# Chapter 2: Number Systems and Codes

## Lesson 1.1: Number Systems

Computer Fundamentals

Second Edition

On completion of this lesson you will know:

- Basic concepts of different number systems
- Characteristics of decimal, binary, octal and hexadecimal numbers
- Conversion of numbers

In logical design, however, it is necessary to perform manipulations in the binary system of numbers because of the on-off nature of the physical devices used. The popular number systems are:

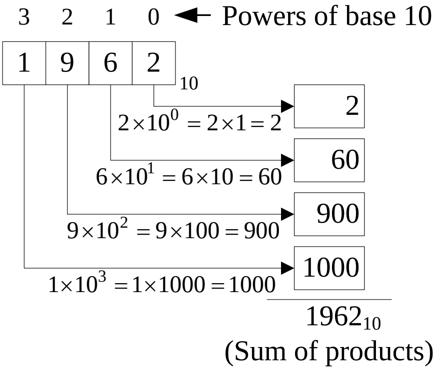
- Decimal number system
- Binary number system
- Octal number system
- Hexadecimal number system

It is the most commonly used number system in real life. It has the following features:

- Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (total ten digits)
- Base (radix): 10 (ten)
- Weights: 1, 10, 100, 1000, (powers of base 10)

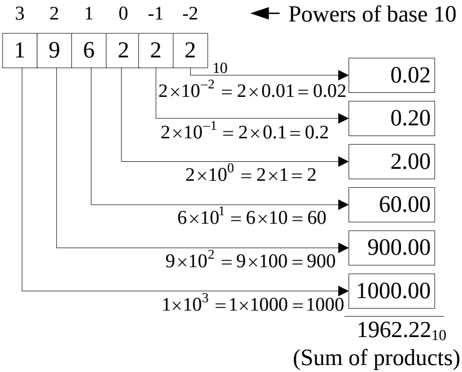
# Example

Find the binary equivalent of the decimal number 1962.



# Example

Find the binary equivalent of the decimal number 1962.22.

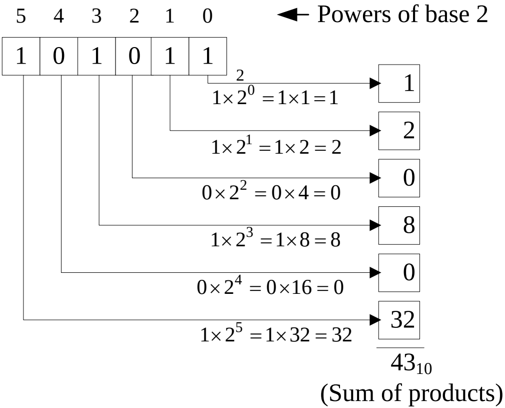


Digital computers use binary numbers for internal operations. It has the following features:

- Digits: 0, 1(total two digits)
- Base (radix): 2
- Weights: 1, 2, 4, 8, 16, and so on (powers of base 2)

# Example

Find the decimal equivalent of the binary 101011.



Octal numbers are used in some programming languages. It has the following features:

- Digits: 0, 1, 2, 3, 4, 5, 6, 7 (total eight digits)
- Base (radix): 8
- Weights: 1, 8, 64, 512, and so on (powers of base 8)

Hexadecimal (also called hex in short) is a propositional numeral system. Hexadecimal is commonly used to represent computer memory addresses. It has the following features:

- Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F (total sixteen digits)
- Base (radix): 16
- Weights: 1, 16, 256, and so on (powers of base 16)

To convert a decimal number to its other equivalent numbers, the remainder method can be used. (This method can be used to convert a decimal number into any other base.) The remainder method involves the following four steps:

- Divide the decimal number by the base (in the case of binary, divide by 2).
- Indicate the remainder to the right.
- Continue dividing into each quotient (and indicating the remainder) until the divide operation produces a zero quotient.
- The base 2 number is the numeric remainder reading from the last division to the first.

# Example 2.1.1

Convert $47_{10}$ to its binary equivalent

| Integer quotient | Reminder | coefficient |
|---|---|---|
| $2 \overline{)47}$ = 23 | 1 | $a_0 = 1$ |
| $2 \overline{)23}$ = 11 | 1 | $a_1 = 1$ |
| $2 \overline{)11}$ = 5 | 1 | $a_2 = 1$ |
| $2 \overline{)5}$ = 2 | 1 | $a_3 = 1$ |
| $2 \overline{)2}$ = 1 | 0 | $a_4 = 0$ |
| $2 \overline{)1}$ = 0 | 1 | $a_5 = 1$ |

Thus, $47_{10}=101111_2$

# Example 2.1.2

Convert $47_{10}$ to its octal equivalent

| Integer quotient | Reminder | coefficient |
|---|---|---|
| $8 \overline{)47}$ = 5 | 7 | $a_0 = 7$ |
| $8 \overline{)5}$ = 0 | 5 | $a_1 = 5$ |

Thus, $47_{10}$=$57_8$

# Example 2.1.3

Convert $47_{10}$ to its hexadecimal equivalent

| Integer quotient | Reminder | coefficient |
| --- | --- | --- |
| $16 \overline{)\,47\,}$ → 2 | F | $a_0 = \text{F}$ |
| $16 \overline{)\,2\,}$ → 0 | 2 | $a_1 = 2$ |

Thus, $47_{10} = 2\text{F}_{16}$

The rule is as follows:

- Starting from the right of the given binary stream into group of three, if leftmost group has fewer bits, attach the required number of leading 0s to complete the group.
- Determine equivalent octal digit for each group.

# Example 2.1.4

Find the octal number of $100110111_2$

Step 1:

100 110 111

Step 2:

$100_2 = (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) = 4_8$

$110_2 = (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 6_8$

$111_2 = (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 7_8$

Thus the octal number of $100110111_2$ is $467_8$

The rule is as follows:

- Find the equivalent binary group of 3 digits for each octal digit.
- Find the results by combining binary groups

# Example 2.1.5

Find the binary number of $467_8$
Step 1:
$4_8 = 100_2 \;\; 6_8 = 110_2 \;\; 7_8 = 111_2$
Step 2:
$100110111_2$ is the binary equivalent of $467_8$

The rule is as follows:

- Starting from the right of the given binary stream into group of four, if leftmost group has fewer bits, attach the required number of leading 0s to complete the group.
- Determine equivalent one hexadecimal digit for each group.

# Example 2.1.6

Find the hexadecimal equivalent number of $10011011_2$
Step 1:1001   1011
Step 2:
$1001_2 = (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 9_{16}$
$1011_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = B_{16}$
Thus the binary number of $9B_{16}$ is $10011011_2$

The rule is as follows:

- Find the equivalent binary group of 4 digits for each hexadecimal digit.
- Find the results by combining binary groups.

# Example 2.1.7

Find the binary equivalent number of $9B_{16}$
Step 1: $9_{16} = 1001_2 \quad B_{16}1011_2$
Step 2: $10011011_2$ is the binary number of $9B_{16}$ is