Multi-Rate Nonlinear Model Predictive Control for Wall-Supported Bipedal Locomotion of Quadrupedal Robots

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Motivation

- Real-time footstep planning is essential for balance and adaptability in agile locomotion.
- On uneven terrain, footholds are limited and constantly changing.
- Conventional MPCs are heuristic, decoupled footstep planning, often leading to instability.

Goal

Design a controller that **unifies robot position, orientation, reaction-force, and footstep planning** within a single optimization loop

- **Multi-Rate Nonlinear MPC (MR-NMPC)** jointly optimizes:
  - **CoM position and orientation trajectories**
  - **Environment reaction forces (ERFs)**
  - **Discrete foot placements**.
- Traditional **Single Rigid Body(SRB)** model is **augmented with foot position and step length**.
- A discrete delta trigger $\delta(t+1)$ enables **multi-rate updates—3Hz step-length planning(slow input) and 50Hz reaction-force updates (fast input)**.

$$\Sigma^a : \begin{cases} x(t+1) = f(x(t), u(t), v(t)), \\ v(t+1) = v(t) + \delta(t+1)\, \Delta v(t), \end{cases}$$

- The upcoming contact configuration is encoded in the MPC via $\delta(t+1)$ ensuring **footstep updates anticipate the next domain.**

Proposed Approach

- Optimum SRB trajectory, GRFs and step lengths from MR-NMPC are tracked by a **Hybrid Zero Dynamics-based Whole-Body Controller(WBC)** running at **500Hz**.

MR-NMPC

$$\begin{aligned} \min_{(x,v,u,\Delta v)} \; & \mathcal{L}_{N,x}\left(x_{t+N|t}\right) + \mathcal{L}_{N,v}\left(v_{t+N|t}\right) \\ & + \sum_{k=0}^{N-1} \mathcal{L}_{stage,x}\left(x_{t+k|t}, u_{t+k|t}\right) \\ & + \sum_{k=0}^{N-1} \mathcal{L}_{stage,v}\left(v_{t+k|t}, \Delta v_{t+k|t}\right) \\ \text{s.t.}\; & x_{t+k+1|t} = f\left(x_{t+k|t}, u_{t+k|t}, v_{t+k|t}\right) \\ & v_{t+k+1|t} = v_{t+k|t} + \delta(t+k+1)\, \Delta v_{t+k|t} \\ & E_u(t+k)\, u_{t+k|t} = 0, \\ & E_{\Delta v}(t+k)\, \Delta v_{t+k|t} = 0 \\ & x_{t+k|t} \in \mathcal{X}, \quad u_{t+k|t} \in \mathcal{U}, \quad v_{t+k|t} \in \mathcal{V} \end{aligned}$$

50Hz

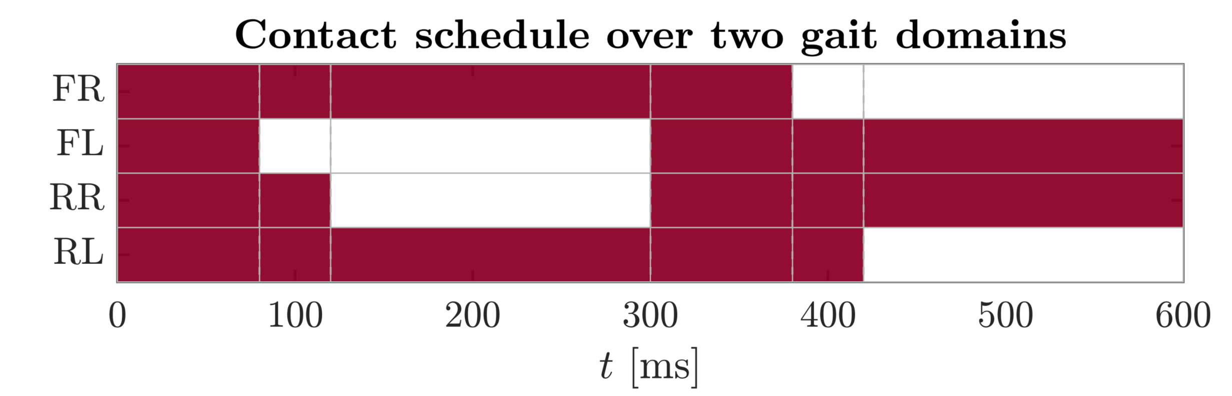
WBC

$$\begin{aligned} \min_{(\tau,F,\epsilon)} \; & \frac{\gamma_1}{2}\|\tau\| + \frac{\gamma_2}{2}\|F-u\| + \frac{\gamma_3}{2}\|\epsilon\| \\ \text{s.t.}\; & \ddot{y} + K_P y + K_D \dot{y} = \epsilon \\ & D(q)\ddot{q} + H(q,\dot{q}) = B\tau + \sum_{\ell\in\mathcal{C}} J_\ell^T F_\ell \\ & J_\ell \ddot{q} + \dot{J}_\ell \dot{q} = 0 \quad \forall \ell \in \mathcal{C} \end{aligned}$$

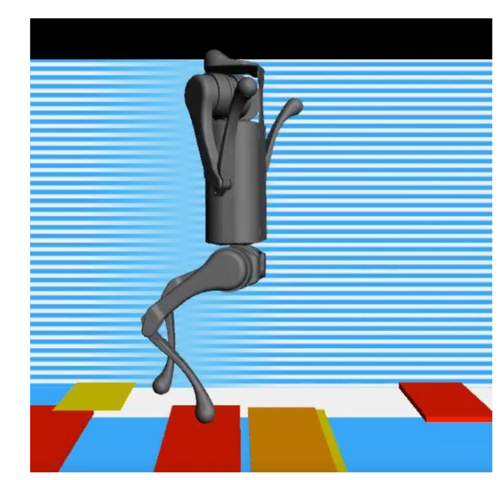
500Hz

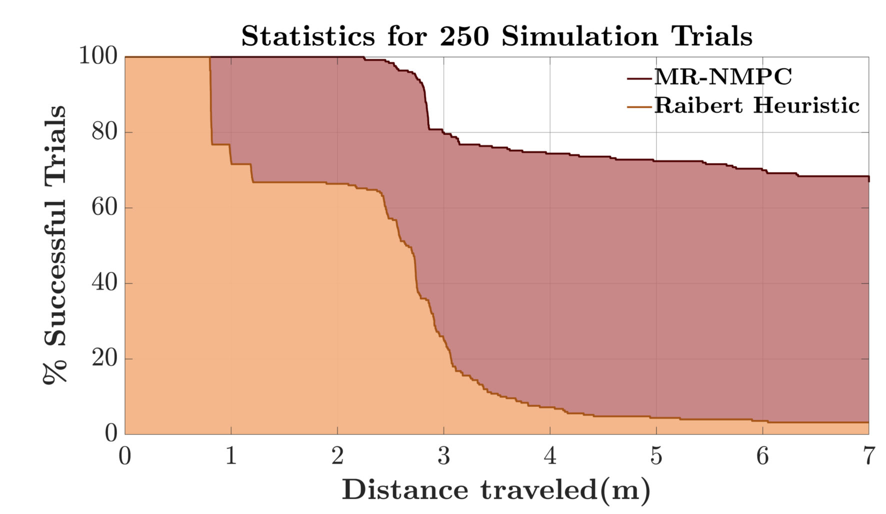
Results

- **MR-NMPC footstep planner** was implemented on the **Unitree Go2** in the **dynamically unstable upright bipedal configuration.**
- The **contact sequence** was inspired by a **monkey-like gait**, in which agile upper-limb interactions stabilize the torso during rapid transitions.

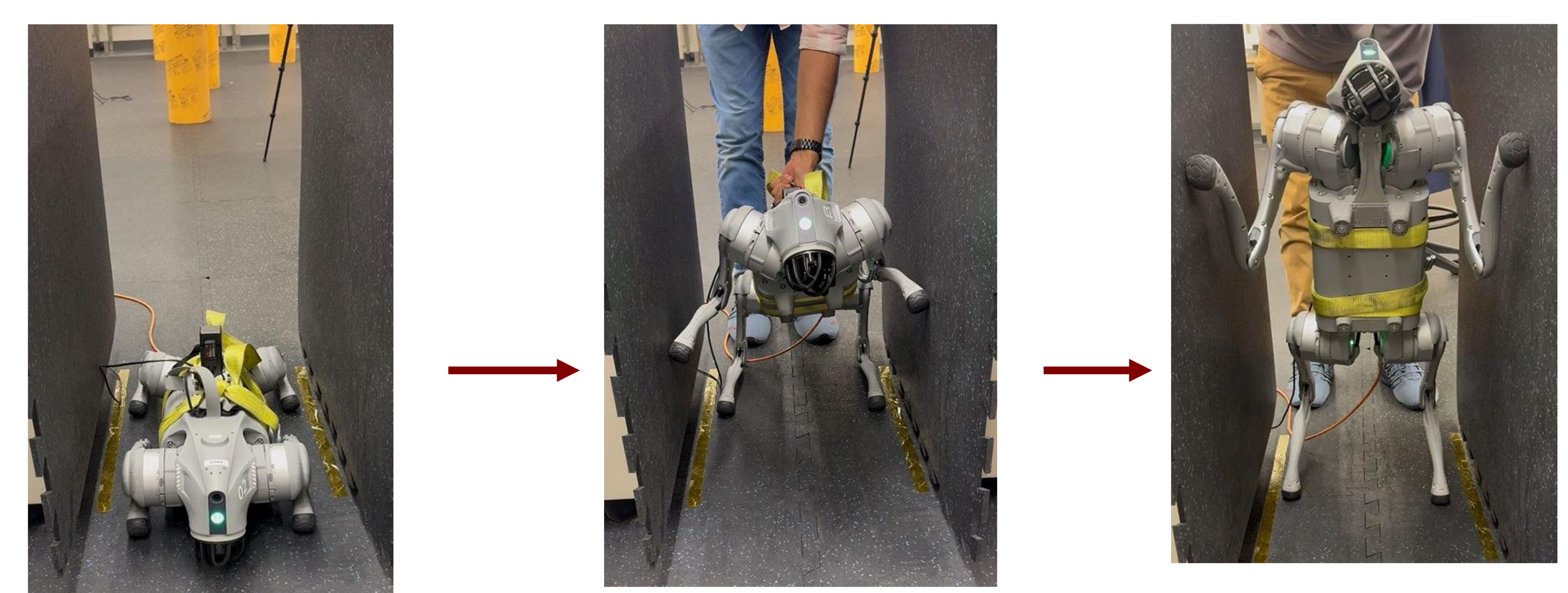


- **Traversal success rate**: **66% (MR-NMPC)** vs **< 5% (Raibert)** on random wooden-block terrain.





Experimental Setup: Unitree Go2 controlled by MR-NMPC to rise from **crouch to stand and trot in place**.



Watch the full simulation and experiment video: