Associated Legendre polynomial

Associated Ledgendre polynomials are defined in terms of derivatives of Legendre polynomial,

$$P_l^m(x) := (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x).$$

Legendre polynomial again defined in terms of derivatives is,

$$P_l(x) := \frac{1}{2^l l!} \frac{d^l}{dx^l} \left[(x^2-1)^l \right].$$

- Reference
- wikipedia/Associated_Legendre_polynomials

Spherical harmonics

Using the associated Legendre polynomials defined above, general definition of spherical harmonics is given as

$$Y_l^m(\theta, \phi) := \sqrt{\frac{(2l+1)}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{im\phi}.$$

Then, Real form basis of spherical harmonics we are to use is

$$Y_l^m := \begin{cases} \sqrt{2}(-1)^m \mathrm{Im}[Y_l^{|m|}] & \text{if } m < 0 \\ Y_l^0 & \text{if } m = 0 \\ \sqrt{2}(-1)^m \mathrm{Re}[Y_l^m] & \text{if } m > 0. \end{cases}$$

- Reference
- wikipedia/Spherical_harmonics

Implementation

```
#include <cmath>
#include <boost/math/constants/constants.hpp>
#include <boost/math/special_functions/spherical_harmonic.hpp>

namespace sph
{

    using real_t = double;

    real_t sph_harm(const unsigned int& l, const int& m, const real_t&
theta, const real_t& phi) {
        using namespace boost::math;
        constexpr real_t sqrt2{double_constants::root_two};

        if (m > 0) {
            return sqrt2 * (m % 2 == 0? 1: -1) * spherical_harmonic_r(l, m,
theta, phi);
```

```
        }
        else if (m < 0) {
            return sqrt2 * (m % 2 == 0? 1: -1) * spherical_harmonic_i(l, -
m, theta, phi);
        }
        else {// m == 0 {
            return spherical_harmonic_r(l, 0, theta, phi);
        }
    }

}
```