Session type inference examples

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1 Introduction

This documents contains the outputs of Algorithm W (adapted to session types) for the examples presented in [1]. The first level of type inference algorithm returns the following information:

- Expression: the input expression plus automatically generated labels l on channel names;
- Type: the type T of the expression;
- Behaviour: the inferred behaviour b of the expression;
- Constraints: the set C of inferred constraints. Notice that session constraints (i.e. $c \sim \eta$) may be duplicated.

2 Examples

2.1 Swap service

This is the result of analysing Example 2.1 from [1].

`Inference complete.`
`Expression:` $\mathsf{let}\, coord = \mathsf{fun}\, coord(z) = \mathsf{let}\, p1 = (\mathsf{acc}\text{-}swap^{l_1}\ ())\ \mathsf{in}\ \mathsf{let}\, v1 = (\mathsf{recv}\ \ p1)\ \mathsf{in}\ \mathsf{let}\, p2 = (\mathsf{acc}\text{-}swap^{l_2}\ ())\ \mathsf{in}\ \mathsf{let}\, v2 = (\mathsf{recv}\ \ p2)\ \mathsf{in}\ \mathsf{let}\, \# = (\mathsf{send}\ \ (p2, v1))\ \mathsf{in}\ \mathsf{let}\, \# = (\mathsf{send}\ \ (p1, v2))\ \mathsf{in}\ (coord\ \ ())\ \mathsf{in}\ \mathsf{let}\, swap = \mathsf{fn}\, x \Rightarrow \mathsf{let}\, p = (\mathsf{req}\text{-}swap^{l_3}\ ())\ \mathsf{in}\ \mathsf{let}\, \# = (\mathsf{send}\ \ (p, x))\ \mathsf{in}\ (\mathsf{recv}\ \ p)\ \mathsf{in}\ \mathsf{let}\, \# = \mathsf{spawn}\, (coord)\ \mathsf{in}\ \mathsf{let}\, \# = \mathsf{spawn}\, (\mathsf{fn}\, z \Rightarrow \mathsf{let}\, \# = (swap\ \ 1)\ \mathsf{in}\ ())\ \mathsf{in}\ \mathsf{spawn}\, (\mathsf{fn}\, z \Rightarrow \mathsf{let}\, \# = (swap\ \ 2)\ \mathsf{in}\ ())$
`Type:` Unit
`Behaviour:` $\tau; \tau; \tau; \mathsf{spawn}\, (\beta_{95}); \tau; \mathsf{spawn}\, (\beta_{112}); \tau; \mathsf{spawn}\, (\beta_{128})$
`Constraints:`

- $\overline{swap^{l_1}} \sim \psi_{99}$
- $\overline{swap^{l_2}} \sim \psi_{100}$
- $swap^{l_3} \sim \psi_{120}$

- $\mathsf{push}(l_1 : \psi_{99}) \subseteq \beta_{102}$
- $\mathsf{push}(l_2 : \psi_{100}) \subseteq \beta_{104}$
- $\mathsf{push}(l_3 : \psi_{120}) \subseteq \beta_{122}$
- $\mathsf{push}(l_3 : \psi_{120}) \subseteq \beta_{138}$
- $\rho_{110}!\alpha_{97} \subseteq \beta_{106}$
- $\rho_{111}!\alpha_{98} \subseteq \beta_{107}$
- $\rho_{125}!\mathsf{Int} \subseteq \beta_{123}$
- $\rho_{141}!\mathsf{Int} \subseteq \beta_{139}$
- $\rho_{108}?\alpha_{97} \subseteq \beta_{103}$
- $\rho_{109}?\alpha_{98} \subseteq \beta_{105}$
- $\rho_{126}?\alpha_{119} \subseteq \beta_{124}$
- $\rho_{142}?\alpha_{135} \subseteq \beta_{140}$
- $\tau;\tau;\beta_{117};\tau \subseteq \beta_{112}$
- $\tau;\tau;\beta_{122};\tau;\tau;\tau;\beta_{123};\tau;\tau;\beta_{124} \subseteq \beta_{117}$
- $\tau;\tau;\beta_{133};\tau \subseteq \beta_{128}$
- $\tau;\tau;\beta_{138};\tau;\tau;\tau;\beta_{139};\tau;\tau;\beta_{140} \subseteq \beta_{133}$
- $\mathsf{rec}_{\beta_{95}}\,(\tau;\tau;\beta_{102};\tau;\tau;\beta_{103};\tau;\tau;\beta_{104};\tau;\tau;\beta_{105};\tau;\tau;\tau;\beta_{106};\tau;\tau;\tau;\beta_{107};\tau;\tau;\beta_{95}) \subseteq \beta_{95}$
- $l_1 \sim \rho_{108}$
- $l_1 \sim \rho_{111}$
- $l_2 \sim \rho_{109}$
- $l_2 \sim \rho_{110}$
- $l_3 \sim \rho_{125}$
- $l_3 \sim \rho_{126}$
- $l_3 \sim \rho_{141}$
- $l_3 \sim \rho_{142}$

2.2 Delegation for Efficiency

This is the result of analysing Example 2.2 from [1].

`Inference complete.`
`Expression:` $\mathsf{let}\, coord = \mathsf{fun}\, coord(z) = \mathsf{let}\, p1 = (\mathsf{acc}\text{-}swap^{l_1}\ ())\ \mathsf{in}\ \mathsf{let}\, \# = \mathsf{sel}\text{-}SWAP\ p1\ \mathsf{in}\ \mathsf{let}\, p2 = (\mathsf{acc}\text{-}swap^{l_2}\ ())\ \mathsf{in}\ \mathsf{let}\, \# = \mathsf{sel}\text{-}LEAD\ p2\ \mathsf{in}\ \mathsf{let}\, \# = (\mathsf{deleg}\ (p2, p1))\ \mathsf{in}\ (coord\ ())\ \mathsf{in}\ \mathsf{let}\, swap = \mathsf{fn}\, x \Rightarrow \mathsf{let}\, p = (\mathsf{req}\text{-}swap^{l_3}\ ())\ \mathsf{in}\ \mathsf{case}\, p\, \{SWAP : \mathsf{let}\, \# = (\mathsf{send}\ (p, x))\ \mathsf{in}\ (\mathsf{recv}\ p), LEAD : \mathsf{let}\, q = (\mathsf{resume}^{l_4}\ p)\ \mathsf{in}\ \mathsf{let}\, y = (\mathsf{recv}\ q)\ \mathsf{in}\ \mathsf{let}\, \# = (\mathsf{send}\ (q, x))\ \mathsf{in}\ y\}\ \mathsf{in}\ \mathsf{let}\, \# = \mathsf{spawn}\, (coord)\ \mathsf{in}\ \mathsf{let}\, \# = \mathsf{spawn}\, (\mathsf{fn}\, z \Rightarrow \mathsf{let}\, \# = (swap\ 1)\ \mathsf{in}\ ())\ \mathsf{in}\ \mathsf{spawn}\, (\mathsf{fn}\, z \Rightarrow \mathsf{let}\, \# = (swap\ 2)\ \mathsf{in}\ ())$
`Type:` Unit
`Behaviour:` $\tau; \tau; \tau; \mathsf{spawn}\, (\beta_{112}); \tau; \mathsf{spawn}\, (\beta_{126}); \tau; \mathsf{spawn}\, (\beta_{157})$
`Constraints:`
Expression:

- $\overline{swap^{l_1}} \sim \psi_{114}$
- $\overline{swap^{l_2}} \sim \psi_{115}$
- $swap^{l_3} \sim \psi_{136}$
- $\mathsf{push}(l_1 : \psi_{114}) \subseteq \beta_{117}$
- $\mathsf{push}(l_2 : \psi_{115}) \subseteq \beta_{119}$
- $\mathsf{push}(l_3 : \psi_{136}) \subseteq \beta_{138}$
- $\mathsf{push}(l_3 : \psi_{136}) \subseteq \beta_{169}$
- $\rho_{145}!\mathsf{Int} \subseteq \beta_{139}$
- $\rho_{149}!\mathsf{Int} \subseteq \beta_{143}$
- $\rho_{176}!\mathsf{Int} \subseteq \beta_{170}$
- $\rho_{180}!\mathsf{Int} \subseteq \beta_{174}$
- $\rho_{146}?\alpha_{154} \subseteq \beta_{140}$
- $\rho_{148}?\alpha_{151} \subseteq \beta_{142}$
- $\rho_{177}?\alpha_{185} \subseteq \beta_{171}$
- $\rho_{179}?\alpha_{182} \subseteq \beta_{173}$
- $\rho_{124}!\rho_{125} \subseteq \beta_{121}$
- $\rho_{147}?l_4 \subseteq \beta_{141}$
- $\rho_{178}?l_4 \subseteq \beta_{172}$
- $\rho_{122}!SWAP \subseteq \beta_{118}$

- $\rho_{123}!LEAD \subseteq \beta_{120}$
- $\tau;\tau;\beta_{131};\tau \subseteq \beta_{126}$
- $\tau;\tau;\beta_{138};\tau;+\{\rho_{144}?SWAP.\tau;\tau;\tau;\beta_{139};\tau;\tau;\beta_{140},\ \rho_{144}?LEAD.\tau;\tau;\beta_{141};\tau;\tau;\beta_{142};\tau;\tau;\tau;\beta_{143};\tau\} \subseteq \beta_{131}$
- $\tau;\tau;\beta_{162};\tau \subseteq \beta_{157}$
- $\tau;\tau;\beta_{169};\tau;+\{\rho_{175}?SWAP.\tau;\tau;\tau;\beta_{170};\tau;\tau;\beta_{171},\ \rho_{175}?LEAD.\tau;\tau;\beta_{172};\tau;\tau;\beta_{173};\tau;\tau;\tau;\beta_{174};\tau\} \subseteq \beta_{162}$
- $\mathsf{rec}_{\beta_{112}}(\tau;\tau;\beta_{117};\tau;\beta_{118};\tau;\tau;\beta_{119};\tau;\beta_{120};\tau;\tau;\tau;\beta_{121};\tau;\tau;\beta_{112}) \subseteq \beta_{112}$
- $l_1 \sim \rho_{122}$
- $l_1 \sim \rho_{125}$
- $l_2 \sim \rho_{123}$
- $l_2 \sim \rho_{124}$
- $l_3 \sim \rho_{144}$
- $l_3 \sim \rho_{145}$
- $l_3 \sim \rho_{146}$
- $l_3 \sim \rho_{147}$
- $l_3 \sim \rho_{175}$
- $l_3 \sim \rho_{176}$
- $l_3 \sim \rho_{177}$
- $l_3 \sim \rho_{178}$
- $l_4 \sim \rho_{148}$
- $l_4 \sim \rho_{149}$
- $l_4 \sim \rho_{179}$
- $l_4 \sim \rho_{180}$

2.3 A Database Library

This is the result of analysing Example 2.2 from [1].

`Inference complete.`
`Expression:` $\mathsf{let}\, process = \mathsf{fn}\, x \Rightarrow x \;\mathsf{in}\; \mathsf{let}\, coord = \mathsf{fun}\, coord(z) = \mathsf{let}\, p = (\mathsf{acc}\text{-}db^{l_1}\;\;()) \;\mathsf{in}$ $\mathsf{let}\, loop = \mathsf{fun}\, loop(z) = \mathsf{case}\, p\, \{QRY : \mathsf{let}\, sql = (\mathsf{recv}\;\; p) \;\mathsf{in}\; \mathsf{let}\, res = (process\;\; sql) \;\mathsf{in}$ $\mathsf{let}\, \# = (\mathsf{send}\;\; (p, res)) \;\mathsf{in}\; (loop\;\; ()), END : ()\} \;\mathsf{in}\; \mathsf{let}\, \# = \mathsf{spawn}\, (coord) \;\mathsf{in}$ $(loop\;\; ()) \;\mathsf{in}\; \mathsf{let}\, \# = \mathsf{spawn}\, (coord) \;\mathsf{in}\; \mathsf{let}\, clientInit = \mathsf{fn}\, z \Rightarrow \mathsf{let}\, con = (\mathsf{req}\text{-}db^{l_2}\;\;()) \;\mathsf{in}$ $\mathsf{let}\, f1 = \mathsf{fn}\, sql \Rightarrow \mathsf{let}\, \# = \mathsf{sel}\text{-}QRY\;\; con \;\mathsf{in}\; \mathsf{let}\, \# = (\mathsf{send}\;\; (con, sql)) \;\mathsf{in}\; (\mathsf{recv}\;\; con) \;\mathsf{in}$ $\mathsf{let}\, f2 = \mathsf{fn}\, z \Rightarrow \mathsf{sel}\text{-}END\;\; con \;\mathsf{in}\; (f1, f2) \;\mathsf{in}\; \mathsf{let}\, dbInit = (clientInit\;\; ()) \;\mathsf{in}\; \mathsf{let}\, qry =$ $(\mathtt{fst}\;\; dbInit) \;\mathsf{in}\; \mathsf{let}\, close = (\mathtt{snd}\;\; dbInit) \;\mathsf{in}\; \mathsf{let}\, \# = (qry\;\; 1) \;\mathsf{in}\; \mathsf{let}\, \# = (qry\;\; 2) \;\mathsf{in}$ $(close\;\; ())$
`Type:` Unit
`Behaviour:` $\tau; \tau; \tau; \mathsf{spawn}\, (\beta_{64}); \tau; \tau; \tau; \beta_{155}; \tau; \tau; \beta_{179}; \tau; \tau; \beta_{205}; \tau; \tau; \beta_{230}; \tau; \tau; \beta_{242}; \tau; \tau; \beta_{254}$
`Constraints:`

- $\overline{db^{l_1}} \sim \psi_{66}$
- $db^{l_2} \sim \psi_{160}$
- $\tau \subseteq \beta_{71}$
- $\tau \subseteq \beta_{179}$
- $\tau \subseteq \beta_{205}$
- $\mathsf{push}(l_1 : \psi_{66}) \subseteq \beta_{67}$
- $\mathsf{push}(l_2 : \psi_{160}) \subseteq \beta_{162}$
- $\rho_{75}!\alpha_{65} \subseteq \beta_{72}$
- $\rho_{219}!\alpha_{209} \subseteq \beta_{213}$
- $\rho_{238}!\mathsf{Int} \subseteq \beta_{234}$
- $\rho_{250}!\mathsf{Int} \subseteq \beta_{246}$
- $\rho_{74}?\alpha_{65} \subseteq \beta_{70}$
- $\rho_{220}?\alpha_{210} \subseteq \beta_{214}$
- $\rho_{239}?\alpha_{232} \subseteq \beta_{235}$
- $\rho_{251}?\alpha_{244} \subseteq \beta_{247}$
- $\rho_{195}!END \subseteq \beta_{189}$
- $\rho_{218}!QRY \subseteq \beta_{212}$
- $\rho_{237}!QRY \subseteq \beta_{233}$

- $\rho_{249}!QRY \subseteq \beta_{245}$
- $\rho_{258}!END \subseteq \beta_{256}$
- $\tau; \beta_{189} \subseteq \beta_{200}$
- $\tau; \beta_{256} \subseteq \beta_{254}$
- $\tau; \beta_{212}; \tau; \tau; \tau; \beta_{213}; \tau; \tau; \beta_{214} \subseteq \beta_{224}$
- $\tau; \beta_{233}; \tau; \tau; \tau; \beta_{234}; \tau; \tau; \beta_{235} \subseteq \beta_{230}$
- $\tau; \beta_{245}; \tau; \tau; \tau; \beta_{246}; \tau; \tau; \beta_{247} \subseteq \beta_{242}$
- $\tau; \tau; \beta_{162}; \tau; \tau; \tau; \tau \subseteq \beta_{155}$
- $\mathsf{rec}_{\beta_{69}} (\tau; +\{\rho_{73}?QRY.\tau; \tau; \beta_{70}; \tau; \tau; \beta_{71}; \tau; \tau; \tau; \beta_{72}; \tau; \tau; \beta_{69},\ \rho_{73}?END.\tau\}) \subseteq \beta_{69}$
- $\mathsf{rec}_{\beta_{64}} (\tau; \tau; \beta_{67}; \tau; \tau; \mathsf{spawn}\,(\beta_{64}); \tau; \tau; \beta_{69}) \subseteq \beta_{64}$
- $l_1 \sim \rho_{73}$
- $l_1 \sim \rho_{74}$
- $l_1 \sim \rho_{75}$
- $l_2 \sim \rho_{195}$
- $l_2 \sim \rho_{218}$
- $l_2 \sim \rho_{219}$
- $l_2 \sim \rho_{220}$
- $l_2 \sim \rho_{237}$
- $l_2 \sim \rho_{238}$
- $l_2 \sim \rho_{239}$
- $l_2 \sim \rho_{249}$
- $l_2 \sim \rho_{250}$
- $l_2 \sim \rho_{251}$
- $l_2 \sim \rho_{258}$

References

[1] C. Spaccasassi, V. Koutavas, *Type-Based Analysis for Session Inference* 2016, http://arxiv.org/abs/1510.03929.

3 Session sub-typing system in Gay & Hole

$$\frac{C \vdash T_2 <: T_1 \qquad C \vdash \eta_1 <: \eta_2}{C \vdash !T_1.\eta_1 <: !T_2.\eta_2}$$
$$\frac{C \vdash T_1 <: T_2 \qquad C \vdash \eta_1 <: \eta_2}{C \vdash ?T_1.\eta_1 <: ?T_2.\eta_2}$$
$$\frac{m \leq n \qquad \forall i \leq n.C \vdash \eta_i <: \eta_j}{C \vdash \underset{i \in [1,m]}{\oplus}\{L_i : \eta_i\} <: \underset{j \in [1,n]}{\oplus}\{L_j : \eta_j\}}$$
$$\frac{I_1 \subseteq J_1, J_1 \cup J_2 \subseteq I_1 \cup I_2, \forall (i \in J_1 \cup J_2).\ C \vdash \eta_i <: \eta_i'}{C \vdash \&\{L_i : \eta_i'\}_{i \in (I_1, I_2)} <: \&\{L_i : \eta_i'\}_{i \in (J_2, J_2)}}$$

3.1 Bounded quantification - removing C constraints

Types: $XTopT \overset{T}{\to} b\forall X <: T.T\forall X <: \eta.T$

$\mathsf{req}\text{-} : \forall \chi.\forall (X <: \eta).\eta\ \mathsf{chan} \overset{\mathsf{push}(\chi:X)}{\to} \mathsf{Ses}^{\chi}$

$\mathsf{send} : \forall \chi.\forall (X <: T).(X, \mathsf{Ses}^{\chi}) \overset{\chi!T}{\to} \mathsf{Unit}$

$\mathsf{recv}\ : \forall \chi.\forall (X <: T).\mathsf{Ses}^{\chi} \overset{\chi?X}{\to} T$

Rule Ins is type application.
Rule Gen becomes:

$$\frac{\Gamma, X <: U \vdash t : (T, b)}{\Gamma \vdash \lambda X <: U.t : (\forall X <: U.T, b)}$$

$\forall X <: U.T$ is well-formed (see N.&N.)
$X \notin FV(b, \Gamma)$