

Lecture-03: Circuit Laws- KVL and KCL

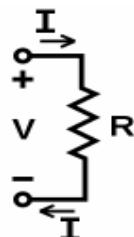
Circuit Law

Introduction:

- An electric circuit or network consists of a number of interconnected single circuit elements. The circuit will generally contain at least one voltage or current source.
- The arrangement of circuit elements results in a new set of constraints and their corresponding equations, added to the current-voltage relationships of the individual elements, provide the solution of the network.
- The underlying purpose of defining the individual elements, connecting them in network, and solving the equations is to analyze the performance of such electric devices as motors, generators, transformers etc. The solution generally answers necessary questions about the operation of the device under conditions applied by a source of energy.

Ohm's Law:

- Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference across the two points.
- Mathematically, $I=V/R$, where I is the current through the conductor in units of amperes, V is the potential difference measured across the conductor in units of volts, and R is the resistance of the conductor in units of ohms.



- More specifically, Ohm's law states that the R in this relation is constant, independent of the current.
- The law was named after the German physicist George Ohm.
- In circuit analysis, three equivalent expressions of Ohm's law are used interchangeably:

$$I = \frac{V}{R} \quad \text{or} \quad V = IR \quad \text{or} \quad R = \frac{V}{I}$$

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Kirchhoff's Circuit Law:

- A single equivalent resistance (R_T) can be found when two or more resistors are connected together either in series, parallel or combinations of both, and that these circuits obey *Ohm's Law*.
 - ◎ However, sometimes in complex circuits such as bridge or T networks, we can not simply use Ohm's Law alone to find the voltages or currents circulating within the circuit.
 - ◎ For these types of calculations, we need certain rules which allow us to obtain the circuit equations and for this we can use Kirchhoff's Circuit Law.
- In 1845, a German physicist, **Gustav Kirchhoff** developed a pair or set of rules or laws which deal with the conservation of current and energy within electrical circuits. The rules are commonly known as: *Kirchoff's Circuit Laws*:
 - One law dealing with current flow around a closed circuit, known as Kirchhoff's Current Law, (KCL)
 - Other law deals with the voltage around a closed circuit, known as Kirchhoff's Voltage Law, (KVL).
- Kirchhoff's laws are particularly useful:
 - In determining the equivalent resistance of a complicated network of conductors.
 - For calculating the currents flowing in the various conductors.

Kirchhoff's First Law - The Current Law or Point Law, (KCL):

Kirchhoff's Current Law or KCL, states that-

the "total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node".

In other words, the algebraic sum of all the currents entering and leaving a node must be equal to zero, i.e., $I_{(exiting)} + I_{(entering)} = 0$.

This idea by Kirchhoff is known as the **Conservation of Charge**.

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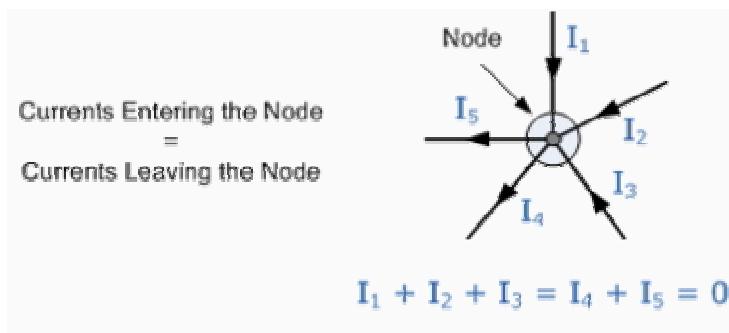


Figure: Kirchhoff's Current Law

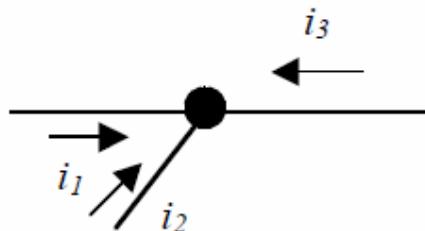
- Here, the 3 currents I_1, I_2, I_3 entering the node are all positive in value and the 2 currents I_4 and I_5 leaving the node are negative in value. Then this means we can also rewrite the equation as;

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

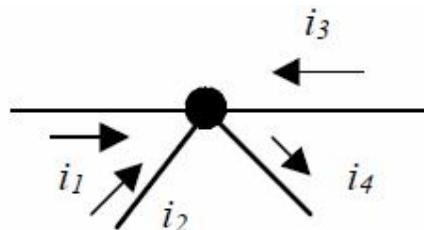
$$\text{Or, } I_1 + I_2 + I_3 = I_4 + I_5$$

Example-1:

Write the KCL equation for the principal node shown in the figures below.



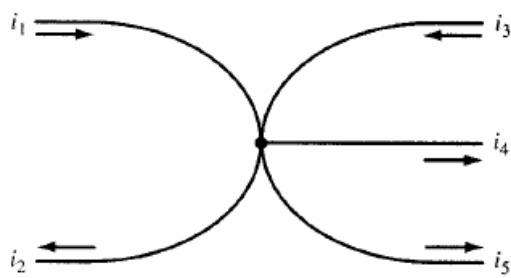
$$i_1 + i_2 + i_3 = 0$$



$$i_1 + i_2 + i_3 + (-i_4) = 0$$

$$i_1 + i_2 + i_3 = i_4$$

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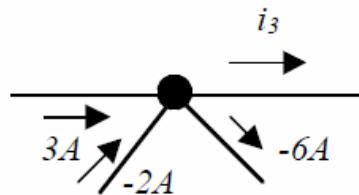


$$i_1 - i_2 + i_3 - i_4 - i_5 = 0$$

$$i_1 + i_3 = i_2 + i_4 + i_5$$

Example-2:

What is the value of i_3 ?

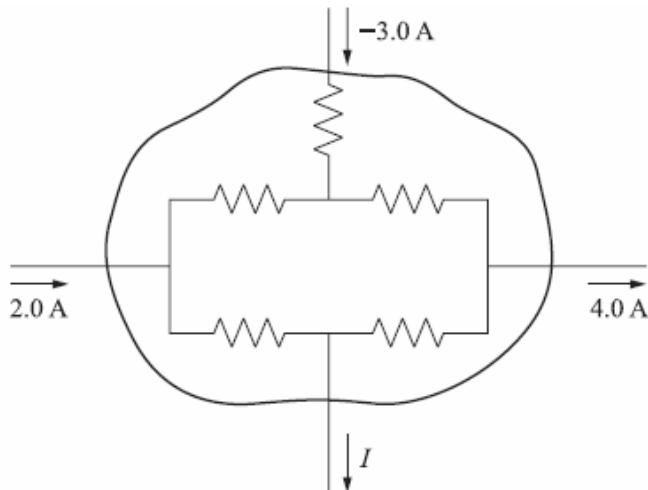
**Solution:**

$$3 + (-2) - (-6) - i_3 = 0$$

Therefore, $i_3 = 7\text{A}$

Example-3:

Find the current I for the circuit shown in the figure below.



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Solution:

The branch currents within the enclosed area cannot be calculated since no values of the resistors are given. However, KCL can be applied to the network taken it as a single node. Thus,

$$2.0 - 3.0 - 4.0 - I = 0$$

$$\text{Or, } I = -5.0 \text{ A}$$

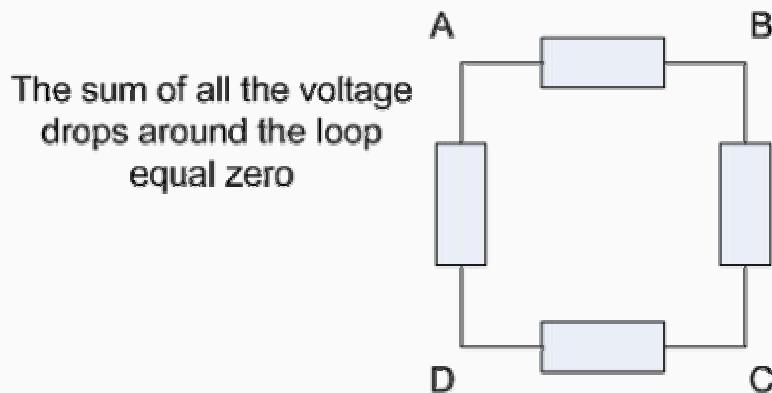
Kirchhoff's Second Law - The Voltage Law or Mesh Law, (KVL):

Kirchhoff's Voltage Law or KVL, states that-

"in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop".

In other words the algebraic sum of all voltages within the loop must be equal to zero.

This idea by Kirchhoff is known as the **Conservation of Energy**.



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

Figure: Kirchhoff's Voltage Law

- Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point.
- It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero.

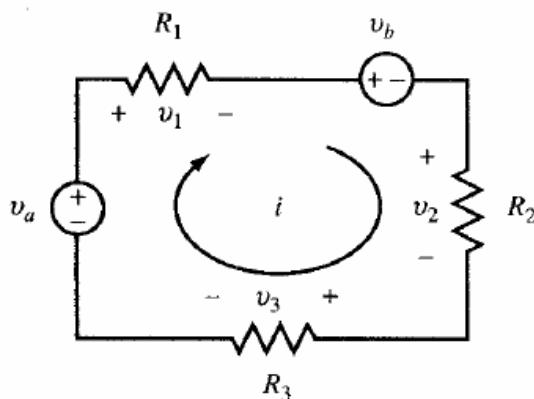
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Note:

- ⑤ KVL applies equally well to circuits driven by constant voltage or current sources, and time variable voltage or current sources.
- ⑥ The mesh current analysis method is based on the KVL.
- ⑦ We can use KVL when analyzing series circuits.

Example-1:

Write the KVL equation for the circuit given below.



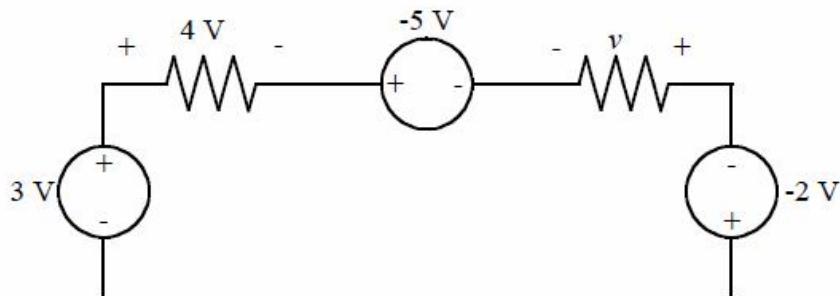
Solution:

Starting at the lower left corner of the circuit for clockwise current direction as shown, we have-

$$\begin{aligned} +v_a - v_1 - v_b - v_2 - v_3 &= 0 \\ v_a - iR_1 - v_b - iR_2 - iR_3 &= 0 \\ v_a - v_b &= i(R_1 + R_2 + R_3) \end{aligned}$$

Example-2:

What's the value of v for the circuit given below?



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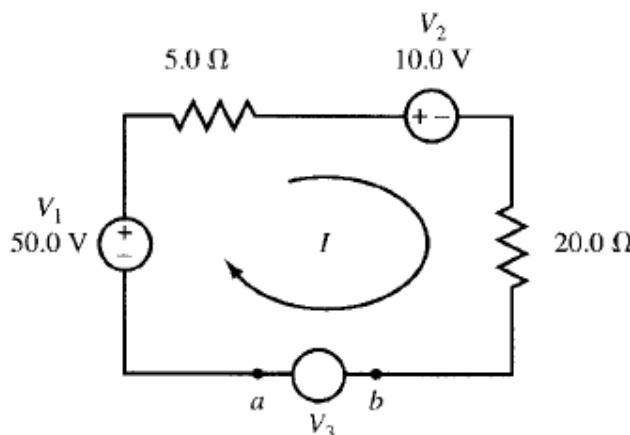
Solution:

Write KVL clockwise starting at lower left:

$$+3 - 4 - (-5) + v + (-2) = 0, \quad v = -2 \text{ V}$$

Example-3:

Find V_3 and its polarity if the current I in the circuit given below is 0.40 A.



Solution:

Assume that V_3 has the same polarity as V_1 (i.e. terminal *b* is negative and terminal *a* is positive). Applying KVL clockwise starting at lower left corner,

$$V_1 - I(5.0) - V_2 - I(20.0) + V_3 = 0$$

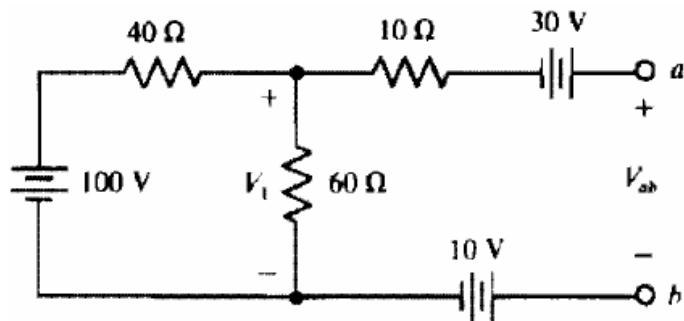
$$50.0 - 2.0 \cdot 10.0 - 8.0 + V_3 = 0$$

Therefore, $V_3 = -30.0 \text{ V}$

Since the result is negative, terminal *b* is positive with respect to terminal *a*.

Example-4:

Find the voltage V_{ab} across the open circuit shown below.



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Solution:

The $10\ \Omega$ resistor has zero current flowing through it because it is in series with an open circuit. Also it has zero volts across it. Consequently, voltage division can be used to obtain V_1 . The result is

$$V_1 = 100 \times \frac{60}{60+40} = 60V$$

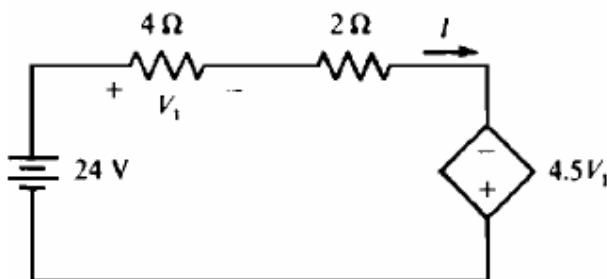
Then, applying KVL at the right-hand loop gives-

$$0 + 30 - V_{ab} - 10 + 60 = 0$$

Therefore, $V_{ab} = 80\text{ V}$

Example-5:

Calculate I for the circuit shown below.



Solution:

At first, we solve for the controlling quantity V_1 in terms of I. Applying Ohm's law to the $4\ \Omega$ resistor gives

$$V_1 = 4xI$$

Consequently, the voltage rise across the dependent source is-

$$4.5(4xI) = 18I$$

Then by KVL,

$$24 - 4I - 2I + 18I = 0$$

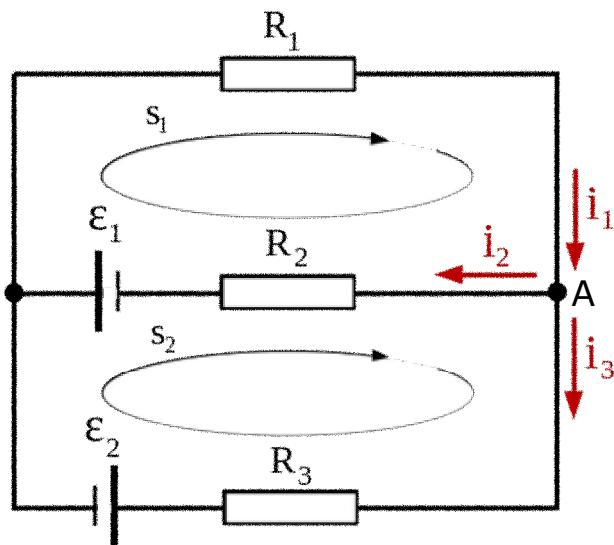
So, $I = -2\text{ A}$

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Applying KVL and KCL to Solve Circuit Parameters:

Example-1:

Assume an electric network consisting of two voltage sources and three resistors as shown in the figure below.



Solution:

Applying KCL at **node A**, we have-

$$i_1 - i_2 - i_3 = 0 \quad \dots \dots \dots \text{(i)}$$

Applying KVL to the loop s_1 gives-

$$-R_2 i_2 + \epsilon_1 - R_1 i_1 = 0 \quad \dots \dots \dots \text{(ii)}$$

Applying KVL to the loop s_2 gives-

$$-R_3 i_3 - \epsilon_2 - \epsilon_1 + R_2 i_2 = 0 \quad \dots \dots \dots \text{(iii)}$$

Thus we get a linear system of equations in i_1, i_2, i_3 :

$$\begin{cases} i_1 - i_2 - i_3 = 0 \\ -R_2 i_2 + \epsilon_1 - R_1 i_1 = 0 \\ -R_3 i_3 - \epsilon_2 - \epsilon_1 + R_2 i_2 = 0 \end{cases}$$

Assuming

$$R_1 = 100, R_2 = 200, R_3 = 300 \text{ (ohms)}; \epsilon_1 = 3, \epsilon_2 = 4 \text{ (volts)}$$

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the solution is

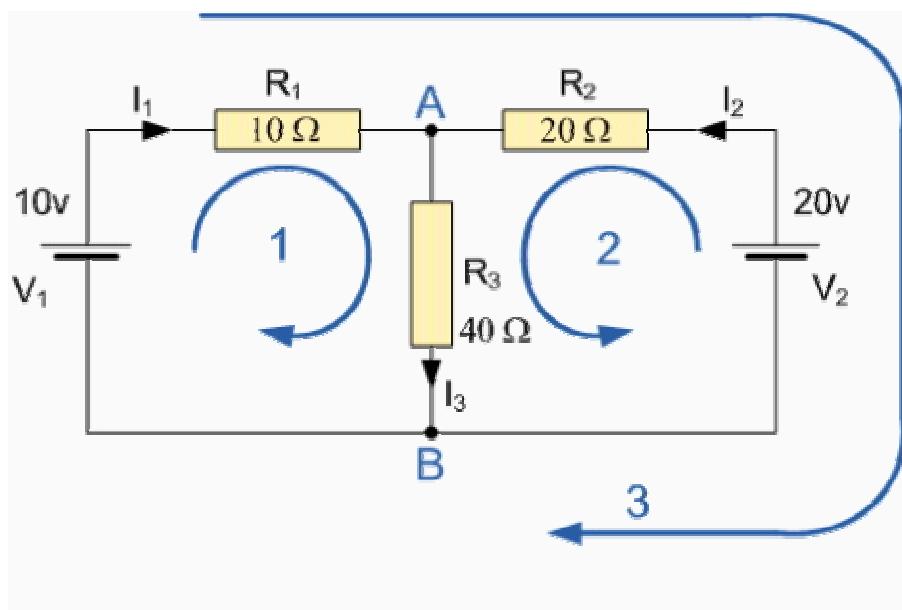
$$\begin{cases} i_1 = \frac{1}{1100} \\ i_2 = \frac{4}{275} \\ i_3 = -\frac{3}{220} \end{cases}$$

Note:

- i_3 has a negative sign, which means that the direction of i_3 is opposite to the assumed direction (the direction defined in the picture).

Example-2:

Find the current flowing in the 40Ω Resistor R_3 .



Solution:

The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using Kirchhoff's Current Law, KCL the equations are given as;

$$\text{At node A : } I_1 + I_2 = I_3 \quad \dots\dots\dots(1)$$

$$\text{At node B : } I_3 = I_1 + I_2 \quad \dots\dots\dots(2)$$

Using Kirchhoff's Voltage Law, KVL the equations are given as;

$$\text{Loop 1 is given as : } 10 = R_1 \times I_1 + R_3 \times I_3 = 10I_1 + 40I_3 \quad \dots\dots\dots(3)$$

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Loop 2 is given as : $20 = R_2 \times I_2 + R_3 \times I_3 = 20I_2 + 40I_3$ (4)

Loop 3 is given as : $10 - 20 = 10I_1 - 20I_2$ (5)

As I_3 is the sum of $I_1 + I_2$ we can rewrite equations (3) and (4) as;

$$\text{Eq. No 3 : } 10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2 \quad \dots\dots\dots(6)$$

$$\text{Eq. No 4 : } 20 = 20I_2 + 40(I_1 + I_2) = 40I_1 + 60I_2 \quad \dots\dots\dots(7)$$

We now have two "Simultaneous Equations" that can be reduced to give us the value of both I_1 and I_2 .

Substitution of I_1 in terms of I_2 gives us the value of I_1 as -0.143 Amps

Substitution of I_2 in terms of I_1 gives us the value of I_2 as +0.429 Amps

$$\text{As : } I_3 = I_1 + I_2$$

The current flowing in resistor R_3 is given as :

$$-0.143 + 0.429 = 0.286 \text{ Amps}$$

and the voltage across the resistor R_3 is given as :

$$0.286 \times 40 = 11.44 \text{ volts}$$

The negative sign for I_1 means that the direction of current flow initially chosen was wrong, but never the less still valid. In fact, the 20v battery is charging the 10v battery.

Application of Kirchhoff's Circuit Laws:

These two laws enable the Currents and Voltages in a circuit to be found, ie, the circuit is said to be "Analysed", and the basic procedure for using Kirchoff's Circuit Laws is as follows:

1. Assume all voltage sources and resistances are given. (If not, label them V_1 , V_2 ..., R_1 , R_2 etc)
 2. Label each branch with a branch current. (I_1 , I_2 , I_3 etc)
 3. Find Kirchoff's first law equations for each node.
 4. Find Kirchoff's second law equations for each of the independent loops of the circuit.
 5. Use Linear simultaneous equations as required to find the unknown currents.

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Determination of algebraic sign:

In applying Kirchhoff's laws to specific problem, particular attention should be paid to the algebraic signs of voltage drops and emf, otherwise results will come out to be wrong. Following sign conventions is suggested.

Sign of Battery EMF:

- A rise (or increase) in voltage should be given a +ve sign and a fall (or decrease) in voltage should be given a -ve sign.
- While going round a loop (in a direction of our own choice) if we go from the -ve terminal of battery to its +ve terminal , there is rise in potential , hence this EMF should be given as + ve sign .On the other hand if we go from its + ve terminal to its -ve terminal , then there is a fall in potential , hence this battery EMF should be given as -ve sign .
- It is important to note that algebraic sign of battery EMF is independent of the direction of current flow. (Whether clockwise or in anticlockwise) through the branch which the battery is connected.

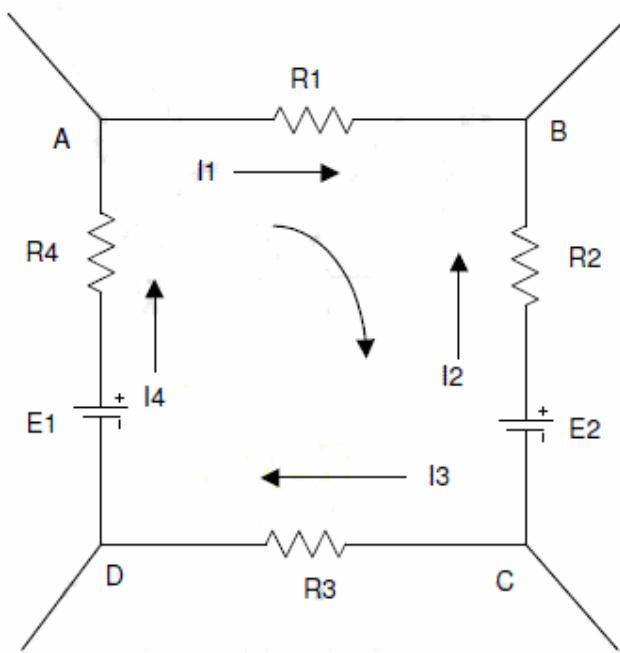
Sign of IR Drops:

- If we go through a resistor in the same direction as its current, then there is a fall or decrease in potential for the simple reason that current always flow from higher to lower potential. Hence this IR drop (voltage fall) should be taken as -ve.
- However, if we go around the loop in a direction opposite to that of the current, there is a rise in voltage. Hence these IR should be taken as +ve.
- It clears that the algebraic sign of IR drop across a resister depends on the direction of current through that resistor but is independent of the polarity of any other source of emf in the circuit under consideration.

Example:

- Consider a loop, for example, ABCDA shown in the figure below:

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- As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs:

I_1R_1 ----- is -ve (fall in potential)

I_2R_2 ----- - is +ve (rise in potential)

E_2 ----- is -ve (Fall in potential)

I_3R_3 ----- is -ve (fall in potential)

E_1 ----- is +ve (rise in potential)

I_4R_4 ----- is -ve (fall in potential)

According to KVL

$$-I_1R_1 + I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

$$\text{Or, } -I_1R_1 + I_2R_2 - I_3R_3 - I_4R_4 = E_2 - E_1$$

$$\text{Or, } I_1R_1 - I_2R_2 + I_3R_3 + I_4R_4 = E_1 - E_2$$

$$\text{Or, } V_{R1} - V_{R2} + V_{R3} + V_{R4} = E_1 - E_2$$

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Assumed Direction of Current:

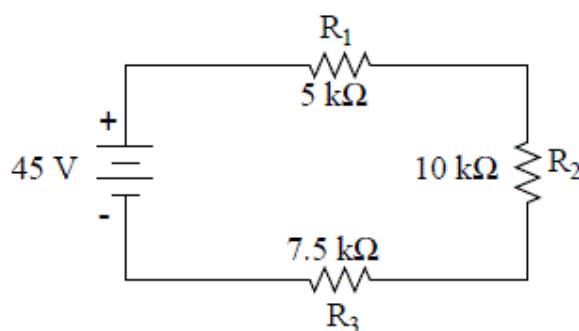
- The direction of current flow may be assumed either clockwise or anticlockwise.
- If the assumed direction of current is not the actual direction, then on solving the question, this current will be found to have a minus sign. If the answer is positive, then assumed direction is the actual direction.
- Once a particular direction has been assumed, the same should be used throughout the solution of the question.

Voltage-Divider Rule (VDR):

- A voltage divider circuit is a series network which gives the voltage across any resistor in terms of the resistances and the total voltage across the series combination without finding the resistor current.
- Voltage drop across each resistor in a series circuit can be calculated according to Voltage Divider Rule, as:
 - (i) Find equivalent resistance R of the series combination: $R = R_1 + R_2 + R_3$
 - (ii) According to VDR, voltage drop across n th resistor is $V_n = V \cdot \frac{R_n}{R}$

Example-1:

Find voltage across all resistors for the circuit shown below.



Solution:

Since all resistors are in series combination, therefore equivalent resistance is $R = R_1 + R_2 + R_3 = (5 + 10 + 7.5) \text{ Ohm} = 22.5 \text{ Ohm}$

According to VDR,

$$v_1 = v \left(\frac{R_1}{R_1 + R_2 + R_3} \right) = v \frac{R_1}{R} = 45 \times \frac{5}{22.5} = 10V$$

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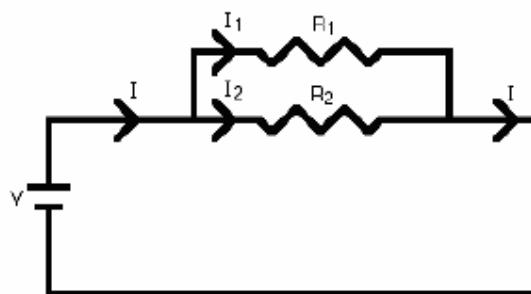
Similarly,

$$v_2 = v \frac{R_2}{R} = 45 \times \frac{10}{22.5} = 20V$$

$$v_3 = v \frac{R_3}{R} = 45 \times \frac{7.5}{22.5} = 15V$$

Current-Divider Rule:

- A parallel arrangement of resistors as shown in the figure below results in a current divider.



- The current-divider rule gives the current through any resistor in terms of the conductance and the current into the parallel combination, thus eliminating the finding of resistor voltage.
- The ratio of the branch current I_1 to the total current I illustrates the operation of the current divider.

$$I = \frac{V}{R_1} + \frac{V}{R_2} \text{ and } I_1 = \frac{V}{R_1}$$

$$\text{Then } \frac{I_1}{I} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$\frac{I_1}{I} = \frac{\frac{1}{R_1}}{\frac{R_1 + R_2}{R_1 R_2}}$$

$$\frac{I_1}{I} = \frac{R_2}{R_1 + R_2}$$

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Division of Current for Two Resistors in Parallel:

$$I_1 = I \cdot \frac{R_2}{R_1 + R_2}$$

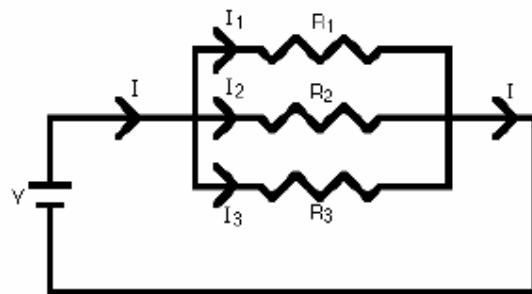
$$I_2 = I \cdot \frac{R_1}{R_1 + R_2}$$

In terms of conductance

$$I_1 = I \cdot \frac{G_1}{G_1 + G_2}$$

$$I_2 = I \cdot \frac{G_2}{G_1 + G_2}$$

In a similar fashion, the ratio of the branch current I_1 to the total current I for the circuit shown below also illustrates the operation of the current divider.



Division of Current for Three Resistors in Parallel:

$$I_1 = I \left(\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \right)$$

$$I_2 = I \left(\frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \right)$$

$$I_3 = I \left(\frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} \right)$$

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In terms of conductance

$$I_1 = I \left(\frac{G_1}{G_1 + G_2 + G_3} \right)$$

$$I_2 = I \left(\frac{G_2}{G_1 + G_2 + G_3} \right)$$

$$I_3 = I \left(\frac{G_3}{G_1 + G_2 + G_3} \right)$$

Example-1:

Find all branch currents in the network shown below.

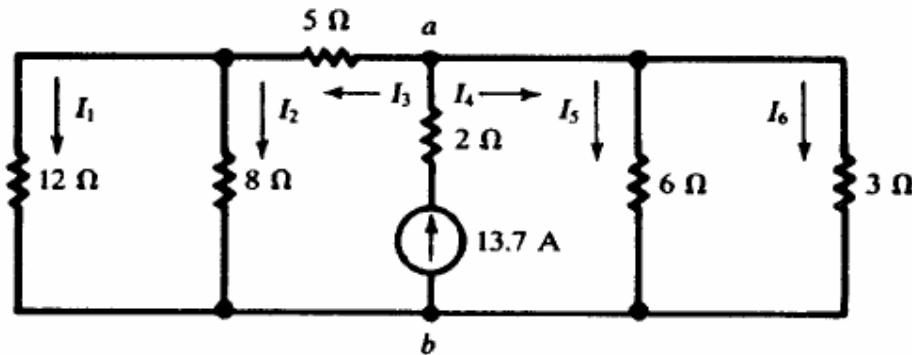


Figure: Given network

Solution:

The equivalent resistances to the left and right of nodes **a** and **b** are

$$R_{\text{eq(left)}} = 5 + \frac{(12)(8)}{20} = 9.8 \Omega$$

$$R_{\text{eq(right)}} = \frac{(6)(3)}{9} = 2.0 \Omega$$

The circuit is reduced to the network shown below.

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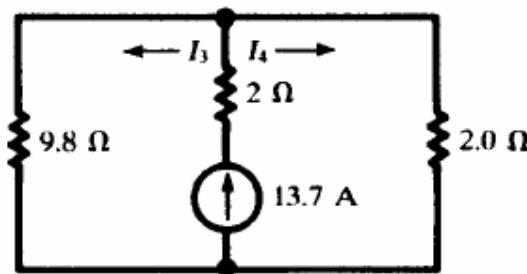


Figure: Reduced Network

Now applying current-divider rule to the reduced network we get,

$$I_3 = \frac{2.0}{11.8} (13.7) = 2.32 \text{ A}$$

$$I_4 = \frac{9.8}{11.8} (13.7) = 11.38 \text{ A}$$

Then referring to the original network,

$$I_1 = \frac{8}{20} (2.32) = 0.93 \text{ A} \quad I_2 = 2.32 - 0.93 = 1.39 \text{ A}$$

$$I_5 = \frac{3}{9} (11.38) = 3.79 \text{ A} \quad I_6 = 11.38 - 3.79 = 7.59 \text{ A}$$

Constant-Current Sources:

- All the circuits presented so far have used voltage sources as the means of providing power.
- However, the analysis of certain circuits is easier if you work with current rather than with voltage.
- Unlike a voltage source, a constant-current source maintains the same current in its branch of the circuit regardless of how components are connected external to the source.
- The magnitude and the direction of current through a voltage source vary according to the size of the circuit resistances and how other voltage sources

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are connected in the circuit. For current sources, the voltage across the current source depends on how the other components are connected.

- The symbol for a constant-current source is shown in the figure below. The direction of the current source arrow indicates the direction of conventional current in the branch.

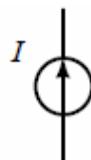
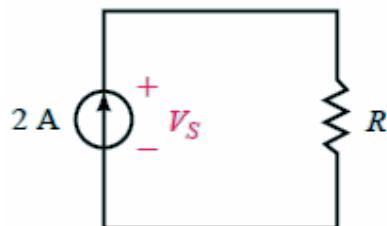


Figure: Symbol of ideal constant current source

- Voltage sources always have some series resistance, although in some cases this resistance is so small in comparison with other circuit resistance that it may effectively be ignored when determining the operation of the circuit. Similarly, a constant-current source will always have some shunt (or parallel) resistance. If this resistance is very large in comparison with the other circuit resistance, the internal resistance of the source may once again be ignored. An ideal current source has an infinite shunt resistance.

Example:

Calculate the voltage V_S across the current source if the resistor is 100Ω .



Solution:

The current source maintains a constant current of 2 A through the circuit. Therefore,
 $V_S = V_R = (2 \text{ A})(100 \Omega) = 200 \text{ V}$.

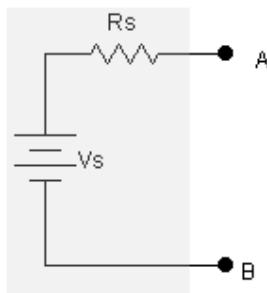
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Source Conversion:

- When we are trying to analyze complicated circuits, a few tricks can greatly simplify and ease our task. One of the tricks is source conversion which lets us replace a voltage source with a current source, or vice versa.
- If the internal resistance of a source is considered, the source (whether it is a voltage source or a current source) is easily converted to the other type.
 - A given voltage source in series with a resistance can be converted or replaced by an equivalent current source in parallel with the resistance.
 - Conversely, a current source in parallel with a resistance can be converted into a voltage source in series with the resistance.
- Two sources are equivalent if, for any load resistor connected to the two sources, they produce the same voltage across that resistor and the same current through it.

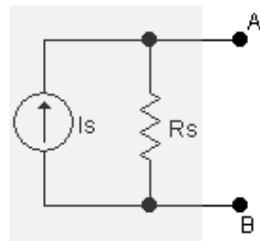
Converting a Voltage Source to a Current Source:

- A given voltage source in series with a resistance can be converted or replaced by an equivalent current source in parallel with the resistance.
- Suppose we are analyzing a circuit that contains a practical voltage source with source voltage V_s and internal resistance R_s . We can replace this voltage source with a practical current source having the same internal resistance and having a source current of $I_s = V_s / R_s$. In other words, the practical voltage source shown here



can be replaced by the practical current source shown here

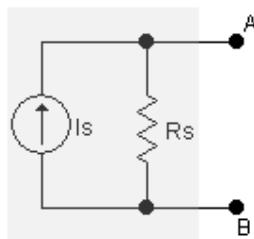
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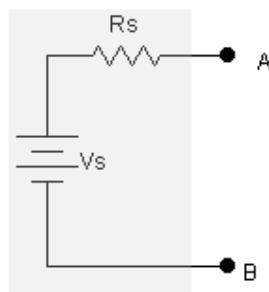
if both have the same internal resistance R_s and $I_s = V_s/R_s$.

Converting a Current Source to a Voltage Source:

- A current source in parallel with a resistance can be converted into a voltage source in series with the resistance.
- Now let's go in the other direction. Suppose we are analyzing a circuit that contains a practical current source with source current I_s and internal resistance R_s . We can replace this current source with a practical voltage source having **the same internal resistance** and having a source voltage of $V_s = I_s \times R_s$. In other words, the practical current source shown here



can be replaced by the practical voltage source shown here



if both have the same internal resistance R_s and $V_s = I_s \times R_s$.

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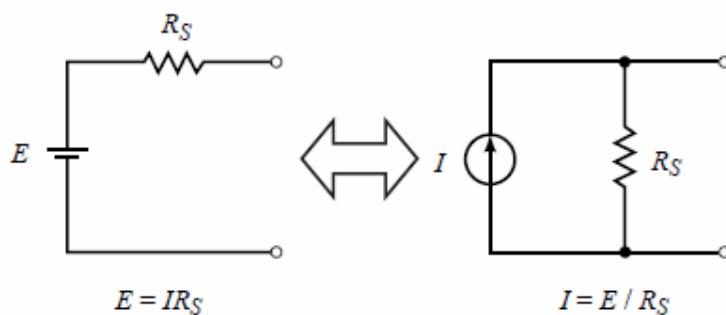
Proof:

- Two sources are equivalent if, for any load resistor connected to the two sources, they produce the same voltage across that resistor and the same current through it. That is if

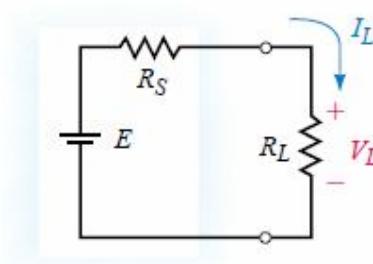
$$E = IR_S \quad \dots \dots \dots (1)$$

and

$$I = E / R_S \quad \dots \dots \dots (2)$$



- These results may be easily verified by connecting an external resistance, R_L , across each source. The sources can be equivalent only if the voltage across R_L is the same for both sources. Similarly, the sources are equivalent only if the current through R_L is the same when connected to either source.
- Consider the circuit shown in figure below.



- The voltage across the load resistor is given as-

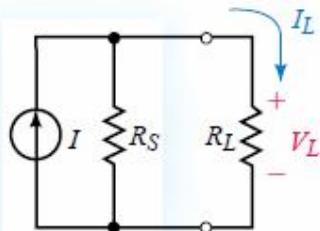
$$V_L = \frac{R_L}{R_L + R_S} E \quad \dots \dots \dots (3)$$

- The current through the resistor R_L is given as

$$I_L = \frac{E}{R_L + R_S} \quad \dots \dots \dots (4)$$

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- Next, consider an equivalent current source connected to the same load as shown in figure below.



- The current through the resistor R_L is given by-

$$I_L = \frac{R_s}{R_s + R_L} I \quad (\text{according to current-divider rule})$$

- But, when converting the source, we get

$$I = \frac{E}{R_s}$$

- And so

$$I_L = \left(\frac{R_s}{R_s + R_L} \right) \left(\frac{E}{R_s} \right)$$

- This result is equivalent to the current obtained in Equation 4.

- The voltage across the resistor is given as

$$\begin{aligned} V_L &= I_L R_L \\ &= \left(\frac{E}{R_s + R_L} \right) R_L \end{aligned}$$

- The voltage across the resistor is precisely the same as the result obtained in Equation 3.

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- We therefore conclude that the load current and voltage drop are the same whether the source is a voltage source or an equivalent current source.

Example-1:

Convert the voltage source of figure-A below into a current source and verify that the current, I_L , through the load is the same for each source.

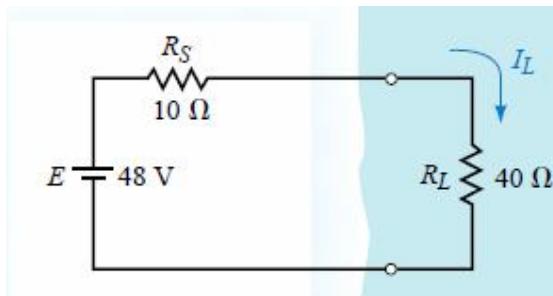


Figure-A

Solution:

The equivalent current source will have a current magnitude given as

$$I = \frac{48 \text{ V}}{10 \Omega} = 4.8 \text{ A}$$

The resulting circuit is shown in the figure-B below.

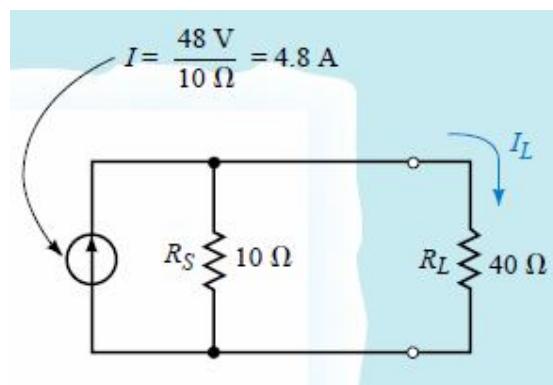


Figure-B

For the circuit of figure-A, the current through the load is found as-

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$$I_L = \frac{48 \text{ V}}{10 \Omega + 40 \Omega} = 0.96 \text{ A}$$

For the equivalent circuit of Figure-B, the current through the load is-

$$I_L = \frac{(4.8 \text{ A})(10 \Omega)}{10 \Omega + 40 \Omega} = 0.96 \text{ A}$$

Clearly the results are the same.

Example-2:

Convert the current source of figure-C below into a voltage source and verify that the voltage, V_L , across the load is the same for each source.

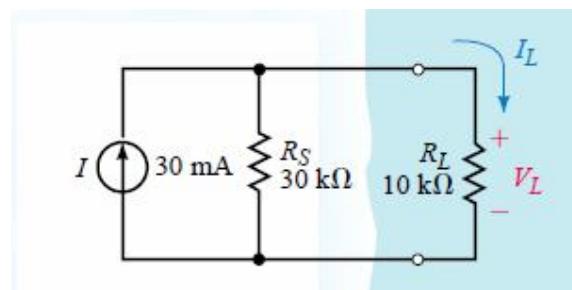


Figure-C

Solution:

The equivalent voltage source will have a magnitude given as

$$E = (30 \text{ mA})(30 \text{ k}\Omega) = 900 \text{ V}$$

The resulting circuit is shown in Figure-D.

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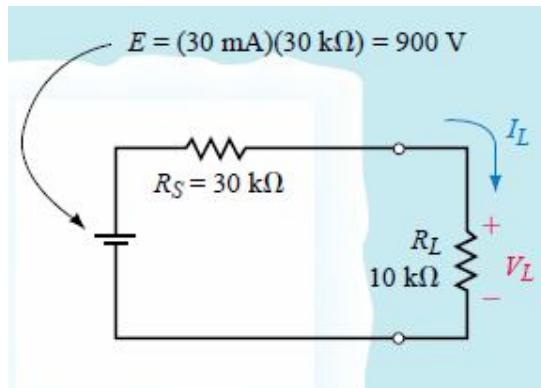


Figure-D

For the circuit of Figure-C, the voltage across the load is determined as

$$I_L = \frac{(30 \text{ k}\Omega)(30 \text{ mA})}{30 \text{ k}\Omega + 10 \text{ k}\Omega} = 22.5 \text{ mA}$$

$$V_L = I_L R_L = (22.5 \text{ mA})(10 \text{ k}\Omega) = 225 \text{ V}$$

For the equivalent circuit of Figure-D, the voltage across the load is

$$V_L = \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 30 \text{ k}\Omega} (900 \text{ V}) = 225 \text{ V}$$

Once again, we see that the circuits are equivalent.

More Examples: 2.41, 2.42, 2.43 (Page: 94-95)

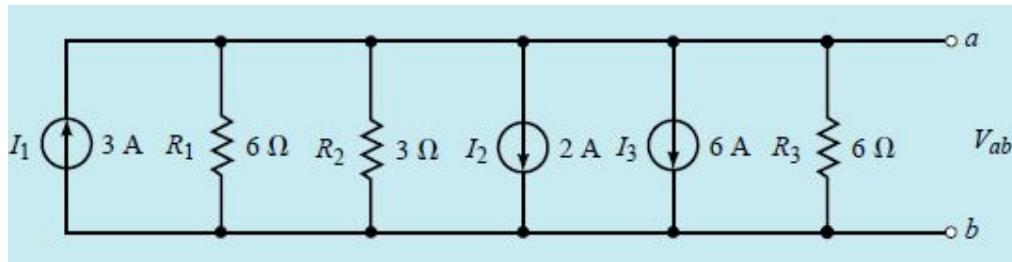
Current Sources in Parallel and Series:

When several current sources are placed in parallel, the circuit may be simplified by combining the current sources into a single current source. The magnitude and direction of this resultant source is determined by adding the currents in one direction and then subtracting the currents in the opposite direction.

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Example-3:

Simplify the circuit of Figure below and determine the Voltage V_{ab} .



Solution:

- Since all of the current sources are in parallel, they can be replaced by a single current source.
- The equivalent current source will have a direction which is the same as both I_2 and I_3 , since the magnitude of current in the downward direction is greater than the current in the upward direction.
- The equivalent current source has a magnitude of

$$I = 2 \text{ A} + 6 \text{ A} - 3 \text{ A} = 5 \text{ A}$$

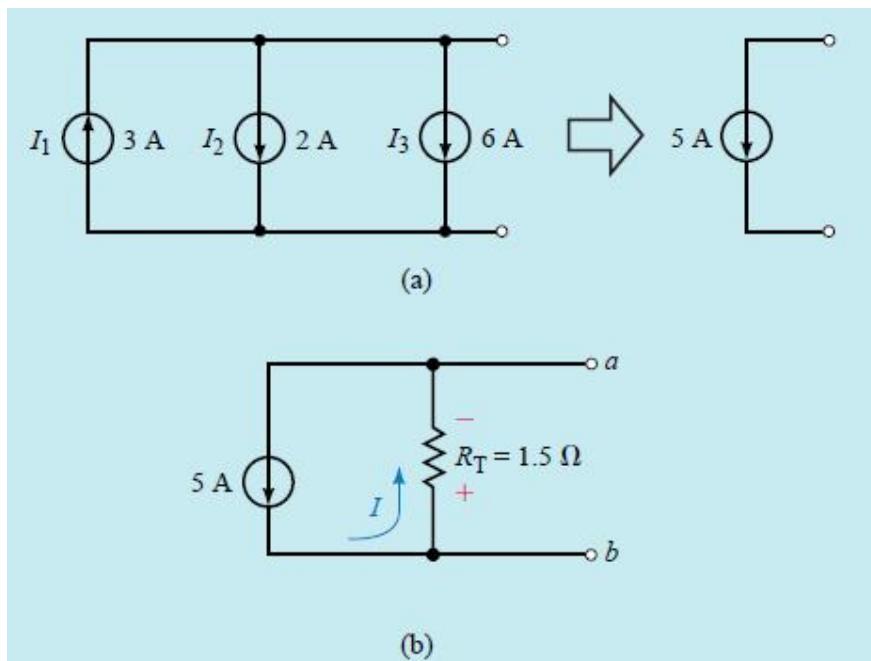
as shown in Figure (a).

- The circuit is further simplified by combining the resistors into a single value:

$$R_T = 6 \Omega \parallel 3 \Omega \parallel 6 \Omega = 1.5 \Omega$$

- The equivalent circuit is shown in Figure (b).

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- The voltage V_{ab} is found as

$$V_{ab} = -(5 \text{ A})(1.5 \Omega) = -7.5 \text{ V}$$
