AC Fundamentals: Resistance, Reactance and Impedance

Review of R, X, and Z:

- Before exploring the effects of resistors, inductors, and capacitors connected together in the same AC circuits, let us briefly review some basic terms and facts.

Resistance:

- Resistance is an impediment that slows the flow of electrons. It is like friction against the wall of holes of a pipe flowing water that slows the flow of water.
- It is defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (i.e. electrons) through it.
- It is present in all conductors to some extent (except *super*conductors!), most notably in resistors.
- When alternating current goes through a resistance, a voltage drop is produced that is in-phase with the current.
- Resistance is mathematically symbolized by the letter "R" and measured in ohms, Ω.
- A conductor is said to have a resistance of 1 Ω if it permits 1 A current to flow through it when 1 V is impressed across its terminal. R=V/I
- Resistance increases, current decreases.

Reactance:

- Reactance is essentially inertia against the motion of electrons. It is a measure of the opposition of capacitance and inductance to current.
- Perfect resistors possess resistance, but not reactance. Whereas, perfect inductors and perfect capacitors possess reactance but no resistance.
- It is present anywhere electric or magnetic fields are developed in proportion to applied voltage or current, respectively; but most notably in capacitors and inductors.

- When alternating current goes through a pure reactance, a voltage drop is produced that is 90^{o} out of phase with the current.
- Reactance varies with the frequency of the electrical signal.
- It is mathematically symbolized by the letter "X" and measured in the unit of ohms.
- There are two types of reactance:
  - Capacitive reactance (X_C) and
  - Inductive reactance (X_L)
- The **total reactance (X)** is the *difference* between the two: $\mathbf{X = X_L - X_C}$

Capacitive Reactance, X_C

- *Capacitive reactance* is the opposition that a capacitor offers to alternating current due to its phase-shifted storage and release of energy in its electric field.
- Capacitive reactance is symbolized by the capital letter "X_C" and is measured in ohms just like resistance (R).
- Capacitive reactance can be calculated using this formula:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad (unit\ is\ in\ \ \Omega)$$

Where,

X_C = reactance in ohms (Ω)

f = frequency in hertz (Hz)

C = capacitance in farads (F)

- Capacitive reactance *decreases* with increasing frequency. In other words, the higher the frequency, the less it opposes (the more it "conducts") the AC flow of electrons.

Example:

Determine the reactance of a 1μF capacitor for a 50 Hz signal. Also determine its reactance when frequency is 10 kHz.

Solution:

Calculate capacitive reactance using this formula: $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Ans. 3.2kΩ (at 50 Hz), 16Ω (at 10 kHz)

Inductive reactance, X_L

- *Inductive reactance* is the opposition that an inductor offers to alternating current due to its phase-shifted storage and release of energy in its magnetic field.
- Inductive reactance is symbolized by the capital letter "X_L" and is measured in ohms just like resistance (R).
- Inductive reactance can be calculated using this formula:

$$X_L = \omega L = 2\pi f L \quad (unit\ is\ in\ \ \Omega)$$

Where,

X_L = reactance in ohms (Ω)

f = frequency in hertz (Hz)

L = inductance in henrys (H)

- Inductive reactance *increases* with increasing frequency. In other words, the higher the frequency, the more it opposes the AC flow of electrons.

Example:

Determine the reactance of a 1mH inductor for a 50 Hz signal. Also determine its reactance when frequency is 10 kHz.

Solution:

Calculate inductive reactance using this formula: $X_L = \omega L = 2\pi f L$

Ans. 0.3Ω (at 50 Hz), 63Ω (at 10 kHz)

Impedance

- Impedance is a measure of the overall opposition of a circuit to current, in other words: how much the circuit **impedes** the flow of current. It is like resistance, but it also takes into account the effects of capacitance and inductance.
- Impedance is more complex than resistance because, the effects of capacitance and inductance vary with the frequency of the current passing through the circuit and this means **impedance varies with frequency**! The effect of resistance is constant regardless of frequency.
- It is present in all circuits, and in all components.
- When alternating current goes through an impedance, a voltage drop is produced that is somewhere between 0^o and 90^o out of phase with the current.
- It is the complex (vector) sum of ("real") resistance and ("imaginary") reactance. Therefore, impedance can be split into two parts:
    - **Resistance R** (the part which is constant regardless of frequency)
    - **Reactance X** (the part which varies with frequency due to capacitance and inductance)
- Impedance is mathematically symbolized by the letter "Z" and measured in ohms, just like resistance (R) and reactance (X), in complex form, e.g. Z=R+JX.
- Parallel impedances are managed just like resistances in series circuit analysis, but in complex form: series impedances add to form the total impedance. $Z_{Total} = Z_1 + Z_2 + \ldots Z_n$.
- Parallel impedances are manipulated just like resistances in parallel circuit analysis, but in complex form. $1/Z=1/Z_1+1/Z_2+\ldots\ldots+1/Z_n$.
- Perfect resistors possess resistance, but not reactance.

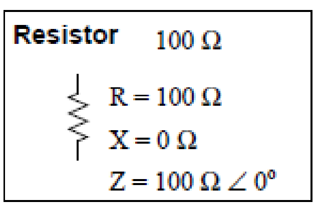


Figure: Perfect resistor

- Perfect inductors and perfect capacitors possess reactance but no resistance.

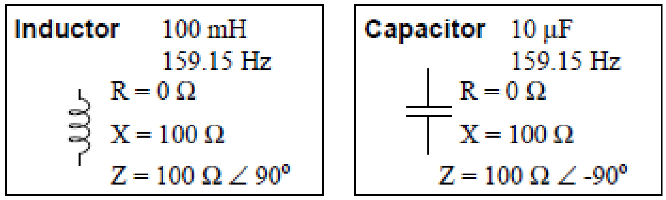
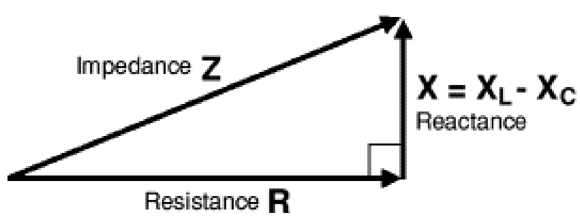


Figure: Perfect inductor and capacitor

- All components (resistor, capacitor and inductor) possess impedance, and because of this universal quality, it makes sense to translate all component values into common terms of impedance as the first step in analyzing an AC circuit.
- The impedance phase angle for any component is the phase shift between voltage across that component and current through that component.
- For a perfect resistor, the voltage drop and current are *always* in phase with each other, and so the impedance angle of a resistor is said to be 0^{o}.
- For a perfect inductor, voltage drop always leads current by 90^{o}, and so an inductor's impedance phase angle is said to be $+90^{o}$.
- For a perfect capacitor, voltage drop always lags current by 90^{o}, and so a capacitor's impedance phase angle is said to be -90^{o}.
- In an RLC circuit, the resistance causes no **phase shift** (i.e. 0^{0} phase angle) between voltage and current, but the capacitance and inductance cause a **phase shift** between the current and voltage. Therefore, the resistance and reactance cannot be simply added up to give impedance. Instead they must be **added as vectors** with reactance at right angles to resistance as shown in the diagram.



$$\text{Impedance,} \quad Z = \sqrt{R^2 + X^2}$$

Impedance for a series RL Circuit:

$Z = \sqrt{(R^2 + X_L^2)}$ =R+JX_L

Impedance for a series RC Circuit:

$Z = \sqrt{(R^2 + X_C^2)}$ =R+JX_C

Impedance for a series RLC Circuit:

$Z = \sqrt{(R^2 + X^2)} = \sqrt{(R^2 + (X_L - X_C)^2}$ where X= X_L-X_C

Ohm's Law for AC circuits:

$$\mathbf{E = IZ} \qquad \mathbf{I = \frac{E}{Z}} \qquad \mathbf{Z = \frac{E}{I}}$$

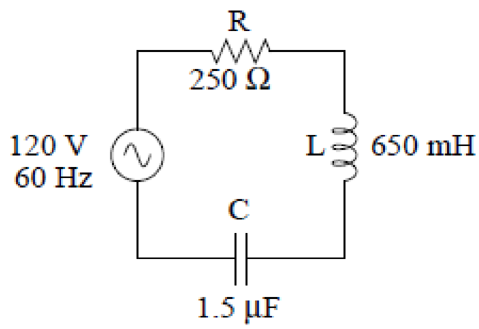
All quantities expressed in complex, not scalar, form

- Kirchhoff 's Laws and all network analysis methods and theorems are true for AC circuits as well, so long as quantities are represented in complex rather than scalar form.
- The only real difference between DC and AC circuit calculations is in regard to power. Because reactance doesn't dissipate power as resistance does, the concept of power in AC circuits is radically different from that of DC circuits.

Solving Series RLC Circuits

Example:

Determine the total impedance, current and voltage drop in each component of a series RLC circuit connected with a 60Hz supply shown below.



Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi fL$$

$$X_L = (2)(\pi)(60\ \text{Hz})(650\ \text{mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{(2)(\pi)(60\ \text{Hz})(1.5\ \mu\text{F})}$$

$$X_C = 1.7684\ \text{k}\Omega$$

Step-2:

- Express all resistances and reactances in a mathematically common form: impedance.

$$Z_R = 250 + j0\ \Omega \quad \textit{or} \quad 250\ \Omega \angle 0^\circ$$

$$Z_L = 0 + j245.04\ \Omega \quad \textit{or} \quad 245.04\ \Omega \angle 90^\circ$$

$$Z_C = 0 - j1.7684\text{k}\ \Omega \quad \textit{or} \quad 1.7684\ \text{k}\Omega \angle -90^\circ$$

Note:

- Remember that an inductive reactance translates into a positive imaginary impedance (or an impedance at +90°), while a capacitive reactance translates into a negative imaginary impedance (impedance at -90°). Resistance, of course, is still regarded as a purely "real" impedance (polar angle of 0°):

Step-3:

- Express the given source voltage in rectangular or polar form:

$$V=120+j0 \text{ V or } 120 \text{ V} \angle 0^0$$

Note:

➢ Unless otherwise specified, the source voltage will be our reference for phase shift, and so it is written at an angle of 0°.

- All given quantities are shown in the circuit below.

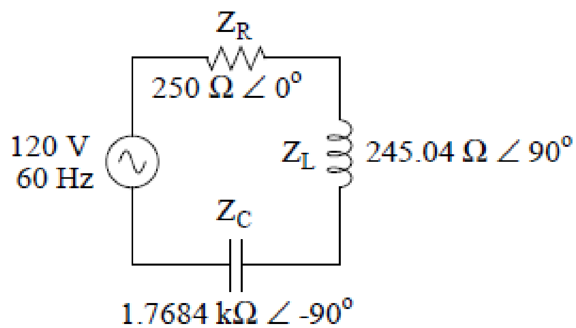


Figure: Series RLC circuit with component values replaced by impedances.

Step-4:

Calculate the total impedance of the circuit.

$$Z_{total} = Z_R + Z_L + Z_C$$

$$Z_{total} = (250 + j0 \ \Omega) + (0 + j245.04 \ \Omega) + (0 - j1.7684k \ \Omega)$$

$$Z_{total} = 250 - j1.5233k \ \Omega \quad \textit{or} \quad 1.5437 \text{ k}\Omega \angle -80.680^\circ$$

Step-5:

Determine circuit current using Ohm's law.

$$I = \frac{V}{Z} = \frac{120\angle 0^{\circ}}{(1.5437 \times 1000)\angle -80.680^{\circ}} = 0.07773\angle 80.680^{\circ} = 12.589 + j76.708$$

Note:

- Being a series circuit, current must be equal through all components of the given circuit.

Step-6:

Determine voltage drop across resistor, capacitor and inductor.

$$V_R = IZ_R = 0.07773\angle 80.680^{\circ} \times 250\angle 0^{\circ} = 19.4325\angle 80.680^{\circ}$$

$$V_L = IZ_L = 0.07773\angle 80.680^{\circ} \times 245.04\angle 90^{\circ} = 19.046\angle 170.68^{\circ}$$

$$V_C = IZ_C = 0.07773\angle 80.680^{\circ} \times (1.7684 \times 1000)\angle -90^{\circ} = 137.46\angle -9.3199^{\circ}$$

Note:

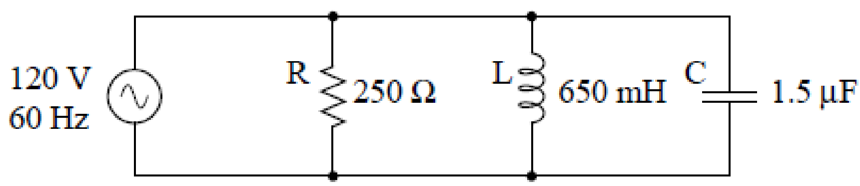
- Although our supply voltage is only 120 volts, the voltage across the capacitor is 137.46 volts! How can this be? The answer lies in the interaction between the inductive and capacitive reactances. Expressed as impedances, we can see that the inductor opposes current in a manner precisely opposite that of the capacitor. Expressed in rectangular form, the inductor's impedance has a positive imaginary term and the capacitor has a negative imaginary term. When these two contrary impedances are added (in series), they tend to cancel each other out! Although they're still added together to produce a sum, that sum is actually less than either of the individual (capacitive or inductive) impedances alone.
- Although impedances add in series, the total impedance for a circuit containing both inductance and capacitance may be less than one or more of the individual impedances, because series inductive and capacitive impedances tend to cancel each other out.
- If the total impedance in a series circuit with both inductive and capacitive elements is less than the impedance of either element separately, then the total current in that circuit must be greater than what it would be with only the inductive or only the capacitive elements there.

- With this abnormally high current through each of the components, voltages greater than the source voltage may be obtained across some of the individual components!

Solving Parallel RLC Circuits

Example:

Determine the total impedance, current and voltage drop in each component of a parallel RLC circuit connected with a 60Hz supply shown below.



Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi fL$$

$$X_L = (2)(\pi)(60\ \text{Hz})(650\ \text{mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{(2)(\pi)(60\ \text{Hz})(1.5\ \mu\text{F})}$$

$$X_C = 1.7684\ \text{k}\Omega$$

Step-2:

- Express all resistances and reactances in a mathematically common form: impedance.

$$Z_R = 250 + j0\ \Omega \quad or \quad 250\ \Omega \angle 0^o$$

$$Z_L = 0 + j245.04\ \Omega \quad or \quad 245.04\ \Omega \angle 90^o$$

$$Z_C = 0 - j1.7684k\ \Omega \quad or \quad 1.7684\ k\Omega \angle -90^o$$

Step-3:

- Express the given source voltage in rectangular or polar form:

$$V=120+j0\ V \text{ or } 120\ V \angle 0^0$$

- All given quantities are shown in the circuit below.

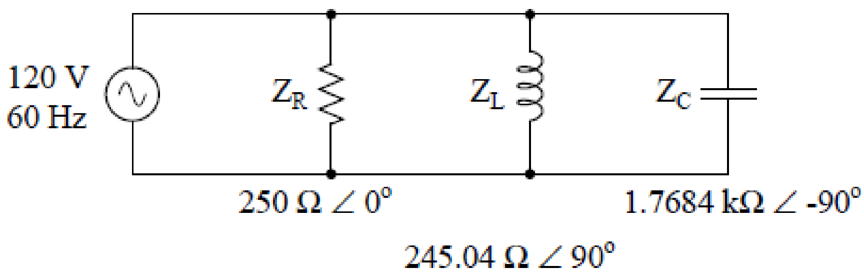


Figure: Parallel RLC circuit with component values replaced by impedances.

Step-4:

Calculate the total impedance of the parallel circuit.

$$\frac{1}{Z_{Total}} = \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$= \frac{1}{250\angle 0°} + \frac{1}{245.04\angle 90°} + \frac{1}{(1.7684 \times 1000)\angle -90°}$$

$$Z_{Total} = 187.79\angle 41.311° = 141.05 + j123.96$$

Step-5:

Determine voltage drop across resistor, capacitor and inductor.

Being a parallel circuit, voltage must be equally shared by all components of the given circuit. Therefore voltage drop across each component is the same as the supply voltage which is V=120+j0 V or 120 V$\angle 0^0$

$$V_R = V_L = V_C = V = 120 + j0 = 120\angle 0°$$

Step-6:

Determine total current and current flowing through each component using Ohm's law.

$$I = \frac{V}{Z} = \frac{120\angle 0^{\circ}}{187.79\angle 41.311°} = 639.03mA\angle -41.311° = 480mA - j421.85mA$$

$$I_R = \frac{V_R}{Z_R} = \frac{120\angle 0^{\circ}}{250\angle 0°} = 480mA\angle 0° = 480mA + j0$$

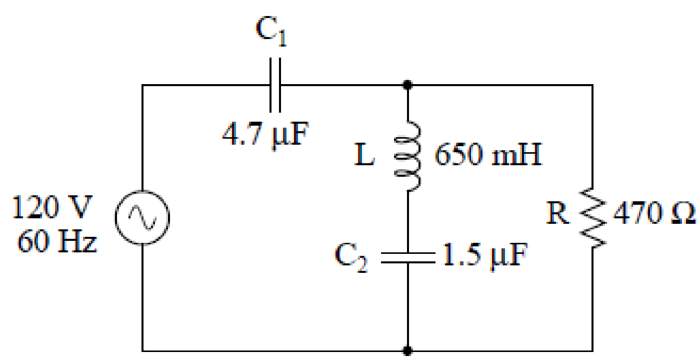
$$I_L = \frac{V_L}{Z_L} = \frac{120\angle 0^{\circ}}{245.04\angle 90°} = 489.71mA\angle -90° = 0 - j489.71mA$$

$$I_C = \frac{V_C}{Z_C} = \frac{120\angle 0^{\circ}}{(1.7684 \times 1000)\angle -90°} = 67.858mA\angle 90° = 0 - j67.858mA$$

Solving Series-Parallel RLC Circuits

Example:

Determine the total impedance, current and voltage drop in each component of a series-parallel RLC circuit connected with a 60Hz supply shown below.



Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi fL$$

$$X_L = (2)(\pi)(60\text{ Hz})(650\text{ mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_{C1} = \frac{1}{(2)(\pi)(60Hz)(4.7\mu F)} = 564.38\Omega$$

$$X_{C2} = \frac{1}{(2)(\pi)(60Hz)(1.5\mu F)} = 1.7684k\Omega$$

Step-2:

- Express all resistances and reactances in a mathematically common form: impedance.

$$Z_R = 470 + j0 = 470\Omega\angle 0^\circ$$
$$Z_{C1} = 0 - j564.38 = 564.38\Omega\angle -90^\circ$$
$$Z_{C2} = 0 - j1.7684 = 1.7684\Omega\angle -90^\circ$$
$$Z_L = 0 + j245.04 = 245.04k\Omega\angle 90^\circ$$

Step-3:

- Express the given source voltage in rectangular or polar form:

V=120+j0 V or $120\angle 0^\circ$

- All given quantities are shown in the circuit below.

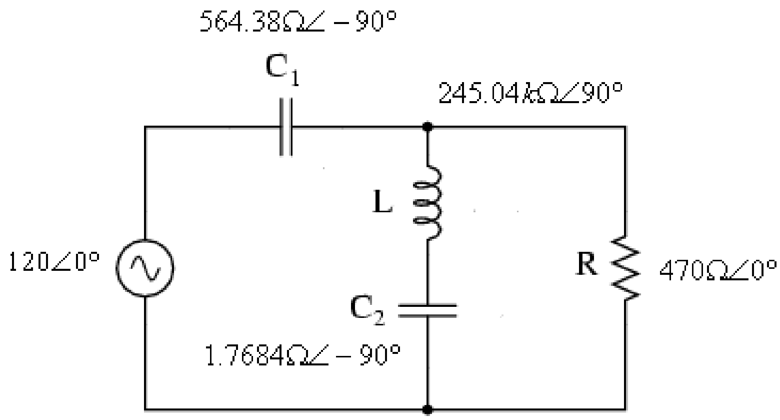


Figure: Series-Parallel RLC circuit with component values replaced by impedances.

Step-4:

Calculate the total impedance of the parallel circuit.

The first step is to combine L and C_2 as a series combination of impedances, by adding their impedances together.

Then, that impedance will be combined in parallel with the impedance of the resistor R, to arrive at another combination of impedances. Finally, that quantity will be added to the impedance of C_1 to arrive at the total impedance.

$$Z_{L-C2} = Z_{C2} + Z_L = 0 - j1.7684 + 0 + j245.04 = 0 - j1.5233 = 1.5233k\Omega\angle -90°$$

$$Z_{R\|(L-C2)} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_{L-C2}}} = 429.15 - j132.41 = 449.11\angle -17.147°$$

$$Z_{Total} = Z_{C1} + Z_{R\|(L-C2)} = 429.15 - j696.79 = 818.34\angle -58.371°$$

Step-5:

Determine total current using Ohm's law.

$$I = \frac{V}{Z} = \frac{120\angle 0^{\circ}}{818.34\angle -58.371} = 146.64mA\angle 58.371° = 76.899 + 124.86$$

We observe that total current $I = I_{C1}$

Total current is divided into two parts: one flowing through L and C_2, and another one passing through R.

We can calculate the branch currents using current-divider rule.

And finally, we can calculate voltage drops across each component since, current and impedance values are known.