Cities' Demand-driven Industrial Composition*

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Abstract

I show that large cities specialize in income-elastic industries. To explain this pattern, I develop a model of cities with industries that differ in their tradability and income elasticity of demand. Industry-neutral productivity differences generate the specialization pattern through the home-market effect. A more productive city is larger, pays higher wages, and specializes in income-elastic industries. When the industries are gross complements, or the tradable industries are income-elastic, the tradable industries command greater shares of employment and expenditure in the productive and larger city, amplifying regional income inequality.

Keywords: Home-market effect, Non-homothetic preference, Cities, Industrial composition, Comparative advantage **JEL classification:** F12, F14, R12, R13, R23

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1 Introduction

Industrial composition varies greatly across cities: Detroit, for example, is synonymous with cars, and Silicon Valley with computers. Figure 1 shows that much of the variation in employment composition across U.S. Metropolitan Statistical Areas (MSAs) is linked to the income elasticity of demand for that industry's output. Employment shares of industries with high income elasticities of demand, such as air transport services and insurance are higher in large MSAs like New York-Newark-Jersey City Metropolitan Area. Industries with low income elasticities of demand, such as sugar manufacturing and processed rice manufacturing, have larger employment shares in small MSAs.

This paper investigates this new stylized fact and its implications by developing a model that can generate this specialization pattern as an equilibrium outcome. My model is based on Matsuyama (2019), who theoretically studies international trade patterns with heterogeneous industrial income elasticities and differentiated goods within an industry. I extend Matsuyama (2019) by introducing worker mobility, land consumption, and non-tradable industries (e.g., retail). In an equilibrium, a fundamental productivity difference between cities generates asymmetric population allocation, nominal income inequality, and a trade pattern consistent with the specialization patterns in Figure 1. In addition to explaining the specialization pattern, the model implies that the share of the tradable sector, which my model endogenously generates as the sum of tradable industries' employment shares, can amplify cross-location income patterns. When the industries are gross complements or when the tradable industries are income-elastic, the large city commands a greater tradable sector share, amplifying the cross-location income inequality.

The home-market effect plays a vital role in the production patterns explained as equilibrium outcomes of my model. First formally theorized by Krugman (1980), the effect is of two types, each of which shares the mechanism from trade costs and an increasing return to scale production. The first affects the wage rate. Other things being equal, the wage rate is higher in larger markets. When firms are exposed to competition with firms in different locations and sharing demands, differences in access to markets due to trade costs drive the difference in the input costs so that the firms' profits are equalized at zero in any location. The second effect affects trade patterns by generating comparative advantage. When the relative market size of industries varies across regions, regions export goods for which they have relatively large domestic markets.

In the presence of trade costs, local firms are incentivized to operate in an industry with a relatively larger home market, and this incentive is strong enough to amplify the demand pattern to the production pattern.

In my model, a difference in cities' fundamental productivity generates these two home-market effects, eventually producing the specialization pattern consistent with Figure 1. First, a city with better fundamental productivity attracts workers, resulting in a large population, which generates the home-market effect on the wage rate. Second, due to the higher wage, residents in the large city spend relatively more on income-elastic industries, generating the home-market effect on the trade pattern. Hence, in the equilibrium, the fundamentally productive city becomes larger, offers a higher wage, and specializes in income-elastic industries.

In addition to explaining the production pattern, the model elucidates that endogenous tradable sector shares can amplify cross-location income inequality. In equilibrium, the wage increases not only with city size but also with the tradable sector share. This result reflects that the home-market effect works only through the tradable sector. The source of the home-market effect is competition between firms in different locations, which does not exist in the non-tradable sector. Thus, the market size that drives the wage is the size of the tradable sector, which is the product of the overall market size and the tradable sector share. In my model, the tradable sector share of a city is endogenously determined, as the aggregate share of tradable industries, by two factors: the prices and the income elasticities of the tradable industries relative to the non-tradable industries. The prices of the tradable industries are, other things being equal, relatively expensive in the large city, which makes the tradable sector share greater when the industries are gross complements and smaller when gross substitutes. The tradable sector share of the large city becomes greater also when the tradable industries are relatively income-elastic. Hence, tradable sector shares amplify regional income inequality when the industries are gross complements or the tradable industries are income-elastic. These results provide a new perspective on cross-city and cross-country income patterns and their evolution; these patterns should evolve when a change in the economic environment affects tradable sector shares.

Industrial composition is important from several perspectives. First, it is a critical factor for local economies, and not only because a tradable sector share is a driver of local wages through the home-market effect but also because local economic performance is significantly affected by the industries that locate in a city (e.g., Autor et al. 2013; Mian and Sufi 2014). To fully understand variation in local economic performance, it is necessary, first, to understand what determines the mix of industry in cities. Second, it relates to heterogeneous returns to experience across cities. Eckert et al. (2022) find that workers' sorting into the industries typically found in big cities substantially explains faster wage growth in big cities. This finding suggests that understanding the determinants of industrial composition is essential in studying productivity growth taking place in cities. Third, the mechanism that drives industrial composition also is crucial for researchers who want to exploit regional variation in the size of industries. In this paper, I report that industrial composition is related to city size. Given this relationship, regressing dependent variables on industrial sizes or shares, while not controlling for the city size of examined locations, might lead to an omitted variable bias problem because the trend of a dependent variable could be driven by the city size of the locations rather than by industrial sizes or shares. Understanding the mechanism can help researchers avoid this endogeneity issue.

This paper is the first research on cross-city inter-industry specialization patterns from the demand-side perspective. Most previous works on the cross-city specialization pattern focus on non-demand side factors such as functions in production (Duranton and Puga 2005; Henderson and Ono 2008), the strength of the agglomeration economy (Behrens and Robert-Nicoud 2015), and the skill supply (Davis and Dingel 2019). A few works focus on the demand side's effect on cross-city differences (e.g., Handbury 2019). The closest of these is Dingel (2017), who quantifies the contributions made to the quality specialization of cities by the heterogeneous demand factor and the skill supply factor, and he creates a model to guide the quantification. There are two significant differences between his model and mine. First, we focus on different types of specialization and trade patterns. His work, therefore, his model, examines vertical specialization and the intra-industry trade, where different quality goods are gross substitutes. In contrast, my model looks into horizontal specialization and inter-industry trade where goods in different industries can be either gross complements or substitutes. Second, my model has mobile agents and land consumption, which his model does not. These assumptions fit the urban economy environment, enabling us to study the relationship between the size of a city and its industrial composition. This paper, the first work on cross-city inter-industry specialization patterns undertaken from the demand-side perspective, contributes to the literature by reporting the stylized fact and providing the theoretical framework for urban economies.

Furthermore, my work broadly contributes to studies of the demand-side effect in trade patterns. The international trade literature shows theoretically that the demand-side effect plays an important role in determining trade patterns and specializations (e.g., Flam and Helpman 1987; Stokey 1991; Matsuyama 2019; Fajgelbaum et al. 2011). Matsuyama (2019) is the closest to mine because he analyzes horizontal specialization and inter-sectoral trade. My model extends Matsuyama (2019) by introducing non-tradable industries, land consumption, and mobile agents, and it reveals a new effect that heterogeneous demand across locations generates. This is the effect on cross-location income through endogenous tradable sector shares, and it is relevant both in international and regional income inequalities.

The rest of this paper is organized as follows. Section (2) introduces the model, and Section (2) illustrates how the cities differ in the equilibrium in population, wages, employment, expenditures, and prices and provides its implications on regional income inequality. Section (4) implements robustness checks of the production pattern. It also shows that the demand-side effect explains a significant variation in cities' production patterns. Section (5) concludes.

2 Model

In this section, I introduce the model of two cities ($i \in \{1, 2\}$). The cities fundamentally differ in productivity and amenities. A mass of N workers are freely mobile and homogeneous except for inherent taste over cities. Conditional on location, individual labor supply is inelastic. There are K goods-producing industries, and the industries differ in relative income elasticity (ϵ_k) and tradability (τ_k) as well as productivities (ϕ_k, ψ_k) and preference shifters (β_k). In a city, land (L_i) is inelastically supplied and consumed by households. I start by explaining the household problem.

Household

The problem for a worker θ is given by

$$\max_{i\in\{1,2\},U_i,C_i,H_i\{q_{ik}(\nu)\}_{\nu\in\Omega_{ik},k\in\mathcal{K}},\{Q_{ik}\}_{k\in\mathcal{K}}} U_i \cdot a_i \cdot \delta(\theta, i) \tag{1}$$

$$\begin{aligned}
s.t.\ U_i &= C_i^{1-\alpha} H_i^{\alpha} \\
C_i &= \left[\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{ik}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
Q_{ik} &= \left[\int_{\Omega_{ik}} q_{ik}(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}} \\
E_i &= \sum_{k\in\mathcal{K}} \int_{\Omega_{ik}} p_{ik}(\nu) q_{ik}(\nu) d\nu + P_{iH} H_i
\end{aligned}$$

where i is the city to reside in, $q_{ik}(\nu)$ is the consumption of variety ν in industry k, Ω_{ik} is the set of available varieties of industry k in city c, $\mathcal{K}$ is the set of industries, Q_{ik} is the composite consumption of industry k, β_k is the preference shifter of industry k, E_i is the income in city i, P_{iH} is the land rent in city i, $a_i > 0$ for $i \in \{1, 2\}$, $0 < \eta < \sigma$, $\eta \neq 1$, $1 + (1-\alpha)\min_{k\in\mathcal{K}}\{\epsilon_k/(1-\eta)\} > 0$, and $\sigma > 1$.[1] The utility consists of three factors: U_c, a_c, and $\delta(i, c)$.

The real consumption, U_i, is defined over goods consumption (C_i) and land consumption (H_i) as a Cobb-Douglass utility function. The functional form of goods consumption captures the non-homothetic preference of the consumer. When $\epsilon_k = 0$ for all $k \in \mathcal{K}$, this C_i becomes a standard homothetic CES function. When ϵ_k varies across industries, a higher ϵ_k corresponds to a more income-elastic industry; the weight, $\beta_k^{1/\eta} U_i^{\epsilon_k/\eta}$, grows relatively more when the real consumption grows. This functional form follows Hoelzlein (2019), who incorporates housing consumption into a non-homothetic utility function that Comin et al. (2021), Matsuyama (2019), and others use. It requires $1 + (1-\alpha)\min_{k\in\mathcal{K}}\{\epsilon_k/(1-\eta)\} > 0$ to ensure global monotonicity of C_i. When solving this household problem, a convenient property of the functional form of C_i appears. The demand function derived from this preference becomes

$$Q_{ik} = \beta_l U_i^{\frac{\epsilon_k}{\eta}} P_{ik}^{-\eta} C_i^{1-\eta} E_{iC}^{\eta} \tag{2}$$

[1] I exclude $\eta = 1$ to simplify the algebra and the analysis.

where E_{iC} and P_{ik} are the expenditure on goods consumption and the price index for industry k, respectively, in city i and P_{ik} is defined as $P_{ik} = \left[\int_{\nu\in\Omega_{ik}} p_{ik}(\nu)^{1-\sigma} d\nu\right]^{1/(1-\sigma)}$. This demand function shows that the relative income elasticity of demand (ϵ_k) and price elasticity (η) at the industry level are separated. Price elasticities are common among industries, and one at the industry level is lower than at the variety level ($\eta < \sigma$). Equation (2) implies the industries are gross complements when $\eta < 1$ and gross substitutes when $\eta > 1$. Also, the expenditure share of industry k is obtained as follows (see Appendix B for the derivation):

$$m_{ik} \equiv \frac{P_{ik}Q_{ik}}{\sum_{\ell\in\mathcal{K}} P_{i\ell}Q_{i\ell}} = \frac{\beta_k P_{ik}^{1-\eta} U_i^{\epsilon_k}}{\sum_{\ell\in\mathcal{K}} \beta_\ell P_{i\ell}^{1-\eta} U_i^{\epsilon_\ell}} \tag{3}$$

This term is log-supermodular in U_i andϵ_k, and it shows that, holding the price indices $\{P_k\}_{k\in\mathcal{K}}$ constant, agents with higher real consumption (U_i) spend relatively more on industries with high ϵ_k.[2] Another result is the price index for goods consumption ($P_i \equiv E_{iC}/C_i$), which is expressed as

$$P_{iC} = \left(\sum_{k\in\mathcal{K}} \beta_k U_i^{\epsilon_k} P_{ik}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{4}$$

(see Appendix B for the derivation). This illustrates that as the real consumption U_i rises, agents care particularly about the prices of high ϵ_k goods, on which they spend relatively more.

A worker consumes land for her residential use. L_i unit of land is inelastically supplied in city i, and I assume $L_1 = L_2 = 1$. This is without loss of generality because changing L_i for city i is isomorphic to changing the utility level from amenities a_i for city i. The land rent revenues in a city are uniformly redistributed to the residents, which implies that a worker's income is proportional to her wage in an equilibrium.

Workers homogeneously appreciate amenities offered by city i, such as weather, landscape, and historic heritage, the overall level of which a_i measures. While "amenity" generally refers to access to local services and consumer goods (e.g., restaurants), called consumption amenities, in this model, those local services and goods contribute to C_i when consumed.

Workers additionally have heterogeneous tastes in cities (following Tabuchi and Thisse (2002), Redding (2016), and others); the worker θ and city i pair receives the idiosyncratic utility shock $\delta(\theta, i)$. I assume $\delta(\theta, i)$ is distributed i.i.d. across workers and cities according to the Fréchet distribution with shape param-

[2] $f(x, y)$ is log-supermodular in x and y iff $\partial^2 \log f(x, y)/\partial x \partial y > 0$.

eter $1/\gamma$ ($Pr[\delta < x] = e^{-x^{-1/\gamma}}$). Each worker chooses the city that offers the higher utility, considering her consumption optimization. Thus, given the products of the real consumption and the utility from amenities in the two cities, a_1U_1 and a_2U_2, the probability of choosing city 1 for a given agent is derived as $Pr[a_1U_1\delta(i) > a_2U_2\delta(i)] = (a_1U_1)^{1/\gamma} / \left\{(a_1U_1)^{1/\gamma} + (a_2U_2)^{1/\gamma}\right\}$. Since the shock is i.i.d., the cities' population ratio follows.

$$\frac{N_1}{N_2} = \left(\frac{a_1U_1}{a_2U_2}\right)^{1/\gamma} \tag{5}$$

Given the ratio of the product of the real consumption and the utility from amenities (a_1U_1/a_2U_2), the lower γ is, the greater the population inequality is; thereby, the parameter γ measures the dispersion force from the heterogenous tastes.

Production

The production in my model is based on Krugman (1980). For all industries $k \in \mathcal{K}$ there are endogenous sets of varieties, homogeneous firms, and monopolistic competition. Each industry is either tradable with iceberg trade cost $\tau > 1$ or non-tradable. Let $\mathcal{T}$ be the set of tradable industries (tradable sector) and $\mathcal{N}$ be the set of non-tradable industries (non-tradable sector) ($\mathcal{K} = \mathcal{T} \cup \mathcal{N}$ and $\mathcal{T} \cap \mathcal{N} = \emptyset$). Each worker chooses the location of labor they supply. Conditional on location, individual labor supply is inelastic. I let w_i denote the wage in city i. Each firm in industry k in city i needs to employ ϕ_k/λ_i units of labor as the fixed cost and ψ_k/λ_i as the variable cost to produce a unit of variety. To let city 1 be fundamentally more productive than city 2, I assume that $\lambda = \lambda_1 > \lambda_2 = 1$. The problem for a firm that produces variety ν in industry k in city i is

$$\begin{aligned}\pi_{ik}(\nu) = \max_{p_{iik}(\nu), q_{iik}(\nu), p_{ijk}(\nu), q_{ijk}(\nu)} & [p_{iik}(\nu) q_{iik}(\nu) - \frac{\psi_k}{\lambda_i} q_{iik}(\nu) w_i] \\ & + \mathbb{1}\{k \in \mathcal{T}\} \left[p_{ijk}(\nu) q_{ijk}(\nu) - \tau \frac{\psi_k}{\lambda_i} q_{ijk}(\nu) w_i\right] - \frac{\phi_k}{\lambda_i} w_i \\ s.t.\ q_{ii}(\nu) &= p_{iik}(\nu)^{-\sigma} P_{ik}^{\sigma} Q_{ik} \\ q_{ijk}(\nu) &= p_{ijk}(\nu)^{-\sigma} P_{jk}^{\sigma} Q_{jk}\end{aligned} \tag{6}$$

where $\pi_{ik}(\nu)$ is the profit by optimized production; $(i,j) \in \{(1,2),(2,1)\}$; $p_{iik}(\nu)$ and $p_{ijk}(\nu)$ are the prices for the markets in city i and j, respectively; and $q_{iik}(\nu)$ and $q_{ijk}(\nu)$ are the quantities for the markets in city i and j, respectively. In the following part, I omit ν unless it is confusing. The terms in the first bracket are the variable profits from selling products in city i, while those in the second are those in city j, which is zero if $k \in \mathcal{N}$. If industry k in city i has non-zero production in an equilibrium, π_{ik} must be zero such that there is no entrant. Similarly, if industry k in city i has zero production in an equilibrium, π_{ik} must be non-positive, which is the zero-profit condition in industry k in city i.

Definition of Competitive Equilibrium

A competitive equilibrium is $\{N_i, U_i, C_i, H_i, w_i, E_i, P_{iH}\}_{i\in\{1,2\}}$, $\{p_{ijk}, q_{ijk}\}_{(i,j,k)\in\{1,2\}^2\times\mathcal{K}}$, and $\{\Omega_{ik}\}_{(i,k)\in\{1,2\}\times\mathcal{K}}$ such that

1. workers optimize consumption and locational choice as eq. (1) for $i \in \{1,2\}$,
2. workers' income is given by $E_i = w_i + (N_i P_{iH} H_i)/N_i$ for $i \in \{1,2\}$,
3. land clearing condition holds such that $N_i H_i = L_i(=1)$ for $i \in \{1,2\}$,
4. producers optimize production as eq. (6) for all $k \in \mathcal{K}$ and $i \in \{1,2\}$,
5. the zero-profit condition holds such that $\pi_{ik} \leq 0$ for all $k \in \mathcal{K}$ and $i \in \{1,2\}$ where equality holds if $q_{iik} + \tau q_{ijk} > 0$,
6. the national labor market clearing condition that $N_1 + N_2 = N$ holds, and
7. the local labor market clearing conditions,

$$\sum_{k\in\mathcal{N}} \int_{\Omega_{1k}} \left(\frac{\psi_k}{\lambda_1} q_{11k} + \frac{\phi_k}{\lambda_1} \right) d\nu + \sum_{k\in\mathcal{T}} \int_{\Omega_{1k}} \left(\frac{\psi_k}{\lambda_1} q_{11k} + \tau \frac{\psi_k}{\lambda_1} q_{12k} + \frac{\phi_k}{\lambda_1} \right) d\nu = N_1$$

   and

$$\sum_{k\in\mathcal{N}} \int_{\Omega_{2k}} \left(\frac{\psi_k}{\lambda_2} q_{22k} + \frac{\phi_k}{\lambda_2} \right) d\nu + \sum_{k\in\mathcal{T}} \int_{\Omega_{2k}} \left(\frac{\psi_k}{\lambda_2} q_{22k} + \tau \frac{\psi_k}{\lambda_2} q_{21k} + \frac{\phi_k}{\lambda_2} \right) d\nu = N_2,$$

   hold.

Equilibrium Conditions

I now characterize an equilibrium by obtaining simplified conditions. I focus on equilibria where all industries have nonzero output in both cities and, given the optimized production and the demand function, I impose the zero-profit condition on each sector in each city. The first result concerns non-tradable industries.

Proposition 1. *Given an equilibrium, a non-tradable industry's expenditure share equals its employment share in a city.*

$$\forall i \in \{1, 2\},\ \forall k \in \mathcal{N} \quad m_{ik} = x_{ik} \tag{7}$$

where x_{ik} is the employment share of industry k in city i.

Proof. See Appendix D . □

This equalization follows from the zero-profit condition at the industry level. Each firm has zero profit; therefore, the industry has zero aggregate profit. Since the revenues in the non-tradable industries are only received from the agents in the same city, and the factor payments are made only to the same agents, the zero-profit condition boils down to equation (7). Corollary 1 follows.

Corollary 1. *Given an equilibrium, the expenditure share equals the employment shares for a city's non-tradable and tradable sectors.*

$$\forall i \in \{1, 2\},\ \sum_{k \in \mathcal{N}} m_{ik} = \sum_{k \in \mathcal{N}} x_{ik} \ and \ \sum_{k \in \mathcal{T}} m_{ik} = \sum_{k \in \mathcal{T}} x_{ik}$$

Proof. $\sum_{k \in \mathcal{N}} m_{ck} = \sum_{k \in \mathcal{N}} x_{ck}$ follows from equation (7). $\sum_{k \in \mathcal{T}} m_{ck} = 1 - \sum_{k \in \mathcal{N}} m_{ck} = 1 - \sum_{k \in \mathcal{N}} x_{ck} = \sum_{k \in \mathcal{T}} x_{ck}$. □

The equalization for the tradable sector reflects that a city's trade balance must be zero in an equilibrium. This equalization between the expenditure share and the employment share of the tradable sector is crucial to the tractability of the model. In the rest of the paper, I refer to them interchangeably as tradable sector shares. Another result is Proposition 2.

Proposition 2 (Home-Market Effect on Wage)**.** *Given an equilibrium, the city sizes (N_1 and N_2), the tradable sector shares ($m_{1\mathcal{T}} \equiv \sum_{k\in\mathcal{T}} m_{1k}$, and $m_{2\mathcal{T}} \equiv \sum_{k\in\mathcal{T}} m_{2k}$), and the relative wage ($\omega = w_1/w_2$) satisfy the following equation.*

$$\frac{m_{1\mathcal{T}}}{m_{2\mathcal{T}}}\frac{N_1}{N_2} = \omega^{2\sigma-1}\left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{\lambda^{\sigma}-\rho\omega^{\sigma}}\right] \tag{8}$$

Proof. See Appendix D . □

This equation summarizes the industry-level zero-profit condition for the tradable industries at the city level. The LHS is the relative tradable market size of city 1 to city 2. The righthand-side (RHS), which increases with the relative wage of city 1 (ω), shows two things. First, the sector-level force--a large local market accompanies a higher input cost--is carried over to the city level. Second, the relative tradable sector share matters. For the zero-profit condition to hold in each sector-city pair, the larger city's advantage of lower trade costs for selling to consumers must be exactly offset by higher input costs. This force does not appear in the non-tradable sector because the local wage rate change affects both the demand and the input cost at the same rate.[3] Next, the expenditure shares in an equilibrium are obtained.

Lemma 1. *Given an equilibrium, the expenditure shares of industry k in city 1 and city 2 can be expressed as follows:*

$$m_{1k} = \left[\frac{\lambda^{-\sigma}-\rho_k\omega^{-\sigma}}{(1-\rho_k^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}}\left(\tilde{\beta}_k U_1^{\epsilon_k}\right)^{\mu}\left[U_1^{\frac{1}{1-\alpha}}N_1^{\frac{\alpha}{1-\alpha}}\right]^{(1-\eta)\mu} \tag{9}$$

$$m_{2k} = \left[\frac{1-\rho_k\lambda^{-\sigma}\omega^{\sigma}}{(1-\rho_k^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}}\left(\tilde{\beta}_k U_2^{\epsilon_k}\right)^{\mu}\left[U_2^{\frac{1}{1-\alpha}}N_2^{\frac{\alpha}{1-\alpha}}\right]^{(1-\eta)\mu} \tag{10}$$

where

$$\mu = \frac{\sigma-1}{\sigma-\eta} > 0,\ \rho_k = \begin{cases} 0 & k\in\mathcal{N} \\ \rho = \tau^{1-\sigma} & k\in\mathcal{T}\end{cases},\ \textit{and}\ \tilde{\beta}_k = \beta_k\left(\frac{\phi_k^{\frac{1}{\sigma-1}}\psi_k}{\sigma^{\frac{\sigma}{\sigma-1}}(\sigma-1)}\right)^{1-\eta}$$

[3]In Onoda (2022), I empirically verify that the local wage is positively associated with the tradable sector share.

Proof. See Appendix D . □

Using Lemma 1 and $\sum_{k\in\mathcal{K}} m_{ik} = 1$, the real consumption in an equilibrium is obtained as Proposition 3.

Proposition 3. *Given an equilibrium, the real consumptions in city 1 and city 2 (U_1 and U_2) can be implicitly expressed as follows:*

$$U_1 = \left\{ \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{N}} \left(\tilde{\beta}_k U_i^{\epsilon_k}\right)^{\mu} + \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{(1-\rho^2)}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \left(\tilde{\beta}_k U_i^{\epsilon_k}\right)^{\mu} \right\}^{-\frac{(1-\alpha)}{(1-\eta)\mu}} N_1^{\frac{1-\alpha}{\sigma-1}-\alpha} \tag{11}$$

$$U_2 = \left\{ \sum_{k\in\mathcal{N}} \left(\tilde{\beta}_k U_1^{\epsilon_k}\right)^{\mu} + \left[\frac{1 - \rho\lambda^{-\sigma}\omega^{\sigma}}{(1-\rho^2)}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \left(\tilde{\beta}_k U_2^{\epsilon_k}\right)^{\mu} \right\}^{-\frac{(1-\alpha)}{(1-\eta)\mu}} N_2^{\frac{1-\alpha}{\sigma-1}-\alpha} \tag{12}$$

Proof. It follows from $1 = \sum_{k\in\mathcal{K}} m_{1k}$ and Lemma 1. □

Holding the relative wage constant, the elasticity of the real consumption with respect to the local population is given by

$$\frac{\partial U_i/U_i}{\partial N_i/N_i} = \frac{(1-\alpha)/(\sigma-1) - \alpha}{1 + (1-\alpha)\tilde{\epsilon}_k/(1-\eta)}$$

where $\tilde{\epsilon}_c = \sum_{k\in\mathcal{K}} m_{ik}\epsilon_k$. The factor $1/(\sigma-1)$ in the numerator is the elasticity of the agglomeration economy or the positive externality in Krugman-type models with homothetic preference. The mass of varieties in a location increases with market size, and consumers have a love-of-variety preference. This elasticity is multiplied by $1-\alpha$ because it is the share of goods consumption. The inelastic supply of land causes the negative externality α. As the local population grows, the size of land that a resident can consume becomes smaller, which appears as a negative externality. The externality has an additional term, $(1-\alpha)(1-\eta)/\tilde{\epsilon}_c$. To understand this, suppose all the industries have homogeneous income elasticity, $\bar{\epsilon}$, and increase Q_{ik} for all $k \in \mathcal{K}$ by the same proportion ($dQ_{ik}/Q_{ik} = dQ_{i\ell}/Q_{i\ell}$ for all $(i,\ell) \in \mathcal{K}^2$) . The elasticity of the real consumption is given by:

$$\frac{dU_i}{U_i} / \sum_{k\in\mathcal{K}} \frac{dQ_{ik}}{Q_{ik}} = \left[\frac{1}{(1-\alpha)} + \frac{\bar{\epsilon}}{1-\eta}\right]^{-1}$$

This equation shows that the greater $\bar{\epsilon}$ is, the smaller the elasticity of the real consumption is. This relationship between the elasticity of real consumption and $\bar{\epsilon}$ carries over to my heterogeneous-income-elasticity model. The average of ϵ_k weighted with the expenditure shares summarizes the effect on the marginal utility from aggregate consumption.

The negative relationship between the relative wage ($\omega = w_1/w_2$) and the real consumption in city 1 (U_1) relates to the home-market effect on the wage. Given w_1, a higher w_2 implies greater demand for the tradable sector in city 2 according to the home-market effect on the wage equation (8). The greater demand is accompanied by richer varieties produced in city 2 in the tradable sector because the trade balance must be zero. In turn, the richer tradable varieties from city 2 reduce the price index for goods consumption in city 1 and raise the real consumption.

Finally, an equilibrium is characterized by the seven conditions obtained above: equations (5), (8), (11), (12), the national labor clearing condition ($N_1 + N_2 = N$), and the tradable sector shares as follows:

$$m_{1\mathcal{T}} = \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \left(\tilde{\beta}_k U_1^{\epsilon_k}\right)^{\mu} \left[U_1^{\frac{1}{1-\alpha}} N_1^{\frac{\alpha}{1-\alpha}}\right]^{\frac{(1-\alpha)}{(1-\eta)\mu}} \tag{13}$$

$$m_{2\mathcal{T}} = \left[\frac{1 - \rho\lambda^{-\sigma}\omega^{\sigma}}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \left(\tilde{\beta}_k U_2^{\epsilon_k}\right)^{\mu} \left[U_2^{\frac{1}{1-\alpha}} N_2^{\frac{\alpha}{1-\alpha}}\right]^{\frac{(1-\alpha)}{(1-\eta)\mu}} \tag{14}$$

where $m_{1\mathcal{T}} = \sum_{k\in\mathcal{T}} m_{1k}$, and $m_{2\mathcal{T}} = \sum_{k\in\mathcal{T}} m_{2k}$. These shares follow from individual tradable sector's share (eq. (9) and (10)). There are seven unknown variables $\{U_1, U_2, \omega, N_1, N_2, m_{1\mathcal{T}}, m_{2\mathcal{T}}\}$ for these seven equations. Using the zero-profit conditions for all city-sector pairs, the number of endogenous variables here is fewer than that of the equilibrium definition. Based on these conditions, I now provide some properties of the equilibrium.

The Existence, the Uniqueness, and the Stability of an Equilibrium

There exists a stable equilibrium subject to parameter conditions. First, I impose Assumption 1, which I assume in the rest of the paper.

Assumption 1. $\gamma > \frac{(1-\alpha)/(\sigma-1)-\alpha}{1+(1-\alpha)\min_{k\in\mathcal{K}}\{\epsilon_k/(1-\eta)\}}$

Assumption 1 is sufficient for an equilibrium to exist as Proposition 4.

Proposition 4 (Existence of equilibrium)**.** *Given Assumption 1, there exists an equilibrium.*

Proof. See Appendix D. □

Assumption 1 ensures that whatever the expenditure composition is, the agglomeration force on the real consumption is weaker than the aggregate dispersion force by inelastic land supply and heterogenous locational taste. This relationship prevents a city from attracting all the workers. Without the term $\min_{k\in\mathcal{K}}\{\epsilon_k/(1-\eta)\}$, this becomes the condition for a unique stable equilibrium in Redding (2016), who also has the dispersion forces from the heterogeneous taste over cities and inelastic land supply. Unlike Redding (2016), Proposition 4 concerns only the existence because the uniqueness requires additional assumptions in my model due to the endogenous tradable sector shares. Even without the uniqueness, as I explain in the next section, cross-city production, consumption, and wage patterns become qualitatively the same in all the possible equilibria. In Appendix C, I explain sufficient conditions to have uniqueness.

Next, I analyze whether there exists a stable equilibrium. When U_1 and U_2 can be expressed as functions of only N_1 from equations (8), (11), and (12), a stable equilibrium is defined as follows.

Definition. A competitive equilibrium is a stable equilibrium if and only if

$$\frac{d\left(U_1(N_1)a_1/U_2(N_1)a_2\right)^{1/\gamma}}{dN_1} < \frac{d\left(N_1/(N-N_1)\right)}{dN_1}$$

This inequality guarantees that, with the migration of infra-marginal agents who are indifferent between two cities ($U_1(N_1)a_1\delta(\theta,2) = U_2(N_1)a_2\delta(\theta,2)$), the expanding city would not experience enough of a relative gain in real consumption to support the post-migration population allocation.[4][5] With this condition, the infra-marginal agents in the expanding city find the shrinking one preferable and return to their original location. Consequently, the economy converges back to its original state. For stability, Proposition 5 is obtained.

Proposition 5 (Stability of equilibrium)**.** *Given Assumption 1, there exists a stable equilibrium.*

[4]With V_1 and V_2 s.t. $V_1 = a_1U_1N_1^{-\gamma}$ and $V_2 = a_2U_2N_2^{-\gamma}$, the condition of the stable equilibrium becomes $dV_1/dN_1 < dV_2/dN_1$. In the homogeneous agent's interpretation of the model, this guarantees that the expanding city does not offer a higher utility than the shrinking city.

[5]The product of the relative real consumption and amenity level after the migration of ε_1 agents to city 1 becomes $\{U_1(N_1{+}\varepsilon_1)a_1\}/\{U_2(N_1{+}\varepsilon_1)a_2\}$. This new ratio supports population increase by ε_2 such that $(N_1{+}\varepsilon_2)/(N_2{-}\varepsilon_2) = [\{U_1(N_1+\varepsilon_1)a_1\}/\{U_2(N_1+\varepsilon_1)a_2\}]^{1/\gamma}$ from equation (5). If $\epsilon_2 < \epsilon_1$, the new relative utility cannot support the migration. When $\epsilon_1 \to 0$, this is equivalent to the condition of the stability definition.

Proof. See Appendix D. □

When the dispersion force is strong enough, an equilibrium is stable. In the rest of the paper, I assume that Assumption 1 holds and focus on a stable equilibrium.

3 Cross-City Analysis

In this section, I illustrate how the two cities differ in an equilibrium depending on their fundamental productivity and amenities. I start with two partial equilibrium analyses to show the directions of the forces that the fundamental differences generate. Then, I lay out general equilibrium results where the cities have the same amenity level and differ only in fundamental productivity ($a_1 = a_2$). The results of different amenities with the same productivity are briefly explained (A detailed explanation is provided in Appendix E).

Partial Equilibrium Analyses

In the first partial equilibrium analysis, I illustrate how a difference in fundamentals generates a force that produces asymmetric population allocation and nominal wage inequality. I explain the interplay between this force and the endogenous tradable sector shares in the second partial analysis. The second analysis elucidates that the endogenous tradable sector shares amplify the nominal wage inequality and generate a dispersion force when the industries are gross complements.

Population and Wage

The first analysis uses two equilibrium conditions relating the relative wage to the relative population. Using the population allocation with Fréchet utility shock (eq. (5)), the real consumption in city 1 in equilibrium (eq. (11)), and the home-market effect (HME) on the wage rate (eq. (8)) can be rewritten as follows:

$$
\begin{aligned}
1 = \left[\frac{1}{\lambda^\sigma}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{N}} \tilde{\beta}_k^\mu & \left[\left(\frac{V}{a_1}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)} N_1^{\left\{\gamma\left(\epsilon_k+\frac{1}{1-\alpha}\right)+\frac{\alpha}{1-\alpha}-\frac{1}{\sigma-1}\right\}}\right]^{(1-\eta)\mu} \\
& + \left[\frac{1}{\left(\lambda^\sigma+\rho\omega^{\sigma-1}\left(N_2/N_1\right)\left(m_{2\mathcal{T}}/m_{1\mathcal{T}}\right)\right)}\right]^{\frac{1-\eta}{\sigma-\eta}} \cdot \\
& \sum_{k\in\mathcal{K}} \tilde{\beta}_k^\mu \left[\left(\frac{V}{a_1}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)} N_1^{\left\{\gamma\left(\epsilon_k+\frac{1}{1-\alpha}\right)+\frac{\alpha}{1-\alpha}-\frac{1}{\sigma-1}\right\}}\right]^{(1-\eta)\mu}
\end{aligned}
\tag{15}
$$

$$
N_1 = \frac{m_{2\mathcal{T}}}{m_{1\mathcal{T}}}\omega^{2\sigma-1}\left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{\lambda^\sigma-\rho\omega^\sigma}\right] N_2 \tag{16}
$$

where $V = a_2U_2N_2^\gamma$. In this partial equilibrium analysis, I focus on city 1 and fix $V(= a_2U_2N_2^\gamma)$, N_2, and the relative tradable sector share $(m_{1\mathcal{T}}/m_{2\mathcal{T}})$. Then, equation (15) can be interpreted as the labor supply curve in city 1, and, given Assumption 1, N_1 increases in ω; as the wage increases, the city attracts more workers. Similarly, N_1 increases in ω from the HME on the wage rate (eq. (16)); a large local market requires a higher input cost to keep the profit at zero. These two curves are depicted in Figure 3a. [6]

When city 1 becomes more productive ($\lambda \uparrow$), the HME curve shifts up, and the labor supply curve shifts to the right, as in Figure 3b. The HME curve shifts for two reasons. First, the local wage linearly increases in local productivity. Consequently, the local market size expands, and the input cost, w_1, needs to rise to keep the zero profit. Meanwhile, additional local varieties drive the shift of the labor supply curve. Given the labor supply, higher productivity increases the mass of local varieties. Given the prices of varieties, which are linear in the wages w_1, the variety increase reduces the price indices and raises the real consumption, attracting more workers. Because of the shifts of the two curves, the new intersection is located where the population and the wage are higher than before.

When city 1 has better amenities ($a_1 \uparrow$), the labor supply curve shifts to the right, as in Figure 3c, because the better amenities attract more people. However, the HME curve is unaffected since amenities do not affect production. As a result, the intersection moves along the HME curve, and the population and the wage are

[6] How the curves intersect is not easy to tell from the equations in this part. The depiction here is based on the theoretical results in the general equilibrium. For cities with asymmetric productivity, the one with higher productivity offers a higher local wage and a larger population. For cities with asymmetric amenities, the one with better amenities offers a higher local wage and has a larger population if $(1-\alpha)/(\sigma-1) > \alpha$ and $\eta < 1$. These are consistent with the labor supply curve intersecting the home-market effect curve from below.

higher than before. [7]

This analysis, which ignores what is taking place in the other city (N_2, U_2) and the effect through the relative tradable sector share ($\sum_{k\in\mathcal{T}} x_{2k}/\sum_{k\in\mathcal{T}} x_{1k}$), shows the main forces generated by heterogeneous fundamental productivity and amenities. In the following analysis, I show how the relative tradable sector share behaves and relates to the relative wage when the population increases.

Wage and Tradable Sector Share

In the second partial equilibrium analysis, I focus on the relationship between the relative wage (ω) and the tradable sector share ($m_{1\mathcal{T}}$). It uses, again, two equilibrium conditions. First, the home-market effect on wage (eq. (8)) positively associates the relative wage and the tradable sector share, depicted as the HME-by-$m_{\mathcal{T}}$ curve in Figure 4. Second, the tradable sector share increases and decreases with the relative wage when the industries are gross complements ($\eta < 1$) and gross substitutes ($\eta > 1$), respectively (eq. (13) and (14)). The ZPC+Substitution curve in Figure 4 depicts this relationship.

The ZPC+Substitution curve reflects two economic forces. The first is the zero profit conditions (ZPCs) in the tradable and non-tradable sectors. These conditions can be written as follows: (see eq. (30) in Appendix D for the derivation)

$$\text{For } k \in \mathcal{T},\ \frac{(w_i/\lambda_i)^\sigma - \rho\,(w_j/\lambda_j)^\sigma}{1-\rho^2} = N_i \tilde{\beta}_k \tilde{P}_{ik}^{\sigma-\eta} U_i^{\epsilon_k} C_i^{1-\eta} E_{iC}^{\eta}$$

$$\text{For } k \in \mathcal{N},\ \left(\frac{w_i}{\lambda_i}\right)^\sigma = N_i \tilde{\beta}_k \tilde{P}_{ik}^{\sigma-\eta} U_i^{\epsilon_k} C_i^{1-\eta} E_{iC}^{\eta}$$

By combining these conditions, it follows

$$\text{For } k \in \mathcal{T},\ \ell \in \mathcal{N},\ \frac{1-\rho\omega^{-\sigma}\lambda^\sigma}{1-\rho^2} = \frac{\tilde{\beta}_k}{\tilde{\beta}_\ell}\left(\frac{\tilde{P}_{1k}}{\tilde{P}_{1\ell}}\right)^{\sigma-\eta} U_1^{\epsilon_k-\epsilon_\ell}$$

This equation shows that ignoring the effect from the heterogeneous income elasticities ($U_1^{\epsilon_k-\epsilon_\ell}$), a higher relative wage ω requires a higher price index in a tradable industry relative to a non-tradable industry, $\tilde{P}_{1k}/\tilde{P}_{1\ell}$.

[7] While this result is in contrast to what a Rosen-Roback model (Rosen 1979; Roback 1982) implies, this is not new in the literature. For example, Glaeser and Gottlieb (2009) point out that rising amenities can increase wages because of agglomeration economy.

A higher relative wage is disadvantageous for firms only in the tradable industries, and, therefore, it must entail a change in price indices relatively favorable for the tradable industries to keep zero profits in both sectors. The second force is the substitution between the tradable and non-tradable sectors. The higher price indices of the tradable industries imply a higher tradable sector share when the industries are gross complements and a higher tradable sector share when gross substitutes.

When city 1 attracts workers ($N_1 \uparrow$), whether the industries are gross complements or gross substitutes determines how it affects the relative wage and the tradable sector share. The larger population shifts the HME-by-$m_{\mathcal{T}}$ curve upwards because of the home-market effect (eq. (8)). When the industries are gross complements (Figure 4a), workers spend more on the tradable industries, which become relatively more expensive, raising the relative wage further in the new intersection by the home-market effect through the expenditure share. When the industries are gross substitutes (Figure 4b), the residents spend less on the tradable industries. The reduced expenditure shares attenuate the wage increase.

The endogenous tradable sector share generates an additional dispersion force when the industries are gross complements and an additional agglomeration force when gross substitutes. As a result of the increasing tradable sector share in the gross-complement case, the mass of available varieties in the non-tradable industries decreases, and the non-tradable industries become expensive in terms of the price index. [8] The tradable industries become even more expensive because the price index rises relative to that of the non-tradable industries due to the zero profit conditions. [9] Thus, the tradable and non-tradable industries become expensive, and the real consumption decreases. [10] This effect can be interpreted as a force that counteracts the population increase–a dispersion force. On the other hand, the tradable sector share decreases when the industries are gross substitutes. This drop in the expenditure share attenuates the relative wage increase and increases the real consumption. This effect can be interpreted as an agglomeration force that does not appear when we fix the tradable sector share.

[8] This result can be verified with the price index of a non-tradable sector relative to income $\tilde{P}_{ik}/\{w_{ik}/(1-\alpha)\} = (1-\alpha)\lambda_i^{\sigma/(1-\sigma)} x_{ik}^{1/1-\sigma} N_i^{1/1-\sigma}$, which decreases in the employment share x_{ik}.

[9] It does so despite the increase in workers in the tradable industries. This result is possible if the mass of varieties shipped from city 2 drops, which indeed happens in the general equilibrium.

[10] The change in the tradable sector shares increases the relative wage and reduces the real consumption. This relationship between the relative wage and the real consumption can be seen in equation (11). Holding N_1 constant, with the assumption for global monotonicity of the utility function $1 + (1-\alpha)\min_{k\in\mathcal{K}}\{\epsilon_k/(1-\eta)\} > 0$, the real consumption U_1 decreases with the relative wage ω.

In the next section, I provide the results in general equilibrium for the case in which cities differ only in terms of productivity. Appendix E provides the results for the case with asymmetric amenities and common productivity. The city with better fundamental characteristics (productivity or amenities) becomes larger.[11] In the asymmetric productivity case, the large city offers a higher wage and specializes in income-elastic industries. In the asymmetric-amenities case, the same results are obtained with additional assumptions such that the industries are gross complements or that the agglomeration force is stronger than the dispersion force from the inelastic land supply.

General Equilibrium with Productivity Difference

In this section, I consider cities that differ only in fundamental productivity and provide a cross-city analysis for a stable equilibrium. The first comparison concerns population allocation and relative nominal income. The partial equilibrium analysis (Figure 3b) suggests that city 1 has a larger population, and the residents receive the higher nominal income, which is proportional to the wage in an equilibrium. When we consider a general equilibrium and allow additional variables, including N_2, to move, the movement of N_2 attenuates the agglomeration economy in city 1 because of the decrease in the mass of varieties shipped from city 2. As a result, this shortens the shift of the labor supply curve in Figure 3b. On the other hand, the shrinking population in city 2 amplifies the home-market effect, shifting further up the HME curve in Figure 3b. After all, the qualitative result stays the same as long as the price elasticity is not too high and Proposition 6 is obtained.

Proposition 6 (Cross-City Population and Wage Patterns with Gross Complements)**.** *Suppose that the productivities are asymmetric ($\lambda_1 > \lambda_2$), the amenities are symmetric ($a_1 = a_2$), and the industries are gross complements ($\eta < 1$). Then, given an equilibrium with Assumption 1, the fundamentally productive city is larger and the relative wage is greater than the relative fundamental productivity.*

$$N_1 > N_2, \; \omega > \lambda > 1$$

[11] The asymmetric-amenities case requires either the industries to be gross complements or the industries to be gross substitutes with Assumption 2, which appears below in the main text.

Proof. See Appendix D. □

Although Assumption 1 does not ensure a unique equilibrium, this result applies to all possible equilibria. The second partial equilibrium analysis (Figure 4) suggests an additional dispersion force from the tradable sector share when the industries are gross complements. This force and Assumption 1 ensure that the aggregate dispersion force is stronger than the agglomeration force globally, and the qualitative results become the same as the first partial equilibrium analysis. In contrast, when the industries are gross substitutes, there is an additional agglomeration force from the tradable sector share. Despite this additional force, Assumption 2 is sufficient to ensure that the aggregate dispersion force is globally stronger than the agglomeration force.

Assumption 2. $\eta < 1 + (1-\alpha)\left(1 + \gamma \min_{k\in\mathcal{T}} \epsilon_k\right)/(\gamma+\alpha)$

It is reasonable to impose Assumption 2 for the case of gross substitutes when the number of industries is comparable to ten. [12]The RHS of the inequality is greater than one unless the lowest income elasticity ϵ_k in the tradable sector is too small. In contrast, the inter-industry price elasticity of substitution η in the LHS is smaller than one. Moreover, Comin et al. (2021) estimate the inter-industry price elasticity to be 0.07-0.13 with ten industries. Given Assumption 2, the same population and wage patterns can be obtained as the gross-complement case.

Proposition 7 (Cross-City Population and Wage Patterns with Gross Substitutes)**.** *Suppose that the productivities are asymmetric ($\lambda_1 > \lambda_2$), the amenities are symmetric ($a_1 = a_2$), the industries are gross substitutes ($\eta > 1$), and Assumption 2 holds. Then, given an equilibrium with Assumption 1, the fundamentally productive city is larger, and the relative wage is greater than the relative fundamental productivity.*

$$N_1 > N_2,\ \omega > \lambda > 1$$

Proof. See Appendix D. □

[12]Also, it does not contradict Assumptions 1. These assumptions can be rewritten as:

$$\frac{(\gamma+\alpha)(\eta-1)-(1-\alpha)}{(1-\alpha)\gamma} < \min_{k\in\mathcal{T}} \epsilon_k, \max_{k\in\mathcal{K}} \epsilon_k < \frac{(\gamma+\alpha)(\eta-1)-(1-\alpha)(\eta-1)/(\sigma-1)}{(1-\alpha)\gamma}$$

Given the assumption of $\eta < \sigma$, this can be simultaneously satisfied.

The results of Propositions 6 and 7 are consistent with a stylized fact that nominal income is higher in larger cities, even when we control for observable or unobservable workers' characteristics (Behrens and Robert-Nicoud 2015; Glaeser and Mare 2001). The following result concerns expenditure shares.

Proposition 8 (Cross-City Expenditure Pattern)**.** *Suppose that the productivities are asymmetric ($\lambda_1 > \lambda_2$), the amenities are symmetric ($a_1 = a_2$), and the industries are either gross complements ($\eta < 1$) or gross substitutes ($\eta > 1$) with Assumption 2 holding. Then, given an equilibrium with Assumption 1, the industrial expenditure share ratio of city 1 to city 2 (m_{1k}/m_{2k}) increases in income elasticity (ϵ_k) within the corresponding sector.*

Proof. Equations (9) and (10) provide the expenditure share ratio of a non-tradable industry and a tradable industry, respectively,

$$\forall k \in \mathcal{N}, \quad \frac{m_{1k}}{m_{2k}} = \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \left(\frac{U_1}{U_2}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \left(\frac{N_1}{N_2}\right)^{\left(\frac{\alpha}{1-\alpha}-\frac{1}{\sigma-1}\right)(1-\eta)\mu}$$

$$\forall k \in \mathcal{T}, \quad \frac{m_{1k}}{m_{2k}} = \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{1-\rho\lambda^{-\sigma}\omega^{\sigma}}\right]^{\frac{1-\eta}{\sigma-\eta}} \left(\frac{U_1}{U_2}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \left(\frac{N_1}{N_2}\right)^{\left(\frac{\alpha}{1-\alpha}-\frac{1}{\sigma-1}\right)(1-\eta)\mu}$$

$N_1 > N_2$ in the equilibrium implies $U_1 > U_2$ according to equation (5), and the expenditure share ratio increases in ϵ_k within the corresponding sector. □

The large city spends more on the income-elastic industries. This difference in the expenditure shares generates the following results in the employment shares.

Proposition 9 (Home-Market Effect on Industrial Specialization)**.** *Given an equilibrium, for a tradable industry, there is a relationship between the within-tradable employment share ratio and the within-tradable expenditure share ratio such that*

$$\forall k \in \mathcal{T}, \ \frac{\tilde{x}_{1k}}{\tilde{x}_{2k}} = \frac{(\tilde{m}_{1k}/\tilde{m}_{2k}) - \rho\lambda^{\sigma}\omega^{-\sigma}}{1-\rho\lambda^{\sigma}\omega^{-\sigma}} \frac{1-\rho\omega^{\sigma}\lambda^{-\sigma}}{1-\rho\omega^{\sigma}\lambda^{-\sigma}(\tilde{m}_{1k}/\tilde{m}_{2k})}$$

where $\tilde{m}_{ik} = m_{ik}/\sum_{k\in\mathcal{T}} m_{ik}$ and $\tilde{x}_{ck} = x_{ik}/\sum_{k\in\mathcal{T}} x_{ik}$. This equation shows $\tilde{x}_{1k}/\tilde{x}_{2k}$ increases in $\tilde{m}_{1k}/\tilde{m}_{2k}$; that is, the greater an industry's the within-tradable expenditure share difference, the more significant that industry's within-tradable employment share difference. Also, this implies that a city becomes *the net exporter in industries for which the city has a greater within-tradable expenditure share as follows:*

$$\forall k \in \mathcal{T},\ \tilde{m}_{1k} > \tilde{m}_{2k} \iff x_{1k} > m_{1k}$$

Proof. See Appendix D. □

Then, Corollary 2 follows.

Corollary 2 (Cross-City Trade Pattern)**.** *Suppose that the productivities are asymmetric ($\lambda_1 > \lambda_2$), the amenities are symmetric ($a_1 = a_2$), and the industries are either gross complements ($\eta < 1$) or gross substitutes ($\eta > 1$) with Assumption 2 holding. Then, given an equilibrium with Assumption 1, there exists tradable industry $s \in \mathcal{T}$ such that the fundamentally productive city becomes a net exporter for all tradable industry $k \in \mathcal{T}$ whose income elasticity is not smaller than that of industry s ($\epsilon_k \geq \epsilon_s$). Also, there exists tradable industry $s \in \mathcal{T}$ such that the fundamentally productive city becomes a net importer for all tradable industry $k \in \mathcal{T}$ whose income elasticity is not greater than that of tradable industry $s \in \mathcal{T}$ ($\epsilon_k \leq \epsilon_s$).*

$$\exists s \in \mathcal{T}, \forall k \in \mathcal{T}\ s.t.\ \epsilon_k \geq \epsilon_s,\ x_{1k} > m_{1k}$$

$$\exists s \in \mathcal{T}, \forall k \in \mathcal{T}\ s.t.\ \epsilon_k \leq \epsilon_s,\ x_{1k} < m_{1k}$$

This result is the home-market effect in the industrial specialization in my model. The difference in the expenditure pattern generates comparative advantages and is amplified to that of the employment pattern. Unlike Krugman (1980), who assumes an exogenous taste difference to generate the heterogeneous relative demand, that difference arises endogenously from the non-homothetic preference. The endogenous formation of the relative demand is the same as in Matsuyama (2019), but my definition of the relative demand is different. The result of Proposition 9 demonstrates that when non-tradable industries exist, the relative size of demand should be measured within the tradable sector. In Matsuyama (2019), all industries are tradable, and the relative demand size is the same whether within the overall economy or the tradable sector.

Next, I analyze the tradable and non-tradable sector expenditure shares, equal to their employment shares from Corollary 1. In the equilibrium, the ratios of these shares follow from equations (9) and (10) as

$$\frac{m_{1\mathcal{T}}}{m_{1\mathcal{N}}} = \left[\frac{1-\rho\lambda^{\sigma}\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \frac{\sum_{k\in\mathcal{T}}\left(\tilde{\beta}_k U_1^{\epsilon_k}\right)^{\mu}}{\sum_{k\in\mathcal{N}}\left(\tilde{\beta}_k U_1^{\epsilon_k}\right)^{\mu}}$$

$$\frac{m_{2T}}{m_{2N}} = \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \frac{\sum_{k\in\mathcal{T}}\left(\tilde{\beta}_k U_2^{\epsilon_k}\right)^{\mu}}{\sum_{k\in\mathcal{N}}\left(\tilde{\beta}_k U_2^{\epsilon_k}\right)^{\mu}}$$

where $m_{i\mathcal{N}} = \sum_{k\in\mathcal{N}} m_{ik}$ for $i \in \{1,2\}$. Two forces determine the tradable sector share. The first is the relative price of goods in the tradable sector, captured by the first factor in each RHS. Other things being equal, this force makes the large city has a greater tradable sector share when the industries are gross complements and a smaller tradable sector when gross substitutes, as discussed in the second partial equilibrium analysis (Figure 4). This effect is captured by $1-\rho\lambda^{\sigma}\omega^{-\sigma} > 1-\rho\lambda^{-\sigma}\omega^{\sigma}$ as $\omega > \lambda$ from Propositions 6 and 7. The second force is the income elasticities of the two sectors. Because the real consumption is higher in city 1 ($U_1 > U_2$) in the equilibrium, the higher the income elasticities of the tradable industries as a whole are relative to those of the non-tradable industries, the higher the tradable sector share in city 1 is relative to that in city 2, other things being equal.[13]

Finally, the price index of an industry differs between locations. The price index ratio is given by

$$\frac{P_{1k}}{P_{2k}} = \begin{cases} \frac{w_1}{w_2}\left(\frac{\lambda N_1}{N_2}\frac{x_{1k}}{x_{2k}}\right)^{-\frac{1}{\sigma-1}} & k\in\mathcal{N} \\ \frac{1}{\tau}\left(1+\frac{\tau^{2(\sigma-1)}-1}{(\tau w_2/w_1)^{\sigma-1}(\lambda N_1/N_2)(x_{1k}/x_{2k})+1}\right)^{\frac{1}{\sigma-1}} & k\in\mathcal{T} \end{cases}$$

The price index ratio in both sectors decreases in the employment share ratio (x_{1k}/x_{2k}). As the employment share ratio increases in income elasticity within a sector, the price index ratio decreases in income elasticity within a sector. In city 1, the higher expenditure shares on income-elastic goods attract firms in those industries, and the price indices in those industries become relatively inexpensive, reflecting relatively more varieties.

[13]This observation becomes critical when we consider the effect of sector-specific trade cost reductions. When a non-tradable industry becomes tradable, it changes the income elasticities of the two sectors and, thereby, the tradable sector share. Onoda (2022) examines how business services' trade cost reduction has affected cross-city income inequality by this mechanism.

Connection to Stylized Fact

In summary, the fundamentally productive location becomes a large and high-income city, and within each sector (tradable and non-tradable), the residents allocate their expenditure relatively more towards income-elastic industries, which offer relatively richer varieties. On the supply side, the large and high-income city specializes in income-elastic industries (within each sector), replicating the industrial specialization pattern seen in Figure 1. While what we observe in the real world are the city size and the industrial employment of a city, the fundamental productivity generates the relationship between them in this model.

Additionally, the specialization pattern is consistent with the amplification mechanism in the model. The model predicts that the employment shares of income-elastic industries are relatively higher in large cities and that they are even higher for tradable industries because the home-market effect amplifies the demand pattern as Proposition 9 and Corollary 2.[14] To see if there is such a difference between tradable and non-tradable industries in the data, I classify industries in Figure 1 as non-tradable that have export shares smaller than 0.03 on average in the international trade data in Caron et al. (2020) and the others as tradable.[15] The regression line between the two elasticities in Figure 1 is re-estimated for each sector separately and shown in Figure 2. The positive relationship is steeper for the tradable industries; the slope of the regression line for the tradable industries (solid) is 1.06, while that of the non-tradable industries (dashed) is 0.30. The employment share of an income-elastic industry increases with the city size faster for a tradable industry than for a non-tradable industry, consistent with the amplification mechanism in the tradable industries.

4 Robustness Check

In this section, I provide two robustness checks; the first addresses an alternative explanation of the cities' specialization patterns in Figure 1, and the second tests the pattern with an alternative specification. I implement regressions for multiple years separately in each test, which serves as additional robustness checks. Furthermore, I show with the alternative specification that the demand effect has economically significant

[14] This prediction is consistent with a common presumption that the geographical concentration of an industry serves as a tradability index (e.g., Mian and Sufi 2014).

[15] Non-tradable industries are "Water", "Electricity", "Construction", "Trade" (including wholesale trade, retail sales, hotels, and restaurants), "Recreational and other services", "Public Administration, Defense, Education, Health", and "Financial services nec".

explanatory power in cities' specialization patterns.

Controlling for Skill Intensities in Figure 1

An omitted variable—skilled-labor supply in cities—possibly drives the positive relationship. This concern is reasonable given that skilled workers, who are at the same time high-income earners, tend to reside in large cities, and those cities tend to host skill-intensive industries (Davis and Dingel 2020). Also, and as is well known, there is a positive correlation between skill intensities and income elasticities of industries (Caron et al. 2014, 2020).

To test if the positive relationship in Figure 1 is robust to controlling for the skill intensities of industries, I regress the elasticities of employment share on the income elasticities of demand and the skill intensities. The regression model is given by

$$\xi_k = \alpha + \beta \epsilon_k + \gamma \theta_k + e_k$$

where

ξ_k : elasticity of employment share of industry k with respect to MSA's population

ϵ_k : income elasticity of demand for industry k output

θ_k : skill intensity of industry k

e_k : error term for industry k

The population elasticity of employment share ξ_k is the y-axis variable in Figure 1. I obtain each ξ_k by regressing the log of employment shares of industry k in MSAs on the log of MSAs' population sizes, controlling for the region of an MSA. I use the population data from the Bureau of Economic Analysis (BEA) and the employment data from the Country Business Patterns (CBP). The income elasticity ϵ_k is the x-axis variable in Figure 1, and the skill intensity θ_k is the control. For these two variables, I use the estimates by Caron et al. (2020), who obtain the estimates through a structural estimation with international trade data with heterogeneous skill intensities and income elasticities. If the cities' specialization pattern in Figure 1 only reflects that large cities specialize in skill-intensive industries by the supply side mechanism (Davis and Dingel 2020), $\beta = 0$ and $\gamma > 0$ are expected. I implement the cross-sectional regression for three years;[16] Table 1 shows the results.

The coefficient β is significantly positive in all three years, whereas γ is not. A caveat is that all of ξ_k, ϵ_k, and θ_k used in the regressions are estimated values; therefore, the standard errors here are not precise for the hypothesis test. Nevertheless, these results suggest that the positive relationship in Figure 1 is robust to the supply-side explanation.

Alternative Specification and Explanatory Power

The second robustness check tests the specialization pattern with an alternative specification that follows Nunn (2007). The regression model is given by

$$\begin{aligned} y_{mk} =& \alpha \cdot \exp(\beta_{demand} \cdot \epsilon_k \cdot \log(Population_m) + \beta_{skill} \cdot \theta_k \cdot \log(College_m) \\ &+ \sum_m \gamma_m D_m + \sum_{region} \gamma_{k,region} D_{k,region})) \cdot e_{mk} \end{aligned} \tag{17}$$

[16]The latest year in this study is 2016 because the data source of employment, the CBP datasets, does not contain employment-size information for MSA-NAICS (MSA-industry) pairs with fewer than three establishments from 2017. These pairs account for a significant portion of the MSA-NAICS sample I use to construct the MSA-GTAP sample for regressions, accounting for 28% in 2016. Since small employment shares are crucial in detecting specialization patterns, I use employment data up to 2016. See Appendix A for additional data details.

where

$$
\begin{aligned}
y_{mk} &: \text{employment level of industry } k \text{ in MSA } m \\
\epsilon_k &: \text{income elasticity of demand in industry } k \\
Population_m &: \text{population in MSA } m \\
\theta_k &: \text{skill intensity of industry } k \\
College_m &: \text{college employment ratio in MSA } m \\
D_m &: \text{MSA dummy variable} \\
D_{k,region} &: (\text{industry } k \times \text{region} \in \{\text{Northeast, Midwest, South, West}\}) \text{ dummy variable} \\
e_{mk} &: \text{error term for MSA } m \times \text{industry } k
\end{aligned}
$$

The coefficient of interest is β_{demand}. When β_{demand} is positive, the employment level rises more for high ϵ_k as the population rises, consistent with the pattern in Figure 1 and the model prediction. Two points are noteworthy. First, as the regression model shows, I implement level regressions by the Poisson Pseudo Maximum Likelihood (PPML) estimation. As discussed in Silva and Tenreyro (2006), log-linear regressions require a very specific condition on error terms to obtain consistent estimators. Moreover, it is problematic in log-linear estimations when zeros are contained in the data. On the other hand, PPML provides consistent estimators that do not require this condition, and it is efficient with various error term patterns. For this reason, PPML is very common in gravity equation estimations in international trade where zeros are prevalent and error terms show heteroskedasiticy. In my dataset, 17% of the sample is zero. To address these zeros, I use PPML and set employment levels (instead of employment shares) as the dependent variable so that the error terms show a heteroskedasticity pattern suitable for PPML in terms of efficiency. Second, I control for the supply-side effect using the skill intensities of industries. This time, the interaction term, $\theta_k \cdot \log(College_m)$, achieves the control. I borrow the estimates of skill intensities again from Caron et al. (2020), who estimate them through a structural estimation that had both heterogeneous skill intensities and income elasticities. I calculate the college employment ratios for full-time workers in MSAs from US Census data via Integrated Public Use Microdata Series (IPUMS) (Ruggles et al. 2023). (See Appendix A for further data details.)

The regression results are consistent with the pattern in Figure 1 and the model prediction. Because this regression is cross-sectional, I implement it for three different years separately. Table 2 shows the results. The three left columns contain results with PPML, while the three right columns contain those with log-linear regressions after 1 is added to every observation to take care of zeros. In all of the results, β_{demand} is significantly positive. The size of β_{demand} is between 0.30 and 0.37 in PPML. To gauge the economic significance of this β_{demand} using 2006 data, suppose, first, that City A has a population that is greater than the sample mean in log point by one standard deviation, corresponding to a population of 1,201,486 in level, second, that City B has a population of the sample mean in log point, which is 415,057, and, third, that City A and City B have the same college employment ratio. Then, the employment ratio of an industry with $\epsilon_k = 1.2$ over that with $\epsilon_k = 0.8$ ($N_{\epsilon_k=1.2}/N_{\epsilon_k=0.8}$) is 1.14 times greater in City A than in City B.

The income elasticities have a significant explanatory power even when controlling for the skill supply. It is not straightforward to measure the explanatory power of the PPML estimators because, technically, they are obtained by a maximum likelihood estimation. To construct a measurement for the PPML estimators, I calculate the estimated employment shares implied by the fitted values for the employment levels and obtain the residual sum of squares (RSS) from the difference between the estimated shares and the actual data. Using this RSS, a measurement analogous to R^2 in linear regressions is constructed and summarized in Table 3. In the first row, RSS_{FE} is the RSS when the regression has only the fixed effects of MSA and industry $\times$ region, and total sum of squares (TSS) is the residual sum of squares by the unconditional mean of the employment shares. Table 3 shows that the fixed effects explain over 90% of TSS in all three years, which is natural given the rich fixed effects and substantial variation in average employment shares across industries. In the second row, $RSS_{FE+skill}$ is obtained by the regression with the skill interaction term as an additional control. Given an industry and a region, the skill supply effect explains at most 12% of the variation. Similarly, in the third row, $RSS_{FE+demand}$ is obtained by the regression, here with the income-elasticity interaction term instead of the skill-intensity interaction term, and the income-elasticity interaction term explains 13%-17% of the variation Finally, in the last row, $RSS_{FE+skill+demand}$ is by the regression with the same controls as (17), and the skill and the demand effects jointly capture 19%-27% of the variation. These results suggest that, given an industry and a region, the demand effect can explain a significant portion of the variation in the employment share, even after controlling for the skill supply.

5 Conclusion

Beyond explaining the cities' specialization pattern from the demand-side perspective, my model implies that endogenous tradable sector shares can amplify cross-location income inequality. Because the tradable sector share substantially varies across cities, this implication provides a new reason why we should understand the mechanism that drives the industrial composition. As a related effort, Onoda (2022) examines how a disproportionate trade cost reduction for business services, an income-elastic industry, since 1980 has affected regional income inequality by raising the tradable sector shares in large cities relative to small cities. The insight of the tradable sector share applies not only to spatial economics but also to international trade. The endogenous and heterogeneous share tradable sector and its implication on income inequality deserve further investigation.

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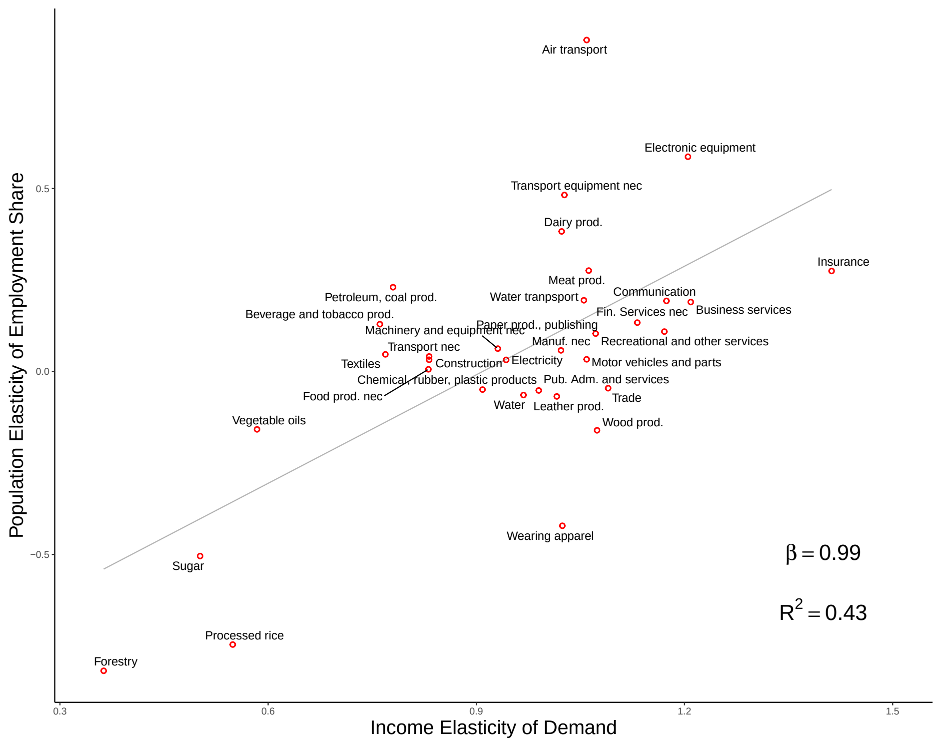
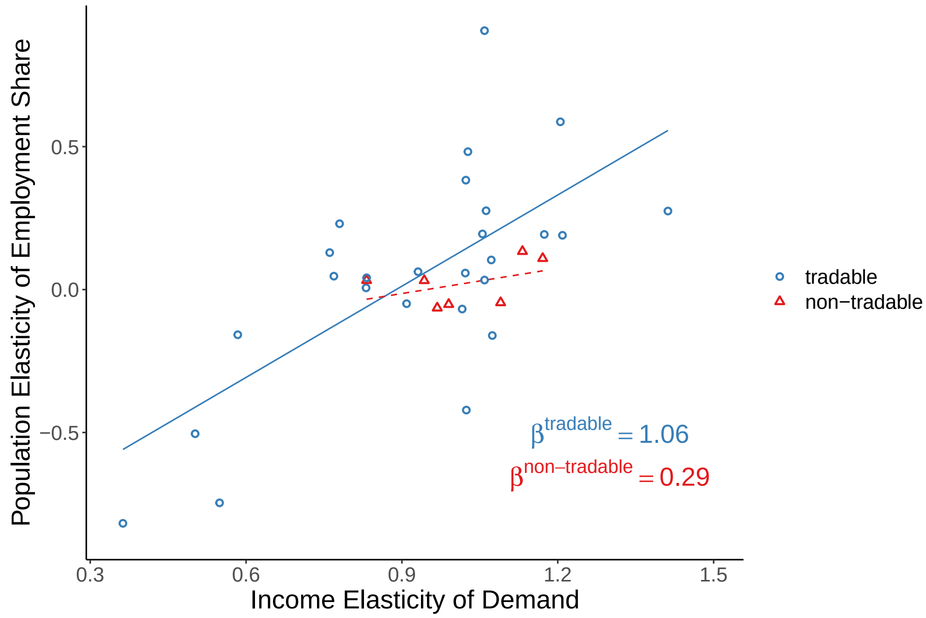
Figures and Tables

Figure 1: Elasticity of Employment Share with respect to MSA's Population and Elasticity of Demand with respect to Income



Notes: MSA population is 2016 data from the Bureau of Economic Analysis. I calculate employment shares by mapping the NAICS-based 2016 County Business Pattern data to GTAP sectors and obtain the population elasticity of employment share controlling for region $\in$ {Northeast, Midwest, South, West}. Income elasticity estimates are from Caron et al. (2020). The upward-sloping line is the unweighted regression line. For details of data construction, see Appendix A.

Figure 2: Elasticity of Employment Share with respect to MSA's Population and Income Elasticity of Demand for Tradable and Non-tradable Industries



Notes: MSA population is 2016 data from the Bureau of Economic Analysis. I calculate employment shares by mapping the NAICS-based 2016 County Business Pattern data to GTAP sectors and obtain the population elasticity of employment share controlling for region $\in$ {Northeast, Midwest, South, West}. Income elasticity estimates are from Caron et al. (2020). The two upward-sloping lines are the unweighted regression line for tradable industries (blue solid) and that for non-tradable industries (red dashed). Non-tradable industries are "Water", "Electricity", "Construction", "Trade" (including wholesale trade, retail sales, hotels, and restaurants), "Recreational and other services", "Public Administration, Defense, Education, Health", and "Financial services nec". For details of data construction, see Appendix A.

Figure 3: Partial Equilibrium Analysis on N_1 and ω

(a) Original State (b) Higher Productivity ($\lambda \uparrow$) (c) Better Amenities ($a_1 \uparrow$)

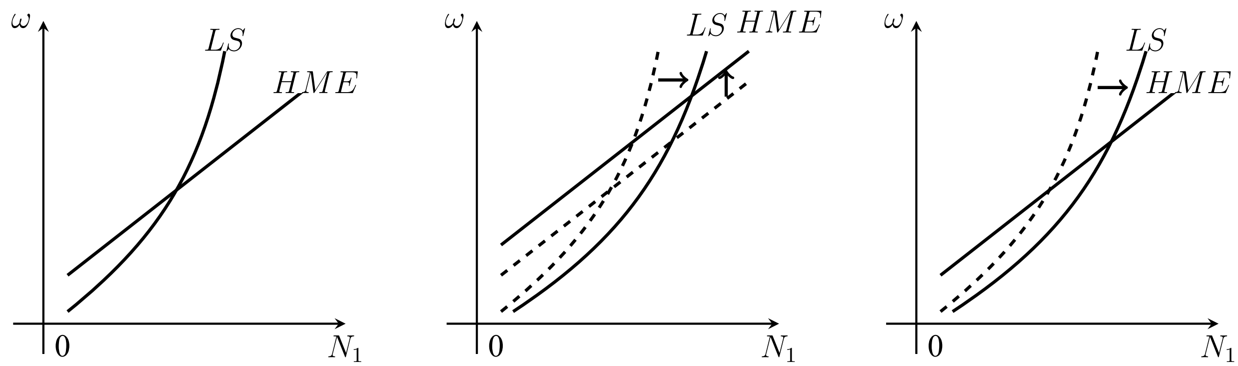


Figure 4: Partial Equilibrium Analysis on ω and $m_{1\mathcal{T}}$ when N_1 increases

(a) Gross Complements ($\eta < 1$) (b) Gross Substitutes ($\eta > 1$)

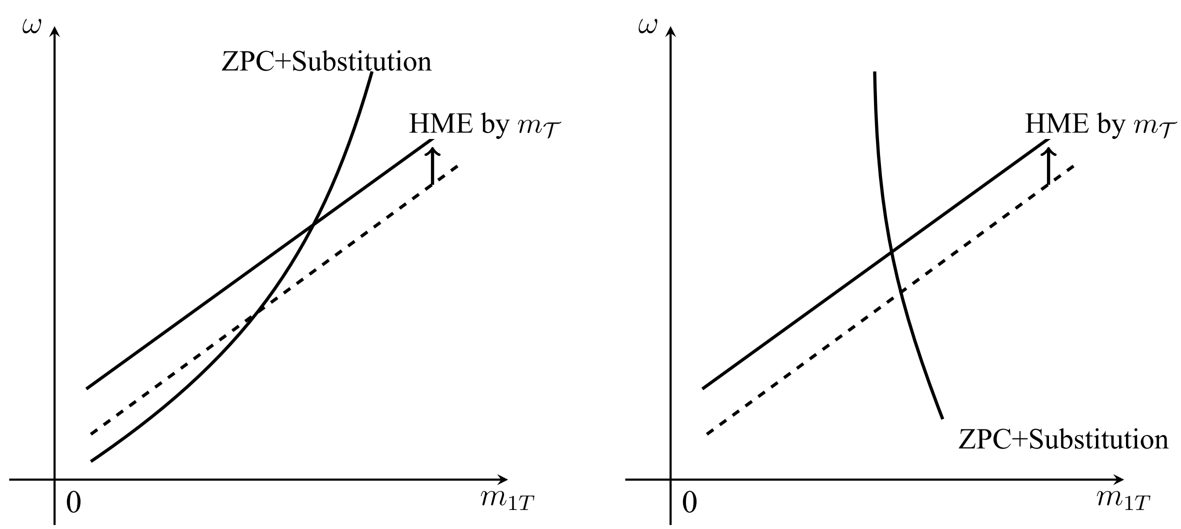


Table 1: Results with Controlling Skill Intensities

	Population elasticity of employment share		
	2006	2011	2016
	(1)	(2)	(3)
Income elasticity	1.176***	1.086***	0.951***
	(0.331)	(0.312)	(0.287)
Skill intensity	−0.165	0.101	0.110
	(0.679)	(0.640)	(0.589)
Observations	33	33	33
R^2	0.423	0.452	0.431
Adjusted R^2	0.385	0.416	0.394
F Statistic (df = 2; 30)	11.007***	12.382***	11.385***

Notes: ***,**, and * denote significance at the 1%, 5%, and 10% levels, respectively. Standard errors in parentheses. This table displays the coefficients of regressing industries' elasticities of employment share with respect to MSA's population on their income elasticities and skill intensities, controlling for industry-region and MSA fixed effects. When obtaining the elasticities of employment share with respect to population, region $\in$ {Northeast, Midwest, South, West} is controlled for. Employment data are from Country Business Pattern data, income elasticities and skill intensities are from Comin et al. (2020), and MSAs' population data are from the Bureau of Economic Analysis.

Table 2: Regression Result of Alternative Specification

	employment (PPML)			log(employment+1)		
	2006	2011	2016	2006	2011	2016
	(1)	(2)	(3)	(4)	(5)	(6)
Income elas.$\times$ $\log(Pop.)$ (β_{demand})	0.30***	0.37***	0.33***	1.05***	1.04***	0.90***
	(0.03)	(0.03)	(0.03)	(0.08)	(0.08)	(0.07)
Skill int.$\times$ $\log(College_m)$ (β_{skill})	2.00***	2.20***	2.23***	1.31***	1.33***	1.68***
	(0.15)	(0.14)	(0.14)	(0.36)	(0.35)	(0.38)
Observations	8,316	8,349	7,920	8,316	8,349	7,920
R^2				0.86	0.86	0.86
Adjusted R^2				0.85	0.85	0.85

Notes: ***,**, and * denote significance at the 1%, 5%, and 10% levels, respectively. Standard errors in parentheses. The three left columns display estimated coefficients for the interaction terms by PPML regressions of employment level. The three right columns display estimated coefficients for interaction terms by regressions of log of (employment +1). All the regressions include industry-region ($\in$ {Northeast, Midwest, South, West}) and MSA fixed effects. Income elasticities and skill intensities are from Comin et al. (2020), MSAs' population data are from BEA, and the college employment ratios are for full-time workers in MSAs from US Census data via IPUMS.

Table 3: Explanatory Power of Demand Effect, Skill Supply, and MSA and Industry-Region FEs in MSAs' Employment Shares

	2006	2011	2016
$1 - RSS_{FE}/TSS$	0.92	0.92	0.92
$1 - RSS_{FE+skill}/RSS_{FE}$	0.10	0.12	0.10
$1 - RSS_{FE+demand}/RSS_{FE}$	0.13	0.17	0.17
$1 - RSS_{FE+skill+demand}/RSS_{FE}$	0.20	0.27	0.23

Notes: This table shows measurements analogous to R^2 of four derivatives of the PPML model (17). A residual used in calculating the residual sum of squares (RSS) is the difference between the employment share for a given MSA-industry pair and a predicted employment share I construct using the fitted employment level by the PPML. RSS_{FE} is the RSS with only the MSA and the industry-region ($\in$ {Northeast, Midwest, South, West}) fixed effects. The models of $RSS_{FE+skill}$ and $RSS_{FE+demand}$ additionally have the skill-supply and the income-elasticity interaction terms, respectively. The model of $RSS_{FE+skill+demand}$ is the PPML model (17) and has both interaction terms. I calculate employment shares by mapping the NAICS-based 2016 County Business Pattern data to GTAP sectors. Income elasticities and skill intensities are from Caron et al. (2020). MSAs' population data are from the Bureau of Economic Analysis.

Appendix

A Data

To create Figure 1, I borrow estimates of income elasticities from Caron et al. (2020). Using 1997 international trade data for 109 countries, these scholars estimate the elasticities for 49 industries. The elasticity varies from 0.137 for "Processed rice" to 1.311 for "Financial services nec". I use datasets from Country Business Patterns (CBP) for employment data. CBP provides employment data of MSAs annually for industries classified according to the North American Industry Classification System (NAICS). The classification that Caron et al. (2020) use is Global Trade Analysis Project (GTAP) sectors, which differs from NAICS. In most cases, one GTAP sector code corresponds to multiple 3-digit or 4-digit NAICS codes. Following Carrico et al. (2012) and mapping NAICS data to GTAP sectors, I create employment data by GTAP sectors. CBP datasets do not contain employment-size information for MSA-NAICS pairs with fewer than three establishments from 2017. CBP provides employment size class data (e.g., "25,000-49,999") for these pairs before 2017, and these pairs account for a significant portion of the MSA-NAICS data I map to MSA-GTAP pairs, accounting for 28% in 2016. Since small employment shares are crucial in detecting specialization patterns, I use employment data up to 2016 throughout the paper and the midpoint of the employment size class for pairs only with employment size class information, including these.

A robustness check in Section 4 uses college employment ratios that I measure by college workers to non-college workers working in a given MSA. I use the US census data via IPUMS on 25-55-years-olds worker whose "Usual hours worked per week " are at least 35 hours and exclude individuals living in group quarters (Ruggles et al. 2023). I classify workers who have completed at least four years of college as college workers, while all other workers as non-college workers.

Table 4 displays the distribution of population and college employment ratio in 2016. The largest MSA in 2016 in this sample is New York-Newark-Jersey City (NY-NJ-PA), which had a population of 19,943,198, while the smallest is Parkersburg-Vienna, WV, which had a population of 91,940. The college employment ratio, which I use in a robustness check, varies substantially across MSAs: 1.19 in San Jose-Sunnyvale-Santa Clara, CA, is the highest and 0.18 in Hanford-Corcoran, CA, is the lowest.

Table 4: Distribution of Population and College Employment Ratio across MSAs in 2016

	Min	Q1	Median	Mean	Q3	Max
Population	91,940	174,538	380,010	989,159	847,835	19,943,198
College employment ratio	0.18	0.32	0.44	0.46	0.549	1.19

Table 5 summarizes the elasticities of employment share with respect to MSA's population that I estimated and the income elasticities and skill intensities that I borrow from Caron et al. (2020).

Table 5: Estimates for Industries

GTAP code	Industry	Income elasticity	Skill intensity	Population elasticity of employment share	
				Estimate	S.E.
atp	Air transport	1.06	0.30	0.91	0.09
ele	Electronic equipment	1.21	0.38	0.59	0.11
otn	Transport equipment nec	1.03	0.34	0.49	0.12
mil	Dairy prod.	1.02	0.24	0.38	0.13
omt+cmt	Meat prod.	1.06	0.22	0.28	0.12
isr	Insurance	1.41	0.52	0.27	0.04
p_c	Petroleum, coal prod.	0.78	0.35	0.23	0.11
cmn	Communication	1.17	0.50	0.19	0.03
obs	Business services	1.21	0.49	0.19	0.01
wtp	Water transport	1.06	0.32	0.19	0.11
b_t	Beverage and tobacco prod.	0.76	0.28	0.14	0.08
ofi	Fin. Services nec	1.13	0.53	0.13	0.02
ros	Recreational and other services	1.17	0.48	0.11	0.02
ppp	Paper prod., publishing	1.07	0.34	0.10	0.03
ome	Machinery and equipment nec	0.93	0.37	0.06	0.06
omf	Manuf. nec	1.02	0.27	0.06	0.05
tex	Textiles	0.77	0.23	0.05	0.07
otp	Transport nec	0.83	0.29	0.04	0.03
ely	Electricity	0.94	0.37	0.03	0.06
cns	Construction	0.83	0.30	0.03	0.02
mvh	Motor vehicles and parts	1.06	0.34	0.03	0.11
ofd	Food prod. nec	0.83	0.26	0.02	0.06
trd	Trade	1.09	0.30	-0.05	0.01
crp	Chemical, rubber, plastic products	0.91	0.36	-0.05	0.06
osg	Pub. Adm. and services	0.99	0.50	-0.05	0.01
lea	Leather prod.	1.02	0.20	-0.07	0.10
wtr	Water	0.97	0.37	-0.07	0.08
vol	Vegetable oils	0.58	0.22	-0.15	0.11
lum	Wood prod.	1.07	0.25	-0.17	0.07
wap	Wearing apparel	1.02	0.23	-0.42	0.08
sgr	Sugar	0.50	0.20	-0.51	0.08
pcr	Processed rice	0.55	0.12	-0.74	0.06
frs	Forestry	0.36	0.14	-0.82	0.10

B Derivation of Demand Function and Indirect Utility

Derivation of Demand Function

The household problem is given by

$$\begin{aligned}
\max_{U_i,H_i,C_i,\left\{Q_{i,k}\right\}_{k\in\mathcal{K}},\left\{q_{i,k}(\nu)\right\}_{\nu\in\Omega_{i,k},(i,k)\in(1,2)\times\mathcal{K}}} \quad & U_i + \lambda\left(U_i - H_i^{\alpha}C_i^{1-\alpha}\right) \\
& +\pi\left(C_i - \left[\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{i,k}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\right) \\
& +\sum_{k\in\mathcal{K}}\xi_k\left(Q_{i,k} - \left[\int_{\Omega_{i,k}} q_{i,k}(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}}\right) \\
& +\phi\left(E_i - \sum_{k\in\mathcal{K}}\int_{\Omega_{i,k}} p_{i,k}(\nu)q_{i,k}(\nu)d\nu - P_i(H)H_i\right)
\end{aligned}$$

where λ, π, $\{\xi_k\}_{k\in\mathcal{K}}$, and ϕ are the lagrange multipliers. The FOCs are:

$$\begin{aligned}
U_i : 1 + \lambda - \mu\frac{\eta}{\eta-1}\left(\sum_{k\in\mathcal{K}}\beta_k^{\frac{1}{\eta}}U_i^{\frac{\epsilon_k}{\eta}}Q_{i,k}\right)^{\frac{\eta}{\eta-1}-1}\left(\sum_{k\in\mathcal{K}}\epsilon_k\frac{1}{\eta}\beta_k^{\frac{1}{\eta}}U_i^{\frac{\epsilon_k}{\eta}}U_i^{-\frac{1}{\eta}}Q_{i,k}\right) &= 0 \\
C_i : \lambda(1-\alpha)H^{\alpha}C_i^{-\alpha} - \pi &= 0 \\
Q_{i,k} : \pi\left(\sum_{k\in\mathcal{K}}\beta_k^{\frac{1}{\eta}}U_i^{\frac{\epsilon_k}{\eta}}Q_{i,k}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1}\left(\beta_k^{\frac{1}{\eta}}U_i^{\frac{\epsilon_k}{\eta}}Q_{i,k}^{\frac{\eta-1}{\eta}-1}\right) - \xi_k &= 0 \\
q_{i,k}(\nu) : \xi_k\left[\int_{\Omega_{i,k}} q_{i,k}(\nu)^{\frac{\sigma-1}{\sigma}}d\mu\right]^{\frac{\sigma}{\sigma-1}-1} q_{i,k}(\nu)^{\frac{\sigma-1}{\sigma}-1} - \phi p_{i,k}(\nu) &= 0 \\
H_i : \lambda\alpha H_i^{\alpha-1}C_i^{1-\alpha} - \phi P_{i,H} &= 0
\end{aligned}$$

First, derive the usual result with CES from the FOC w.r.t. $q_{i,k}(\nu)$ and the definition of $Q_{i,k}$.

$$P_{i,k}Q_{i,k}^{\frac{1}{\sigma}} = p_{i,k}(\nu)q_{i,k}(\nu)^{\frac{1}{\sigma}}$$

where $P_{i,k} = \left[\int_{\Omega_{i,k}(j)} \left(p_{i,k}(\nu)\right)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$It follows from this result, the FOC w.r.t $Q_{i,k}$, and that w.r.t $q_{i,k}(\nu)$,

$$\pi C_i \frac{\left(\beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{i,k}^{\frac{\eta-1}{\eta}}\right)}{\left(\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{i,k}^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}}} = \phi P_{i,k} Q_{i,k} \implies \frac{\beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{i,k}^{\frac{\eta-1}{\eta}}}{\beta_l^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{i,\ell}^{\frac{\eta-1}{\eta}}} = \frac{P_{i,k} Q_{i,k}}{P_{i,l} Q_{i,\ell}} \tag{18}$$

Next, I obtain the expenditure on good consumption by using eq. (18).

$$\begin{aligned}
E_{iC} \equiv \sum_{k\in\mathcal{K}} \int_{\Omega_{ik}} p_{ik}(\nu) q_{ik}(\nu) d\nu &= \sum_{k\in\mathcal{K}} P_{ik} Q_{ik} \\
&= \sum_{k\in\mathcal{K}} P_{i\ell} Q_{i\ell} \frac{\beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{ik}^{\frac{\eta-1}{\eta}}}{\beta_l^{\frac{1}{\eta}} U_i^{\frac{\epsilon_l}{\eta}} Q_{i\ell}^{\frac{\eta-1}{\eta}}} \\
&= \frac{P_{i\ell} Q_{i\ell}}{\beta_l^{\frac{1}{\eta}} U_i^{\frac{\epsilon_l}{\eta}} Q_{i\ell}^{\frac{\eta-1}{\eta}}} \sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k(1-\eta)}{\eta}} Q_{ik}^{\frac{\eta-1}{\eta}} \\
&= \frac{P_{i\ell} Q_{i\ell}}{\beta_l^{\frac{1}{\eta}} U_i^{\frac{\epsilon_l}{\eta}} Q_{i\ell}^{\frac{\eta-1}{\eta}}} C_i^{\frac{\eta-1}{\eta}}
\end{aligned}$$

The demand function immediately follows as:

$$Q_{i\ell} = \beta_\ell U_i^{\epsilon_k} P_{i\ell}^{-\eta} C_i^{1-\eta} E_{iC}^{\eta} \tag{19}$$

The expenditure on goods consumption follows from multiplying eq. (19) by $P_{i\ell}$ and aggregating it across industries.

$$E_{iC} = \sum_{k\in\mathcal{K}} \beta_\ell U_i^{\epsilon_k} P_{i\ell}^{1-\eta} C_i^{1-\eta} E_{iC}^{\eta}$$

Then,

$$m_{ik} = \frac{\beta_k U_i^{\epsilon_k} P_{ik}^{1-\eta}}{\sum_{\ell\in\mathcal{K}} \beta_\ell U_i^{\epsilon_k} P_{i\ell}^{1-\eta}}$$

Derivation of Price Index for Goods Consumption

Substitute eq. 19 for $Q_{i,\ell}$ in the definition of C_i.

$$
\begin{aligned}
C_i &= \left(\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} \left[\beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} P_{ik}^{-1} C_i^{\frac{1-\eta}{\eta}} E_{iC} \right]^{\eta-1} \right)^{\frac{\eta}{\eta-1}} \\
\iff E_{iC} &= \left(\sum_{k\in\mathcal{K}} \beta_k U_i^{\epsilon_k} P_{ik}^{1-\eta} \right)^{\frac{1}{1-\eta}} C_i \\
&= P_{iC} C_i
\end{aligned}
$$

where $P_{i,C} \equiv \left(\sum_{k\in\mathcal{K}} \beta_k U_c^{\epsilon_k} P_{c,k}^{1-\eta} \right)^{1/(1-\eta)}$, which is the price index for goods consumption.

Derivation of Indirect Real Consumption

It follows from the FOC w.r.t. $Q_{i,k}$ and $P_{i,k} Q_{i,k}^{\frac{1}{\sigma}} = p_{i,k}(\nu) q_{i,k}(\nu)^{\frac{1}{\sigma}}$ that

$$
\xi_k P_{ck}^{-1} = \phi
$$

The FOC w.r.t $q_{i,k}(\nu)$ can be transformed with this equation.

$$
\begin{aligned}
\xi_k =& \pi C_i^{\frac{1}{\eta}} \left(\beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{ik}^{\frac{\eta-1}{\eta}-1} \right) \\
\implies -\phi \sum_{k\in\mathcal{K}} P_{ik} Q_{ik} =& \pi C_i^{\frac{1}{\eta}} \sum_{k\in\mathcal{K}} \left(\beta_k^{\frac{1}{\eta}} U_i^{\frac{\epsilon_k}{\eta}} Q_{ik}^{\frac{\eta-1}{\eta}} \right) \\
\iff -\phi E_{iC} =& \pi C_i
\end{aligned}
$$

Then, I obtain the usual Cobb-Douglas result by combining this with the FOCs w.r.t. C_i and H_i.

$$
\frac{\alpha}{1-\alpha} = \frac{P_{iH} H_i}{P_{iC} C_i}
$$

I can implicitly express the indirect real consumption by plugging this into the definition of U_i,

$$
U_i = \alpha^{\alpha} (1-\alpha)^{1-\alpha} \frac{E_i}{P_{iC}^{1-\alpha} P_{iH}^{\alpha}} = \alpha^{\alpha} (1-\alpha)^{1-\alpha} \frac{E_i}{P_{iH}^{\alpha}} \left(\sum_{k\in\mathcal{K}} \beta_k U_i^{\epsilon_k} P_{ik}^{1-\eta} \right)^{\frac{\alpha-1}{1-\eta}}
$$

Derivation of Price Index for Good Consumption as Function of E_i, L_i, N_i, and U_i

It can be obtained from the real good consumption with the land clearing condition.

$$U_i = (1-\alpha)^{1-\alpha} \left(\frac{L_i}{N_i}\right)^{\alpha} \left(\frac{E_i}{P_{iC}}\right)^{1-\alpha}$$
$$\implies P_{iC} = (1-\alpha) E_i \left(\frac{L_i}{N_i}\right)^{\frac{\alpha}{1-\alpha}} U_i^{-\frac{1}{1-\alpha}} \tag{20}$$

C Uniqueness of Equilibrium

As I explain in the main text, the uniqueness of equilibrium is unnecessary in obtaining cross-city patterns consistent with empirical facts. That said, additional assumptions provide the uniqueness. I start with the case of gross complements. Proposition 10 requires an additional assumption.

Assumption 3. $(1-\alpha)/(\sigma-1) > \alpha$

Proposition 10 (Uniqueness of equilibrium with gross complements)**.** *Given Assumptions 1 and 3, if the industries are gross complements ($\eta < 1$), then there exists a unique equilibrium*

Proof. See Appendix D. □

Proposition 10 concerns only gross complements, and this is related to the endogenous tradable sector shares. When a city attracts new workers, the mass of locally produced varieties increases both in the non-tradable and tradable industries. However, the effects on the price index are different. The mass of varieties produced in and shipped from the other city in the tradable industries decreases. Also, the share of local varieties in the price index is smaller than 1. For these reasons, the price index reduction is relatively minor in the tradable industries. When the industries are gross complements, this relatively minor reduction of the price index increases the tradable sector share in the city that attracts new workers. This force is captured by the first term of the RHS in eq. (8) and (9). A larger population N_1, holding the expenditure shares constant, increases the relative wage ω by the home-market effect (eq. (8)), increasing the first term of eq. 9 only for the tradable industries.

The increasing tradable sector shares work as a dispersion force. When residents spend more on the tradable industries, workers move from the non-tradable industries to the tradable industries as Corollary 1. This relocation reduces the price index of the city's tradable industries by producing additional varieties, whereas it raises that of the non-tradable industries by losing varieties. However, there is another effect. The new varieties in the tradable industries become available in the other city, reducing the tradable industries' price index without costing locally-produced varieties in the non-tradable industries. By this mechanism, the increase of the tradable sector shares decreases the relative real consumption of the city (U_i/U_j). This additional dispersion force only strengthens the aggregate dispersion force that is already stronger than the agglomeration force by Assumption 1. Thus, the uniqueness can be obtained with a relatively weak additional assumption. Assumption 3 ensures that the agglomeration force is stronger than the dispersion force by inelastic land supply, and it is a sufficient condition to obtain a relative wage that increases in population, which provides tractability.

When the industries are gross substitutes, tradable sector shares work as an agglomeration force. Given $\eta > 1$, expenditures and employment relocate to non-tradable industries when a city attracts workers because they become relatively inexpensive. This relocation generates a force that raises the relative real consumption. With heterogeneous income elasticities, it is infeasible to prove that this force does not reverse the relationship, globally, between the agglomeration force and the dispersion force in Assumption 1. Hence, I impose Assumption 4 to obtain the uniqueness.

Assumption 4. $|\mathcal{N}| = |\mathcal{T}|$ *, and* $\forall k \in \mathcal{T}$*,* $\exists \ell \in \mathcal{N}$ *such that* $\epsilon_l = \epsilon_k$*,* $\beta_\ell = \kappa_1 \beta_k$*,* $\phi_\ell = \kappa_2 \phi_k$*, and* $\psi_\ell = \kappa_3 \psi_k$ *where* $0 \leq \kappa_1$ *and* $0 < \kappa_2, \kappa_3$

Proposition 11 (Uniqueness of equilibrium with symmetric industries)**.** *Given Assumptions 1, and 4, there exists a unique equilibrium.*

Proof. See Appendix D. □

In Proposition 11, κ_1 can be 0, corresponding to no non-tradable sector. Also, there is no condition on η in Assumption 4. In other words, industries can be either gross substitutes or gross complements. Finally, Proposition 11 does not use Assumption 3. This result can be obtained because the tradable sector share becomes tractable with Assumption 4.

D Proof of Propositions and Lemma

Proof of Proposition 1

I begin by substituting the optimized production into the zero-profit condition. As for the optimized production, the optimized prices are as follows.

$$\forall k \in \mathcal{K},\ p_{ij,k} = p_{ik} = \frac{\sigma}{\sigma-1}\frac{\psi_k}{\lambda_i}w_i$$
$$\forall k \in \mathcal{T},\ p_{ij,k} = \tau p_{ik} = \frac{\sigma}{\sigma-1}\tau\frac{\psi_k}{\lambda_i}w_i$$

Then, the zero-profit condition implies $\pi_{i,k} = 0$ for all k in $\mathcal{K}$ and $i \in \{1,2\}$. It follows for all (i,j) in $\{(1,2),(2,1)\}$,

$$\forall k \in \mathcal{N},\ q_{ii,k}\frac{\psi_k}{\lambda_i}w_i\left[\frac{1}{\sigma-1}\right] - \frac{\phi_k}{\lambda_i}w_i = 0$$
$$\forall k \in \mathcal{T},\ \left(q_{ii,k} + q_{ij,k}\tau\right)\frac{\psi_k}{\lambda_i}w_i\left[\frac{1}{\sigma-1}\right] - \frac{\phi_k}{\lambda_i}w_i = 0$$

The total labor demand by a firm producing variety ν in industry k in city i, $N_{ik}(\nu)$, is pinned down as

$$N_{ik}(\nu) = \begin{cases} q_{ii,k}\frac{\psi_k}{\lambda_i} + \frac{\phi_k}{\lambda_i} = \sigma\frac{\phi_k}{\lambda_i} & k \in \mathcal{N} \\ \left(q_{ii,k} + q_{ij,k}\tau\right)\frac{\psi_k}{\lambda_i} + \frac{\phi_k}{\lambda_i} = \sigma\frac{\phi_k}{\lambda_i} & k \in \mathcal{T} \end{cases} \tag{21}$$

The fixed cost and the productivity levels determine the labor demand. Now, I use normalization. It can be shown that β_k, ϕ_k, and ψ_k affect the equilibrium values of $N_1, N_2, w_1, w_2, C_1, C_2,\ H_1, H_2, U_1$, and U_2 only through $\beta_k^{\frac{1}{1-\eta}}\phi_k^{\frac{1}{\sigma-1}}\psi_k$. Therefore, given any set of parameters, replacing $\{\beta_k, \phi_k, \psi_k\}_{k\in\mathcal{K}}$ by $\left\{\tilde{\beta}_k, 1/\sigma, (\sigma-1)/\sigma\right\}_{k\in\mathcal{K}}$ where $\tilde{\beta}_k = \left[\beta_k^{\frac{1}{1-\eta}}\phi_k^{\frac{1}{\sigma-1}}\psi_k / \left\{(1/\sigma)^{\frac{1}{\sigma-1}}(\sigma-1)/\sigma\right\}\right]^{1-\eta}$ does not affect the equilibrium values of those variables. One caveat is that the price index is affected by this change. Let $\tilde{P}_k$ be the new price index given $\left\{\tilde{\beta}_k, 1/\sigma, (\sigma-1)/\sigma\right\}_{k\in\mathcal{K}}$. Then, $P_k = (1/\sigma)^{\frac{1}{\sigma-1}}\{\sigma/(\sigma-1)\}\tilde{P}_k$. Following Matsuyama (2019), I set $\psi_k = (\sigma-1)/\sigma$ and $\phi_k = 1/\sigma$ so that $p_{ik} = w_i/\lambda_i$ for all $k \in \mathcal{K}$ and $N_{i,k}(\nu) = 1/\lambda_i$, which requires that β_k is replaced by $\tilde{\beta}_k = \left[\beta_k^{\frac{1}{1-\eta}}\phi_k^{\frac{1}{\sigma-1}}\psi_k / \left\{(1/\sigma)^{\frac{1}{\sigma-1}}(\sigma-1)/\sigma\right\}\right]^{1-\eta}$. Then, it follows from eq. (21) and the normalization that the aggregate supply of goods by a firm is given by

$$\forall k \in \mathcal{N},\, q_{ii,k} = 1$$
$$\forall k \in \mathcal{T},\, q_{ij,k} + q_{ij,k}\tau = 1$$

Next, to equate demand to supply, I derive the aggregate demand for a variety in industry k in city i. I let D_{ck} denote the aggregate demand, and it is as follows:

$$D_{ik} = p_k^{-\sigma} A_{ik} \quad \text{where } A_{ik} = N_i \tilde{P}_{ik}^{\sigma} Q_{ik} + \rho_k N_j \tilde{P}_{jk}^{\sigma} Q_{jk}, \text{ and } \rho_k = \begin{cases} 0 & k \in \mathcal{N} \\ \rho = \tau^{1-\sigma} & k \in \mathcal{T} \end{cases}$$

Equating demand (D_{ik}) and supply ($q_{ii,k}$ for $k \in \mathcal{N}$ and $q_{ii,k} + q_{ij,k}\tau$ for $k \in \mathcal{T}$) with $p_{ik} = w_i/\lambda_i$ gives, $\forall i \in \{1,2\}, \forall k \in \mathcal{K}$

$$1 = \left(\frac{w_i}{\lambda_i}\right)^{-\sigma} A_{ik} \tag{22}$$

This (22) equates supply to demand, and it reflects the zero-profit condition. Given an industry (k) and a productivity level (λ_i), the city with greater aggregate demand A_{ik} has a higher wage. Also, given a city (i), this (22) equalizes the aggregate demands across industries. To make use of eq. (22), I use two different expressions of the demand function to substitute for the Q_{ik} contained in A_{ik}.

$$Q_{ik} = \begin{cases} \tilde{P}_{ik}^{-1} E_{iC} m_{ik} \\ \tilde{\beta}_k \tilde{P}_{ik}^{-\eta} U_i^{\epsilon_k} C_i^{1-\eta} E_{iC}^{\eta} \end{cases} \tag{23}$$

The first follows from eq. (3) and the second from eq. (2). With the first expression, eq. (22) becomes

$$\left(\frac{w_i}{\lambda_i}\right)^{-\sigma} = N_i \tilde{P}_{ik}^{\sigma-1} E_{iC} m_{ik} + \rho_k N_j \tilde{P}_{jk}^{\sigma-1} E_{iC} m_{jk} \tag{24}$$

Proposition 1 follows from eq. (24) for a non-tradable industry k

$$\left(\frac{w_i}{\lambda_i}\right)^{\sigma} = N_i \tilde{P}_{ik}^{-1} E_{iC} m_{ik} = \frac{N_i w_i m_{ik}}{\lambda_i^{\sigma} x_{ik} N_i w_i^{1-\sigma}}$$
$$\implies x_{1k} = m_{1k} \tag{25}$$

where I use $E_{iC} = w_i$ and the price index $\tilde{P}_{ik}^{1-\sigma} = \lambda_i^{\sigma} x_{ik} N_i w_i^{1-\sigma}$ for $k \in \mathcal{N}$ where x_{ik} is the employment share in industry k in city i.

Proof of Corollary 1

This corollary follows from eq. (25) as follows:

$$\sum_{k\in\mathcal{N}} x_{ik} = \sum_{k\in\mathcal{N}} m_{ik} \iff 1 - \sum_{k\in\mathcal{T}} x_{ik} = 1 - \sum_{k\in\mathcal{T}} m_{ik} \iff \sum_{k\in\mathcal{T}} x_{ik} = \sum_{k\in\mathcal{T}} m_{ik} \tag{26}$$

Proof of Proposition 2

It follows from zero-profit conditions for a tradable industry k (eq. (24)) for the two cities that

$$\frac{(w_i/\lambda_i)^{\sigma} - \rho\,(w_j/\lambda_j)^{\sigma}}{1-\rho^2} = \tilde{P}_{ik}^{\sigma-1} N_i E_{iC} m_{ik} \tag{27}$$
$$= \frac{N_i w_i m_{ik}}{\lambda_i^{\sigma} x_{ik} N_i w_i^{1-\sigma} + \rho \lambda_j^{\sigma} x_{jk} N_j w_j^{1-\sigma}}$$

where I use the price index $\tilde{P}_{ik}^{1-\sigma} = \lambda_i^{\sigma} x_{ik} N_i w_i^{1-\sigma} + \rho \lambda_j^{\sigma} x_{jk} N_j w_j^{1-\sigma}$, $E_{iC} = (1-\alpha)E_i = w_i$, and x_{ik} is the employment share in industry k in city i. It follows that

$$\lambda_i^{\sigma} x_{ik} N_i w_i^{1-\sigma} + \rho \lambda_j^{\sigma} x_{jk} N_j w_j^{1-\sigma} = (1-\rho^2)\frac{N_i w_i m_{ik}}{(w_i/\lambda_i)^{\sigma} - \rho\,(w_j/\lambda_j)^{\sigma}} \tag{28}$$

For city 1, aggregate over tradable industries and use the income (wage) ratio $\omega = E_1/E_2 = w_1/w_2$,

$$\sum_{k\in\mathcal{T}} x_{1k} \lambda^{\sigma} N_1 \omega^{1-\sigma} + \rho \sum_{k\in\mathcal{T}} x_{2k} N_2 = (1-\rho^2)\frac{N_1 \omega \sum_{k\in\mathcal{T}} m_{1k}}{\lambda^{-\sigma}\omega^{\sigma} - \rho}$$

By transforming this equation, making use of eq. (26), the condition that characterizes the home-market effect on wage is obtained.

$$\frac{\sum_{k\in\mathcal{T}} m_{1k}N_1}{\sum_{k\in\mathcal{T}} m_{2k}N_2} = \omega^{2\sigma-1}\left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{\lambda^{\sigma}-\rho\omega^{\sigma}}\right]$$

Proof of Lemma 1

I use the second form of the demand function (23). eq. (22) becomes

$$\left(\frac{w_i}{\lambda_i}\right)^{\sigma} = N_i\tilde{\beta}_k\tilde{P}_{ik}^{\sigma-\eta}U_i^{\epsilon_k}C_i^{1-\eta}E_{iC}^{\eta} + N_j\rho_k\tilde{\beta}_k\tilde{P}_{jk}^{\sigma-\eta}U_j^{\epsilon_k}C_j^{1-\eta}E_{jC}^{\eta} \tag{29}$$

It follows from eq. 29 of industry $k \in \mathcal{K}$ for the two cities (eq. (29)),

$$\frac{(w_i/\lambda_i)^{\sigma} - \rho_k\,(w_j/\lambda_j)^{\sigma}}{1-\rho_k^2} = N_i\tilde{\beta}_k\tilde{P}_{ik}^{\sigma-\eta}U_i^{\epsilon_k}C_i^{1-\eta}E_{iC}^{\eta} \tag{30}$$

I eliminate $\tilde{P}_{ik}$ and C_i. First, from eq. (3) and (20),

$$\begin{aligned}
\tilde{P}_{ik}^{1-\eta} &= \frac{m_{ik}\sum_{\ell\in\mathcal{K}}\beta_l U_i^{\epsilon_k}\tilde{P}_{i\ell}^{-\eta}}{\beta_k U_i^{\epsilon_k}} \\
&= \frac{m_{ik}\left[(1-\alpha)E_i\,(L_i/N_i)^{\frac{\alpha}{1-\alpha}}\,U_i^{-\frac{1}{1-\alpha}}\right]^{1-\eta}}{\beta_k U_c^{\epsilon_k}} \\
\iff \tilde{P}_{ik} &= \left(\frac{m_{ik}}{\beta_k U_i^{\epsilon_k}}\right)^{\frac{1}{1-\eta}}(1-\alpha)E_i\left(\frac{L_i}{N_i}\right)^{\frac{\alpha}{1-\alpha}}U_i^{-\frac{1}{1-\alpha}}
\end{aligned}$$

Next, I derive $\tilde{P}_{iC}$ by aggregating $\tilde{P}_{ik}$ to obtain C_i.

$$
\begin{aligned}
\tilde{P}_{iC} &= \left(\sum_{k\in\mathcal{K}} \beta_k U_i^{\epsilon_k} \tilde{P}_{ik}^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= \left(\sum_{k\in\mathcal{K}} \beta_k U_i^{\epsilon_k} \left(\frac{m_{i,k}}{\beta_k U_i^{\epsilon_k}}\right) \left[(1-\alpha)E_i \left(\frac{L_i}{N_i}\right)^{\frac{\alpha}{1-\alpha}} U_i^{-\frac{1}{1-\alpha}}\right]^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
&= (1-\alpha)E_i \left(\frac{L_i}{N_i}\right)^{\frac{\alpha}{1-\alpha}} U_i^{-\frac{1}{1-\alpha}} \\
\iff C_i &= U_i^{\frac{1}{1-\alpha}} \left(\frac{L_i}{N_i}\right)^{-\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

By plugging these into eq. (30),

$$
\begin{aligned}
\frac{(w_i/\lambda_i)^\sigma - \rho_k (w_j/\lambda_j)^\sigma}{1-\rho_k^2} &= N_i \left(\tilde{\beta}_k U_i^{\epsilon_k}\right)^{\frac{1-\sigma}{1-\eta}} m_{ik}^{\frac{\sigma-\eta}{1-\eta}} \left[U_i^{\frac{1}{1-\alpha}} \left(\frac{L_i}{N_i}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{1-\sigma} w_i^\sigma \\
\iff m_{ik} &= \left[\frac{\lambda_i^{-\sigma} - \rho_k \lambda_j^{-\sigma} (w_j/w_i)^\sigma}{(1-\rho_k^2)N_i}\right]^{\frac{1-\eta}{\sigma-\eta}} \left(\tilde{\beta}_k U_i^{\epsilon_k}\right)^{\frac{\sigma-1}{\sigma-\eta}} \left[U_i^{\frac{1}{1-\alpha}} \left(\frac{N_i}{L_i}\right)^{\frac{\alpha}{1-\alpha}}\right]^{(\sigma-1)\frac{1-\eta}{\sigma-\eta}}
\end{aligned}
$$

Proof of Proposition 9

For a tradable industry in city 1, it follows from (28) that

$$
x_{1k}\lambda^\sigma N_1 + \rho x_{2k} N_2 \omega^{\sigma-1} = (1-\rho^2)\frac{N_1 m_{1k}}{\lambda^{-\sigma} - \rho\omega^{-\sigma}} \tag{31}
$$

I obtain the counterpart with m_{2k} as

$$
\rho x_{1k}\lambda^\sigma N_1 + x_{2k} N_2 \omega^{\sigma-1} = (1-\rho^2)\frac{N_2 m_{2k}}{\omega^{-\sigma} - \rho\lambda^{-\sigma}}\omega^{-1} \tag{32}
$$

Solve (31) and (32) for x_{1k} and x_{2k} using 8.

$$\frac{x_{1k}}{\sum_{k\in\mathcal{T}} x_{1k}} = \frac{\left(m_{1k}/\sum_{k\in\mathcal{T}} m_{1k}\right) - \rho\lambda^{\sigma}\omega^{-\sigma}\left(m_{2k}/\sum_{k\in\mathcal{T}} m_{2k}\right)}{1-\rho\lambda^{\sigma}\omega^{-\sigma}}$$
$$\frac{x_{2k}}{\sum_{k\in\mathcal{T}} x_{2k}} = \frac{\left(m_{2k}/\sum_{k\in\mathcal{T}} m_{2k}\right) - \rho\omega^{\sigma}\lambda^{-\sigma}\left(m_{1k}/\sum_{k\in\mathcal{T}} m_{1k}\right)}{1-\rho\omega^{\sigma}\lambda^{-\sigma}}$$

The employment share ratio immediately follows from this.

Proof of Proposition 4 and Proposition 5

I introduce new variables, V_1 and V_2, defined as

$$V_1 = a_1 U_1 N_1^{-\gamma}, \text{ and } V_2 = a_2 U_2 N_2^{-\gamma}$$

Using them, the equilibrium conditions can be rewritten as follows:

$$V_1 = V_2$$
$$N = N_1 + N_2$$

$$N_1^{\left(\frac{1}{1-\sigma}-\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} = \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{N}} \tilde{\beta}_k^{\mu}\left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} + \left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu}\left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \tag{33}$$

$$N_2^{\left(\frac{1}{1-\sigma}-\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} = \sum_{k\in\mathcal{N}} \tilde{\beta}_k^{\mu}\left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} + \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu}\left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \tag{34}$$

$$\left(\frac{N_2}{N_1}\right)^{\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} = \frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu}\left(V_1 N_1^{\gamma}/a_1\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu}\left(V_2 N_2^{\gamma}/a_2\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}} \lambda^{\sigma}\left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\mu}\omega^{1-2\sigma} \tag{35}$$

I prove the existence of an equilibrium by showing that V_1 and V_2 can be expressed as continuous functions of N_1 and that they have an intersection by the intermediate value theorem.

(i) V_1 and V_2 can be expressed as functions of N_1 First, given the assumption for global monotonicity of C_i, $1 + (1-\alpha)\min_{k\in\mathcal{K}}\{\epsilon_k/(1-\eta)\} > 0$, it follows from eq. (33) and (34) that

$$\frac{\partial V_1(N_1,\omega)}{\partial\omega} < 0, \frac{\partial V_2(N_2,\omega)}{\partial\omega} > 0$$

Then, notice that, given N_1, N_2, the RHS of eq. (35) decreases in ω, taking into account $\partial V_1(\omega,N_1)/\partial\omega$ and $\partial V_2(\omega,N_2)/\partial\omega$. Also, given N_1, N_2,

$$\lim_{\omega\to\rho^{\frac{1}{\sigma}}\lambda} \frac{\sum_{k\in\mathcal{T}}\tilde{\beta}_k^{\mu}\left(V_1N_1^{\gamma}/a_1\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}}\tilde{\beta}_k^{\mu}\left(V_2N_2^{\gamma}/a_2\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}\lambda^{\sigma}\left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\mu}\omega^{1-2\sigma}\to+\infty \tag{36}$$

$$\lim_{\omega\to\rho^{-\frac{1}{\sigma}}\lambda} \frac{\sum_{k\in\mathcal{T}}\tilde{\beta}_k^{\mu}\left(V_1N_1^{\gamma}/a_1\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}}\tilde{\beta}_k^{\mu}\left(V_2N_2^{\gamma}/a_2\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}\lambda^{\sigma}\left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\mu}\omega^{1-2\sigma}\to 0 \tag{37}$$

Therefore, by the intermediate value theorem, given (N_1, N_2), ω_1 is unique; consequently, V_1 and V_2 are also unique from eq. (33) and (34), respectively. With the worker clearing condition, V_1 and V_2 are functions of N_1.

(ii) V_1 and V_2 are continuous in N_1 Given $N_1 \in (0, N)$ (and therefore N_2), eq. (33) and (34) imply that $V_1N_1^{\gamma}$ and $V_2N_2^{\gamma}$ are continuous in ω on $(\rho^{\frac{1}{\sigma}}\lambda, \rho^{-\frac{1}{\sigma}})$, decrease and increase in ω, respectively, and are nonzero. Then, given $N_1 \in (0, N)$, the RHS of eq. (35) is a continuous decreasing function of ω. Combined with eq. (36) and (37), it follows that for all $N_1 \in (0, N)$ there exists $\omega \in (\rho^{\frac{1}{\sigma}}\lambda, \rho^{-\frac{1}{\sigma}}\lambda)$. Also, $\lim_{N_1\to x+}\omega = \lim_{N_1\to x-}\omega$ for all $x \in (0, N)$. This result implies that ω is continuous on $N_1 \in (0, N)$. It immediately follows from eq. (33) and (34) that V_1 and V_2 are continuous in $N_1 \in (0, N)$.

(iii) $V_1 > V_2$ when $N_1 \to 0$ and $V_1 < V_2$ when $N_1 \to N$ and intersection exits Transform eq. (33) and (34),

$$
\begin{aligned}
1 = &\sum_{k\in\mathcal{N}} \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\mu} \left(\frac{V_1}{a_1}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_1^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\frac{\alpha}{1-\alpha}-\frac{1}{1-\sigma}\right](1-\eta)\mu} \\
&+ \sum_{k\in\mathcal{T}} \left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\mu} \left(\frac{V_1}{a_1}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_1^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\frac{\alpha}{1-\alpha}-\frac{1}{1-\sigma}\right](1-\eta)\mu}
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
1 = &\sum_{k\in\mathcal{N}} \tilde{\beta}_k^{\mu} \left(\frac{V_2}{a_2}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_2^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\frac{\alpha}{1-\alpha}-\frac{1}{1-\sigma}\right](1-\eta)\mu} \\
&+ \sum_{k\in\mathcal{T}} \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\mu} \left(\frac{V_2}{a_2}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_2^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\frac{\alpha}{1-\alpha}-\frac{1}{1-\sigma}\right](1-\eta)\mu}
\end{aligned} \tag{39}
$$

Notice $0 \leq (\lambda^{-\sigma}-\rho\omega^{-\sigma})/(1-\rho^2),\ (1-\rho\lambda^{-\sigma}\omega^{\sigma})/(1-\rho^2) < \infty$. and, for all $k \in \mathcal{K}$, $0 < \tilde{\beta}_k$. Given Assumption 1, $\gamma\left(\epsilon_k/(1-\eta)+1/(1-\alpha)\right)+\alpha/(1-\alpha)-1/(1-\sigma) > 0$. Also, remember $\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha} > 0$ for all $k \in \mathcal{K}$. Thus, eq. (38) implies $\lim_{N_1\to 0} V_1 = \infty$, and eq. (39) implies $\lim_{N_1\to 0} V_2 < \infty$. The same can be done when $N_1 \to N$ or $N_2 \to 0$.

Since V_1 and V_2 are continuous functions of N_1, respectively, there exists an intersection at $N_1 \in (0, N)$ by the intermediate value theorem. At the intersection, it satisfies the condition of a stable equilibrium as

$$
\begin{aligned}
&\frac{dV_1}{dN_1} < \frac{dV_2}{dN_1} \implies \frac{d\ln V_1}{dN_1} < \frac{d\ln V_2}{dN_1}\ (\because V_1 = V_2) \\
\iff &\frac{d\ln\left(U_1(N_1)a_1/U_2(N_1)a_2\right)}{dN_1} < \frac{d\ln\left(N_1/(N-N_1)\right)^{\gamma}}{dN_1} \\
\implies &\frac{d\left(U_1(N_1)a_1/U_2(N_1)a_2\right)^{1/\gamma}}{dN_1} < \frac{d\left(N_1/(N-N_1)\right)}{dN_1}\ (\because U_1(N_1)a_1N_1^{-\gamma} = U_2(N_1)a_2N_2^{-\gamma})
\end{aligned}
$$

Proof of Proposition 10

I prove this in the following two steps.

(i) ω increases in N_1 Given Assumption 3 and $\eta < 1$, the LHS of eq. (33) and (34) strictly increase and strictly decreases in N_1, respectively. Suppose, for $\exists x \in [0, N]$, ω weakly decreases in N_1 at $N_1 = n$. Then, at n, $V_1 N_1^{\gamma}$ must strictly increase, and $V_2 N_2^{\gamma}$ strictly decreases in N_1 to satisfy eq. (33) and (34), respectively. Then, ω needs to strictly increase in N_1 to satisfy eq. (35). This contradicts that ω weakly decreases in N_1. Therefore, ω strictly increases in N_1 ($d\omega/dN_1 > 0$) for all $N_1 \in [0, N]$.

(ii) V_1 decreases and V_2 increases in N_1 and unique intersection Given Assumption 1, it follows that

$$\frac{\partial V_1(N_1, \omega)}{\partial N_1} < 0, \ \frac{\partial V_2(N_2, \omega)}{\partial N_2} < 0$$

Combining the results so far, the signs of the total derivatives can be obtained.

$$\begin{aligned}
\frac{dV_1(N_1, \omega)}{dN_1} &= \frac{\partial V_1(N_1, \omega)}{\partial N_1} + \frac{\partial V_1(N_1, \omega)}{\partial \omega_1} \frac{d\omega}{dN_1} < 0 \\
\frac{dV_2(N_2, \omega)}{dN_1} &= \frac{\partial V_2(N_2, \omega)}{\partial N_2} \frac{dN_2}{dN_1} + \frac{\partial V_2(N_2, \omega)}{\partial \omega} \frac{d\omega}{dN_1} > 0
\end{aligned}$$

Since V_1 and V_2 are monotonically decreasing and increasing continuous functions of N_1, respectively, the intersection is unique in the proof of Proposition 4.

Proof of Proposition 11

It follows from eq. (9) and (10)

$$\begin{aligned}
\sum_{k \in \mathcal{T}} m_{1k} &= \frac{\left\{(\lambda^{-\sigma} - \rho\omega^{-\sigma})/(1-\rho^2)\right\}^{\frac{1-\eta}{\sigma-\eta}}}{\kappa^{\mu} \lambda^{-\sigma \frac{1-\eta}{\sigma-\eta}} + \left\{(\lambda^{-\sigma} - \rho\omega^{-\sigma})/(1-\rho^2)\right\}^{\frac{1-\eta}{\sigma-\eta}}} \\
\sum_{k \in \mathcal{T}} m_{2k} &= \frac{\left[(1 - \rho\lambda^{-\sigma}\omega^{\sigma})/(1-\rho^2)\right]^{\frac{1-\eta}{\sigma-\eta}}}{\kappa^{\mu} + \left[(1 - \rho\lambda^{-\sigma}\omega^{\sigma})/(1-\rho^2)\right]^{\frac{1-\eta}{\sigma-\eta}}}
\end{aligned}$$

where $\kappa = \kappa_1 \kappa_2^{\frac{1-\eta}{\sigma-1}} \kappa_3^{1-\eta}$. Substituting these for eq. (8) gives:

$$\frac{N_2}{N_1} = \frac{\kappa^\mu \left(1-\rho\lambda^{-\sigma}\omega^\sigma\right)^\mu + \left(1-\rho^2\right)^{-\frac{1-\eta}{\sigma-\eta}} \left(1-\rho\lambda^{-\sigma}\omega^\sigma\right)}{\kappa^\mu \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \left(\lambda^{-\sigma}-\rho\omega^{-\sigma}\right)^\mu + \left(1-\rho^2\right)^{-\frac{1-\eta}{\sigma-\eta}} \left(\lambda^{-\sigma}-\rho\omega^{-\sigma}\right)} \lambda^\sigma \omega^{1-2\sigma}$$

This equation shows that ω increases in N. Equilibrium conditions can be rewritten as follows:

$$1 = \Phi_1(\omega) \sum_{k\in\mathcal{N}} \tilde{\beta}_k^\mu \left(\frac{V_1}{a_1}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_1^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\frac{\alpha}{1-\alpha}-\frac{1}{1-\sigma}\right](1-\eta)\mu} \tag{40}$$

$$1 = \Phi_2(\omega) \sum_{k\in\mathcal{N}} \tilde{\beta}_k^\mu \left(\frac{V_2}{a_2}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_2^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\frac{\alpha}{1-\alpha}-\frac{1}{1-\sigma}\right](1-\eta)\mu} \tag{41}$$

where $\Phi_1(\omega) \equiv \kappa^\mu \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} + \left[(\lambda^{-\sigma}-\rho\omega^{-\sigma})/(1-\rho^2)\right]^{\frac{1-\eta}{\sigma-\eta}}$ and $\Phi_2(\omega) \equiv \kappa^\mu + [(1-\rho\lambda^{-\sigma}\omega^\sigma)/(1-\rho^2)]^{\frac{1-\eta}{\sigma-\eta}}$. Given ω increasing in N_1, the assumption of $1+(1-\alpha)\min_{k\in\mathcal{K}}\{\epsilon_k/(1-\eta)\} > 0$ for global monotonicity of C_i, and Assumption (1), eq. (40) and (41) imply V_1 and V_2 decreases and increases in N_1, respectively. The rest of the proof follows Proof of Proposition 4 and Proposition 5.

Proof of Proposition of 6

I prove this by contradiction. Suppose $N_1 \le N_2$ in an equilibrium. Then, eq. (38) and (39) and $V_1 = V_2 = V$ imply that it is necessary

$$\lambda^{-\sigma} - \rho\omega^{-\sigma} > 1 - \rho\lambda^{-\sigma}\omega^\sigma$$

It follows $\omega > \lambda$. Then, it follows from Equation 35

$$1 = \frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^\mu \left(VN_1^\gamma\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_1^{\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^\mu \left(VN_2^\gamma\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_2^{\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu}} \lambda^\sigma \left[\frac{1-\rho\lambda^{-\sigma}\omega^\sigma}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^\mu \omega^{1-2\sigma} \tag{42}$$

$$< \frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^\mu V^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_1^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)\right](1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^\mu V^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_2^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)\right](1-\eta)\mu}} \lambda^{-\sigma} \tag{43}$$

When $\eta < 1$, it is obvious that the RHS of the inequality is smaller than 1 when $N_1 \leq N_2$. Thus, a contradiction. When $\eta > 1$, given Assumption 2

$$\frac{\min_{k\in\mathcal{T}} \epsilon_k + 1}{1-\eta} + \frac{\gamma+\alpha}{1-\alpha} < 0$$

It follows that

$$\forall k \in \mathcal{T},\ \left[\gamma\left(\frac{\epsilon_k}{1-\eta} + \frac{1}{1-\alpha}\right) + \left(\frac{1}{1-\eta} + \frac{\alpha}{1-\alpha}\right)\right](1-\eta)\mu > 0$$

This inequality implies that the RHS of inequality (43) is smaller than 1 when $N_1 \leq N_2$. Thus, a contradiction.

As for the wage level, think about eq. (42) in the equilibrium and evaluate the RHS with $\omega = \lambda$

$$\frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(VN_1^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_1^{\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(VN_2^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_2^{\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu}} \lambda^{\eta\mu}$$

With either (i) $\eta < 1$ or (ii) $\eta > 1$ and Assumption 2, this is greater than 1 with $N_1 > N_2$. The RHS of eq. (42) decreases in ω, and, therefore, $\omega > \lambda$ in the equilibrium.

E Equilibrium with Asymmetric Amenities

In this appendix, I impose symmetric fundamental productivity ($\lambda = \lambda_1 = \lambda_2 = 1$) and show how the asymmetric amenities ($a_1 \neq a_2$) generate a cross-city difference. I let city 1 have better amenities ($a_1 > a_2$) without loss of generality. The same result on the population pattern can be obtained as the asymmetric productivity case.

Proposition 12 (Cross-City Population Patterns with Asymmetric Amenities)**.** *Suppose that the amenities are asymmetric ($a_1 > a_2$), the productivities are symmetric ($\lambda_1 = \lambda_2$), and the industries are either gross complements ($\eta < 1$), or gross substitutes ($\eta > 1$) with Assumption 2 holding. Then, given an equilibrium with Assumption 1, the city with better amenities are larger ($N_1 > N_2$).*

Proof. Suppose $N_1 \leq N_2$ in an equilibrium. Then, from eq. (38) and (39), it is necessary that $1 - \rho\omega^{-\sigma} > 1 - \rho\omega^{\sigma}$. This inequality implies $\omega > 1$. Then, it follows from eq. 35

$$1 = \frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(VN_1^{\gamma}/a_1\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_1^{\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(VN_2^{\gamma}/a_2\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_2^{\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu}} \left[\frac{1-\rho\omega^{\sigma}}{1-\rho\omega^{-\sigma}}\right]^{\mu} \omega^{1-2\sigma} \tag{44}$$

$$< \frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(V/a_1\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_1^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)\right](1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(V/a_2\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} N_2^{\left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)\right](1-\eta)\mu}} \tag{45}$$

When $\eta < 1$, it is obvious that the RHS of eq. (45) is smaller than 1 when $N_1 \leq N_2$. Thus, a contradiction. When $\eta > 1$, given Assumption 2

$$\frac{\min_{k\in\mathcal{T}} \epsilon_k + 1}{1-\eta} + \frac{\gamma+\alpha}{1-\alpha} < 0$$

It follows that

$$\forall k \in \mathcal{T},\ \left[\gamma\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)+\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)\right](1-\eta)\mu > 0$$

Thus, the RHS of eq. (45) is smaller than 1. Therefore, a contradiction. □

Next, the wage pattern requires an additional assumption introduced in Appendix C for equilibrium uniqueness.

Proposition 13 (Cross-City Wage Patterns with Asymmetric Amenities)**.** *Suppose that the amenities are asymmetric ($a_1 > a_2$), the productivities are symmetric ($\lambda_1 = \lambda_2$), the agglomeration force is stronger than the dispersion force from the inelastic land supply (Assumption 3), and the industries are gross complements ($\eta < 1$). Then, given an equilibrium with Assumptions 1, the city with better amenities offers a higher wage ($w_1 > w_2$).*

Proof. The equilibrium conditions can be rewritten as

$$
\begin{aligned}
V_1 &= V_2 \\
N &= N_1 + N_2 \\
N_1^{\left(\frac{1}{\sigma-1}-\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} &= \sum_{k\in\mathcal{N}} \tilde{\beta}_k^{\mu} \left(\frac{V_1}{a_1} N_1^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \\
&\quad + \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(\frac{V_1}{a_1} N_1^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}
\end{aligned}
$$

$$
\begin{aligned}
N_2^{\left(\frac{1}{\sigma-1}-\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} &= \sum_{k\in\mathcal{N}} \tilde{\beta}_k^{\mu} \left(\frac{V_2}{a_2} N_2^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \\
&\quad + \left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(\frac{V_2}{a_2} N_2^{\gamma}\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \\
\left(\frac{N_2}{N_1}\right)^{\left(\frac{1}{1-\eta}+\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} &= \frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(V N_1^{\gamma}/a_1\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} \left(V N_2^{\gamma}/a_2\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}} \left[\frac{1-\rho\omega^{\sigma}}{1-\rho\omega^{-\sigma}}\right]^{\mu} \omega^{1-2\sigma}
\end{aligned}
$$

Suppose $\omega < 1$ at $N_1 = N_2 = N/2$. Given Assumption 3 and $\eta < 1$, the third and the fourth equations imply $V_1/a_1 > V_2/a_2$. However, the fifth implies $V_1/a_1 < V_2/a_2$. These contradict each other. So, when $N_1 = N_2 = N/2$, the last three equations imply $\omega \geq 1$. Given Assumption 3 and $\eta < 1$, ω increases in N_1 as Proof of Proposition 10, and, therefore, the wage at an equilibrium, where $N_1 > N/2$, is higher in city 1 than in city 2 ($\omega = w_1/w_2 > 1$) □

Assumption 3 ensures that the larger city has a greater goods consumption (C_1), which increases the relative wage through the home-market effect.

Similarly, the expenditure pattern generally depends on which force is stronger, the agglomeration force or the dispersion force by inelastic land supply. First, I analyze the case without tradable industries, which requires no additional strong assumption to obtain a clear result.

Proposition 14 (Cross-City Expenditure Pattern with Asymmetric Amenities and without Tradable industries)**.** *Suppose that the amenities are asymmetric ($a_1 > a_2$), the productivities are symmetric ($\lambda_1 = \lambda_2$),* *there is no tradable sector ($\mathcal{K} = \mathcal{N}$), and either the industries are gross complements ($\eta < 1$) or the industries are gross substitutes ($\eta > 1$) with Assumption 2 holding. Then, given an equilibrium with Assumption 1, if Assumption 3 holds, the industrial expenditure share ratio of city 1 to city 2 (m_{1k}/m_{2k}) increases in the income elasticity (ϵ_k) and decreases otherwise.*

Proof. First, Proposition 12 applies, and $N_1 > N_2$. Second, the equilibrium conditions characterizing real consumption become

$$N_1^{\left(\frac{1}{\sigma-1}-\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} = \sum_{k\in\mathcal{N}} \tilde{\beta}_k^{\mu} \left(U_1\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}$$
$$N_2^{\left(\frac{1}{\sigma-1}-\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} = \sum_{k\in\mathcal{N}} \tilde{\beta}_k^{\mu} \left(U_2\right)^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}$$

Given $N_1 > N_2$, these immediately imply the results. □

The large city offers a relatively higher real consumption when the agglomeration is relatively stronger, and this relative real consumption determines the relative expenditure pattern.

When the model has both non-tradable and tradable industries, it becomes difficult to obtain results because of the endogenous tradable sector share, which is related to the relative wage. In the rest of the paper, I focus on the cases without non-tradable industries and provide two contrasting results in special cases. The first case is given by Proposition 15.

Proposition 15 (Cross-City Expenditure Pattern with Asymmetric Amenities without Non-Tradable industries Case 1)**.** *Suppose that the amenities are asymmetric ($a_1 > a_2$), the productivities are symmetric ($\lambda_1 = \lambda_2$), there is no non-tradable sector ($K = \mathcal{T}$), and there is no housing expenditure ($\alpha = 0$). Then, given an equilibrium with Assumption 1, the industrial expenditure share ratio of city 1 to city 2 (m_{1k}/m_{2k}) increases in income elasticity (ϵ_k).*

Proof. The three of equilibrium conditions are given by

$$N_1^{\frac{1-\eta}{\sigma-\eta}} = \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_1^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \tag{46}$$

$$N_2^{\frac{1-\eta}{\sigma-\eta}} = \left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_2^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \tag{47}$$

$$\frac{N_1}{N_2} = \omega^{2\sigma-1}\left[\frac{1-\rho\omega^{-\sigma}}{1-\rho\omega^{\sigma}}\right] \tag{48}$$

It follows from eq. (46) and (47) that

$$\frac{N_1^{\frac{1-\eta}{\sigma-\eta}}}{N_2^{\frac{1-\eta}{\sigma-\eta}}}\left[\frac{1-\rho\omega^{\sigma}}{1-\rho\omega^{-\sigma}}\right]^{\frac{1-\eta}{\sigma-\eta}} = \frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_1^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_2^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}$$

With eq. (48),

$$\omega^{(2\sigma-1)\frac{1-\eta}{\sigma-\eta}} = \frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_1^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_2^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}$$

In the equilibrium, $\omega > 1$ as Proposition 13. It implies $U_1 > U_2$, and the expenditure share pattern immediately follows. □

The housing consumption is eliminated here to ensure that the real consumption (U) becomes higher with a larger population. When the industries are tradable, residents in the small city can enjoy the benefit of an agglomeration economy in the large city, although partially, as they can access the rich varieties in the large city by paying the trade cost. Therefore, it is impossible to generally show that the real consumption becomes higher even with Assumption 3[17]. However, when the model does not have the dispersion force from the inelastic land supply, the real consumption necessarily increases with population by the agglomeration force, and the city with better amenities offers a higher real consumption. Consequently, the workers spend relatively more on income-elastic industries.

On the other hand, when the dispersion force from the inelastic land supply is stronger than the agglomeration force, the opposite result is obtained as Proposition 16.

[17] When the driver of the city size difference is a difference in productivity, it is possible as in the main text.

Proposition 16 (Cross-City Expenditure Pattern with Asymmetric Amenities Case 2)**.** *Suppose that the amenities are asymmetric ($a_1 > a_2$), the productivities are symmetric ($\lambda_1 = \lambda_2$), there is no non-tradable sector ($\mathcal{K} = \mathcal{T}$), the dispersion force from the inelastic land supply is stronger than the agglomeration force ($(1-\alpha)/(\sigma-1) < \alpha$), and either the industries are gross complements ($\eta < 1$) or the industries are gross substitutes ($\eta > 1$) with Assumption 2 holding. Then, given an equilibrium with Assumption 1, the industrial expenditure share ratio of city 1 to city 2 (m_{1k}/m_{2k}) decreases in the income elasticity (ϵ_k).*

Proof. First, Proposition 12 applies, and $N_1 > N_2$. Second, three equilibrium conditions are given by

$$N_1^{\left(\frac{1}{\sigma-1}-\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} = \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_1^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \tag{49}$$

$$N_2^{\left(\frac{1}{\sigma-1}-\frac{\alpha}{1-\alpha}\right)(1-\eta)\mu} = \left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_2^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu} \tag{50}$$

$$\frac{N_1}{N_2} = \omega^{2\sigma-1}\left[\frac{1-\rho\omega^{-\sigma}}{1-\rho\omega^{\sigma}}\right] \tag{51}$$

It follows from eq. 51 that $\omega > 1$. Finally, it follows from eq. (49) and (50) that

$$\left(\frac{N_1}{N_2}\right)^{\left(\frac{1}{\sigma-1}-\frac{\alpha}{1-\alpha}\right)\mu}\left[\frac{1-\rho\omega^{\sigma}}{1-\rho\omega^{-\sigma}}\right]^{\frac{1}{\sigma-\eta}} = \left[\frac{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_1^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}{\sum_{k\in\mathcal{T}} \tilde{\beta}_k^{\mu} U_2^{\left(\frac{\epsilon_k}{1-\eta}+\frac{1}{1-\alpha}\right)(1-\eta)\mu}}\right]^{1/(1-\eta)}$$

Given$(1-\alpha)/(\sigma-1) \leq \alpha$ and $N_1 > N_2$, the LHS is smaller than 1, which implies $U_1 \leq U_2$. The expenditure share pattern immediately follows. $\square$

In this case, a majority of workers choose to live in a city with better amenities, but they have a lower real consumption because of the high land price and the high expenditure share of land consumption. Consequently, the residents spend relatively more on income-inelastic industries.