AC Fundamentals: Series AC Circuit

Linear Vs. Complex Numbers:

- Complex numbers are useful for AC circuit analysis because they provide a convenient method of symbolically denoting phase shift between AC quantities like voltage and current.
- Without the use of vector (complex number) notation to describe AC quantities, it would be *very* difficult to perform mathematical calculations for AC circuit analysis.
- If you were asked, “What is the distance between two cities (e.g Dhaka and Pabna)?” The answer is a single number in miles, kilometers, or some other unit of linear measurement (e.g. 140 km).
- But if you were asked about how to travel from one city to another, the answer is not a single number only. Information about the direction to travel is also to be provided as well.
- The kind of information that expresses a single dimension, such as linear distance, is called a *scalar* quantity in mathematics. Scalar numbers are the kind of numbers we have used in most of our mathematical applications so far. For example, the voltage produced by a battery is a scalar quantity. So is the resistance of a piece of wire (ohms), or the current through it (amps).
- However, when we begin to analyze alternating current circuits, we find that quantities of voltage, current, and even resistance (called *impedance* in AC) are not the familiar one dimensional quantities we’re used to measuring in DC circuits. Rather, these quantities, because they’re dynamic (alternating in direction and amplitude), possess other dimensions that must be taken into account. Frequency and phase shift are two of these dimensions that come into play. Even with relatively simple AC circuits, where we’re only dealing with a single frequency, we still have the dimension of phase shift to contend with in addition to the amplitude.
- In order to successfully analyze AC circuits, we need *complex numbers* which is capable of representing multi-dimensional quantities.

- Just like the example of giving directions from one city to another, AC quantities in a single-frequency circuit have both amplitude (analogy: distance) and phase shift (analogy: direction). A complex number is a single mathematical quantity able to express these two dimensions of amplitude and phase shift at once.
- One-dimensional, scalar numbers are perfectly adequate for counting beads, representing weight, or measuring DC battery voltage, but they fall short of being able to represent something more complex like the distance *and* direction between two cities, or the amplitude *and* phase of an AC waveform. To represent these kinds of quantities, we need multidimensional representations. In other words, we need a number line that can point in different directions, and that's exactly what a vector is.
- A *scalar* number is a mathematical quantity representing one-dimension of magnitude like temperature, length, weight, etc.
- A *complex number* is a mathematical quantity representing two dimensions of magnitude and direction.
- A *vector* is a graphical representation of a complex number. It has both magnitude and direction. It looks like an arrow, with a starting point, a tip, a definite length, and a definite direction. Sometimes the word *phasor* is used in electrical applications where the angle of the vector represents phase shift between waveforms.

Vectors and AC waveforms

- How can we represent AC quantities (e.g. voltage and current) in the form of a vector?
- When used to describe an AC quantity, the length of a vector represents the amplitude of the waveform (shown in the figure below). The greater the amplitude of the waveform, the greater the length of its corresponding vector.

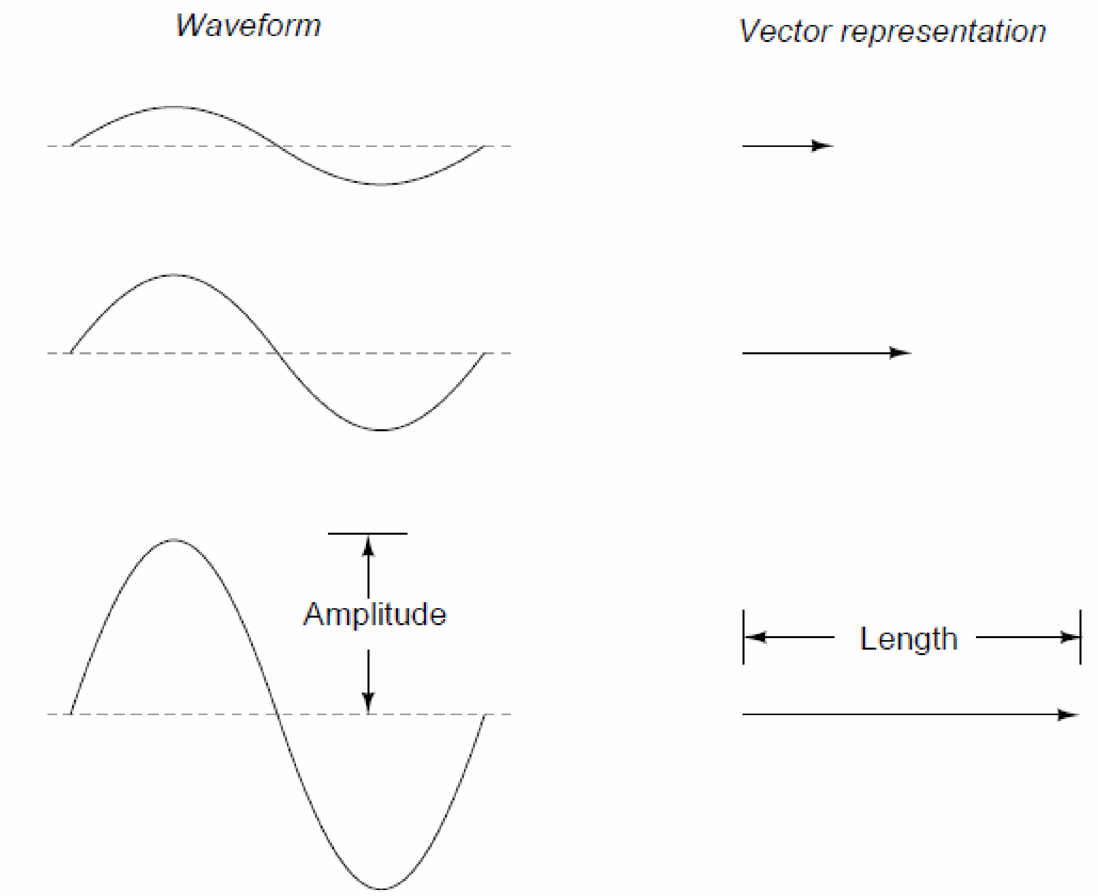


Figure: Vector length represents AC voltage magnitude.

- When used to describe an AC quantity, the angle of the vector represents the phase shift or phase angle (in degrees) between the waveform in question and another waveform acting as a “reference” in time (Shown in the figure below).

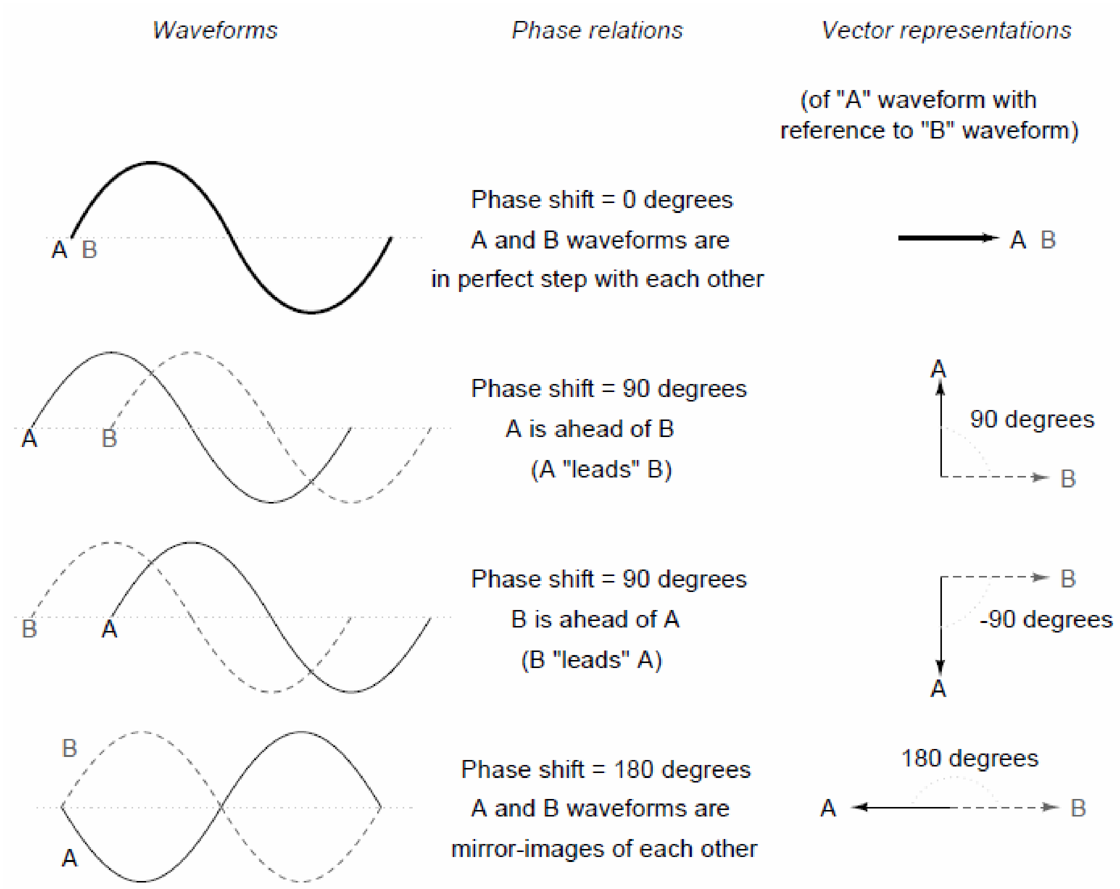


Figure: Vector angle is the phase with respect to another waveform

- Usually, when the phase of a waveform in a circuit is expressed, it is referenced to the power supply voltage waveform. Remember that phase is always a *relative* measurement between two waveforms rather than an absolute property.
- The greater the phase shift in degrees between two waveforms, the greater the angle difference between the corresponding vectors.
- Being a relative measurement, like voltage, phase shift (vector angle) only has meaning in reference to some standard waveform. Generally this "reference" waveform is the main AC power supply voltage in the circuit.
- If there is more than one AC voltage source, then one of those sources is arbitrarily chosen to be the phase reference for all other measurements in the circuit (shown in the figure below).

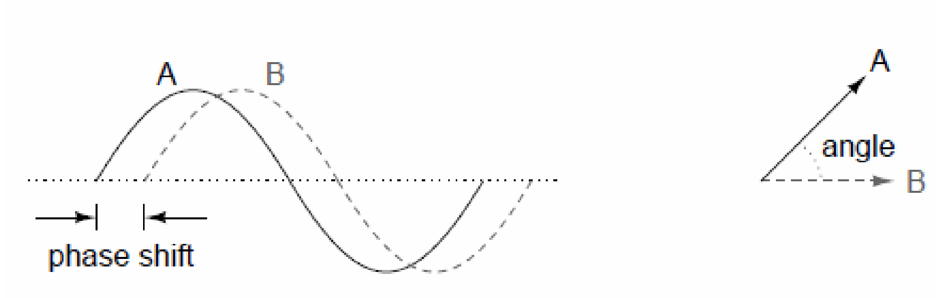


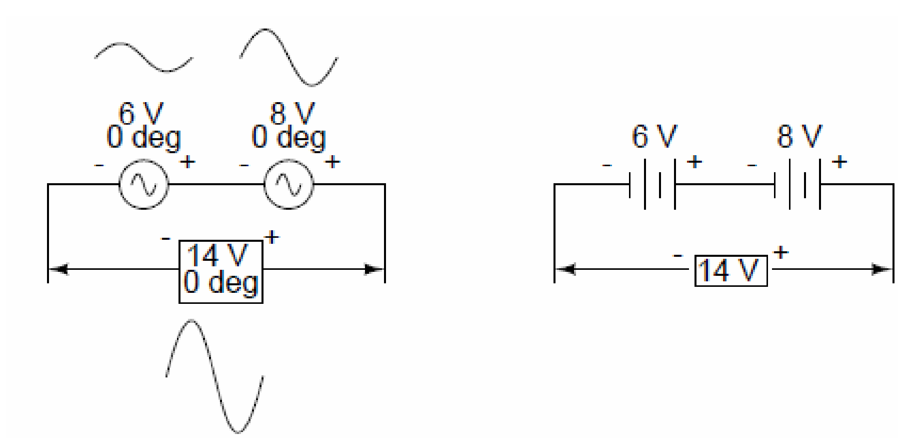
Figure: Phase shift between waves and vector phase angle

- If the current in an AC circuit is described as "24.3 milliamps at -64 degrees," it means that the current waveform has an amplitude of 24.3 mA, and it lags 64^{o} behind the reference waveform, usually assumed to be the main source voltage waveform.

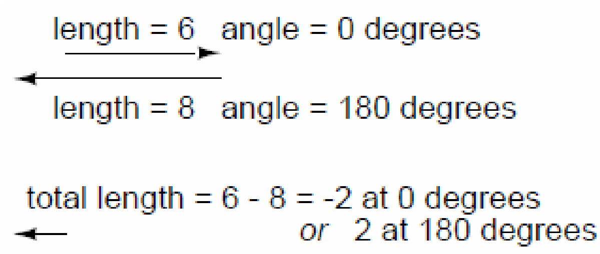
Simple Vector Addition (Vectors with Common Angles):

- As mathematical objects, vectors can be added, subtracted, multiplied, and divided.
- If vectors with **common angles** are added, their magnitudes (lengths) add up just like regular scalar quantities (shown below):

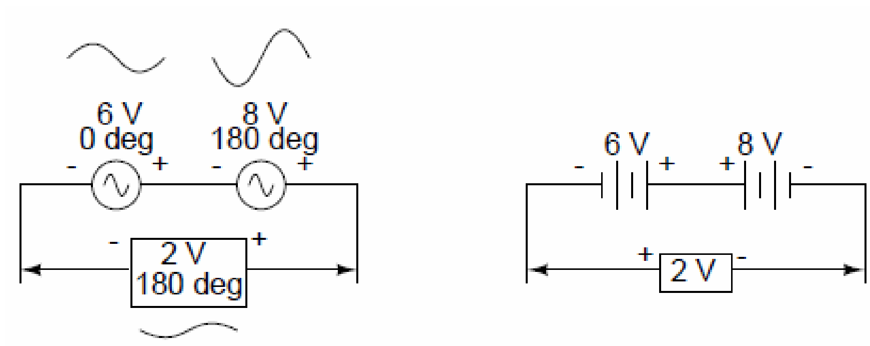
- Similarly, if AC voltage sources with the same phase angle are connected together in series, their voltages add just as with DC batteries (shown in the figure below).



- If vectors directly opposing each other (180o out of phase) are added together, their magnitudes (lengths) subtract just like positive and negative scalar quantities subtract when added (shown below):

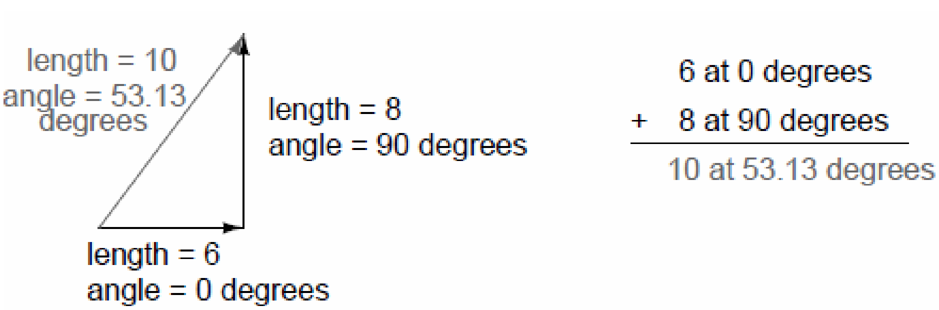


- Similarly, if opposing AC voltage sources are connected in series, their voltages subtract as with DC batteries connected in an opposing fashion (shown below):

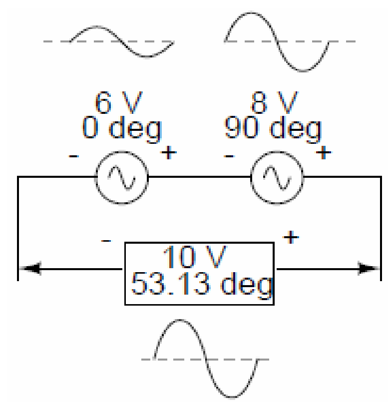


Complex Vector Addition (Vectors with Uncommon Angles)

- If vectors with uncommon angles are added, their magnitudes (lengths) add up quite differently than that of scalar magnitudes:



- Similarly, if two AC voltages, 90o out of phase, are added together by being connected in series, their voltage magnitudes do not directly add or subtract as with scalar voltages in DC. Instead, these voltage quantities are complex quantities. They add up with the help of trigonometry. For example, a 6 volt source at 0^{o} added to an 8 volt source at 90^{o} results in 10 volts at a phase angle of 53.13^{o} (shown below):



Representing Vector Quantities in Polar and Rectangular Notation:

- Previously, vector quantities are represent in graphical form.
- In order to work with complex numbers without drawing vectors, we need some kind of standard mathematical notation. There are two basic forms of complex number notation:
  - *Polar Notation*
  - *Rectangular Notation*

- Either method of notation is valid for complex numbers. The primary reason for having two methods of notation is for ease of longhand calculation:
  - Rectangular form lending itself to addition and subtraction
  - Polar form lending itself to multiplication and division.

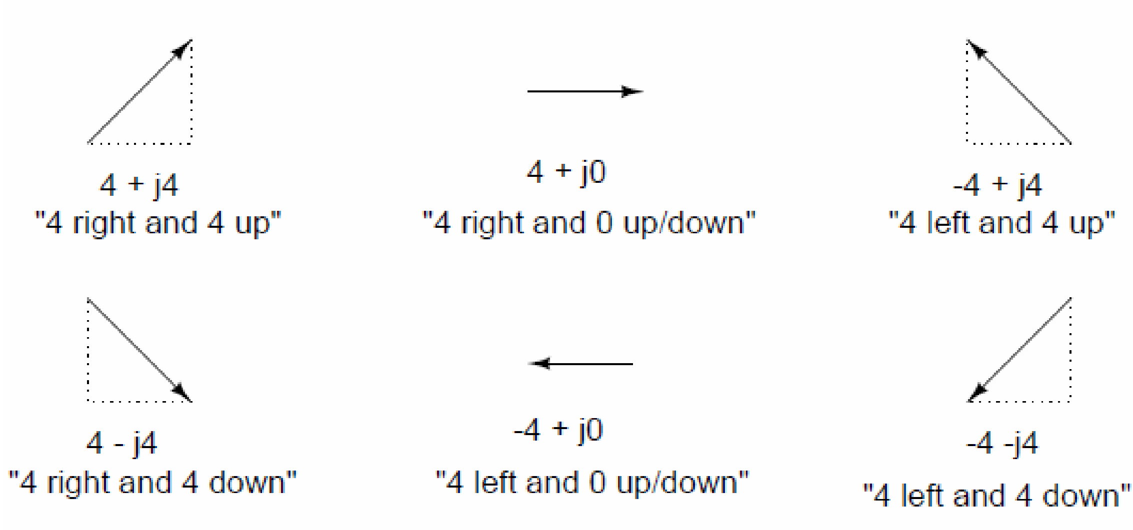
Polar Form:

- In polar form, a complex number is denoted by the *length* (otherwise known as the *magnitude*, *absolute value*, or *modulus*) and the *angle* of its vector (usually denoted by an angle symbol that looks like this: ∠). Example: fly 45 miles ∠203° (West by Southwest).
- Standard orientation for vector angles in AC circuit calculations defines 0° as being to the right (horizontal), making 90° straight up, 180° to the left, and 270° straight down. Please note that vectors angled "down" can have angles represented in polar form as positive numbers in excess of 180, or negative numbers less than 180. For example, a vector angled ∠ 270° (straight down) can also be said to have an angle of -90°. The vector 7.81∠ 230.19° can also be denoted as 7.81∠ -129.81°.

Rectangular Form:

- In rectangular form, a complex number is denoted by its respective horizontal and vertical components. In essence, the angled vector is taken to be the hypotenuse of a right triangle, described by the lengths of the adjacent and opposite sides. Example: drive 41 miles West, then turn and drive 18 miles South.
- Rather than describing a vector's length and direction by denoting magnitude and angle, it is described in terms of "how far left/right" and "how far up/down."
- These two dimensional figures (horizontal and vertical) are symbolized by two numerical figures. In order to distinguish the horizontal and vertical dimensions from each other, the vertical is prefixed with a lower-case "i" (in pure mathematics) or "j" (in electronics). These lower-case letters do not represent a physical variable (such as instantaneous current, also symbolized by a lower-case letter "i"), but rather are mathematical *operators* used to distinguish the vector's vertical component from its horizontal component. As a complete complex number, the horizontal and vertical quantities are written as a sum (shown below).
- In "rectangular" form the vector's length and direction are denoted in terms of its horizontal and vertical span, the first number representing the

the horizontal ("real") and the second number (with the "j" prefix) representing the vertical ("imaginary") dimensions.

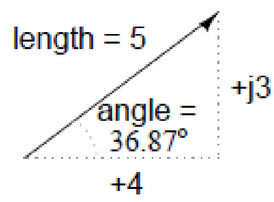


- The horizontal component is referred to as the *real* component, since that dimension is compatible with normal, scalar ("real") numbers. The vertical component is referred to as the *imaginary* component, since that dimension lies in a different direction, totally alien to the scale of the real numbers.

Conversion between Polar and Rectangular Form

- To convert from polar to rectangular form:
  - Find the real component by multiplying the polar magnitude by the cosine of the angle
  - Find the imaginary component by multiplying the polar magnitude by the sine of the angle.

- This may be understood more readily by drawing the quantities as sides of a right triangle, the hypotenuse of the triangle representing the vector itself (its length and angle with respect to the horizontal constituting the polar form), the horizontal and vertical sides representing the "real" and "imaginary" rectangular components, respectively (shown in the figure below).



Example:

$5 \angle 36.87°$	**(polar form)**
$(5)(\cos 36.87°) = 4$	(real component)
$(5)(\sin 36.87°) = 3$	(imaginary component)
$4 + j3$	**(rectangular form)**

- To convert from rectangular to polar Form:
  - Find the polar magnitude through the use of the Pythagorean Theorem (the polar magnitude is the hypotenuse of a right triangle, and the real and imaginary components are the adjacent and opposite sides, respectively).
  - Find the angle by taking the arctangent (tan inverse) of the imaginary component divided by the real component:

Example:

$4 + j3$ **(rectangular form)**

$c = \sqrt{a^2 + b^2}$ (pythagorean theorem)

$\text{polar magnitude} = \sqrt{4^2 + 3^2}$

$\text{polar magnitude} = 5$

$\text{polar angle} = \arctan \frac{3}{4}$

$\text{polar angle} = 36.87^\circ$

$5 \angle 36.87^\circ$ **(polar form)**

Complex number arithmetic:

- In order to manipulate complex numbers in the analysis of AC circuits, we need to know some basic operations.
- Either polar or rectangular form is valid for complex numbers arithmetic. The primary reason for having two methods of notation is for ease of longhand calculation:
    - Rectangular form lending itself to addition and subtraction
    - Polar form lending itself to multiplication and division. For longhand multiplication and division, polar form is the favored notation to work with.

Addition:

- To add complex numbers in rectangular form, simply add up the real components of the complex numbers to determine the real component of the sum, and add up the imaginary components of the complex numbers to determine the imaginary component of the sum.
- Example:

$$\begin{array}{r} 2 + j5 \\ +\ 4 - j3 \\ \hline \mathbf{6 + j2} \end{array} \qquad \begin{array}{r} 175 - j34 \\ +\ 80 - j15 \\ \hline \mathbf{255 - j49} \end{array} \qquad \begin{array}{r} -36 + j10 \\ +\ 20 + j82 \\ \hline \mathbf{-16 + j92} \end{array}$$

Subtraction:

- To subtract one complex number from another in rectangular form, simply subtract the real component of the second complex number from the real component of the first to arrive at the real component of the difference, and subtract the imaginary component of the second complex number from the imaginary component of the first to arrive the imaginary component of the difference.
- Example:

$$\begin{array}{r} 2 + j5 \\ -\ (4 - j3) \\ \hline \mathbf{-2 + j8} \end{array} \qquad \begin{array}{r} 175 - j34 \\ -\ (80 - j15) \\ \hline \mathbf{95 - j19} \end{array} \qquad \begin{array}{r} -36 + j10 \\ -\ (20 + j82) \\ \hline \mathbf{-56 - j72} \end{array}$$

Multiplication:

- To multiply complex numbers in polar form, multiply the magnitudes and add the angles.
- When multiplying complex numbers in polar form, simply *multiply* the polar magnitudes of the complex numbers to determine the polar magnitude of the product, and *add* the angles of the complex numbers to determine the angle of the product.
- Example:

$$(35 \angle 65^{\circ})(10 \angle -12^{\circ}) = \mathbf{350 \angle 53^{\circ}}$$

$$(124 \angle 250^{\circ})(11 \angle 100^{\circ}) = \mathbf{1364 \angle -10^{\circ}}$$

or

$$\mathbf{1364 \angle 350^{\circ}}$$

$$(3 \angle 30^{\circ})(5 \angle -30^{\circ}) = \mathbf{15 \angle 0^{\circ}}$$

Division:

- To divide a complex number by another in polar form, simply divide the polar magnitude of the first complex number by the polar magnitude of the second complex number to arrive at the polar magnitude of the quotient, and subtract the angle of the second complex number from the angle of the first complex number to arrive at the angle of the quotient.
- Example:

$$\frac{35 \angle 65^{o}}{10 \angle -12^{o}} = \mathbf{3.5 \angle 77^{o}}$$

$$\frac{124 \angle 250^{o}}{11 \angle 100^{o}} = \mathbf{11.273 \angle 150^{o}}$$

$$\frac{3 \angle 30^{o}}{5 \angle -30^{o}} = \mathbf{0.6 \angle 60^{o}}$$

- To obtain the reciprocal, or "invert" (1/x), of a complex number, simply divide the number (in polar form) into a scalar value of 1, which is nothing more than a complex number with no imaginary component (angle = 0).
- Example:

$$\frac{1}{35 \angle 65^{o}} = \frac{1 \angle 0^{o}}{35 \angle 65^{o}} = \mathbf{0.02857 \angle -65^{o}}$$

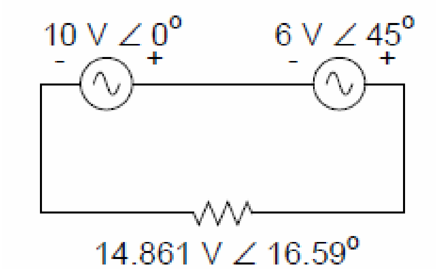
$$\frac{1}{10 \angle -12^{o}} = \frac{1 \angle 0^{o}}{10 \angle -12^{o}} = \mathbf{0.1 \angle 12^{o}}$$

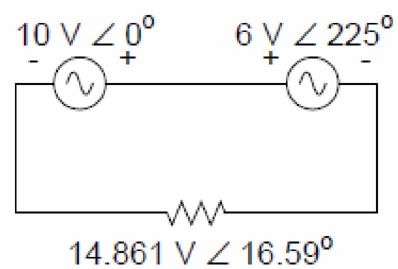
$$\frac{1}{0.0032 \angle 10^{o}} = \frac{1 \angle 0^{o}}{0.0032 \angle 10^{o}} = \mathbf{312.5 \angle -10^{o}}$$

About Polarity of AC

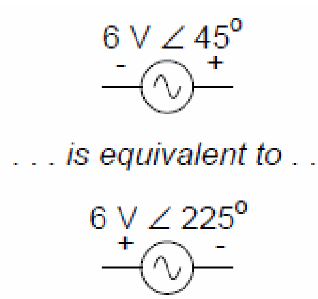
- In AC circuits, we don't deal with "negative" quantities of voltage, i.e. we never describe an AC voltage as being negative in. Instead, we describe to what degree one voltage aids or opposes another by *phase*: the time-shift between two waveforms.
- If one AC voltage directly opposes another AC voltage, we simply say that one is 180^{o} out of phase with the other.

- In DC voltage sources, polarity is defined symbolically by means of short and long lines.
- But, AC voltage symbols have no intrinsic polarity marking. Therefore, any polarity marks must be included as additional symbols on the diagram, and there is no one “correct” way in which to place them. They must, however, correlate with the given phase angle to represent the true phase relationship of that voltage with other voltages in the circuit.



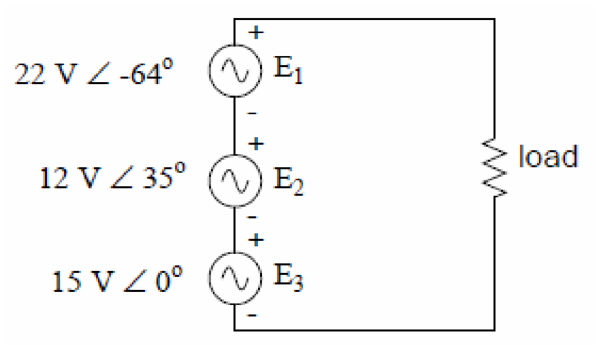


- Consider the above two AC circuits.
- 6 V $\angle 45^o$ with negative on the left and positive on the right is exactly the same as 6 V $\angle 225^o$ with positive on the left and negative on the right: the reversal of polarity markings perfectly adds 180^o to phase angle



Some examples with AC circuits

Let us connect three AC voltage sources in series and use complex numbers to determine additive voltages.



Here we will apply DC rules (e.g. KVL) to AC circuit for the addition of complex voltages. But we have to consider two points:

- all variables *must* be expressed in complex form, taking into account phase as well as magnitude.
- all voltages and currents must be of the same frequency.
- From the above figure, we see that the polarity marks for all three voltage sources are oriented in such a way that their stated voltages should add to make the total voltage across the load resistor.
- Notice that although magnitude and phase angle is given for each AC voltage source, no frequency value is specified. If this is the case, it is assumed that all frequencies are equal, thus meeting our qualifications
- for applying DC rules to an AC circuit (all figures given in complex form, all of the same
- frequency).
- The setup of our equation to find total voltage appears as such:

$$E_{total} = E_1 + E_2 + E_3$$

$$E_{total} = (22\text{ V} \angle -64^\circ) + (12\text{ V} \angle 35^\circ) + (15\text{ V} \angle 0^\circ)$$

➢ Graphically, the vectors add up as shown in the figure below:

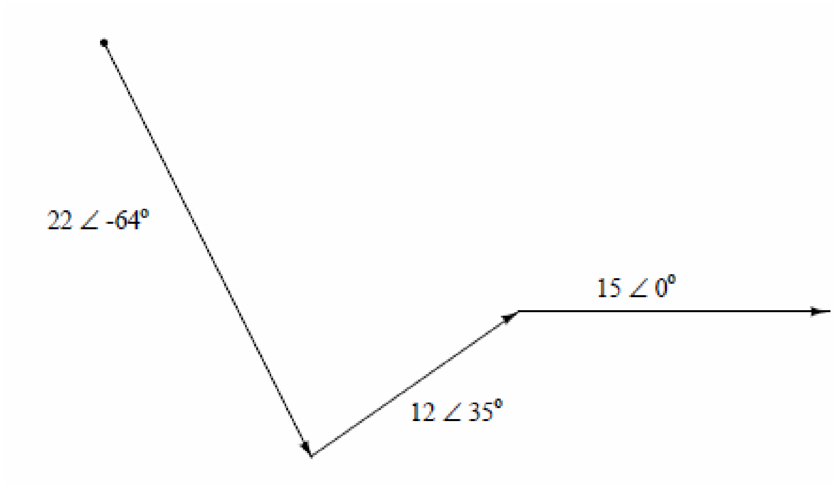


Figure: Graphic addition of vector voltages

- The sum of these vectors will be a resultant vector originating at the starting point for the 22 volt vector (dot at upper-left of diagram) and terminating at the ending point for the 15 volt vector (arrow tip at the middle-right of the diagram):

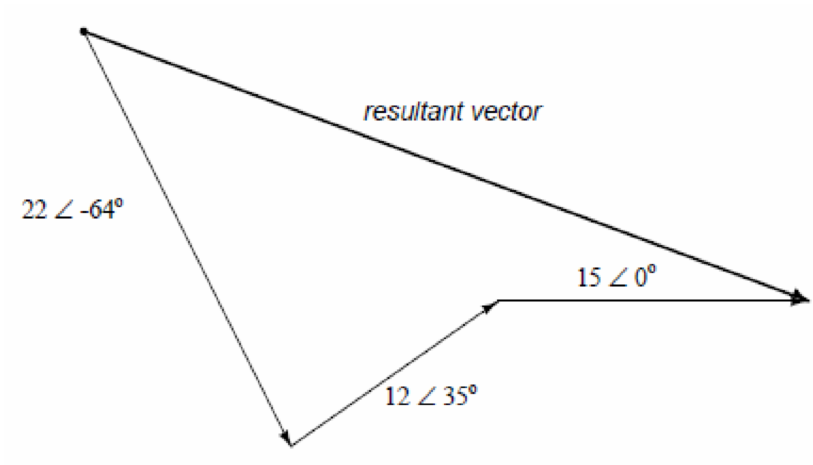


Figure: Resultant is equivalent to the vector sum of the three original voltages.

- In order to determine what the resultant vector's magnitude and angle are without resorting to graphic images, we can convert each one of these polar-form complex numbers into rectangular form and add.
- Remember, we're *adding* these figures together because the polarity marks for the three voltage sources are oriented in an additive manner:

$$15\text{ V} \angle 0^{o} = 15 + j0\text{ V}$$

$$12\text{ V} \angle 35^{o} = 9.8298 + j6.8829\text{ V}$$

$$22\text{ V} \angle -64^{o} = 9.6442 - j19.7735\text{ V}$$

$$\begin{array}{rll} & 15 & +j0\text{ V} \\ & 9.8298 & +j6.8829\text{ V} \\ + & 9.6442 & -j19.7735\text{ V} \\ \hline & \mathbf{34.4740} & \mathbf{-\,j12.8906\text{ V}} \end{array}$$

- In polar form, this equates to 36.8052 volts$\angle$-20.5018^{o}. What this means in real terms is that the voltage measured across these three voltage sources will be 36.8052 volts, lagging the 15 volt (0^{o} phase reference) by 20.5018^{o}.

- A voltmeter connected across these points in a real circuit would only indicate the polar magnitude of the voltage (36.8052 volts), not the angle. An oscilloscope could be used to display two voltage waveforms and thus provide a phase shift measurement, but not a voltmeter. The same principle holds true for AC ammeters: they indicate the polar magnitude of the current, not the phase angle.

Reactance:

Reactance is a measure of the opposition of capacitance and inductance to current.

Reactance varies with the frequency of the electrical signal.

It is symbolized by the letter "X" and measured in ohms,

There are two types of reactance: capacitive reactance (Xc) and inductive reactance (X_L).

The **total reactance (X)** is the *difference* between the two: $\mathbf{X = X_L - Xc}$

Capacitive Reactance, Xc

- *Capacitive reactance* is the opposition that a capacitor offers to alternating current due to its phase-shifted storage and release of energy in its electric field.
- Capacitive reactance is symbolized by the capital letter "X_C" and is measured in ohms just like resistance (R).
- Capacitive reactance can be calculated using this formula:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad (unit\ is\ in\ \ \Omega)$$

Where,

X_C = reactance in ohms (Ω)

f = frequency in hertz (Hz)

C = capacitance in farads (F)

- Capacitive reactance *decreases* with increasing frequency. In other words, the higher the frequency, the less it opposes (the more it "conducts") the AC flow of electrons.

Example:

Determine the reactance of a 1μF capacitor for a 50Hz signal. Also determine its reactance when frequency is 10kHz.

Ans. 3.2kΩ (at 50 Hz), 16Ω (at 10 kHz)

Inductive reactance, X_L

- *Inductive reactance* is the opposition that an inductor offers to alternating current due to its phase-shifted storage and release of energy in its magnetic field.
- Inductive reactance is symbolized by the capital letter "X_L" and is measured in ohms just like resistance (R).
- Inductive reactance can be calculated using this formula:

$$X_L = \omega L = 2\pi fL \quad (unit\, is\, in \;\; \Omega)$$

Where,

X_L = reactance in ohms (Ω)

f = frequency in hertz (Hz)

L = inductance in henrys (H)

- Inductive reactance *increases* with increasing frequency. In other words, the higher the frequency, the more it opposes the AC flow of electrons.

Example:

Determine the reactance of a 1mH inductor for a 50Hz signal. Also determine its reactance when frequency is 10kHz.

Ans. 0.3Ω (at 50 Hz), 63Ω (at 10 kHz)

Impedance

- Impedance is a measure of the overall opposition of a circuit to current, in other words: how much the circuit **impedes** the flow of current. It is like resistance, but it also takes into account the effects of capacitance and inductance.
- *It* is the complex (vector) sum of ("real") resistance and ("imaginary") reactance.
- Impedance can be split into two parts:
    - **Resistance R** (the part which is constant regardless of frequency)
    - **Reactance X** (the part which varies with frequency due to capacitance and inductance)

- It is symbolized by the letter "Z" and measured in ohms, just like resistance (R) and reactance (X).

- Impedance is more complex than resistance because the effects of capacitance and inductance vary with the frequency of the current passing through the circuit and this means **impedance varies with frequency**! The effect of resistance is constant regardless of frequency.

- Impedances (Z) are managed just like resistances (R) in series circuit analysis: series impedances add to form the total impedance. Just be sure to perform all calculations in complex (not scalar) form! ZTotal = Z1 + Z2 + . . . Zn

- Perfect resistors possess resistance, but not reactance.

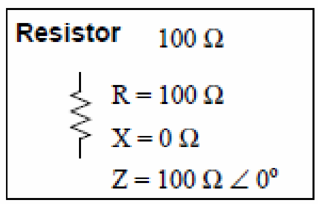


Figure: Perfect resistor

- Perfect inductors and perfect capacitors possess reactance but no resistance.

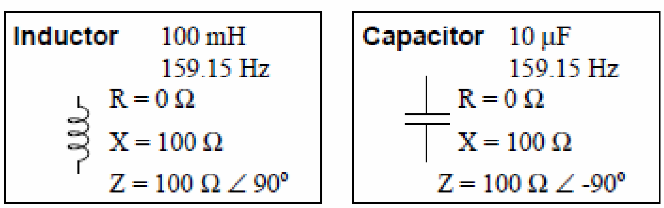
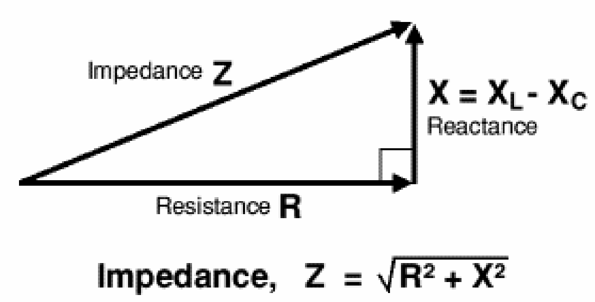


Figure: Perfect inductor and capacitor

- All components (resistor, capacitor and inductor) possess impedance, and because of this universal quality, it makes sense to translate all component values into common terms of impedance as the first step in analyzing an AC circuit.

- The impedance phase angle for any component is the phase shift between voltage across that component and current through that component.
- For a perfect resistor, the voltage drop and current are *always* in phase with each other, and so the impedance angle of a resistor is said to be 0^o.
- For an perfect inductor, voltage drop always leads current by 90^o, and so an inductor's impedance phase angle is said to be $+90^o$.
- For a perfect capacitor, voltage drop always lags current by 90^o, and so a capacitor's impedance phase angle is said to be -90^o.

- The capacitance and inductance cause a **phase shift** between the current and voltage which means that the resistance and reactance cannot be simply added up to give impedance. Instead they must be **added as vectors** with reactance at right angles to resistance as shown in the diagram.



Impedance for a series RL Circuit:

$Z = \sqrt{(R^2 + X_L^2)}$ $= R + JX_L$

Impedance for a series RC Circuit:

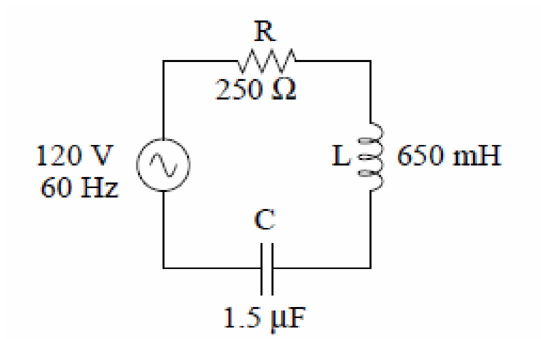
$Z = \sqrt{(R^2 + X_C^2)}$ $= R + JX_C$

Impedance for a series RLC Circuit:

$$Z = \sqrt{(R^2 + X^2)} = \sqrt{(R^2 + (X_L - X_C)^2}$$ where $X = X_L - X_C$

Example:

Determine the impedance of a series RLC circuit shown below connected with a 60Hz supply.



Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi fL$$

$$X_L = (2)(\pi)(60\ \text{Hz})(650\ \text{mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{(2)(\pi)(60\ \text{Hz})(1.5\ \mu\text{F})}$$

$$X_C = 1.7684\ \text{k}\Omega$$

Step-2:

Express all resistances and reactances in a mathematically common form: impedance.

$$Z_R = 250 + j0\ \Omega \quad or \quad 250\ \Omega \angle 0^\circ$$

$$Z_L = 0 + j245.04\ \Omega \quad or \quad 245.04\ \Omega \angle 90^\circ$$

$$Z_C = 0 - j1.7684k\ \Omega \quad or \quad 1.7684\ k\Omega \angle -90^\circ$$

- Remember that an inductive reactance translates into a positive imaginary impedance (or an impedance at $+90^o$), while a capacitive reactance translates into a negative imaginary impedance (impedance at -90^o). Resistance, of course, is still regarded as a purely "real" impedance (polar angle of 0^o):

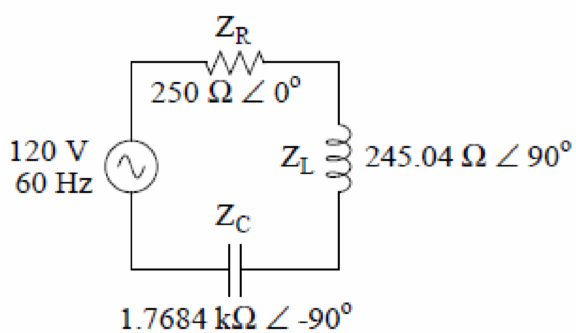


Figure: Series R, L, and C circuit with component values replaced by impedances.

Step-3:

Calculate total impedance of the circuit.

$$Z_{total} = Z_R + Z_L + Z_C$$

$$Z_{total} = (250 + j0\ \Omega) + (0 + j245.04\ \Omega) + (0 - j1.7684k\ \Omega)$$

$$Z_{total} = 250 - j1.5233k\ \Omega \quad \textit{or} \quad 1.5437\ k\Omega \angle -80.680^{\circ}$$

	R	L	C	Total	
E				**120 + j0** **120 ∠ 0°**	Volts
I					Amps
Z	**250 + j0** **250 ∠ 0°**	**0 + j245.04** **245.04 ∠ 90°**	**0 - j1.7684k** **1.7684k ∠ -90°**		Ohms

Analysis table for the above circuit. All the "given" figures are shown (total voltage, and the impedances of the resistor, inductor, and capacitor).

	R	L	C	Total	
E				120 + j0 120 ∠ 0°	Volts
I					Amps
Z	250 + j0 250 ∠ 0°	0 + j245.04 245.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	**250 - j1.5233k** **1.5437k ∠ -80.680°**	Ohms

Rule of series circuits:
$Z_{total} = Z_R + Z_L + Z_C$

	R	L	C	Total	
E				120 + j0 120 $\angle$ 0°	Volts
I				**12.589m + 76.708m** **77.734m $\angle$ 80.680°**	Amps
Z	250 + j0 250 $\angle$ 0°	0 + j245.04 245.04 $\angle$ 90°	0 - j1.7684k 1.7684k $\angle$ -90°	250 - j1.5233k 1.5437k $\angle$ -80.680°	Ohms

Ohm's Law

$I = \frac{E}{Z}$

	R	L	C	Total	
E				120 + j0 120 $\angle$ 0°	Volts
I	**12.589m + 76.708m** **77.734m $\angle$ 80.680°**	**12.589m + 76.708m** **77.734m $\angle$ 80.680°**	**12.589m + 76.708m** **77.734m $\angle$ 80.680°**	12.589m + 76.708m 77.734m $\angle$ 80.680°	Amps
Z	250 + j0 250 $\angle$ 0°	0 + j245.04 245.04 $\angle$ 90°	0 - j1.7684k 1.7684k $\angle$ -90°	250 - j1.5233k 1.5437k $\angle$ -80.680°	Ohms

Rule of series circuits:

$I_{total} = I_R = I_L = I_C$

	R	L	C	Total	
E	**3.1472 + j19.177** **19.434 $\angle$ 80.680°**	**-18.797 + j3.0848** **19.048 $\angle$ 170.68°**	**135.65 - j22.262** **137.46 $\angle$ -9.3199°**	120 + j0 120 $\angle$ 0°	Volts
I	12.589m + 76.708m 77.734m $\angle$ 80.680°	12.589m + 76.708m 77.734m $\angle$ 80.680°	12.589m + 76.708m 77.734m $\angle$ 80.680°	12.589m + 76.708m 77.734m $\angle$ 80.680°	Amps
Z	250 + j0 250 $\angle$ 0°	0 + j245.04 245.04 $\angle$ 90°	0 - j1.7684k 1.7684k $\angle$ -90°	250 - j1.5233k 1.5437k $\angle$ -80.680°	Ohms

Ohm's Law $E = IZ$	*Ohm's Law* $E = IZ$	*Ohm's Law* $E = IZ$

$E_R + E_L + E_C$ *should equal* E_{total}

$$\begin{array}{rll} & 3.1472 + j19.177\ \text{V} & E_R \\ & -18.797 + j3.0848\ \text{V} & E_L \\ + & 135.65 - j22.262\ \text{V} & E_C \\ \hline & 120 + j0\ \ \text{V} & E_{total} \end{array}$$

Ohm's Law for AC circuits:

$$\mathbf{E = IZ} \qquad \mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} \qquad \mathbf{Z} = \frac{\mathbf{E}}{\mathbf{I}}$$

All quantities expressed in complex, not scalar, form