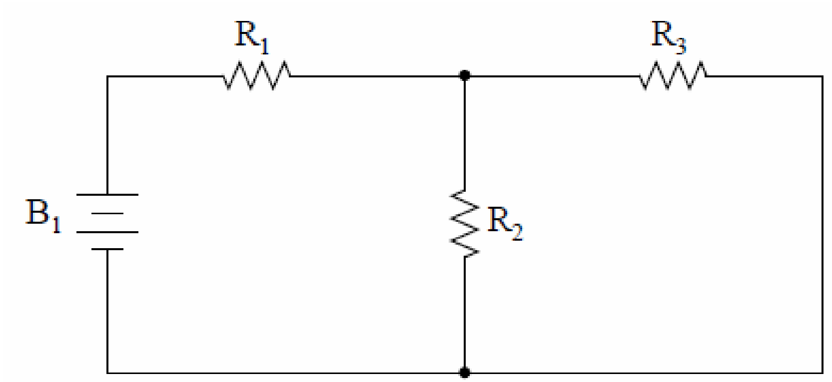
Lecture-08: Methods of Circuit Analysis: Branch Current Method

****

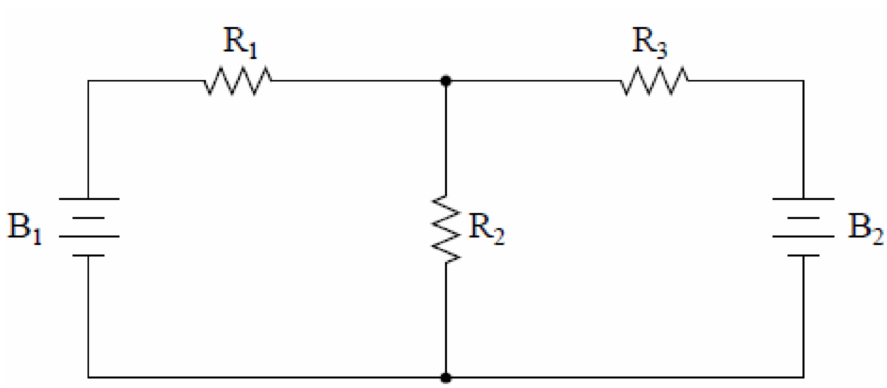
Introduction:

- The circuits having a single voltage source as the means of providing power could be easily analyzed using techniques such as Ohm's and Kirchhoff's laws. For example, consider the simple circuit shown below:



- To analyze the above circuit, one would first find the equivalent resistance of R_2 and R_3 which are in parallel, then add R_1 in series to arrive at a total resistance. Then, taking the voltage of battery B_1 with that total circuit resistance, the total current could be calculated through the use of Ohm's Law (I=E/R), then that current figure is used to calculate voltage drops in the circuit. All in all, solution to this circuit is fairly simple.

- But, circuits more than one voltage source can not be easily analyzed using these techniques. For example, consider the above circuit again with an extra battery shown below:



****

- In the above circuit, resistors R_2 and R_3 are no longer in parallel with each other, because another battery B_2 has been inserted into R_3's branch of the circuit.
- Upon closer inspection, it appears, there are no two resistors in this circuit directly in series or parallel with each other. This is the crux of our problem: in series-parallel analysis, we started off by identifying sets of resistors that were directly in series or parallel with each other, reducing them to single equivalent resistances. If there are no resistors in a simple series or parallel configuration with each other, then what can we do?
- The methods used in determining the operation of such complex networks will include **branch-current analysis**, **mesh (or loop) analysis**, and **nodal analysis**.
- These circuit analysis methods allow us to find two or more unknown currents or voltages by solving simultaneous equations.
- Although any of the above methods may be used, we will find that certain circuits are more easily analyzed using one particular approach. The advantages of each method will be discussed in the appropriate section.

Branch-Current Analysis Method:

- The first and most straightforward network analysis technique is called the Branch Current Method that allows us to directly calculate the current in each branch of a circuit having more than one source.
- In this method, we assume directions of currents in a network, then write equations describing their relationships to each other through Kirchhoff 's and Ohm's Laws.
- Once we have one equation for every unknown current, we can solve the simultaneous equations and determine all currents, and therefore all voltage drops in the network.

Lecture-08: Methods of Circuit Analysis: Branch Current Method

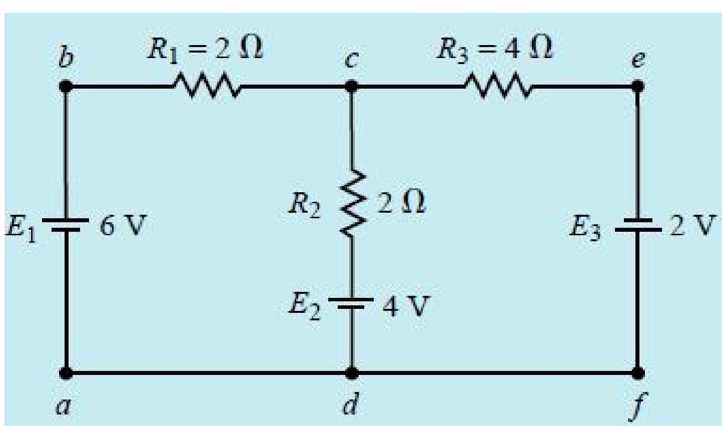
****

The steps used in solving a circuit using branch-current analysis:

1. The first step is to choose a node in the circuit to use as a point of reference for unknown currents. Label the currents (as I_1, I_2 etc.) and arbitrarily assign their directions entering in or exiting from this reference node. If a particular branch has a current source, then this step is not necessary since you already know the magnitude and direction of the current in this branch.
2. Using the assumed directions of currents, label the polarities of the voltage drops across all resistors in the circuit.
3. Apply Kirchhoff's voltage law around each of the closed loops. Write just enough equations to include all branches in the loop equations. If a branch has only a current source and no series resistance, it is not necessary to include it in the KVL equations.
4. Apply Kirchhoff's current law at enough nodes to ensure that all branch currents have been included. In the event that a branch has only a current source, it will need to be included in this step.
5. Solve the linear equations resulting from steps 3 and 4 for the branch current values.

Example-1:

Find the current in each branch of the circuit shown below.

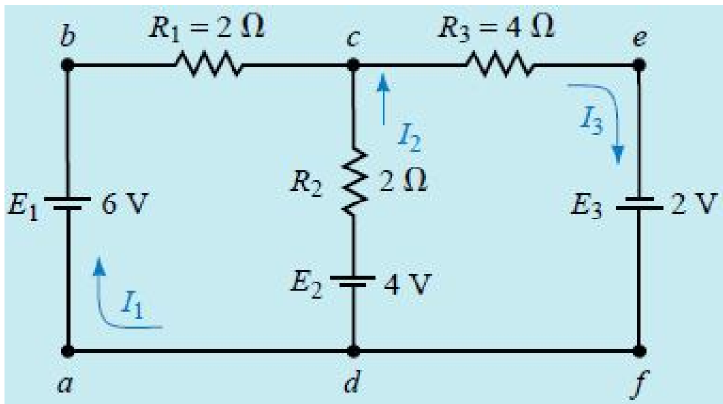


****

Solution:

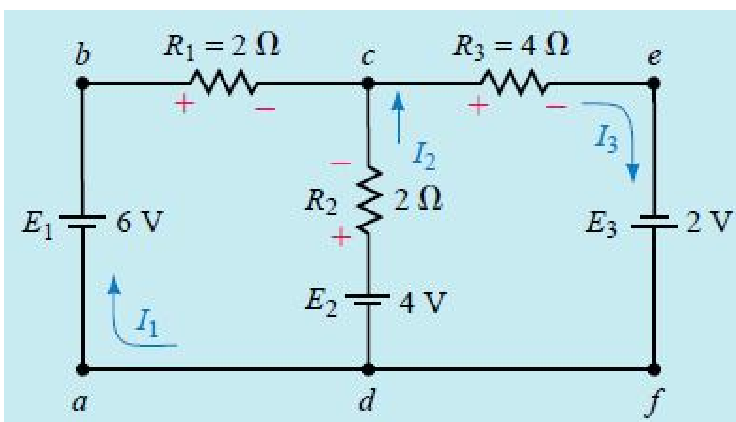
Step-1:

Choose a node in the circuit to use as a point of reference for unknown currents. Here node ***c*** is chosen. Label the currents as I_1, I_2 and I_3. Arbitrarily assign their directions entering in or exiting from this reference node.



Step-2:

Using the assumed current direction, label the polarities of the voltage drops across all resistors in the circuit.



Step-3:

Apply Kirchhoff's voltage law around each of the closed loops. Write just enough equations to include all branches in the loop equations.

Loop *abcda*:

$$6V-(2\Omega)I_1+(2\Omega)I_2-4V=0V$$
$$\Rightarrow \quad -(2\Omega)I_1+(2\Omega)I_2+2V=0V$$
$$\Rightarrow \quad -2I_1+2I_2=-2$$

$$\Rightarrow \quad 2I_1+(-2)I_2+0I_3=2 \qquad \text{............(1)}$$

Loop *cefdc*:

$$4V-(2\Omega)I_2-(4\Omega)I_3+2V=0V$$
$$\Rightarrow \quad -(2\Omega)I_2-(4\Omega)I_3+6V=0V$$
$$\Rightarrow \quad -2I_2-4I_3=-6$$

$$\Rightarrow \quad 0I_1+(-2)I_2+(-4)I_3=(-6) \qquad \text{............(2)}$$

Step-4:

Apply Kirchhoff's current law at enough nodes to ensure that all branch currents have been included.

By applying KCL at node ***c***, all branch currents in the network are included:

$$I_3=I_1+I_2$$
$$1I_1+1I_2+(-1)I_3=0 \qquad \text{............(3)}$$

Step-5:

Solve the resulting simultaneous linear equations.

- To simplify the solution of the simultaneous linear equations we write them as follows:

$$2I_1+(-2)I_2+0I_3=2 \qquad \text{............(1)}$$

****

$$0I_1 + (-2)I_2 + (-4)I_3 = (-6) \qquad \text{............(2)}$$

$$1I_1 + 1I_2 + (-1)I_3 = 0 \qquad \text{............(3)}$$

- We can solve the above equations by the use of determinants and Cramer's rule:

The above equations can be put in the matrix form as under:

$$\begin{bmatrix} 2 & -2 & 0 \\ 0 & -2 & -4 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 0 \end{bmatrix}$$

The value of the common determinant is given by:

$$\Delta = \begin{bmatrix} 2 & -2 & 0 \\ 0 & -2 & -4 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= 2\{[(-2)\times(-1)] - [(-4)\times(1)]\} - (-2)\{[(0)\times(-1)] - [(-4)\times(1)]\} + (0)\{[(0)\times(1)] - [(-2)\times(1)]\}$$

$$= 2\{[2] - [-4]\} - (-2)\{[0] - [-4]\} + (0)\{[0] - [-2]\}$$

$$= 2\{2 + 4\} - (-2)\{0 + 4\} + (0)\{0 + 2\}$$

$$= 2\times 6 - (-2\times 4) + 0\times 2$$

$$= 12 + 8 + 0$$

$$= 20$$

The determinant for I_1 can be found by replacing coefficients of I_1 in the original matrix by the three constants:

$$\Delta_1 = \begin{vmatrix} 2 & -2 & 0 \\ -6 & -2 & -4 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 2\{[(-2)\times(-1)] - [(-4)\times(1)]\} - (-2)\{[(-6)\times(-1)] - [(-4)\times(0)]\} + (0)\{[(-6)\times(1)] - [(-2)\times(0)]\}$$
$$= 24$$

****

Lecture-08: Methods of Circuit Analysis: Branch Current Method

****

Similarly, the determinant for I_2 can be found by replacing coefficients of I_2 in the original matrix by the three constants:

$$\Delta_2 = \begin{vmatrix} 2 & 2 & 0 \\ 0 & -6 & -4 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= 2\{[(-6)\times(-1)] - [(-4)\times(0)]\} - (2)\{[(0)\times(-1)] - [(-4)\times(1)]\} + (0)\{[(0)\times(0)] - [(-6)\times(1)]\} \quad = 4$$

And the same way, the determinant for I_3 can be found by replacing coefficients of I_3 in the original matrix by the three constants:

$$\Delta_3 = \begin{vmatrix} 2 & -2 & 2 \\ 0 & -2 & -6 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2\{[(-2)\times(0)] - [(-6)\times(1)]\} - (2)\{[(0)\times(0)] - [(-6)\times(1)]\} + (2)\{[(0)\times(1)] - [(-2)\times(1)]\}$$

$$= 4$$

Therefore, according to Cramer's rule:

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{24}{20} = 1.2\,A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{4}{20} = 0.20\,A$$

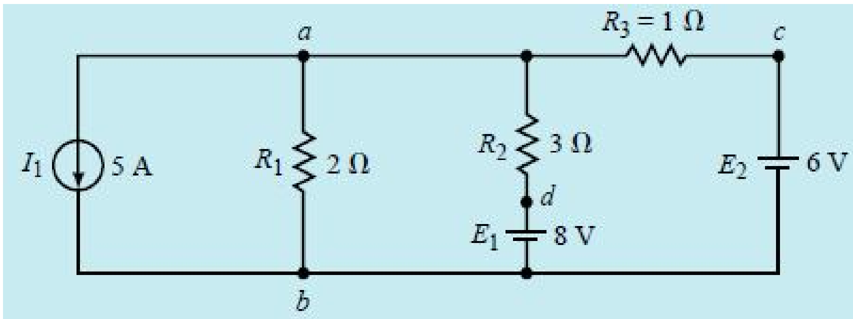
$$I_3 = \frac{\Delta_3}{\Delta} = \frac{4}{20} = 0.20\,A$$

Lecture-08: Methods of Circuit Analysis: Branch Current Method

****

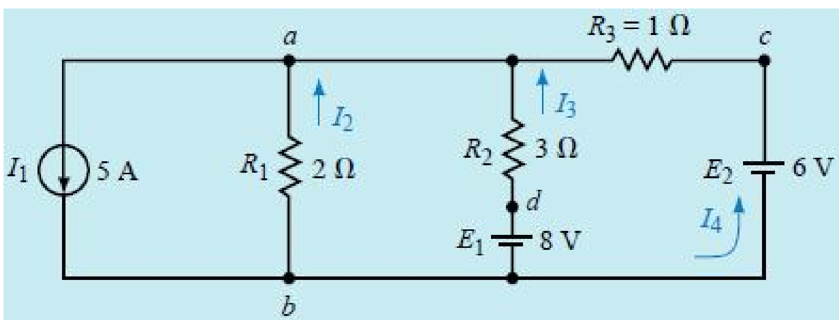
Example-2:

Find the currents in each branch of the circuit shown in figure below. Solve for the voltage V_{ab}.



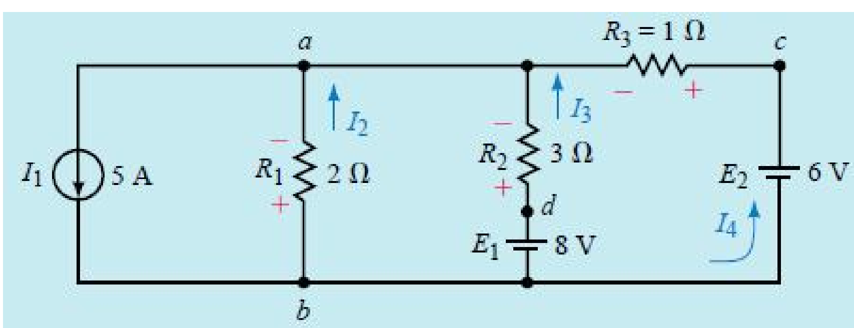
Step-1:

Arbitrarily assign current directions to each branch in the network. Since there is a current source at I_1 branch, hence this step is not necessary for this branch as you already know the magnitude and direction of the current in this branch.



Step-2:

Using the assumed current direction, label the polarities of the voltage drops across all resistors in the circuit.



****

Step-3:

Apply Kirchhoff's voltage law around each of the closed loops. Write just enough equations to include all branches in the loop equations.

Loop *badb*:

$$-(2\Omega)I_2 + (3\Omega)I_3 - 8V = 0V$$
$$\Rightarrow \quad (-2)I_2 + 3I_3 + 0I_4 = 8 \qquad \text{............}(1)$$

Loop *bacb*:

$$-(2\Omega)I_2 + (1\Omega)I_4 - 6V = 0V$$
$$\Rightarrow \quad (-2)I_2 + 0I_3 + 1I_4 = 6 \qquad \text{............}(2)$$

Step-4:

Apply Kirchhoff's current law at enough nodes to ensure that all branch currents have been included.

By applying KCL at node ***a***, all branch currents in the network are included:

$$I_2 + I_3 + I_4 = 5$$
$$\Rightarrow \quad 1I_2 + 1I_3 + 1I_4 = 5 \quad \text{............}(3)$$

Step-5:

Solve the resulting simultaneous linear equations.

- To simplify the solution of the simultaneous linear equations we write them as follows:

$$(-2)I_2 + 3I_3 + 0I_4 = 8 \qquad \text{............}(1)$$

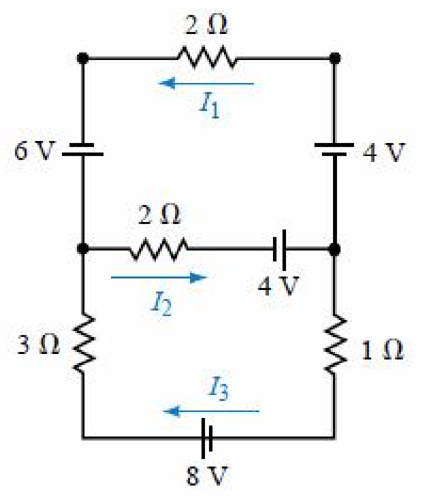
$$(-2)I_2 + 0I_3 + 1I_4 = 6 \qquad \text{............}(2)$$

$$1I_2 + 1I_3 + 1I_4 = 5 \qquad \text{............}(3)$$

- We can solve the above equations by the use of determinants and Cramer's rule as explained earlier.

Exercise:

Use branch-current analysis to solve for the indicated currents in the circuit of figure given below.



Answers: $I_1 = 3.00$ A, $I_2 = 4.00$ A, $I_3 = 1.00$ A