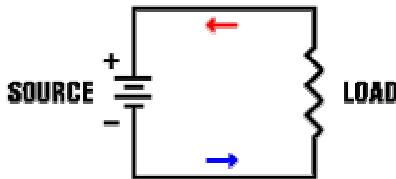


Lecture-11: AC Fundamentals: Basic AC Theory

Alternating Current vs. Direct Current

- DC stands for "Direct Current," meaning voltage or current that maintains constant polarity or direction, respectively, over time.
- The figure below shows the schematic diagram of a very basic DC circuit.

**Figure: Direct Current (DC)**

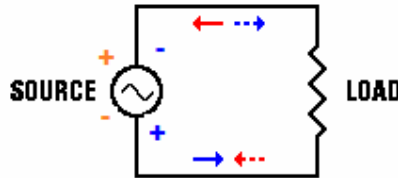
- It consists of nothing more than a source (a producer of electrical energy) and a load (whatever is to be powered by that electrical energy).

The source can be any electrical source: a chemical battery, an electronic power supply, a mechanical generator, or any other possible continuous source of electrical energy. For simplicity, we represent the source in this figure as a battery.

At the same time, the load can be any electrical load: a light bulb, electronic clock or watch, electronic instrument, or anything else that must be driven by a continuous source of electricity. The figure here represents the load as a simple resistor.

- Regardless of the specific source and load in this circuit, electrons leave the negative terminal of the source, travel through the circuit in the direction shown by the arrows, and eventually return to the positive terminal of the source. This action continues for as long as a complete electrical circuit exists.
- Now consider the same circuit with a single change, as shown in the figure below.

Lecture-11: AC Fundamentals: Basic AC Theory

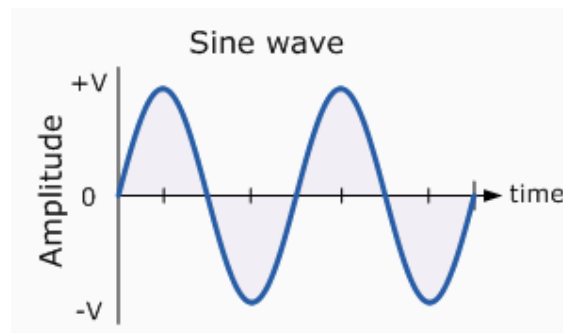
**Figure: Alternating Current (AC)**

- This time, the energy source is constantly changing.
 - It begins by building up a voltage which is **positive on top** and **negative on the bottom**, and therefore pushes electrons through the circuit in the direction shown by the solid arrows. However, then **the source voltage starts to fall off, and eventually reverse polarity**.
 - This time, the voltage is **negative on top** and **positive on the bottom**, and therefore pushes electrons through the circuit in the direction shown by the dotted arrows.
- This cycle repeats itself endlessly, and as a result the **current through the circuit reverses direction repeatedly**. This is known as an **alternating current (AC)**.
- This kind of reversal makes no difference to some kinds of loads. For example, the light bulbs in your home don't care which way current flows through them. When you close the circuit by turning on the light switch, the light turns on without regard for the direction of current flow.
- Of course, there are some kinds of loads that require current to flow in only one direction. In such cases, we often need to convert alternating current such as the power provided at your wall socket to direct current for use by the load. There are several ways to accomplish this.
- Whereas the familiar battery symbol is used as a generic symbol for any DC voltage source ($\text{---}\frac{\text{+}}{\text{---}}\text{---}$), the circle with the wavy line inside is the generic symbol for any AC voltage source ($\text{---}\bigcirc\text{---}$).
- A major advantage of alternating current is that the voltage can be increased and decreased by a transformer for more efficient transmission over long distances. **Direct current cannot use transformers to change voltage**.

Lecture-11: AC Fundamentals: Basic AC Theory

What is AC?

- AC stands for “alternating current” meaning voltage or current that changes polarity or direction, respectively, over time. In AC, amplitude of the signal also varies over time. Therefore, AC voltage alternates in polarity and AC current alternates in direction.
- Each cycle of alternating current consists of two half cycles. During one half cycle, the current or voltage acts in one direction while during the other half cycle in opposite direction.
- It starts from zero, grows to a maximum, decreases to zero, reverses, reaches a maximum in the opposite direction, returns again to zero, and repeats the cycle indefinitely.
- Its value can be positive, zero, negative, maximum or in between zero and maximum.
- The usual waveform of an AC power circuit is a sine wave.

**Generation of Alternating Voltage and Current:**

- A sinusoidal AC voltage can be produced by rotating a coil in a uniform magnetic field or by rotating a magnetic field within a stationary coil.
- The value of the voltage generated depends upon the number of turns in the coil, strength of the field, and the speed at which the coil or magnetic field rotates.

Lecture-11: AC Fundamentals: Basic AC Theory

Mathematical Equations of Alternating Voltages and Currents:

- Alternating currents are caused by alternating voltages. The sinusoidal AC voltage can be expressed mathematically as a function of time by the following equation:

$$e = E_M \sin \omega t$$

where,

e = Instantaneous value of alternating voltage, i.e. time-varying value of voltage.

E_M = Peak or Maximum value (magnitude or amplitude) of alternating voltage.

ω = Angular frequency of the coil and $\omega = 2\pi f$ (unit: radians per second).

t = Time

- Sinusoidal voltage always produces sinusoidal currents. Therefore, a sinusoidal current can be expressed in the same way as voltage:

$$i = I_M \sin \omega t$$

- The above equations of AC voltage and currents are assumed to have zero phase angles. In general, the phase of the voltage or current may have a value other than zero. [The equations of voltage and current with a phase angle are:](#)

$$e = E_M \sin(\omega t + \theta)$$

$$i = I_M \sin(\omega t + \theta)$$

- In a linear circuit excited by sinusoidal sources, in the steady-state, all voltages and currents are sinusoidal and have the same frequency. However, there may be a phase difference between the voltage and current depending on the type of load used.

Lecture-11: AC Fundamentals: Basic AC Theory

Different Forms of EMF Equation:

We know that the standard form of an alternating voltage is $e = E_M \sin\theta$. Using this equation, we may derive the other forms of emf equation:

$$(i) \quad e = E_M \sin\theta$$

$$(ii) \quad e = E_M \sin\omega t$$

$$(iii) \quad e = E_M \sin 2\pi f t \quad (\text{since, } \omega = 2\pi f)$$

$$(iv) \quad e = E_M \sin \frac{2\pi}{T} t \quad (\because f = \frac{1}{T})$$

By closely looking at the above equations, we find that

- The maximum value or peak value or amplitude of an alternating voltage is given by the coefficient of the **sine** of the time angle.
- The frequency is given by the coefficient of time divided by 2π .

Example-1:

Determine the maximum value and frequency for the each of the alternating voltage equation given below:

$$(i) \quad e = 50 \sin 314t$$

$$(ii) \quad e = I_M \sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\omega t$$

Solution:

$$(i) \quad e = 50 \sin 314t$$

$$\text{Therefore } V_{MAX} = 50 \text{ V}$$

$$\text{Frequency } f = \frac{314}{2\pi} = \frac{100 \times 3.14}{2 \times 3.14} = 50 \text{ Hz}$$

$$(ii) \quad e = I_M \sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\omega t$$

$$\text{Therefore, } V_{MAX} = I_M \sqrt{(R^2 + 4\omega^2 L^2)}$$

$$\text{Frequency } f = \frac{2\omega}{2\pi} = \frac{\omega}{\pi} \text{ Hz}$$

Lecture-11: AC Fundamentals: Basic AC Theory

Example-2:

For a circuit connected to 50 Hz supply, the maximum values of the alternating voltage and current are 400 V and 20 A respectively. The instantaneous values of voltage and current are 283 V and 10 A respectively at $t=0$ both increasing positively.

- (i) Write down the expression for voltage and current at time t .
- (ii) Determine the power consumed in the circuit.

Solution:

(i) The general expression for an AC voltage is $v = V_M \sin(\omega t + \phi)$, where ϕ is the phase difference with respect to the point where $t=0$.

Given, $v=283$ V, $V_M=400$ V. Substituting $t=0$ in the above equation, we get

$$283 = 400 \sin(\omega \times 0 + \phi)$$

$$\Rightarrow \sin \phi = \frac{283}{400} = 0.707$$

$$\Rightarrow \phi = \sin^{-1} 0.707 = 45^\circ \text{ or } \frac{\pi}{4} \text{ radian.}$$

Hence, general expression for voltage is

$$\begin{aligned} v &= 400 \sin(\omega t + \phi) \\ &= 400 \sin(2\pi f t + \pi/4) \\ &= 400 \sin(2\pi \times 50 \times t + \pi/4) \\ v &= 400 \sin(100\pi t + \pi/4) \end{aligned}$$

Again, the general expression for alternating current is $i = I_M \sin(\omega t + \phi)$

Given $i=10$ A, $I_M=20$ A. Substituting $t=0$ in the above equation, we get

Lecture-11: AC Fundamentals: Basic AC Theory

$$10 = 20 \sin(\omega \times 0 + \phi)$$

$$\Rightarrow \sin \phi = \frac{10}{20} = 0.5$$

$$\Rightarrow \phi = \sin^{-1} 0.5 = 30^\circ \text{ or } \frac{\pi}{6} \text{ radian.}$$

Hence, the general expression for current is

$$i = 20 \sin(100\pi t + \pi/6)$$

(ii) $P = VI \cos \theta$, where V and I are RMS values and θ is the phase difference between voltage and current. $\theta = 45 - 30 = 15^\circ$

$$\begin{aligned} \text{Now, } V_{RMS} &= \frac{V_{MAX}}{\sqrt{2}} \text{ and } I_{RMS} = \frac{I_{MAX}}{\sqrt{2}} \\ &= \frac{400}{\sqrt{2}} \qquad \qquad = \frac{20}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} P &= VI \cos \theta \\ &= \frac{400}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos 15^\circ \\ &= 3864 \text{ Watt} \end{aligned}$$

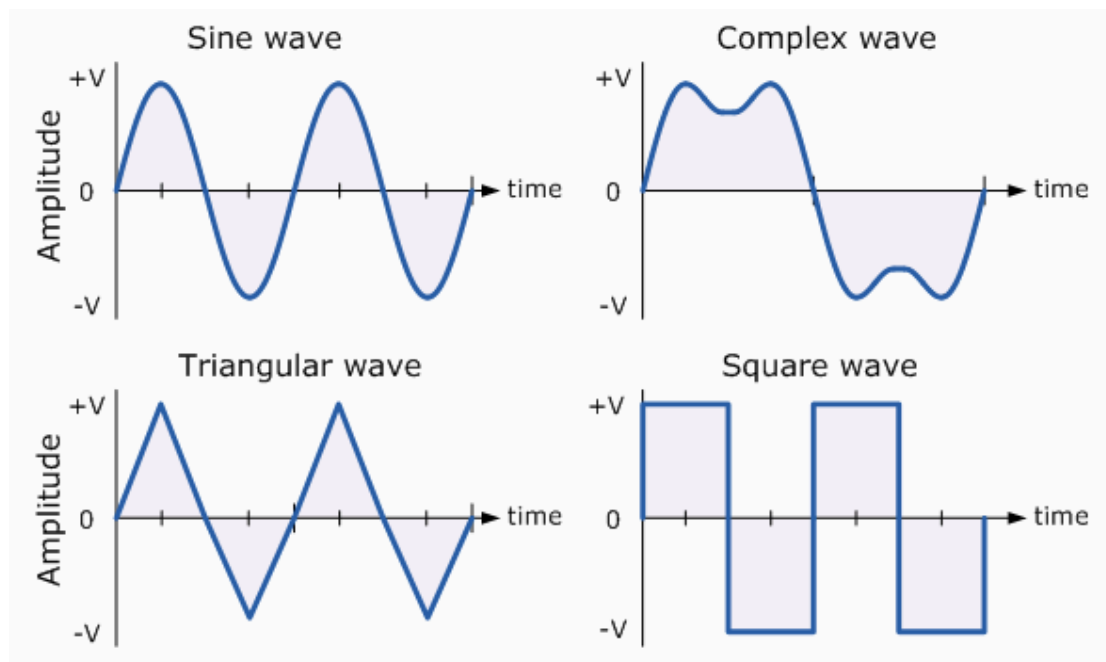
AC Waveform

- Waveforms are basically a visual representation of the variation of a voltage or current plotted to a base of time.
- The AC Waveform is a "time-dependent signal" which may be periodic or non-periodic.
 - A periodic signal completes a pattern within a measurable time frame, called a period, and repeats that pattern over subsequent identical periods. The completion of one full pattern is called a cycle.
 - A nonperiodic signal changes without exhibiting a pattern or cycle that repeats over time.

Lecture-11: AC Fundamentals: Basic AC Theory

Types of Periodic Waveform

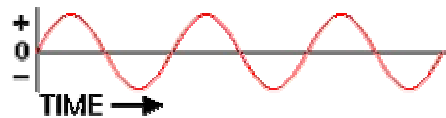
- The most common periodic signal waveforms that are used in Electrical and Electronic Engineering are the *Sinusoidal Waveforms*.
- However, an alternating AC waveform may not always take the shape of a smooth shape based around the trigonometric sine or cosine function.
- AC waveforms can also take the shape of either *Complex Waves*, *Square Waves* or *Triangular Waves* and these are shown below.

**Figure: Various types of AC waveform****Properties of Alternating Current**

- A DC power source, such as a battery, outputs a constant voltage over time, as depicted in the figure above. Of course, once the chemicals in the battery have completed their reaction, the battery will be exhausted and cannot develop any output voltage. But until that happens, the output

Lecture-11: AC Fundamentals: Basic AC Theory

voltage will remain essentially constant. The same is true for any other source of DC electricity: the output voltage remains constant over time.



- By contrast, an AC source of electrical power changes constantly in amplitude and regularly changes polarity, as shown in the figure above. The changes are smooth and regular, endlessly repeating in a succession of identical cycles, and form a sine wave as depicted here.
- Alternating current starts from zero, grows to a maximum, decreases to zero, reverses, reaches a maximum in the opposite direction, returns again to zero, and repeats the cycle indefinitely.
- Because the changes are so regular, alternating voltage and current have a number of properties associated with any such waveform. These basic properties include the following list:

Frequency:

- One of the most important properties of any regular waveform is frequency.
- It is the number of oscillations per unit time (usually seconds). In other words, frequency is the rate of change with respect to time.
- Frequency is formally expressed in Hertz (Hz), which is cycle per second.
- A 40-Hz signal has one-half the frequency of an 80-Hz signal; it completes 1 cycle in twice the time of the 80-Hz signal, so each cycle also takes twice as long to change from its lowest to its highest voltage levels.
- Change in a short span of time means high frequency.
- Change over a long span of time means low frequency.
- If a signal does not change at all, its frequency is zero.
- If a signal changes instantaneously, its frequency is infinite

Lecture-11: AC Fundamentals: Basic AC Theory

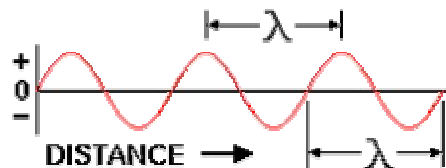
- The frequency of the electrical system varies by country. In North America (primarily the US and Canada), the AC power system operates at a frequency of 60 Hz. In Europe, including the UK, Ireland, and Scotland, the power system operates at a frequency of 50 Hz. Some countries have a mixture of 50 Hz and 60 Hz supplies, notably Japan.
- Low frequencies (50 – 60 cycles per second) are used for domestic and commercial power, but frequencies of around 100 million cycles per second (100 megahertz) are used in television and of several thousand megahertz in radar and microwave communication

Period:

- Sometimes we need to know the amount of time required to complete one cycle of the waveform, rather than the number of cycles per second of time.
- Period refers to the amount of time, in seconds, a signal needs to complete 1 cycle. The time taken to complete one cycle is called the period.
- Period is formally expressed in seconds per cycle.
- Period and frequency are the inverse of each other:

$$f = 1/T \quad \text{and} \quad T = 1/f$$

- AC power at 50 Hz will have a period of $1/50 = 0.02$ seconds/cycle. A 60 Hz power system has a period of $1/60 = 0.016667$ seconds/cycle. These are often expressed as 20 ms/cycle or 16.6667 ms/cycle, where 1 ms is 1 millisecond = 0.001 second (1/1000 of a second).



Lecture-11: AC Fundamentals: Basic AC Theory

Wavelength:

- Because an AC wave moves physically as well as changing in time, sometimes we need to know **how far it moves in one cycle of the wave**, rather than how long that cycle takes to complete. This of course depends on how fast the wave is moving as well.
- Electrical signals travel through their wires at nearly the speed of light, which is very nearly 3×10^8 meters/second, and is represented mathematically by the letter 'c.' Since we already know the frequency of the wave in Hz, or cycles/second, we can perform the division of c/f to obtain a result in units of meters/cycle, which is what we want.
- The Greek letter λ (lambda) is used to represent wavelength in mathematical expressions. Thus, $\lambda = c/f$.
- As shown in the figure above, wavelength can be measured from any part of one cycle to the equivalent point in the next cycle. Wavelength is very similar to period as discussed above, except that **wavelength is measured in distance per cycle** where **period is measured in time per cycle**.

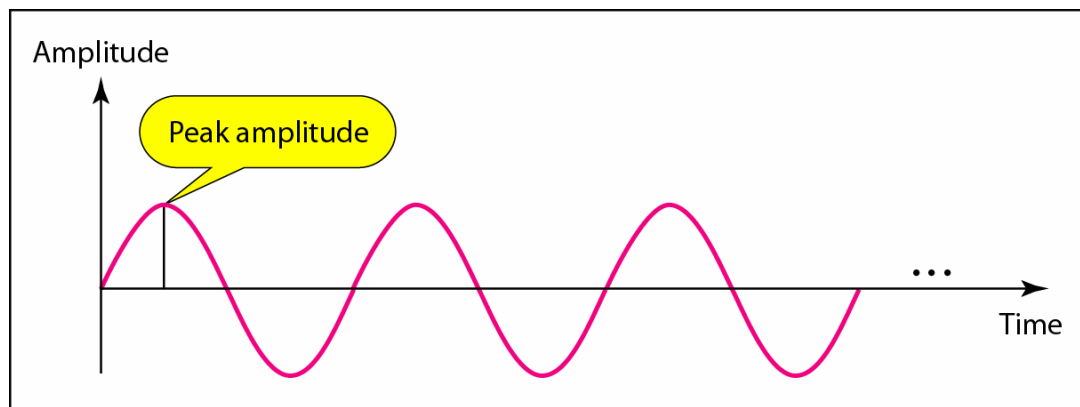
Amplitude:

- Another thing we have to know is just how positive or negative the voltage is, with respect to some selected neutral reference.
- With DC, this is easy; the voltage is constant at some measurable value.
- But AC is constantly changing, and yet it still powers a load.
- Mathematically, **the amplitude of a sine wave is the value of that sine wave at its peak**. This is the maximum value, positive or negative, that it can attain.
- It refers to the difference between the maximum and minimum signal heights.
- However, when we speak of an AC power system, it is more useful to refer to the **effective voltage or current**. This is the rating that would cause the same amount of work to be done (the same effect) as the same value of DC voltage or current would cause.
- For a sine wave, **the effective voltage (or, RMS value) of the AC power system is 0.707 times the peak voltage**. Thus, when we say that the AC

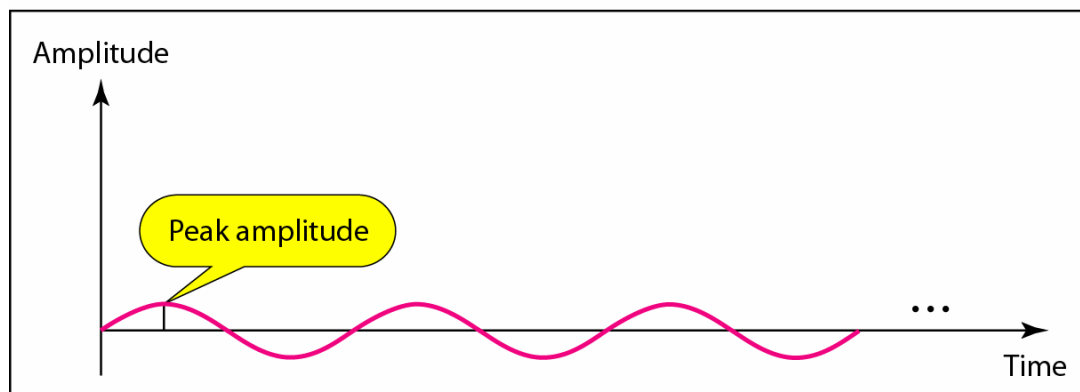
Lecture-11: AC Fundamentals: Basic AC Theory

line voltage in the US is 120 volts, we are referring to the voltage amplitude, but we are describing the effective voltage, not the peak voltage of nearly 170 volts. The effective voltage is also known as the RMS voltage.

- The **peak-to-peak value of an AC voltage** is defined as the difference between its positive peak and its negative peak. Since the maximum value of $\sin(x)$ is +1 and the minimum value is -1, an AC voltage swings between + V_{peak} and - V_{peak} . The peak-to-peak voltage, usually written as V_{pp} or $V_{\text{p-p}}$, is therefore $V_{\text{peak}} - (-V_{\text{peak}}) = 2V_{\text{peak}}$.
- Figure below shows two signals and their peak amplitudes.



a. A signal with high peak amplitude



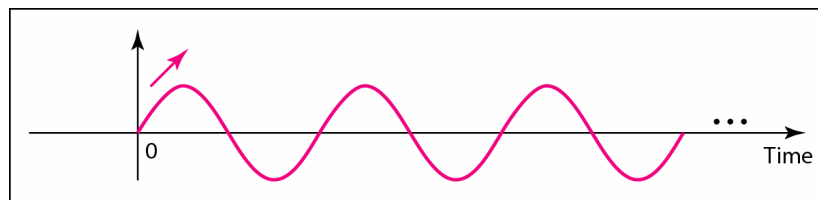
b. A signal with low peak amplitude

Figure: Amplitude of AC quantity

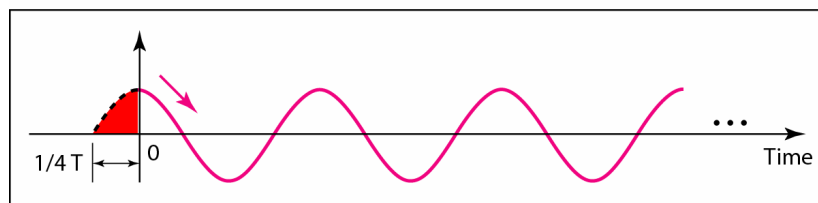
Lecture-11: AC Fundamentals: Basic AC Theory

Phase:

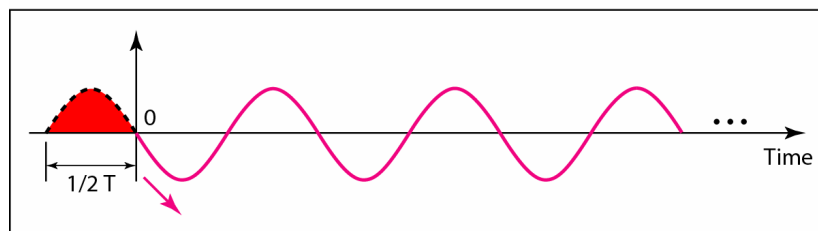
- Phase describes the position of the waveform relative to time 0. That is, it refers to how far the start of the sine wave is shifted from a reference time.
- If we think of the wave as something that can be shifted backward or forward along the time axis, phase describes the amount of that shift. It indicates the status of the first cycle.
- Phase is measured in degrees or radians [360° is 2π rad; 1 degree is $2\pi/360$ rad, and 1 rad is $360/(2\pi)$].
- A phase shift of 360° corresponds to a shift of a complete period; a phase shift of 180° corresponds to a shift of one-half of a period; and a phase shift of 90° corresponds to a shift of one-quarter of a period.
- Figure shows three sine waves with the same amplitude and frequency, but different phases.



a. 0 degrees



b. 90 degrees



c. 180 degrees

Figure: Phase difference

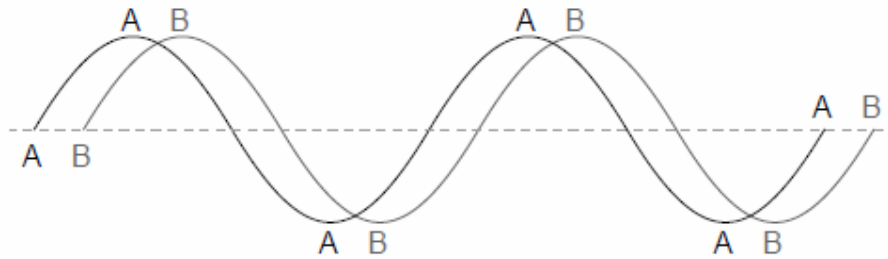
Lecture-11: AC Fundamentals: Basic AC Theory

- Looking at figure above, we can say that
 - A sine wave with a phase of 0° starts at time 0 with a zero amplitude. The amplitude is increasing.
 - A sine wave with a phase of 90° starts at time 0 with a peak amplitude. The amplitude is decreasing.
 - A sine wave with a phase of 180° starts at time 0 with a zero amplitude. The amplitude is decreasing.
- Another way to look at the phase is in terms of **shift** or offset. We can say that
 - A sine wave with a phase of 0° is not shifted.
 - A sine wave with a phase of 90° is shifted to the left by $1/4$ cycle. However, note that the signal does not really exist before time 0.
 - A sine wave with a phase of 180° is shifted to the left by $1/2$ cycle. However, note that the signal does not really exist before time 0.

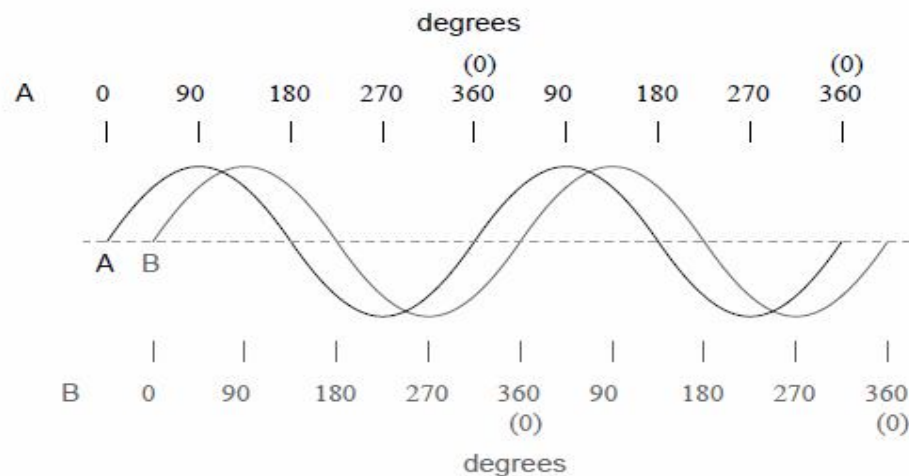
Phase Shift:

- In a linear circuit excited by sinusoidal sources, in the steady-state, all voltages and currents are sinusoidal and have the same frequency. However, there may be a phase difference between the voltage and current depending on the type of load used.
- Phase shift is where two or more waveforms are out of step with each other.
- The amount of phase shift between two waves can be expressed in terms of degrees.
- Consider two alternating waveforms shown below (A versus B). They are of the same amplitude and frequency, but they are out of step with each other, i.e. they are not synchronized: their peaks and zero points do not match up at the same points in time. In technical terms, this is called a phase shift.

Lecture-11: AC Fundamentals: Basic AC Theory

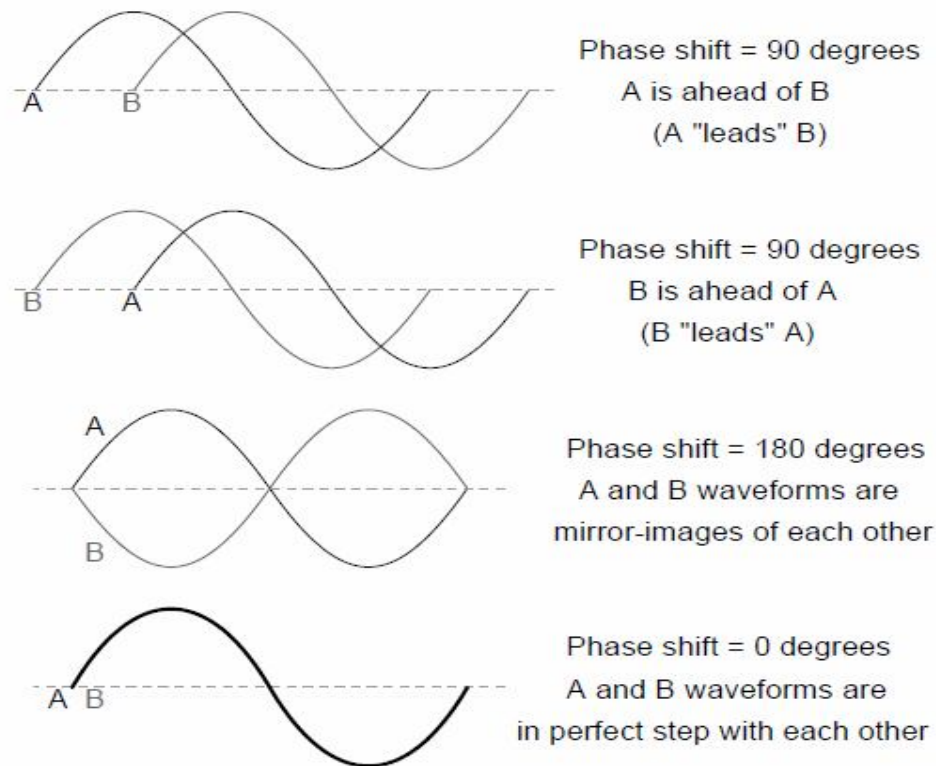
**Figure: Phase difference**

- The starting point of a sine wave was zero amplitude at zero degrees, progressing to full positive amplitude at 90 degrees, zero at 180 degrees, full negative at 270 degrees, and back to the starting point of zero at 360 degrees.

**Figure: Phase difference**

- The shift between these two waveforms is about 45 degrees, the "A" wave being ahead of the "B" wave.
- A sampling of different phase shifts is given in the graphs shown below to better illustrate this concept:

Lecture-11: AC Fundamentals: Basic AC Theory

**Figure: Phase shift**

- Suppose "voltage 'A' is 45 degrees out of phase with voltage 'B'. Whichever waveform is ahead in its evolution is said to be leading and the one behind is said to be lagging.
- A leading waveform is defined as one waveform that is ahead of another in its evolution.
- A lagging waveform is one that is behind another. Example:



- Note that the current and voltage waveforms in the resistor are in phase, while inductances and capacitors both have a 90° phase shift between voltage and current. The inductor current waveform lags the inductor

Lecture-11: AC Fundamentals: Basic AC Theory

voltage waveform by 90° , while in the capacitor, the current leads the voltage by 90° .

Example-1:

A sine wave is offset $1/6$ cycle with respect to time 0. What is its phase in degrees and radians?

Solution:

We know that 1 complete cycle is 360° . Therefore, $1/6$ cycle is

$$\frac{1}{6} \times 360 = 60^\circ = 60 \times \frac{2\pi}{360} \text{ rad} = \frac{\pi}{3} \text{ rad} = 1.046 \text{ rad}$$

Instantaneous Value

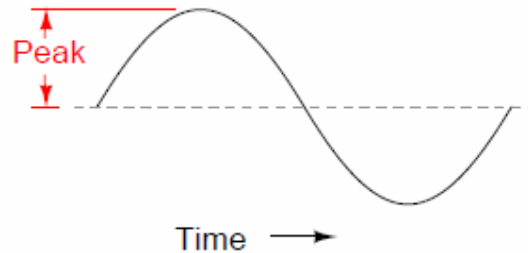
- The instantaneous value of an alternating voltage or current is the value of voltage or current at one particular instant.
 - The value may be zero if the particular instant is the time in the cycle at which the polarity of the voltage is changing.
 - It may also be the same as the peak value, if the selected instant is the time in the cycle at which the voltage or current stops increasing and starts decreasing.
- There are actually an infinite number of instantaneous values between zero and the peak value.

Peak Value or Crest Value of an AC Waveform:

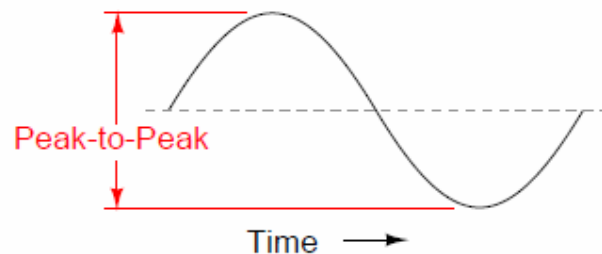
- One way to express the intensity, or magnitude (also called the *amplitude*), of an AC quantity is to measure its peak height on a waveform graph. This is known as the *peak* or *crest* value of an AC waveform (shown in the figure below).

Lecture-11: AC Fundamentals: Basic AC Theory

- *Peak* amplitude is the height of an AC waveform as measured from the zero mark to the highest positive or lowest negative point on a graph. Also known as the *crest* amplitude of a wave.

**Figure: Peak voltage of a waveform.****Peak-to-Peak Value of an AC Waveform:**

- Another way to express the magnitude is to measure the total height between opposite peaks. This is known as the *peak-to-peak* (P-P) value of an AC waveform (shown in the figure below).
- *Peak-to-peak* amplitude is the total height of an AC waveform as measured from maximum positive to maximum negative peaks on a graph. Often abbreviated as "P-P".

**Figure: Peak-to-peak voltage of a waveform.****Average Value:**

- Unfortunately, either one of the expressions (peak value and peak-to-peak value) of waveform amplitude can be misleading when comparing two different types of waves. For example, a square wave peaking at 10 volts is obviously a greater amount of voltage for a greater amount of

Lecture-11: AC Fundamentals: Basic AC Theory

time than a triangle wave peaking at 10 volts. The effects of these two AC voltages powering a load would be quite different which is illustrated in the figure below.

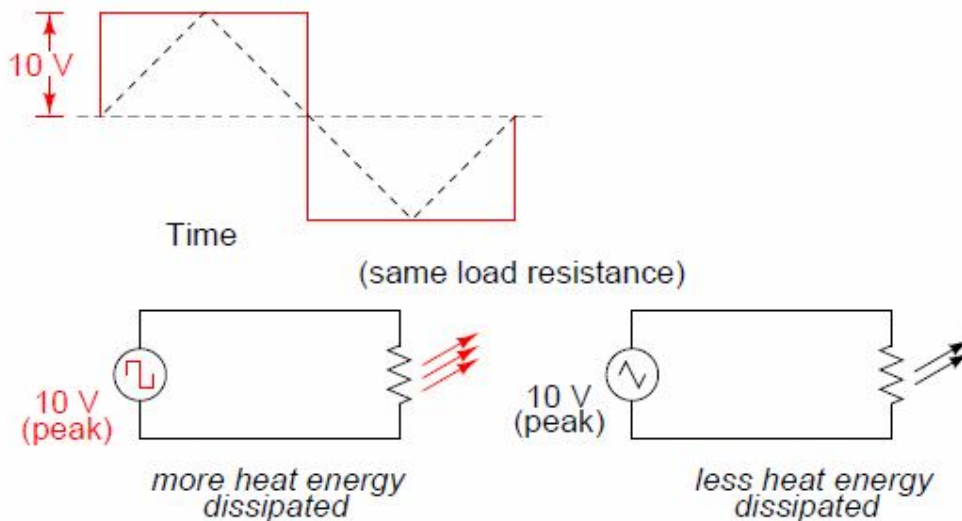
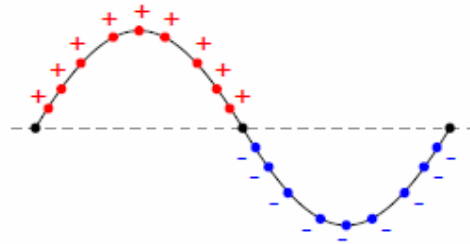


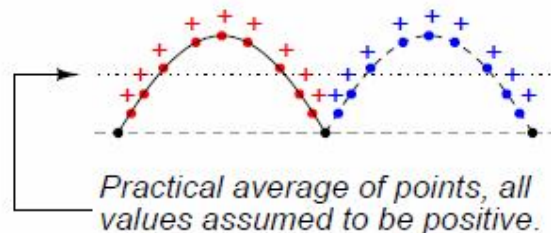
Figure: A square wave produces a greater heating effect than the same peak voltage triangle wave.

- One way of expressing the amplitude of different waveshapes in a more equivalent fashion is to mathematically average the values of all the points on a waveform's graph to a single, aggregate number. This amplitude measure is known simply as the *average* value of the waveform.
- In the case of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), **the average value over a complete cycle is zero** as the positive and negative halves will cancel each other out (shown in the figure below). **Hence, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only.**

Lecture-11: AC Fundamentals: Basic AC Theory

**Figure: The average value of a sinewave is zero**

- But, in the case of an asymmetrical alternating current (like half-wave rectified current), the average value must always be taken over the whole cycle.
- Average value or amplitude is the mathematical “mean” of all the waveform’s points over the period of one cycle. Technically, the average amplitude of any waveform with equal-area portions above and below the “zero” line on a graph is zero. However, as a practical measure of amplitude, a waveform’s average value is often calculated as the mathematical mean of all the points’ *absolute values* (taking all the negative values and considering them as positive). For a sine wave, the average value so calculated is approximately 0.637 of its peak value.

**Figure: Waveform seen by AC “average responding” meter**

- The average value I_{AVG} of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.

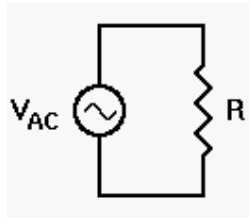
$$I_{AVG} = \frac{I_{MAX}}{\frac{1}{2}\pi} = 0.637 I_{MAX}$$

Lecture-11: AC Fundamentals: Basic AC Theory

- The average or mean value of a continuous DC voltage will always be equal to its maximum peak value as a DC voltage is constant.
- Note that RMS value is always greater than the average value except in the case of a rectangular wave when both are equal.

RMS Value

- Consider the circuit below with applied AC voltage with a resistor only.



- When we apply an ac voltage to the resistor, current will flow through that resistor. This much seems logical and reasonable. But how much current? Can we apply Ohm's Law as we did with DC? How can we even specify the ac voltage, since it is constantly changing? To answer these questions, we must first define how we will specify the ac voltage applied to the resistor.
- In most cases, it makes a lot of sense to describe ac voltage and current in terms of a "dc equivalent," such that the actual ac power delivered to the load does exactly the same amount of work as the same value of dc voltage and current applied to the same load.
- To do this, we need to find some sort of "average ac power" over the entire cycle. Unfortunately, the actual average voltage of the applied ac is zero. The same is true of the alternating current flowing back and forth through the circuit. Yet we know that real power is used, because light bulbs turn on, clocks run, electric motors work, etc. How do we resolve this?
- The answer is to identify the power dissipated by the resistor, in terms of either the ac voltage by itself, or the current by itself. This is easy enough; we already know that we can use either of two expressions for this:

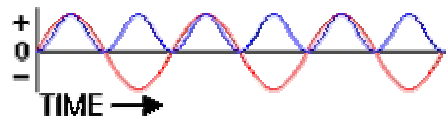
$$P = I^2R = E^2/R$$

- Since squaring a number always results in a positive result (or zero, and we are not using imaginary numbers here), we should be able to find a

Lecture-11: AC Fundamentals: Basic AC Theory

real average of the squared value. Then we can take the square root of that average to get the effective current or voltage.

- If we plot a unit sine wave and its square, we will get the graph shown below.



- Here, the sine wave (in red) varies in the range ± 1 , while its square (in blue) is a smaller sinusoidal waveform that varies from 0 to 1.
- Mathematically:

$$\sin^2(x) = \frac{1}{2} - [\cos(2x)]/2$$

- Since the average value of any sine (or cosine) wave by itself is zero, the average value of the above expression is simply $1/2$. This is the average of the squared sine wave. Therefore we must take its square root to get the effective value, which is $1/\sqrt{2} = 0.707$. This factor gives us the square root of the mean (or average) of the squared value of the sine wave. Therefore, the effective value of the waveform is also known as the *root-mean-square*, or *rms* value.
- It might seem difficult to describe an ac signal in terms of a specific value, since an ac signal is not constant. Therefore, when a voltage or a current is described simply as ac, we will refer to its RMS or effective value, not its maximum value, which simplifies the description of ac signal.
- “RMS” stands for *Root Mean Square*, and is a way of expressing an AC quantity of voltage or current in terms functionally equivalent to DC. For example, 10 volts AC RMS is the amount of voltage that would produce the same amount of heat dissipation across a resistor of given value as a 10 volt DC power supply. Also known as the “equivalent” or “DC equivalent” value of an AC voltage or current. For a sine wave, the RMS value is approximately 0.707 of its peak value.

$$V_{\text{RMS}} = V_{\text{peak}} / \sqrt{2} = 0.707 \times V_{\text{peak}} \text{ and}$$

$$V_{\text{peak}} = \sqrt{2} \times V_{\text{RMS}} = 1.414 \times V_{\text{RMS}}$$

Lecture-11: AC Fundamentals: Basic AC Theory

Form Factor

- The *form factor* of an AC waveform is the ratio of its RMS value to its average value.
- The form factor of an alternating current waveform (signal) is the ratio of the RMS (Root Mean Square) value to the average value (mathematical mean of absolute values of all points on the waveform).
- In case of a sinusoidal wave, i.e. an analogue wave, the form factor is approximately 1.11.

$$V_{\text{RMS}} = V_{\text{peak}} / \sqrt{2} = 0.707 \times V_{\text{peak}} \text{ and}$$

$$V_{\text{AVERAGE}} = (V_{\text{peak}})(2/\pi) = 0.637 \times V_{\text{peak}} \text{ and}$$

$$K_f = \frac{\text{RMS}}{\text{Average}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\pi}} = \frac{\pi}{2\sqrt{2}} \approx 1.110720735$$

- In the case of a square wave, i.e. a digital wave, the RMS and the average value are equal; therefore, the form factor is 1.

$$K_f = \frac{\text{RMS}}{\text{Average}} = 1, \text{RMS} = \text{Average}$$

Crest or Peak or Amplitude Factor K_a :

- The *crest factor* of an AC waveform is the ratio of its peak (crest) to its RMS value.
- It is defined as the ratio of maximum value to the RMS value.
- The crest factor varies for different waveforms.
- For a sinusoidal alternating voltage:

$$K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{V_{\text{MAX}}}{V_{\text{MAX}} / \sqrt{2}} = \sqrt{2} = 1.414$$

- For a sinusoidal alternating current:

$$K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_{\text{MAX}}}{I_{\text{MAX}} / \sqrt{2}} = \sqrt{2} = 1.414$$

Lecture-11: AC Fundamentals: Basic AC Theory

- For a triangle wave form centered about zero

$$K_a = \sqrt{3} = 1.732$$

- For a square wave form centered about zero

$$V_{rms} = V_{peak}$$

$$K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{V_{MAX}}{V_{RMS}} = \frac{V_{MAX}}{V_{MAX}} = 1$$
