Chapter.1 p.15

例1 (p.14)

$$
\begin{aligned}
p(x|m) &= \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2\sigma^2}(x-m)^2\right) \\
&= \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{x^2}{2\sigma^2}\right)\exp\left(\frac{m}{\sigma^2}x-\frac{m^2}{2\sigma^2}\right) \\
&= v(x)\exp(f(w)\cdot g(x)).
\end{aligned}
$$

よって

$$
\begin{aligned}
v(x) &= \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{x^2}{2\sigma^2}\right) \\
f(w) &= \left(\frac{m}{\sigma^2}, -\frac{m^2}{2\sigma^2}\right) \\
g(x) &= (x, 1) \quad \Rightarrow \phi = (\phi_1, \phi_2) \text{ と置き換えると共役分布}
\end{aligned}
$$

$$
\begin{aligned}
\varphi(m|\phi) &= \frac{1}{z(\phi)}\exp\left(\frac{m}{\sigma^2}\phi_1 - \frac{m^2}{2\sigma^2}\phi_2\right) \quad \Rightarrow m\text{について平方完成} \\
&= \frac{1}{z(\phi)}\exp\left\{-\frac{\phi_2}{2\sigma^2}\left(m-\frac{\phi_1}{\phi_2}\right)^2+\frac{\phi_1^2}{2\phi_2\sigma^2}\right\}
\end{aligned}
$$

これはmに関して正規分布。また積分して1であるから、

$$
\begin{aligned}
z(\phi) &= \int \exp\left\{-\frac{\phi_2}{2\sigma^2}\left(m-\frac{\phi_1}{\phi_2}\right)^2+\frac{\phi_1^2}{2\phi_2\sigma^2}\right\}dm \\
&= \exp\left(\frac{\phi_1^2}{2\phi_2\sigma^2}\right)\int \exp\left\{-\frac{\phi_2}{2\sigma^2}\left(m-\frac{\phi_1}{\phi_2}\right)^2\right\}dm \\
&= \exp\left(\frac{\phi_1^2}{2\phi_2\sigma^2}\right)\sqrt{\frac{2\pi\sigma^2}{\phi_2}}
\end{aligned}
$$

p.13の1行目の式

$$
\hat{\phi} = \phi + \sum_i \beta g(X_i)
$$

を用いると、

$$
\begin{aligned}
\hat{\phi}_1 &= \phi + \sum_i \beta X_i \\
\hat{\phi}_2 &= \phi + \beta n
\end{aligned}
$$

予測分布式(1.24)より

$$
\begin{aligned}
p(x|X^n) &= v(x)\frac{z(\hat{\phi} + g(x))}{z(\hat{\phi})} \propto v(x)z(\hat{\phi} + g(x)) \quad \Rightarrow x\text{依存性のみ残す} \\
&\propto \exp\left(-\frac{x^2}{2\sigma^2}\right)\exp\left(\frac{(\hat{\phi}_1 + x)^2}{2\sigma^2(\hat{\phi}_2 + 1)}\right) \\
&= \frac{1}{2\sigma^2}\frac{\hat{\phi}_2}{\hat{\phi}_2 + 1}\left\{-x^2 + 2\frac{\hat{\phi}_1}{\hat{\phi}_2}x + \text{const.}\right\} \quad \Rightarrow \{\ldots\}\text{を平方完成する} \\
&\propto \exp\left\{-\frac{1}{2\hat{\sigma}^2}\left(x - \frac{\hat{\phi}_1}{\hat{\phi}_2}\right)^2\right\} \\
&= \frac{1}{\sqrt{2\pi\hat{\sigma}^2}}\exp\left\{-\frac{1}{2\hat{\sigma}^2}\left(x - \frac{\phi_1 + \beta\sum X_i}{\phi_2 + \beta n}\right)^2\right\} \quad \text{ただし} \\
\hat{\sigma}^2 &= \frac{\hat{\phi}_2 + 1}{\hat{\phi}_2}\sigma^2 = \frac{\phi_2 + n\beta + 1}{\phi_2 + n\beta}\sigma^2
\end{aligned}
$$

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In [ ]:
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