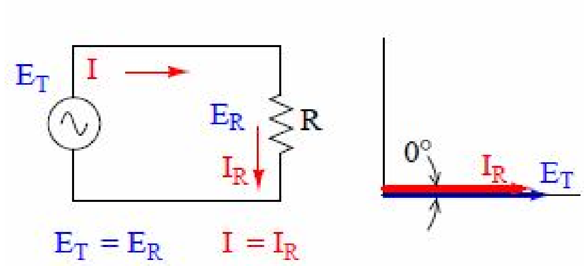
Lecture-14: AC Fundamentals: AC Through PURE R, L and C Circuits

****

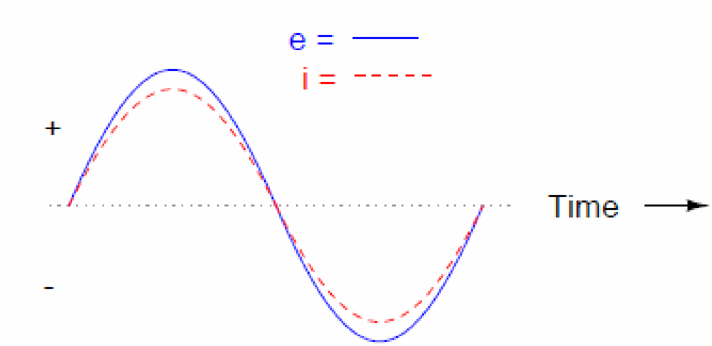
AC Through Pure Ohmic Resistance Alone:

- A pure resistive AC circuit consisting of a source and a resistor is shown below.



- Because the resistor simply and directly resists the flow of electrons at all periods of time, the waveform for the voltage drop across the resistor is exactly in phase with the waveform for the current through it. That is, resistor voltage and current are in phase.

- If we were to plot the current and voltage for this type of simple AC circuit, it would look something like this:



- It is seen that alternating voltage and current are in phase with each other in a pure resistive AC circuit. This means that when the instantaneous value for current is zero, the instantaneous voltage across the resistor is also zero. Likewise, at the moment in time where the current through the resistor is at its positive peak, the voltage across the resistor is also at its positive peak, and so on. At any given point in time along the waves, Ohm's Law holds true for the instantaneous values of voltage and current.

****

- We can also calculate the power dissipated by this resistor. Let the applied voltage be given by the equation $v = V_m Sin\theta = V_m Sin\omega t$, therefore the equation of current is given by $i = I_m Sin\theta = I_m Sin\omega t$, since voltage and current are in phase with each other in a pure resistive AC circuit.

  Therefore, the equation of instantaneous power is:

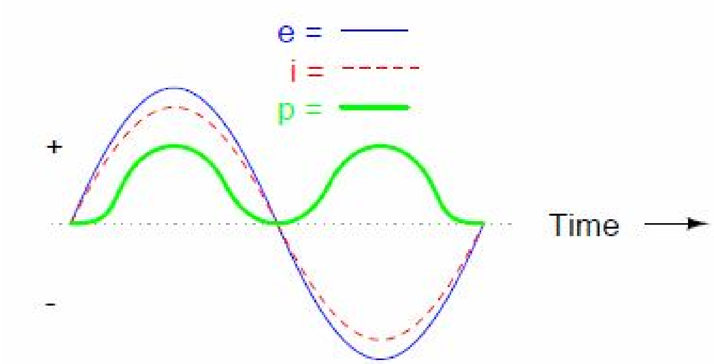
$$p = vi = V_m Sin\omega t \times I_m Sin\omega t = V_m I_m Sin^2\omega t = \frac{V_m I_m}{2} 2Sin^2\omega t$$

$$= \frac{V_m I_m}{2}(1 - Cos2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} Cos2\omega t$$

- From the above equation we see that power consists of constant part $\frac{V_m I_m}{2}$ and fluctuating part $\frac{V_m I_m}{2} Cos2\omega t$. For a complete cycle, the average value of $\frac{V_m I_m}{2} Cos2\omega t$ is zero. Hence, power for the whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{RMS} \times I_{RMS} = VI$$

- We can plot the power on the same graph as the voltage and current shown below.



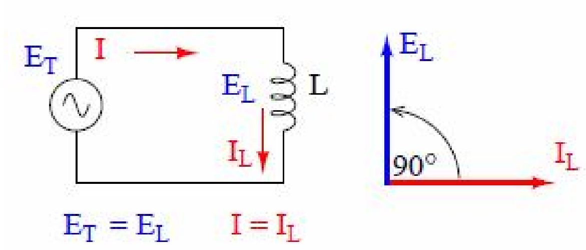
- It is seen from the above figure that no part of the power cycle becomes negative at any point. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive. In other words, in a pure resistive AC circuit, power is always positive. This consistent "polarity" of power tells us that the resistor is always dissipating power, taking it from the source and releasing it in the form of heat energy. Whether the current is positive or negative, a resistor still dissipates energy.

Lecture-14: AC Fundamentals: AC Through PURE R, L and C Circuits

****

AC Through Pure Inductance Alone:

- A pure inductive AC circuit consisting of a source and an inductor is shown below.



- Inductors do not behave the same as resistors. Whereas resistors simply oppose the flow of electrons through them (by dropping a voltage directly proportional to the current), inductors oppose *changes* in current through them, by dropping a voltage directly proportional to the *rate of change* of current.

- According to *Lenz's Law*, this induced voltage is always of such a polarity as to try to maintain current at its present value. That is, if current is increasing in magnitude, the induced voltage will "push against" the electron flow; if current is decreasing, the polarity will reverse and "push with" the electron flow to oppose the decrease.

- Expressed mathematically, the relationship between the voltage dropped across the inductor and rate of current change through the inductor is as such:

$$v = L\frac{di}{dt}$$

Now, $v = V_m Sin\omega t$

Therefore, $V_m Sin\omega t = L\frac{di}{dt}$

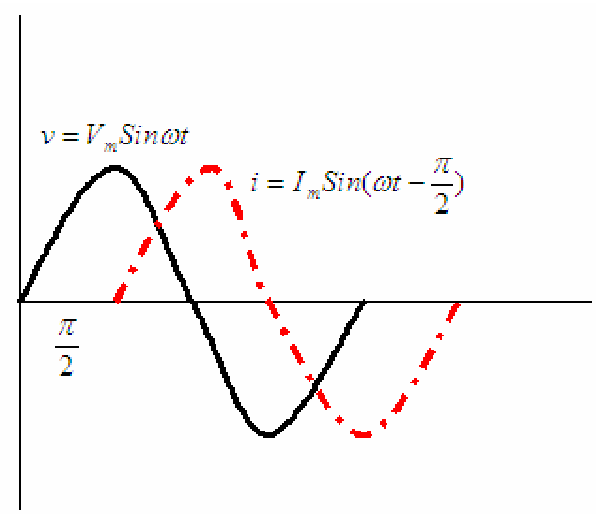
$$\therefore \quad di = \frac{V_m}{L} Sin\omega t \; dt$$

- Integrating both sides, we get,

$$i = \frac{V_m}{L}\int Sin\omega t \; dt = \frac{V_m}{\omega L}(-Cos\omega t) = \frac{V_m}{\omega L} Sin(\omega t - \frac{\pi}{2}) = \frac{V_m}{X_L} Sin(\omega t - \frac{\pi}{2}),$$

Where $X_L = \omega L = 2\pi f L$ is called the inductive reactance.

- Maximum value of I is $I_m = \frac{V_m}{\omega L}$ when $Sin(\omega t - \frac{\pi}{2})$ is unity.
- Hence, the equation of current becomes, $i = I_m Sin(\omega t - \frac{\pi}{2})$
- So, we find that for a pure inductive circuit, if applied voltage is represented by $v = V_m Sin\omega t$, then current flowing through the inductor is given by $i = I_m Sin(\omega t - \frac{\pi}{2})$
- If we were to plot the current and voltage for this type of simple AC circuit, it would look something like this:



- We see from the equations of voltage and current that, in a pure inductive circuit, inductor current lags inductor voltage by 90° or a quarter cycle. This means that, the instantaneous voltage is at a peak wherever the instantaneous current is at zero and the instantaneous voltage is zero whenever the instantaneous current is at a peak.
- We can also calculate the power. Let the applied voltage be given by the equation $v = V_m Sin\omega t$, therefore the equation of current is given by $i = I_m Sin(\omega t - \frac{\pi}{2})$, since current lags voltage by 90^0 in a pure inductive circuit.

  Therefore, the equation of instantaneous power is:

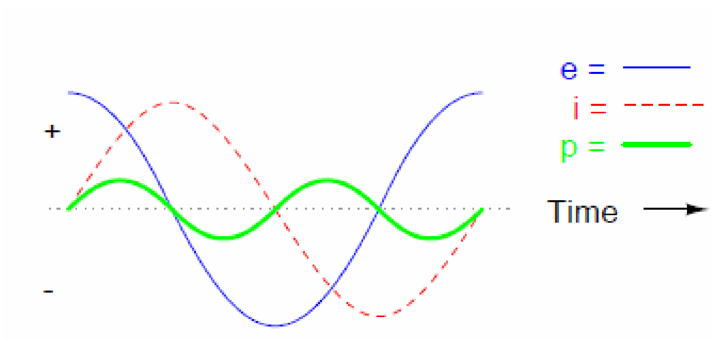
Lecture-14: AC Fundamentals: AC Through PURE R, L and C Circuits

****

$$p = vi = V_m Sin\omega t \times I_m Sin(\omega t - \frac{\pi}{2}) = -V_m I_m Sin\omega t . Cos\omega t = -\frac{V_m I_m}{2} Sin 2\omega t$$

Power for a whole cycle is $P = -\frac{V_m I_m}{2} \int_0^{2\pi} Sin 2\omega t \, dt = 0$

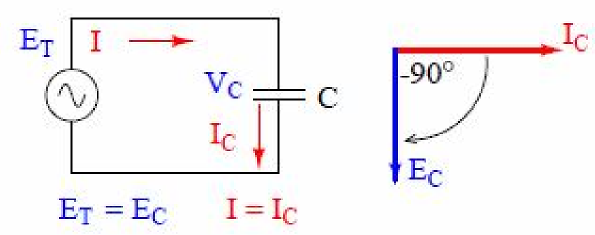
- We can plot the power on the same graph as the voltage and current shown below.



- It is seen from the above figure that, in a pure inductive circuit, instantaneous power may be positive or negative. Power wave is a sine wave of frequency double that of the voltage and current.
- Because instantaneous power is the product of the instantaneous voltage and the instantaneous current (p=vi), the power equals zero whenever the instantaneous current *or* voltage is zero. Whenever the instantaneous current and voltage are both positive (above the line), the power is positive. The power is also positive when the instantaneous current and voltage are both negative (below the line). However, because the current and voltage waves are 90° out of phase, there are times when one is positive while the other is negative, resulting in equally frequent occurrences of *negative instantaneous power*.
- But what does *negative* power mean? It means that the inductor is releasing power back to the circuit, while a positive power means that it is absorbing power from the circuit. Since the positive and negative power cycles are equal in magnitude and duration over time, the inductor releases just as much power back to the circuit as it absorbs over the span of a complete cycle. What this means in a practical sense is that the reactance of an inductor dissipates a net energy of zero, quite unlike the resistance of a resistor, which dissipates energy in the form of heat.

AC Through Pure Capacitance Alone:

- A pure capacitive AC circuit consisting of a source and a capacitor is shown below.



- Capacitors do not behave the same as resistors. Whereas resistors allow a flow of electrons through them directly proportional to the voltage drop, capacitors oppose *changes* in voltage by drawing or supplying current as they charge or discharge to the new voltage level. The flow of electrons "through" a capacitor is directly proportional to the *rate of change* of voltage across the capacitor. This opposition to voltage change is another form of *reactance*, but one that is precisely opposite to the kind exhibited by inductors.
- Expressed mathematically, the relationship between the rate of voltage change across the capacitor and the current through the capacitor is as such:

$$i = C\frac{dv}{dt}$$

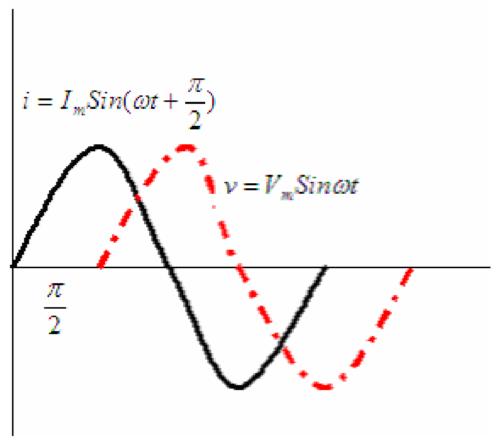
Now, $v = V_m Sin\omega t$

Therefore, $i = C\frac{d}{dt}V_m Sin\omega t = \frac{V_m}{1/\omega C}Cos\omega t = \frac{V_m}{1/\omega C}Sin(\omega t + \frac{\pi}{2}) = \frac{V_m}{X_C}Sin(\omega t + \frac{\pi}{2})$

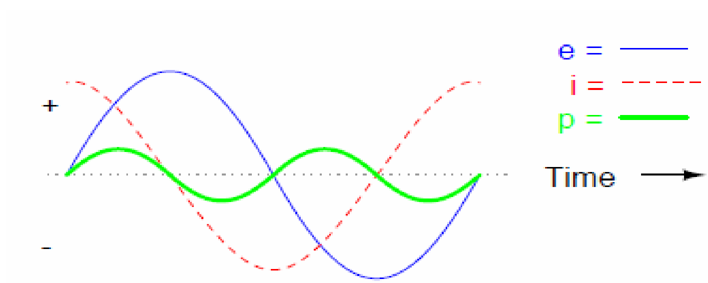
Where $X_C = 1/\omega C = 1/2\pi fC$ is called the capacitive reactance.

- Maximum value of I is $I_m = \frac{V_m}{1/\omega C}$ when $Sin(\omega t + \frac{\pi}{2})$ is unity.
- Hence, the equation of current becomes, $i = I_m Sin(\omega t + \frac{\pi}{2})$

- So, we find that for a pure capacitive circuit, if applied voltage is represented by $v = V_m Sin\omega t$, then current flowing through the inductor is given by $i = I_m Sin(\omega t + \frac{\pi}{2})$
- If we were to plot the current and voltage for this type of simple AC circuit, it would look something like this:



- We see from the equations of voltage and current that, in a pure capacitive circuit, capacitor current leads capacitor voltage by 90^o or a quarter cycle. This means that, the instantaneous voltage is at a peak wherever the instantaneous current is at zero and the instantaneous voltage is zero whenever the instantaneous current is at a peak.
- We can also calculate the power. The same unusual power wave that we saw with the simple inductor circuit is present in the simple capacitor circuit, too:



- As with the simple inductor circuit, the 90 degree phase shift between voltage and current results in a power wave that alternates equally between positive and negative. This means that a capacitor does not dissipate power as it reacts against changes in voltage; it merely absorbs and releases power, alternately.

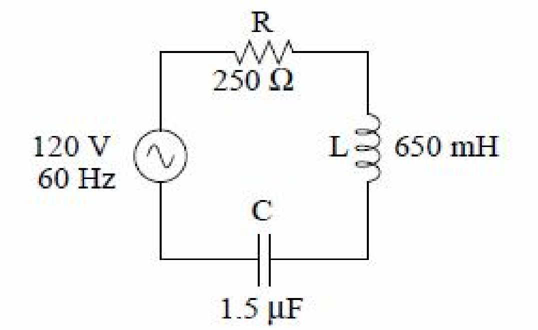
Lecture-14: AC Fundamentals: AC Through PURE R, L and C Circuits

****

Solving Series RLC Circuits

Example:

Determine the total impedance, current and voltage drop in each component of a series RLC circuit connected with a 60Hz supply shown below.



Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi f L$$

$$X_L = (2)(\pi)(60\ \text{Hz})(650\ \text{mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{(2)(\pi)(60\ \text{Hz})(1.5\ \mu\text{F})}$$

$$X_C = 1.7684\ \text{k}\Omega$$

Step-2:

- Express all resistances and reactances in a mathematically common form: impedance.

$$Z_R = 250 + j0\ \Omega \quad or \quad 250\ \Omega \angle 0^\circ$$

$$Z_L = 0 + j245.04\ \Omega \quad or \quad 245.04\ \Omega \angle 90^\circ$$

$$Z_C = 0 - j1.7684k\ \Omega \quad or \quad 1.7684\ k\Omega \angle -90^\circ$$

Note:

- Remember that an inductive reactance translates into a positive imaginary impedance (or an impedance at +90°), while a capacitive reactance translates into a negative imaginary impedance (impedance at -90°). Resistance, of course, is still regarded as a purely "real" impedance (polar angle of 0°):

Step-3:

- Express the given source voltage in rectangular or polar form:

$$V=120+j0\ V \text{ or } 120\ V \angle 0^0$$

Note:

- Unless otherwise specified, the source voltage will be our reference for phase shift, and so it is written at an angle of 0°.

- All given quantities are shown in the circuit below.

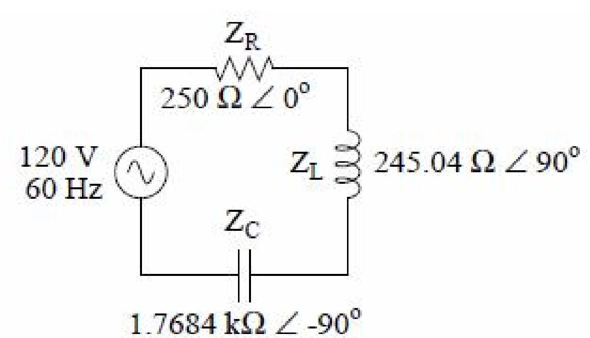


Figure: Series RLC circuit with component values replaced by impedances.

Step-4:

Calculate the total impedance of the circuit.

$$Z_{total} = Z_R + Z_L + Z_C$$

$$Z_{total} = (250 + j0\ \Omega) + (0 + j245.04\ \Omega) + (0 - j1.7684k\ \Omega)$$

$$Z_{total} = 250 - j1.5233k\ \Omega \quad or \quad 1.5437\ k\Omega \angle -80.680^\circ$$

Step-5:

Determine circuit current using Ohm's law.

$$I = \frac{V}{Z} = \frac{120\angle 0^\circ}{(1.5437 \times 1000)\angle -80.680^\circ} = 0.07773\angle 80.680^\circ = 12.589 + j76.708$$

Note:

- Being a series circuit, current must be equal through all components of the given circuit.

Step-6:

Determine voltage drop across resistor, capacitor and inductor.

$$V_R = IZ_R = 0.07773\angle 80.680^\circ \times 250\angle 0^\circ = 19.4325\angle 80.680^\circ$$

$$V_L = IZ_L = 0.07773\angle 80.680^\circ \times 245.04\angle 90^\circ = 19.046\angle 170.68^\circ$$

$$V_C = IZ_C = 0.07773\angle 80.680^\circ \times (1.7684 \times 1000)\angle -90^\circ = 137.46\angle -9.3199^\circ$$

Note:

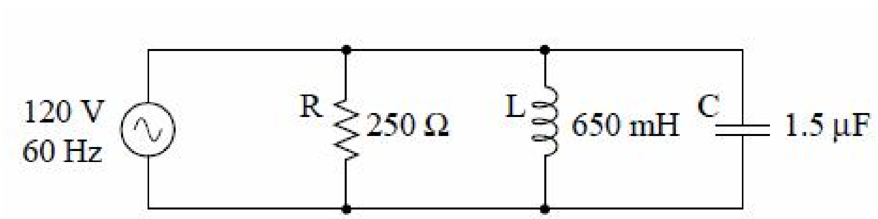
- Although our supply voltage is only 120 volts, the voltage across the capacitor is 137.46 volts! How can this be? The answer lies in the interaction between the inductive and capacitive reactances. Expressed as impedances, we can see that the inductor opposes current in a manner precisely opposite that of the capacitor. Expressed in rectangular form, the inductor's impedance has a positive imaginary term and the capacitor has a negative imaginary term. When these two contrary impedances are added (in series), they tend to cancel each other out! Although they're still added together to produce a sum, that sum is actually less than either of the individual (capacitive or inductive) impedances alone.

- Although impedances add in series, the total impedance for a circuit containing both inductance and capacitance may be less than one or more of the individual impedances, because series inductive and capacitive impedances tend to cancel each other out.
- If the total impedance in a series circuit with both inductive and capacitive elements is less than the impedance of either element separately, then the total current in that circuit must be greater than what it would be with only the inductive or only the capacitive elements there.
- With this abnormally high current through each of the components, voltages greater than the source voltage may be obtained across some of the individual components!

Solving Parallel RLC Circuits

Example:

Determine the total impedance, current and voltage drop in each component of a parallel RLC circuit connected with a 60Hz supply shown below.



Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi f L$$

$$X_L = (2)(\pi)(60\ \text{Hz})(650\ \text{mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{(2)(\pi)(60\ \text{Hz})(1.5\ \mu\text{F})}$$

$$X_C = 1.7684\ \text{k}\Omega$$

Step-2:

- Express all resistances and reactances in a mathematically common form: impedance.

$$Z_R = 250 + j0\ \Omega \quad \textit{or} \quad 250\ \Omega \angle 0^\circ$$

$$Z_L = 0 + j245.04\ \Omega \quad \textit{or} \quad 245.04\ \Omega \angle 90^\circ$$

$$Z_C = 0 - j1.7684\text{k}\ \Omega \quad \textit{or} \quad 1.7684\ \text{k}\Omega \angle -90^\circ$$

Step-3:

- Express the given source voltage in rectangular or polar form:

$$V = 120 + j0\ \text{V or } 120\ \text{V} \angle 0^0$$

- All given quantities are shown in the circuit below.

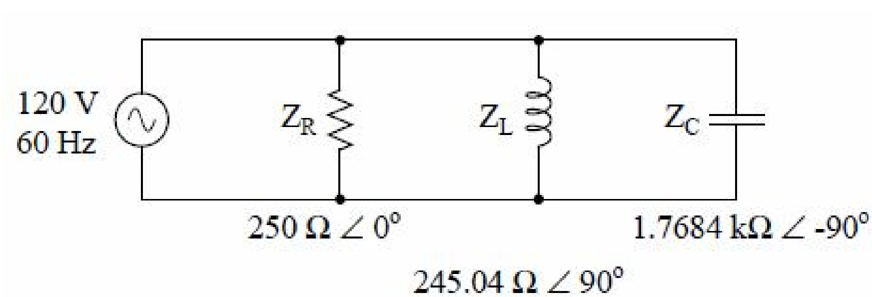


Figure: Parallel RLC circuit with component values replaced by impedances.

Step-4:

Calculate the total impedance of the parallel circuit.

$$\frac{1}{Z_{Total}} = \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$
$$= \frac{1}{250\angle 0°} + \frac{1}{245.04\angle 90°} + \frac{1}{(1.7684\times 1000)\angle -90°}$$
$$Z_{Total} = 187.79\angle 41.311° = 141.05 + j123.96$$

Step-5:

Determine voltage drop across resistor, capacitor and inductor.

Being a parallel circuit, voltage must be equally shared by all components of the given circuit. Therefore voltage drop across each component is the same as the supply voltage which is V=120+j0 V or 120 V$\angle 0^0$

$$V_R = V_L = V_C = V = 120 + j0 = 120\angle 0°$$

Step-6:

Determine total current and current flowing through each component using Ohm's law.

$$I = \frac{V}{Z} = \frac{120\angle 0^{\circ}}{187.79\angle 41.311°} = 639.03mA\angle -41.311° = 480mA - j421.85mA$$

$$I_R = \frac{V_R}{Z_R} = \frac{120\angle 0^{\circ}}{250\angle 0°} = 480mA\angle 0° = 480mA + j0$$

$$I_L = \frac{V_L}{Z_L} = \frac{120\angle 0^{\circ}}{245.04\angle 90°} = 489.71mA\angle -90° = 0 - j489.71mA$$

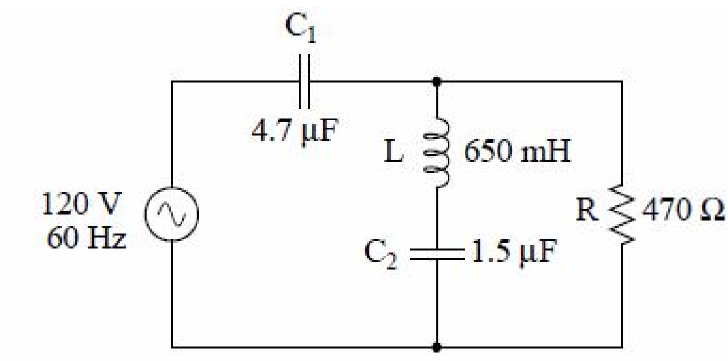
$$I_C = \frac{V_C}{Z_C} = \frac{120\angle 0^{\circ}}{(1.7684\times 1000)\angle -90°} = 67.858mA\angle 90° = 0 - j67.858mA$$

****

Solving Series-Parallel RLC Circuits

Example:

Determine the total impedance, current and voltage drop in each component of a series-parallel RLC circuit connected with a 60Hz supply shown below.



Step-1:

Determine the capacitive and inductive reactance with the help of the following formulas:

$$X_L = 2\pi f L$$

$$X_L = (2)(\pi)(60\text{ Hz})(650\text{ mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_{C1} = \frac{1}{(2)(\pi)(60Hz)(4.7\mu F)} = 564.38\Omega$$

$$X_{C2} = \frac{1}{(2)(\pi)(60Hz)(1.5\mu F)} = 1.7684k\Omega$$

Step-2:

- Express all resistances and reactances in a mathematically common form: impedance.

$$Z_R = 250 + j0\ \Omega \quad or \quad 250\ \Omega \angle 0^\circ$$

$$Z_L = 0 + j245.04\ \Omega \quad or \quad 245.04\ \Omega \angle 90^\circ$$

$$Z_C = 0 - j1.7684k\ \Omega \quad or \quad 1.7684\ k\Omega \angle -90^\circ$$

Step-3:

- Express the given source voltage in rectangular or polar form:

$$V=120+j0\ V \text{ or } 120\ V \angle 0^0$$

- All given quantities are shown in the circuit below.

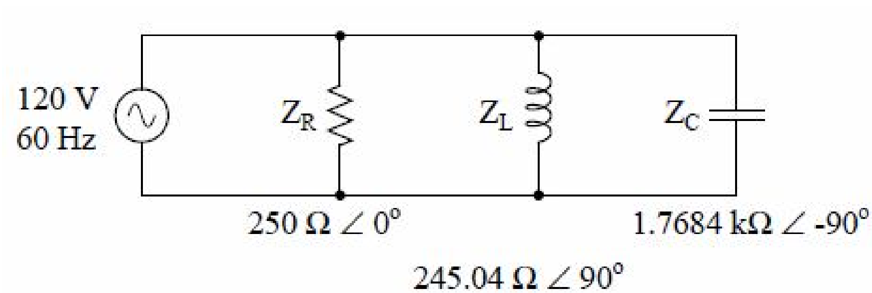


Figure: Parallel RLC circuit with component values replaced by impedances.

Step-4:

Calculate the total impedance of the parallel circuit.

$$\frac{1}{Z_{Total}} = \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$= \frac{1}{250\angle 0°} + \frac{1}{245.04\angle 90°} + \frac{1}{(1.7684\times 1000)\angle -90°}$$

$$Z_{Total} = 187.79\angle 41.311° = 141.05 + j123.96$$

Step-5:

Determine voltage drop across resistor, capacitor and inductor.

Being a parallel circuit, voltage must be equally shared by all components of the given circuit. Therefore voltage drop across each component is the same as the supply voltage which is V=120+j0 V or 120 V$\angle 0^0$

$$V_R = V_L = V_C = V = 120 + j0 = 120\angle 0°$$

Step-6:

Determine total current and current flowing through each component using Ohm's law.

$$I = \frac{V}{Z} = \frac{120\angle 0^{\circ}}{187.79\angle 41.311°} = 639.03mA\angle -41.311° = 480mA - j421.85mA$$

$$I_R = \frac{V_R}{Z_R} = \frac{120\angle 0^{\circ}}{250\angle 0°} = 480mA\angle 0° = 480mA + j0$$

$$I_L = \frac{V_L}{Z_L} = \frac{120\angle 0^{\circ}}{245.04\angle 90°} = 489.71mA\angle -90° = 0 - j489.71mA$$

$$I_C = \frac{V_C}{Z_C} = \frac{120\angle 0^{\circ}}{(1.7684\times 1000)\angle -90°} = 67.858mA\angle 90° = 0 - j67.858mA$$