Image Restoration

Preview

- Goal of image restoration
  - Improve an image in some predefined sense
  - Difference with image enhancement ?
- Features
  - Image restoration v.s image enhancement
  - Objective process v.s. subjective process
  - A prior knowledge v.s heuristic process
  - A prior knowledge of the degradation phenomenon is considered
  - Modeling the degradation and apply the inverse process to recover the original image

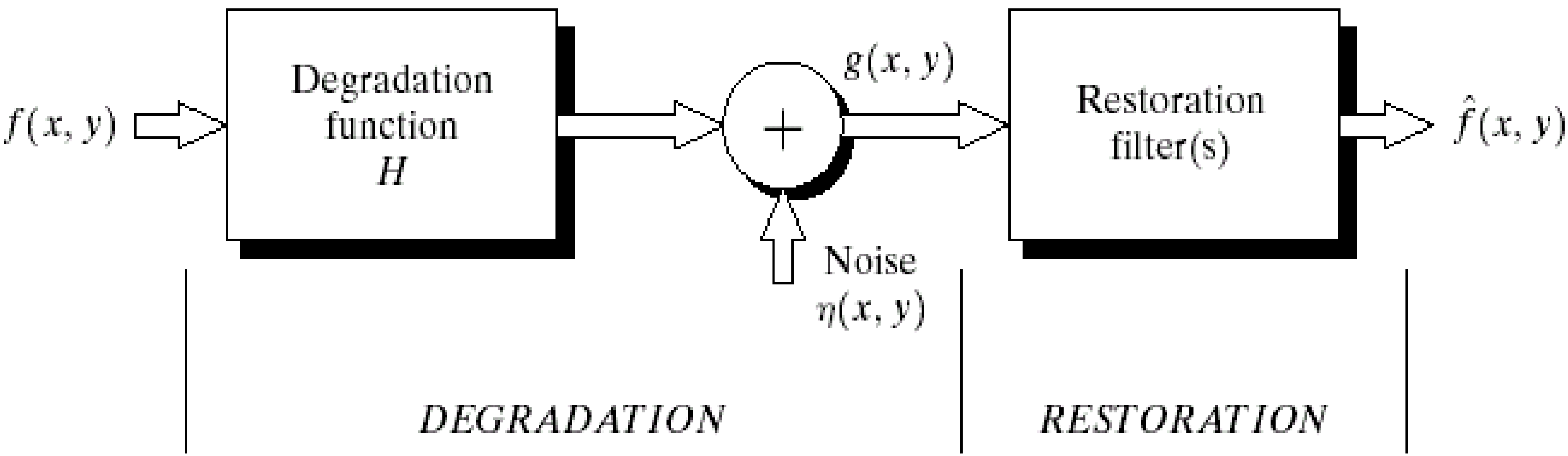
Preview (cont.)

- Target
  - Degraded digital image
  - Sensor, digitizer, display degradations are less considered
- Spatial domain approach
- Frequency domain approach

Outline

- A model of the image degradation / restoration process
- Noise models
- Restoration in the presence of noise only – spatial filtering
- Periodic noise reduction by frequency domain filtering
- Linear, position-invariant degradations
- Estimating the degradation function
- Inverse filtering

A model of the image degradation/restoration process



$$
\begin{cases}
g(x,y)=f(x,y)*h(x,y)+\eta(x,y) \\
G(u,v)=F(u,v)H(u,v)+N(u,v)
\end{cases}
$$

Noise models

- Source of noise
  - Image acquisition (digitization)
  - Image transmission
- Spatial properties of noise
  - Statistical behavior of the gray-level values of pixels
  - Noise parameters, correlation with the image
- Frequency properties of noise
  - Fourier spectrum
  - Ex. white noise (a constant Fourier spectrum)

Noise probability density functions

- Noises are taken as random variables
- Random variables
  - Probability density function (PDF)

Gaussian noise

- Math. tractability in spatial and frequency domain
- Electronic circuit noise and sensor noise

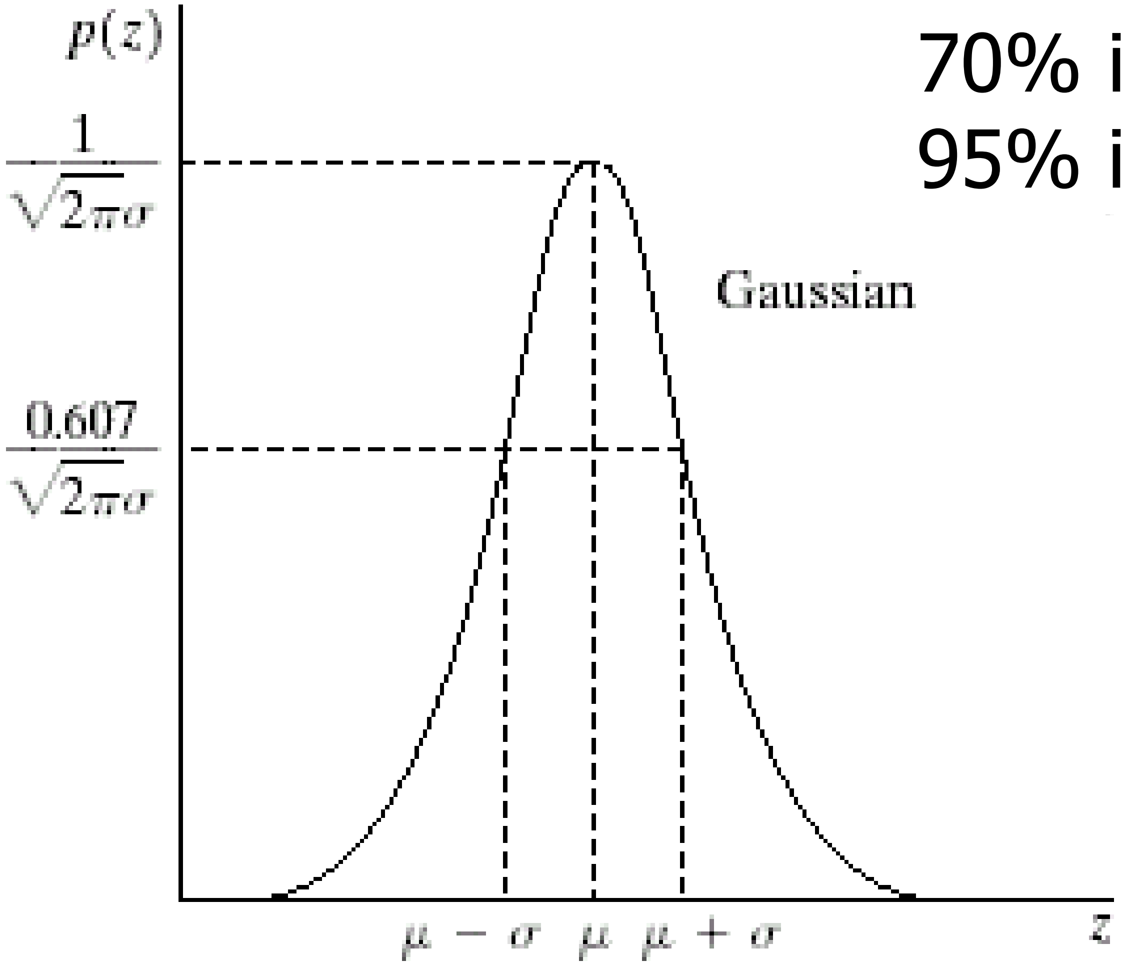
$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$

mean

variance

Note: $\int_{-\infty}^{\infty} p(z)dz = 1$

Gaussian noise (PDF)



70% in [(μ–σ), (μ+σ)]
95% in [(μ–2σ), (μ+2σ)]

Uniform noise

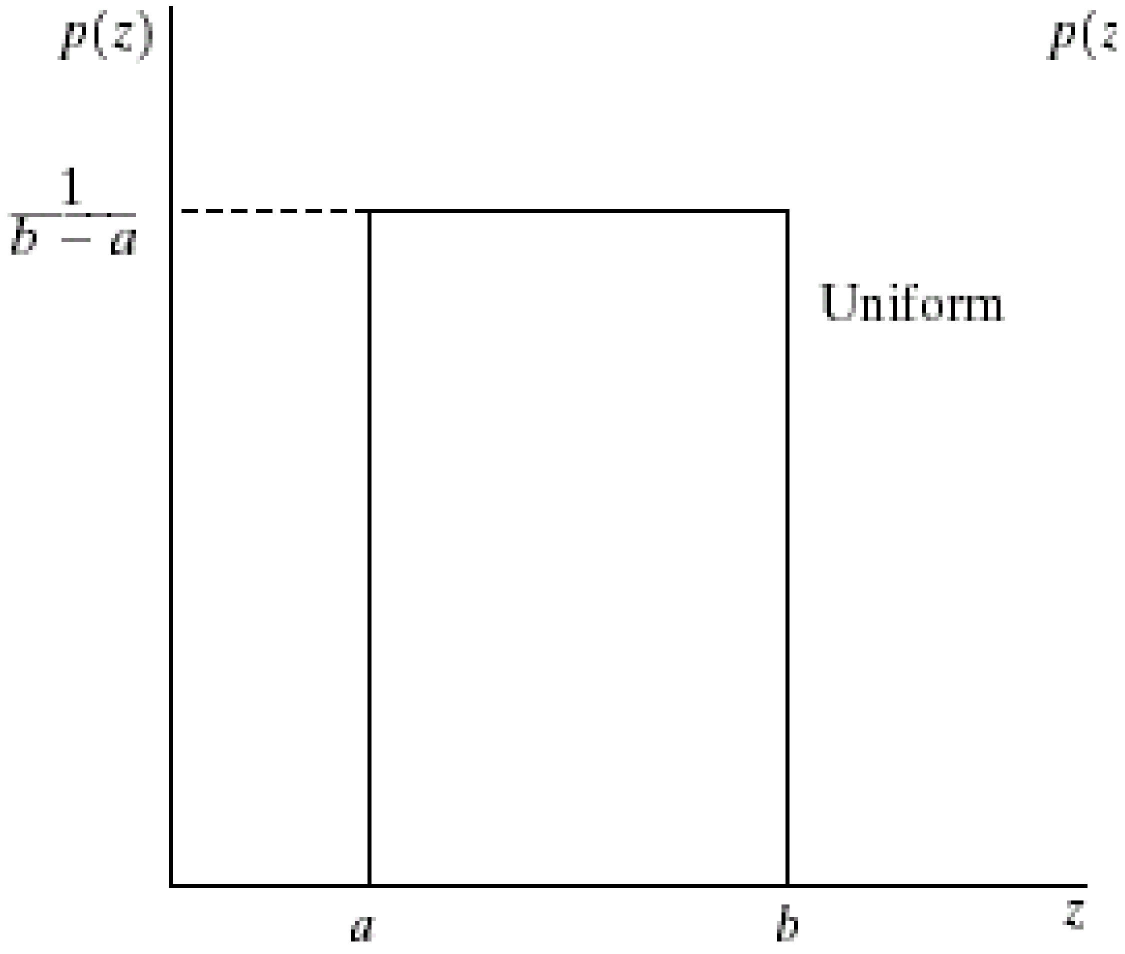
- Less practical, used for random number generator

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $$\mu = \frac{a+b}{2}$$

Variance: $$\sigma^2 = \frac{(b-a)^2}{12}$$

Uniform PDF



Impulse (salt-and-pepper) nosie

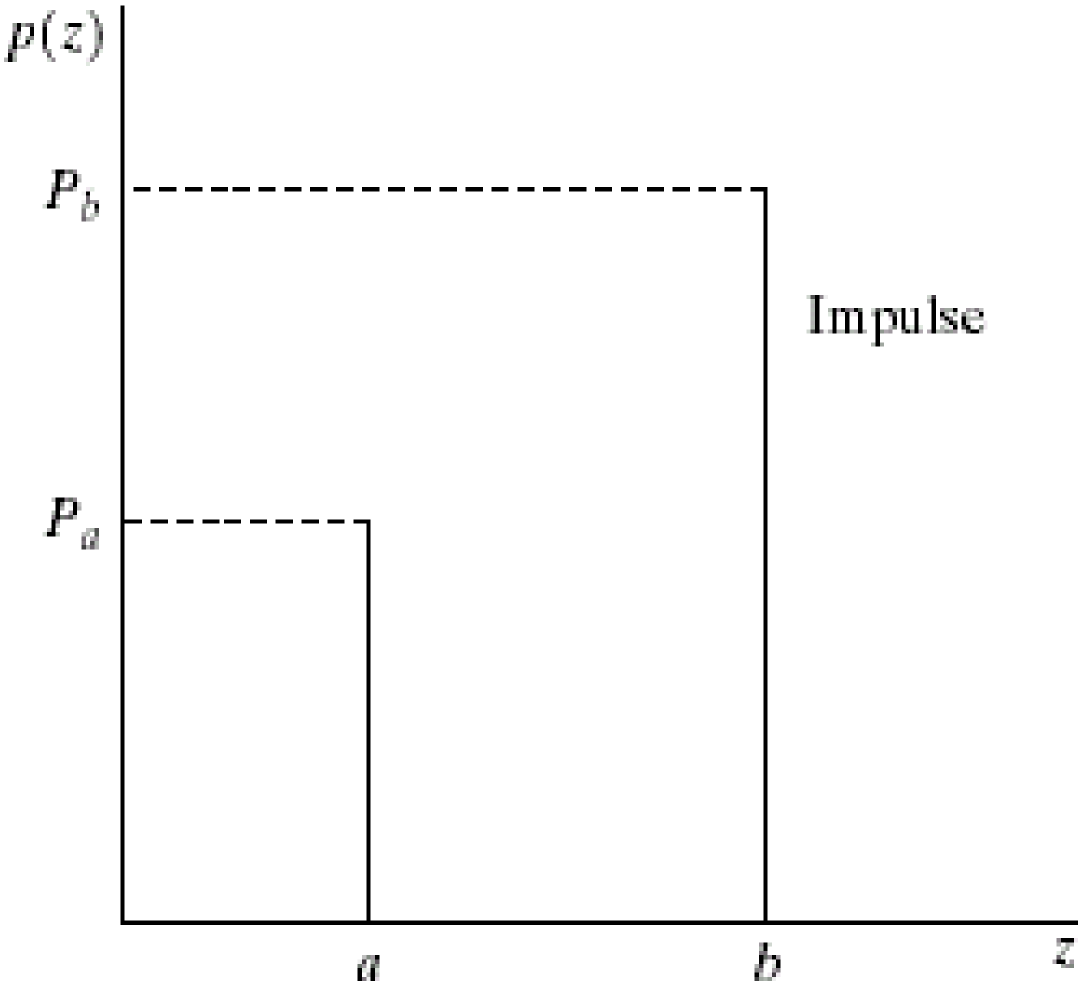
- Quick transients, such as faulty switching during imaging

$$p(z)=\begin{cases} P_a & \text{for } z=a \\ P_b & \text{for } z=b \\ 0 & \text{otherwise} \end{cases}$$

If either P_a or P_b is zero, it is called *unipolar.* Otherwise, it is called bipoloar.

•In practical, impulses are usually stronger than image signals. Ex., a=0(black) and b=255(white) in 8-bit image.

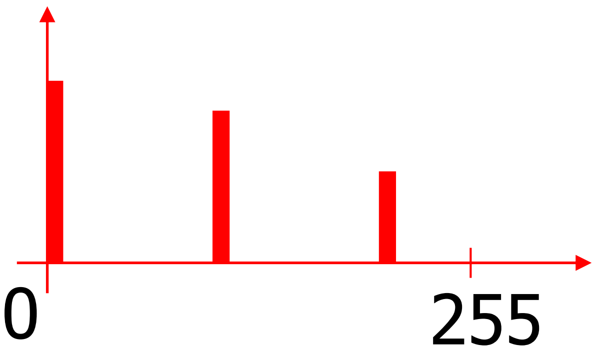
Impulse (salt-and-pepper) nosie PDF

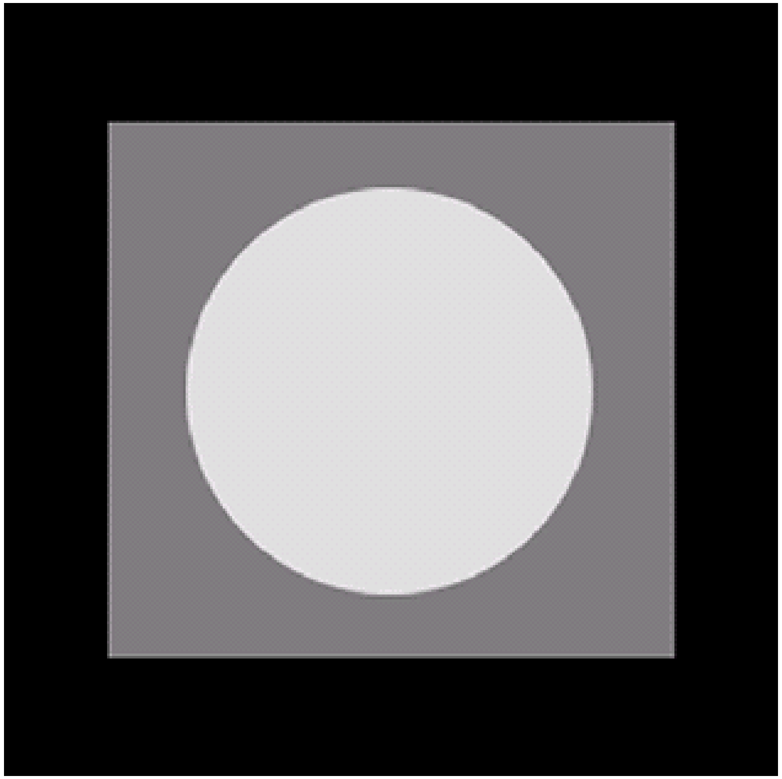


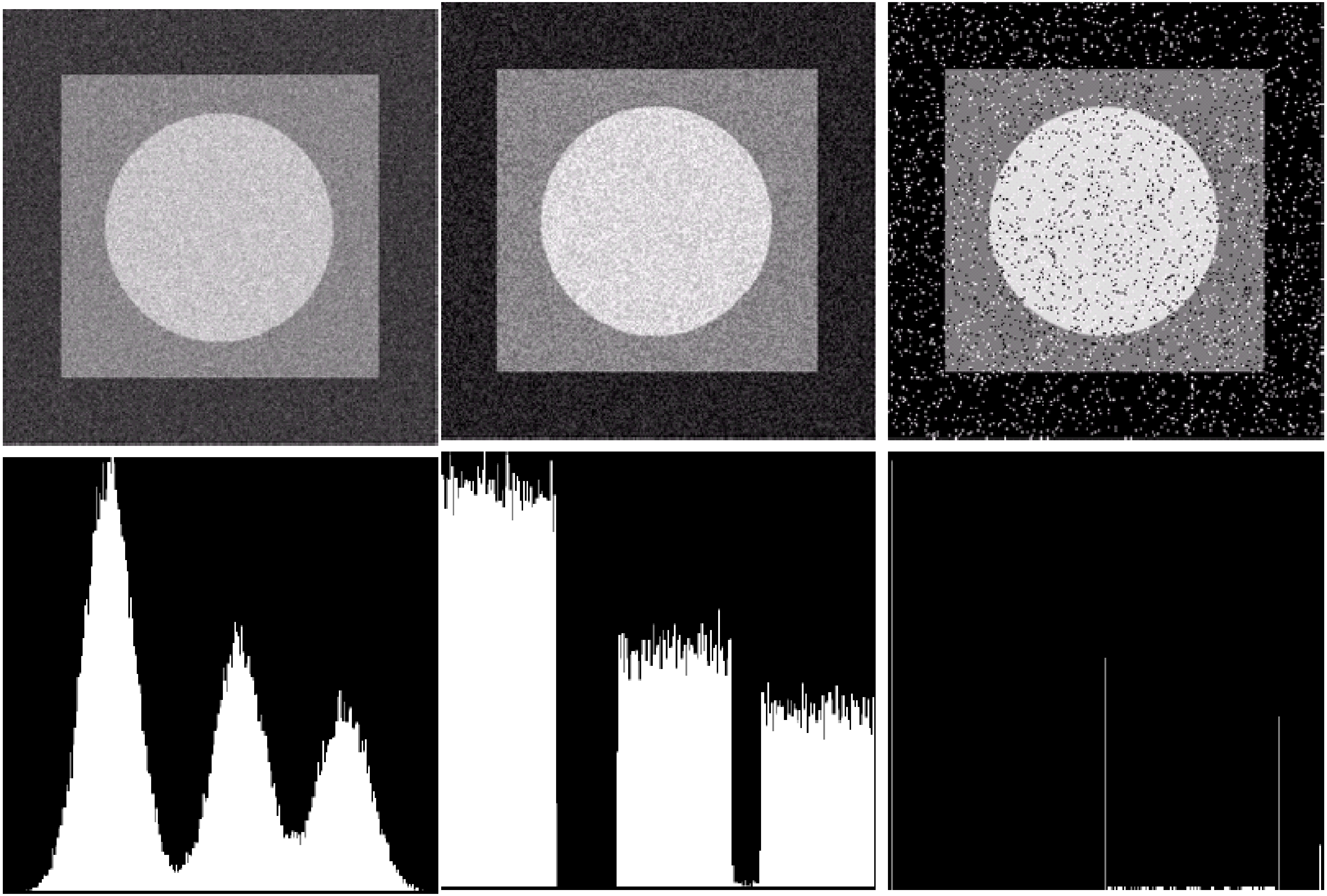
Test for noise behavior

- Test pattern

Its histogram:



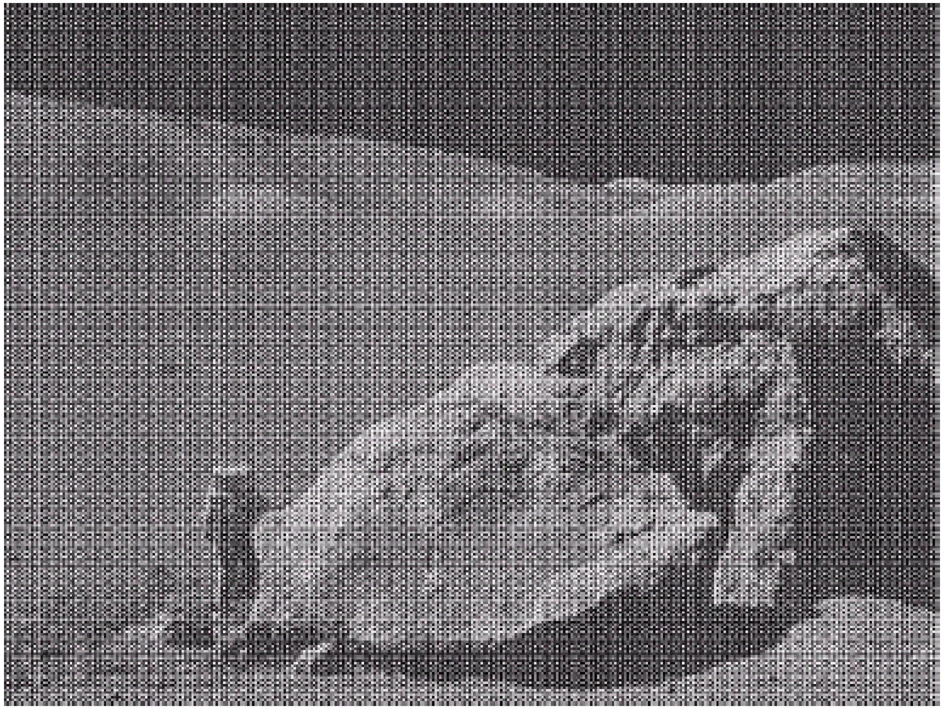


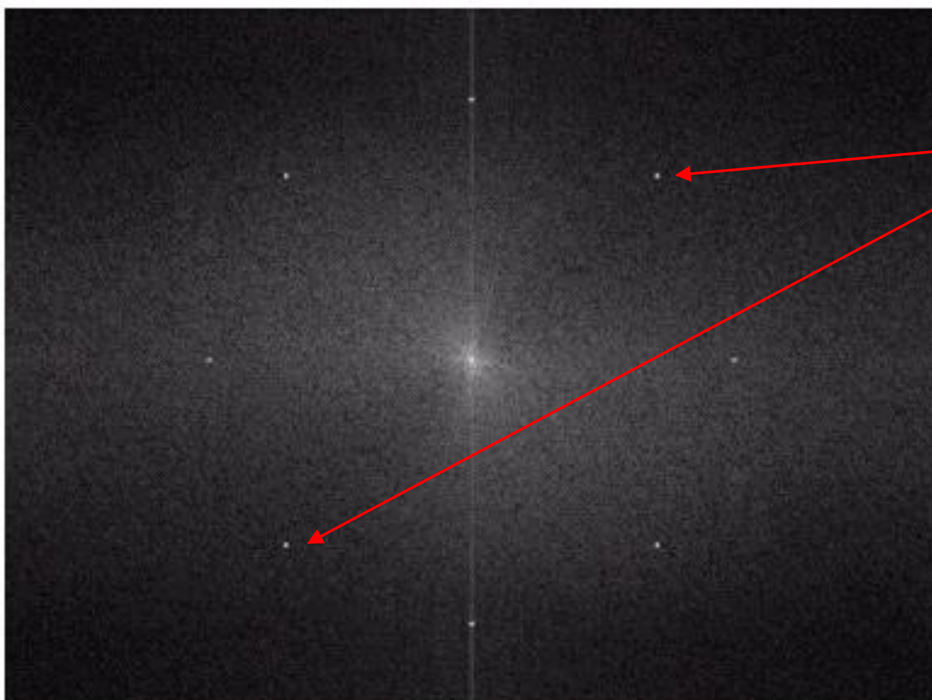


Gaussian | Uniform | Salt & Pepper

Periodic noise

- Arise from electrical or electromechanical interference during image acquisition
- Spatial dependence
- Observed in the frequency domain



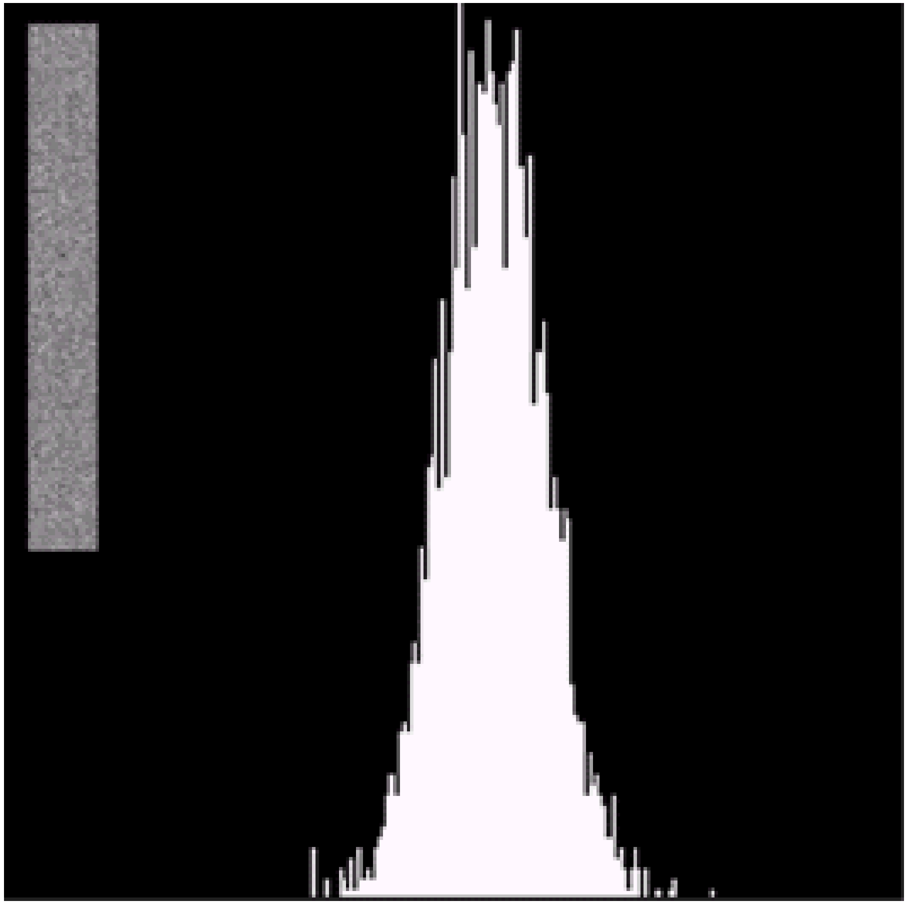


Sinusoidal noise: Complex conjugate pair in frequency domain

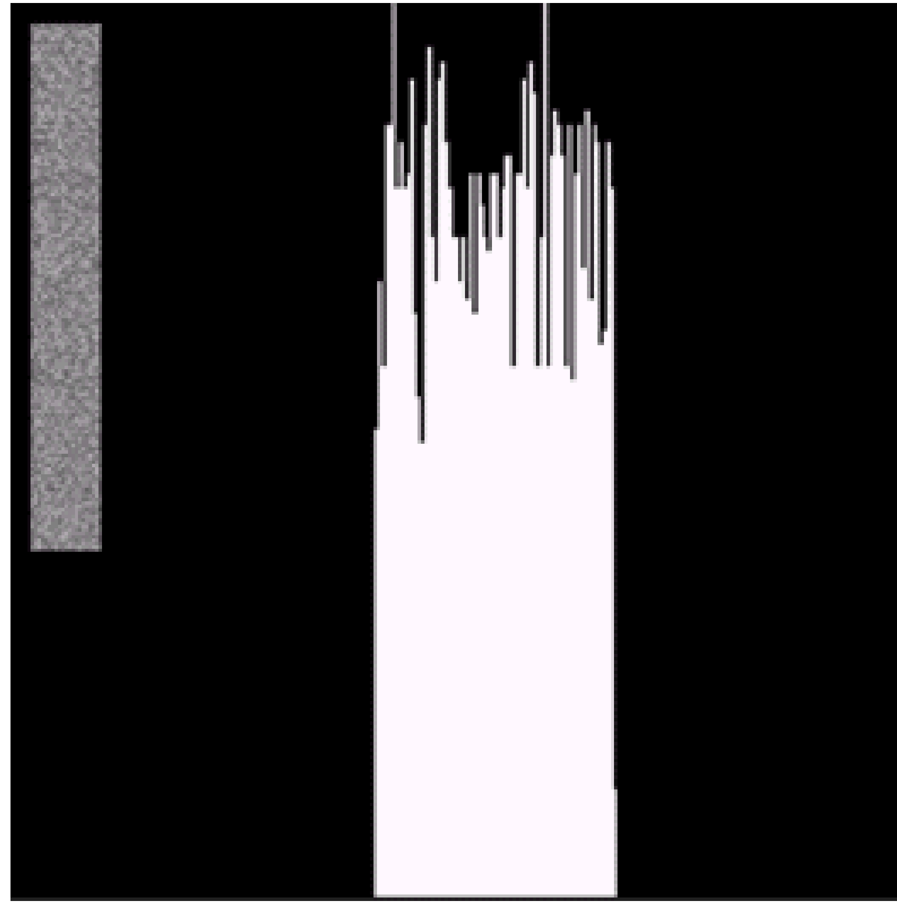
Estimation of noise parameters

- Periodic noise
  - Observe the frequency spectrum
- Random noise with unknown PDFs
  - Case 1: imaging system is available
    - Capture images of "flat" environment
  - Case 2: noisy images available
    - Take a strip from constant area
    - Draw the histogram and observe it
    - Measure the mean and variance

Observe the histogram



Gaussian



uniform

Measure the mean and variance

- Histogram is an estimate of PDF

$$\begin{cases} \mu = \sum_{z_i \in S} z_i \, p(z_i) \\ \sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 \, p(z_i) \end{cases}$$

$\Leftrightarrow$ $\begin{cases} \text{Gaussian: } \mu, \sigma \\ \text{Uniform: } a, b \end{cases}$

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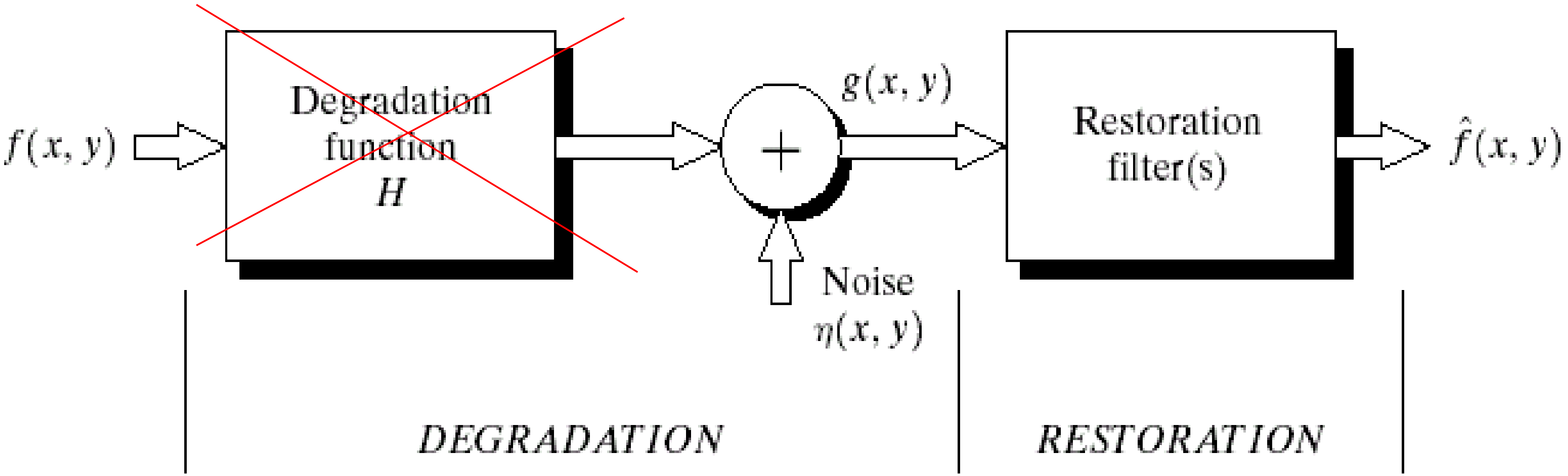
Additive noise only



$$\begin{cases} g(x,y)=f(x,y)+\eta(x,y) \\ G(u,v)=F(u,v)+N(u,v) \end{cases}$$

Spatial filters for de-noising additive noise

- Skills similar to image enhancement
- Mean filters
- Order-statistics filters
- Adaptive filters

Mean filters

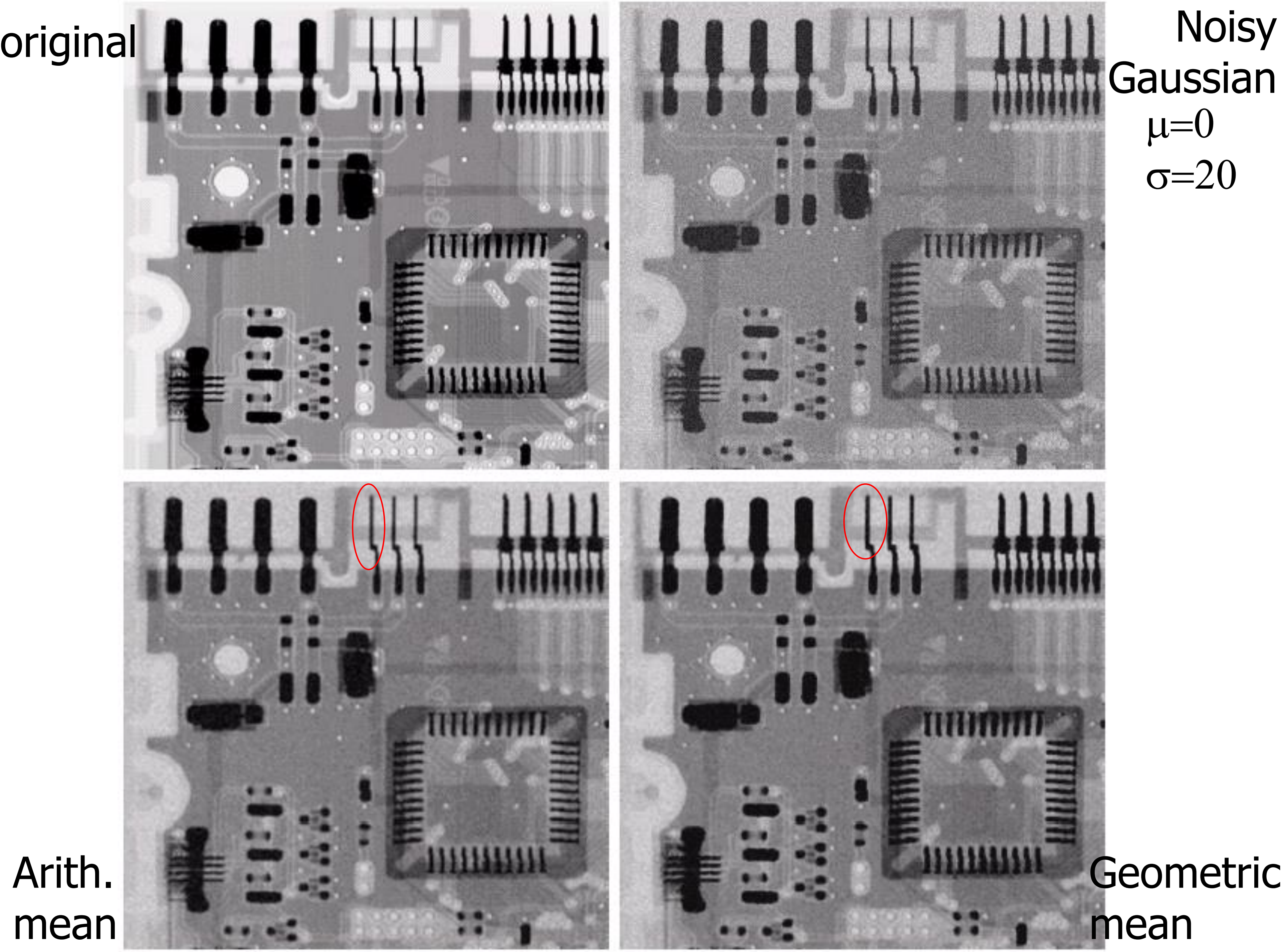
- Arithmetic mean

$$\hat{f}(x,y)=\frac{1}{mn}\sum_{(s,t)\in S_{xy}} g(s,t)$$

Window centered at (x,y)

- Geometric mean

$$\hat{f}(x,y)=\left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{1/mn}$$



original
Noisy
Gaussian
μ=0
σ=20
Arith.
mean
Geometric
mean

Mean filters (cont.)

- Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\displaystyle\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- Contra-harmonic mean filter

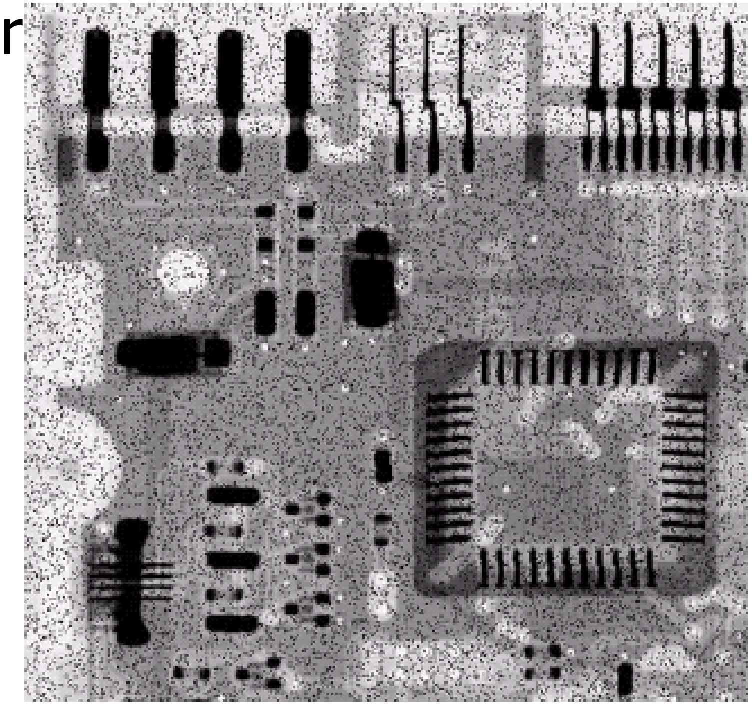
$$\hat{f}(x,y) = \frac{\displaystyle\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\displaystyle\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

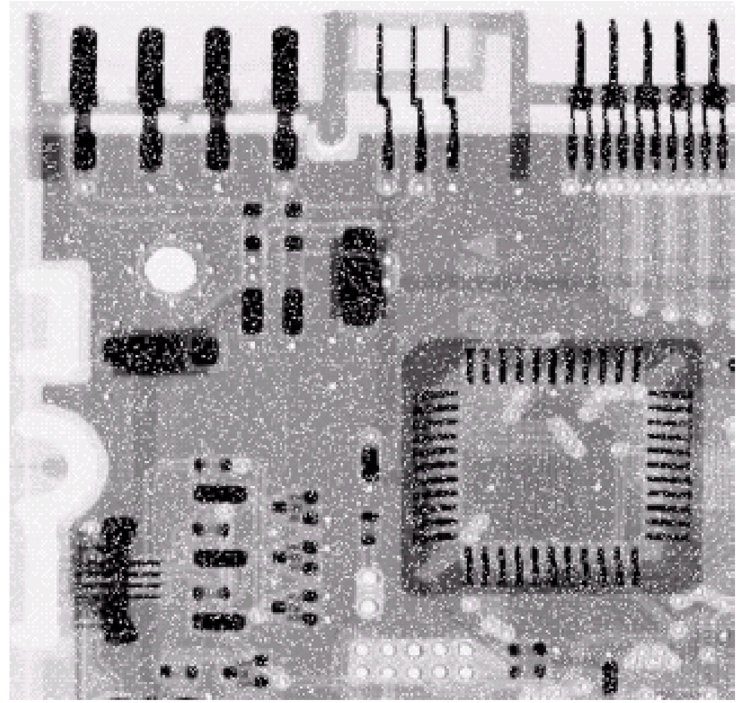
Q=-1, harmonic

Q=0, airth. mean

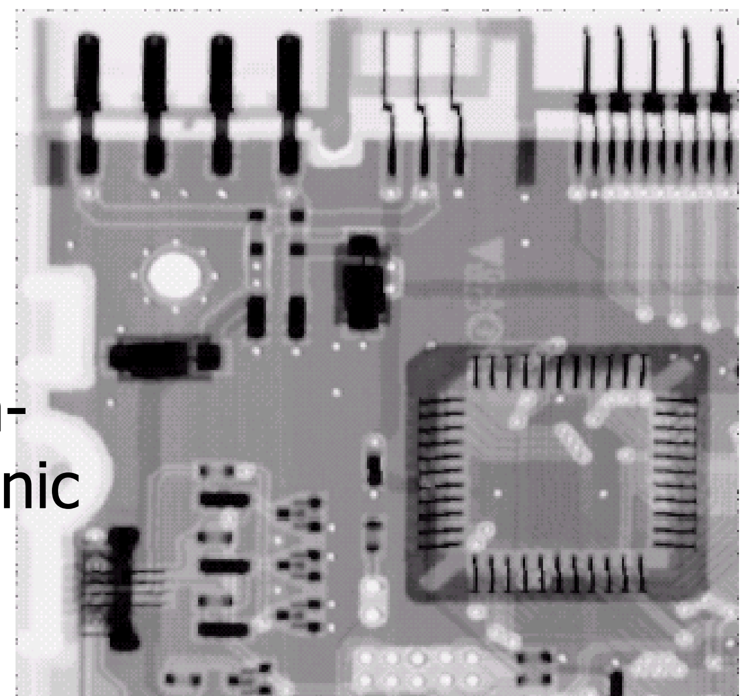
Q=+, ?

Pepper Noise
黑點

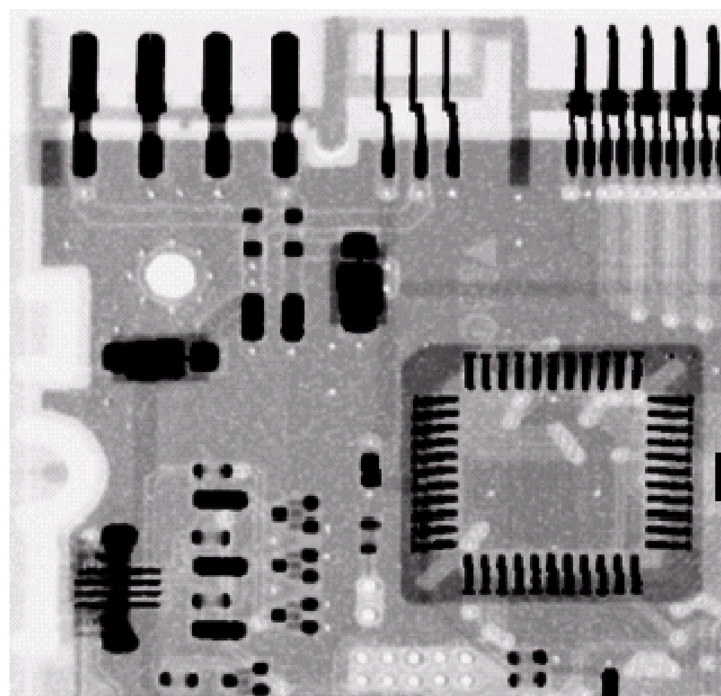




Salt Noise
白點

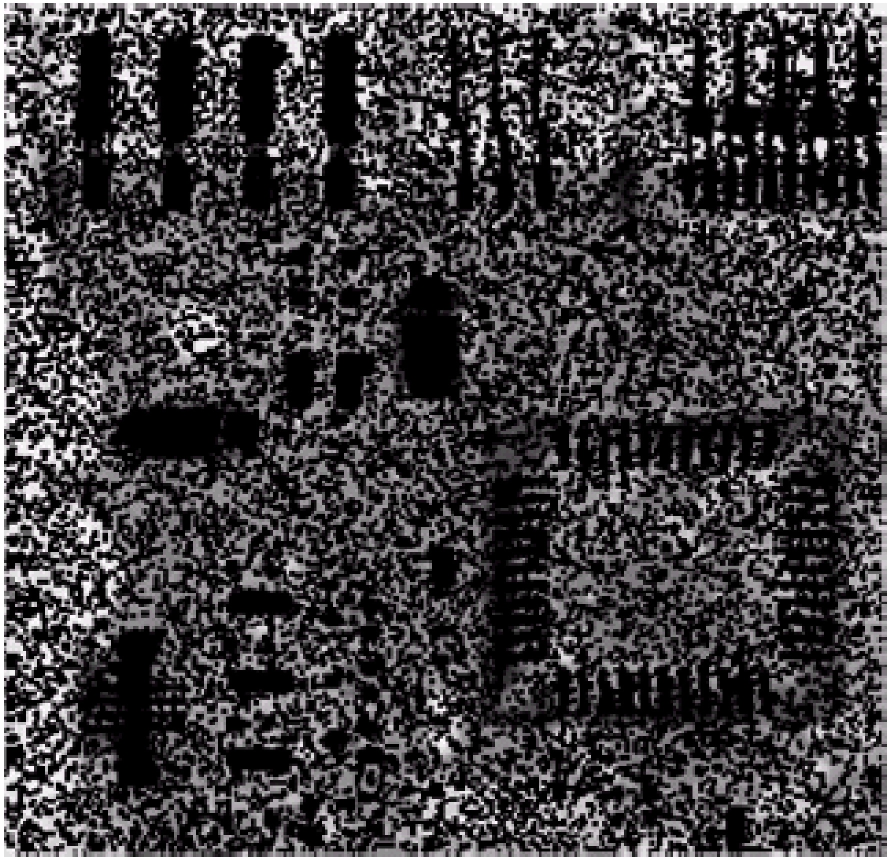


Contra-harmonic Q=1.5

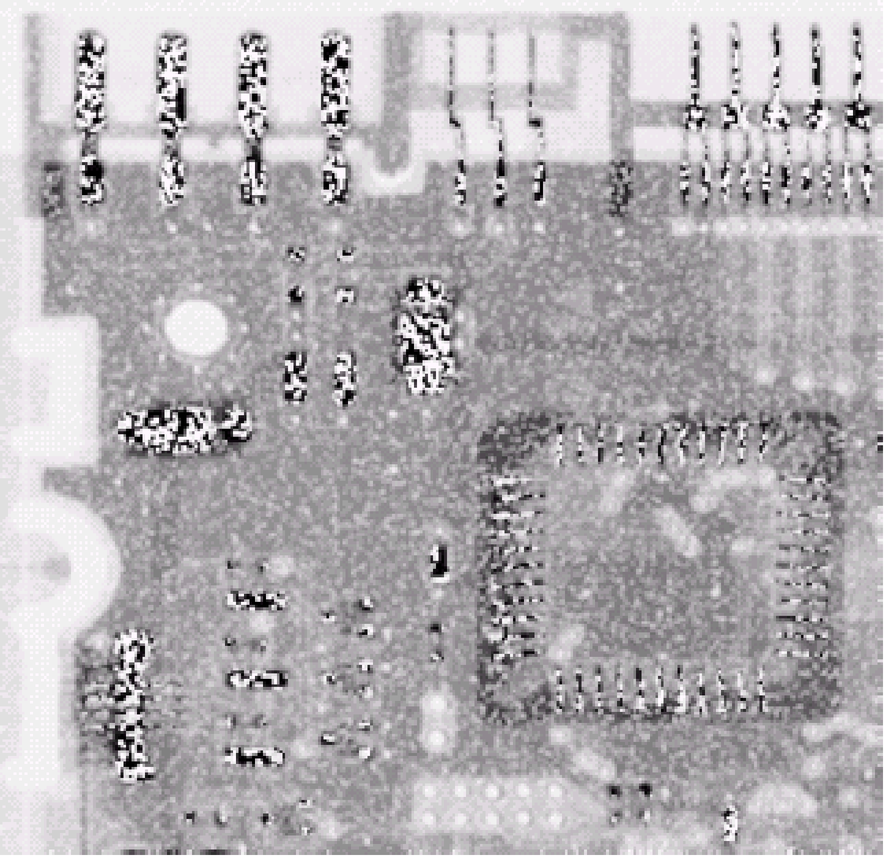


Contra-harmonic Q=-1.5

Wrong sign in contra-harmonic filtering



Q=-1.5



Q=1.5

Order-statistics filters

- Based on the ordering(ranking) of pixels
  - Suitable for unipolar or bipolar noise (salt and pepper noise)
- Median filters
- Max/min filters
- Midpoint filters
- Alpha-trimmed mean filters

Order-statistics filters

- Median filter

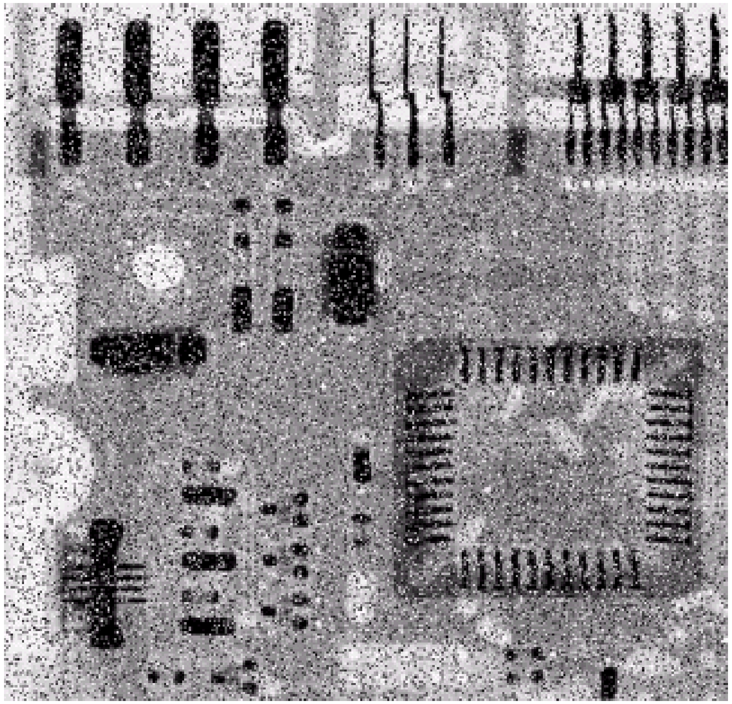
$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

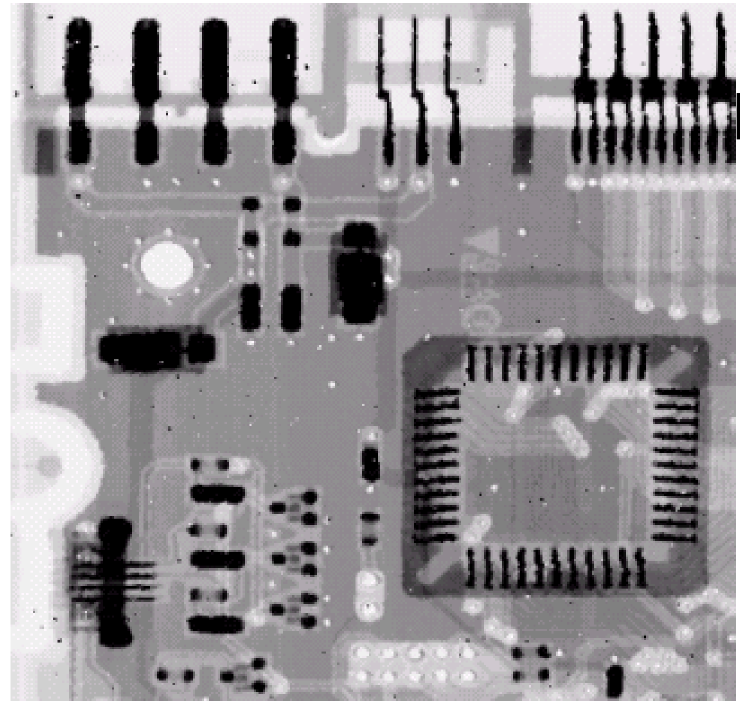
- Max/min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

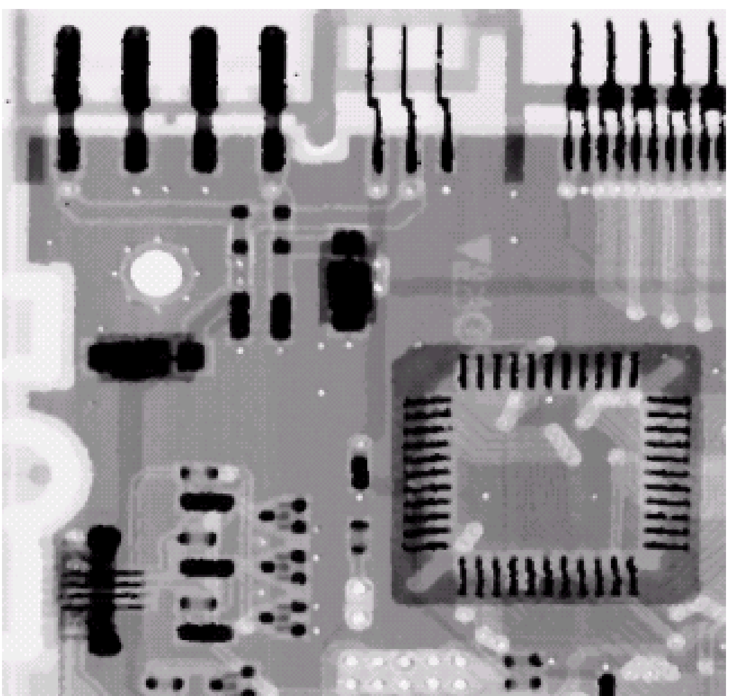
bipolar Noise
$P_a = 0.1$
$P_b = 0.1$

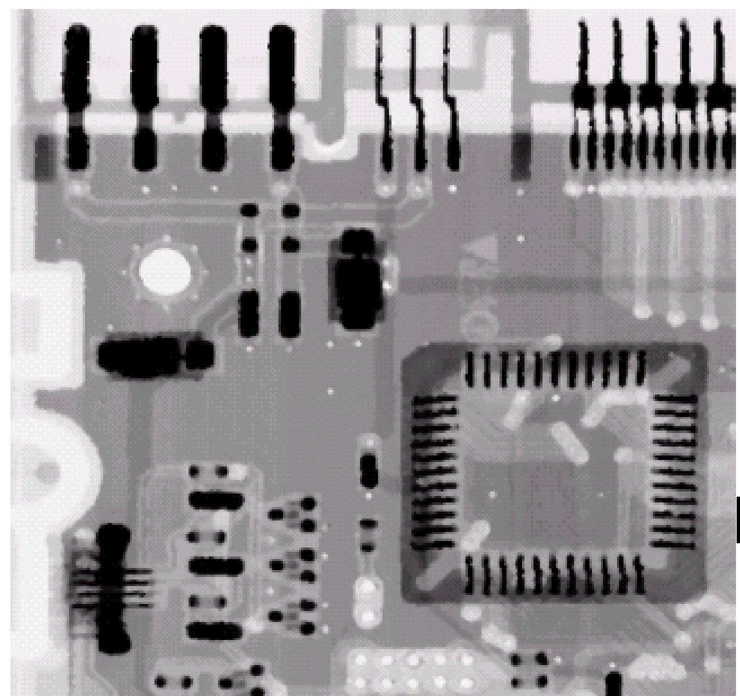




3x3 Median Filter Pass 1

3x3 Median Filter Pass 2

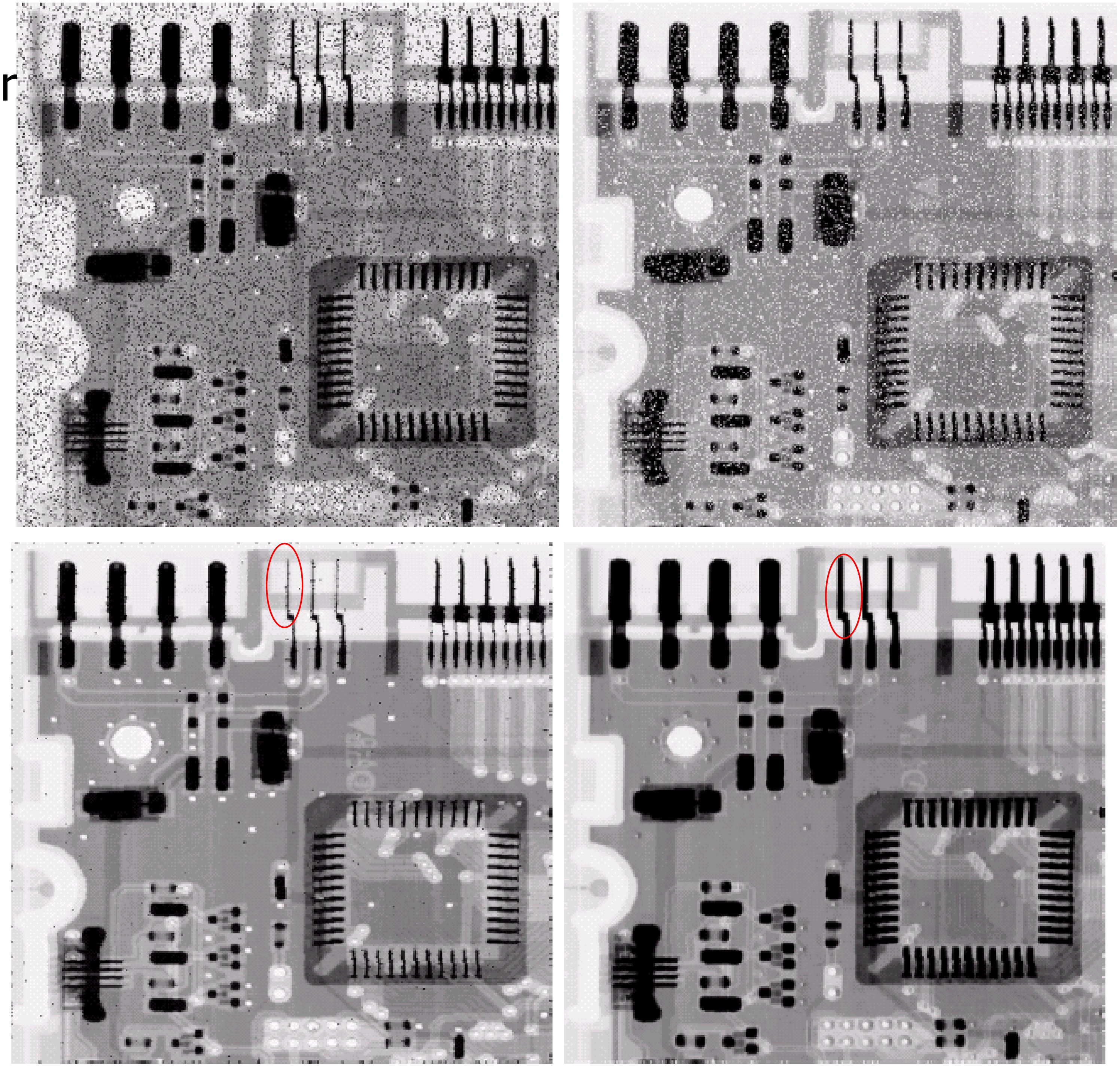




3x3 Median Filter Pass 3

Pepper noise

Salt noise



Max filter

Min filter

Order-statistics filters (cont.)

- Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\} \right]$$

- Alpha-trimmed mean filter
  - Delete the d/2 lowest and d/2 highest gray-level pixels

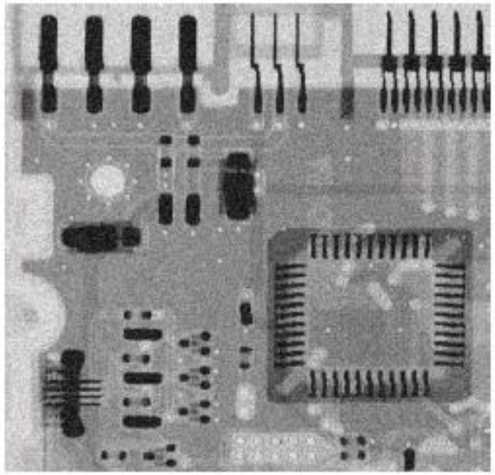
$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

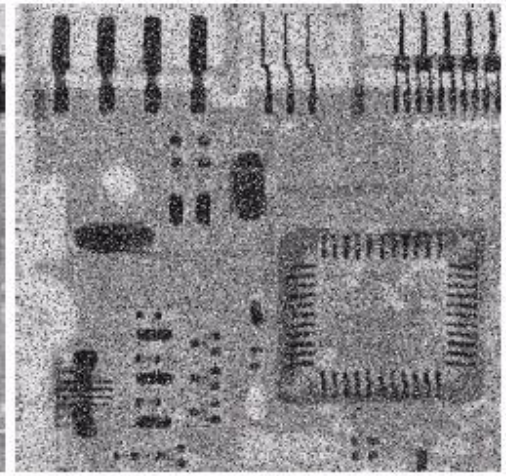
Middle (mn-d) pixels

Uniform noise

$\mu=0$

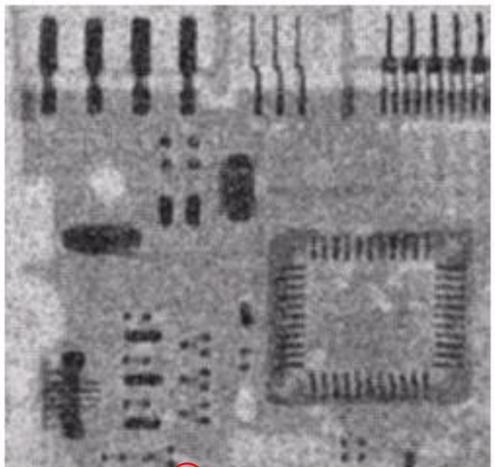
$\sigma^2=800$

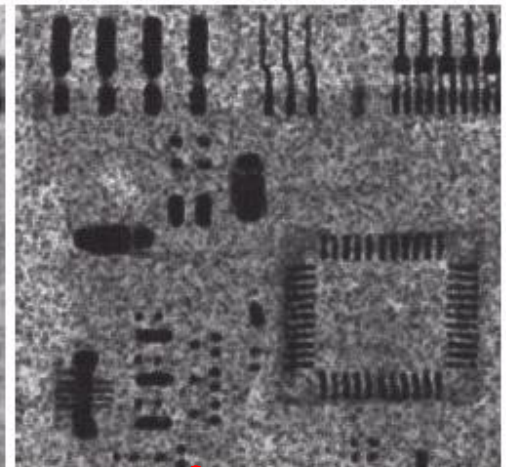




Left +
Bipolar Noise
$P_a = 0.1$
$P_b = 0.1$

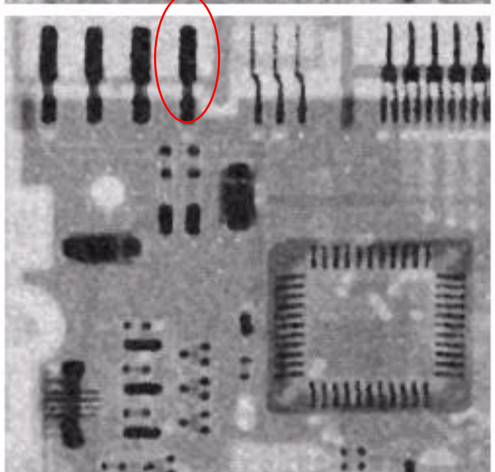
5x5
Arith. Mean
filter

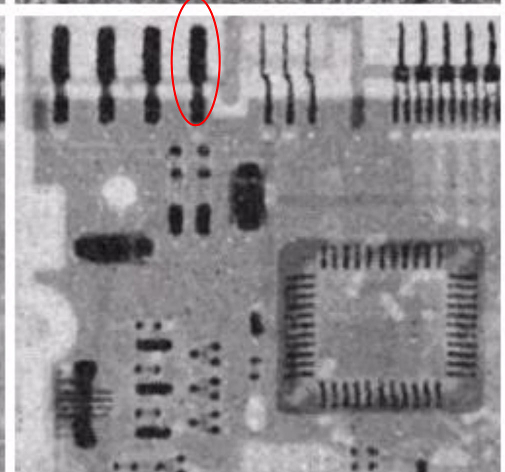




5x5
Geometric
mean

5x5
Median
filter





5x5
Alpha-trim.
Filter
d=5

Adaptive filters

- Adapted to the behavior based on the statistical characteristics of the image inside the filter region S_{xy}
- Improved performance v.s increased complexity
- Example: Adaptive local noise reduction filter

Adaptive local noise reduction filter

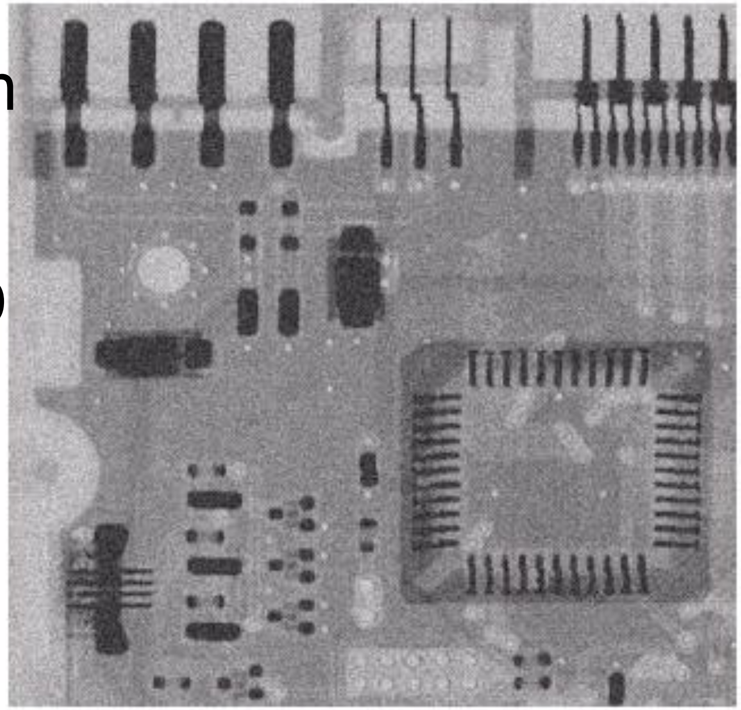
- Simplest statistical measurement
  - Mean and variance
- Known parameters on local region S_{xy}
  - g(x,y): noisy image pixel value
  - σ^2_η: noise variance (assume known a prior)
  - m_L : local mean
  - σ^2_L : local variance

Adaptive local noise reduction filter (cont.)

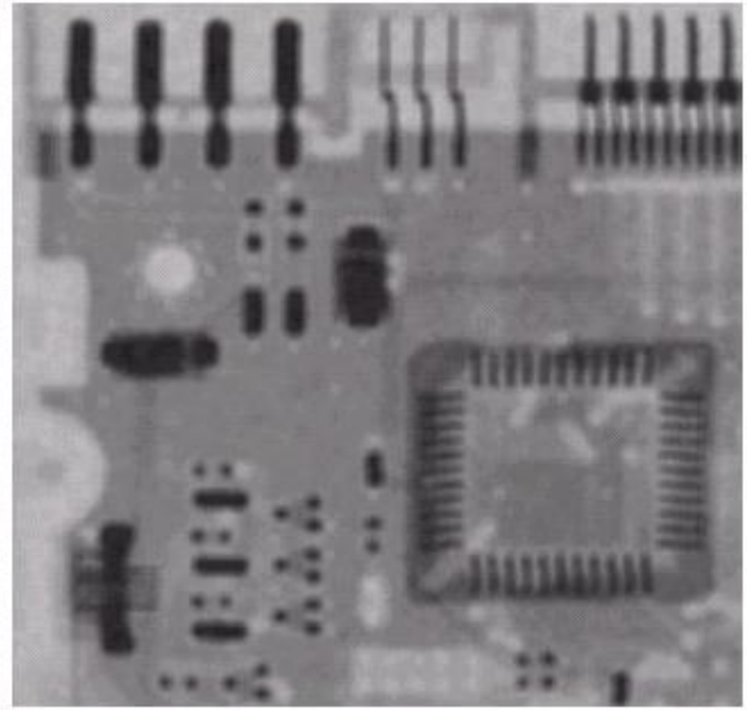
- Analysis: we want to do
  - If σ^2_η is zero, return g(x,y)
  - If $\sigma^2_L > \sigma^2_\eta$, return value close to g(x,y)
  - If $\sigma^2_L = \sigma^2_\eta$, return the arithmetic mean m_L
- Formula

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

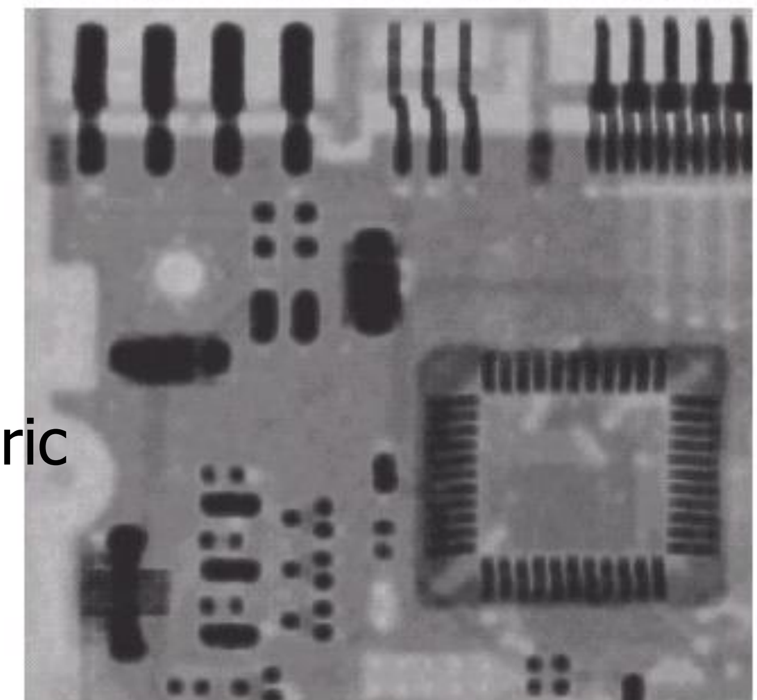
Gaussian noise $\mu=0$ $\sigma^2=1000$

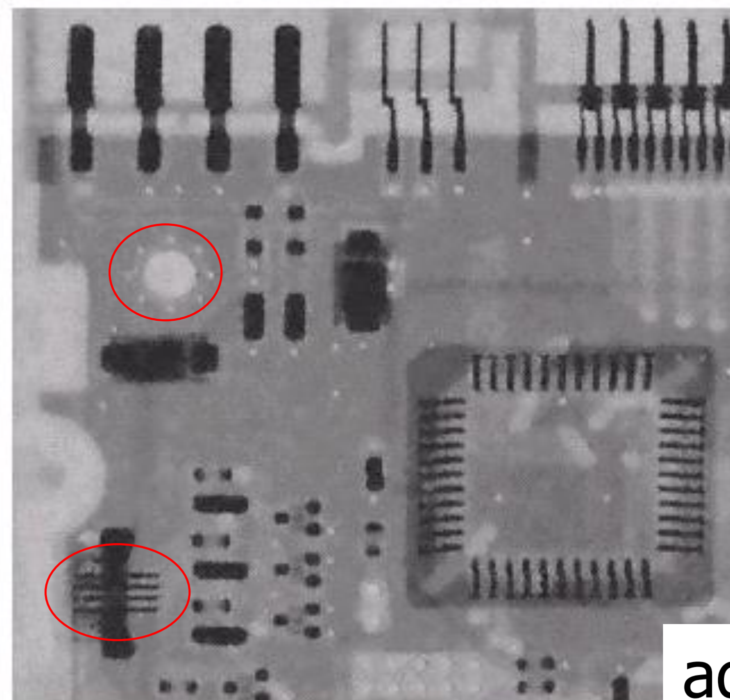


Arith. mean 7x7



Geometric mean 7x7





adaptive

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Periodic noise reduction

- Pure sine wave
  - Appear as a pair of impulse (conjugate) in the frequency domain

$$\begin{cases} f(x,y) = A\sin(u_0 x + v_0 y) \\ F(u,v) = -j\dfrac{A}{2}\left[\delta(u - \dfrac{u_0}{2\pi}, v - \dfrac{v_0}{2\pi}) - \delta(u + \dfrac{u_0}{2\pi}, v + \dfrac{v_0}{2\pi})\right] \end{cases}$$

Periodic noise reduction (cont.)

- Bandreject filters
- Bandpass filters
- Notch filters
- Optimum notch filtering

Bandreject filters

* Reject an isotropic frequency

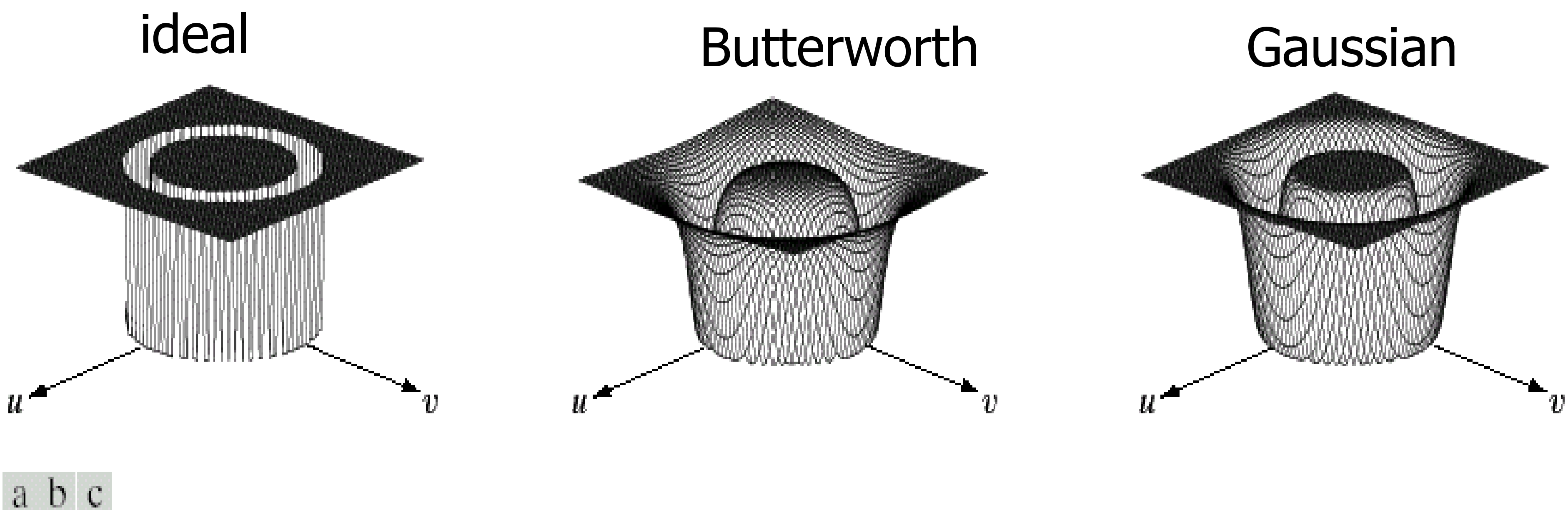
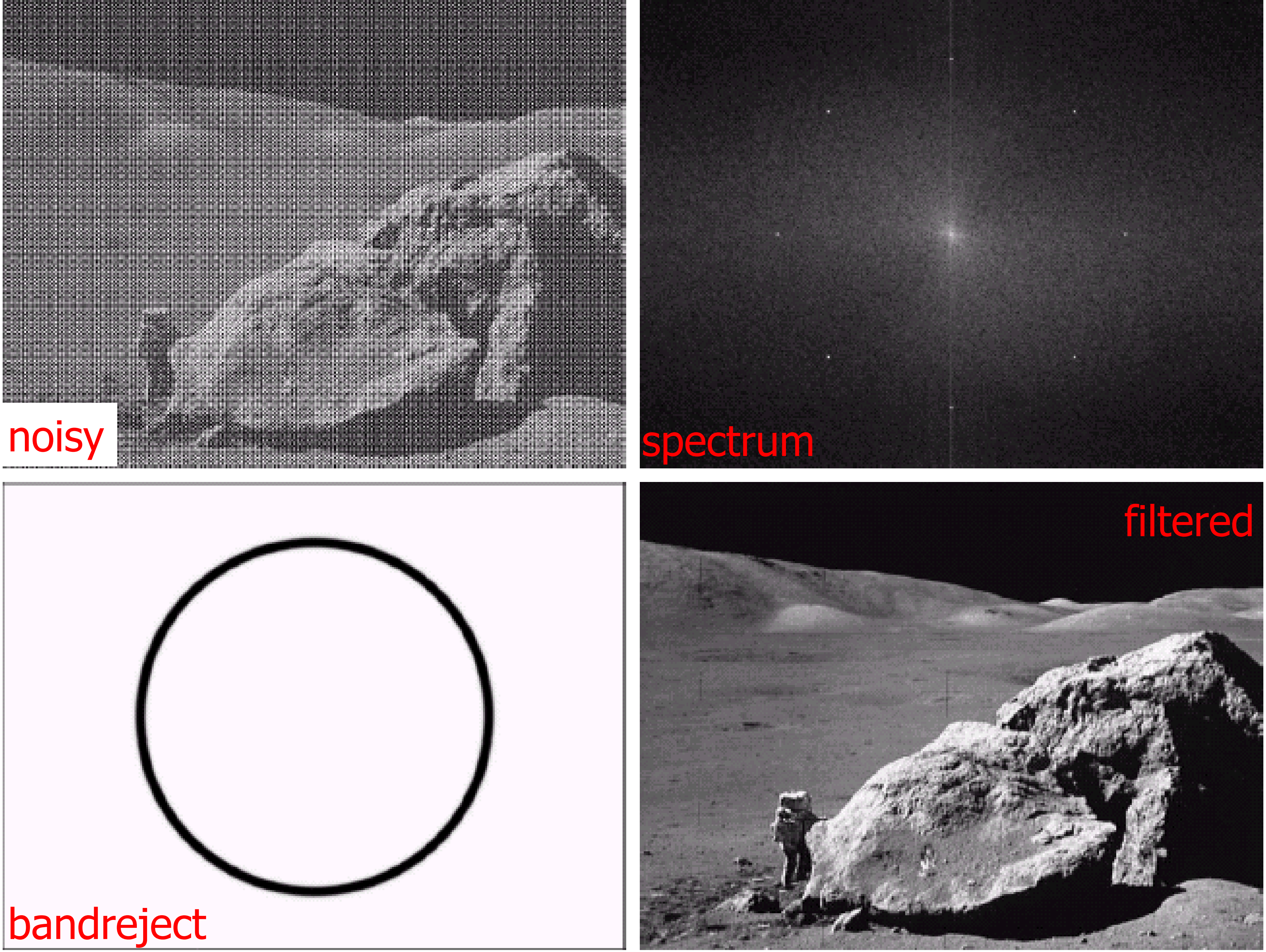
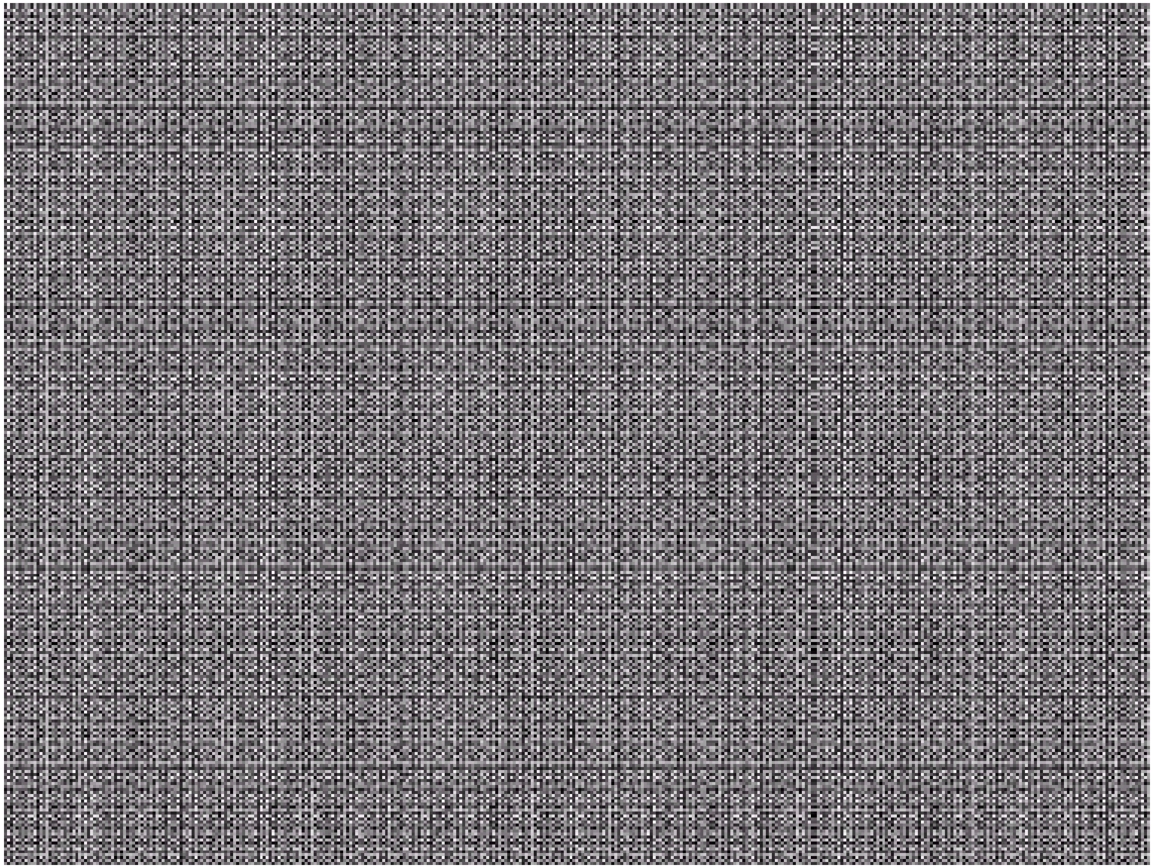


FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.


noisy
spectrum
bandreject
filtered

Bandpass filters

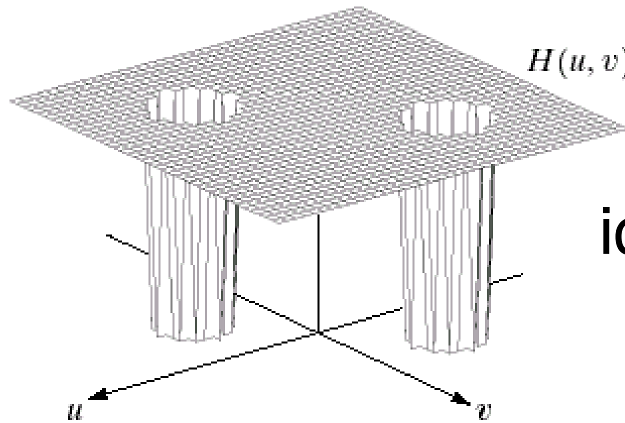
- $H_{bp}(u,v)=1- H_{br}(u,v)$



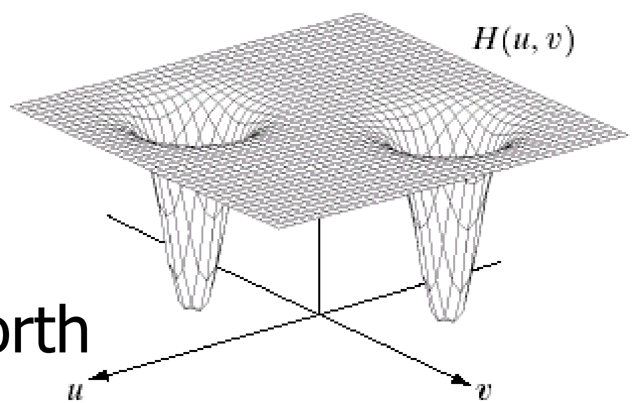
$$\Im^{-1}\left\{G(u,v)H_{bp}(u,v)\right\}$$

Notch filters

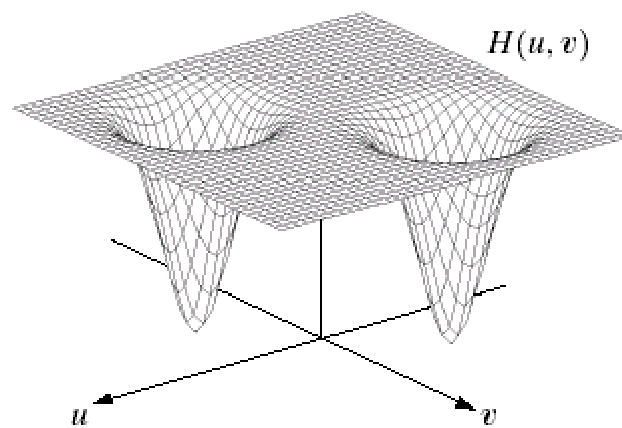
- Reject(or pass) frequencies in predefined neighborhoods about a center frequency



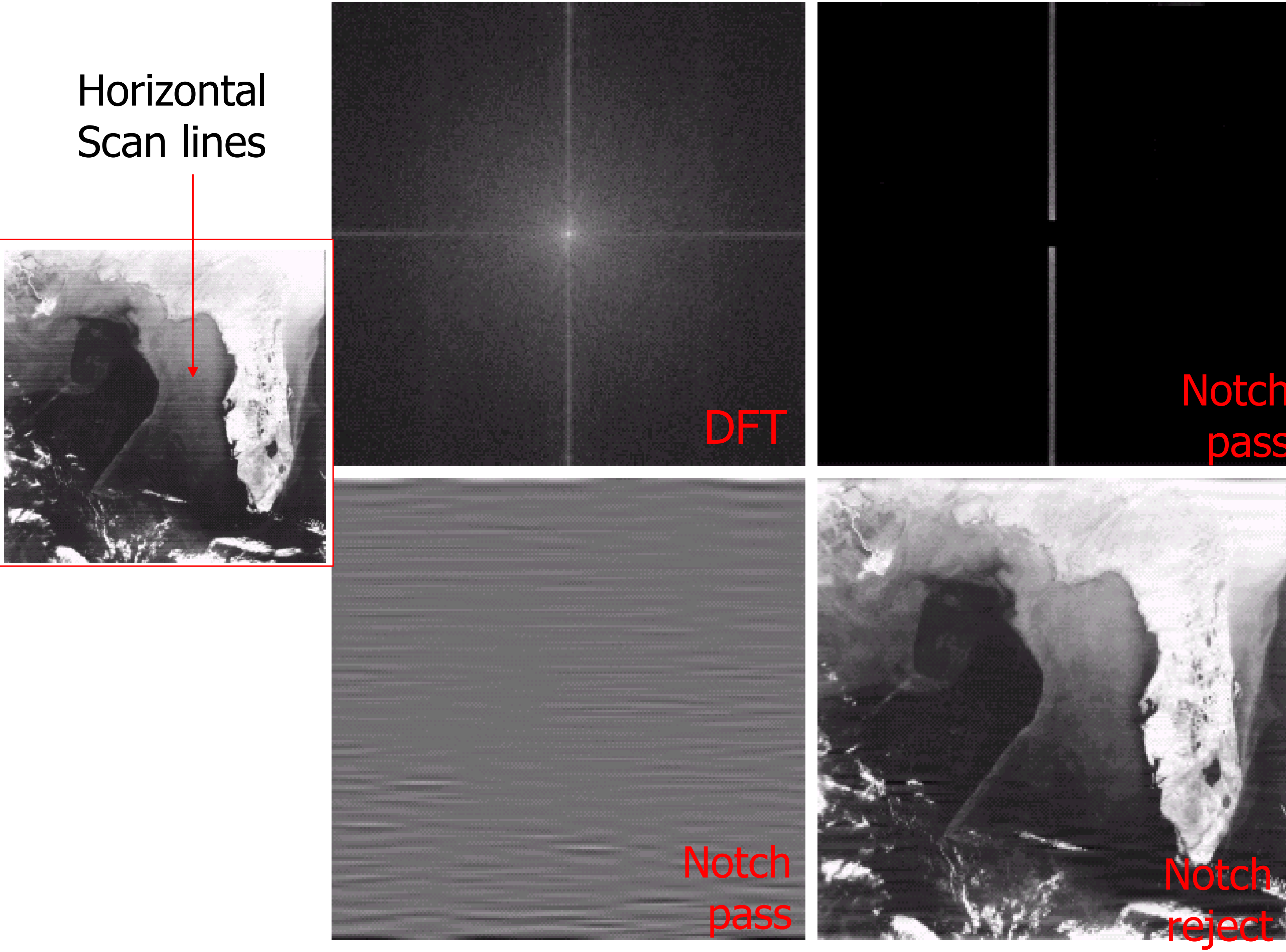
ideal



Butterworth



Gaussian


Horizontal
Scan lines
DFT
Notch
pass
Notch
pass
Notch
reject

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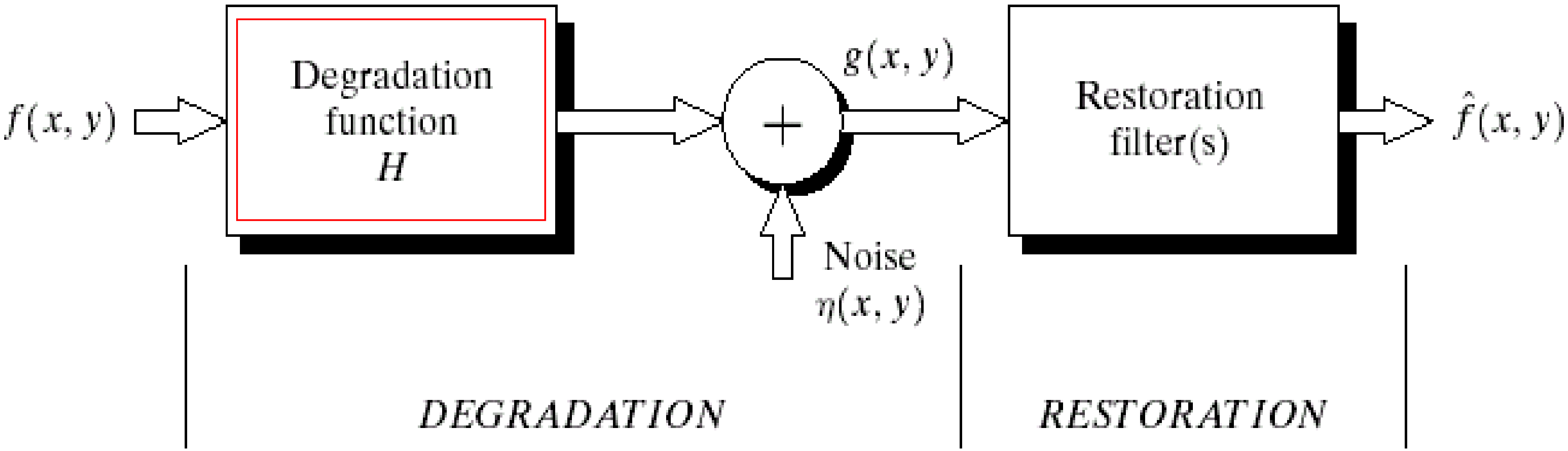
A model of the image degradation /restoration process



$$\begin{cases} g(x,y)=f(x,y)*h(x,y)+\eta(x,y) \\ G(u,v)=F(u,v)H(u,v)+N(u,v) \end{cases}$$

If linear, position-invariant system

Linear, position-invariant degradation

Properties of the degradation function H

- Linear system
  - $H[af_1(x,y)+bf_2(x,y)]=aH[f_1(x,y)]+bH[f_2(x,y)]$
- Position(space)-invariant system
  - $H[f(x,y)]=g(x,y)$
  - $\Leftrightarrow H[f(x-\alpha, y-\beta)]=g(x-\alpha, y-\beta)$
- c.f. 1-D signal
  - LTI (linear time-invariant system)

Linear, position-invariant degradation model

- Linear system theory is ready
- Non-linear, position-dependent system
  - May be general and more accurate
  - Difficult to solve compuatationally
- Image restoration: find H(u,v) and apply inverse process
  - Image deconvolution

Estimating the degradation function

- Estimation by Image observation
- Estimation by experimentation
- Estimation by modeling

Estimation by image observation

- Take a window in the image
  - Simple structure
  - Strong signal content
- Estimate the original image in the window

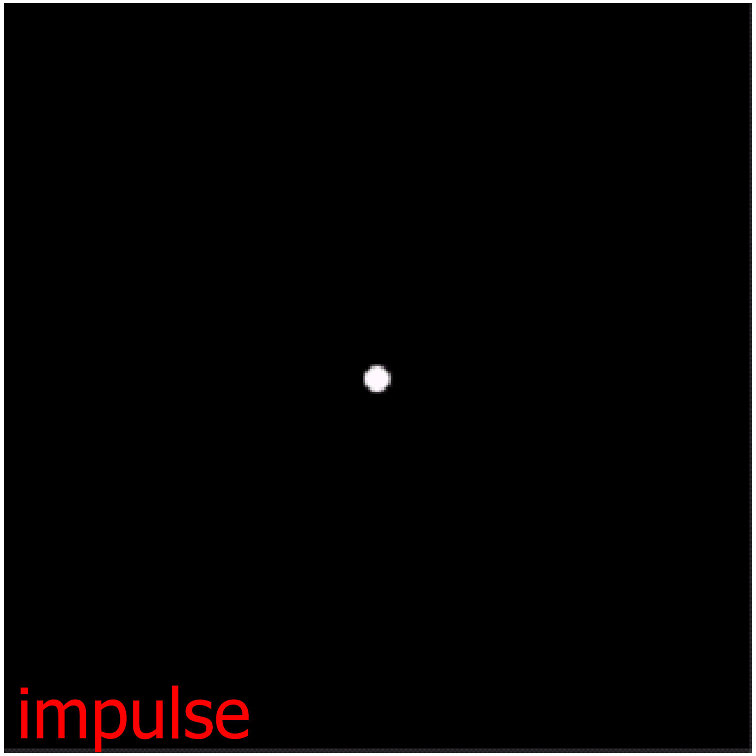
known

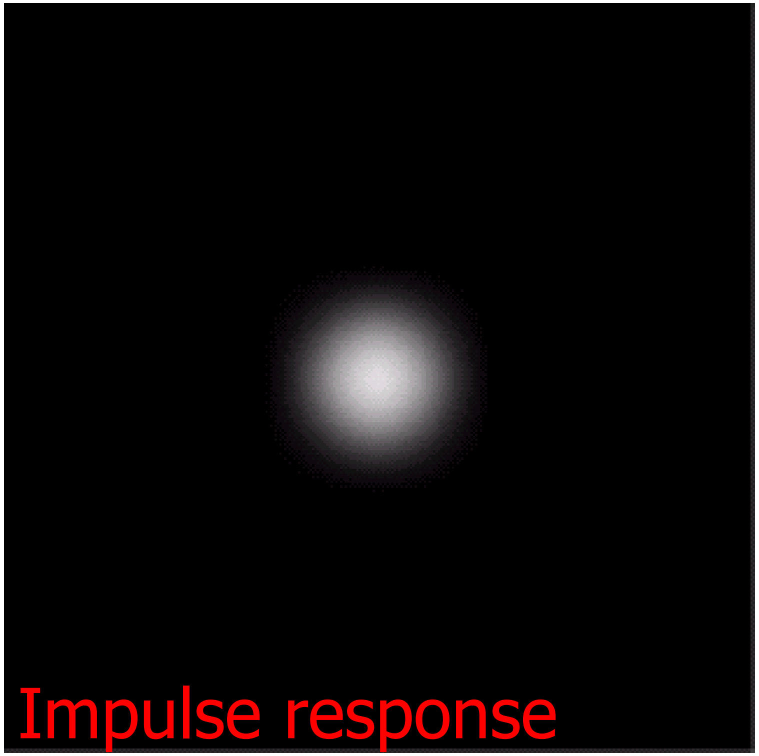
$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

estimate

Estimation by experimentation

- If the image acquisition system is ready
- Obtain the impulse response





Estimation by modeling (1)

- Ex. Atmospheric model $H(u,v) = e^{-k(u^2+v^2)^{5/6}}$

original

k=0.0025

k=0.001

k=0.00025

Estimation by modeling (2)

- Derive a mathematical model
- Ex. Motion of image

$$g(x,y)=\int_0^T f(x-x_0(t),y-y_0(t))dt$$

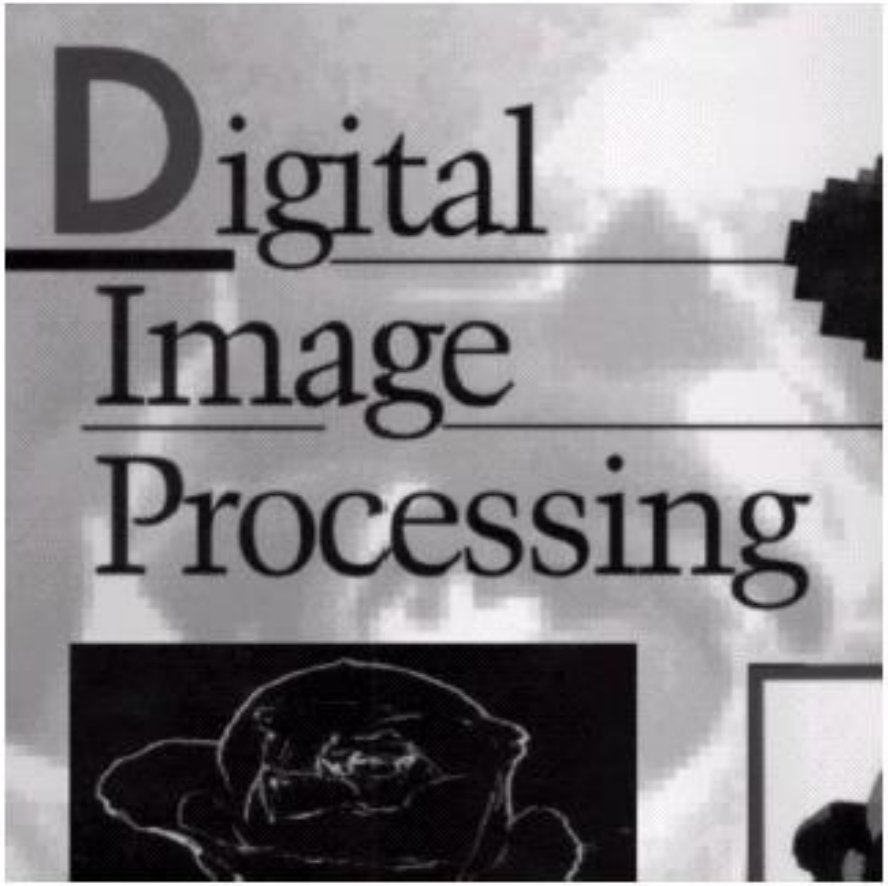
Planar motion

Fourier transform

$$G(u,v)=F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

Estimation by modeling: example

original



Apply motion model

Inverse filtering

- With the estimated degradation function H(u,v)

Unknown noise

G(u,v)=F(u,v)H(u,v)+N(u,v)

=> $$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

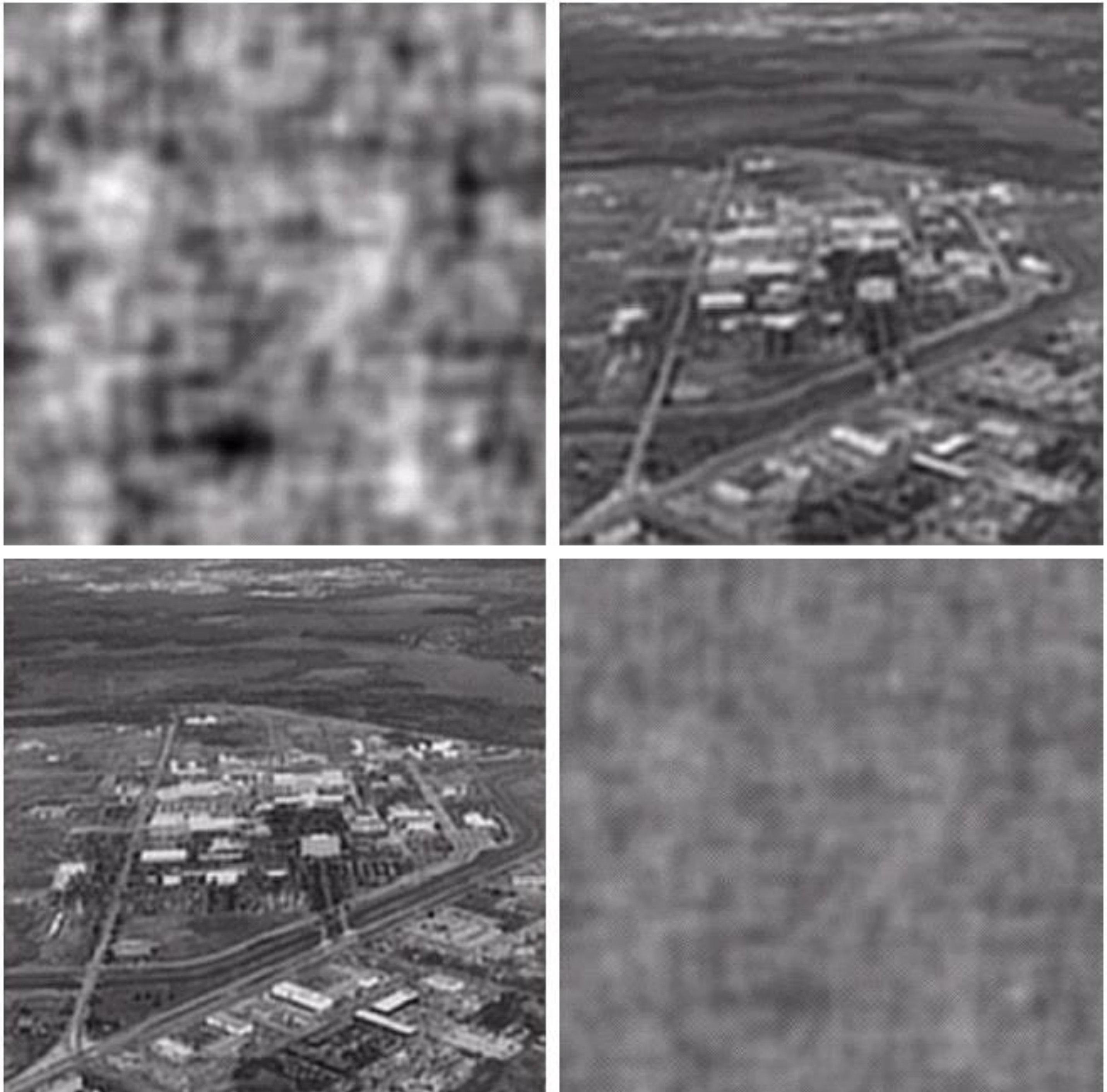
Estimate of original image

Problem: 0 or small values

Sol: limit the frequency around the origin

Full inverse filter for k=0.0025

Cut Outside 40%



Cut Outside 70%

Cut Outside 85%