# Relations and Their Properties

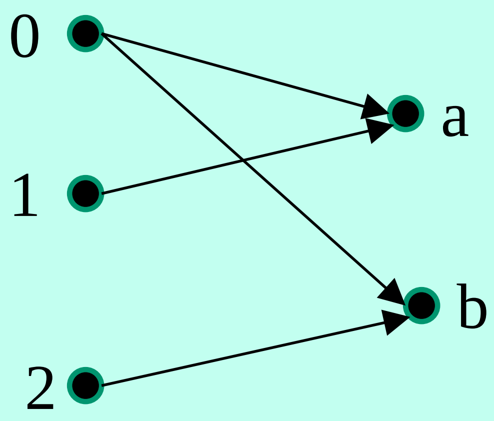
IT-209: Discrete Mathematics

# Binary Relations

- Let A and B be sets. A **binary relation** from A to B is a subset of $A \times B$.
- A binary relation from A to B is a set R of ordered pairs where the first element of each ordered pair comes from A and the second element comes from B.
- If **$(a,b) \in R$**, then we say a is related to b by R. This is sometimes written as ***a R b***.

# Binary Relations - Example

- Let A = {0, 1, 2} and B = {a, b}. Then relation R from A to B is defined as:
  - R = {(0, a), (0, b), (1, a), (2, b)}
  - Relations can be represented graphically
    - Arrow representation
    - Table representation




| R | a | b |
|---|---|---|
| 0 | × | × |
| 1 | × | |
| 2 | | × |

# Relations on a set

- A relation on the set A is a relation from A to A.
- A relation on a set is a subset of $A \times A$
  - Example: Consider the following relations on set of integers:
    - $R_1 = \{(a, b) \mid a \leq b\}$
    - $R_2 = \{(a, b) \mid a > b\}$
    - $R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$
    - $R_4 = \{(a, b) \mid a = b\}$
    - $R_5 = \{(a, b) \mid a = b + 1\}$
    - $R_6 = \{(a, b) \mid a + b \leq 3\}$
  - Which of these relation contain each pairs (1, 1), (1, 2), (2, 1), (1, -1), and (2, 2)?
  - Answer:
    - (1, 1) is in $R_1$, $R_3$, $R_4$, and $R_6$
    - (1, 2) is in $R_1$, and $R_6$
    - (2, 1) is in $R_2$, $R_5$, and $R_6$
    - (1, -1) is in $R_2$, $R_3$, and $R_6$
    - (2, 2) is in $R_1$, $R_3$, and $R_4$

# Properties on Relations

- There are several properties that are used to classify relations on a set.
  - Reflexive
  - Symmetric
  - Antisymmetric
  - Transitive

# Reflexive

- A relation R on a set A is called **reflexive** if $(a,a) \in R$ for <u>**every**</u> element $a \in A$.
- $\forall a\ ((a, a) \in R)$, where the u. of d. is the set of all elements in the set.
- A relation R on a set A is reflexive if every element of A is related to itself.
  - Example: Is the "divides" relation i.e. $R = \{(a, b) \mid a \text{ divides } b\}$ on the set of integers is reflexive?
  - Answer: Yes, because $a \mid a$ is true for all positive integers. So the "divides" relation is reflexive.

# Symmetric

- A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for some $a, b \in A$.
- A relation R on a set A such that $(a, b) \in R$ and $(b, a) \in R$ only if $a = b$ for $a, b \in A$ is called **antisymmetric**.
  - Note that antisymmetric is not the opposite of symmetric. A relation can be both.
- A relation R on a set A is called **asymmetric** if $(a, b) \in R \rightarrow (b, a) \notin R$.
  - Example: Is the "divides" relation i.e. $R = \{(a, b) \mid a \text{ divides } b\}$ on the set of integers is symmetric? Is it antisemmetric?
  - Answer: The relation is not symmetric because $(1, 2) \in R$ but $(2, 1) \notin R$.
  - The relation is antisymmetric, if a and b are positive integer s with $a \mid b$ and $b \mid a$, then $a = b$.

# Transitive

- A relation R on a set A, is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$ , for $(a, b, c) \in A$.
  - Example: Is the "divides" relation i.e. $R = \{(a, b) \mid a \text{ divides } b\}$ on the set of integers is transitive?
  - Answer: Suppose a divides b and b divides c. Then there are positive integers k and l such that $b = ak$ and $c = bl$. Hence $c = akl = a(kl)$, so a divides c. It follows that the relation is transitive.
  - Suppose $3 \mid 6$ and $6 \mid 12$ then $3 \mid 12$.

# List of Examples

If R is a relation on Z where $(x, y) \in R$ when $x \neq y$.

Is R reflexive?

  No, $x = x$ is not included.

Is R symmetric?

  Yes, if $x \neq y$, then $y \neq x$.

Is R antisymmetric?

  No, $x \neq y$ and $y \neq x$ does not imply $x = y$.

Is R transitive?

  No, $(1,2) \in R$ and $(2,1) \in R$ but $(1,1) \notin R$.

# List of Examples

If R is a relation on Z where $(x, y) \in R$ when $xy \geq 1$

Is R reflexive?

  No, $0*0 \geq 1$ is not true.

Is R symmetric?

  Yes, if $xy \geq 1$, then $yx \geq 1$. ($1*2 \geq 1$; $2*1 \geq 1$)

Is R antisymmetric?

  No, $1*2 \geq 1$ and $2*1 \geq 1$, but $1 \neq 2$.

Is R transitive?

  Yes, $xy \geq 1$ and $yz \geq 1$ implies $xz \geq 1$ ($1*2 \geq 1$; $2*3 \geq 1$; $1*3 \geq 1$)

  (x, y and z can't be zero and must be all positive or all negative.)

# List of Examples

If R is a relation on Z where $(x, y) \in R$ when $x = y + 1$ or $x = y - 1$

Is R reflexive?

No, $(2,2) \notin R$. $2 \neq 2+1$ and $2 \neq 2-1$.

Is R symmetric?

Yes, if $(x, y) \in R$, $x = y + 1 \rightarrow y = x - 1$ or

$x = y - 1 \rightarrow y = x + 1$. So $(y, x) \in R$. $(3 = 2 + 1; 2 = 3 - 1)$

Is R antisymmetric?

No, $(2,1) \in R$ and $(1,2) \in R$, but $1 \neq 2$.

Is R transitive?

No, (1,2) and $(2,3) \in R$ , but $(1,3) \notin R$.

$1 \neq 3 + 1$ and $1 \neq 3 - 1$.

# List of Examples

If R is a relation on Z where $(x, y) \in R$ when x is a multiple of y.

Is R reflexive?

Yes, $(x, x) \in R$ for all x, because x is a multiple of itself.

Is R symmetric?

No, $(4,2) \in R$, but $(2,4) \notin R$.

Is R antisymmetric?

No, $(2,-2) \in R$ and $(-2,2) \in R$, but $2 \neq -2$.

Is R transitive?

Yes, if $(x, y) \in R$ and $(y, z) \in R$, $x = k*y$ and $y = j*z$  $j,k \in Z$.

$x = kj*z$ and $kj \in Z$, thus x is a multiple of z and $(x, z) \in R$.

# List of Examples

If R is a relation on Z where $(x, y) \in R$ when x and y are both negative or both nonnegative

Is R reflexive?

Yes, x has the same sign as itself so $(x, x) \in R$ for all x.

Is R symmetric?

Yes, if $(x, y) \in R$ then x and y are both negative or both nonnegative. It follows that y and x are as well.

Is R antisymmetric?

No, $(99,132) \in R$ and $(132,99) \in R$, but $99 \neq 132$.

Is R transitive?

Yes, if $(x, y) \in R$ and $(y, z) \in R$, then x, y and z are all negative or all nonnegative. Thus $(x, z) \in R$. $((-2, -3), (-3, -4), (-2, -4) \in R)$

# List of Examples

If R is a relation on Z where $(x, y) \in R$ when $x = y^2$

Is R reflexive?

No, $(2,2) \notin R$. $2 \neq 2^2$.

Is R symmetric?

No, $(4,2) \in R$, but $(2,4) \notin R$.

Is R antisymmetric?

Yes, if $(x, y) \in R$ and $(y, x) \in R$ then $x = y^2$ and $y = x^2$. The only time this holds true is when $x = y$ (and more specifically when $x = y = 1$ or 0).

Is R transitive?

No, $(16,4) \in R$ and $(4,2) \in R$, but $(16,2) \notin R$.

# List of Examples

If R is a relation on Z where $(x, y) \in R$ when $x \geq y^2$

Is R reflexive?

No, $(2,2) \notin R$. $2 < 2^2$.

Is R symmetric?

No, $(10,3) \in R$, but $(3,10) \notin R$.

Is R antisymmetric?

Yes, $(x, y) \in R$ and $(y, x) \in R$ implies that $x \geq y^2$ and $y \geq x^2$. The only time this holds true is when $x = y$ (=1 or 0).

Is R transitive?

Yes, if $(x, y) \in R$ and $(y, z) \in R$, then $x \geq y^2$ and $y \geq z^2$.

$x \geq y^2 \geq (z^2)^2 (\geq z^2)$. Thus $(x, z) \in R$.

# Combining Relations

- Example: Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. The relation $R_1 = \{(1, 1), (2, 2), (3, 3)\}$ and $R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$ can be combine to obtain
  - $R_1 \cup R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (3, 3)\}$
  - $R_1 \cap R_2 = \{(1, 1)\}$
  - $R_1 - R_2 = \{(2, 2), (3, 3)\}$
  - $R_2 - R_1 = \{(1, 2), (1, 3), (1, 4)\}$
  - $R_1 \oplus R_2 = R_1 \cup R_2 - R_1 \cap R_2 = \{(1, 2), (1, 3), (1, 4), (2, 2), (3, 3)\}$

# Combining Relations

***the composite of R and S***

- Let R be a relation from a set A to a set B and S a relation from set B to a set C. The **composite** of R and S is the relation consisting of ordered pairs (a,c) where $a \in A$, $c \in C$, and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.
- The composite of R and S is written $S \circ R$.
  - Example: What is the composite of the relations R and S, where R is the relation from {1, 2, 3} to {1, 2, 3, 4} and S is a relation from {1, 2, 3, 4} to {0, 1, 2} with R = {(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)} and S = {(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)}?
  - Answer: $S \circ R$ is constructed using all ordered pair in R and S, where the second element of the ordered pair in R agrees with the first element of the ordered pair in S.
  - $S \circ R$ = {(1, 0), (1, 1), (2, 1), (2, 2), (3, 0), (3, 1)}

# The powers of R, $R^n$

- Let R be a relation on the set A. The powers $R^n$, $n = 1, 2, 3, \ldots$, are defined inductively by
- $R^1 = R$ and $R^{n+1} = R^n \circ R$
- Thus the definition shows that:
  - $R^2 = R \circ R$
  - $R^3 = R^2 \circ R = (R \circ R) \circ R$ and so on.

# Theorem 1

**Prove:** The relation R on a set A is transitive if and only if $R^n \subseteq R$ for n = 1,2,3 . . .

**Proof:** We must prove this in two parts:

1) (R is transitive) $\rightarrow$ ($R^n \subseteq R$ for n = 1,2,3 . . . )

2) ($R^n \subseteq R$ for n = 1,2,3 . . . )$\rightarrow$ (R is transitive).

# The Proof – Part 1

Assume R is transitive. We must show that this implies that $R^n \subseteq R$ for n = 1,2,3 . . . .

To do this, we'll use induction.

**Basis Step:** $R^1 \subseteq R$ is trivially true ($R^1 = R$).

# The Proof – Part 1 (continued)

**Inductive Step:** Assume that $R^n \subseteq R$.

We must show that this implies that $R^{n+1} \subseteq R$.

Assume $(a, b) \in R^{n+1}$.

Then since $R^{n+1} = R^n \circ R$, there is an element x in A such that $(a, x) \in R$ and $(x, b) \in R^n$.

By the inductive hypothesis, i. e. $R^n \subseteq R$; $(x, b) \in R$.

Since R is transitive and $(a, x) \in R$ and $(x, b) \in R$, $(a, b) \in R$. Thus $R^{n+1} \subseteq R$.

# The Proof – Part 2

Now we must show that

$R^n \subseteq R$ for $n = 1, 2, 3 \ldots \rightarrow R$ is transitive.

**Proof:** Assume $R^n \subseteq R$ for $n = 1, 2, 3 \ldots$.

In particular, $R^2 \subseteq R$.

This means that if $(a, b) \in R$ and $(b, c) \in R$, then by the definition of composition, $(a, c) \in R^2$. Since $R^2 \subseteq R$, $(a, c) \in R$.

Hence R is transitive.

# Thank You

- Study all the solved problem from your text book.
- Try to solve related problems from exercise.
- Text from Rosen 8.1

# Representing Relation(7.3)

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# Matrix Representation of Relations

- A relation between sets can be presented using zero-one matrix.
- Suppose A relation R from set A = {$a_1$, $a_2$, ….., $a_m$} to B = {$b_1$, $b_2$, ….., $b_n$} can be represented as matrix $M_R = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R. \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

  – Example: A relation R between set A={1, 2, 3} and B={1, 2}. Let R contains (a, b) if a ∈ A, b ∈ B and a > b. What would be the matrix representation of R if $a_1 = 1$, $a_2 = 2$, $a_3 = 3$ and $b_1 = 1$, $b_2 = 2$?

  – Solution: R = {(2, 1), (3, 1), (3, 2)}. The matrix for R is:

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

# Matrix Representation of Properties of Relations

$$\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & & 1 & \\ & 1 & & 0 \\ 1 & & 1 & \\ & 0 & & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & & 1 & \\ & 1 & & 0 \\ 0 & & 1 & 0 \\ & 0 & 1 & 1 \end{bmatrix}$$

Reflexive

Symmetric

Antisymmetric

# Example on Properties

- Suppose that the relation R on a set is represented by the matrix $M_R$ , Is R reflexive, symmetric and/or antisymmetric?

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- Solution:
  - As the diagonal elements of the matrix is 1, so R is reflexive.
  - It is also symmetric as the opposite positions are 1, so R is symmetric.
  - So we can say that R is not antisymmetric.

# Combining Relations into Matrix form

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2}$$

$$M_{S \circ R} = M_R \odot M_S$$

$$M_{R^n} = M_R^{[n]}$$

# Examples

- Suppose $R_1$ and $R_2$ on set A are represented by the matrices given below. Find $R_1 \cup R_2$ and $R_1 \cap R_2$.

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

# Examples

- Find the matrix representing the relation S ○ R, where S and R is given as:

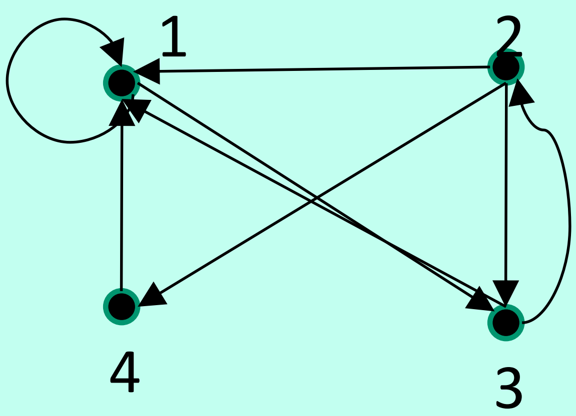
$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad M_{S \circ R} = M_R \odot M_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

- Find the matrix representing the relation $R^2$, where R is given as:

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad M_{R^2} = M_R^{[2]} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

# Representing Relation Using Diagraphs

- A directed graph or diagraph consists of set *V* of vertices (nodes) together with a set *E* of edges (arcs). The vertex a is called the initial vertex and b is called the terminal vertex of edge (a, b).
- A edge from vertex a to a is called a loop, represented in form (a, a).
  - Example: The directed graph of the relation R on set {1, 2, 3, 4} shown as: where, R = {(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)}



# Thank You

- Study all the solved problem from your text book.
- Try to solve related problems from exercise.
- Text from Rosen 8.3

# Equivalence Relations(7.5)

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# Definition of Equivalence Relations

- A relation *R* on a set *A* is an ***equivalence relation*** iff *R* is
  - Reflexive
  - Symmetric
  - Transitive
- Two elements related by an equivalence relation are called equivalent.
- The notation a ~ b is often used to denote that a and b are equivalent elements w. r. t. a particular equivalence relation.
- Example: Consider relation *R* = { (*a,b*) | *len*(*a*) = *len*(*b*) }, where *len*(*a*) means the length of string *a*
  - It is reflexive: *len*(*a*) = *len*(*a*)
  - It is symmetric: if *len*(*a*) = *len*(b), then *len*(*b*) = *len*(*a*)
  - It is transitive: if *len*(*a*) = *len*(b) and *len*(b) = *len*(c), then *len*(a) = *len*(c)
  - Thus, *R* is a equivalence relation

# Equivalence relation example

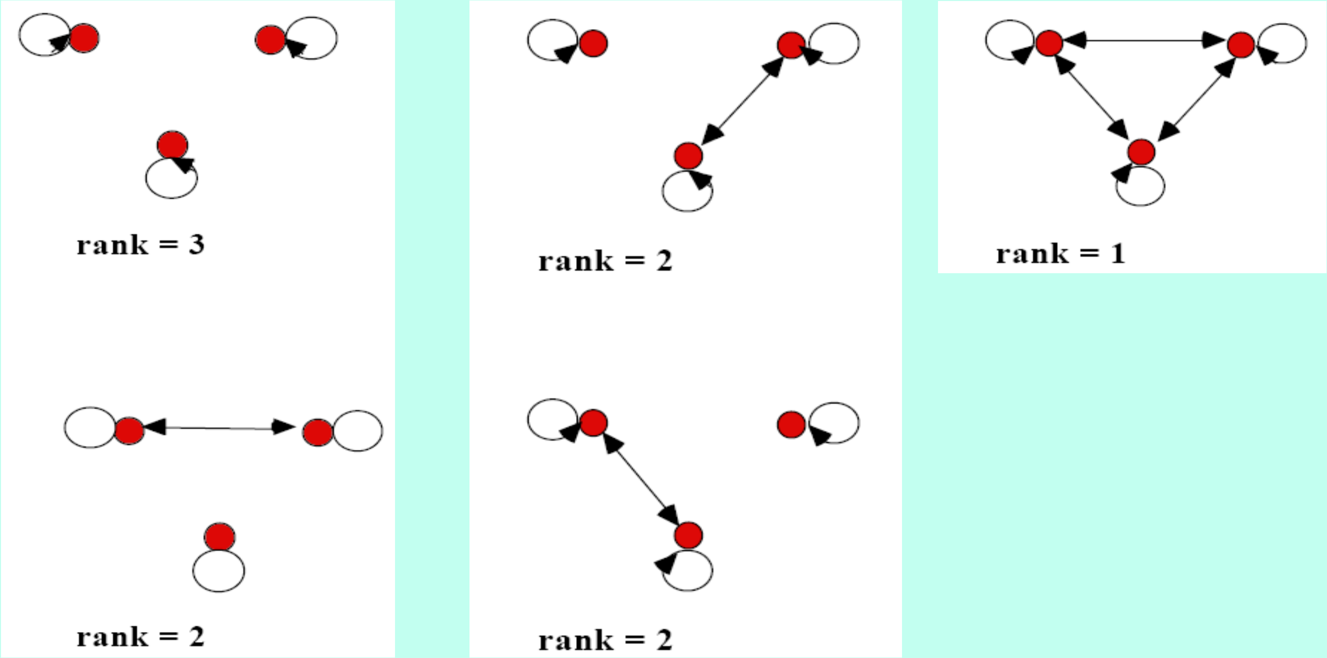
- Consider the relation $R = \{ (a,b) \mid a \equiv b \pmod{m} \}$
  - Remember that this means that $m \mid a - b$
  - Called “congruence modulo *m*”
- Is it reflexive: $(a, a) \in R$ means that $m \mid a - a$
  - $a - a = 0$, which is divisible by $m$
- Is it symmetric: if $(a,b) \in R$ then $(b,a) \in R$
  - $(a,b)$ means that $m \mid a - b$
  - Or that $km = a - b$. Negating that, we get $b - a = -km$
  - Thus, $m \mid b - a$, so $(b,a) \in R$
- Is it transitive: if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$
  - $(a,b)$ means that $m \mid a-b$, or that $km = a - b$
  - $(b,c)$ means that $m \mid b-c$, or that $lm = b - c$
  - $(a,c)$ means that $m \mid a-c$, or that $nm = a - c$
  - Adding these two, we get $km+lm = (a - b) + (b - c)$
  - Or $(k+l)m = a-c$
  - Thus, $m$ divides $a - c$, where $n = k+l$
- Thus, congruence modulo *m* is an equivalence relation

# Equivalence Class

- Let R be an equivalence relation on A. If $a \in A$, we define the set $[a]=\{b \in A : bRa\}$, called the *equivalence class containing a*.
- That is [a] is the set of all elements of A that are related to a.
- We also define the set $[A]_R=\{[a] : a \in A\}$, the set of all equivalence classes of A under equivalence relation R.
- The element in the bracket is called a ***representative*** of the equivalence class. We could have chosen any one.
- The number of equivalence classes is called the *rank* of the equivalence relation.

# More on Equivalence Relations

- It is easy to recognize equivalence relations using digraphs.
- The equivalence class of a particular element forms a universal relation (contains all possible edges) between the elements in the equivalence class.
- Example: All possible equivalence relations on a set *A* with 3 elements:



# Theorem on Equivalence Relation

- **Theorem:** Let R be an equivalence relation on *A*. Then either i) $a\ R\ b$ or ii) $[a] = [b]$ or iii) $[a] \cap [b] \neq \emptyset$
- **Proof:** We first show that i) → ii). Assume that $a\ R\ b$. We will prove that [a] = [b] by showing $[a] \subseteq [b]$ and $[b] \subseteq [a]$.
  - Suppose $c \in [a]$. Then $a\ R\ c$. Because R is symmetric, $a\ R\ b$ implies that there is $b\ R\ a$.
  - Furthermore, because R is transitive and $b\ R\ a$ and $a\ R\ c$, it follows that $b\ R\ c$. Hence $c \in [b]$. Which prove that $[a] \subseteq [b]$. Similarly we can prove $[b] \subseteq [a]$.
- Second, we show ii) → iii). Assume that [a] = [b]. It follows that $[a] \cap [b] \neq \emptyset$ because [a] is non-empty ($a \in [a]$ because R is reflexive)
- Next we will show that iii) → i).
  - Suppose that $[a] \cap [b] \neq \emptyset$. Then there is an element c with $c \in [a]$ and $c \in [b]$. In other words, $a\ R\ c$ and $b\ R\ c$. By the symmetric property $c\ R\ b$. Then by transitivity, because $a\ R\ c$ and $c\ R\ b$, then $a\ R\ b$.
- Because i) → ii); ii) → iii) and iii) → i), these three statements are equivalent.

# Example equivalence classes

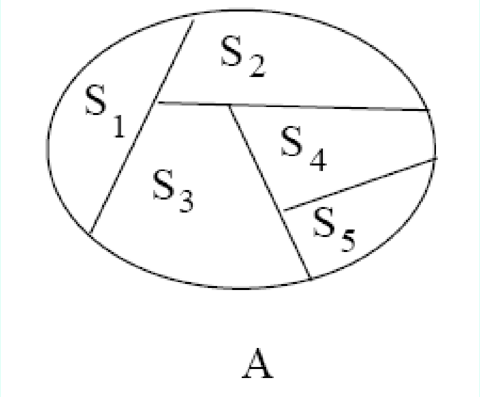
- Consider the relation $R = \{ (a,b) \mid \text{a mod } 2 = \text{b mod } 2 \}$ on the set of integers
  - Thus, all the even numbers are related to each other
  - As are the odd numbers
- The even numbers form an equivalence class
  - As do the odd numbers
- The equivalence class for the even numbers is denoted by [2] (or [4], or [784], etc.)
  - $[2] = \{ \ldots, -4, -2, 0, 2, 4, \ldots \}$
  - 2 is a *representative* of its equivalence class
- There are only 2 equivalence classes formed by this equivalence relation

# Example on equivalence classes

- Consider the relation
  $R = \{ (a,b) \mid a = b \text{ or } a = -b \}$
  - Thus, every number is related to additive inverse
- The equivalence class for an integer $a$:
  - $[7] = \{ 7, -7 \}$
  - $[0] = \{ 0 \}$
  - $[a] = \{ a, -a \}$
- There are an infinite number of equivalence classes formed by this equivalence relation

# Partition of a Set

- **Definition:** Let $S_1, S_2, \ldots, S_n$ be a collection of subsets of a set $A$. Then the collection forms a ***partition*** of $A$ if the subsets are nonempty, disjoint and *exhaust* $A$:
  - $S_i \neq \varnothing$
  - $S_i \cap S_j = \varnothing$ if $i \neq j$
  - $\cup S_i = A$




Note that { {}, {1,3}, {2} } is not a partition (it contains the empty set).
{ {1,2}, {2, 3} } is not a partition because the subsets are not disjoint.
{ {1}, {2} } is not a partition of {1, 2, 3} because none of its blocks contains 3.

# Rosen, section 8.5, question 1

- Which of these relations on {0, 1, 2, 3} are equivalence relations? Determine the properties of an equivalence relation that the others lack

a) { (0,0), (1,1), (2,2), (3,3) }
  - Has all the properties, thus, is an equivalence relation

b) { (0,0), (0,2), (2,0), (2,2), (2,3), (3,2), (3,3) }
  - Not reflexive: (1,1) is missing
  - Not transitive: (0,2) and (2,3) are in the relation, but not (0,3)

c) { (0,0), (1,1), (1,2), (2,1), (2,2), (3,3) }
  - Has all the properties, thus, is an equivalence relation

d) { (0,0), (1,1), (1,3), (2,2), (2,3), (3,1), (3,2) (3,3) }
  - Not transitive: (1,3) and (3,2) are in the relation, but not (1,2)

e) { (0,0), (0,1) (0,2), (1,0), (1,1), (1,2), (2,0), (2,2), (3,3) }
  - Not symmetric: (1,2) is present, but not (2,1)
  - Not transitive: (2,0) and (0,1) are in the relation, but not (2,1)

# Rosen, Section 8.5, question 9

- Suppose that $A$ is a non-empty set, and $f$ is a function that has $A$ as its domain. Let $R$ be the relation on $A$ consisting of all ordered pairs $(x,y)$ where $f(x) = f(y)$
  - Meaning that $x$ and $y$ are related if and only if $f(x) = f(y)$
- Show that $R$ is an equivalence relation on $A$

- Reflexivity: $f(x) = f(x)$
  - True, as given the same input, a function always produces the same output
- Symmetry: if $f(x) = f(y)$ then $f(y) = f(x)$
  - True, by the definition of equality
- Transitivity: if $f(x) = f(y)$ and $f(y) = f(z)$ then $f(x) = f(z)$
  - True, by the definition of equality

# Rosen, section 8.5, question 44

- Which of the following are partitions of the set of integers?

a) The set of even integers and the set of odd integers
   - Yes, it's a valid partition

b) The set of positive integers and the set of negative integers
   - No: 0 is in neither set

c) The set of integers divisible by 3, the set of integers leaving a remainder of 1 when divided by 3, and the set of integers leaving a remaineder of 2 when divided by 3
   - Yes, it's a valid partition

d) The set of integers less than -100, the set of integers with absolute value not exceeding 100, and the set of integers greater than 100
   - Yes, it's a valid partition

e) The set of integers not divisible by 3, the set of even integers, and the set of integers that leave a remainder of 3 when divided by 6
   - The first two sets are not disjoint (2 is in both), so it's not a valid partition

# Thank You

- Study all the solved problem from your text book.
- Try to solve related problems from exercise.
- Text from Rosen 8.5