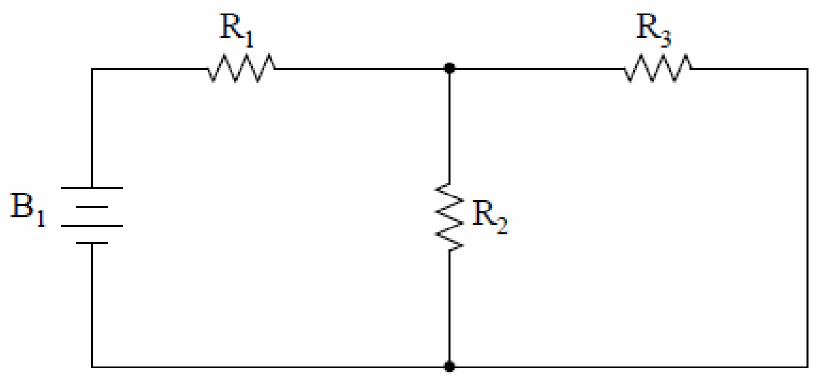
Methods of Circuit Analysis

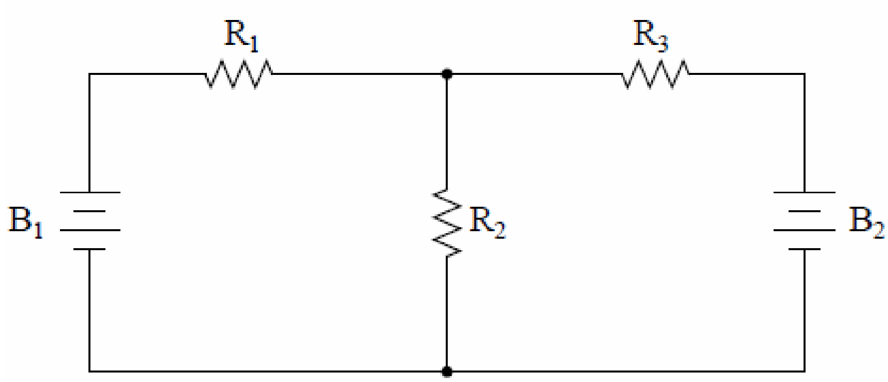
(DC Network Theorem. continue…)

- All the circuits presented so far have generally had a single voltage source as the means of providing power and could be easily analyzed using techniques such as Ohm's and Kirchhoff's laws. For example, consider the simple circuit shown below:



- To analyze the above circuit, one would first find the equivalent resistance of R_2 and R_3 which are in parallel, then add R_1 in series to arrive at a total resistance. Then, taking the voltage of battery B_1 with that total circuit resistance, the total current could be calculated through the use of Ohm's Law ($I=E/R$), then that current figure is used to calculate voltage drops in the circuit. All in all, solution to this circuit is fairly simple.

- But, circuits more than one voltage source can not be easily analyzed using these techniques. For example, consider the above circuit again with an extra battery shown below:



- In the above circuit, resistors R_2 and R_3 are no longer in parallel with each other, because another battery B_2 has been inserted into R_3's branch of the circuit.
- Upon closer inspection, it appears, there are no two resistors in this circuit directly in series or parallel with each other. This is the crux of our problem: in series-parallel analysis, we started off by identifying sets of resistors that were directly in series or parallel with each other, reducing them to single equivalent resistances. If there are no resistors in a simple series or parallel configuration with each other, then what can we do?
- The methods used in determining the operation of such complex networks will include **branch-current analysis**, **mesh (or loop) analysis**, and **nodal analysis**.
- These circuit analysis methods allow us to find two or more unknown currents or voltages by solving simultaneous equations.
- Although any of the above methods may be used, we will find that certain circuits are more easily analyzed using one particular approach. The advantages of each method will be discussed in the appropriate section.

Branch-Current Analysis Method:

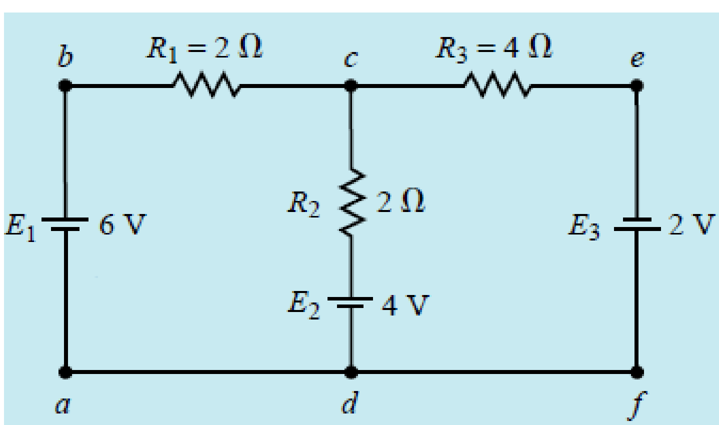
- The first and most straightforward network analysis technique is called the Branch Current Method that allows us to directly calculate the current in each branch of a circuit having more than one source.
- In this method, we assume directions of currents in a network, then write equations describing their relationships to each other through Kirchhoff 's and Ohm's Laws.
- Once we have one equation for every unknown current, we can solve the simultaneous equations and determine all currents, and therefore all voltage drops in the network.

The steps used in solving a circuit using branch-current analysis:

1. The first step is to choose a node (junction of wires) in the circuit to use as a point of reference for unknown currents. Label the currents (as I_1, I_2 etc) and arbitrarily assign their directions entering in or exiting from this reference node. If a particular branch has a current source, then this step is not necessary since you already know the magnitude and direction of the current in this branch.
2. Using the assumed directions of currents, label the polarities of the voltage drops across all resistors in the circuit.
3. Apply Kirchhoff's voltage law around each of the closed loops. Write just enough equations to include all branches in the loop equations. If a branch has only a current source and no series resistance, it is not necessary to include it in the KVL equations.
4. Apply Kirchhoff's current law at enough nodes to ensure that all branch currents have been included. In the event that a branch has only a current source, it will need to be included in this step.
5. Solve the linear equations resulting from steps 3 and 4 for the branch current values.

Example-1:

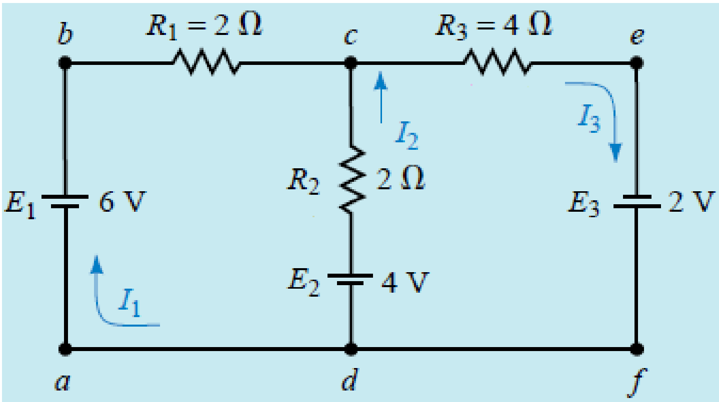
Find the current in each branch of the circuit shown below.



Solution:

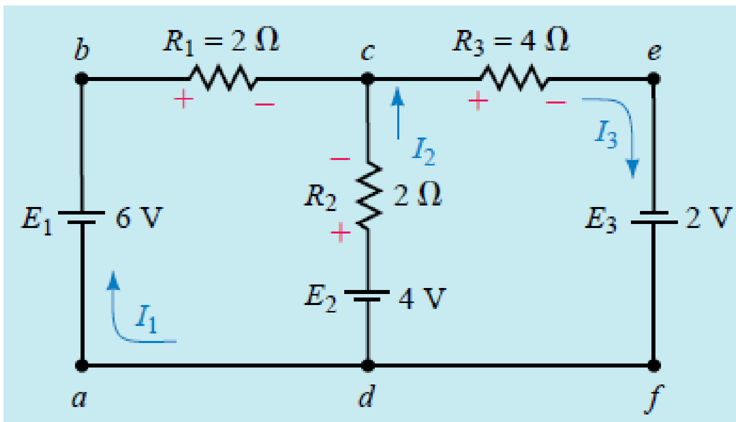
Step-1:

Choose a node in the circuit to use as a point of reference for unknown currents. Here node ***c*** is chosen. Label the currents as I_1, I_2 and I_3. Arbitrarily assign their directions entering in or exiting from this reference node.



Step-2:

Using the assumed current direction, label the polarities of the voltage drops across all resistors in the circuit.



Step-3:

Apply Kirchhoff's voltage law around each of the closed loops. Write just enough equations to include all branches in the loop equations.

Loop *abcda*:

$$6V-(2\Omega)I_1+(2\Omega)I_2-4V=0V$$
$$\Rightarrow \quad -(2\Omega)I_1+(2\Omega)I_2+2V=0V$$
$$\Rightarrow \quad -2I_1+2I_2=-2$$

$$\Rightarrow \quad 2I_1+(-2)I_2+0I_3=2 \qquad ....(1)$$

Loop *cefdc*:

$$4V-(2\Omega)I_2-(4\Omega)I_3+2V=0V$$
$$\Rightarrow \quad -(2\Omega)I_2-(4\Omega)I_3+6V=0V$$
$$\Rightarrow \quad -2I_2-4I_3=-6$$

$$\Rightarrow \quad 0I_1+(-2)I_2+(-4)I_3=(-6) \qquad ....(2)$$

Step-4:

Apply Kirchhoff's current law at enough nodes to ensure that all branch currents have been included.

By applying KCL at node ***c***, all branch currents in the network are included:

$$I_3=I_1+I_2$$
$$1I_1+1I_2+(-1)I_3=0 \qquad ....(3)$$

Step-5:

Solve the resulting simultaneous linear equations.

- To simplify the solution of the simultaneous linear equations we write them as follows:

$$2I_1+(-2)I_2+0I_3=2 \qquad ....(1)$$

$$0I_1+(-2)I_2+(-4)I_3=(-6) \qquad ....(2)$$

$$1I_1+1I_2+(-1)I_3=0 \qquad ....(3)$$

- We can solve the above equations by the use of determinants and Cramer's rule:

The above equations can be put in the matrix form as under:

$$\begin{bmatrix} 2 & -2 & 0 \\ 0 & -2 & -4 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 0 \end{bmatrix}$$

The value of the common determinant is given by:

$$\Delta = \begin{bmatrix} 2 & -2 & 0 \\ 0 & -2 & -4 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= 2\{[(-2)\times(-1)] - [(-4)\times(1)]\} - (-2)\{[(0)\times(-1)] - [(-4)\times(1)]\} + (0)\{[(0)\times(1)] - [(-2)\times(1)]\}$$

$$= 2\{[2] - [-4]\} - (-2)\{[0] - [-4]\} + (0)\{[0] - [-2]\}$$

$$= 2\{2+4\} - (-2)\{0+4\} + (0)\{0+2\}$$

$$= 2\times 6 - (-2\times 4) + 0\times 2$$

$$= 12+8+0$$

$$= 20$$

The determinant for I_1 can be found by replacing coefficients of I_1 in the original matrix by the three constants:

$$\Delta_1 = \begin{vmatrix} 2 & -2 & 0 \\ -6 & -2 & -4 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 2\{[(-2)\times(-1)] - [(-4)\times(1)]\} - (-2)\{[(-6)\times(-1)] - [(-4)\times(0)]\} + (0)\{[(-6)\times(1)] - [(-2)\times(0)]\}$$
$$= 24$$

Similarly, the determinant for I_2 can be found by replacing coefficients of I_2 in the original matrix by the three constants:

$$\Delta_2 = \begin{vmatrix} 2 & 2 & 0 \\ 0 & -6 & -4 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= 2\{[(-6)\times(-1)] - [(-4)\times(0)]\} - (2)\{[(0)\times(-1)] - [(-4)\times(1)]\} + (0)\{[(0)\times(0)] - [(-6)\times(1)]\}$$
$$= 4$$

And the same way, the determinant for I_3 can be found by replacing coefficients of I_3 in the original matrix by the three constants:

$$\Delta_3 = \begin{vmatrix} 2 & -2 & 2 \\ 0 & -2 & -6 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2\{[(-2)\times(0)] - [(-6)\times(1)]\} - (2)\{[(0)\times(0)] - [(-6)\times(1)]\} + (2)\{[(0)\times(1)] - [(-2)\times(1)]\}$$

$$= 4$$

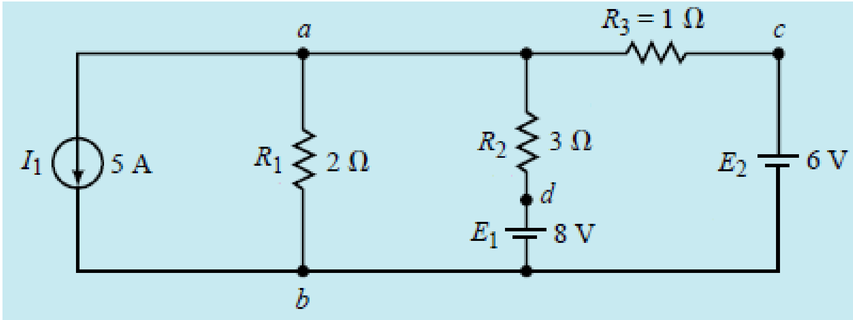
Therefore, according to Cramer's rule:

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{24}{20} = 1.2\,A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{4}{20} = 0.20\,A$$

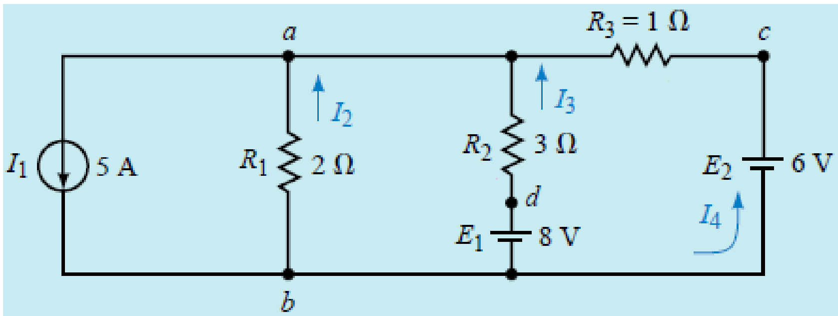
$$I_3 = \frac{\Delta_3}{\Delta} = \frac{4}{20} = 0.20\,A$$

Example-2:
Find the currents in each branch of the circuit shown in figure below. Solve for the voltage *Vab*.



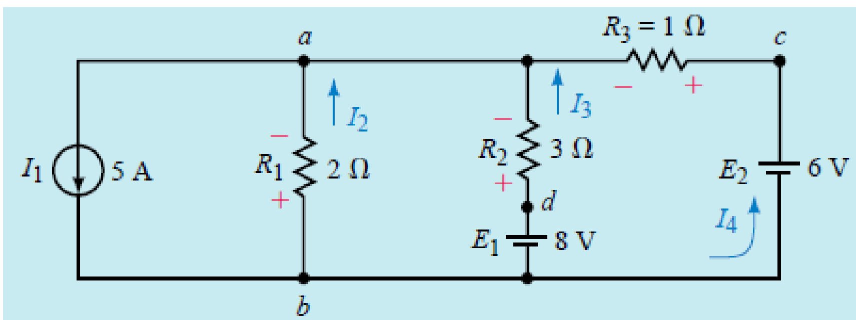
Step-1:

Arbitrarily assign current directions to each branch in the network. Since there is a current source at I_1 branch, hence this step is not necessary for this branch as you already know the magnitude and direction of the current in this branch.



Step-2:

Using the assumed current direction, label the polarities of the voltage drops across all resistors in the circuit.



Step-3:

Apply Kirchhoff's voltage law around each of the closed loops. Write just enough equations to include all branches in the loop equations.

$$-(2\ \Omega)(I_2) + (3\ \Omega)(I_3) - 8\ \text{V} = 0\ \text{V}$$
$$-(2\ \Omega)(I_2) + (1\ \Omega)(I_4) - 6\ \text{V} = 0\ \text{V}$$

Loop *badb*:

$$-(2\Omega)I_2 + (3\Omega)I_3 - 8V = 0V$$
$$\Rightarrow \quad (-2)I_2 + 3I_3 + 0I_4 = 8 \qquad \text{.............(1)}$$

Loop *bacb*:

$$-(2\Omega)I_2 + (1\Omega)I_4 - 6V = 0V$$
$$\Rightarrow \quad (-2)I_2 + 0I_3 + 1I_4 = 6 \qquad \text{.............(2)}$$

Step-4:

Apply Kirchhoff's current law at enough nodes to ensure that all branch currents have been included.

By applying KCL at node ***a***, all branch currents in the network are included:

$$I_2 + I_3 + I_4 = 5$$
$$\Rightarrow \quad 1I_2 + 1I_3 + 1I_4 = 5 \quad \text{.............(3)}$$

Step-5:

Solve the resulting simultaneous linear equations.

- To simplify the solution of the simultaneous linear equations we write them as follows:

$$(-2)I_2 + 3I_3 + 0I_4 = 8 \qquad \text{.............(1)}$$

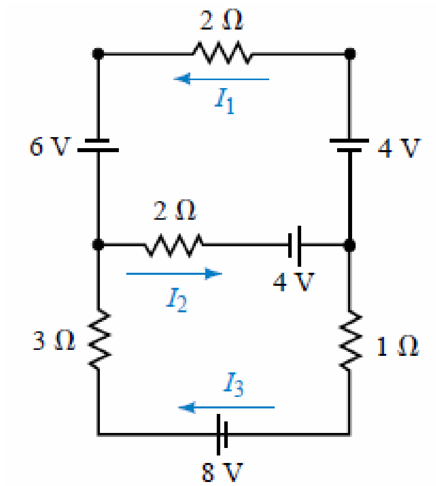
$$(-2)I_2 + 0I_3 + 1I_4 = 6 \qquad \text{.............(2)}$$

$$1I_2 + 1I_3 + 1I_4 = 5 \qquad \text{.............(3)}$$

- We can solve the above equations by the use of determinants and Cramer's rule as explained earlier.

Exercise:

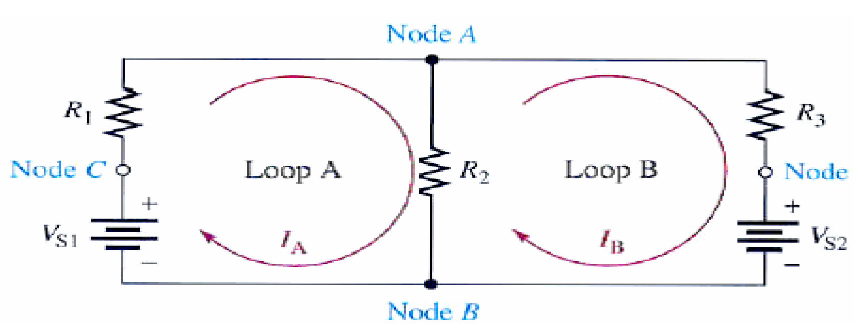
Use branch-current analysis to solve for the indicated currents in the circuit of figure given below.



Answers: $I_1 = 3.00$ A, $I_2 = 4.00$ A, $I_3 = 1.00$ A

Mesh-Current (Loop-Current) Analysis

- In branch-current analysis, we used Kirchhoff's voltage and current laws to solve for the current in each branch of a given network.
- Mesh-current method is quite similar to the Branch Current method in that it uses simultaneous equations, Kirchhoff 's Voltage Law, and Ohm's Law to determine unknown currents in a network.
- While the methods used were relatively simple, branch-current analysis is awkward to use because it generally involves solving several simultaneous linear equations. The number of equations may be prohibitively large even for a relatively simple circuit.
- A better and extensively used approach in analyzing linear bilateral networks is called **mesh** (or **loop**) **analysis** which is used to solve currents flowing around complex circuit.
- While the technique is similar to branch-current analysis, the number of simultaneous linear equations tends to be less.
- The principal difference between mesh analysis and branch-current analysis is that we simply need to apply Kirchhoff's voltage law around closed loops without the need for applying Kirchhoff's current law for mesh analysis. In branch-current method, we work with branch currents of a network, but in mesh analysis method, we will work with loop currents instead of branch currents.
- Unlike branch currents, loop currents are mathematical quantities, rather than actual physical currents, that are used to make circuit analysis somewhat easier than with the branch-current method.
- Note that a loop is a complete current path within a circuit. Whereas, a branch is a path that connects two nodes. (A node is a point where two or more components are connected). Figure below illustrates loop, branch and nodes.

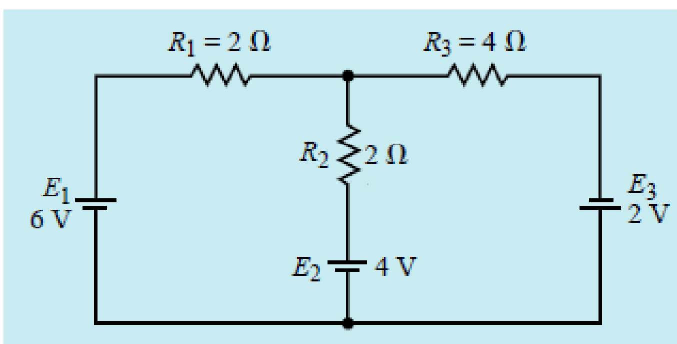


The steps used in solving a circuit using mesh analysis are as follows:

1. The first step in the Mesh Current method is to identify “loops” within the circuit encompassing all components.
2. Arbitrarily assign a current in the clockwise (CW) direction around each interior closed loop in the network. Although the assigned current may be in any direction, a clockwise direction is used to make later work simpler.
3. Based on the assigned loop current directions, indicate the voltage drop polarities across all resistors in each loop in the circuit. For a resistor which is common to two loops, the polarities of the voltage drop due to each loop current should be indicated on the appropriate side of the component.
4. Applying Kirchhoff’s voltage law, write the loop equations for each loop in the network. Do not forget that resistors which are common to two loops will have two voltage drops, one due to each loop.
5. Solve the resultant simultaneous linear equations using substitution or determinant method.
6. Branch currents are determined by algebraically combining the loop currents which are common to the branch.

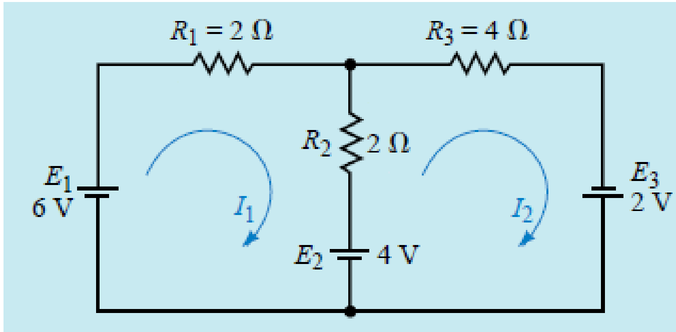
Example-1:

Using mesh analysis, find the current in each branch for the circuit shown in the figure below.



Step-1:

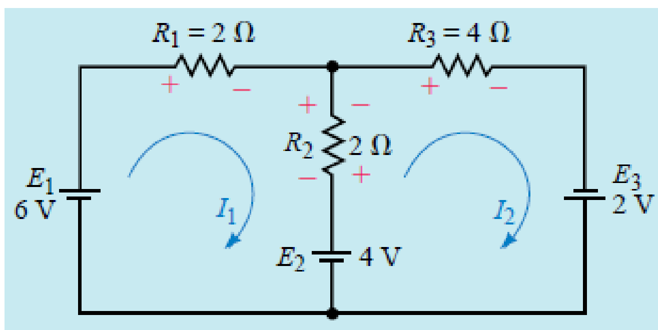
Identify “loops” within the circuit encompassing all components and then arbitrarily assign a current in the clockwise (CW) direction around each interior closed loop in the network.



Loop currents are designated clockwise as I_1 and I_2 in the figure above.

Step-2:

Based on the assigned loop-current directions, indicate the voltage drop polarities across all resistors in each loop in the circuit. For a resistor which is common to two loops, the polarities of the voltage drop due to each loop current should be indicated on the appropriate side of the component.



Notice that the resistor R_2 has two different voltage polarities due to the different loop currents.

Step-3:

Applying Kirchhoff's voltage law, write the loop equations for each loop in the network. Do not forget that resistors which are common to two loops will have two voltage drops, one due to each loop.

The loop equations are as follows:

$$Loop-1: \quad E_1 - R_1 I_1 - R_2(I_1 - I_2) - E_2 = 0$$
$$\Rightarrow \quad 6V - (2\Omega)I_1 - (2\Omega)I_1 + (2\Omega)I_2 - 4V = 0$$
$$\Rightarrow \quad 2V - (4\Omega)I_1 + (2\Omega)I_2 = 0$$
$$\Rightarrow \quad 4I_1 - 2I_2 = 2$$
$$\Rightarrow \quad 4I_1 + (-2)I_2 = 2 \quad ....(1)$$

$$Loop-2: \quad E_2 - R_2(I_2 - I_1) - R_3 I_2 + E_3 = 0$$
$$\Rightarrow \quad 4V - (2\Omega)I_2 + (2\Omega)I_1 - (4\Omega)I_2 + 2V = 0$$
$$\Rightarrow \quad 6V + (2\Omega)I_1 - (6\Omega)I_2 = 0$$
$$\Rightarrow \quad 2I_1 - 6I_2 = -6$$
$$\Rightarrow \quad 2I_1 + (-6)I_2 = -6 \quad ....(2)$$

Step-4:

Solve the resultant simultaneous linear equations.

- To simplify the solution of the simultaneous linear equations we write them as follows:

$$4I_1 + (-2)I_2 = 2 \quad ....(1)$$

$$2I_1 + (-6)I_2 = -6 \quad ....(2)$$

- We can solve the above equations by the use of determinants and Cramer's rule:

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 2 & -2 \\ -6 & -6 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 2 & -6 \end{vmatrix}} = \frac{[(2)\times(-6)]-[(-2)\times(-6)]}{[(4)\times(-6)]-[(-2)\times(2)]} = \frac{-12-12}{-24+4} = \frac{-24}{-20} = \frac{12}{10} = 1.2\,A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 4 & 2 \\ 2 & -6 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 2 & -6 \end{vmatrix}} = \frac{[(4)\times(-6)]-[(2)\times(2)]}{[(4)\times(-6)]-[(-2)\times(2)]} = \frac{-24-4}{-24+4} = \frac{-28}{-20} = \frac{14}{10} = 1.4\,A$$

- From the above results, we see that the currents through resistors R_1 and R_3 are I_1 and I_2 respectively.
- The branch current for R_2 is found by combining the loop currents through this resistor:

$$I_{R2} = I_2 - I_1 = 1.4 - 1.2 = 0.2\,A$$

- Since, the direction of I_1 is downward and I_2 is upward; and I_2 is greater than I_1, hence, the direction of I_{R2} is upward.

Note:

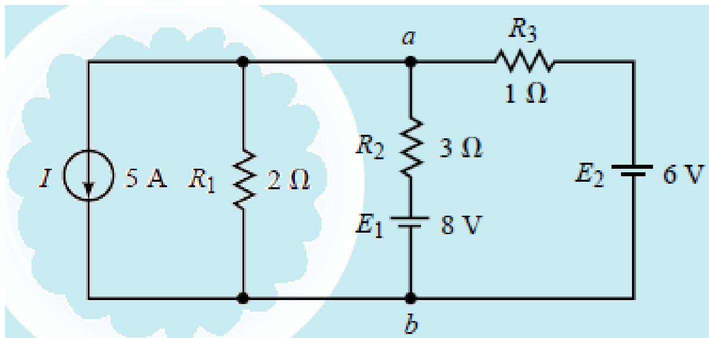
- The results obtained by using mesh analysis are exactly the same as those obtained by branch-current analysis.
- Whereas branch-current analysis required three equations, this approach requires the solution of only two simultaneous linear equations.
- Mesh analysis also requires that only Kirchhoff's voltage law be applied and clearly illustrates why mesh analysis is preferred.

Solving Circuit Containing Current Source:

- If the circuit being analyzed contains current sources, the procedure is a bit more complicated. The circuit may be simplified by converting the current source(s) to voltage sources and then solving the resulting network using the procedure shown in the previous example.
- If you do not wish to convert the circuit, in that case the current source will provide one of the loop currents.

Example-2:

Determine the current through the 8-V battery for the circuit shown in the figure below.



Solution:

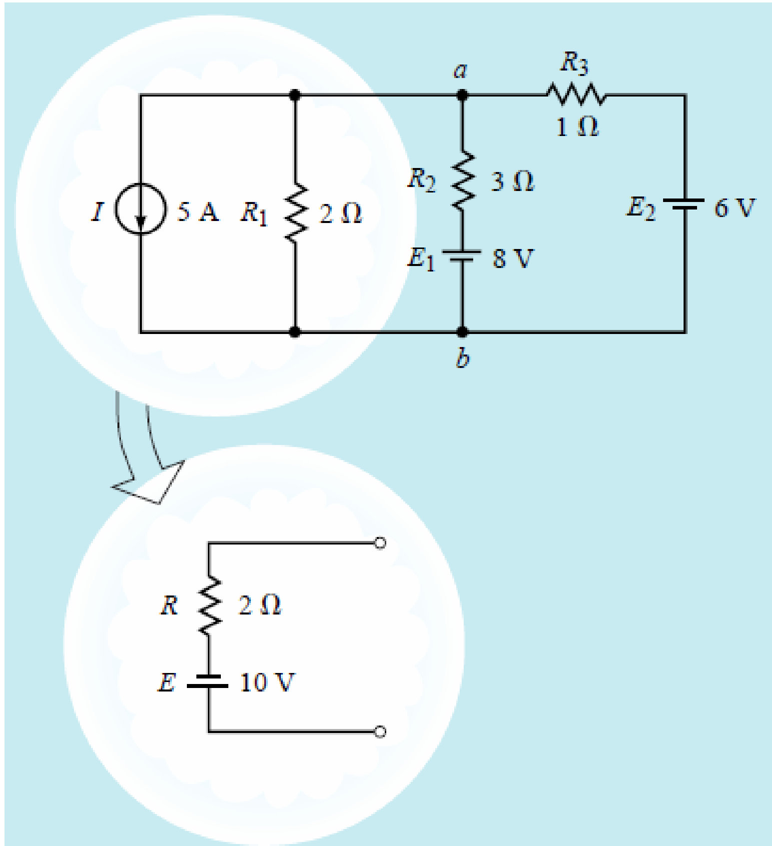
We can solve the above circuit in either two ways:

- Converting current source into voltage source
- Without not converting

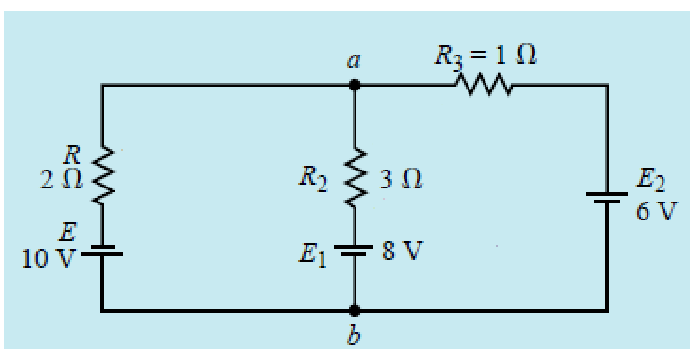
The steps of solution for the first method are discussed here.

Step-1:

This circuit contains a current source. So, convert the current source into a voltage source.

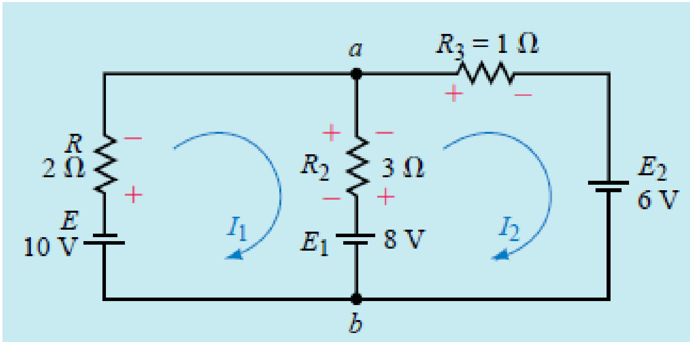


After converting, the circuit becomes:



Step-2:

Arbitrarily assign a clockwise current to each interior closed loop in the network.

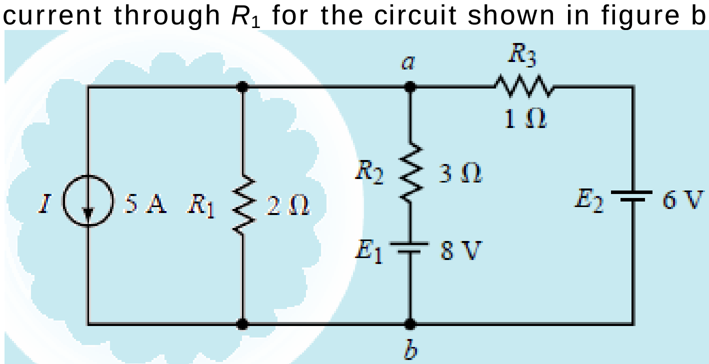


Now solve the circuit using Mesh analysis explained earlier.

Alternative Solution:

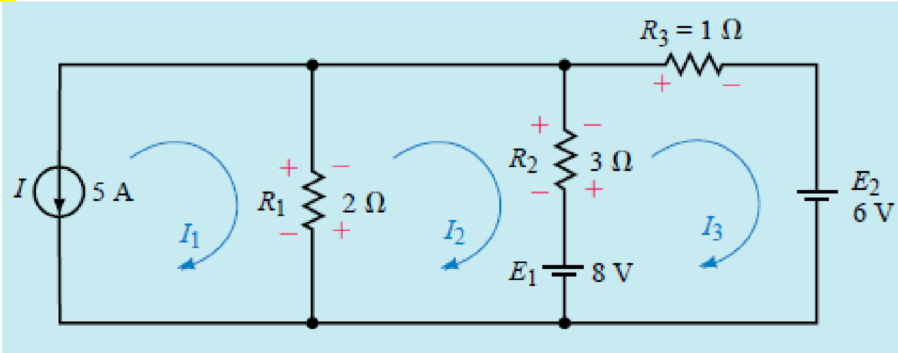
Solution without converting current source into voltage source:

Determine the current through R_1 for the circuit shown in figure below.



The circuit of the figure above may also be analyzed without converting the current source to a voltage source. Although the approach is generally not used, the following example illustrates the technique.

Solution:



By inspection, we see that the loop current I_1=-5 A

The mesh equations for the other two loops are as follows:

Loop 2: $-(2\ \Omega)I_2 + (2\ \Omega)I_1 - (3\ \Omega)I_2 + (3\ \Omega)I_3 - 8\text{ V} = 0$
Loop 3: $8\text{ V} - (3\ \Omega)I_3 + (3\ \Omega)I_2 - (1\ \Omega)I_3 - 6\text{ V} = 0$

$$Loop-2: \quad -(2\Omega)I_2 + (2\Omega)I_1 - (3\Omega)I_3 - 8V = 0$$
$$\Rightarrow \quad (2\Omega)I_1 - (5\Omega)I_2 + (3\Omega)I_3 = 8$$
$$\Rightarrow \quad 2I_1 + (-5)I_2 + 3I_3 = 8 \qquad ....(1)$$

$$Loop-3: \quad 8V - (3\Omega)I_3 + (3\Omega)I_2 - (1\Omega)I_3 - 6V = 0$$
$$\Rightarrow \quad (3\Omega)I_2 - (4\Omega)I_3 = -2$$
$$\Rightarrow \quad 3I_2 + (-4)I_3 = -2 \qquad ....(1)$$

Now the three linear equations are:

$$I_1 = -5 \qquad ....(1)$$

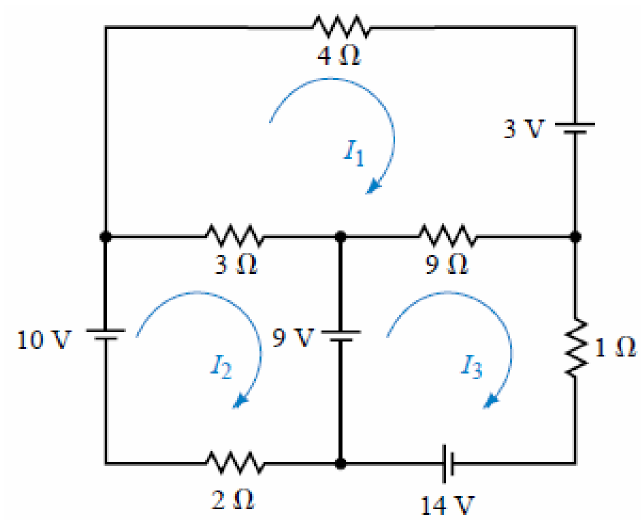
$$2I_1 + (-5)I_2 + 3I_3 = 8 \qquad ....(2)$$

$$3I_2 + (-4)I_3 = -2 \qquad ....(3)$$

Substituting the value of I_1 from equation-1 to equation-2, there are only two linear equations of two unknown values I_2 and I_3. So, it can be easily solved.

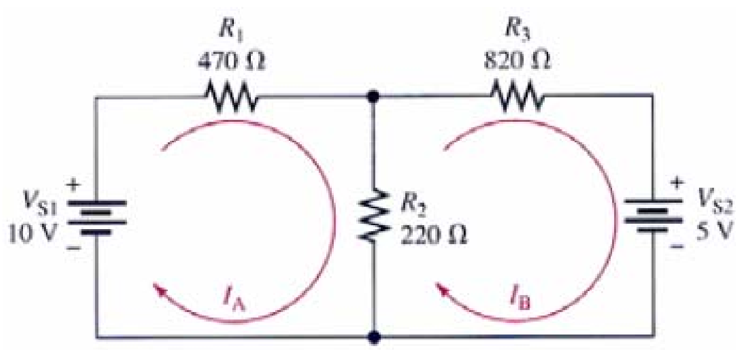
Exercise:

1. Use mesh analysis to find the loop currents in the circuit shown in the figure below.



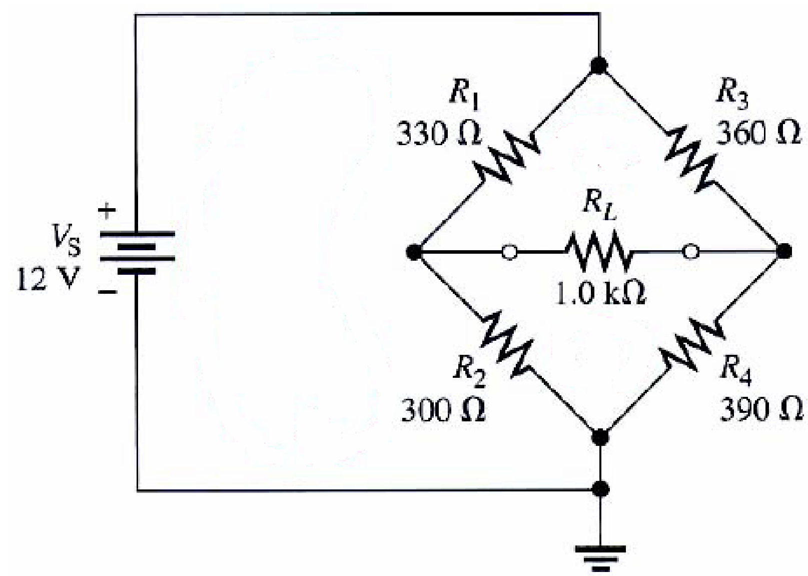
Answers: $I_1 = 3.00$ A, $I_2 = 2.00$ A, $I_3 = 5.00$ A

2. Using loop-current (mesh analysis) method, find the branch currents of the network shown below:



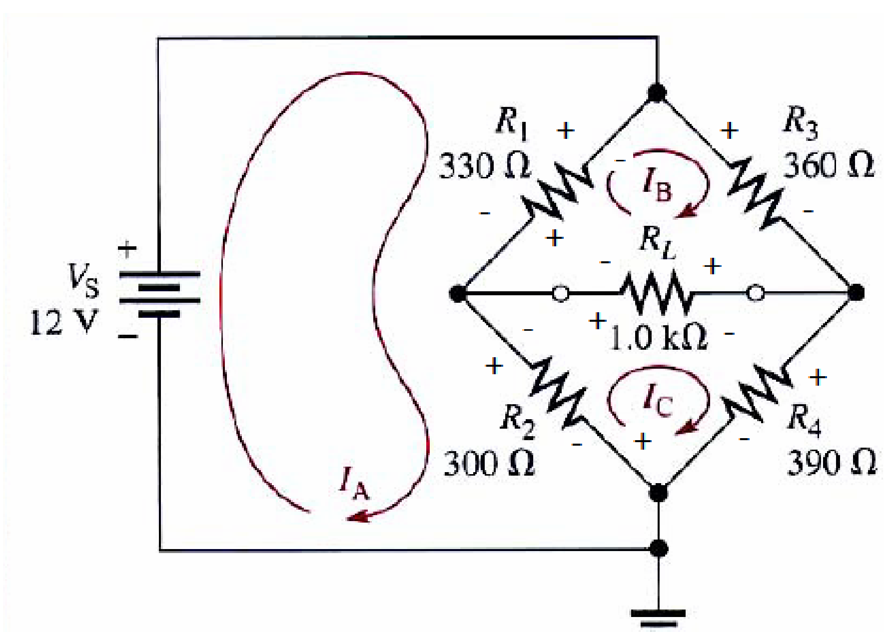
Answer $I_A = I_1 = 13.9\,mA \quad I_B = I_3 = -1.87\,mA \quad I_2 = I_A - I_B = 13.9 - (-1.87) = 15.8\,mA$

3. Using loop-current (mesh analysis) method, find the branch currents of the Wheatstone bridge network shown below:



Hints:

- Assign three clockwise loop currents I_A, I_B and I_C.
- Indicate the voltage drop polarities across all resistors in each loop in the circuit based on the assigned loop-current directions.
- Notice that the resistor R_1, R_2 and R_L has two different voltage polarities due to the different loop currents.
- Applying Kirchhoff's voltage law, write the loop equations for each loop in the network.



- The loop equations are as follows:

$$KVL \ \ at\ loop\ A: \qquad V_S - R_1(I_A - I_B) - R_2(I_A - I_C) = 0$$

$$KVL \ \ at\ loop\ B: \qquad -R_3 I_B - R_L(I_B - I_C) - R_1((I_B - I_A) = 0$$

$$KVL \ \ at\ loop\ C: \qquad -R_4 I_c - R_2(I_C - I_A) - R_L(I_C - I_B) = 0$$

- Now rearrange the equations and solve them using determinant and Cramer's rule.

Nodal Voltage Analysis/ Node Voltage Method:

- In Mesh (loop) analysis, Kirchhoff's voltage law is applied to arrive at loop currents in a network.
- As its name implies, Nodal analysis uses Kirchhoff's current law to determine the potential difference (voltage) at any node with respect to some arbitrary reference point in a network.
- Once the potentials of all nodes are known, it is a simple matter to determine other quantities such as current and power within the network.
- Using node analysis, we can calculate the voltages around the loops that reduce the amount of mathematics required using just Kirchoff's laws.

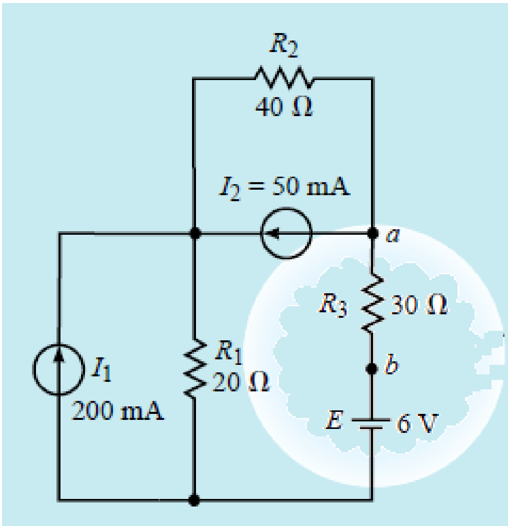
The steps used in solving a circuit using nodal analysis are as follows:

1. Determine the number of nodes within the given circuit.
2. Arbitrarily select one node as a reference node and indicate this node as **ground**. The reference node is usually located at the bottom of the circuit, although it may be located anywhere.
3. Convert each voltage source in the network to its equivalent current source. This step, although not absolutely necessary, makes further calculations easier to understand.
4. Arbitrarily assign voltages (V_1, V_2, . . . , V_n) to the remaining nodes in the circuit. (Remember that all voltages to the remaining nodes will be relative to the chosen reference node).
5. Arbitrarily assign a current direction to each branch in which there is no current source.

6. Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.
7. With the exception of the reference node (ground), apply Kirchhoff's current law at each of the nodes. If a circuit has a total of *n+1* nodes (including the reference node), there will be *n* simultaneous linear equations.
8. Rewrite each of the arbitrarily assigned currents in terms of the potential difference across a known resistance.
9. Solve the resulting simultaneous linear equations for the voltages (V_1, V_2, . . . , V_n).

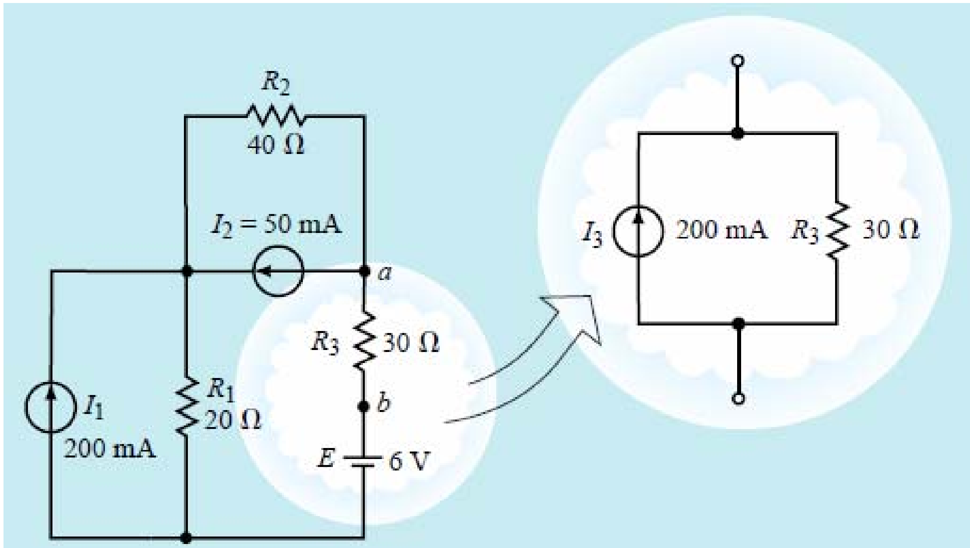
Example-1:

Use nodal analysis to solve for the voltage V_{ab} from the circuit given below.

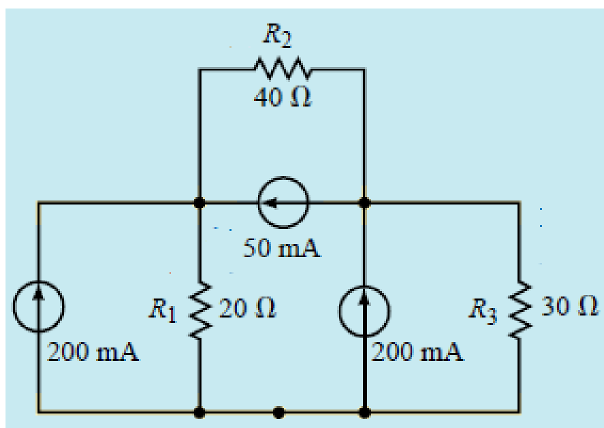


Step-1:

The given circuit has a voltage source. So, convert it to its equivalent current source.

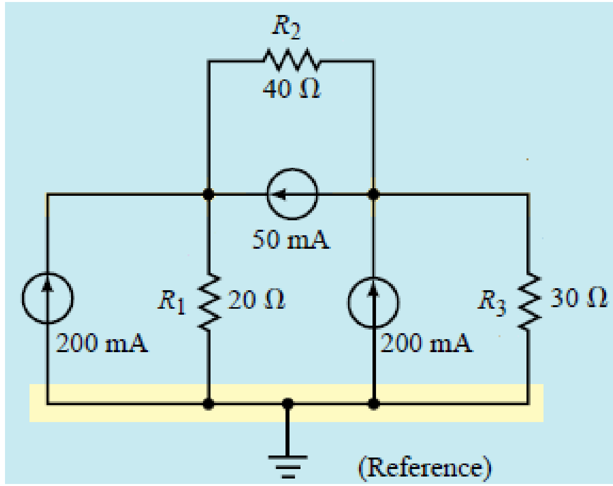


After converting the voltage source to its equivalent current source, the equivalent circuit is shown in figure below.



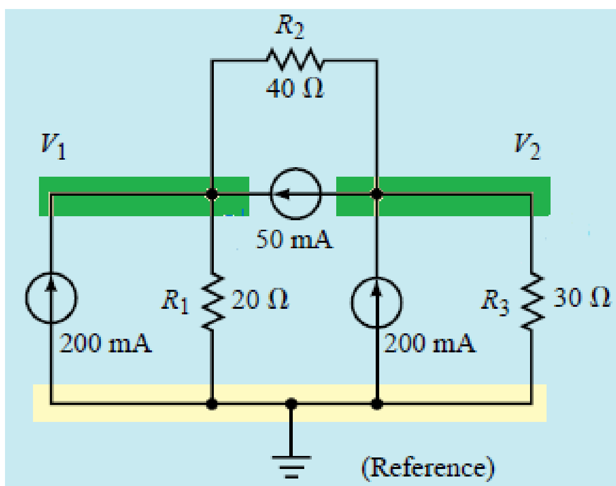
Step-2:

Arbitrarily select a convenient reference node within the circuit and indicate this node as **ground**. The circuit with the reference node (shaded as white rectangle) is given below.



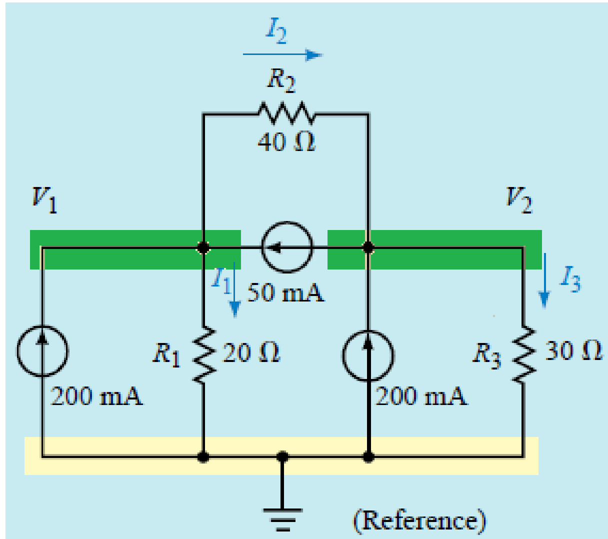
Step-3:

After assigning a reference node, arbitrarily assign voltages (V_1, V_2, . . . , V_n) to the remaining nodes in the circuit. These voltages will all be with respect to the chosen reference. (Note that the remaining nodes are shaded as green rectangles).



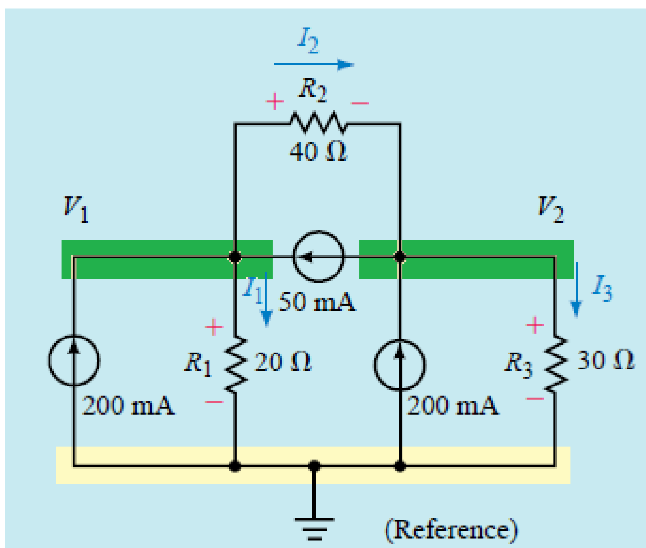
Step-4:

Arbitrarily assign a current direction to each branch in which there is no current source.



Step-5:

Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.



Step-6:

With the exception of the reference node (ground), apply Kirchhoff's current law at each of the other nodes. As there are 2 nodes (labeled as V_1 and V_2) except the reference node, we now apply Kirchhoff's current law at these two nodes.

$$KCL\ at\ Node\ V_1 \qquad \sum I_{Entering} = \sum I_{Leaving}$$

$$200\,mA + 50\,mA = I_1 + I_2$$

$$\Rightarrow \qquad I_1 + I_2 = 250\,mA \qquad ....(1)$$

$$KCL\ at\ Node\ V_2 \qquad \sum I_{Entering} = \sum I_{Leaving}$$

$$200\,mA + I_2 = 50\,mA + I_3$$

$$\Rightarrow \qquad I_2 - I_3 = -150\,mA + \qquad ....(2)$$

Step-7:

Rewrite each of the arbitrarily assigned currents in terms of the potential difference across a known resistance. Here, the currents are rewritten in terms of the voltages across the resistors as follows:

$$I_1 = \frac{V_1}{20\ \Omega}$$

$$I_2 = \frac{V_1 - V_2}{40\ \Omega}$$

$$I_3 = \frac{V_2}{30\ \Omega}$$

By substituting the values of I_1, I_2 and I_3 to equation (1) and (2), the nodal equations become:

$$I_1 + I_2 = 250\,mA \qquad ....(1)$$

$$\Rightarrow \qquad \frac{V_1}{20} + \frac{V_1 - V_2}{40} = 250$$

$$\Rightarrow \qquad \frac{2V_1 + V_1 - V_2}{40} = 250$$

$$\Rightarrow \qquad 3V_1 - V_2 = 10000 \qquad ....(3)$$

$$I_2 - I_3 = -150\,mA + \qquad ....(2)$$

$$\Rightarrow \quad \frac{V_1 - V_2}{40} - \frac{V_2}{30} = -150$$

$$\Rightarrow \quad \frac{30(V_1 - V_2) - 40V_2}{1200} = -150$$

$$\Rightarrow \quad \frac{30V_1 - 30V_2 - 40V_2}{1200} = -150$$

$$\Rightarrow \quad 30V_1 - 70V_2 = -150 \times 1200$$

$$\Rightarrow \quad 3V_1 - 7V_2 = -18000 \qquad ....(5)$$

- To simplify the solution of the simultaneous linear nodal equations (3) and (5), we write them as follows:

$$3V_1 + (-1)V_2 = 10000 \qquad ....(3)$$

$$3V_1 + (-7)V_2 = -18000 \qquad ....(5)$$

- We can solve the above equations by the use of determinants and Cramer's rule or as just substitution method:

$$3V_1 - V_2 = 10000 \qquad ....(3)$$

$$3V_1 - 7V_2 = -18000 \qquad ....(5)$$

$$6V_2 = 28000$$

$$\Rightarrow \quad V_2 = \frac{28000}{6} = 4666.66\,mV = 4.67V$$

Substituting the value of V_2 in equation (3), Value of V_1 will be found:

$$3V_1 - V_2 = 10000 \qquad ....(3)$$

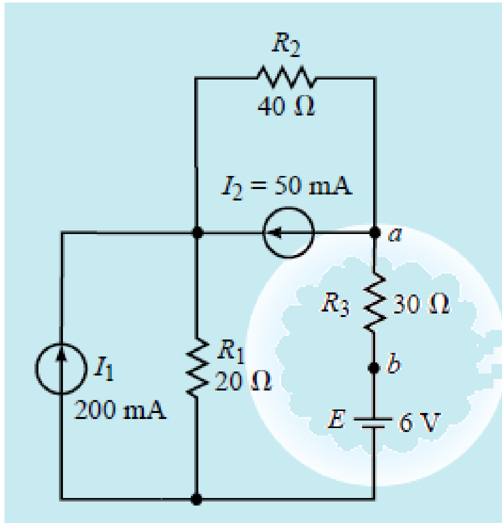
$$3V_1 = 10000 + V_2$$

$$= 10000 + 4666.66$$

$$= 14666.66$$

$$V_1 = \frac{14666.66}{3} = 4888.88\,mV = 4.88V$$

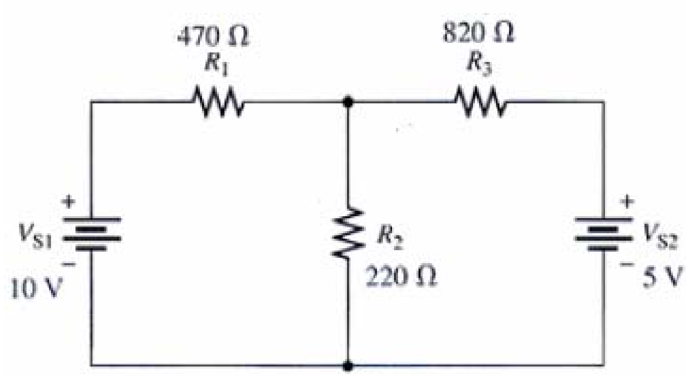
If we go back to the given original circuit (shown below again), we see that the voltage V_2 is the same as the voltage V_a.



$$V_a = V_2 = 4.67V = 6.0V + V_{ab}$$
$$\Rightarrow V_{ab} = 4.67 - 6.0 = -1.33V$$

Example-2:

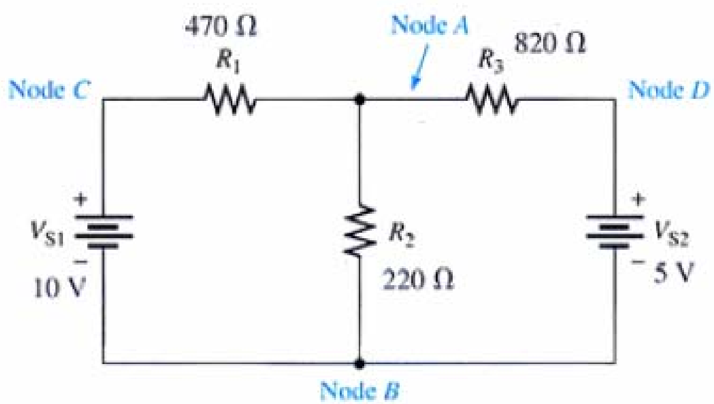
Determine the nodal voltages for the circuit shown in the figure below.



Solution:

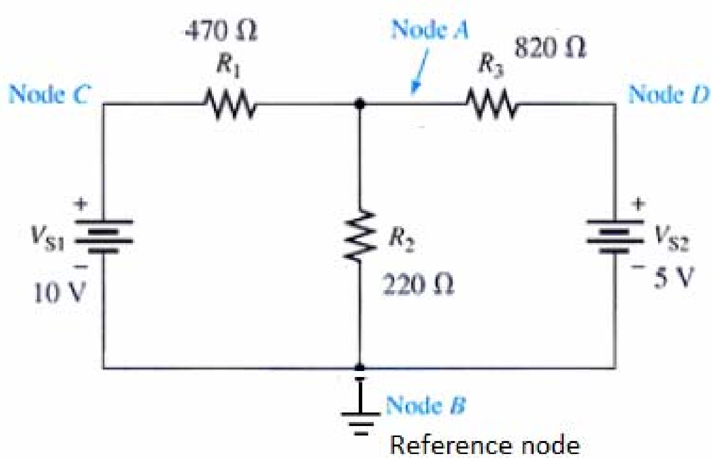
Step-1:

Determine the number of nodes. In this case, there are four nodes: Node A, B, C and D.



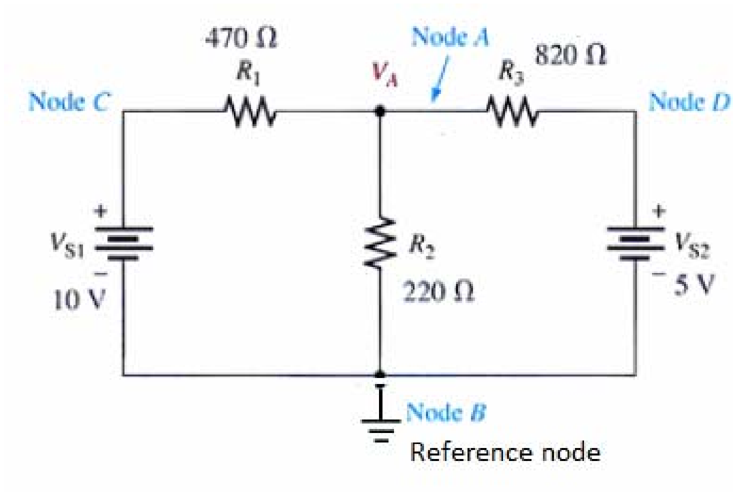
Step-2:

Arbitrarily select one node as a reference node and indicate this node as **ground**. In this case, let us assume node B as the reference node.



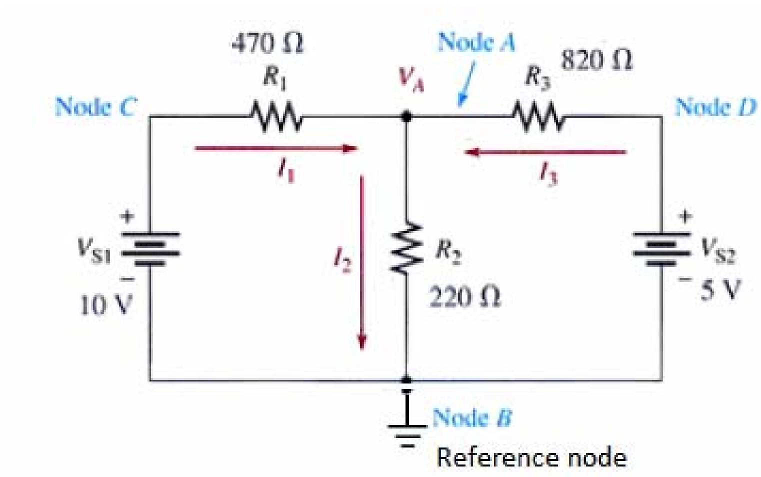
Step-3:

Except the reference node, arbitrarily assign voltages (V_A, V_C and V_D) to the remaining nodes in the circuit. Here node voltage C and D are already known to be the source voltages, i.e. V_C=10V, V_D=5V. The only unknown voltage is at node A, i.e. V_A.



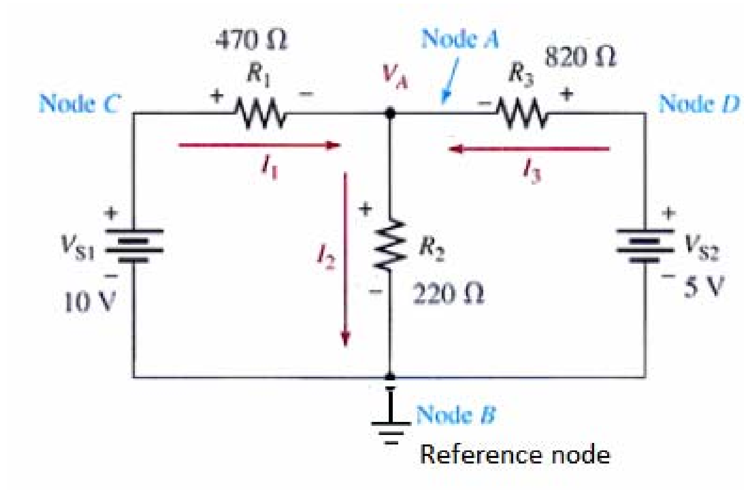
Step-4:

Arbitrarily assign branch currents along with directions at each node where the voltage is unknown, except at the reference node. Here, voltage at node A is unknown.



Step-5:

Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.



Step-6:

With the exception of the reference node (ground), apply Kirchhoff's current law at each of the nodes.

As there are 3 nodes (labeled as Node C, Node A, and Node D) except the reference node, and as the only unknown voltage is at node A, we now apply Kirchhoff's current law at node A.

$$KCL \text{ at Node } A \qquad \sum I_{Entering} = \sum I_{Leaving}$$

$$\Rightarrow \qquad I_1 + I_3 = I_2$$

$$\Rightarrow \qquad I_1 - I_2 + I_3 = 0 \qquad ....(1)$$

Step-7:

Express the current equation(s) in terms of the potential difference across a known resistance.

Here, the currents are rewritten in terms of the voltages across the resistors as follows:

$$I_1 = \frac{V_1}{R_1} = \frac{V_{S1} - V_A}{R_1} \qquad ....(2)$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_A}{R_2} \qquad ....(3)$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_{S2} - V_A}{R_3} \qquad .... \qquad ..(4)$$

By substituting the values of I_1, I_2 and I_3 to equation (1), the nodal equations become:

$$I_1 - I_2 + I_3 = 0 \qquad \text{.........(1)}$$

$$\Rightarrow \quad \frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0 \qquad \text{.........(5)}$$

Given, V_{S1}=10V, V_{S2}=5V, R_1=470 ohm, R_2=220 ohm, R_3=820 ohm. The only unknown term is V_A. So, putting the known values, we can easily calculate the value of unknown term V_A.

Step-8:

Solve the resulting linear equation for the unknown node voltages using substitution or determinant method.

Here the equation is:

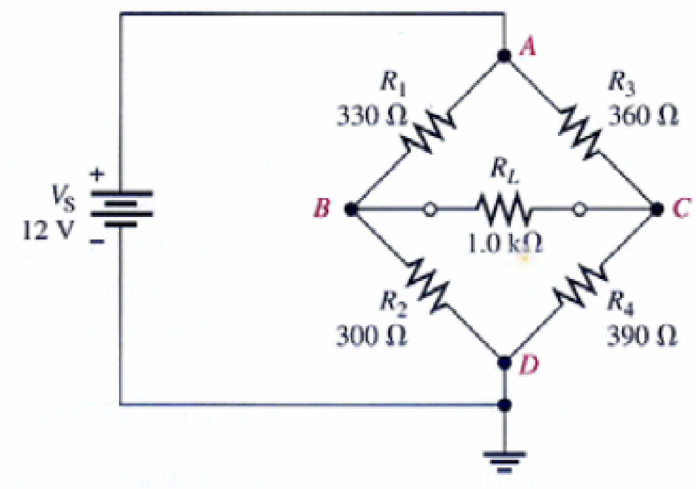
$$\frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0 \qquad \text{.........(5)}$$

Given, V_{S1}=10V, V_{S2}=5V, R_1=470 ohm, R_2=220 ohm, R_3=820 ohm. The only unknown term is V_A. So, putting the known values, we can easily calculate the value of unknown term V_A.

After determining V_A, we can easily calculate the branch current I_1, I_2 and I_3 using equations (2), (3) and (4).

Example-2:

Find the node voltages at node B and C for the Wheatstone bridge circuit given below:



Solution:

Among the four nodes, D is designated as reference node. Node A has the same voltage as the source. Voltage at nodes B and C is unknown. Hence we need two linear equations to solve the problem.

Node B:

$$I_1 + I_L = I_2$$

$$\frac{V_A - V_B}{R_1} + \frac{V_C - V_B}{R_L} = \frac{V_B}{R_2}$$

$$\frac{12 - V_B}{0.330\,\text{k}\Omega} + \frac{V_C - V_B}{1.0\,\text{k}\Omega} = \frac{V_B}{0.300\,\text{k}\Omega}$$

Node C:

$$I_3 = I_L + I_4$$

$$\frac{V_A - V_C}{R_3} = \frac{V_C - V_B}{R_L} + \frac{V_C}{R_4}$$

$$\frac{12\text{ V} - V_C}{0.360\,\text{k}\Omega} = \frac{V_C - V_B}{1.0\,\text{k}\Omega} + \frac{V_C}{0.390\,\text{k}\Omega}$$