Algorithms and Data Structures

Spring 2019

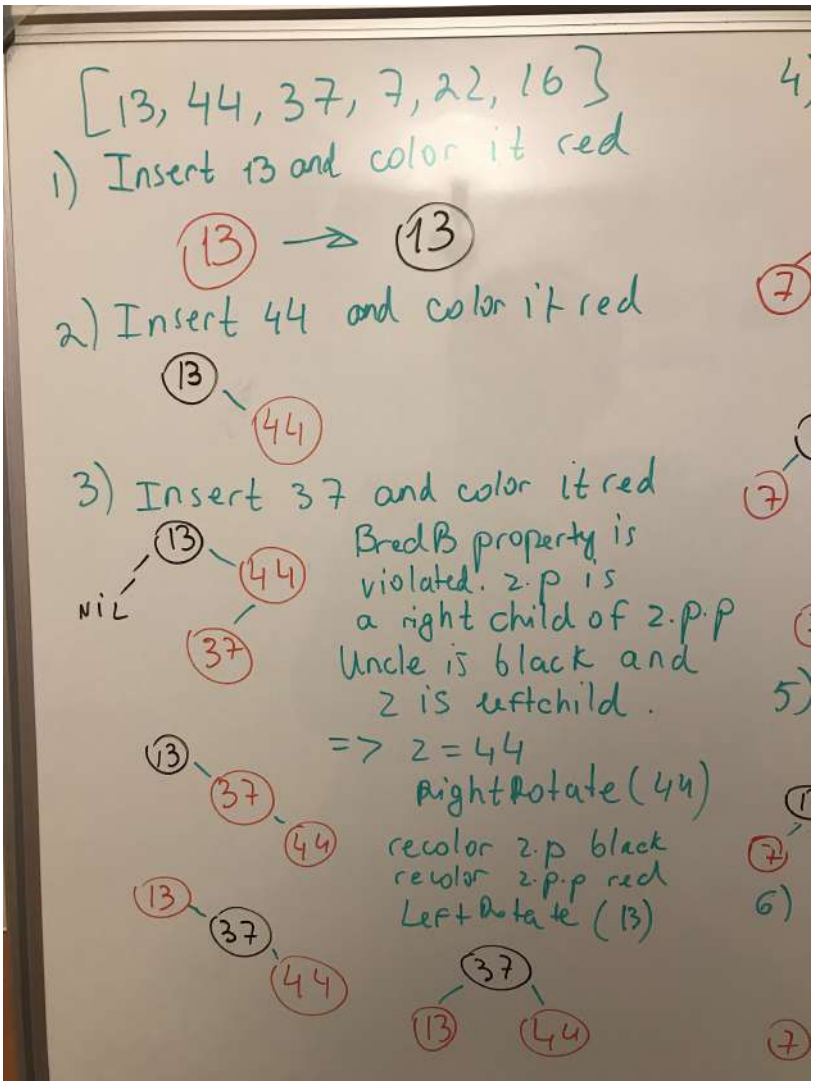
Assignment 8

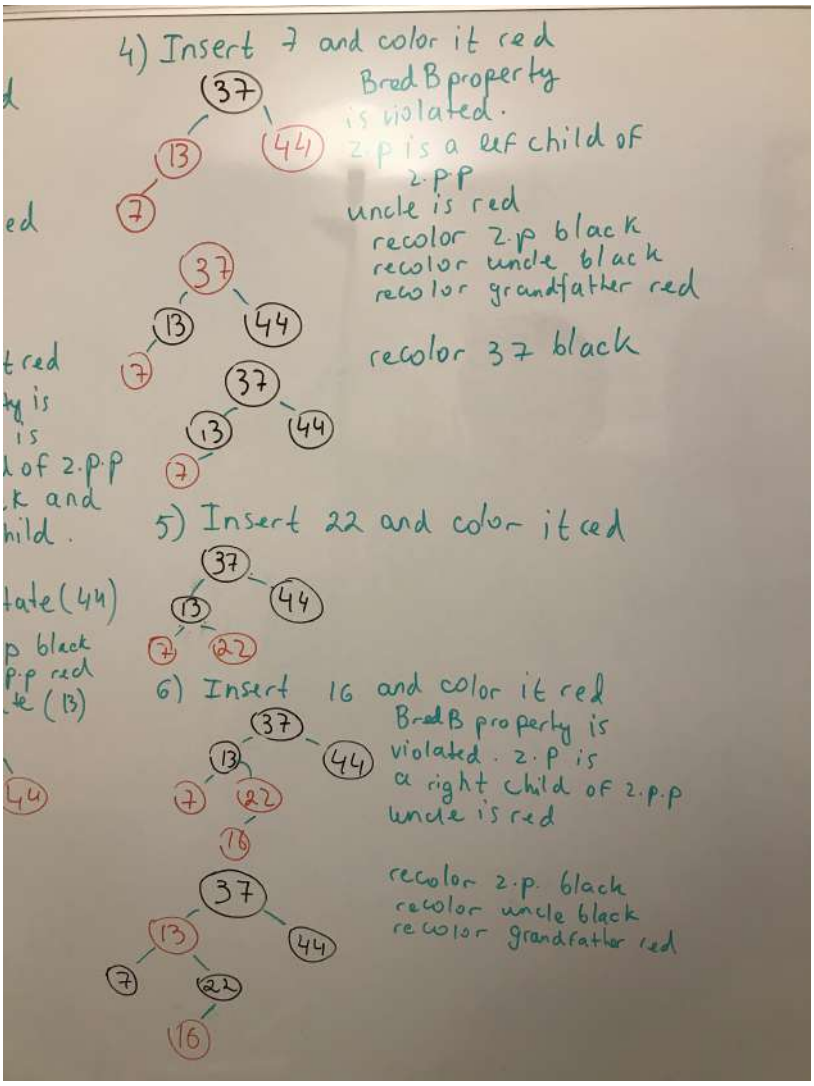
Date: April 11, 2019

Taiyr Begeyev

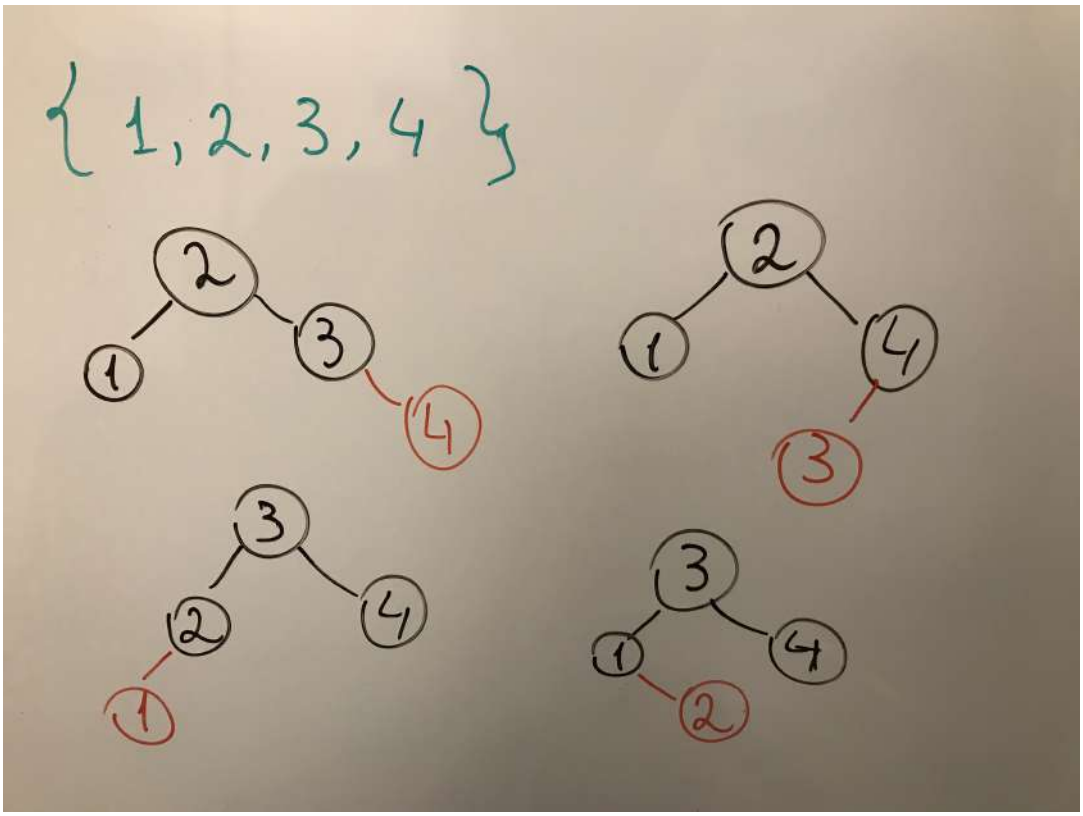
Problem 8.1 Understanding Red Black Trees

(a) Draw (or describe by using preorder traversal) the red-black trees that result after successively inserting the values step by step in the following order [13, 44, 37, 7, 22, 16] into an empty red-black tree. You are required to draw (or describe by using preorder traversal) the tree after each insertion, as well as any additional recoloring and balancing.





(b) Draw (or describe by using preorder traversal) all valid red-black trees that store the values $1, 2, 3, 4$



For better quality: `https://drive.google.com/open?id=1fvYadnBuZFbWED30nD1HkPLmC-tFYrt_`

(c) Consider a red-black tree formed by inserting n nodes with the algorithm described in the lecture slides. Prove that if $n > 1$, the tree contains at least one red node.

Proof by induction:

1. Base Case

$n = 2$. There are two nodes. One is black and another one is red.

2. Induction Hypotheses:

Assume that there exists at least one *Red Node* in a red-black tree, consisting of n nodes, where $1 < n \leq N$

3. Inductive Step:

Prove that there exists at least one *Red One* in a red-black tree of $N+1$ nodes. Now let's consider two cases:

Case 1:

We have the best case when the Red Node is inserted as a child of the Black Node. This happens to be a trivial case.

Case 2:

Insertion Case 1: Z remains red after the recoloring parent, uncle and grandfather. Therefore, we have at least one red node.

Insertion Case 2: $Z.p$ remains red after recoloring Z and grandparent. Consequently, we have at least one red node.

Insertion Case 3: Z remains red after rotating of $Z.p$ and it remains red after recoloring $Z.p$ and $Z.p.p$.

Problem 8.2 Implementing Red Black Trees

Implement a red black tree (with integer nodes), closely following the specifications and algorithms from the lecture. Make sure you handle errors appropriately by printing messages or throwing exceptions. Your implementation has to be along the interface below with the following or equivalent components:

To execute run: **make**