Mean Curvature Skeletons

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1) SKELETONIZATION

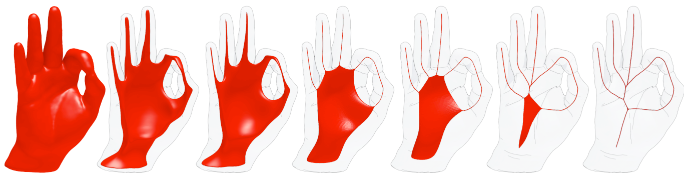
S $\qquad \dot{S} = -\vec{n}$ $\qquad \dot{S} = -H\vec{n}$

Skeletons as singularities of surface flows

2) CONTRIBUTIONS

- Anisotropy analysis of MCF
- Polished FEM discretization
- Medial skeletonization flow

$$\dot{S} = -H\vec{n} - \omega \vec{N}^{t=0}$$



3) MEAN CURVATURE FLOW

$$\dot{S} = -H\,\vec{n}$$

Each point on the surface moves...

... with a velocity equal to its local mean curvature: $H = (\kappa_1 + \kappa_2)/2$

in a direction normal to the surface...

More Information

Andrea Tagliasacchi (andrea.tagliasacchi@gmail.com), Ibraheem Alhashim, Matt Olson, Hao Zhang, "Mean Curvature Skeletons", Computer Graphics Forum (Proceedings of the Symposium on Geometry Processing), 2012. Project Homepage: https://code.google.com/p/starlab-mcfskel

4) AREA MINIMIZATION

Skeleton Extraction by Mesh Contraction [SIGGRAPH'08]

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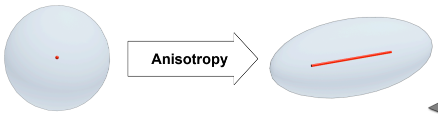
Original shape:	$A > 0$	$V > 0$
Curvature Flow:	$\dot{A} < 0$ (necessary)	$\dot{V} < 0$
Curve-Skeleton:	$A = 0$	$V = 0$

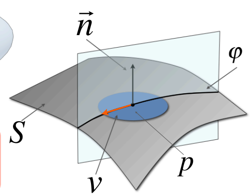
Contraction

5) MCF - ANISOTROPY



$$\langle \dot{v}, v \rangle = -H(p)\,\Pi_p(v)$$

Second Fundamental Form



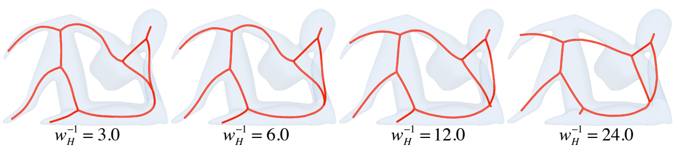
6) MCF DISCRETIZATION (FEM)

Iteratively minimize the "contraction" energy:

$$E_{MCF} = \left\| L^t V^{t+1} \right\|^2 + w_H \sum \left\| v_i^{t+1} - v_i^t \right\|^2$$

Approximates Laplacian $L^{t+1} \approx L^t$

Bounds Approximation (by bounding velocity)



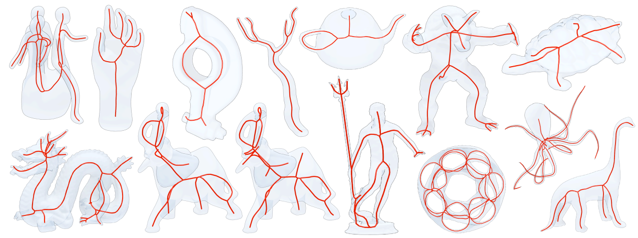
7) MEDIAL CENTERING

$$E = E_{MCF} + \sum \left\| v_i^{t+1} - \mu(v_i^t) \right\|^2$$

Corresponding Medial Pole

- Poles approximate normals
- Maintains flow centred-ness
- Still converges to curve
- Boost performance
  - ➡ external v.s. internal forces
  - ➡ weaker velocity control w_H

8) RESULTS (CURVE SKELETON)



9) RESULTS (MESO SKELETON)

Intermediate states of the medial flow: mixture of curvilinear and surface elements