Lecture-05: DC Network Theorem: Thevenin's & Norton's Theorems

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Introduction:

- In today's lecture, we will explain two theorems:

  3. Thevenin's theorem

  4. Norton's theorem

3. Thevenin's Theorem:

- Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex it is, to an equivalent circuit with just a **single voltage source (V_{TH})** and **series resistance (R_{TH})** which is **connected to a load (R_L)** which is looked like this:

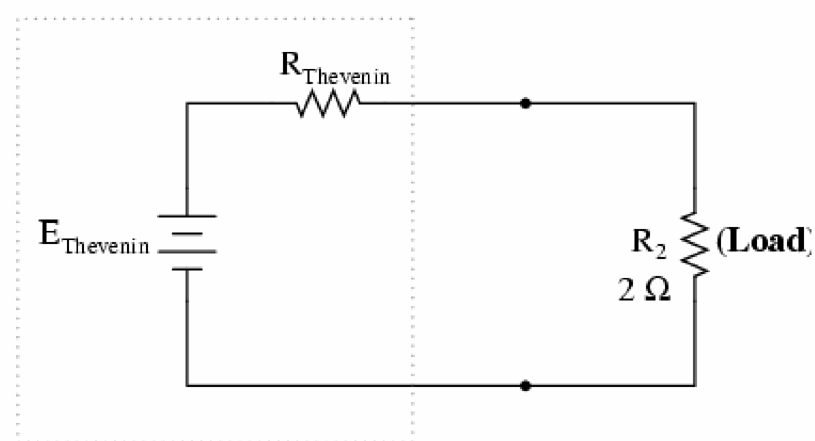

Figure: Thevenin equivalent Circuit

- Thevenin's Theorem is especially useful in analyzing power systems and other circuits where one particular resistor in the circuit (called the "load" resistor) is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it.
- Consider the circuit given below. We apply Thevenin's Theorem to it:

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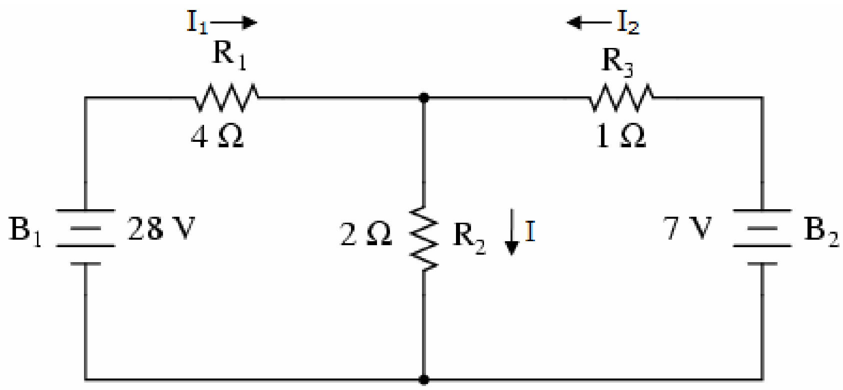
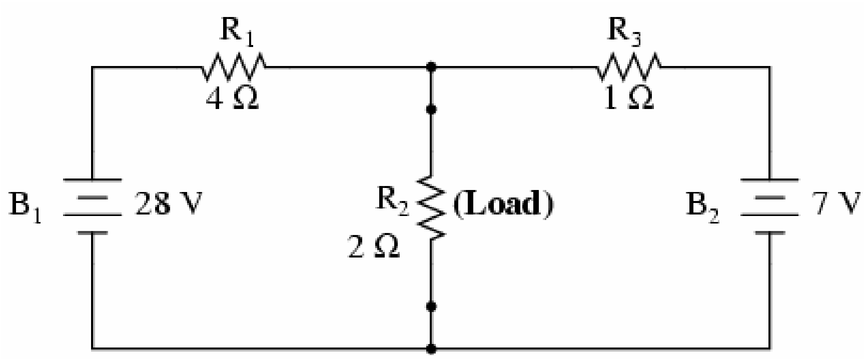


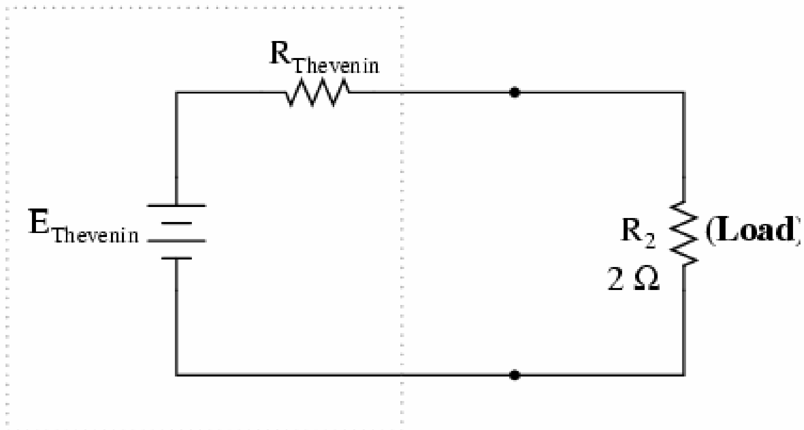
Figure-1: Original Circuit

- In the circuit above, I_1, I_2 and I represent the values of currents which are due to the simultaneous action of the two sources of emf in the network.
- Let us assume that we decide to designate R_2 as the "load" resistor in this circuit.



- We already have discussed Superposition Theorem by which we can determine voltage across R_2 and current through R_2, but this method is time-consuming. Imagine repeating this method over and over again to find what would happen if the load resistance changed (changing load resistance is very common in power systems, as multiple loads get switched on and off as needed, the total resistance of their parallel connections changing depending on how many are connected at a time). This could potentially involve a lot of work!
- Thevenin's Theorem makes this easy by temporarily removing the load resistance from the original circuit and reducing what is left to an equivalent circuit composed of a **single voltage source** and **series resistance**. The **load resistance** can then be re-connected to this "Thevenin equivalent circuit" and calculations carried out as if the whole network were nothing but a simple series circuit.

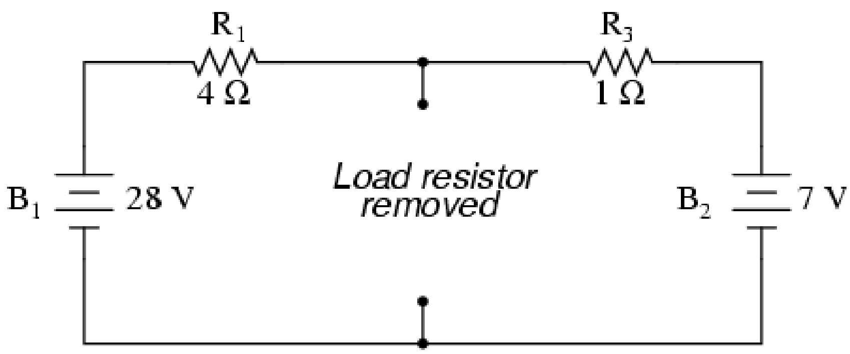
- After Thevenin conversion, the Thevenin equivalent circuit is:



- The "Thevenin Equivalent Circuit" is the electrical equivalent of B_1, R_1, R_3, and B_2 as seen from the two points where our load resistor (R_2) connects.
- The Thevenin equivalent circuit, if correctly derived, will behave exactly the same as the original circuit formed by B_1, R_1, R_3, and B_2. In other words, the load resistor (R_2) voltage and current should be exactly the same for the same value of load resistance in the two circuits. The load resistor R_2 cannot "tell the difference" between the original network of B_1, R_1, R_3, and B_2, and the Thevenin equivalent circuit of $E_{Thevenin}$, and $R_{Thevenin}$, provided that the values for $E_{Thevenin}$ and $R_{Thevenin}$ have been calculated correctly.
- The advantage in performing the "Thevenin conversion" to the simpler circuit, of course, is that it makes load voltage and load current so much easier to solve than in the original network. Calculating the equivalent Thevenin source voltage and series resistance is actually quite easy.

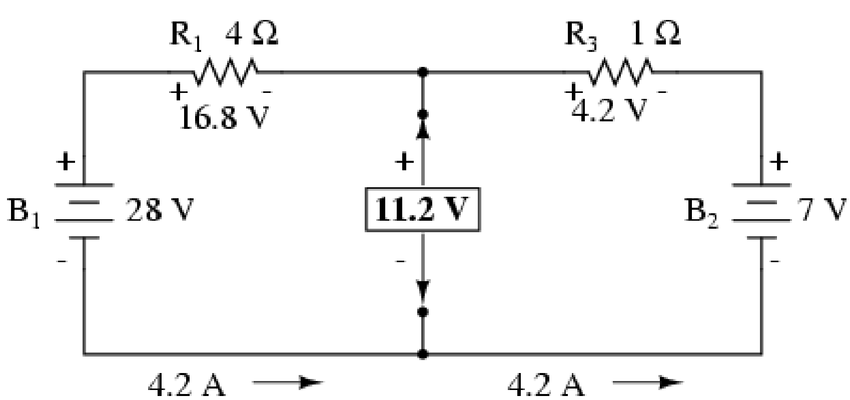
Performing Thevenin Conversion:

- First, remove load resistor R_L: the chosen load resistor is removed from the original circuit, replaced with a break (open circuit):

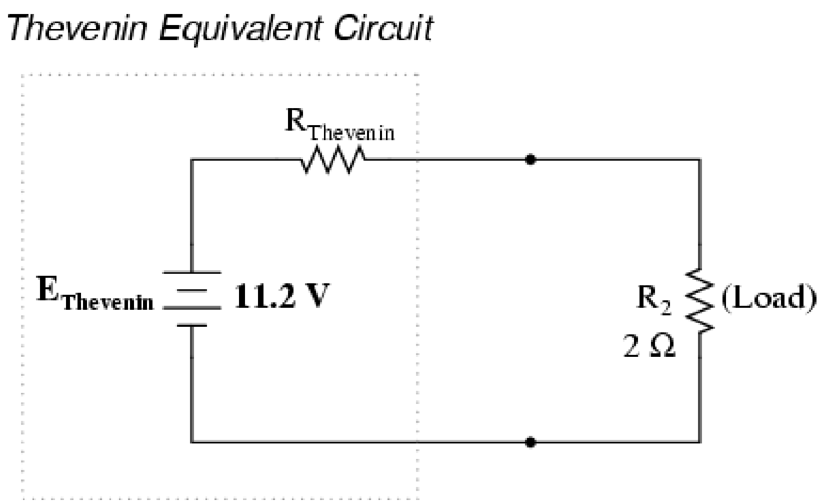


- Next, determine Thevenin voltage $E_{Thevenin}$: calculate the voltage between the two points where the load resistor used to be attached is determined. Use whatever analysis methods are at your disposal to do this. In this case, the original circuit with the load resistor removed is nothing more than a simple series circuit with opposing batteries, and so we can determine the voltage across the open load terminals by applying the rules of series circuits, Ohm's Law, and Kirchhoff's Voltage Law:

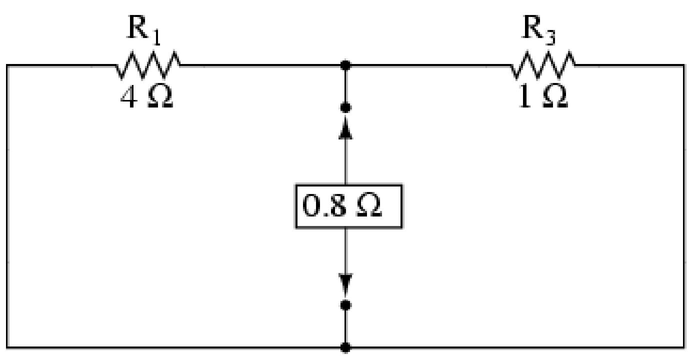
	R_1	R_3	Total	
E	16.8	4.2	21	Volts
I	4.2	4.2	4.2	Amps
R	4	1	5	Ohms



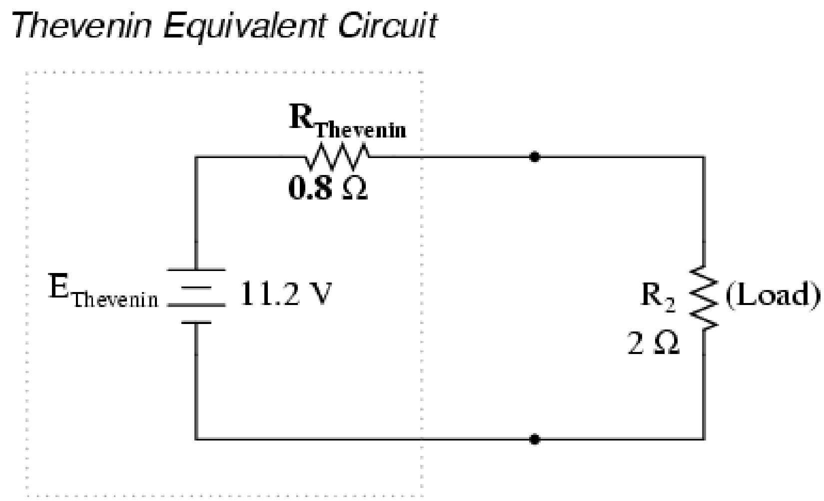
- The voltage between the two load connection points can be figured from the one of the battery's voltage and one of the resistor's voltage drops, and comes out to 11.2 volts. This is our "Thevenin voltage" ($E_{Thevenin}$) in the equivalent circuit:



- Then, determine Thevenin resistance $R_{Thevenin}$: To find the Thevenin series resistance for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure the resistance from one load terminal to the other:



- With the removal of the two batteries, the total resistance measured at this location is equal to R_1 and R_3 in parallel: 0.8 Ω. This is our "Thevenin resistance" ($R_{Thevenin}$) for the equivalent circuit:



- Finally, determine the current through and voltage across R_L: With the load resistor (2 Ω) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing more than a simple series circuit:

	$R_{Thevenin}$	R_{Load}	Total	
E	3.2	8	11.2	Volts
I	4	4	4	Amps
R	0.8	2	2.8	Ohms

- Notice that the voltage and current figures for R_2 (8 volts, 4 amps) are identical to those found using Superposition Theorem and Kirchhoff's Circuit laws.
- Also notice that the voltage and current figures for the Thevenin series resistance and the Thevenin source (total) do not apply to any component in the original, complex circuit. Thevenin's Theorem is only useful for determining what happens to a single resistor in a network: the load.
- The advantage, of course, is that you can quickly determine what would happen to that single resistor if it were of a value other than 2 Ω without having to go through a lot of analysis again. Just plug in that other value for the load resistor into the Thevenin equivalent circuit and a little bit of series circuit calculation will give you the result.

Lecture-05: DC Network Theorem: Thevenin’s & Norton’s Theorems

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Steps to follow for Thevenin’s Theorem/How to Thevenize a Given Circuit

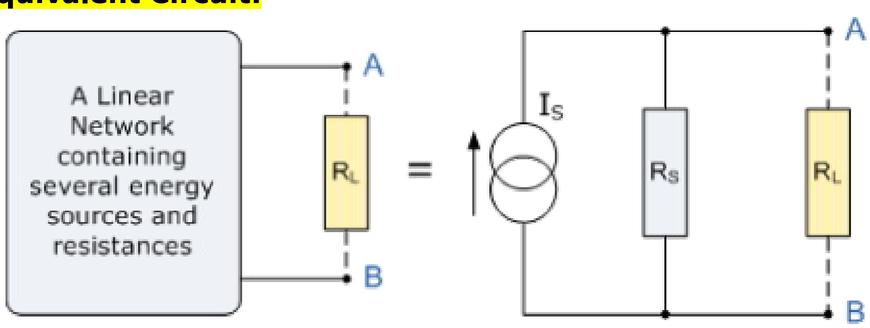
Thevenin's Theorem is a way to reduce a network to an equivalent circuit composed of a single voltage source, series resistance, and series load. Steps to follow for Thevenin's Theorem are:

1. Find the Thevenin source voltage by removing the load resistor from the original circuit and calculating voltage across the open connection points where the load resistor used to be.
2. Find the Thevenin resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.
3. Draw the Thevenin equivalent circuit, with the Thevenin voltage source in series with the Thevenin resistance. The load resistor re-attaches between the two open points of the equivalent circuit.
4. Analyze voltage and current for the load resistor following the rules for series circuits.

4. Norton's Theorem:

- Norton's Theorem is a way to reduce a network to an equivalent circuit composed of a single current source, parallel resistance, and parallel load.
- In some ways **Norton's Theorem** can be thought of as the opposite to "Thevenins Theorem", in that Thevenin reduces his circuit down to a single resistance in series with a single voltage. Norton on the other hand reduces his circuit down to a single resistance in parallel with a constant current source.
- **Norton's Theorem** states that "*Any linear circuit containing several energy sources and resistances can be replaced by a single constant current source in parallel with a single resistor*".
- As far as the load resistance R_L is concerned, this single resistance R_S is the value of the resistance looking back into the network with all the current sources open circuited and I_S is the short circuit current at the output terminals as shown below.

Norton's Equivalent Circuit:



- The value of this "constant current" is one which would flow if the two output terminals where shorted together while the source resistance would be measured looking back into the terminals, (the same as Thevenin).
- Consider the circuit shown in the figure below. We apply Norton's Theorem to it:

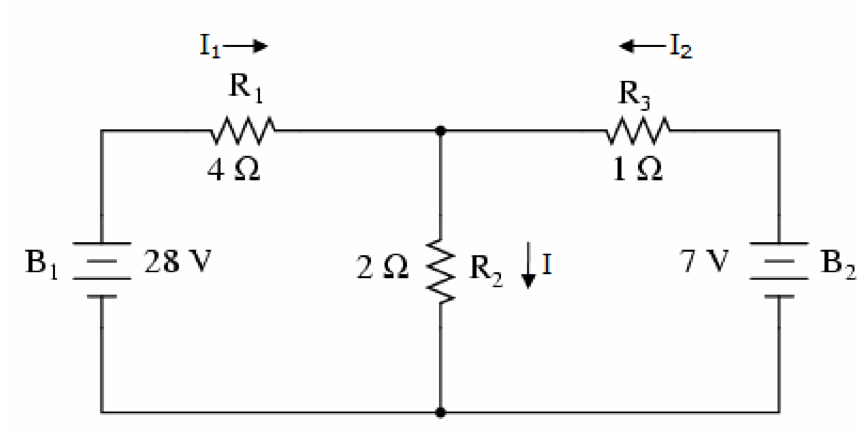


Figure-1: Original Circuit

- In the circuit above, I_1, I_2 and I represent the values of currents which are due to the simultaneous action of the two sources of emf in the network.
- After Norton conversion, the Norton's equivalent circuit will be looked like this:

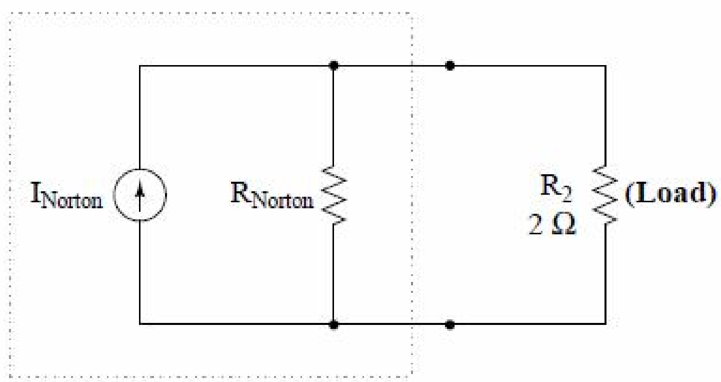
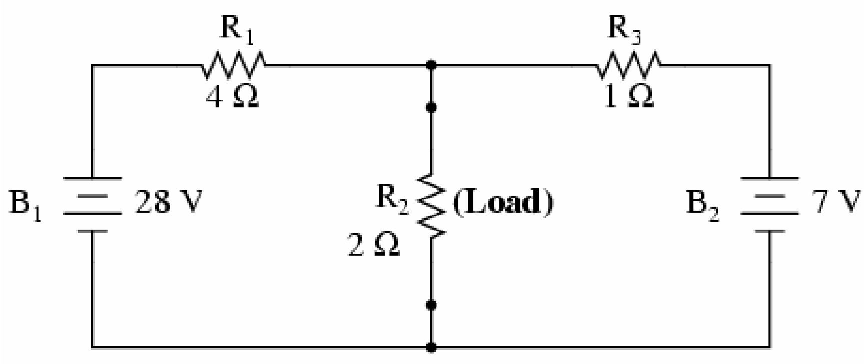


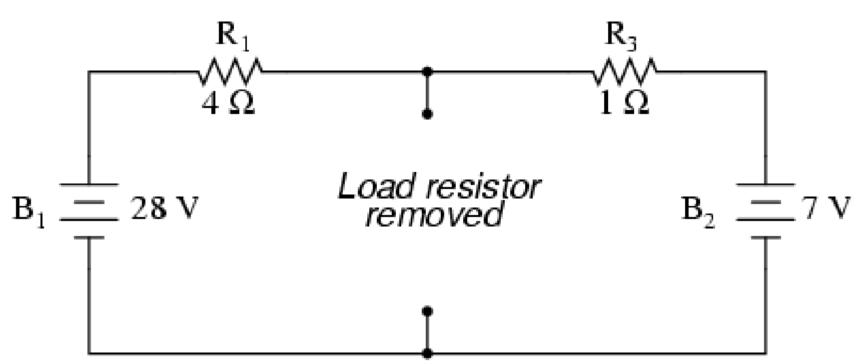
Figure: Assumed Norton's equivalent circuit before the calculation

- Let us assume that we decide to designate R_2 as the "load" resistor in this circuit.



Finding Norton's Equivalent of the Circuit:

- To find the Norton's equivalent of the above circuit, the first step is to identify the load resistance R_L and remove it from the original circuit:



- Next, determine Norton current $\mathbf{I_{Norton}}$: To find the Norton current I_{Norton} (for the current source in the Norton equivalent circuit), short out the terminals ***A*** and ***B*** (i.e. place a direct wire connection between the load points). Note that this step is exactly opposite the respective step in Thevenin's Theorem, where we replaced the load resistor with a break (open circuit). Now the circuit looks like this:

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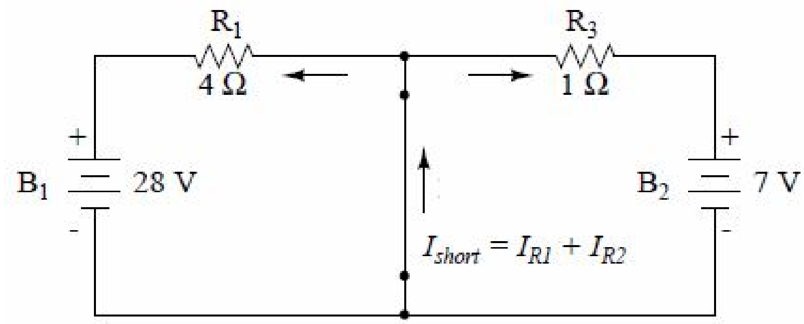


- When the terminals ***A*** and ***B*** are shorted together, voltage drop between load resistor connection point is zero. Then the current through R_1 is strictly a function of B_1's voltage and R_1's resistance:

$$I_{R1}=E_1/R_1=28/4=7 \text{ A.}$$

- Similarly, the current through R_3 is now strictly a function of B_2's voltage and R_3's resistance:

$$I_{R2}=E_2/R_3=7/1=7 \text{ A.}$$

- The total current through the short between the load connection points is the sum of these two currents: $I_{short}=I_{R1}+I_{R3}= 7 \text{ A} + 7 \text{ A} = 14 \text{ A}$.
- This value of ***14 A*** current becomes the Norton source current (I_{Norton}) in our equivalent circuit which now looks like:

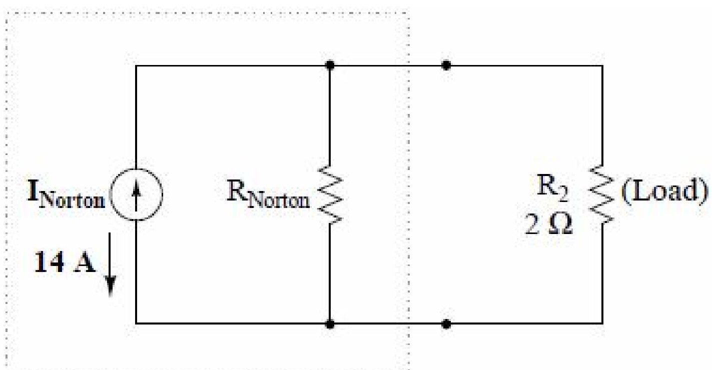
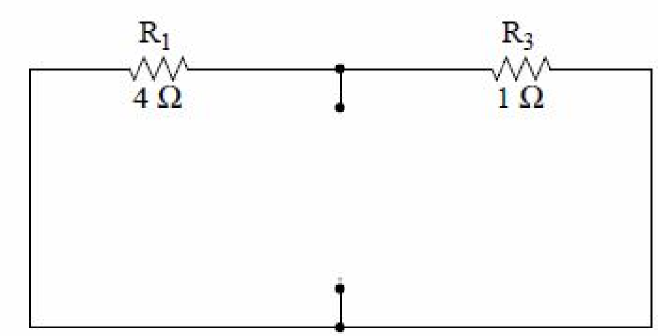


Figure: Norton's equivalent circuit after calculating I_{Norton}

- Next, determine the Norton resistance **R_{Norton}**: To find the Norton resistance (parallel to current source) for our equivalent circuit, we need to take the original circuit (with the load resistor still removed), remove the power sources (in the same style as we did with the Superposition Theorem: voltage sources replaced with wires and current sources replaced with breaks), and figure the total resistance from one load connection point to the other:



- With the removal of the two batteries, the total resistance measured at this location is equal to R_1 and R_3 in parallel:

  $R_T=R_1||R_3=(4x1)/(4+1)=4/5=0.8\ \Omega$.

  This is our "Norton resistance" (R_{Norton}) for the equivalent circuit:

- This value of 0.8 Ω resistance becomes the Norton resistance (R_{Norton}) in our equivalent circuit which now looks like:

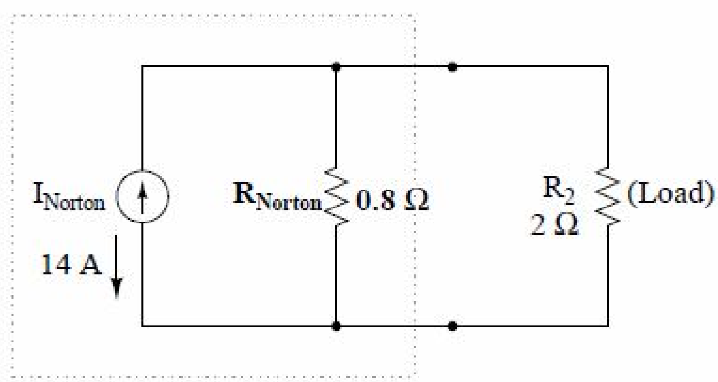


Figure: Norton's equivalent circuit after calculating I_{Norton} and R_{Norton}

- Finally, determine the current through and voltage across R_L: With the load resistor (2 Ω) attached between the connection points, we can determine voltage across it and current through it as though the whole network were nothing more than a simple parallel circuit with a current source:

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	R_{Norton}	R_{Load}	Total	
E	8	8	8	Volts
I	10	4	14	Amps
R	0.8	2	571.43m	Ohms

In Norton equivalent circuit, R_{Norton} and R_L (in this case R_2) are connected in parallel across the load terminals ***A*** and ***B***. According to current-divider rule, current through load resistor and Norton resistor can be found as:

$$I_{Load} = I_{RL} = I\frac{R_{Norton}}{R_{Norton} + R_{Load}}$$

$$= 14\frac{0.8}{0.8+2} = 4.03A$$

$$I_{R-Norton} = I\frac{R_{Load}}{R_{Norton} + R_{Load}}$$

$$= 14\frac{2}{0.8+2} = 10A$$

Therefore, total current $I=I_{Norton}=I_L+I_{R\text{-}Norton}=10+4.03=14.03$ A

Voltage drop across load is $V_L=I_L x R_L=4.03x2=8.06$ V

Voltage drop across R_{Norton} is $V_{R\text{-}Norton}=I_{R\text{-}Norton} x R_{Norton}=10x0.8=8$ V

Steps to follow for Norton's Theorem:

1. Find the Norton source current by removing the load resistor from the original circuit and calculating current through a short (wire) jumping across the open connection points where the load resistor used to be.
2. Find the Norton resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.
3. Draw the Norton equivalent circuit, with the Norton current source in parallel with the Norton resistance. The load resistor re-attaches between the two open points of the equivalent circuit.
4. Analyze voltage and current for the load resistor following the rules for parallel circuits.

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Thevenin-Norton equivalencies:

- Since Thevenin's and Norton's Theorems are two equally valid methods of reducing a complex network down to something simpler to analyze, there must be some way to convert a Thevenin equivalent circuit to a Norton equivalent circuit, and vice versa.
- Let us consider the same circuit shown in the figure below on which we have applied Thevenin's and Norton's theory.

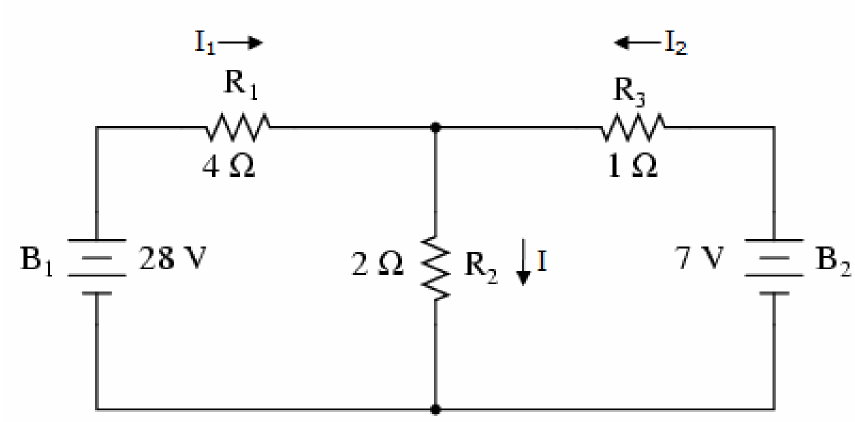
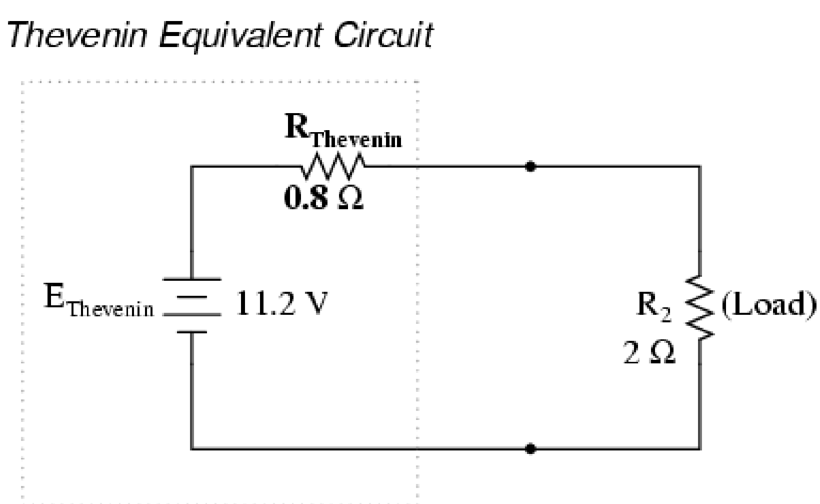
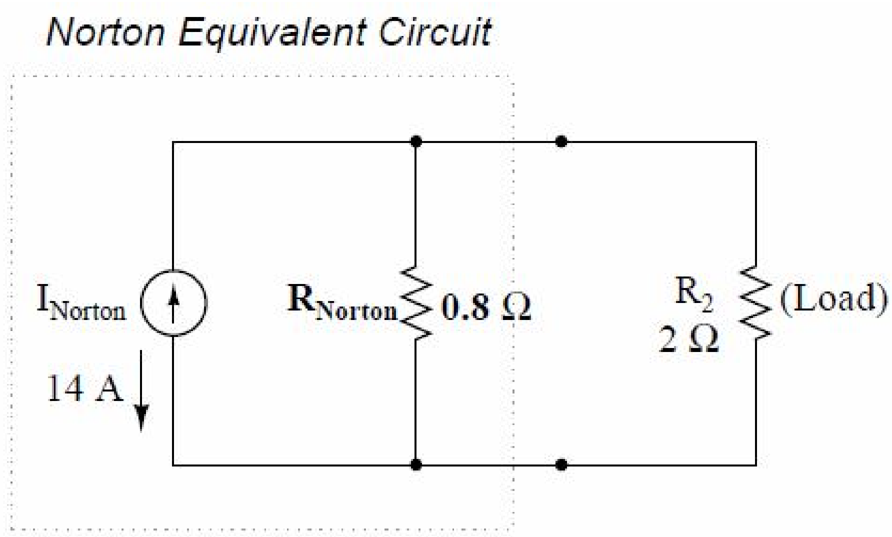


Figure-1: Original Circuit

- After Thevenin conversion, the Thevenin's equivalent circuit will be looked like this:



- And after Norton conversion, the Norton's equivalent circuit will be looked like this:



- You may have noticed that the procedure for calculating Thevenin resistance is identical to the procedure for calculating Norton resistance:
    - Remove all power sources and determine resistance between the open load connection points. As a result, the Thevenin and Norton resistances for the same original network must be equal.
    - We see from the above two circuits that the two resistances are indeed equal: $\mathbf{R_{Thevenin}=R_{Norton}=0.8}$ **Ohm.**

- Both the Thevenin and Norton equivalent circuits, having been derived from the same original network should behave identically. This means that both Thevenin and Norton equivalent circuits should produce the same voltage across the load terminals with no load resistor attached.
    - With the Thevenin equivalent, the open-circuited voltage would be equal to the Thevenin source voltage (no circuit current present to drop voltage across the series resistor), which is 11.2 volts in this case.
    - With the Norton equivalent circuit, all 14 amps from the Norton current source would have to flow through the 0.8 Norton resistance, producing the exact same voltage, 11.2 volts (E=IR).

- Thus, we can say that the Thevenin voltage is equal to the Norton current times the Norton resistance: $\mathbf{E_{Thevenin} = I_{Norton}R_{Norton}}$

- Conversely, both Thevenin and Norton equivalent circuits should generate the same amount of current through a short circuit across the load terminals.
  - With the Norton equivalent, the short-circuit current would be exactly equal to the Norton source current, which is 14 amps in this case.
  - With the Thevenin equivalent, all 11.2 volts would be applied across the 0.8. Thevenin resistance, producing the exact same current through the short, 14 amps (I=E/R).
  - Thus, we can say that the Norton current is equal to the Thevenin voltage divided by the Thevenin resistance: $I_{Norton} = \dfrac{E_{Thevenin}}{R_{Thevenin}}$