Theory of Computation (CIT3006)

Tutorial Sheet (no. 1)

1. Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.
   a. {-2, 4, -8, 16, ...}– the number in -2 power of k (-2^k), where k∈N
   b. {1, 1, 2, 3, 5, ...}– fibonacci numbers
   c. {1,3,5,7, ... }– odd numbers
   d. {n|n = 2m for some m in N}– even natural numbers
   e. {w|w is a string of 0's and 1's and w equals the reverse of w} – palindrome
   f. {n| n is an integer and n = n + 1}– sum of arithmetic series

2. Write formal descriptions of the following sets.
   a. The set containing the numbers 1, 10 and 100.- A = {n | n =10^m for m∈{0,1,2}}
   b. The set containing all integers that are greater than 5.- A = {n | n ∈Z and n > 5}
   c. The set containing all natural numbers that are less than 5.- A = {n | n∈N and n < 5}
   d. The set containing all natuaral numbers less than -2.- A = {ε}

3. Let A be the set {x, y, z} and B be the set {x, y}
   a. Is A a subset of B? FALSE
   b. Is B a subset of A? TRUE
   c. What is A ∪B? A
   d. What is A ∩ B? B
   e. What is A × B?{(x,x), (x,y), (y,x), (y, y), (z, x), (z, y)}
   f. What is the power set of B?{∅, (x), (y), (x,y)}

4. Let X be the set {1, 2, 3, 4, 5} and Y be the set {6, 7, 8, 9, 10}. The unary function $f: X \longrightarrow Y$ and the binary function $g: X \times Y \longrightarrow Y$ are described in the followingtables.

n	f(n)
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

   a. What is the value of f (2)? 7
   b. What are the range and domain of f? R={6,7} D={1,2,3,4,5}
   c. What is the value of g(2, 10)? 6
   d. What are the range and domain of g? R={6,7,8,9,10} D={(1,6), (1,7), (1,8), (1,9), (1,10), …, (5,10)}
   e. What is the value of g(4, f(4))? 8

5. Let A be the set {0, 1, 2, 3} let two binary functions f and g, using $A \times A$ as their domain, be described as follows:
   a. $f: addition$
   b. $g: addition\ modulu$ 4

   What is the closure property for each operation?

f	0	1	2	3
0	0	1	2	3
1	1	2	3	4
2	2	3	4	5
3	3	4	5	6

Not closed

f	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

closed

6. For each part, give a relations that satises the condition.
   a. Reflexive and symmetric but not transitive

      iRj where i;jare people and they share at least one biological parent

      - reflexive, everyone has their own parents forparents.
      - symmetric, if personxshares a parent with persony, then personyshares that same parent with personx
      - not transitive. Assume a personmwhoshares exactly one parent (a mother) with persongand exactly oneparent with persons(a father).g's father is nots's father, andg'smother is nots's mother. Hence, whilegRmandmRsare true,gRsis false.

   b. Reflexive and transitive but not symmetric

      iRj where i;j∈Nandi–j ≤ 0

      - reflexive, because x-x = 0.
      - transitive, because ifxRythenx≥yand ifyRztheny≥z,thusxRzbecausex≥y≥z.
      - not symmetric, because 5-3≥0, but3-5<0.

   c. Symmetric and transitive but not reflexive

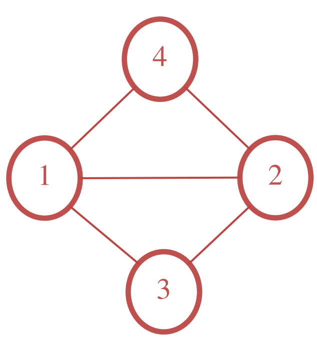
      iRj where i;j∈Zandi*j >0 **Set={0,1}**

      symmetric, because multiplication is symmetric.

      transitive, as well, since ifxRythen neitherxnoryis zero andifyRz, then neitherynorzis zero, thusxRzbecause neitherxnorzis zero.

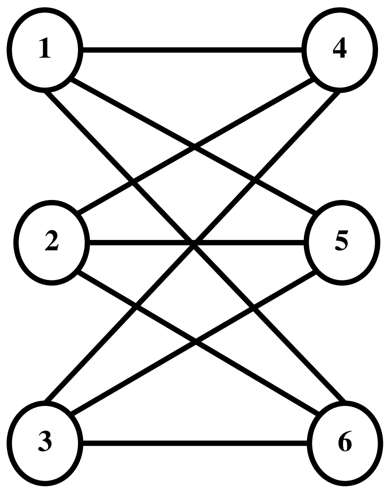
      not reflexive, because 0*0 = 0

7. Consider the undirected graph G = (V,E) where V, the set of nodes, is {1, 2, 3, 4} and E, the set of edges, is {{1,2}, {2,3}, {1,3}, {2,4}, {1,4}}. Draw the graph G. What is the degree of node 1? of node 3? Indicate a path from node 3 to node 4 on your drawing of G.



node1 degree=3, node3 degree=2, 3-2-4 or 3,1,4

8. Write a formal description of the following graph



$G = \{\{1,2,3,4,5,6\}, \{\{1,4\}, \{1,5\}, \{1,6\}, \{2,4\}, \{2,5\}, \{2,6\}, \{3,4\}, \{3,5\}, \{3,6\}\}\}$

9. **Find the error in the following proof that 2 = 1**.
   Consider the equation a=b.
   a. Multiply both sides by ato obtain $\mathbf{a^2=ab}$.
   b. Subtract b^2 from both sides to get $\mathbf{a^2-b^2=ab-b^2}$.
   c. Now factor each side, **(a+b)(a–b) = b(a–b)**, and
   d. Divide each side by (a–b), to get **a+b=b**.
   e. Finally, let a and b equal 1, which shows that 2 = 1.

   Dividing by (a – b) is illegal, because a = b hence a – b = 0 and division by 0 is undefined

10. **Find the error in the following proof that all horses are the same colour.**
    Claim: In any set of h horses, all horses are the same colour.

    Basis: For h = 1. Then all horses are clearly of the same colour.
    Induction step: For **k ≥ 1** assume that the claim is true for **h = k** and prove that it is true for **h = k+1**.
    - Take any set H of k+1 horses. Remove one horse from the set to obtain a new set H1 with k horses. Hypothesis, all the horses in H1 are of the same colour.
    - Now replace the removed horse and remove a different one to obtain set H2. By the same argument, all horses in H2 are the same colour.
    - Therefore, all horses in H must be of the same colour, and the proof is complete.

    The error is in the two statements “all horses in H1 are the same colour” and “all horses in H2 are the same colour”.
    For our choice of H1 and H2, imply that all the horses in $H = H1 \cup H2$ are the same colour only if $H1 \cap H2$ is nonempty, but for H contains exactly two horses (h = 2). Then H1 and H2 each will consist of exactly one of the two horses. Obviously the single horse in H1 has its own colour and the single horse in H2 has its own colour but this gives us no means to conclude that they both have the same colour.