Chapter.1 章末問題3

$$
\begin{aligned}
p(x|m,s) &= \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{s}{2}x^2 + msx - \left(\frac{m^2 s}{2} - \frac{1}{2}\log s\right)\right\} \\
&= v(x)\exp(f(w)\cdot g(x)) \\
v(x) &= \frac{1}{\sqrt{2\pi}} \\
f(w) &= \left(-\frac{s}{2},\ ms,\ -\frac{m^2 s}{2} + \frac{1}{2}\log s\right) \\
g(x) &= \left(x^2,\ x,\ 1\right) \quad \Rightarrow \phi = (\phi_1, \phi_2, \phi 3)\text{で置き換える}
\end{aligned}
$$

$$
\begin{aligned}
\varphi(m,s|\phi) &= \frac{1}{z(\phi)}\exp\left\{-\frac{s}{2}\phi_1 + ms\phi_2 - \left(\frac{m^2 s}{2} - \frac{1}{2}\log s\right)\phi_3\right\} \\
&= \frac{1}{z(\phi)}s^{\frac{\phi_3}{2}}\exp\left\{-\frac{s}{2}\phi_1 + ms\phi_2 - \frac{m^2 s}{2}\phi_3\right\}
\end{aligned}
$$

これは、

- m に関しては正規分布
- s に関してはガンマ分布($\propto s^{\phi_3/2}e^{-s/\alpha} = \mathrm{Gamma}(\frac{\phi_3}{2}, \alpha)$) の形をしている。

$$
\begin{aligned}
z(\phi) &= \int s^{\frac{\phi_3}{2}}ds\int \exp\left\{-\frac{s}{2}\phi_1 + ms\phi_2 - \frac{m^2 s}{2}\phi_3\right\}dm \quad \Rightarrow \text{先に}m\text{に関するガウス積分を実行} \\
&= \int s^{\frac{\phi_3}{2}}ds\int \exp\left\{-\frac{s\phi_3}{2}\left(m^2 - 2\frac{\phi_2}{\phi_3}m\right) - \frac{s\phi_1}{2}\right\}dm \\
&= \int s^{\frac{\phi_3}{2}}ds\int \exp\left\{-\frac{s\phi_3}{2}\left(m - \frac{\phi_2}{\phi_3}m\right)^2 + \frac{s\phi_3}{2}\frac{\phi_2^2}{\phi_3^2} - \frac{s\phi_1}{2}\right\}dm \\
&= \int s^{\frac{\phi_3}{2}}\exp\left\{\frac{s\phi_3}{2}\frac{\phi_2^2}{\phi_3^2} - \frac{s\phi_1}{2}\right\}ds\underbrace{\int \exp\left\{-\frac{s\phi_3}{2}\left(m - \frac{\phi_2}{\phi_3}m\right)^2\right\}dm}_{\sqrt{2\pi/(s\phi_3)}} \\
&= \sqrt{\frac{2\pi}{\phi_3}}\int s^{\frac{\phi_3-1}{2}}\exp\left\{-\frac{\phi_1\phi_3 - \phi_2^2}{2\phi_3}s\right\}ds
\end{aligned}
$$

ここでガンマ分布が

$$
\mathrm{Gamma}(x|k,\theta) = \frac{x^{k-1}e^{-\frac{x}{\theta}}}{\Gamma(k)\theta^k}
$$

を用いると $\int s^{k-1}\exp\{-\frac{s}{\theta}\}ds = \Gamma(k)\theta^k$ ゆえ

$$z(\phi) = \sqrt{\frac{2\pi}{\phi_3}} \left(\frac{2\phi_3}{\phi_1\phi_3 - \phi_2^2} \right)^{\frac{\phi_3+1}{2}} \Gamma(\frac{\phi_3 + 1}{2})$$

In [ ]:

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