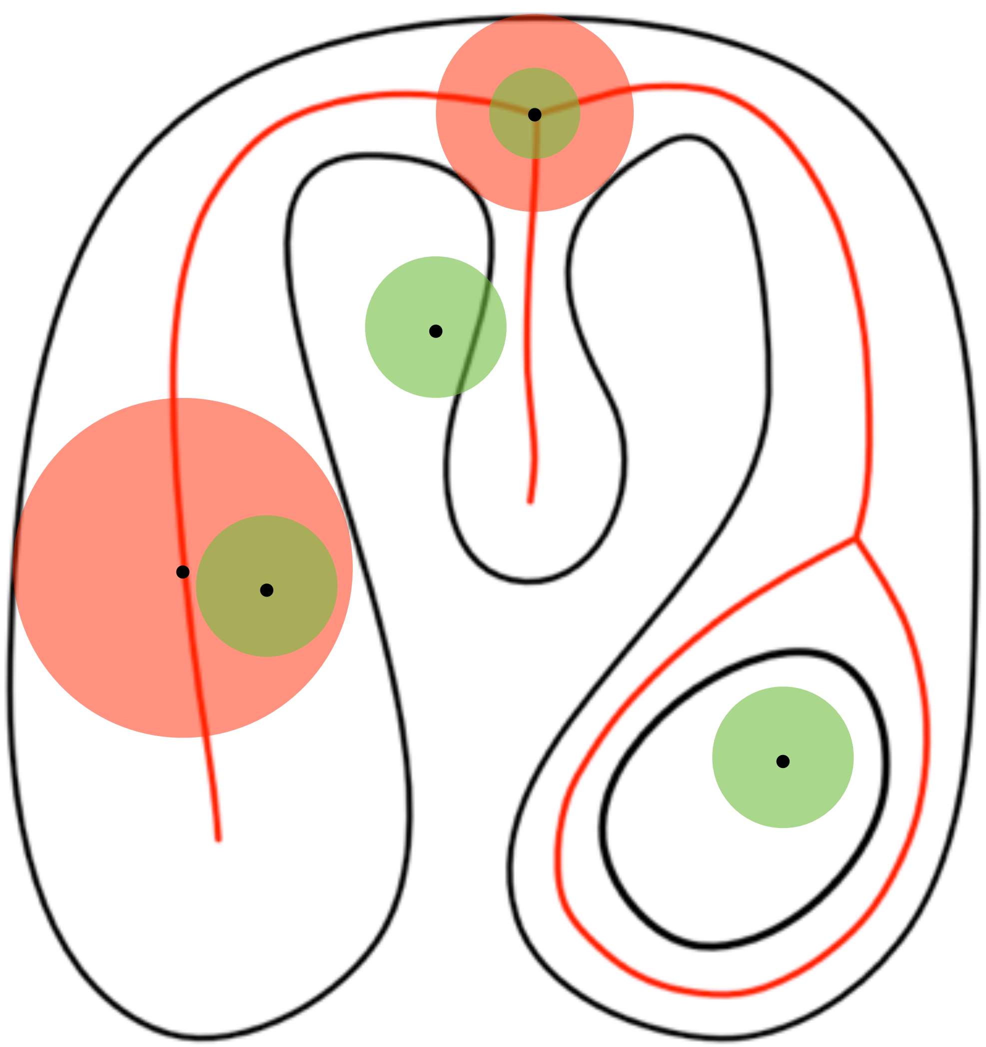
Medial Axis Transform

- Generalize the problem from 2D to 3D



Defining the MAT (1/4)

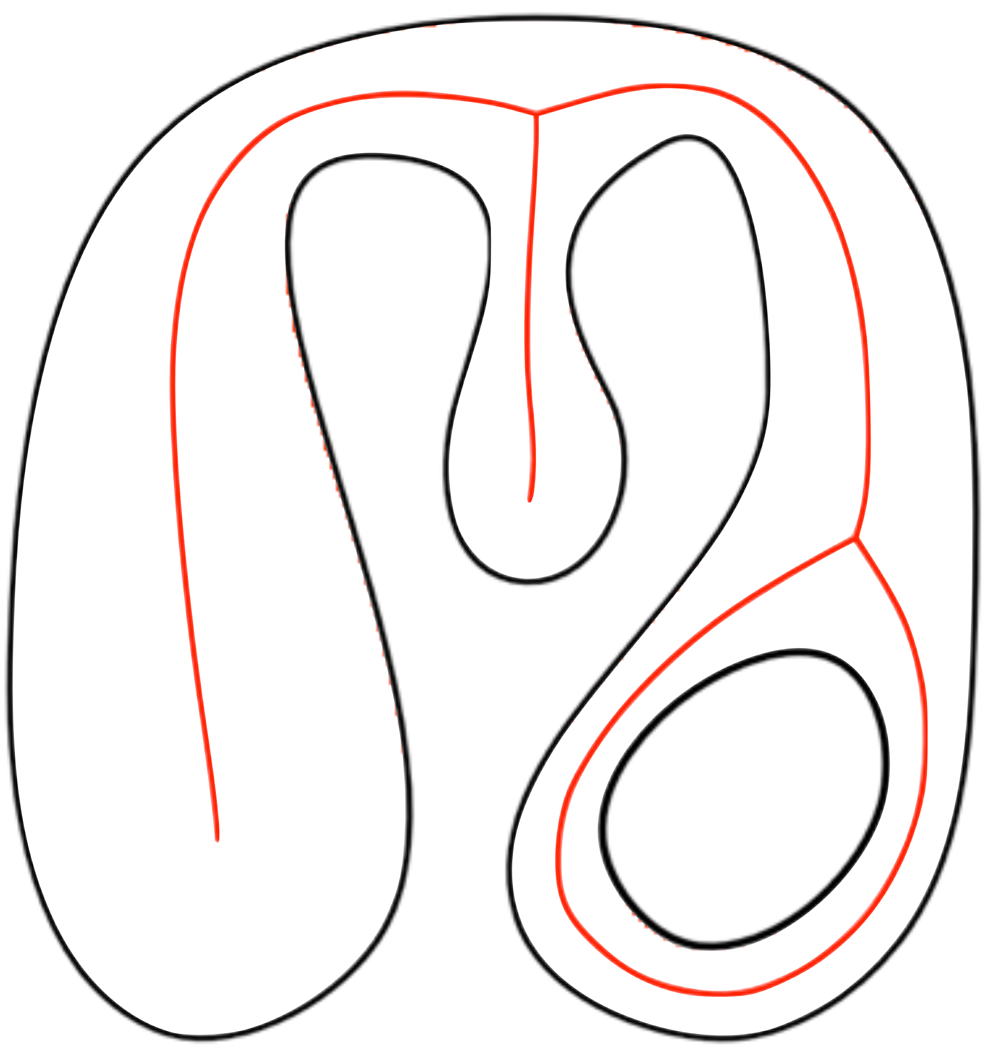
Definition 2.1 The Medial Axis Transform $MAT(\mathcal{O})$ of $\mathcal{O}$ is the set of centers $\mathcal{M}$ and corresponding radii $\mathcal{R}$ of all maximal inscribed balls in $\mathcal{O}$.

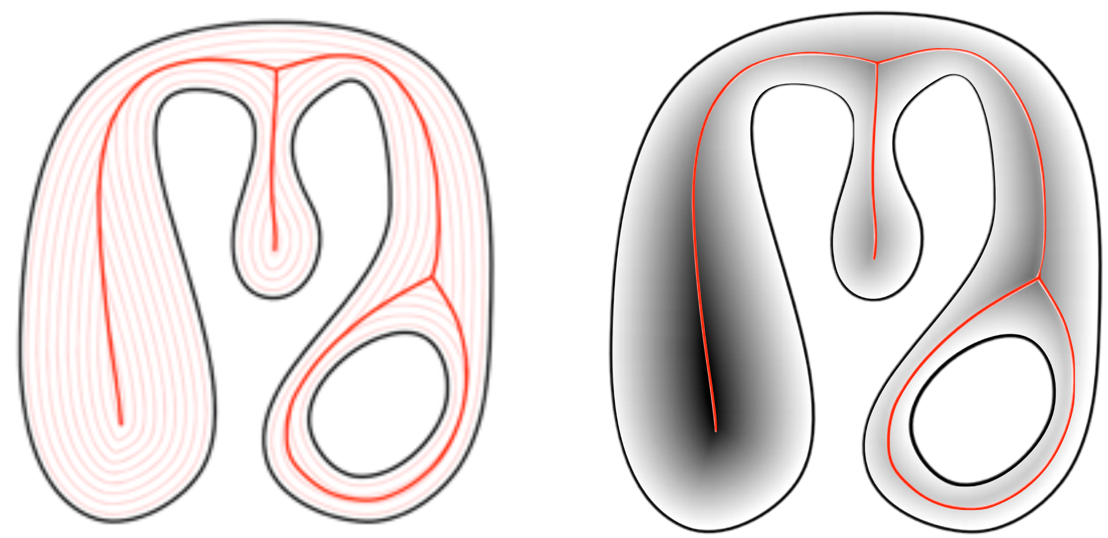


$$\mathcal{O} = \bigcup_{(\mathbf{x},r)\in \mathrm{MAT}(\mathcal{O})} B(\mathbf{x},r)$$

Defining the MAT (2/4)

Definition 2.2 The Medial Axis Transform of $\mathcal{O}$ with boundary $\mathcal{S}$ is given by the shock graph of the motion $\dot{\mathcal{S}}(t) = -\mathbf{n}(t)$ and the time t when a shock is formed.





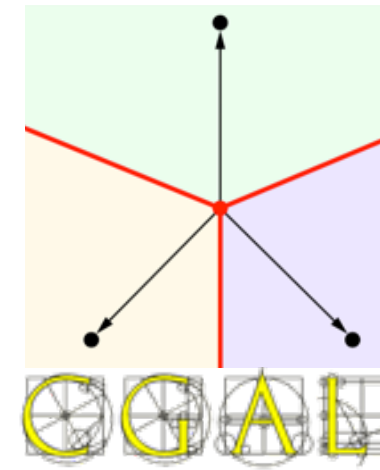
$$\mathcal{S}(t) = \{\mathbf{x} \in \mathcal{O} | T(\mathbf{x}) = t\}$$

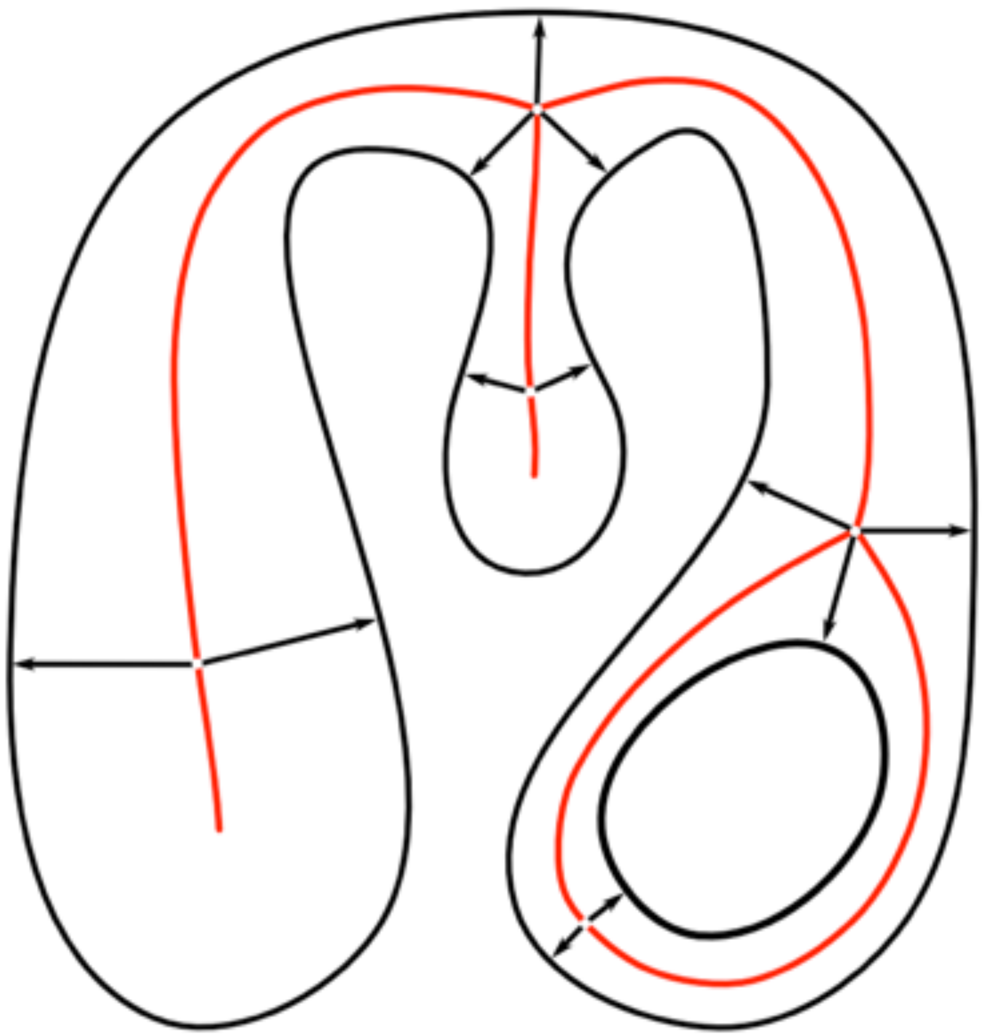
$$\|\nabla T\| = 1 \text{ s.t. } T(\mathbf{x} \in \mathcal{S}) = 0$$

[Felzenszwalb ToC'12] Distance transform of sampled functions **(exact and linear time)**

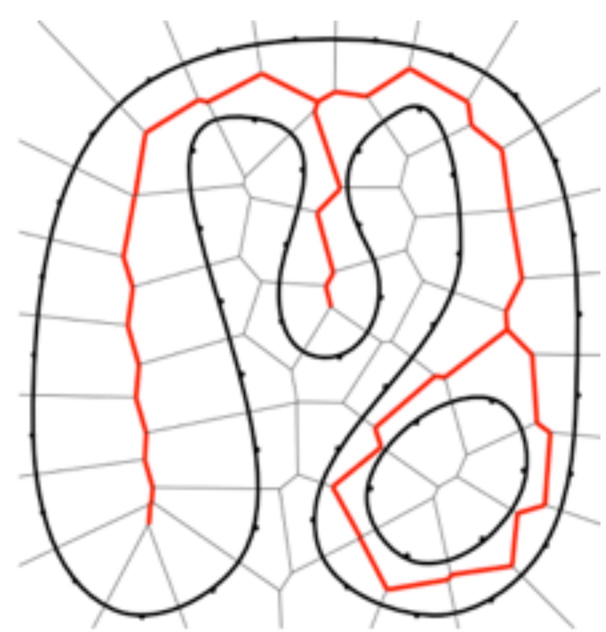
Defining the MAT (3/4)

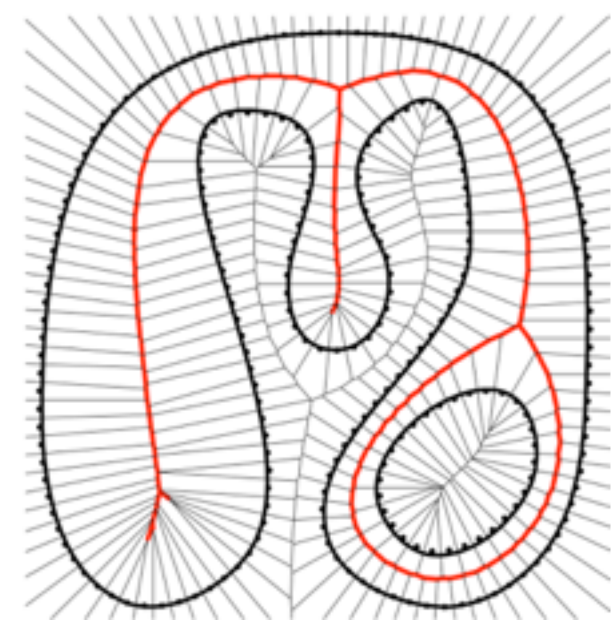
Definition 2.3 The Medial Axis Transform associates to a shape $\mathcal{O}$ the set of locations $\mathcal{M} \in \mathcal{O}$ with more than one corresponding closest point on the boundary $\mathcal{S}$ of $\mathcal{O}$ and their respective distances $\mathcal{R}$ to $\mathcal{S}$.





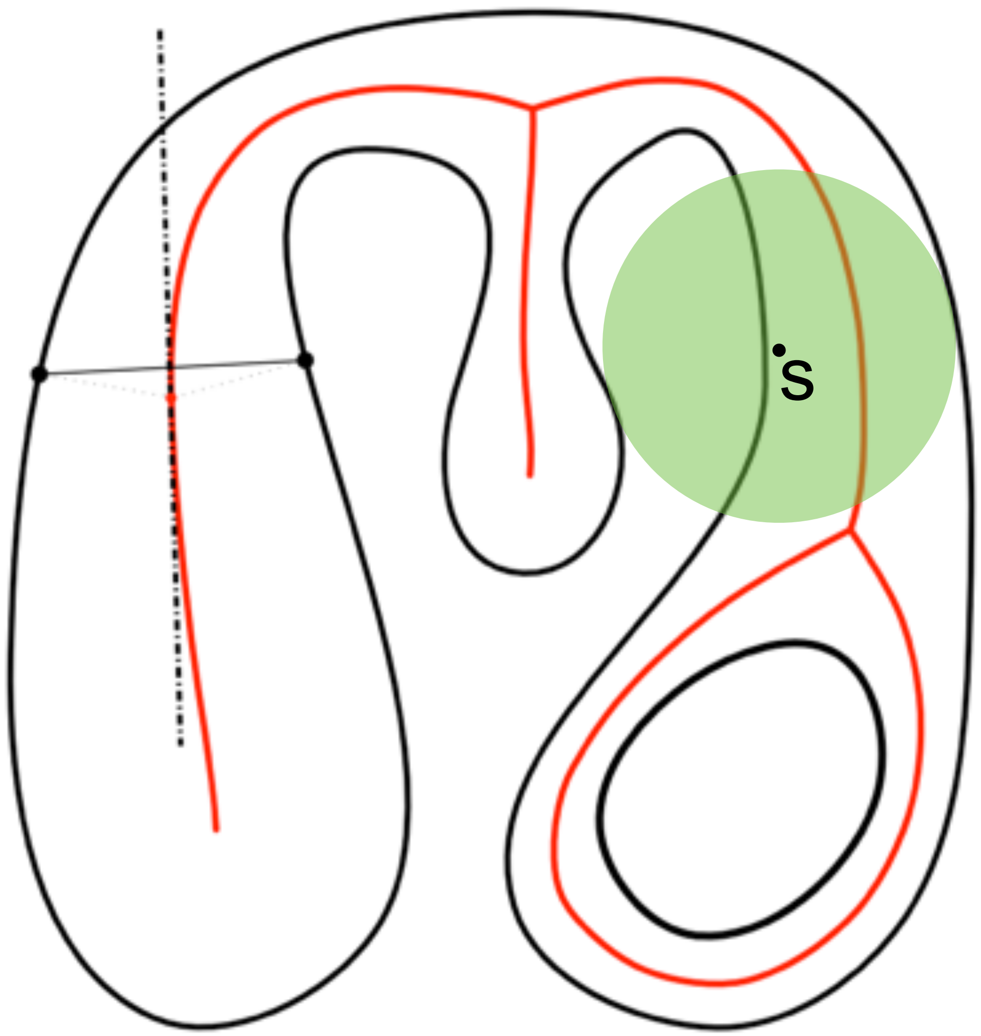
"Medial Spokes"



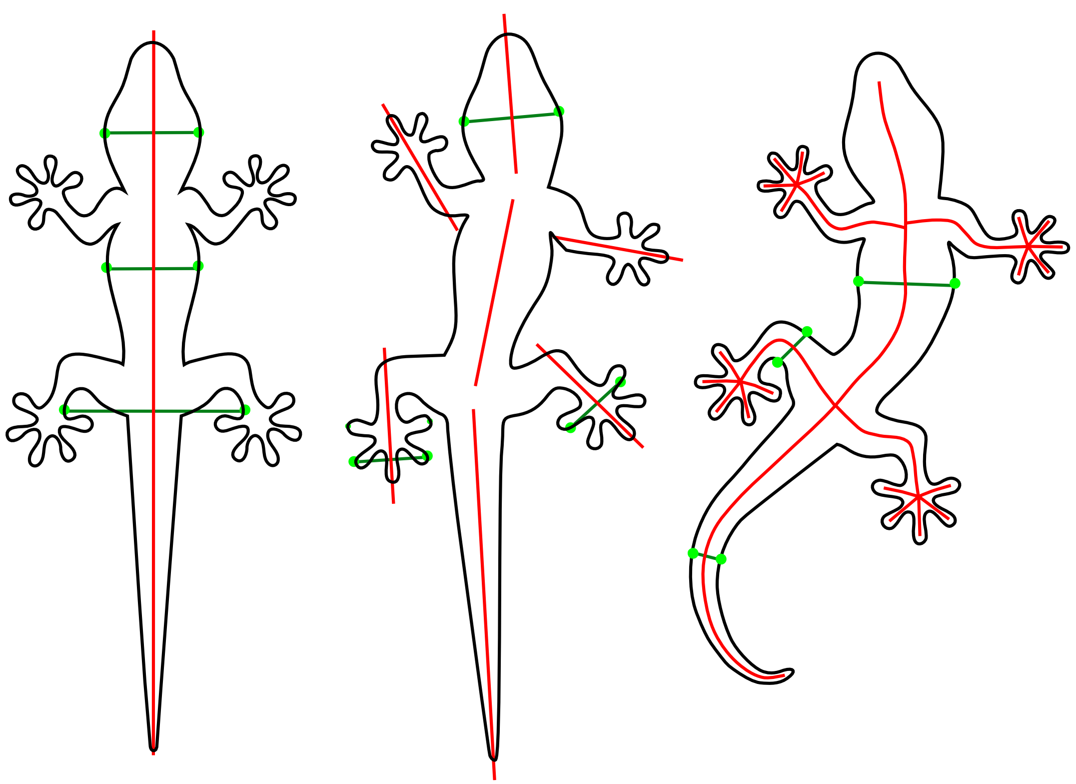


Defining the MAT (4/4)

Definition 2.4 The Medial Axis Transform associates to $\mathcal{O}$ the set of centers $\mathcal{M}$ and radii $\mathcal{R}$ of all inscribed balls in $\mathcal{O}$ which are bi-tangent to its boundary $\mathcal{S}$.



Symmetry at different scales:



note: **s** is on the symmetry set, but not the medial axis

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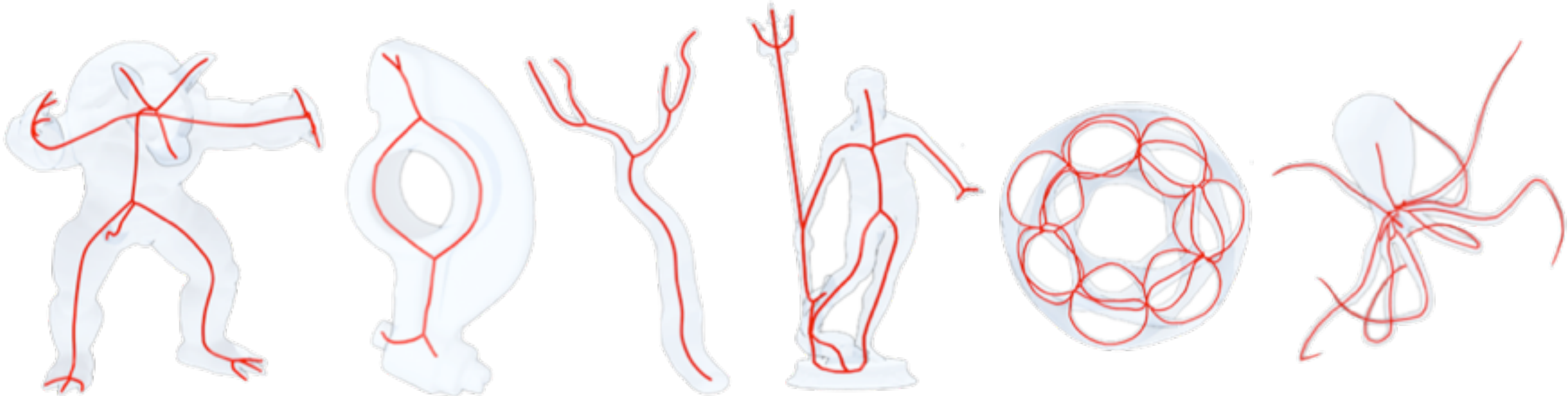
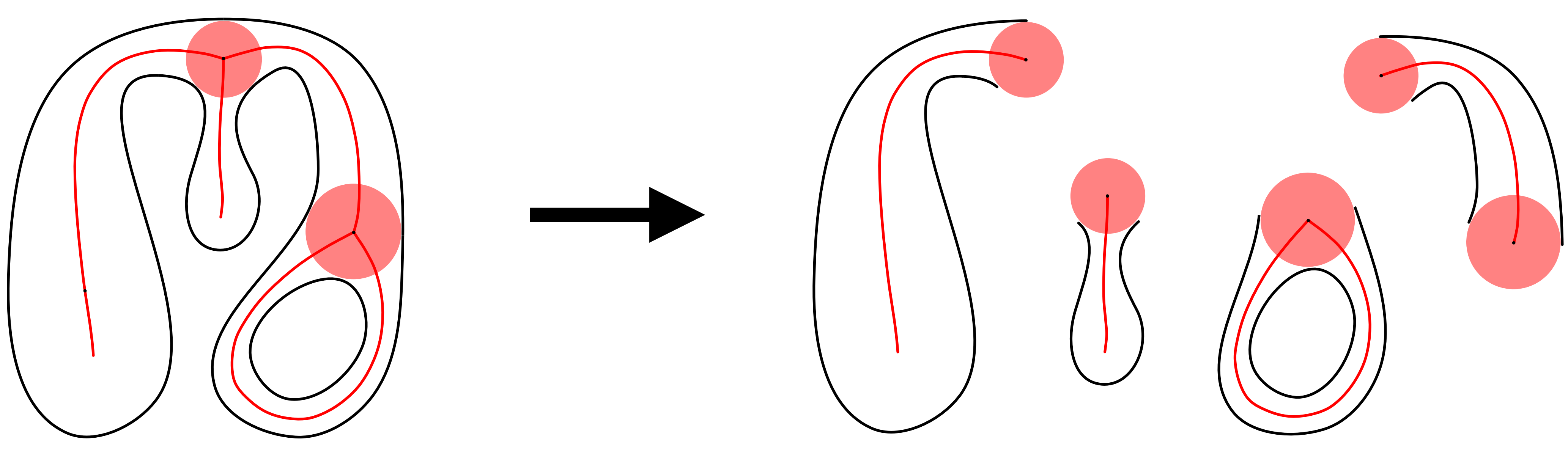
Topology Encoding

- Skeleton have the same homotopy as objects

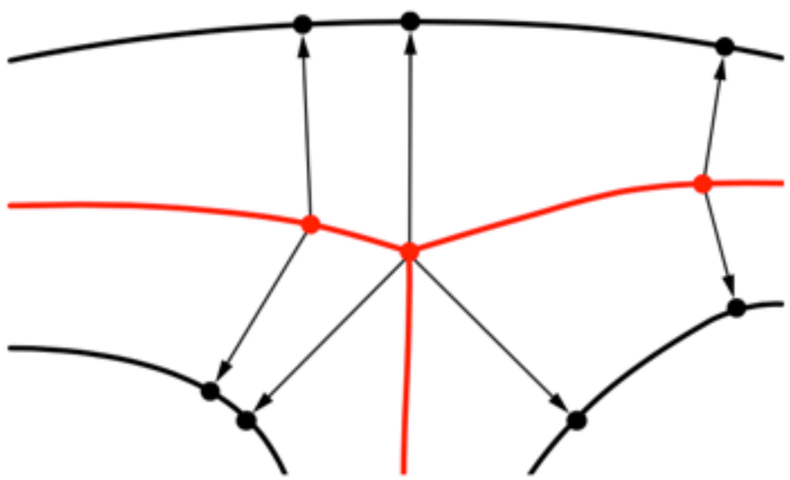


- Induces natural part decomposition



Surface-Skeleton Correspondences

- The *Feature Transform* (FT)



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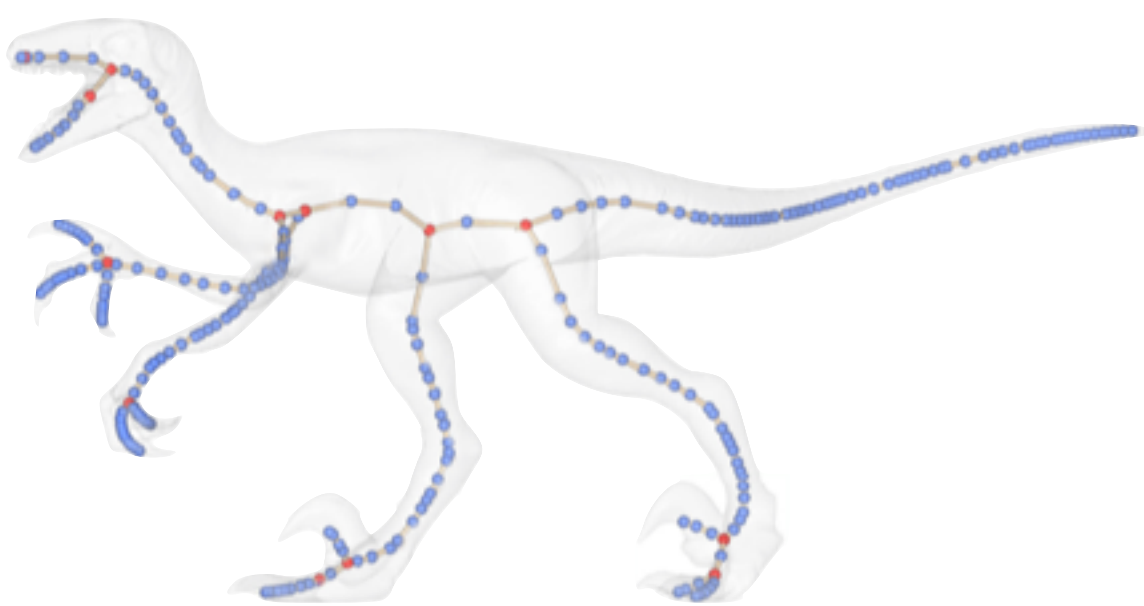
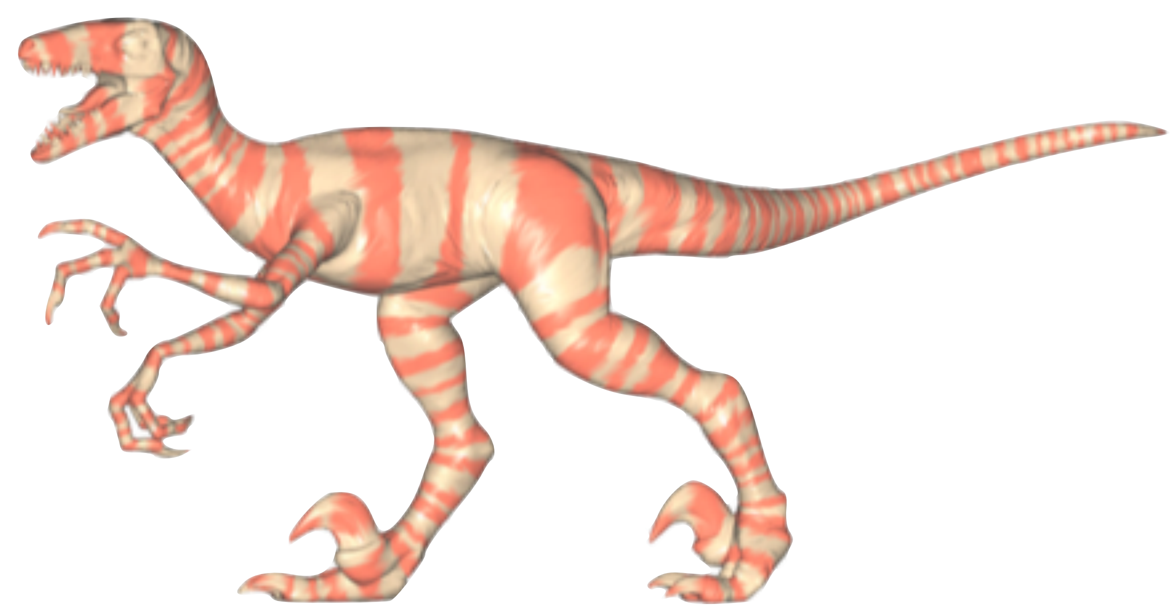
$$FT(\mathbf{x} \in \mathcal{O}) = \{\mathbf{y} \in \mathcal{S} \mid \|\mathbf{x}-\mathbf{y}\| = DT_{\mathcal{S}}(\mathbf{x})\} = \arg\min_{\mathbf{y}\in\mathcal{S}} \|\mathbf{x}-\mathbf{y}\|$$

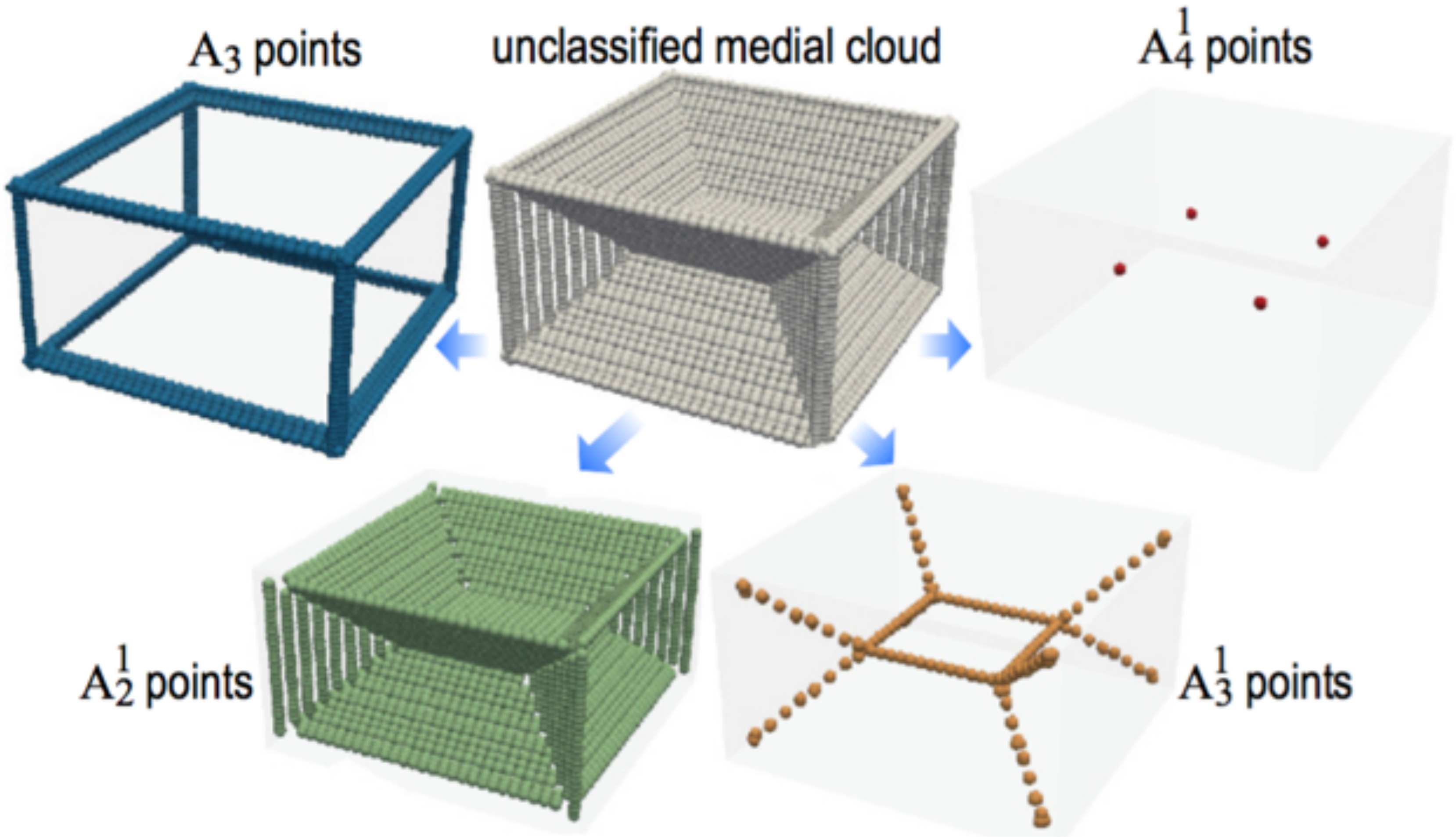




[CGAL 2014] http://doc.cgal.org/latest/Surface_mesh_skeletonization/index.html
[Maurer TPAMI'03] A linear time algorithm for computing exact Euclidean distance transforms of binary images

Skeletal Structure of MAT

- Induced by the cardinality of the mapping



[Giblin PAMI'04] A formal classification of {3D} medial axis points and their local geometry

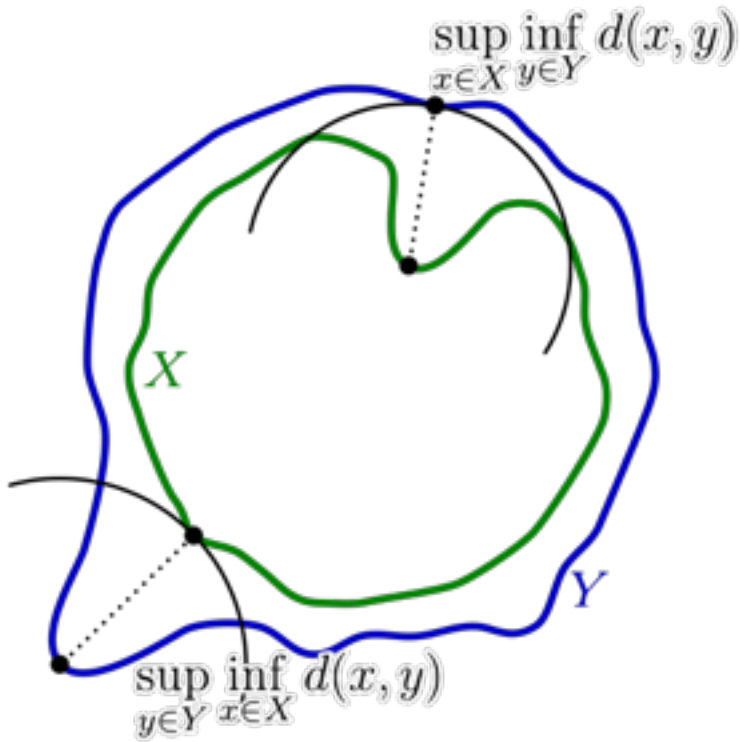
Semi-Continuity (or Instability)

- Hausdorff Distance

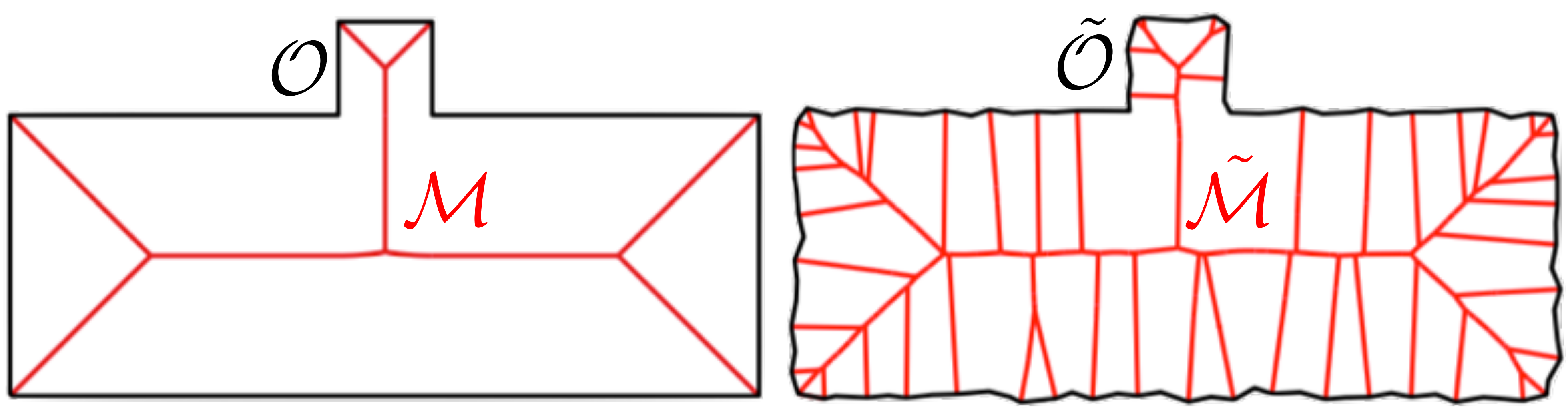
$$d_{X \to Y} = \max_{x \in X} \left[\min_{y \in Y} d(x,y) \right]$$

$$d_{Y \to X} = \max_{y \in Y} \left[\min_{x \in X} d(x,y) \right]$$

$$d_{X \leftrightarrow Y} = max\{d_{X \to Y}, d_{Y \to X}\}$$



- Semi-continuity



$$\exists \epsilon, \delta \quad | \quad d_{\mathcal{O} \leftrightarrow \tilde{\mathcal{O}}} < \epsilon \implies d_{\mathcal{M} \to \tilde{\mathcal{M}}} < \delta$$

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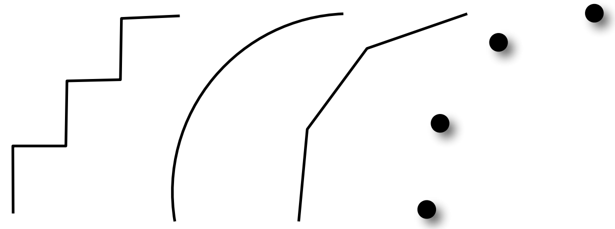
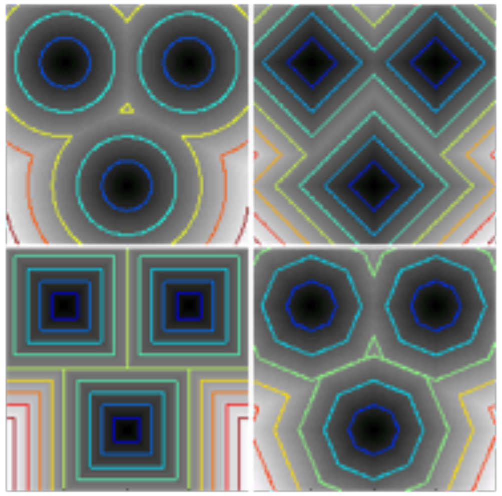
Challenges

- Which object representation?
  - voxels, surface, rational, points, …



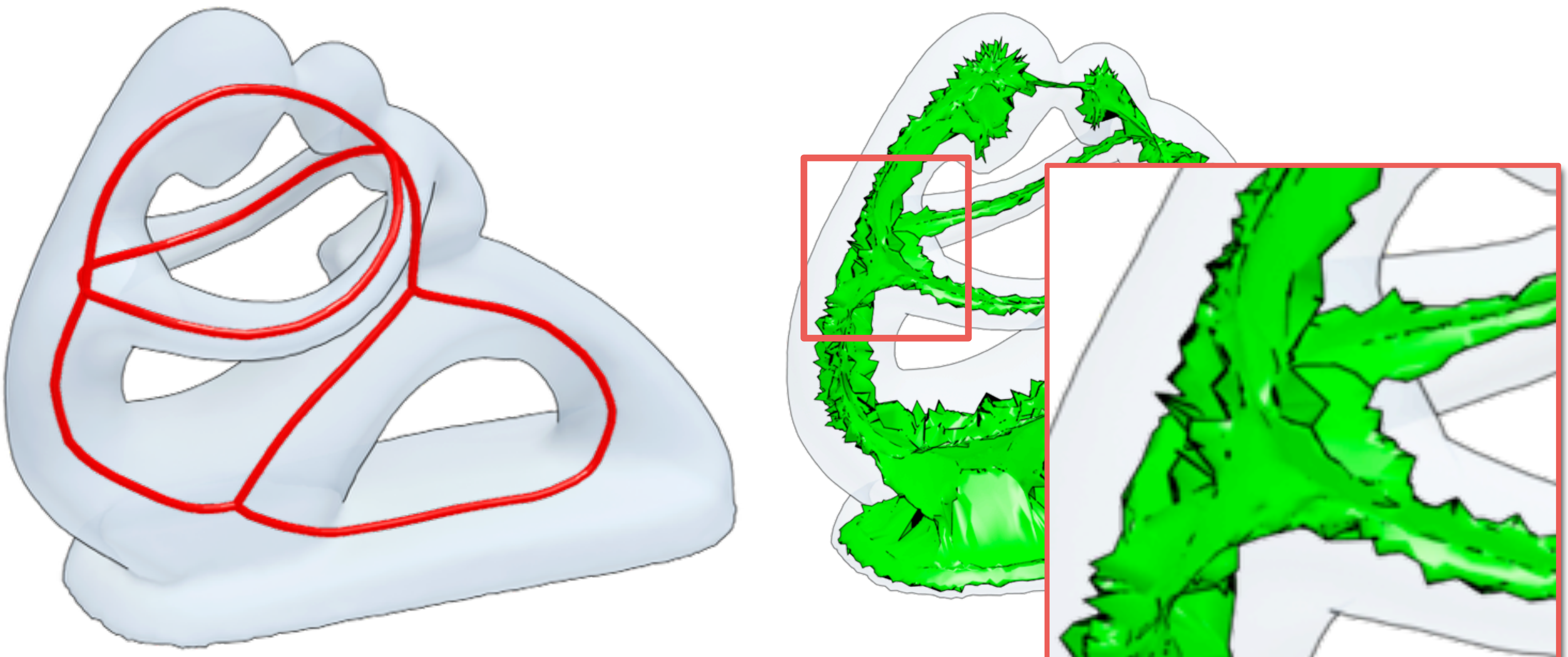
- How to compare boundary points?
  - essential in filtering (stability)
  - different alternative (euclidean, angle, …)

- How to compute distances?
  - marching grid, exact, etc…



Core Challenge

- Does not directly generalize to 3D
  - (piecewise manifold) surfaces instead of curves :(
  - noise even more challenging :(



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Surface Skeletons

- Computed by <u>approximating</u> one of the various medial skeleton definition discussed earlier
- Two classes of methods
  - **Analytic** Surface Skeletons (ASS)
  - **Image** Surface Skeletons (ISS)

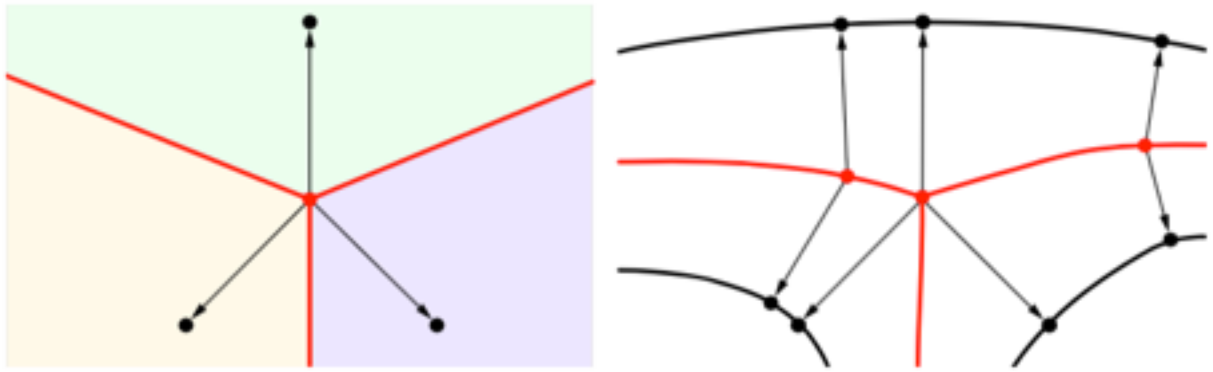
Analytic Surface Skeletons

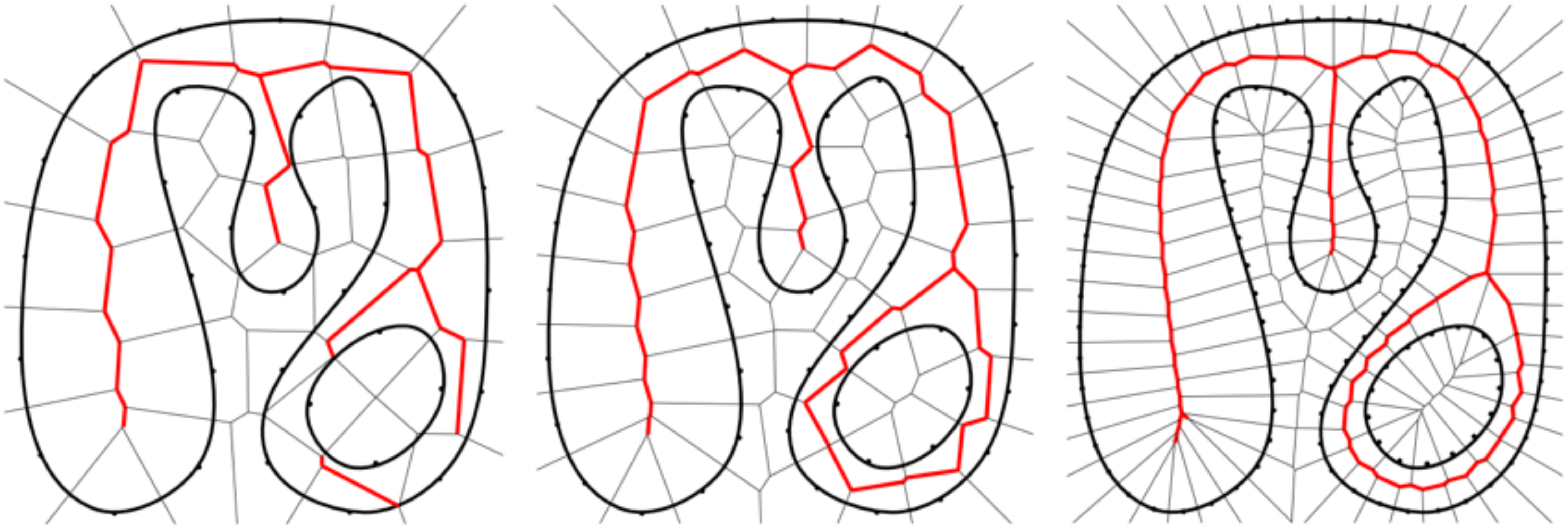
- Both input shape and skeleton are approximated analytically (points as floating point)
  - high accuracy and level-of-detail
  - high computational & implementation complexity
- Four methods in this class
  1. Voronoi
  2. bisector & medial scaffold
  3. shrinking-ball
  4. spherical collapse*

*missing from STAR paper

Voronoi (2D)

- Sample the surface
- Voronoi diagram

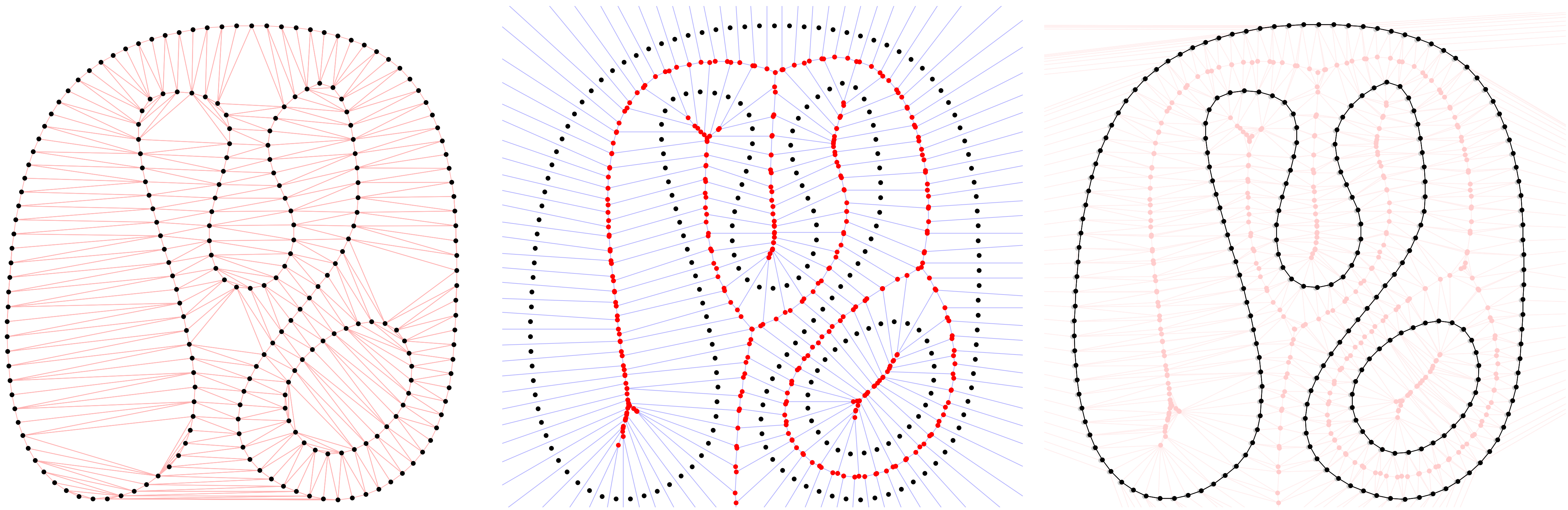




[Brandt et al. CVGIP'92] Continuous skeleton computation by Voronoi diagram

Voronoi (2D)

- Interpolatory reconstruction: **"Crust"**
  - LFS(p∈S) = distance from p → medial
  - ϵ-sampling : no p∈S further than ϵ LFS(p)

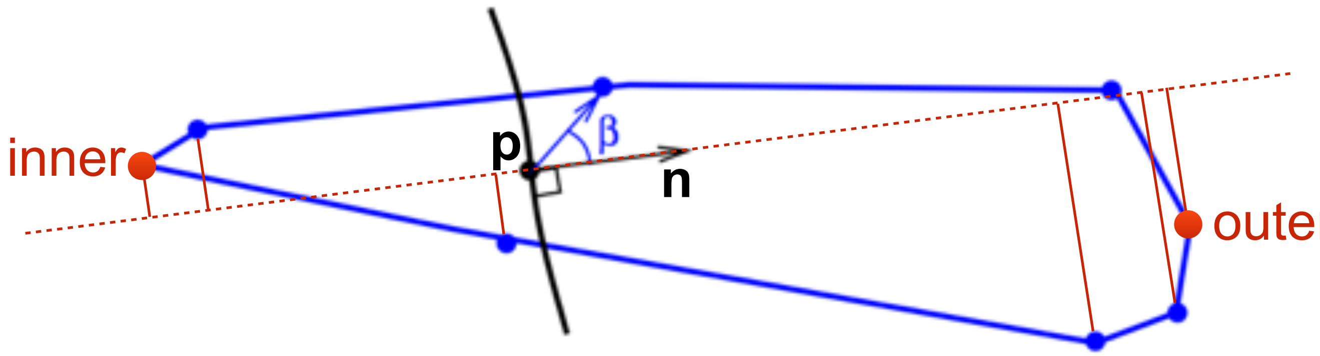


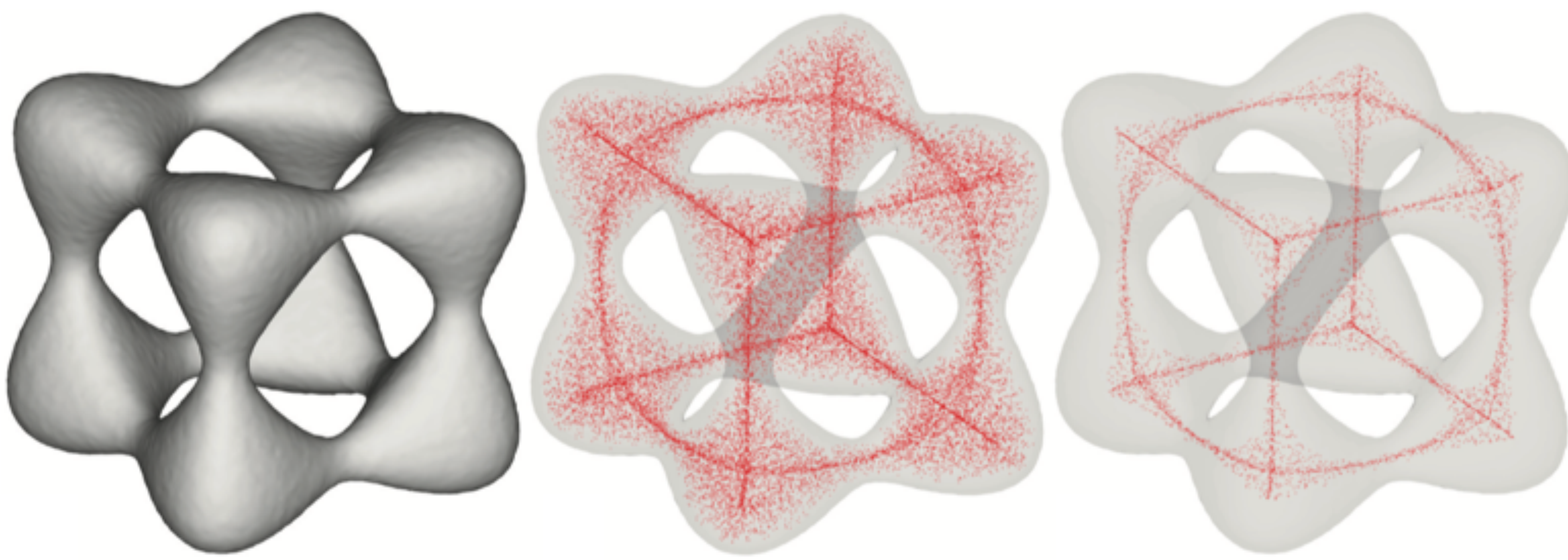
Delaunay(P) M=Voronoi(P) Delaunay(P∪M)

[Amenta et al. SCG'98] Surface Reconstruction by Voronoi Filtering

Voronoi (3D)

- Voronoi poles (subset of Voronoi)



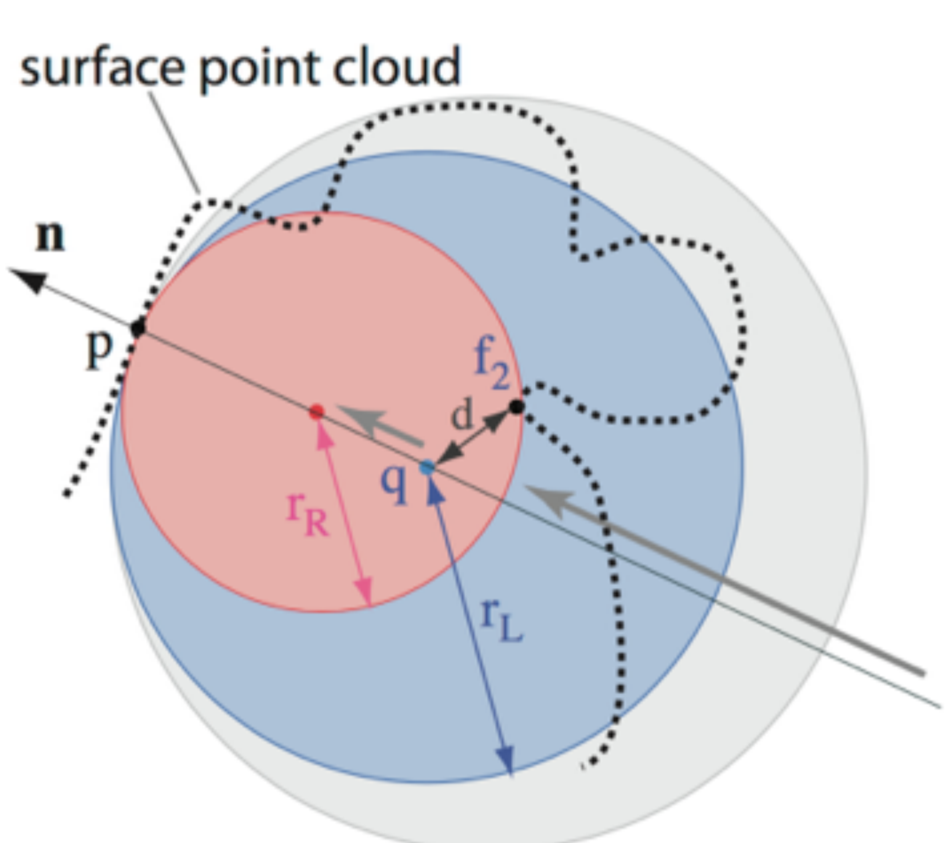


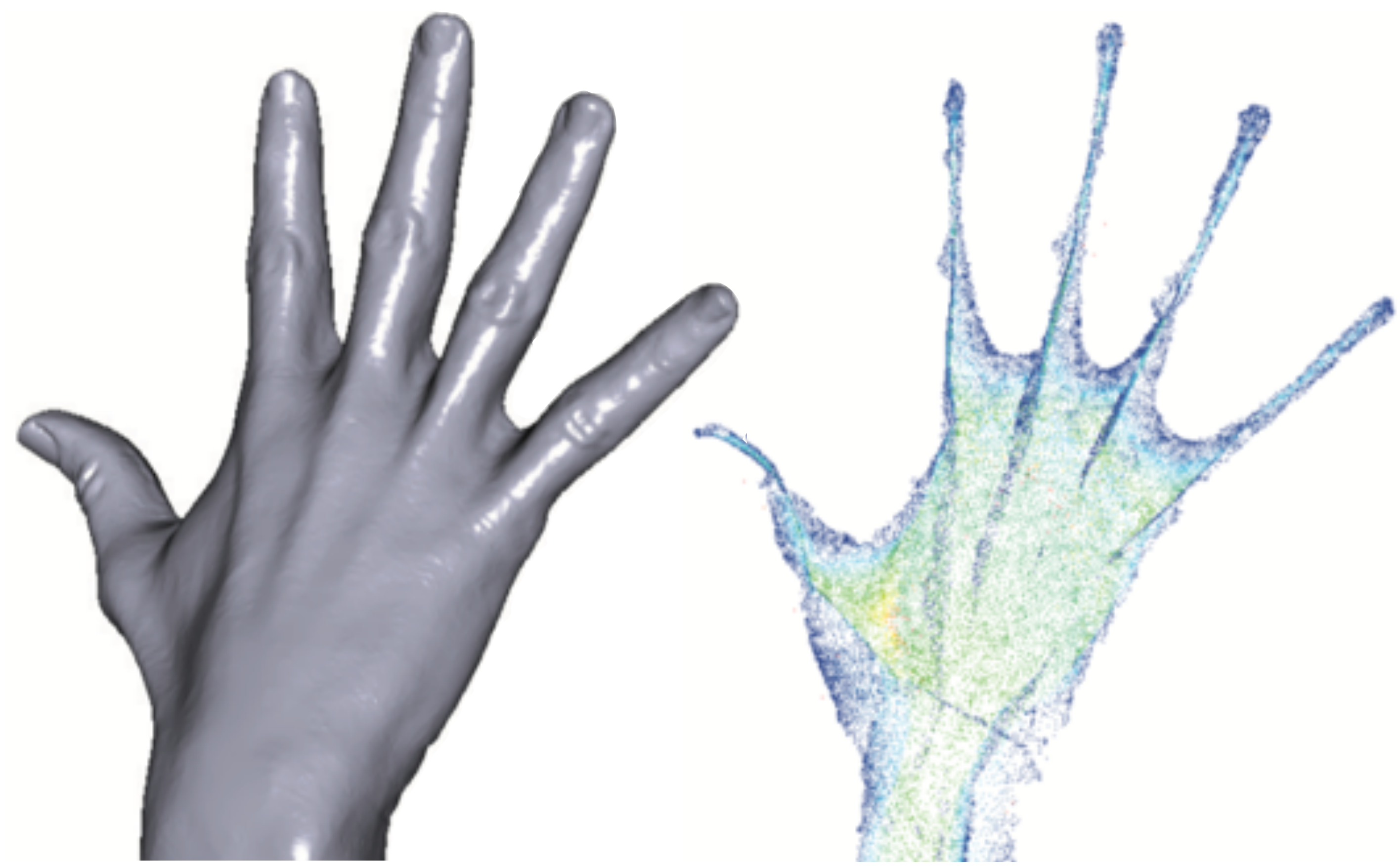
input surface (noisy)	Voronoi vertices (interior)	Voronoi poles (interior)

[Amenta et al. SMA'01] The Power Crust
[Dey et al. CAD'04] Approximate medial axis as a Voronoi subcomplex **(filtering)**

Shrinking ball methods

- Medial balls are {tangent, maximal, empty}
  - Start with a large ball and “shrink” it until empty

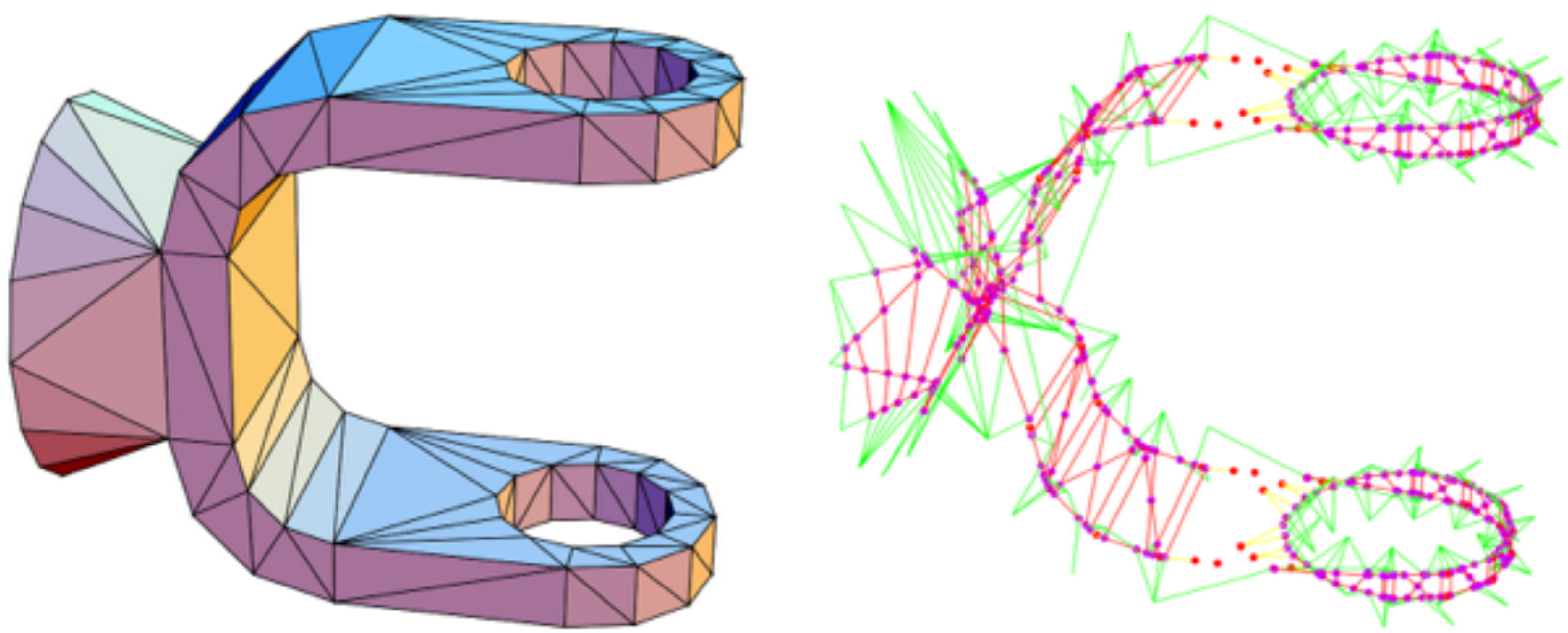




[Ma et al. VC’12] 3D Medial axis point approximation using nearest neighbors and the normal
[Jalba et al. TPAMI’13] Surface and curve skeletonization of large 3D models on the GPU

Bisector & Scaffold Methods

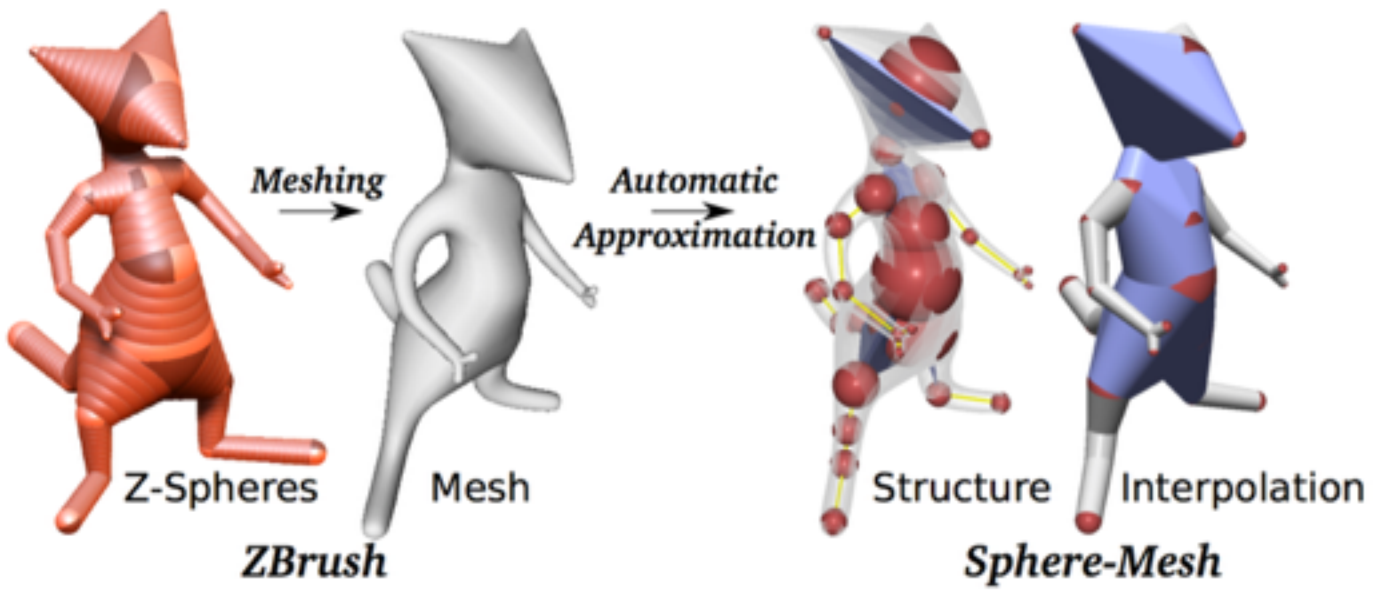
- Post-process the Maxwell interpretation
  - compute all bisectors B for all point-pairs (p,q)
  - **union** of **subsets** of B closest to (p,q) and inside S

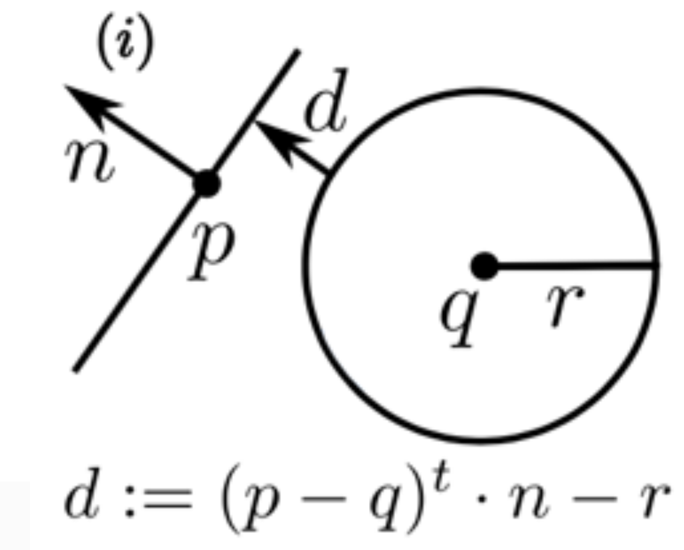


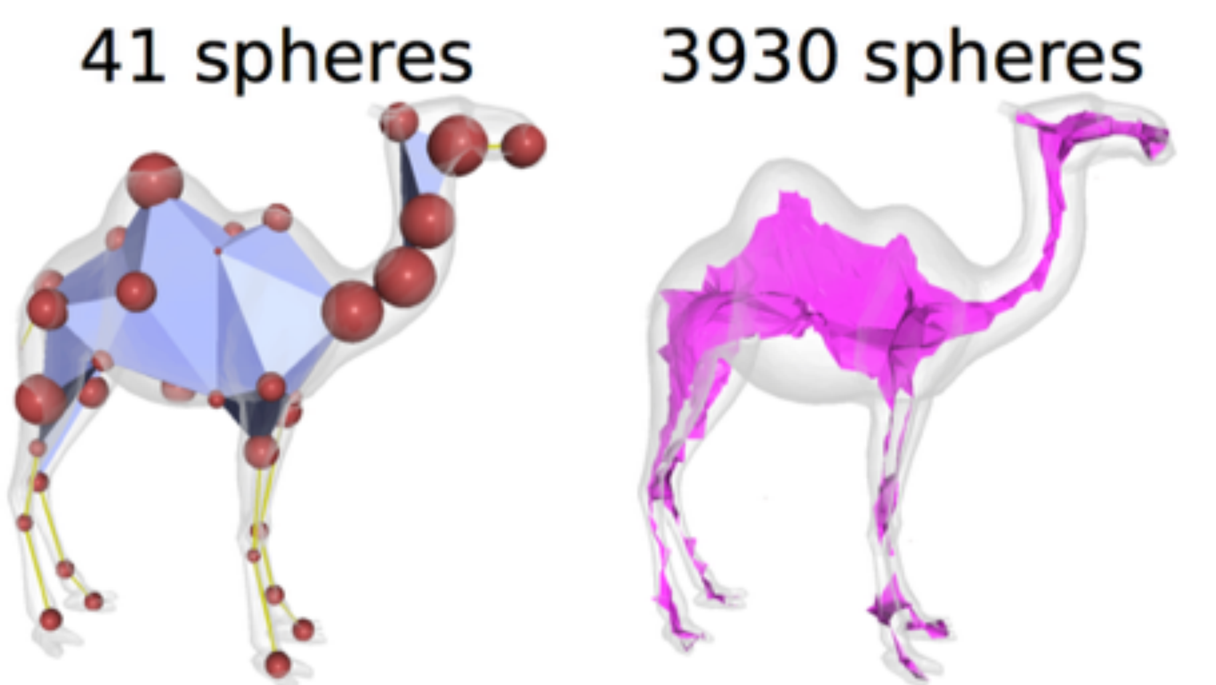
  - absurdly inefficient

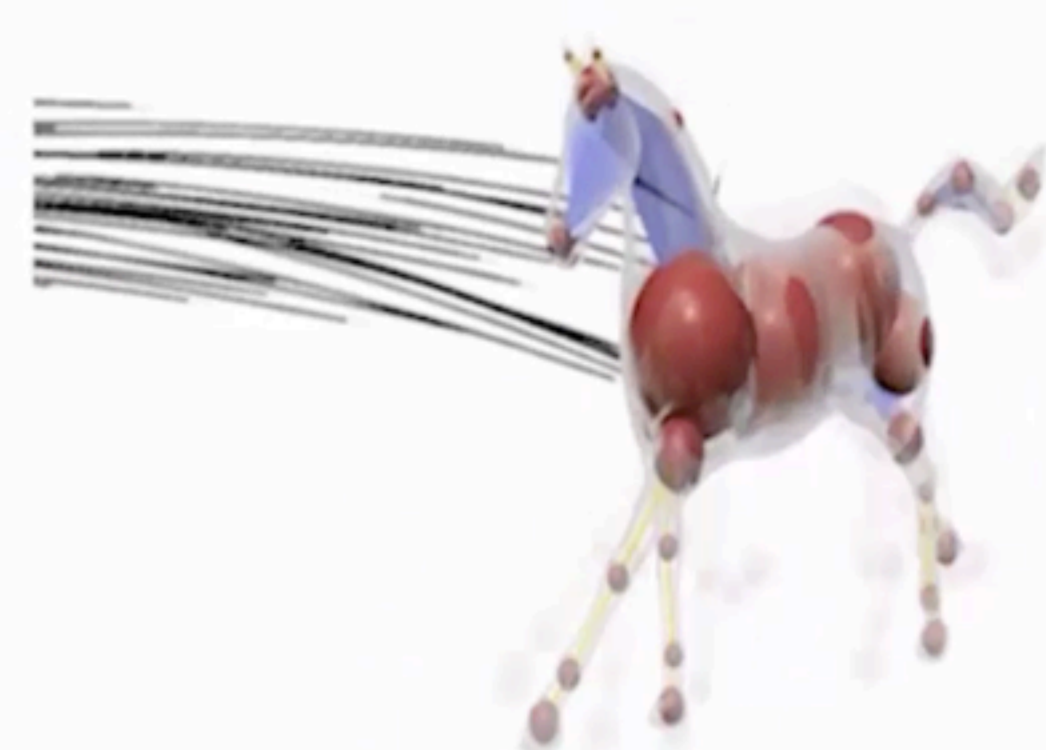
[Culver et al. CAGD'04] Exact computation of a medial axis of a polyhedron
[Leymarie et al. VF'01] The shock scaffold for representing 3D shape

Spherical Edge Collapse









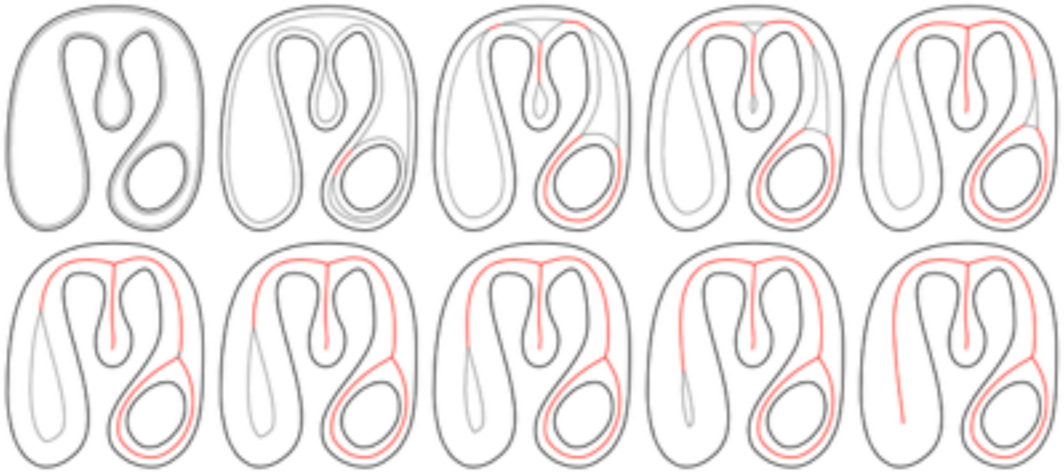
[Thiery et al. TOG'16] Animated Mesh Approximation With Sphere-Meshes
[Thiery et al. SIG'13] **Sphere-Meshes**: Shape Approximation using Spherical Quadric Error Metrics

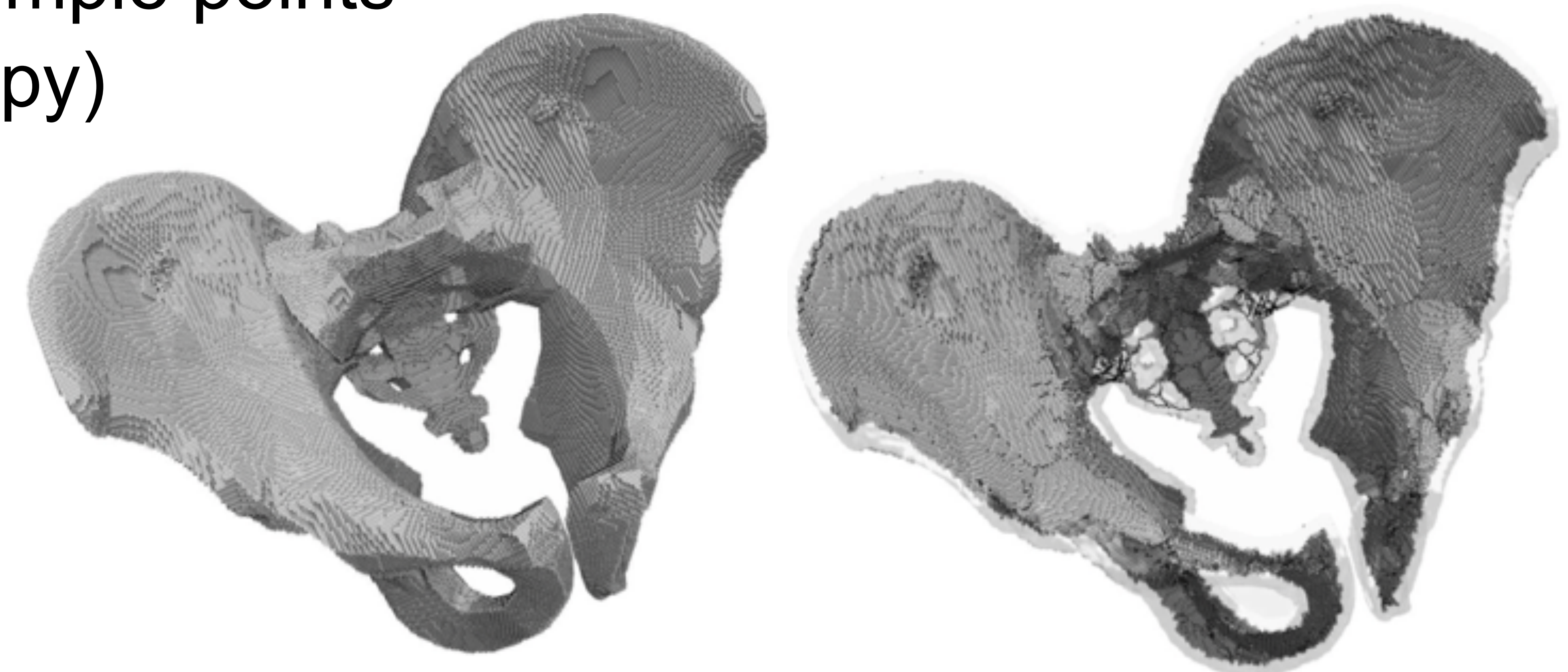
Image Surface Skeletons

- Input shape and skeleton are approximated using nearest-neighbor sampling (images in 2D, voxel volumes in 3D)
  – relatively simple to implement
  – higher memory requirements, lower resolution
- Four methods in this class
  1. topological thinning
  2. distance field
  3. mass advection
  4. feature field

Topological Thinning

- Discrete grassfire
  - iteratively remove voxels until a one-voxel thin result is obtained
  - avoid "simple points" (homotopy)

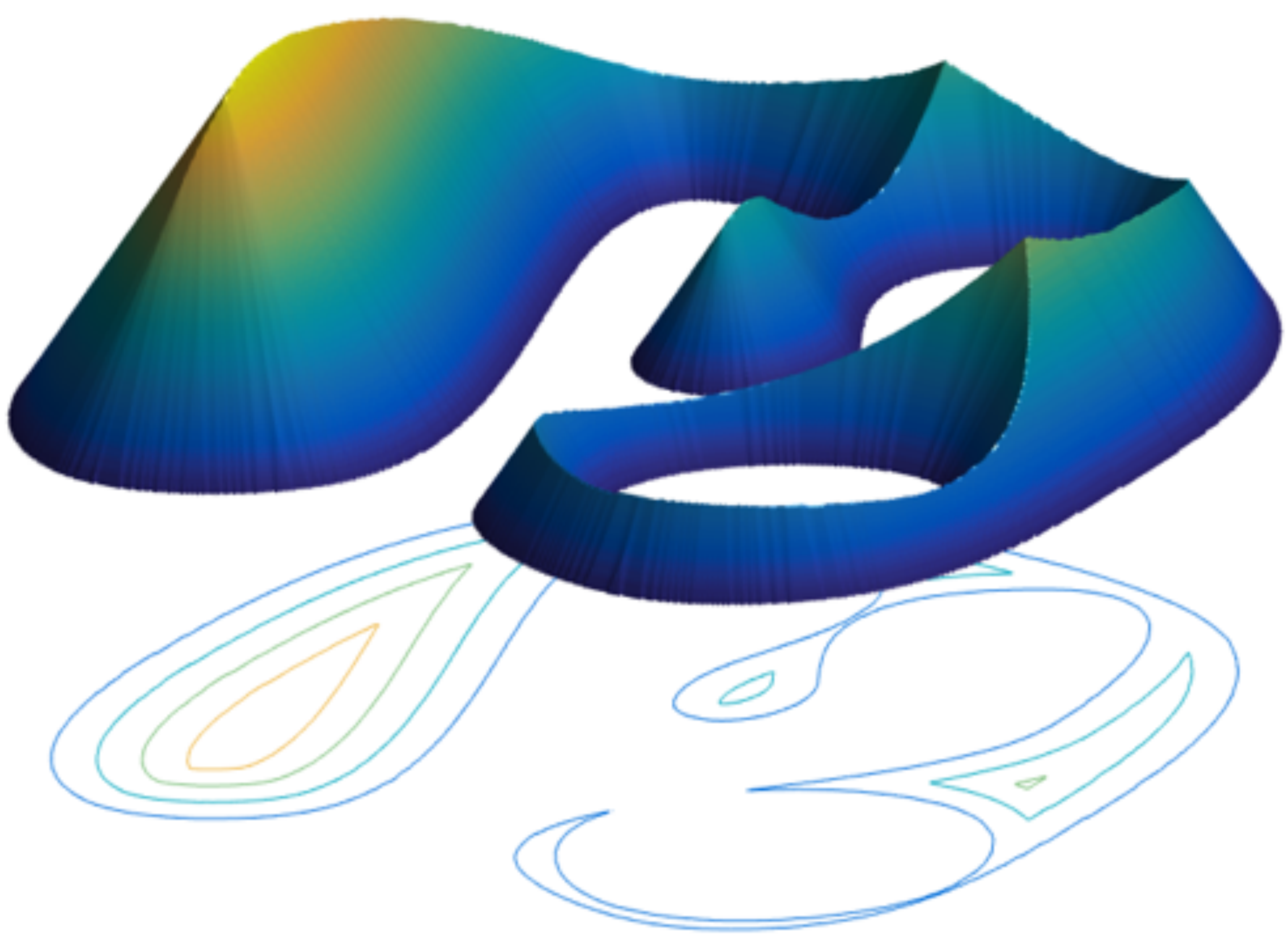


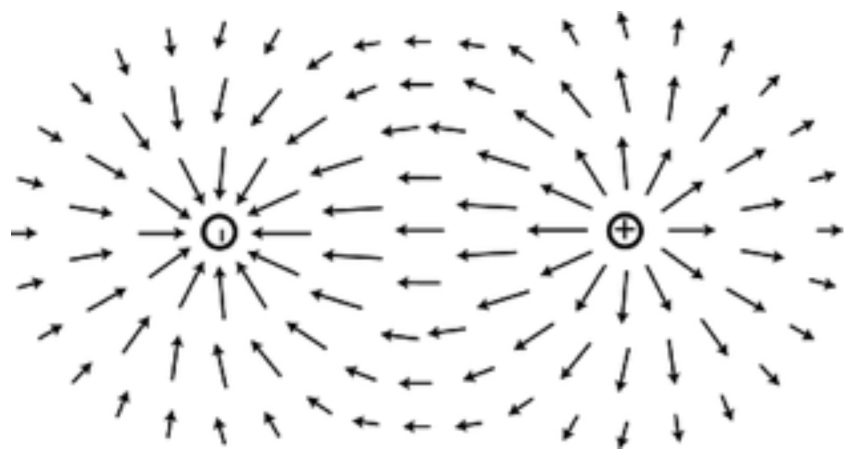


[Arcelli et al. TPAMI'11] **Distance-driven** skeletonization in voxel images
[Palagyi et al. GMIP'99] Directional 3D thinning using 8 sub-iterations **(*thinvox* software)**

Distance Field Methods

- Detect singularities of 3D distance transform
  - divergence of gradient of DT
  - first order moments

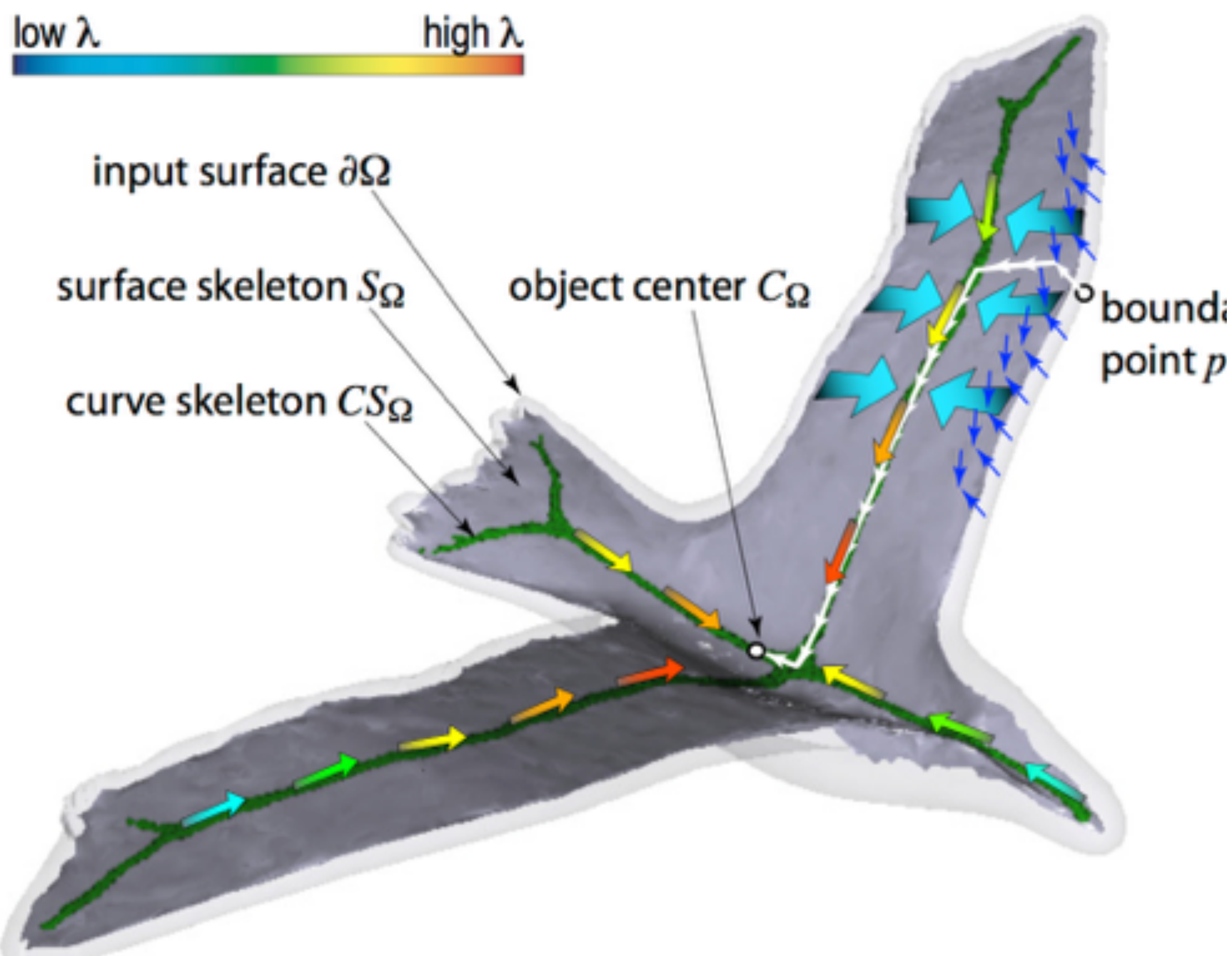


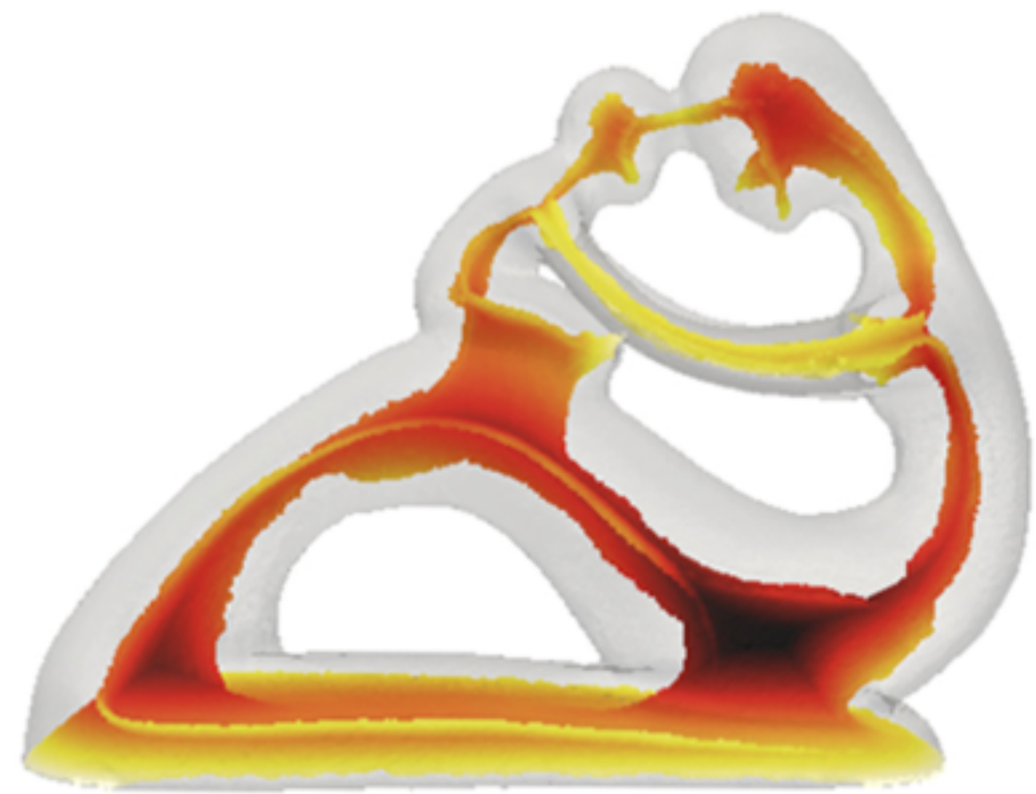


[Siddiqi et al. IJCV’02] Hamilton-Jacobi Skeletons **(divergence)**
[Rumpf et al. ’02] A continuous skeletonization method based on level-sets

Mass Advection Methods

- Singularities of a 3D mass advection process
  - momentum conservation

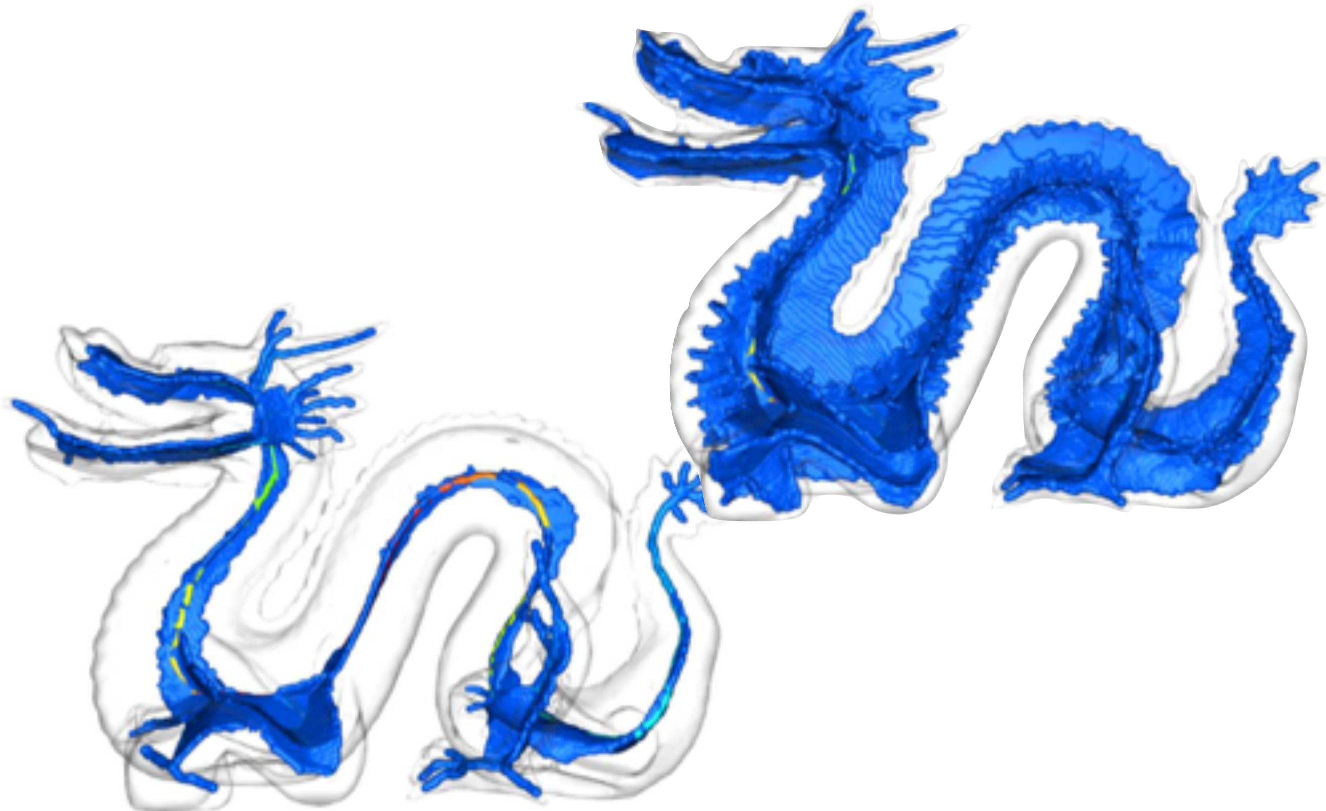


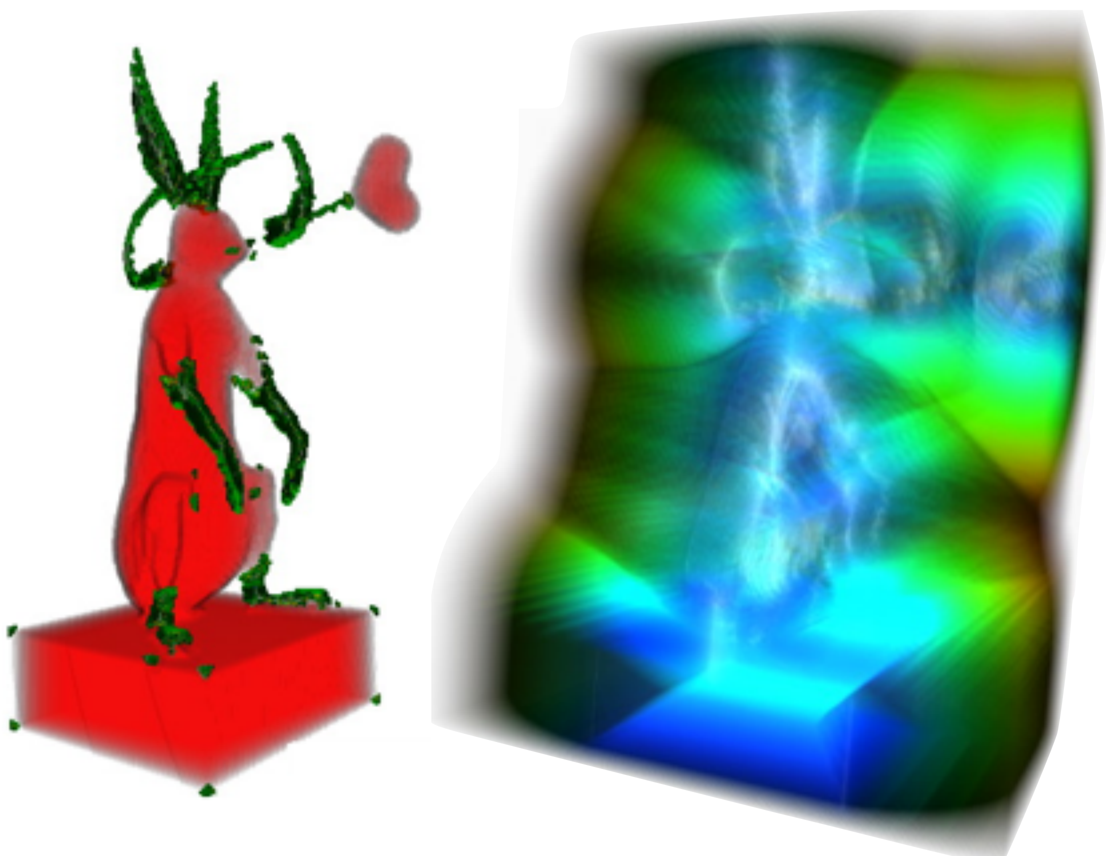


[Rossi CVIU'14] Coarse-to-fine skeleton extraction for high resolution 3D meshes
[Jalba TVCG'15] A unified multi-scale framework for surface and curve-skeletonization (mass advection)

Feature Field Methods

- Exploit the single-point "*Feature Transform*"
  - *neighbors have "different" image points*
  - *linear time and parallelizable*





[Dortmont et al. DGCI'06] Skeletonization and distance transforms of 3D volumes using graphics hardware
[Reniers et al. TVCG'08] Multi-scale skeletons of genus-0 objects using a global importance measure

Overview

- Theory and Background (Andrea, 15m)
  - Questions (5m)
- Properties and Taxonomy (Thomas, 12m)
- Skeletonization Methods (Andrea, 12m)
  - Surface Skeletons
  - **Curve Skeletons**
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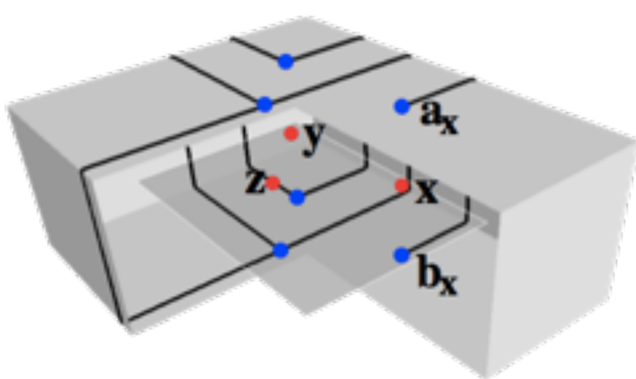
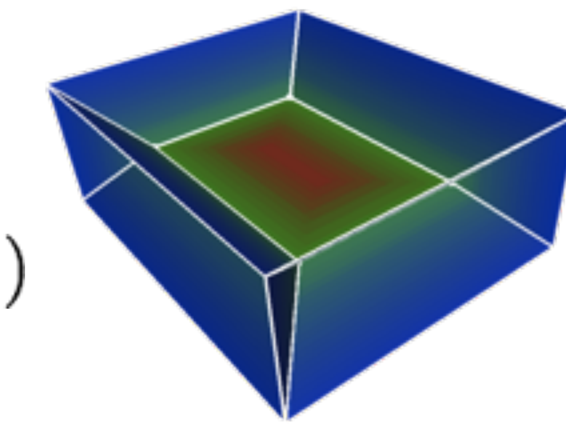
Medial Geodesic Function

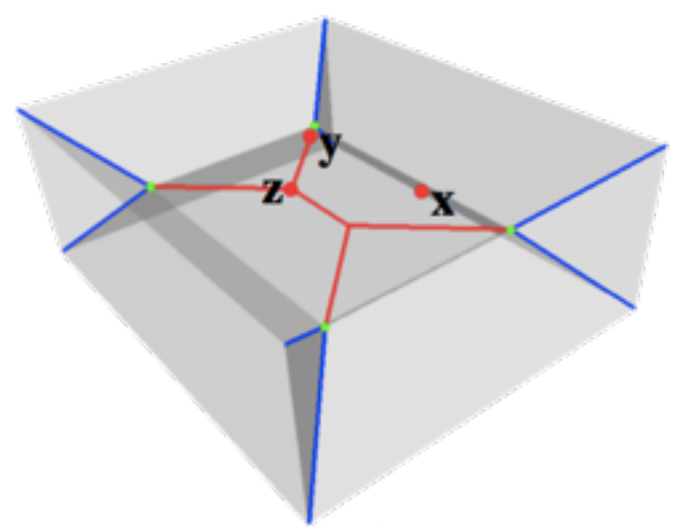
- Define a scalar function on the medial surface
  - *geodesic* distance between image points

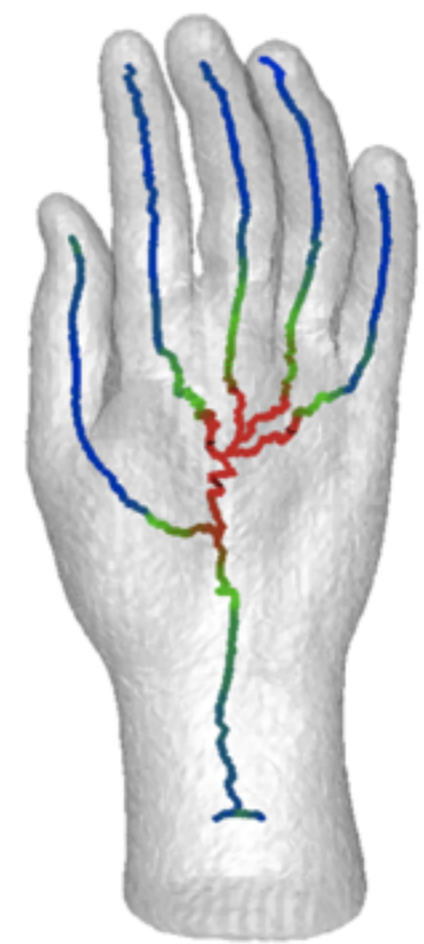


$$MGF(\mathbf{x}) = d_S(\mathbf{a_x}, \mathbf{b_x})$$



  - The skeleton is the shock graph of MGF
    - “skeleton of skeletons”





[Dey SGP’07] Defining and computing curve skeletons with MGF
[Jalba TPAMI’13] Surface and curve skeletonization of large models on the GPU

Generalized Fields

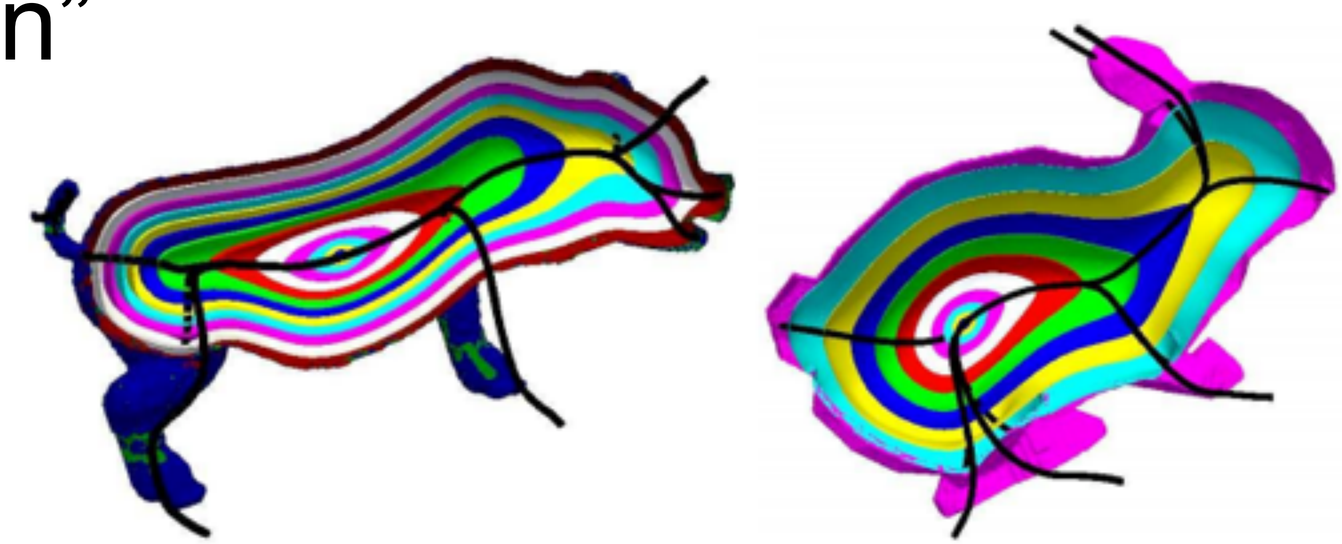
- MAT used the euclidean distance transform
  - Generalize with “non-local” distance

- Smooth vector field V from a variational problem

$$E(V(x)) = \iiint \underbrace{|\nabla V_x(\mathbf{x})|^2 + |\nabla V_y(\mathbf{x})|^2 + |\nabla V_z(\mathbf{x})|^2}_{\text{smooth vector field (volume)}} + \mu \underbrace{(\nabla f(\mathbf{x}))^2 (V(\mathbf{x}) - \nabla f(\mathbf{x}))^2}_{\text{aligned with surface normals}} d\mathbf{x}$$

- Define “Medial Function”

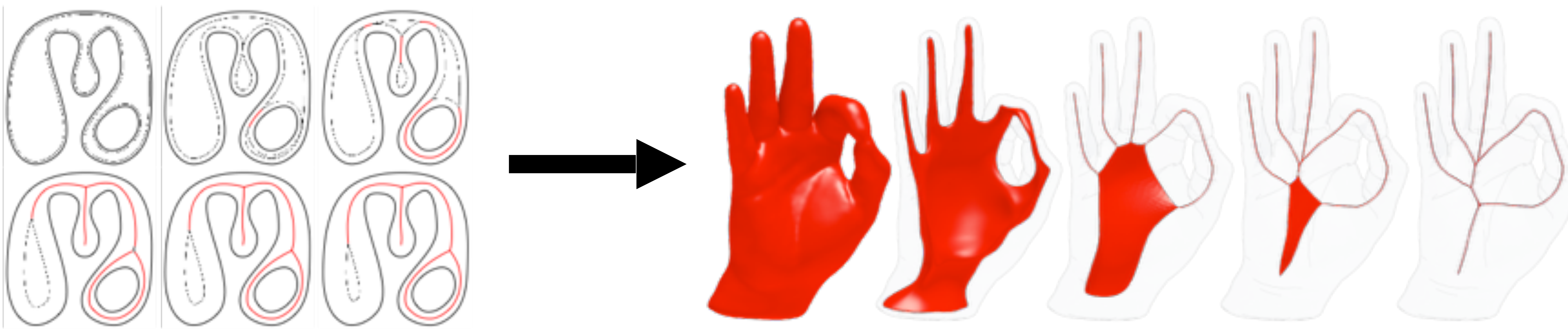
$$\lambda(\mathbf{x}) = 1 - \left(\frac{V(\mathbf{x} - min|V|)}{max|V| - min|V|} \right)^{\gamma}$$



[Hassouna TPAMI’09] Curve skeleton extraction using iterative LS optimization
[Cornea TVCG’07] Curve-skeleton properties, applications and algorithms

Contraction Methods - Flows

- Generalize MAT's *grassfire* to *curve* skeletons



$$\dot{\mathcal{S}}(t) = -\mathbf{n}(t)$$

$$\begin{cases} \dot{\mathcal{S}}(t) = -H(t)\mathbf{n}(t) & \text{if } H(t) < \infty \\ \dot{\mathcal{S}}(t) = 0 & \text{otherwise} \end{cases}$$

Mean Curvature Flow

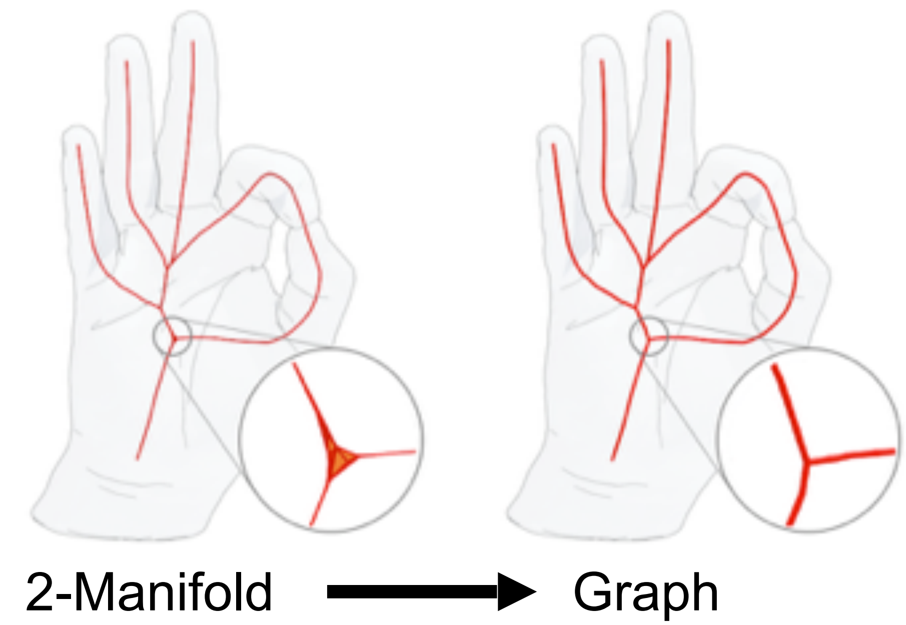
- Combine the two flow for centered skeletons!

[Tagliasacchi SGP'12] Mean Curvature Skeletons
[Chuang EG'11] Fast MCF via finite element tracking
[Au SIG'08] Skeleton extraction by mesh contraction
[Wang TVCG'08] Curve skeleton extraction using iterative LS optimization

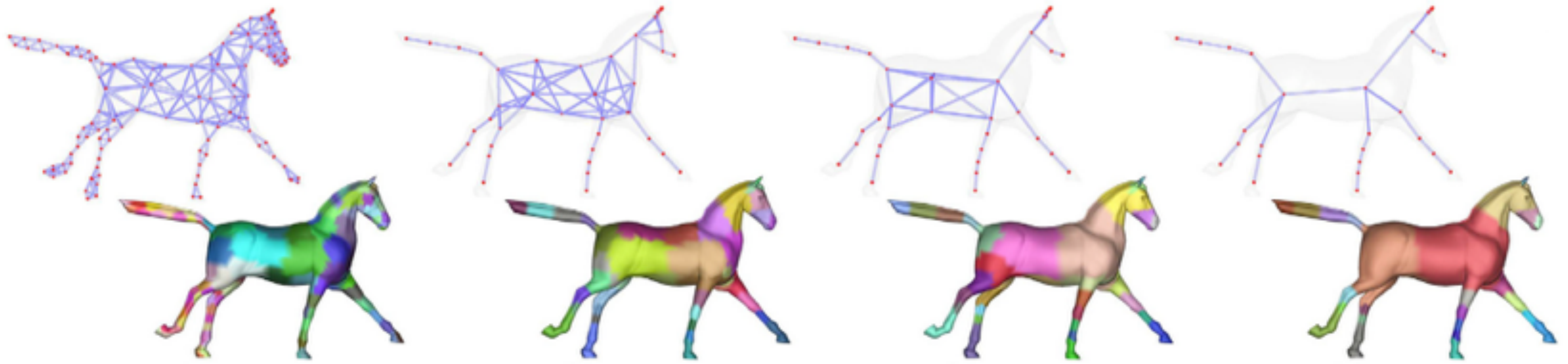
[Tagliasacchi SGP'12] available

Mesh Decimation

- Way to convert (contracted) manifold geometry into a graph



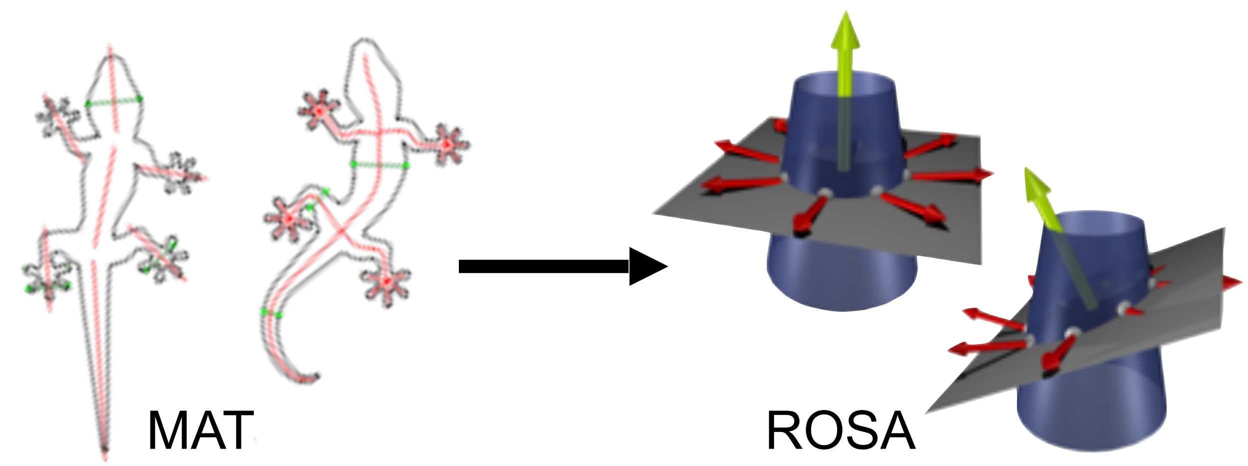
- Extension to skeletonization methods (attempt)



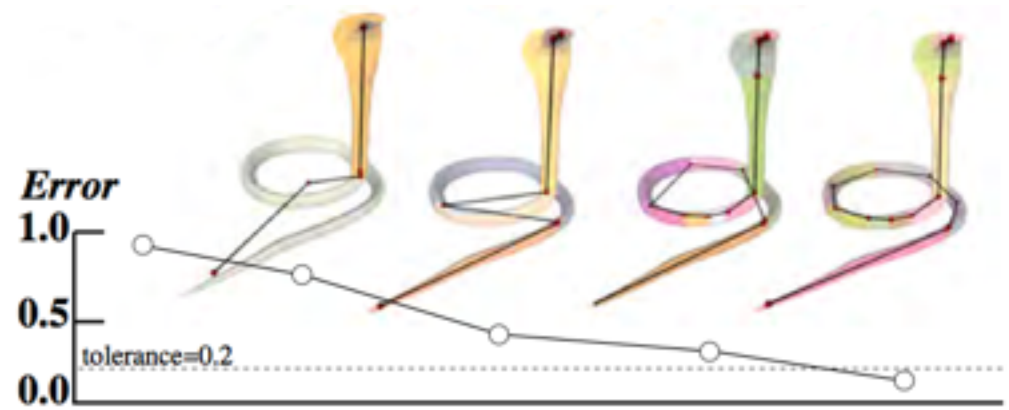
[Li i3D'01] Decomposing polygon meshes for interactive applications
[Jiang GM'12] Curve skeleton extraction by coupled graph contraction and surface clustering

Property Grouping

- Generalize Symmetry
  - 2D: reflectional (MAT)
  - 3D: rotational (ROSA)



- Convex decomposition
  - PCA line fitting
  - recursive split



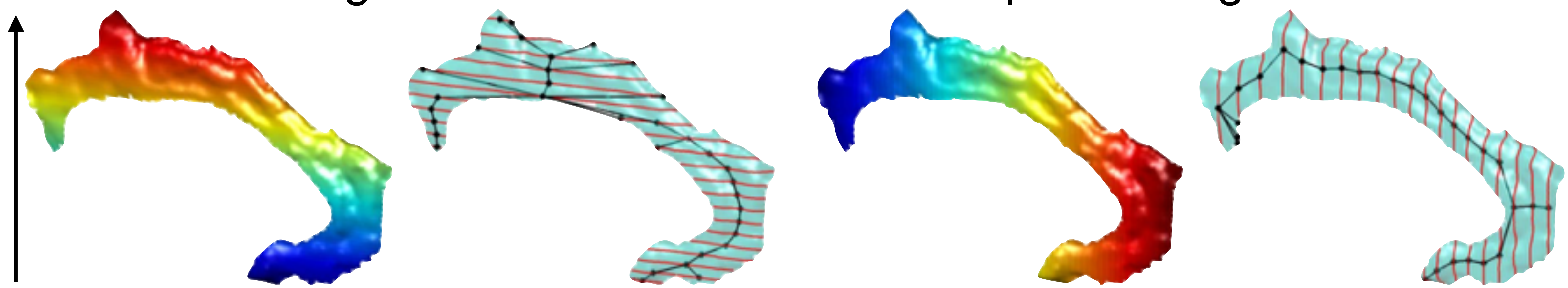
- Kinematic clustering
  - similar transformations

[Tagliasacchi SIG'09] Skeleton extraction from incomplete point clouds
[Lien SPM'06] Simultaneous shape decomposition and skeletonization
[Anguelov UAI'04] Recovering articulated object models from 3D range data

Topology Driven

- Generalize Symmetry

Height Field | Laplacian Eigenvector



[He GM'09] Harmonic 1-form based skeleton extraction from examples
[Shi CVPR'08] Anisotropic Laplace-Beltrami eigenmaps: Bridging Reeb graphs and skeletons
[Biasotti SMI'03] An overview on properties and efficacy of topological skeletons in shape modelling

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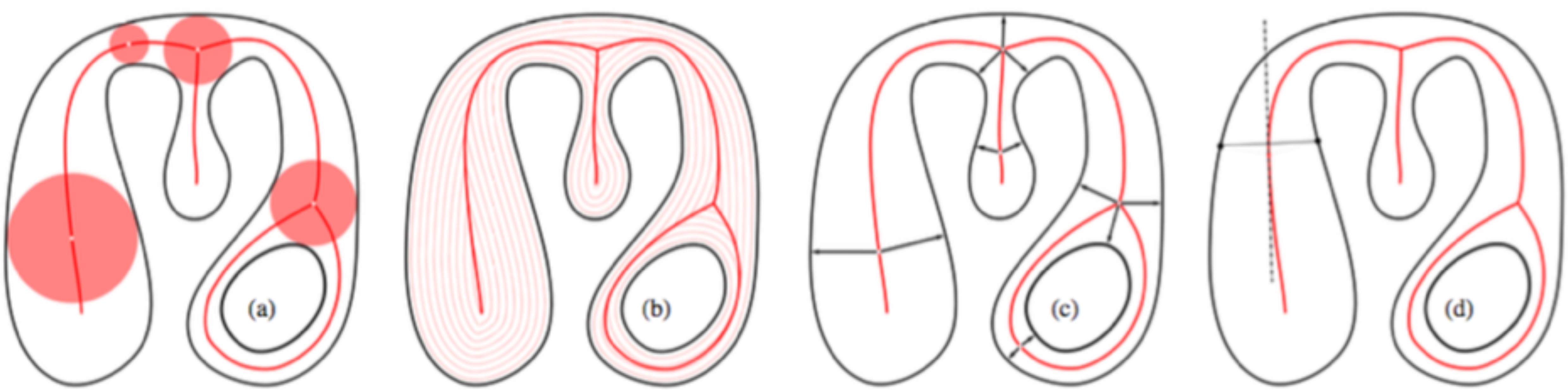
Conclusions

- Unifying characterization of skeletons
  - tied by the alternative definitions of MAT
  - the tighter a method is to one of these definition, the better it, in *our* experience, performs.
- Lightweight entry point to skeleton techniques



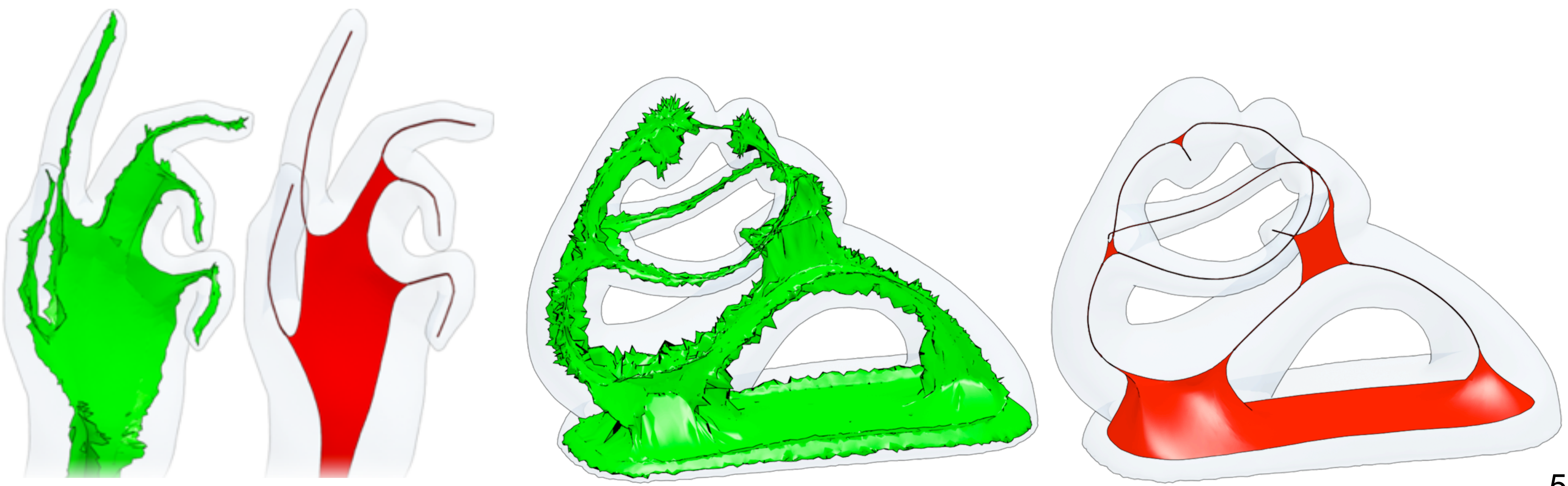
Conclusions

- Medial structures and skeletons are used in a variety of application domains
  - still, the preferred usage domain of skeletons seems to be the one of *organic* shapes, just as it was intended by [Blum'67]
- Advancements at the theoretical and algorithmic levels have made them also a practical solution for several tasks
  - processing: reconstruction, repairing, features
  - segmentation, deformation, matching, fabrication

Unifying Continuous Formulation

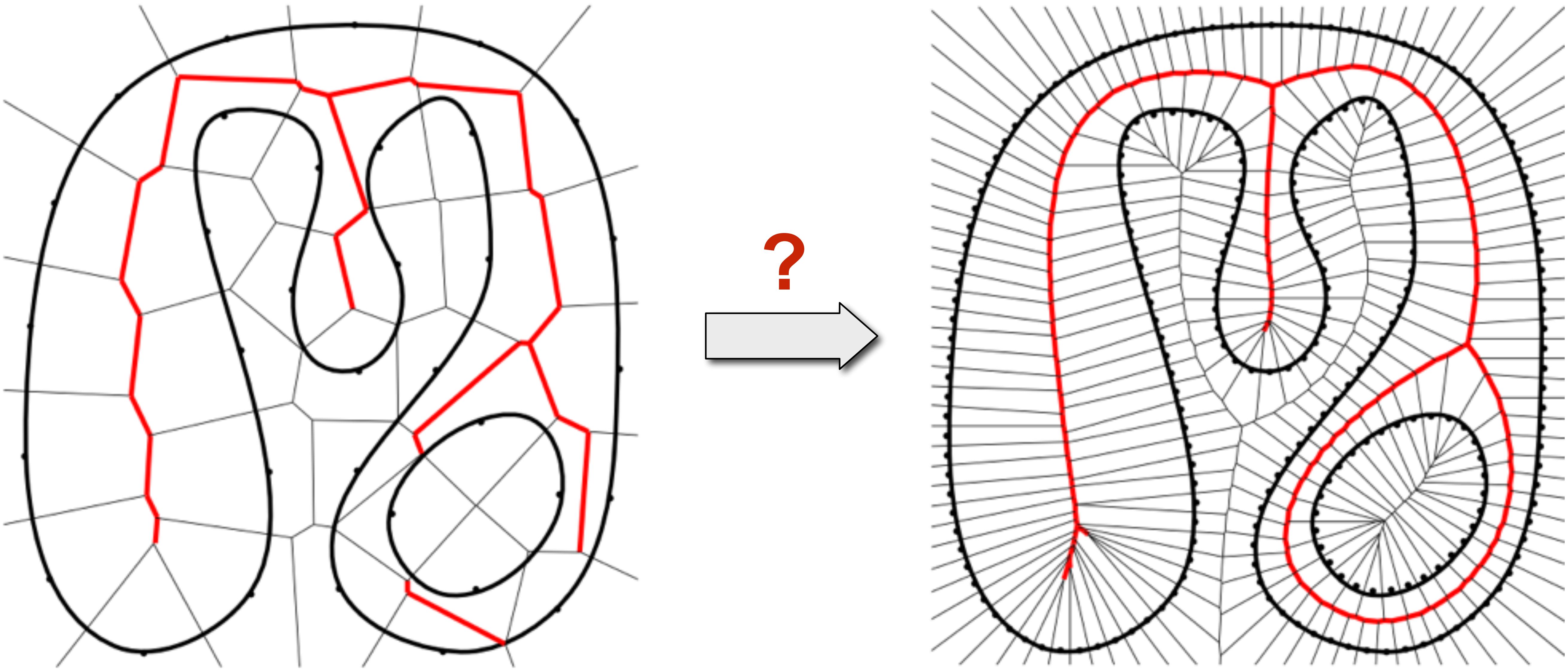
- Surface and curve skeletons
  - – a unifying definition is missing… yet:

$$\dot{\mathcal{S}}(t) = -\mathbf{n}(t) \quad \overset{?}{+} \quad \begin{cases} \dot{\mathcal{S}}(t) = -H(t)\mathbf{n}(t) & \text{if } H(t) < \infty \\ \dot{\mathcal{S}}(t) = 0 & \text{otherwise} \end{cases}$$



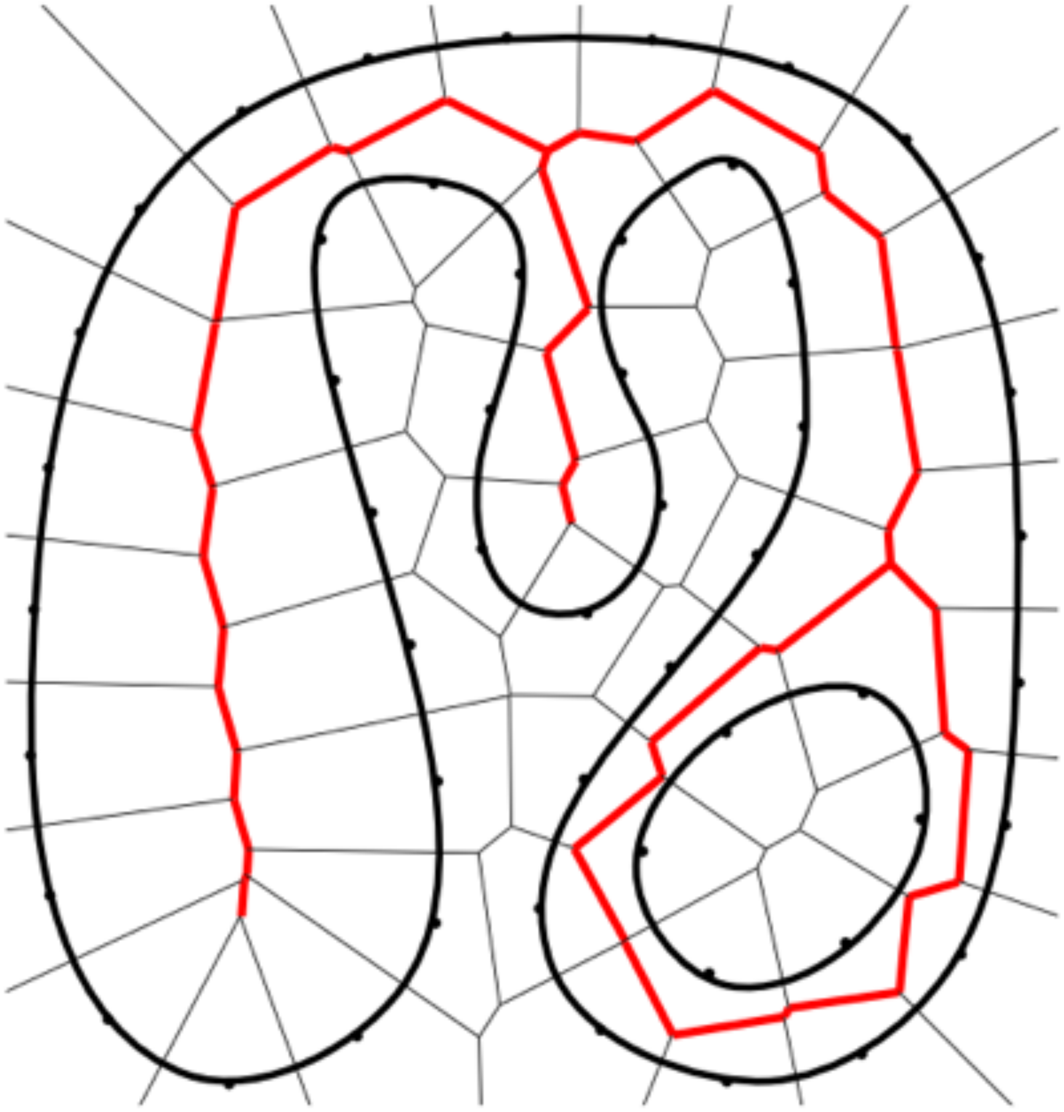
Medial Axis Transform

- Can we generate an ϵ-sampling? (watertight)
  - (e.g. assume oracle for inside/outside queries)



Medial Axis Transform

- Can we optimize sample locations?
  - (assume *sparsest* ϵ-sampling)



Beyond the STAR

- The power of 3D skeletons lies in their nature:
  - integrating information of **volume** and **surface**
  - preserving important global information - topology

- How to formalize 3D skeletons to become a new modelling framework for 3D shapes?

- Design operators for 3D skeletons that support easy modelling operations?

Beyond the STAR

- Benchmarks for 3D skeletonization methods
  - quantitative and qualitative performance metrics
  - shape categories
  - application tracks

- External resources
  - http://gfx.uvic.ca/pubs/2016/skelstar/slides
  - http://www.cs.rug.nl/svcg/Shapes/SkelBenchmark
  - http://www.imati.cnr.it

Thank you for listening

Questions?