

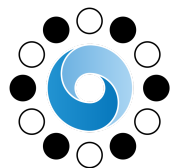
# Robust probabilistic target-oriented exploration with reliability approximation

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# Background

Reinforcement learning (RL) agents have reached superhuman levels in games such as Go and chess.



AlphaGo

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# Task complexity

In terms of the complexity of the environment, real-world tasks are more challenging than games.



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# Reinforcement learning and human learning

[Problem] RL is still too costly to use in the real-world tasks.

- The required amount of sampling time for optimization is not feasible in a realistic time frame.
- The required amount of exploration time for optimization is not feasible in a realistic time frame.

[Idea] Can we solve this by imitating human learning?

- We refer to **satisficing**, which is a learning tendency of humans.
- We introduce the concept of an **aspiration level** into reinforcement learning.
- We generalize the goal of reinforcement learning to satisficing rather than optimization (but optimization is also possible).

We implemented **target-oriented exploration**  
in order to achieve the aspiration level through learning.

# Objectives of this study

## **[Main objective]**

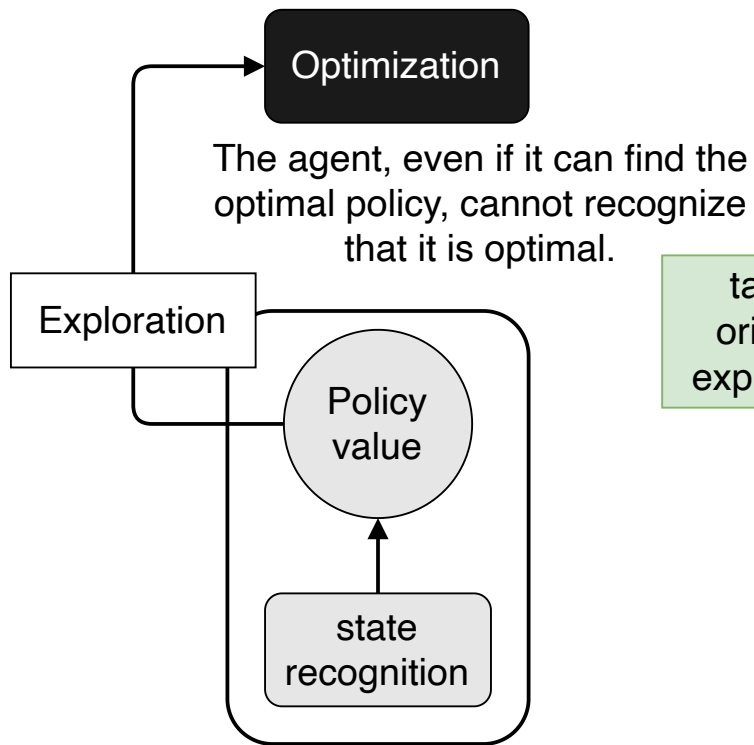
Application of target-oriented exploration to deep reinforcement learning

## **[Sub objective]**

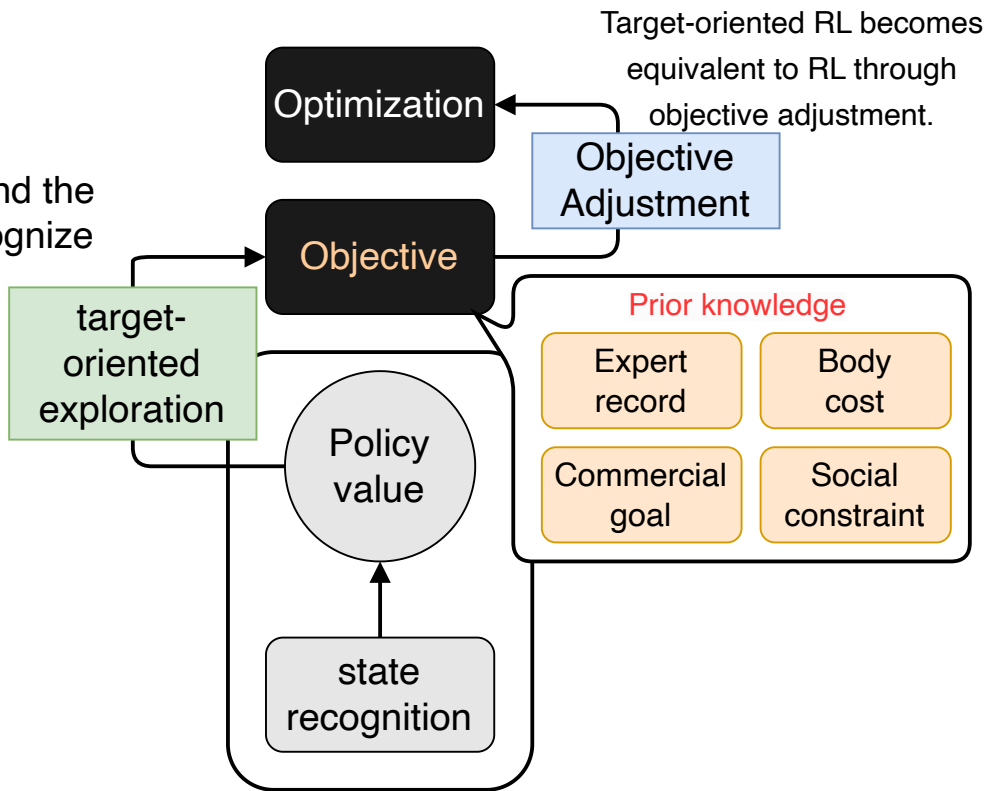
Generalization of the action selection of target-oriented exploration methods stochastically

- We generalize the methods that can approximate states.
- We show that the new method can be generalized stochastically without performance degradation.
- We aim to show performance equal to or better than that of representative methods.

## Conventional RL



## Target oriented RL



# Related research

## **Risk-sensitive Satisficing (RS)**

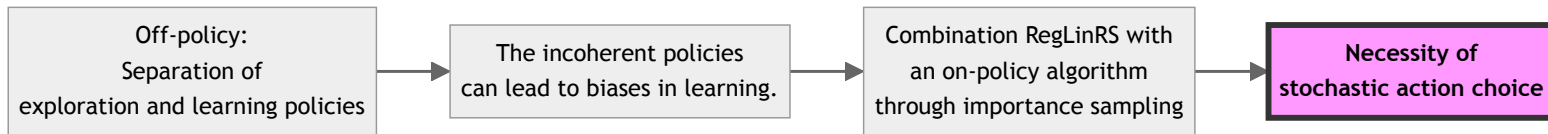
- Overview: RS is a method that incorporates an aspiration level into RL (target-oriented RL).
- Properties: Off-policy, Deterministic action selection
- Features: RS showed better performance than other methods in bandit problems and RL problems.
  - Takahashi et al., 2016
  - Tamatsukiri et al., 2019

# Related research

## Regional Linear RS (RegLinRS)

- Overview: RegLinRS is one of the RS methods that can identify states.
- Properties: Off-policy, Deterministic action selection
- Features: RegLinRS showed better performance than LinUCB and LinTS in contextual bandit problems.
  - Tsuboya et al., 2023

We want to generalize deterministic action selection to stochastic action selection.





# Contextual Bandit Problems

- Experiment task: Linear contextual bandit problems
- Determination of reward expectation value  $p_{t,i}$  of each action  $a_i$  by context  $\mathbf{x}_t$  and weight  $\theta_i$
- The agent observes the context  $\mathbf{x}_t$  at time  $t$  and chooses an action  $a_i$ .
  - As a result, the agent observes the reward  $r_t$  (in this study, the reward expectation value  $p_{t,i}$ ).
- The calculation method of  $p_{t,i}$  is as follows.

$$p_{t,i} = \mathbf{x}_t^T \theta_i + \epsilon_t$$

- $\theta_i$ : The parameter of the reward expectation value
- $\epsilon_t$ : The error term with an expected value of 0

Methods performing well in contextual bandit problems

- LinUCB (Li et al., 2010)
- LinTS (Riquelme et al., 2018)

# Regret

- We use **regret** as an evaluation index.
  - Expected reward loss

$$\text{regret} = \sum_{t=1}^T (p_{\max} - p_{t, \text{chosen}})$$

- $p_{\max}$ : The highest reward expectation value
- $p_{t, \text{chosen}}$ : The reward expectation of the action chosen in the  $t$ -th step

- Properties of regret
  - Regret is the loss expectation value of the agent, which is a weakly increasing function.
  - The minimum value of regret is 0 (when the agent continues to choose the optimal action).

# Subjective regret

Implementation of target-oriented exploration

- If we use target-oriented exploration, we can use a subjective index instead of regret.
  - We call this **subjective regret** (SR).

$$I_i^{\text{SR}} = \sum_{t=1}^T (\aleph - p_{t, \text{chosen}})$$

- $\aleph$ : Aspiration level
- Properties of SR
  - If the agent newly acquired reward is
    - greater than or equal to  $\aleph$  (i.e., sufficient),  $I_i^{\text{SR}}$  decreases.
    - less than  $\aleph$  (i.e., insufficient),  $I_i^{\text{SR}}$  increases.
  - We can interpret this index as a risk-sensitive value function.

# Risk-sensitive Satisficing (RS)

Implementation of target-oriented exploration

- The formula for the core metric of target-oriented exploration
  - We define the RS value function by  $I_i^{\text{RS}} := -I_i^{\text{SR}}$ .
  - The agent chooses an action by taking the argmax from  $I_i^{\text{RS}}$ .

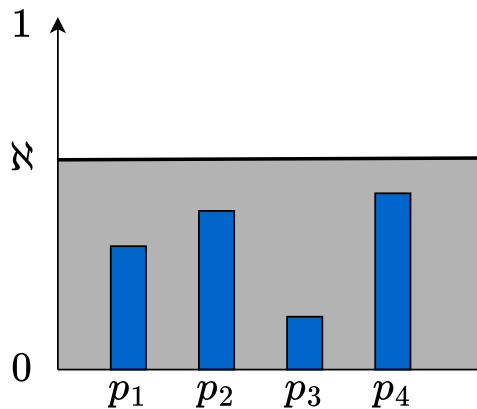
$$I_i^{\text{RS}} = \frac{n_i}{N}(p_i - \aleph) = \frac{n_i}{N}\delta_i$$

- $p_i$ : Reward expectation value of action  $a_i$
  - $n_i$ : The number of times the agent chose action  $a_i$
  - $N$ : The total number of times the agent chose an action
  - $n_i/N$ : Reliability (Choice probability) of action  $a_i$
- $\delta_i$ : Reflection effect of prospect theory  $\rightarrow$  Difference between aspiration level  $(p_i - \aleph)$ 
  - By multiplying the reliability and  $\delta_i$ , the agent makes optimistic or pessimistic action choices depending on the situation.

# Under-archieved and Over-archieved situation

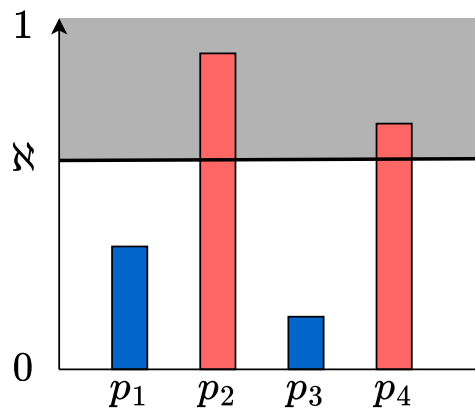
## Under-archieved situation

All reward expectation values are less than  $\aleph$ .



## Over-achieved situation

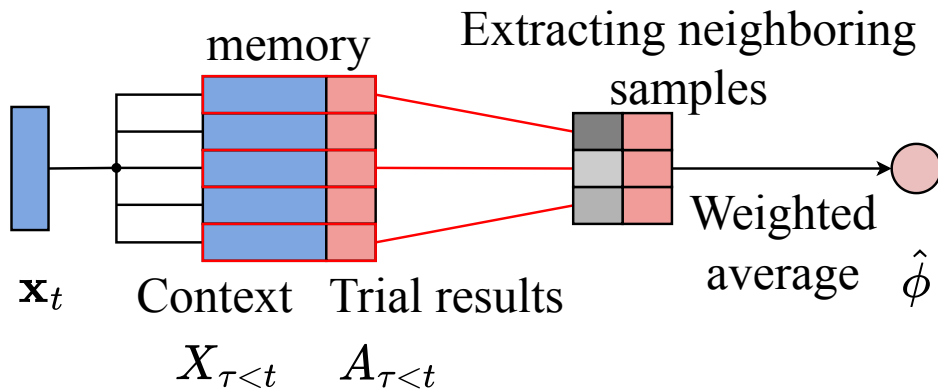
At least one reward expectation value is greater than or equal to  $\aleph$ .



■ Exploration area

# Local approximation of the reliability

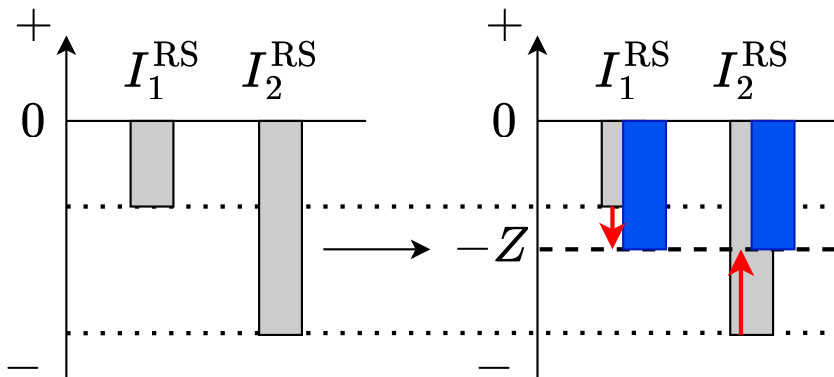
- The agent approximates and estimates the reliability using episodic memory and k-nearest neighbor.
- **Regional Linear RS** (RegLinRS)
  - Tsuboya et al., 2023



# Inversely calculating of the choice distribution in RS

- RS is estimable for internal choice ratio in under-achieved situation.
- The agent generates a probability distribution from the difference between the estimated reliability and the actual reliability.
  - The agent generates the estimated reliability  $\rho_i^z$  using the RS equilibrium value  $-Z$ .
- **Stochastic RS** (SRS)

RS equilibrium value  $-Z$



$$I_i^{\text{RS}} = -Z$$

$$\rho_i = n_i/N = Z/(\aleph - p_i)$$

$$\sum_{i=1}^K \rho_i = \sum_{i=1}^K Z/(\aleph - p_i) = 1$$

$$Z = 1 / \sum_{i=1}^K \frac{1}{\aleph - p_i}$$

# Calculate policy of SRS

## Under-achieved situation

- We can calculate  $-Z$ .

$$b_i = \frac{n_i}{\rho_i^z} - N + \epsilon$$

$$I_i^{\text{SRS}} = (N + \max_i(b_i))\rho_i^z - n_i > 0$$

$$\pi_i = I_i^{\text{SRS}} / \sum_{i=1}^K I_i^{\text{SRS}}$$

- $b$ : Adjustment parameter for preventing negative ratios
- $\epsilon$ : An extremely small value to prevent zero division

## Over-achieved situation

- We cannot calculate  $-Z$ .
- We calculate the probability of selecting an action that has a reward expectation value greater than  $\aleph$ .

$$I_i^{\text{RS}'} = \begin{cases} I_i^{\text{RS}} + \epsilon, & \text{if } p_i \geq \aleph \\ 0, & \text{if } p_i < \aleph \end{cases}$$

$$\pi_i = I_i^{\text{RS}'} / \sum_{j=1}^K I_j^{\text{RS}'}$$



# Regional Linear SRS

- **New method**
- This method is a combination of the reliability estimation part of RegLinRS and SRS.
  - The formula of SRS contains  $n_i$  and  $N$ .
  - We can approximate  $n_i/N$ , but we cannot approximate  $n_i$  and  $N$ .

## Transformation of the equations in SRS

$$\begin{aligned}\frac{b_i}{N_x} &= \frac{1}{\rho_i^z} \cdot \frac{n_i}{N_x} - 1 + \epsilon = \frac{\rho_i}{\rho_i^z} - 1 + \epsilon \\ \frac{I_i^{\text{SRS}}}{N_x} &= \left( \frac{N_x + \max_i(b_i)}{N_x} \right) \\ &= \left\{ \max_i \left( \frac{\rho_i}{\rho_i^z} \right) + \epsilon \right\} \rho_i^z - \rho_i\end{aligned}$$

$$\begin{aligned}\frac{I_i^{\text{SRS}}}{N_x} &= \left\{ \max \left( \frac{\hat{\phi}_i}{\rho_i^z} \right) + \epsilon \right\} \rho_i^z - \hat{\phi}_i \\ \pi_i &= \frac{I_i^{\text{SRS}}}{N_x} / \sum_{j=1}^K \frac{I_j^{\text{SRS}}}{N_x} = I_i^{\text{SRS}} / \sum_{j=1}^K I_j^{\text{SRS}}\end{aligned}$$

→ We can extend SRS to RegLinSRS.

# Artificial dataset

- We created an artificial dataset in which the aspiration level  $\mathfrak{N}$  is always constant.
  - The reason is to compare RegLinRS, RegLinSRS with other methods using the same evaluation index.
- We used the same dataset as Tsuboya et al., 2023.
- We designed the dataset so that the optimal action would not be biased in order to properly evaluate the balance between exploration and exploitation.

Configuration Item	Configuration Value
Feature vector dimension $d$	128
Number of actions $K$	8
Optimal Aspiration level $\mathfrak{N}_{\text{opt}}$	0.7
Data size $N$	100,000

\*  $\mathfrak{N}_{\text{opt}}$  is the reference value set between the optimal and suboptimal.

# Experiment 1

## Comparison methods

LinUCB, LinTS, RS methods (RegLinRS, RegLinSRS)

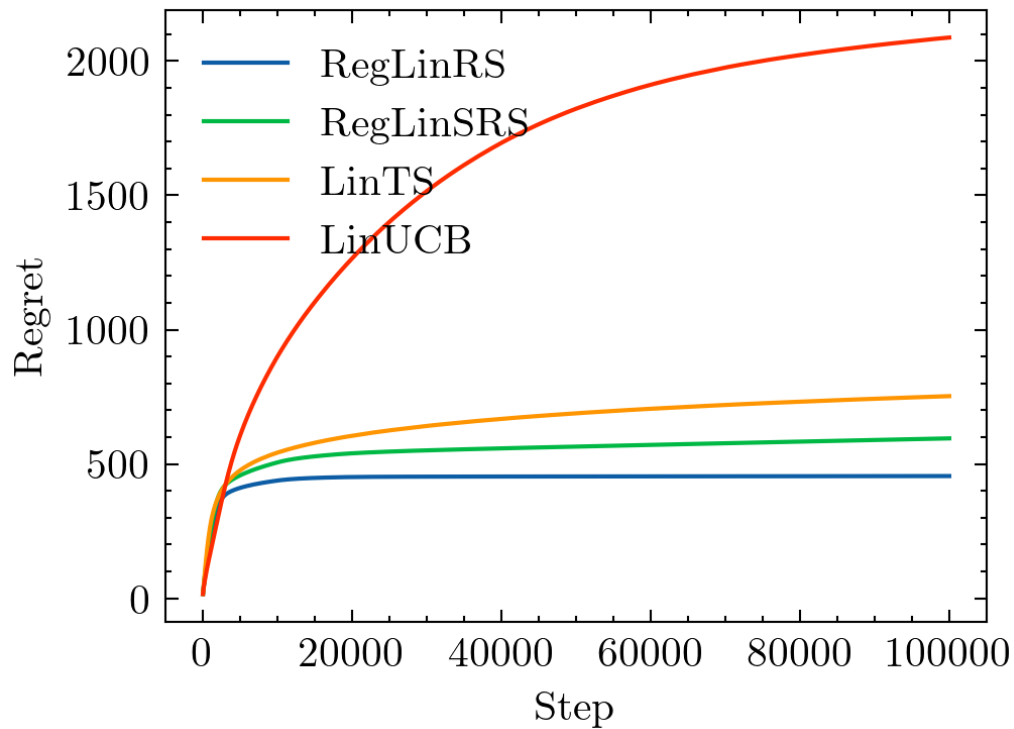
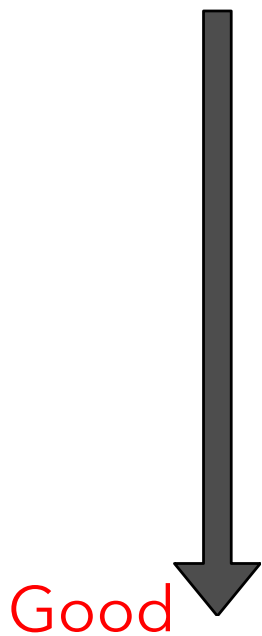
## Experiment settings

- We ran 1,000 simulations with 100,000 steps per simulation.
  - We calculated the average value and used it as the result.
- The agent initially selects each action 10 times.
  - This setting is necessary for parameter initialization.
- We set the batch size to 20 for all methods.

Value Name	Value
$\epsilon$	sys.float_info.epsilon in Python
episodic memory size	10,000
$k$ of $K$ -nearest neighbors	50
$\eta$	0.6
$\alpha$ of LinUCB	0.1
$\lambda$ of LinTS	0.25
$\alpha$ of LinTS	6
$\beta$ of LinTS	6
$\mathbf{b}_i$	All 0
$\mathbf{A}_i$	Identity matrix $\mathbf{I}$

# Result

Experiment 1



# Result

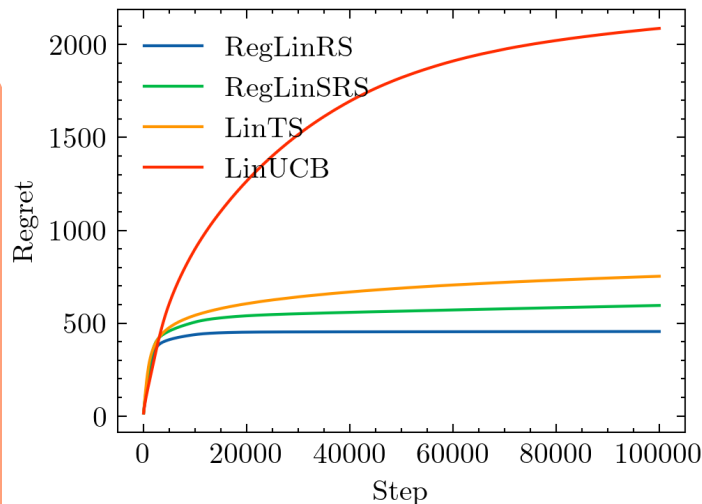
## Experiment 1

- RegLinRS, RegLinSRS, LinTS, LinUCB performed well in that order.
- LinTS, LinUCB have a logarithmic increase in regret.
- RegLinRS, RegLinSRS have almost converged regret.

- LinTS is one of the state-of-the-art methods (Agrawal et al., 2019).
- LinTS is inferior to RegLinRS, RegLinSRS in the initial rise of regret.



RegLinRS, RegLinSRS can stop learning faster than LinTS and can select the truly optimal action even in situations where accurate approximation has not yet been achieved?

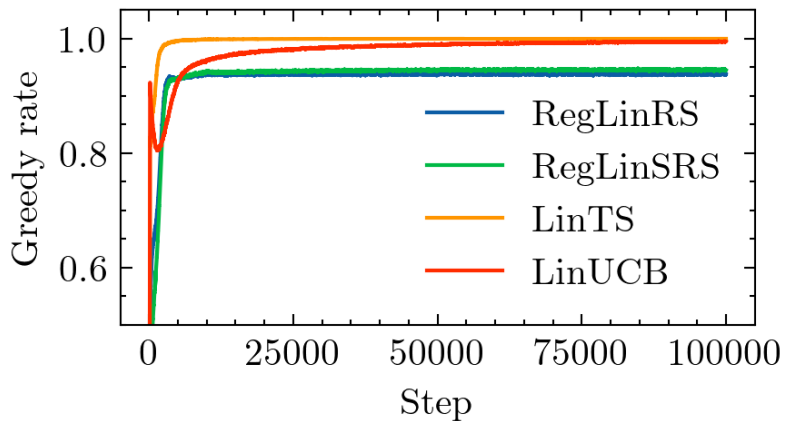


# Discussions

## Experiment 1

- LinTS, LinUCB have reached a Greedy rate of 1.0.
- RegLinRS, RegLinSRS have stopped at a Greedy rate of over 0.9.
  - Not greedy about 1 in 10 times

→ RegLinRS, RegLinSRS can choose the truly optimal action even if the action is overestimated.



# Hypothesis

## Experiment 1

**[Fact] RegLinRS, RegLinSRS can partially mitigate the effects of approximation errors.**

- These methods can choose the truly optimal action even if the action is overestimated due to approximation errors.

**[Hypothesis] Are RegLinRS and RegLinSRS achieving this by reliability?**

- RS does not necessarily make greedy action selection due to the reflection effect of reliability.
- This property is rational in the sense that the agent can choose a satisfactory action with certainty.

→ We conducted an experiment to verify this property by intentionally adding noise to the reward expectation value.

# Experiment 2

## Experimental purpose

- **To verify the robustness of RS to approximation errors**
  - We added noise to the estimated reward expectation value.
  - We intentionally created a situation where it is easy to select a non-optimal action.

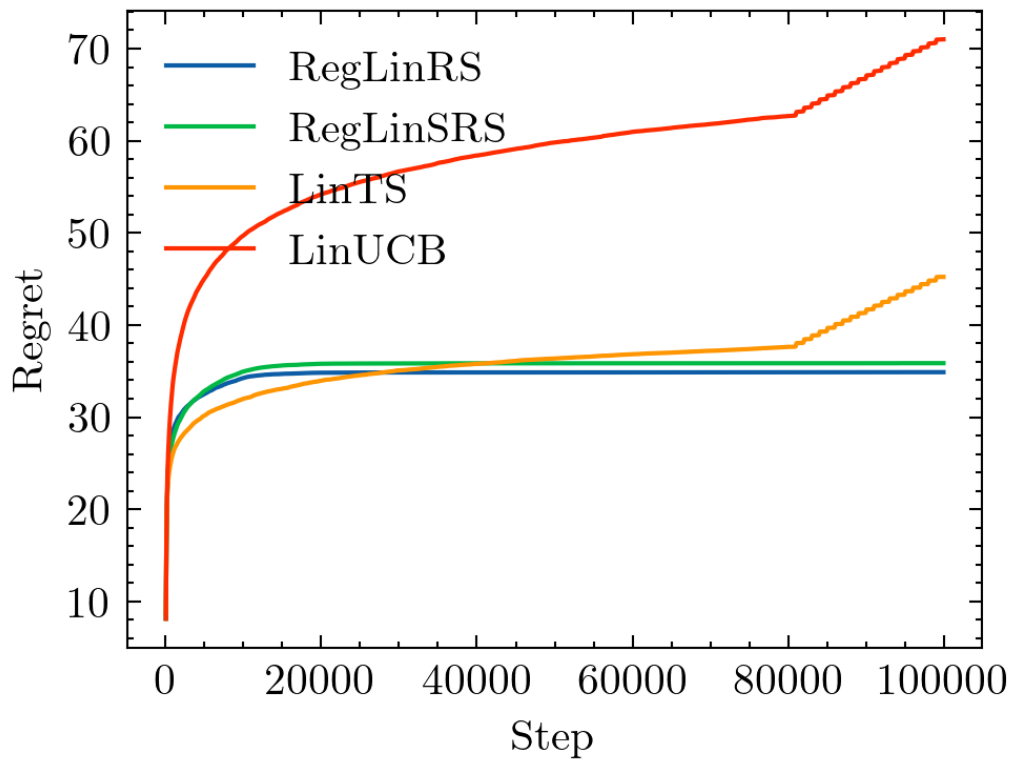
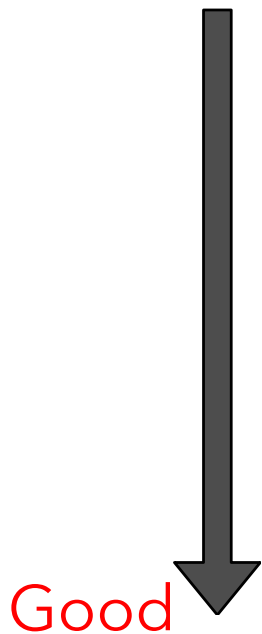
## Experimental settings

- We set the number of actions to  $K = 2$  to simplify the experiment in Experiment 1.
- We added noise to the estimated reward expectation value at equal intervals after 80,000 steps.
- We set the other settings the same as in Experiment 1.



# Result

Experiment 2

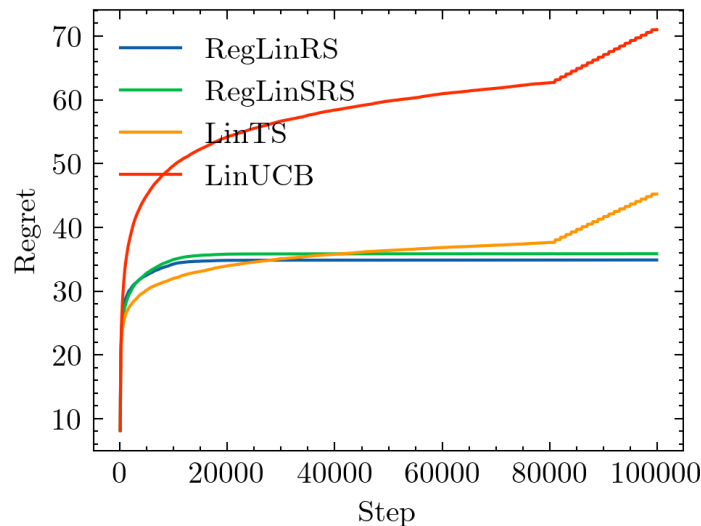


# Result

## Experiment 2

- After the step on which the noise is added, LinTS, LinUCB have a sharp increase in regret.
- RegLinRS, RegLinSRS have no increase in regret.
  - These methods can partially mitigate the effects of approximation errors by the reflection effect of reliability.

→ We showed the robustness of RS to approximation errors.



# Conclusion

- We generalized deterministic action selection (RegLinRS) to **stochastic action selection (RegLinSRS)**.
- We showed that
  - RegLinSRS **does not have a significant performance degradation** compared to RegLinRS.
  - RegLinSRS **has better performance than LinTS and LinUCB**.
- We showed **the robustness of RS to approximation errors**.
  - This property is considered to be useful for extending RS to deep RL in the future.

→ We prepared for the application of RS to deep RL.

