Modern Techniques and Applications for Real-Time Non-rigid Registration

Andrea Tagliasacchi Hao Li

Sicily 2003, Level 1
09:00 — 10:45

SIGGRAPH ASIA 2016 MACAO

http://gfx.uvic.ca/pubs/2016/registration/slides

Presenters

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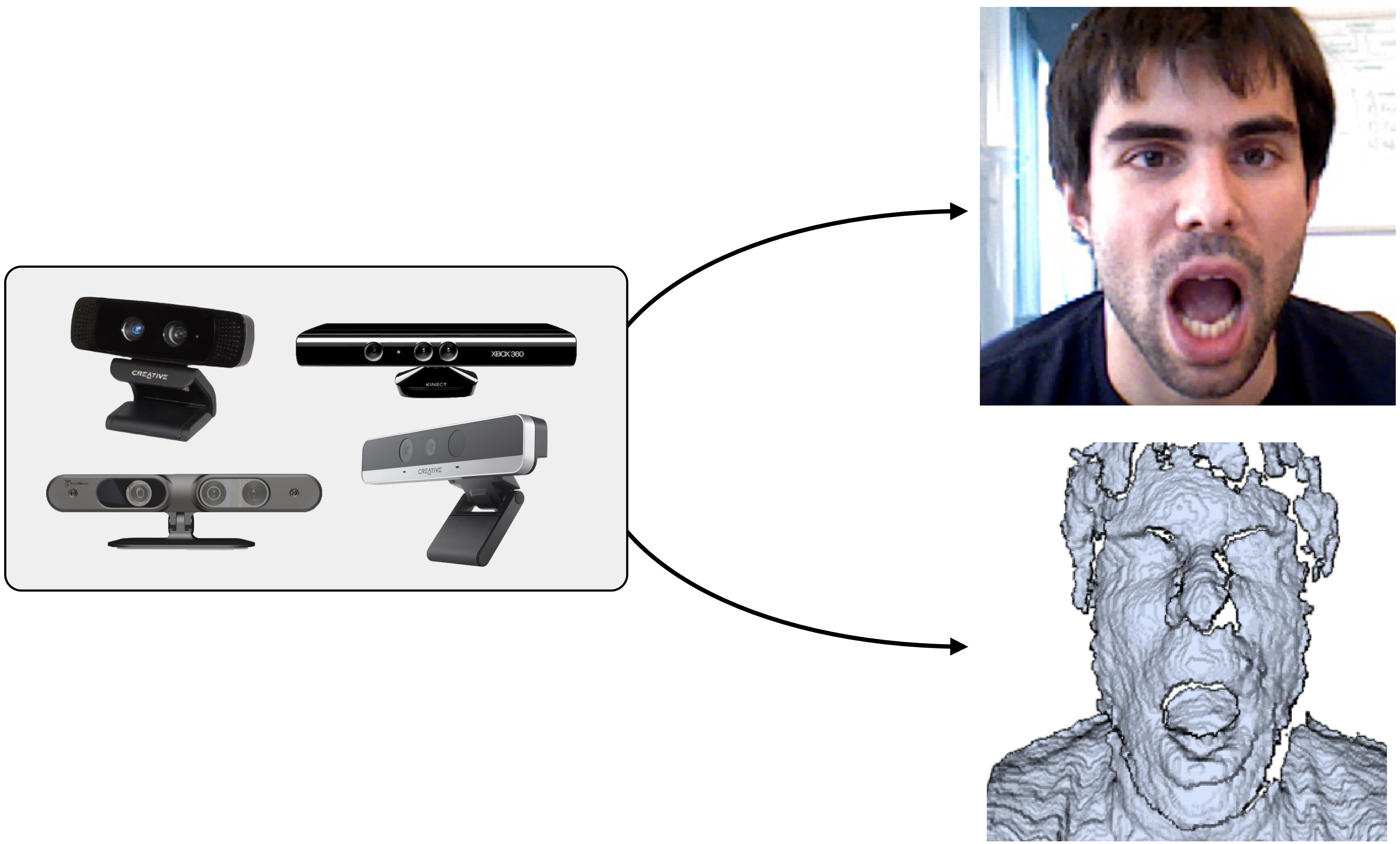
Chair, Intel/Google Industrial Research in 3D Sensing

Overview

- Part 1 - presented by Andrea Tagliasacchi
  - Introduction, motivation and sensing hardware (5 min)
  - Rigid registration, ICP, Hausdorff distances (20 min)
  - Robust registration, articulated registration (20 min)
- Part 2 - presented by Hao Li
  - Non-rigid registration and face tracking (20 min)
  - Correspondences with Convolutional Neural Networks (20 min)
  - Conclusions and Q&A (10 min)

RGB-D Sensors



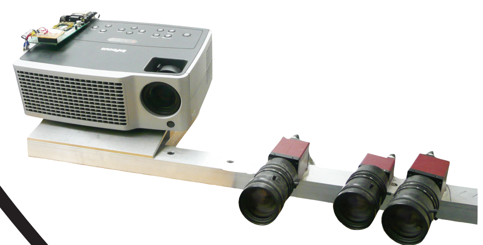
RGB-D Sensors

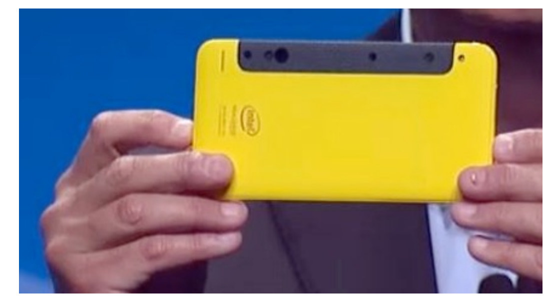
2008

2010

2013

2015

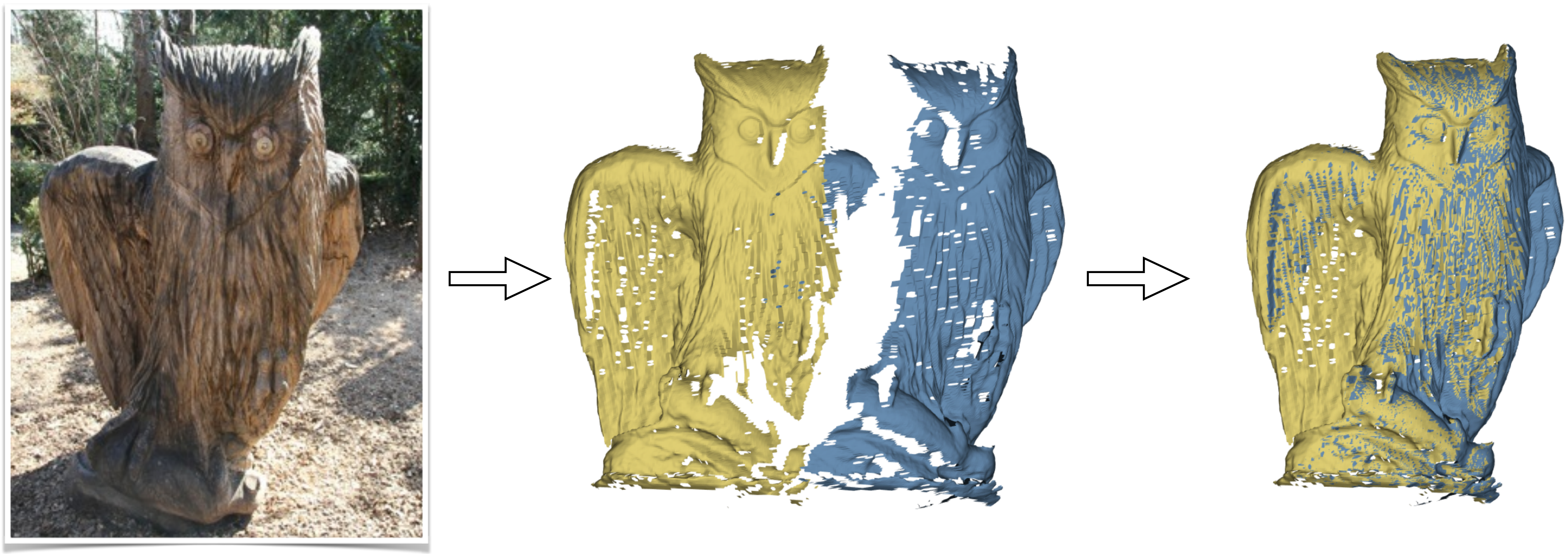




size, cost

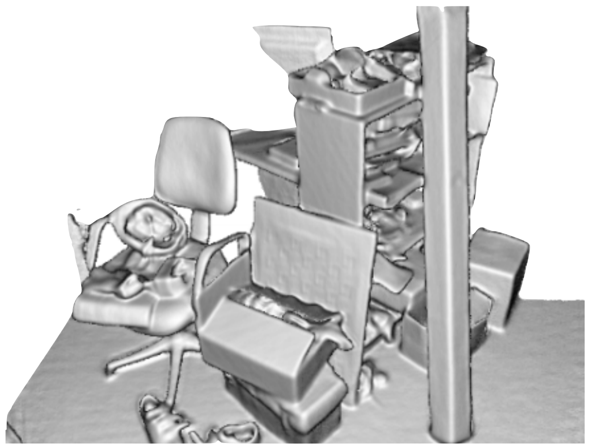
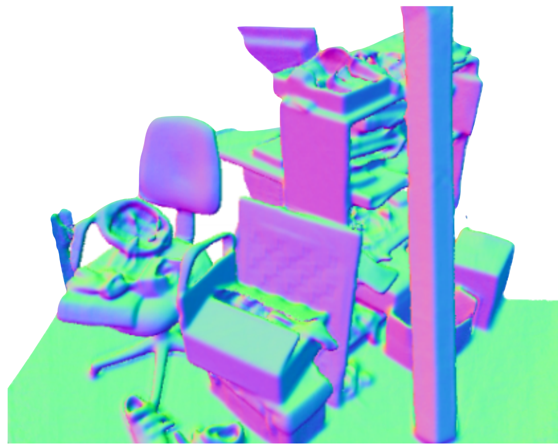
Registration - Examples

- Scan to scan



Registration - Examples

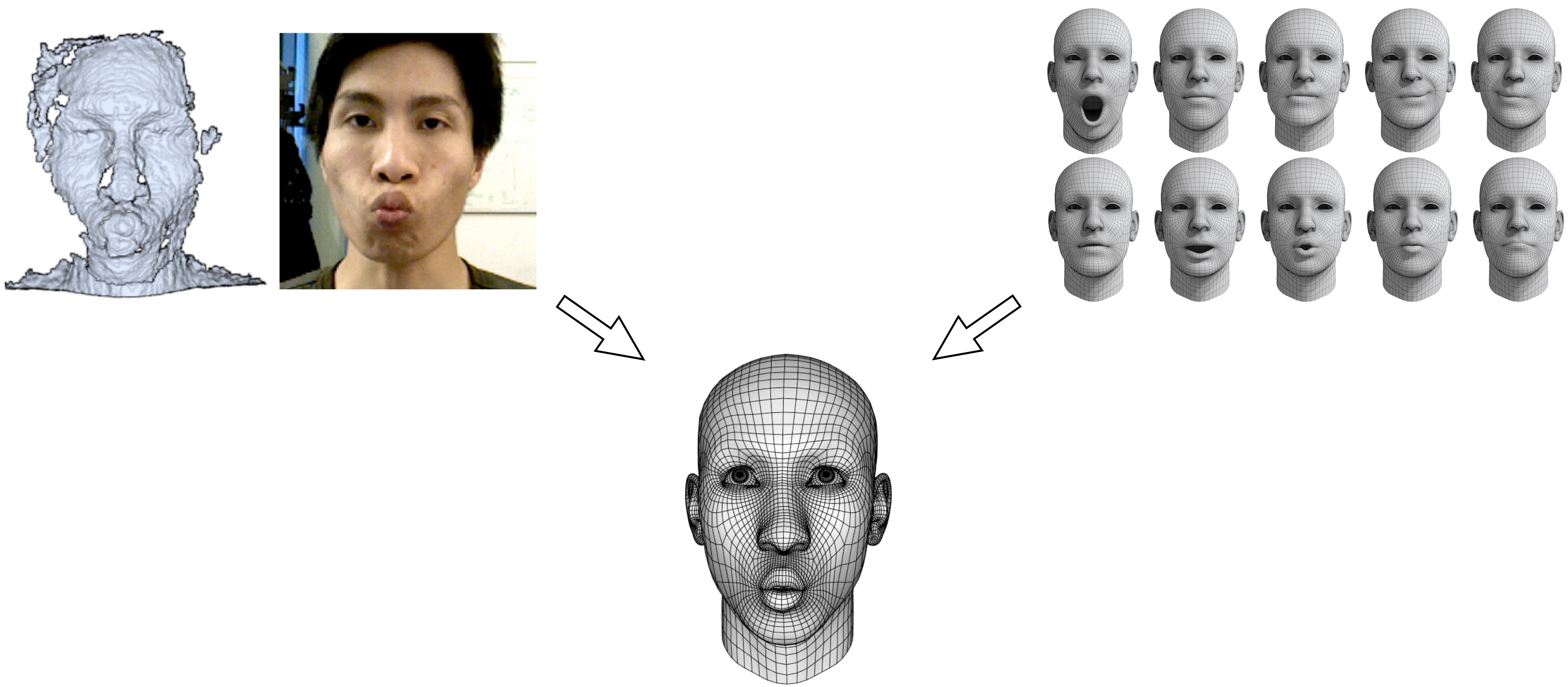
- Scan to scene

Newcombe et al. KinectFusion: Real-Time Dense Surface Mapping and Tracking, ISMAR 2011

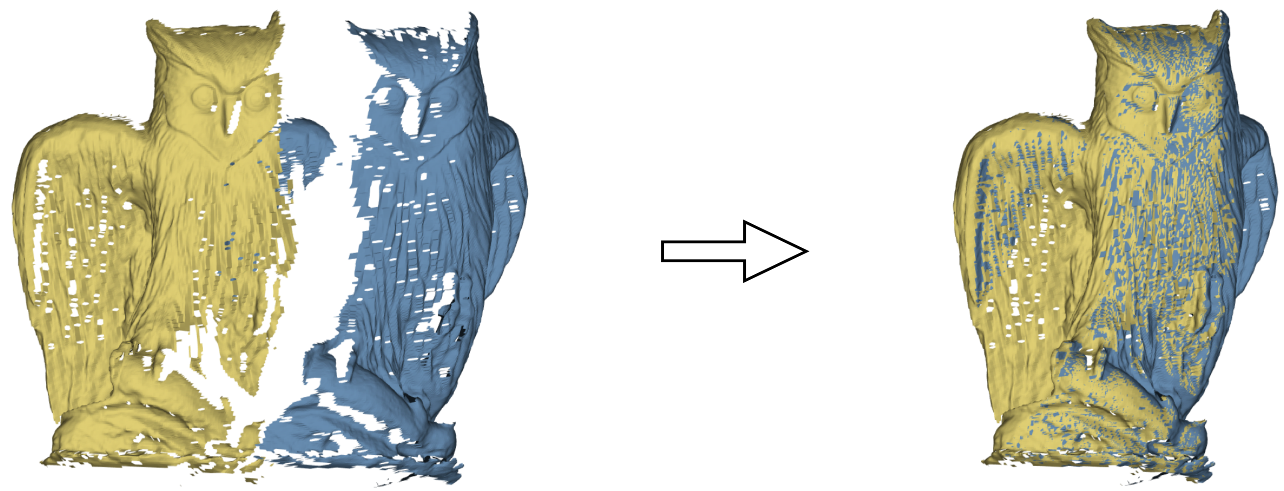
Registration - Examples

- Scan to parameterized template



Pairwise Registration

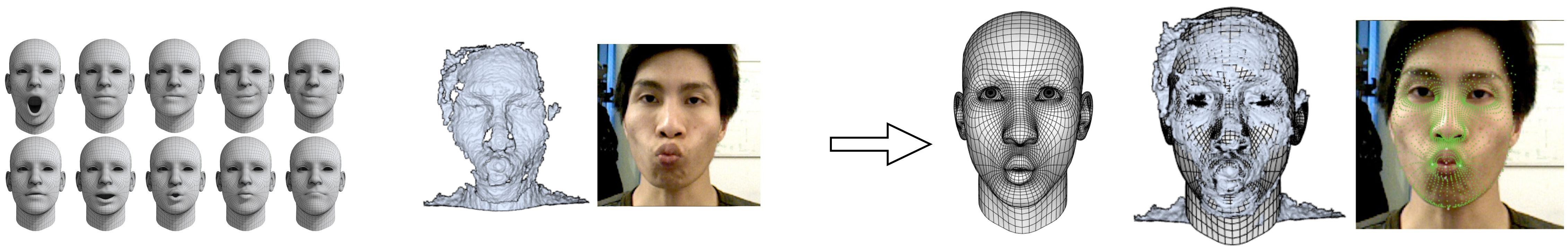
- Align a *source model* $\mathcal{X}$ onto a *target model* $\mathcal{Y}$
  - find a *transformation* $T(\mathcal{X})$ that brings $\mathcal{X}$ into alignment with $\mathcal{Y}$



$\mathcal{X}$ $\quad$ $\mathcal{Y}$ $\quad$ $\mathcal{Z} = T(\mathcal{X})$ $\quad$ $\mathcal{Y}$

Pairwise Registration

- Align a *source model* $\mathcal{X}$ onto a *target model* $\mathcal{Y}$
  - find a *transformation* $T(\mathcal{X})$ that brings $\mathcal{X}$ into alignment with $\mathcal{Y}$



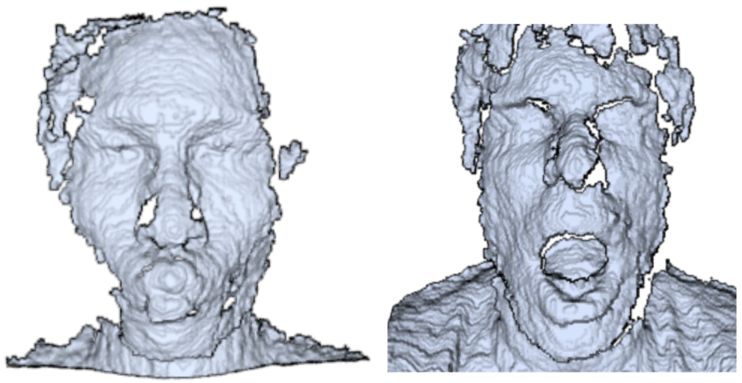
$\mathcal{X}$ $\qquad$ $\mathcal{Y}$ $\qquad$ $\mathcal{Z} = T(\mathcal{X})$

Pairwise Registration

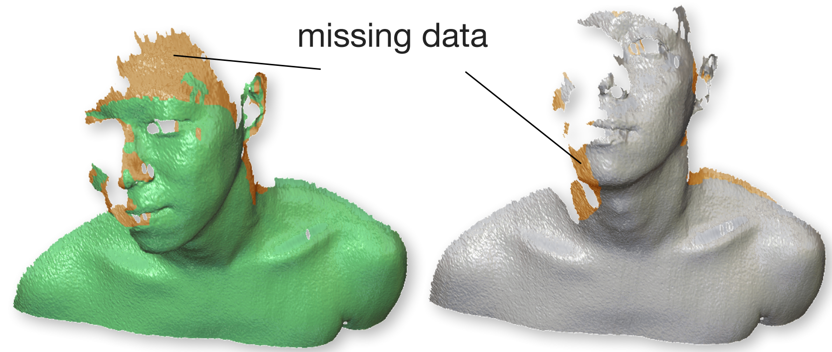
- Align a *source model* $\mathcal{X}$ onto a *target model* $\mathcal{Y}$
  - find a *transformation* $T(\mathcal{X})$ that brings $\mathcal{X}$ into alignment with $\mathcal{Y}$
- Two main questions:
  - How do we measure the <u>quality</u> of the alignment?
  - What <u>transformations</u> are acceptable?

Pairwise Registration

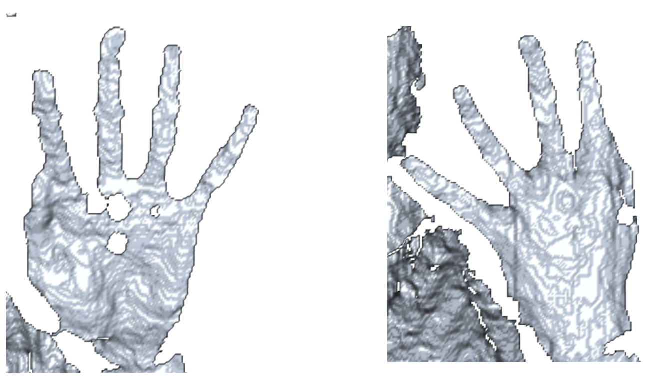
- Fundamental issues



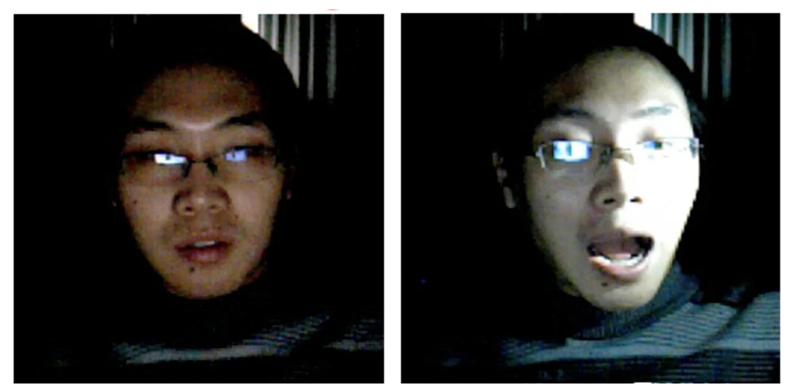
Noise



Partial matching



Ambiguity



Illumination changes

Registration

- Registration as energy minimization

source

$$\underset{T}{\arg\min}\ E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

transformation

target

Registration

- Registration as energy minimization

$$\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

$$E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y}) = E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) + E_{\text{prior}}(T)$$

Alignment Error

How do we measure the quality of the alignment?

Registration

- Registration as energy minimization

$$\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

$$E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y}) = E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) + E_{\text{prior}}(T)$$

Alignment Error

How do we measure the quality of the alignment?

Transformation Error

What transformations are allowed / good?

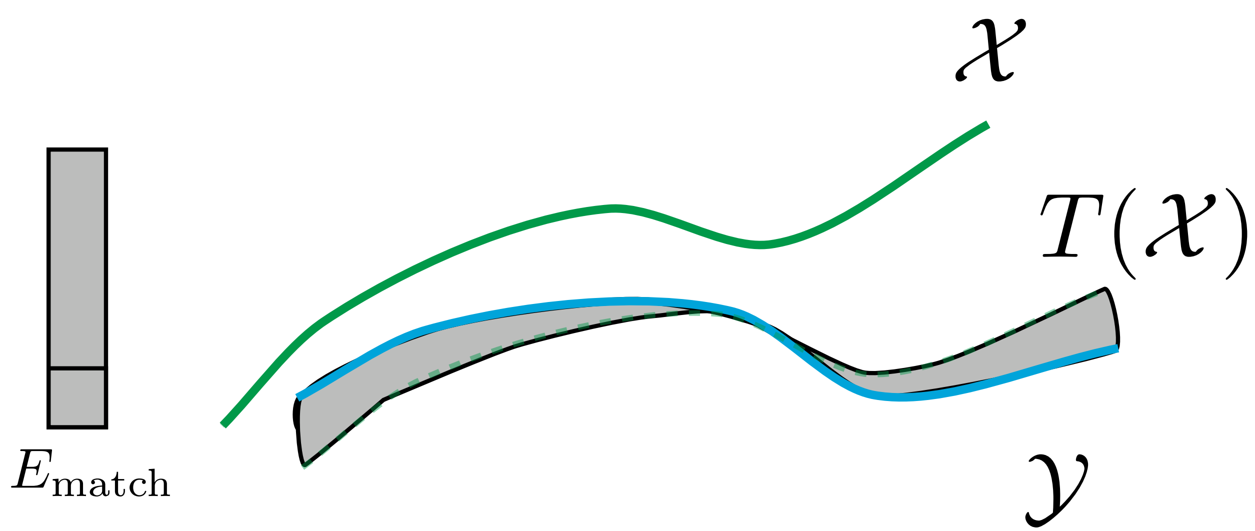
Registration

- Alignment Error

$$E_{\text{reg}} = \boxed{E_{\text{match}}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



Registration

$$E_{\text{reg}} = E_{\text{match}} + \boxed{E_{\text{prior}}}$$

rigid

composite

articulated

elastic

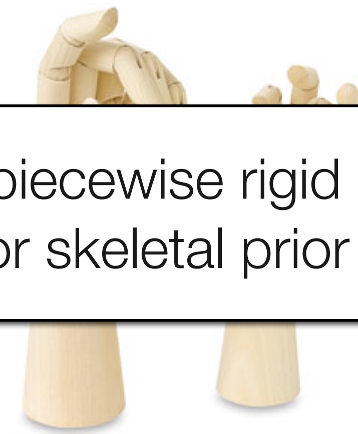
fluid

Registration

$$E_{\text{reg}} = E_{\text{match}} + \boxed{E_{\text{prior}}}$$

rigid motion prior

rigid

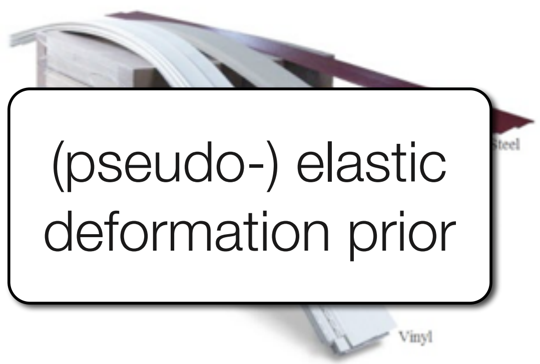


piecewise rigid or skeletal prior

articulated

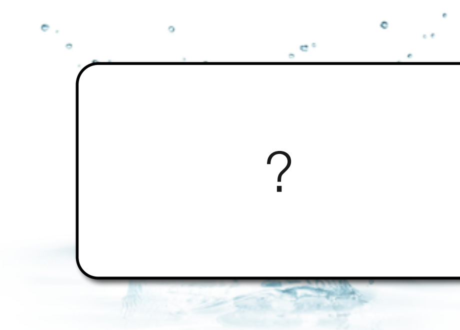
data-driven prior

composite



(pseudo-) elastic deformation prior

elastic



?

fluid

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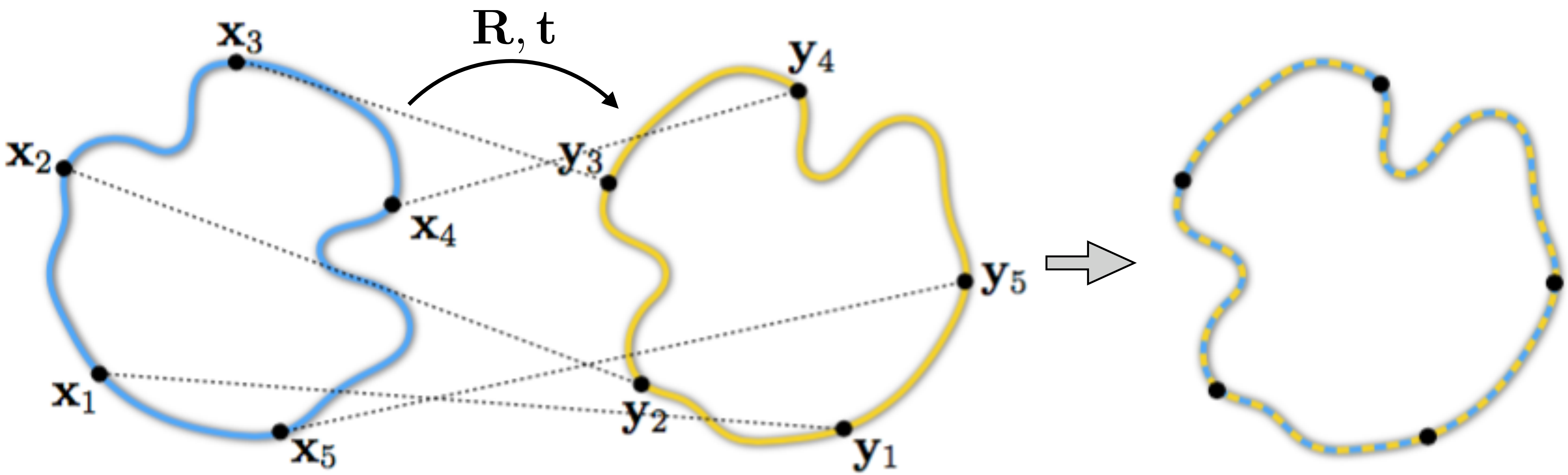
Shape Matching Problem

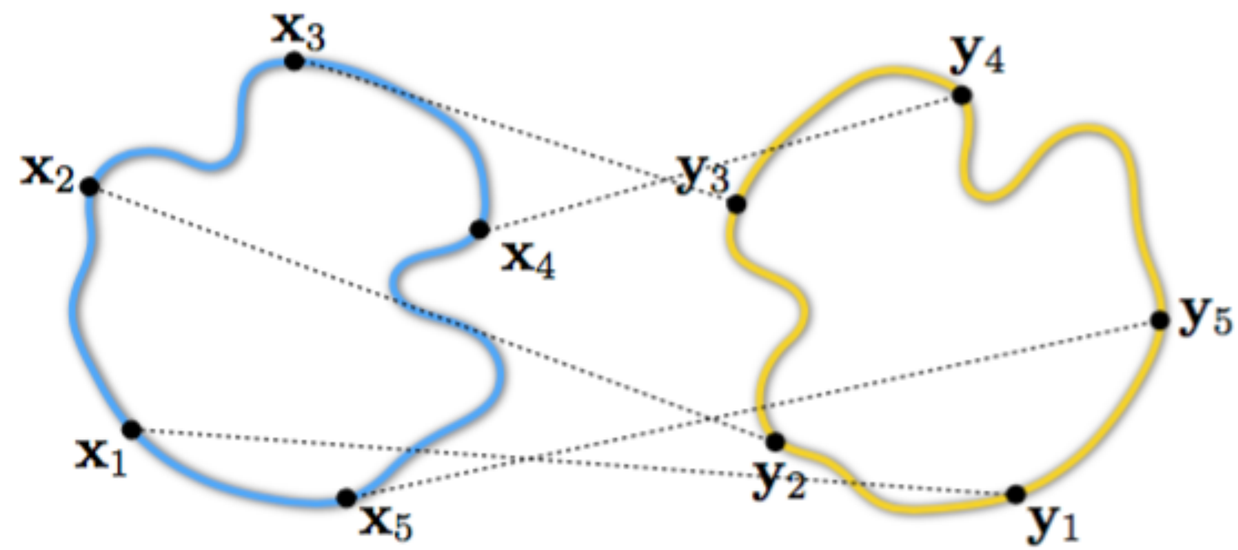
- Given "exact" pairwise correspondences
  - what is the optimal <u>rigid</u> transformation of X into Y?



Shape Matching Problem

- Given "exact" pairwise correspondences
  - what is the optimal (rigid) transformation of X into Y?
  - let us write this problem as an <u>energy minimization</u>

$$\arg\min_{\mathbf{R},\mathbf{t}} \sum_{n=1}^{N} \|(\mathbf{R}\mathbf{x}_n + \mathbf{t}) - \mathbf{y}_n\|_2^2 + I_{SO(3)}(\mathbf{R})$$



Sorkine "Least-Squares Rigid Motion Using SVD" Technical Report 2009

Shape Matching Problem

- What is the optimal translation aligning X into Y?
  - Compute barycenters of the two sets

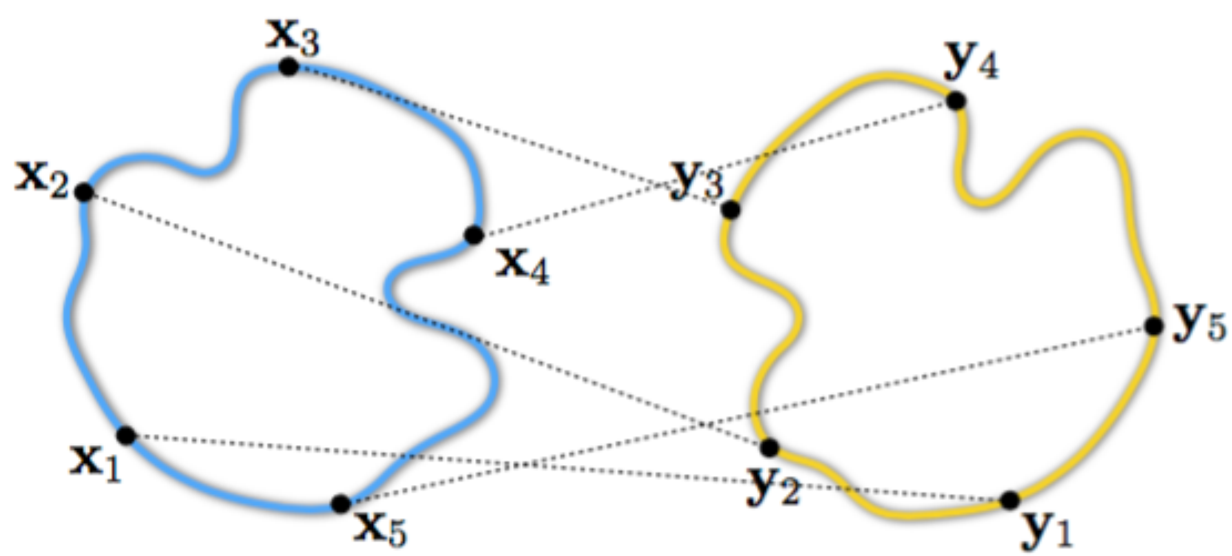
$$\hat{\mathbf{x}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}_n, \qquad \hat{\mathbf{y}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{y}_n$$

  - Define "barycentered" point sets

$$\bar{\mathbf{x}}_i = \mathbf{x}_i - \hat{\mathbf{x}} \qquad \bar{\mathbf{y}}_i = \mathbf{y}_i - \hat{\mathbf{y}}$$

  - Translation $\mathbf{t}$ maps barycenters

$$\mathbf{t} = \hat{\mathbf{y}} - \mathbf{R}\hat{\mathbf{x}}$$



Sorkine "Least-Squares Rigid Motion Using SVD" Technical Report 2009

Shape Matching Problem

- What is the optimal rotation aligning X into Y?
  - Given the "barycentered" point sets

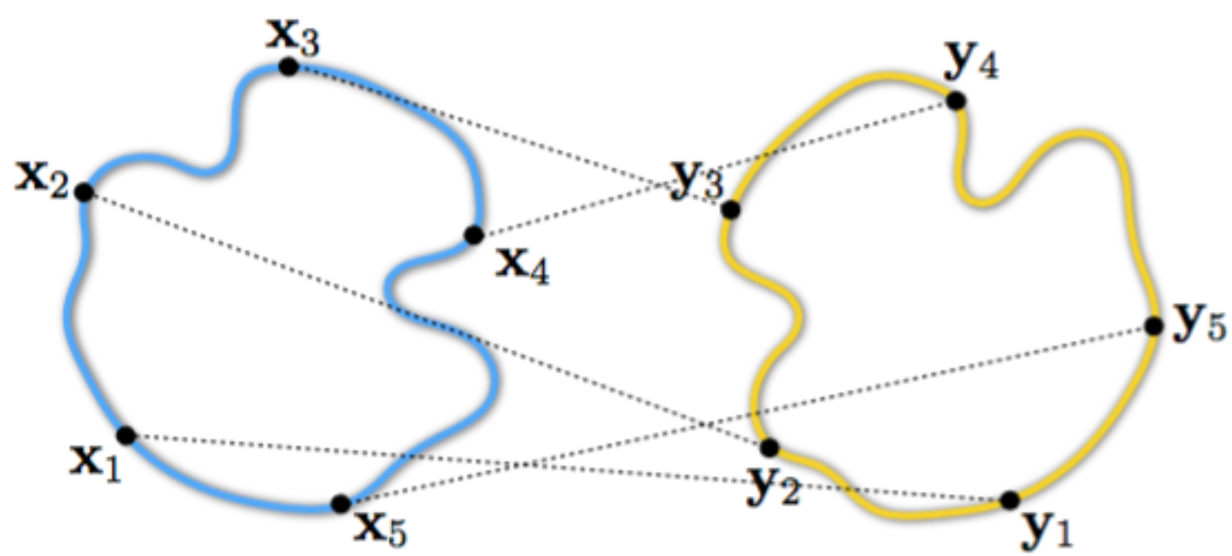
    $$\bar{X} = [\bar{\mathbf{x}}_1, \dots, \bar{\mathbf{x}}_N] \quad \bar{Y} = [\bar{\mathbf{y}}_1, \dots, \bar{\mathbf{y}}_N]$$

  - Compute covariance matrix (and SVD)

    $$[U \Sigma V^T] = \mathrm{SVD}(\bar{X}\bar{Y}^T)$$

  - Optimal rotation given by:

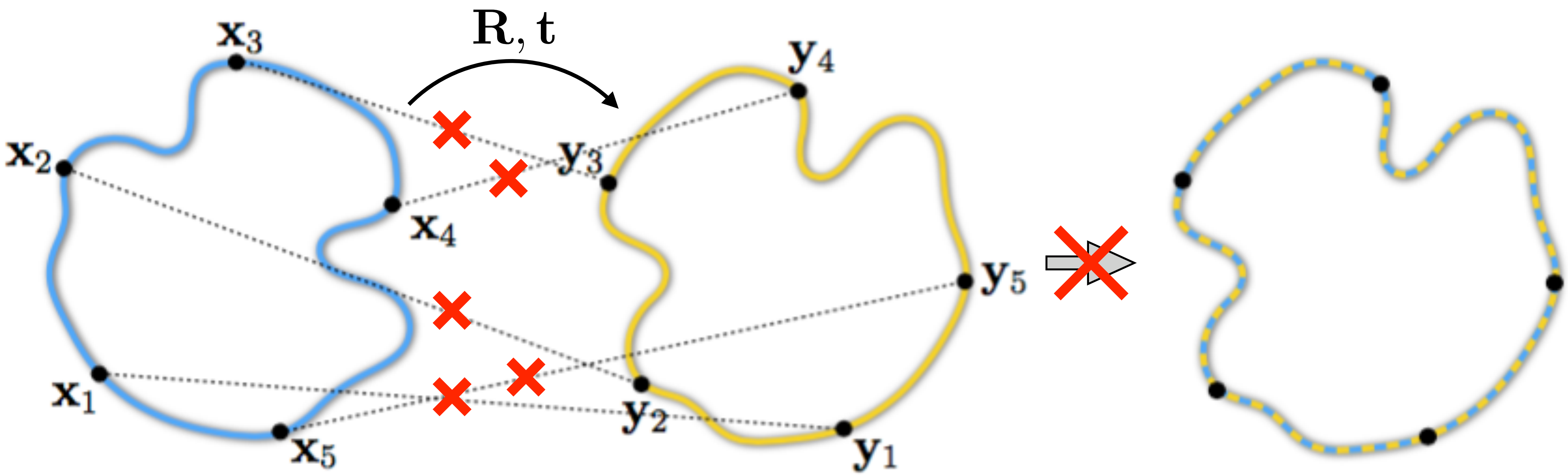
    $$\mathbf{R} = V \begin{pmatrix} 1 & & \\ & 1 & \overset{\text{reflections}}{\searrow} \\ & & \boxed{\det(VU^T)} \end{pmatrix} U^T$$



Sorkine "Least-Squares Rigid Motion Using SVD" Technical Report 2009

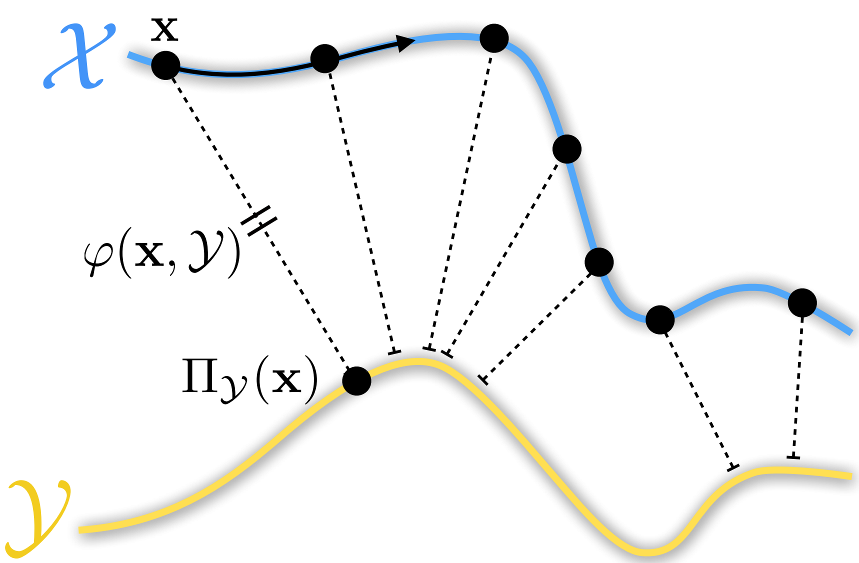
Shape Matching Problem

- ~~Given "exact" pairwise correspondences~~ not available :(
  - what is the optimal (rigid) transformation of X into Y?



Iterative Closest Point

- We don't have "exact" correspondences…
  - But we can design a <u>metric</u> that measures <u>alignment</u> quality!



$$E_{\text{match}}(\mathcal{X}) = \int_{\mathcal{X}} \varphi(\mathbf{x}, \mathcal{Y}) d\mathbf{x}$$

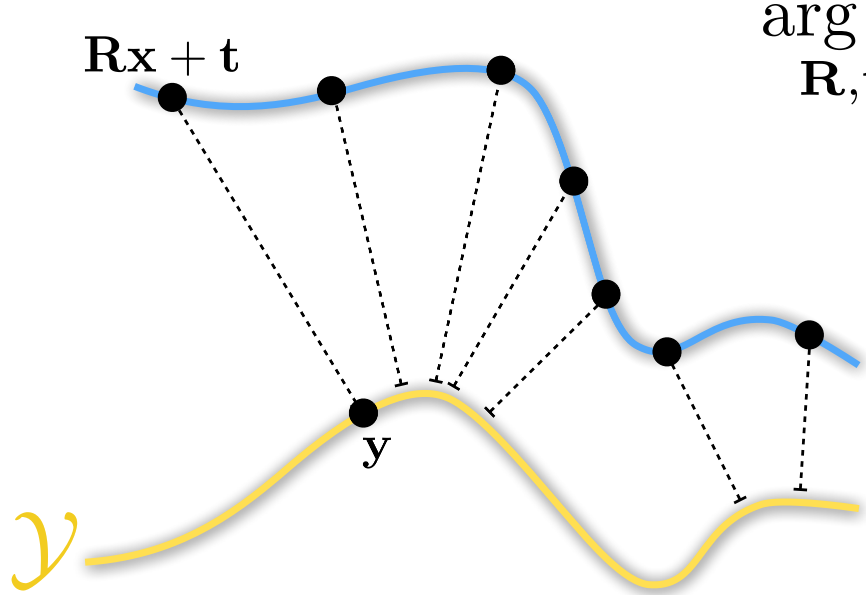
$$\approx \sum_{n=1}^{N} \varphi(\mathbf{x}_n, \mathcal{Y})$$

Euclidean metric

$$\approx \sum_{n=1}^{N} \|\mathbf{x}_n - \Pi_{\mathcal{Y}}(\mathbf{x}_n)\|_2^2$$

Iterative Closest Point

- We don't have "exact" correspondences…
  - Introduce <u>auxiliary</u> variables Y (closest point correspondences)



$$\arg\min_{\mathbf{R},\mathbf{t},Y} \sum_{n=1}^{N} \|(\mathbf{R}\mathbf{x}_n + \mathbf{t}) - \mathbf{y}_n\|_2^2 + I_{SO(3)}(\mathbf{R})$$

$$\mathbf{y}_n = \Pi_{\mathcal{Y}}(\mathbf{R}\mathbf{x}_n + t)$$

Iterative Closest Point

- Initialize and solve by alternating minimization
  - **Step 1**: compute correspondences (closest point)

$$\arg\min_{Y} \sum_{n=1}^{N} \|(\mathbf{R}\mathbf{x}_n + \mathbf{t}) - \boxed{\mathbf{y}_n}\|_2^2 + \boxed{I_{\mathcal{Y}}(\mathbf{y}_n)}$$

  - **Step 2**: find optimal transformation (shape matching)

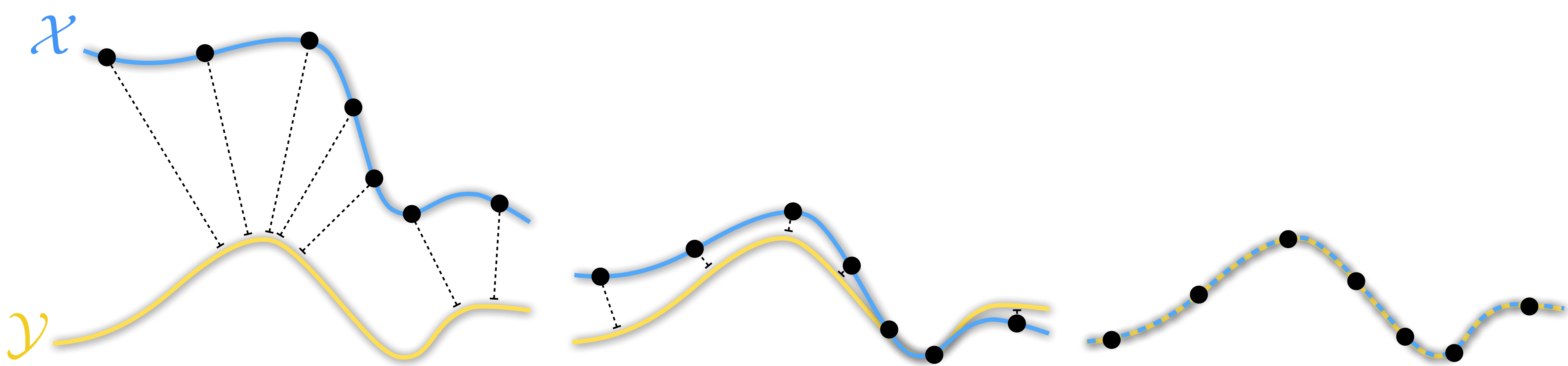
$$\arg\min_{\mathbf{R},\mathbf{t}} \sum_{n=1}^{N} \|(\boxed{\mathbf{R}}\mathbf{x}_n + \boxed{\mathbf{t}}) - \mathbf{y}_n\|_2^2 + \boxed{I_{SO(3)(\mathbf{R})}}$$

Besl et al. "A method for registration of 3D shapes" TPAMI 1992

Iterative Closest Point

- Initialize and solve by alternating minimization
  - **Step 1**: compute correspondences (closest point)
  - **Step 2**: find optimal transformation (shape matching)

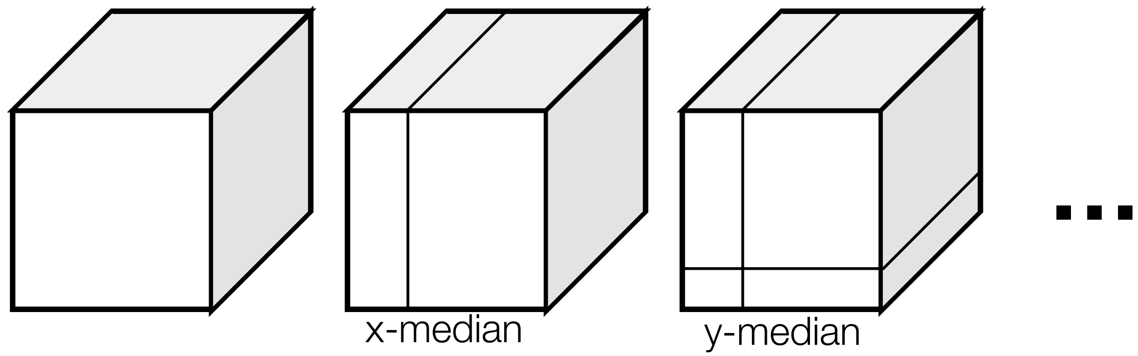
DEMO



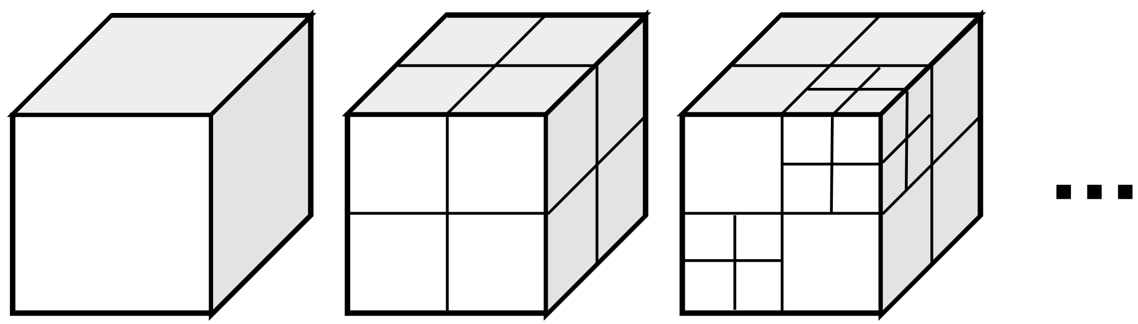
How to compute correspondences?

- Spatial data structures
  - O(n log n) build and O(log n) per each query point

- kd-tree



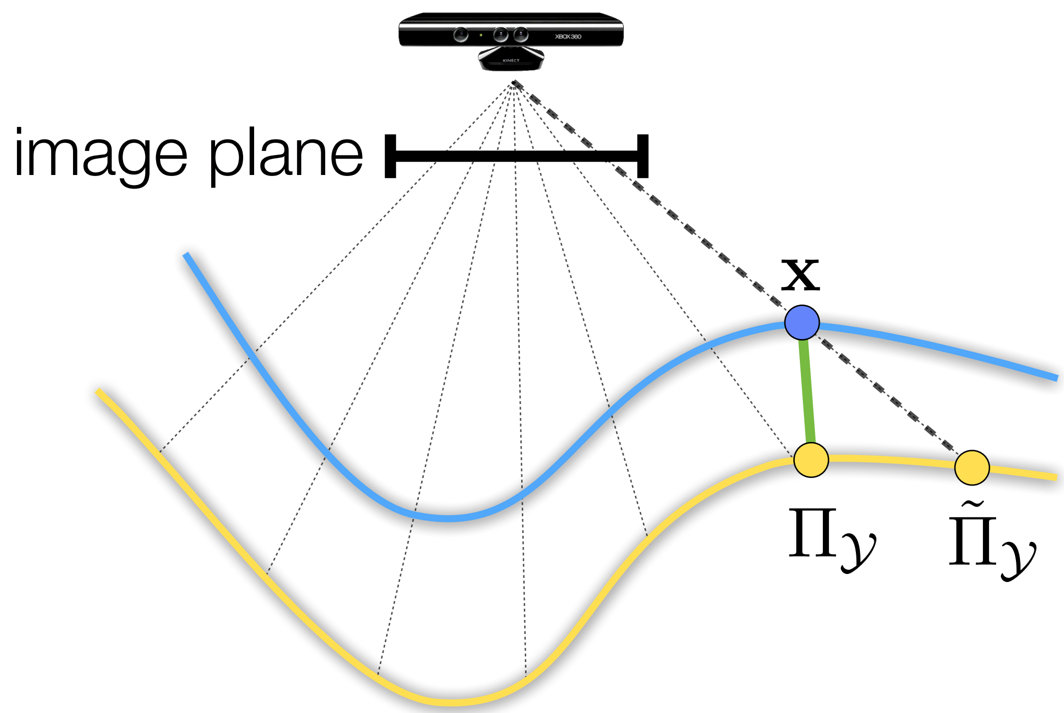
- octree



FLANN (C++), NanoFlann (C++) or KDTreeSearcher (MATLAB)

How to compute correspondences?

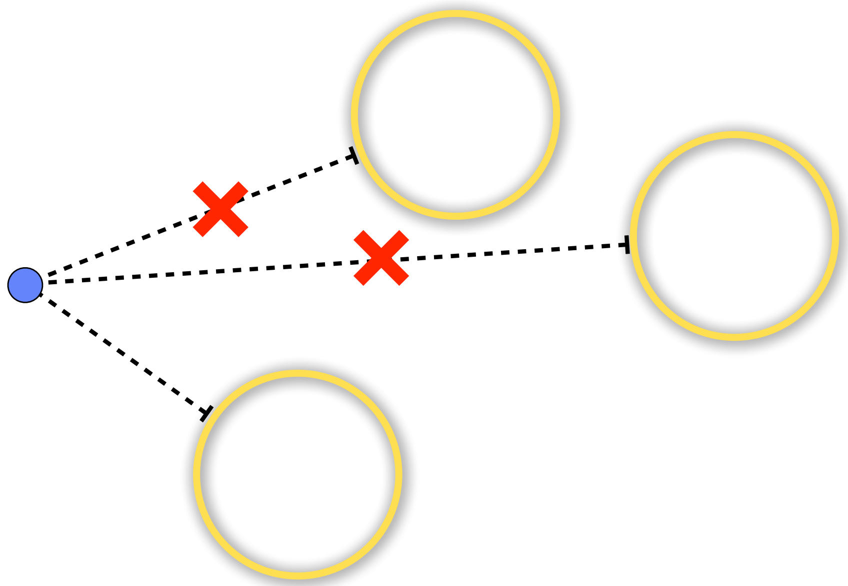
- Projective lookup (approximation)
  - O(1) build (render) and O(1) per each query point (texture lookup)

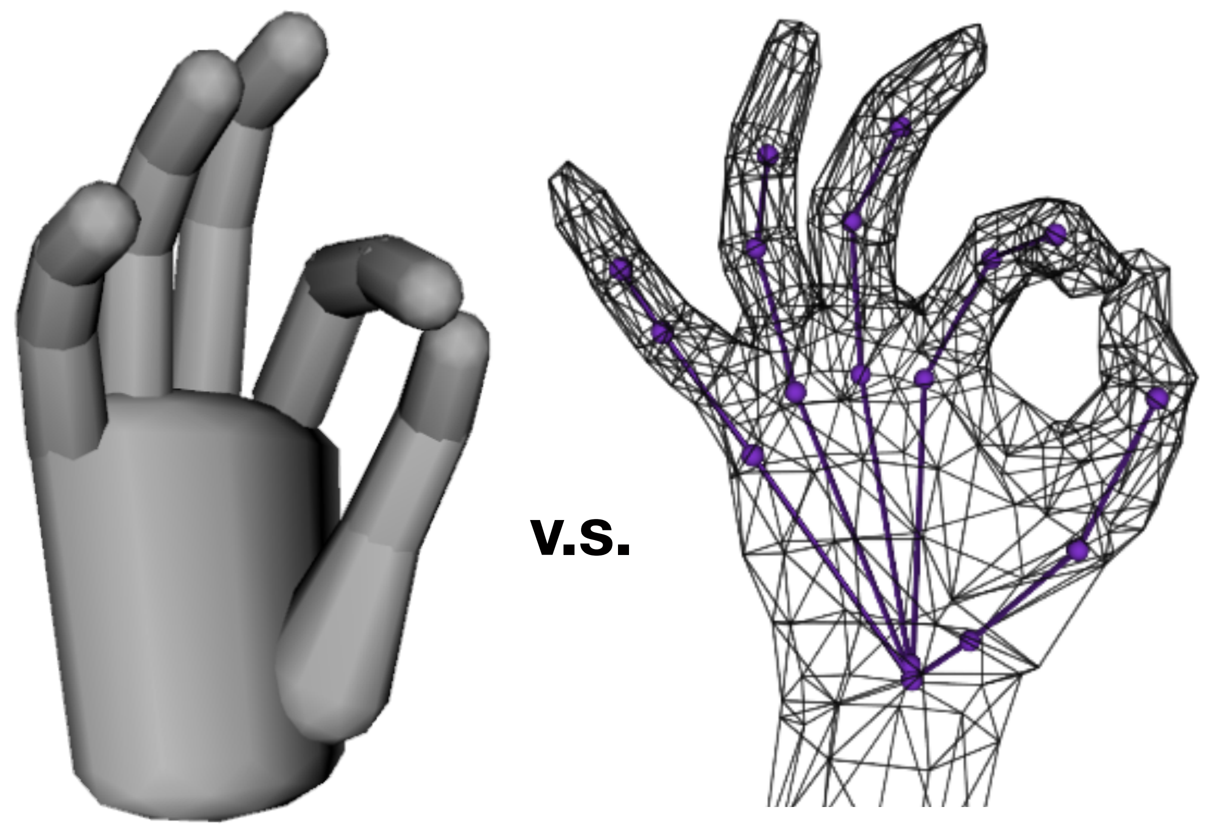


Newcombe et al. “KinectFusion: Real-Time Dense Surface Mapping and Tracking” ISMAR 2011

How to compute correspondences?

- Brute force (given *m* proxy shapes)
  - O(m) per each query point
  - trivially parallelizable (GPU)

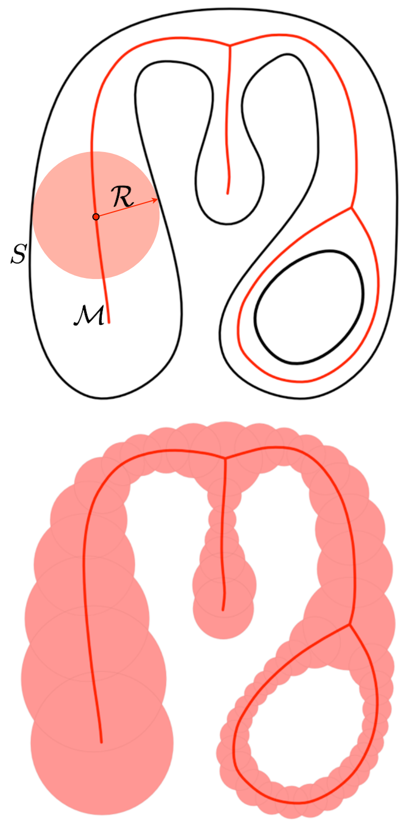




Tagliasacchi et al. "Robust Articulated ICP for Real Time Hand Tracking" SGP 2015

How to compute correspondences?

- Leverage medial axis transform (MAT) and sphere-meshes

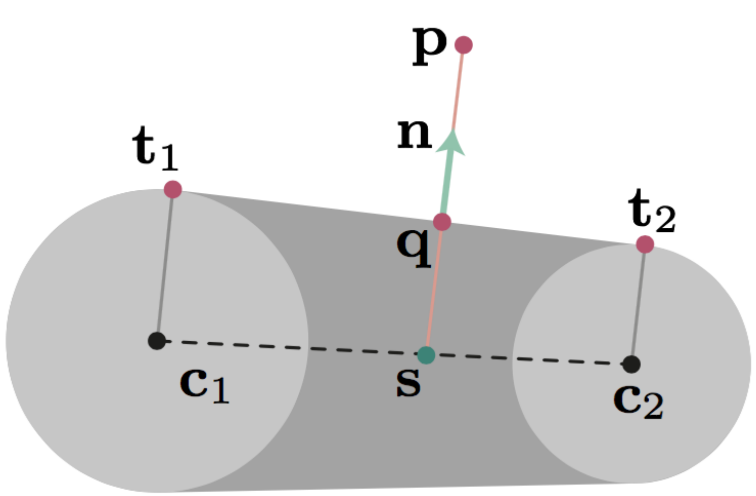


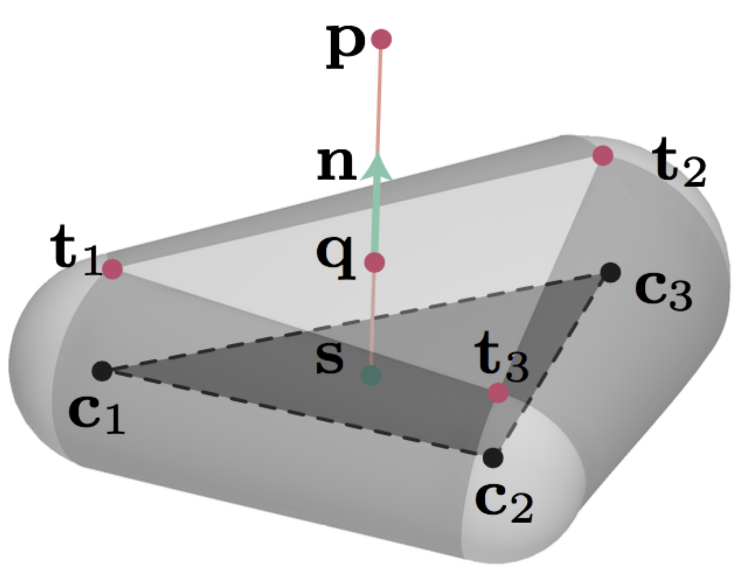
Tagliasacchi et al. “3D Skeletons: A State-of-the-Art Report” EG 2016
Thiery et al. “Animated Mesh Approximation With Sphere-Meshes” SIGGRAPH 2016

Input Anim.	Animated Sphere-Mesh (#S / #E / #T)	H	M12	M21	QEM Simplification (#V / #T)	H	M12	M21
Capoeira	20 / 9 / 12	**2.45**	**0.46**	**0.47**	20 / 31	6.05	1.58	1.38

How to compute correspondences?

- Sphere-meshes and correspondences
  - Brute force computation as ray casting



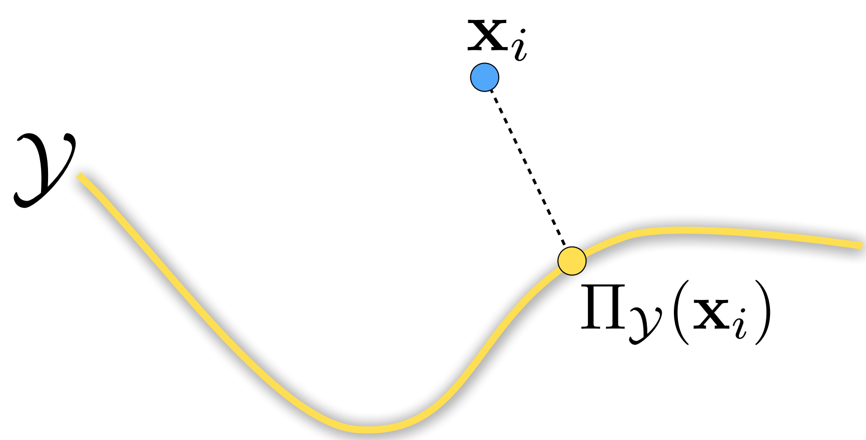


Tkach et al. “Sphere-Meshes for Real-Time Hand Modeling and Tracking” SIGGRAPH Asia 2016

Point to {Point, Plane}

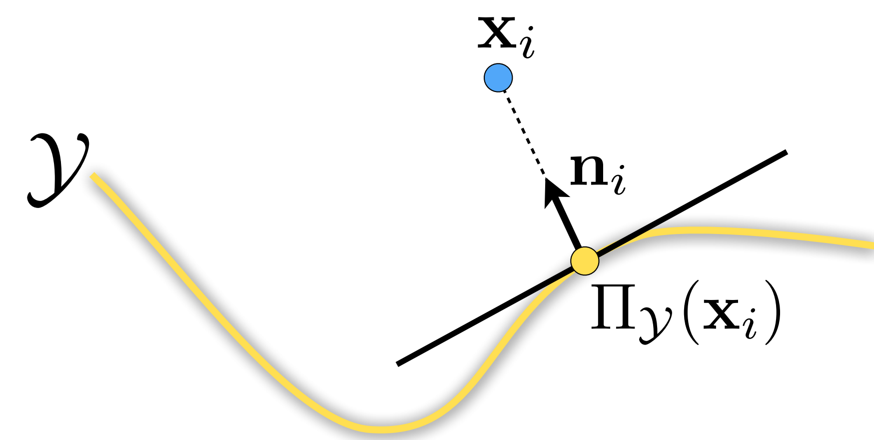
- How many iterations are necessary? (i.e. convergence speed)

Point-to-Point



$$\|\mathbf{x}_i - \Pi_{\mathcal{Y}}(\mathbf{x}_i)\|_2^2$$

Point-to-Plane

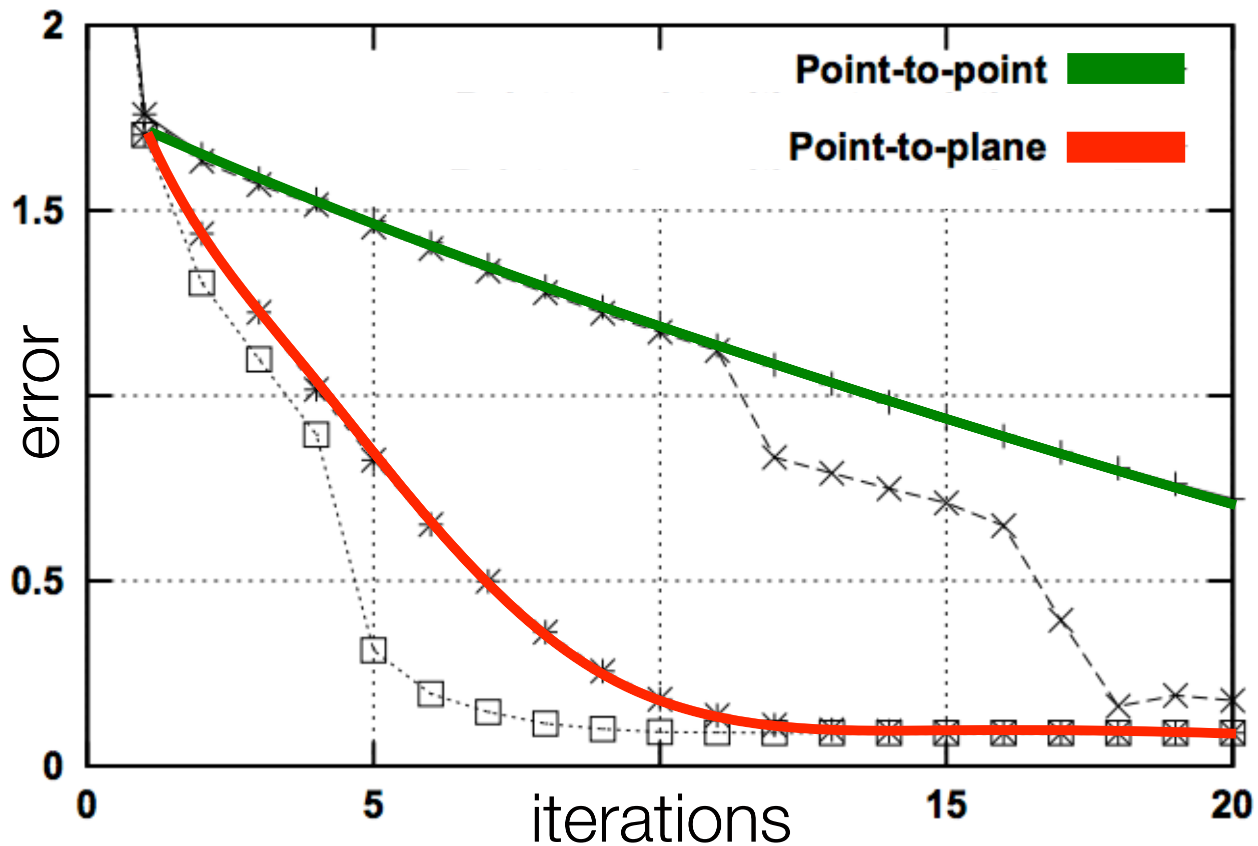


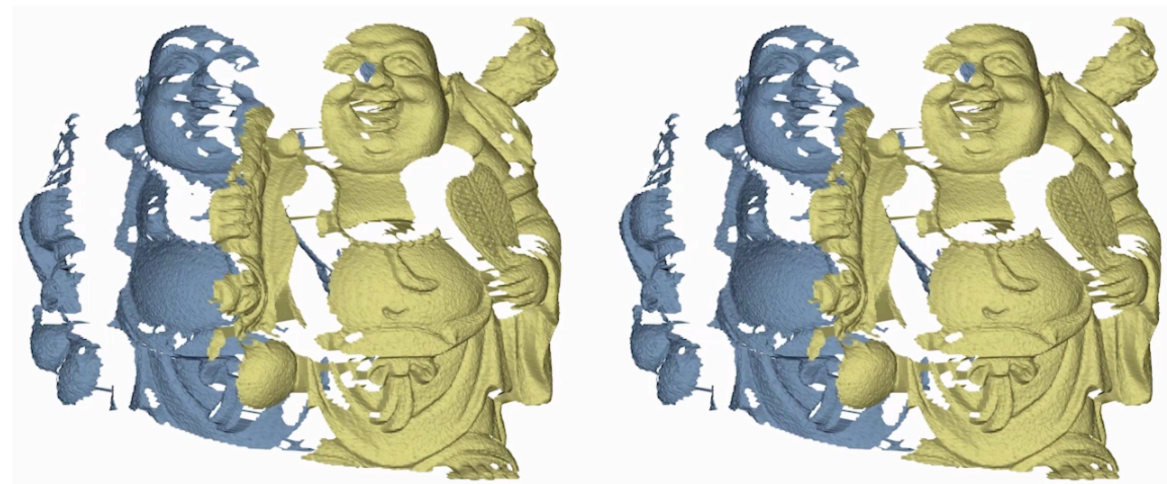
$$\|\mathbf{n}_i \cdot (\mathbf{x}_i - \Pi_{\mathcal{Y}}(\mathbf{x}_i))\|^2$$

Pottmann et al. “Geometry and convergence analysis of algorithms for registration of 3D shapes” IJCV 2006

Point to {Point, Plane}

- How many iterations are necessary? (convergence speed)



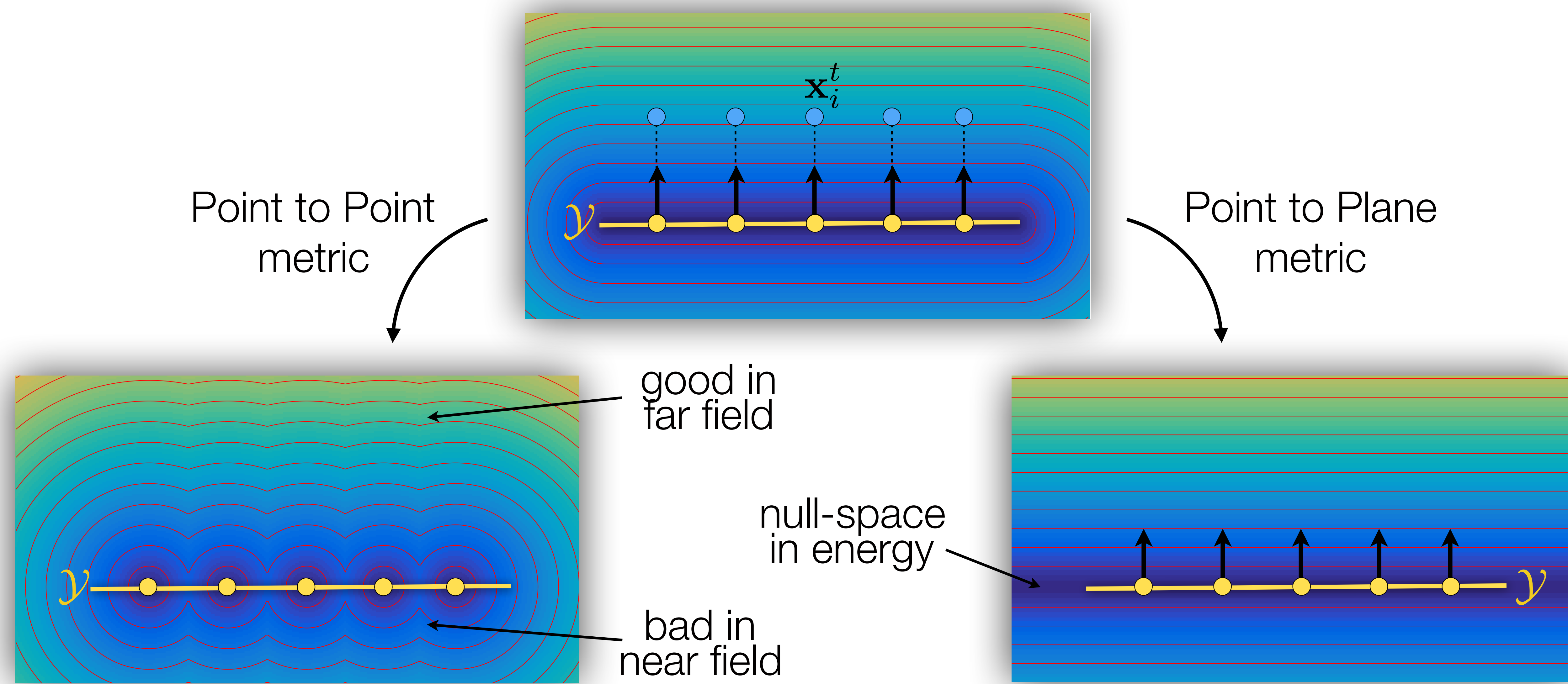


point-to-point
(linear)

point-to-plane
(quadratic)

Rusinkiewicz and Levoy "Efficient variants of the ICP algorithm" 3DDIM 01

Point to {Point, Plane}



Flory and Hofer “Surface fitting and registration of point clouds using approximations of the unsigned distance function” CAGD 2010

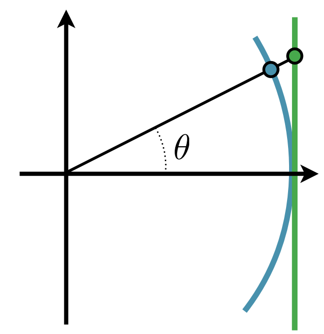
Shape Matching Problem

- Alternative (to SVD): linearize the objective function (2D example)

$$\arg\min_{\tilde{\mathbf{R}},\mathbf{t}} \sum_{n=1}^{N} \|(\tilde{\mathbf{R}}\mathbf{R}^{t-1}\mathbf{x}_n + \mathbf{t}) - \mathbf{y}_n\|_2^2$$

refine previous

$$\tilde{\mathbf{R}} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \approx \begin{bmatrix} 1 & -\theta \\ \theta & 1 \end{bmatrix}$$



- Solve as damped least-squares (Levenberg-Marquardt)

$$\arg\min_{\theta,\mathbf{t}} \sum_{n=1}^{N} \left\| \left(\begin{bmatrix} 1 & -\theta \\ \theta & 1 \end{bmatrix} \mathbf{R}^{t-1}\mathbf{x}_n + \mathbf{t} \right) - \mathbf{y}_n \right\|_2^2 + \omega\|\theta\|^2$$

respect Taylor assumption

Rusinkiewicz et al. “Derivation of Point-to-Plane Minimization” Technical Report 2003

Registration Framework

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

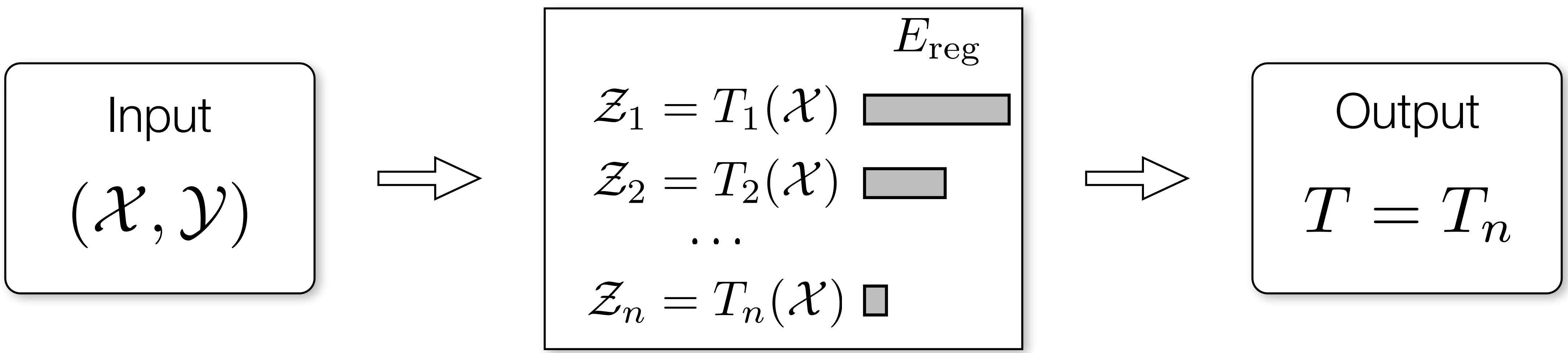
- Registration as energy minimization
  - typically non-linear… linearize and iterate

$$\arg\min_{T} E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

source

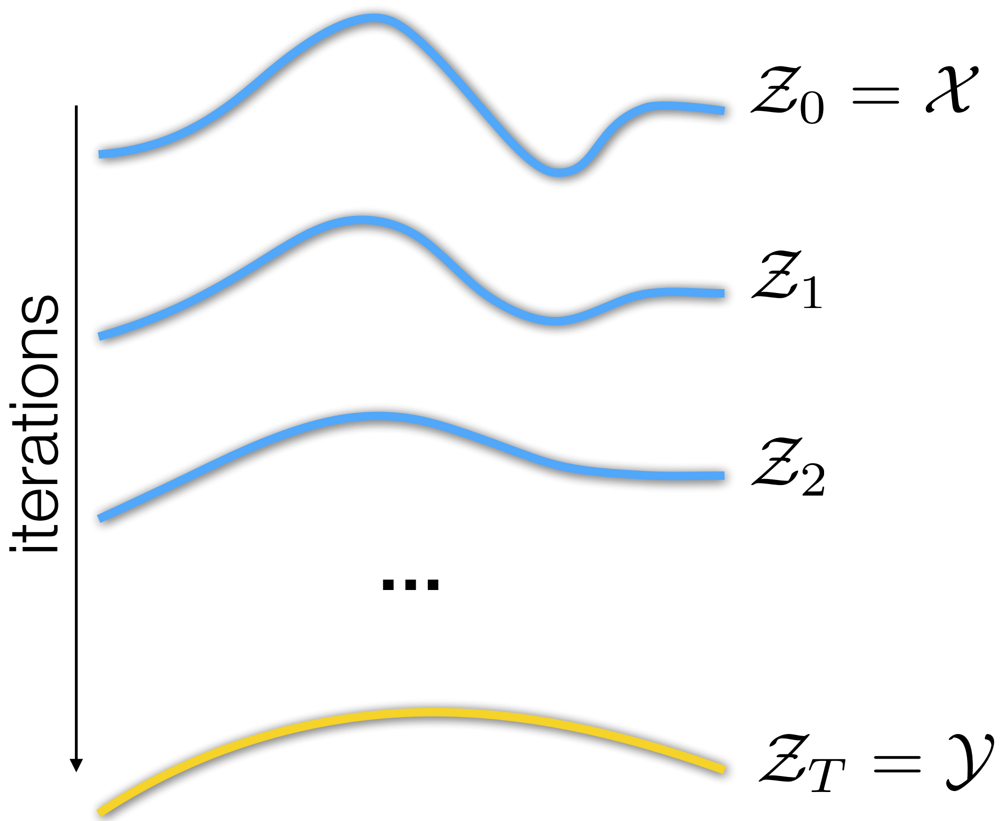
transformation

target



Registration Framework

- How to generalize ICP into a framework?



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$\arg\min_{Z^t, \mathbf{R}^t, \mathbf{t}^t} \; w_1 \sum_{i=1}^{n} \boxed{\|\mathbf{z}_i^t - \Pi_{\mathcal{Y}}(\mathbf{z}_i^{t-1})\|_2^2} + \quad \text{matching}$$

$$w_2 \sum_{i=1}^{n} \boxed{\|\mathbf{z}_i^t - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2} \quad \text{prior (rigid)}$$

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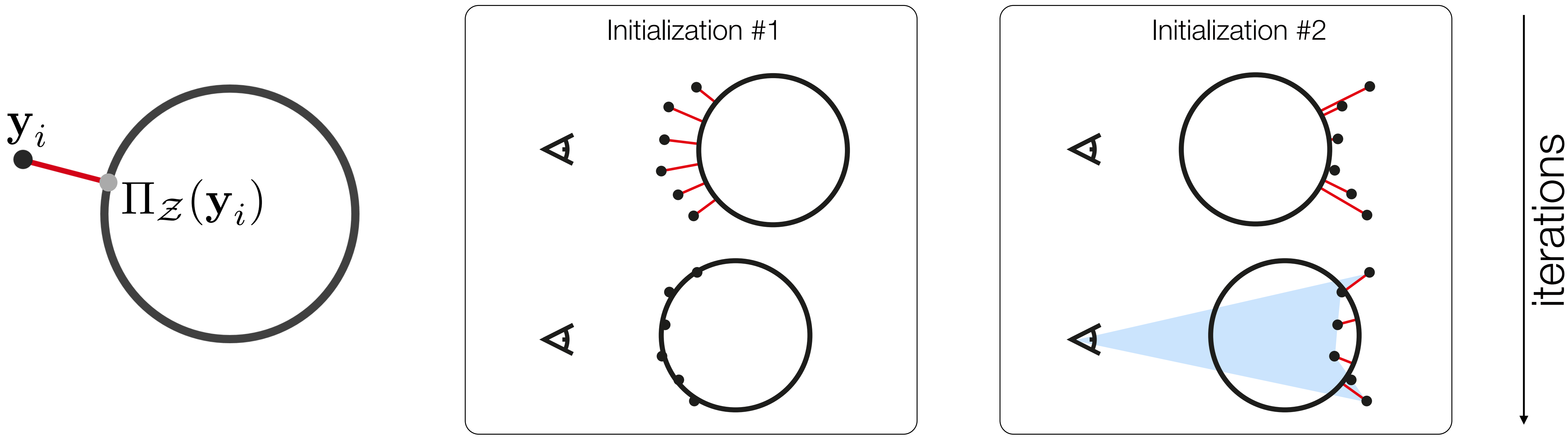
Registration and Visibility

- In monocular acquisition, does ICP have bad local minima?

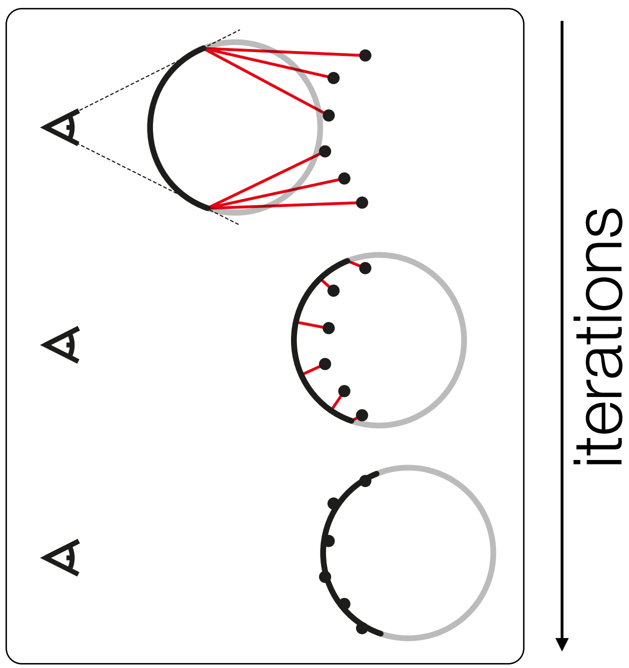
$$\arg\min_{\mathbf{t}} \; w_1 \sum_{i=1}^{n} \|\mathbf{y}_i - \Pi_{\mathcal{Z}}(\mathbf{y}_i)\|_2^2 + w_2 \sum_{i=1}^{n} \|\mathbf{z}_i - (\mathbf{x}_i + \mathbf{t})\|_2^2$$



Tagliasacchi et al. “Robust Articulated ICP for Real Time Hand Tracking” SGP 2015

Registration and Visibility

$$\sum_{i=1}^{n} \|\mathbf{y}_i - \Pi_{\hat{\mathcal{Z}}}(\mathbf{y}_i)\|_2^2$$

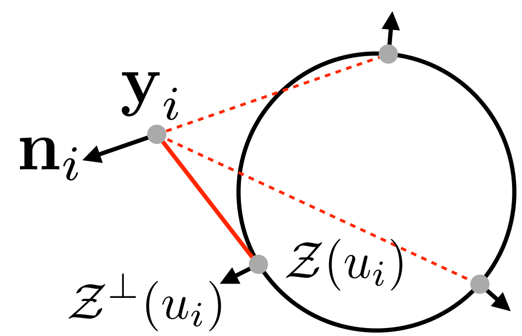


$$\sum_{i=1}^{n} \|\mathbf{y}_i - \mathcal{Z}(u_i)\|_2^2 + \|\mathbf{n}_i - \mathcal{Z}^{\perp}(u_i)\|_2^2$$

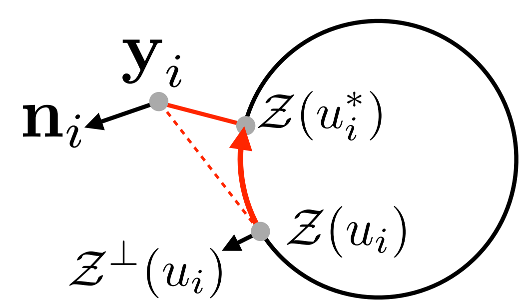
parameterized model point

parameterized model normal

1) discrete search



2) continuous search



Tagliasacchi et al. “Robust Articulated ICP for Real Time Hand Tracking” SGP 2015

Taylor et al. “Hand Tracking […] Joint Optimization of Pose and Correspondences”, SIGGRAPH 2016

Hausdorff Metric

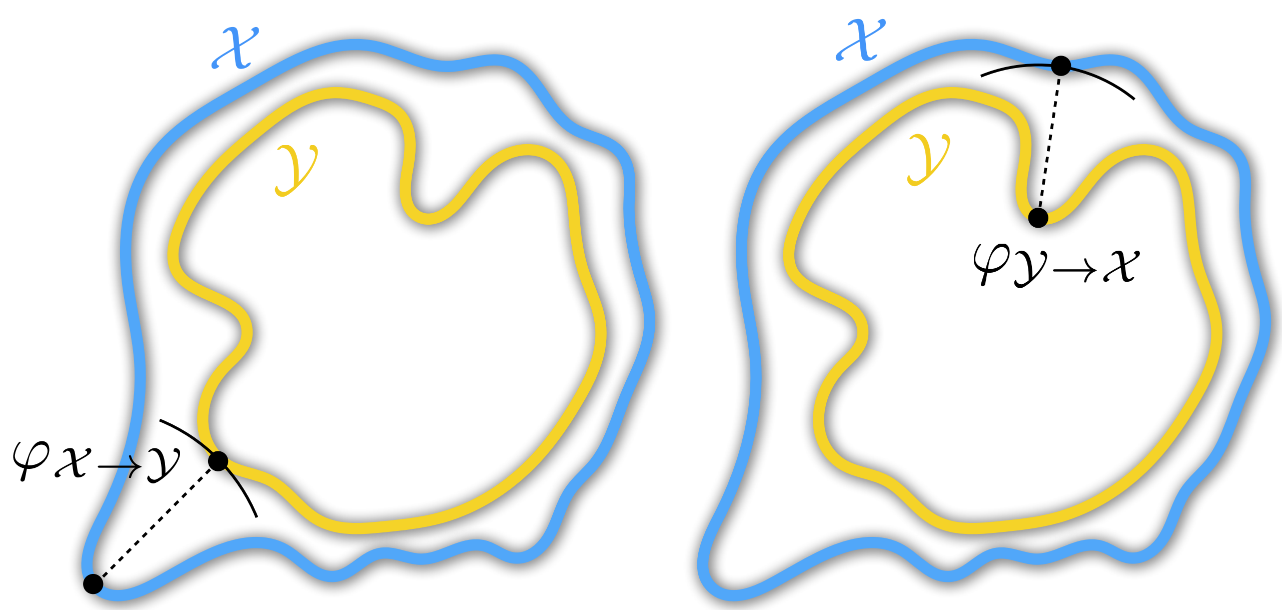
- Is our metric <u>sufficient</u> to measure alignment success?
  - two models are <u>identical</u> if their Hausdorff distance is zero

$$\varphi_{\mathcal{X}\leftrightarrow\mathcal{Y}} = \max\left\{\varphi_{\mathcal{Y}\to\mathcal{X}}, \varphi_{\mathcal{Y}\to\mathcal{X}}\right\}$$

$$\varphi_{\mathcal{X}\to\mathcal{Y}} = \max_{\mathbf{x}\in\mathcal{X}}\left[\min_{\mathbf{y}\in\mathcal{Y}} \varphi(\mathbf{x},\mathbf{y})\right]$$

$$\varphi_{\mathcal{Y}\to\mathcal{X}} = \max_{\mathbf{y}\in\mathcal{Y}}\left[\min_{\mathbf{x}\in\mathcal{X}} \varphi(\mathbf{x},\mathbf{y})\right]$$

closest point correspondences



Differentiable Hausdorff

- Is our metric sufficient to measure alignment success?

$$\varphi_{\mathcal{X}\leftrightarrow\mathcal{Y}} = \max\left\{ \max_{\mathbf{x}\in\mathcal{X}} \left[\min_{\mathbf{y}\in\mathcal{Y}} \varphi(\mathbf{x},\mathbf{y}) \right],\; \max_{\mathbf{y}\in\mathcal{Y}} \left[\min_{\mathbf{x}\in\mathcal{X}} \varphi(\mathbf{x},\mathbf{y}) \right] \right\}$$

(2) (1) (2) (1)

$$\varphi_{\mathcal{X}\leftrightarrow\mathcal{Y}} \approx \max\left\{ \sum_{\mathbf{x}\in X} \|\mathbf{x} - \Pi_{\mathcal{Y}}(\mathbf{x})\|_2^2,\; \sum_{\mathbf{y}\in Y} \|\mathbf{y} - \Pi_{\mathcal{X}}(\mathbf{y})\|_2^2 \right\}$$

(3)

$$\varphi_{\mathcal{X}\leftrightarrow\mathcal{Y}} \approx \boxed{E_{\text{match}}} = \omega_1 \boxed{\sum_{\mathbf{x}\in X} \|\mathbf{x} - \Pi_{\mathcal{Y}}(\mathbf{x})\|_2^2} + \omega_2 \boxed{\sum_{\mathbf{y}\in Y} \|\mathbf{y} - \Pi_{\mathcal{X}}(\mathbf{y})\|_2^2}$$

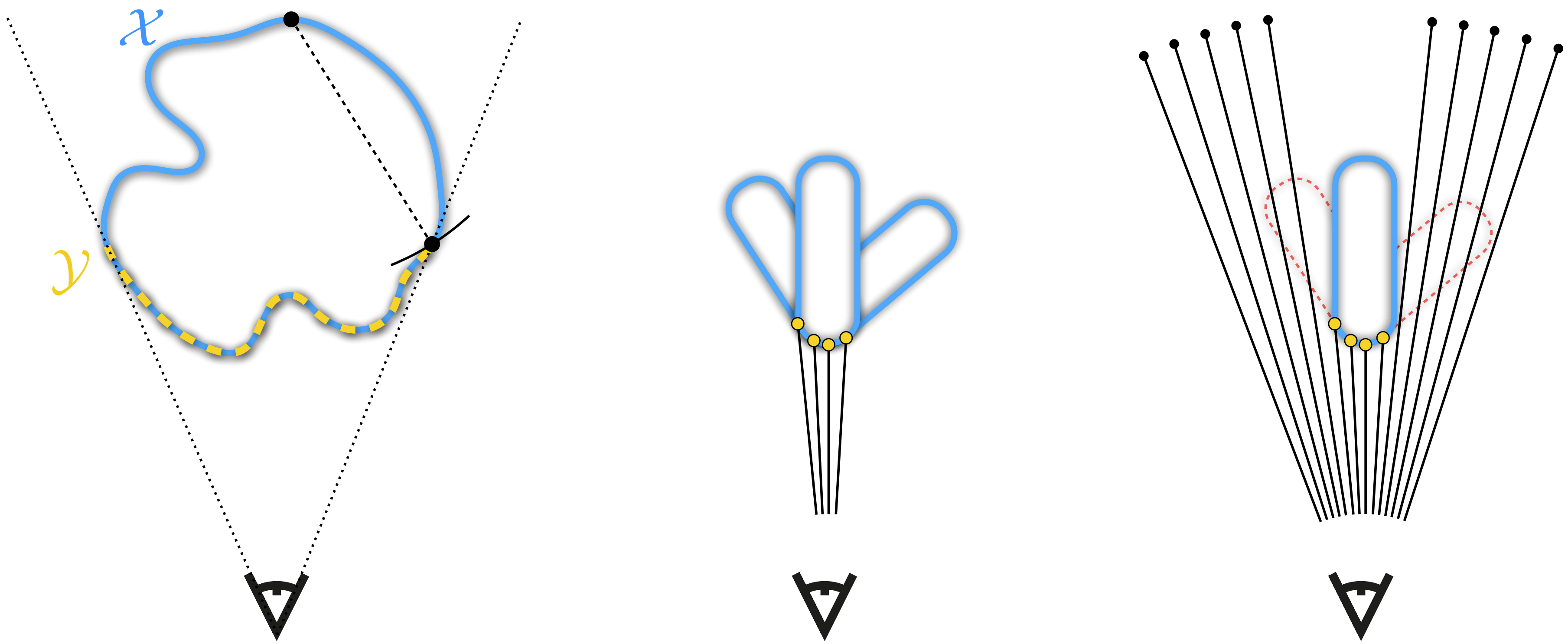
model-to-data data-to-model

Hausdorff in Monocular Sensing

$$\varphi_{\mathcal{X}\leftrightarrow\mathcal{Y}} \approx E_{\text{match}} = \omega_1 \boxed{\sum_{\mathbf{x}\in X} \|\mathbf{x} - \Pi_{\mathcal{Y}}(\mathbf{x})\|_2^2} + \omega_2 \sum_{\mathbf{y}\in Y} \|\mathbf{y} - \Pi_{\mathcal{X}}(\mathbf{y})\|_2^2$$

model-to-data ≠ 0

data-to-model = 0

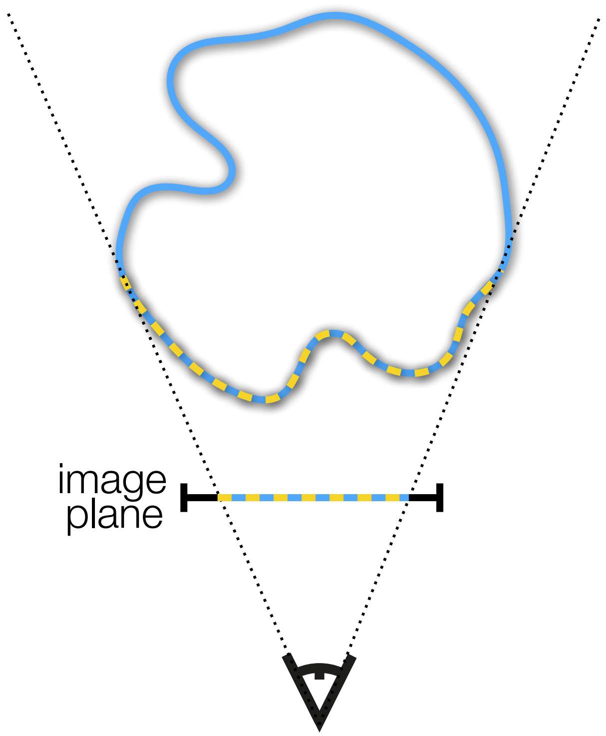


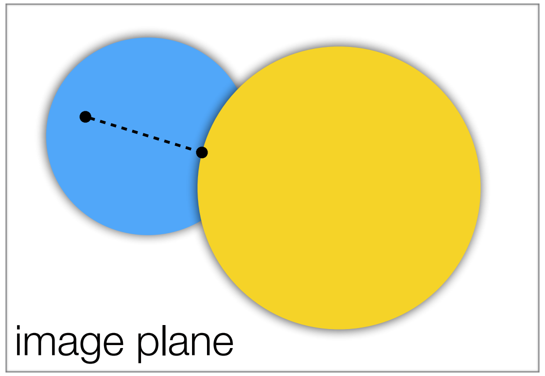
Monocular Hausdorff

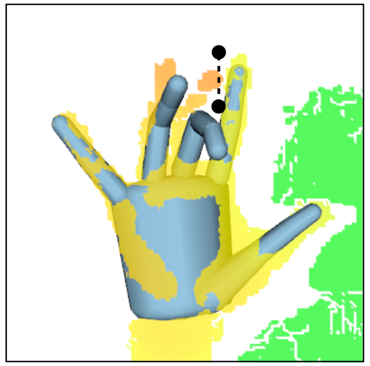
$$\varphi_{\mathcal{X}\leftrightarrow\mathcal{Y}} \approx E_{\text{match}} = \omega_1 \sum_{\mathbf{x}\in X} \|\mathbf{x} - \Pi_{\mathcal{Y}}(\mathbf{x})\|_2^2 \; + \; \omega_2 \sum_{\mathbf{y}\in Y} \|\mathbf{y} - \Pi_{\mathcal{X}}(\mathbf{y})\|_2^2$$

$f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ perspective projection

$$\approx \omega_1 \sum_{\mathbf{z}\in f(Z)} \|\mathbf{z} - \Pi_{f(\mathcal{Y})}(\mathbf{z})\|_2^2 \; + \; \omega_2 \sum_{\mathbf{y}\in Y} \|\mathbf{y} - \Pi_{\mathcal{X}}(\mathbf{y})\|_2^2$$







model-to-data (2D) = 0

data-to-model (3D) = 0

Overview

- Part 1 - presented by Andrea Tagliasacchi
  - Introduction, motivation and sensing hardware (5 min)
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Registration Priors

$$E_{\text{reg}} = E_{\text{match}} + \boxed{E_{\text{prior}}}$$

rigid

composite

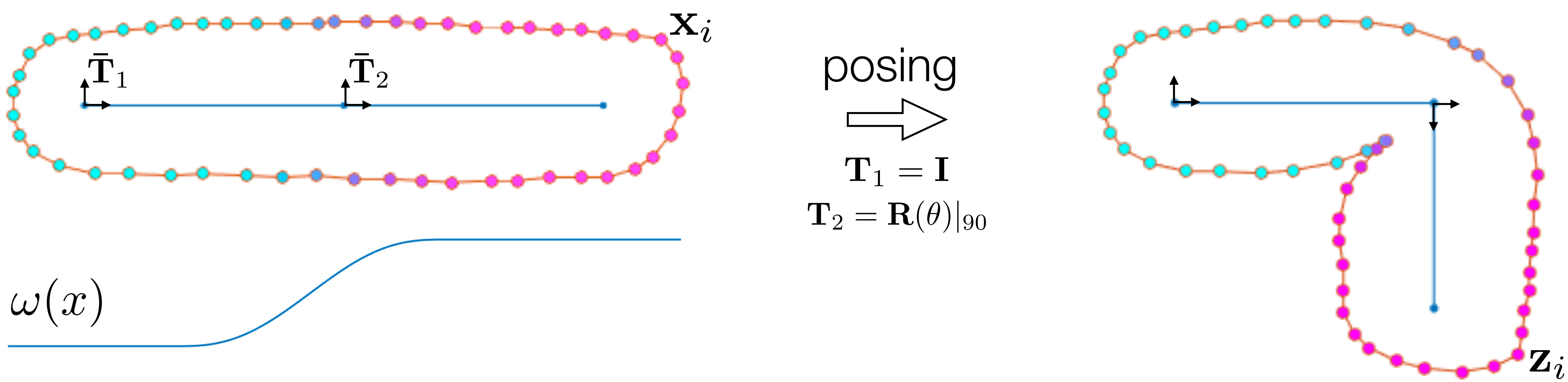
articulated

elastic

fluid

Registration Priors: Articulated (v1)

$$E_{prior}(Z, \underbrace{\{\mathbf{T}_*\}}_{\text{pose}}, \underbrace{\{\bar{\mathbf{T}}_*\}}_{\text{posture}}, \underbrace{\{\omega_*\}}_{\text{weights}}) = \sum_{i=1}^{N} \|\mathbf{z}_i - [\underbrace{(1-\omega_i)}_{\text{linearly blended}}\overbrace{\bar{\mathbf{T}}_1\mathbf{T}_1\bar{\mathbf{T}}_1^{-1}}^{\text{transformations}} + \underbrace{(\omega_i)}_{\text{linearly blended}}\overbrace{\bar{\mathbf{T}}_2\mathbf{T}_2\bar{\mathbf{T}}_2^{-1}}^{\text{transformations}}]\,\mathbf{x}_i\|_2^2$$



Jacobson et al. “Skinning: Real-time Shape Deformation”, SIGGRAPH’Asia 2014

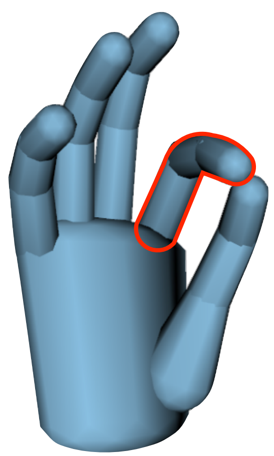
Articulated Tracking

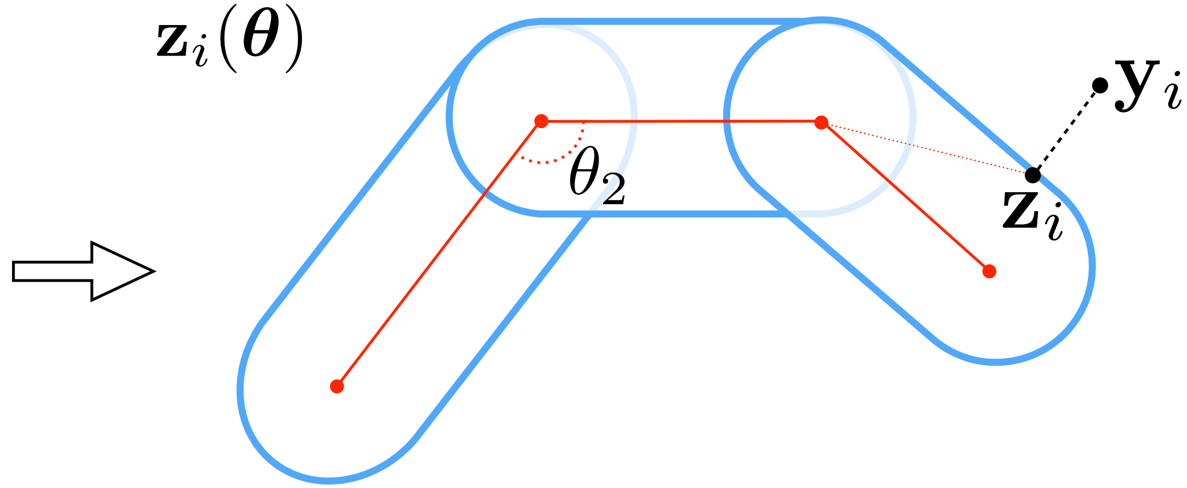
$$E_{prior}(Z, \boxed{\{\mathbf{T}_*\}}) = \|\mathbf{z}_i - \boxed{\prod_{k \in K(\mathbf{x})}} \bar{\mathbf{T}}_k \mathbf{T}_k \bar{\mathbf{T}}_k^{-1} \mathbf{x}_i\|_2^2$$

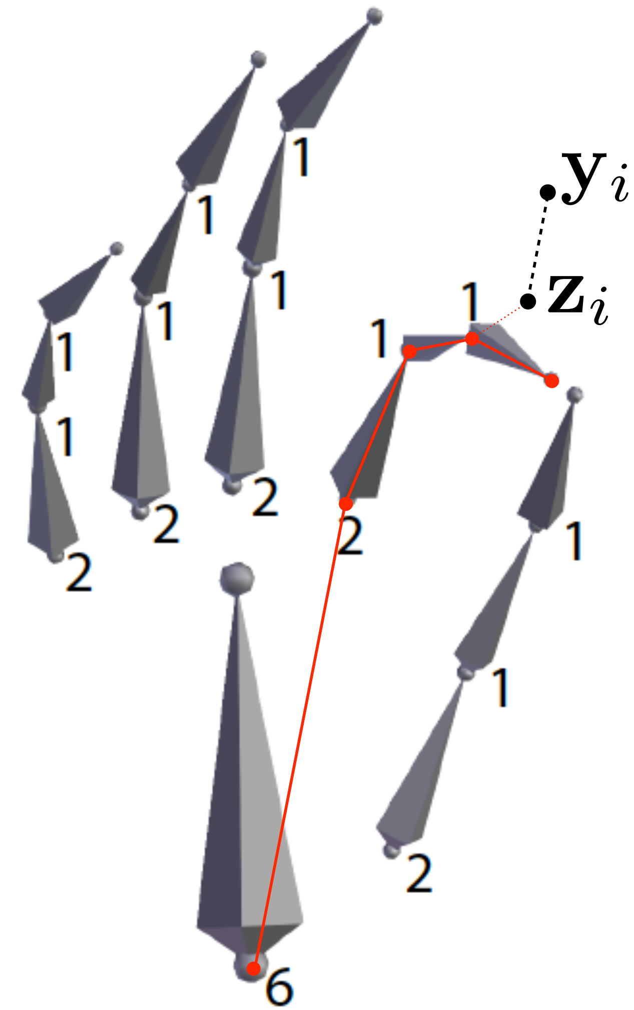
only pose

kinematic chain

$$E_{\text{match}} = \sum_{i=1}^{N} \|\mathbf{y}_i - \underbrace{\boxed{\Pi_{\mathcal{Z}(\boldsymbol{\theta})}(\mathbf{y}_i)}}_{\mathbf{z}_i(\boldsymbol{\theta})}\|_2^2$$







Tagliasacchi et al. “Robust Articulated ICP for Real Time Hand Tracking” SGP 2015

Articulated Tracking

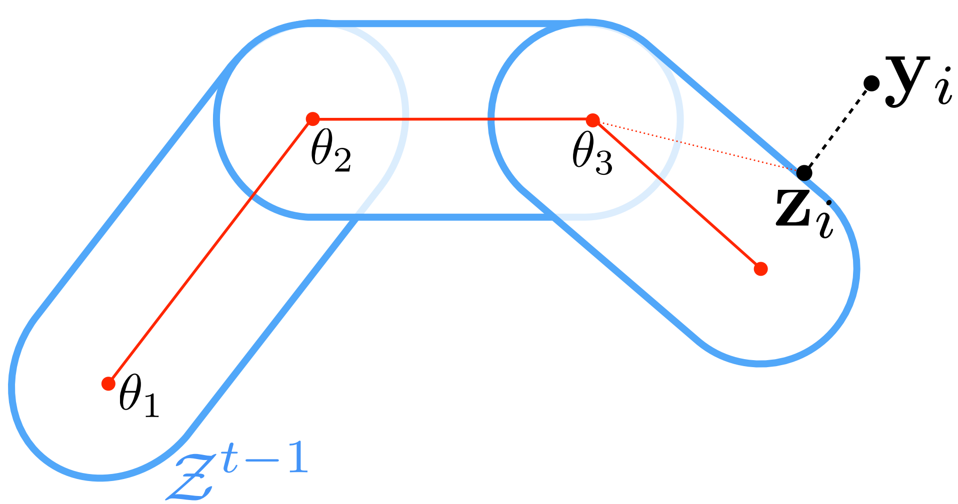
$$E_{\text{match}}(\boldsymbol{\theta}) = \sum_{i=1}^{N} \|\mathbf{y}_i - \boxed{\mathbf{z}_i(\boldsymbol{\theta})}\|_2^2$$

non-linear function

differential

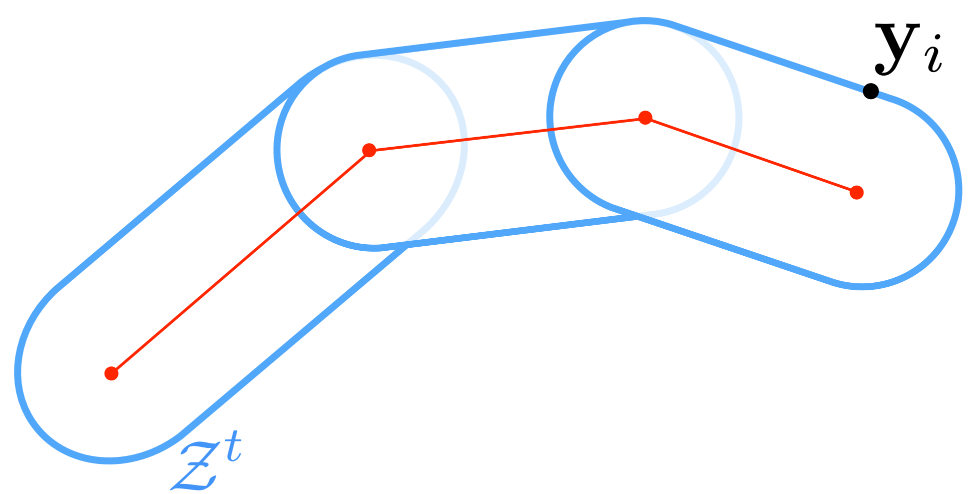
Taylor expansion

$$E_{\text{match}}(\delta\boldsymbol{\theta}) \approx \sum_{i=1}^{N} \|\mathbf{y}_i - \boxed{\left[\mathbf{z}_i^{t-1} + \mathbf{J}_{\text{skel}}^{t-1}\delta\boldsymbol{\theta}\right]}\|_2^2 + \omega\|\delta\boldsymbol{\theta}\|_2^2$$



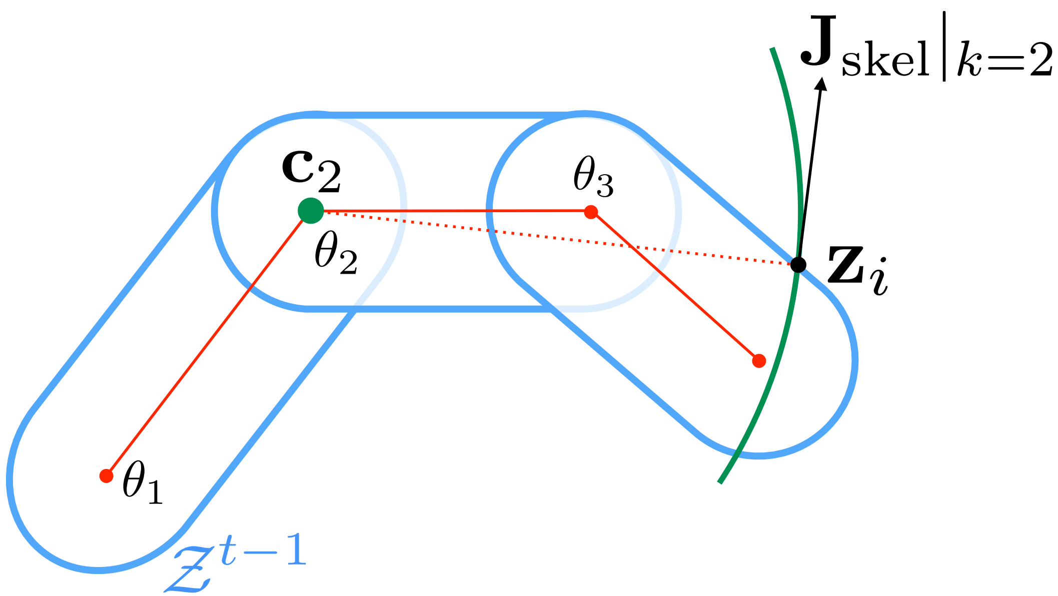
$$\boldsymbol{\theta}^t = \boldsymbol{\theta}^{t-1} + \delta\boldsymbol{\theta}$$

solve + update



Articulated Tracking

$$E_{\text{match}}(\delta\boldsymbol{\theta}) \approx \sum_{i=1}^{N} \|\mathbf{y}_i - [\mathbf{z}_i^{t-1} + \mathbf{J}_{\text{skel}}^{t-1}\delta\boldsymbol{\theta}]\|_2^2 + \omega\|\delta\boldsymbol{\theta}\|_2^2$$



$$\mathbf{J}_{\text{skel}} = \left[\frac{\partial \mathbf{z}_i}{\partial \theta_k} = \mathbf{v_k} \times (\mathbf{z}_i - \mathbf{c_k})\right]_{k \in K(\mathbf{z}_i)}$$

joint rotation center, e.g. [0,0,1]

joint rotation center

DEMO

Buss "Introduction to Inverse Kinematics with […] Damped Least Squares methods" Technical Report 2004

Articulated Tracking Solver

$$\min_{\boldsymbol{\theta}} \underbrace{E_{\text{m2d}} + E_{\text{d2m}}}_{E_{\text{match}}} + \underbrace{E_{\text{pose-space}} + E_{\text{angle-bounds}} + E_{\text{collision}} + E_{\text{smooth}}}_{E_{\text{prior}}}$$

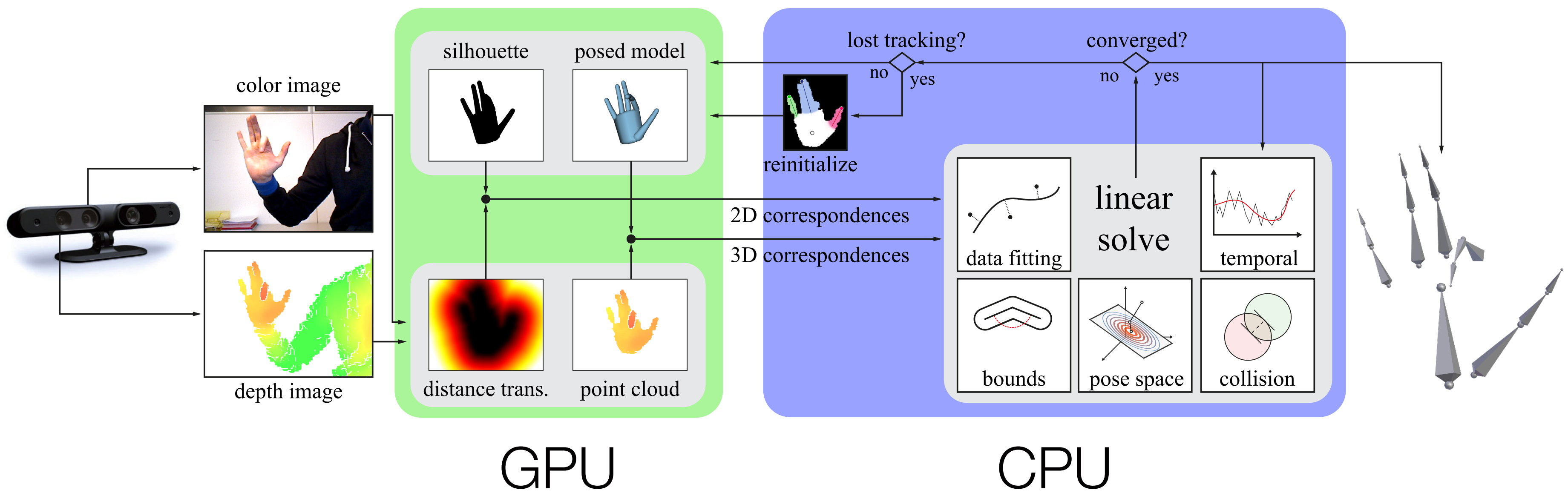
monocular Hausdorff

database similarity

physiological constraints

no self intersections

smooth motion



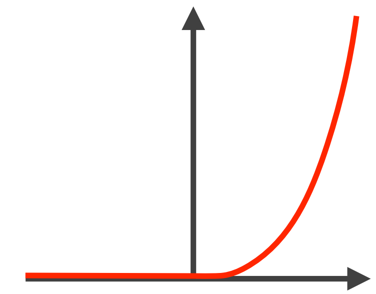
Tagliasacchi et al. “Robust Articulated ICP for Real Time Hand Tracking” SGP 2015

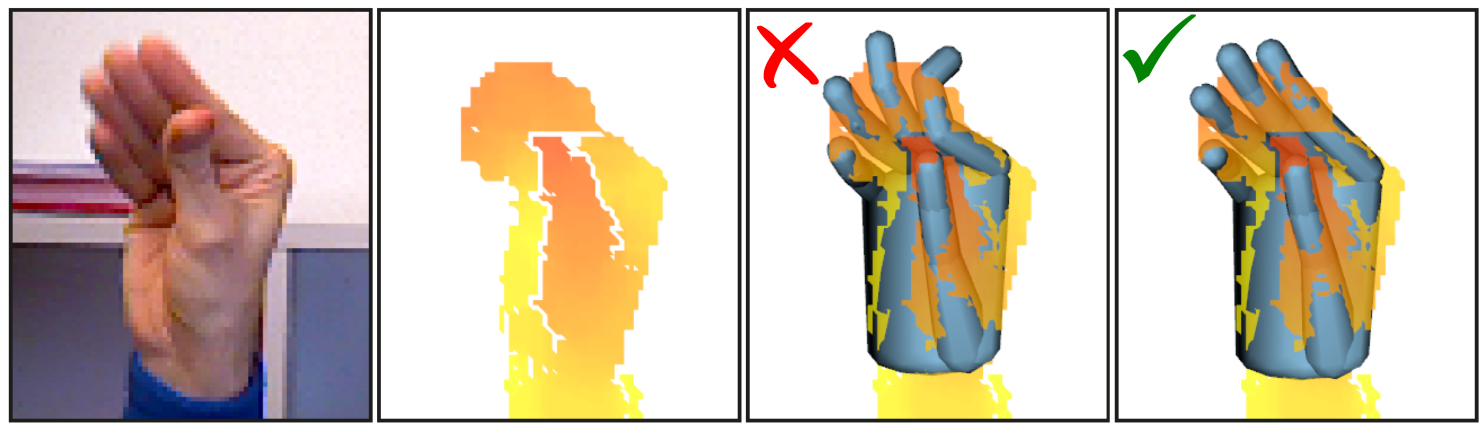
Articulated Tracking Solver

$$E_{\text{angle-bounds}}(\boldsymbol{\theta}) = \sum_{n=7}^{26} \| \max(\theta_i - \bar{\theta}_i, 0) + \min(\theta_i - \underline{\theta}_i, 0) \|_2^2$$

quadratic barrier

only for rotations

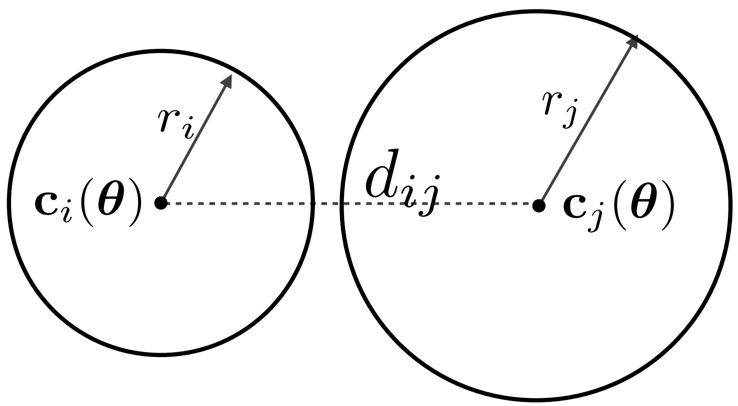


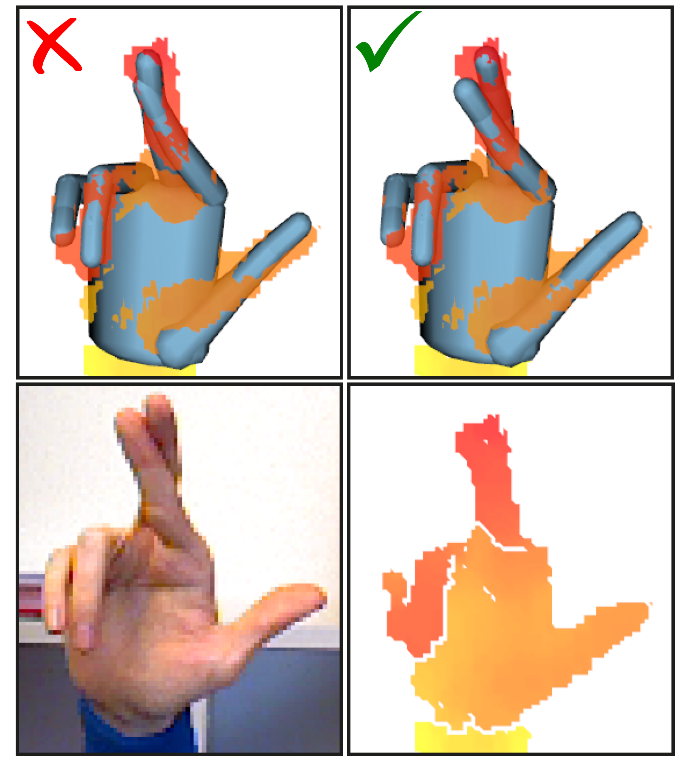


Articulated Tracking Solver

bone distance

$$E_{\text{collision}}(\boldsymbol{\theta}) = \sum_{\{(i,j)\}} \| \min(d_{ij}(\boldsymbol{\theta}) - (r_i + r_j), 0)\|^2$$





Articulated Tracking Solver

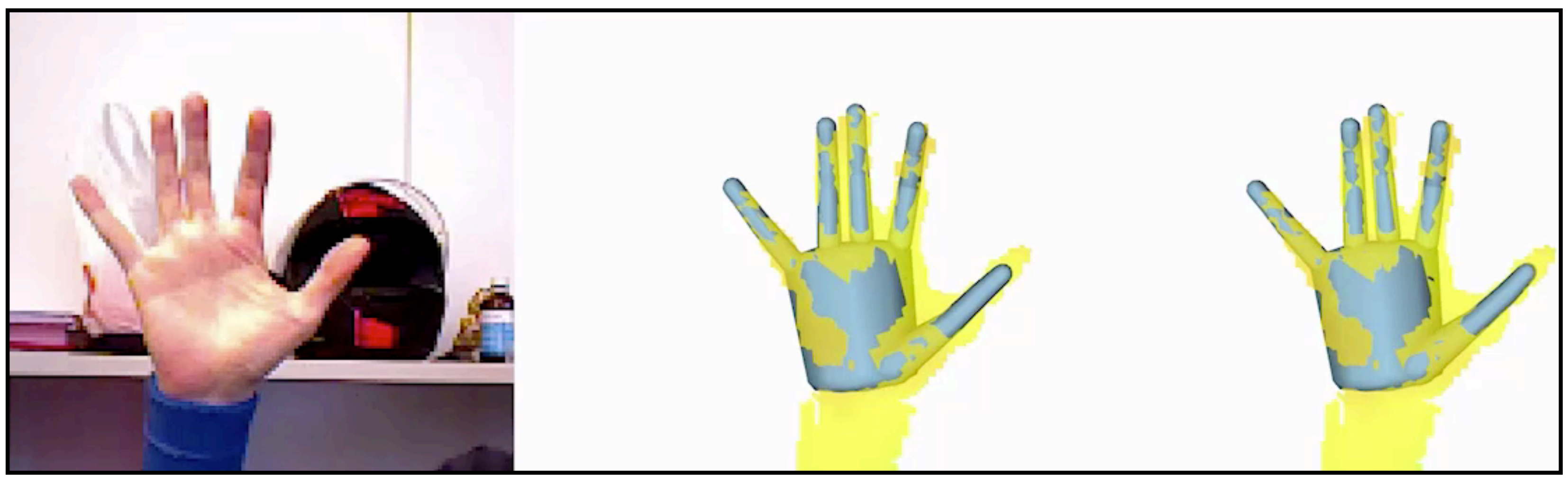
velocity

acceleration

$$E_{\text{smooth}} = \sum_{\mathbf{k}_i \in \mathcal{K}} \|\dot{\mathbf{k}}(\boldsymbol{\theta})\|_2^2 + \sum_{\mathbf{k}_i \in \mathcal{K}} \|\ddot{\mathbf{k}}(\boldsymbol{\theta})\|_2^2$$

skeleton vertices

PrimeSense



enabled disabled

Articulated Tracking

$$E_{\text{pose-space}} = (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T H (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})$$

latent metric

$\Theta = [\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_M]$ 1) collect pose dataset

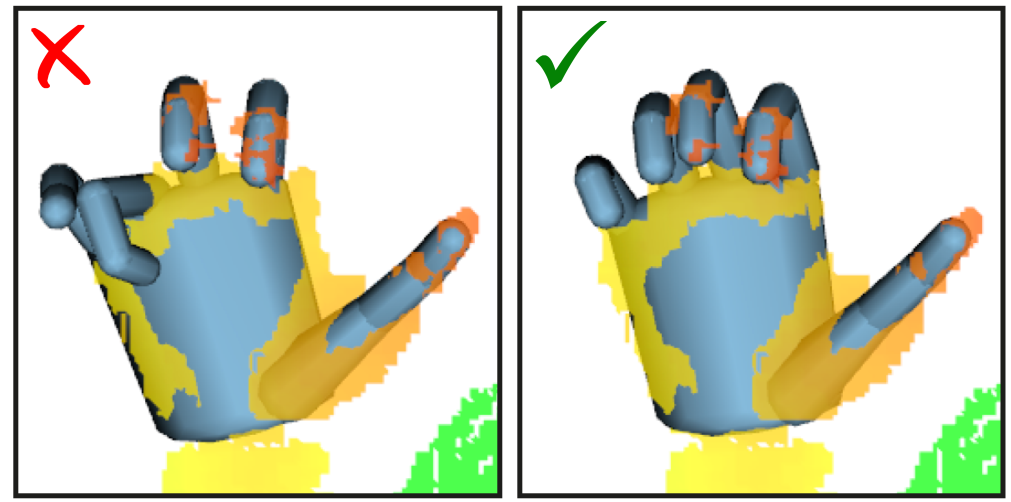
$\hat{\boldsymbol{\theta}} = \sum \boldsymbol{\theta}_m / M$ 2) compute mean

$C = (\Theta - \hat{\boldsymbol{\theta}})^T (\Theta - \hat{\boldsymbol{\theta}})$ 3) compute covariance

$[USU^T] = \mathrm{SVD}(C)$ 4) eigen-decomposition

$H = U(*, 1..m)\, S(1..m, 1..m)\, U(*, 1..m)^T$

latent projection residual



Tagliasacchi et al. “Robust Articulated ICP for Real Time Hand Tracking” SGP 2015
Taylor et al. “Hand Tracking […] Joint Optimization of Pose and Correspondences”, SIGGRAPH 2016

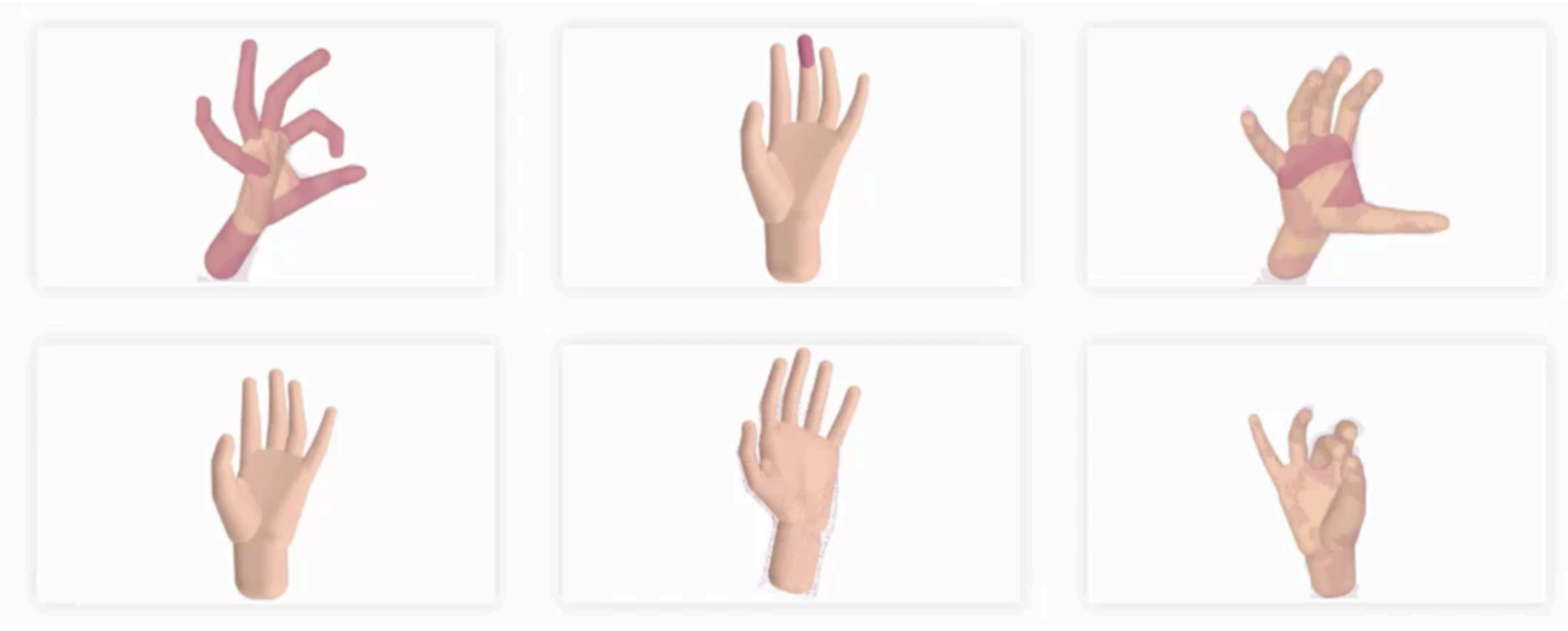
Articulated Tracking Solver

GitHub

Tagliasacchi et al. "Robust Articulated ICP for Real Time Hand Tracking" SGP 2015

http://github.com/OpenGP/htrack

Sphere-Meshes Modeling+Tracking



Tkach, Pauly and Tagliasacchi “Sphere-Meshes for Real-Time […] Tracking” SIGGRAPH Asia 2016 http://github.com/OpenGP/htrack

Sphere-Meshes Modeling+Tracking

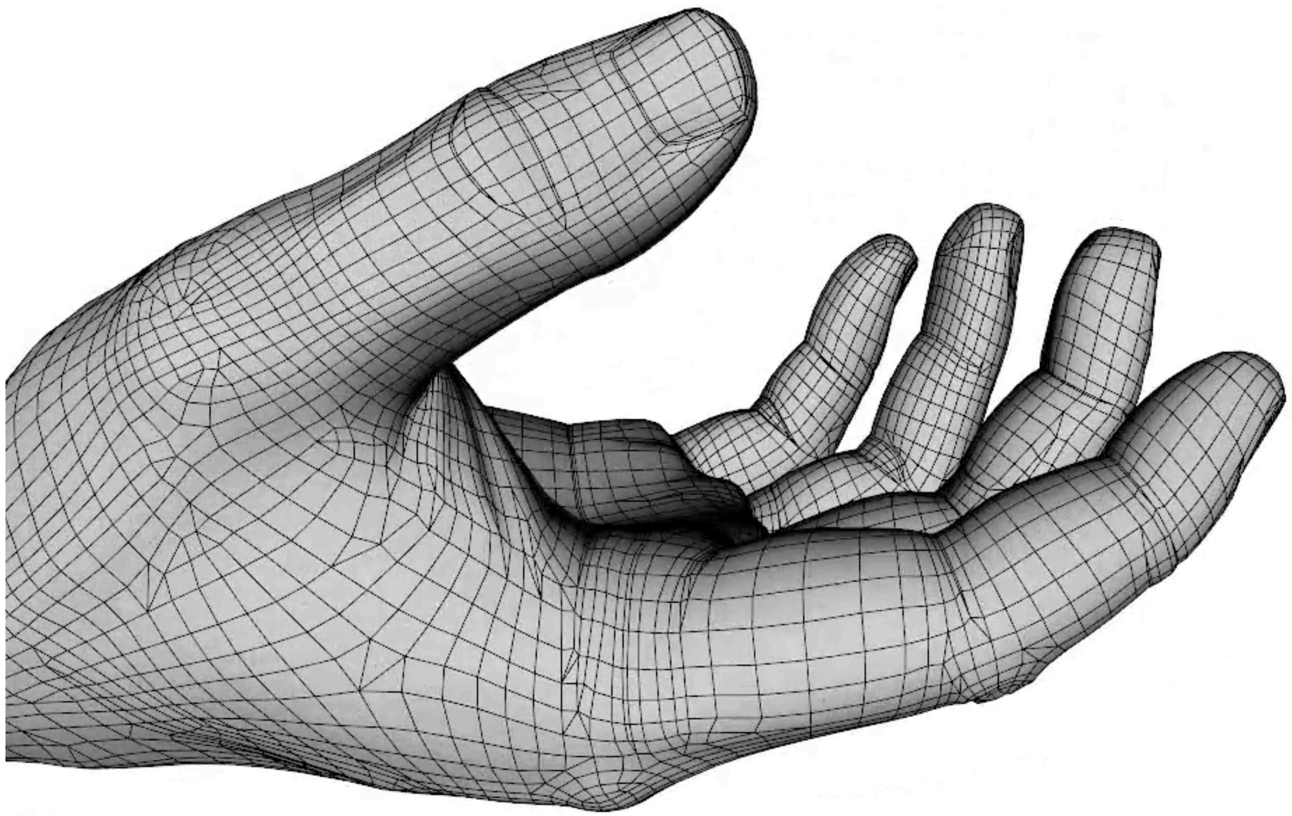
GitHub

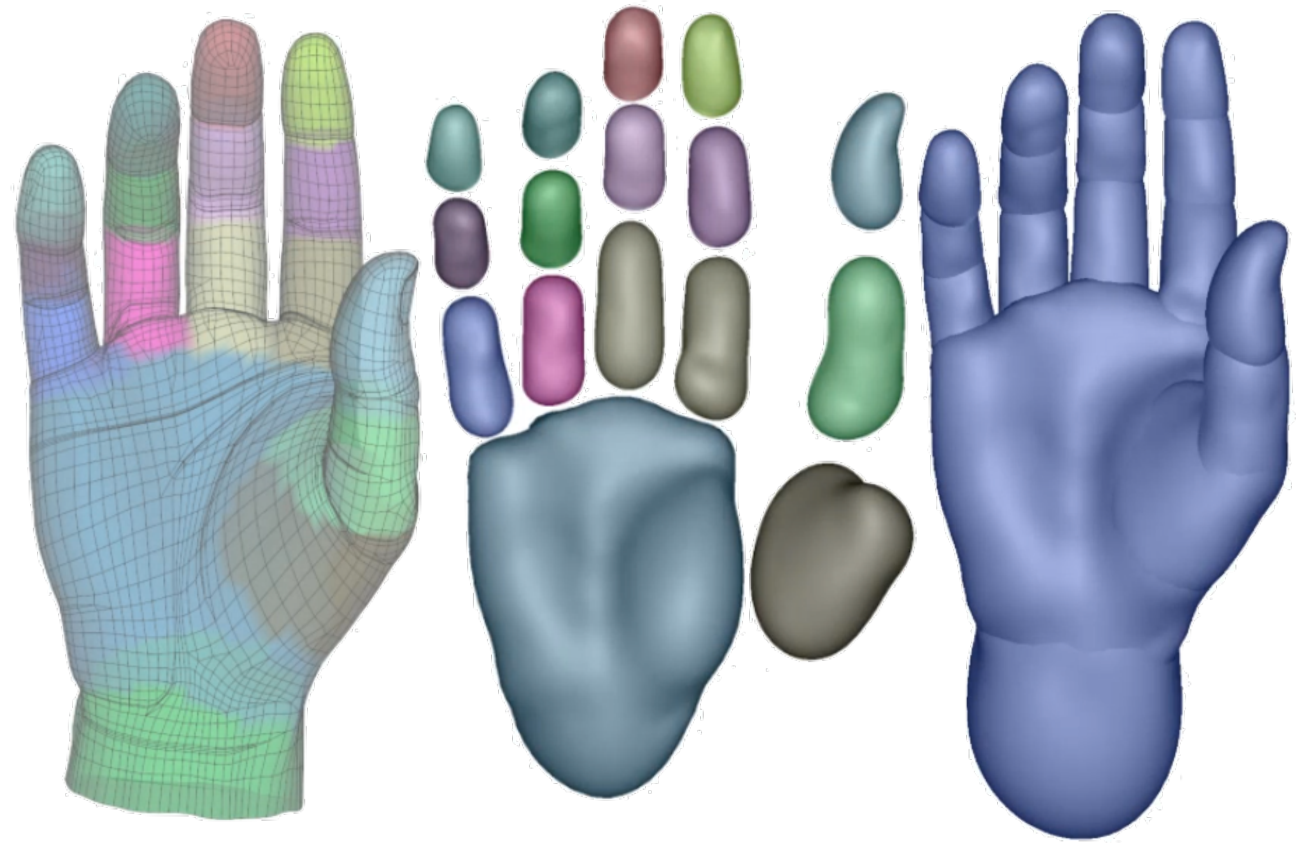
Tkach, Pauly and Tagliasacchi “Sphere-Meshes for Real-Time […] Tracking” SIGGRAPH Asia 2016

http://github.com/OpenGP/hmodel

Implicit Skinning

- Highest quality real-time articulated deformation





Vaillant et al. “Implicit Skinning: Real-Time Skin Deformation with Contact Modeling” SIGGRAPH’13

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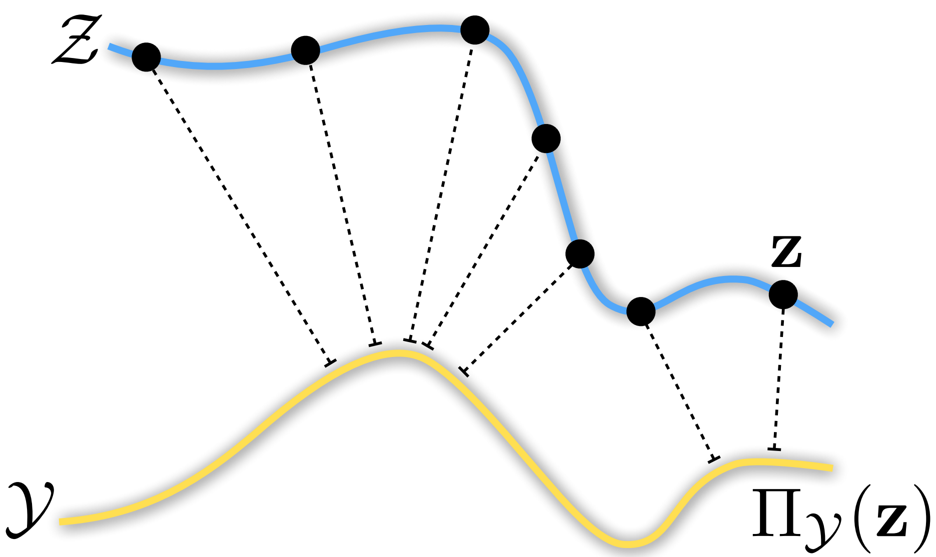
Matching Energy



correspondence weight

corresponding point on target

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - \Pi_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

transformed point on source

Matching Energy

- Side Remark: **Error norm**
  - squared Euclidean distance is sensitive to outliers

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} w_i \|\mathbf{z}_i - \Pi_{\mathcal{Y}}(\mathbf{z}_i)\|_2^{\boxed{2}}$$

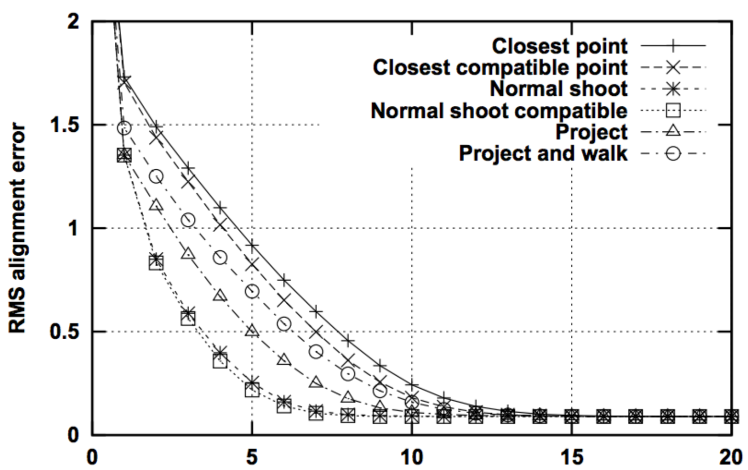
Zach "Robust Bundle Adjustment Revisited" ECCV 2014
Fitzgibbon "Robust registration of 2d and 3d point sets" IVC 2003
Bouaziz, Tagliasacchi and Pauly"Sparse Iterative Closest Point" SGP 2013
Flory and Hofer "[…] registration of point clouds using approximations of the unsigned distance function" CAGD 2010

Iterative Closest Point

- **Weight** controls the importance (i.e.) confidence of correspondence

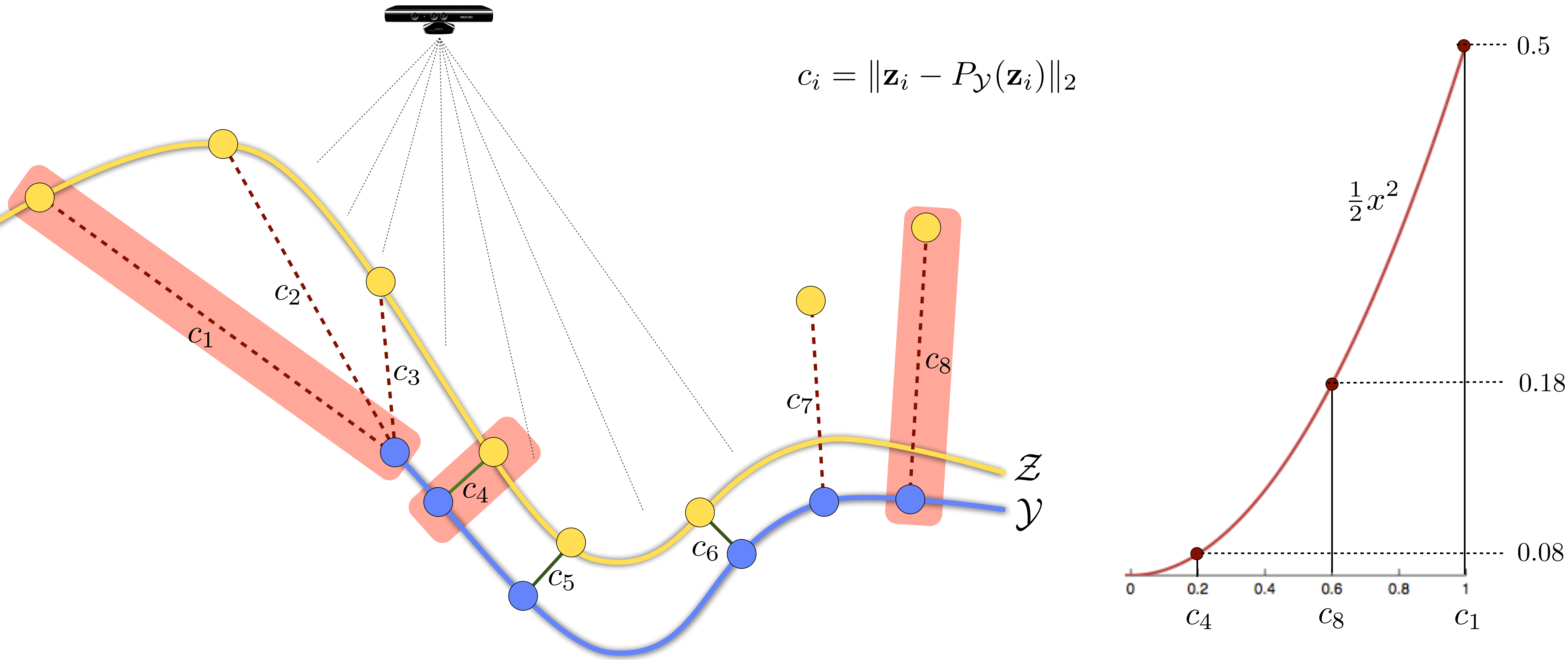
$$E_{\text{match}}(Z) = \sum_{i=1}^{n} \boxed{w_i} \|\mathbf{z}_i - \Pi_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

- **heuristics** to down-weigh (or prune) bad correspondences, e.g.
  - mutual correspondences
  - boundary rejection
  - normal rejection



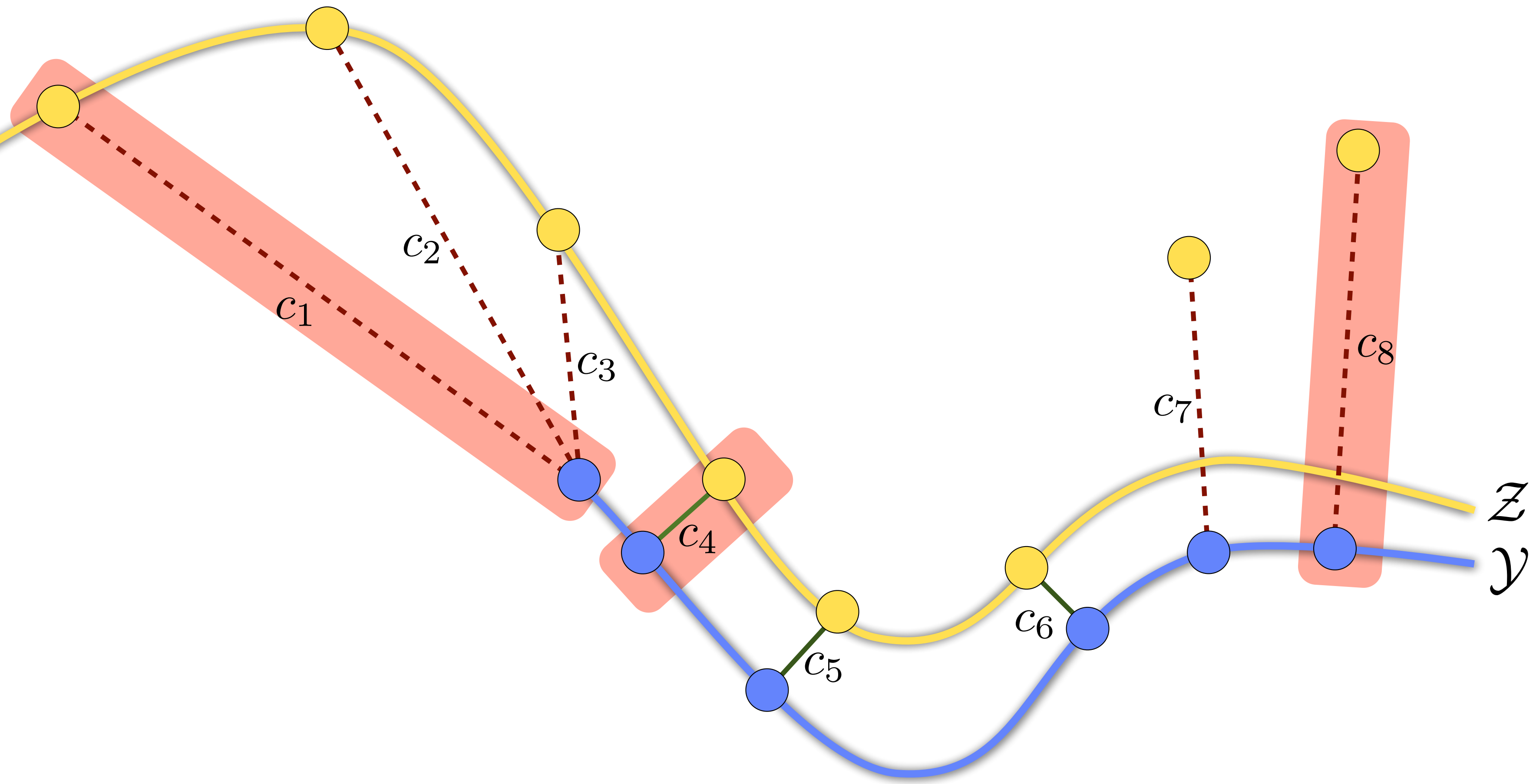
Rusinkiewicz et al. “Efficient Variants of the ICP Algorithm” 3DDIM, 2001

Robust Registration

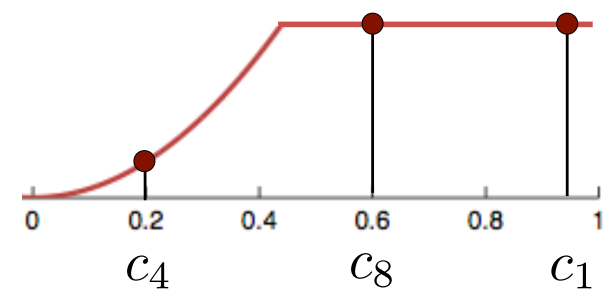


Robust Registration

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} c_i^2, \quad c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$

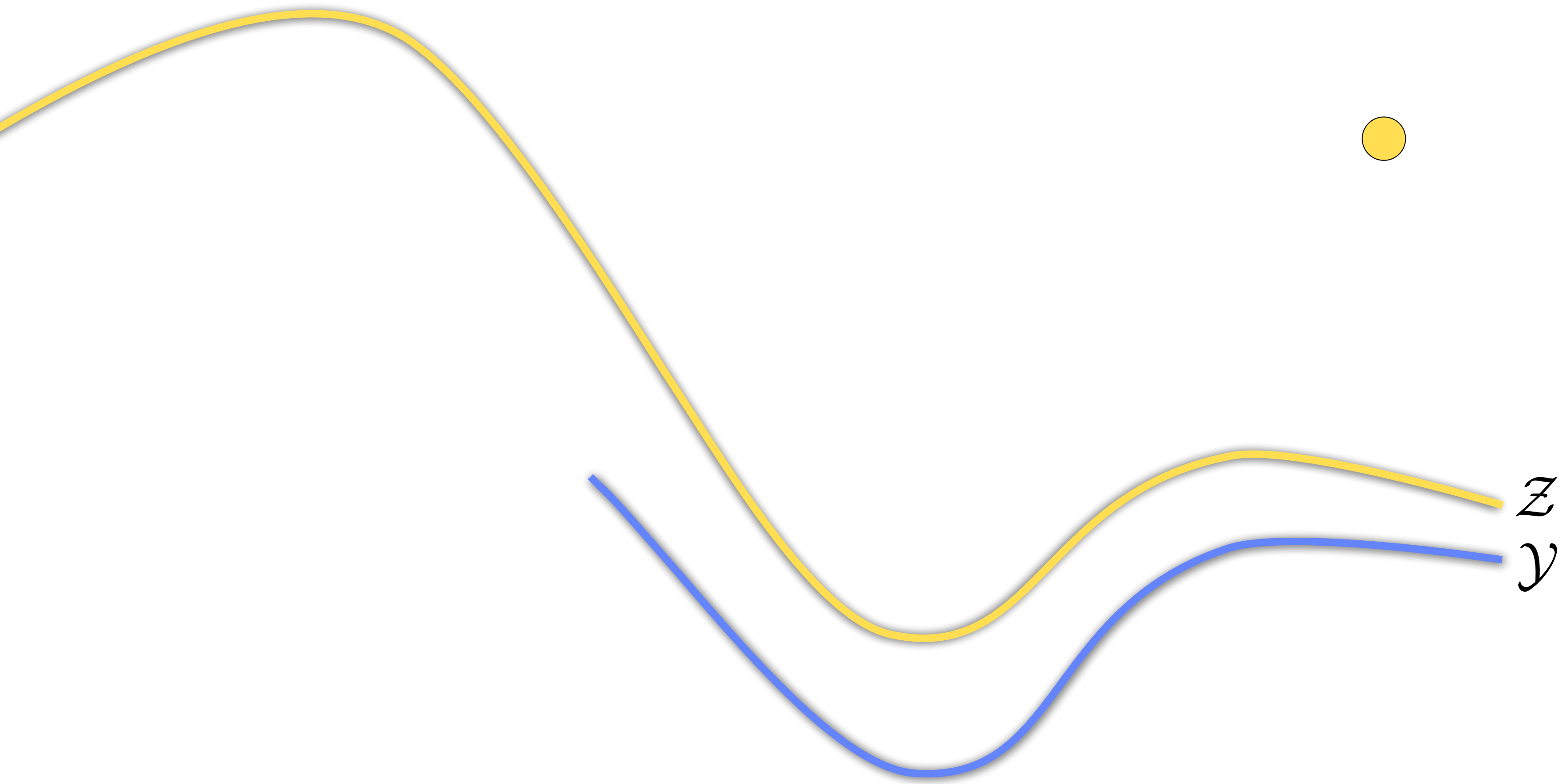


$$\begin{cases} \frac{1}{2}x^2 \text{ if } |x| \leqslant \tau \\ \frac{1}{2}\tau^2 \text{ otherwise} \end{cases}$$

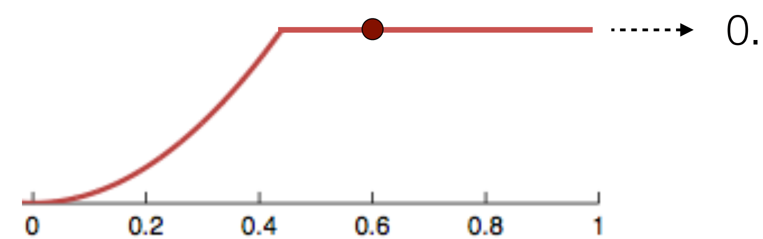


Robust Registration

$$E_{\text{match}}(Z) = \sum_{i=1}^{n} c_i^2, \quad c_i = \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2$$



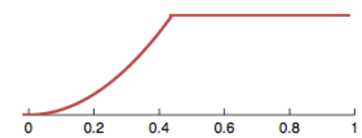
$$E_{\text{match}}(\mathbf{z}) = cst$$
$$\nabla E_{\text{match}}(\mathbf{z}) = 0$$



Robust Registration

$$\arg\min_{Z} \sum_{i=1}^{n} c_i^2 \Longrightarrow \arg\min_{Z} \sum_{i=1}^{n} \varphi(c_i)$$

robust function



IRLS

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

weight function

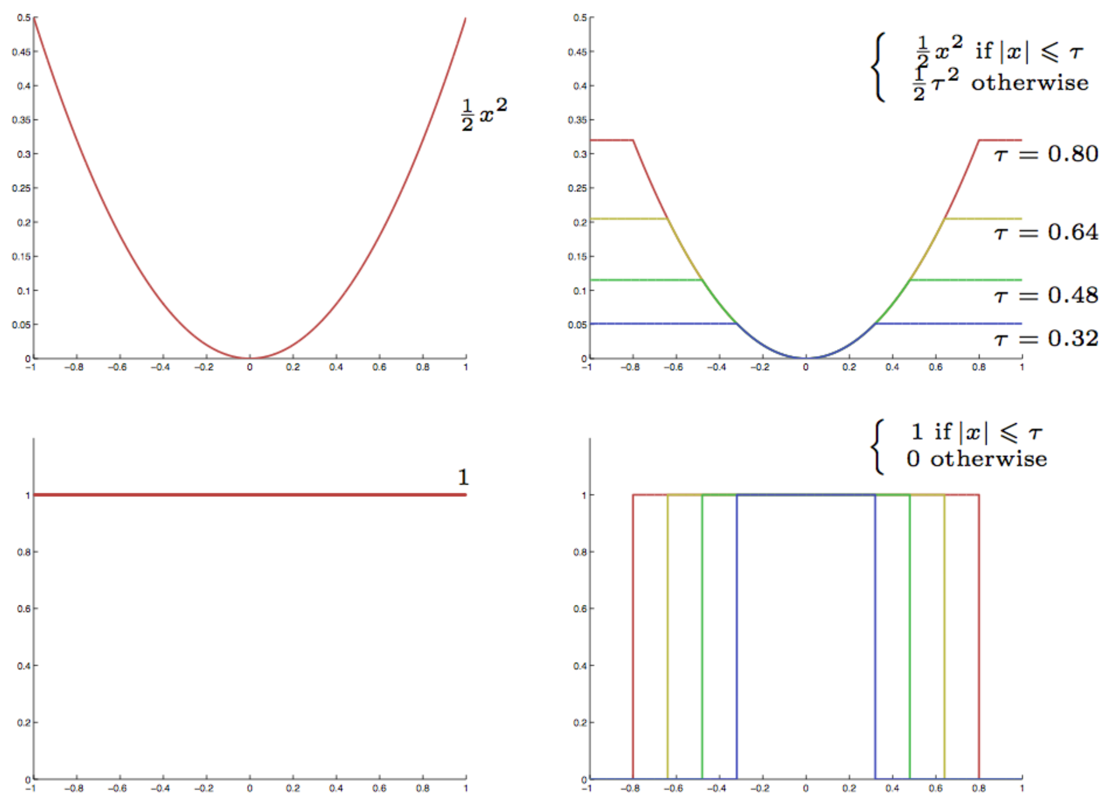
previous iteration residuals

Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

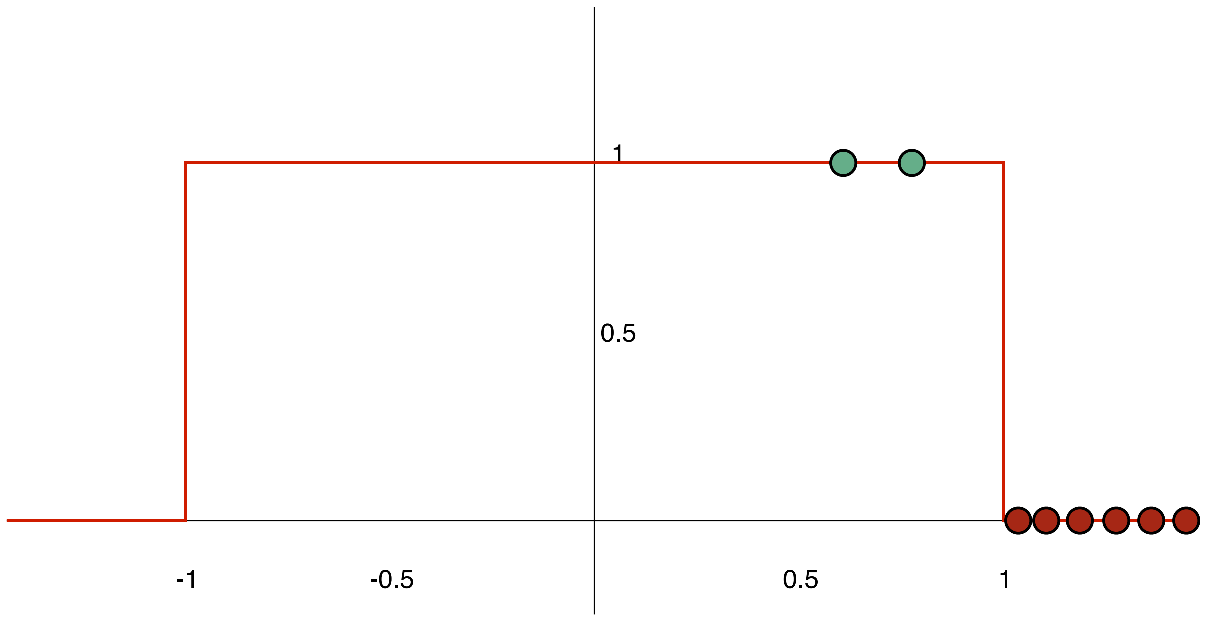
$\varphi(x)$

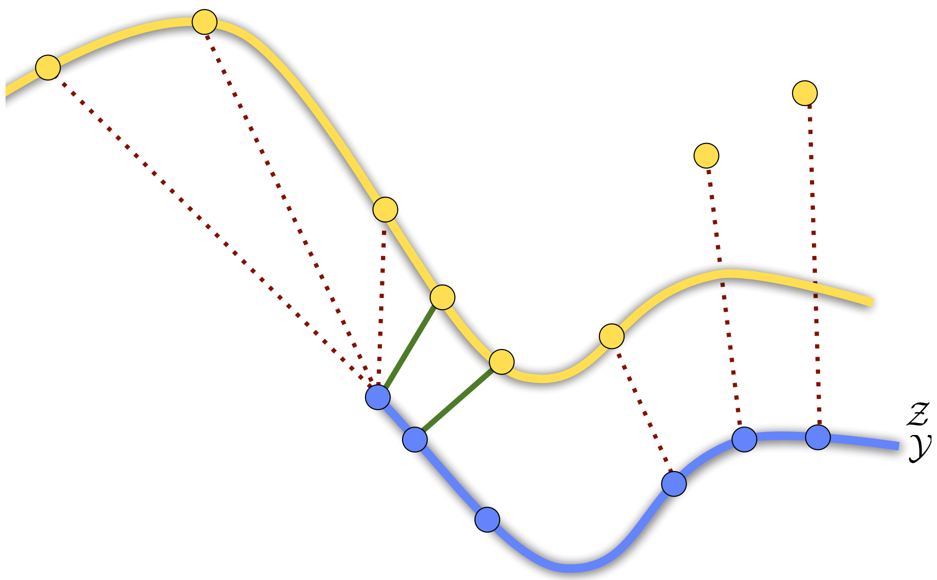
$w(x)$



Robust Registration

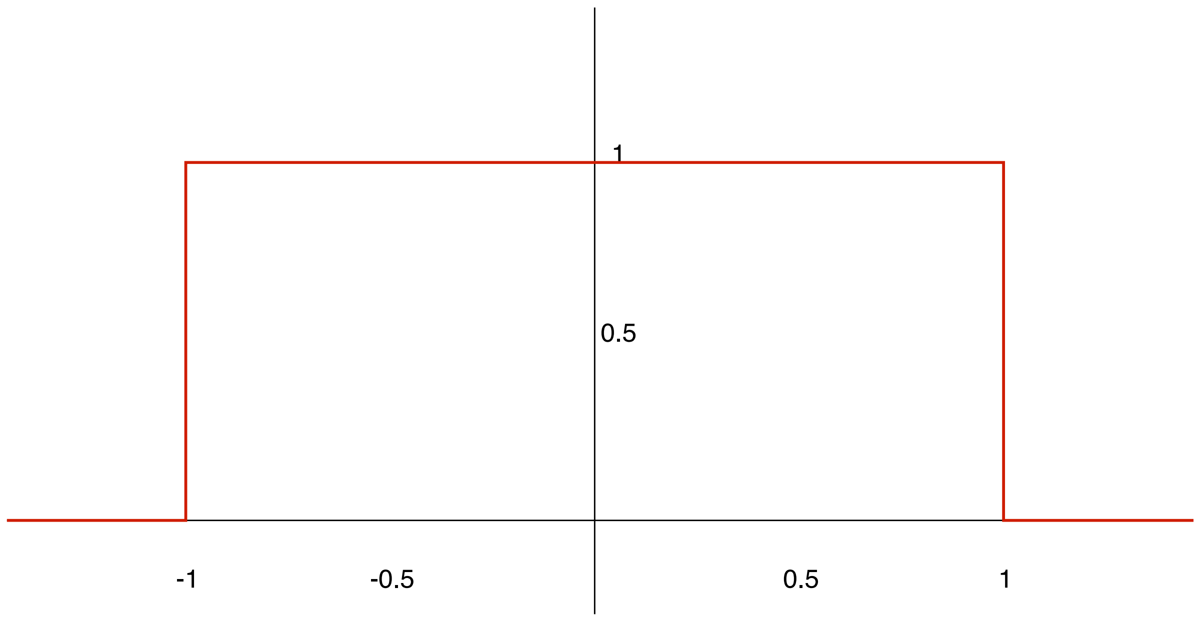
$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

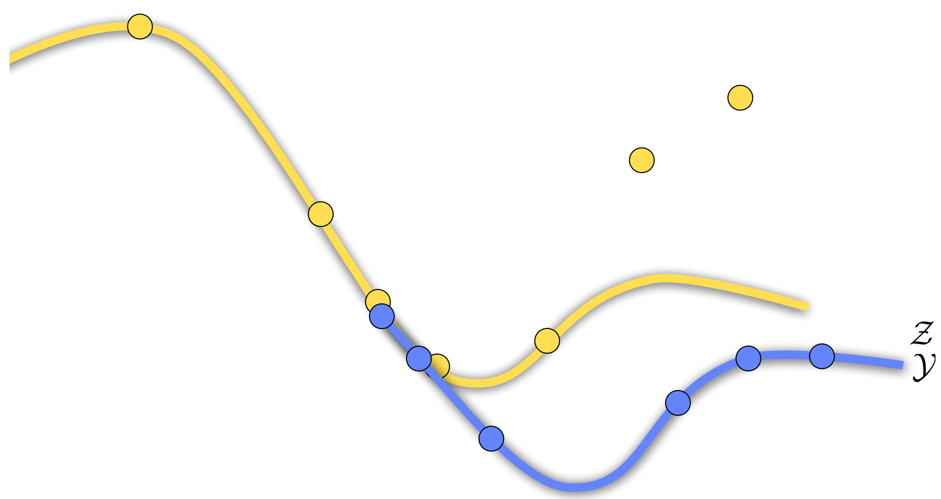




Robust Registration

$$\arg\min_{Z^t} \sum_{i=1}^{n} w(c_i^{t-1})(c_i^t)^2$$

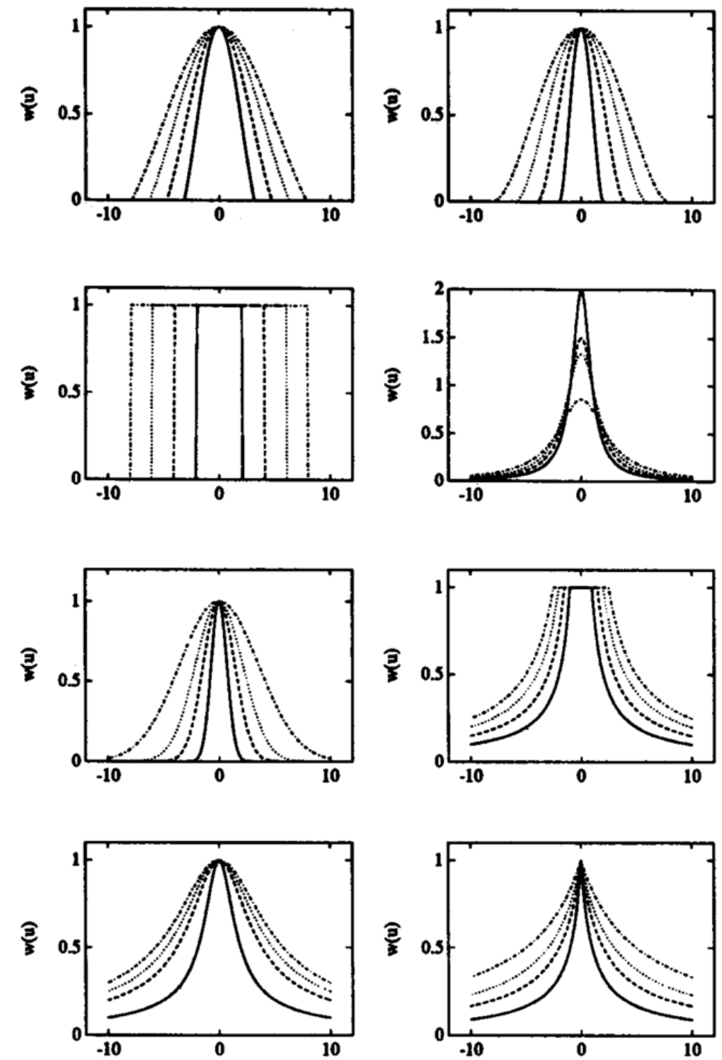




Robust Registration

TABLE I
WEIGHT FUNCTION

Estimator with Tuning Constant	Objective Function ρ	ψ-Function	Weight Function w	Range of u	Comments
Andrews' wave (a)	$a^2[1-\cos(u/a)]$ $2a^2$	$a\sin(u/a)$ 0	$(u/a)^{-1}\sin(u/a)$ 0	$\|u\| \le \pi a$ $\|u\| > \pi a$	$1.0 \le a \le 2.5$
Tukey's Biweight (b)	$(b^2/2)[1-(1-(u/b)^2)]^3$ $b^2/2$	$u[1-(u/b)^2]^2$ 0	$[1-(u/b)^2]^2$ 0	$\|u\| \le b$ $\|u\| > b$	$2.0 \le b \le 8.0$
Talwar (t)	$u^2/2$ $t^2/2$	u 0	1 0	$\|u\| \le t$ $\|u\| > t$	$2.0 \le t \le 8.0$
Student-t(f)			$(1+f)/(f+u^2)$		Derived in [1] by ML analysis
Cauchy(c)	$(c^2/2)\log[1+(u/c)^2]$	$u[1+(u/c)^2]^{-1}$	$[1+(u/c)^2]^{-1}$		special case of t-distribution
Welsch(w)	$(w^2/2)[1-\exp[-(u/w)^2]]$	$u\exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \le w \le 5.0$
Huber(h)	$u^2/2$ $h\|u\| - h^2/2$	u h sig(u)	1 $h\|u\|^{-1}$	$\|u\| \le h$ $\|u\| > h$	$1.0 \le h \le 2.5$
Logistic(l)	$l^2\log[\cosh(u/l)]$	$l\tanh(u/l)$	$(u/l)^{-1}\tanh(u/l)$		$1.0 \le l \le 3.0$
Fair (f)	$f^2[\|u\|/f - \log(1+\|u\|/f)]$	$u(1+\|u\|/f)^{-1}$	$(1+\|u\|/f)^{-1}$		$1.0 \le f \le 5.0$



Mirza et al. "Performance Evaluation of a Class of M-Estimators […]" IEEE Transactions on Robotics and Automation 1993

Robust Registration

TABLE I
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Estimator with Tuning Constant	Objective Function ρ	ψ-Function	Weight Function w	Range of u	Comments
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Talwar (t)	$u^2/2$ $t^2/2$	u 0	1 0	$\lvert u\rvert \le t$ $\lvert u\rvert > t$	$2.0 \le t \le 8.0$
Student-t(f)			$(1+f)/($		
Cauchy(c)	$(c^2/2)\log[1+(u/c)^2]$	$u[1+(u/c)^2]^{-1}$	$[1+(u/c)^2]^{-1}$		special case of t-distribution
Welsch(w)	$(w^2/2)[1-\exp[-(u/w)^2]]$	$u\exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \le w \le 5.0$
Huber(h)	$u^2/2$ $h\lvert u\rvert - h^2/2$	u $h\ \mathrm{sig}(u)$	1 $h\lvert u\rvert^{-1}$	$\lvert u\rvert \le h$ $\lvert u\rvert > h$	$1.0 \le h \le 2.5$
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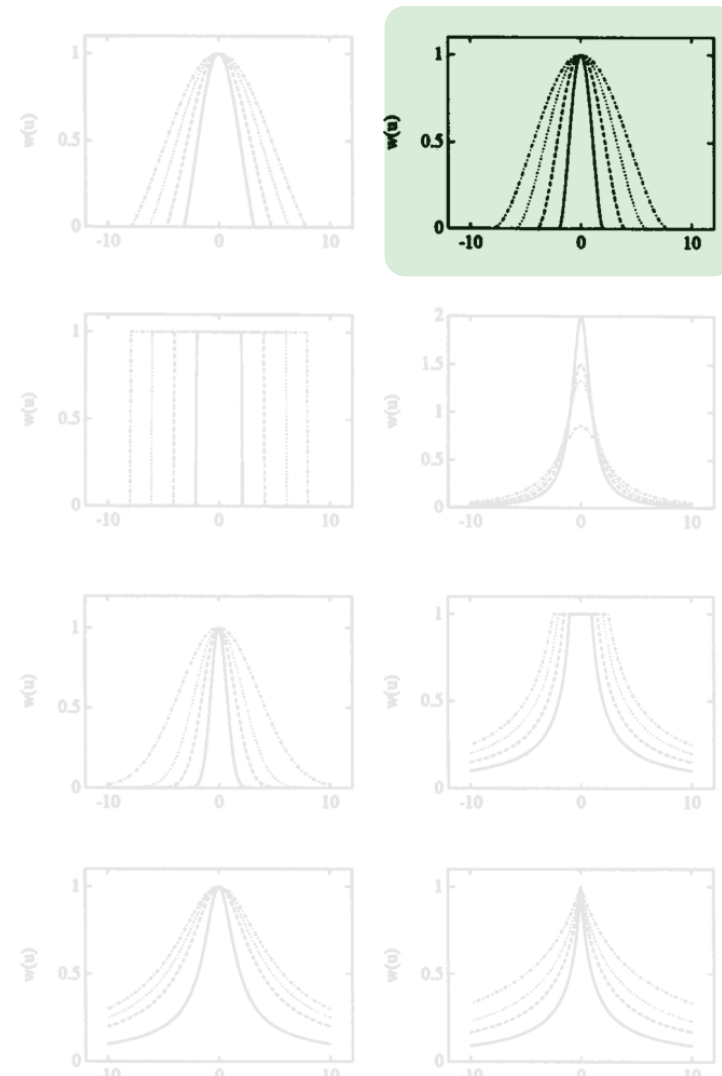
Fair (f)	$f^2[\lvert u\rvert/f - \log(1+\lvert u\rvert/f)]$	$u(1+\lvert u\rvert/f)^{-1}$	$(1+\lvert u\rvert/f)^{-1}$		$1.0 \le f \le 5.0$

DEMO



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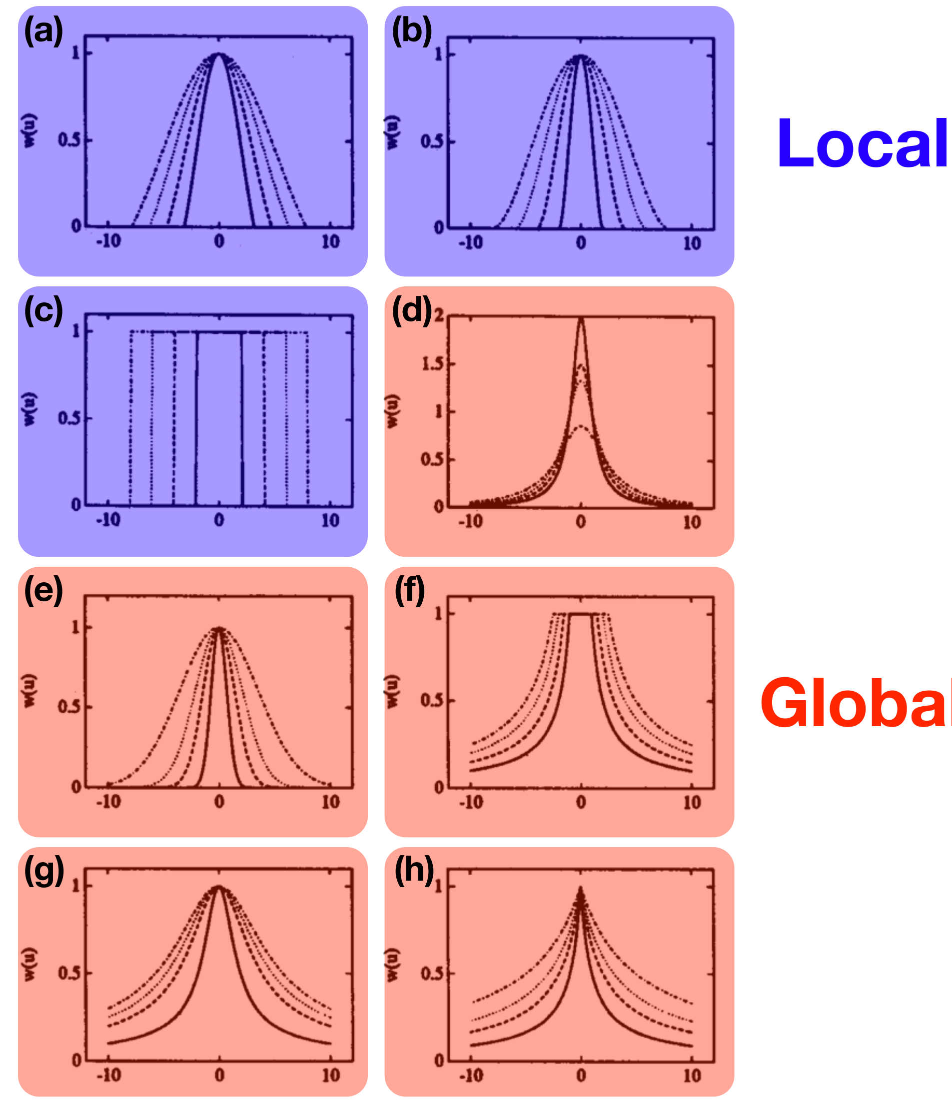
Robust Registration

TABLE I
WEIGHT FUNCTION

Estimator with Tuning Constant	Objective Function ρ	ψ-Function	Weight Function w	Range of u	Comments	
						(a)
Andrews' wave (a)	$a^2[1-\cos(u/a)]$ $2a^2$	$a\sin(u/a)$ 0	$(u/a)^{-1}\sin(u/a)$ 0	$\lvert u\rvert \le \pi a$ $\lvert u\rvert > \pi a$	$1.0 \le a \le 2.5$	(b)
Tukey's Biweight (b)	$(b^2/2)[1-(1-(u/b)^2)]^3$ $b^2/2$	$u[1-(u/b)^2]^2$ 0	$[1-(u/b)^2]^2$ 0	$\lvert u\rvert \le b$ $\lvert u\rvert > b$	$2.0 \le b \le 8.0$	(c)
Talwar (t)	$u^2/2$ $t^2/2$	u 0	1 0	$\lvert u\rvert \le t$ $\lvert u\rvert > t$	$2.0 \le t \le 8.0$	
Student-t(f)			$(1+f)/(f+u^2)$		Derived in [1] by ML analysis	(d)
Cauchy(c)	$(c^2/2)\log[1+(u/c)^2]$	$u[1+(u/c)^2]^{-1}$	$[1+(u/c)^2]^{-1}$		special case of t-distribution	
Welsch(w)	$(w^2/2)[1-\exp[-(u/w)^2]]$	$u\exp[-(u/w)^2]$	$\exp[-(u/w)^2]$		$1.0 \le w \le 5.0$	(e)
Huber(h)	$u^2/2$ $h\lvert u\rvert - h^2/2$	u h sig(u)	1 $h\lvert u\rvert^{-1}$	$\lvert u\rvert \le h$ $\lvert u\rvert > h$	$1.0 \le h \le 2.5$	(f)
Logistic(l)	$l^2\log[\cosh(u/l)]$	$l\tanh(u/l)$	$(u/l)^{-1}\tanh(u/l)$		$1.0 \le l \le 3.0$	(g)
Fair (f)	$f^2[\lvert u\rvert/f - \log(1+\lvert u\rvert/f)]$	$u(1+\lvert u\rvert/f)^{-1}$	$(1+\lvert u\rvert/f)^{-1}$		$1.0 \le f \le 5.0$	(h)

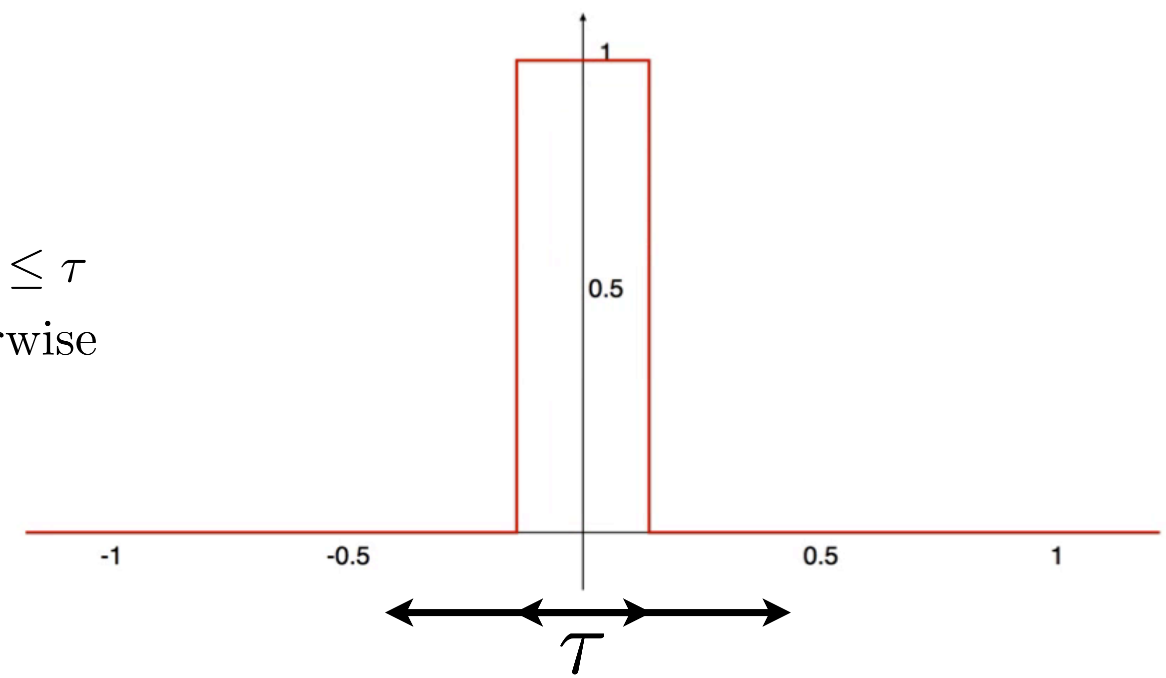


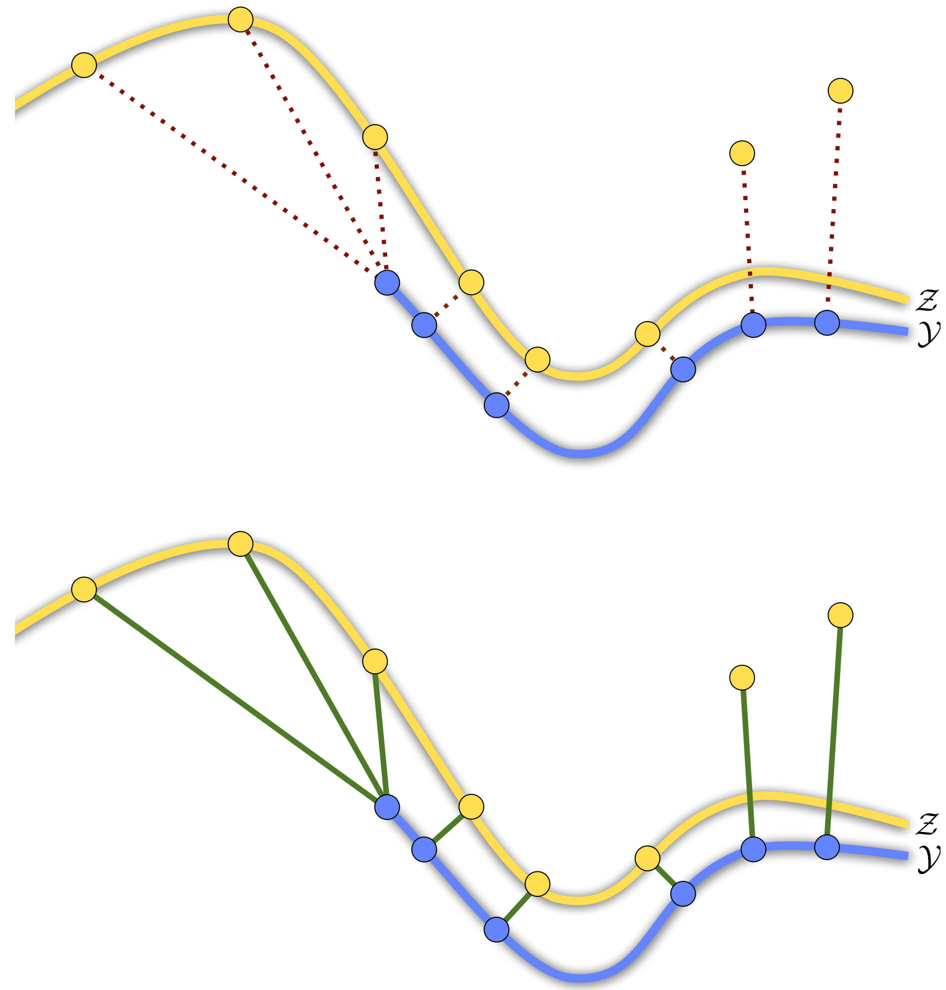
Mirza et al. "Performance Evaluation of a Class of M-Estimators […]" IEEE Transactions on Robotics and Automation 1993

Robust Registration

- Local Support

$$w(x) = \begin{cases} 1 & \text{if } |x| \leq \tau \\ 0 & \text{otherwise} \end{cases}$$



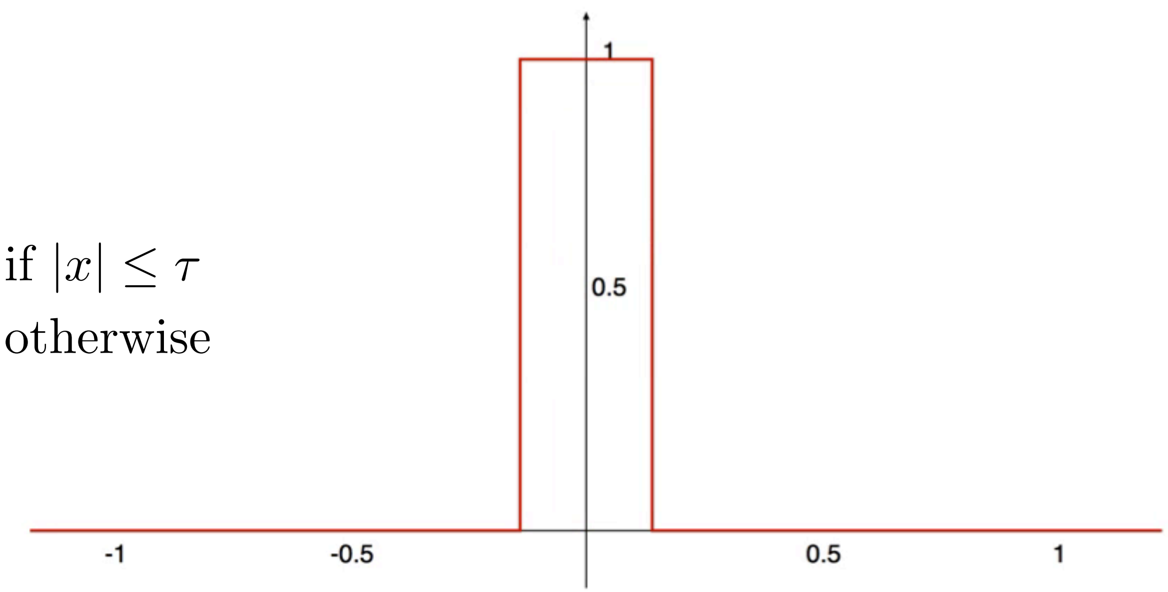


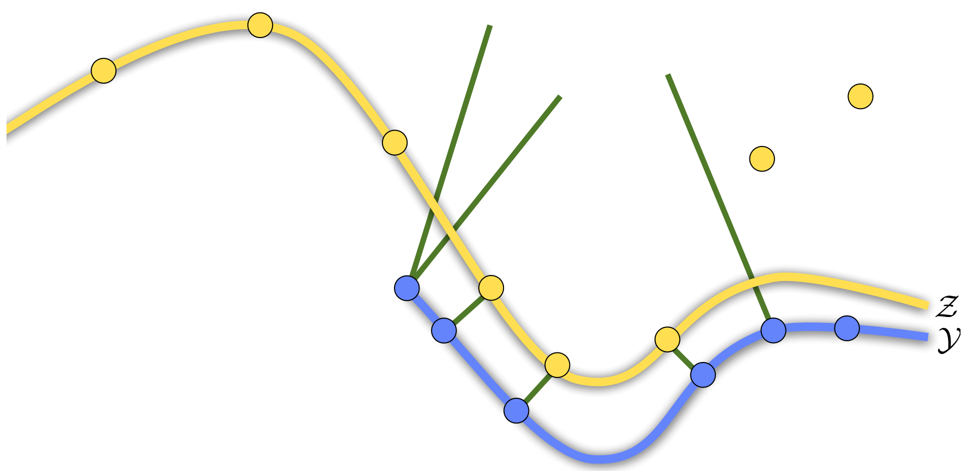
Robust Registration

- Trimmed Metrics
  - *Known number of inlier N*

N=3

$$w(x) = \begin{cases} 1 & \text{if } |x| \leq \tau \\ 0 & \text{otherwise} \end{cases}$$



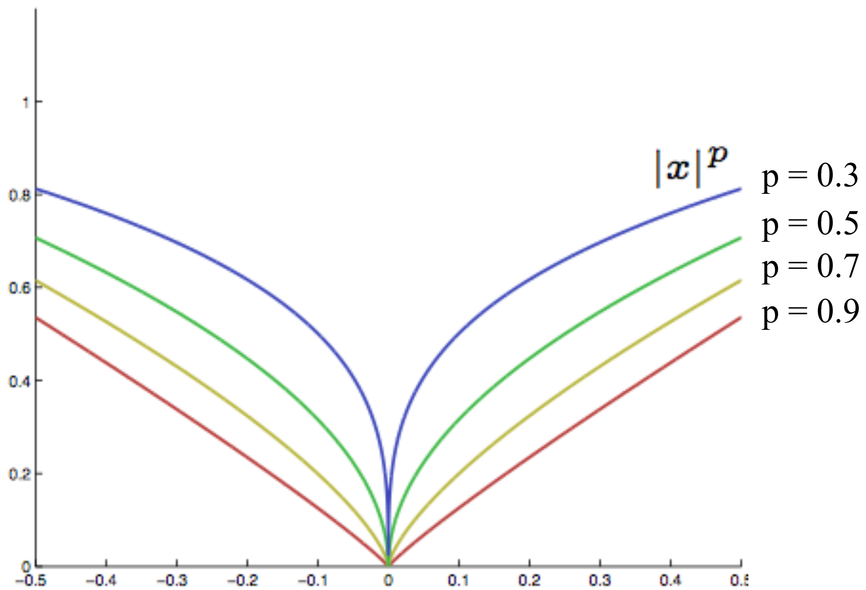


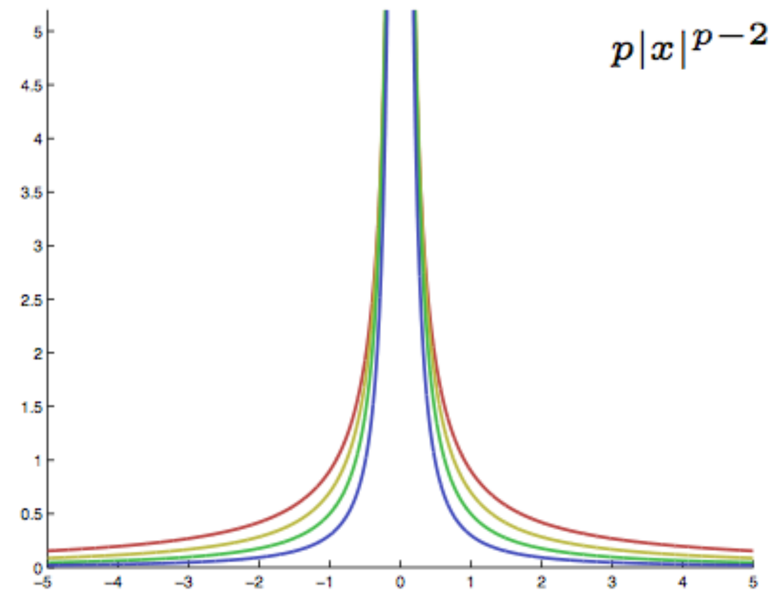
Chetverikov et al. “The Trimmed Iterative Closest Point Algorithm”, ICPR 2002

Robust Registration

- Sparse Metrics

$$\varphi(x) = |x|^p \qquad\qquad w(x) = p|x|^{p-2}$$



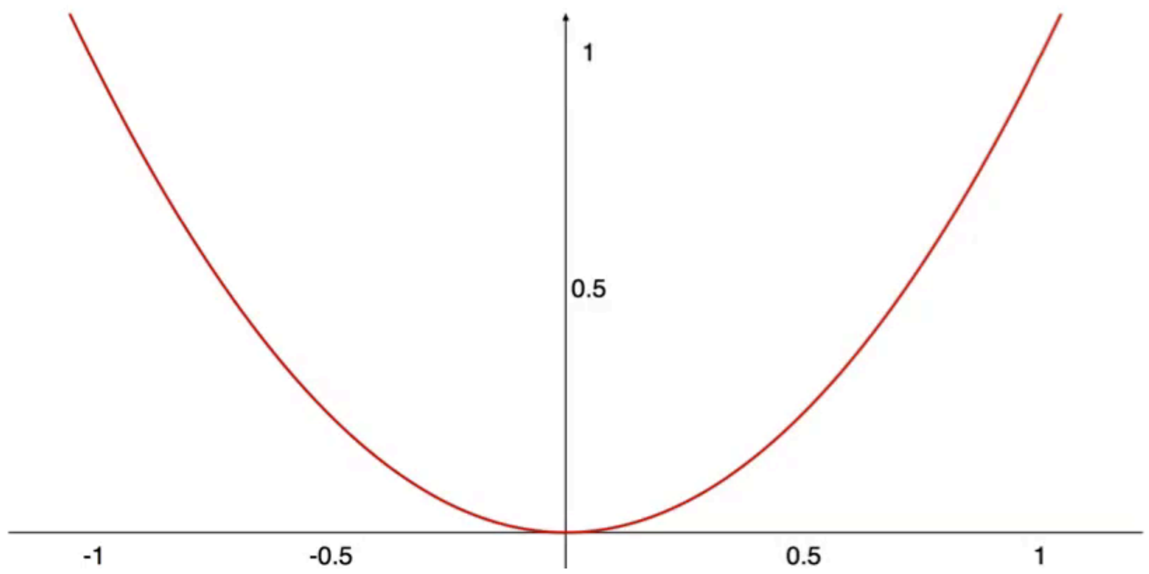


Bouaziz, Tagliasacchi and Pauly “Sparse Iterative Closest Point” SGP 2013

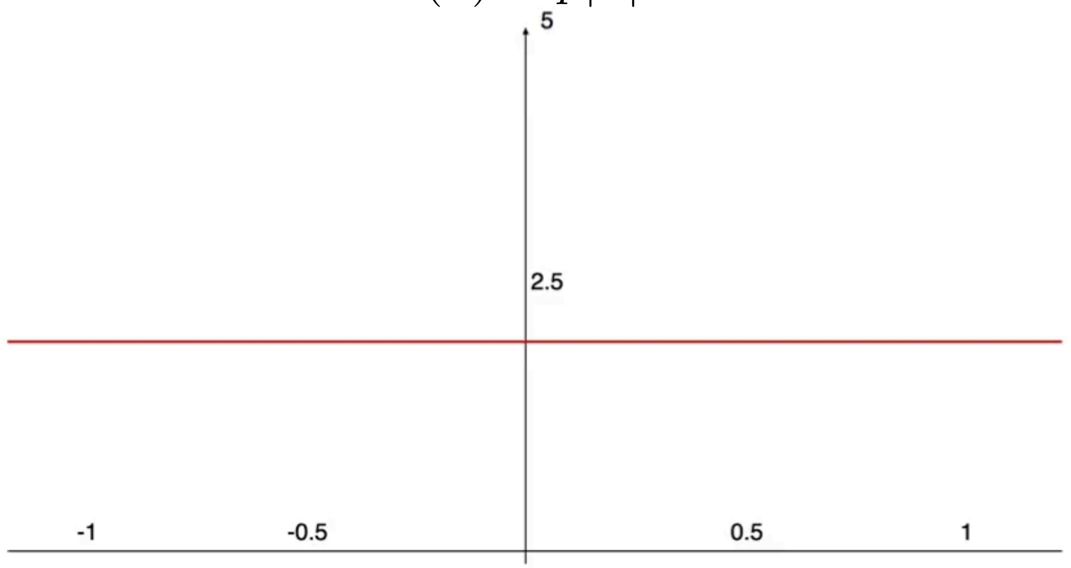
Robust Registration

- Sparse Metrics

$$\varphi(x) = |x|^p$$



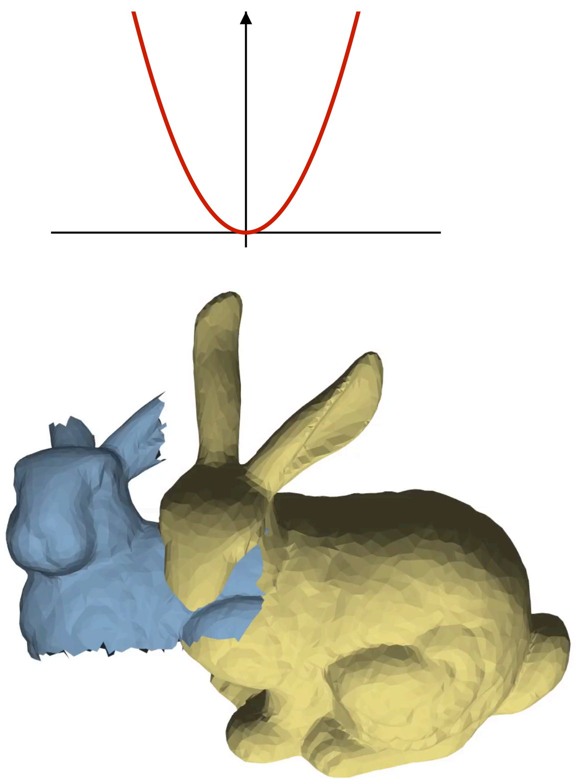
$$w(x) = p|x|^{p-2}$$



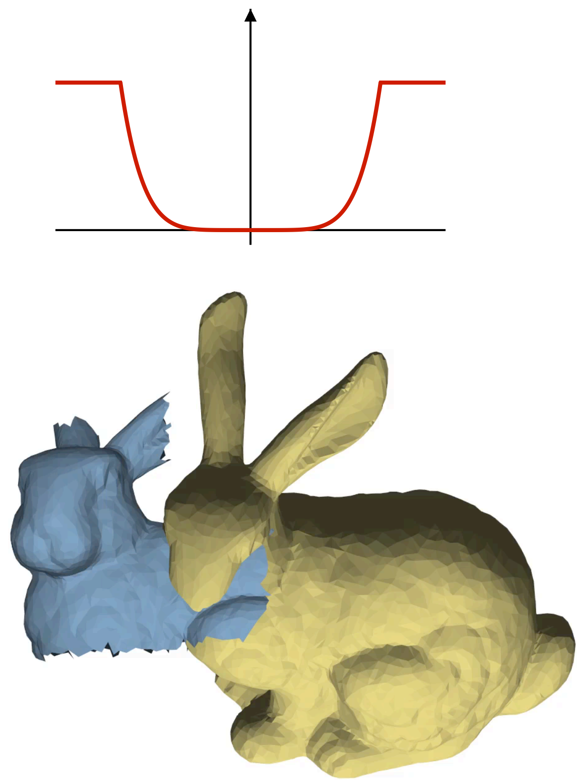
Bouaziz, Tagliasacchi and Pauly “Sparse Iterative Closest Point” SGP 2013

Robust Registration

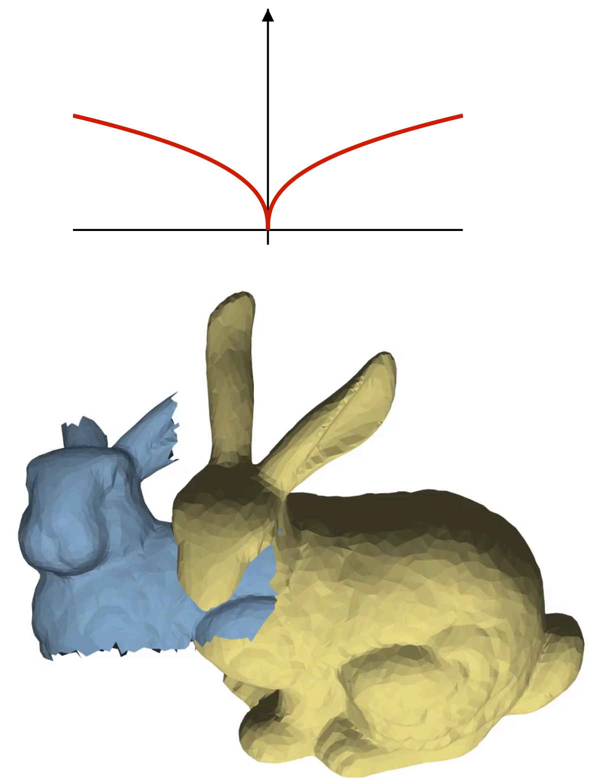
p=2



Tukey



p=0.4



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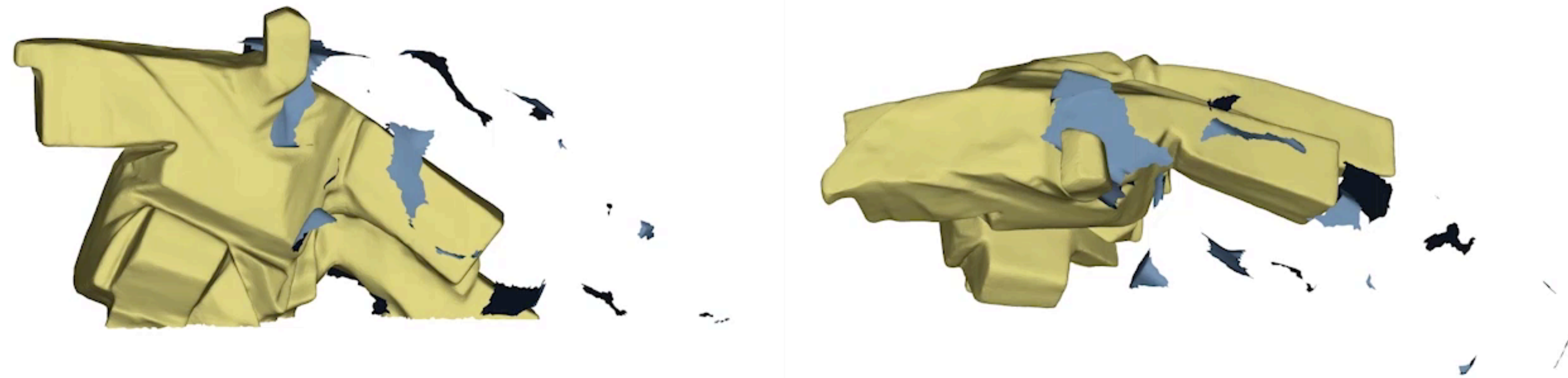
Robust Registration



Bouaziz, Tagliasacchi and Pauly "Sparse Iterative Closest Point" SGP 2013

Overview

- Part 1 - presented by Andrea Tagliasacchi
  - Introduction, motivation and sensing hardware (5 min)
  - Rigid registration, ICP, Hausdorff distances (20 min)
  - Robust registration, articulated registration (20 min)

10m break!

- Part 2 - presented by Hao Li
  - Non-rigid registration and face tracking (20 min)
  - Correspondences with Convolutional Neural Networks (20 min)
  - Conclusions and Q&A (10 min)

http://gfx.uvic.ca/pubs/2016/registration/slides