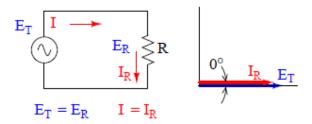
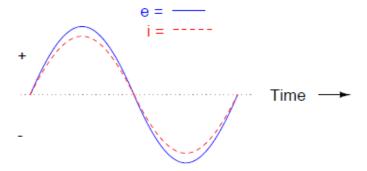
## AC Fundamentals: Resistance, Reactance and Impedance

# **AC Through Pure Ohmic Resistance Alone:**

■ A pure resistive AC circuit consisting of a source and a resistor is shown below.



- Because the resistor simply and directly resists the flow of electrons at all periods of time, the waveform for the voltage drop across the resistor is exactly in phase with the waveform for the current through it. That is, resistor voltage and current are in phase.
- If we were to plot the current and voltage for this type of simple AC circuit, it would look something like this:



■ It is seen that alternating voltage and current are in phase with each other in a pure resistive AC circuit. This means that when the instantaneous value for current is zero, the instantaneous voltage across the resistor is also zero. Likewise, at the moment in time where the current through the resistor is at its positive peak, the voltage across the resistor is also at its positive peak, and so on. At any given point in time along the waves, Ohm's Law holds true for the instantaneous values of voltage and current.

■ We can also calculate the power dissipated by this resistor. Let the applied voltage be given by the equation  $v = V_m Sin\theta = V_m Sin\omega t$ , therefore the equation of current is given by  $i = I_m Sin\theta = I_m Sin\omega t$ , since voltage and current are in phase with each other in a pure resistive AC circuit.

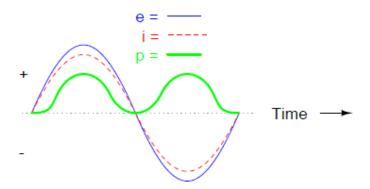
Therefore, the equation of instantaneous power is:

$$p = vi = V_m Sin\omega t \times I_m Sin\omega t = V_m I_m Sin^2 \omega t = \frac{V_m I_m}{2} 2Sin^2 \omega t$$
$$= \frac{V_m I_m}{2} (1 - Cos2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} Cos2\omega t$$

From the above equation we see that power consists of constant part  $\frac{V_m I_m}{2}$  and fluctuating part  $\frac{V_m I_m}{2} Cos2\omega t$ . For a complete cycle, the average value of  $\frac{V_m I_m}{2} Cos2\omega t$  is zero. Hence, power for the whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{RMS} \times I_{RMS} = VI$$

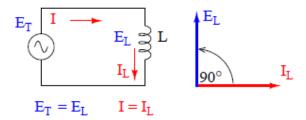
■ We can plot the power on the same graph as the voltage and current shown below.



■ It is seen from the above figure that no part of the power cycle becomes negative at any point. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive. In other words, in a pure resistive AC circuit, power is always positive. This consistent "polarity" of power tells us that the resistor is always dissipating power, taking it from the source and releasing it in the form of heat energy. Whether the current is positive or negative, a resistor still dissipates energy.

### **AC Through Pure Inductance Alone:**

■ A pure inductive AC circuit consisting of a source and an inductor is shown below.



- Inductors do not behave the same as resistors. Whereas resistors simply oppose the flow of electrons through them (by dropping a voltage directly proportional to the current), inductors oppose *changes* in current through them, by dropping a voltage directly proportional to the *rate of change* of current.
- According to *Lenz's Law*, this induced voltage is always of such a polarity as to try to maintain current at its present value. That is, if current is increasing in magnitude, the induced voltage will "push against" the electron flow; if current is decreasing, the polarity will reverse and "push with" the electron flow to oppose the decrease.
- Expressed mathematically, the relationship between the voltage dropped across the inductor and rate of current change through the inductor is as such:

$$v = L\frac{di}{dt}$$

Now, 
$$v = V_m Sin\omega t$$

Therefore, 
$$V_m Sin\omega t = L \frac{di}{dt}$$

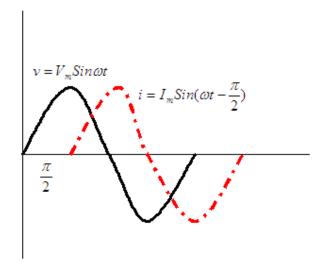
$$\therefore di = \frac{V_m}{L} Sin\omega t dt$$

Integrating both sides, we get,

$$i = \frac{V_m}{L} \int Sin\omega t \ dt = \frac{V_m}{\omega L} (-Cos\omega t) = \frac{V_m}{\omega L} Sin(\omega t - \frac{\pi}{2}) = \frac{V_m}{X_L} Sin(\omega t - \frac{\pi}{2}),$$

Where  $X_L = \omega L = 2\pi f L$  is called the inductive reactance.

- Maximum value of I is  $I_m = \frac{V_m}{\omega L}$  when  $Sin(\omega t \frac{\pi}{2})$  is unity.
- Hence, the equation of current becomes,  $i = I_m Sin(\omega t \frac{\pi}{2})$
- So, we find that for a pure inductive circuit, if applied voltage is represented by  $v = V_m Sin\omega t$ , then current flowing through the inductor is given by  $i = I_m Sin(\omega t \frac{\pi}{2})$
- If we were to plot the current and voltage for this type of simple AC circuit, it would look something like this:



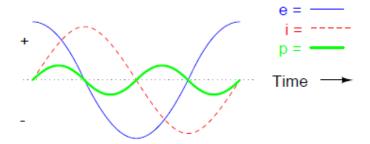
- We see from the equations of voltage and current that, in a pure inductive circuit, inductor current lags inductor voltage by 90° or a quarter cycle. This means that, the instantaneous voltage is at a peak wherever the instantaneous current is at zero and the instantaneous voltage is zero whenever the instantaneous current is at a peak.
- We can also calculate the power. Let the applied voltage be given by the equation  $v = V_m Sin\,\omega t$ , therefore the equation of current is given by  $i = I_m Sin(\omega t \frac{\pi}{2})$ , since current lags voltage by  $90^{\circ}$  in a pure inductive circuit.

Therefore, the equation of instantaneous power is:

$$p = vi = V_{m}Sin\omega t \times I_{m}Sin(\omega t - \frac{\pi}{2}) = -V_{m}I_{m}Sin\omega t.Cos\omega t = -\frac{V_{m}I_{m}}{2}Sin2\omega t$$

Power for a whole cycle is 
$$P = -\frac{V_m I_m}{2} \int_{0}^{2\pi} Sin 2\omega t \, dt = 0$$

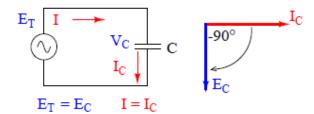
■ We can plot the power on the same graph as the voltage and current shown below.



- It is seen from the above figure that, in a pure inductive circuit, instantaneous power may be positive or negative. Power wave is a sine wave of frequency double that of the voltage and current.
- Because instantaneous power is the product of the instantaneous voltage and the instantaneous current (p=vi), the power equals zero whenever the instantaneous current *or* voltage is zero. Whenever the instantaneous current and voltage are both positive (above the line), the power is positive. The power is also positive when the instantaneous current and voltage are both negative (below the line). However, because the current and voltage waves are 90° out of phase, there are times when one is positive while the other is negative, resulting in equally frequent occurrences of *negative instantaneous power*.
- But what does *negative* power mean? It means that the inductor is releasing power back to the circuit, while a positive power means that it is absorbing power from the circuit. Since the positive and negative power cycles are equal in magnitude and duration over time, the inductor releases just as much power back to the circuit as it absorbs over the span of a complete cycle. What this means in a practical sense is that the reactance of an inductor dissipates a net energy of zero, quite unlike the resistance of a resistor, which dissipates energy in the form of heat.

### **AC Through Pure Capacitance Alone:**

■ A pure capacitive AC circuit consisting of a source and an capacitor is shown below.



- Capacitors do not behave the same as resistors. Whereas resistors allow a flow of electrons through them directly proportional to the voltage drop, capacitors oppose *changes* in voltage by drawing or supplying current as they charge or discharge to the new voltage level. The flow of electrons "through" a capacitor is directly proportional to the *rate of change* of voltage across the capacitor. This opposition to voltage change is another form of *reactance*, but one that is precisely opposite to the kind exhibited by inductors.
- Expressed mathematically, the relationship between the rate of voltage change across the capacitor and the current through the capacitor is as such:

$$i = C \frac{dv}{dt}$$

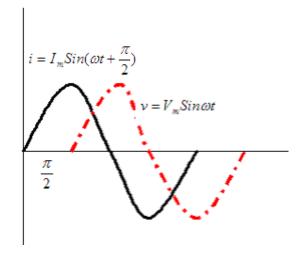
Now, 
$$v = V_m Sin\omega t$$

Therefore, 
$$i = C \frac{d}{dt} V_m Sin\omega t = \frac{V_m}{1/\omega C} Cos\omega t = \frac{V_m}{1/\omega C} Sin(\omega t + \frac{\pi}{2}) = \frac{V_m}{X_C} Sin(\omega t + \frac{\pi}{2})$$

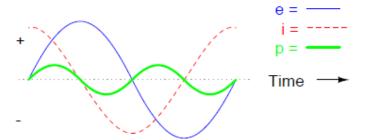
Where  $X_C = \frac{1}{2} \sqrt{2\pi fC}$  is called the capacitive reactance.

- Maximum value of I is  $I_m = \frac{V_m}{1/\omega C}$  when  $Sin(\omega t + \frac{\pi}{2})$  is unity.
- Hence, the equation of current becomes,  $i = I_m Sin(\omega t + \frac{\pi}{2})$
- So, we find that for a pure capacitive circuit, if applied voltage is represented by  $v = V_m Sin\omega t$ , then current flowing through the inductor is given by  $i = I_m Sin(\omega t + \frac{\pi}{2})$

■ If we were to plot the current and voltage for this type of simple AC circuit, it would look something like this:



- We see from the equations of voltage and current that, in a pure capacitive circuit, capacitor current leads capacitor voltage by 90° or a quarter cycle. This means that, the instantaneous voltage is at a peak wherever the instantaneous current is at zero and the instantaneous voltage is zero whenever the instantaneous current is at a peak.
- We can also calculate the power. The same unusual power wave that we saw with the simple inductor circuit is present in the simple capacitor circuit, too:



■ As with the simple inductor circuit, the 90 degree phase shift between voltage and current results in a power wave that alternates equally between positive and negative. This means that a capacitor does not dissipate power as it reacts against changes in voltage; it merely absorbs and releases power, alternately.

#### Resistance:

- Resistance is an impediment that slows the flow of electrons. It is like friction against the wall of holes of a pipe flowing water that slows the flow of water.
- It is defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (i.e. electrons) through it.
- It is present in all conductors to some extent (except *super*conductors!), most notably in resistors.
- When alternating current goes through a resistance, a voltage drop is produced that is in-phase with the current.
- $\blacksquare$  Resistance is mathematically symbolized by the letter "R" and measured in ohms,  $\Omega$ .
- A conductor is said to have a resistance of 1  $\Omega$  if it permits 1 A current to flow through it when 1 V is impressed across its terminal. R=V/I
- Resistance increases, current decreases.

#### Reactance:

- Reactance is essentially inertia against the motion of electrons. It is a measure of the opposition of capacitance and inductance to current.
- Perfect resistors possess resistance, but not reactance. Whereas, perfect inductors and perfect capacitors possess reactance but no resistance.
- It is present anywhere electric or magnetic fields are developed in proportion to applied voltage or current, respectively; but most notably in capacitors and inductors.
- When alternating current goes through a pure reactance, a voltage drop is produced that is 90° out of phase with the current.
- Reactance varies with the frequency of the electrical signal.
- It is mathematically symbolized by the letter "X" and measured in the unit of ohms.
- There are two types of reactance:
  - > Capacitive reactance (X<sub>C</sub>) and

- ➤ Inductive reactance (X<sub>L</sub>)
- The total reactance (X) is the difference between the two:  $X = X_L X_C$

# Capacitive Reactance, X<sub>c</sub>

- Capacitive reactance is the opposition that a capacitor offers to alternating current due to its phase-shifted storage and release of energy in its electric field.
- Capacitive reactance is symbolized by the capital letter " $X_C$ " and is measured in ohms just like resistance (R).
- Capacitive reactance can be calculated using this formula:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$
 (unit is in  $\Omega$ )

Where,

 $X_C$  = reactance in ohms ( $\Omega$ )

f = frequency in hertz (Hz)

C = capacitance in farads (F)

■ Capacitive reactance *decreases* with increasing frequency. In other words, the higher the frequency, the less it opposes (the more it "conducts") the AC flow of electrons.

# **Example:**

Determine the reactance of a  $1\mu F$  capacitor for a 50 Hz signal. Also determine its reactance when frequency is 10 kHz.

### Solution:

Calculate capacitive reactance using this formula:  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$ 

Ans.  $3.2k\Omega$  (at 50 Hz),  $16\Omega$  (at 10 kHz)

# Inductive reactance, X<sub>L</sub>

- Inductive reactance is the opposition that an inductor offers to alternating current due to its phase-shifted storage and release of energy in its magnetic field.
- Inductive reactance is symbolized by the capital letter " $X_L$ " and is measured in ohms just like resistance (R).
- Inductive reactance can be calculated using this formula:

$$X_L = \omega L = 2\pi f L$$
 (unit is in  $\Omega$ )

Where,

 $X_L$  = reactance in ohms ( $\Omega$ )

f = frequency in hertz (Hz)

L = inductance in henrys (H)

■ Inductive reactance *increases* with increasing frequency. In other words, the higher the frequency, the more it opposes the AC flow of electrons.

## Example:

Determine the reactance of a 1mH inductor for a 50 Hz signal. Also determine its reactance when frequency is 10 kHz.

### Solution:

Calculate inductive reactance using this formula:  $X_L = \omega L = 2\pi f L$ 

**Ans.**  $0.3\Omega$  (at 50 Hz),  $63\Omega$  (at 10 kHz)

### **Impedance**

- Impedance is a measure of the overall opposition of a circuit to current, in other words: how much the circuit **impedes** the flow of current. It is like resistance, but it also takes into account the effects of capacitance and inductance.
- Impedance is more complex than resistance because, the effects of capacitance and inductance vary with the frequency of the current passing through the circuit and this means **impedance varies with frequency**! The effect of resistance is constant regardless of frequency.

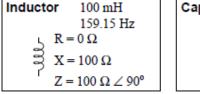
- It is present in all circuits, and in all components.
- When alternating current goes through an impedance, a voltage drop is produced that is somewhere between 0° and 90° out of phase with the current.
- It is the complex (vector) sum of ("real") resistance and ("imaginary") reactance. Therefore, impedance can be split into two parts:
  - Resistance R (the part which is constant regardless of frequency)
  - Reactance X (the part which varies with frequency due to capacitance and inductance)
- Impedance is mathematically symbolized by the letter "Z" and measured in ohms, just like resistance (R) and reactance (X), in complex form, e.g. Z=R+JX.
- Parallel impedances are managed just like resistances in series circuit analysis, but in complex form: series impedances add to form the total impedance.  $Z_{Total} = Z_1 + Z_2 + ... Z_n$ .
- Parallel impedances are manipulated just like resistances in parallel circuit analysis, but in complex form.  $1/Z=1/Z_1+1/Z_2+.....+1/Z_n$ .
- Perfect resistors possess resistance, but not reactance.

Resistor 
$$100 \Omega$$

$$\begin{cases} R = 100 \Omega \\ X = 0 \Omega \\ Z = 100 \Omega \angle 0^{\circ} \end{cases}$$

Figure: Perfect resistor

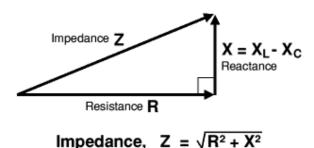
■ Perfect inductors and perfect capacitors possess reactance but no resistance.



Capacitor 10  $\mu$ F 159.15 Hz  $\underline{\qquad} R = 0 \Omega$   $X = 100 \Omega$   $Z = 100 \Omega \angle -90^{\circ}$ 

Figure: Perfect inductor and capacitor

- All components (resistor, capacitor and inductor) possess impedance, and because of this universal quality, it makes sense to translate all component values into common terms of impedance as the first step in analyzing an AC circuit.
- The impedance phase angle for any component is the phase shift between voltage across that component and current through that component.
- For a perfect resistor, the voltage drop and current are *always* in phase with each other, and so the impedance angle of a resistor is said to be 0°.
- For a perfect inductor, voltage drop always leads current by 90°, and so an inductor's impedance phase angle is said to be +90°.
- For a perfect capacitor, voltage drop always lags current by 90°, and so a capacitor's impedance phase angle is said to be -90°.
- In an RLC circuit, the resistance causes no phase shift (i.e. 0° phase angle) between voltage and current, but the capacitance and inductance cause a phase shift between the current and voltage. Therefore, the resistance and reactance cannot be simply added up to give impedance. Instead they must be added as vectors with reactance at right angles to resistance as shown in the diagram.



### Impedance for a series RL Circuit:

$$Z = \sqrt{(R^2 + X_L^2)} = R + JX_L$$

### Impedance for a series RC Circuit:

$$Z = \sqrt{(R^2 + X_C^2)} = R + JX_C$$

### Impedance for a series RLC Circuit:

$$Z = \sqrt{(R^2 + X^2)} = \sqrt{(R^2 + (X_L - X_C)^2)}$$
 where X= X<sub>L</sub>-X<sub>C</sub>

### Ohm's Law for AC circuits:

$$E = IZ$$
  $I = \frac{E}{Z}$   $Z = \frac{E}{I}$ 

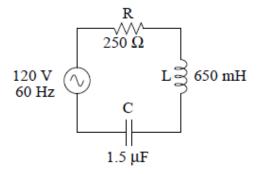
All quantities expressed in complex, not scalar, form

- > Kirchhoff 's Laws and all network analysis methods and theorems are true for AC circuits as well, so long as quantities are represented in complex rather than scalar form.
- The only real difference between DC and AC circuit calculations is in regard to power. Because reactance doesn't dissipate power as resistance does, the concept of power in AC circuits is radically different from that of DC circuits.

### Solving Series RLC Circuits

# **Example:**

Determine the total impedance, current and voltage drop in each component of a series RLC circuit connected with a 60Hz supply shown below.



# Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi f L$$

$$X_L = (2)(\pi)(60 \text{ Hz})(650 \text{ mH})$$

$$X_T = 245.04 \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{(2)(\pi)(60 \text{ Hz})(1.5 \mu\text{F})}$$

$$X_C = 1.7684 \text{ k}\Omega$$

# Step-2:

■ Express all resistances and reactances in a mathematically common form: impedance.

$$Z_{\rm R} = 250 + {\rm j}0~\Omega$$
 or  $250~\Omega \angle 0^{\rm o}$  
$$Z_{\rm L} = 0 + {\rm j}245.04~\Omega$$
 or  $245.04~\Omega \angle 90^{\rm o}$  
$$Z_{\rm C} = 0 - {\rm j}1.7684 {\rm k}~\Omega$$
 or  $1.7684~{\rm k}\Omega \angle -90^{\rm o}$ 

### Note:

■ Remember that an inductive reactance translates into a positive imaginary impedance (or an impedance at +90°), while a capacitive reactance translates into a negative imaginary impedance (impedance at -90°). Resistance, of course, is still regarded as a purely "real" impedance (polar angle of 0°):

### Step-3:

■ Express the given source voltage in rectangular or polar form:

$$V=120+j0 \ V \ or \ 120 \ V \ \angle 0^{0}$$

### Note:

 $\succ$  Unless otherwise specified, the source voltage will be our reference for phase shift, and so it is written at an angle of 0°.

All given quantities are shown in the circuit below.

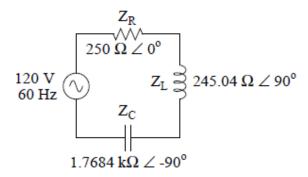


Figure: Series RLC circuit with component values replaced by impedances.

# Step-4:

Calculate the total impedance of the circuit.

$$Z_{total} = Z_R + Z_L + Z_C$$

$$Z_{total} = (250 + j0 \Omega) + (0 + j245.04 \Omega) + (0 - j1.7684k \Omega)$$

$$Z_{total} = 250 - j1.5233k \Omega \quad or \quad 1.5437 k\Omega \angle -80.680^{\circ}$$

### Step-5:

Determine circuit current using Ohm's law.

$$I = \frac{V}{Z} = \frac{120 \angle 0^{\circ}}{(1.5437 \times 1000) \angle -80.680^{\circ}} = 0.07773 \angle 80.680^{\circ} = 12.589 + j76.708$$

#### Note:

■ Being a series circuit, current must be equal through all components of the given circuit.

### Step-6:

Determine voltage drop across resistor, capacitor and inductor.

$$V_R = IZ_R = 0.07773 \angle 80.680^{\circ} \times 250 \angle 0^{\circ} = 19.4325 \angle 80.680^{\circ}$$

$$V_L = IZ_L = 0.07773 \angle 80.680^\circ \times 245.04 \angle 90^\circ = 19.046 \angle 170.68^\circ$$
 
$$V_C = IZ_C = 0.07773 \angle 80.680^\circ \times (1.7684 \times 1000) \angle -90^\circ = 137.46 \angle -9.3199^\circ$$

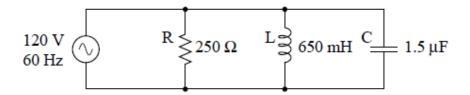
### Note:

- Although our supply voltage is only 120 volts, the voltage across the capacitor is 137.46 volts! How can this be? The answer lies in the interaction between the inductive and capacitive reactances. Expressed as impedances, we can see that the inductor opposes current in a manner precisely opposite that of the capacitor. Expressed in rectangular form, the inductor's impedance has a positive imaginary term and the capacitor has a negative imaginary term. When these two contrary impedances are added (in series), they tend to cancel each other out! Although they're still added together to produce a sum, that sum is actually less than either of the individual (capacitive or inductive) impedances alone.
- Although impedances add in series, the total impedance for a circuit containing both inductance and capacitance may be less than one or more of the individual impedances, because series inductive and capacitive impedances tend to cancel each other out.
- If the total impedance in a series circuit with both inductive and capacitive elements is less than the impedance of either element separately, then the total current in that circuit must be greater than what it would be with only the inductive or only the capacitive elements there.
- With this abnormally high current through each of the components, voltages greater than the source voltage may be obtained across some of the individual components!

### Solving Parallel RLC Circuits

# **Example:**

Determine the total impedance, current and voltage drop in each component of a parallel RLC circuit connected with a 60Hz supply shown below.



# Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi f L$$

$$X_L = (2)(\pi)(60 \text{ Hz})(650 \text{ mH})$$

$$X_T = 245.04 \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{(2)(\pi)(60 \text{ Hz})(1.5 \text{ uF})}$$

$$X_C = 1.7684 \text{ k}\Omega$$

# Step-2:

■ Express all resistances and reactances in a mathematically common form: impedance.

$$Z_{\rm R} = 250 + {\rm j}0~\Omega$$
 or  $250~\Omega \angle 0^{\rm o}$  
$$Z_{\rm L} = 0 + {\rm j}245.04~\Omega$$
 or  $245.04~\Omega \angle 90^{\rm o}$  
$$Z_{\rm C} = 0 - {\rm j}1.7684 {\rm k}~\Omega$$
 or  $1.7684~{\rm k}\Omega \angle -90^{\rm o}$ 

# Step-3:

Express the given source voltage in rectangular or polar form:

$$V=120+j0 \text{ V or } 120 \text{ V } \angle 0^0$$

■ All given quantities are shown in the circuit below.

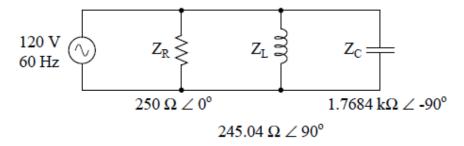


Figure: Parallel RLC circuit with component values replaced by impedances.

# Step-4:

Calculate the total impedance of the parallel circuit.

$$\begin{split} \frac{1}{Z_{Total}} &= \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C} \\ &= \frac{1}{250 \angle 0^{\circ}} + \frac{1}{245.04 \angle 90^{\circ}} + \frac{1}{(1.7684 \times 1000) \angle -90^{\circ}} \\ Z_{Total} &= 187.79 \angle 41.311^{\circ} = 141.05 + j123.96 \end{split}$$

# Step-5:

Determine voltage drop across resistor, capacitor and inductor.

Being a parallel circuit, voltage must be equally shared by all components of the given circuit. Therefore voltage drop across each component is the same as the supply voltage which is V=120+j0 V or 120 V $\angle 0^0$ 

$$V_R = V_L = V_C = V = 120 + j0 = 120 \angle 0^\circ$$

# Step-6:

Determine total current and current flowing through each component using Ohm's law.

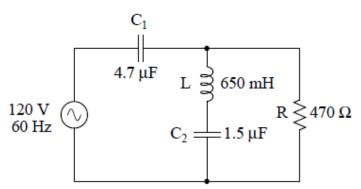
$$I = \frac{V}{Z} = \frac{120 \angle 0^{\circ}}{187.79 \angle 41.311^{\circ}} = 639.03 \text{mA} \angle -41.311^{\circ} = 480 \text{mA} - j421.85 \text{mA}$$

$$\begin{split} I_R = & \frac{V_R}{Z_R} = \frac{120 \angle 0^\circ}{250 \angle 0^\circ} = 480 \text{mA} \angle 0^\circ = 480 \text{mA} + j0 \\ \\ I_L = & \frac{V_L}{Z_L} = \frac{120 \angle 0^\circ}{245.04 \angle 90^\circ} = 489.71 \text{mA} \angle -90^\circ = 0 - j489.71 \text{mA} \\ \\ I_C = & \frac{V_C}{Z_C} = \frac{120 \angle 0^\circ}{(1.7684 \times 1000) \angle -90^\circ} = 67.858 \text{mA} \angle 90^\circ = 0 - j67.858 \text{mA} \end{split}$$

# **Solving Series-Parallel RLC Circuits**

# **Example:**

Determine the total impedance, current and voltage drop in each component of a series-parallel RLC circuit connected with a 60Hz supply shown below.



# Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi f L$$

$$X_L = (2)(\pi)(60 \text{ Hz})(650 \text{ mH})$$

$$X_T = 245.04 \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_{C1} = \frac{1}{(2)(\pi)(60Hz)(4.7\mu F)} = 564.38\Omega$$

$$X_{C2} = \frac{1}{(2)(\pi)(60Hz)(1.5\mu F)} = 1.7684k\Omega$$

### Step-2:

■ Express all resistances and reactances in a mathematically common form: impedance.

$$Z_R = 250 + j0 \Omega$$
 or  $250 \Omega \angle 0^\circ$    
 $Z_L = 0 + j245.04 \Omega$  or  $245.04 \Omega \angle 90^\circ$    
 $Z_C = 0 - j1.7684k \Omega$  or  $1.7684 k\Omega \angle -90^\circ$ 

### Step-3:

■ Express the given source voltage in rectangular or polar form:

$$V=120+j0 \text{ V or } 120 \text{ V } \angle 0^0$$

■ All given quantities are shown in the circuit below.

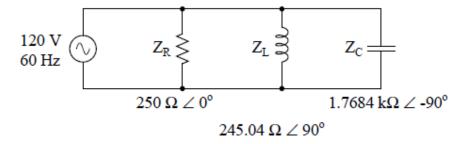


Figure: Parallel RLC circuit with component values replaced by impedances.

## Step-4:

Calculate the total impedance of the parallel circuit.

$$\begin{split} \frac{1}{Z_{Total}} &= \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C} \\ &= \frac{1}{250 \angle 0^{\circ}} + \frac{1}{245.04 \angle 90^{\circ}} + \frac{1}{(1.7684 \times 1000) \angle -90^{\circ}} \\ Z_{Total} &= 187.79 \angle 41.311^{\circ} = 141.05 + j123.96 \end{split}$$

## Step-5:

Determine voltage drop across resistor, capacitor and inductor.

Being a parallel circuit, voltage must be equally shared by all components of the given circuit. Therefore voltage drop across each component is the same as the supply voltage which is V=120+j0 V or 120 V $\angle 0^0$ 

$$V_R = V_L = V_C = V = 120 + j0 = 120 \angle 0^\circ$$

# Step-6:

Determine total current and current flowing through each component using Ohm's law.

$$I = \frac{V}{Z} = \frac{120 \angle 0^{\circ}}{187.79 \angle 41.311^{\circ}} = 639.03 \text{mA} \angle -41.311^{\circ} = 480 \text{mA} - j421.85 \text{mA}$$

$$I_R = \frac{V_R}{Z_R} = \frac{120\angle 0^\circ}{250\angle 0^\circ} = 480mA\angle 0^\circ = 480mA + j0$$
 
$$I_L = \frac{V_L}{Z_L} = \frac{120\angle 0^\circ}{245.04\angle 90^\circ} = 489.71mA\angle - 90^\circ = 0 - j489.71mA$$
 
$$I_C = \frac{V_C}{Z_C} = \frac{120\angle 0^\circ}{(1.7684\times 1000)\angle - 90^\circ} = 67.858mA\angle 90^\circ = 0 - j67.858mA$$