A **random vector** is a list of mathematical variables each of whose value is unknown, either because the value has not yet occurred or because there is imperfect knowledge of its value. The individual variables in a random vector are grouped together because they are all part of a single mathematical system — often they represent different properties of an individual statistical unit. For example, while a given person has a specific age, height and weight, the representation of these features of *an unspecified person* from within a group would be a random vector. Normally each element of a random vector is a real number.

A **probability space** or a **probability triple** is a mathematical construct that models a real-world process (or "experiment") consisting of states that occur randomly. A probability space is constructed with a specific kind of situation or experiment in mind. One proposes that each time a situation of that kind arises, the set of possible outcomes is the same and the probabilities are also the same.

A probability space consists of three parts:

1. A sample space, , which is the set of all possible outcomes.
2. A set of events , where each event is a set containing zero or more outcomes.
3. The assignment of probabilities to the events; that is, a function from events to probabilities.

Convergence, in mathematics, property (exhibited by certain infinite series and functions) of approaching a limit more and more closely as an argument (variable) of the function increases or decreases or as the number of terms of the series increases. For example, the function y = 1/x converges to zero as x increases.

Convergence of Random Variables. ... Certain processes, distributions and events can result in **convergence**— which basically mean the values will get closer and closer together. Random variables can **converge** on a single number. They may not be exactly that number, but they come very, very close.

The convergence of sequences of random variables to some limit random variable is an important concept in probability theory, and its applications to statistics and stochastic processes. The same concepts are known in more general mathematics as **stochastic convergence** and they formalize the idea that a sequence of essentially random or unpredictable events can sometimes be expected to settle down into a behaviour that is essentially unchanging when items far enough into the sequence are studied. The different possible notions of convergence relate to how such a behaviour can be characterised: two readily understood behaviours are that the sequence eventually takes a constant value, and that values in the sequence continue to change but can be described by an unchanging probability distribution.

Different Types of Convergence for Sequences of Random Variables

Here, we would like to provide definitions of different types of convergence and discuss how they are related. Consider a sequence of random variables X_1

, X_2, X_3, $\cdots$, i.e, $\{X_n, n \in \mathbb{N}\}$. This sequence might "converge" to a random variable X

. There are four types of convergence that we will discuss in this section:

1. Convergence in distribution,
2. Convergence in probability,
3. Convergence in mean,
4. Almost sure convergence.

These are all different kinds of convergence. A sequence might converge in one sense but not another. Some of these convergence types are "stronger" than others and some are "weaker." By this, we mean the following: If Type A convergence is stronger than Type B convergence, it means that Type A convergence implies Type B convergence. Figure 7.4 summarizes how these types of convergence are related. In this figure, the stronger types of convergence are on top and, as we move to the bottom, the convergence becomes weaker. For example, using the figure, we conclude that if a sequence of random variables converges *in probability* to a random variable X

, then the sequence converges *in distribution* to X

as well.

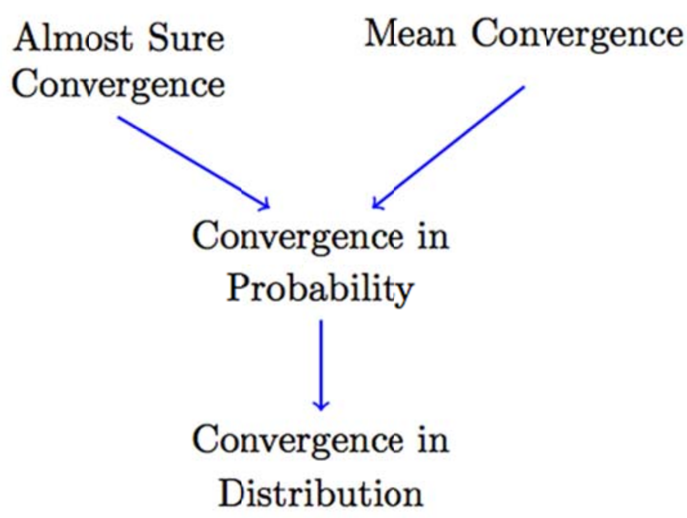

Fig. - Relations between different types of convergence