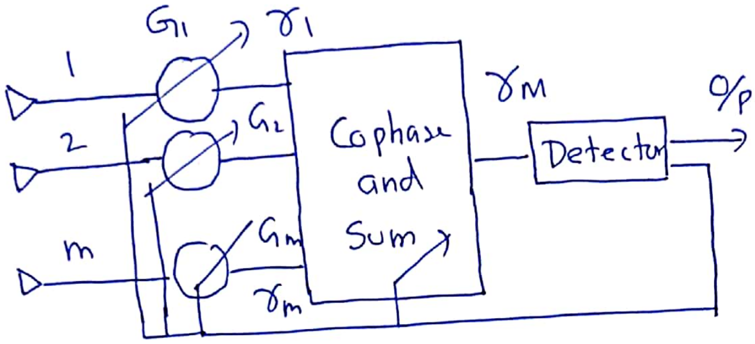
Maximul Ratio Combining Diversity



m branches are weighte
according to individual $\frac{S}{N}$

The SNRi are summed
to provide every signal

Coming from each branch is under co-phased conditin

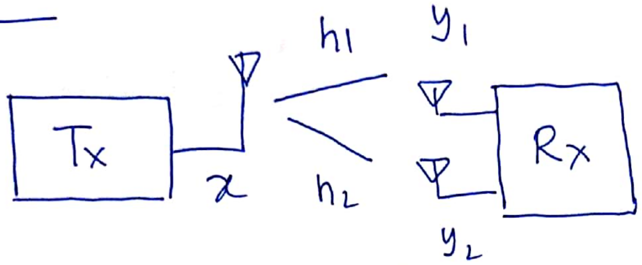
$$\gamma_{MRC} = \sum_i SNR_i = \sum_i \gamma_i$$

Hense even if one signal is below threshold, th
summation will be beyond threshold

Equal Gain Combining Diversity

- It is not always convenient to provide weigh
capacity for MRC. In such cases branches are given
equal unity gain
- Signals are co-phased
- Possibility of achiveng an acceptable γ_0
from a number of unacceptable inpuls
- Performance is marginally poor.

MRC Maximul Ratio combining



$$h_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j$$

$$h_2 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j$$

$$y_1 = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j\right) x + n_1 \quad \text{— ①}$$

$$y_2 = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j\right) x + n_2 \quad \text{— ②}$$

Ave noise power $E\left[|n_1|^2\right] = E\left[|n_2^2|\right] = \sigma^2 = \frac{1}{2}$ — ③

Eqns ① and ② can be written as —

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\bar{y} = \bar{h} x + \bar{n} \quad \text{—— ④}$$

Optimal MRC vector $= \dfrac{\bar{h}}{\|\bar{h}\|} = \bar{w}$ — ⑤

where $\|\bar{h}\| = \sqrt{|h_1|^2 + |h_2|^2} = \sqrt{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}}$

$= \sqrt{2}$ — ⑥

Using ⑤ and ⑥ —

Thus $\bar{w} = \dfrac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix}$

$$\text{SNR} = \frac{|\bar{w}^H \bar{h}|^2 P}{\sigma^2 \|\bar{w}\|^2} \quad \text{—— ⑦}$$

$$\bar{w}^H \bar{h} = \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \quad \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j\right] \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right] = \sqrt{2} \quad \text{— ⑧}$$

Thus $|\bar{w}^H \bar{h}|^2 = 2$

$$\|\bar{w}\| = |w_1|^2 + |w_2|^2 = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right) = 1 \quad (9)$$

Using (7) - (8) we have

$$SNR = \frac{P\,|w^H \bar{h}|^2}{\sigma^2\,\|\bar{w}\|^2} = \frac{P \cdot 2}{1/2 \cdot 1} = 4P$$

If there are L received antennas then

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \\ n_L \end{bmatrix} \Rightarrow \bar{y} = \bar{h}x + \bar{n}$$

Noise power $E\left[|n_i|^2\right] = \sigma^2$

$E\left[n_i n_j\right] = 0 \quad \forall i \neq j$

$$\bar{y} = w_1^* y_1 + w_2^* y_2 + \cdots + w_L^* y_L$$

$$\bar{w}^H \bar{y} = w^H(\bar{h}x + \bar{n})$$

$$= \underbrace{w^H \bar{h} x}_{\text{Signal}} + \underbrace{\bar{w}^H \bar{n}}_{\text{Noise}}$$

For max SNR

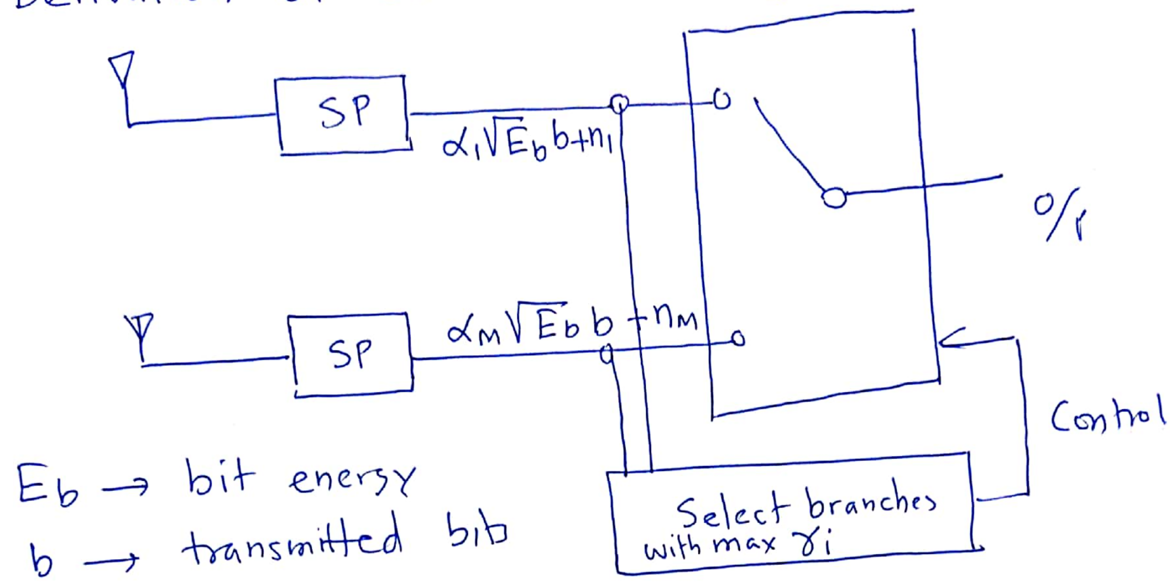
$$\bar{w} = \frac{\bar{h}}{\|\bar{h}\|} = \frac{1}{\|h\|}\begin{bmatrix} h_1 \\ h_2 \\ \\ h_L \end{bmatrix}$$

$$\|\bar{h}\| = \sqrt{|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2}$$

$$\text{SNR with MRC} = \frac{\|\bar{h}\|^2 P}{\sigma^2}$$

$$= \frac{P\left(|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2\right)}{\sigma^2}$$

Derivation of Selection Diversity



$E_b \rightarrow$ bit energy

$b \rightarrow$ transmitted bit

$\alpha_i \rightarrow$ Fading coefficient

$n_i \longrightarrow$ AWGN of ith Branch

Average SNR $= \Gamma = \dfrac{E_b}{N_0}\overline{\alpha^2}$

$N_0 \longrightarrow$ noise power spectral density

If Rayleight fading is considered then PDF of γ_i is

$$p(\gamma_i) = \frac{1}{\Gamma} e^{-\gamma_i/\Gamma}$$

Outage occurs when

$$P_r[\gamma_i \leq \gamma] = \int_0^{\gamma} p(\gamma_i)\, d\gamma_i$$

$$= \int_0^{\gamma} \frac{1}{\Gamma} e^{-\frac{\gamma_i}{\Gamma}} = 1 - e^{-\gamma/\Gamma}$$

For M independent branches

$$P_r[\gamma_1 \gamma_2 \ldots \gamma_M \leq \gamma] = \left(1 - e^{-\gamma_i/\Gamma}\right)^M = P_M(\gamma)$$

If one branch is sucessful then

$$P_r[\gamma_i > \gamma] = 1 - \left(1 - e^{-\gamma/\Gamma}\right)^M$$

The mean SNR

$$\bar{\gamma} = \int_0^\infty \gamma P_M(\gamma)\, d\gamma = \Gamma \int_0^\infty M x \left(1 - e^{-x}\right)^{M-1} e^{-x}\, dx$$

where $x = \frac{\gamma}{\Gamma}$

Mean SNR / Ave SNR $= \frac{\bar{\gamma}}{\Gamma} \sum_{k=1}^{M} \frac{1}{k}$

---

4 diversity branch where each branch receives an independend Rayleigh Fading Signal. If Γ ~~γ_{ave}~~ $= 20\,dB$ Determine the probability that the SNR will drop below 10 dB. Compare this with the case of a Single Rx without diversity

---

$M = 4 \qquad \Gamma = 20\,dB \qquad \gamma = 10\,dB$

$$P(\gamma_i < \gamma) = \left(1 - e^{-\gamma/\Gamma}\right)^M$$

$$= \left(1 - e^{-0.1}\right)^4$$

$$= 8.2 \times 10$$

Consider a single branch Rayleigh fading signal has a 20% chance of being 6 dB below some mean SNR threshold

(a) Determine the mean of the Rayleigh fading signal as referenced to threshold

(b) Find the likelyhood that a 2/3/4 branch selection diversity receiver will be 6 dB below the mean threshold

$$\Pr[\gamma_i \le \gamma] = (1-e^{-x}) = 0.2$$

$$\left(1-e^{-\gamma/\Gamma}\right)^M = 0.2$$

$-6\,dB = 1/4$ thus $\gamma' = \gamma/4$

$$\left(1-e^{-\gamma/4\Gamma}\right) = 0.2$$

$$\Rightarrow e^{-\gamma/4\Gamma} = 0.8$$

$$\Rightarrow \gamma/4\Gamma = -\ln(0.8)$$

$$\Rightarrow \Gamma/\gamma = \frac{1}{-4\ln(0.8)} = 1.12$$

$$\left(1-e^{-1/1.12}\right)^{2/3/4}$$