Chapter 2: Number Systems and Codes

Lesson 2.2: Binary Arithmetic

Computer Fundamentals

Second Edition

On completion of this lesson you will know:

- Basic concepts of binary arithmetic
- Details of step by step binary addition, subtraction, multiplication and division
- Additive method of binary subtraction

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Binary Addition Rule

Two input binary addition

Input	Input 2	Sum	Carry
0	0	0	0 (No Carry)
0	1	0	0 (No Carry)
1	0	0	0 (No Carry)
1	1	0	1 (Carry)

Three input binary addition

Input	Input 2	Input 3	Sum	Carry
1	1	1	0	1

Binary Addition Rule

Addition of the binary numbers involves the following steps-

1. Start addition by adding the bits in unit column (the rightmost column). Use the rules of binary addition.
2. The result of adding bits of a column is a sum with or without a carry.
3. Write the sum in the result of that column. If carry is present, the carry is carried-over to the addition of the next left column.
4. Repeat steps 2-4 for each column and so on.

Example 2.2.1

Add 10 and 01. Verify the answer with the help of decimal addition.

	Binary addition	Decimal addition
	10	2
	+ 01	+ 1
Result	11	3

$$\boxed{\text{Thus, } 11_2 = 3_{10}}$$

Example 2.2.2

Add 01 and 11. Verify the answer with the help of decimal addition.

	Binary addition	Decimal addition
Carry →	11	
	01	1
	+ 11	+ 3
Result	100	4

$\boxed{\text{Thus, } 100_2 = 4_{10}}$

Example 2.2.3

Add 11 and 11. Verify the answer with the help of decimal addition.

Binary addition		Decimal addition
Carry	$\longrightarrow$ 11	
	11	3
	+ 11	+ 3
Result	110	6

$$\boxed{\text{Thus, } 110_2 = 6_{10}}$$

Example 2.2.4

Add 10111, 11100 and 11. Verify the answer with the help of decimal addition.

	Binary addition		Decimal addition
	11111	→ Carry	
	10111		23
	11100		28
+	11		+ 3
Result	110110		54

$$\boxed{\text{Thus, } 110110_2 = 54_{10}}$$

Binary Subtraction Rule

Two input binary subtraction

Input	Input 2	Difference	Borrow
0	0	0	0 (No Borrow)
0	1	1	1 (Borrow)
1	0	1	0 (No Borrow)
1	1	0	0 (No Borrow)

Binary Subtraction Rule

The steps for performing subtraction of the binary numbers are as follows

1. Start subtraction by subtracting the bit in the lower row from the upper row, in the unit column.
2. Use the binary subtraction rules. If the bit in the upper row is less than lower row, borrow 1 from the upper row of the next column (on the left side). The result of subtraction of two bits is the difference.
3. Write the difference in the result of that column.
4. Repeat step 2-3 for each column and so on.

Example 2.2.5

Subtract 01 from 11. Verify the answer with the help of decimal subtraction.

	Binary subtraction	Decimal subtraction
	11	3
	- 01	- 1
Result	10	2

$\boxed{\text{Thus, } 10_2 = 2_{10}}$

Example 2.2.6

Subtract 10011111 from 10101001. Verify the answer with the help of decimal subtraction.

	Binary Subtraction	Decimal Subtraction
Borrow	01 01	
	10101001	169
	- 10011111	- 159
Result	00001010	10

$$\boxed{\text{Thus, } 1010_2 = 10_{10}}$$

Additive Method of Binary Subtraction

Additive Method of Binary Subtraction: This method is called complement method. The following steps are involved:

- Find the complement of subtrahend.
- Add results of step 1 to the minuend.
- If a carry is obtained, add it to obtain the result, else recomplement the sum and attach a negative sign to obtain the result.

Example 2.2.7

Subtract 110111 from 11001 using additive approach.

Step 1: Here the complement of subtrahend 10111 is 01000
Step 2: Here the minuend is 11001.

$$
\begin{array}{rl}
 & 11001 \\
+ & 01000 \\
\hline
 & 00001 \text{ (with carry of 1)}
\end{array}
$$

Step 3: Add carry to obtain the result, i.e., $1+1=10_2$

The whole process is shown as

The complement of 10111 is 01000

$$
\begin{array}{rl}
 & 11001 \\
+ & 01000 \\
\hline
 & 00001 \text{ (with carry of 1)} \\
+ & 1 \\
\hline
 & 10
\end{array}
$$

It is actually much simpler than decimal multiplication. In the case of decimal multiplication, we need to remember $3 \times 9 = 27$, $7 \times 8 = 56$, and so on.
In binary multiplication, we only need to remember the following Table

Input	Input 2	Multiplication
0	0	0
0	1	0
1	0	0
1	1	1

Find the binary multiplication of 101 and 11,

$$\begin{array}{r} 101 \\ \times 11 \\ \hline \end{array}$$

First we multiply 101 by 1, which produces 101. Then we put a 0 as a placeholder as v
multiplication, and multiply 101 by 1, which produces 101.

$$\begin{array}{r} 101 \\ \times 11 \\ \hline 101 \\ 101\boxed{0} \\ \hline \end{array} \longrightarrow \text{Place holder}$$

The next step is to add. The result(s) from our previous step indicates that we must ad
sum of which is 1111.

$$\begin{array}{r} 101 \\ \times 11 \\ \hline 101 \\ 1010 \\ \hline 1111 \end{array}$$

Rules for performing binary division

The method for binary division is as follows:

- $\frac{0}{0} = 0$
- $\frac{1}{1} = 1$

1. Starting from left compare divisor with dividend
2. If dividend is greater, take value of the quotient 1 and subtract the divisor from the corresponding digits of dividend.
3. If dividend is less, take value of the quotient 0 and repeat whole process till sufficient digits in dividend

Example 2.2.9

Find $\frac{110011}{101}$

$$
\begin{array}{llll}
 & & 1010 & \text{(Quotient)} \\
\cline{3-3}
\text{(Divisor)} & 101 & \multicolumn{1}{|l}{110011} & \text{(Dividend)} \\
\cline{2-2}
 & & 101 & \\
\cline{3-3}
 & & 0010 & \\
 & & 000 & \\
\cline{3-3}
 & & 00101 & \\
 & & 101 & \\
\cline{3-3}
 & & 0001 & \text{(Remainder)}
\end{array}
$$