Convergence
Divergence

infinity is not Convergence

- probability
- distribution
- mean

Renewal process — problem

Markov chain ~~s~~ states

Non-Homogeneous problem

1 Random vector

probability space denoted by $P(\Omega, E, P)$

Ω — Sample Space

E — Event Space

P — prob space

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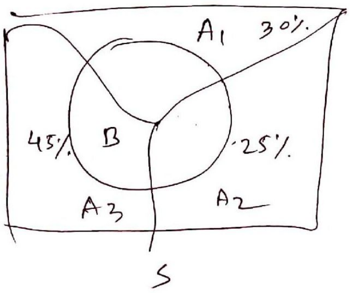
prior probabilitis (BRK ← William Hine)

Bayes theorem states that posterior probability is propotional to prior probabilitis multiplied by its likelihood.

$p(A_i/B) = ?$

Bayes theorem



$P(A_1) = .3$

$P(A_2) = .25$

$P(A_3) = .45$

$B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B)$

$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$

$= P(B/A_1)P(A_1) + P(B/A_2)P(A_2) + P(B/A_3)P(A_3)$

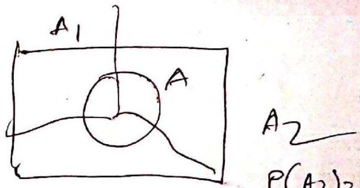
$= \sum_{i=1}^{3} P(B/A_i)P(A_i)$

$\therefore P(A_i/B) = \dfrac{P(B/A_i)P(A_i)}{\sum_{i=1}^{3} P(B/A_i)P(A_i)}$

Ex- 1.40

~~#~~ Bayes theorem

$P(A_1) = 0.20$



$P(A_2) = .30$

$P(A_3) = .5$

$P(A) = P(A_1 \cap A) + P(A_2 \cap A) + P(A_3 \cap A)$

Stochastic process, types with graphical presentation.

Chapman Kolmogorov equation

~~probability~~ Construction transition Matrix

Limiting probability

Mean recurrence time

Markov chain → def definition
- transient
- recurrence

Convergent, divergent

exponential distribution

Definition

* Independent, stationary process

* Markov process, chain, $\boxed{\text{white noise}}$, normal distribution

~~strict~~ sense stationary

weakly stationary / wide sense

Covariance stationary

probability, probability space

sample space

range, parameter space

Random variable

Conditional probability

Additional probability

first tutorial

$$X(t) = A\cos(\omega_0 t + \theta)$$

uniformly distributed

$$\theta \sim U(0, 2\pi)$$

$$f(\theta) = \frac{1}{2\pi - 0} = \frac{1}{2\pi}$$

$$0 \le \theta \le 2\pi$$

$$E(X(t)) = \frac{A}{2\pi}\int \cos(\omega_0 t + \theta)\, d\theta$$

$$E(X(t)^2) = \frac{A}{2\pi}\int \cos^2(\omega_0 t + \theta)\, d\theta$$

$$V(x(t)) = E(x(t)^2) - (E x(t))^2$$

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probability & stochastic process

Date: 26-09-18

* Destructive → uses given information/Rules to reach conclusion.
* Inductive → Involves making generalizations based upon behavior observed in specific cases.

Experiment → gives outcome
Control condition
- Deterministic
- Nondeterministic/ Random/ stochastic

probability is the measure of chance.

* Sample space
  All possible outcome of an experiment
* Sampling (subset)
  possible outcome of an experiment.

Or, either, neither ~~co~~ atleast — Union

Either/or → Union
Neither/nor → Intersection

St probability & Stochastic process

— Date: 05/11/18

Statistics [~~Destructive~~ Deductive / Inductive

Destructive ———— Inductive

↑

probability

~~or~~ Conditional Mean

probability is measure of change
(Mathematical)

Set — Collection well defined distinct elements

Sample space — is the collection of all ~~outcome~~ el of an experiment.

Ex. ~~Roll a die~~ dice Rolling experiment

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$ $C = \{1, 3, 5\}$

$B = \{2, 3, 5\}$

Events: is the collection of possible outcomes of an experiment.

Mutually exclusive events: No common points

$P(A) = 3/6$ $\quad P(B) = 3/6$ $\quad P(C) = 3/6$

$P(A \cup C) = P\{1, 2, 3, 4, 5, 6\}$

$= P(A) + P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Addition laws probabi

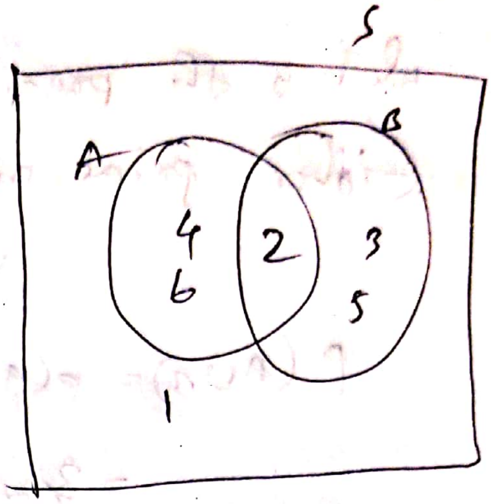
Introduction to probability — Roc

probability & stochastic process

Date: 12/12/18

Union

Intersection



Exactly, Atleast, At most

Either, neither, ~~or~~ not

Exactly $A = \{A \cap \bar{B}\}$

Exactly $B = \{\bar{A} \cap B\}$

Exactly one of 2 events = Exactly A or Exactly B

$= \{(A \cap \bar{B}) \cup (\bar{A} \cap B)\}$

$$\boxed{\begin{array}{l} P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ \quad P(A \cap B) = 0 \text{ if } A \text{ and } B \text{ are mutually exclusive} \end{array}}$$

$P(\{(A \cap \bar{B}) \cup (\bar{A} \cap B)\}) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$

Because they are mutually exclusive $(A \cap \bar{B})$ & $(\bar{A} \cap B)$

$= 3/6 + 3/6$

what is the prob. of
Either prime number or even number will occure ?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 3/6 + 3/6 - 1/6 = 5/6$

Conditional probability

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{3/6} = \frac{1}{3}$

given

Independent:

Two events are said to be independent if
~~probability~~ $P(A \cap B) = P(A)\, P(B)$

Mutually exclusive: they have no common point

If A and B are indepeadent then Conditional probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,\cancel{P(B)}}{\cancel{P(B)}}$$

$$= P(A)$$

N.B

when Computing 'at least' & 'at most' probabilitis, it is necessary to consider in addition to the given probability

* all probabilitis larger than the given probability (at least)
* all probabilitis smaller than the given probability (at most)

probality and stochastic process

10/12/18

Book
* probability & statistics with Reliability, Queuing & CS Application — Trivedi
* Introduction to probability model — Sheldon M. Ross

Stochastic process
- time [discrete / continuous
- state [discrete / continuous

graphical presentation

time can be discrete/contin.

page- 270

* Markov chain
probability of future state depends on the present state not on the past states.

$$\{ X(t), t \in T \}$$

$$P(X_{n+1} = j / X_n = i, X_{n-1} = i-1 \;---)$$

$$P_{ij} = P(X_{n+1} = j / X_n = i)$$

(Transition prob.)

* Limiting probability

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States → 0 1 0 1 0 0 1 1 0 0 1 1 2 1 0 2 1 1 1 2 2 0

Transition Matrix

Future →

Present ↓

$$T = \begin{array}{c|ccc|l} & 0 & 1 & 2 & \\ \hline 0 & 2 & 4 & 1 & \rightarrow 7 \\ 1 & 4 & 4 & 2 & \rightarrow 10 \\ 2 & 1 & 2 & 1 & \rightarrow 4 \end{array}$$

$$P = \begin{bmatrix} 2/7 & 4/7 & 1/7 \\ 4/10 & 4/10 & 2/10 \\ 1/4 & 2/4 & 1/4 \end{bmatrix}$$

$P_{00}^{(1)} = 2/7$

* recurrent

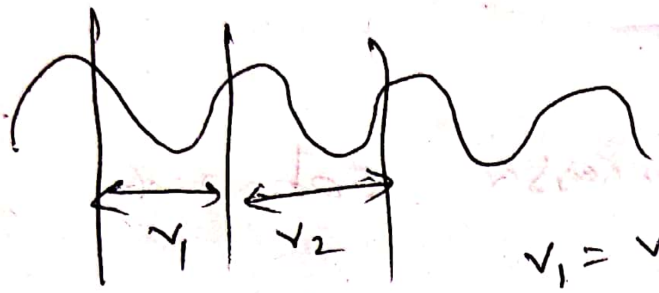
* state space, state, time

mean/variance $\xrightarrow{\text{depends}}$ on time t → Nonstationary

Independent → Stationary Series

Covariace stationary $\longrightarrow$ best signal

expected signal



$v_1 = v_2$

strict sense stationary

Renewal process

$\{X_n, n \geq 0\}$ $\quad\quad$ $\{X(t), t \in T\}$

Compilation of n steps tradition metrix

$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0.6 & .4 \\ .4 & .6 \end{pmatrix} \end{array}$$

$P \cdot P = P^2$

$P \cdot P \cdot P = P^3$

$P_{00}^1 = 0.6$ $\quad\quad$ $P_{00}^2 =$ $\quad\quad$ $P_{00}^3 =$

$P(X_2 = c / X_1 = c) = P_{cc} = 0.5$

stochastic & probability process

14.01.19

RVS book

Example

Problem 4.3

			0	1	2
			C	S	G
0	C	(	0.5	.4	.1
1	S		0.3	.4.	.3
2	G		.2	.3	.5)

$\{X_n, n = 0, 1, 2 \dots\}$

$X_1 = S, C, G$

$P(X_3 = c / X_1 = c) = P_{cc}^2 = (.5 \; .4 \; .1)\begin{pmatrix} .5 \\ .3 \\ .2 \end{pmatrix}$

$= 0.25 + .12 + 0.02$

$= 0.39$

probability of being cheerful ~~to~~day after twomorrow if the person is cheerful today

* Communicating

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & & & \\ 1 & & & \\ 2 & & & \end{array} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} T & 0 & 0 \\ 0 & T & T \\ 0 & 0 & T \end{pmatrix} \begin{pmatrix} T & 0 & 0 \\ 0 & T & T \\ 0 & 0 & T \end{pmatrix}$$

Not Communicating $= \begin{pmatrix} T & 0 & 0 \\ 0 & T & T \\ 0 & 0 & T \end{pmatrix}$

<u>4.15, 4.14</u>

if possible to transit from one stage all other stats then marcov chain is irreducible

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 2 & 0 & \frac{1}{3} & \frac{2}{3} \end{array}$$

$0 \to 1 \to 2 \to 1$

$\to$ irreducible

* Random walk Model

* Gambling

Limiting probability

$$\lim_{n \to \infty} P_{ij}^{n} = \pi_j$$

if all the values of column is/are ~~equal~~ similar

after x multiplication then, $n = x$

Example 4.22

$$\boxed{P' \cdot \pi = \pi} \qquad \sum \pi_j = 1$$

if we have solution then p. limit is exist.

$$P = \begin{vmatrix} 0.45 & 0.48 & 0.07 \\ 0.05 & .70 & .25 \\ .01 & .50 & .49 \end{vmatrix}$$

$$\boxed{P' \cdot \pi = \pi}$$

$$\boxed{\sum \pi_j = 1}$$

$\pi_0 + \pi_1 + \pi_2 = 1$

$\pi_0 = (1 - \pi_1 - \pi_2)$

Probability & stochastic process

11/02/19

Chap -5

Exponential distn

$X \sim \exp(\lambda)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Toss a coin 3 time

$2^3 = 8 \rightarrow \{\underbrace{HHH}_{3}, \underbrace{HHT, HTH, THH}_{2}, \underbrace{TTH, THT, HTT}_{3}, \underbrace{TTT}_{0}\}$

X	0	1	2	3
$f(x)$	$1/8$	$3/8$	$3/8$	$1/8$

$E(x) = \sum x p(x)$

$= 0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right)$

$= \frac{3+6+3}{8} = 1.5$

$V(X) = E(x^2) - (E(x))^2$

$E(x^2) = \sum x^2 p(x)$

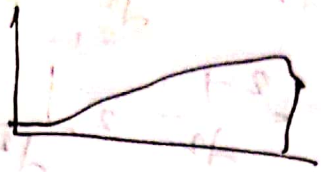
$= 0^2\left(\frac{1}{8}\right) + 1^2\left(\frac{3}{8}\right) + 2^2\left(\frac{3}{8}\right) + 3^2\left(\frac{1}{8}\right)$

$=$

n_e

Binomial distribution

$f(x) = 1$



$E(x) = \int x \, f(x) \, dx$

$F(x) = P(X \leq x)$ * failure ~~Survival~~ function

$P(X > x)$ ~~* Reliable~~ Survival function

$= \cancel{\int_0^{\infty} x \left(\lambda e^{-\lambda x}\right) dx}$

$= \lambda \int_0^{\infty} x e^{-\lambda x} \, dx$

$F(x) = P(X \leq x)$ Failure function

$= \int_0^{x} \lambda e^{-\lambda x} \, dx$

$\left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^{\infty}$

$= \left[-\frac{1}{\cancel{\lambda}} \cancel{\lambda} e^{-\lambda x}\right]_0^{x}$

$= \frac{1}{\lambda}$

$= 1 - e^{-\lambda x}$

$V(x) = E(X^2) - (E(x))^2$

Survival fun.

$P(X > x) = 1 - P(X \leq x)$

$E(X^2) = \int_0^{\infty} x^2 \lambda e^{-\lambda x} \, dx$

$= 1 - (1 - e^{-\lambda x})$

$= \frac{2}{\lambda^2}$

$= 1 - 1 + e^{-\lambda x}$

$= e^{-\lambda x}$

$$v(x) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$= \frac{2-1}{\lambda^2} = \frac{1}{\lambda^2}$$

* Math problem

Moment generating function

$$M_X(t) = E(e^{tx})$$

$$= \int_0^\alpha e^{tx} \lambda e^{-\lambda x} dx$$

$$= \int_0^\alpha \lambda e^{-(\lambda - t)x} dx$$

$$= \lambda \int_0^\alpha e^{-(\lambda - t)x} dx$$

$$= \left[\lambda \frac{-1 e^{-(\lambda - t)}}{\lambda - t} \right]_0^\alpha$$

$$= \left[\frac{-\lambda e^{-(\lambda - t)}}{(\lambda - t)} \right]_0^\alpha = \frac{\lambda}{\lambda - t}$$

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$$M'_x(t)\Big]_{t=0} = \frac{\lambda}{(\lambda-t)^2}\Big]_{t=0}$$

$$= \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}$$

$$M''_x(t)\Big]_{t=0} = \frac{2}{\lambda^2} = E(x^2)$$

(Edition - 11)
Pages → 288

$X_1 \cdot \text{---} \; x_n$

$x_i \sim \mathcal{E}(\mu_i)$

$$P_r\left(\min(X_1, \ldots, X_n) > x\right)$$

$$= \exp\left\{-\sum_{i=1}^{n} \mu_i\right\} x$$

$$= e^{-\sum \mu_i x}$$

$$f\left(\min(x_1 \ldots x_n)\right) = \left(\sum \mu_i\right) e^{-\sum \mu_i x}$$

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$$Pr\left(\min(x_1, \dots x_n) > x\right)$$

$$= p(x_1 > x, x_2 > x, \dots x_n > x)$$

$$= e^{-\mu_1 x} e^{-\mu_2 x} \dots e^{-\mu_n x}$$

$$= \sum_{i=1}^{n} e^{-\Sigma \mu_i x}$$

$$P(\min(x_1 \dots x_n) = x) = \left(\Sigma \mu_i\right) e^{-\Sigma \mu_i x}$$

$x > 0$

probability & stochastic process

Date - 04.03.19

Chap-5 RBS

Counting process

$N(t), t \geq 0$

- Independent increment

$\underbrace{t_1 - -}\ \underbrace{t_2 -}\ \underbrace{- t_3}$

- Stationary Counting process

- $f(\cdot) - - o(h)$

$$\lim_{h \to 0} \frac{f(h)}{h} = 0$$

$f(x) = x^2$ is $o(h)$

$$\lim_{h \to 0} \frac{f(h)}{h} = \frac{h^2}{h} = 0$$

pro

$$\lim_{h \to 0} \frac{P(t < X < t+h)}{f(t)}$$

$X \longrightarrow$ Random variable

$\lambda(t)$ — failure rate

$$P(t < x < t+h / x > t) \approx \lambda(t) h$$

$$= \lambda(t) h + o(h)$$

<u>poission process</u>

from book

Definition - 5.2

$\lambda h \rightarrow$ rate in specify interval

(need add wise defi - 5.2)

The number of events in any interval of length t is poission distributed with mean λ, that is, for all $s, t \geq 0$

$$\underbrace{P\{N(t+s) - N(s) = n\}} = e^{-\lambda t} \frac{(\lambda t)^n}{n!} \quad n = 0, 1, \ldots$$

dependent on t,

independent on s

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* If arrival time is distributes, what is the distribution of Inter arrival time?

$\boxed{\text{poisson process}}$

Slide - 11, 12 — 23, 25 (Erlang distribution), 52 (Non homogeneous process)

~~Example~~ 5.13 Suppose the people immigrate into a

~~Ex~~ 5.24 Let 8.am = 0 (non Homogen)

So, t =

probability & stochastic process

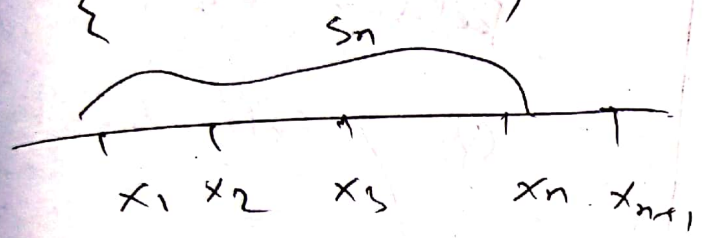
er 7

Definition 7.1

$N(t) \geq n \Leftrightarrow S_n \leq t$

$N(t) = n$

$\{ N(t) \cdot t \geq 0 \}$



7.2 Distribution of N(t)

Number of trial required to get 1st success is called geometric distr.

$P(X=x) =$

$x \rightarrow$ Number of attempt $(1-p)^{x-1} p$

$x = 1, 2, \ldots$

Another expression

$P(N(t)$

proposition 7.1   Law of large numbers

$$\frac{N(t)}{t} \rightarrow \frac{1}{\mu} \text{ as } t \rightarrow \infty$$

Example - 7.4

$E(N(t)) = \mu$

$E(X_i) = \mu$

$S_n = \frac{\Sigma X_n}{n}$

poisson
Counting
~~Re~~newal process