EUROGRAPHICS 2014
Strasbourg, France

State of the Art in Surface Reconstruction from Point Clouds

Matthew Berger **Andrea Tagliasacchi** Lee M. Seversky

Pierre Alliez Joshua A. Levine Andrei Sharf Claudio T. Silva

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

CLEMSON UNIVERSITY

informatiques mathématiques

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Ben-Gurion University of the Negev

NYU

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State of the Art in Surface R s

PART #2: Smooth Priors

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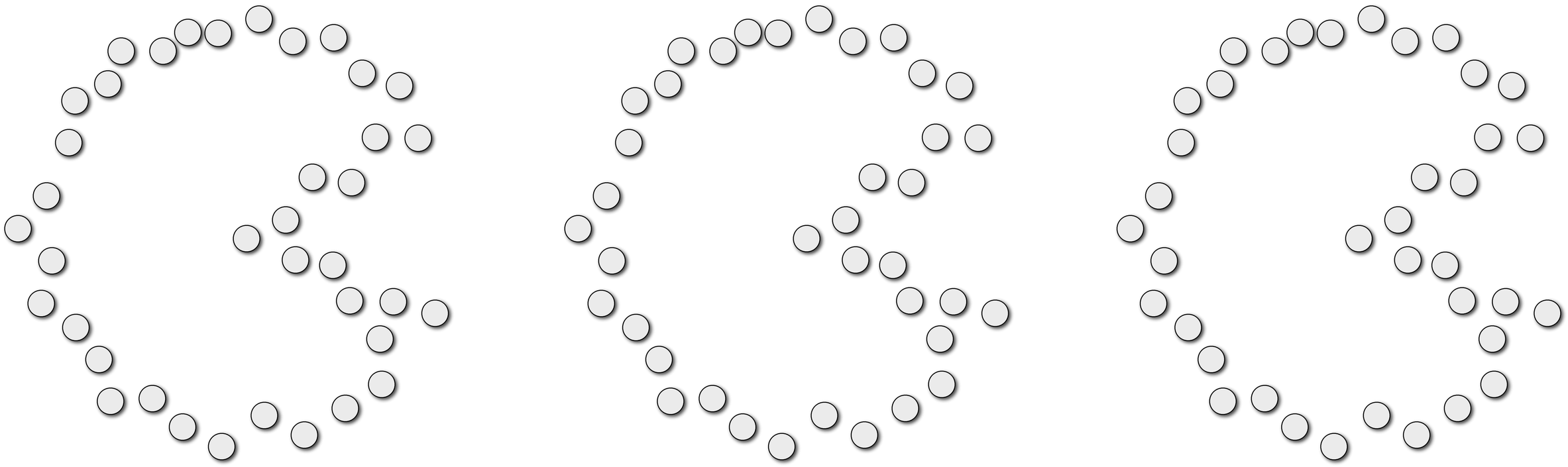
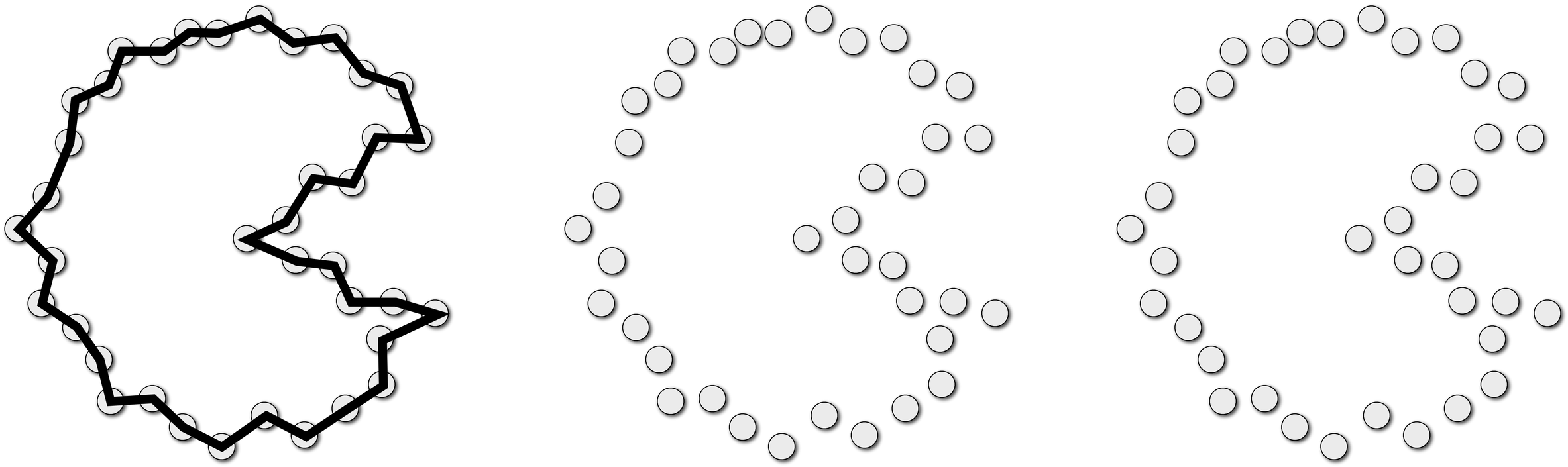
Smooth Priors



Many candidate surfaces for the reconstruction problem!

Smooth Priors



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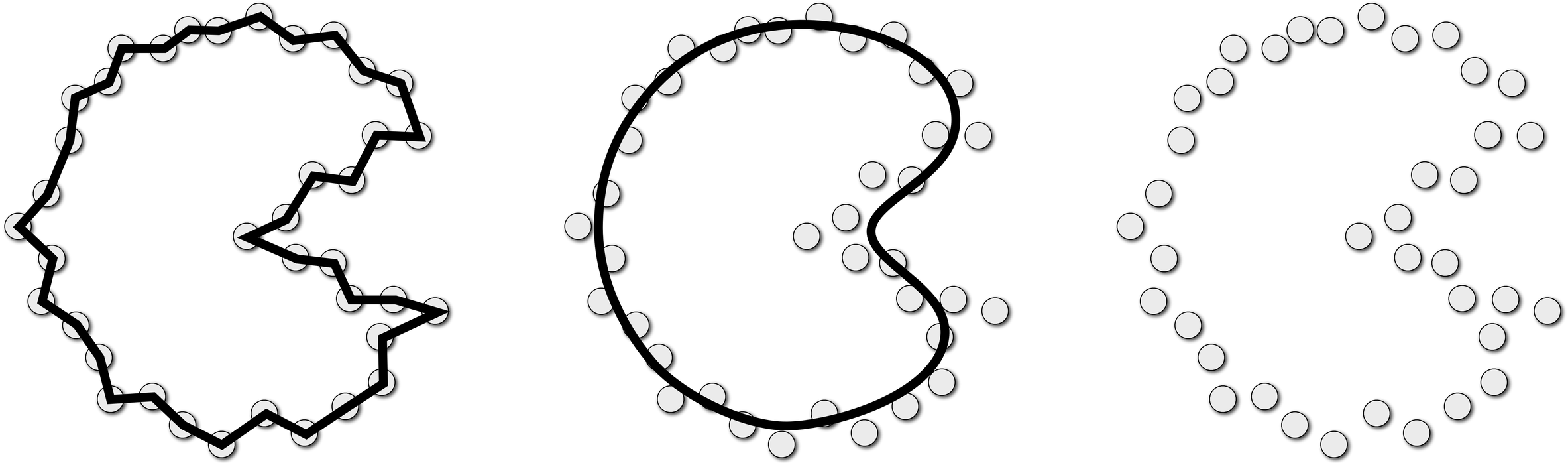

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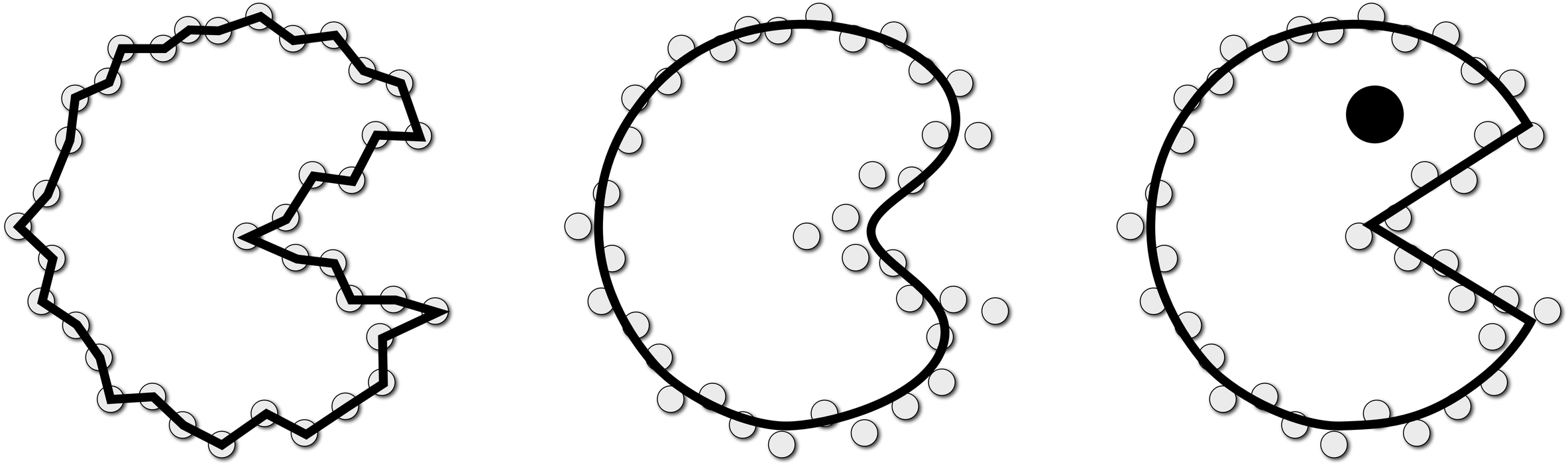

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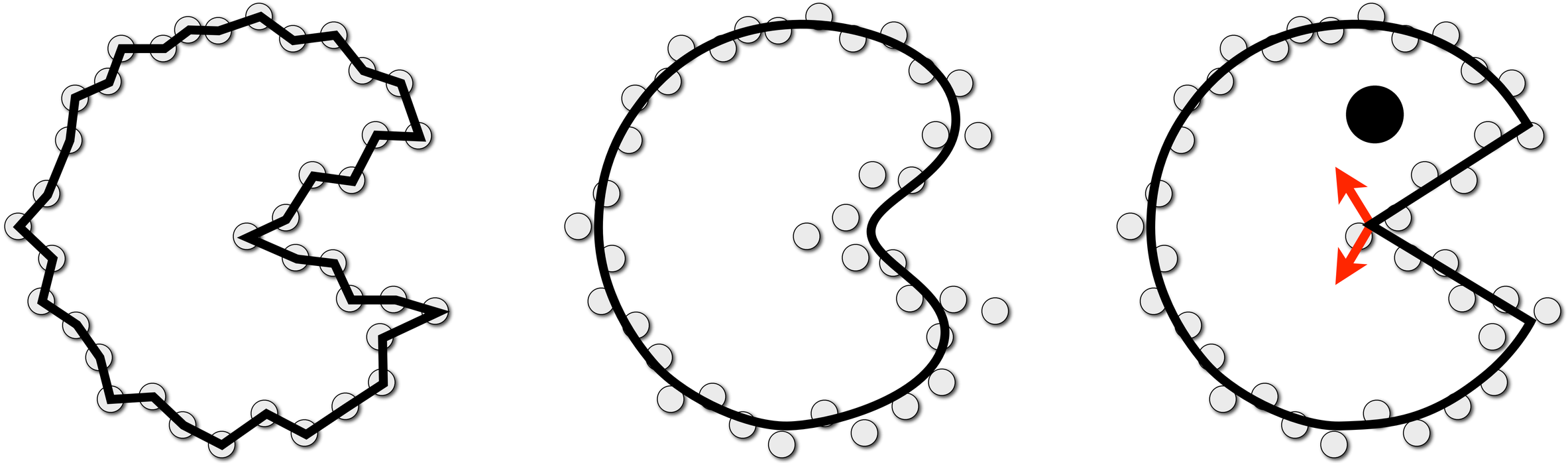

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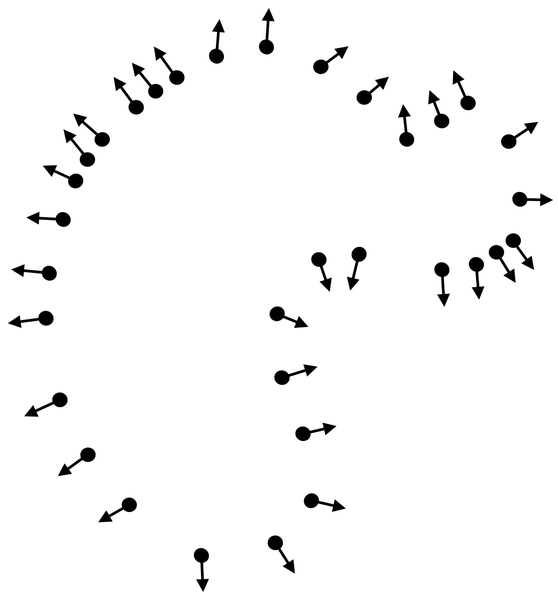
Notation

Notation

Point Cloud



$$P = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$$
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Notation

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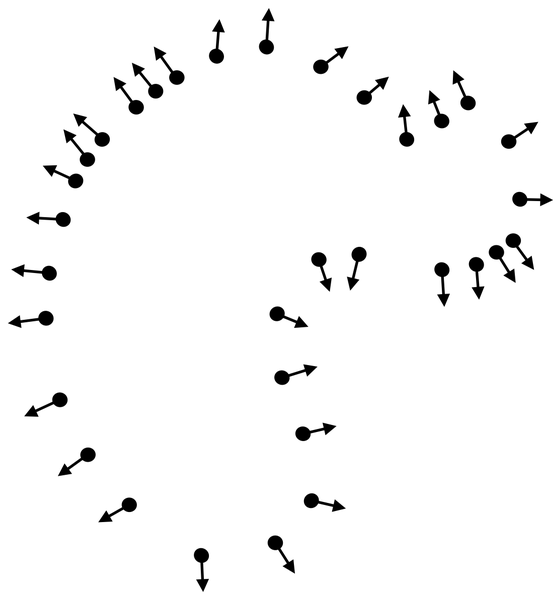
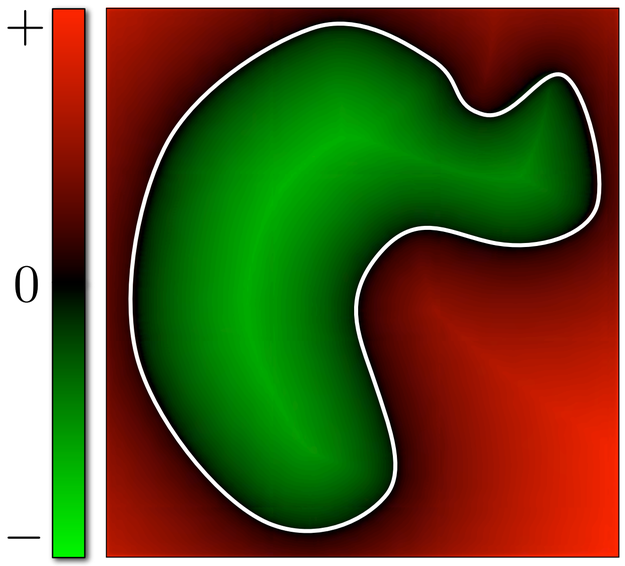


Signed Distance Function



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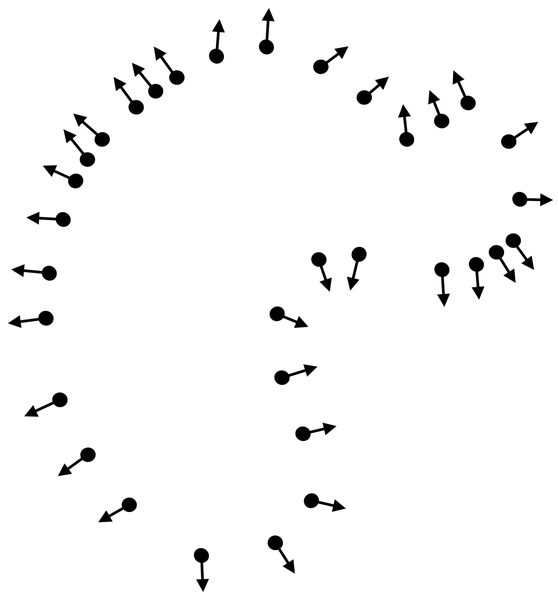

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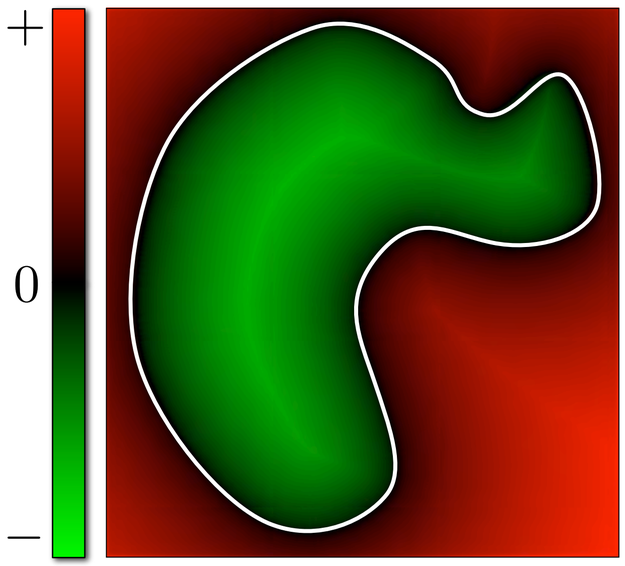

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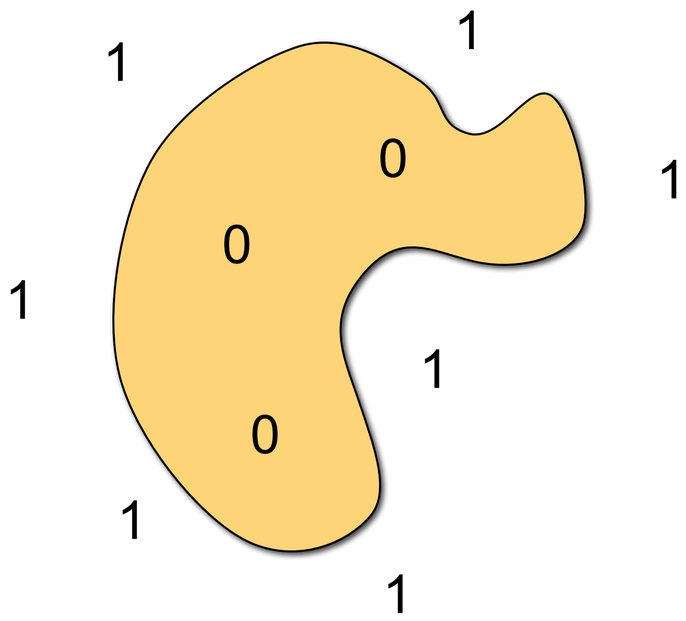
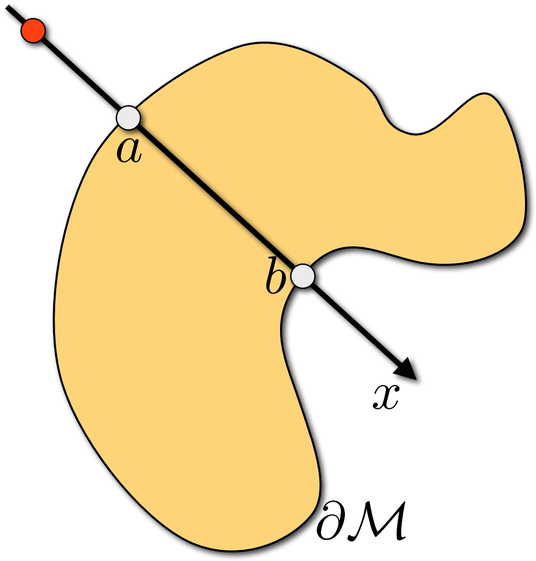
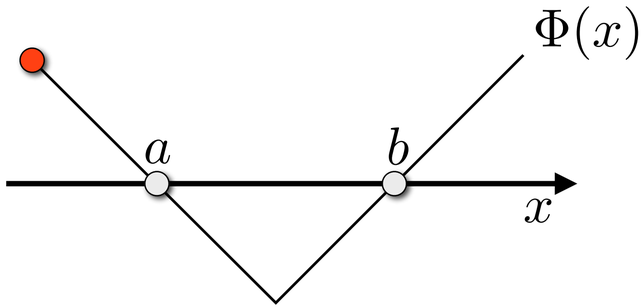
Indicator Function

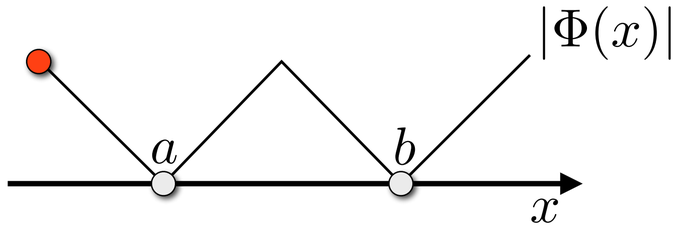


$$\chi(\mathbf{x}) = \begin{cases} 0 \text{ if } \mathbf{x} \in \mathcal{M} \\ 1 \text{ if } \mathbf{x} \notin \mathcal{M} \end{cases}$$

Notation







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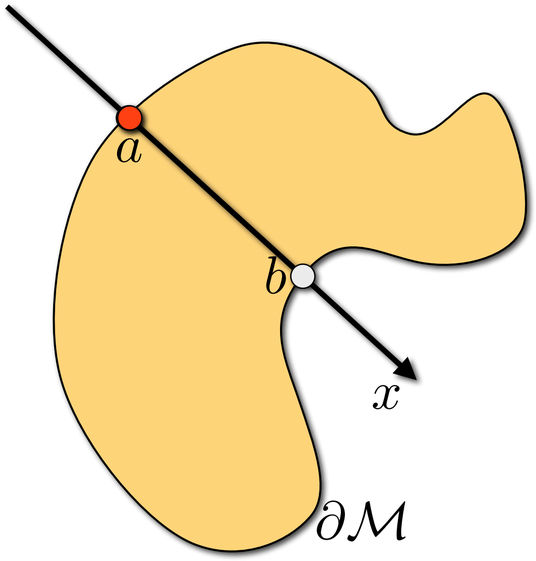

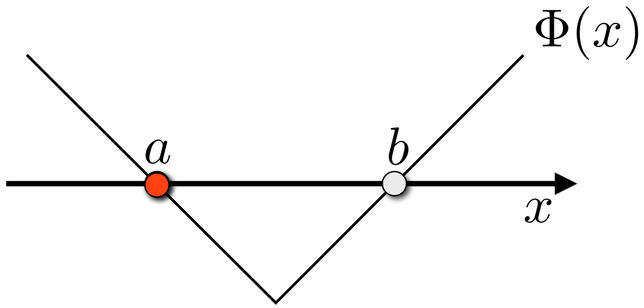

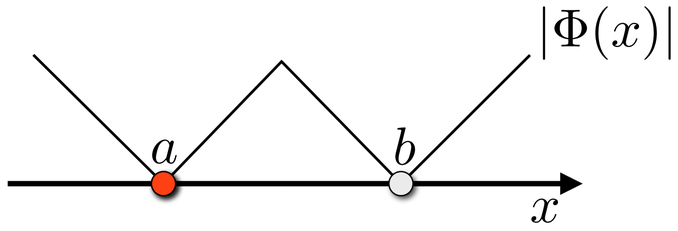

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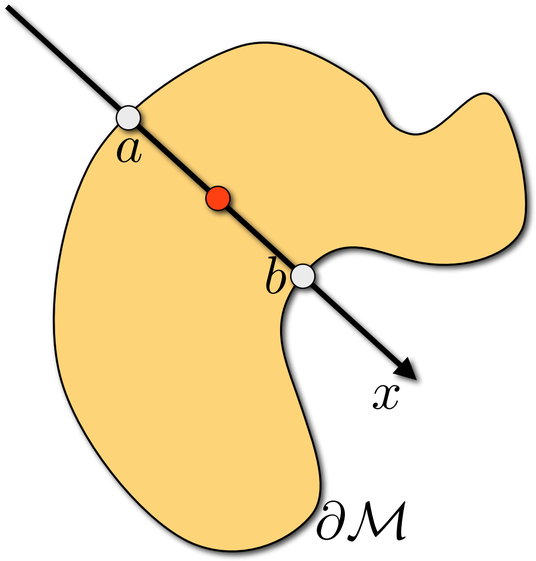

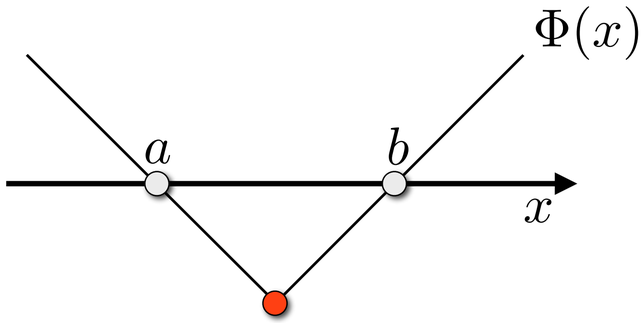

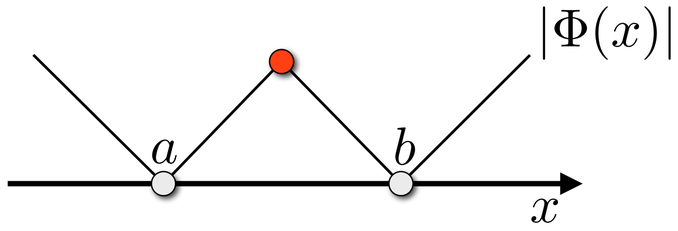

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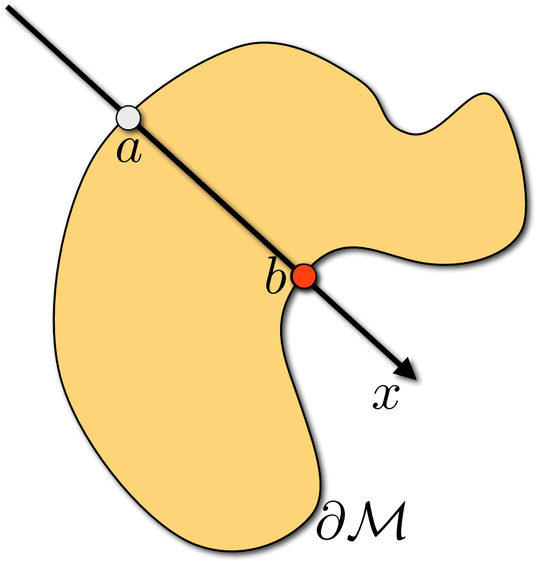

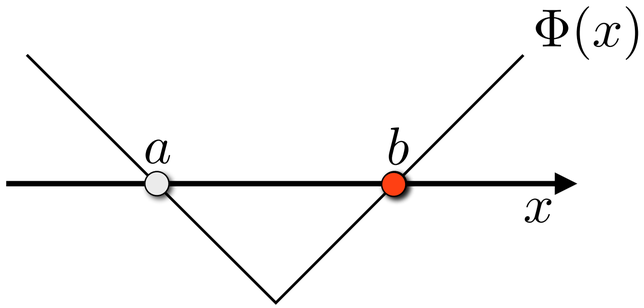

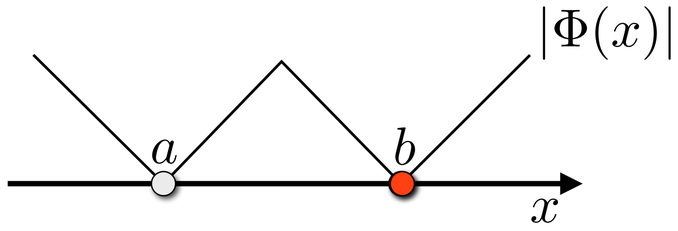

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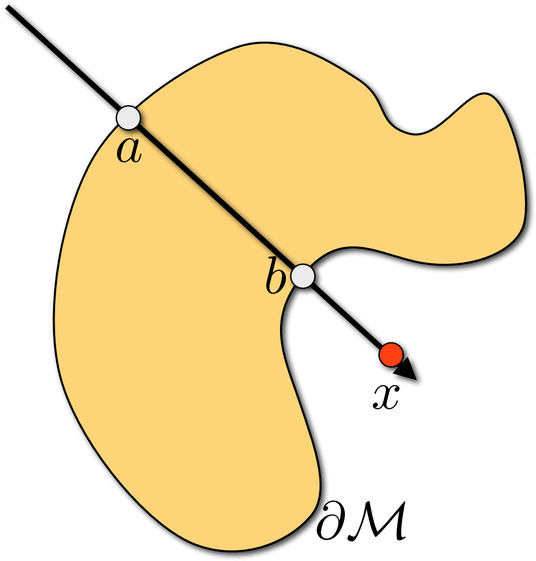

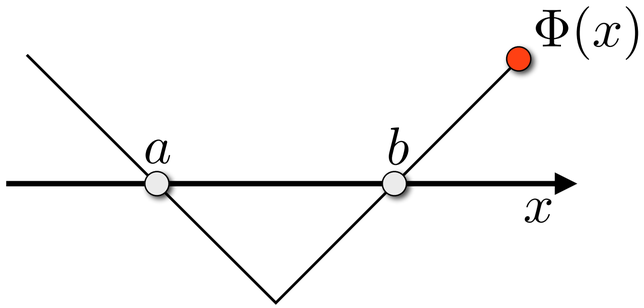

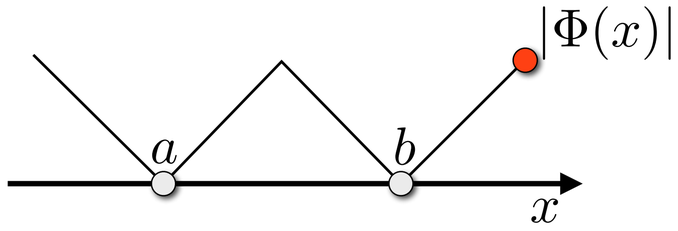

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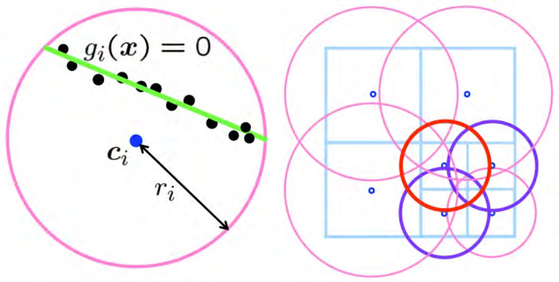
Surface Smoothness Priors

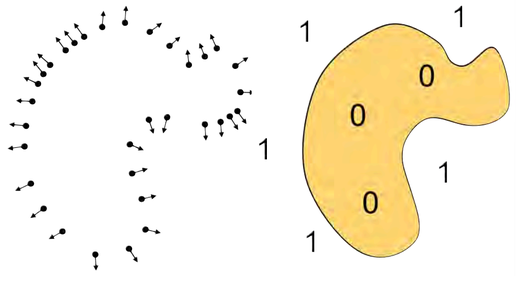
Local Smoothness



$$g_i(\mathbf{x}) = \alpha_i(\mathbf{x} - \mathbf{c}_i) + \beta_i$$

- Local solve ---> local fitting
- no control away from data
- solution by interpolation

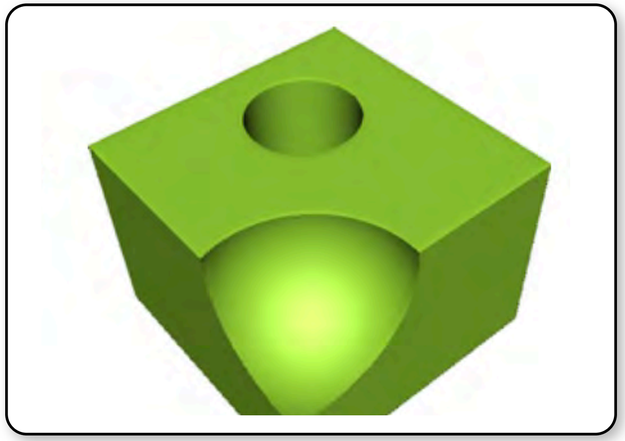
Global Smoothness



$$\min_{\chi} \int \|\nabla\chi(\mathbf{x}) - \mathbf{v}(\mathbf{x})\|^2 \, \mathrm{d}\mathbf{x}$$

- Global solve ---> global fitting
- linear problem, eigenvalue problem, graph cut
- Missing data in-painting

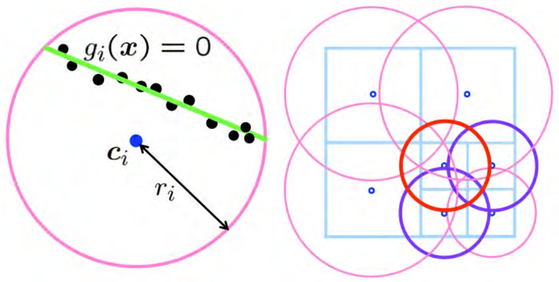
Piecewise Smoothness



- Not smooth everywhere!
- Sharp near features
- Sparse discontinuities
- Non-continuous problem

Surface Smoothness Priors

Local Smoothness



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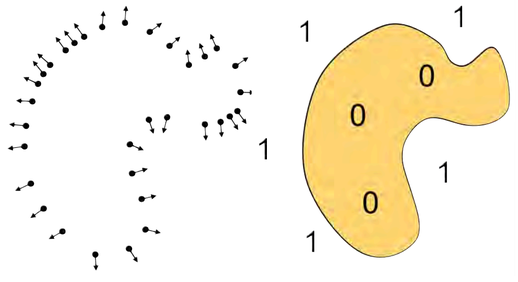

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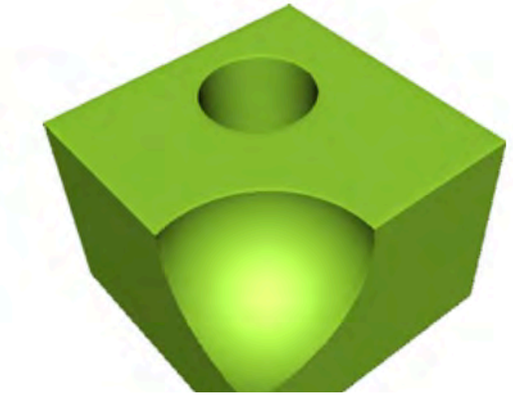

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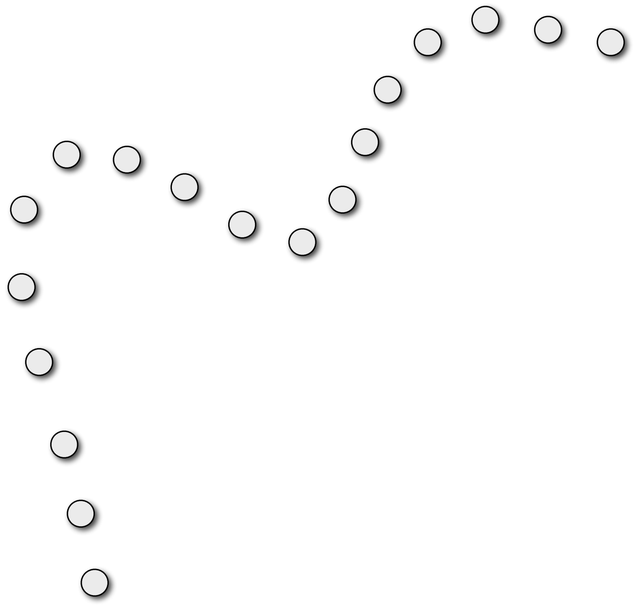
“Classical” Reconstruction

- **Construct SDF from point samples**
  - Estimate un-oriented normals

$$\arg\min_{\mathbf{n},\mathbf{c}} \sum_{i=1}^{m} (\mathbf{n}^T(\mathbf{p}_i - \mathbf{c}))^2 \text{ s.t. } \|\mathbf{n}\|_2^2 = 1$$



[Hoppe et al. SIG’92]

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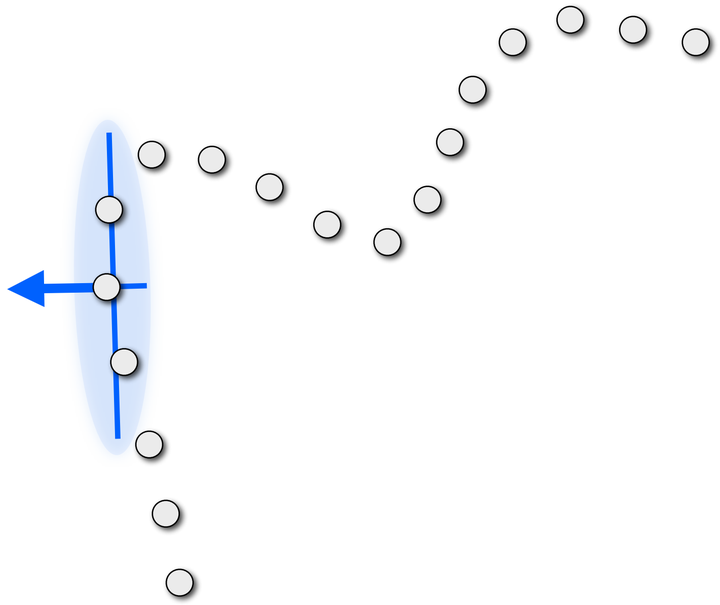

[Hoppe et al. SIG’92]

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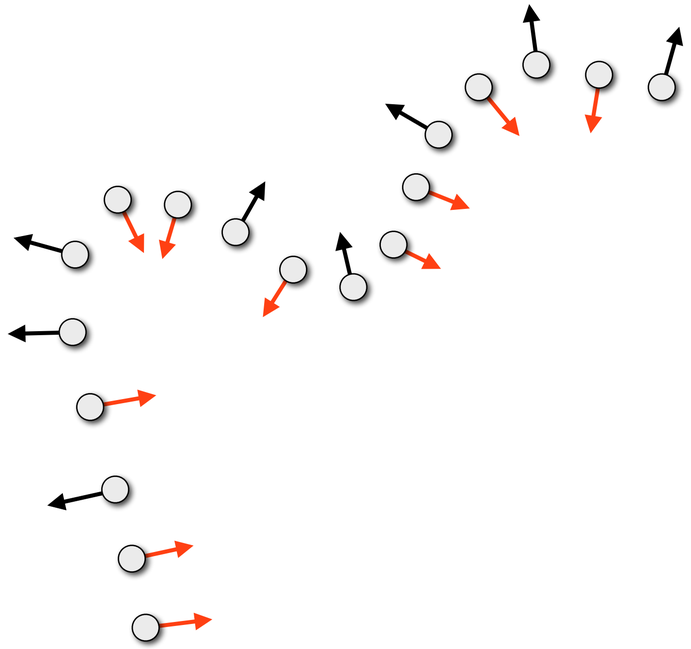

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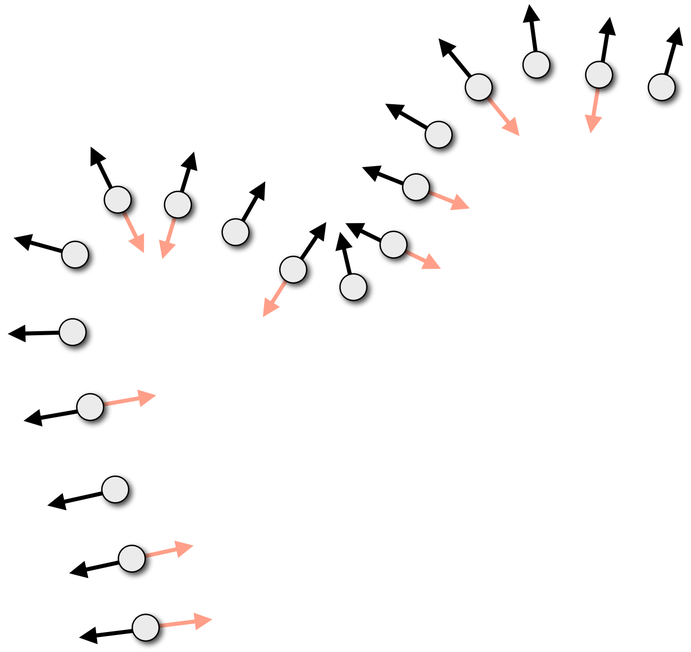

[Hoppe et al. SIG’92]

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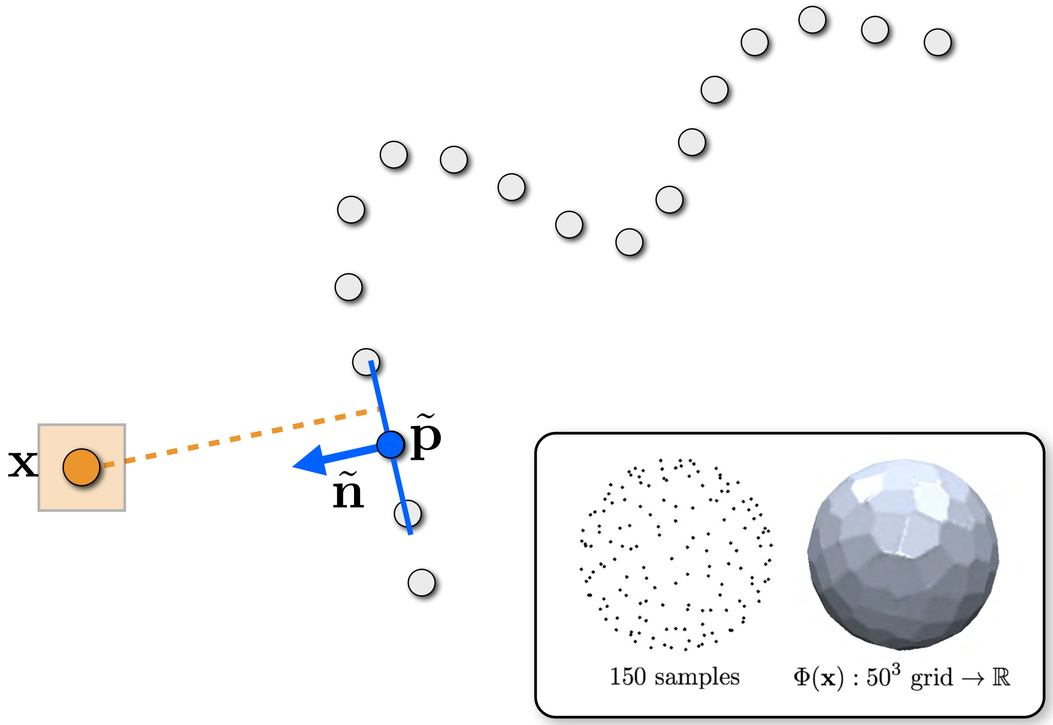

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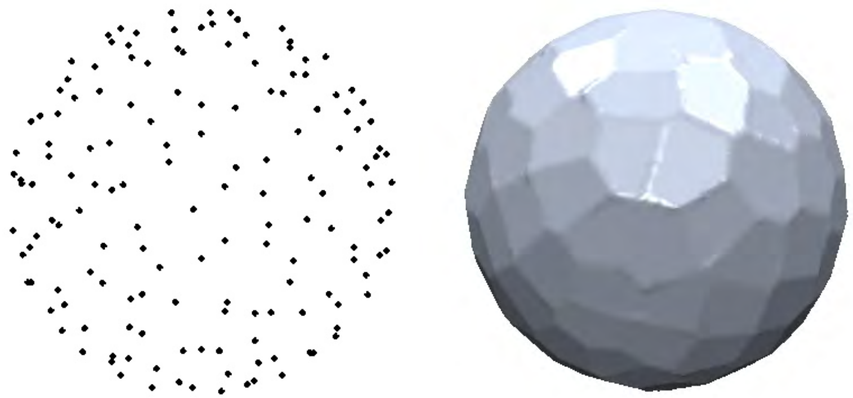
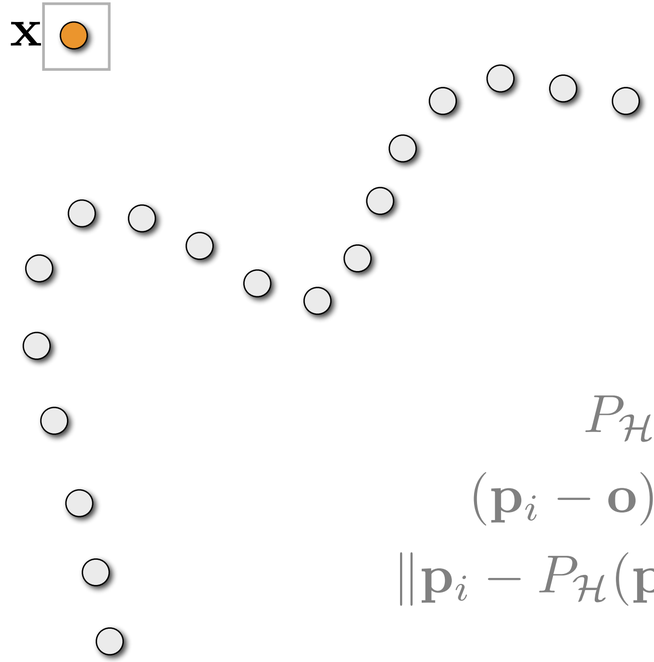


150 samples $\Phi(\mathbf{x}) : 50^3 \text{ grid} \rightarrow \mathbb{R}$

[Hoppe et al. SIG’92]

Moving Least Squares

- **Construct SDF from “ local parameterizations”**



$$P_{\mathcal{H}}(\mathbf{x}) = \mathbf{o}$$

$$(\mathbf{p}_i - \mathbf{o}) \cdot \mathbf{n} = x_i$$

$$\|\mathbf{p}_i - P_{\mathcal{H}}(\mathbf{p}_i)\| = y_i$$

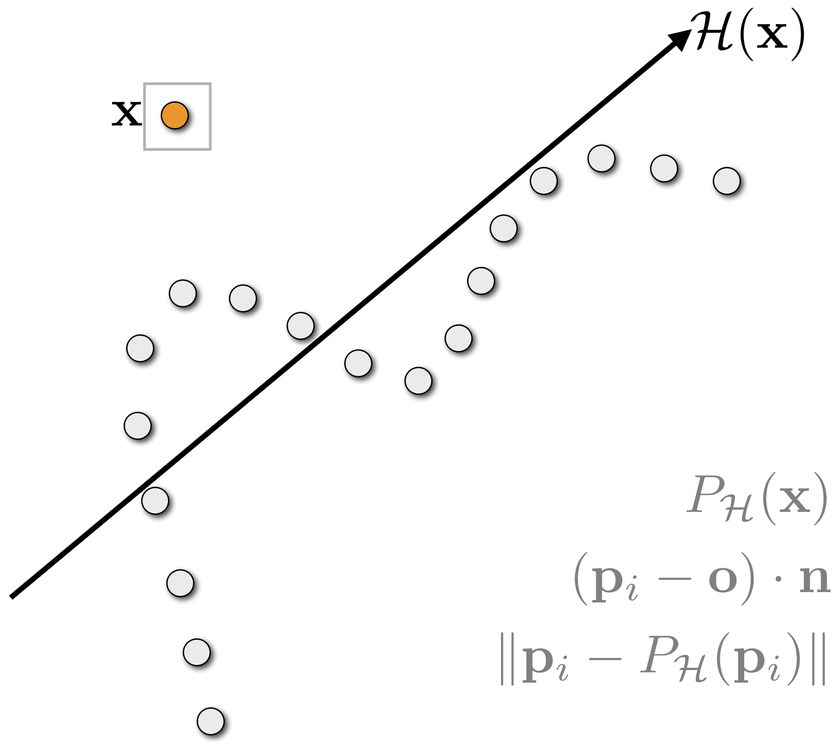
[Alexa et al. VIS’01]

Moving Least Squares

- **Construct SDF from " local parameterizations"**

  - Estimate parameterization $\mathcal{H}(\mathbf{x})$

$$\mathcal{H}(\mathbf{x}) = \arg\min_{\mathbf{n},D} \sum_i (\mathbf{n}\cdot\mathbf{p}_i - D)^2 \theta(\|\mathbf{p}_i - P_{\mathcal{H}}(\mathbf{x})\|)$$



$$P_{\mathcal{H}}(\mathbf{x}) = \mathbf{o}$$
$$(\mathbf{p}_i - \mathbf{o})\cdot\mathbf{n} = x_i$$
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[Alexa et al. VIS'01]

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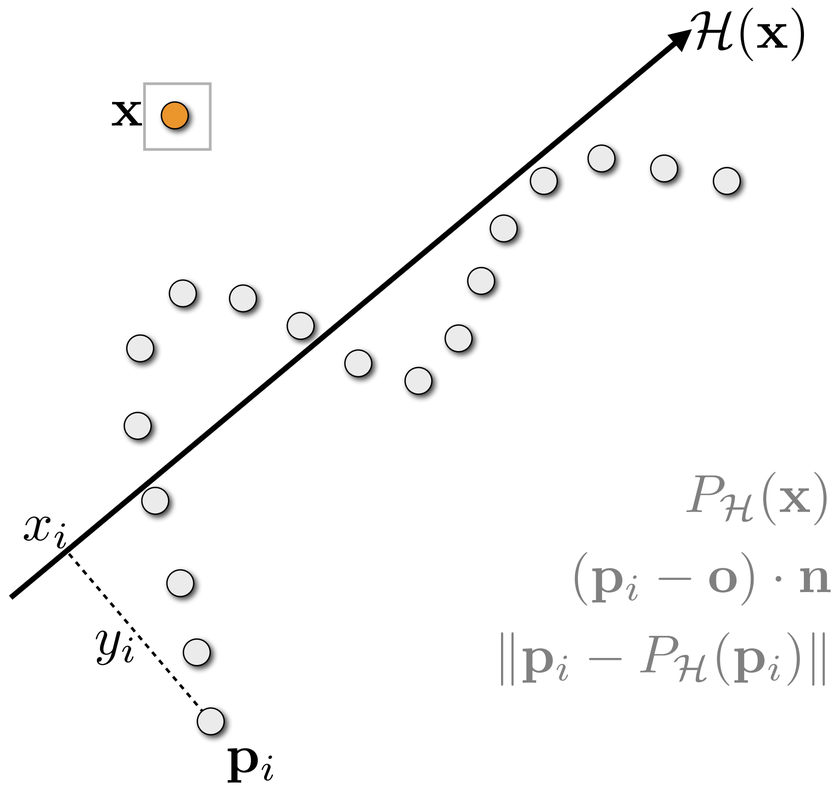

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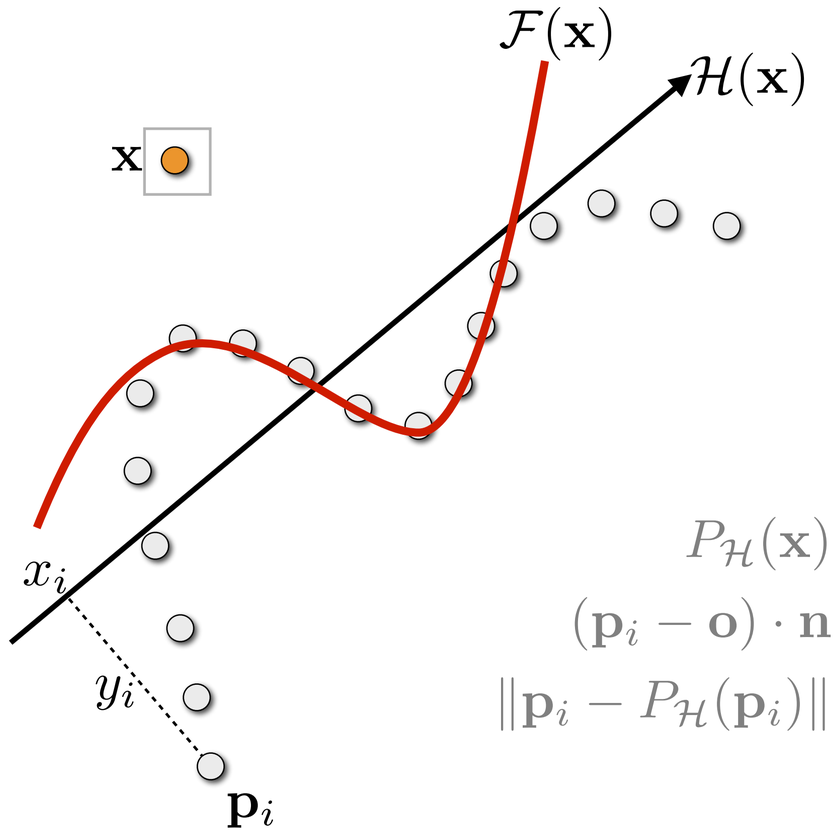

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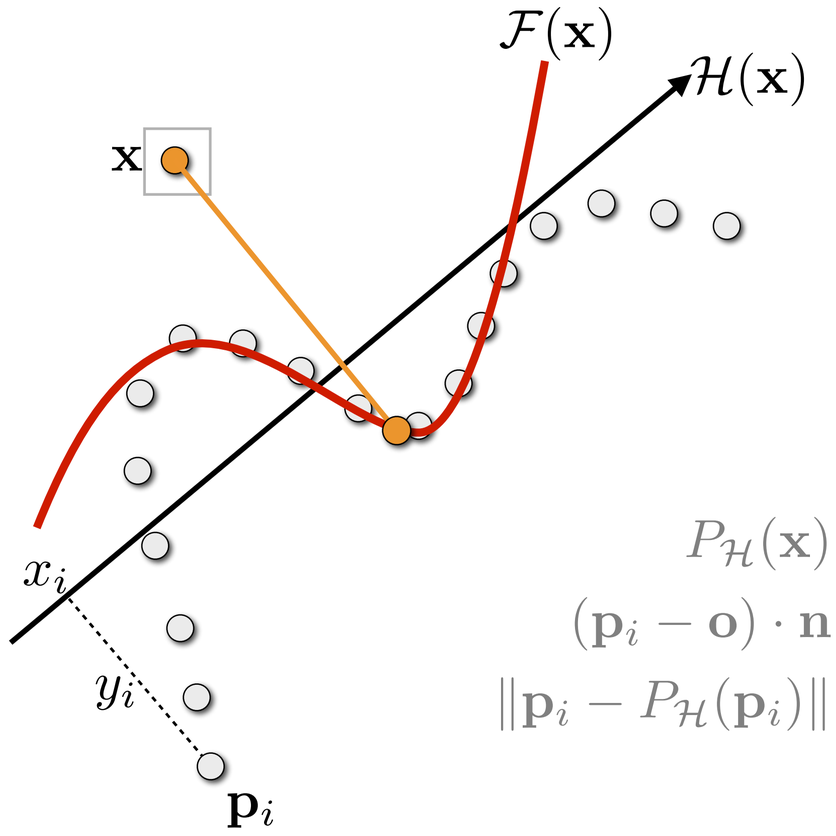

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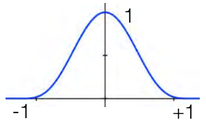
[Alexa et al. VIS'01]

Moving Least Squares

- **Construct SDF from " local parameterizations"**
  - Pick radial weight function $\theta(d)$

    $\theta(d) = (|d| < 1)?(1 - d^2)^4 : 0$

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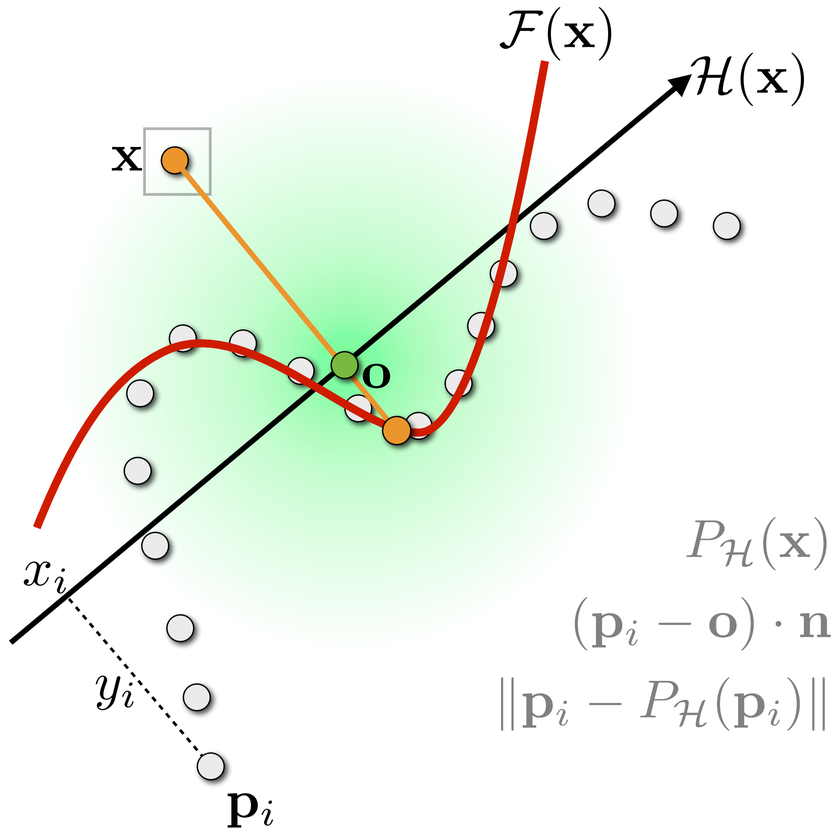

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[Alexa et al. VIS'01]

Interpolatory* MLS

- Blend point-to-plane distances

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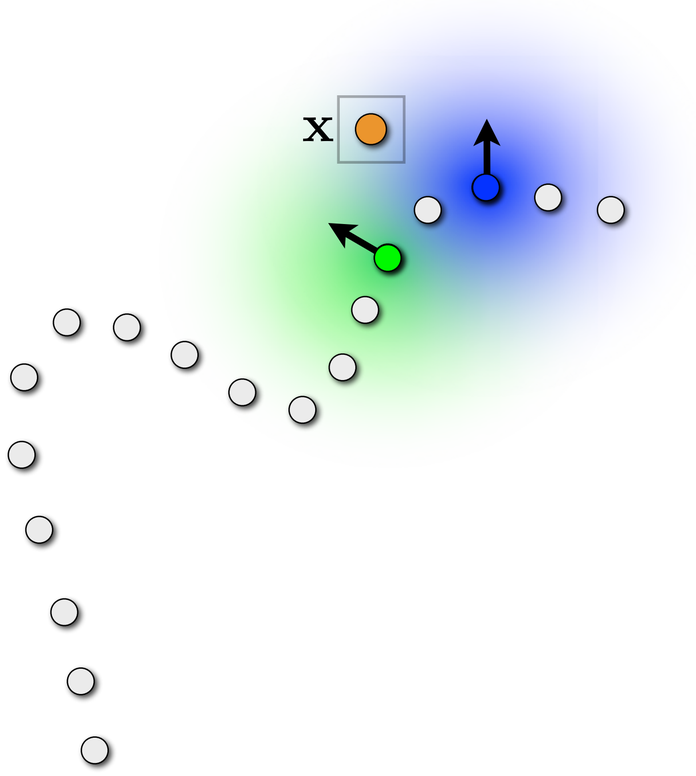

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[Shen et al. SIG'04]

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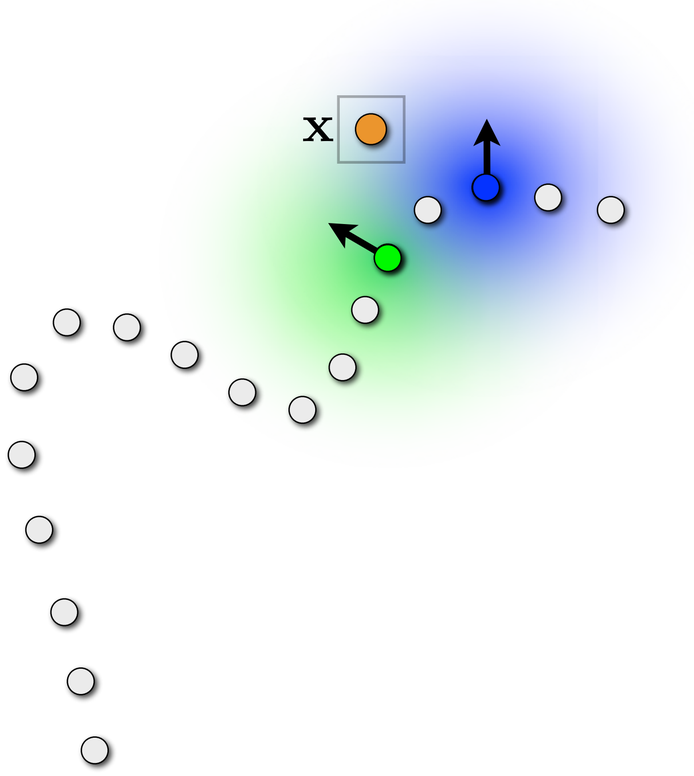

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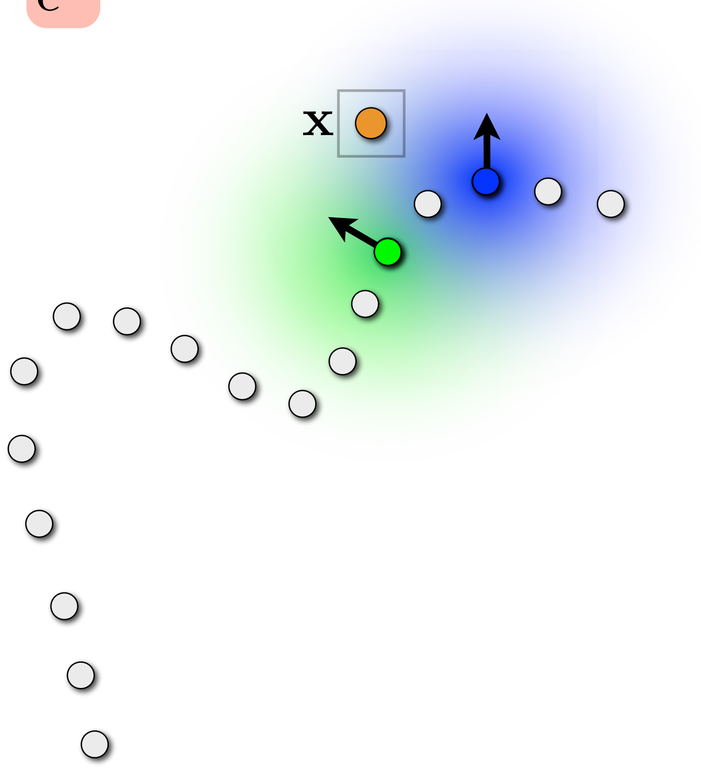

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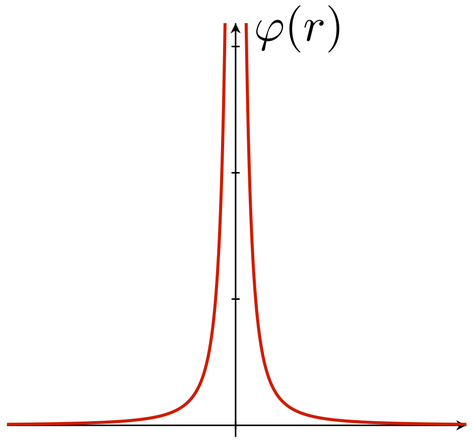

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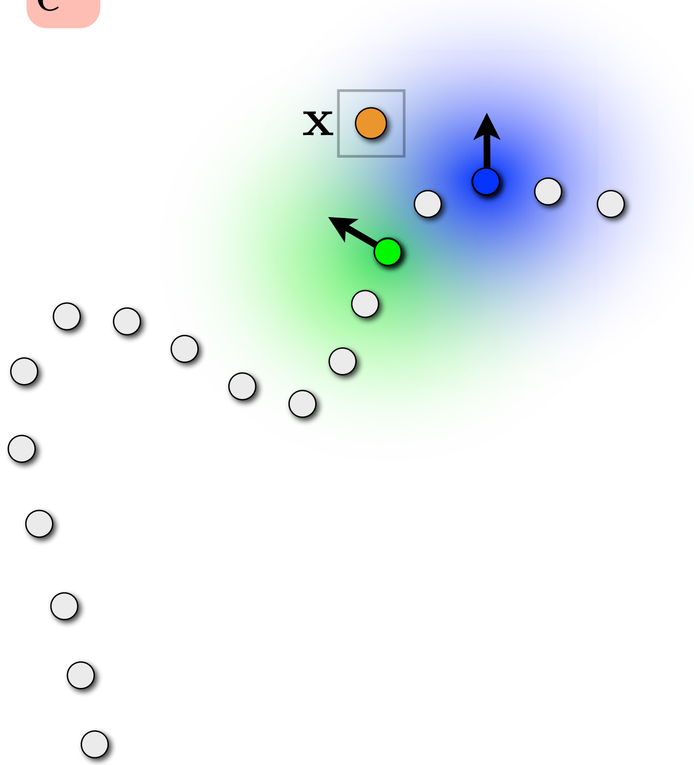

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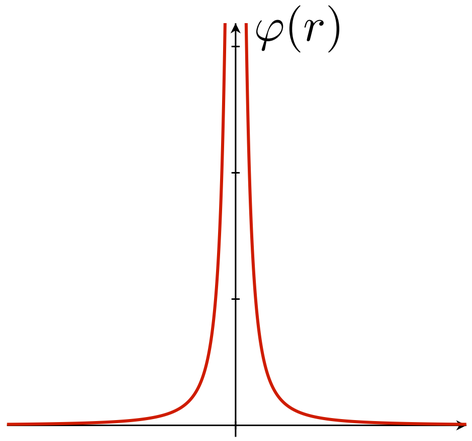

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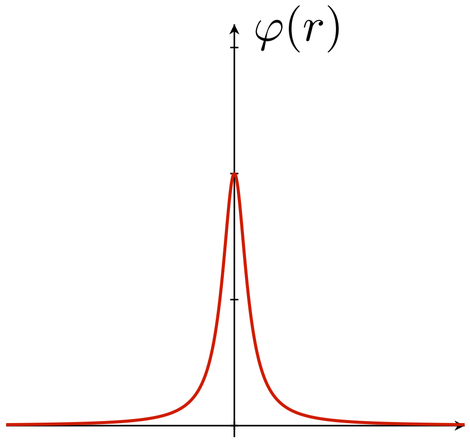

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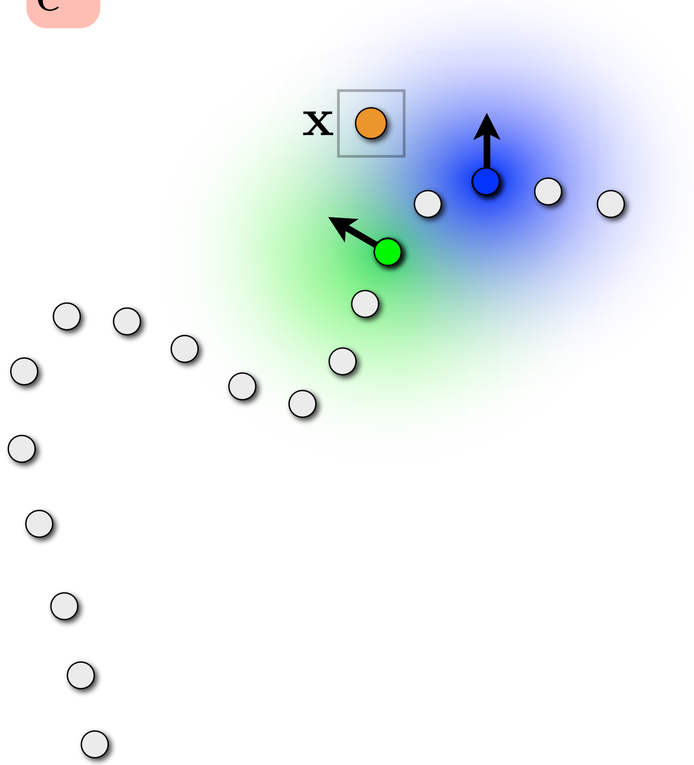

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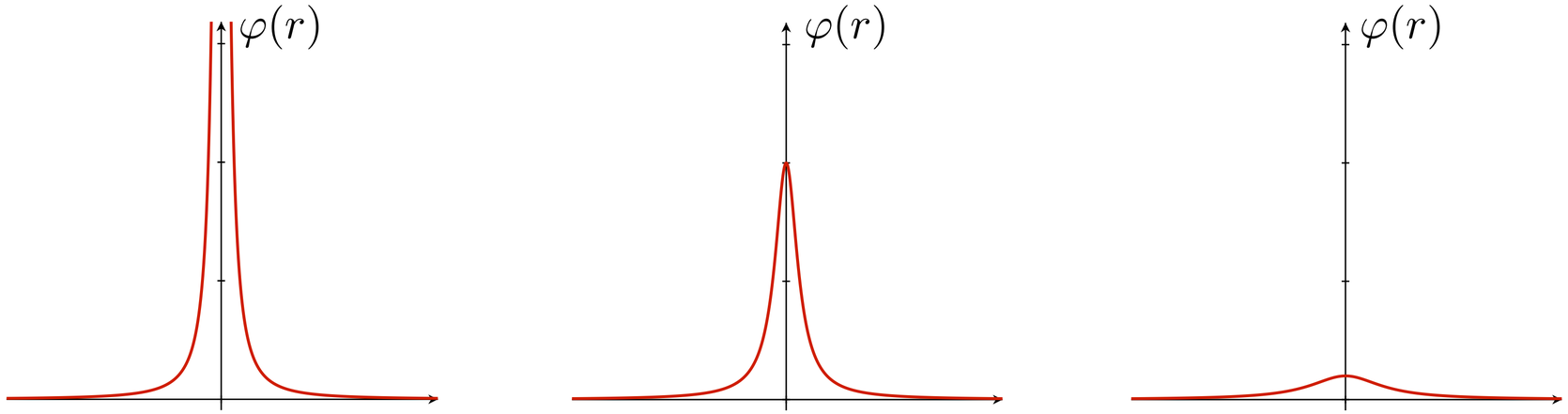

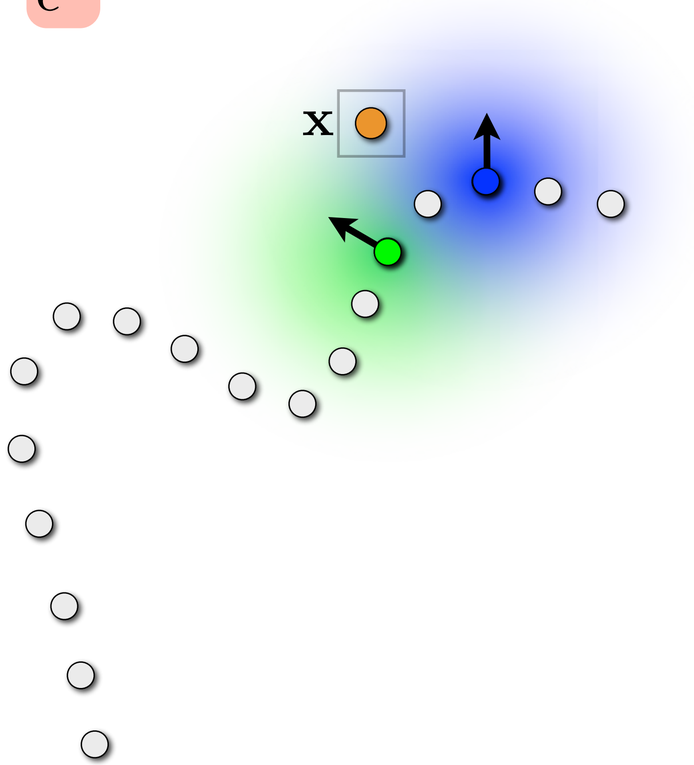

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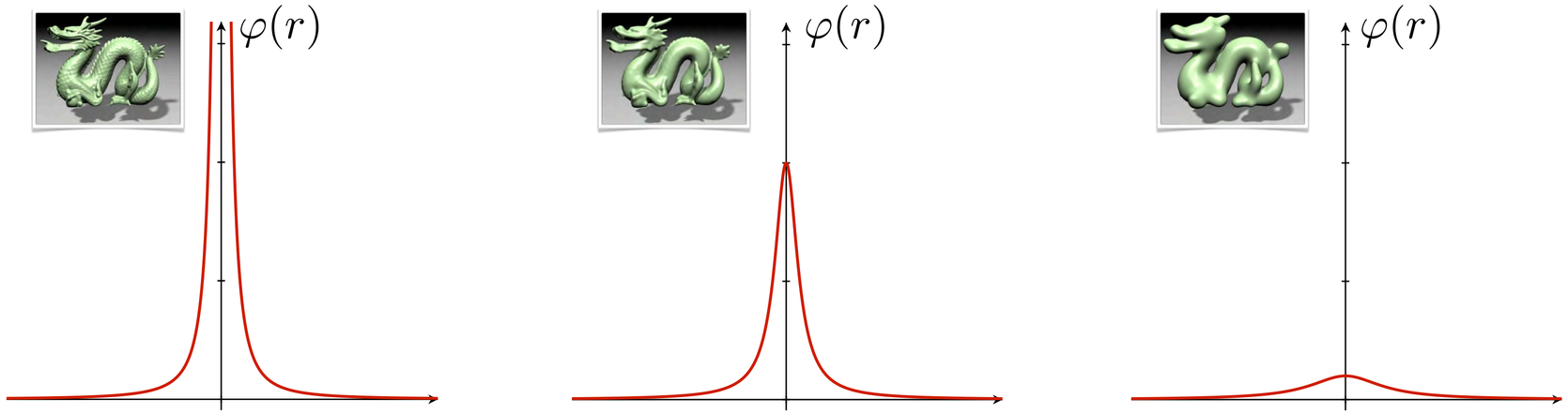

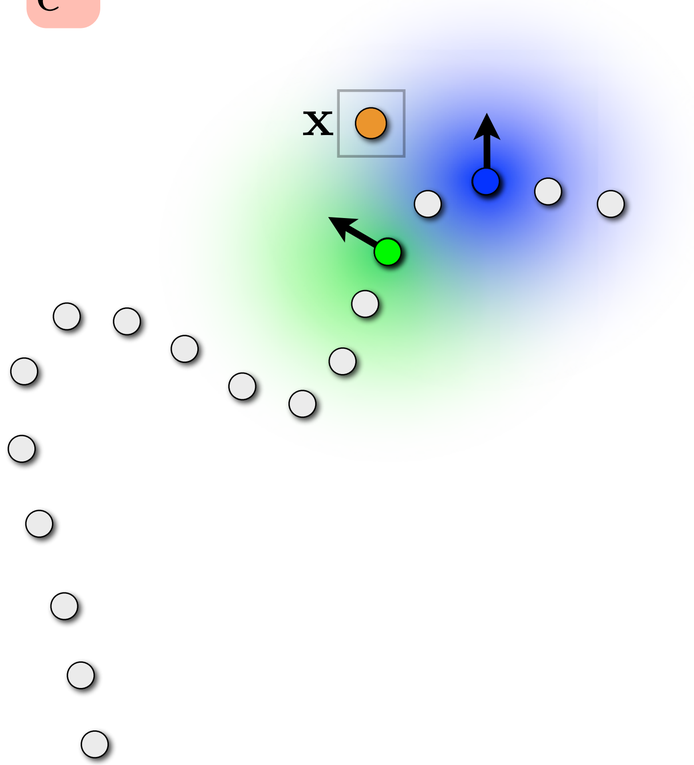

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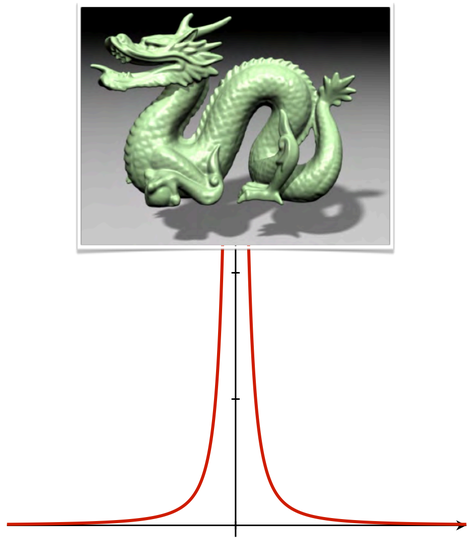

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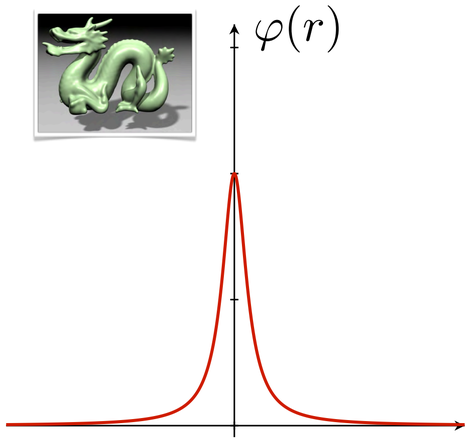

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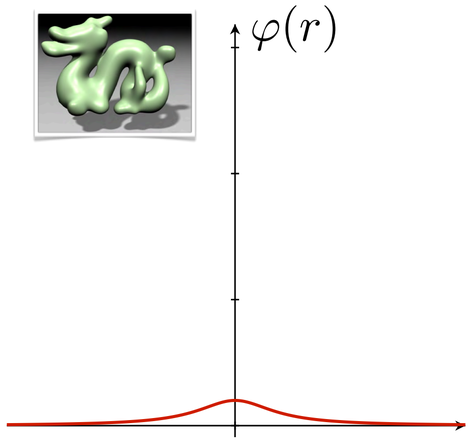

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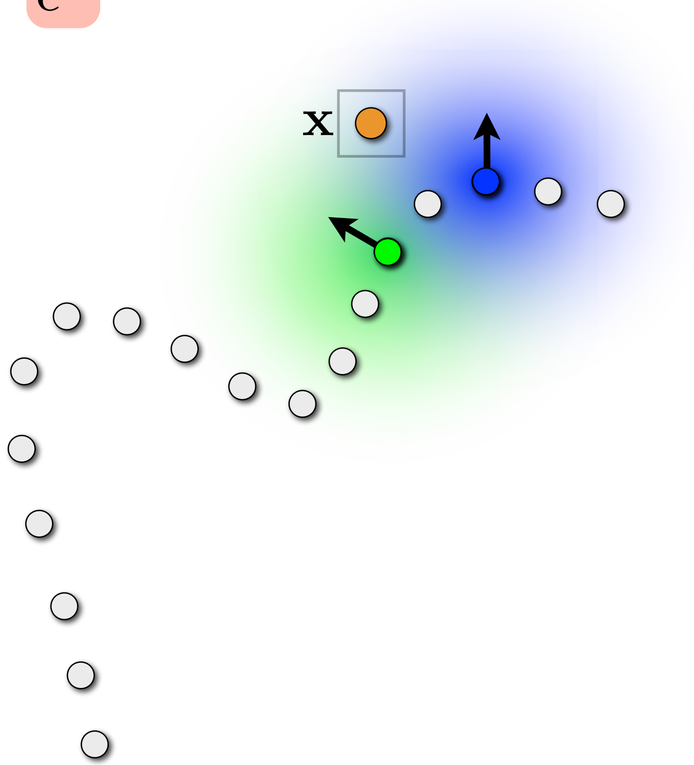

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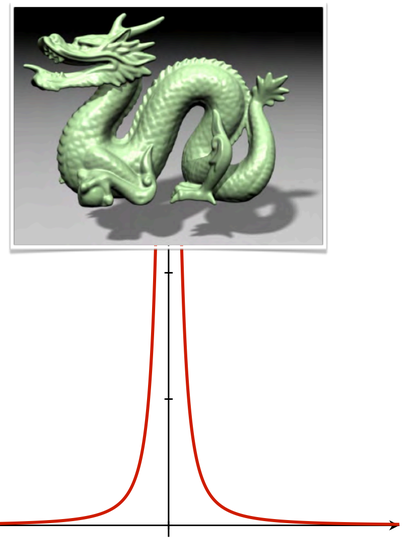

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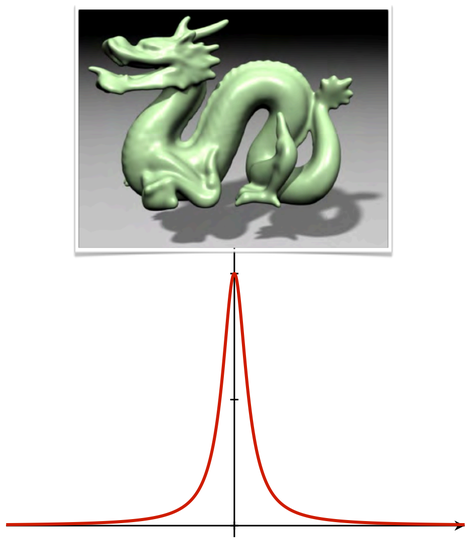

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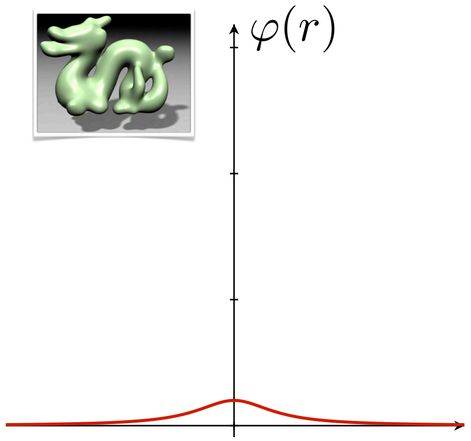

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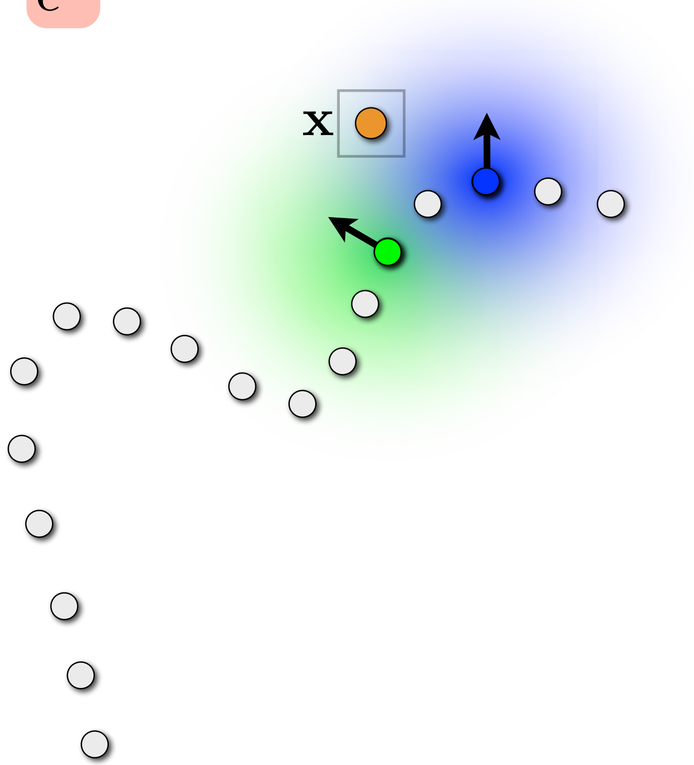

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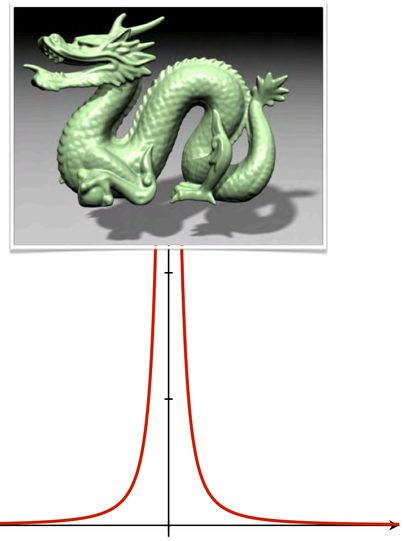

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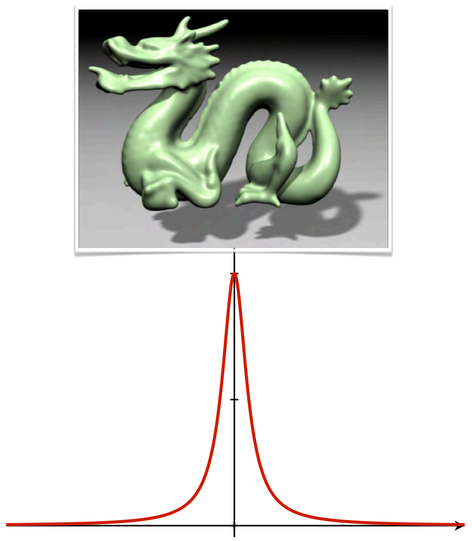

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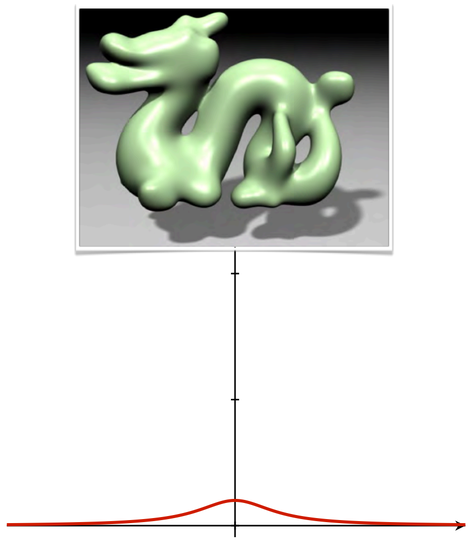

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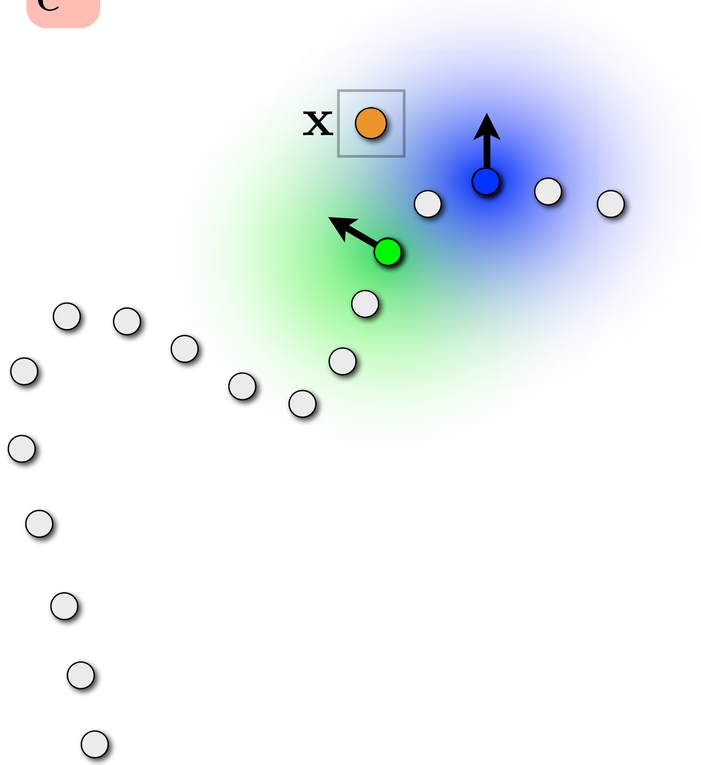

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Parameterization Free

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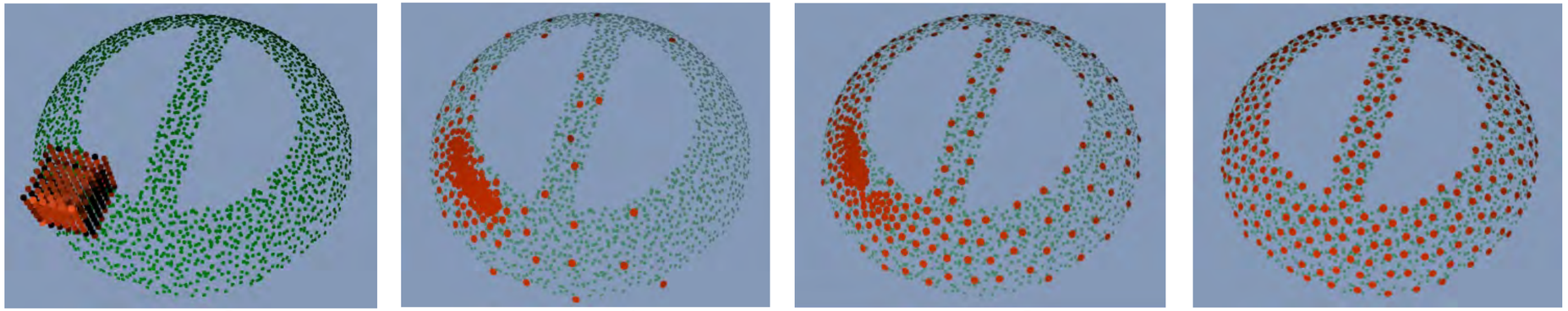
$$Q^t = \arg\min_Q \quad E_{data}(Q^{t-1}, P) + E_{spread}(Q^{t-1}, Q)$$

P



$Q^{t=0}$ $Q^{t=1}$ $Q^{t=2}$

[Lipman et al. SIG'07]

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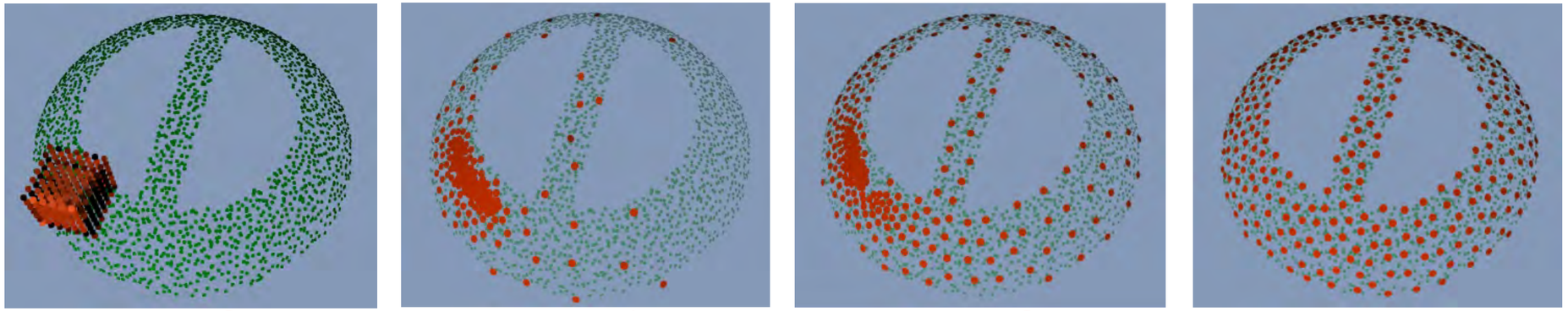

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ℓ_2^1 fitting

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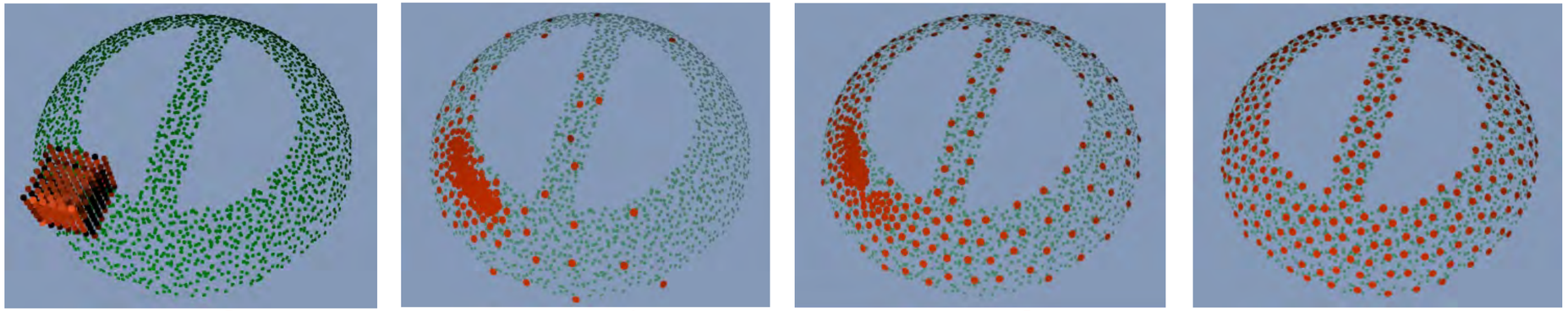

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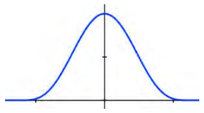

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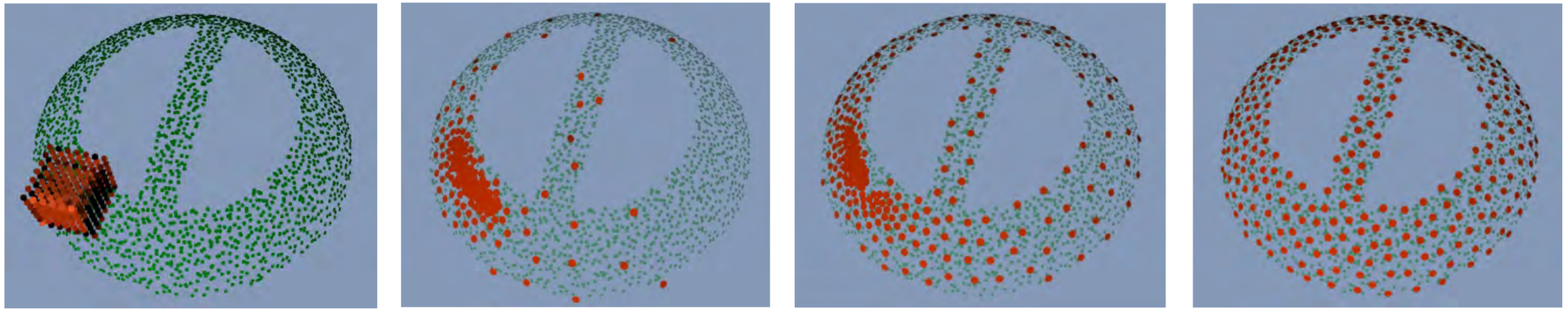
ℓ_2^1 fitting

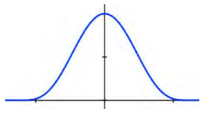
P



$Q^{t=0}$ $Q^{t=1}$ $Q^{t=2}$

[Lipman et al. SIG'07]

Parameterization Free



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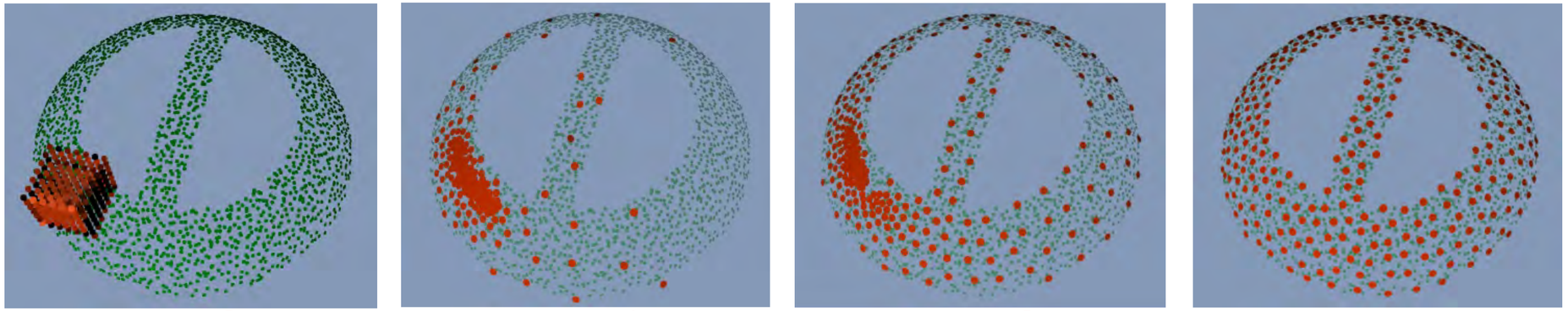

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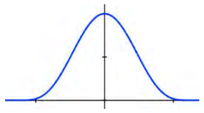

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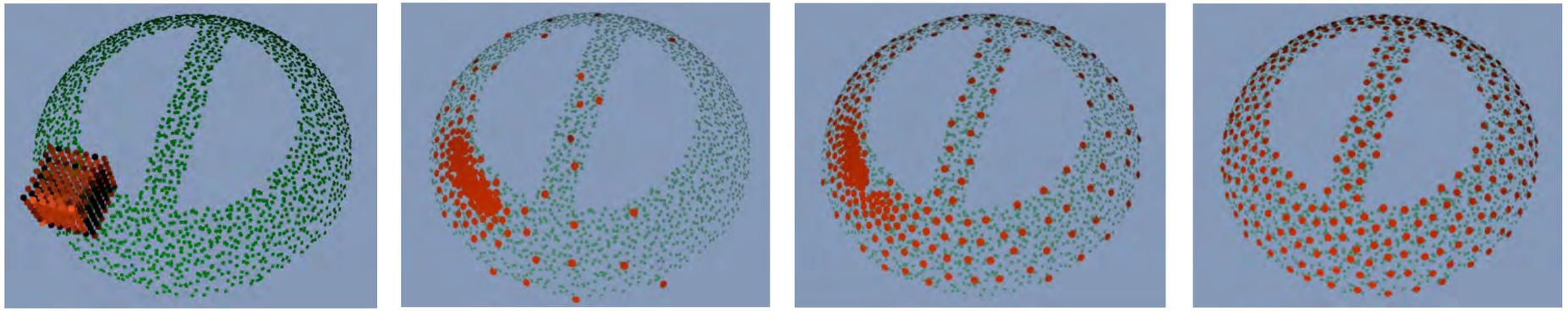

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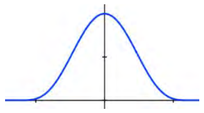

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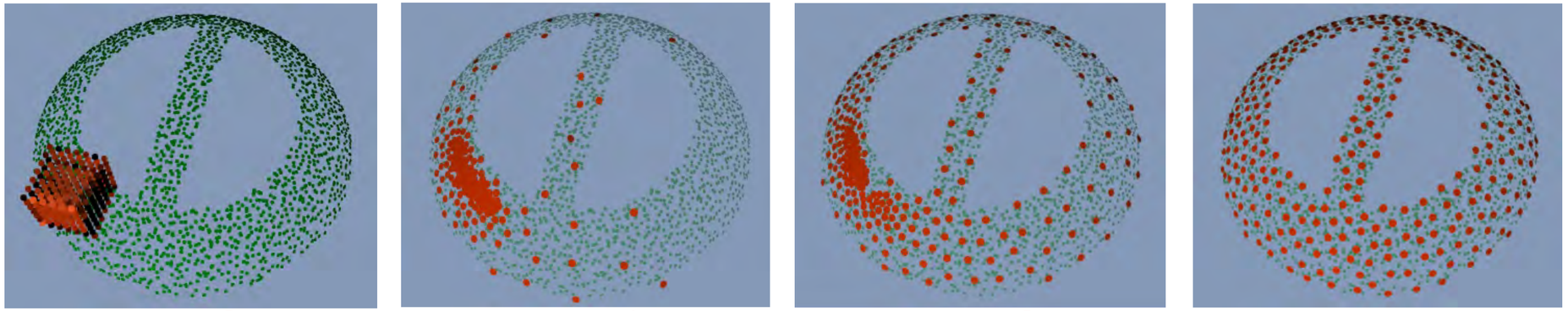

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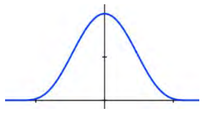

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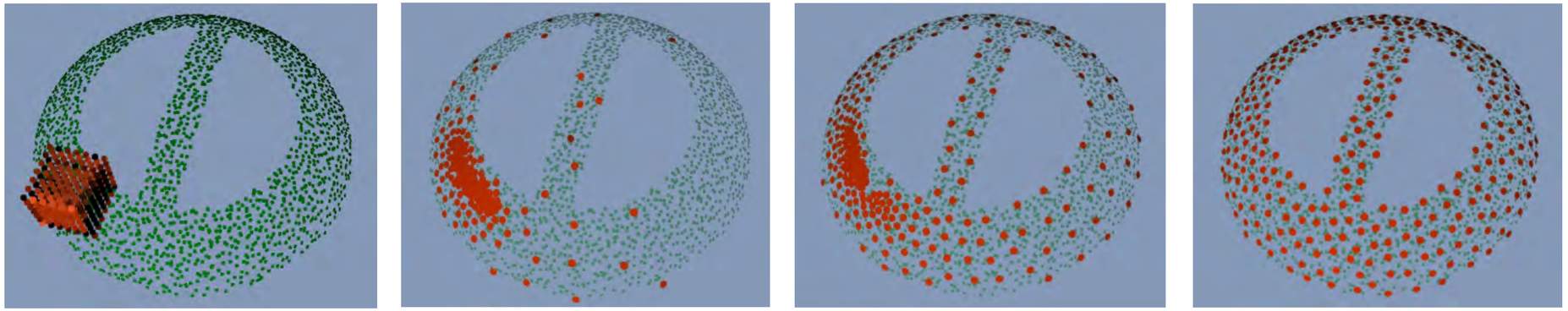
$\eta(r) = 1/r^3$

P



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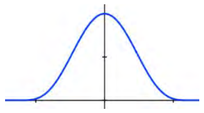

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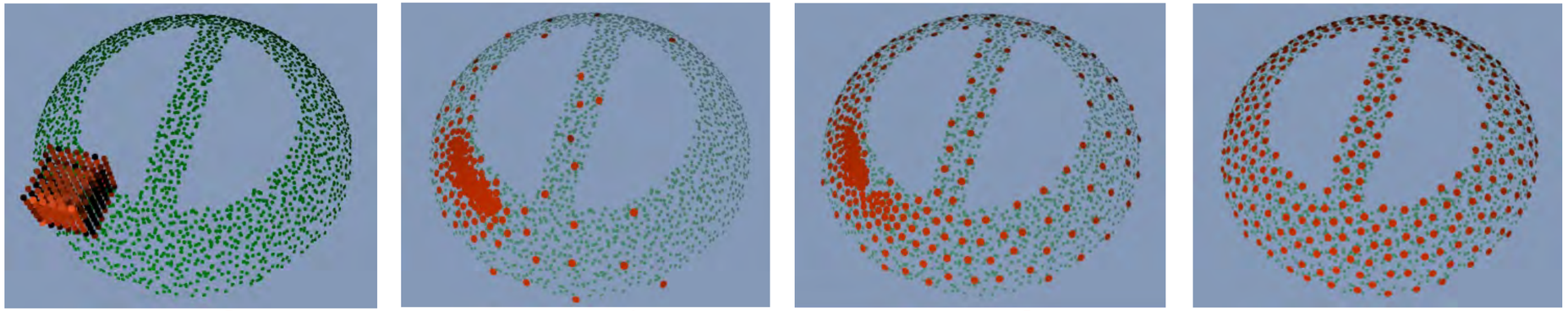

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[Lipman et al. SIG'07]

(recap) Local Smoothness

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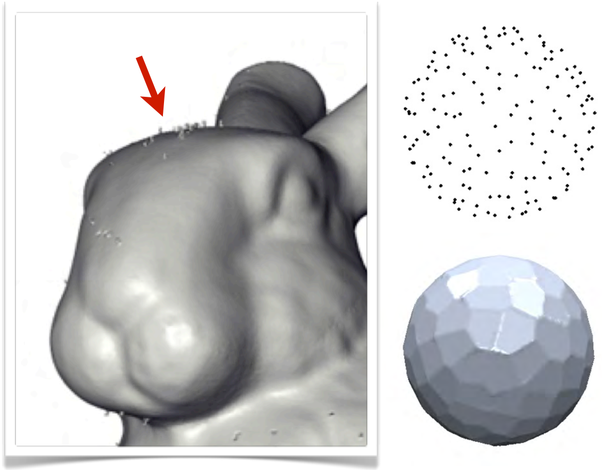
Surface Reconstruction from Unorganized Points



seminal paper :)
discontinuous solution :(

[Hoppe et al. SIG'92]

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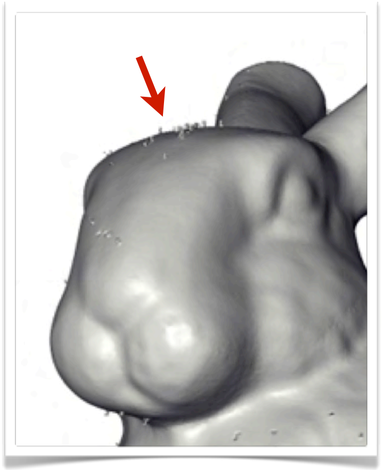

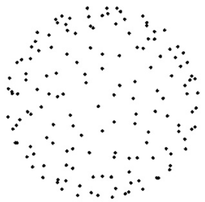

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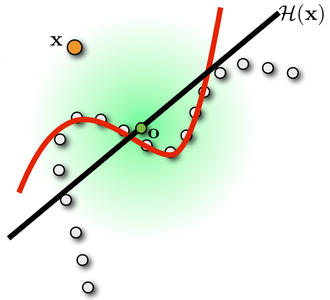
Point Set Surfaces



parameterization :(
provide initial point-set :(

[Alexa et al. VIS'01]

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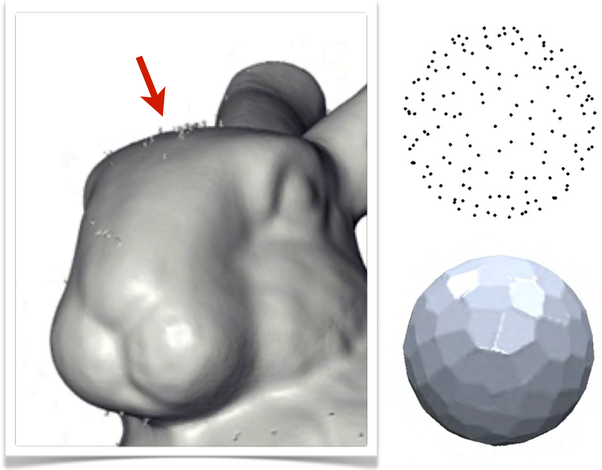

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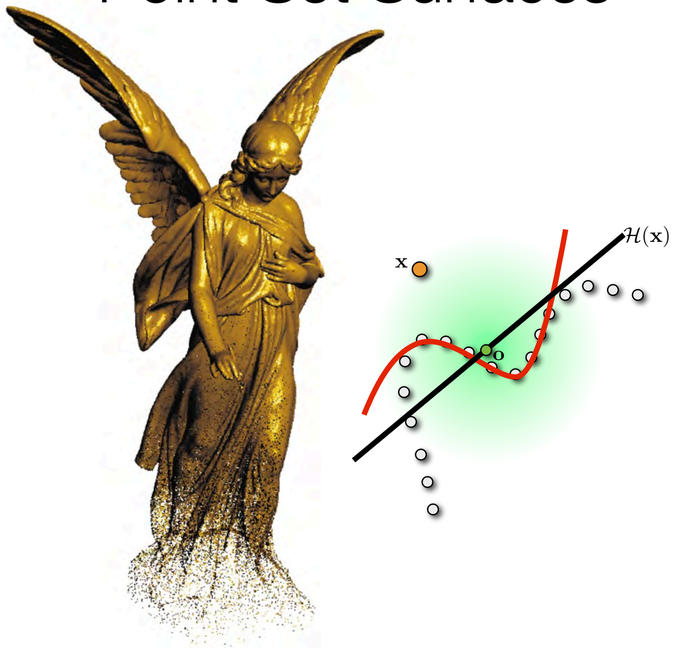

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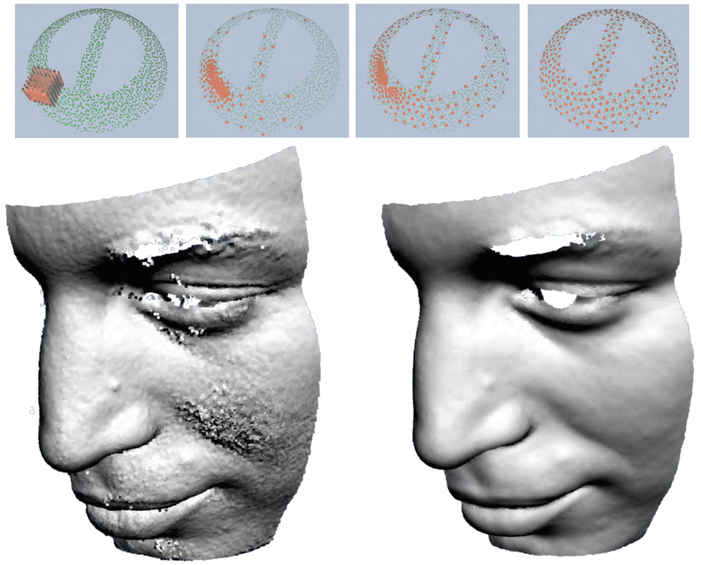
Locally Optimal Projection



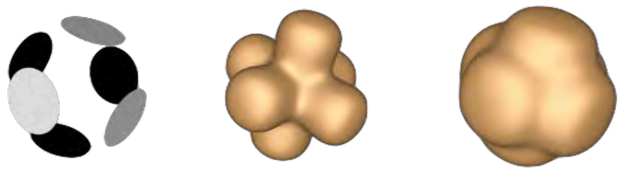
no parameterization :)
self-adaptive :)

[Lipman et al. SIG'07]

(recent) Local Smoothness

(recent) Local Smoothness

Defining Point Set Surfaces (PSS)



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polished formulation :)
oriented samples :)

[Amenta et al. TOG'04]

(recent) Local Smoothness

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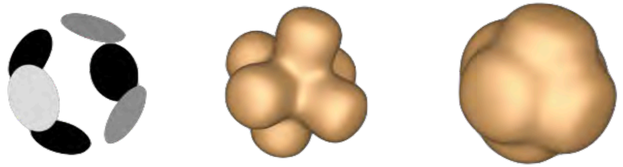

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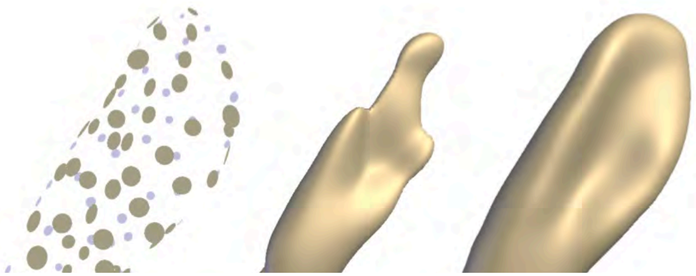
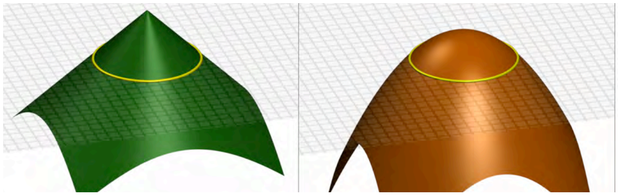
Algebraic Point Set Surfaces (APSS)



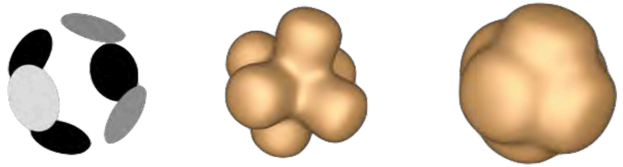


spherical fits (curvature)
robust to low sampling

[Guennebaud et al. SIG'07]

(recent) Local Smoothness

Defining Point Set Surfaces (PSS)



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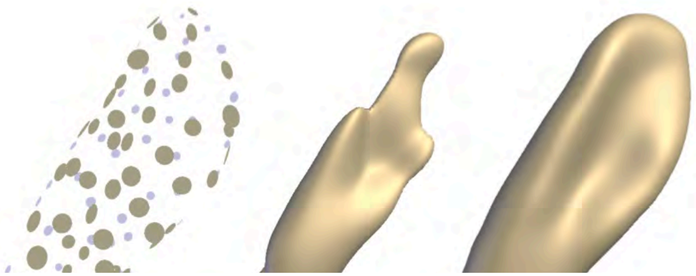

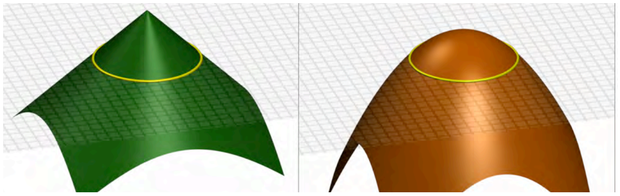

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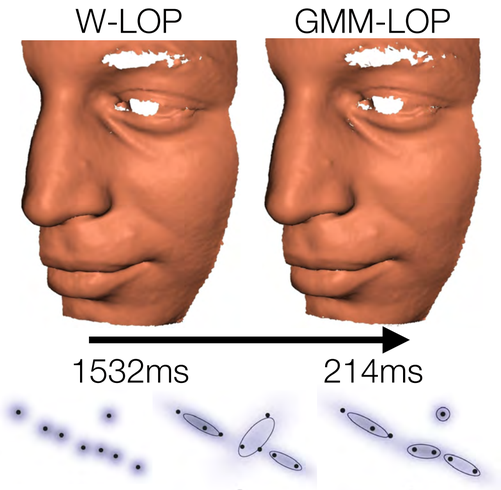
Continuous Projection for L1 Reconstruction

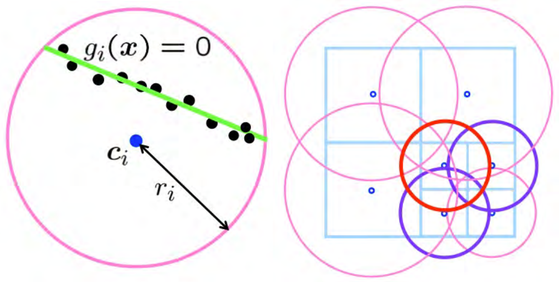


continuous formulation :)
improved performance :)

[Preiner et al. SIG'14]

Surface Smoothness Priors

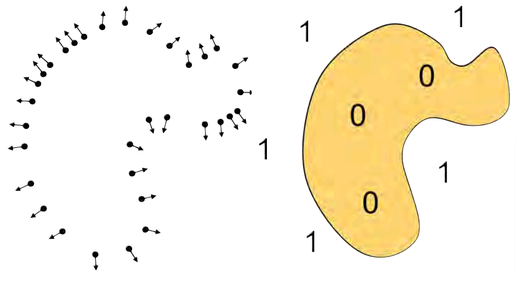
Local Smoothness



$$g_i(\mathbf{x}) = \alpha_i(\mathbf{x} - \mathbf{c}_i) + \beta_i$$

- Local solve ---> local fitting
- no control away from data
- solution by interpolation

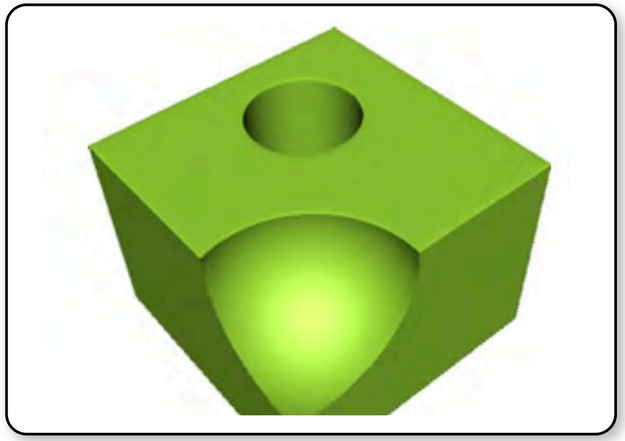
Global Smoothness



$$\min_{\chi} \int \|\nabla\chi(\mathbf{x}) - \mathbf{v}(\mathbf{x})\|^2 \, \mathrm{d}\mathbf{x}$$

- Global solve ---> global fitting
- linear problem, eigenvalue problem, graph cut
- Missing data in-painting

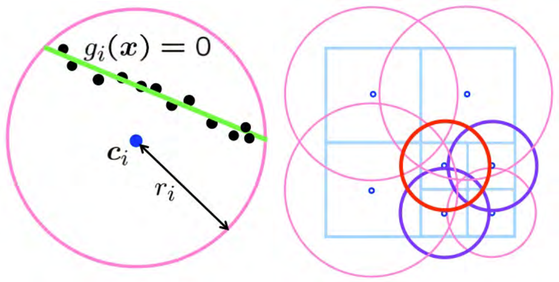
Piecewise Smoothness



- Not smooth everywhere!
- Sharp near features
- Sparse discontinuities
- Non-continuous problem

Surface Smoothness Priors

Local Smoothness



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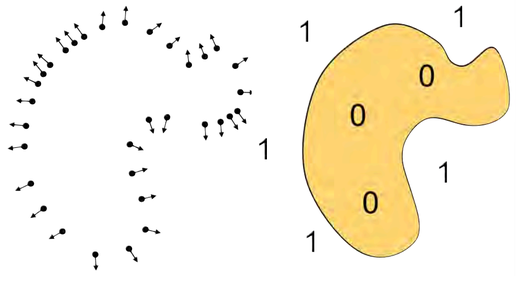

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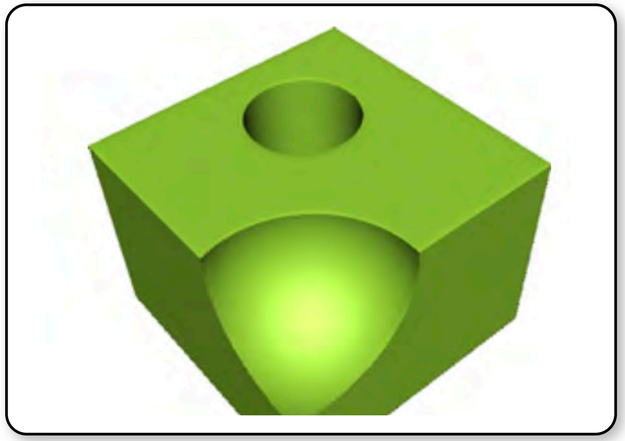

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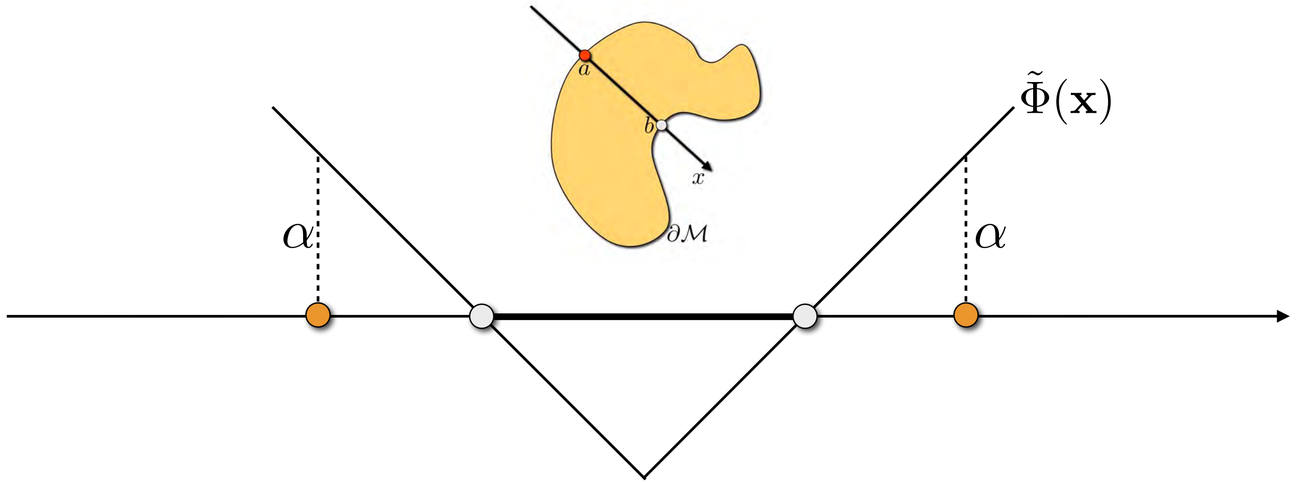
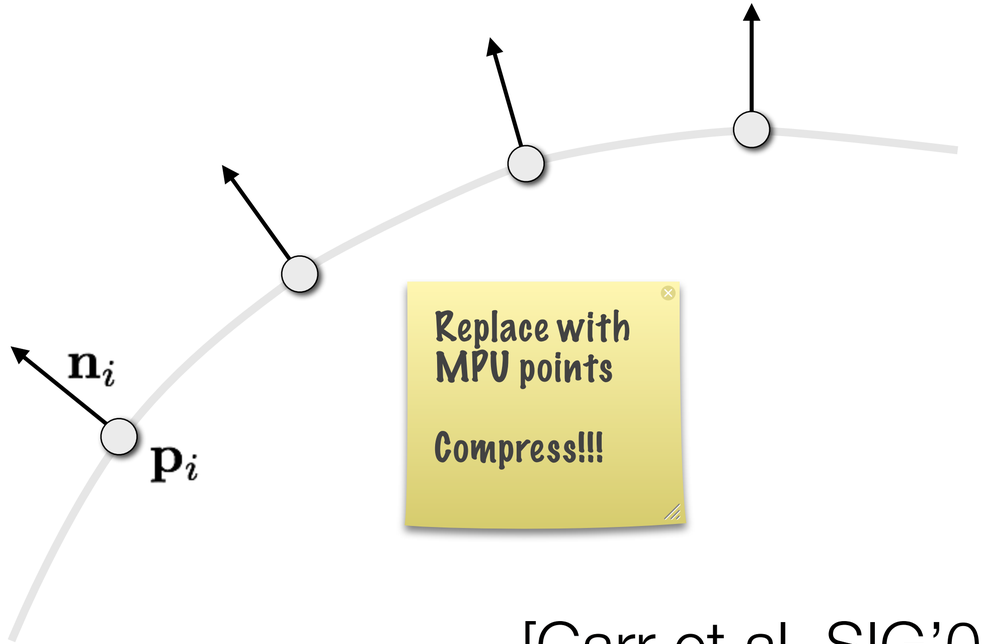
Radial Basis Functions

- Scattered interpolation $$\Phi(\mathbf{x}) = \sum_i w_i \cdot \varphi(\|\mathbf{x} - \mathbf{c}_i\|)$$





[Carr et al. SIG'01]

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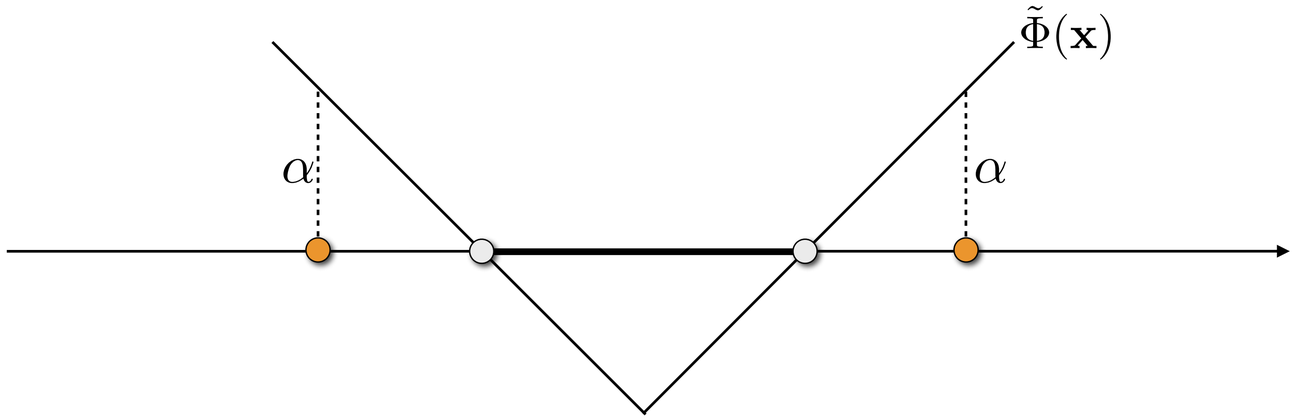

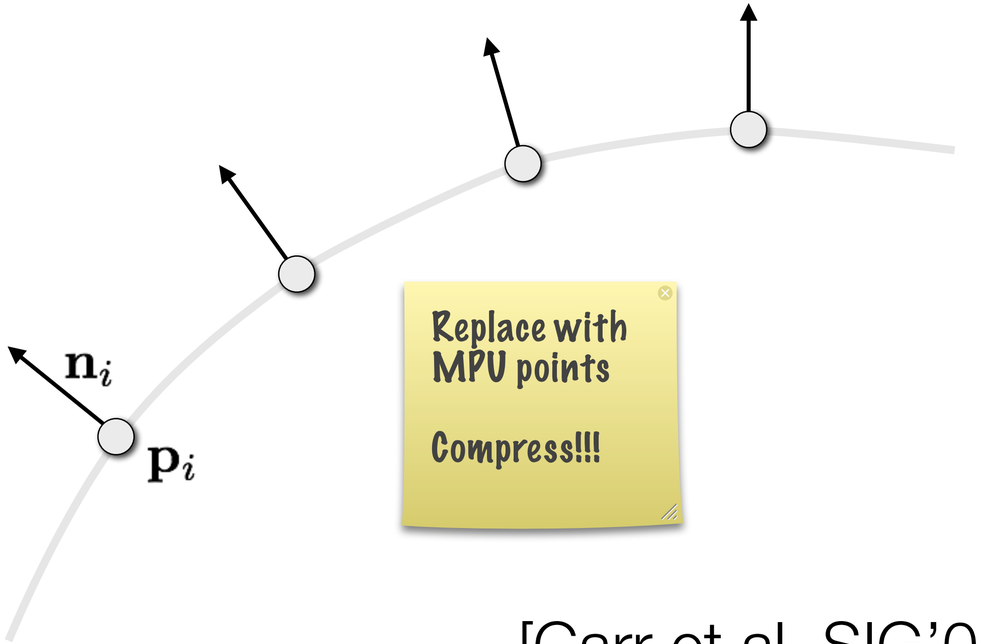

[Carr et al. SIG'01]

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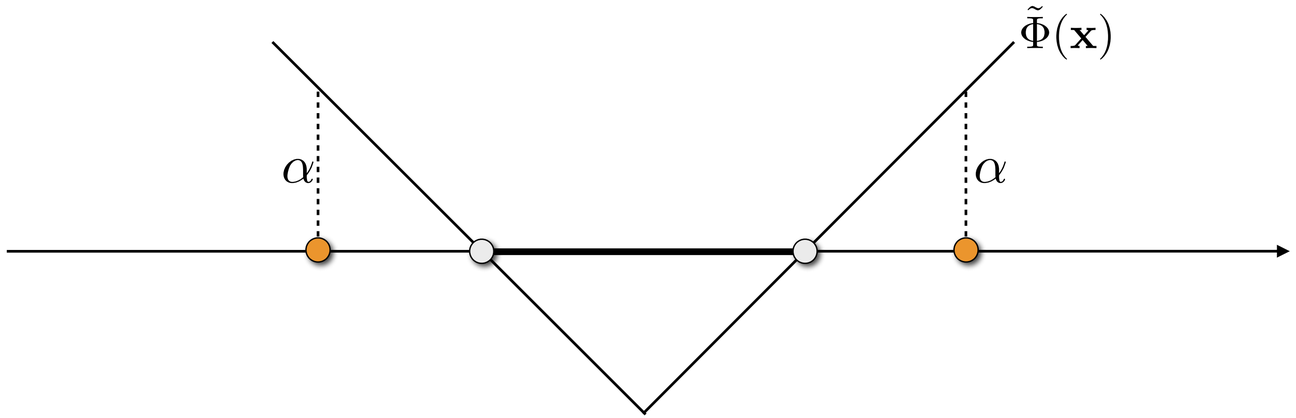

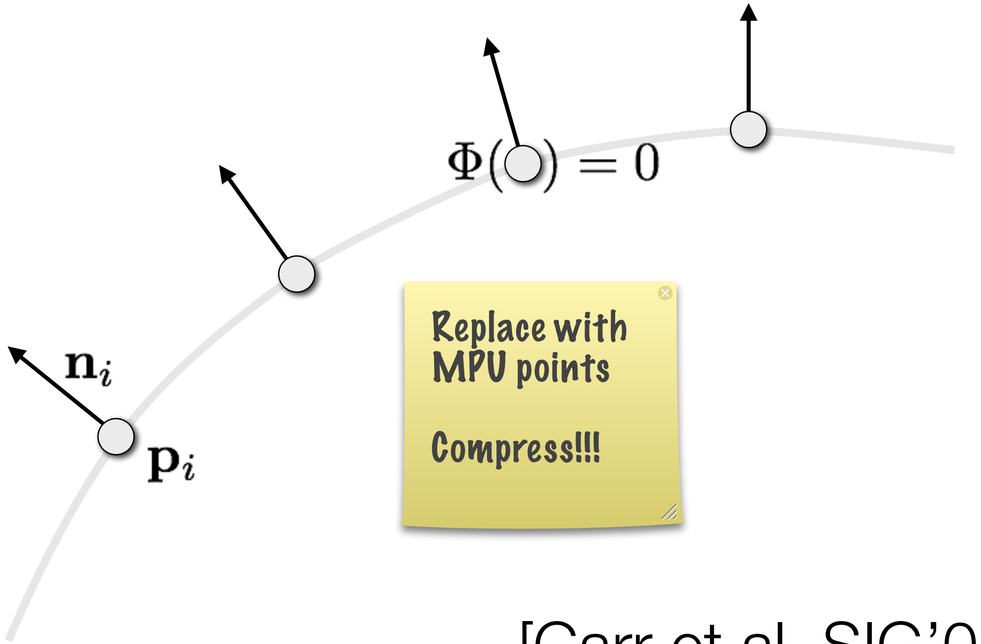

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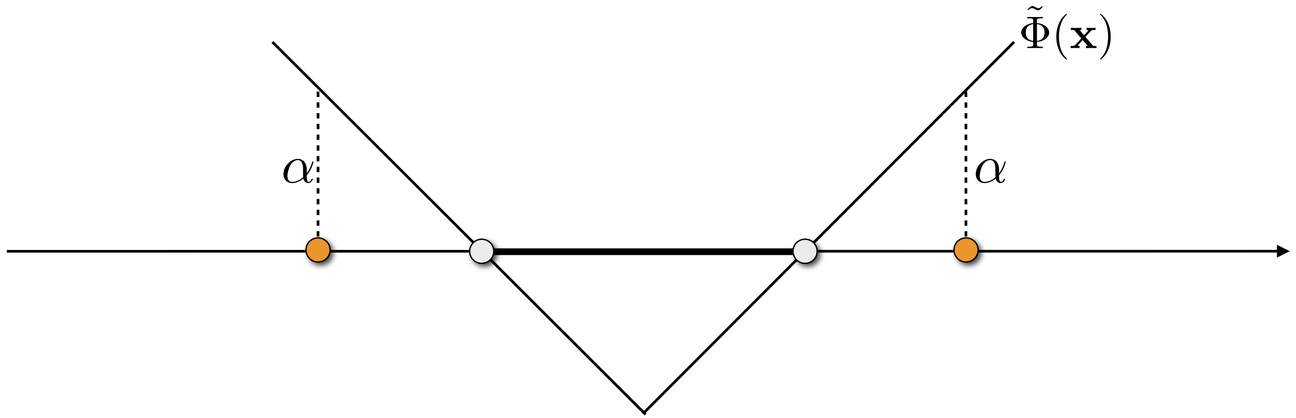

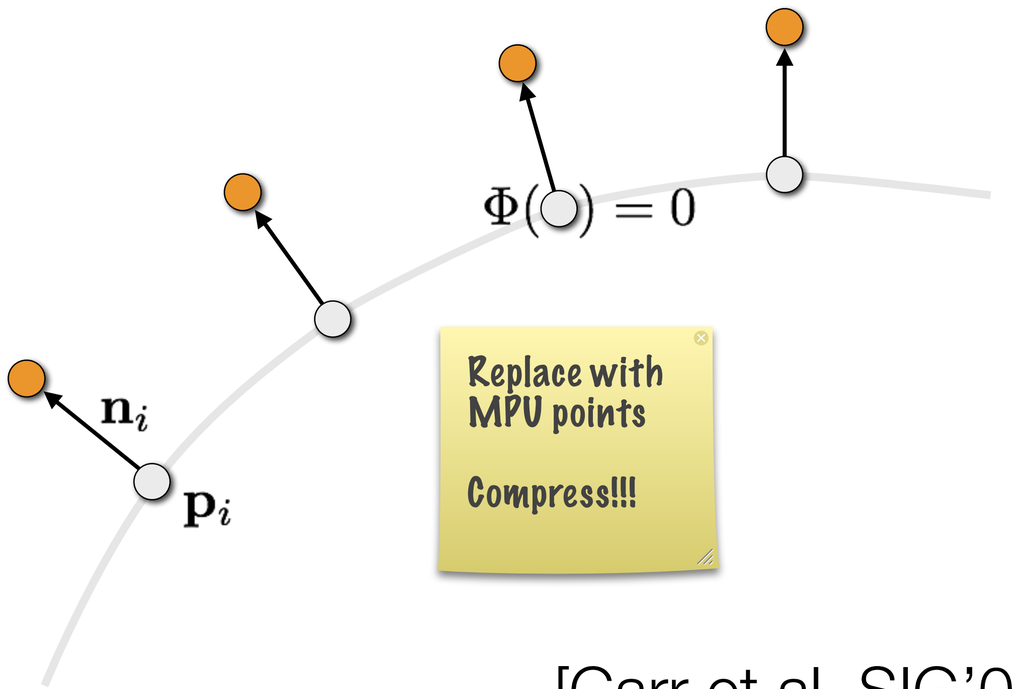

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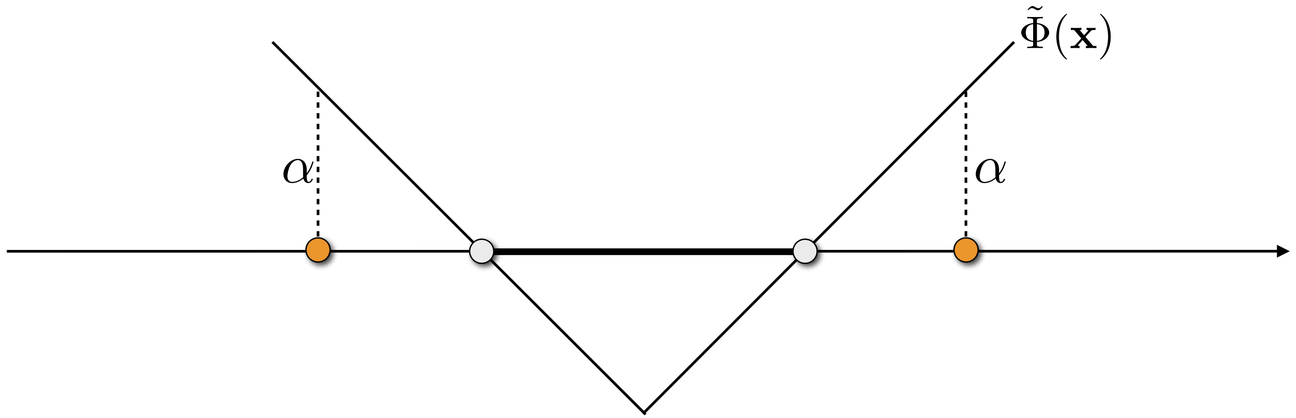

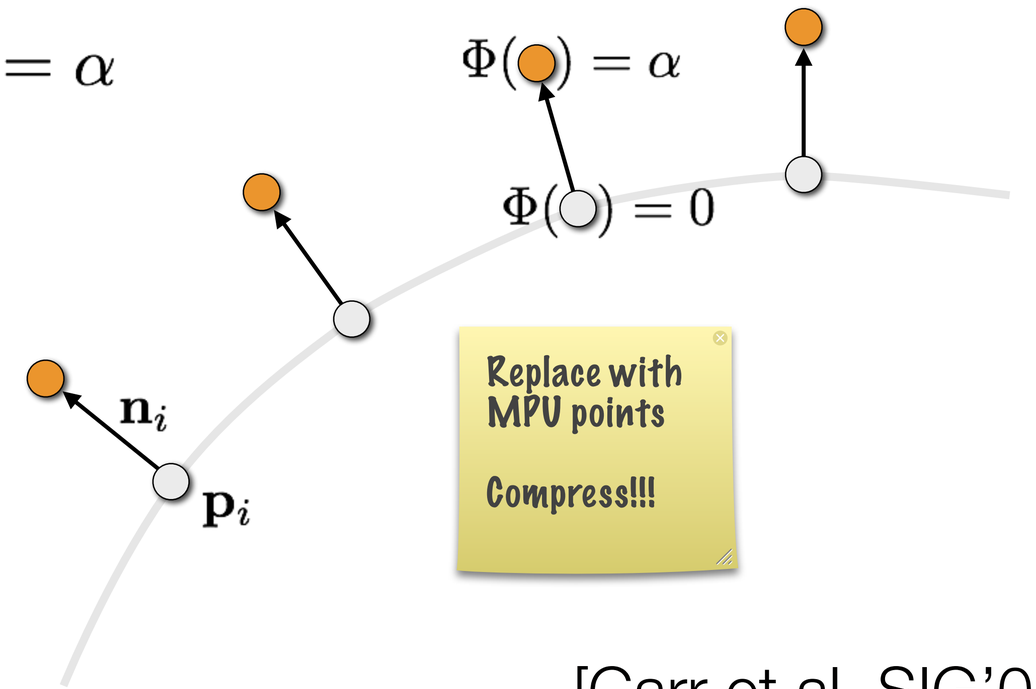

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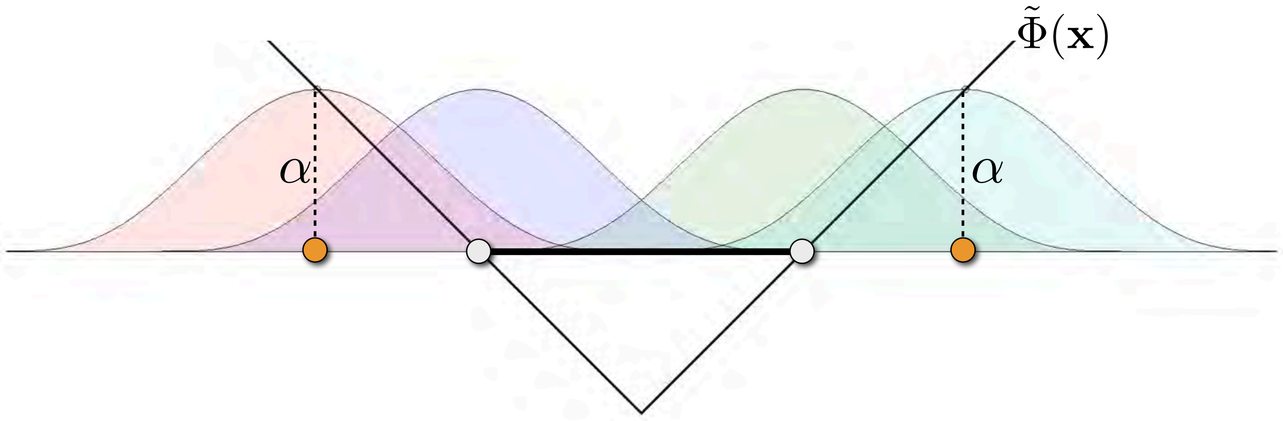

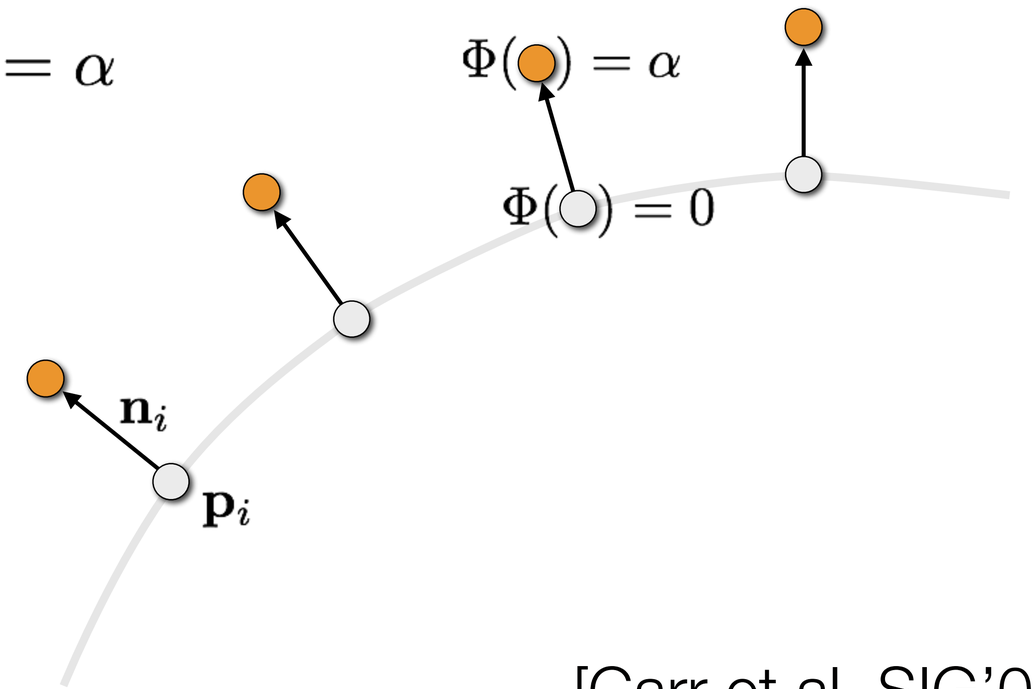

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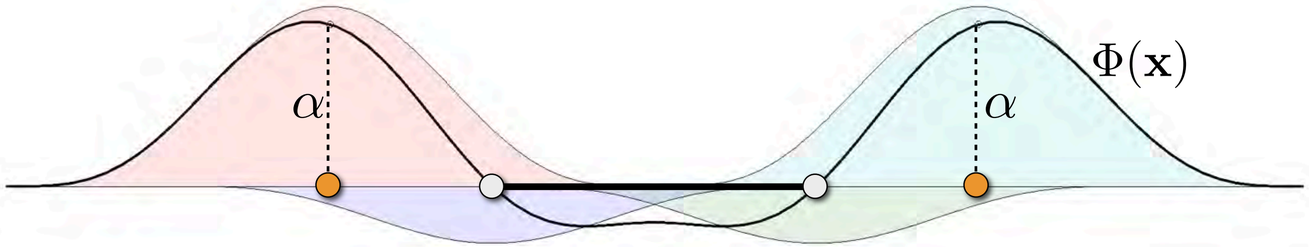
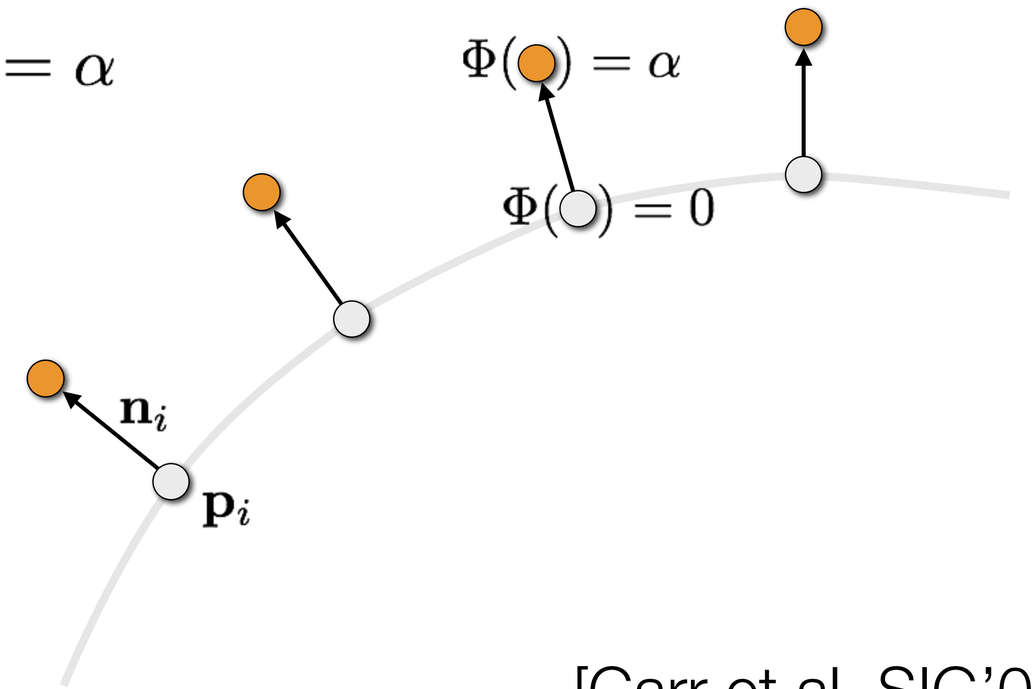
$\mathbf{w} = [1.11, -.35, -.35, 1.11]$





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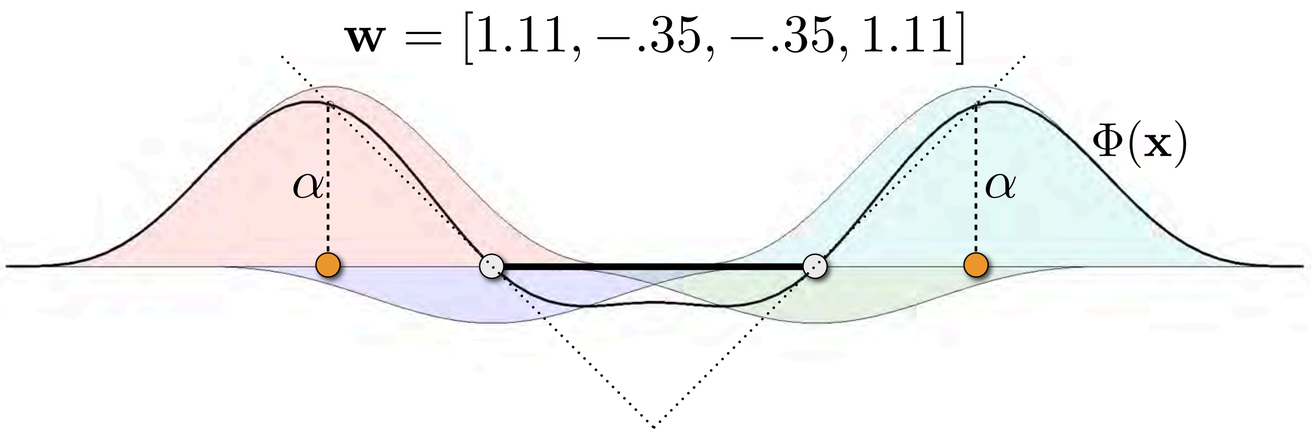

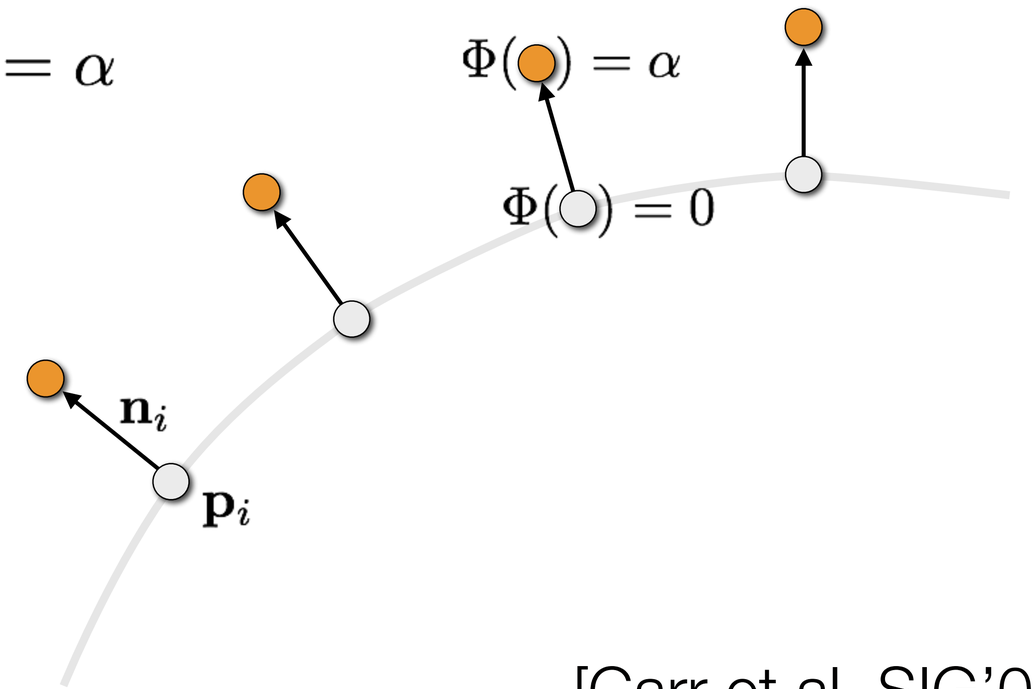

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Radial Basis Functions

- Scattered interpolation $$\Phi(\mathbf{x}) = \sum_i w_i \cdot \varphi(\|\mathbf{x} - \mathbf{c}_i\|)$$

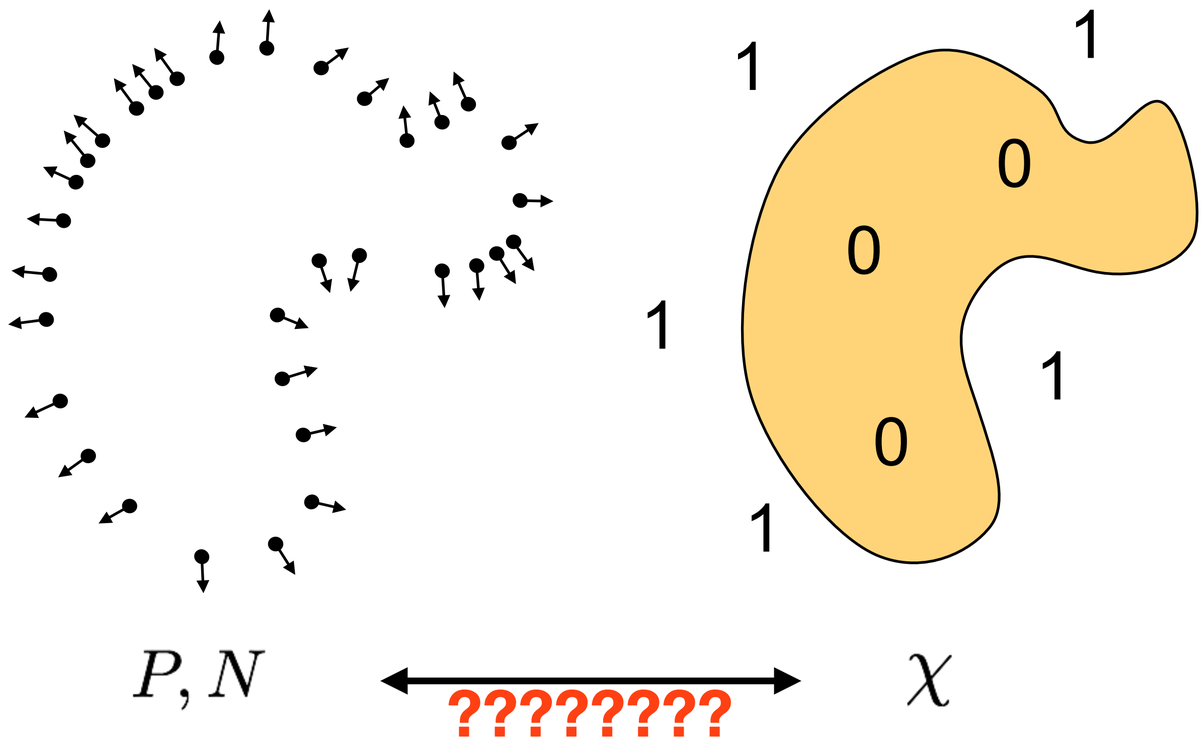
- Wendland
  - **compact** support
  - **sparse** SPD system
  - C^2 smooth

- Triharmonic $\varphi(r) = r^3$
  - **global** support
  - **dense** linear system
  - provably **fair**

[Carr et al. SIG'01]

Indicator Functions

- Construct indicator function from point samples



[Kazhdan et al. SGP'06]

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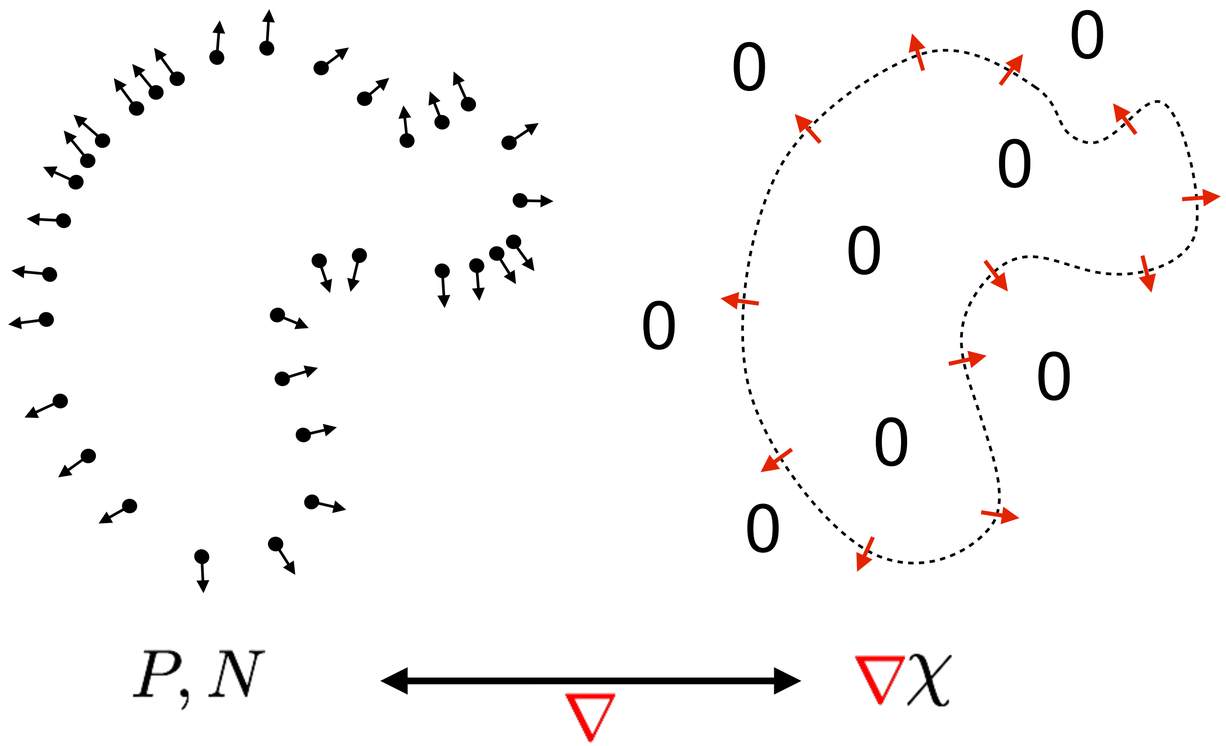

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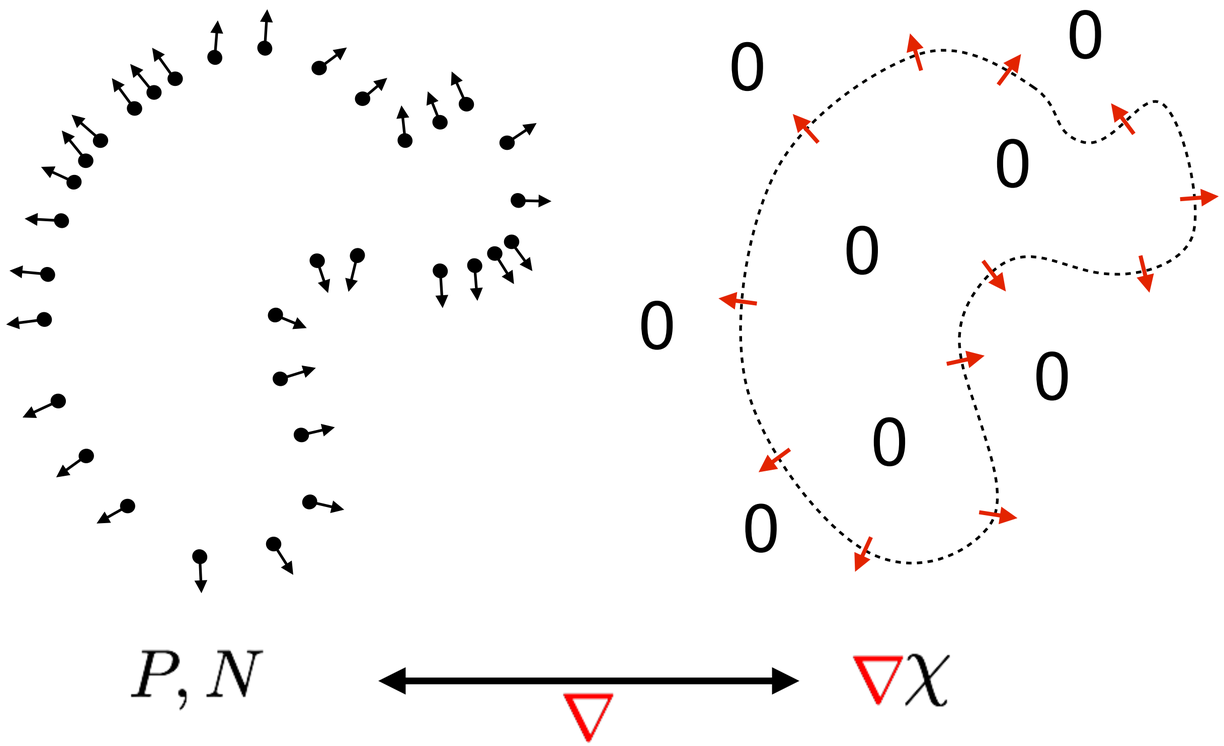

$$\min_{\chi} \int \|\nabla\chi(\mathbf{x}) - \mathcal{N}(\mathbf{x})\|_2^2 \, dx$$

[Kazhdan et al. SGP'06]

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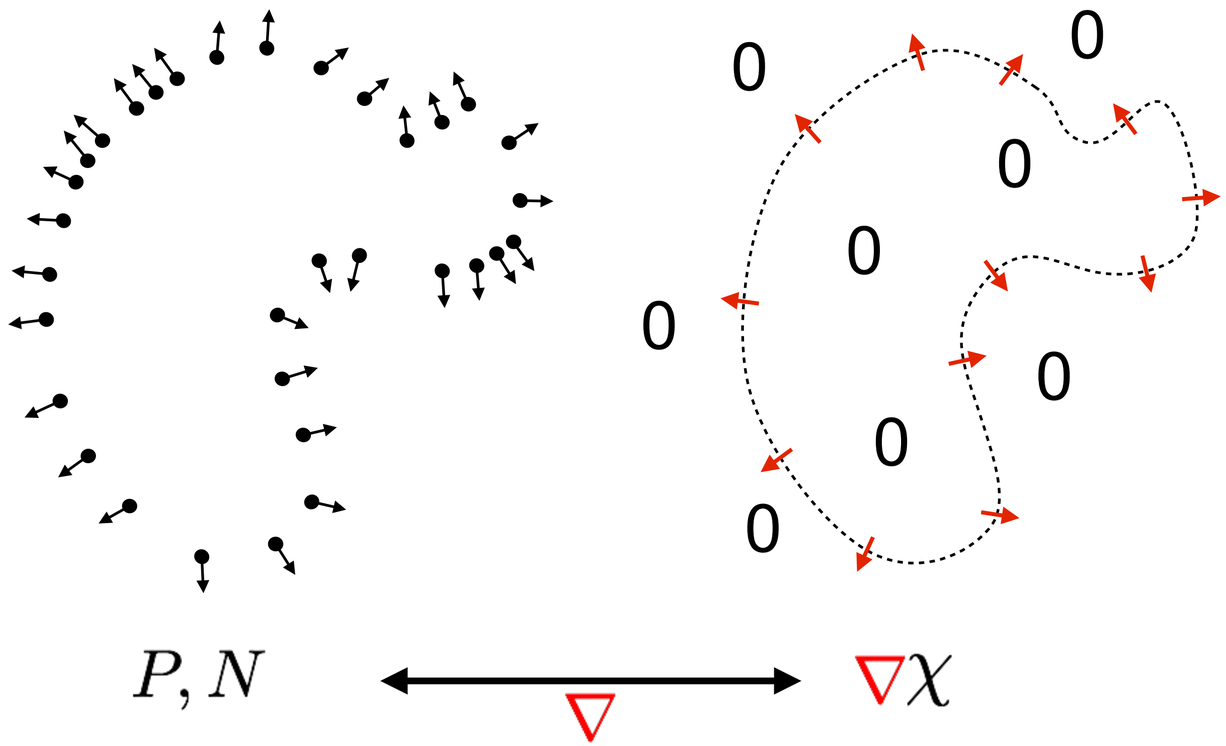

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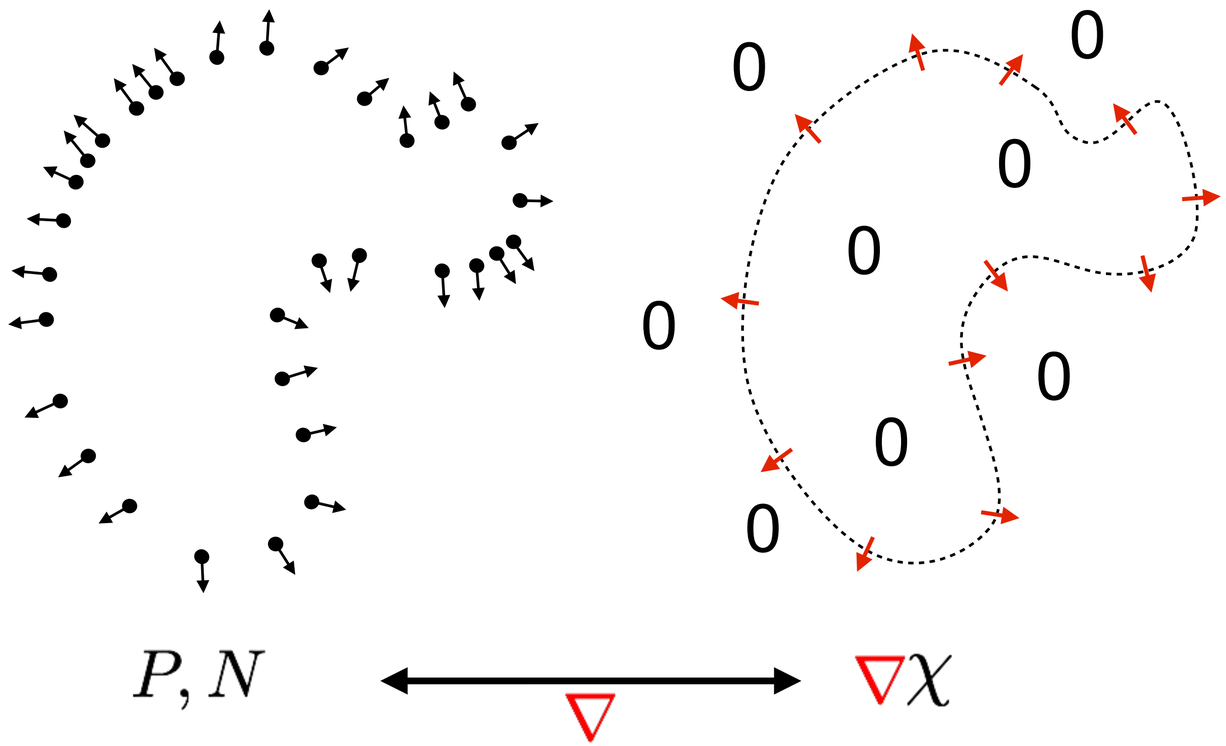

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variational calculus

$$\Delta\chi = \nabla \cdot \mathcal{N}$$

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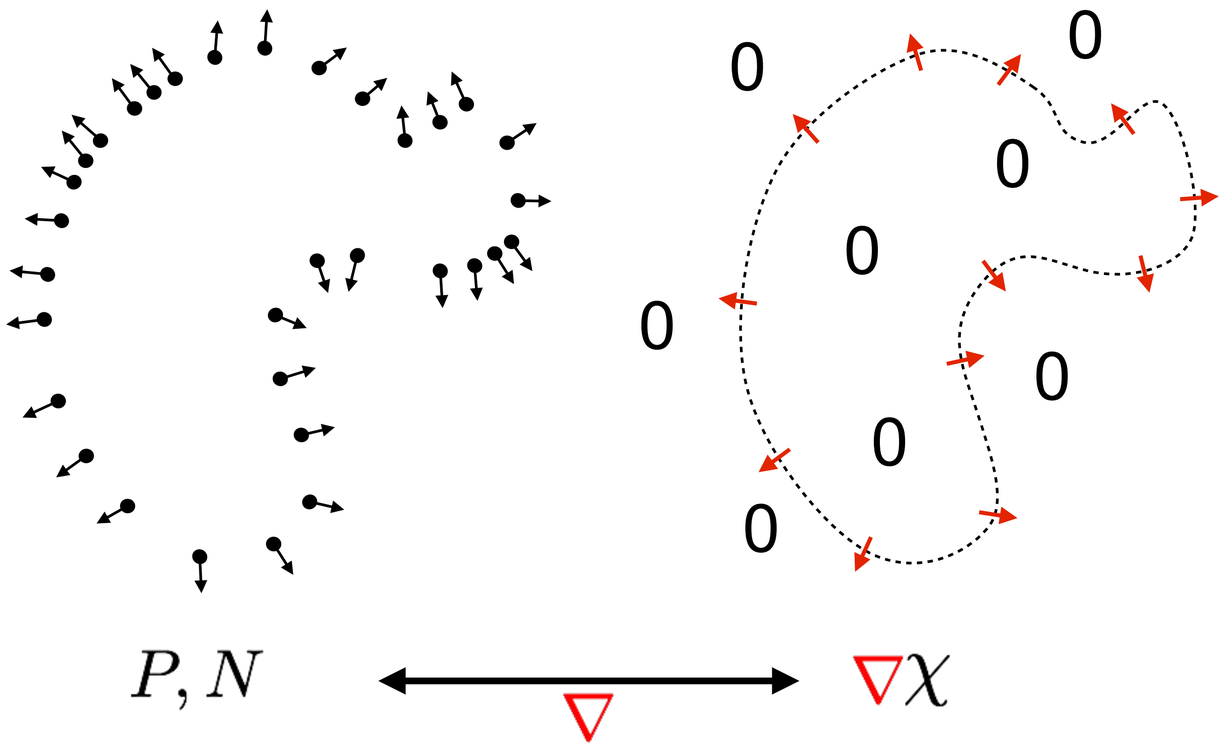

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sparse linear system

[Kazhdan et al. SGP'06]

Volumetric Segmentation

- Surface as an “optimal cut” of an embedded graph

[Hornung et al. SGP’06]

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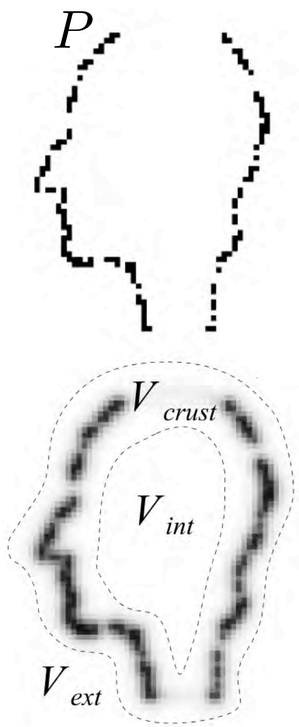

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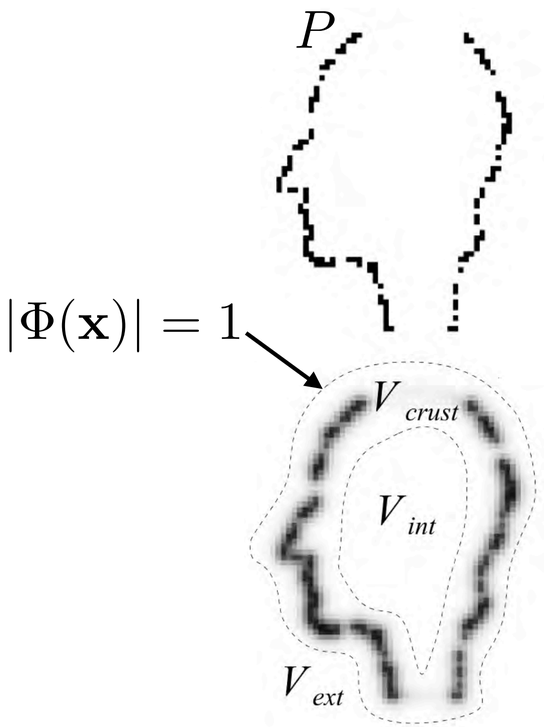

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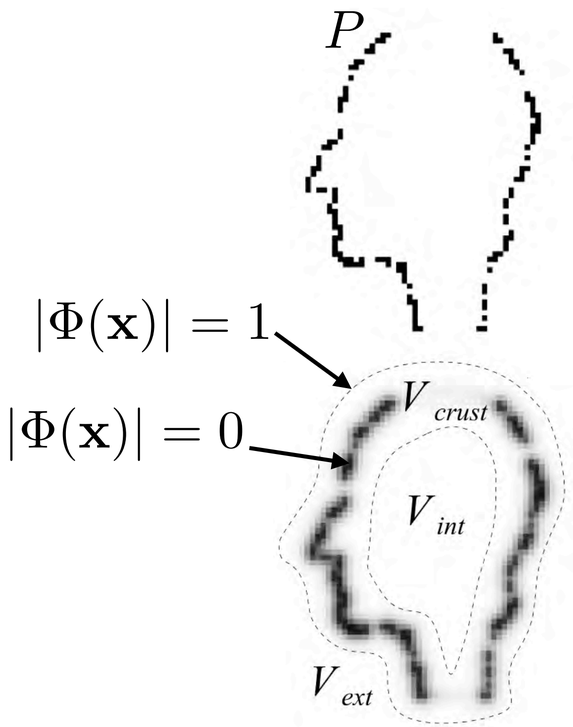

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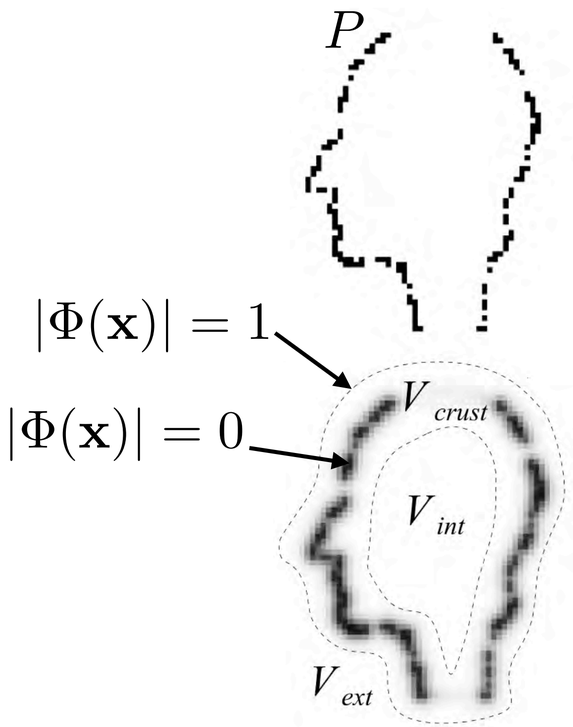

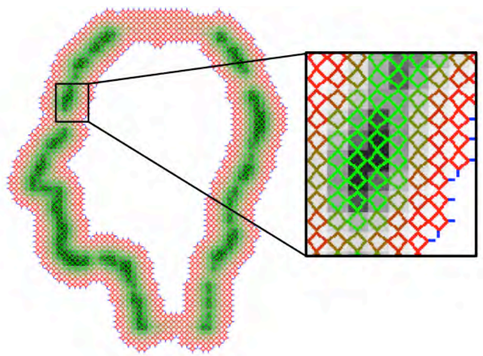

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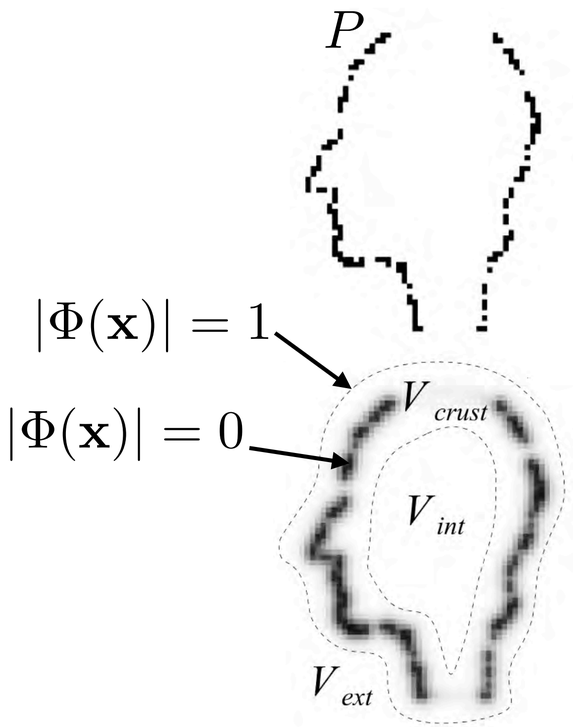

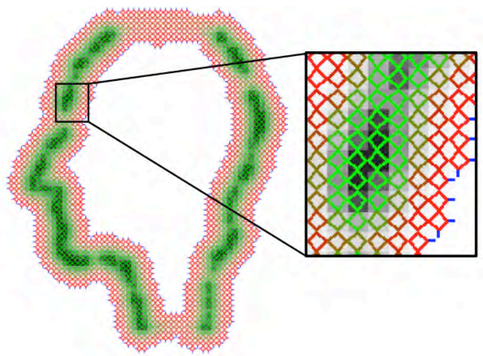

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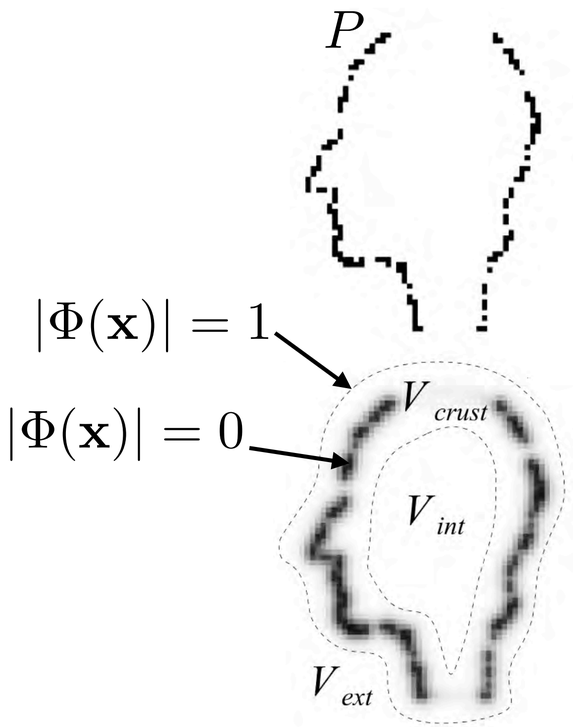

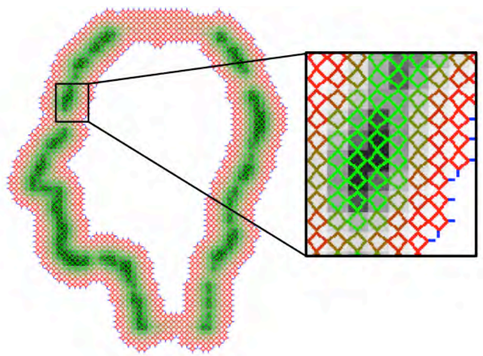

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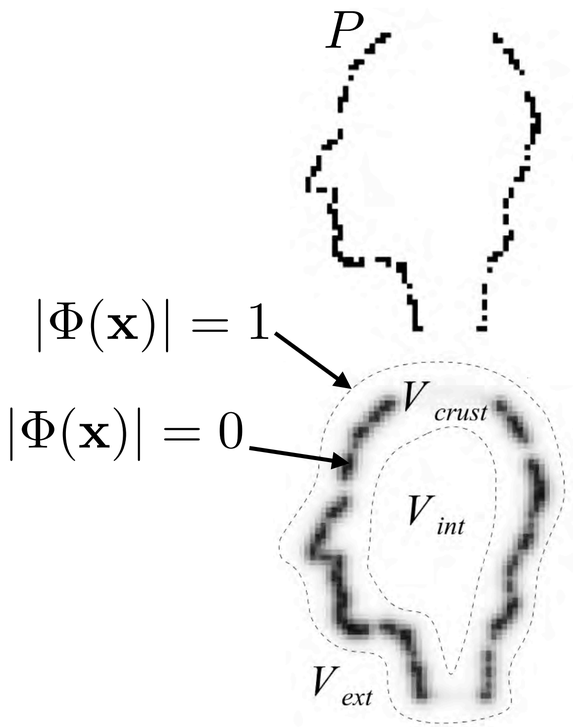

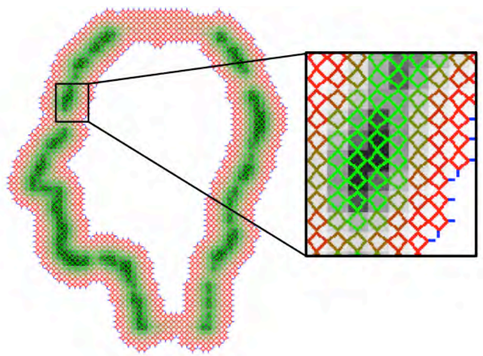

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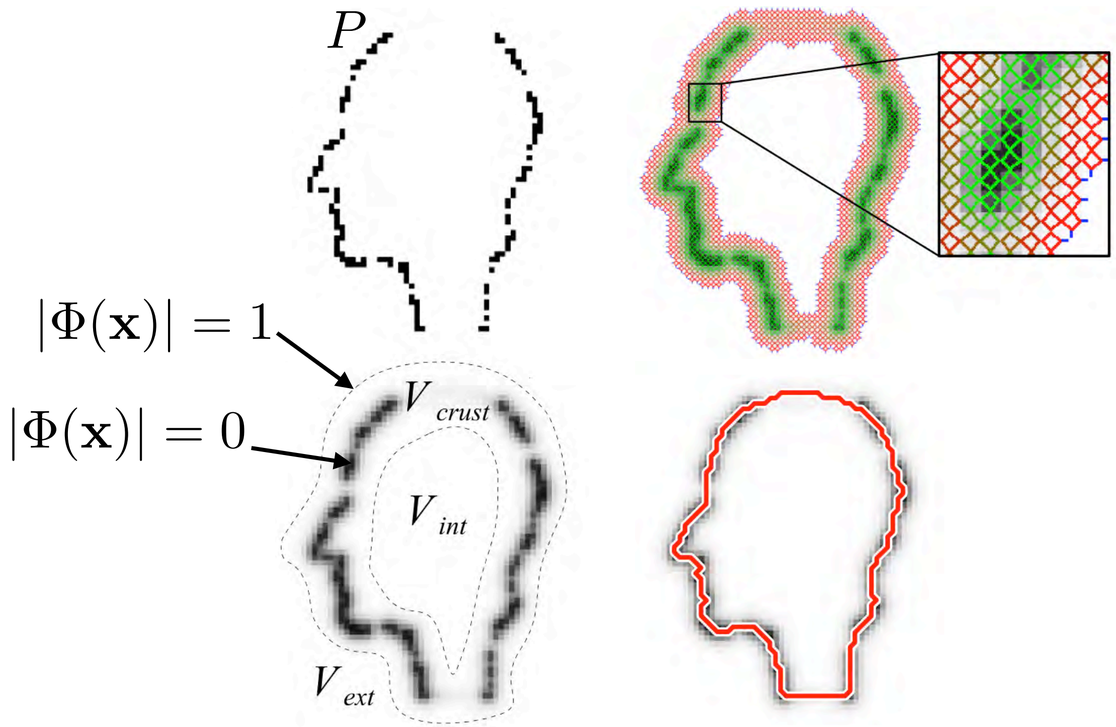

[Hornung et al. SGP’06]

(recap) Global Smoothness

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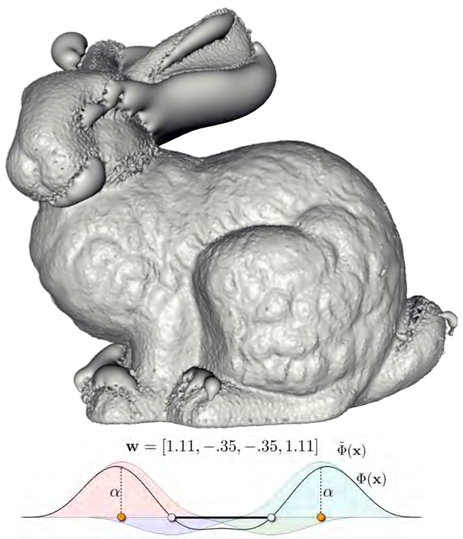

controllable fairness :)
difficult multipole :(

[Carr et al. SIG'01]

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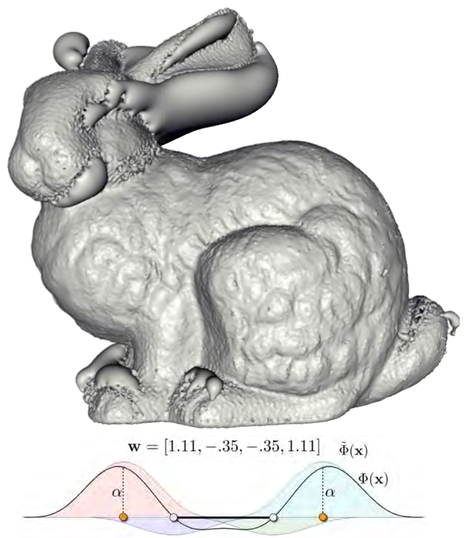

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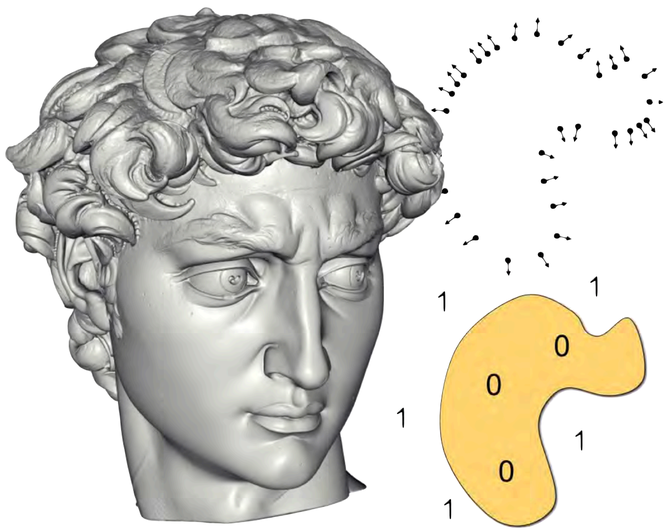
Poisson Reconstruction



gradient domain :)
hierarchical solve :)

[Kazhdan et al. SGP'06]

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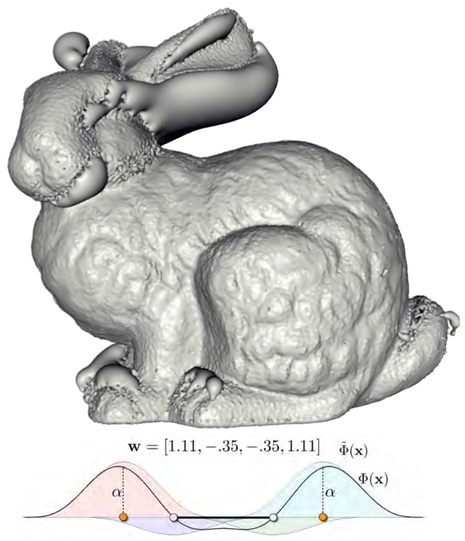

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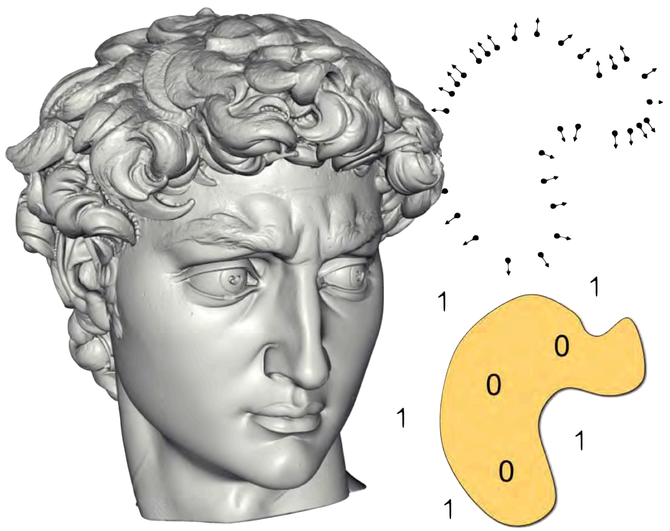

gradient domain :)
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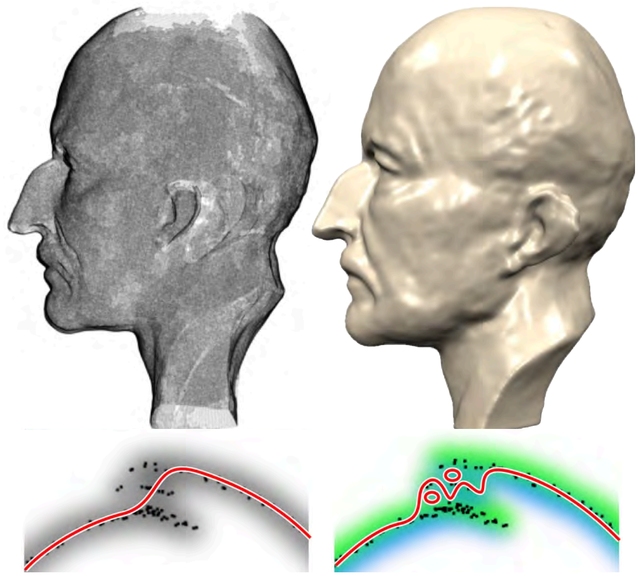
[Kazhdan et al. SGP'06]

(Smooth) Graph Cut



no normals! :D
crust == topology :(

[Hornung et al. SGP'06]

(recent) Global Smoothness

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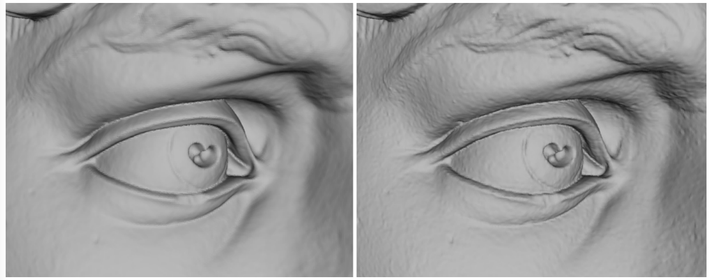
Smooth Signed Distance Surface Reconstruction



poisson SSD

$$\int_{\mathcal{S}} \varphi(\mathbf{x})^2 + \omega_1 \|\nabla\varphi(\mathbf{x}) - \mathcal{N}(\mathbf{x})\|_2^2 d\mathbf{x}$$

$$+\omega_2 \int_{\Omega} \|H_\varphi(\mathbf{x})\|_F^2 \, d\mathbf{x}$$

constant gradient away from data

[Calakli et al. PG'11]

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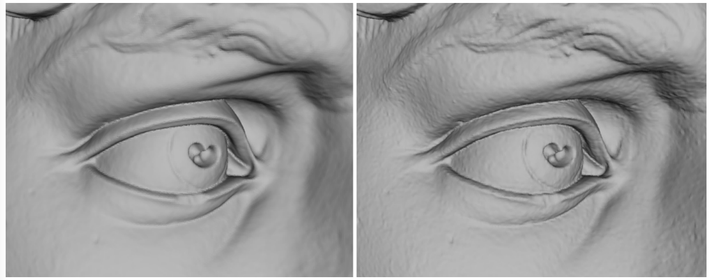

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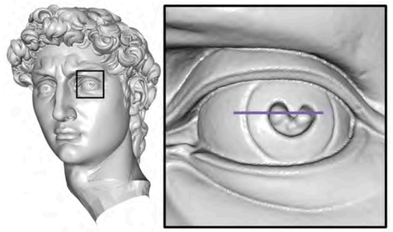
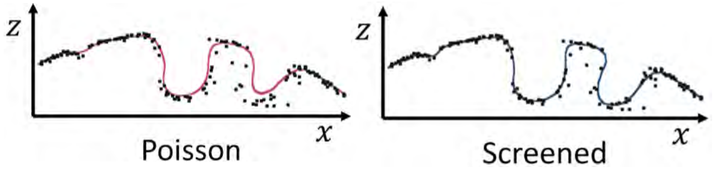
[Calakli et al. PG'11]

Screened Poisson Reconstruction





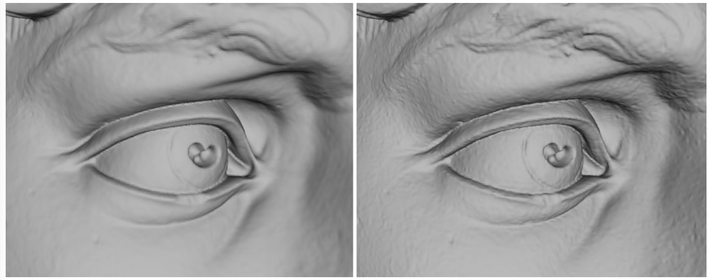
$$\arg\min_{\chi} \int \|\nabla\chi(\mathbf{x}) - \mathcal{N}(\mathbf{x})\|_2^2$$

$$+\lambda \sum_{\mathbf{p}_i \in P} \chi^2(\mathbf{p}_i)$$

[Kazhdan et al. TOG'12]

(recent) Global Smoothness

Smooth Signed Distance Surface Reconstruction



poisson SSD

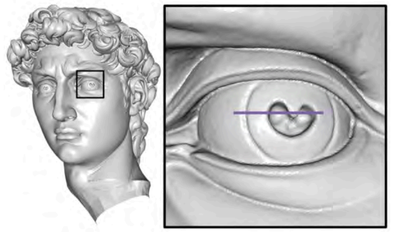
$$\int_{\mathcal{S}} \varphi(\mathbf{x})^2 + \omega_1 \|\nabla\varphi(\mathbf{x}) - \mathcal{N}(\mathbf{x})\|_2^2 d\mathbf{x}$$

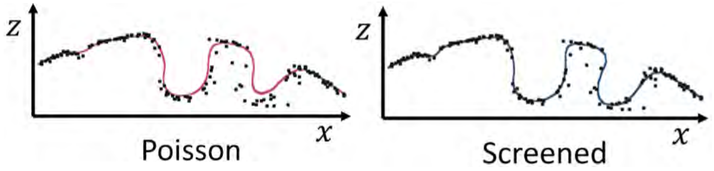
$$+\omega_2 \int_{\Omega} \|H_\varphi(\mathbf{x})\|_F^2 \, d\mathbf{x}$$

constant gradient away from data

[Calakli et al. PG'11]

Screened Poisson Reconstruction





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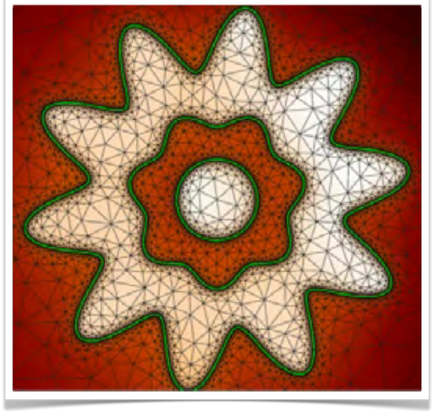
[Kazhdan et al. TOG'12]

Variational Reconstruction of Oriented Point Sets



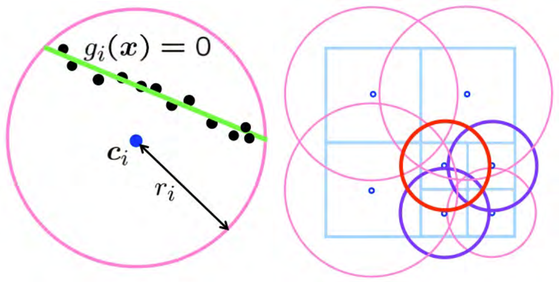
$$\arg\min \int \nabla\Phi^T \mathbf{C} \nabla\Phi$$

$$\text{s.t.} \int \left[(\Delta\Phi)^2 + \epsilon(\Phi)^2\right] = 1$$

[Alliez et al. SGP'07]

Surface Smoothness Priors

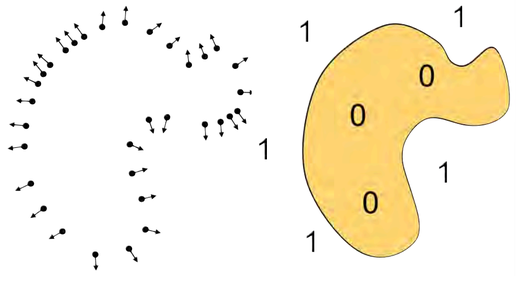
Local Smoothness



$$g_i(\mathbf{x}) = \alpha_i(\mathbf{x} - \mathbf{c}_i) + \beta_i$$

- Local solve ---> local fitting
- no control away from data
- solution by interpolation

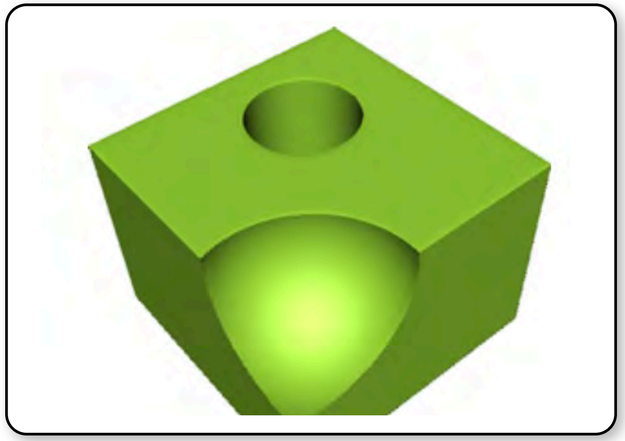
Global Smoothness



$$\min_{\chi} \int \|\nabla\chi(\mathbf{x}) - \mathbf{v}(\mathbf{x})\|^2 \, \mathrm{d}\mathbf{x}$$

- Global solve ---> global fitting
- linear problem, eigenvalue problem, graph cut
- Missing data in-painting

Piecewise Smoothness



- Not smooth everywhere!
- Sharp near features
- Sparse discontinuities
- Non-continuous problem

Surface Smoothness Priors

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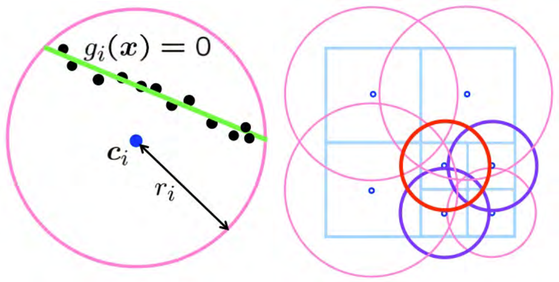

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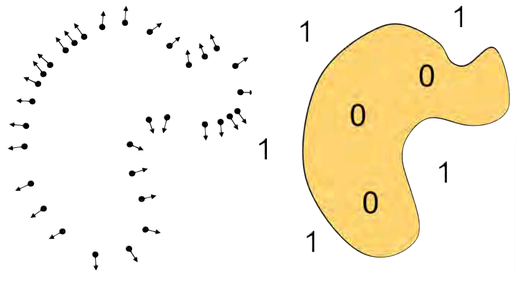

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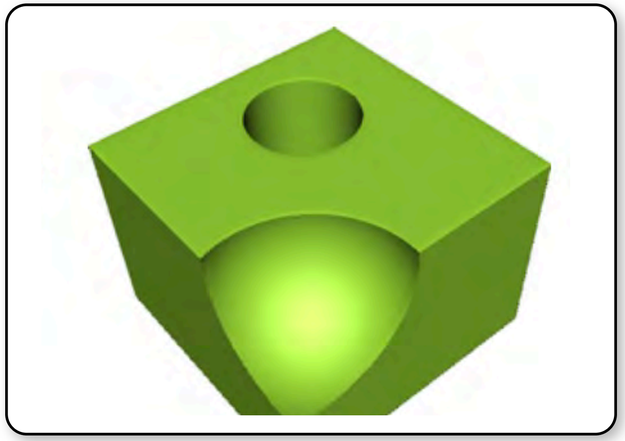

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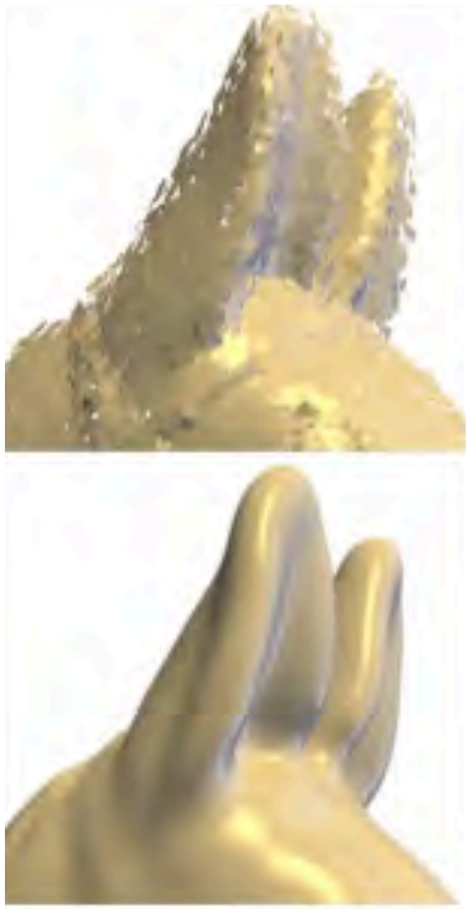
Re-Weighted Interpolatory MLS

- Blend point-to-plane distances

$$\Phi(\mathbf{x}) = \frac{\sum_i \mathbf{n}_i \cdot (\mathbf{p}_i - \mathbf{x})\varphi(\|\mathbf{p}_i - \mathbf{x}\|)}{\sum_i \varphi(\|\mathbf{p}_i - \mathbf{x}\|)}$$



[Oztireli et al. EG'09]

Re-Weighted Interpolatory MLS

- Blend point-to-plane distances

iterative correction

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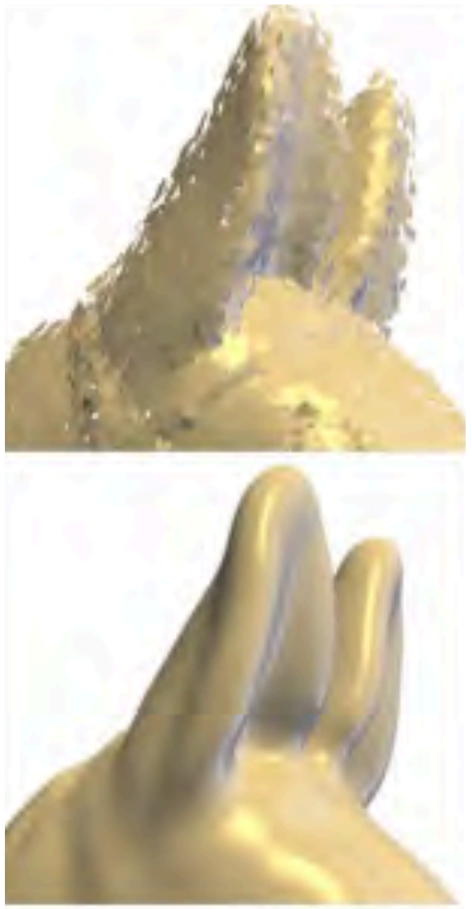

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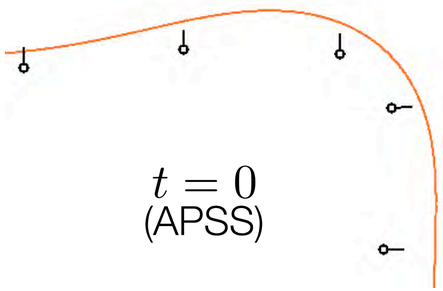

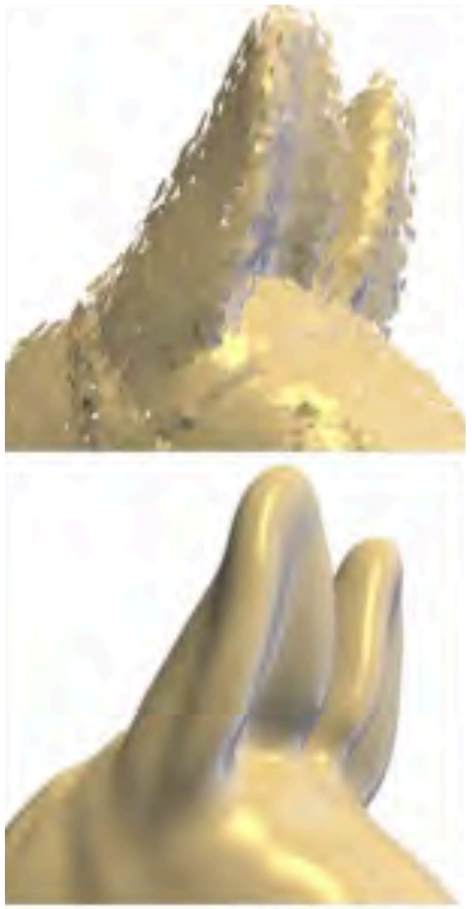

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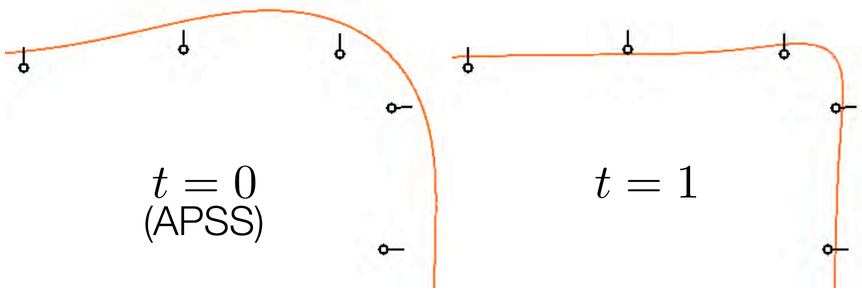

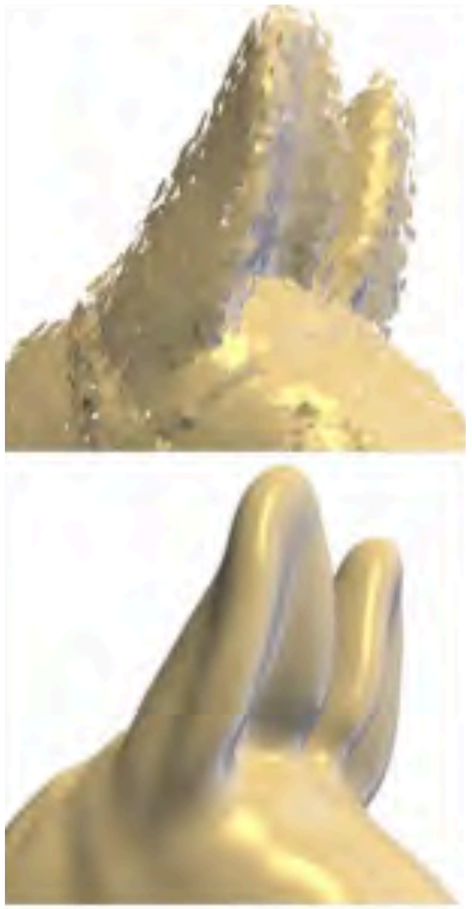

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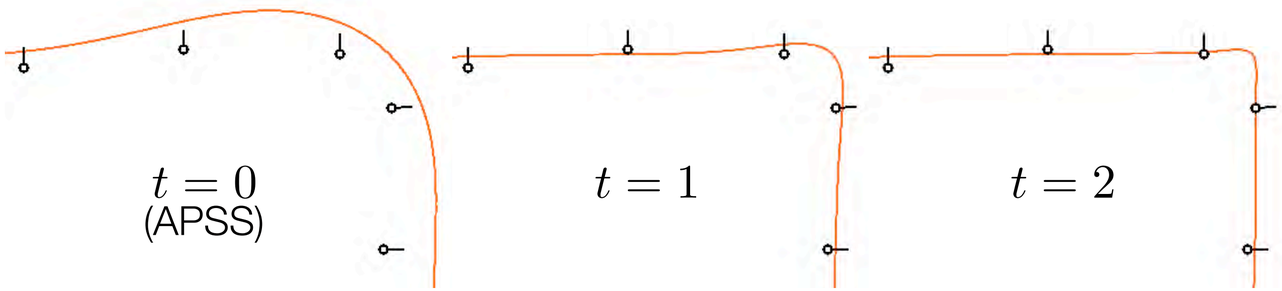

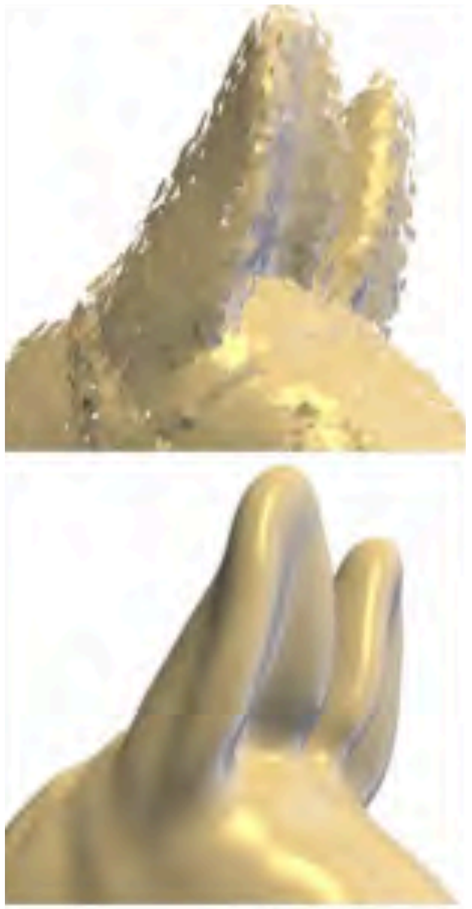

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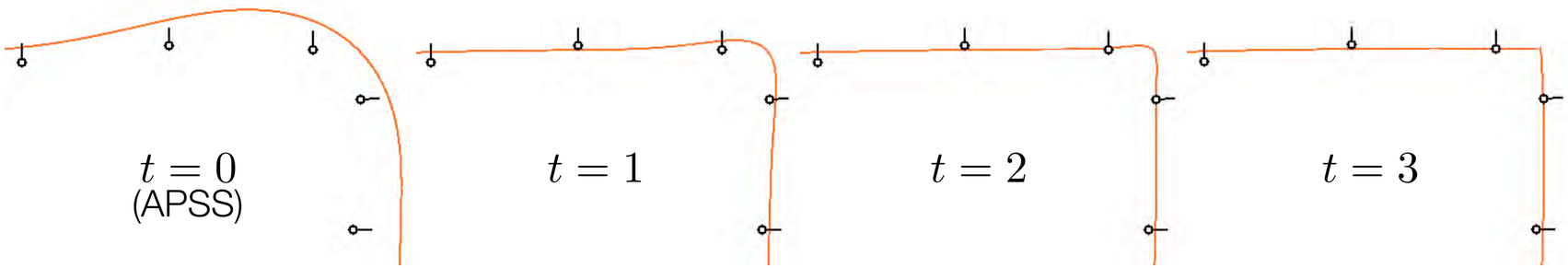

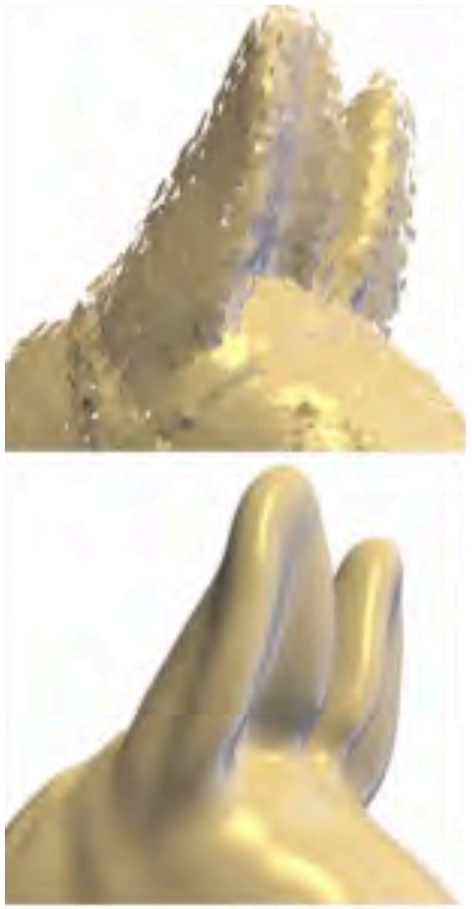

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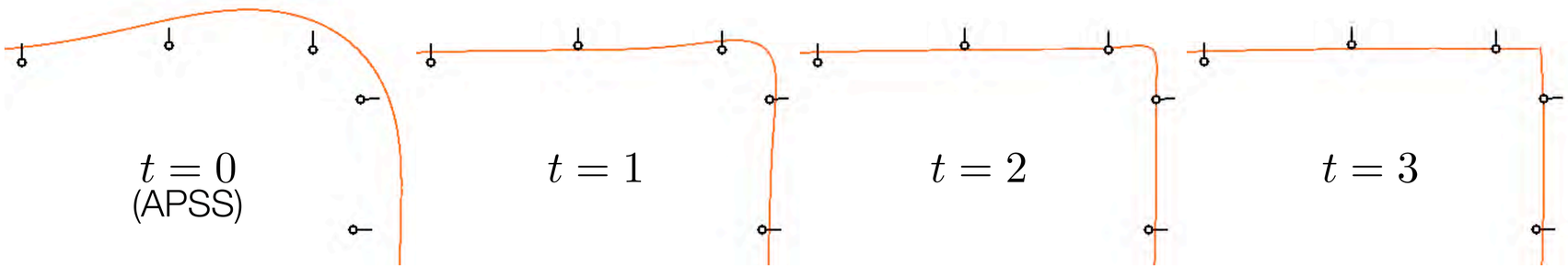
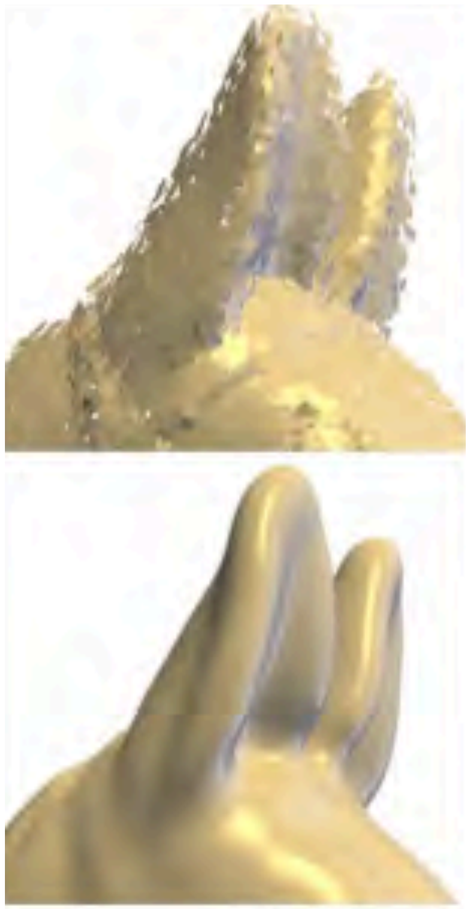
$$w(\mathbf{n}_i) = e^{-(\|\nabla\Phi^t(\mathbf{x}) - \mathbf{n}_i\|/\sigma_\mathbf{n})^2}$$

bilateral filtering





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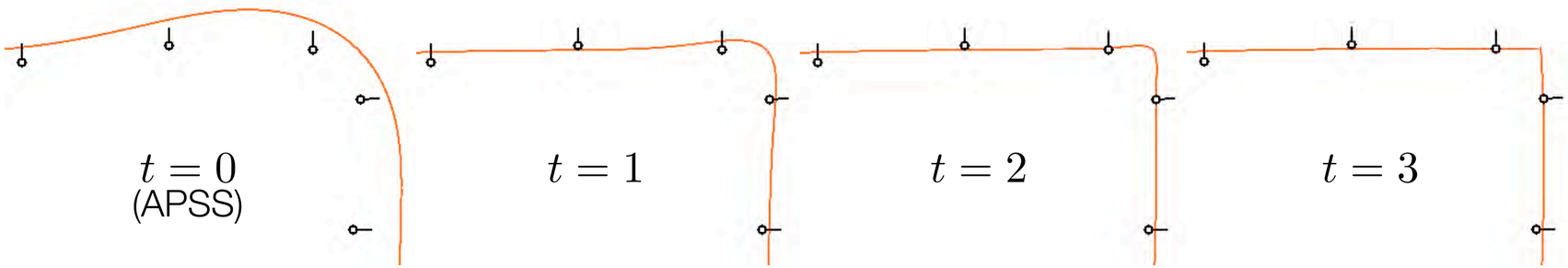
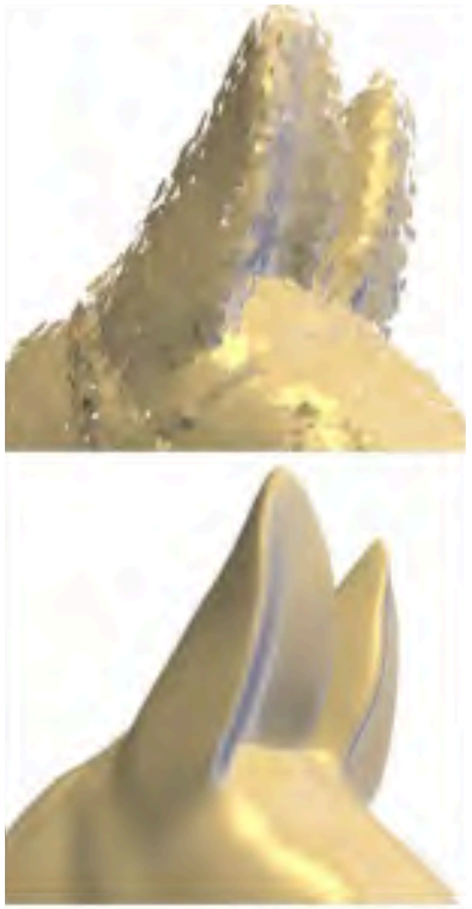


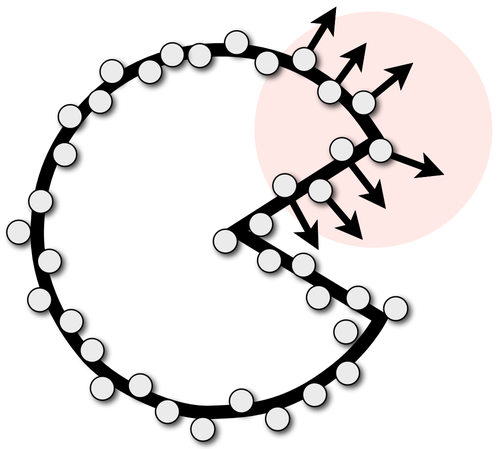


[Oztireli et al. EG'09]

Locally Sharp

- Combine local approximations (MPU)
  - Identify sharp features

$$\min_{i,j}(\mathbf{n}_i \cdot \mathbf{n}_j) < 0.9$$ [Kobbelt et al. SIG 01]



[Ohtake et al. SIG'03]
[Fleishman et al. SIG'04]

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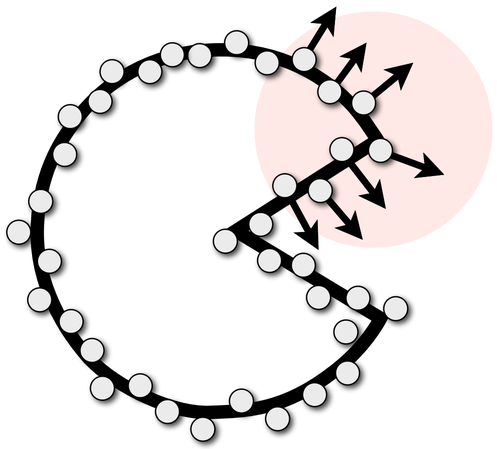

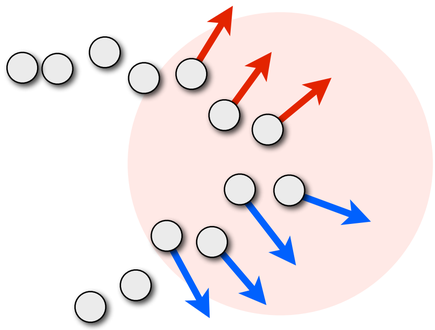

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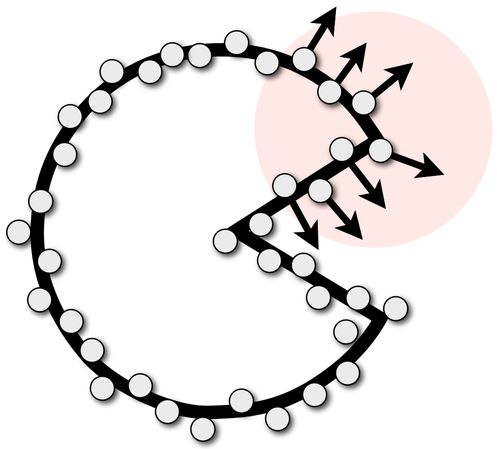

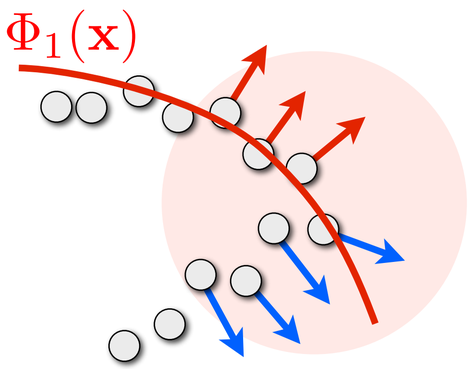

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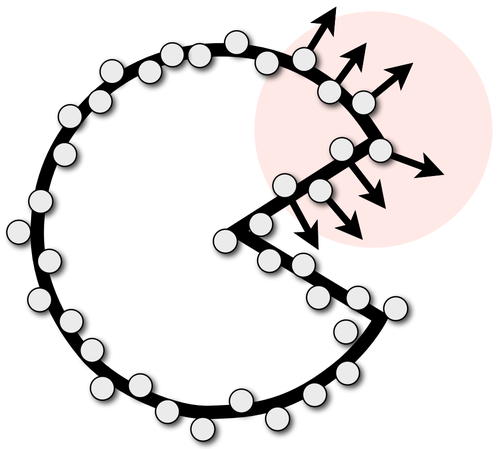

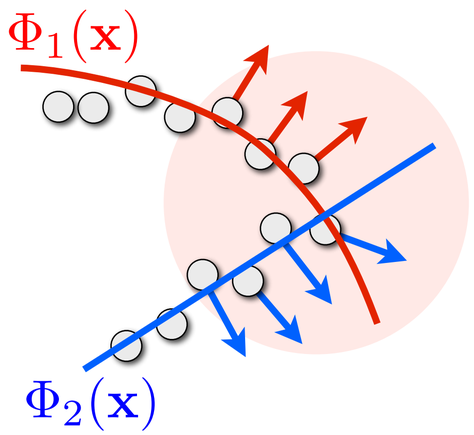

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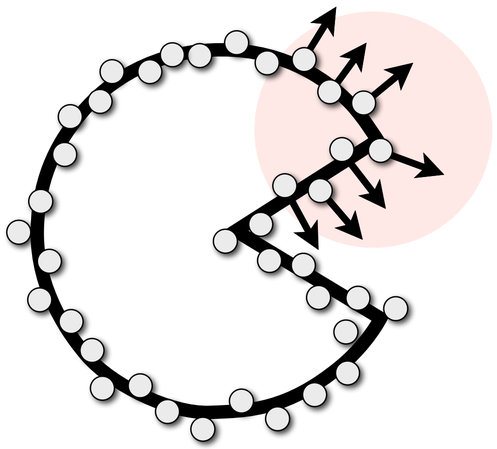
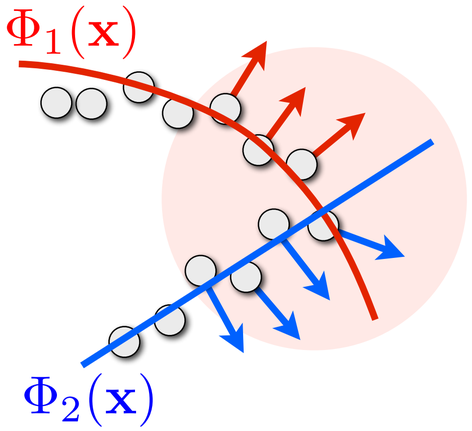
  - Min-Max to form implicit

    $\Phi(\mathbf{x}) = \min(\Phi_1(\mathbf{x}), \Phi_2(\mathbf{x}))$ [Ricci et al. TVC'73]





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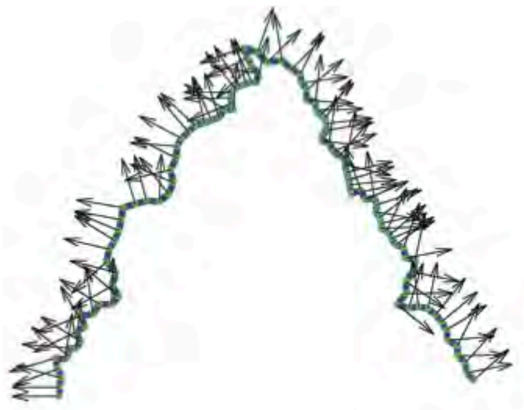
Globally Sharp (variational)

- Global (convex) formulation

Mention w_ij is theta of angles??



[Avron et al. TOG'10]

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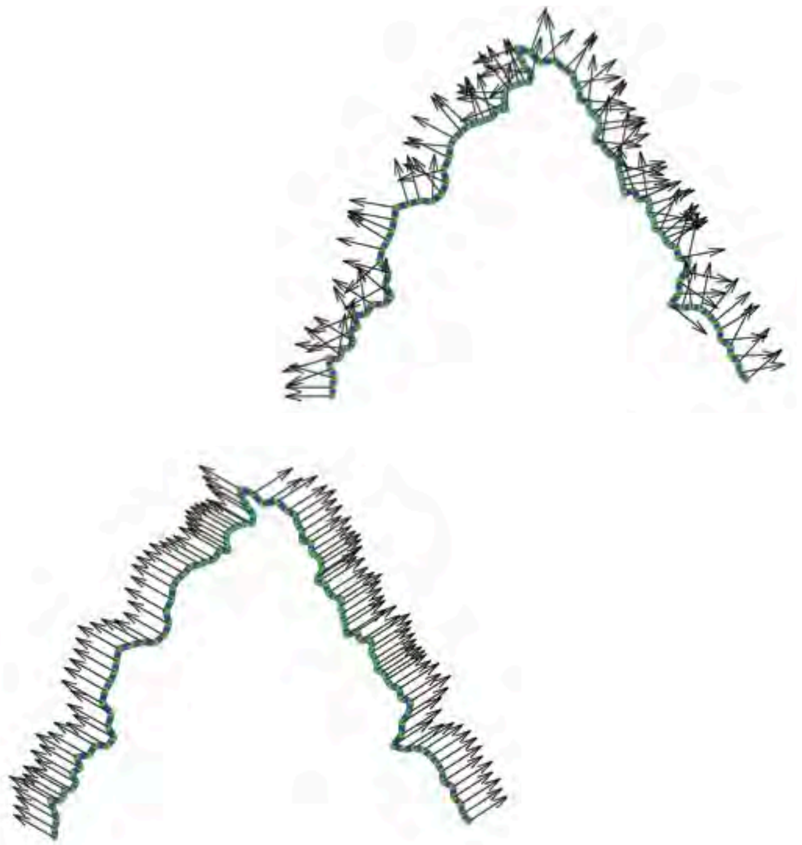

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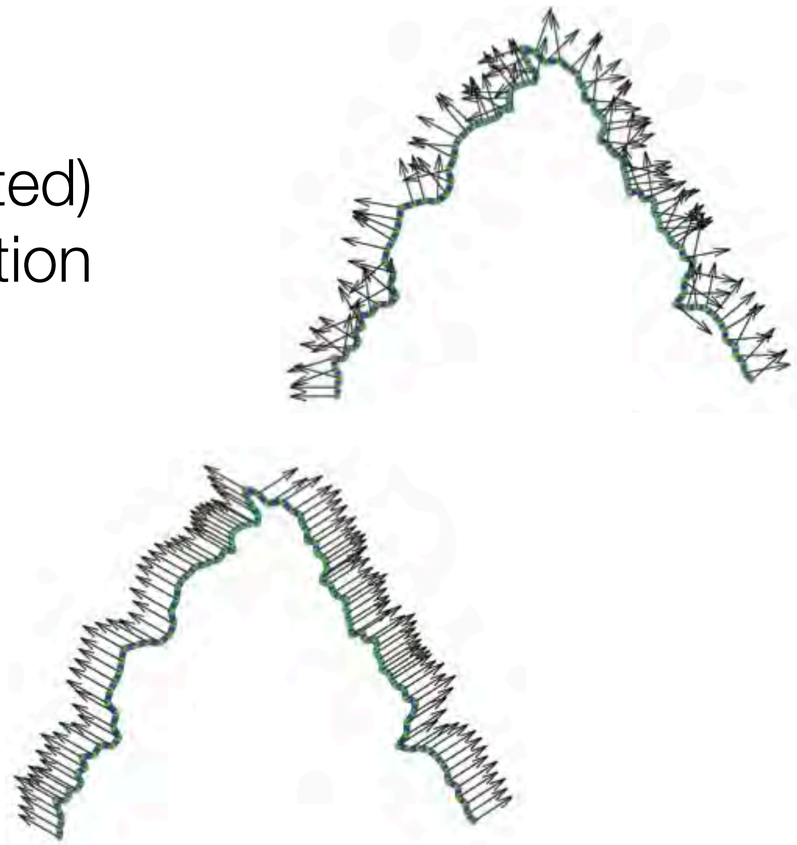

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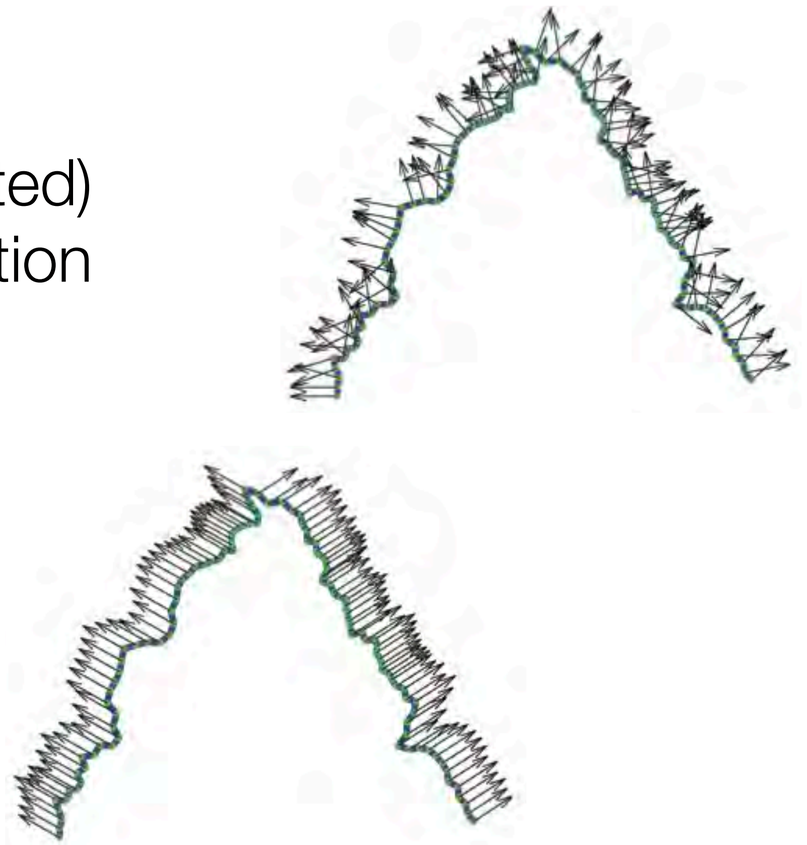
close to initial



[Avron et al. TOG'10]

Globally Sharp (variational)

- Global (convex) formulation
  - Normal smoothing

Mention w_ij is theta of angles??

ℓ_1 (re-weighted) regularization

$$N = \arg\min_{N} \sum_{e_{ij}} w_{ij} \|\mathbf{n}_i - \mathbf{n}_j\|_2$$

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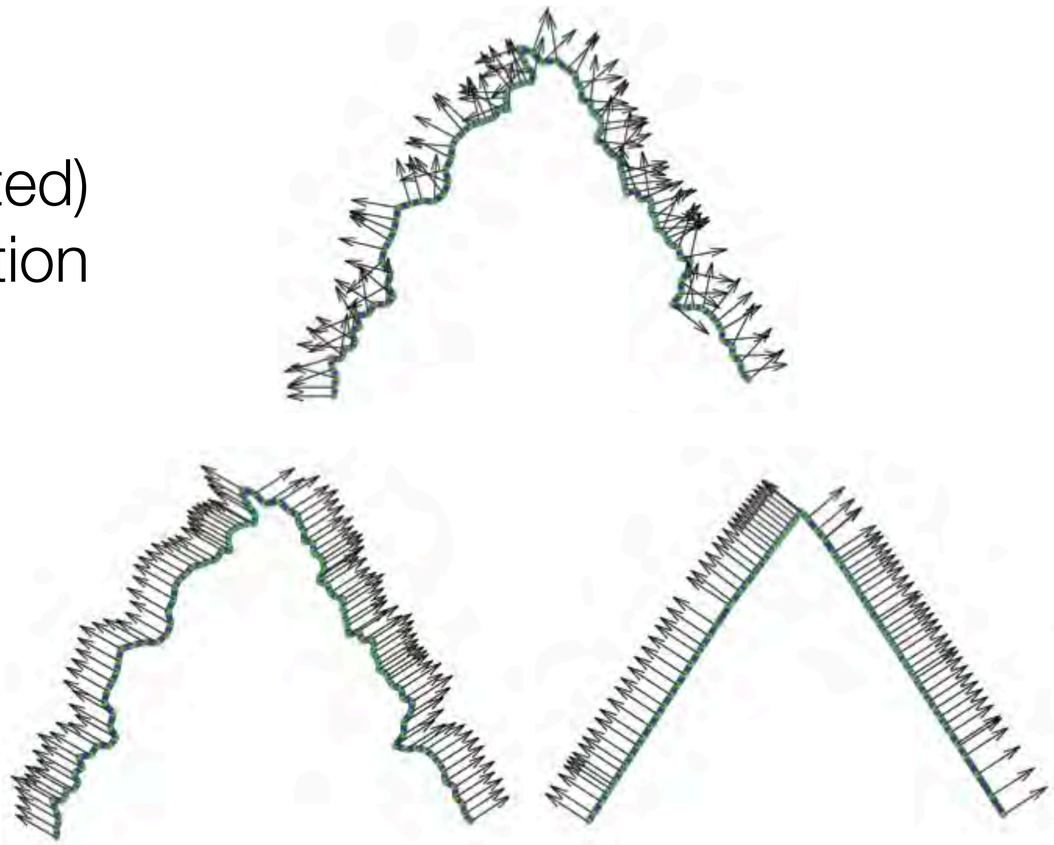
close to initial



  - Offsetting smoothing

$$P = \arg\min_{\mathbf{p}_i = \bar{\mathbf{p}}_i + \alpha \mathbf{n}_i} \sum_{e_{ij}} w_{ij} \left|\mathbf{n}_{i,j} \cdot (\mathbf{p}_i - \mathbf{p}_j)\right| \text{ s.t. } \alpha \in \mathbb{R}$$

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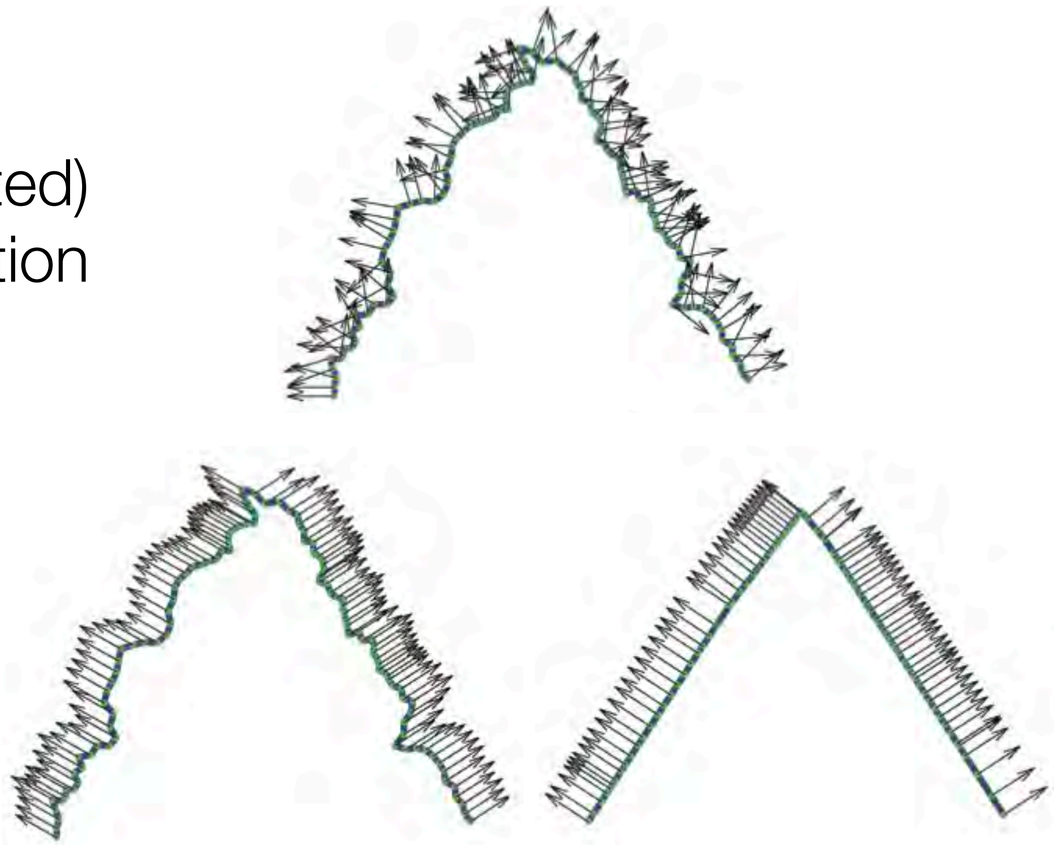

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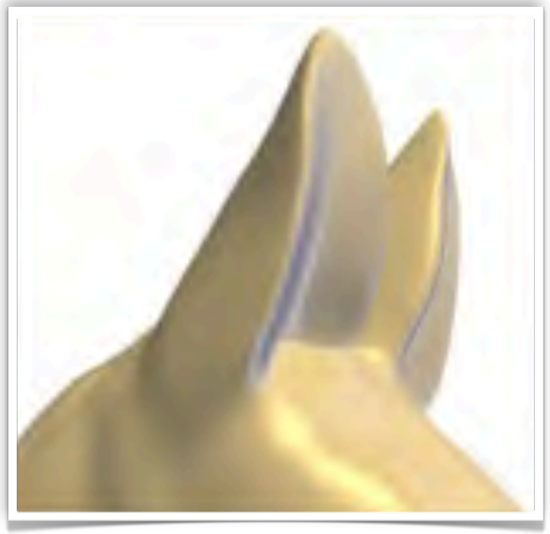
[Avron et al. TOG'10]

(recap) Piecewise Smooth

Ask Gael thoughts

(recap) Piecewise Smooth

Re-weighted IMLS



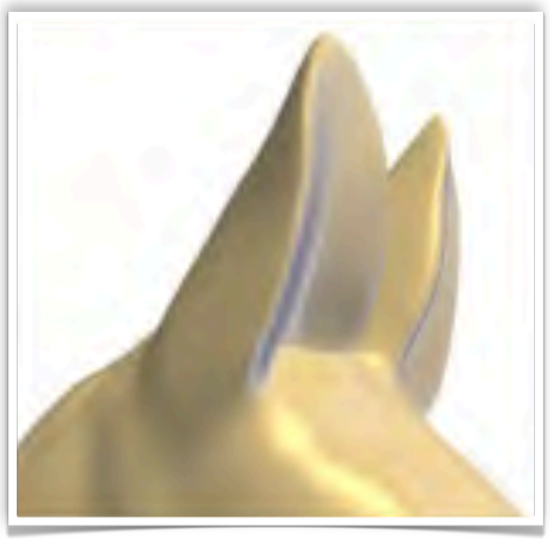
robust fitting + sharp :)
sharp... is still smooth :(

[Oztireli et al. EG'09]

Ask Gael thoughts

(recap) Piecewise Smooth

Re-weighted IMLS



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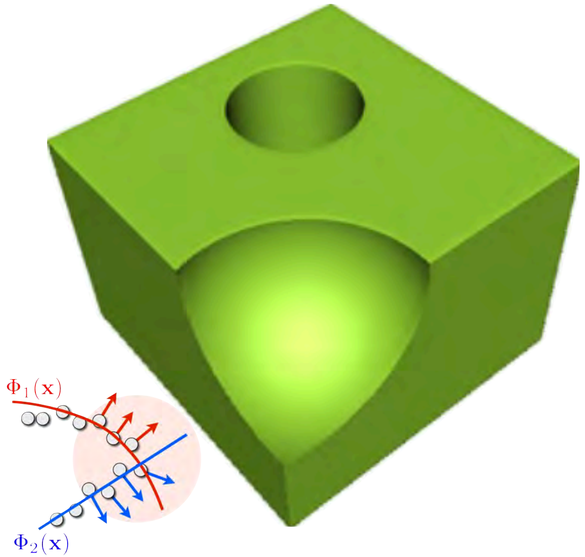
[Oztireli et al. EG'09]

Normal Clustering



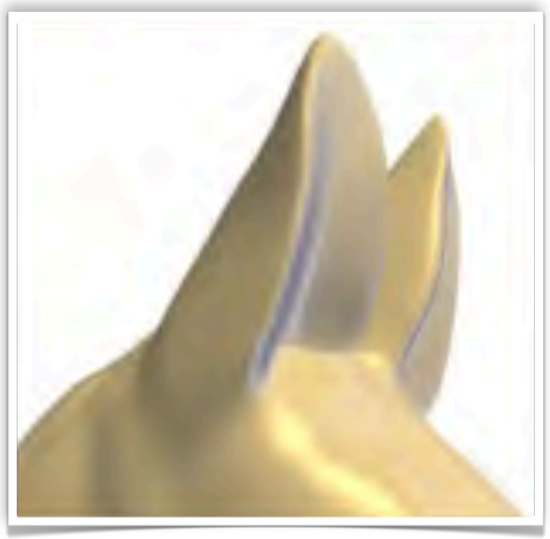
identify & fit
"delicate"

Ask Gael thoughts

[Fleishman et al. SIG'04]

(recap) Piecewise Smooth

Re-weighted IMLS



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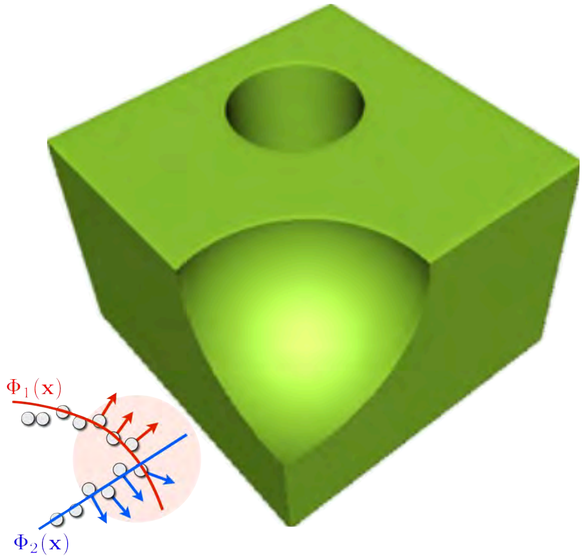
Normal Clustering



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[Fleishman et al. SIG'04]

Sparse Reconstruction

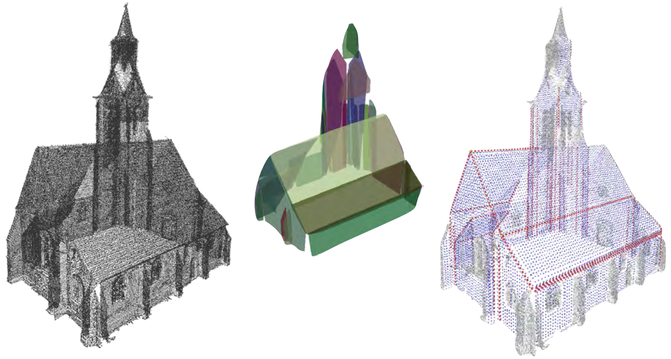
global cvx problem
robust & capture sharp

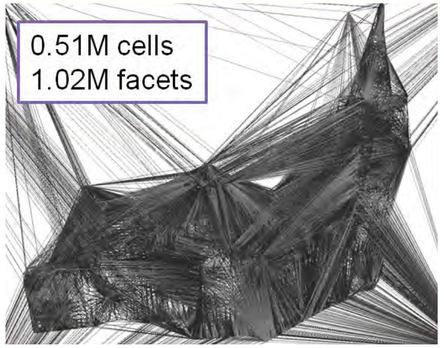
[Avron et al. TOG'10]

(extensions) Piecewise Smooth

(extensions) Piecewise Smooth

Surface Reconstruction by Point Set Structuring

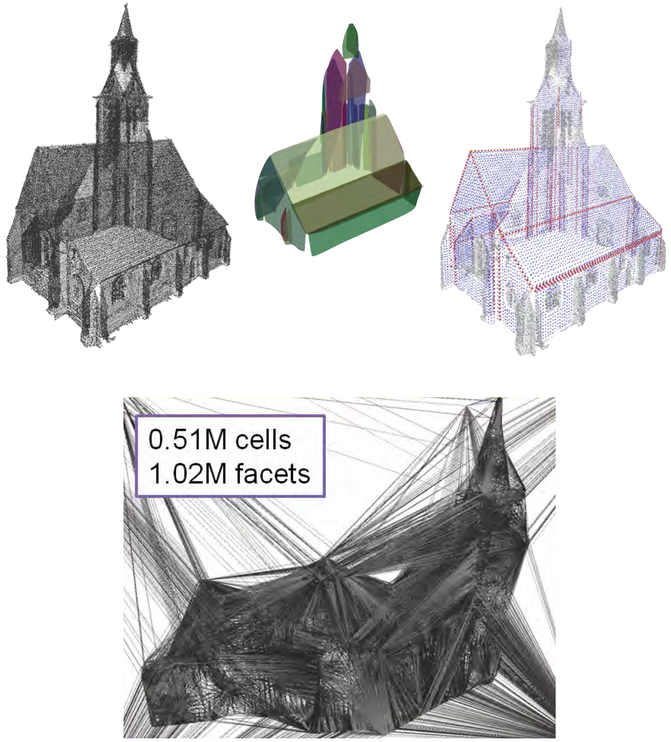




[Lafarge et al. EG'13]

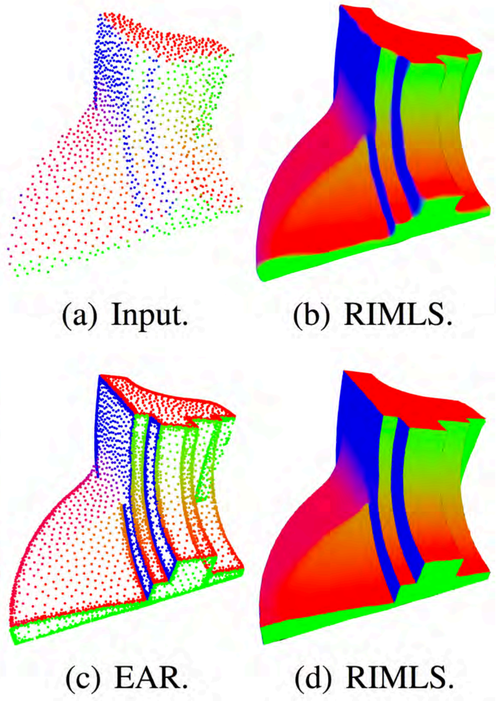
(extensions) Piecewise Smooth

Surface Reconstruction by Point Set Structuring



[Lafarge et al. EG'13]

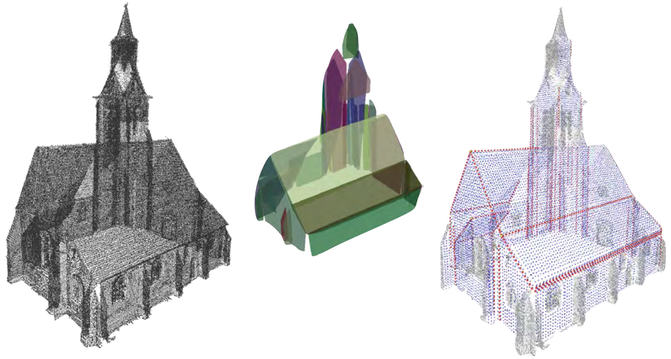
Edge **A**ware Point Set **R**esampling

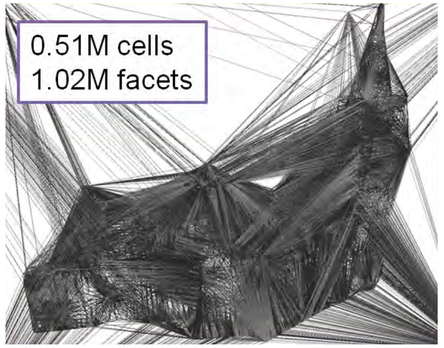


[Huang et al. TOG'13]

(extensions) Piecewise Smooth

Surface Reconstruction by Point Set Structuring





[Lafarge et al. EG'13]

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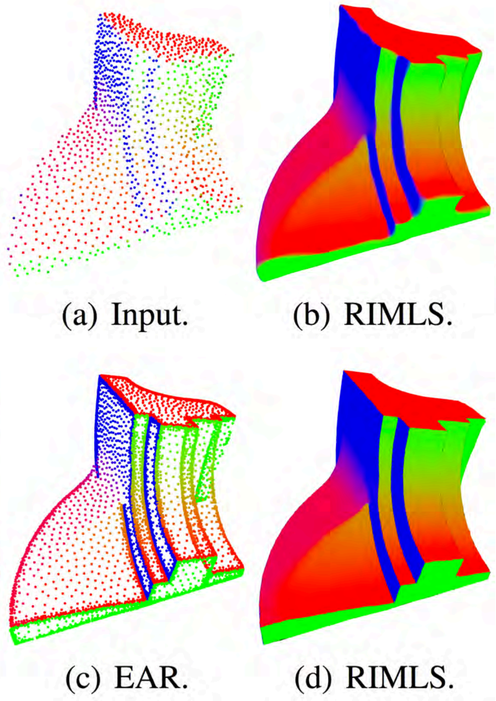
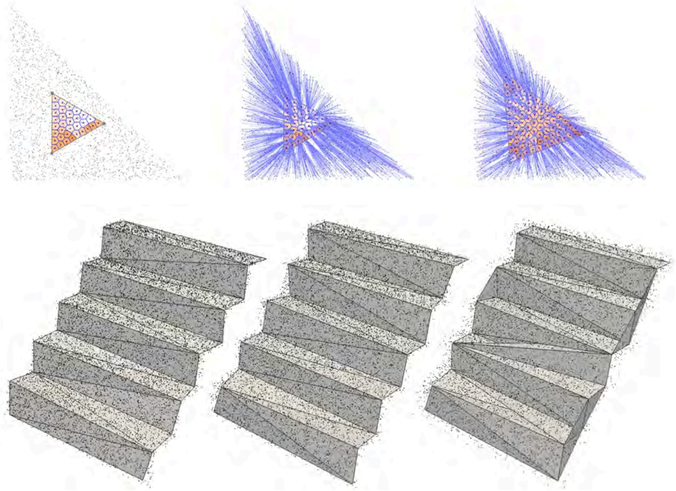


[Huang et al. TOG'13]

Feature-Preserving (Optimal Transport)

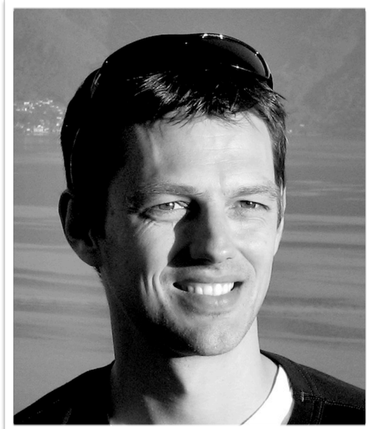


simultaneous recon./simplification

[Digne et al. JMIV'13]

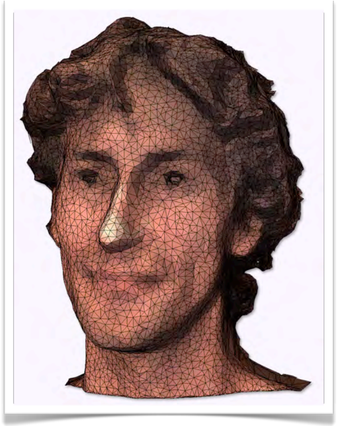
Acknowledgements

Misha Kazhdan


Mark Pauly

Mario Botsch

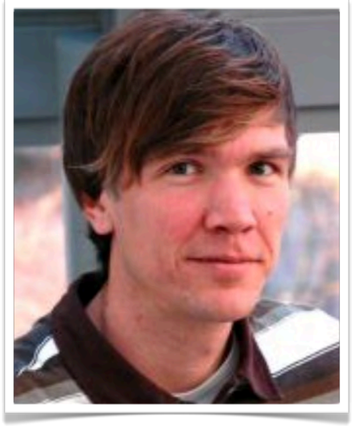
Yaron Lipman


Florent Lafarge

Gaël Guennebaud

Pierre Alliez

Oliver Mattausch


Alexandre Sorkine-Hornung