DC Network Theorem

Kirchhoff's Circuit Law

- A single equivalent resistance, (R_T) can be found when two or more resistors are connected together in either series, parallel or combinations of both, and that these circuits obey *Ohm's Law*.
- However, sometimes in complex circuits such as bridge or T networks, we can not simply use Ohm's Law alone to find the voltages or currents circulating within the circuit. For these types of calculations we need certain rules which allow us to obtain the circuit equations and for this we can use Kirchhoff's Circuit Law.
- Kirchhoff's laws are particularly useful:
  - In determining the equivalent resistance of a complicated network of conductors.
  - For calculating the currents flowing in the various conductors.
- In 1845, a German physicist, Gustav Kirchhoff developed a pair or set of rules or laws which deal with the conservation of current and energy within electrical circuits. The rules are commonly known as: *Kirchoff's Circuit Laws:*
  - One law dealing with current flow around a closed circuit, known as Kirchhoff's Current Law, (KCL)
  - Other law deals with the voltage around a closed circuit, known as Kirchhoff's Voltage Law, (KVL).

Kirchhoff's First Law - The Current Law or Point Law, (KCL)

Kirchhoff's Current Law or KCL, states that-

the "*total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node*". In other words, *the algebraic sum of all the currents entering and leaving a node must be equal to zero, i.e.,* $I_{(exiting)} + I_{(entering)} = 0$. This idea by Kirchhoff is known as the **Conservation of Charge**.

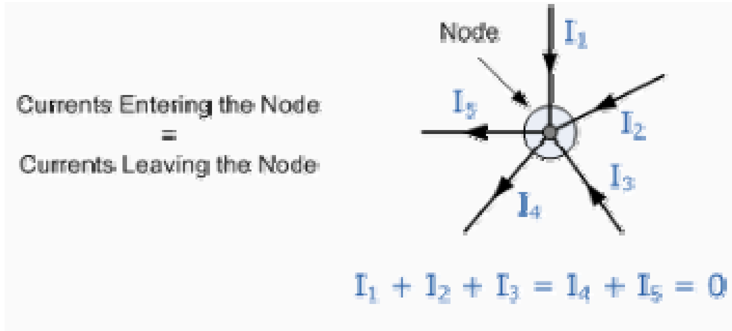
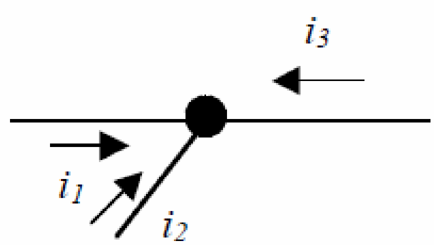


Figure-: Kirchhoff's Current Law

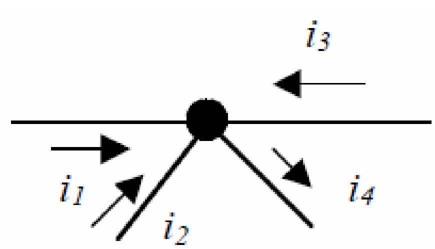
- Here, the 3 currents entering the node, I_1, I_2, I_3 are all positive in value and the 2 currents leaving the node, I_4 and I_5 are negative in value. Then this means we can also rewrite the equation as;

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

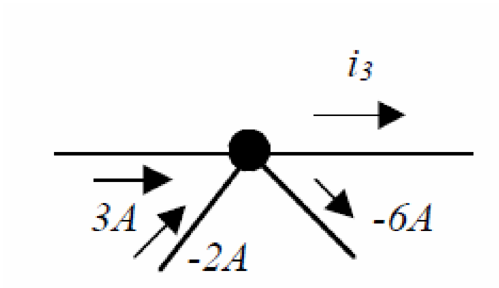
More Examples:



$$i_1 + i_2 + i_3 = 0$$



$$i_1 + i_2 + i_3 + (-i_4) = 0$$
$$i_1 + i_2 + i_3 = i_4$$



What is the value of i_3?

$3+(-2)-(-6)-i_3 = 0$, $i_3 = 7A$

Note:

- We can use Kirchoff's current law when analyzing parallel circuits.

Kirchhoff's Second Law - The Voltage Law or Mesh Law, (KVL)

Kirchhoff's Voltage Law or KVL, states that-

> "*in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop*" which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchhoff is known as the **Conservation of Energy**.

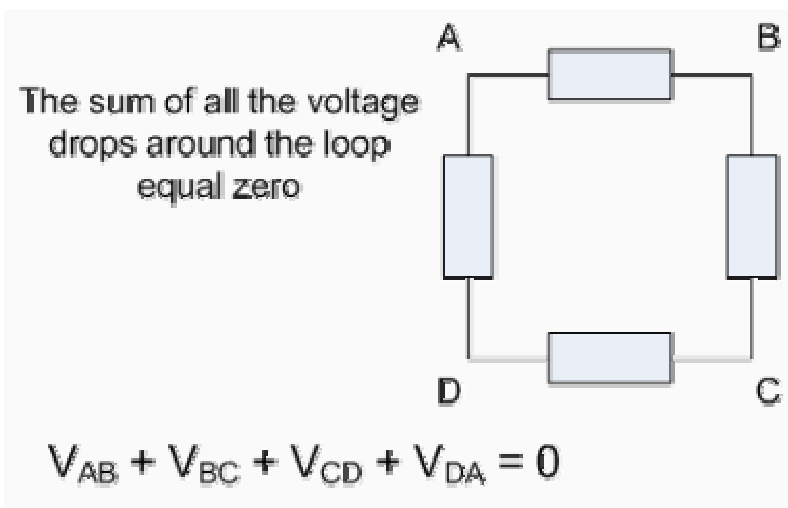


Figure-: Kirchhoff's Voltage Law

- Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point.

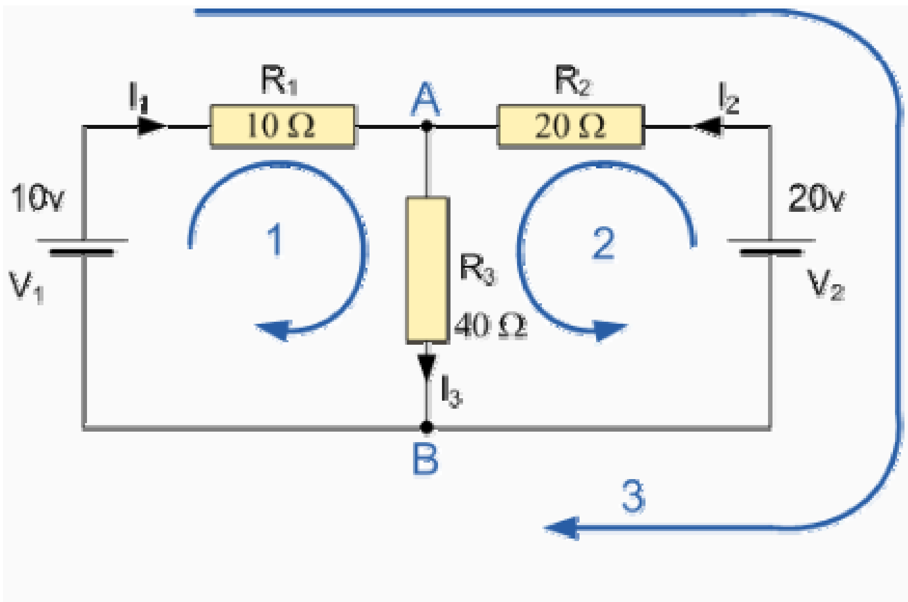
- It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero.

Note:

- We can use Kirchoff's voltage law when analyzing series circuits.

Example-1:

Find the current flowing in the 40Ω Resistor, R_3



The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using Kirchhoff's Current Law, KCL the equations are given as;

At node A : $I_1 + I_2 = I_3$ ....(1)

At node B : $I_3 = I_1 + I_2$ ....(2)

Using Kirchhoff's Voltage Law, KVL the equations are given as;

Loop 1 is given as : $10 = R_1 \times I_1 + R_3 \times I_3 = 10I_1 + 40I_3$ ....(3)

Loop 2 is given as : $20 = R_2 \times I_2 + R_3 \times I_3 = 20I_2 + 40I_3$ ....(4)

Loop 3 is given as : $10 - 20 = 10I_1 - 20I_2$ ....(5)

As I_3 is the sum of $I_1 + I_2$ we can rewrite equations (3) and (4) as;

Eq. No 3 : $10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$ ....(6)

Eq. No 4 : $20 = 20I_2 + 40(I_1 + I_2) = 40I_1 + 60I_2$ ....(7)

We now have two "Simultaneous Equations" that can be reduced to give us the value of both I_1 and I_2.

Substitution of I_1 in terms of I_2 gives us the value of I_1 as -0.143 Amps

Substitution of I_2 in terms of I_1 gives us the value of I_2 as +0.429 Amps

As : $I_3 = I_1 + I_2$

The current flowing in resistor R_3 is given as :

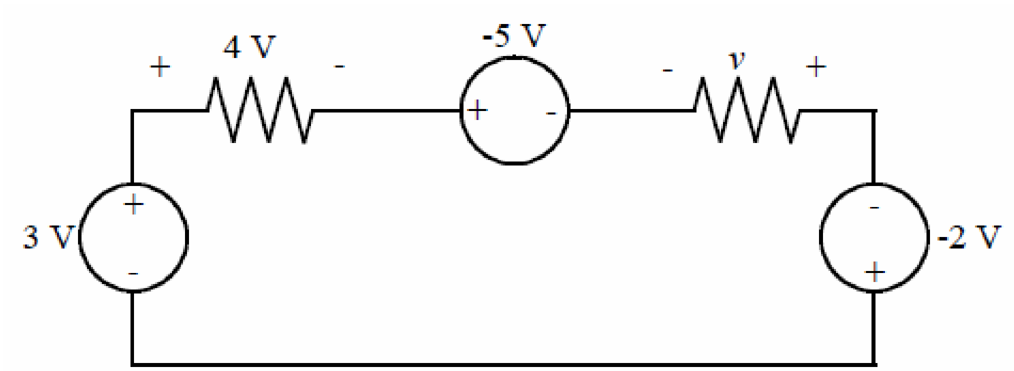
$$-0.143 + 0.429 = 0.286 \text{ Amps}$$

and the voltage across the resistor R_3 is given as :

$$0.286 \times 40 = 11.44 \text{ volts}$$

The negative sign for I_1 means that the direction of current flow initially chosen was wrong, but never the less still valid. In fact, the 20v battery is charging the 10v battery.

Example-2:



What's the value of v?
Write KVL clockwise starting at lower left:
$+3 - 4 - (-5) + v + (-2) = 0, \quad v = -2$ V

Application of Kirchhoff's Circuit Laws

These two laws enable the Currents and Voltages in a circuit to be found, ie, the circuit is said to be "Analysed", and the basic procedure for using Kirchoff's Circuit Laws is as follows:

1. Assume all voltage sources and resistances are given. (If not, label them V_1, V_2 ..., R_1, R_2 etc)

2. Label each branch with a branch current. (I_1, I_2, I_3 etc)
3. Find Kirchoff's first law equations for each node.
4. Find Kirchoff's second law equations for each of the independent loops of the circuit.
5. Use Linear simultaneous equations as required to find the unknown currents.

Determination of algebraic sign

In applying Kirchhoff's laws to specific problem, particular attention should be paid to the algebraic signs of voltage drops and emf, otherwise results will come out to be wrong. Following sign conventions is suggested.

Sign of Battery EMF

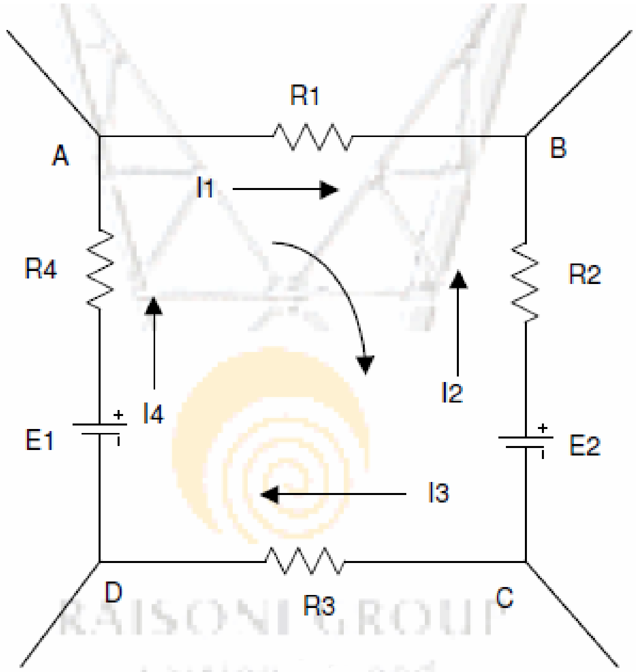
- A rise (or increase) in voltage should be given a +ve sign and a fall (or decrease) in voltage should be given a -ve sign.
- While going round a loop (in a direction of our own choice) if we go from the –ve terminal of battery to its +ve terminal , there is rise in potential , hence this EMF should be given as + ve sign .On the other hand if we go from its + ve terminal to its –ve terminal , then there is a fall in potential , hence this battery EMF should be given as –ve sign .
- It is important to note that algebraic sign of battery EMF is independent of the direction of current flow. (Whether clockwise or in anticlockwise) through the branch which the battery is connected.

Sign of IR Drops:

- If we go through a resistor in the same direction as its current, then there is a fall or decrease in potential for the simple reason that current always flow from higher to lower potential. Hence this IR drop (voltage fall) should be taken as –ve.
- However, if we go around the loop in a direction opposite to that of the current, there is a rise in voltage. Hence these IR should be taken as +ve.
- It clears that the algebraic sign of IR drop across a resister depends on the direction of current through that resistor but is independent of the polarity of any other source of emf in the circuit under consideration.

Example:

- Consider a loop, for example, ABCDA shown in the figure below:



- As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs:

I_1R_1 ------ is –ve (fall in potential)

I_2R_2 ----- - is +ve (rise in potential)

E_2 ------ is –ve (Fall in potential

I_3R_3 ------ is –ve (fall in potential)

E_1--------- is +ve (rise in potential)

I_4R_4--------- is –ve (fall in potential)

According to KVL

$- I_1R_1 + I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$

Or, $- I_1R_1 + I_2R_2 - I_3R_3 - I_4R_4 = E_2 - E_1$

Or, $I_1R_1 - I_2R_2 + I_3R_3 + I_4R_4 = E_1 - E_2$

Or, $V_{R1} - V_{R2} + V_{R3} + V_{R4} = E_1 - E_2$

Assumed Direction of Current:

- The direction of current flow may be assumed either clockwise or anticlockwise.
- If the assumed direction of current is not the actual direction, then on solving the question, this current will be found to have a minus sign. If the answer is positive, then assumed direction is the actual direction.
- Once a particular direction has been assumed, the same should be used throughout the solution of the question.

Superposition Theorem:

- If there are a number of e.m.fs acting simultaneously in any linear bilateral network, then the strategy used in the Superposition Theorem is to eliminate all but one source of power within a network at a time.
- After that, voltage drops (and/or currents) within the modified network for each power source are determined separately using series/parallel analysis.
- Then, the values are all "superimposed" on top of each other (added algebraically) to find the actual voltage drops/currents with all sources active.

This theorem may be stated as follows:

- In a linear bilateral network containing more than one generator (source of emf), the current which flows at any point is the sum of all the currents which would follow at that point if each generator where considered separately and all the other generators replaced for the time being by resistances equal to their internal resistances.

- Consider the circuit given below. We apply Superposition Theorem to it:

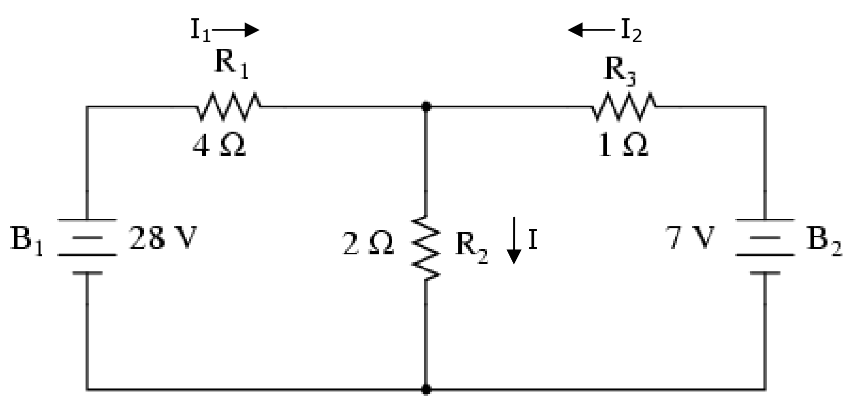


Figure-1: Original Circuit

In the circuit above, I_1, I_2 and I represent the values of currents which are due to the simultaneous action of the two sources of emf in the network.

- Since we have two sources of power in this circuit, we will have to calculate two sets of values for voltage drops and/or currents:

- One for the circuit with only the 28 volt battery in effect which is shown below.

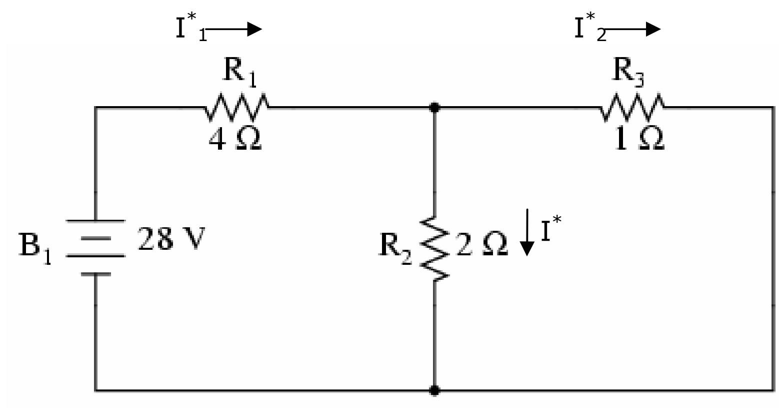


Figure-2: Circuit only 28 V battery in effect

- Another for the circuit with only the 7 volt battery in effect which is shown below.

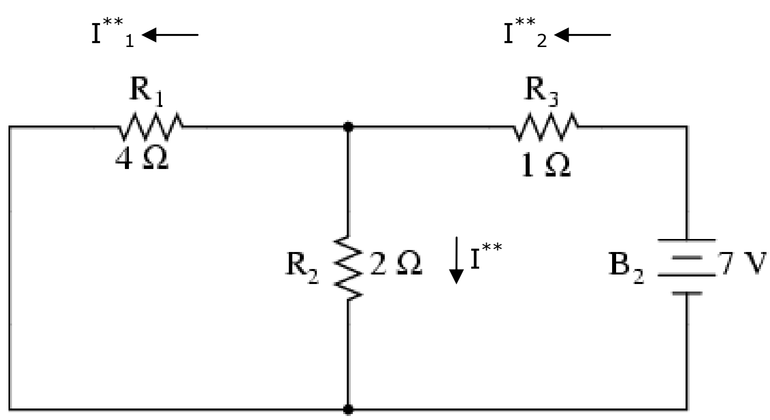


Figure-3: Circuit only 7 V battery in effect

- By combining the current values of figure-2 and figure-3, the actual values of figure-1 can be obtained:

$I_1 = I^*_1 - I^{**}_1$, $I_2 = I^{**}_2 - I^*_2$, $I = I_1 + I_2$

Note:

- When we re-draw the circuit for series/parallel analyses with one source, all other voltage sources are replaced by wires (shorts), and all current sources are replaced with open circuits (breaks).
- Since we only have voltage sources (batteries) in our example circuit, we will replace every inactive source during analysis with a wire.

- Analyzing the circuit with only the 28 volt battery, we obtain the following values for voltage and current:

	R_1	R_2	R_3	$R_2//R_3$	$R_1 + R_2//R_3$ Total	
E	24	4	4	4	28	Volts
I	6	2	4	6	6	Amps
R	4	2	1	0.667	4.667	Ohms

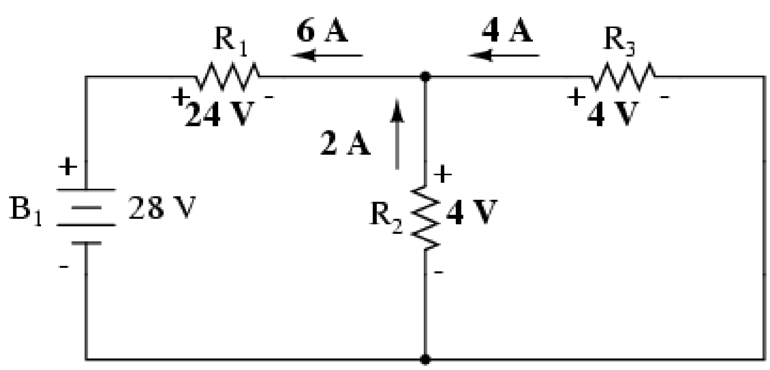


Figure-4: Analysis of the circuit only 28 V battery in effect

Analyzing the circuit with only the 7 volt battery, we obtain another set of values for voltage and current:

	R_1	R_2	R_3	$R_1//R_2$	$R_3 + R_1//R_2$ Total	
E	4	4	3	4	7	Volts
I	1	2	3	3	3	Amps
R	4	2	1	1.333	2.333	Ohms

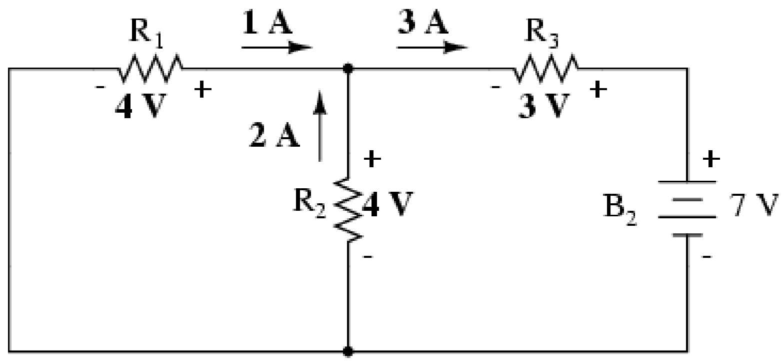


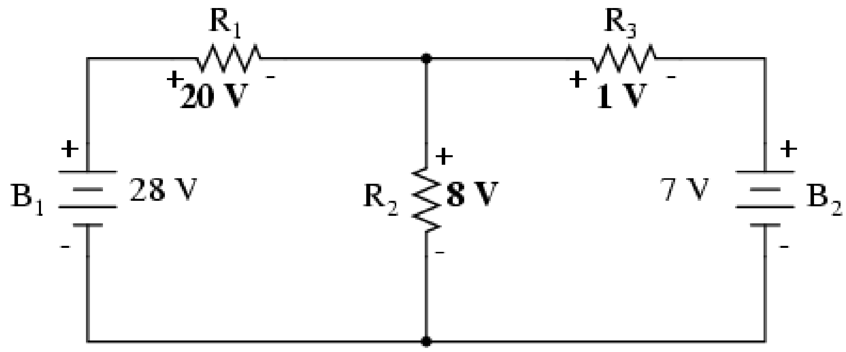
Figure-5: Analysis of the circuit only 28 V battery in effect

Note:

- When superimposing these values of voltage and current, we have to be very careful to consider polarity (voltage drop) and direction (electron flow), as the values have to be added *algebraically*.

With 28 V battery	*With 7 V battery*	*With both batteries*
24 V, + −, E_{R1}	4 V, − +, E_{R1}	**20 V**, + −, E_{R1} *24 V - 4 V = 20 V*
E_{R2} + 4 V −	E_{R2} + 4 V −	E_{R2} + **8 V** − *4 V + 4 V = 8 V*
4 V, + −, E_{R3}	3 V, − +, E_{R3}	**1 V**, + −, E_{R3} *4 V - 3 V = 1 V*

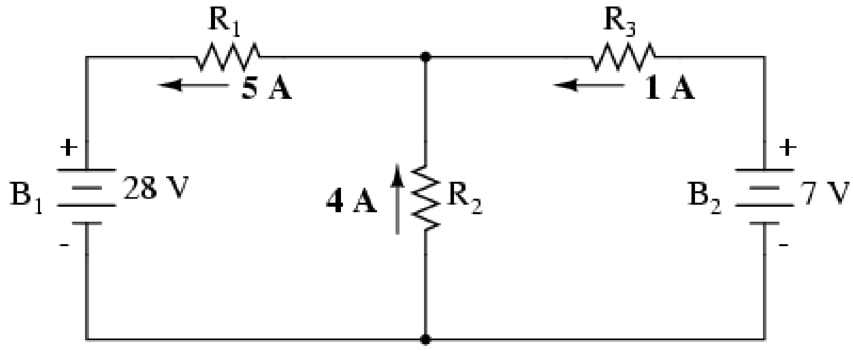
- Applying these superimposed voltage figures to the circuit, the end result looks something like this:



- Currents add up algebraically as well, and can either be superimposed as done with the resistor voltage drops, or simply calculated from the final voltage drops and respective resistances (I=E/R). Either way, the answers will be the same. Here we will show the superposition method applied to current:

With 28 V battery	*With 7 V battery*	*With both batteries*
← 6 A I_{R1}	→ 1 A I_{R1}	I_{R1} ← **5 A** *6 A - 1 A = 5 A*
I_{R2} ↑ 2 A	I_{R2} ↑ 2 A	I_{R2} ↑ **4 A** *2 A + 2 A = 4 A*
← 4 A I_{R3}	→ 3 A I_{R3}	I_{R3} ← **1 A** *4 A - 3 A = 1 A*

- Once again applying these superimposed figures to our circuit:



Note:

- Superposition Theorem works only for circuits that are reducible to series/parallel combinations for each of the power sources at a time; thus, this theorem is useless for analyzing an unbalanced bridge circuit, and it only works to linear networks where current is linearly related to voltage as per Ohm's law.
- The requisite of linearity means that Superposition Theorem is only applicable for determining voltage and current, *not power!!!* Power dissipations, being nonlinear functions, do not algebraically add to an accurate total when only one source is considered at a time. The need for linearity also means this Theorem cannot be applied in circuits where the resistance of a component changes with voltage or current. Hence, networks containing components like lamps (incandescent or gas-discharge) or varistors could not be analyzed.
- Another prerequisite for Superposition Theorem is that all components must be "bilateral," meaning that they behave the same with electrons flowing either direction through them.