linearized pressure diffusion

volume rate

$$\frac{\partial(\rho\phi w 2\pi r)}{\partial t} + \frac{\partial(\rho q)}{\partial r} = 0 , \qquad r \ge R , \qquad q [=] \frac{m^3}{s}$$

$$q = w 2\pi r \left(-\frac{k}{\mu}\frac{\partial p}{\partial r}\right) , \qquad w 2\pi r = \text{flow cross-sectional area}$$

$$\underbrace{\cancel{\rho}\phi w 2\pi r (\beta_f + \beta_\phi)}_{b w 2\pi r}\frac{\partial p}{\partial t} - \cancel{\rho}\frac{\partial}{\partial r}\left(\frac{k}{\mu} w 2\pi r \frac{\partial p}{\partial r}\right) = 0$$

$b w 2\pi r$, $b = \phi(\beta_f + \beta_\phi)$

BC: $q(R,t) = w 2\pi r \left(-\frac{k}{\mu}\frac{\partial p}{\partial r}\right)\Big|_{r=R} = q_0(t)$

semi-discretization :

SAT vector

$$B\frac{d\vec{p}}{dt} - D_2\vec{p} = \vec{S} q_0 \qquad (1)$$

$\mathrm{diag}\{b w 2\pi r\}$ $\qquad \approx \frac{\partial}{\partial r}\left(\frac{k}{\mu} w 2\pi r \frac{\partial}{\partial r}\right)$

well $\quad S_w \frac{dp_w}{dt} = Q(t) - q_0(t) \Rightarrow q_0 = Q - S_w\frac{dp_w}{dt} \qquad (2)$

injection rate

coupling: $p(R,t) = p_w \qquad (3)$

combine (1)-(3) :

$$B\frac{d\vec{p}}{dt} - D_2\vec{p} = \vec{S}\left(Q - S_w \vec{e}^{\,T}\vec{p}\right)$$

$\vec{e}^{\,T} = [1 \ 0 \cdots 0]$

$$= \vec{S} Q - S_w s \vec{p}$$

$s = \vec{S} e^T$ (matrix)

backward Euler

$$B\frac{\vec{p}_{n+1} - \vec{p}_n}{\Delta t} - D_2\vec{p}_{n+1} = \vec{s} Q_{n+1} - S_w s \vec{p}_{n+1}$$

$$(B - \Delta t\, D_2 + S_w \mathbf{s})\, \vec{p}_{n+1} = B\vec{p}_n + \Delta t\, \vec{S} Q_{n+1}$$

reference case with no flow out of well

$$S_w \frac{dp_w}{dt} = Q$$

$$p_w^{n+1} = p_w^n + \Delta t\, Q^{n+1} / S_w$$