DC Network Theorem (Continue….)

Constant-Current Sources:

- All the circuits presented so far have used voltage sources as the means of providing power.
- However, the analysis of certain circuits is easier if you work with current rather than with voltage.
- Unlike a voltage source, a constant-current source maintains the same current in its branch of the circuit regardless of how components are connected external to the source.
- The magnitude and the direction of current through a voltage source vary according to the size of the circuit resistances and how other voltage sources are connected in the circuit. For current sources, the voltage across the current source depends on how the other components are connected.
- The symbol for a constant-current source is shown in the figure below. The direction of the current source arrow indicates the direction of conventional current in the branch.

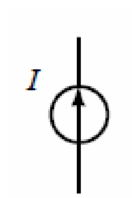


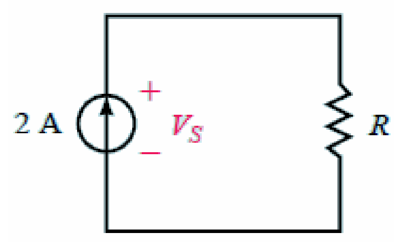
Figure: Symbol of ideal constant current source

(An ideal current source has an infinite shunt resistance)

- Voltage sources always have some series resistance, although in some cases this resistance is so small in comparison with other circuit resistance that it may effectively be ignored when determining the operation of the circuit. Similarly, a constant-current source will always have some shunt (or parallel) resistance. If this resistance is very large in comparison with the other circuit resistance, the internal resistance of the source may once again be ignored. An ideal current source has an infinite shunt resistance.

Example:

Calculate the voltage V_S across the current source if the resistor is 100 Ω.



Solution:
The current source maintains a constant current of 2 A through the circuit. Therefore, $V_S=V_R=(2\ A)(100\ \Omega)= 200\ V$.

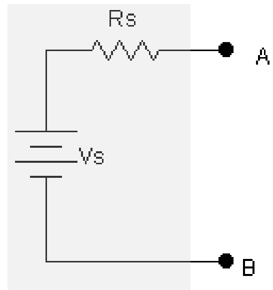
Source Conversion:

- When we are trying to analyze complicated circuits, a few tricks can greatly simplify and ease our task. One of the tricks is source conversion which lets us replace a voltage source with a current source, or vice versa.
- If the internal resistance of a source is considered, the source, whether it is a voltage source or a current source, is easily converted to the other type.
  - A given voltage source in series with a resistance can be converted or replaced by an equivalent current source in parallel with the resistance.
  - Conversely, a current source in parallel with a resistance can be converted into a voltage source in series with the resistance.
- Two sources are equivalent if, for any load resistor connected to the two sources, they produce the same voltage across that resistor and the same current through it.

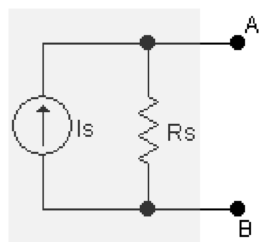
Converting a Voltage Source to a Current Source:

- A given voltage source in series with a resistance can be converted or replaced by an equivalent current source in parallel with the resistance.
- Suppose we are analyzing a circuit that contains a practical voltage source with source voltage V_S and internal resistance R_S. We can replace this voltage source with a practical current source having the same internal

resistance and having a source current of $I_S = V_S / R_S$. In other words, the practical voltage source shown here



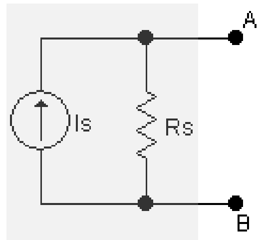
can be replaced by the practical current source shown here



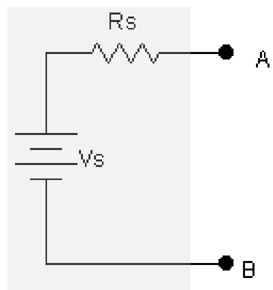
if both have the same internal resistance R_S and $I_S = V_S/R_S$.

Converting a Current Source to a Voltage Source:

- A current source in parallel with a resistance can be converted into a voltage source in series with the resistance.
- Now let's go in the other direction. Suppose we are analyzing a circuit that contains a practical current source with source current I_S and internal resistance R_S. We can replace this current source with a practical voltage source having **the same internal resistance** and having a source voltage of $V_s = I_S \times R_S$. In other words, the practical current source shown here



can be replaced by the practical voltage source shown here



if both have the same internal resistance R_S and $V_S = I_S \times R_S$.

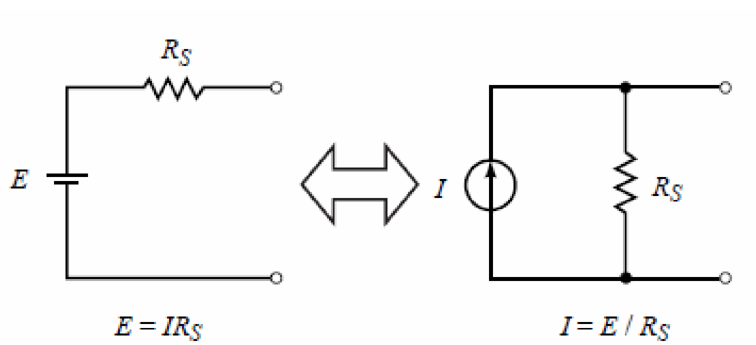
Proof:

- Two sources are equivalent if, for any load resistor connected to the two sources, they produce the same voltage across that resistor and the same current through it. That is if

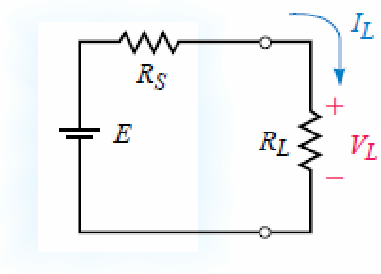
$$E=IR_S \quad \text{...............(1)}$$

and

$$I=E/R_S \quad \text{............ (2)}$$



- These results may be easily verified by connecting an external resistance, R_L, across each source. The sources can be equivalent only if the voltage across R_L is the same for both sources. Similarly, the sources are equivalent only if the current through R_L is the same when connected to either source.
- Consider the circuit shown in figure below.



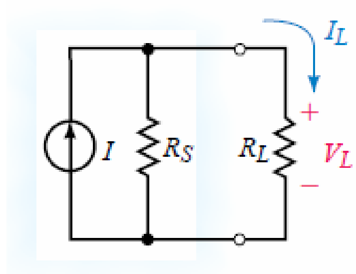
- The voltage across the load resistor is given as-

$$V_L = \frac{R_L}{R_L + R_S} E \qquad \text{....... (3)}$$

- The current through the resistor R_L is given as

$$I_L = \frac{E}{R_L + R_S} \qquad \text{....... (4)}$$

- Next, consider an equivalent current source connected to the same load as shown in figure below.



- The current through the resistor R_L is given by-

$$I_L = \frac{R_S}{R_S + R_L} I$$ (according to current-divider rule)

- But, when converting the source, we get

$$I = \frac{E}{R_S}$$

- And so

$$I_L = \left(\frac{R_S}{R_S + R_L}\right)\left(\frac{E}{R_S}\right)$$

- This result is equivalent to the current obtained in Equation 4.

- The voltage across the resistor is given as

$$V_L = I_L R_L$$
$$= \left(\frac{E}{R_S + R_L}\right) R_L$$

- The voltage across the resistor is precisely the same as the result obtained in Equation 3.

- We therefore conclude that the load current and voltage drop are the same whether the source is a voltage source or an equivalent current source.

Example-1:

Convert the voltage source of figure-A below into a current source and verify that the current, I_L, through the load is the same for each source.

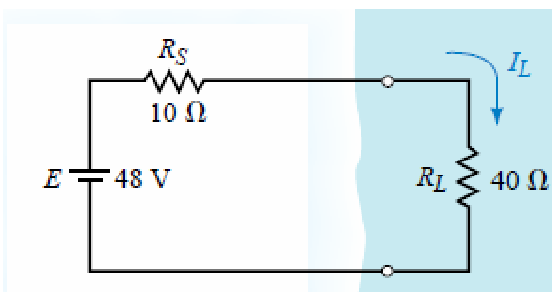


Figure-A

Solution:

The equivalent current source will have a current magnitude given as

$$I = \frac{48\ \text{V}}{10\ \Omega} = 4.8\ \text{A}$$

The resulting circuit is shown in the figure-B below.

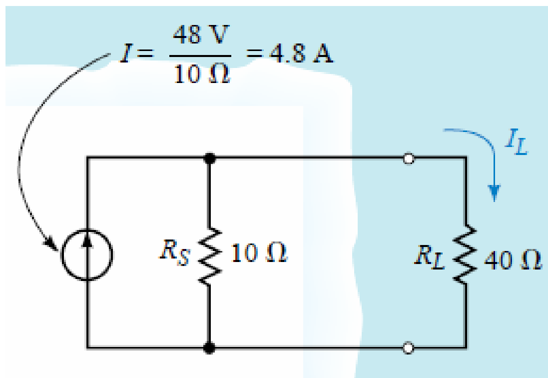


Figure-B

For the circuit of figure-A, the current through the load is found as-

$$I_L = \frac{48\ \text{V}}{10\ \Omega + 40\ \Omega} = 0.96\ \text{A}$$

For the equivalent circuit of Figure-B, the current through the load is-

$$I_L = \frac{(4.8\ \text{A})(10\ \Omega)}{10\ \Omega + 40\ \Omega} = 0.96\ \text{A}$$

Clearly the results are the same.

Example-2:

Convert the current source of figure-C below into a voltage source and verify that the voltage, V_L, across the load is the same for each source.

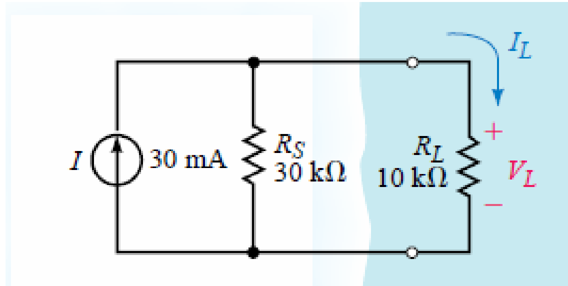


Figure-C

Solution:

The equivalent voltage source will have a magnitude given as

$$E = (30 \text{ mA})(30 \text{ k}\Omega) = 900 \text{ V}$$

The resulting circuit is shown in Figure-D.

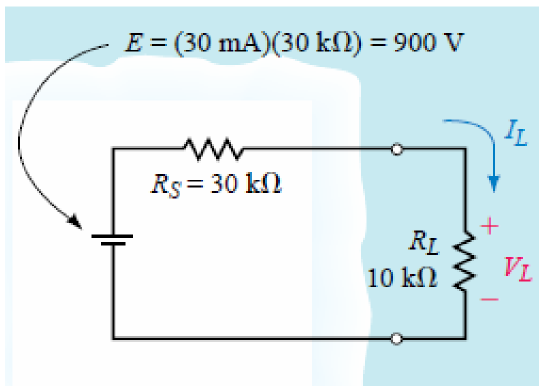


Figure-D

For the circuit of Figure-C, the voltage across the load is determined as

$$I_L = \frac{(30 \text{ k}\Omega)(30 \text{ mA})}{30 \text{ k}\Omega + 10 \text{ k}\Omega} = 22.5 \text{ mA}$$
$$V_L = I_L R_L = (22.5 \text{ mA})(10 \text{ k}\Omega) = 225 \text{ V}$$

For the equivalent circuit of Figure-D, the voltage across the load is

$$V_L = \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 30 \text{ k}\Omega}(900 \text{ V}) = 225 \text{ V}$$

Once again, we see that the circuits are equivalent.

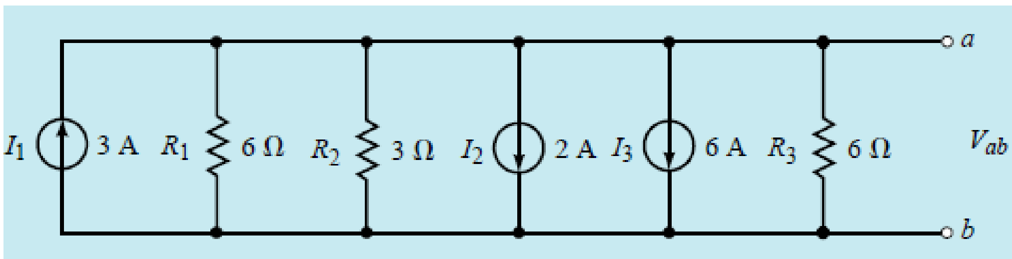
More Examples: 2.41, 2.42, 2.43 (Page: 94-95)

Current Sources in Parallel and Series

When several current sources are placed in parallel, the circuit may be simplified by combining the current sources into a single current source. The magnitude and direction of this resultant source is determined by adding the currents in one direction and then subtracting the currents in the opposite direction.

Example:

Simplify the circuit of Figure below and determine the Voltage *Vab*.



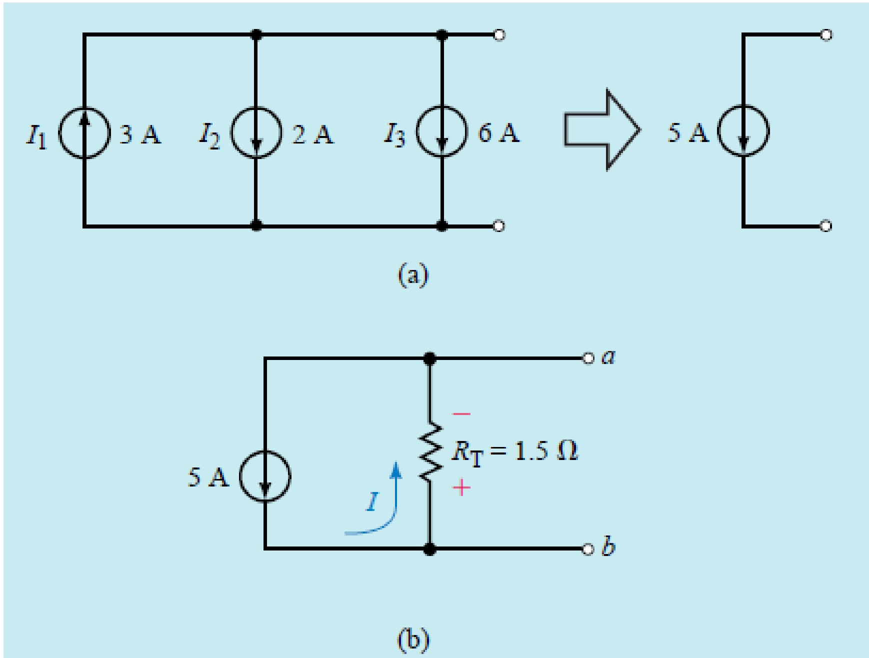
Solution:

- Since all of the current sources are in parallel, they can be replaced by a single current source.
- The equivalent current source will have a direction which is the same as both I_2 and I_3, since the magnitude of current in the downward direction is greater than the current in the upward direction.
- The equivalent current source has a magnitude of

  $$I = 2\text{ A} + 6\text{ A} - 3\text{ A} = 5\text{ A}$$

  as shown in Figure (a).
- The circuit is further simplified by combining the resistors into a single value:

  $$R_T = 6\ \Omega \| 3\ \Omega \| 6\ \Omega = 1.5\ \Omega$$
- The equivalent circuit is shown in Figure (b).

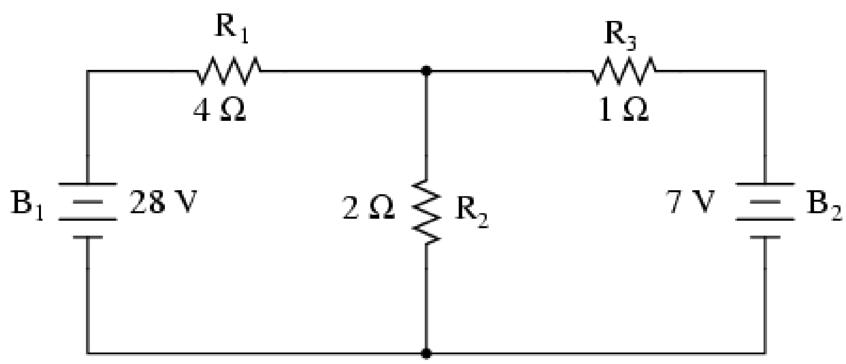


- The voltage *Vab* is found as

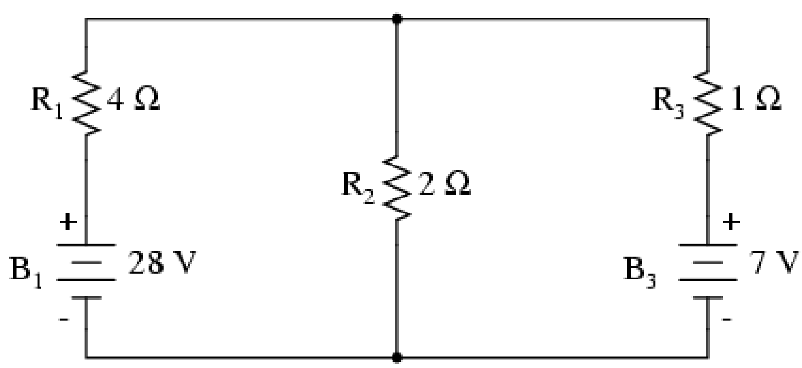
$$V_{ab} = -(5\text{ A})(1.5\ \Omega) = -7.5\text{ V}$$

Millman's Theorem:

- Millman's Theorem treats circuits as a set of parallel-connected branches, each branch with its own voltage source and series resistance. That is, the given circuit is re-drawn as a parallel network of branches, each branch containing a resistor or series battery/resistor combination.
- This theorem is used for finding the common voltage across each parallel branch.
- Millman's Theorem is applicable only to those circuits which can be re-drawn accordingly. Here again is our example circuit used for the last two analysis methods:



- And here is that same circuit, re-drawn for the sake of applying Millman's Theorem:



- By considering the supply voltage within each branch and the resistance within each branch, Millman's Theorem will tell us the common voltage across all branches. (N.B. Please note that I have labeled the battery in the rightmost branch as "B_3" to clearly denote it as being in the third branch, even though there is no "B_2" in the circuit).
- The value of the common voltage across each branch is determined by the Millman's equation:

$$\text{Voltage across all branches} = \frac{\dfrac{E_{B1}}{R_1} + \dfrac{E_{B2}}{R_2} + \dfrac{E_{B3}}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

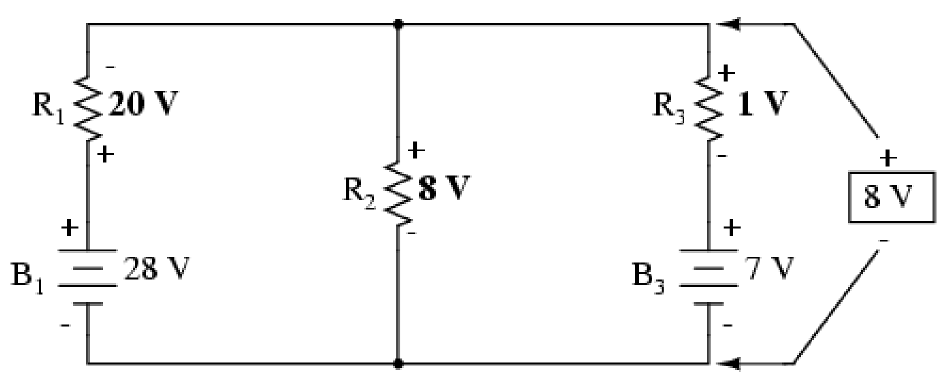
$$= \frac{I_{R1} + I_{R2} + I_{R3}}{G_1 + G_2 + G_3}$$

$$= \frac{\Sigma I}{\Sigma G}$$

- Substituting actual voltage and resistance figures from our example circuit for the variable terms of this equation, we get the following expression:

$$\frac{\dfrac{28\text{ V}}{4\,\Omega} + \dfrac{0\text{ V}}{2\,\Omega} + \dfrac{7\text{ V}}{1\,\Omega}}{\dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega} + \dfrac{1}{1\,\Omega}} = 8\text{ V}$$

- The final answer of 8 volts is the voltage seen across all parallel branches, like this:



- The polarity of all voltages in Millman's Theorem are referenced to the same point. In the example circuit above, I used the bottom wire of the parallel circuit as my reference point, and so the voltages within each branch (28 for the R_1 branch, 0 for the R_2 branch, and 7 for the R_3 branch) were inserted into the equation as positive numbers.

- Likewise, when the answer came out to 8 volts (positive), this meant that the top wire of the circuit was positive with respect to the bottom wire (the original point of reference). If both batteries had been connected backwards (negative ends up and positive ends down), the voltage for branch 1 would have been entered into the equation as a -28 volts, the voltage for branch 3 as -7 volts, and the resulting answer of -8 volts would have told us that the top wire was negative with respect to the bottom wire (our initial point of reference).

- To solve for resistor voltage drops, the Millman voltage (across the parallel network) must be compared against the voltage source within each branch, using the principle of voltages adding in series to determine the magnitude and polarity of voltage across each resistor:

$$E_{R1} = 8\text{ V} - 28\text{ V} = -20\text{ V} \quad \text{(negative on top)}$$

$$E_{R2} = 8\text{ V} - 0\text{ V} = 8\text{ V} \quad \text{(positive on top)}$$

$$E_{R3} = 8\text{ V} - 7\text{ V} = 1\text{ V} \quad \text{(positive on top)}$$

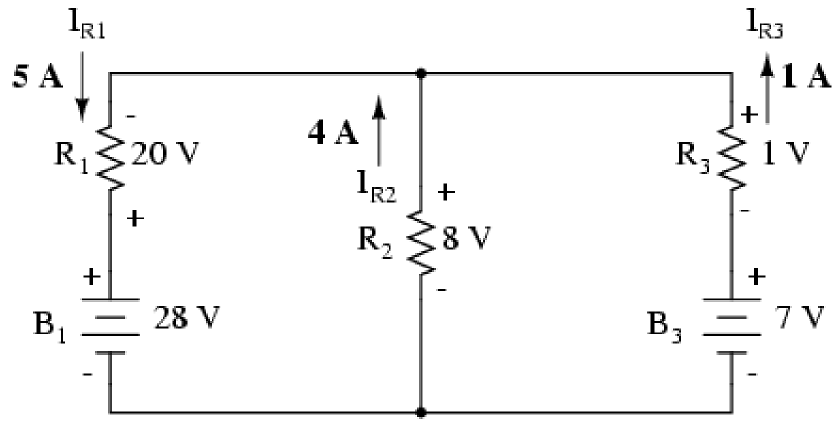
- To solve for branch currents, each resistor voltage drop can be divided by its respective resistance (I=E/R):

$$I_{R1} = \frac{20\text{ V}}{4\ \Omega} = 5\text{ A}$$

$$I_{R2} = \frac{8\text{ V}}{2\ \Omega} = 4\text{ A}$$

$$I_{R3} = \frac{1\text{ V}}{1\ \Omega} = 1\text{ A}$$

- The direction of current through each resistor is determined by the polarity across each resistor, *not* by the polarity across each battery, as current can be forced backwards through a battery, as is the case with B_3 in the example circuit. This is important to keep in mind, since Millman's Theorem doesn't provide as direct an indication of "wrong" current direction as does the Branch Current or Mesh Current methods. You must pay close attention to the polarities of resistor voltage drops as given by Kirchhoff's Voltage Law, determining direction of currents from that.



- Millman's Theorem is very convenient for determining the voltage across a set of parallel branches, where there are enough voltage sources present to preclude solution via regular series-parallel reduction method.
- It also is easy in the sense that it doesn't require the use of simultaneous equations.
- However, it is limited in that it only applied to circuits which can be re-drawn to fit this form. It cannot be used, for example, to solve an unbalanced bridge circuit. And, even in cases where Millman's Theorem can be applied, the solution of individual resistor voltage drops can be a bit daunting to some, the Millman's Theorem equation only providing a single figure for branch voltage.

Thevenin's Theorem:

- Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex it is, to an equivalent circuit with just a **single voltage source (V_{TH})** and **series resistance (R_{TH})** which is **connected to a load (R_L)** which is looked like this:

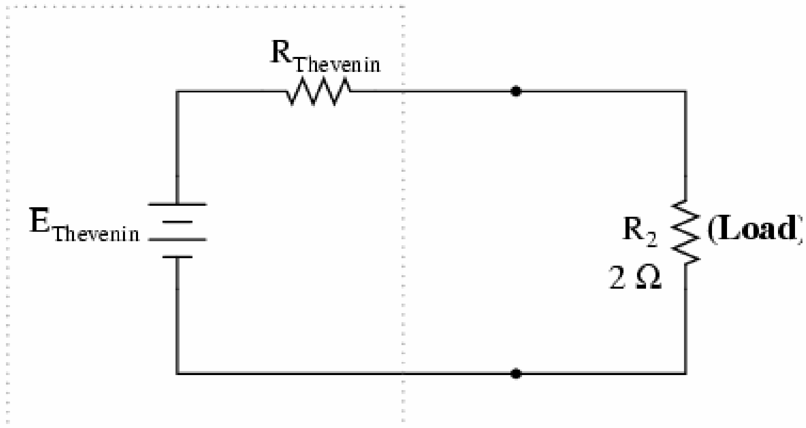


Figure: Thevenin equivalent Circuit

- Thevenin's Theorem is especially useful in analyzing power systems and other circuits where one particular resistor in the circuit (called the "load" resistor) is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it.
- Consider the circuit given below. We apply Thevenin's Theorem to it:

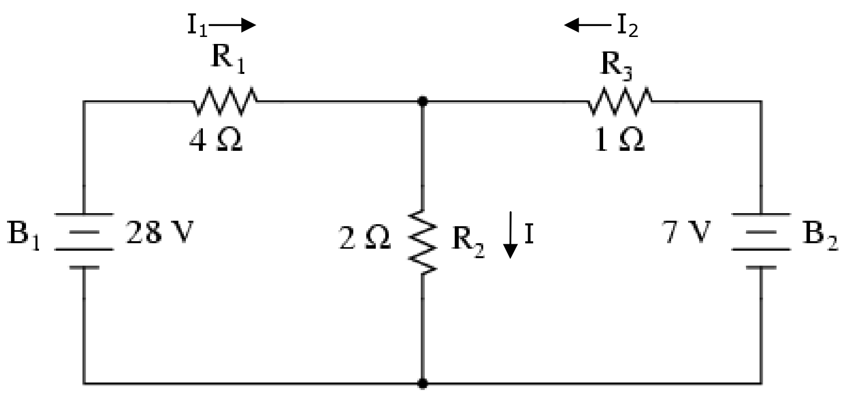
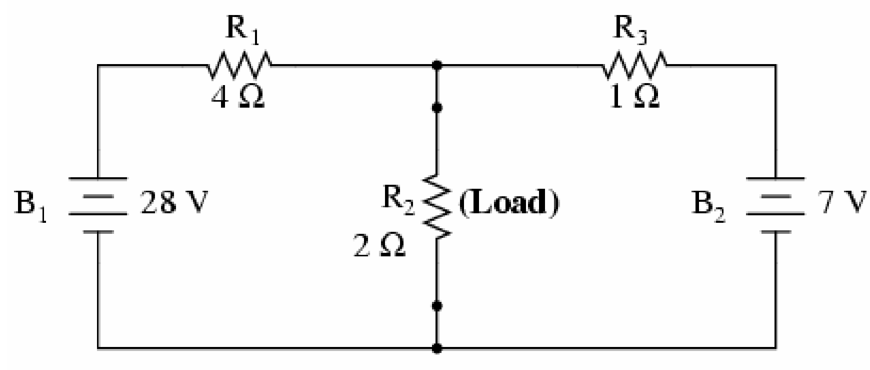


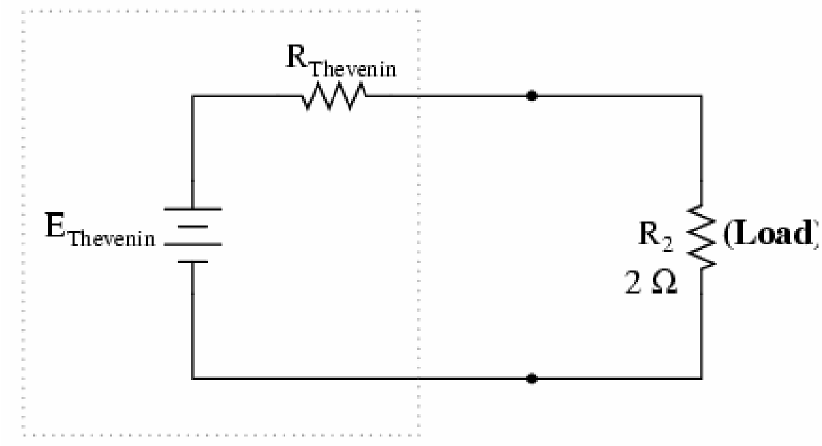
Figure-1: Original Circuit

- In the circuit above, I_1, I_2 and I represent the values of currents which are due to the simultaneous action of the two sources of emf in the network.

- Let us assume that we decide to designate R2 as the "load" resistor in this circuit.



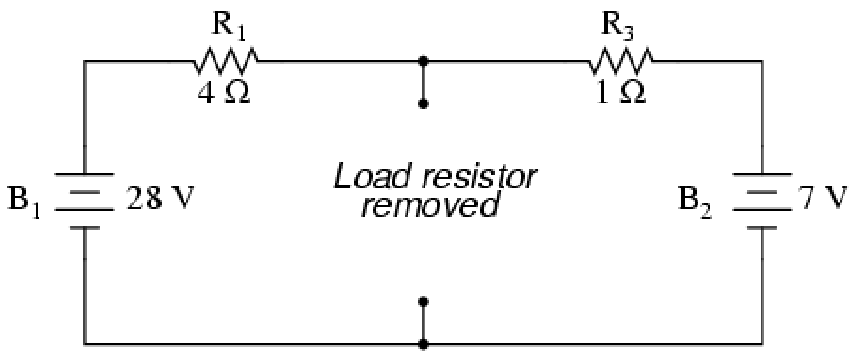
- We already have discussed Superposition Theorem by which we can determine voltage across R2 and current through R2, but this method is time-consuming. Imagine repeating this method over and over again to find what would happen if the load resistance changed (changing load resistance is very common in power systems, as multiple loads get switched on and off as needed, the total resistance of their parallel connections changing depending on how many are connected at a time). This could potentially involve a lot of work!
- Thevenin's Theorem makes this easy by temporarily removing the load resistance from the original circuit and reducing what is left to an equivalent circuit composed of a single voltage source and series resistance. The load resistance can then be re-connected to this "Thevenin equivalent circuit" and calculations carried out as if the whole network were nothing but a simple series circuit.
- After Thevenin conversion, the Thevenin equivalent circuit is:



- The "Thevenin Equivalent Circuit" is the electrical equivalent of B1, R1, R3, and B2 as seen from the two points where our load resistor (R2) connects.
- The Thevenin equivalent circuit, if correctly derived, will behave exactly the same as the original circuit formed by B1, R1, R3, and B2. In other words, the load resistor (R2) voltage and current should be exactly the same for the same value of load resistance in the two circuits. The load resistor R2 cannot "tell the difference" between the original network of B1, R1, R3, and B2, and the Thevenin equivalent circuit of $E_{Thevenin}$, and $R_{Thevenin}$, provided that the values for $E_{Thevenin}$ and $R_{Thevenin}$ have been calculated correctly.
- The advantage in performing the "Thevenin conversion" to the simpler circuit, of course, is that it makes load voltage and load current so much easier to solve than in the original network. Calculating the equivalent Thevenin source voltage and series resistance is actually quite easy.

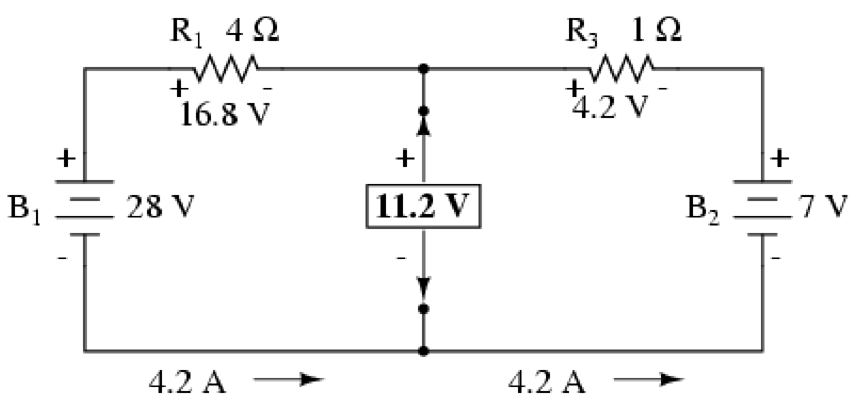
Performing Thevenin Conversion:

- First, remove load resistor R_L: the chosen load resistor is removed from the original circuit, replaced with a break (open circuit):

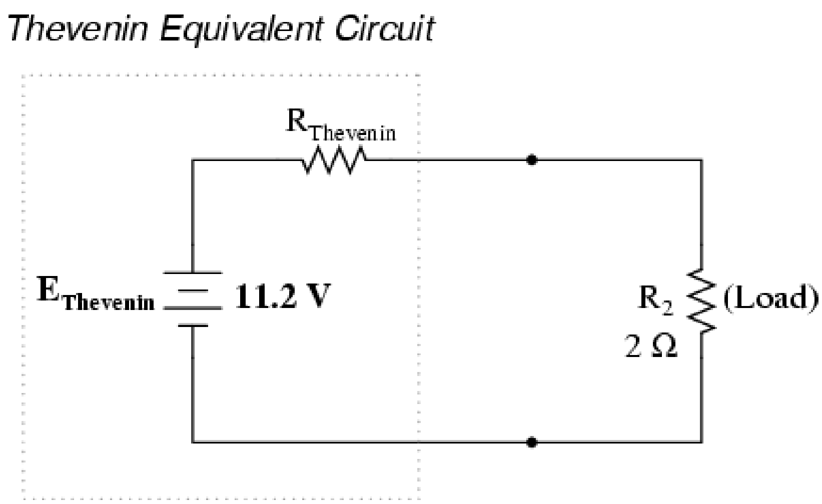


- Next, determine Thevenin voltage $E_{Thevenin}$: calculate the voltage between the two points where the load resistor used to be attached is determined. Use whatever analysis methods are at your disposal to do this. In this case, the original circuit with the load resistor removed is nothing more than a simple series circuit with opposing batteries, and so we can determine the voltage across the open load terminals by applying the rules of series circuits, Ohm's Law, and Kirchhoff's Voltage Law:

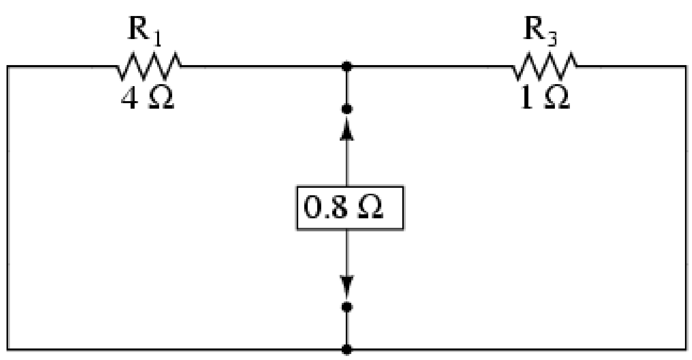
	R_1	R_3	Total	
E	16.8	4.2	21	Volts
I	4.2	4.2	4.2	Amps
R	4	1	5	Ohms



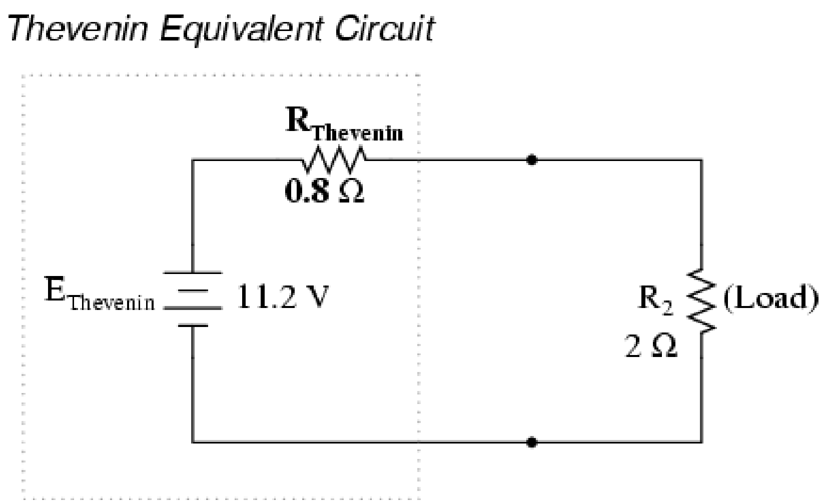
- The voltage between the two load connection points can be figured from the one of the battery's voltage and one of the resistor's voltage drops, and comes out to 11.2 volts. This is our "Thevenin voltage" ($E_{Thevenin}$) in the equivalent circuit:



- Then, determine Thevenin resistance $R_{Thevenin}$: To find the Thevenin series resistance for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure the resistance from one load terminal to the other:



- With the removal of the two batteries, the total resistance measured at this location is equal to R1 and R3 in parallel: 0.8 Ω. This is our "Thevenin resistance" ($R_{Thevenin}$) for the equivalent circuit:



- Finally, determine the current through and voltage across R_L: With the load resistor (2 Ω) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing more than a simple series circuit:

	$R_{Thevenin}$	R_{Load}	Total	
E	3.2	8	11.2	Volts
I	4	4	4	Amps
R	0.8	2	2.8	Ohms

- Notice that the voltage and current figures for R2 (8 volts, 4 amps) are identical to those found using Superposition Theorem and Kirchhoff's Circuit laws.
- Also notice that the voltage and current figures for the Thevenin series resistance and the Thevenin source (total) do not apply to any component in the original, complex circuit. Thevenin's Theorem is only useful for determining what happens to a single resistor in a network: the load.

- The advantage, of course, is that you can quickly determine what would happen to that single resistor if it were of a value other than 2 Ω without having to go through a lot of analysis again. Just plug in that other value for the load resistor into the Thevenin equivalent circuit and a little bit of series circuit calculation will give you the result.

REVIEW:

- Thevenin's Theorem is a way to reduce a network to an equivalent circuit composed of a single voltage source, series resistance, and series load.
- Steps to follow for Thevenin's Theorem:
- Find the Thevenin source voltage by removing the load resistor from the original circuit and calculating voltage across the open connection points where the load resistor used to be.
- Find the Thevenin resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.
- Draw the Thevenin equivalent circuit, with the Thevenin voltage source in series with the Thevenin resistance. The load resistor re-attaches between the two open points of the equivalent circuit.
- Analyze voltage and current for the load resistor following the rules for series circuits.