Lecture-04: DC Network Theorem: Superposition & Millman's Theorem

****

Introduction:

- Network theorems are often important means for network analysis.
- Some theorems apply only to linear, bilateral circuits, or portions of them.
- Some theorems require deactivation of independent sources. That is, all independent sources are replaced by their internal resistances. In other words, all ideal voltage sources are replaced by short circuits, and all ideal current sources are replaced by open circuits.
- Dependent sources are never deactivated in the application of any theorem.
- In today's lecture, we will explain two theorems:
    1. Superposition theorem
    2. Millman's theorem

1. Superposition Theorem:

- If there are a number of e.m.fs acting simultaneously in any linear bilateral network, then the strategy used in the Superposition Theorem is to eliminate all but one source of power within a network at a time.
- After that, voltage drops (and/or currents) within the modified network for each power source are determined separately using series/parallel analysis.
- Then, the values are all "superimposed" on top of each other (added algebraically) to find the actual voltage drops/currents with all sources active.

This theorem may be stated as follows:

- In a linear bilateral network containing more than one independent source, the current or voltage of a circuit element equals the algebraic sum of the component voltages or currents produced by the independent sources acting alone.
- In other words, the voltage or current contribution from each independent source can be found separately, and then all the contributions are added algebraically to obtain the actual voltage or current with all independent sources present in the circuit.

Important Points to Remember about Superposition Theorem:

- This theorem applies only to independent sources, not to dependent ones.
- This theorem is applicable only for linear circuit which means it applies only for determining voltage and current, *not power* in the DC circuits*!!!* Power dissipations, being nonlinear functions; do not algebraically add to an accurate total when only one source is considered at a time.
- Another prerequisite for Superposition theorem is that all components must be "bilateral," meaning that they behave the same with electrons flowing either direction through them.
- Superposition theorem works only for circuits that are reducible to series/parallel combinations for each of the power sources acting alone at a time, which means that the other independent sources must be deactivated; thus, this theorem is useless for analyzing an unbalanced bridge circuit, and it only works to linear networks where current is linearly related to voltage as per Ohm's law.

Explanation for Superposition Theorem:

- Consider the circuit given below. We apply Superposition Theorem to it:

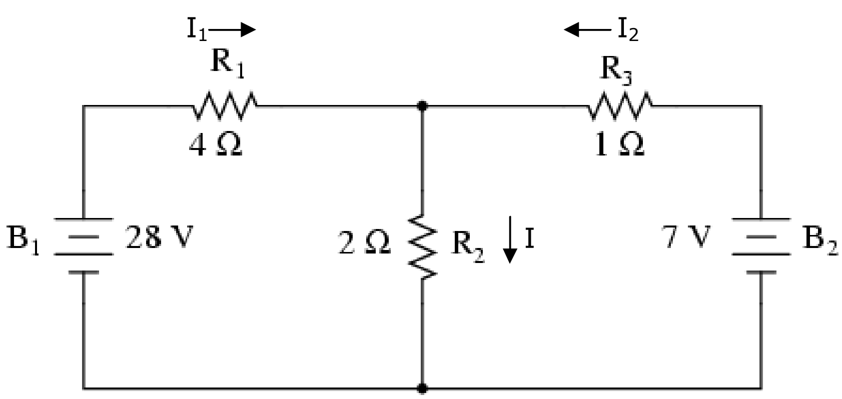


Figure-1: Original Circuit

In the circuit above, I_1, I_2 and I represent the values of currents which are due to the simultaneous action of the two sources of emf in the network.

Lecture-04: DC Network Theorem: Superposition & Millman's Theorem

****

- Since we have two sources of power in this circuit, we will have to calculate two sets of values for voltage drops and/or currents:
  - One for the circuit with only the 28 volt battery in effect which is shown below.

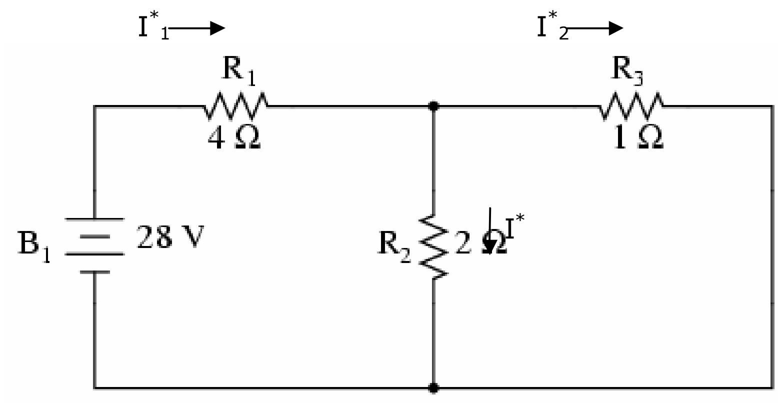


Figure-2: Circuit only 28 V battery in effect

  - Another for the circuit with only the 7 volt battery in effect which is shown below.

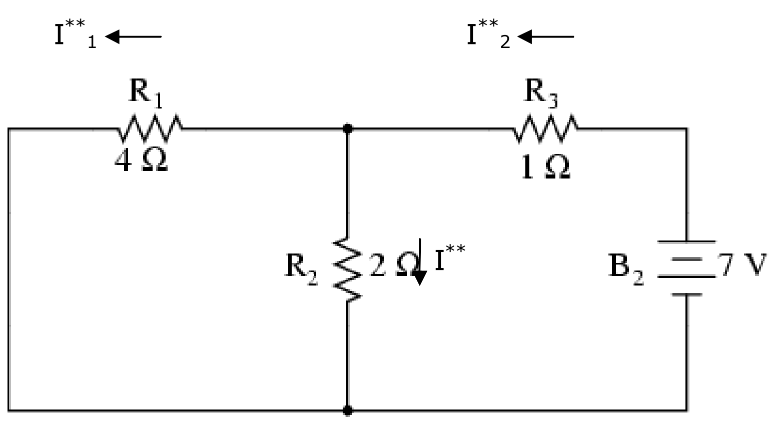


Figure-3: Circuit only 7 V battery in effect

- By combining the current values of figure-2 and figure-3, the actual values of figure-1 can be obtained:

$I_1 = I^*_1 - I^{**}_1$, $\quad I_2 = I^{**}_2 - I^*_2$, $\quad I = I_1 + I_2$

****

Note to Remember:

- When we re-draw the circuit for series/parallel analyses with one source, all other voltage sources are replaced by wires (shorts), and all current sources are replaced with open circuits (breaks).
- Since we only have voltage sources (batteries) in our example circuit, we will replace every inactive source during analysis with a wire.

- Analyzing the circuit with only the 28 volt battery, we obtain the following values for voltage and current:

	R_1	R_2	R_3	$R_2//R_3$	$R_1 + R_2//R_3$ Total	
E	24	4	4	4	28	Volts
I	6	2	4	6	6	Amps
R	4	2	1	0.667	4.667	Ohms

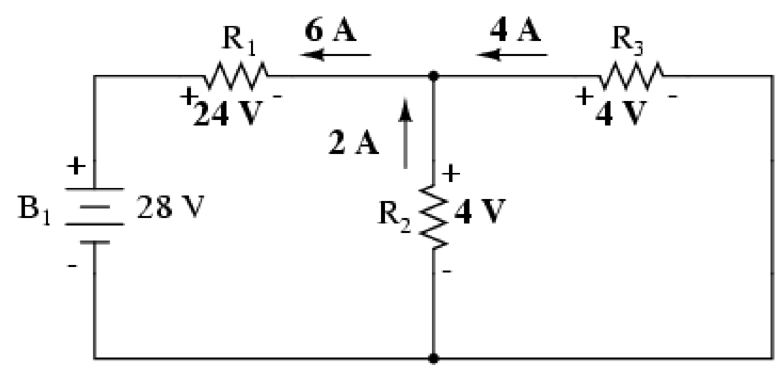


Figure-4: Analysis of the circuit only 28 V battery in effect

- Analyzing the circuit with only the 7 volt battery, we obtain another set of values for voltage and current:

	R_1	R_2	R_3	$R_1//R_2$	$R_3 + R_1//R_2$ Total	
E	4	4	3	4	7	Volts
I	1	2	3	3	3	Amps
R	4	2	1	1.333	2.333	Ohms

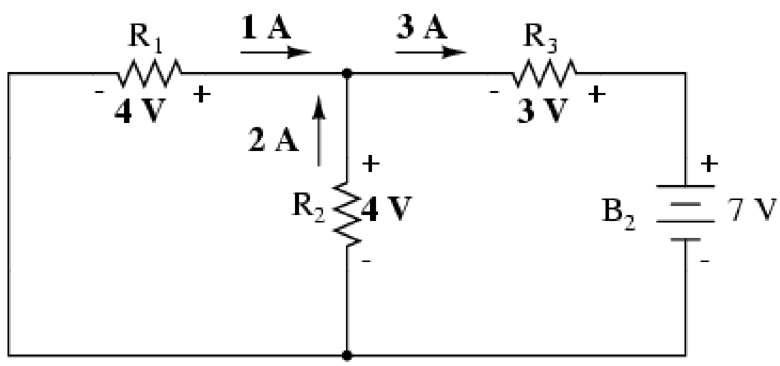


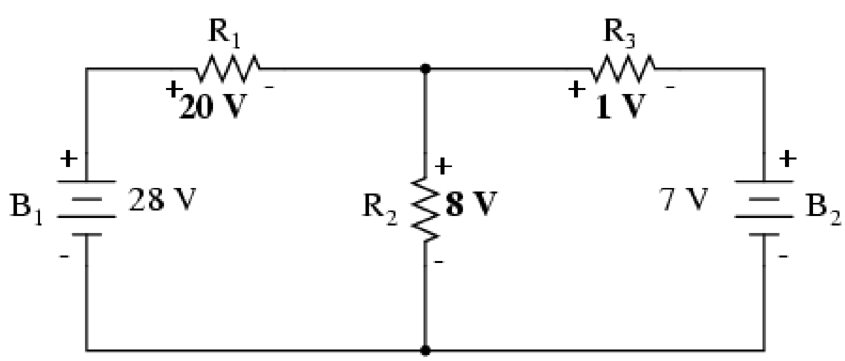
Figure-5: Analysis of the circuit only 28 V battery in effect

Note:

- When superimposing these values of voltage and current, we have to be very careful to consider polarity (voltage drop) and direction (electron flow), as the values have to be added *algebraically*.

With 28 V battery	*With 7 V battery*	*With both batteries*
24 V + - E_{R1}	4 V - + E_{R1}	**20 V** E_{R1} + - *24 V - 4 V = 20 V*
E_{R2} + - 4 V	E_{R2} + - 4 V	E_{R2} + - **8 V** *4 V + 4 V = 8 V*
4 V + - E_{R3}	3 V - + E_{R3}	**1 V** E_{R3} + - *4 V - 3 V = 1 V*

- Applying these superimposed voltage figures to the circuit, the end result looks something like this:

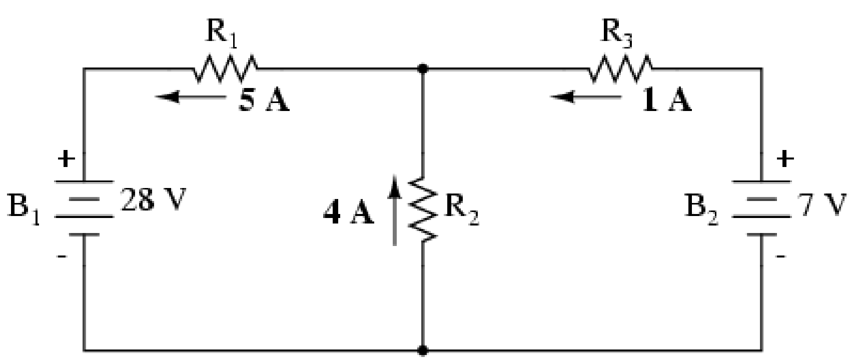


- Currents add up algebraically as well, and can either be superimposed as done with the resistor voltage drops, or simply calculated from the final voltage drops and respective resistances (I=E/R). Either way, the answers will be the same. Here we will show the superposition method applied to current:

****

With 28 V battery	*With 7 V battery*	*With both batteries*
← 6 A, I_{R1}	→ 1 A, I_{R1}	I_{R1} ← **5 A** *6 A - 1 A = 5 A*
I_{R2} ↑ 2 A	I_{R2} ↑ 2 A	I_{R2} ↑ **4 A** *2 A + 2 A = 4 A*
← 4 A, I_{R3}	→ 3 A, I_{R3}	I_{R3} ← **1 A** *4 A - 3 A = 1 A*

- Once again applying these superimposed figures to our circuit:

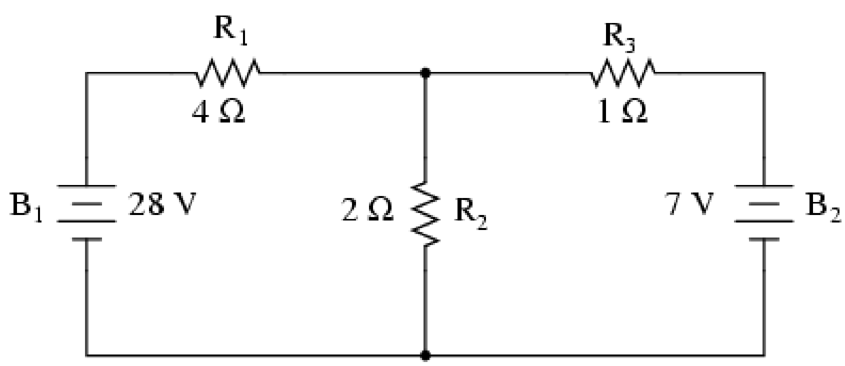


2. Millman's Theorem:

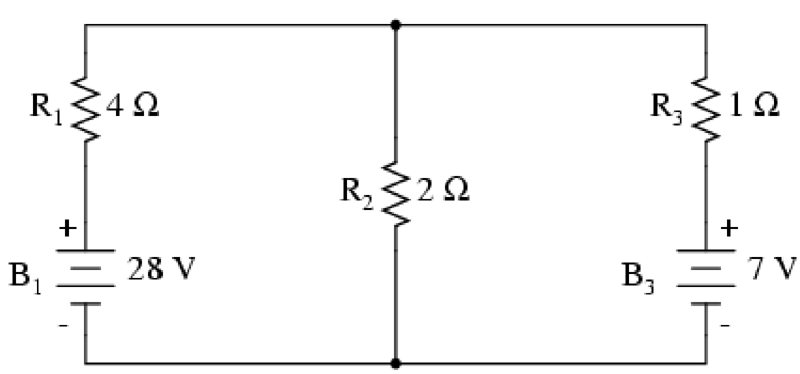
- Millman's theorem is a method for reducing a circuit by combining parallel voltage sources into a single voltage source.
- This theorem treats circuits as a set of parallel-connected branches, each branch with its own voltage source and series resistance. That is, the given circuit is re-drawn as a parallel network of branches, each branch containing a resistor or series battery/resistor combination.
- This theorem is used for finding the common voltage across each parallel branch.
- Millman's Theorem is applicable only to those circuits which can be re-drawn accordingly.

Explanation for Millman's Theorem:

- Consider the circuit given below. We apply Millman's theorem to it:



- The above circuit is re-drawn below for the sake of applying Millman's Theorem:



****

- By considering the supply voltage within each branch and the resistance within each branch, Millman's Theorem will tell us the common voltage across all branches. (N.B. The battery in the rightmost branch is labeled as "B_3" to clearly denote it as being in the third branch, even though there is no "B_2" in the circuit).
- The value of the common voltage across each branch is determined by the Millman's equation:

$$\text{Voltage across all branches} = \frac{\frac{E_{B1}}{R_1} + \frac{E_{B2}}{R_2} + \frac{E_{B3}}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

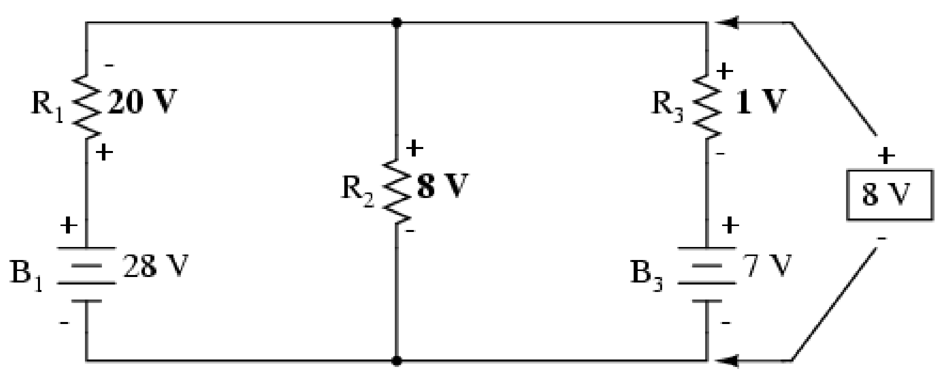
$$= \frac{I_{R1} + I_{R2} + I_{R3}}{G_1 + G_2 + G_3}$$

$$= \frac{\Sigma I}{\Sigma G}$$

- Substituting actual voltage and resistance figures from our example circuit for the variable terms of this equation, we get the following expression:

$$\frac{\frac{28\text{ V}}{4\,\Omega} + \frac{0\text{ V}}{2\,\Omega} + \frac{7\text{ V}}{1\,\Omega}}{\frac{1}{4\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{1\,\Omega}} = 8\text{ V}$$

- The final answer of 8 volts is the voltage seen across all parallel branches, like this:



****

- The polarity of all voltages in Millman's Theorem are referenced to the same point. In the example circuit above, the bottom wire of the parallel circuit is used as reference point, and so the voltages within each branch (28 for the R_1 branch, 0 for the R_2 branch, and 7 for the R_3 branch) were inserted into the equation as positive numbers.
- Likewise, when the answer came out to 8 volts (positive), this meant that the top wire of the circuit was positive with respect to the bottom wire (the original point of reference). If both batteries had been connected backwards (negative ends up and positive ends down), the voltage for branch 1 would have been entered into the equation as a -28 volts, the voltage for branch 3 as -7 volts, and the resulting answer of -8 volts would have told us that the top wire was negative with respect to the bottom wire (our initial point of reference).

- To solve for resistor voltage drops, the Millman voltage (across the parallel network) must be compared against the voltage source within each branch, using the principle of voltages adding in series to determine the magnitude and polarity of voltage across each resistor:

$$E_{R1} = 8\ \text{V} - 28\ \text{V} = -20\ \text{V} \quad \text{(negative on top)}$$

$$E_{R2} = 8\ \text{V} - 0\ \text{V} = 8\ \text{V} \quad \text{(positive on top)}$$

$$E_{R3} = 8\ \text{V} - 7\ \text{V} = 1\ \text{V} \quad \text{(positive on top)}$$

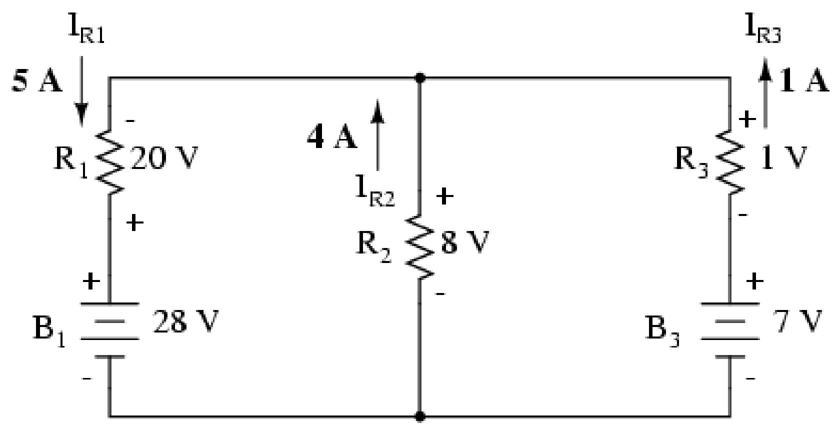
- To solve for branch currents, each resistor voltage drop can be divided by its respective resistance (I=E/R):

$$I_{R1} = \frac{20\ \text{V}}{4\ \Omega} = 5\ \text{A}$$

$$I_{R2} = \frac{8\ \text{V}}{2\ \Omega} = 4\ \text{A}$$

$$I_{R3} = \frac{1\ \text{V}}{1\ \Omega} = 1\ \text{A}$$

- The direction of current through each resistor is determined by the polarity across each resistor, *not* by the polarity across each battery, as current can be forced backwards through a battery, as is the case with B_3 in the example circuit. This is important to keep in mind, since Millman's Theorem doesn't provide as direct an indication of "wrong" current direction as does the Branch Current or Mesh Current methods. You must pay close attention to the polarities of resistor voltage drops as given by Kirchhoff's Voltage Law, determining direction of currents from that.



- Millman's Theorem is very convenient for determining the voltage across a set of parallel branches, where there are enough voltage sources present to preclude solution via regular series-parallel reduction method.
- It also is easy in the sense that it doesn't require the use of simultaneous equations.
- However, it is limited in that it only applied to circuits which can be re-drawn to fit this form. It cannot be used, for example, to solve an unbalanced bridge circuit. And, even in cases where Millman's Theorem can be applied, the solution of individual resistor voltage drops can be a bit daunting to some, the Millman's Theorem equation only providing a single figure for branch voltage.