DC Network Theorem (Continue....)

Thevenin's Theorem:

- Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex it is, to an equivalent circuit with just a **single voltage source (V_{TH})** in **series with a single resistor (R_{TH})** which is **connected to a load (R_L).** The Thevenin's equivalent network looks like this:

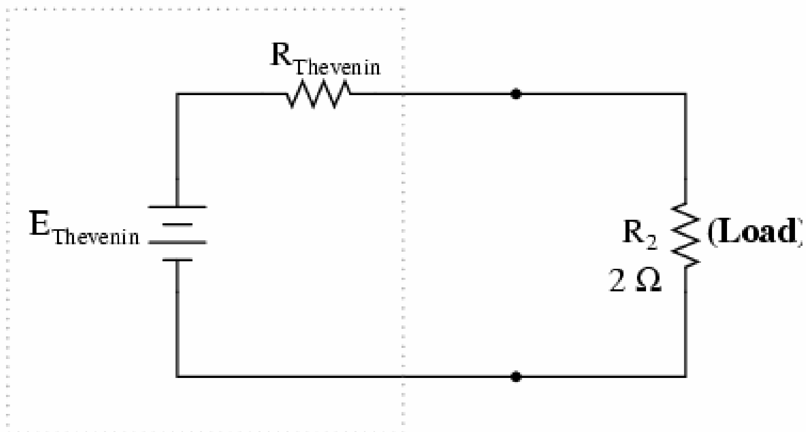


Figure: Thevenin equivalent Circuit

- Thevenin's Theorem is especially useful in analyzing power or battery systems and other circuits where one particular resistor in the circuit (called the "load" resistor) is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance- to determine voltage across it and current through it.
- Consider the circuit given below. We apply Thevenin's Theorem to it:

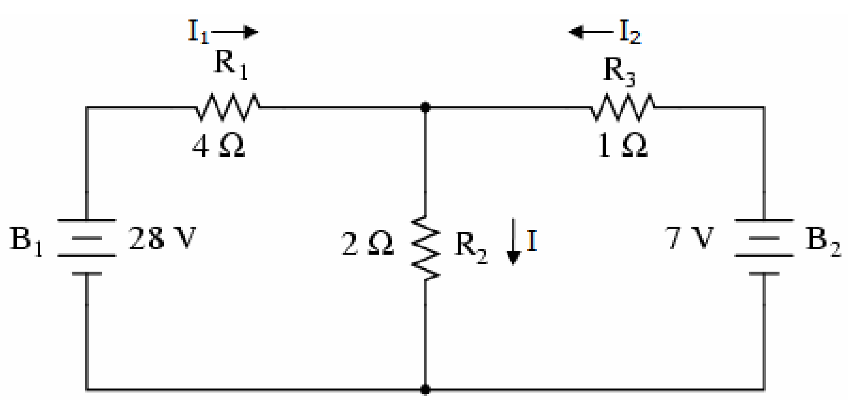
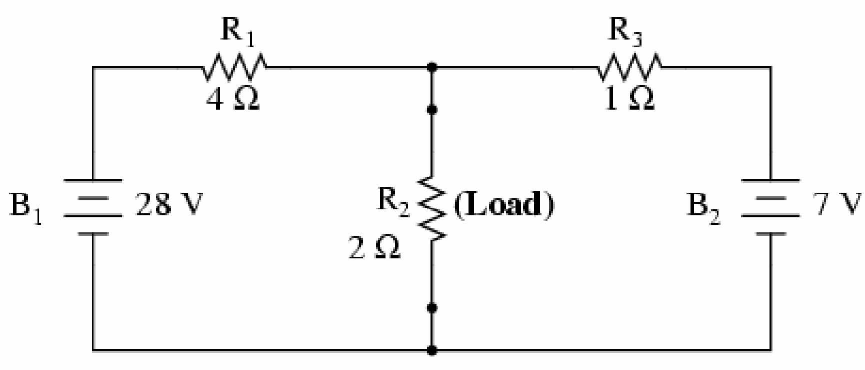
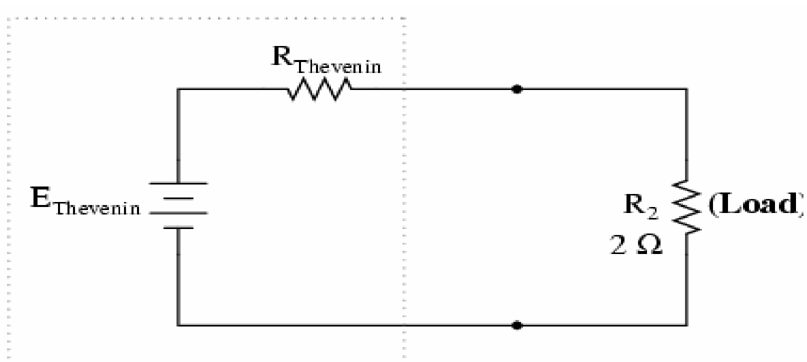


Figure-1: Original Circuit

- In the circuit above, I_1, I_2 and I represent the values of currents which are due to the simultaneous action of the two sources of emf in the network.
- Let us assume that we decide to designate R_2 as the "load" resistor in this circuit.



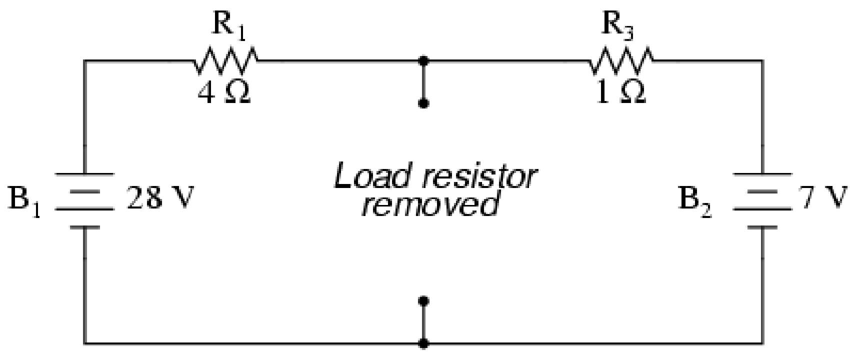
- We already have discussed Superposition Theorem by which we can determine voltage across R2 and current through R2, but this method is time-consuming. Imagine repeating this method over and over again to find what would happen if the load resistance changed (changing load resistance is very common in power systems, as multiple loads get switched on and off as needed, the total resistance of their parallel connections changing depending on how many are connected at a time). This could potentially involve a lot of work.
- Thevenin's Theorem makes this easy by temporarily removing the load resistance from the original circuit and reducing what is left to an equivalent circuit composed of a **single voltage source** and **series resistance**. The **load resistance** can then be re-connected to this "Thevenin equivalent circuit" and calculations carried out as if the whole network were nothing but a simple series circuit.
- After Thevenin conversion, the Thevenin equivalent circuit is:



- The "Thevenin Equivalent Circuit" is the electrical equivalent of B1, R1, R3, and B2 as seen from the two points where our load resistor (R2) connects.
- The Thevenin equivalent circuit, if correctly derived, will behave exactly the same as the original circuit formed by B1, R1, R3, and B2. In other words, the load resistor (R2) voltage and current should be exactly the same for the same value of load resistance in the two circuits. The load resistor R2 cannot "tell the difference" between the original network of B1, R1, R3, and B2, and the Thevenin equivalent circuit of $E_{Thevenin}$, and $R_{Thevenin}$, provided that the values for $E_{Thevenin}$ and $R_{Thevenin}$ have been calculated correctly.
- The advantage in performing the "Thevenin conversion" to the simpler circuit, of course, is that it makes load voltage and load current so much easier to solve than in the original network. Calculating the equivalent Thevenin source voltage and series resistance is actually quite easy.

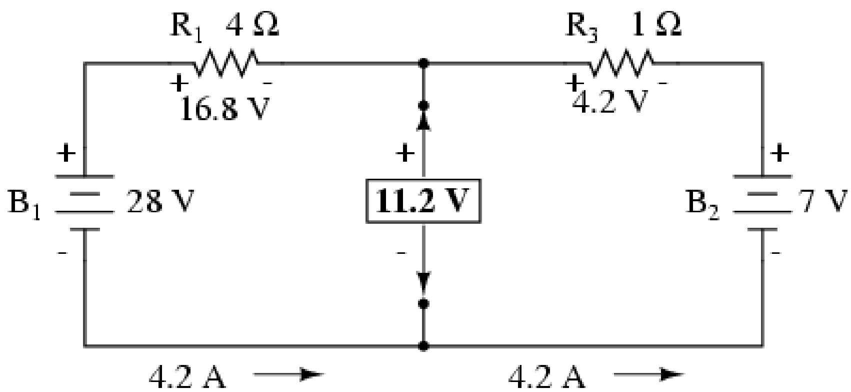
Performing Thevenin Conversion:

- First, remove load resistor R_L: the chosen load resistor is removed from the original circuit, replaced with a break (open circuit):

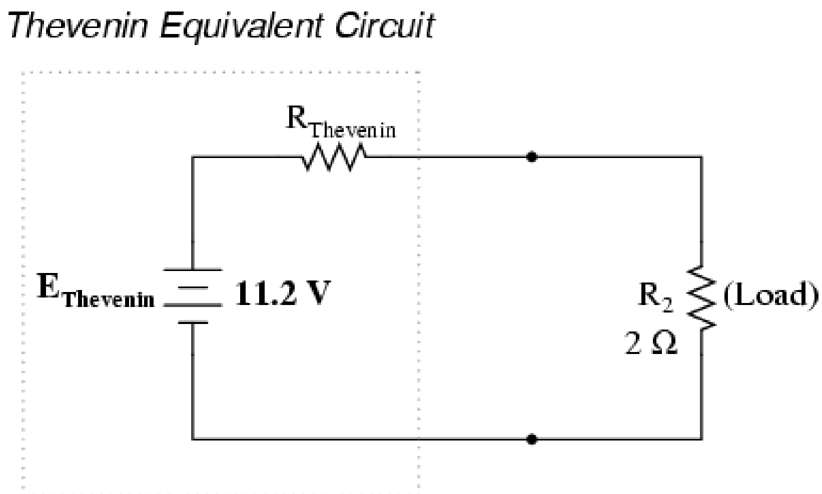


- Next, determine Thevenin voltage $E_{Thevenin}$: calculate the voltage between the two points where the load resistor used to be attached is determined. Use whatever analysis methods are at your disposal to do this. In this case, the original circuit with the load resistor removed is nothing more than a simple series circuit with opposing batteries, and so we can determine the voltage across the open load terminals by applying the rules of series circuits, Ohm's Law, and Kirchhoff's Voltage Law:

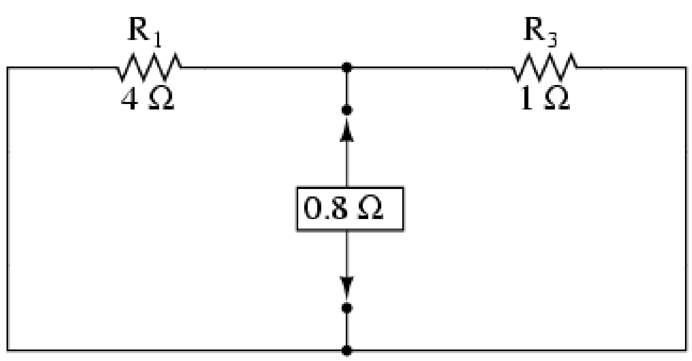
	R_1	R_3	Total	
E	16.8	4.2	21	Volts
I	4.2	4.2	4.2	Amps
R	4	1	5	Ohms



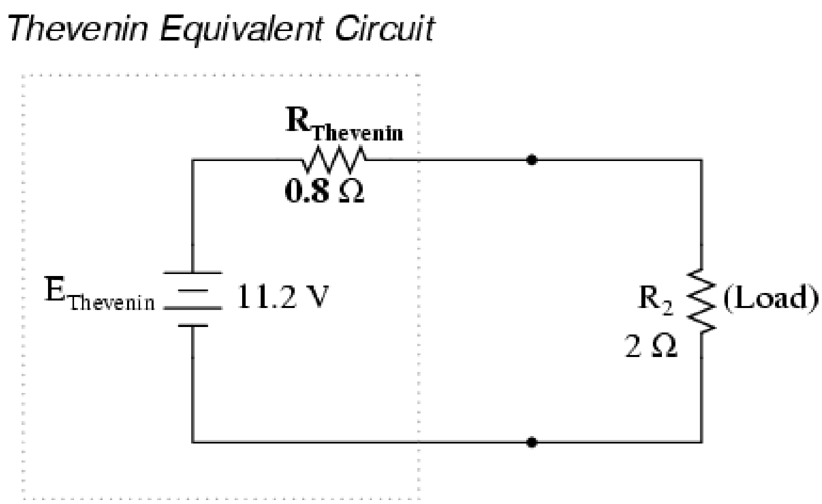
- The voltage between the two load connection points can be figured from the one of the battery's voltage and one of the resistor's voltage drops, and comes out to 11.2 volts. This is our "Thevenin voltage" ($E_{Thevenin}$) in the equivalent circuit:



- Then, determine Thevenin resistance $R_{Thevenin}$: To find the Thevenin series resistance for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure the resistance from one load terminal to the other:



- With the removal of the two batteries, the total resistance measured at this location is equal to R1 and R3 in parallel: 0.8 Ω. This is our "Thevenin resistance" ($R_{Thevenin}$) for the equivalent circuit:



- Finally, determine the current through and voltage across R_L: With the load resistor (2 Ω) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing more than a simple series circuit:

	$R_{Thevenin}$	R_{Load}	Total	
E	3.2	8	11.2	Volts
I	4	4	4	Amps
R	0.8	2	2.8	Ohms

- Notice that the voltage and current figures for R2 (8 volts, 4 amps) are identical to those found using Superposition Theorem and Kirchhoff's Circuit laws.
- Also notice that the voltage and current figures for the Thevenin series resistance and the Thevenin source (total) do not apply to any component in the original, complex circuit. Thevenin's Theorem is only useful for determining what happens to a single resistor in a network: the load.
- The advantage, of course, is that you can quickly determine what would happen to that single resistor if it were of a value other than 2 Ω without having to go through a lot of analysis again. Just plug in that other value for the load resistor into the Thevenin equivalent circuit and a little bit of series circuit calculation will give you the result.

Steps to follow for Thevenin's Theorem / How to Thevenize a Given Circuit

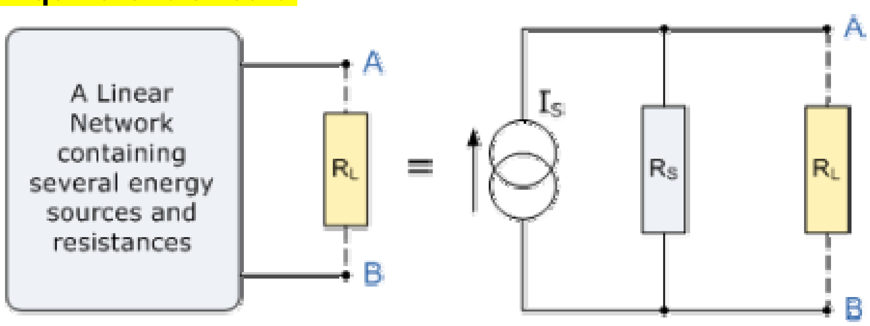
Thevenin's Theorem is a way to reduce a network to an equivalent circuit composed of a single voltage source, series resistance, and series load. Steps to follow for Thevenin's Theorem are:

1. Find the Thevenin source voltage by removing the load resistor from the original circuit and calculating voltage across the open connection points where the load resistor used to be.
2. Find the Thevenin resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.
3. Draw the Thevenin equivalent circuit, with the Thevenin voltage source in series with the Thevenin resistance. The load resistor re-attaches between the two open points of the equivalent circuit.
4. Analyze voltage and current for the load resistor following the rules for series circuits.

Norton's Theorem:

- Norton's Theorem is a way to reduce a network to an equivalent circuit composed of a single current source, parallel resistance, and parallel load.
- In some ways **Norton's Theorem** can be thought of as the opposite to "Thevenins Theorem", in that Thevenin reduces his circuit down to a single resistance in series with a single voltage. Norton on the other hand reduces his circuit down to a single resistance in parallel with a constant current source.
- **Norton's Theorem** states that "*Any linear circuit containing several energy sources and resistances can be replaced by a single constant current source in parallel with a single resistor*".
- As far as the load resistance, R_L is concerned this single resistance, R_S is the value of the resistance looking back into the network with all the current sources open circuited and I_S is the short circuit current at the output terminals as shown below.

Norton's Equivalent Circuit:



- The value of this "constant current" is one which would flow if the two output terminals where shorted together while the source resistance would be measured looking back into the terminals, (the same as Thevenin).
- Consider the circuit shown in the figure below. We apply Norton's Theorem to it:

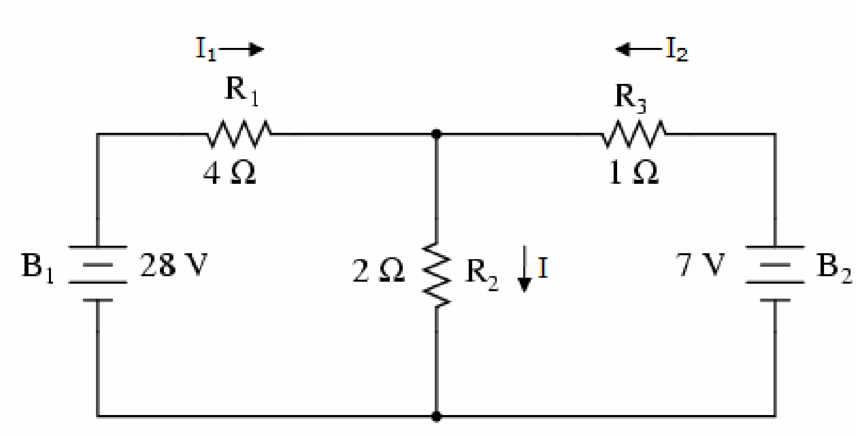


Figure-1: Original Circuit

- After Norton conversion, the Norton's equivalent circuit will be looked like this:

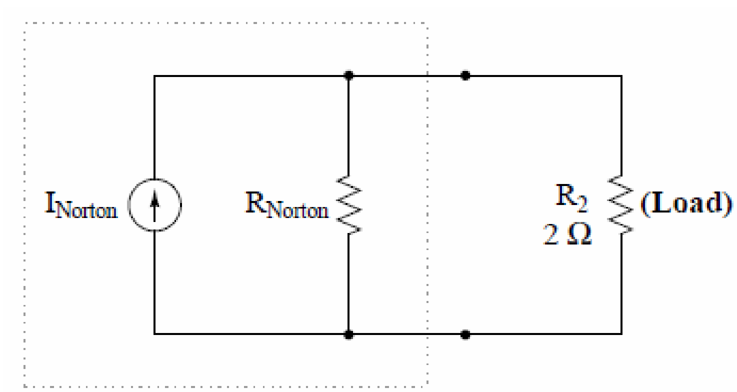
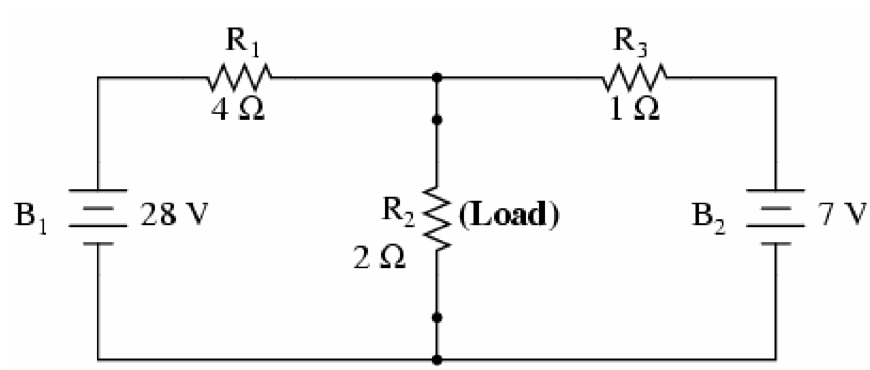


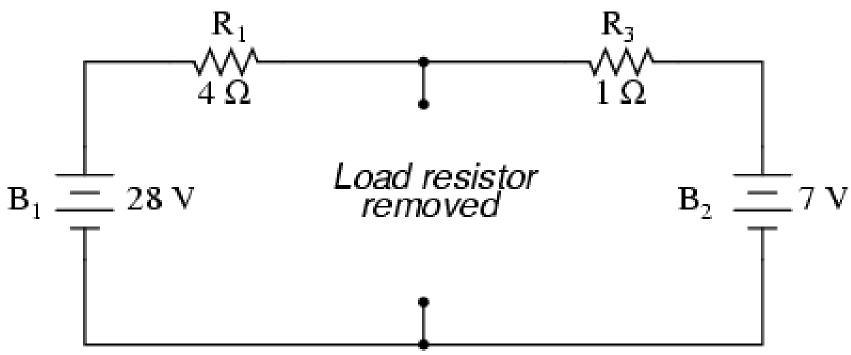
Figure: Assumed Norton's equivalent circuit before the calculation

- Let us assume that we decide to designate R_2 as the "load" resistor in this circuit.

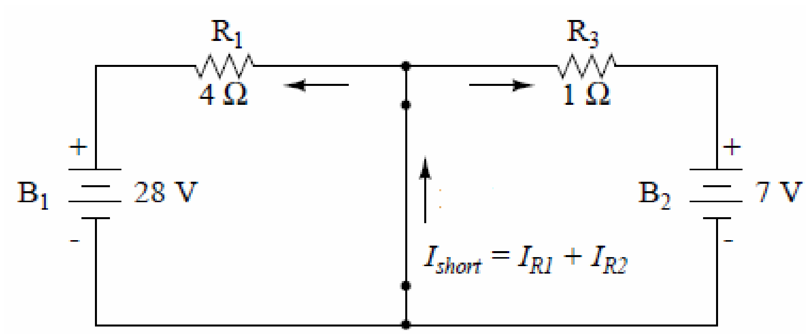


Finding Norton's Equivalent of the Circuit:

- To find the Norton's equivalent of the above circuit, the first step is to identify the load resistance RL and remove it from the original circuit:



- Next, determine Norton current INorton: To find the Norton current INorton (for the current source in the Norton equivalent circuit), short out the terminals A and B (i.e. place a direct wire connection between the load points). Note that this step is exactly opposite the respective step in Thevenin's Theorem, where we replaced the load resistor with a break (open circuit). Now the circuit looks like this:



- When the terminals A and B are shorted together, voltage drop between load resistor connection point is zero. Then the current through R1 is strictly a function of B1's voltage and R1's resistance:

IR1=E1/R1=28/4=7 A.

- Similarly, the current through R3 is now strictly a function of B2's voltage and R3's resistance:

IR2=E2/R3=7/1=7 A.

- The total current through the short between the load connection points is the sum of these two currents: Ishort=IR1+IR3= 7 A + 7 A = 14 A.
- This figure of 14 A becomes the Norton source current (INorton) in our equivalent circuit which now looks like:

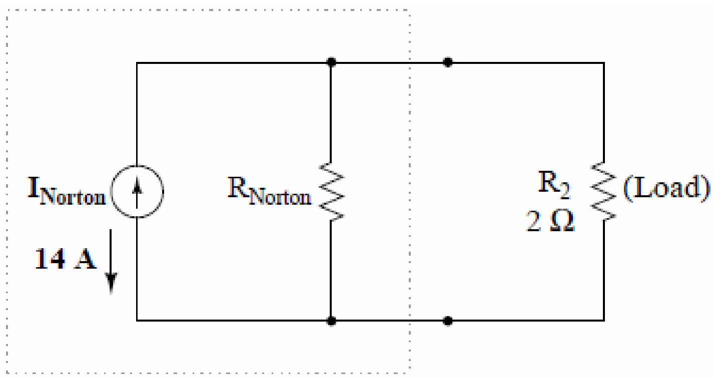
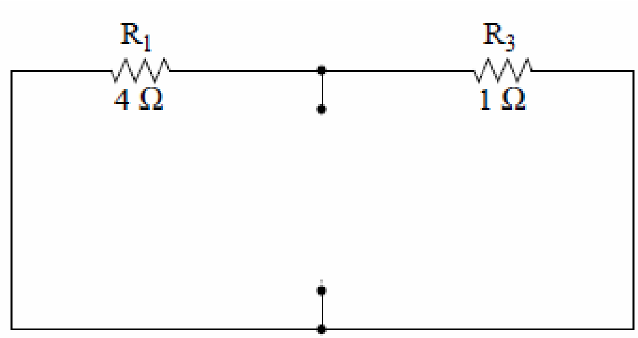


Figure: Norton's equivalent circuit after calculating INorton

- Next, determine the Norton resistance RNorton: To find the Norton resistance (parallel to current source) for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure the total resistance from one load connection point to the other:



- With the removal of the two batteries, the total resistance measured at this location is equal to R1 and R3 in parallel:

RT=R1||R3=(4x1)/(4+1)=4/5=0.8 Ω.

This is our "Norton resistance" (RNorton) for the equivalent circuit:

- This figure of 0.8 Ω resistance becomes the Norton resistance (RNorton) in our equivalent circuit which now looks like:

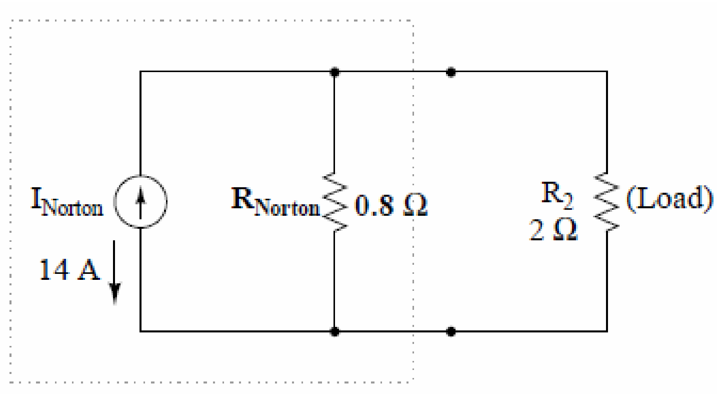


Figure: Norton's equivalent circuit after calculating INorton and RNorton

- Finally, determine the current through and voltage across RL: With the load resistor (2 Ω) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing more than a simple parallel circuit with a current source:

	R_{Norton}	R_{Load}	Total	
E	8	8	8	Volts
I	10	4	14	Amps
R	0.8	2	571.43m	Ohms

In Norton equivalent circuit, RNorton and RL (in this case R2) are connected in parallel across the load terminals A and B. According to current-divider rule, current through load resistor and Norton resistor can be found as:

$$I_{Load} = I_{RL} = I \frac{R_{Norton}}{R_{Norton} + R_{Load}}$$

$$= 14 \frac{0.8}{0.8 + 2} = 4.03A$$

$$I_{R-Norton} = I \frac{R_{Load}}{R_{Norton} + R_{Load}}$$

$$= 14 \frac{2}{0.8 + 2} = 10A$$

Therefore, total current I=INorton=IL+IR-Norton=10+4.03=14.03 A

Voltage drop across load is VL=ILxRL=4.03x2=8.06 V

Voltage drop across RNorton is VR-Norton=IR-NortonxRNorton=10x0.8=8 V

- As with the Thevenin equivalent circuit, the only useful information from this analysis is the voltage and current values for R_2; the rest of the information is irrelevant to the original circuit. However, the same advantages seen with Thevenin's Theorem apply to Norton's as well: if we wish to analyze load resistor voltage and current over several different values of load resistance, we can use the Norton equivalent circuit again and again, applying nothing more complex than simple parallel circuit analysis to determine what's happening with each trial load.

Steps to follow for Norton's Theorem:

1. Find the Norton source current by removing the load resistor from the original circuit and calculating current through a short (wire) jumping across the open connection points where the load resistor used to be.
2. Find the Norton resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.
3. Draw the Norton equivalent circuit, with the Norton current source in parallel with the Norton resistance. The load resistor re-attaches between the two open points of the equivalent circuit.
4. Analyze voltage and current for the load resistor following the rules for parallel circuits.

Reciprocity Theorem:

This theory can be stated as follows:

- In any linear bilateral network, if a source of e.m.f. ***E*** in any branch produces a current ***I*** in any other branch, then the same e.m.f. ***E*** acting in the second branch would produce the same current ***I*** in the first branch.
- In other words, it simply means that ***E*** and ***I*** are mutually transferable.
- It also means that interchange of an ideal voltage source and an ideal ammeter in any network will not change the ammeter reading. Same is

the case with the interchange of an ideal current source and an ideal voltmeter.

- The ratio ***E/I*** is known as transfer resistance.

Example: 2.86 (Page-133)