

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

**Nodal Voltage Analysis/ Node Voltage Method:**

- In Mesh (loop) analysis, Kirchhoff's voltage law is applied to arrive at loop currents in a network.
- As its name implies, Nodal analysis uses Kirchhoff's current law to determine the potential difference (voltage) at any node with respect to some arbitrary reference point in a network.
- Once the potentials of all nodes are known, it is a simple matter to determine other quantities such as current and power within the network.
- Using node analysis, we can calculate the voltages around the loops that reduce the amount of mathematics required using just Kirchhoff's laws.

**The steps for solving a circuit using nodal analysis are as follows:**

1. Determine the number of nodes within the given circuit.
2. Arbitrarily select one node as a reference node and indicate this node as **ground**. The reference node is usually located at the bottom of the circuit, although it may be located anywhere.
3. Convert each voltage source in the network to its equivalent current source. This step, although not absolutely necessary, makes further calculations easier to understand.
4. Arbitrarily assign voltages (such as  $V_1, V_2, \dots, V_n$ ) to the remaining nodes in the circuit. (Remember that all voltages to the remaining nodes will be relative to the chosen reference node).
5. Arbitrarily assign a current direction to each branch in which there is no current source.
6. Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.
7. With the exception of the reference node (ground), apply Kirchhoff's current law at each of the nodes. If a circuit has a total of  $n+1$  nodes (including the reference node), there will be  $n$  simultaneous linear equations.
8. Rewrite each of the arbitrarily assigned currents in terms of the potential difference across a known resistance.
9. Solve the resulting simultaneous linear equations for the voltages ( $V_1, V_2, \dots, V_n$ ).

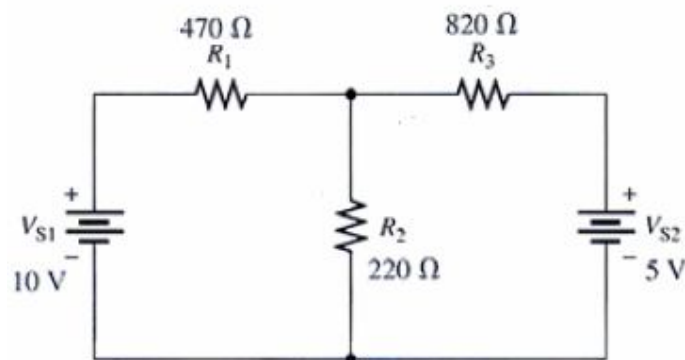
\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

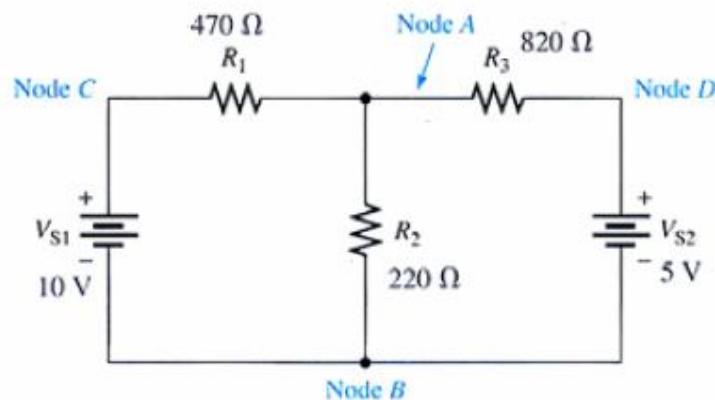
\*\*\*\*\*

**Example-1:**

Using node voltage analysis method, determine the nodal voltages for the circuit shown in the figure below.

**Solution:****Step-1:**

Determine the number of nodes in the given circuit. In this case, there are four nodes: Node A, B, C and D.

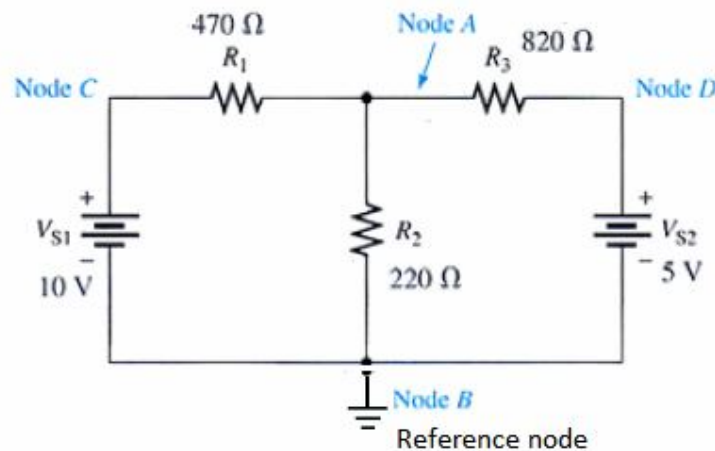
**Step-2:**

Arbitrarily select one node as a reference node and indicate this node as **ground**. In this case, let us assume **node B** as the reference node.

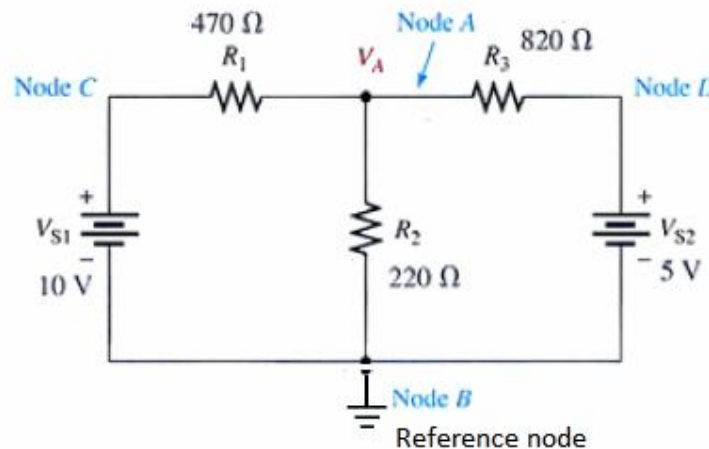
\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

**Step-3:**

Except the reference node, arbitrarily assign voltages ( $V_A$ ,  $V_C$  and  $V_D$ ) to the remaining nodes in the circuit. Here node voltage C and D are already known to be the source voltages, i.e.  $V_C=10\text{V}$ ,  $V_D=5\text{V}$ . The only unknown voltage is at node A, i.e.  $V_A$ . (Note that these voltages will all be with respect to the chosen reference).

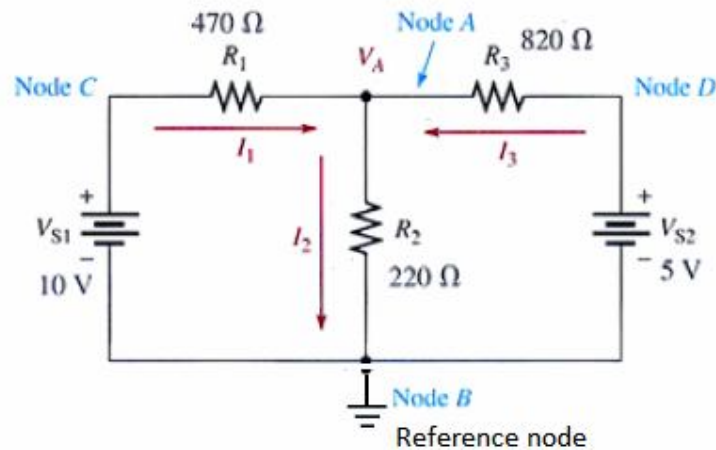
**Step-4:**

Arbitrarily assign branch currents along with directions at each node where the voltage is unknown, except at the reference node. Here, voltage at node A is unknown.

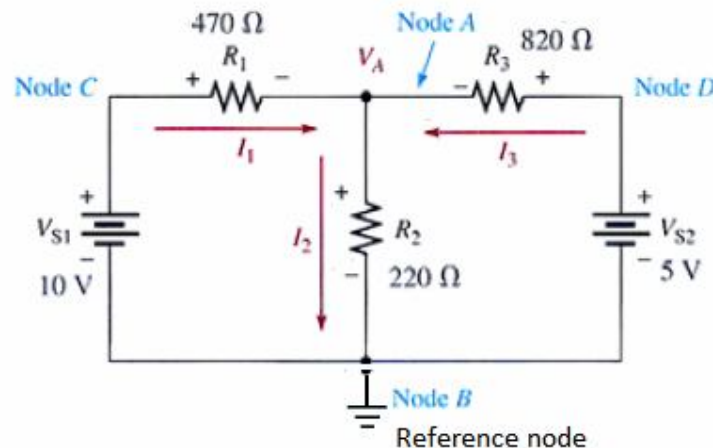
\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

**Step-5:**

Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.

**Step-6:**

With the exception of the reference node (ground), apply Kirchhoff's current law at each of the other nodes.

As there are 3 nodes (labeled as Node C, Node A, and Node D) except the reference node, and as the only unknown voltage is at node A, we now apply Kirchhoff's current law at node A.

$$\begin{aligned}
 \text{KCL at Node A} \quad \sum I_{\text{Entering}} &= \sum I_{\text{Leaving}} \\
 \Rightarrow I_1 + I_3 &= I_2 \\
 \Rightarrow I_1 - I_2 + I_3 &= 0 \quad \dots\dots\dots(1)
 \end{aligned}$$

\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

**Step-7:**

Express the current equation(s) in terms of the potential difference across a known resistance.

Here, the currents are rewritten in terms of the voltages across the resistors as follows:

$$I_1 = \frac{V_1}{R_1} = \frac{V_{S1} - V_A}{R_1} \quad \dots\dots\dots(2)$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_A}{R_2} \quad \dots\dots\dots(3)$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_{S2} - V_A}{R_3} \quad \dots\dots\dots(4)$$

By substituting the values of  $I_1$ ,  $I_2$  and  $I_3$  to equation (1), the nodal equations become:

$$I_1 - I_2 + I_3 = 0 \quad \dots\dots\dots(1)$$

$$\Rightarrow \frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0 \quad \dots\dots\dots(5)$$

**Step-8:**

Solve the resulting linear equation for the unknown node voltages using substitution or determinant method.

Here the equation is:

$$\frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0 \quad \dots\dots\dots(5)$$

Given,  $V_{S1}=10V$ ,  $V_{S2}=5V$ ,  $R_1=470 \text{ ohm}$ ,  $R_2=220 \text{ ohm}$ ,  $R_3=820 \text{ ohm}$ . The only unknown term is  $V_A$ . So, putting the known values, we can easily calculate the value of unknown term  $V_A$ .

\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

$$\begin{aligned}
 \frac{10 - V_A}{470} - \frac{V_A}{220} + \frac{5 - V_A}{820} &= 0 \\
 \frac{220 \times 820(10 - V_A) - 470 \times 820 V_A + 470 \times 220(5 - V_A)}{470 \times 220 \times 820} &= 0 \\
 \frac{1804000 - 180400 V_A - 385400 V_A + 517000 - 103400 V_A}{470 \times 220 \times 820} &= 0 \\
 \frac{11357000 - 669200 V_A}{470 \times 220 \times 820} &= 0 \\
 11357000 - 669200 V_A &= 0 \\
 V_A &= \frac{11357000}{669200} \\
 &= 16.97V
 \end{aligned}$$

After determining  $V_A$ , we can easily calculate the branch current  $I_1$ ,  $I_2$  and  $I_3$  using equations (2), (3) and (4).

$$\begin{aligned}
 I_1 &= \frac{V_1}{R_1} = \frac{V_{S1} - V_A}{R_1} \quad \dots\dots\dots(2) \\
 &= \frac{10 - 16.97}{470} \\
 &= \frac{-6.97}{470} \\
 &= -0.01482A \\
 &= -14.82mA
 \end{aligned}$$

$$\begin{aligned}
 I_2 &= \frac{V_2}{R_2} = \frac{V_A}{R_2} \quad \dots\dots\dots(3) \\
 &= \frac{16.97}{220} \\
 &= 0.07713A \\
 &= 77.13mA
 \end{aligned}$$

$$\begin{aligned}
 I_3 &= \frac{V_3}{R_3} = \frac{V_{S2} - V_A}{R_3} \quad \dots\dots\dots(4) \\
 &= \frac{5 - 16.97}{820} \\
 &= \frac{-11.97}{820} \\
 &= -0.01459A \\
 &= -14.59mA
 \end{aligned}$$

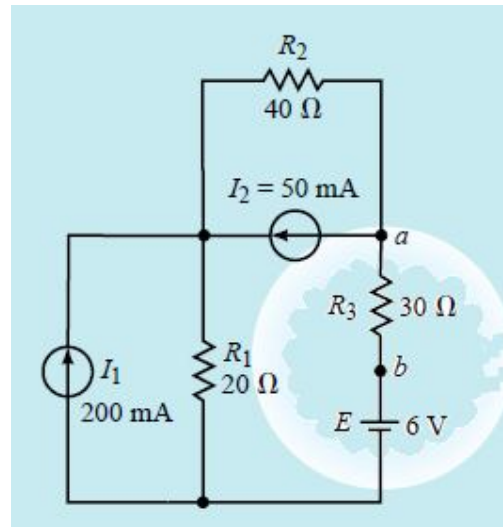
\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

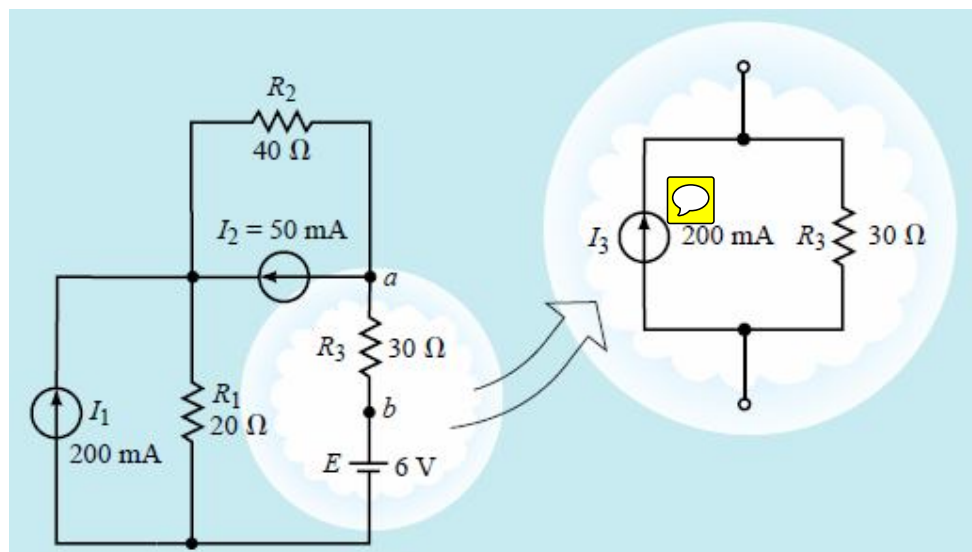
\*\*\*\*\*

**Example-2:**

Use nodal analysis to solve for the voltage  $V_{ab}$  from the circuit given below.

**Step-1:**

The given circuit has a voltage source. So, convert it to its equivalent current source.

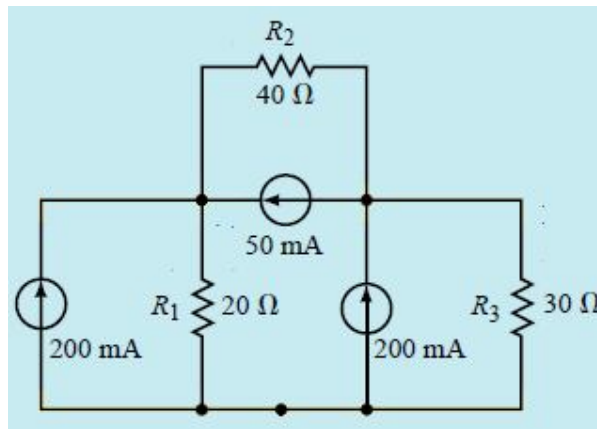


After converting the voltage source to its equivalent current source, the equivalent circuit is shown in figure below.

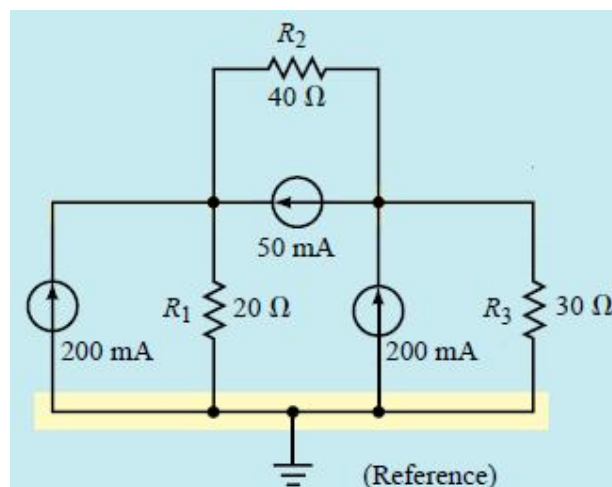
\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

**Step-2:**

Arbitrarily select a convenient reference node within the circuit and indicate this node as **ground**. The circuit with the reference node (shaded as white rectangle) is given below.

**Step-3:**

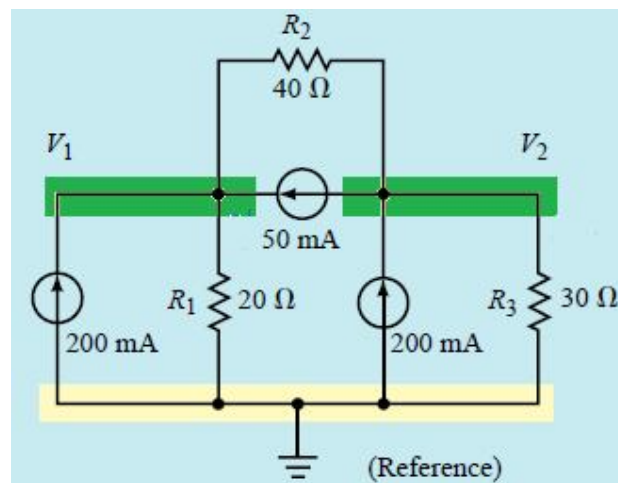
After assigning a reference node, arbitrarily assign voltages ( $V_1, V_2, \dots, V_n$ ) to the remaining nodes in the circuit. These voltages will all be with respect to the chosen reference. (Note that the remaining nodes are shaded as green rectangles).

\*\*\*\*\*

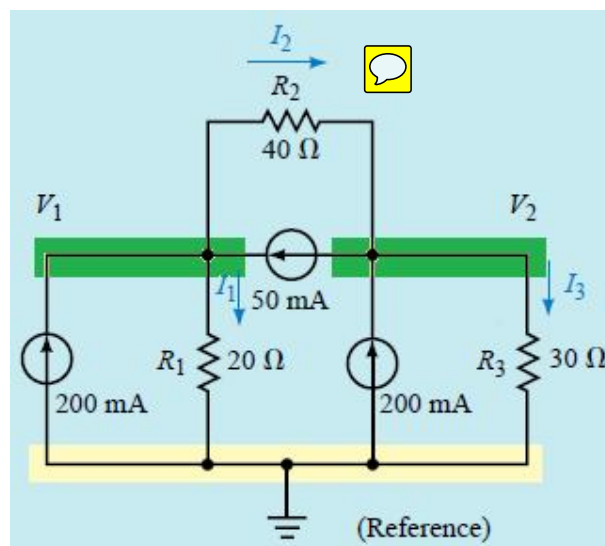


**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

**Step-4:**

Arbitrarily assign a current direction to each branch in which there is no current source.

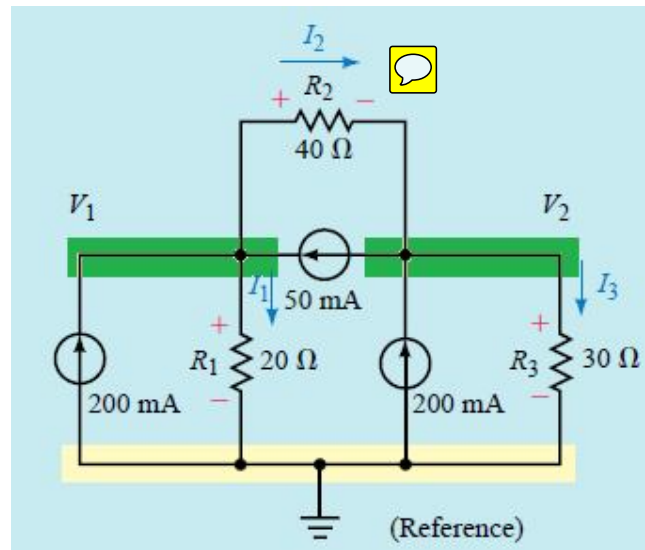
**Step-5:**

Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.

\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

**Step-6:**

With the exception of the reference node (ground), apply Kirchhoff's current law at each of the other nodes. As there are 2 nodes (labeled as  $V_1$  and  $V_2$ ) except the reference node, we now apply Kirchhoff's current law at these two nodes.

$$\begin{aligned} \text{KCL at Node } V_1 \quad \sum I_{\text{Entering}} &= \sum I_{\text{Leaving}} \\ 200 \text{ mA} + 50 \text{ mA} &= I_1 + I_2 \\ \Rightarrow I_1 + I_2 &= 250 \text{ mA} \quad \text{.....(1)} \end{aligned}$$

$$\begin{aligned} \text{KCL at Node } V_2 \quad \sum I_{\text{Entering}} &= \sum I_{\text{Leaving}} \\ 200 \text{ mA} + I_2 &= 50 \text{ mA} + I_3 \\ \Rightarrow I_2 - I_3 &= -150 \text{ mA} \quad \text{.....(2)} \end{aligned}$$

**Step-7:**

Rewrite each of the arbitrarily assigned currents in terms of the potential difference across a known resistance. Here, the currents are rewritten in terms of the voltages across the resistors as follows:

$$\begin{aligned} I_1 &= \frac{V_1}{20 \, \Omega} \\ I_2 &= \frac{V_1 - V_2}{40 \, \Omega} \\ I_3 &= \frac{V_2}{30 \, \Omega} \end{aligned}$$

\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

By substituting the values of  $I_1$ ,  $I_2$  and  $I_3$  to equation (1) and (2), the nodal equations become:

$$\begin{aligned}
 & I_1 + I_2 = 250 \text{ mA} \quad \text{.....(1)} \\
 \Rightarrow & \frac{V_1}{20} + \frac{V_1 - V_2}{40} = 250 \\
 \Rightarrow & \frac{2V_1 + V_1 - V_2}{40} = 250 \\
 \Rightarrow & 3V_1 - V_2 = 10000 \quad \text{.....(3)}
 \end{aligned}$$

$$\begin{aligned}
 & I_2 - I_3 = -150 \text{ mA} \quad \text{.....(2)} \\
 \Rightarrow & \frac{V_1 - V_2}{40} - \frac{V_2}{30} = -150 \\
 \Rightarrow & \frac{30(V_1 - V_2) - 40V_2}{1200} = -150 \\
 \Rightarrow & \frac{30V_1 - 30V_2 - 40V_2}{1200} = -150 \\
 \Rightarrow & 30V_1 - 70V_2 = -150 \times 1200 \\
 \Rightarrow & 3V_1 - 7V_2 = -18000 \quad \text{.....(5)}
 \end{aligned}$$

- To simplify the solution of the simultaneous linear nodal equations (3) and (5), we write them as follows:

$$3V_1 + (-1)V_2 = 10000 \quad \text{.....(3)}$$

$$3V_1 + (-7)V_2 = -18000 \quad \text{.....(5)}$$

- We can solve the above equations by the use of determinants and Cramer's rule or as just substitution method:

$$3V_1 - V_2 = 10000 \quad \text{.....(3)}$$

$$3V_1 - 7V_2 = -18000 \quad \text{.....(5)}$$

---


$$\begin{aligned}
 & \text{eq(3)} - \text{eq(5)} \quad 6V_2 = 28000 \\
 \Rightarrow & V_2 = \frac{28000}{6} = 4666.66 \text{ mV} = 4.67 \text{ V}
 \end{aligned}$$

Substituting the value of  $V_2$  in equation (3), Value of  $V_1$  will be found:

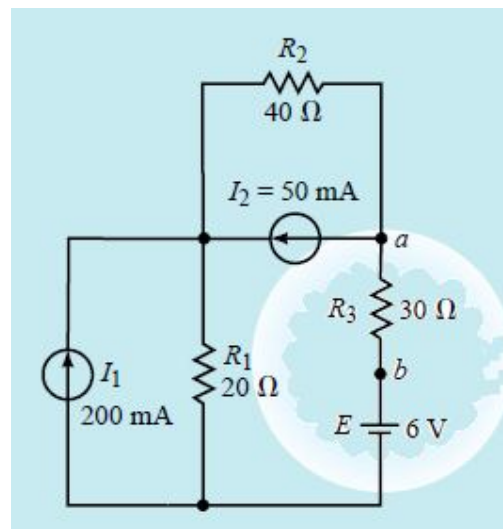
\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

$$\begin{aligned}
 3V_1 - V_2 &= 10000 \quad \dots\dots\dots(3) \\
 3V_1 &= 10000 + V_2 \\
 &= 10000 + 4666.66 \\
 &= 14666.66 \\
 V_1 &= \frac{14666.66}{3} = 4888.88 \text{ mV} = 4.88 \text{ V}
 \end{aligned}$$

If we go back to the given original circuit (shown below again), we see that the voltage  $V_2$  is the same as the voltage  $V_a$ .



$$\begin{aligned}
 V_a = V_2 &= 4.67 \text{ V} = 6.0 \text{ V} + V_{ab} \\
 \Rightarrow V_{ab} &= 4.67 - 6.0 = -1.33 \text{ V}
 \end{aligned}$$

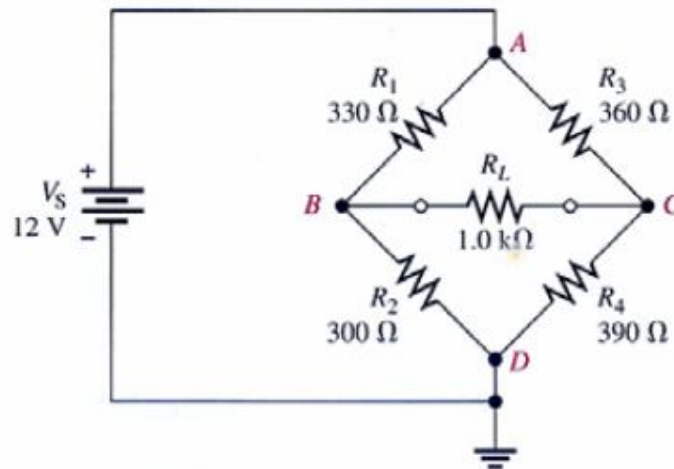
\*\*\*\*\*

**Lecture-10: Methods of Circuit Analysis: Node Voltage Method**

\*\*\*\*\*

**Example-3:**

Find the node voltages at node B and C for the Wheatstone bridge circuit given below:

**Solution:**

Among the four nodes, D is designated as reference node. Node A has the same voltage as the source. Voltage at nodes B and C is unknown. Hence we need two linear equations to solve the problem.

Node B:

$$I_1 + I_L = I_2$$

$$\frac{V_A - V_B}{R_1} + \frac{V_C - V_B}{R_L} = \frac{V_B}{R_2}$$

$$\frac{12 - V_B}{0.330 \text{ k}\Omega} + \frac{V_C - V_B}{1.0 \text{ k}\Omega} = \frac{V_B}{0.300 \text{ k}\Omega}$$

Node C:

$$I_3 = I_L + I_4$$

$$\frac{V_A - V_C}{R_3} = \frac{V_C - V_B}{R_L} + \frac{V_C}{R_4}$$

$$\frac{12 \text{ V} - V_C}{0.360 \text{ k}\Omega} = \frac{V_C - V_B}{1.0 \text{ k}\Omega} + \frac{V_C}{0.390 \text{ k}\Omega}$$

\*\*\*\*\*