Medical Image Processing

IX. Morphological Image Processing

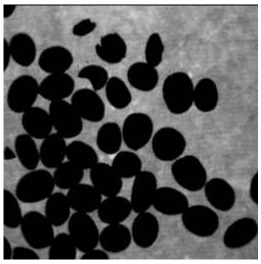
The beginning of mathematical morphology
- Georges Matheron and Jean Serra, 1964

IX. Morphological Image Processing

1. Preliminaries
2. Erosion and Dilation
3. Opening and Closing
4. The Hit-or-Miss Transformation
5. Some Basic Morphological Algorithms
6. Gray Scale Morphology

Preview

- Mathematical morphology
  - A tool for extracting image components that are useful in the presentation and description of region shape, such as boundaries, skeletons and the convex hull
- Morphological techniques for pre- or postprocessing
  - Filtering
  - Thinning
  - Pruning



Can you count the number of the coffee beans?

Preliminaries

- Set theory : Sets in mathematical morphology represent objects in an image
  - binary image (0 = white, 1 = black) : the element of the set is the coordinates (x,y) of a pixel belonging to the object → Z^2
  - gray-scaled image : the element of the set is the coordinates (x,y) of a pixel belonging to the object and the intensity value → Z^3

Definitions and Notations

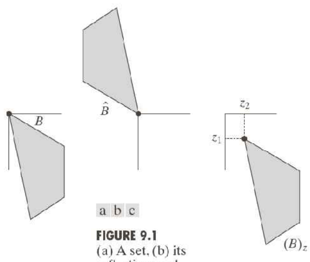
- Structuring elements (SE)
  - Small sets or subimages used to probe an image under study for properties of interest

- Reflection

  $$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

- Translation

  $$(B)_z = \{c \mid c = b + z, \text{ for } b \in B\} \qquad z = (z_1, z_2)$$



a b c

FIGURE 9.1 (a) A set, (b) its reflection, and (c) its translation by z.

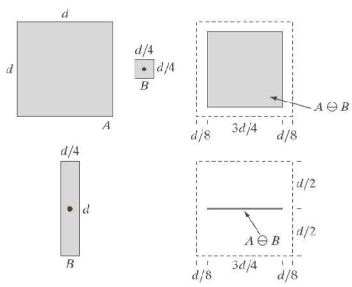
Erosion

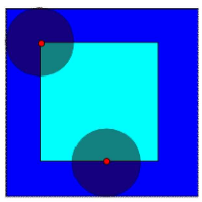
- Definition

  $$A \ominus B = \{z \mid (B)_z \subseteq A\} \qquad A \ominus B = \{z \mid (B)_z \cap A^c = \varnothing\}$$

- The erosion of A by B is the set of all points z such that B, translated by z, in contained in A.

- Shrink or thin objects in a binary image.
- Morphological filter : Image details smaller than the SE are filtered (removed) from the image.

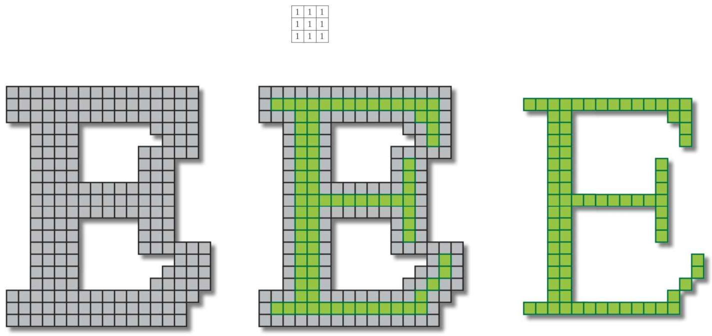




a b c
d e

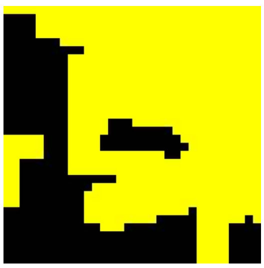
FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Erosion (SE : 3x3)

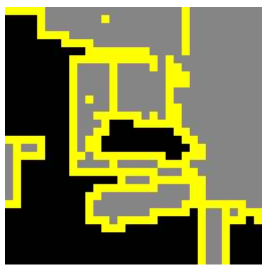


Example
3x3 Erosion (SE : 3x3)

original image

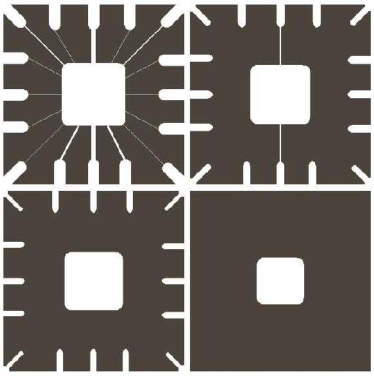


eroded image



original / erosion

Example



a b
c d

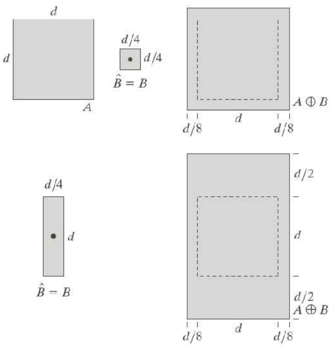
FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Dilation

- Definition

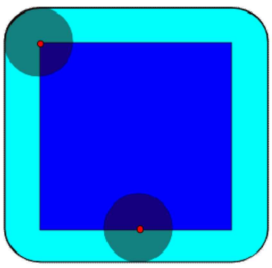
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\} \qquad A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

- The dilation of A by B is the set of all displacements, z, such that $\hat{B}$ and A overlap by at least one element.
- Grow or thicken objects in a binary image.

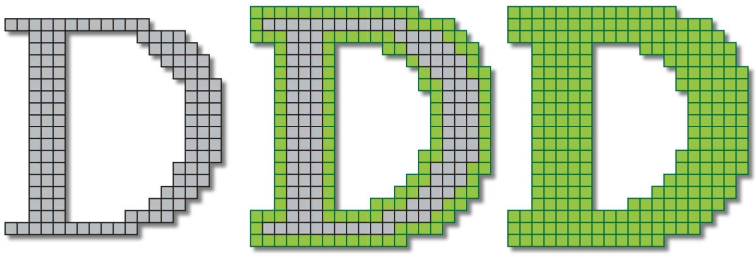


a b c
d e

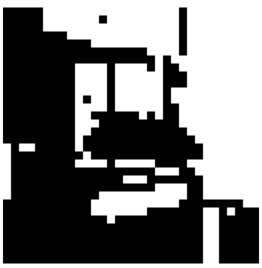
FIGURE 9.6 (a) Set A. (b) Square structuring element (the dot denotes the origin). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference



Dilation



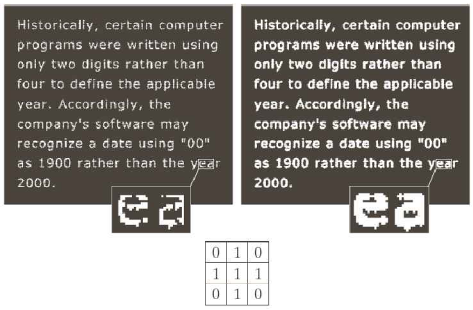
Example
3x3 Dilation (SE : 3x3)


original image


dilated image


dilation / original

Dilation : Bridging gaps



a c
b

FIGURE 9.7 (a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

- Advantage over lowpass filtering : directly in a binary image (LPF starts with a binary image and produces a gray-scale image, which would require thresholding to convert it back to binary form.

How to Implement

- Analogous to spatial convolution
  - The SE B is viewed as a convolution mask (MxN).
  - Slide and compute
- Erosion

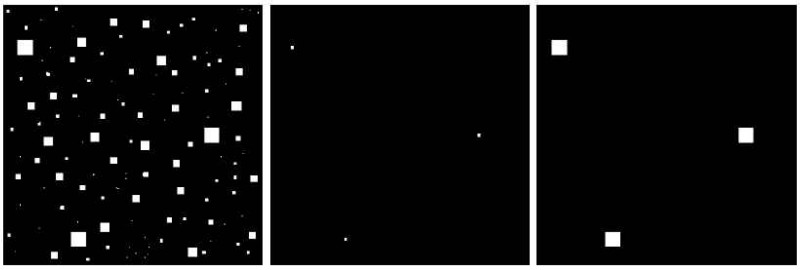
$$A \ominus B = \underset{m=1}{\overset{M}{AND}}\ \underset{n=1}{\overset{N}{AND}} \{\text{if } (\hat{B}), (A)\}$$

$\hat{B}$: flipped B about its origin

- Dilation

$$A \oplus B = \underset{m=1}{\overset{M}{OR}}\ \underset{n=1}{\overset{N}{OR}} \{\text{if } (B), (A)\}$$

Example
- Erosion and Dilation



a b c

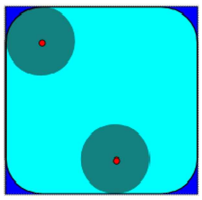
FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element B = 13x13 pixels of gray level 1

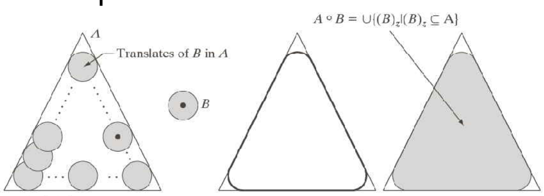
Opening

- Erosion + dilation

$$A \circ B = (A \ominus B) \oplus B = \bigcup \{(B_z) \mid (B_z) \subseteq A\}$$



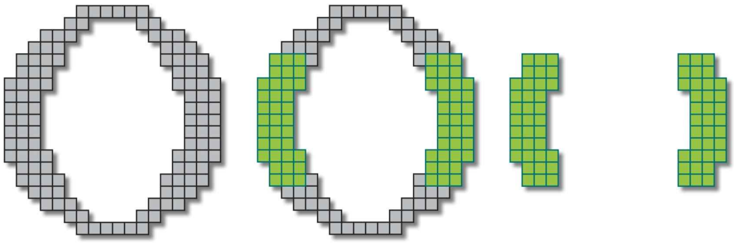
- Smooth the contour of an object
- Break narrow isthmuses
- Eliminate thin protrusions



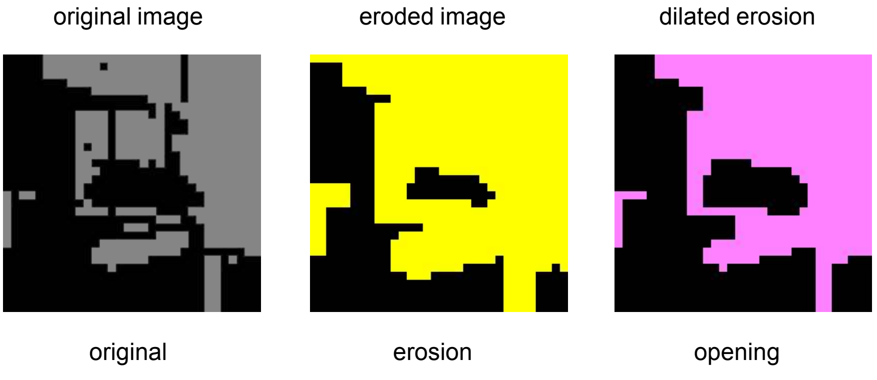
a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Opening



Example (3x3)



Closing

- Dilation + erosion

$$A \bullet B = (A \oplus B) \ominus B$$

- Smooth sections of contours as opposed to opening
- Fuse narrow breaks and long thin gulfs
- Eliminate small holes
- Fill gaps in the contour

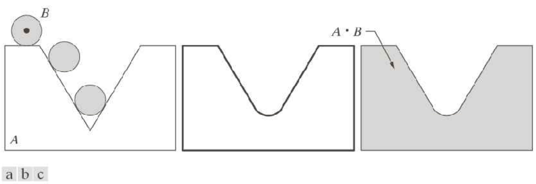
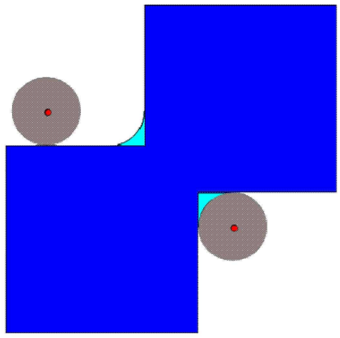
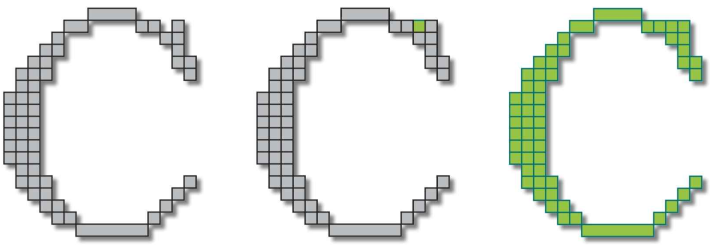


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.



Closing

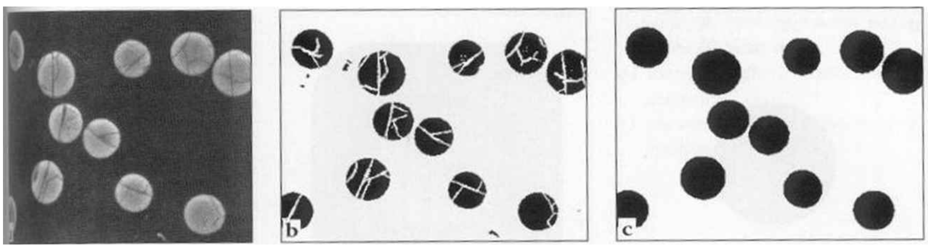


Example (3x3)

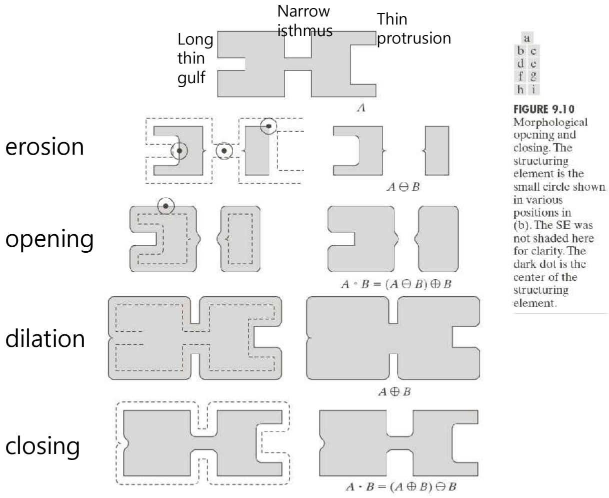
original image | dilated image | eroded dilation

original | dilation | closing

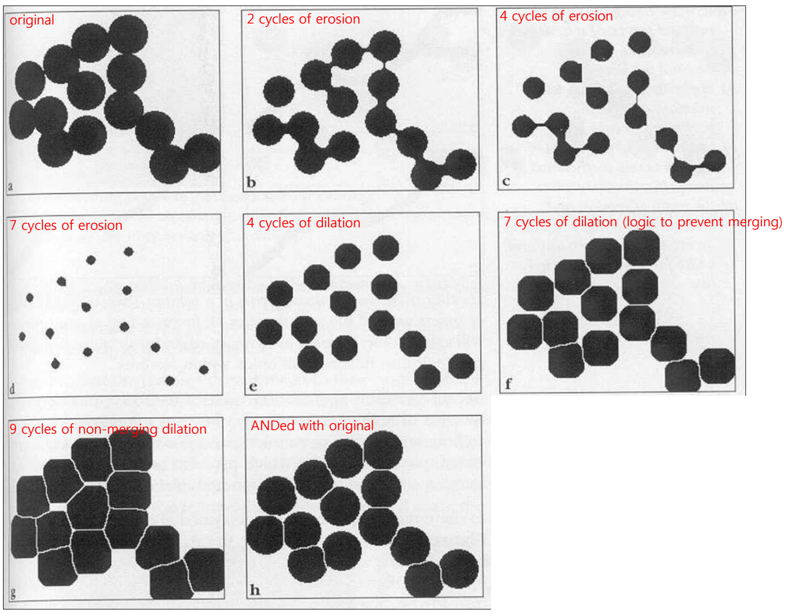
Example
(Cross section of cracked glass fibers)



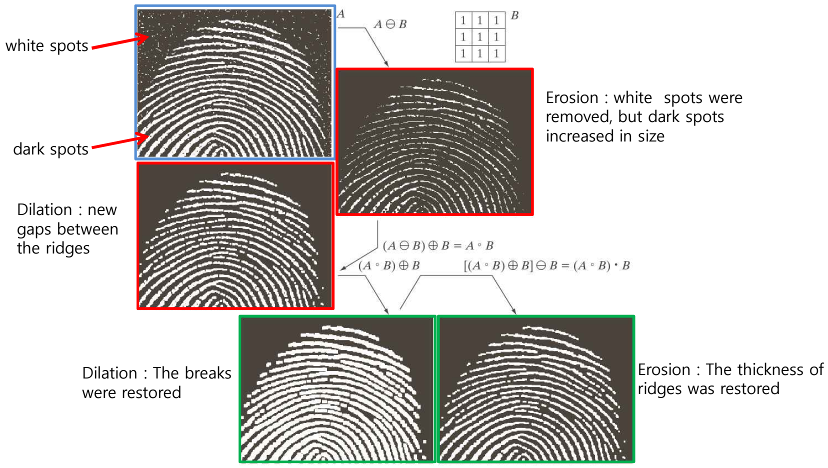
Opening and Closing



Example (Erosion and Dilation)



Morphological Filtering
- Remove noise in a binary image



The Hit-or-Miss Transformation

- Basic tool for shape detection
- Ex) Finding the location of D

  $$A \circledast B = (A \ominus D) \bigcap [A^c \ominus (W - D)]$$
- Hit-or-miss transform

  $B=(B_1, B_2)$

  B_1 : foreground (hit)

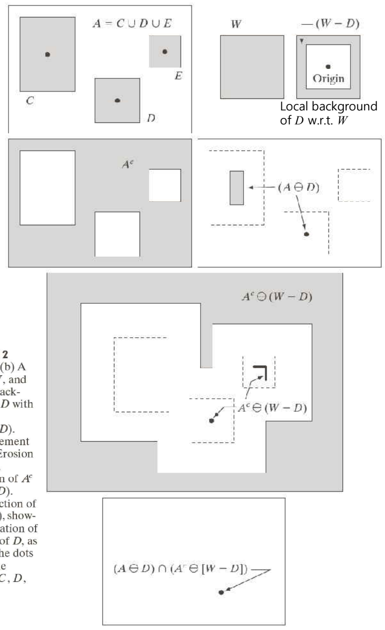
  B_1 finds a match in A.

  B_2: background of B_1 (miss)

  B_2 finds a match in A^c.

  $$A \circledast B = (A \ominus B_1) \bigcap [A^c \ominus B_2]$$

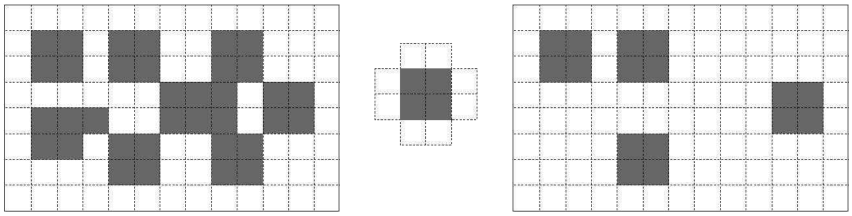
  $$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

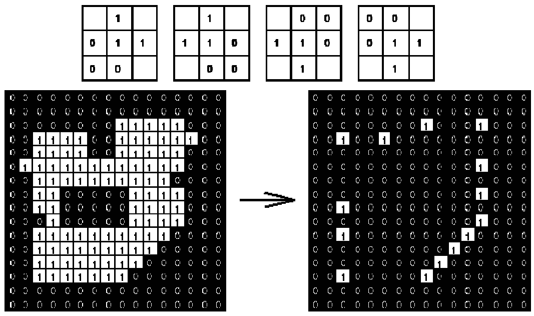


a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

Examples





Some Basic Morphological Algorithms

(1) Boundary extraction
(2) Hole filling
(3) Extraction of connected components
(4) Convex hull
(5) Thinning
(6) Thickening
(7) Skeletons
(8) Pruning
(9) Morphological reconstruction

(1) Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$

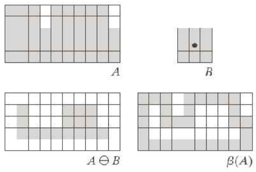
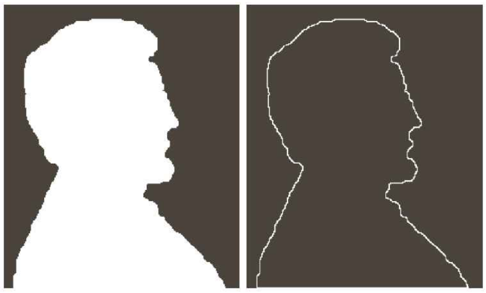


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



(2) Hole Filling

Conditional dilation :

$X_k = (X_{k-1} \oplus B) \bigcap A^c \quad k = 1,2,3,\ldots$, B is the symmetric SE.

(if $X_k = X_{k-1}$, the iteration stops.)

Seed point

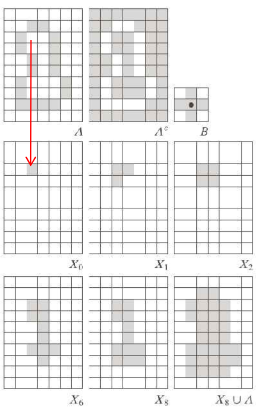


FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

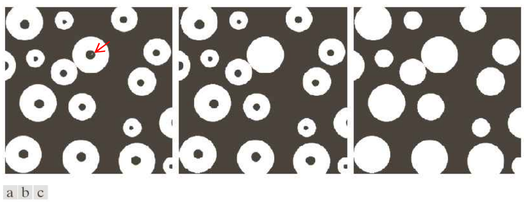


FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

(3) Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \bigcap A \quad k = 1,2,3,\ldots$$

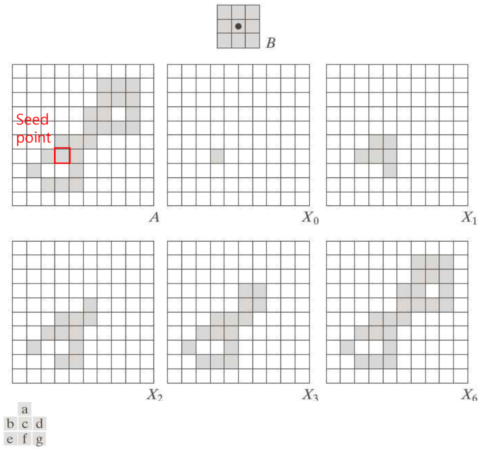
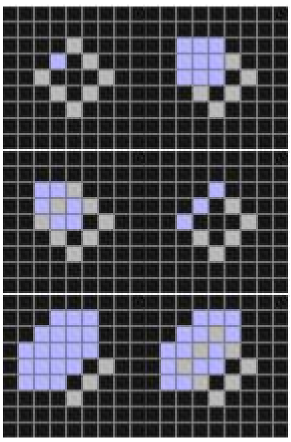
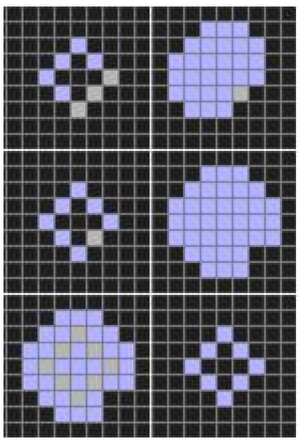


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).

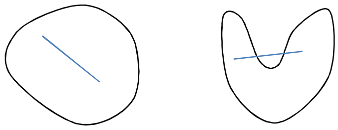
Example



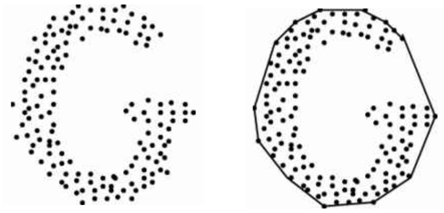


(4) Convex Hull

- Convex : The straight line segment joining any two points in A lies entirely within A

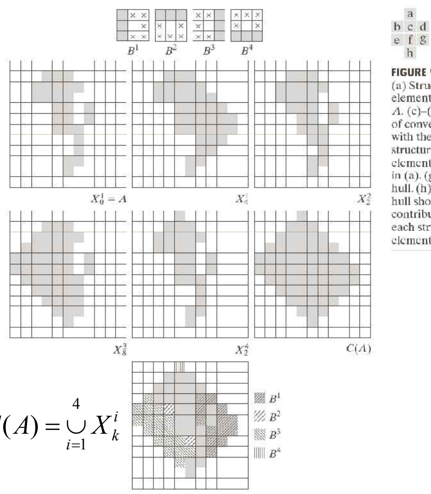


- Convex hull of S : the smallest convex set containing S
- Convex hull problem :



Obtaining the Convex Hull

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A \quad i = 1,2,3,4 \;\text{ and } k = 1,2,3,\ldots$$



$$C(A) = \bigcup_{i=1}^{4} X_k^i$$

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

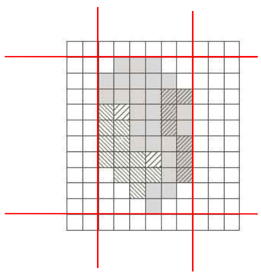


FIGURE 9.20 Result of limiting growth of the convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

(5) Thinning

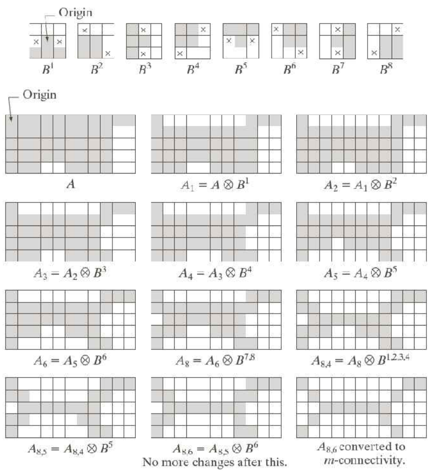
- Used to shrink objects in binary images
- Differs from erosion in that objects are never completely removed
- Thinning is defined as:

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

- More useful expression :

$$\{B\} = \{B^1, B^2, B^3, ..., B^n\} \qquad B^i \text{ is a rotated version of } B^{i-1}$$
$$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$

Example



a
b c d
e f g
h i j
k l m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

(6) Thickening

- Morphological dual of thinning

$$A \odot B = A \cup (A \circledast B)$$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2) \ldots) \odot B^n)$$

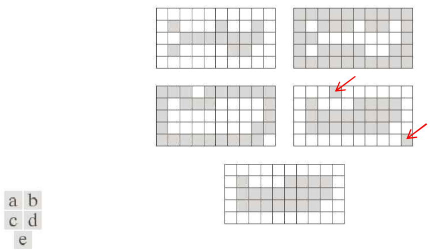
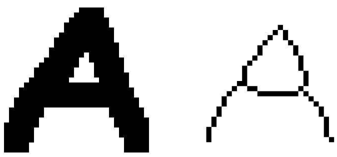


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

(7) Skeletons

- The skeleton of an object is often defined as the medial axis of that object.
  – Pixels are then defined to be skeleton pixels if they have more than one "closest neighbours".



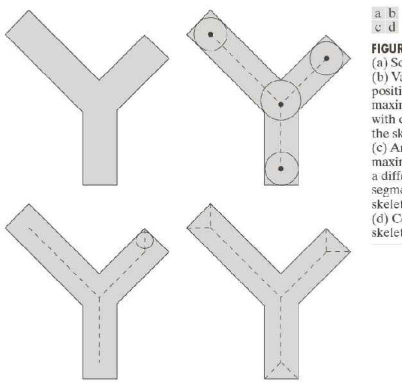


FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

Skeletons

$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$

where $(A \ominus kB) = ((\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B)$: k successive erosions

$S(A) = \bigcup_{k=0}^{K} S_k(A)$ K is the last interative step before A erodes to an empty set.

$K = \max\{k \mid (A \ominus kB) \neq \varnothing\}$

$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$

FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

thick and not connected

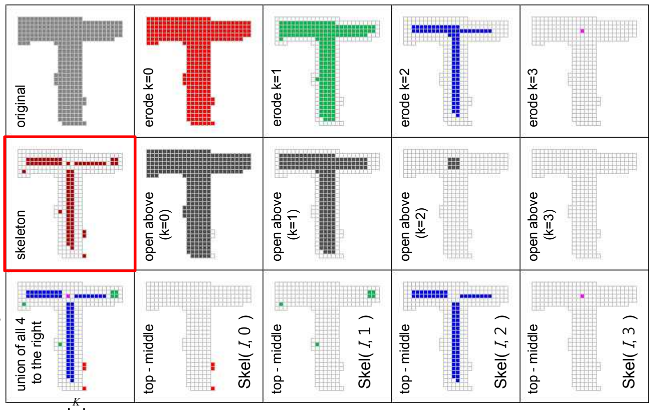
Example

SE :

$(A \ominus kB)$

$(A \ominus kB) \circ B$

$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$

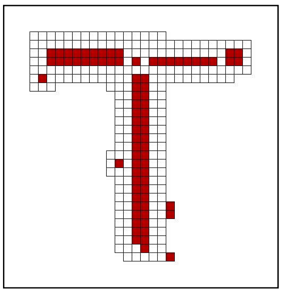


$S(A) = \bigcup_{k=0}^{K} S_k(A)$

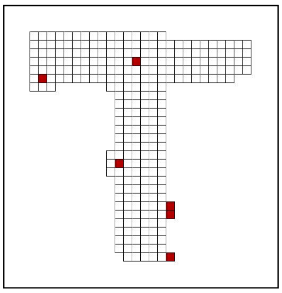
Note that the result is disconnected and has spurious points.
The skeleton must be maximally thin, connected, and minimally eroded.
→ Heuristic formulations are needed.

Skeletonization: Delete Spurious Pixels

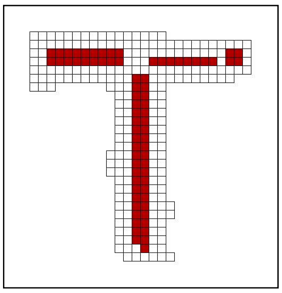
raw skeleton
has spurious pixels



def. spurious pixels as
conn. comp. of < 3 pix.

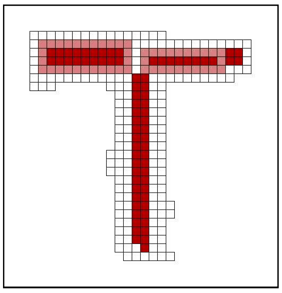


pruned skeleton
raw less spurious

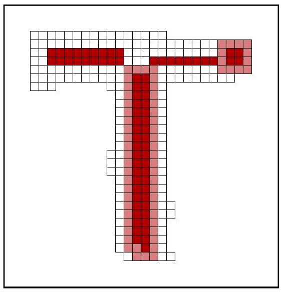


Skeletonization: Reconnect Components

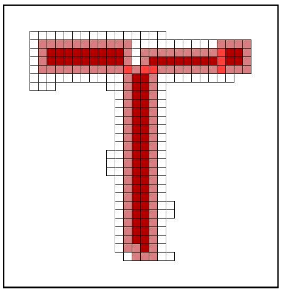
2 components of
pruned skel. dilated.



2 other components
of pruned skel. dilated.

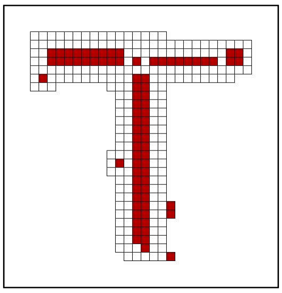


Intersection of
dilated components.

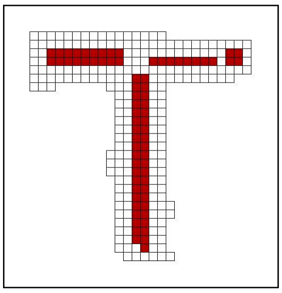


Skeletonization

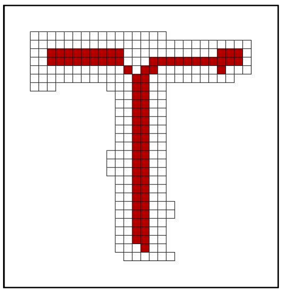
raw skeleton



pruned skeleton



reconnected skeleton



(8) Pruning

- Essential component to thinning and skeletonizing algorithms that leave parasitic components that need to be cleaned up by postprocessing

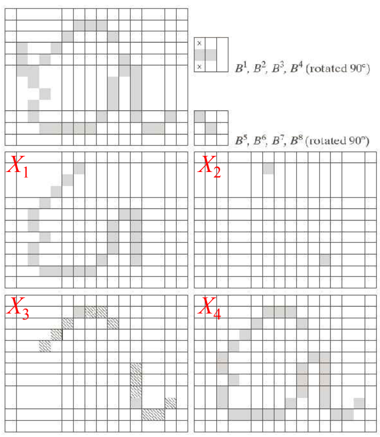
$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

H is a 3x3 SE of 1s.

$$X_4 = X_1 \cup X_3$$

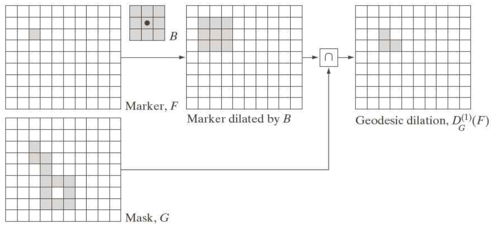


a b
c
d e
f g

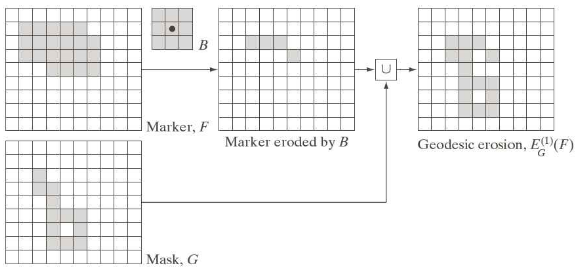
FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

(9) Morphological Reconstruction

- F : marker image, G : mask image
- Geodestic dilation
  - Size 1 : $D_G{}^{(1)}(F)=(F \oplus B) \cap G$
  - Size n : $D_G{}^{(n)}(F)=D_G{}^{(1)}[D_G{}^{(n\text{-}1)}(F)]$ $(D_G{}^{(0)}(F)=F)$



- Geodestic Erosion
  - Size 1 : $E_G{}^{(1)}(F)=(F \ominus B) \cup G$
  - Size n : $E_G{}^{(n)}(F)=E_G{}^{(1)}[E_G{}^{(n\text{-}1)}(F)]$ $(E_G{}^{(0)}(F)=F)$

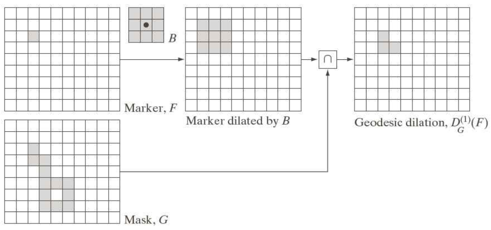


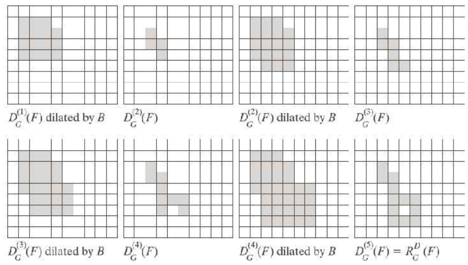
Morphological Reconstruction by Dilation and Erosion

- Morphological reconstruction by dilation

  $R_G^D(F)=D_G^{(k)}(F)$ $D_G^{(k)}(F)=D_G^{(k+1)}(F)$
- Morphological reconstruction by erosion

  $R_G^E(F)=E_G^{(k)}(F)$ $E_G^{(k)}(F)=E_G^{(k+1)}(F)$





Example

Find long vertical strokes

a b
c d

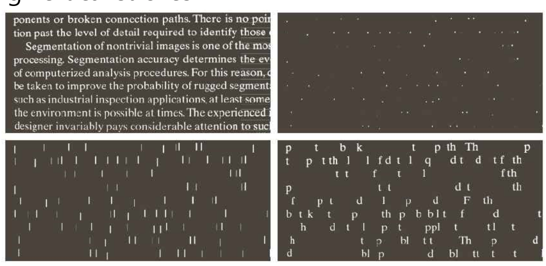
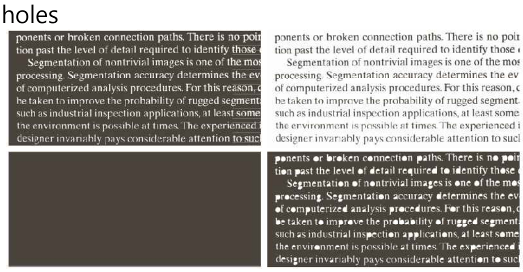


FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

Filling holes



a b
c d

FIGURE 9.31 (a) Text image of size 918 × 2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).

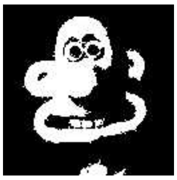
Example

- Remove small regions that are disjoint from larger objects without distorting the small features of the large objects.

1. $J = A \circ B$ (B is a SE)
2. $T = J$
3. $J = J \oplus B$
4. $J = A$ AND J
5. If $J \neq T$, then go to 2
   else stop

original

opened



reconstructed

Summary

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(*Continued*)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) = (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; X_0 = array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$; $A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$; $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$; $A \odot \{B\} = ((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$; $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$; Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

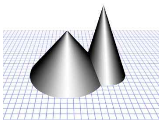
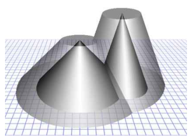
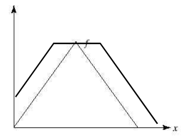
(*Continued*)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$; $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$; $X_3 = (X_2 \oplus H) \cap A$; $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$; $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{F^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.

Gray-Scale Morphology - Dilation

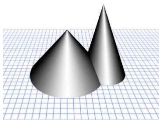
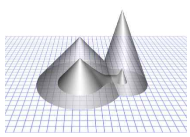
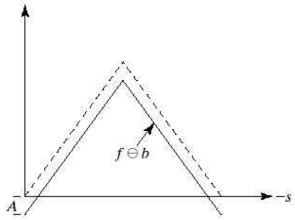
- If all the values of the SE are positive, the output image tends to be **brighter** than the input.
- Dark details either are reduced or eliminated, depending on how their values and shapes relate to the SE used for dilation.
- Like a convolution

$$[f \oplus b](x,y) = \max_{(s,t)\in b}\{f(x-s,y-t)\}$$

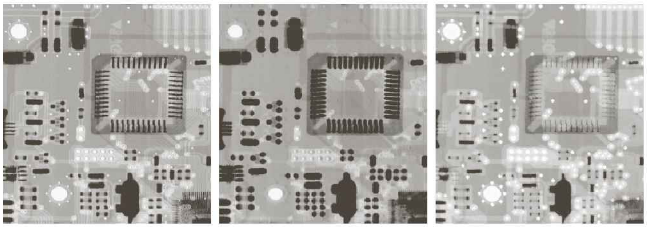




Erosion

- If all the elements of the SE are positive, the output image tends to be **darker** than the input
- The effect of bright details in the input image that are smaller in area than the structuring element is reduced, with the degree of reduction being determined by the gray-level values surrounding the bright detail and by the shape and amplitude values of the structuring element itself
- $[f \ominus b](x,y) = \min_{(s,t)\in b}\{f(x+s,y+t)\}$

Example



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

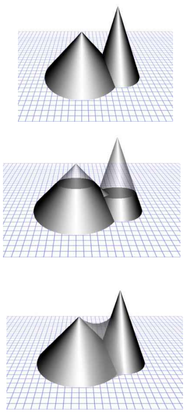
Example : Dilation and Erosion

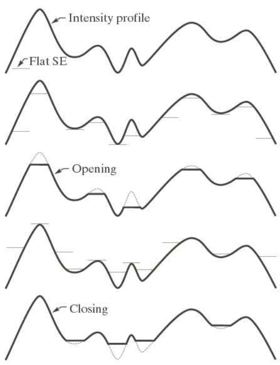
Opening and Closing

- Opening
  - The structuring element is rolled underside the surface of *f*.
  - All the peaks that are narrow with respect to the diameter of the structuring element will be reduced in amplitude and sharpness
  - So, opening is used **to remove small light details**, while leaving the overall gray levels and larger bright features relatively undisturbed.
  - The initial erosion removes the details, but it also darkens the image.
  - The subsequent dilation again increases the overall intensity of the image without reintroducing the details totally removed by erosion.

- Closing
  - The structuring element is rolled on top of the surface of *f*.
  - Peaks essentially are left in their original form (assume that their separation at the narrowest points exceeds the diameter of the structuring element)
  - So, closing is used **to remove small dark details**, while leaving bright features relatively undisturbed.
  - The initial dilation removes the dark details and brightens the image.
  - The subsequent erosion darkens the image without reintroducing the details totally removed by dilation.

Opening and Closing

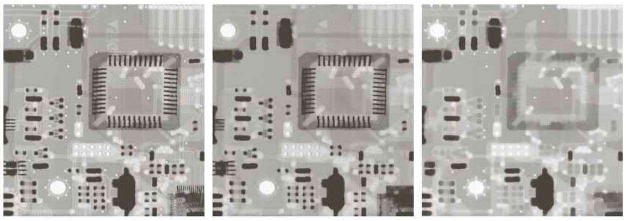




a
b
c
d
e

FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

Example

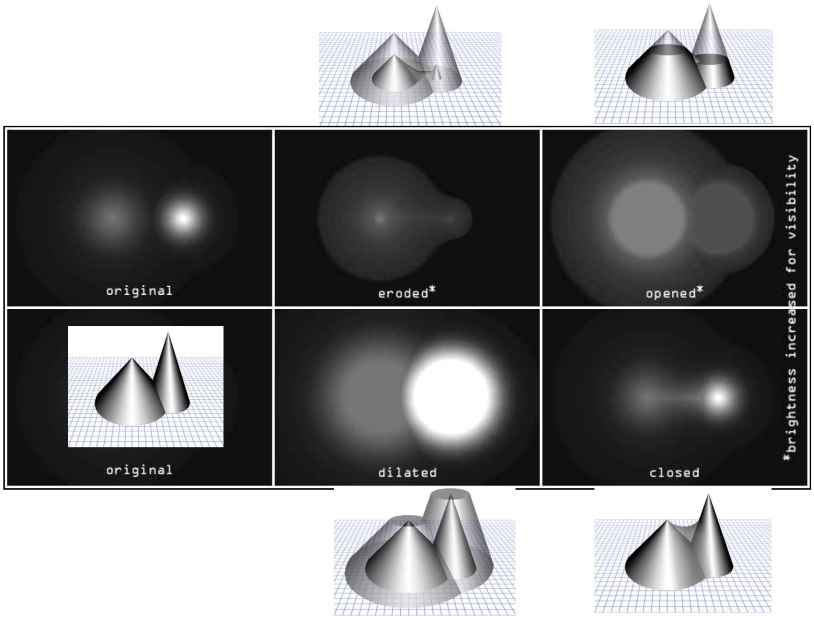


a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

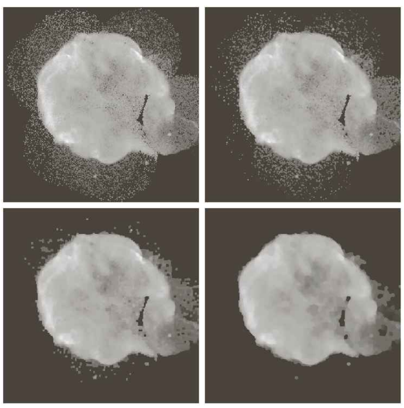
Example : Opening and Closing

Summary of Basic Operations



Some Basic Gray-Scale Morphological Algorithms - Smoothing

- Perform an opening following by a closing
- Effect : remove or attenuate both bright and dark artifacts or noise

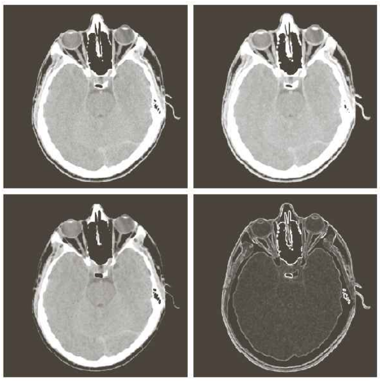


a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Morphological Gradient

- $g = (f \oplus b) - (f \ominus b)$
- The homogeneous areas are suppressed and the edges are enhanced



a b
c d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and Bottom-hat Transformation

- Remove objects
- Top-hat

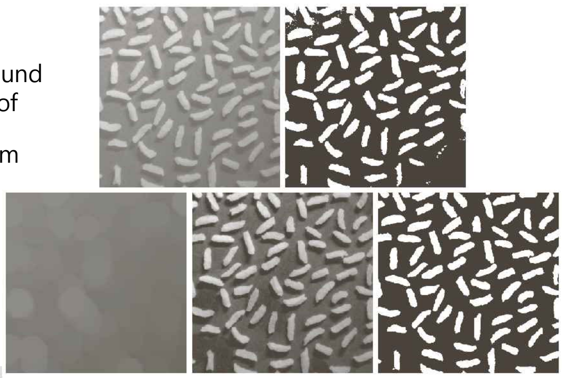
  $T_{hat}(f) = f - (f \circ b)$

  - Light objects on a dark background
  - Enhance detail in the presence of shading
  - Correct the effects of nonuniform illumination

- Bottom-hat

  $T_{bottom}(f) = (f \bullet b) - f$

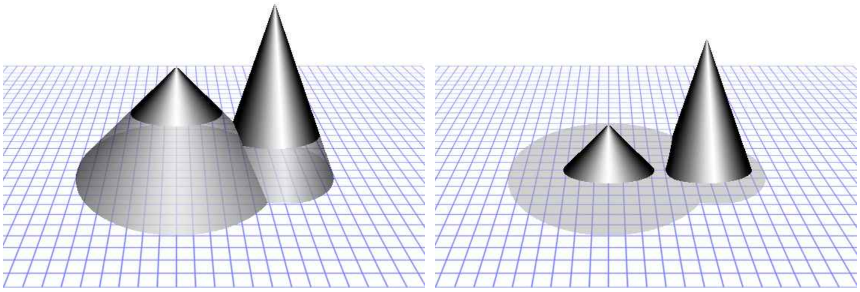
  - Dark objects on a light background (black top-hat)



a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

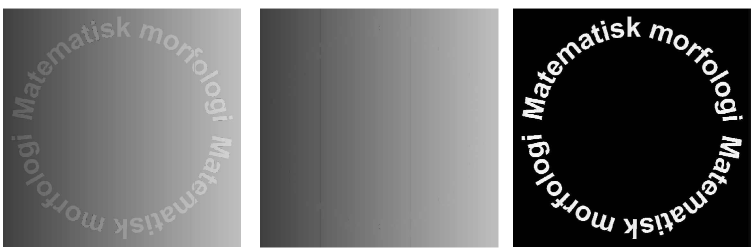
Top-Hat Transformation



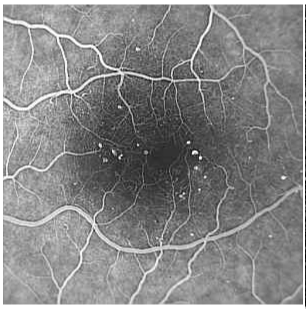
Top-hat + opened = original

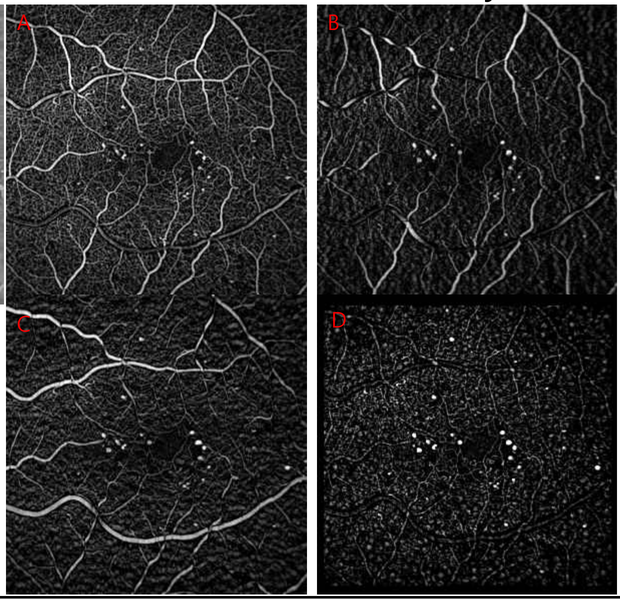
Top-hat=original - opening

Example



Example : Top-Hat
Automated Detection of Microaneurysms





Result of the Top Hat Transform on the image in figure 1 using (A) a circular structuring element (diameter 5 pixels), (B) a horizontal linear element, (C) a vertical linear element, (D) the result of combining linear elements at 8 equally spaced angles. Note the linear elements in (B)-(D) were all 11 pixels in length.

Granulometry

- Determine the size distribution of particles in an image.
- Apply openings with SEs of increasing size
- For each opening, the "surface area" (the sum of the pixel values) in the opening is computed.

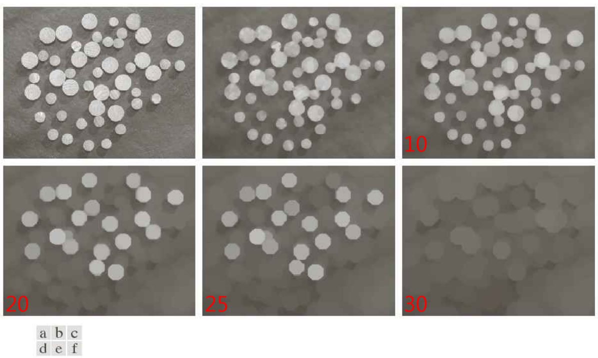


FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

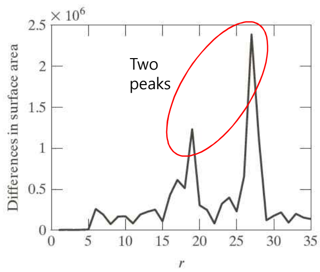
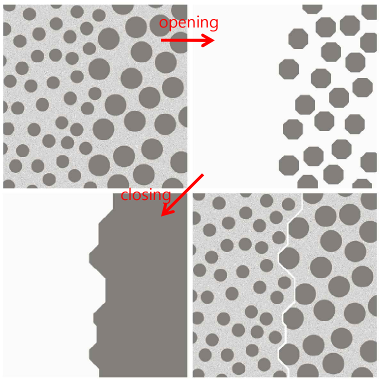


FIGURE 9.42 Differences in surface area as a function of SE disk radius, r. The two peaks are indicative of two dominant particle sizes in the image.

Textural Segmentation

- Subdivide two regions
  - A region composed on large blobs
  - A region composed on smaller blobs



a b
c d

FIGURE 9.43 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

Gray-Scale Morphological Reconstruction

a b c
d e f
g h i

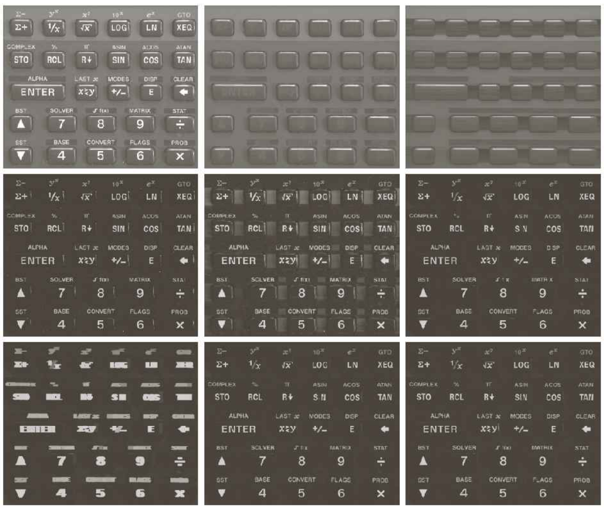


FIGURE 9.44 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a) using a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same line. (d) Top-hat by reconstruction. (e) Top-hat. (f) Opening by reconstruction of (d) using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

Homework #6

- Count the number of the cells

-Remove white and dark noise
-Separate the connected cells by erosion
-Extract connected components and label each cell
-Count the number of the cells

-How can we deal with the cells that touch the border?