Lecture-10: Methods of Circuit Analysis: Node Voltage Method

****

Nodal Voltage Analysis/ Node Voltage Method:

- In Mesh (loop) analysis, Kirchhoff's voltage law is applied to arrive at loop currents in a network.
- As its name implies, Nodal analysis uses Kirchhoff's current law to determine the potential difference (voltage) at any node with respect to some arbitrary reference point in a network.
- Once the potentials of all nodes are known, it is a simple matter to determine other quantities such as current and power within the network.
- Using node analysis, we can calculate the voltages around the loops that reduce the amount of mathematics required using just Kirchoff's laws.

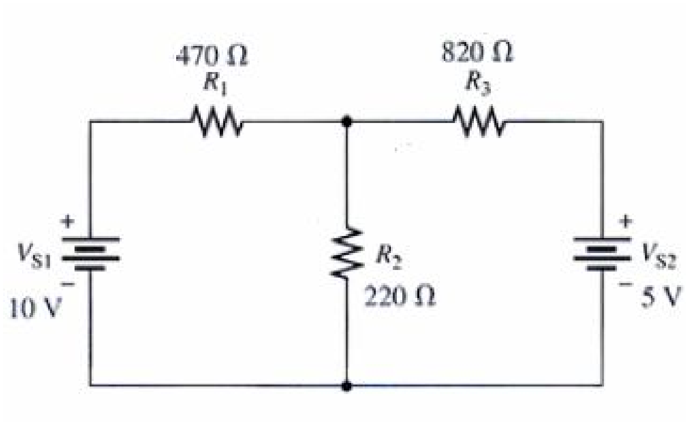
The steps for solving a circuit using nodal analysis are as follows:

1. Determine the number of nodes within the given circuit.
2. Arbitrarily select one node as a **reference node** and indicate this node as **ground**. The reference node is usually located at the bottom of the circuit, although it may be located anywhere.
3. Convert each voltage source in the network to its equivalent current source. This step, although not absolutely necessary, makes further calculations easier to understand.
4. Arbitrarily assign voltages (such as V_1, V_2, . . . , V_n) to the remaining nodes in the circuit. (Remember that all voltages to the remaining nodes will be relative to the chosen reference node).
5. Arbitrarily assign a current direction to each branch in which there is no current source.
6. Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.
7. With the exception of the reference node (ground), apply Kirchhoff's current law at each of the nodes. If a circuit has a total of $n+1$ nodes (including the reference node), there will be n simultaneous linear equations.
8. Rewrite each of the arbitrarily assigned currents in terms of the potential difference across a known resistance.
9. Solve the resulting simultaneous linear equations for the voltages (V_1, V_2, . . . , V_n).

****

Example-1:

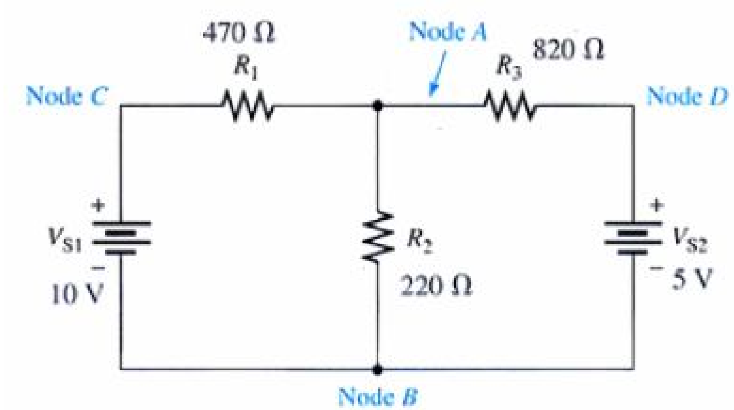
Using node voltage analysis method, determine the nodal voltages for the circuit shown in the figure below.



Solution:

Step-1:

Determine the number of nodes in the given circuit. In this case, there are four nodes: Node A, B, C and D.

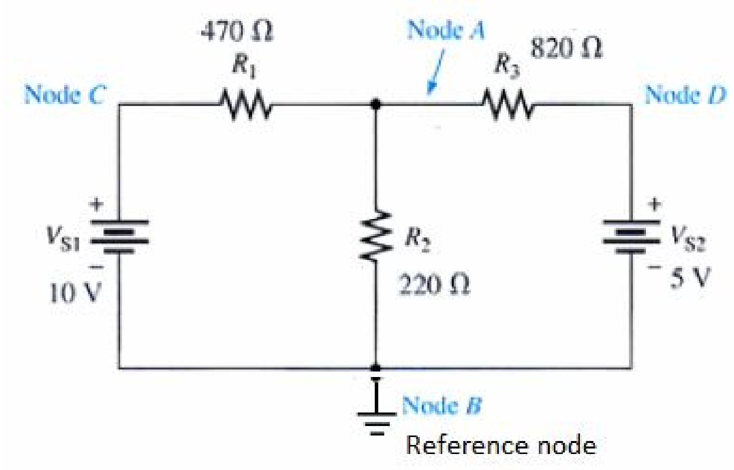


Step-2:

Arbitrarily select one node as a reference node and indicate this node as **ground**. In this case, let us assume **node B** as the reference node.

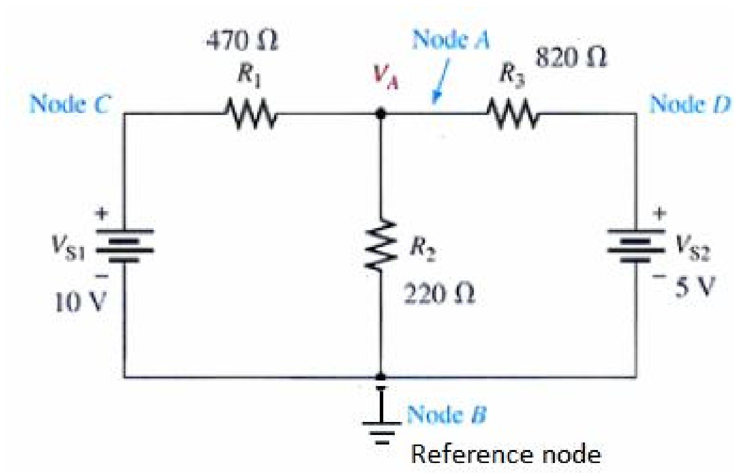
Lecture-10: Methods of Circuit Analysis: Node Voltage Method

****



Step-3:

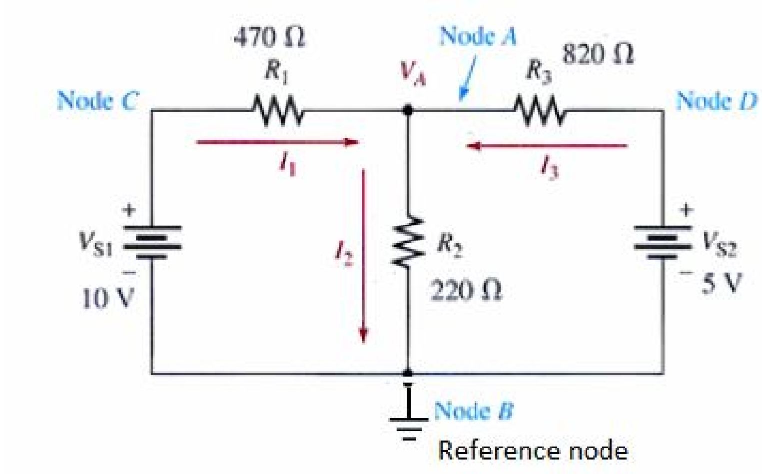
Except the reference node, arbitrarily assign voltages (V_A, V_C and V_D) to the remaining nodes in the circuit. Here node voltage C and D are already known to be the source voltages, i.e. V_C=10V, V_D=5V. The only unknown voltage is at node A, i.e. V_A. (Note that these voltages will all be with respect to the chosen reference).



Step-4:

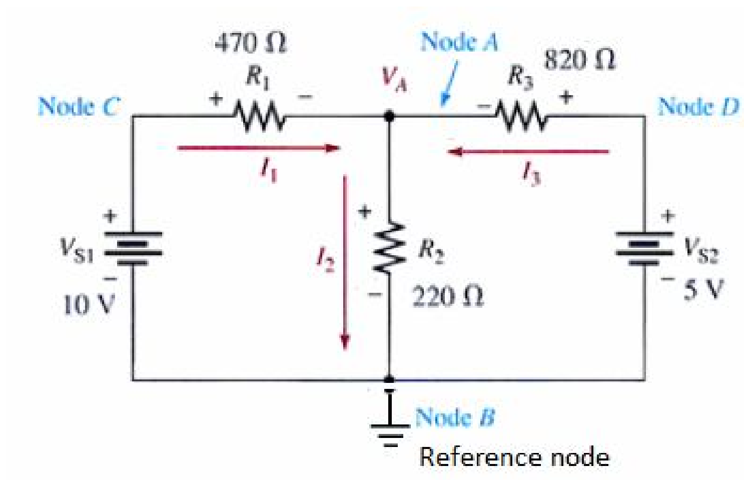
Arbitrarily assign branch currents along with directions at each node where the voltage is unknown, except at the reference node. Here, voltage at node A is unknown.

****



Step-5:

Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.



Step-6:

With the exception of the reference node (ground), apply Kirchhoff's current law at each of the other nodes.

As there are 3 nodes (labeled as Node C, Node A, and Node D) except the reference node, and as the only unknown voltage is at node A, we now apply Kirchhoff's current law at node A.

$$KCL\ at\ Node\ \ A \qquad \sum I_{Entering} = \sum I_{Leaving}$$

$$\Rightarrow \qquad I_1 + I_3 = I_2$$

$$\Rightarrow \qquad I_1 - I_2 + I_3 = 0 \qquad ....(1)$$

****

Step-7:

Express the current equation(s) in terms of the potential difference across a known resistance.

Here, the currents are rewritten in terms of the voltages across the resistors as follows:

$$I_1 = \frac{V_1}{R_1} = \frac{V_{S1} - V_A}{R_1} \qquad ....(2)$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_A}{R_2} \qquad ....(3)$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_{S2} - V_A}{R_3} \quad .... \qquad ..(4)$$

By substituting the values of I_1, I_2 and I_3 to equation (1), the nodal equations become:

$$I_1 - I_2 + I_3 = 0 \qquad ....(1)$$

$$\Rightarrow \quad \frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0 \qquad ....(5)$$

Step-8:

Solve the resulting linear equation for the unknown node voltages using substitution or determinant method.

Here the equation is:

$$\frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0 \qquad ....(5)$$

Given, V_{S1}=10V, V_{S2}=5V, R_1=470 ohm, R_2=220 ohm, R_3=820 ohm. The only unknown term is V_A. So, putting the known values, we can easily calculate the value of unknown term V_A.

$$\frac{10-V_A}{470}-\frac{V_A}{220}+\frac{5-V_A}{820}=0$$
$$\frac{220\times 820(10-V_A)-470\times 820V_A+470\times 220(5-V_A)}{470\times 220\times 820}=0$$
$$\frac{1804000-180400V_A-385400V_A+517000-103400V_A}{470\times 220\times 820}=0$$
$$\frac{11357000-669200V_A}{470\times 220\times 820}=0$$
$$11357000-669200V_A=0$$
$$V_A=\frac{11357000}{669200}$$
$$=16.97V$$

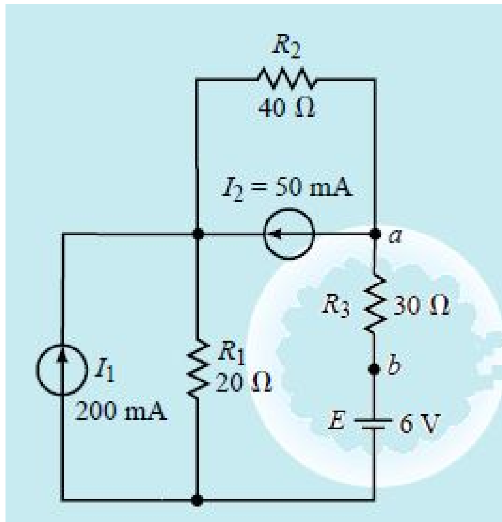
After determining V_A, we can easily calculate the branch current I_1, I_2 and I_3 using equations (2), (3) and (4).

$$I_1=\frac{V_1}{R_1}=\frac{V_{S1}-V_A}{R_1} \qquad \text{..........(2)}$$
$$=\frac{10-16.97}{470}$$
$$=\frac{-6.97}{470}$$
$$=-0.01482A$$
$$=-14.82mA$$

$$I_2=\frac{V_2}{R_2}=\frac{V_A}{R_2} \qquad \text{.........(3)}$$
$$=\frac{16.97}{220}$$
$$=0.07713A$$
$$=77.13mA$$

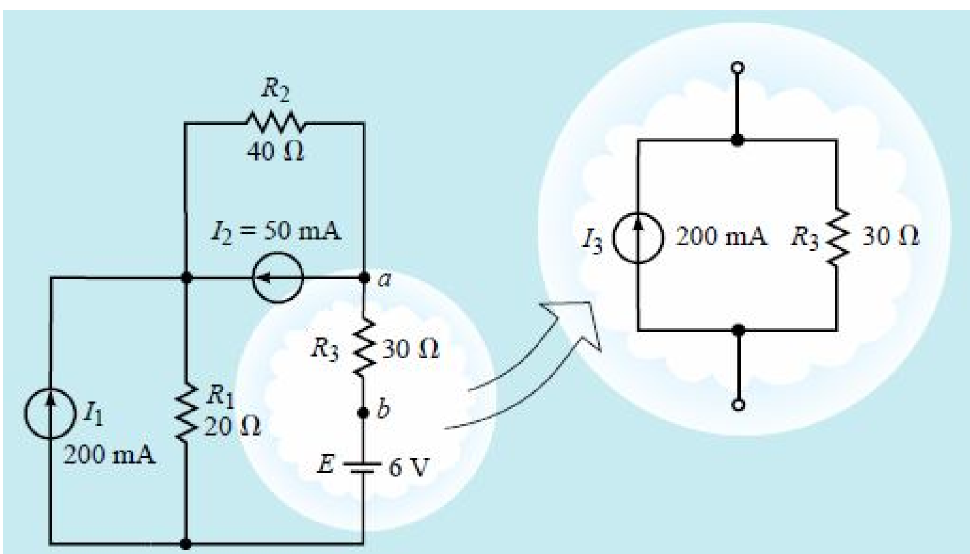
$$I_3=\frac{V_3}{R_3}=\frac{V_{S2}-V_A}{R_3} \qquad \text{....... ..(4)}$$
$$=\frac{5-16.97}{820}$$
$$=\frac{-11.97}{820}$$
$$=-0.01459A$$
$$=-14.59mA$$

Example-2:
Use nodal analysis to solve for the voltage V_{ab} from the circuit given below.

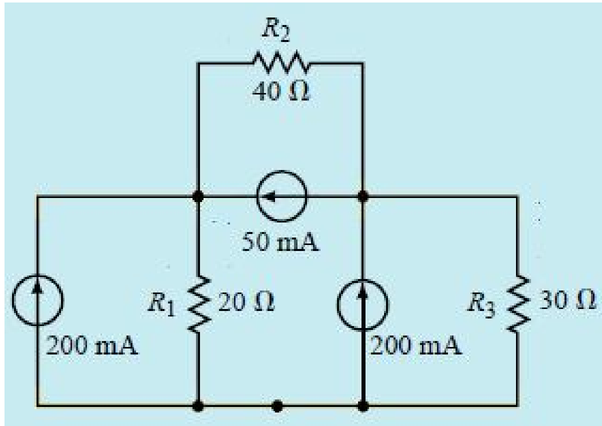


Step-1:

The given circuit has a voltage source. So, convert it to its equivalent current source.

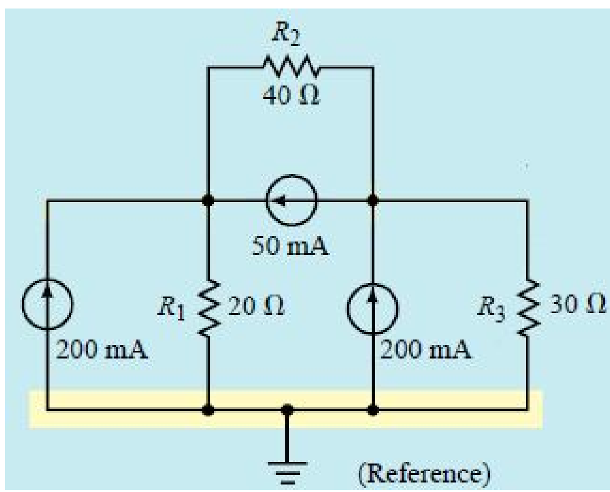


After converting the voltage source to its equivalent current source, the equivalent circuit is shown in figure below.



Step-2:

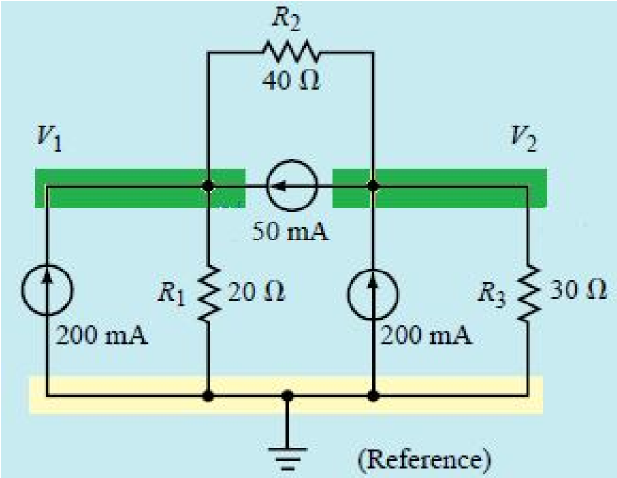
Arbitrarily select a convenient reference node within the circuit and indicate this node as **ground**. The circuit with the reference node (shaded as white rectangle) is given below.



Step-3:

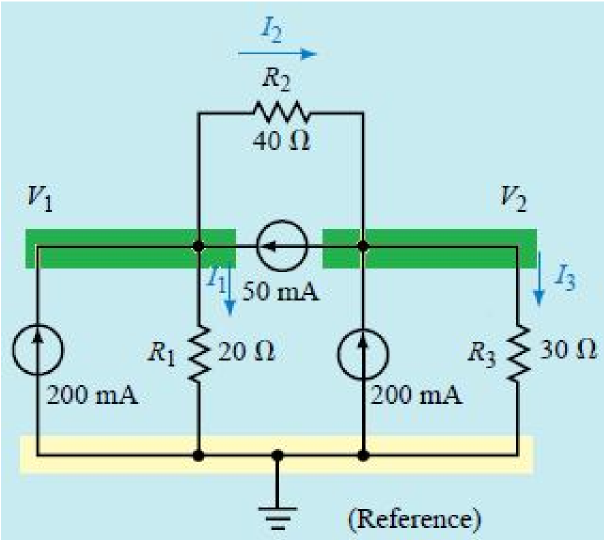
After assigning a reference node, arbitrarily assign voltages (V_1, V_2, . . . , V_n) to the remaining nodes in the circuit. These voltages will all be with respect to the chosen reference. (Note that the remaining nodes are shaded as green rectangles).

Lecture-10: Methods of Circuit Analysis: Node Voltage Method



Step-4:

Arbitrarily assign a current direction to each branch in which there is no current source.

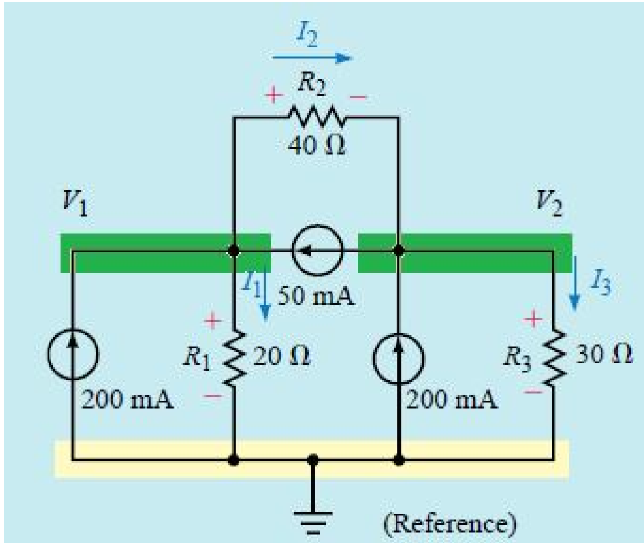


Step-5:

Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.

Lecture-10: Methods of Circuit Analysis: Node Voltage Method

****



Step-6:

With the exception of the reference node (ground), apply Kirchhoff's current law at each of the other nodes. As there are 2 nodes (labeled as V_1 and V_2) except the reference node, we now apply Kirchhoff's current law at these two nodes.

$$KCL\ at\ Node\ V_1 \qquad \sum I_{Entering} = \sum I_{Leaving}$$

$$200mA + 50mA = I_1 + I_2$$

$$\Rightarrow \qquad I_1 + I_2 = 250mA \qquad ....(1)$$

$$KCL\ at\ Node\ V_2 \qquad \sum I_{Entering} = \sum I_{Leaving}$$

$$200mA + I_2 = 50mA + I_3$$

$$\Rightarrow \qquad I_2 - I_3 = -150mA \qquad ....(2)$$

Step-7:

Rewrite each of the arbitrarily assigned currents in terms of the potential difference across a known resistance. Here, the currents are rewritten in terms of the voltages across the resistors as follows:

$$I_1 = \frac{V_1}{20\ \Omega}$$

$$I_2 = \frac{V_1 - V_2}{40\ \Omega}$$

$$I_3 = \frac{V_2}{30\ \Omega}$$

****

By substituting the values of I_1, I_2 and I_3 to equation (1) and (2), the nodal equations become:

$$I_1 + I_2 = 250\,mA \qquad ....(1)$$

$$\Rightarrow \quad \frac{V_1}{20} + \frac{V_1 - V_2}{40} = 250$$

$$\Rightarrow \quad \frac{2V_1 + V_1 - V_2}{40} = 250$$

$$\Rightarrow \quad 3V_1 - V_2 = 10000 \qquad ....(3)$$

$$I_2 - I_3 = -150\,mA \qquad ....(2)$$

$$\Rightarrow \quad \frac{V_1 - V_2}{40} - \frac{V_2}{30} = -150$$

$$\Rightarrow \quad \frac{30(V_1 - V_2) - 40V_2}{1200} = -150$$

$$\Rightarrow \quad \frac{30V_1 - 30V_2 - 40V_2}{1200} = -150$$

$$\Rightarrow \quad 30V_1 - 70V_2 = -150 \times 1200$$

$$\Rightarrow \quad 3V_1 - 7V_2 = -18000 \qquad ....(5)$$

- To simplify the solution of the simultaneous linear nodal equations (3) and (5), we write them as follows:

$$3V_1 + (-1)V_2 = 10000 \qquad ....(3)$$

$$3V_1 + (-7)V_2 = -18000 \qquad ....(5)$$

- We can solve the above equations by the use of determinants and Cramer's rule or as just substitution method:

$$3V_1 - V_2 = 10000 \qquad ....(3)$$

$$3V_1 - 7V_2 = -18000 \qquad ....(5)$$

$$eq(3) - eq(5) \qquad 6V_2 = 28000$$

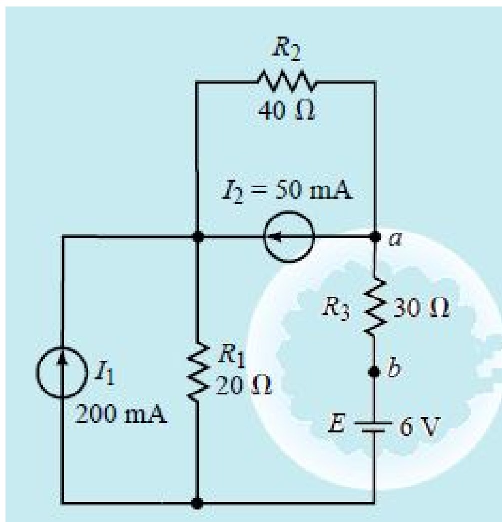
$$\Rightarrow \qquad V_2 = \frac{28000}{6} = 4666.66\,mV = 4.67\,V$$

Substituting the value of V_2 in equation (3), Value of V_1 will be found:

Lecture-10: Methods of Circuit Analysis: Node Voltage Method

$$
\begin{aligned}
3V_1 - V_2 &= 10000 \qquad \text{............(3)}\\
3V_1 &= 10000 + V_2\\
&= 10000 + 4666.66\\
&= 14666.66\\
V_1 &= \frac{14666.66}{3} = 4888.88\,mV = 4.88V
\end{aligned}
$$

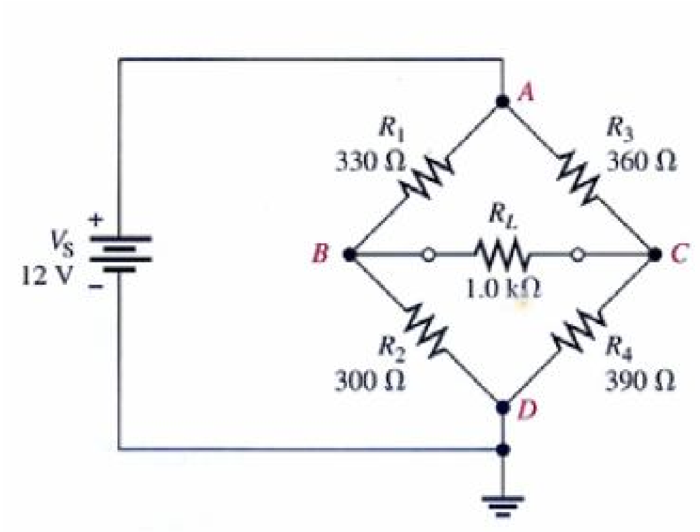
If we go back to the given original circuit (shown below again), we see that the voltage V_2 is the same as the voltage V_a.



$$
\begin{aligned}
&V_a = V_2 = 4.67V = 6.0V + V_{ab}\\
&\Rightarrow V_{ab} = 4.67 - 6.0 = -1.33V
\end{aligned}
$$

Example-3:

Find the node voltages at node B and C for the Wheatstone bridge circuit given below:



Solution:

Among the four nodes, D is designated as reference node. Node A has the same voltage as the source. Voltage at nodes B and C is unknown. Hence we need two linear equations to solve the problem.

Node B:

$$I_1 + I_L = I_2$$

$$\frac{V_A - V_B}{R_1} + \frac{V_C - V_B}{R_L} = \frac{V_B}{R_2}$$

$$\frac{12 - V_B}{0.330\,\text{k}\Omega} + \frac{V_C - V_B}{1.0\,\text{k}\Omega} = \frac{V_B}{0.300\,\text{k}\Omega}$$

Node C:

$$I_3 = I_L + I_4$$

$$\frac{V_A - V_C}{R_3} = \frac{V_C - V_B}{R_L} + \frac{V_C}{R_4}$$

$$\frac{12\text{ V} - V_C}{0.360\,\text{k}\Omega} = \frac{V_C - V_B}{1.0\,\text{k}\Omega} + \frac{V_C}{0.390\,\text{k}\Omega}$$