Central Bank Communication as a Game Theory *

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Abstract

This thesis develops a monetary policy game under the assumption that monetary policy decisions are made not by a single governor but by a policy committee, and that committee members individually obtain private information about the desired rate of inflation. We show that, to make a policy message credible, the central bank has to disclose information about to what extent opinions are dispersed among committee members in addition to the average of them. We also show that information transmission based only on the average of individual opinions, which is often adopted by central banks in industrial countries including the US Federal Reserve and the Bank of England, fails to achieve an equilibrium in our model, so that the central bank cannot convey a credible message to the private sector. This is because the private sector, which recognizes that monetary policy decision differs depending on whether opinions are dispersed or not, does not fully trust the message from the central bank. Our analysis suggests the importance to deliberately consider central banking by committee system.

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1 Introduction

Alan Blinder argues in an influential book on the conduct of monetary policy (Blinder (2008)) that central banks in industrial countries have been experiencing revolution, "quiet revolution" in his terminology, in the following respects: (1) transparency of central bank; (2) communication with the market; (3) central banking by committee. The purpose of this thesis is to provide a theoretical framework to investigate in more detail on the three aspects of the quiet revolution.

One of the major issues central banks have been suffering from is inflation bias. When a central bank (CB) is in a discretionary regime, that is, it operates monetary policy according to the economic conditions at that time, it can create surprise inflation to lower unemployment without raising the inflation rate. The private sector (P), who serves as a rational agent, knows that CB has an incentive to set higher inflation rate than P's expectation, so that P forms a much higher one in advance. In return, CB has to rely on more excessive level of inflation to surprise P and to attain the ideal employment. Consequently, the inflation ascends to the level where central banks can gain no more marginal profit by deviating the private sector's expectation. At this equilibrium, although the employment is the same level as the one that CB initially attempted to achieve, the inflation rate is much higher, which is an undesirable result for both CB and P. Barro and Gordon (1983) suggest a game-theoretical model to explain this dilemma caused by time-inconsistency of the central bank. They conclude that if CB's policy rules determined in advance commit future monetary policy, outcomes improve comparing to discretionary monetary policy.

Although Barro and Gordon (1983) claim that central banks should adopt a rule-based monetary policy, how they credibly commit future policy choices remains to be seen because they are always tempted to choose a time-inconsistent policy as we have seen above. A naive way is to legislate how they make decisions. As Rogoff (1985) points out, however, this resolution is too rigid to effectively respond to unanticipated shocks. Instead, he seeks the possibility of valid commitment in traits of central bankers. According to his analysis, it is reasonable to appoint central bankers who place a large weight on inflation-rate stabilization relative to employment optimization for successful commitment. His study is regarded as providing a theoretical foundation of central bank independence.

As mentioned by Blinder (2008), central banks started to recognize that they need to be transparent to the legislature and the public immediately after they gained independence from political influence, but the theoretical foundation of central bank transparency was not provided until Canzoneri (1985) developed a model in which the central bank has private information. He criticizes the assumption of Barro and Gordon (1983) and Rogoff (1985) that information available to CB is the same as that to P. He claims that if CB has some private information and make a decision based on it, commitment fails regardless of the efforts suggested before. With CB's private information, when a chosen policy is not expected, P does not have a way to distinguish the case CB just reacts to an unanticipated shock from the one that CB indeed attempts to deceive P. Hence, policy rules proposed in old literature are no longer incentive-compatible in his framework.

Moscarini (2007) attempts to build a model concerning central bank communication as well as transparency. Developing Barro and Gordon (1983), Rogoff (1985), and Canzoneri (1985), he introduces cheap talk as a method to communicate with the private sector. So far, discretionary monetary policy is thought to be unpreferable because it causes huge inflation bias. But he shows that if CB is sufficiently competent, its discretion leads to relatively small bias. Such CB can obtain quite precise information about the desired inflation rate and avoid receiving an enormous punishment from inflationary deviation, so that it does not need to surprise P very much. Since P is rational and understands CB's plausible behavior, it forms a modest inflationary expectation, which results in a beneficial equilibrium for both the two agents.

As for the third aspect of the quiet revolution, i.e., central banking by committee, Blinder (2007) suggests the importance of analysis of its pros and cons. Before the prevalence of central bank independence, central banks did not need to organize committee since their job was just to follow governments' orders. Central banks had almost no opportunity to make their own decisions. After the movements towards independence, central banks turned to be required to collect information and execute monetary policy all by themselves. The system of decision-making in central banks is now worthy of considering both theoretically and practically.

The main contribution of this thesis is to propose a model into which we incorporate the idea of committee in addition to transparency and communication in the framework of private information. Based on the model of Moscarini (2007), we consider an information transmission game under the assumption that CB consists of multiple agents, i.e., committee members, who independently acquire information about the desired rate of inflation. In this setting, a conventional policymaker corresponds to CB who commits to making a decision based only on the average of the desired inflation rates observed by individual members. However, as we show later, we prove this cannot be a meaningful equilibrium, i.e., no communication is carried out. The main reason for this is similar to the one pointed out by Canzoneri (1985). That is, all information from individual members is available to CB, so CB could privately enjoy profits from finer information than the average if CB wants. Since P cannot make sure that CB uses only the average for decision-making, P does not take CB at its words. Our analysis implies that CB has to employ the variance of the desired inflation rates observed by individual members for monetary policy decisions and transmit a signal based on it to P to achieve credible communication.

Section 2 reviews the model of Moscarini (2007) and provides the proofs of some propositions which are not mathematically shown in it. Section 3 presents our new models and equilibria. Section 4 numerically studies our models. Section 5 emphasizes the importance to research central banking by committee. Section 6 concludes.

2 The Model of Moscarini (2007) with some Supplements

Before introducing our models, in this section, we first review the model provided in Moscarini (2007). His work focuses on the analysis of transparency and communication of central bank with private information about the desired inflation rate. In his paper, he states some useful propositions, but he just numerically verifies them and omits most of their mathematical proofs. We will show them in the later of this section.

2.1 Model Review

The structure of his model is as follows. CB privately observes a social state and sends its signal to P. Then, P forms an inflationary expectation based on it. P does not literally believe what CB says since CB has an incentive to surprise P to attain a better output gap.

This information transmission game can be viewed as a cheap talk game, which is introduced in Crawford and Sobel (1982). Cheap talk is a game between a sender (S) and a receiver (R). First S observes a state generated from some distribution and sends a signal of the state to R. Then, R takes an action based on the information extracted from the signal. The utilities of S and R depend only on the state and the action. For each state, S's utility is slightly different from R's one, so S has an incentive to tell a small lie. In equilibrium of this game, though S never adopts the fully revealing strategy, information is partitionally conveyed to R. That is, S's signaling rule in equilibrium is characterized to be the one telling R in which partition of the state space the realization lies.

In the model of Moscarini (2007), there are two players, CB and P. The economy is described by the Phillips curve:

$$y = s(\pi - x),$$

where y is the output gap, π is the inflation rate, x is P's expectation of the inflation rate conditional on its available information, and s is the sensitivity of output to inflation forecast error. The desired inflation rate is $\pi^* + \omega$ where π^* is its average level and ω is a random variable drawn from a standard normal distribution $N(0, 1)$. CB observes a noisy signal $\theta = \omega + \varepsilon$ where $\varepsilon \sim N(0, \sigma^2)$. ω and ε are independent, and σ^2 is assumed to be known. CB behaves to minimize the following loss:

$$L = E[(y - b)^2 + \lambda(\pi - \pi^* - \omega)^2 | \theta],$$

where b is the desired output gap and λ is the weight on inflation rate stabilization relative to output gap optimization. CB can send a signal of θ to P, and P forms an expectation x of

π based on it. Although Moscarini (2007) does not explicitly mention this, we define CB's signaling rule as follows for the sake of clarity.

Definition 2.1. *Suppose that $\mathcal{M}$ is a family of nonempty subsets of $\mathbb{R}$ such that $\coprod_{A\in\mathcal{M}} A = \mathbb{R}$. Suppose that $\varsigma : \mathbb{R} \to \mathcal{M}$ is a map which maps θ to $A \in \mathcal{M}$ such that $\theta \in A$. The pair $(\varsigma, \mathcal{M})$ is called a signaling rule. Especially, we call $\mathcal{M}$ a message space.*

The sequence of this game is described as

1. CB determines a signaling rule ς;
2. $\omega \sim N(0, 1)$ realizes;
3. CB observes θ;
4. CB sends a signal $A \in \mathcal{M}$ to P;
5. P formulates an expectation $x(A) = E[\pi|A]$;
6. CB chooses an inflation rate $\pi(\theta, A)$.

When CB observes θ and sends A to P, its loss is

$$E[(s(\pi(\theta, A) - x(A)) - b)^2 + \lambda(\pi(\theta, A) - \pi^* - \omega)^2|\theta].$$

From the first order condition,

$$s(s(\pi(\theta, A) - x(A)) - b) + \lambda(\pi(\theta, A) - \pi^* - E[\omega|\theta]) = 0.$$

Together with $x(A) = E[\pi(\theta, A)|A]$, in equilibrium,

$$\begin{aligned}
x(A) &= \pi^* + \frac{sb}{\lambda} + HE[\theta|\theta \in A],\\
\pi(\theta, A) &= \pi^* + \frac{sb}{\lambda} + H\frac{s^2 E[\theta|\theta \in A] + \lambda\theta}{s^2 + \lambda},\\
y(\theta, A) &= \frac{sH\lambda}{s^2 + \lambda}(\theta - E[\theta|\theta \in A]), \quad \text{and}\\
L(A|\theta) &= \frac{s^2 H^2 \lambda}{s^2 + \lambda}(\theta - E[\theta|\theta \in A])^2 - 2bsH(\theta - E[\theta|\theta \in A]) + b^2(1 + \frac{s^2}{\lambda}) + \lambda(1 - H)
\end{aligned}$$

follow. Here we denote $H \coloneqq \frac{1}{1+\sigma^2}$.

The following is implicitly assumed in Moscarini (2007).

Assumption 1. *For any message space $\mathcal{M}$, $x : \mathcal{M} \to \mathbb{R}$ is one-to-one, i.e., any different two signals do not induce the same expectation.*

CB's signaling rule must satisfy the following incentive compatibility condition:

Definition 2.2. *Let $(\varsigma, \mathcal{M})$ be a signaling rule. $(\varsigma, \mathcal{M})$ is incentive-compatible if it satisfies the following condition:*

$$\textit{for all } \theta \in \mathbb{R} \textit{ and } A \in \mathcal{M}, \quad L(\varsigma(\theta)|\theta) \le L(A|\theta). \tag{I.C.}$$

This condition is necessary because CB has to restrict itself not to tell a lie even after seeing a realized state. In other words, CB can credibly commit itself to adopt a signaling rule under this condition.

Let $\theta \in \mathbb{R}$. Suppose that $(\varsigma, \mathcal{M})$ is an incentive-compatible signaling rule such that $\varsigma(\theta) = A$. (I.C.) implies that for all $A' \in \mathcal{M}$, $L(A|\theta) \leq L(A'|\theta)$, which is reduced to

$$(E[\theta|A] - E[\theta|A'])(E[\theta|A] + E[\theta|A'] - 2\theta + q) \leq 0,$$

where $q := \frac{2b(s^2+\lambda)}{sH\lambda}$. Using this form of incentive compatibility, Moscarini (2007) correctly notices the following property of message spaces satisfying (I.C.).

Proposition 2.1 (Lemma 1 of Moscarini (2007))**.** *Let $(\varsigma, \mathcal{M})$ be a signaling rule satisfying (I.C.). For all $A \in \mathcal{M}$, A is connected.*

Proof. See Moscarini (2007). □

One of the main conclusions of Moscarini (2007) is that all equilibria in this game are partitional, of which decent proof is provided in the next subsection. That is, a credible signaling rule is the one that CB tells P which partition the observation exists for some partition of $\mathbb{R}$ determined in advance. This result is consistent with Crawford and Sobel (1982). The model of Moscarini (2007) corresponds to that of Crawford and Sobel (1982) in the following way. CB and P play the role of sender and receiver, respectively. P is so rational that it tries to expect monetary policy as accurately as possible. Equivalently, if P knew the realized social state, θ, P, of course, predicts it as it is. However, CB's favorite inflationary expectation by P is different from P's. When CB observes θ and sends A to P, we can transform its loss function as follows:

$$L(A|\theta) = \frac{s^2H^2\lambda}{s^2+\lambda}(E[\theta|\theta \in A] - (\theta - \frac{q}{2}))^2 + const.,$$

which implies that CB wants P to predict the social state as $\theta - \frac{q}{2}$, lower than the true one, θ. This comes from CB's motivation to surprise P to achieve a high output gap. As the model of Crawford and Sobel (1982) alludes, this conflict of interests, $\frac{q}{2}$, makes communication coarse. This is how the game of Moscarini (2007) is regarded as cheap talk.

2.2 Supplements for Proofs

Moscarini (2007) concludes that a message space is a countable set of intervals and half-lines as an immediate result of his Lemma 1. However, it only implies that each message is either a point, an interval, or a half-line. To prove what Moscarini (2007) tries to state, it remains to be shown that a message space does not contain any singleton. After confirming it, we also discover that a message space is at most countable. First, we show that each message is uniformly apart from others.

Proposition 2.2. *Let $(\varsigma, \mathcal{M})$ be a signaling rule satisfying (I.C.). Then for any $A, A' \in \mathcal{M}$ such that $A \neq A'$,*

$$|E[\theta|A] - E[\theta|A']| > \frac{q}{2}.$$

Proof. Let $\theta \in A$ and $\theta' \in A'$. We may assume that $\theta > \theta'$. From (I.C.),

$$L(A|\theta) \leq L(A'|\theta),$$

and

$$L(A|\theta') \geq L(A'|\theta').$$

From the continuity of L, there exists $\theta^* \in [\theta', \theta]$ such that

$$L(A|\theta^*) = L(A'|\theta^*).$$

Since A and A' are both connected, $E[\theta|\tilde{A}]$ is increasing in $\tilde{A}$.[1] Hence, from the shape of $L(\cdot|\theta^*)$,

$$E[\theta|A'] \leq \theta^* - \frac{q}{2} \leq E[\theta|A]. \tag{1}$$

If CB observes $\tilde{\theta}$ and informs P of A, $\tilde{\theta} \geq \theta^*$. Otherwise, CB can reduce its loss by sending A'. Therefore,

$$E[\theta|A] \geq \theta^*. \tag{2}$$

From (1) and (2), the statement holds. □

Then, we can confirm that a partition in equilibrium consists of a countable number of intervals and half-lines.

Proposition 2.3 (Part of Lemma 2 of Moscarini (2007))**.** *Let $(\varsigma, \mathcal{M})$ be a signaling rule satisfying (I.C.). $\mathcal{M}$ is a set of at most a countable number of intervals and half-lines.*

Proof. From Proposition 2.2, one can choose $A \in \mathcal{M}$ in order of $|E[\theta|A]|$, so $\mathcal{M}$ is at most countable. We know that each element of $\mathcal{M}$ is connected from Proposition 2.1, and therefore all that remains is to show that $\mathcal{M}$ does not include any singleton. Assume that $\{\theta\} \in \mathcal{M}$. Since $\mathcal{M}$ is at most countable, there exist $A, A' \in \mathcal{M}$ such that $\inf A = \theta = \sup A'$. From (I.C.),

$$L(A|\theta) = L(\{\theta\}|\theta) = L(A'|\theta)$$

holds. Since $L(\cdot|\theta)$ is quadratic, at least two of $E[\theta|A], E[\theta|\theta]$ and $E[\theta|A']$ are equivalent, which contradicts Assumption 1 that different signals imply different expectations. □

Now we are ready to determine the form of a partition in equilibrium.

Proposition 2.4 (Lemma 4 of Moscarini (2007))**.** *Let $(\varsigma, \mathcal{M})$ be a signaling rule satisfying (I.C.). Then there exist $K - 1$ real values $\theta_1 < \cdots < \theta_{K-1}$ such that the message space is written as $\mathcal{M} = ((-\infty, \theta_1), [\theta_1, \theta_2), \cdots, [\theta_{K-1}, \infty))$.*

Proof. What we have to show is the boundedness of the partition. See the appendix of Moscarini (2007) for it. □

[1] Since $\mathcal{M}$ is a family of disjoint subsets of $\mathbb{R}$, one may assume that it is equipped with the canonical order induced by that of $\mathbb{R}$.

So far we have confirmed that any signaling rule in equilibrium can be written in the form stated in Proposition 2.4. Here we will consider how to construct a partition in equilibrium. Let $\mathcal{M} = ((-\infty, \theta_1), [\theta_1, \theta_2), \cdots, [\theta_{K-1}, \infty))$ is a message space that satisfies (I.C.). From (I.C.) at θ_k,

$$L([\theta_{k-1}, \theta_k), |\theta_k) \geq L([\theta_k, \theta_{k+1})|\theta_k).$$

If the inequality strictly holds, from the continuity of L, there exists sufficiently small $\varepsilon > 0$ such that

$$L([\theta_{k-1}, \theta_k)|\theta_k - \varepsilon) > L([\theta_k, \theta_{k+1})|\theta_k - \varepsilon),$$

which contradicts (I.C.). Hence,

$$L([\theta_{k-1}, \theta_k)|\theta_k) = L([\theta_k, \theta_{k+1})|\theta_k)$$

holds. By transforming this equation, we obtain the following *arbitrage condition*:

$$E[\theta|\theta_k < \theta < \theta_{k+1}] = 2\theta_k - q - E[\theta|\theta_{k-1} < \theta < \theta_k] \tag{A.C.}$$

for $k = 1, \cdots, K-1$. ($\theta_0 = -\infty$ and $\theta_K = \infty$.) This formulation implies that if the initial point θ_1 is determined, the other points are automatically determined according to (A.C.). If the condition does not hold for some θ_k, it means that the initial point θ_1 is not an element of a partition of a K-message equilibrium. Conversely, if $\mathcal{M}$ satisfies (A.C.), $\mathcal{M}$ induces a signaling rule of a K-message equilibrium. Therefore, all the equilibria of this game are characterized by sequences that satisfy the arbitrage condition (A.C.) for all elements.

Moscarini (2007) also claims in his Proposition 3 that when the maximum size of a message space is K, for all $k = 1, \cdots, K$, there exists a k-message equilibrium. He omits the proof, but this is not so obvious. Kono and Kandori (2019) point out that this kind of statement does not hold in general. They find some difficulties in the proof of Theorem 1 of Crawford and Sobel (1982) which suggests a similar statement to Proposition 3 in Moscarini (2007). Kono and Kandori (2019) construct a counter-example to it. According to Crawford and Sobel (1982), if there is a partial partition, i.e., there is a sequence of points satisfying the arbitrage condition except for the final point, then one can find the initial point such that its associated sequence satisfies the condition including for the final point. This is because each point is continuous in the initial point. They observe this correctly, but Kono and Kandori (2019) demonstrate that it may happen that before the final point satisfies the arbitrage condition, another point hits its nearby one, which ruins their proof. We will confirm that any size of partition equilibrium exists up to the maximum size based on the proof provided in Kono and Kandori (2019).

First, each element of a sequence satisfying (A.C.) is continuous in its initial point.

Proposition 2.5. *Suppose that $-\infty = \theta_0 < \theta_1 < \cdots, < \theta_{k-1} < \infty$ satisfy*

$$E[\theta|\theta_l < \theta < \theta_{l+1}] = 2\theta_l - q - E[\theta|\theta_{l-1} < \theta < \theta_l] \tag{3}$$

for $l = 0, \cdots k-2$. Then there exist an open neighborhood U of θ_1 and continuous functions $\psi_l : U \to \mathbb{R}$ for $l = 1, \cdots k-1$ such that $\psi_l(\theta_1) = \theta_l$ and $(-\infty = \theta'_0, \psi_1(\theta'_1), \psi_2(\theta'_1), \cdots, \psi_{k-1}(\theta'_1))$ satisfies (A.C.).

Proof. We execute induction on $l = 2, \cdots, k-1$. Let $\psi_0(\theta'_1) \coloneqq -\infty$ and $\psi_1(\theta'_1) \coloneqq \theta'_1$, and $U_0 \coloneqq U_1 \coloneqq \mathbb{R}$. (Note that ψ_0 is improper since it does not take values in $\mathbb{R}$.) Suppose that ψ_{l-1}, ψ_l, and U_l are given. Let

$$h(\theta'_1, \theta'_{l+1}) \coloneqq 2\psi_l(\theta'_1) - \bar{q} - E[\theta|\psi_{l-1}(\theta'_1) < \theta < \psi_l(\theta'_1)] - E[\theta|\psi_l(\theta'_1) < \theta < \theta'_{l+1}]$$

on $U_l \times \{\theta'_{l+1} > \theta'_1\}$. Since h is strictly decreasing in θ'_{l+1}, and $h(\theta_1, \theta_{l+1}) = 0$, we can apply the type of the implicit function theorem proposed in Kono and Kandori (2019). Therefore, there exist an open set U_{l+1} and a continuous function $\psi_{l+1} : U_{l+1} \to \mathbb{R}$ such that $\psi_{l+1}(\theta_1) = \theta_{l+1}$ and $h(\theta'_1, \psi_{l+1}(\theta'_1)) = 0$ for all $\theta'_1 \in U_{l+1}$. Inductively, we can construct ψ_l and U_l for $l = 0, \cdots, k-2$. By letting $U = U_{k-1}$ and restricting ψ_l to U, the conclusion follows. $\square$

What we want follows from the next proposition.

Theorem 2.1 (Part of Proposition 3 in Moscarini (2007)). *Let $(\varsigma, \mathcal{M})$ be a signaling rule satisfying (I.C.) and $\mathcal{M}$ has K elements. Then, for $k = 1, \cdots, K$, there exists a signaling rule $(\varsigma_k, \mathcal{M}_k)$ such that $\mathcal{M}_k$ has k elements.*

Proof. We have nothing to show for $k = 1$, so we may assume that $k \geq 2$. Let

$$D \coloneqq \left\{ \theta_1 \in \mathbb{R} \;\middle|\; \begin{array}{l} \text{There exist } -\infty = \theta_0 < \theta_1 < \cdots < \theta_k < \infty \text{ such that} \\ \begin{cases} E[\theta|\theta_l < \theta < \theta_{l+1}] = 2\theta_l - q - E[\theta|\theta_{l-1} < \theta < \theta_l] \text{ for } l = 0, \cdots k-1 \\ E[\theta|\theta > \theta_k] \geq 2\theta_k - q - E[\theta|\theta_{k-2} < \theta < \theta_{k-1}] \end{cases} \end{array} \right\}.$$

Since we assume that there exists K-partition, D is not empty. Let $\theta^*_1 \coloneqq \sup D$. There exists a sequence $\{\theta^n_1\}_{n \in \mathbb{N}}$ in D such that $\lim_{n\to\infty} \theta^n_1 = \theta^*_1$. From the definition of D, for each n, there exist $-\infty = \theta^n_0 < \theta^n_1 < \cdots < \theta^n_k$ satisfying the conditions of D. Since a k-message partition is bounded[2], $\{(\theta^n_1, \cdots, \theta^n_k)\}_{n\in\mathbb{N}}$ lies in a bounded set of $\mathbb{R}^k$. Therefore, there exists a convergent subsequence $\{(\theta^{n(i)}_1, \cdots, \theta^{n(i)}_k)\}_{i\in\mathbb{N}}$ converging to a k-dimensional vector $(\theta^*_1, \cdots, \theta^*_k)$. We show that $\theta^*_1 \in D$. From the result in Moscarini (2007), for all $i \in \mathbb{N}$ and $l = 1, \cdots, k$,

$$\theta^{n(i)}_l - \theta^{n(i)}_{l-1} > q.$$

By letting $i \to \infty$,

$$\theta^*_l - \theta^*_{l-1} \geq q$$

for $l = 1, \cdots, k$, which especially implies

$$\theta^*_1 < \cdots < \theta^*_k. \tag{4}$$

From the definition of D, for all $i \in \mathbb{N}$ and $l = 0, \cdots, k-1$,

$$E[\theta|\theta^{n(i)}_l < \theta < \theta^{n(i)}_{l+1}] = 2\theta^{n(i)}_l - q - E[\theta|\theta^{n(i)}_{l-1} < \theta < \theta^{n(i)}_l].$$

From the continuity of the expectation operator, by letting $i \to \infty$, we obtain

$$E[\theta|\theta^*_l < \theta < \theta^*_{l+1}] = 2\theta^*_l - q - E[\theta|\theta^*_{l-1} < \theta < \theta^*_l] \tag{5}$$

[2]See the appendix in Moscarini (2007).

for $l = 0, \cdots, k-1$. Similarly,

$$E[\theta|\theta > \theta_k^*] \geq 2\theta_k^* - q - E[\theta|\theta_{k-2}^* < \theta < \theta_{k-1}^*].$$

Therefore, $\theta_1^* \in D$. Now, suppose that this inequality strictly holds. Then there exists an element in D larger than θ_1^*, which contradicts the definition of θ_1^*. That is,

$$E[\theta|\theta > \theta_k^*] = 2\theta_k^* - q - E[\theta|\theta_{k-2}^* < \theta < \theta_{k-1}^*]. \tag{6}$$

These three results (4), (5), and (6) conclude that $\mathcal{M}_k = ((-\infty, \theta_1^*), [\theta_1^*, \theta_2^*), \cdots, [\theta_k^*, \infty))$ induces a k-message signaling rule. $\square$

Throughout this subsection, we have confirmed that all equilibria are partitional and that there exists any size of partition equilibrium up to the maximum size. These conclusions are consistent with Crawford and Sobel (1982). We also have to mention that in the above proofs, we do not use any specific property to the distribution of θ. They are still valid even when θ is drawn from a different distribution from a standard normal distribution. This point theoretically justifies our models in the next section.

3 Models and Results

In Moscarini (2007) model, the sender is CB itself, i.e., it observes only one state while CB, in reality, holds several members in its committee, each of who individually observes a social state and reports it to the board. The abstraction of Moscarini (2007) is too much since decision-making by committee is one of the greatest aspects of central banking, as Blinder (2008) remarks. In this section, we will extend the model of Moscarini (2007) to the one in which CB organizes a committee.

3.1 Foundations of Model

The essence of the game is the same as Moscarini (2007), but we assume that CB has multiple sources of information about the true state. CB holds n members $i = 1, \cdots, n$, and ith member observes $\theta_i := \omega + \varepsilon_i$. Note that $\omega \sim N(0,1), \varepsilon_1, \cdots, \varepsilon_n \sim N(0, \sigma^2)$, and $\omega, \varepsilon_1, \cdots, \varepsilon_n$ are independent. We denote $\boldsymbol{\theta} := (\theta_1, \cdots, \theta_n)$. CB decides its strategy so that it minimizes its loss function:

$$L = E[(y-b)^2 + \lambda(\pi - \pi^* - \omega)^2|\boldsymbol{\theta}].$$

We define signaling rule for CB with committee as follows.

Definition 3.1 (committee version of signaling rule)**.** *Suppose that $\mathcal{M}$ is a family of nonempty subsets of $\mathbb{R}^n$ such that $\coprod_{A\in\mathcal{M}} A = \mathbb{R}^n$. Suppose that $\varsigma : \mathbb{R}^n \to \mathcal{M}$ is a map which maps $\boldsymbol{\theta}$ to $A \in \mathcal{M}$ such that $\boldsymbol{\theta} \in A$. The pair $(\varsigma, \mathcal{M})$ is called a signaling rule.*

The committee version of the incentive compatibility condition is

Definition 3.2 (committee version of incentive compatibility)**.** *Let $(\varsigma, \mathcal{M})$ be a signaling rule. $(\varsigma, \mathcal{M})$ is incentive-compatible if it satisfies the following condition:*

$$\text{for all } \boldsymbol{\theta} \in \mathbb{R}^n \text{ and } A \in \mathcal{M}, \quad L(\varsigma(\boldsymbol{\theta})|\boldsymbol{\theta}) \le L(A|\boldsymbol{\theta}). \tag{C.I.C.}$$

For boundary states of a partition, the nearby two signals are indifferent to CB, so theoretically it may choose either of them at each boundary point. To simply characterize partition in equilibrium, we assume that states on a certain boundary induce the same signal, which is represented in the following assumption.

Assumption 2. *For all $A, A' \in \mathcal{M}$,*

$$\{\boldsymbol{\theta} \in A \cup A' | L(\boldsymbol{\theta}|A) = L(\boldsymbol{\theta}|A')\} \subset A, \text{ or } \{\boldsymbol{\theta} \in A \cup A' | L(\boldsymbol{\theta}|A) = L(\boldsymbol{\theta}|A')\} \subset A',$$

i.e., signals are consistent on each boundary.

This assumption is justified from both theoretical and practical points of view. First, since the probability of CB's observing states on some boundary is zero, what happens there does not matter to the conclusion. Besides, even on states on a boundary, CB indeed prefers either signal to the other for some practical reasons that our model does not capture.

3.2 Model 1: Straightforward Extension

First, we construct a baseline model that directly extends Moscarini (2007). We assume that the variance of noise σ^2 is common knowledge. Let $H_n := \frac{n}{n+\sigma^2}$ and $\bar{\theta}_n := \frac{1}{n}\sum_{i=1}^n \theta_i$. In the same way as Moscarini (2007), we obtain

$$\begin{aligned}
x(A) &= \pi^* + \frac{sb}{\lambda} + H_n E[\bar{\theta}_n|\boldsymbol{\theta} \in A], \\
\pi(\boldsymbol{\theta}, A) &= \pi^* + \frac{sb}{\lambda} + H_n \frac{s^2 E[\bar{\theta}_n|\boldsymbol{\theta} \in A] + \lambda\bar{\theta}_n}{s^2+\lambda}, \\
y(\boldsymbol{\theta}, A) &= \frac{sH_n\lambda}{s^2+\lambda}(\bar{\theta}_n - E[\bar{\theta}_n|\boldsymbol{\theta} \in A]), \quad \text{and} \\
L(A|\boldsymbol{\theta}) &= \frac{s^2H_n^2\lambda}{s^2+\lambda}(\bar{\theta}_n - E[\bar{\theta}_n|\boldsymbol{\theta} \in A])^2 - 2bsH_n(\bar{\theta}_n - E[\bar{\theta}_n|\boldsymbol{\theta} \in A]) + b^2(1+\frac{s^2}{\lambda}) + \lambda(1-H_n)
\end{aligned}$$

for observed states $\boldsymbol{\theta}$ and a signal $A \in \mathcal{M}$. The following proposition specifies the class of signaling rules we have to consider.

Theorem 3.1. *Let $(\varsigma, \mathcal{M})$ be a signaling rule satisfying (C.I.C.) and $A, A' \in \mathcal{M}$. Suppose that two observations $\boldsymbol{\theta} \in A$ and $\boldsymbol{\theta}' \in A'$ have the same sample mean. Then, $A = A'$.*

Proof. When CB observes $\boldsymbol{\theta}$, it prefers A to A', which means

$$L(A|\boldsymbol{\theta}) \le L(A'|\boldsymbol{\theta}).$$

When observing $\boldsymbol{\theta}'$, vice versa:

$$L(A|\boldsymbol{\theta}') \ge L(A'|\boldsymbol{\theta}').$$

From the form of L, $L(\cdot|\boldsymbol{\theta})$ and $L(\cdot|\boldsymbol{\theta}')$ are identical. Therefore,

$$L(A|\boldsymbol{\theta}) = L(A'|\boldsymbol{\theta}) = L(A|\boldsymbol{\theta}') = L(A'|\boldsymbol{\theta}').$$

The equality first equality implies that $\boldsymbol{\theta}$ lies in the border between A and A', and so does $\boldsymbol{\theta}'$ from the third equality. From Assumption 2 we conclude $A = A'$. $\square$

This statement alludes that if two states have the same mean, they belong to the same partition, that is, CB sends the same signal. As long as we analyze signaling rules with (C.I.C.), we only have to consider the following form of message spaces:

$$\mathcal{M} = (\{\boldsymbol{\theta}|\bar{\theta}_n \in B_\lambda\})_{\lambda\in\Lambda} \text{ where } \coprod_{\lambda\in\Lambda} B_\lambda = \mathbb{R}\ .$$

Now we can identify signaling rules satisfying (C.I.C.) as unidimensional ones since we can assume that CB observes $\bar{\theta}_n \in \mathbb{R}$ and sends information about it to P, who forms an expectation based on it. In the same way as in the previous section, we obtain the following arbitrage condition for committee system:

$$E[\bar{\theta}_n|\bar{\theta}_{n,k} < \bar{\theta}_n < \bar{\theta}_{n,k+1}] = 2\bar{\theta}_{n,k} - q - E[\bar{\theta}_n|\bar{\theta}_{n,k-1} < \bar{\theta}_n < \bar{\theta}_{n,k}],$$

for $k = 1, \cdots, K-1$, where $((-\infty = \bar{\theta}_{n,0}, \bar{\theta}_{n,1}), [\bar{\theta}_{n,1}, \bar{\theta}_{n,2}), \cdots, [\bar{\theta}_{n,K-1}, \bar{\theta}_{n,K} = \infty))$ is a partition of $\mathbb{R}$. Then, we can conclude that CB's signaling rule with the incentive compatibility condition is the one that it tells P in which partition the average of its committee members' observations lie.

3.3 Introspection of Model 1

As we have seen above, Model 1 concludes that CB pays attention only to the average of all the members' observations, and therefore, signaling rules in equilibrium are partitional with respect to it. This outcome is essentially the same as the model of Moscarini (2007) since CB eventually cares only about one value, $\bar{\theta}_n$, in spite of the introduction of committee system. No other information than the average contributes to decision-making or communication in Model 1.

Unlike individual decision-making, in a model with committee, agents should be able to extract higher-order information from the members' observations, like variance. Suppose that a central bank has two members in its committee, A and B, and compare the following two situations:

1. A observes 0.1 and B observes -0.1;
2. A observes 100 and B observes -100.

Intuitively, in the first case, people take what CB says more seriously than in the second one. Those who have the former information would suppose that CB's observations are almost pure, that is, the noise is negligible. On the other hand, people with the latter information would cast doubt on the competence of the committee since the observations are too diverse to believe. Rational people would make much account of the signals from CB if the variance is relatively small, and they would almost ignore the signals with a large variance. In spite of

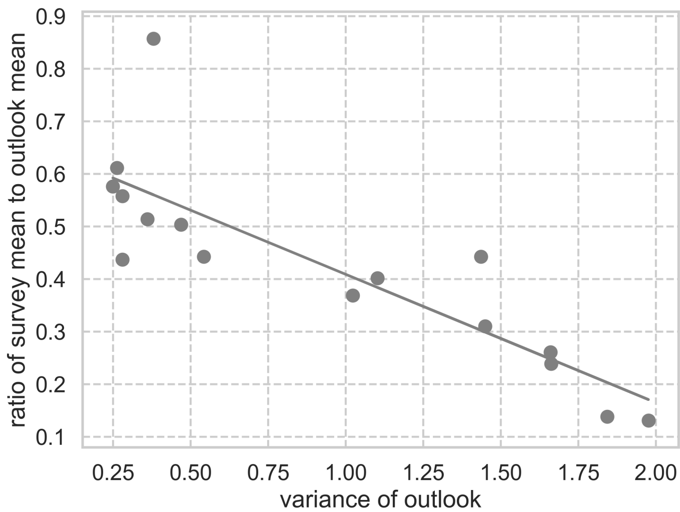

Figure 1: The horizontal axis represents the variance of BOJ's committee members' outlooks. The vertical one represents the ratio of the mean calculated from the survey to that of the committee outlooks.

these intuitions, Model 1 implies that these two situations are completely equivalent because the mean is zero in both cases. The first model fails to take the degree of diversity of the observations into account.

Not only in a thought experiment, but also in the real economy, people pay attention to the discrepancy among the signals. The bank of Japan (BOJ), which has nine members in its committee, quarterly publishes "Outlook for Economic Activity and Prices."[3] In the report, BOJ discloses each member's outlook on the price level in a year. BOJ also opens "Inflation Outlook of Enterprises,"[4] which contains the result of a survey that asks firms how they will set prices in a year. We can reckon BOJ's outlooks as their signal and the firms' prospect of price as an expectation of the inflation rate of the private sector. Under the hypothesis we proposed above, the firms' prediction would be close to the average of BOJ's signals if the variance is sufficiently small. If it is large, to the contrary, firms would make virtually no account of BOJ's signal and their prediction would be near their prior mean, zero. Figure 1 shows that the reasoning seems true. Its horizontal axis represents the variance of BOJ's signals while the vertical one corresponds to the ratio of the expectation of firms to that of BOJ. One can see a negative relation between them. As the variance gets larger, the ratio gets closer to zero, that is, the private sector tends to ignore the information from BOJ.

These examples suggest the necessity to build a model in which the diversity of observations has an impact on results in equilibrium. In the next subsection, we will propose a new model that meets this requirement.

[3]URL: http://www.boj.or.jp/en/mopo/outlook/index.htm/ (visited: 2022-01-14).
[4]URL: https://www.boj.or.jp/en/statistics/tk/bukka/index.htm/ (visited: 2022-01-14).

3.4 Model 2: Variance-based Expectation Model

As we have mentioned in the previous subsection, the goal is to construct a model that implies that the agents' expectations depend on the variance of available information as well as the mean. We give up the assumption of Model 1 that the players exactly know the variance of noise, σ^2. Instead, we assume that σ^2 is a random variable following $IG(\alpha, \beta)$ for some $\alpha, \beta > 0$, which is common knowledge. Note that $IG(\alpha, \beta)$ denotes an inverse gamma distribution with the shape parameter α and the scale parameter β. The probability density function is

$$p(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{1}{x}\right)^{\alpha+1} \exp\left(-\frac{\beta}{x}\right).$$

This prior may seem arbitrary, but it can represent a variety of shapes by varying α and β, so this assumption is not very restrictive.

The game proceeds as follows:

1. CB determines a signaling rule ς;
2. $\omega \sim N(0, 1)$ and $\sigma^2 \sim IG(\alpha, \beta)$ realize.
3. ith committee member observes $\theta_i = \omega + \varepsilon_i$ where $\varepsilon_i \sim N(0, \sigma^2)$;
4. CB sends a signal $A \in \mathcal{M}$ to P;
5. P formulates an expectation $x(A) = E[\pi|A]$;
6. CB chooses an inflation rate $\pi(\boldsymbol{\theta}, A)$.

In a similar way to the previous two models, when CB observes $\boldsymbol{\theta}$ and sends $A \in \mathcal{M}$, its loss is

$$L(A|\boldsymbol{\theta}) = \frac{s^2\lambda}{s^2 + \lambda}(\tau(\boldsymbol{\theta}) - E[\tau(\boldsymbol{\theta})|\boldsymbol{\theta} \in A])^2 - 2bs(\tau(\boldsymbol{\theta}) - E[\tau(\boldsymbol{\theta})|\boldsymbol{\theta} \in A]) + b^2(1 + \frac{s^2}{\lambda}) + \lambda Var(\omega|\boldsymbol{\theta}),$$

where $\tau(\boldsymbol{\theta}) := E[\omega|\boldsymbol{\theta}]$. For further analysis, we need to compute the distribution of $\omega|\boldsymbol{\theta}$. We first enumerate several useful results. Let $V_n := \sum_{i=1}^n (\theta_i - \bar{\theta}_n)^2$.

Lemma 3.1. *The followings hold:*

$$\begin{aligned}
\theta_1, \cdots, \theta_n \mid \omega, \sigma^2 &\sim i.i.d. N(\omega, \sigma^2), \\
\bar{\theta}_n \mid \omega, \sigma^2 &\sim N(\omega, \frac{\sigma^2}{n}), \\
\bar{\theta}_n \mid \sigma^2 &\sim N(0, 1 + \frac{\sigma^2}{n}), \\
\frac{V_n}{\sigma^2} \mid \omega, \sigma^2 &\sim \chi^2(n-1), \\
V_n \mid \sigma^2 &\sim Ga(\frac{n-1}{2}, 2\sigma^2), \quad \textit{and} \\
\bar{\theta}_n \perp\!\!\!\perp V_n &\mid \omega, \sigma^2
\end{aligned}$$

Proof. These are basic results of mathematical statistics. □

Now we are ready to obtain the required distribution:

Proposition 3.1. *The followings hold:*

$$\pi(\omega|\boldsymbol{\theta}) \propto \frac{\exp(-\frac{\omega^2}{2})}{\{n(\bar{\theta}_n - \omega)^2 + V_n + 2\beta\}^{\frac{n}{2}+\alpha}}$$

Proof. See A.1. □

Then we find that

$$\tau(\boldsymbol{\theta}) = \frac{1}{C(\bar{\theta}_n, V_n)} \int_{-\infty}^{\infty} \frac{\omega \exp(-\frac{\omega^2}{2})}{\{n(\bar{\theta}_n - \omega)^2 + V_n + 2\beta\}^{\frac{n}{2}+\alpha}} d\omega,$$

where

$$C(\bar{\theta}_n, V_n) = \int_{-\infty}^{\infty} \frac{\exp(-\frac{\omega^2}{2})}{\{n(\bar{\theta}_n - \omega)^2 + V_n + 2\beta\}^{\frac{n}{2}+\alpha}} d\omega.$$

Since τ depends only on $\bar{\theta}_n$ and V_n, we denote $\tau(\boldsymbol{\theta})$ by $\tau(\bar{\theta}_n, V_n)$ by abusing notation. After some calculus, one can find that τ satisfies the following properties:

1. $\lim_{V_n \to \infty} \tau(\bar{\theta}_n, V_n) = 0$;
2. τ is strictly monotone in V_n.

We can see some intuitions of them in Figure 2.

The following proposition claims that in the present model, we only have to think about message spaces associated with τ for the analysis of equilibrium. Its proof is the same as Theorem 3.1.

Theorem 3.2. *Let $(\varsigma, \mathcal{M})$ be a signaling rule satisfying (C.I.C.) and $A, A' \in \mathcal{M}$. Suppose that $\tau(\boldsymbol{\theta}) = \tau(\boldsymbol{\theta}')$ for two observations $\boldsymbol{\theta} \in A$ and $\boldsymbol{\theta}' \in A'$. Then, $A = A'$.*

Therefore, signaling rules with (C.I.C.) can be written as

$$\mathcal{M} = (\{\boldsymbol{\theta} | \tau(\boldsymbol{\theta}) \in B_\lambda\})_{\lambda \in \Lambda} \text{ where } \coprod_{\lambda \in \Lambda} B_\lambda = \mathbb{R} .$$

In the same way we propose in Section 2, we can find that $(B_\lambda)_{\lambda \in \Lambda}$ is a finite family of intervals and half-lines. Suppose that a partition of $\mathbb{R}$, $((-\infty = \tau_0, \tau_1), [\tau_1, \tau_2), \cdots, [\tau_{K-1}, \tau_K = \infty))$, induces a message space in equilibrium. The arbitrage condition is

$$E[\tau|\tau_k < \tau < \tau_{k+1}] = 2\tau_k - r - E[\tau|\tau_{k-1} < \tau < \tau_k],$$

for $k = 1, \cdots, K-1$ where $r := \frac{2b(s^2+\lambda)}{s\lambda}$. CB's signaling rule with the incentive compatibility condition is the one that it conveys P in which partition τ computed from its committee members' observations lies.

In the last of this subsection, we confirm that this model involves what we want. First of all, $\tau(\boldsymbol{\theta})$ is an estimator of ω and basically takes a close value to $\bar{\theta}_n$. As the variance V_n gets larger, however, τ tends to zero. These properties correspond to the fact that people are likely to neglect the signal from the central bank and not to update their prior very much when the variance is huge. Now the variety of committee members matters in this model.

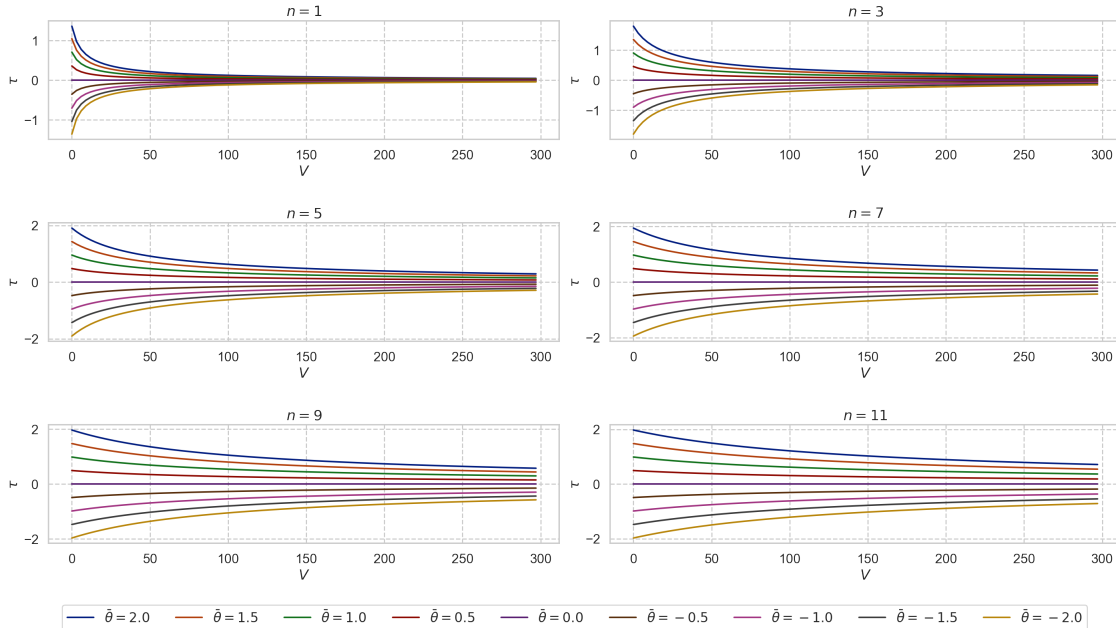

Figure 2: These figures illustrate the property of $\tau(\bar{\theta}, V)$ for $n = 1, 3, 5, 7, 9, 11$. ($\alpha = 3$ and $\beta = 1$.) One can notice that $\tau(\bar{\theta}, V)$ monotonically converges to zero as $V \to 0$.

4 Simulation Study

Based on the theoretical results we have found so far, we numerically compute equilibria of our two models and compare them. First, we propose an algorithm to find a partition in equilibrium based on the one provided in Moscarini (2007). When there exists a K-message equilibrium, we can compute the initial point of the partition by Algorithm 1. This algorithm uses the technique of binary search. It works because the difference between the two sides of the arbitrage condition at the final point of the partition is monotone in the initial point, i.e., $E[\theta|\theta_{K-1} < \theta] - 2\theta_{K-1} - q - E[\theta|\theta_{K-2} < \theta < \theta_{K-1}]$ is monotone in θ_1. Model 1 is essentially the same as the model of Moscarini (2007), so this algorithm is directly applicable to it. Even for Model 2, one can find equilibrium in this way just by regarding θ and q as τ and r, respectively [5].

Using Algorithm 1, we compute a 3-message equilibrium for both Model 1 and Model 2 under the parameters provided in Table 1. To illustrate partitions on a plain later, we set $n = 2$. For s, b and λ, we follow those chosen in Moscarini (2007). We adjust σ^2, α, and β so that the mean of $IG(\alpha, \beta)$ is equal to σ^2 to compare the two models. From the result of numerical computation, we find that the message space $\mathcal{M}_1$ of Model 1 is

$$\mathcal{M}_1 = (\{\boldsymbol{\theta}|\bar{\theta}_2 \leq -0.144\}, \{\boldsymbol{\theta}| -0.144 < \bar{\theta}_2 \leq 1.108\}, \{\boldsymbol{\theta}|1.108 < \bar{\theta}_2\}),$$

and the one $\mathcal{M}_2$ of Model 2 is

$$\mathcal{M}_2 = (\{\boldsymbol{\theta}\ |\tau(\boldsymbol{\theta}) \leq -0.140\}, \{\boldsymbol{\theta}\ | -0.140 < \tau(\boldsymbol{\theta}) \leq 0.973\}, \{\boldsymbol{\theta}\ |0.973 < \tau(\boldsymbol{\theta})\}).$$

[5]Note that since the distribution of τ does not have a closed form, we have to rely on the Monte Carlo method to compute $E[\tau]$.

Algorithm 1 Find K-message partition for Moscarini (2007) model

function Find-Equilibrium-Partition(K, q)
  find θ_1^{low} such that $\theta_1^{low} = E[\theta|\theta < \theta_1^{low}] + q$
  take a sufficiently large θ_1^{high}
  $\theta_1^{curr} = (\theta_1^{low} + \theta_1^{high})/2$
  while true **do**
    make the partial partition $(\theta_1^{curr}, \cdots, \theta_{K-1}^{curr})$ satisfying (A.C.)
    $d = E[\theta|\theta_{K-1}^{curr} < \theta] - 2\theta_{K-1}^{curr} - q - E[\theta|\theta_{K-2}^{curr} < \theta < \theta_{K-1}^{curr}]$
    if $d \approx 0$ **then**
      return θ_1^{curr}
    else if $d > 0$ **then**
      $\theta_1^{curr} = (\theta_1^{curr} + \theta_1^{high})/2$
      $\theta_1^{low} = \theta_1^{curr}$
    else
      $\theta_1^{curr} = (\theta_1^{curr} + \theta_1^{low})/2$
      $\theta_1^{high} = \theta_1^{curr}$
    end if
  end while
end function

Table 1: Parameters of simulation

	n	s	b	λ	σ^2	α	β
Model 1	2	0.25	0.02	5	0.5	–	–
Model 2	2	0.25	0.02	5	–	3	1

.

These message spaces correspond to CB's signal that sends to P either "low", "middle", or "high".

Figure 3 illustrates the boundary lines of $\mathcal{M}_1$ (solid lines) and $\mathcal{M}_2$ (dashed lines). When θ_1 and θ_2 are close to each other, two partitions are similar. When they are apart, on the other hand, the partitions are distinct. Especially, $\mathcal{M}_2$ has a larger "middle" area than $\mathcal{M}_1$ does, which reflects the fact that the expectation tends to zero when CB's committee members' observations are diverse. We now numerically confirm that the variety of opinions certainly matters in our second model. This is what we want to incorporate into our model as we mention two subsections ago.

5 Discussion

In the last section, we develop our second model by considering committee system under the reasonable assumption that no agent exactly knows the variance of noise. In this model, the policymaker considered in previous researches such as Moscarini (2007) can be viewed as CB whose signaling rule is restricted to the one which depends only on the average of the desired rates of inflation observed by the individual committee members. Those researches, of course, investigate communication with respect to just one value, and Model 1, which

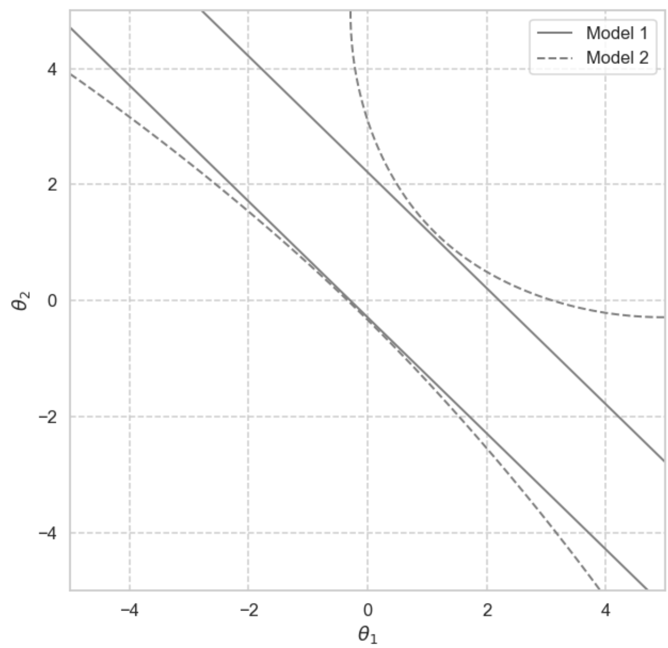

Figure 3: The solid lines illustrate the boundaries of the message for Model 1. The left and right lines correspond to $\bar{\theta}_2 = -0.144$ and $\bar{\theta}_2 = 1.108$, respectively. The dashed lines illustrate the boundaries of the message for Model 2. The left and right lines correspond to $\tau(\boldsymbol{\theta}) = -0.140$ and $\tau(\boldsymbol{\theta}) = 0.973$, respectively.

naturally extends them, also concludes that equilibria depend only on the average.

Our interest is whether signaling rules which tell P information only about the average can be an equilibrium in Model 2. The answer is NO. The intuitive reason for it is similar to the argument in Canzoneri (1985) who claims that commitments by central banks fail if they have private information. Even when CB transmits a signal only about the average to P, CB is motivated to refer the variance to estimate the true social state more precisely. Without information about the variance, P cannot completely monitor CB's decision process, so that P does not believe what CB says at all. To show this, we first prepare two technical statements.

Lemma 5.1. *Suppose that two nonempty subsets A and B of $\mathbb{R}$ satisfy $A \cup B = \mathbb{R}$ and $A \cap B = \emptyset$. If for all $a \in A$ and $b \in B$, $ab \leq 0$, then A and B are essentially equivalent to either of the followings:*

$$A = \mathbb{R} \setminus \{0\} \quad \textit{and} \quad B = \{0\};$$
$$A = \mathbb{R}_{\geq 0} \quad \textit{and} \quad B = \mathbb{R}_{<0};$$
$$A = \mathbb{R}_{>0} \quad \textit{and} \quad B = \mathbb{R}_{\leq 0}.\text{[6]}$$

[6] We denote $\{x \in \mathbb{R} | x > 0\}$ by $\mathbb{R}_{>0}$. The same holds for the other symbols.

Proof. Let $x > 0$. Suppose that $x \in A$. Then due to the assumption $xy \leq 0$ for all $y \in B$, $\mathbb{R}_{>0} \subset A$. If $x \in B$, $\mathbb{R}_{>0} \subset B$ in the same way. Therefore, either $\mathbb{R}_{>0} \subset A$ or $\mathbb{R}_{>0} \subset B$ is true. By considering the same thing for $x < 0$, we obtain the fact that either $\mathbb{R}_{<0} \subset A$ or $\mathbb{R}_{<0} \subset B$ is true. Finally, by considering which A or B contains zero, the statement holds. □

Proposition 5.1. *Let*

$$\tau(\bar{\theta}) := \tau(\bar{\theta}, 0) = \frac{1}{C(\bar{\theta}, 0)} \int_{-\infty}^{\infty} \frac{\omega \exp(-\frac{\omega^2}{2})}{(n(\bar{\theta} - \omega) + 2\beta)^{\alpha + \frac{n}{2}}} d\omega.$$

The followings hold:

$$\begin{aligned} &\textit{(i)}\ \tau(0) = 0; \\ &\textit{(ii) for all}\ \bar{\theta} > 0,\ \ \tau(\bar{\theta}) > 0; \\ &\textit{(iii) for all}\ \bar{\theta} < 0,\ \ \tau(\bar{\theta}) < 0. \end{aligned}$$

Proof. See A.2. □

Now, we are ready to prove the impossibility of average-based signaling rules.

Theorem 5.1. *For any incentive-compatible signaling rule which conveys information only about the average, the message space is trivial, i.e.,* $\mathcal{M} = \{\mathbb{R}\}$.

Proof. Assume that τ and τ' are the induced expectations by $\bar{\theta}$ and $\bar{\theta}'$, respectively, and $\tau \neq \tau'$. From the incentive compatibility, for all $V \geq 0$,

$$\begin{aligned} (\tau' - \tau)(2\tau(\bar{\theta}, V) - \tau - \tau' - r) \leq 0, \quad &\text{and} \\ (\tau - \tau')(2\tau(\bar{\theta}', V) - \tau' - \tau - r) \leq 0. \end{aligned}$$

Let $V \to \infty$, and we obtain

$$\begin{aligned} (\tau' - \tau)(-\tau - \tau' - r) \leq 0, \quad &\text{and} \\ (\tau - \tau')(-\tau' - \tau - r) \leq 0. \end{aligned}$$

Since $\tau \neq \tau'$, we find

$$\tau + \tau' + r = 0. \tag{7}$$

This implies that message spaces in equilibrium have at most two elements. Let $V \to 0$, and we obtain

$$\begin{aligned} (\tau' - \tau)(2\tau(\bar{\theta}) - \tau - \tau' - r) \leq 0, \quad &\text{and} \\ (\tau - \tau')(2\tau(\bar{\theta}') - \tau' - \tau - r) \leq 0. \end{aligned}$$

From Equation 7, we get

$$\tau(\bar{\theta})\tau(\bar{\theta}') \leq 0,$$

and from Proposition 5.1,

$$\bar{\theta}\bar{\theta}' \leq 0$$

holds.

Let $A \coloneqq \{\bar{\theta}; \bar{\theta} \text{ induces } \tau\}$ and $B \coloneqq \{\bar{\theta}'; \bar{\theta}' \text{ induces } \tau'\}$, and we apply Lemma 5.1. In the first case of the lemma, we end up with the conclusion that $\tau = \tau'$, which contradicts the assumption. In the latter two cases, $\tau + \tau' = 0$ holds, which are at odds with the property $\tau + \tau' = r > 0$. Therefore, there exists no 2-message equilibrium. □

This result indicates that communication between a central bank and the market is impossible in the way conventional models suggest in our new model, which is more realistic as we mention in the last section. We cannot reach this conclusion until we deal with decision-making by committee in this model, and we can reconfirm the significance to seriously consider central banking by committee as Blinder (2007) asserts.

6 Conclusion

Blinder (2008) offers transparency of central bank, communication with the market, and central banking by committee, which are called "quiet revolution," as three major issues central banks are facing today. We study those three topics for the central bank with private information, especially focusing on committee system. Our analysis concludes that a central bank has to make a decision based not only on the average of the desired inflation rates observed by individual committee members but also on their variance to avoid inflation bias and to make the private sector trust the central bank. Another important finding is that central banks who publish information only about the average cannot be credible since they could rely on the variance for decision-making in secret to surprise the private sector. This idea that the diversity of observations makes difference is brand new in the field of monetary policy game because it appears only when one incorporates committee system into the model. This thesis theoretically suggests the significance to investigate decision-making by committee more in game-theoretic approaches.

A Technical Proofs

A.1 Proof of Proposition 3.1

Proof. From a basic result of mathematical statistics, $(\bar{\theta}_n, V_n)$ is sufficient for (ω, σ^2), so $\omega|\boldsymbol{\theta}$ has the same distribution as $\omega|\bar{\theta}_n, V_n$. We compute the latter. First

$$\pi(\omega|\bar{\theta}_n, V_n) \propto \pi(\omega) \int_{-\infty}^{\infty} \pi(\bar{\theta}_n, V_n|\omega, \sigma^2)\pi(\sigma^2) d\sigma^2$$

follows from Bayes' theorem. Since $\bar{\theta}_n \perp\!\!\!\perp V_n \mid \omega, \sigma^2$ from Lemma 3.1,

$$
\begin{aligned}
&\pi(\omega)\int_{-\infty}^{\infty}\pi(\bar{\theta}_n, V_n|\omega,\sigma^2)\pi(\sigma^2)d\sigma^2 \\
&= \pi(\omega)\int_{-\infty}^{\infty}\pi(\bar{\theta}_n|\omega,\sigma^2)\pi(V_n|\omega,\sigma^2)\pi(\sigma^2)d\sigma^2 \\
&\propto \exp\left(-\frac{\omega^2}{2}\right)\int_{-\infty}^{\infty}\left(\frac{\sigma^2}{n}\right)^{-\frac{1}{2}}\exp\left(\frac{(\bar{\theta}_n-\omega)^2}{2\frac{\sigma^2}{n}}\right)(\sigma^2)^{\frac{n-1}{2}}\exp\left(-\frac{V_n}{2\sigma^2}\right)(\sigma^2)^{-(\alpha+1)}\exp\left(\frac{\beta}{\sigma^2}\right)d\sigma^2 \\
&= \exp\left(-\frac{\omega^2}{2}\right)\int_{-\infty}^{\infty}\exp\left(\frac{n(\bar{\theta}_n-\omega)^2+2\beta+V_n}{2\sigma^2}\right)(\sigma^2)^{-(\frac{n}{2}+\alpha+1)}d\sigma^2.
\end{aligned}
$$

The integral in the last line is the normalizing constant of $IG(\frac{n}{2}+\alpha, \frac{n(\bar{\theta}_n-\omega)^2+2\beta+V_n}{2})$, so it is equivalent to

$$
\exp\left(-\frac{\omega^2}{2}\right)\frac{\Gamma(\frac{n}{2}+\alpha)}{\{\frac{n(\bar{\theta}_n-\omega)^2+2\beta+V_n}{2}\}^{\frac{n}{2}+\alpha}} \propto \frac{\exp\left(-\frac{\omega^2}{2}\right)}{\{n(\bar{\theta}_n-\omega)^2+2\beta+V_n\}^{\frac{n}{2}+\alpha}}.
$$

□

A.2 Proof of Proposition 5.1

Proof. First, we show that $\tau(\bar{\theta})$ is an odd function. Since C is even in $\bar{\theta}$ and

$$
\int_{-\infty}^{\infty}\frac{\omega\exp(-\frac{\omega^2}{2})}{\{n(-\bar{\theta}-\omega)^2+2\beta\}^{\frac{n}{2}+\alpha}}d\omega = -\int_{-\infty}^{\infty}\frac{\omega\exp(-\frac{\omega^2}{2})}{\{n(-\bar{\theta}+\omega)^2+2\beta\}^{\frac{n}{2}+\alpha}}d\omega,
$$

we obtain $\tau(-\bar{\theta}) = -\tau(\bar{\theta})$, especially (i) $\tau(0) = 0$. To prove (ii), it is sufficient to show that for all $\bar{\theta} > 0$,

$$
\int_{-\infty}^{\infty}\frac{\omega\exp(-\frac{\omega^2}{2})}{\{n(\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}}d\omega > 0,
$$

since $C(\bar{\theta}, 0) > 0$. Suppose that $\bar{\theta} > 0$. For $\omega > 0$,

$$
\{n(\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha} < \{n(-\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}
$$

holds. Hence,

$$
\frac{\omega\exp(-\frac{\omega^2}{2})}{\{n(\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}} > \frac{\omega\exp(-\frac{\omega^2}{2})}{\{n(-\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}},
$$

and

$$
\frac{\omega\exp(-\frac{\omega^2}{2})}{\{n(\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}} + \frac{-\omega\exp(-\frac{(-\omega)^2}{2})}{\{n(-\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}} > 0.
$$

Therefore,

$$
\begin{aligned}
\int_{-\infty}^{\infty}\frac{\omega\exp(-\frac{\omega^2}{2})}{\{n(\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}}d\omega &= \int_{0}^{\infty}\left[\frac{\omega\exp(-\frac{\omega^2}{2})}{\{n(\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}} + \frac{-\omega\exp(-\frac{(-\omega)^2}{2})}{\{n(-\omega-\bar{\theta})^2+2\beta\}^{\frac{n}{2}+\alpha}}\right]d\omega \\
&> 0.
\end{aligned}
$$

(iii) follows from the oddness of $\tau(\bar{\theta})$. □

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