Lecture-09: Methods of Circuit Analysis: Mesh or Loop Current Method

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Mesh-Current (Loop-Current) Analysis

- In branch-current analysis, we used Kirchhoff's voltage and current laws to solve for the current in each branch of a given network.
- Mesh-current method is quite similar to the Branch Current method in that it uses simultaneous equations, Kirchhoff 's Voltage Law, and Ohm's Law to determine unknown currents in a network.
- While the methods used were relatively simple, branch-current analysis is awkward to use because it generally involves solving several simultaneous linear equations. The number of equations may be prohibitively large even for a relatively simple circuit.
- A better and extensively used approach in analyzing linear bilateral networks is called **mesh** (or **loop**) **analysis** which is used to solve currents flowing around complex circuit.
- While the technique is similar to branch-current analysis, the number of simultaneous linear equations tends to be less.
- The principal difference between mesh analysis and branch-current analysis is that we simply need to apply Kirchhoff's voltage law around closed loops without the need for applying Kirchhoff's current law for mesh analysis. In branch-current method, we work with branch currents of a network, but in mesh analysis method, we will work with loop currents instead of branch currents.
    - ◎ Unlike branch currents, loop currents are mathematical quantities, rather than actual physical currents, that are used to make circuit analysis somewhat easier than with the branch-current method.
- Note that a loop is a complete current path within a circuit. Whereas, a branch is a path that connects two nodes. (A node is a point where two or more components are connected). Figure below illustrates loop, branch and nodes.

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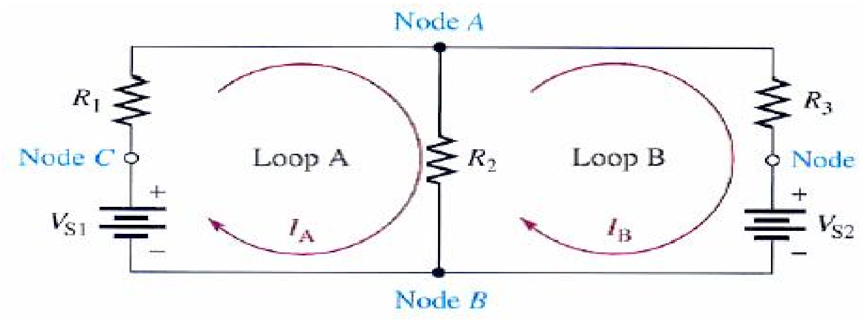
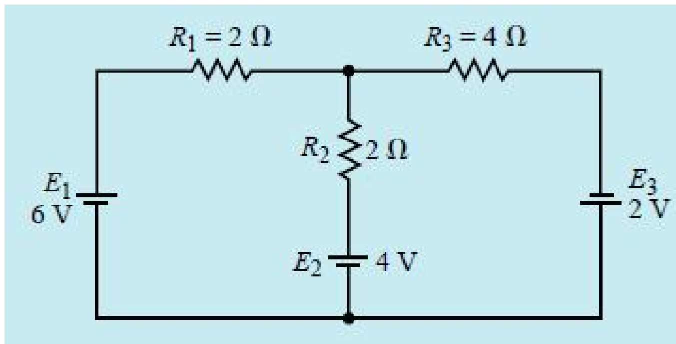


The steps used in solving a circuit using mesh analysis are as follows:

1. The first step in the Mesh Current method is to identify "loops" within the circuit encompassing all components.
2. Arbitrarily assign a current in the clockwise (CW) direction around each interior closed loop in the network. Although the assigned current may be in any direction, a clockwise direction is used to make later work simpler.
3. Based on the assigned loop current directions, indicate the voltage drop polarities across all resistors in each loop in the circuit. For a resistor which is common to two loops, the polarities of the voltage drop due to each loop current should be indicated on the appropriate side of the component.
4. Applying Kirchhoff's voltage law, write the loop equations for each loop in the network. Do not forget that resistors which are common to two loops will have two voltage drops, one due to each loop.
5. Solve the resultant simultaneous linear equations using substitution or determinant method.
6. Branch currents are determined by algebraically combining the loop currents which are common to the branch.

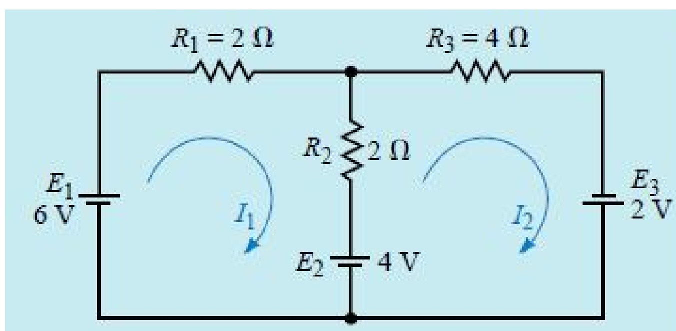
Example-1:

Using mesh analysis, find the current in each branch for the circuit shown in the figure below.



Step-1:

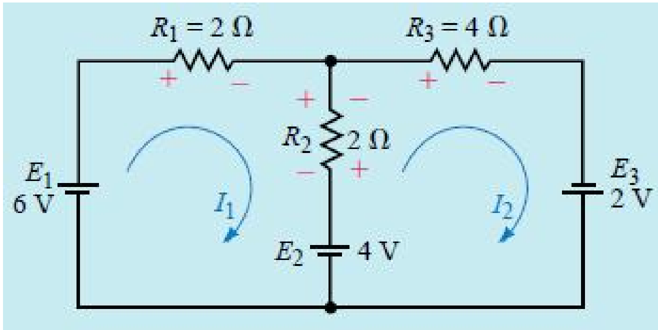
Identify "loops" within the circuit encompassing all components and then arbitrarily assign a current in the clockwise (CW) direction around each interior closed loop in the network.



Loop currents are designated clockwise as I_1 and I_2 in the figure above.

Step-2:

Based on the assigned loop-current directions, indicate the voltage drop polarities across all resistors in each loop in the circuit. For a resistor which is common to two loops, the polarities of the voltage drop due to each loop current should be indicated on the appropriate side of the component.



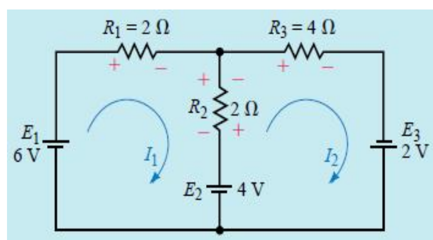
Notice that the resistor R_2 has two different voltage polarities due to the different loop currents.

Step-3:

Applying Kirchhoff's voltage law, write the loop equations for each loop in the network. Do not forget that resistors which are common to two loops will have two voltage drops, one due to each loop.

The loop equations are as follows:

$$Loop-1: \quad E_1 - R_1 I_1 - R_2(I_1 - I_2) - E_2 = 0$$
$$\Rightarrow \quad 6V - (2\Omega)I_1 - (2\Omega)I_1 + (2\Omega)I_2 - 4V = 0$$
$$\Rightarrow \quad 2V - (4\Omega)I_1 + (2\Omega)I_2 = 0$$
$$\Rightarrow \quad 4I_1 - 2I_2 = 2$$
$$\Rightarrow \quad 4I_1 + (-2)I_2 = 2 \qquad \ldots\ldots\ldots\ldots(1)$$



$$Loop-2: \quad E_2 - R_2(I_2 - I_1) - R_3 I_2 + E_3 = 0$$
$$\Rightarrow \quad 4V - (2\Omega)I_2 + (2\Omega)I_1 - (4\Omega)I_2 + 2V = 0$$
$$\Rightarrow \quad 6V + (2\Omega)I_1 - (6\Omega)I_2 = 0$$
$$\Rightarrow \quad 2I_1 - 6I_2 = -6$$
$$\Rightarrow \quad 2I_1 + (-6)I_2 = -6 \qquad \ldots\ldots\ldots\ldots(2)$$

Step-4:

Solve the resultant simultaneous linear equations.

- To simplify the solution of the simultaneous linear equations we write them as follows:

$$4I_1 + (-2)I_2 = 2 \qquad \ldots\ldots\ldots\ldots(1)$$

$$2I_1 + (-6)I_2 = -6 \qquad \ldots\ldots\ldots\ldots(2)$$

- We can solve the above equations by the use of determinants and Cramer's rule:

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 2 & -2 \\ -6 & -6 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 2 & -6 \end{vmatrix}} = \frac{[(2)\times(-6)]-[(-2)\times(-6)]}{[(4)\times(-6)]-[(-2)\times(2)]} = \frac{-12-12}{-24+4} = \frac{-24}{-20} = \frac{12}{10} = 1.2\,A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 4 & 2 \\ 2 & -6 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 2 & -6 \end{vmatrix}} = \frac{[(4)\times(-6)]-[(2)\times(2)]}{[(4)\times(-6)]-[(-2)\times(2)]} = \frac{-24-4}{-24+4} = \frac{-28}{-20} = \frac{14}{10} = 1.4\,A$$

- From the above results, we see that the currents through resistors R_1 and R_3 are I_1 and I_2 respectively.
- The branch current for R_2 is found by combining the loop currents through this resistor:

$$I_{R2} = I_2 - I_1 = 1.4 - 1.2 = 0.2\,A$$

- Since, the direction of I_1 is downward and I_2 is upward; and I_2 is greater than I_1, hence, the direction of I_{R2} is upward.

Note:

- The results obtained by using mesh analysis are exactly the same as those obtained by branch-current analysis.

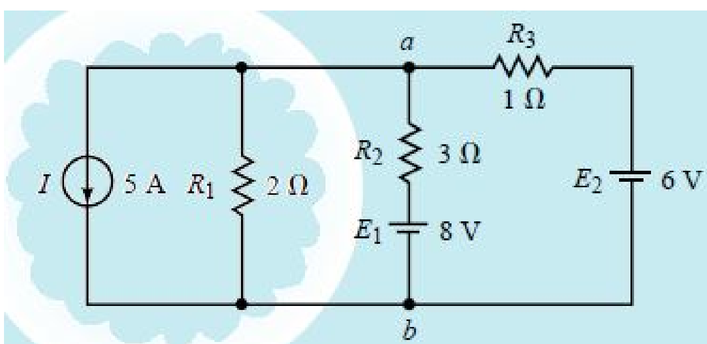
- Whereas branch-current analysis required three equations, this approach requires the solution of only two simultaneous linear equations.
- Mesh analysis also requires that only Kirchhoff's voltage law be applied and clearly illustrates why mesh analysis is preferred.

Solving Circuit Containing Current Source:

- If the circuit being analyzed contains current sources, the procedure is a bit more complicated. The circuit may be simplified by converting the current source(s) to voltage sources and then solving the resulting network using the procedure shown in the previous example.
- If you do not wish to convert the circuit, in that case the current source will provide one of the loop currents.

Example-2:

Determine the current through the 8-V battery for the circuit shown in the figure below.



Solution:

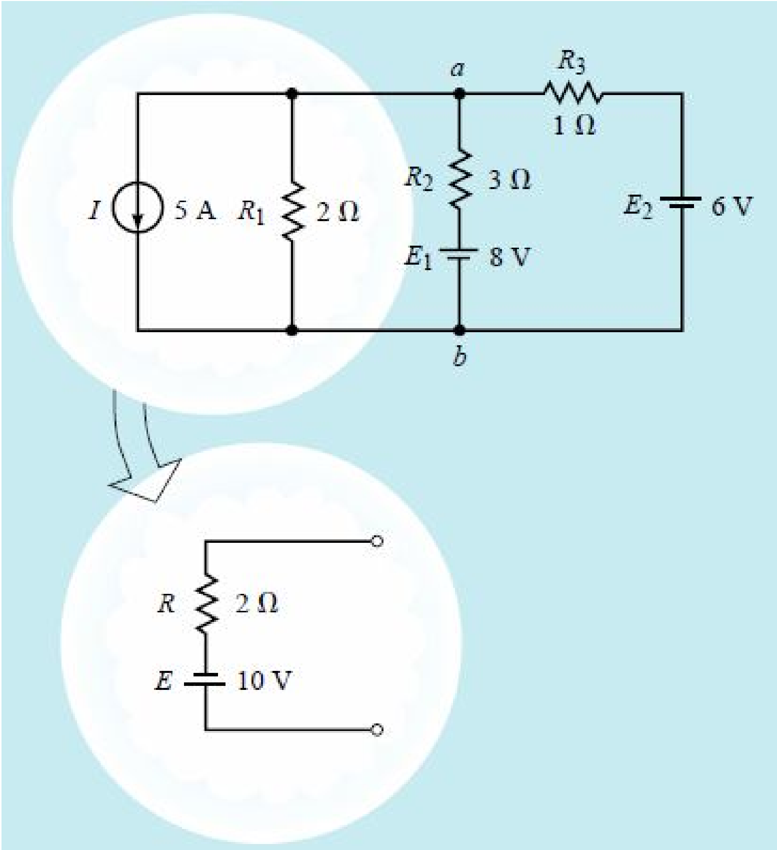
We can solve the above circuit in either two ways:

- Converting current source into voltage source
- Without not converting

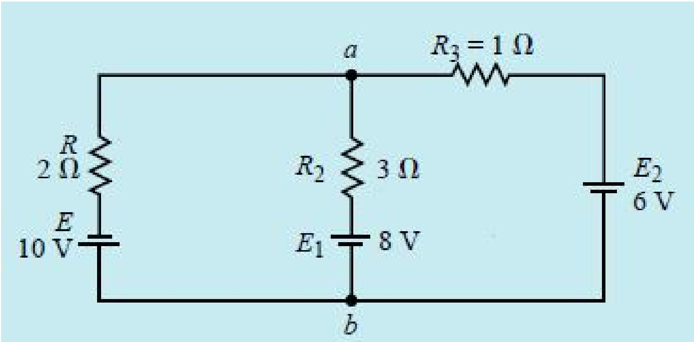
The steps of solution for the first method are discussed here.

Step-1:

This circuit contains a current source. So, convert the current source into a voltage source.

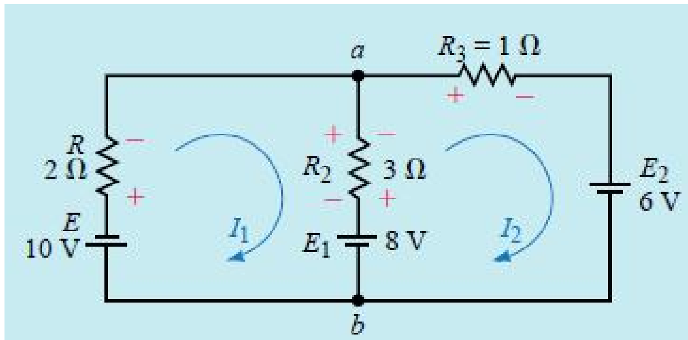


After converting, the circuit becomes:



Step-2:

Arbitrarily assign a clockwise current to each interior closed loop in the network.

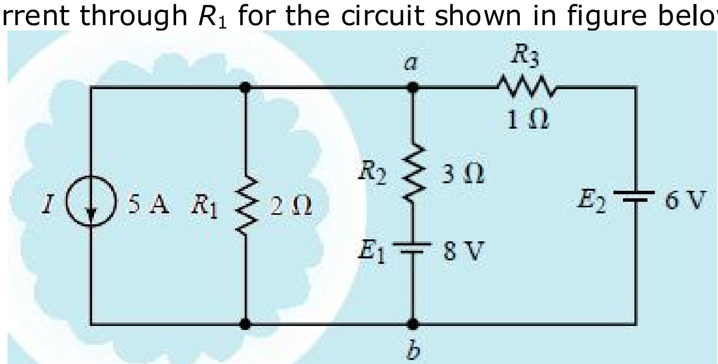


Now solve the circuit using Mesh analysis explained earlier.

Alternative Solution:

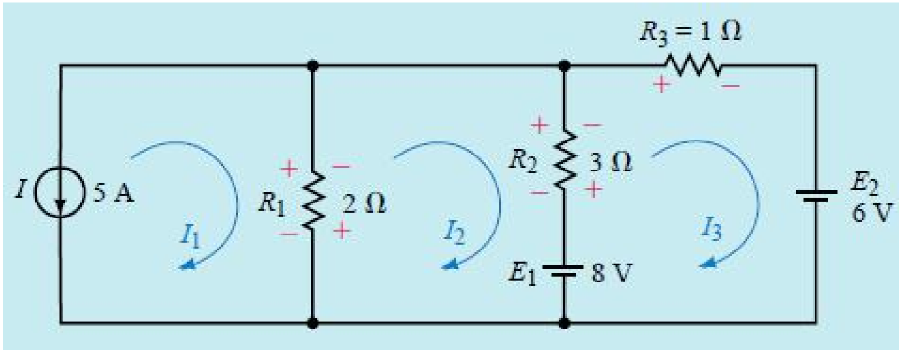
Solution without converting current source into voltage source:

Determine the current through R_1 for the circuit shown in figure below.



The circuit of the figure above may also be analyzed without converting the current source to a voltage source. Although the approach is generally not used, the following example illustrates the technique.

Solution:



By inspection, we see that the loop current I_1=-5 A

The mesh equations for the other two loops are as follows:

$$Loop-2: \quad -(2\Omega)I_2 + (2\Omega)I_1 - (3\Omega)I_3 - 8V = 0$$
$$\Rightarrow \quad (2\Omega)I_1 - (5\Omega)I_2 + (3\Omega)I_3 = 8$$
$$\Rightarrow \quad 2I_1 + (-5)I_2 + 3I_3 = 8 \qquad ....(1)$$

$$Loop-3: \quad 8V - (3\Omega)I_3 + (3\Omega)I_2 - (1\Omega)I_3 - 6V = 0$$
$$\Rightarrow \quad (3\Omega)I_2 - (4\Omega)I_3 = -2$$
$$\Rightarrow \quad 3I_2 + (-4)I_3 = -2 \qquad ....(1)$$

Now the three linear equations are:

$$I_1 = -5 \qquad ....(1)$$
$$2I_1 + (-5)I_2 + 3I_3 = 8 \qquad ....(2)$$
$$3I_2 + (-4)I_3 = -2 \qquad ....(3)$$

Substituting the value of I_1 from equation-1 to equation-2, there are only two linear equations of two unknown values I_2 and I_3. So, it can be easily solved.

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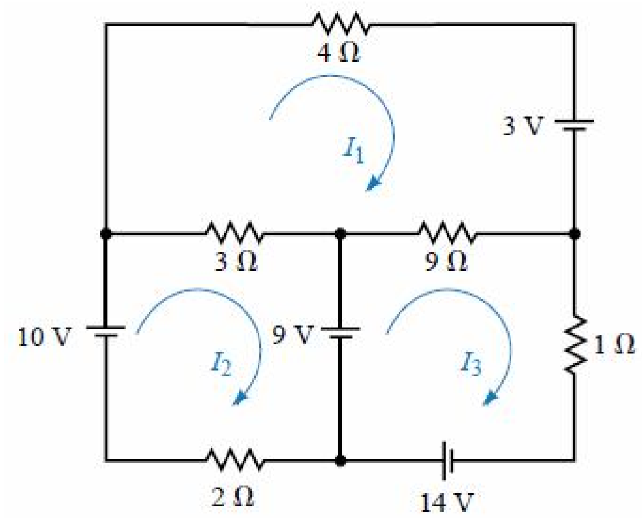
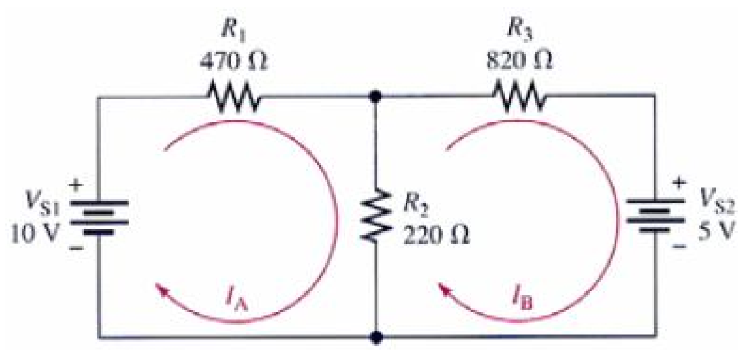
Exercise:

1. Use mesh analysis to find the loop currents in the circuit shown in the figure below.



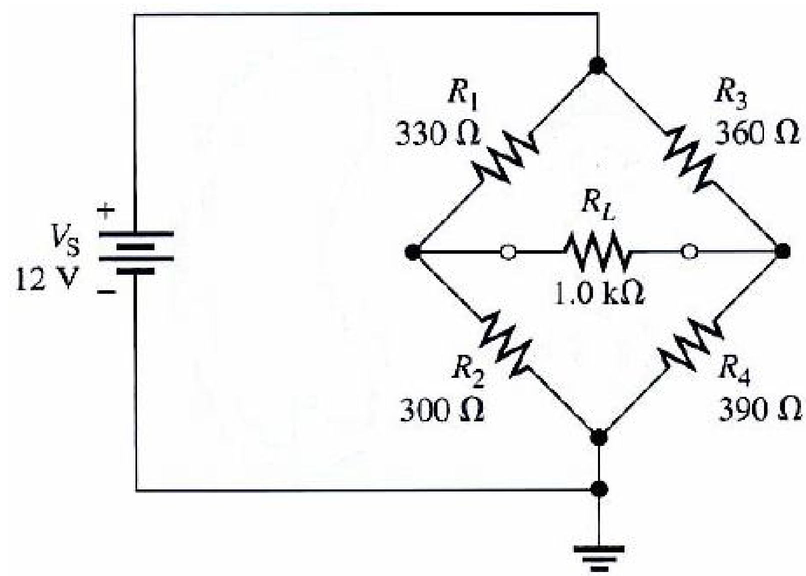
Answers: $I_1 = 3.00$ A, $I_2 = 2.00$ A, $I_3 = 5.00$ A

2. Using loop-current (mesh analysis) method, find the branch currents of the network shown below:



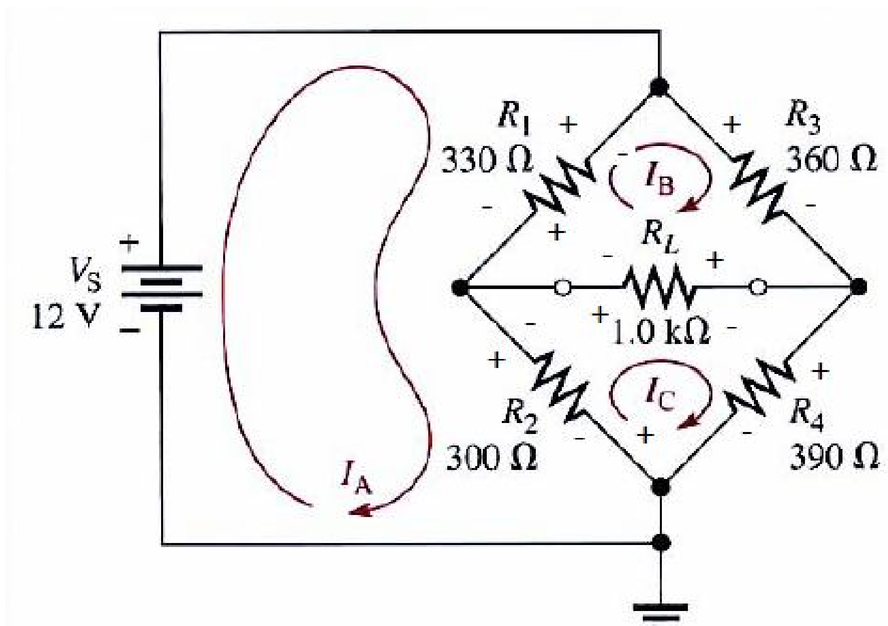
Answer $I_A = I_1 = 13.9mA \quad I_B = I_3 = -1.87mA \quad I_2 = I_A - I_B = 13.9 - (-1.87) = 15.8mA$

3. Using loop-current (mesh analysis) method, find the branch currents of the Wheatstone bridge network shown below:



Hints:

- Assign three clockwise loop currents I_A, I_B and I_C.
- Indicate the voltage drop polarities across all resistors in each loop in the circuit based on the assigned loop-current directions.
- Notice that the resistor R_1, R_2 and R_L has two different voltage polarities due to the different loop currents.
- Applying Kirchhoff's voltage law, write the loop equations for each loop in the network.



- The loop equations are as follows:

$$KVL\ \ at\ loop\ A: \qquad V_S - R_1(I_A - I_B) - R_2(I_A - I_C) = 0$$

$$KVL\ \ at\ loop\ B: \qquad -R_3 I_B - R_L(I_B - I_C) - R_1((I_B - I_A) = 0$$

$$KVL\ \ at\ loop\ C: \qquad -R_4 I_c - R_2(I_C - I_A) - R_L(I_C - I_B) = 0$$

- Now rearrange the equations and solve them using determinant and Cramer's rule.