Cities' Demand-driven Industrial Composition*

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Abstract

I report a new stylized fact: large cities specialize in income-elastic sectors. I explain this by developing a model that has heterogeneous income-elasticities and mobile agents. Either heterogeneous fundamental productivities or heterogeneous amenities generate the specialization pattern through the home market effect. The city with fundamentally better productivity or amenity becomes larger, offers a higher wage, and specialize in income-elastic sectors.

Keywords: Home market effect, Non-homothetic preference, Cities, Industrial composition, Comparative advantage

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1 Introduction

Industrial composition varies greatly across cities: Detroit, for example, is synonymous with cars and Silicon Valley with computers. Figure 1 shows that much of the variation in employment

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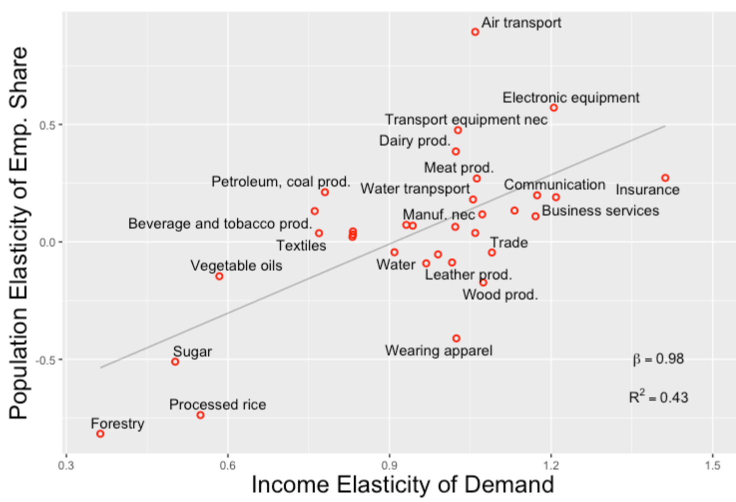
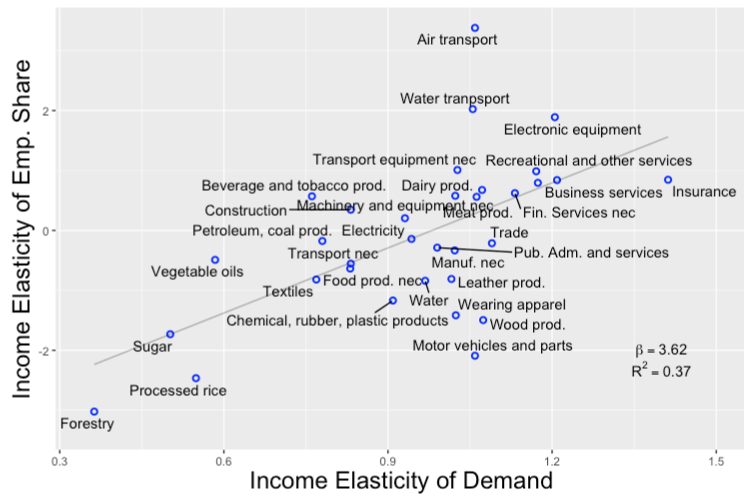
composition across U.S. cities is linked to the income-elasticity of demand for that industries' output. Employment shares of industries with high income-elasticities of demand, such as air transport services and insurance, are higher in large Metropolitan Statistical Areas (MSAs) like New York-Newark-Jersey City Metropolitan Area. Industries with low income-elasticities of demand, such as sugar manufacturing and processed rice manufacturing, have larger employment shares in small MSAs. Figure 2 displays a similar pattern. Employment shares of industries with high income-elasticities are higher in high-income cities and employment shares of industries with low income-elasticities are lower in low-income cities. These two patterns are closely related because large cities tend to be high-income cities.

Figure 1: Elasticity of Employment Share with respect to MSA's Population and Elasticity of Demand with respect to Income



MSA population are 2016 data from the Bureau of Economic Analysis. Employment shares are calculated from 2016 County Business Pattern data. When obtaining the population elasticity of employment share, region $\in \{Northeast, Midwest, South, West\}$ is controlled. Income-elasticity estimates are from Caron et al. (2020). The unweighted regression line is shown. For details of data description, see Appendix A.

Figure 2: Elasticity of Employment Share with respect to MSA's Income Level and Elasticity of Demand with respect to Income



Income is per capita personal income in 2016 from the Bureau of Economic Analysis. Employment shares are calculated from 2016 County Business Pattern data. When obtaining the income-elasticity of employment share, region $\in \{Northeast, Midwest, South, West\}$ is controlled. income-elasticity estimates are from Caron et al. (2020). The unweighted regression line is shown. For details of data description and robustness checks, see Appendix A.

This paper investigates this new stylized fact and the implications. I develop a model that can generate these specialization patterns as equilibrium outcomes. My model is based on Matsuyama (2019), who theoretically studies international trade patterns with heterogeneous sectoral income-elasticities and differentiated goods within a sector. I extend Matsuyama (2019) by introducing worker mobility and non-tradable sectors. In an equilibrium, a fundamental difference between cities either in productivity or amenities generates asymmetric population allocation and the home market effect on the wage rate and on the trade pattern, which is consistent with the specialization patterns in Figure 1 and 2. In addition to explaining the specialization patterns, the model implies that the aggregate share of tradable sectors, which is endogenously determined in my model, has an effect on cross-city wage patterns.

In the production patterns explained as equilibrium outcomes of my model, the home market effect plays a key role. In the home market approach first formally theorized by Krugman (1980), the effect is of two types, each of which shares the mechanism from trade costs and an increasing return to scale production. The first affects the wage rate. Other things being equal, the wage rate tends to be higher in larger markets. When firms are exposed to competition with firms in other locations and sharing demands, differences in access to markets due to trade costs drive the difference in the input costs so that the firms' profits are equalized at zero in any location. The second effect affects trade pattern by generating comparative advantage. When the relative market size of sectors varies across regions, regions export goods for which they have relatively large domestic markets. This is because, in the presence of trade costs, local firms are incentivized to operate in a sector that has a relatively larger home market, and this incentive is strong enough to amplify the demand pattern to the production pattern.

In my model, one fundamental difference between cities either in productivity or amenities generates these two home market effects, and this eventually produces the specialization patterns, which are consistent with Figure 1 and 2. First, a city with better fundamentals (productivity or amenity) attracts workers, which results in a large population, and this generates the home market effect on the wage rate. Second, due to the higher wage, residents in the large city spend relatively more on income-elastic sectors and this generates the home market effect on the trade pattern. Hence, in the equilibrium, the fundamentally attractive city becomes larger, offers a higher wage, and specializes in income-elastic sectors.

In addition to explaining the production pattern, the model highlights the importance of the aggregate tradable sector share when we study cross-location income inequality. In equilibrium, the wage increases not only with city size but also with the aggregate share of tradable sectors. This is because the home market effect works only through tradable sectors. The source of the home market effect is competition between firms in different locations. When a sector is non-tradable, that competition does not exist, and the home market effect does not emerge. Thus, the market size that drives the wage is the size of tradable sectors, which is the product of the overall market size and the aggregate tradable sector share. Because in my model the aggregate tradable sector share of a city is endogenously determined, we can use that share to analyze how the economic environment affects cross-location income patterns. In Onoda (2022), I theoretically and empirically demonstrate that the model can be used to understand the evolution of the income level across cities after 1980.

Industrial composition is an important factor for local economies, and not only because an aggregate tradable sector share is a driver of local wages through the home market effect; it also is important because local economic performance is significantly affected by the industries that locate in a city (e.g., Autor et al. (2013)). To fully understand variation in local economic performance it is necessary, first, to understand what determines the mix of industry in cities. The mechanism that drives industrial composition also is important for researchers who want to exploit regional variation in the size of industries. In this paper, I report that industrial composition is related to population and income level. Given this relationship, regressing dependent variables on sectoral sizes or shares, while not controlling the income level or the city size of examined locations, might lead to an omitted variable bias problem because the trend of a dependent variable could be driven by the income level of the locations rather than by sectoral sizes or shares. Understanding the mechanism can help researchers avoid this endogeneity issue.

This paper is the first research on cross-city inter-sectoral specialization patterns to be undertaken from the demand-side perspective. Most previous works on the cross-city specialization pattern has focus on non-demand side factors such as functions in production, (Duranton and Puga (2005) and Henderson and Ono (2008)), the strength of the agglomeration economy (Behrens and Robert-Nicoud (2015)), and the skill supply (Davis and Dingel (2019)). A few works focus on the demand side's effect on cross-city difference (e.g. Handbury (2019)). The closest of these is Dingel (2017), who quantifies the contributions made to the quality specialization of cities by the heterogeneous demand factor and by the skill supply factor, and he creates a model to guide the quantification. There are two major differences between his model and mine. First, we focus on different types of specialization and trade patterns. His work, and, therefore, his model, examines vertical specialization and the intra-sector trade, where different quality goods are gross substitutes. Whereas, my model examines horizontal specialization and inter-sectoral trade, where goods in different sectors are gross complements. Second, agents are immobile in his model, while in mine they are mobile. The mobile-agent assumption fits the urban economy environment, and, moreover, it enables us to study the relationship between the size of a city and its industrial composition. This paper, the first work on cross-city inter-sectoral specialization patterns undertaken from the demand-side perspective, contributes to the literature by reporting the stylized fact and providing the theoretical framework for urban economies.

Furthermore, my work contributes more broadly to studies of the demand-side effect in trade patterns. The international trade literature shows theoretically that the demand-side effect plays an important role in determining trade patterns and specializations (e.g. Flam and Helpman (1987), Stokey (1991), Matsuyama (2019), Fajgelbaum et al. (2011)). The closest to mine is Matsuyama (2019) in the sense he analyzes horizontal specialization and inter-sectoral trade. My model extends Matsuyama (2019) by introducing non-tradable sectors, and it reveals a new effect that heterogeneous demand across locations generates. This is the effect on cross-location income through endogenous tradable sector shares, and it is relevant both in international trade and in domestic trade.

The rest of this paper is organized as follows. Section 2 develops the model to explain the stylized fact. Section 3 provides robustness checks, and Section 4 concludes.

2 Model

In this section, I introduce the model of two cities ($c \in \{1, 2\}$). The cities fundamentally differ in productivity and amenity. There is a mass of N workers who are freely mobile and homogeneous except for inherent taste over cities. Conditional on location, individual labor supply is inelastic. There are K sectors, and the sectors differ in relative income-elasticity (ϵ_k) and tradability (τ_k) as well as productivities (ϕ_k, ψ_k) and preference shifters (β_k). I start with explaining the household problem.

Household

The problem for a worker i is given by

$$\max_{c\in\{1,2\},\{q_k(\nu)\}_{\nu\in\Omega_{ck},k\in K},\{Q_k\}_{k\in K}} U_c \cdot a_c \cdot \delta(i,c) \tag{1}$$

$$s.t.\ 1 = \left[\sum_{k\in K} \beta_k^{\frac{1}{\eta}} U_c^{\frac{\epsilon_k}{\eta}} Q_{c,k}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$Q_{c,k} = \left[\int_{\nu\in\Omega_{c,k}} q_k(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}}$$

$$E_c = \sum_{k\in K} \int_{\nu\in\Omega_{c,k}} p_{ck}(\nu) q_k(\nu) d\nu$$

where c is the city to reside in, $q_k(\nu)$ is the consumption of variety ν in sector k, Ω_{ck} is the set of available varieties of sector k in city c, K is the set of sectors, $Q_{c,k}$ is the composite consumption of sector k, β_k is the preference shifter of sector k, E_c is the income in city c, and $a_c > 0$ for $c \in \{1,2\}$, $0 < \epsilon_k < 1$ for all $k \in K$, $0 < \eta < 1$, $\sigma > 1$. By setting $\eta < 1$, I assume sectors are gross complements throughout the paper. The utility consists of three factors: U_c, a_c, and $\delta(i,c)$.

U_c is the utility from goods consumption, and the functional form captures the non-homothetic preference of the consumer. This functional form follows Comin et al. (2021). Matsuyama (2019) uses the same form, properties of which he illustrates in detail in the same paper. When solving this household problem, a convenient property of the functional form of U_c can be seen. The demand function derived from this preference becomes

$$Q_{c,k} = \beta_k P_{c,k}^{-\eta} U_c^{\epsilon_k} E_c^{\eta} \tag{2}$$

where $P_{c,k}$ is the price index for sector k in city c and it is defined as $P_{c,k} = \left[\int_{\nu\in\Omega_{c,k}} p_k(\nu)^{1-\sigma} d\nu\right]^{\frac{1}{1-\sigma}}$.

This shows that the relative income-elasticity of demand (ϵ_k) and price elasticity (η) are separated. While the price elasticity is assumed to be common among sectors, I vary the income-elasticity across sectors. Also, the expenditure share of sector k is obtained as follows (see Appendix B for the derivation):

$$m_{c,k} \equiv \frac{P_{c,k}Q_{c,k}}{\sum_{l\in K} P_{c,l}Q_{c,l}} = \frac{\beta_k P_{c,k}^{1-\eta} U_c^{\epsilon_k}}{\sum_{l\in K} \beta_l P_{c,l}^{1-\eta} U_c^{\epsilon_l}} \tag{3}$$

This shows that, holding the price indices $\{P_k\}_{k\in K}$ constant, agents with higher utility from goods consumption (U_c) spend relatively more on sectors with high ϵ_k. Another result is the indirect sub-utility, which is implicitly expressed as

$$E_c^{1-\eta} = \sum_{k\in K} \beta_k P_{ck}^{1-\eta} U_c^{\epsilon_k} \tag{4}$$

(see Appendix B for the derivation). This illustrates that as U_c rises, agents care particularly about the prices of high ϵ_k goods, on which they spend relatively more.

a_c is the utility from an amenity offered by city c, such as weather, landscape, and historic heritage. While "amenity" generally refers to access to local services and consumer goods (e.g., restaurants), in this model, those local services and goods contribute to U_c when they are consumed.

$\delta(i, c)$ is the idiosyncratic utility shock for the worker i and city c pair. This generates the heterogeneous taste over cities (following Tabuchi and Thisse (2002) and others). I assume $\delta(i, c)$ is distributed i.i.d. across workers and cities according to the Fréchet distribution with shape parameter $1/\gamma$ ($Pr[\delta < x] = e^{-x^{-1/\gamma}}$). Each worker chooses the city that offers the higher utility, taking into account her consumption optimization. Thus, given the indirect utility levels in the two cities, a_1U_1 and a_2U_2, the probability of choosing city 1 for a given agent is derived as $Pr[a_1U_1\delta(i) > a_2U_2\delta(i)] = (a_1U_1)^{1/\gamma} / \left\{(a_1U_1)^{1/\gamma} + (a_2U_2)^{1/\gamma}\right\}$. Since the shock is i.i.d., the cities' population ratio follows.

$$\frac{N_1}{N_2} = \left(\frac{a_1U_1}{a_2U_2}\right)^{1/\gamma} \tag{5}$$

Given the same relative utility from goods consumption ($\frac{a_1 U_1}{a_2 U_2}$), the lower γ is, the greater the population inequality is. This illustrates that the γ measures the dispersion force in this economy. [1]

Production

The production in my model is based on Krugman (1980). For all sectors $k \in K$ there are endogenous sets of varieties, homogeneous firms, and monopolistic competition . Each sector is either tradable with iceberg trade cost $\tau > 1$ or it is non-tradable. Let $\mathbb{T}$ be the set of tradable sectors and $\mathbb{N}$ be the set of non-tradable sectors ($K = \mathbb{T} \cup \mathbb{N}$ and $\mathbb{T} \cap \mathbb{N} = \emptyset$). To let city 1 be fundamentally more productive than city 2, I assume that labor supply by a worker in city 1 is λ (≥ 1) efficiency units of labor for all sectors while that by the same worker in city 2 is one unit for all sectors. Each worker chooses the location of labor they supply. Conditional on location, individual labor supply is inelastic. I let w_c denote the wage rate for an efficiency unit in city c. It follows that the income for households in each city becomes

$$E_1 = \lambda w_1$$

$$E_2 = w_2$$

[1] If additional variables V_1 and V_2 are introduced s.t. $V_1 = a_1 U_1 N_1^{-\gamma}, V_2 = a_1 U_2 N_2^{-\gamma}$, (5) becomes $V_1 = V_2$. This shows the isomorphism to agents with homogeneous tastes whose utility function is $V_c = U N_c^{-\gamma}$. N_c^{γ} can be interpreted as a congestion cost from a local population that directly negatively affects the utility and the microfoundation can be provided by a housing sector with an inelastic land supply and a Cobb-Douglas utility function for the fixed expenditure share allocation between housing and goods (Helpman (1998)). However, when the preference is non-homothetic, the price index of aggregate goods consumption depends on the utility level, in which case the fixed expenditure share result does not hold. As a result, the Helpman-type microfoundation does not generate (5) .

Each firm in sector k needs to employ ϕ_k units of labor as the fixed cost and ψ_k as the variable cost to produce a unit of variety. The problem for a firm that produces variety ν in sector k in city c is

$$\begin{aligned}\pi_{ck}(\nu) = \max_{p_{cck}(\nu),q_{cck}(\nu),p_{cc'k}(\nu),q_{cc'k}(\nu)} & [p_{cck}(\nu)q_{ck}(\nu) - \psi_k q_{cck}(\nu)w_c] \\ & + \mathbb{1}\{k \in \mathbb{T}\}\,[p_{cc'k}(\nu)q_{cc'k}(\nu) - \tau\psi_k q_{cc'k}(\nu)w_c] - \phi_k w_c \quad (6) \\ s.t.\ q_{cck}(\nu) &= p_{cck}(\nu)^{-\sigma}P_{ck}^{\sigma}Q_{ck} \\ q_{cc'k}(\nu) &= p_{cc'k}(\nu)^{-\sigma}P_{c'k}^{\sigma}Q_{c'k}\end{aligned}$$

where $\pi_{ck}(\nu)$ is the profit by optimized production; $(c, c') \in \{(1, 2), (2, 1)\}$; $p_{cck}(\nu)$ and $p_{cc'k}(\nu)$ are the prices for the markets in city c and c', respectively; and $q_{cck}(\nu)$ and $q_{cc'k}(\nu)$ are the quantities for the markets in city c and c', respectively. In the following part, I omit ν. The terms in the first bracket are the variable profits from selling products in city c, while those in the second are those in city c', which is zero if $k \in \mathbb{N}$. If sector k in city c has non-zero production in an equilibrium, π_{ck} must be zero such that there is no entrant. Similarly, if sector k in city c has zero production in an equilibrium, π_{ck} must be non-positive. This is the zero-profit condition in sector k in city c.

Definition of Competitive Equilibrium

A competitive equilibrium is
$\{N_1, N_2, U_1, U_2, w_1, w_2, E_1, E_2, \{p_{cc'k}, q_{cc'k}\}_{(c,c',k)\in(1,2)\times(1,2)\times K}, \{\Omega_{1k}, \Omega_{2k}\}_{k\in K}\}$ such that

1. households optimize consumption and locational choice as (1) where $E_1 = \lambda w_1$ and $E_2 = w_2$ for $c \in \{1, 2\}$,
2. producers optimize production as (6) for all $k \in K$ and $c \in \{1, 2\}$,
3. the zero-profit condition holds such that $\pi_{ck} \leq 0$ for all $k \in K$ and $c \in \{1, 2\}$ where equality holds if $q_{cck} + \tau q_{cc'k} \neq \emptyset$,
4. the national labor market clearing condition that $N_1 + N_2 = N$ holds, and

5. the local labor market clearing conditions,

$$\sum_{k\in\mathbb{N}}\int_{\Omega_{1k}}(\psi_k q_{11k}+\phi_k)\,d\nu+\sum_{k\in\mathbb{T}}\int_{\Omega_{1k}}(\psi_k q_{11k}+\tau\psi_k q_{12k}+\phi_k)\,d\nu=\lambda N_1$$

and

$$\sum_{k\in\mathbb{N}}\int_{\Omega_{2k}}(\psi_k q_{22k}+\phi_k)\,d\nu+\sum_{k\in\mathbb{T}}\int_{\Omega_{2k}}(\psi_k q_{22k}+\tau\psi_k q_{21k}+\phi_k)\,d\nu=N_2$$

, hold.

Equilibrium Conditions

I now characterize an equilibrium by obtaining simplified conditions. I focus on on equilibria where all sectors have nonzero output in both cities, and, given the optimized production and the demand function, I impose the zero-profit condition on each sector in each city. The first result concerns non-tradable sectors.

Proposition 1. *Given an equilibrium, the expenditure share and the employment share of a non-tradable sector in a city are equalized.*

$$\forall c\in\{1,2\},\ \forall k\in\mathbb{N}\quad m_{ck}=x_{ck} \tag{7}$$

where x_{ck} is the employment share of sector k in city c (see Appendix C for the proof). This equalization follows from the zero-profit condition in the sector level. That is, each firm has zero profit and therefore the sector has zero aggregate profit. Since the revenues in the non-tradable sectors are only received from the agents in the same city and the factor payments are made only to the same agents, the zero-profit condition boils down to equation (7). Corollary 1 follows.

Corollary 1. *Given an equilibrium, the aggregate expenditure share and the aggregate employment* *share in a city are equalized both for the non-tradable sectors and for the tradable sectors.*

$$\forall c \in \{1,2\},\ \sum_{k\in\mathbb{N}} m_{ck} = \sum_{k\in\mathbb{N}} x_{ck} \ \textit{and} \ \sum_{k\in\mathbb{T}} m_{ck} = \sum_{k\in\mathbb{T}} x_{ck}$$

This equalization between the aggregate expenditure share and the aggregate employment share in the tradable sectors is a key for the tractability of the model. Another result is Proposition 2.

Proposition 2. *Given an equilibrium, the city sizes (N_1 and N_2), the aggregate tradable sector shares ($\sum_{k\in\mathbb{T}} m_{1k}$, and $\sum_{k\in\mathbb{T}} m_{2k}$), and the income ratio ($\omega = \frac{E_1}{E_2} = \frac{\lambda w_1}{w_2}$) satisfy the following equation.*

$$\frac{\sum_{k\in\mathbb{T}} m_{1k}}{\sum_{k\in\mathbb{T}} m_{2k}} \frac{N_1}{N_2} = \omega^{2\sigma-1}\left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{\lambda^{\sigma} - \rho\omega^{\sigma}}\right] \tag{8}$$

This result is obtained by aggregating the zero-profit condition over tradable sectors and making use of $\sum_k m_{ck} = \sum_k x_{ck}$.(see Appendix C for the proof). This summarizes the sector-level zero-profit condition for the tradable sectors into the city level. The LHS is the relative tradable market size of city 1 to city 2. The RHS, which increases in the relative wage of city 1 (ω), shows two things. First, the sector-level force -- that a large local market is accompanied by a higher input cost-- is carried over to the city level. This is the home market effect on the wage rate discussed in Krugman (1980) and Krugman (1991). Second, this home market effect force works only through tradable sectors, and the relative aggregate tradable sector expenditure share matters. For the zero-profit condition to hold in each sector-city pair, the larger city's advantage of lower trade costs for selling to consumers must be exactly offset by higher input costs. This force does not appear in non-tradable sectors because the local wage rate change affects both the demand and the input cost at the same rate. Next, the expenditure shares in an equilibrium are obtained.

Lemma 1. *Given an equilibrium, the expenditure shares of sector $k \in K$ in city 1 and city 2 can* *be expressed as follows:*

$$m_{1k} = \left[\frac{\lambda^{-\sigma} - \rho_k \omega^{-\sigma}}{(1-\rho_k^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \tag{9}$$

$$m_{2k} = \left[\frac{1 - \rho_k \lambda^{-\sigma} \omega^{\sigma}}{(1-\rho_k^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \tag{10}$$

where

$$\rho_k = \begin{cases} 0 & k \in \mathbb{N} \\ \rho = \tau^{1-\sigma} & k \in \mathbb{T} \end{cases}, \ \tilde{\beta}_k = \left(\frac{\beta_k^{\frac{1}{1-\eta}} \phi_k^{\frac{1}{\sigma-1}} \psi_k}{(\frac{1}{\sigma})^{\frac{1}{\sigma-1}} (\frac{\sigma-1}{\sigma})}\right)^{1-\eta}$$

Making use of $\sum_{k \in K} m_{ck} = 1$, Proposition 3 is obtained.

Proposition 3. *Given an equilibrium, the utility from goods consumption in city 1 and city 2 (U_1 and U_2) can be implicitly expressed as follows:*

$$\left[\frac{\lambda^{-\sigma}}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k \in \mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \left[\frac{\lambda^{-\sigma} - \rho \omega^{-\sigma}}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k \in \mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} = 1 \tag{11}$$

$$\left[\frac{1}{N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k \in \mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1 - \rho \lambda^{-\sigma} \omega^{\sigma}}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k \in \mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} = 1 \tag{12}$$

Holding the relative wage constant, the elasticity of utility from goods consumption with respect to the local population is given by

$$\frac{\partial U_c / U_c}{\partial N_c / N_c} = \frac{1}{\sigma - 1} \frac{1-\eta}{\tilde{\epsilon}_c}$$

where $\tilde{\epsilon}_c = \sum_{k \in K} m_{ck} \epsilon_k$. $1/(\sigma-1)$ is the elasticity of the agglomeration economy or the positive externality in Krugman-type models with homothetic preference. The number of varieties in a location increases with market size and consumers have love-of-variety in their preferences. In this non-homothetic model, the positive externality is adjusted by $(1-\eta)/\tilde{\epsilon}_c$. To understand this, suppose all sectors have homogeneous income-elasticity, $\bar{\epsilon}$. The utility becomes explicit, and it is

$$\begin{aligned} U &= \left[\sum_k \tilde{\beta}_k^{\frac{1}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{-\frac{\eta}{\bar{\epsilon}}} \\ &= Y^{\frac{1-\eta}{\bar{\epsilon}}} \end{aligned}$$

where $Y = \left[\sum_k \tilde{\beta}_k^{\frac{1}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$.This shows that, holding Y constant, the greater $\bar{\epsilon}$ is, the smaller the marginal utility is. This relationship between the marginal utility and ϵ_k is carried over to the heterogeneous income-elasticity model. The effect on the marginal utility from aggregate consumption is summarized by the average of ϵ_k weighted with the expenditure shares.

Next, I allow ω to move and see whether it amplifies or attenuates the agglomeration economy. Holding the relative tradable sector share constant, it follows from the home market effect on the wage rate (8) that

$$\frac{\partial \omega}{\partial N_1} > 0$$

When city 1 becomes larger, the home market effects require a higher relative wage rate in city 1. Consequently, holding the relative tradable share constant,

$$\frac{\partial U_c/U_c}{\partial N_c/N_c} + \frac{\partial U_c/U_c}{\partial \omega/\omega}\frac{\partial \omega/\omega}{\partial N_c/N_c} < \frac{\partial U_c/U_c}{\partial N_c/N_c}$$

as $\frac{\partial U_1}{\partial \omega} < 0$ and $\frac{\partial U_2}{\partial \omega} > 0$ from the equilibrium conditions 11 and 12. This shows that the movement of ω attenuates the agglomeration economy. This attenuation effect reflects the fact that the increase in the local population is accompanied by a decrease in the other city's population, and the mass of available varieties shipped from the other city decreases. The direction of the net effect is not obvious. To obtain a clear result about the direction in which U_c moves while taking into account all of the effects, I analyze the case in which the sectors are all tradable ($\mathbb{N} = \emptyset$, $K = \mathbb{T}$).[2]After

[2]When the relative tradable sector share is held constant, there are four effects. First, the increase in local population leads to an increase in locally produced varieties. Second, the mass of available varieties shipped from the other city decreases. Third, the rising share of locally produced varieties makes the average price

tedious calculations, it can be shown that, if $\mathbb{N} = \emptyset$,

$$\forall \tau > 1, \forall N_1 \in (0, N), \quad \frac{dU_1}{dN_1} > 0$$

$$\begin{cases} \forall N_2 \in (0, N), \quad \frac{dU_2}{dN_2} > 0 & if\ \lambda^{\frac{\sigma}{\sigma-1}} \leq \tau \\ \exists \bar{N}_2 > 0\ s.t. \begin{cases} \frac{dU_2}{dN_2} > 0 \quad N_2 \in (0, \bar{N}_2) \\ \frac{dU_2}{dN_2} < 0 \quad N_2 \in (\bar{N}_2, N) \end{cases} & if\ 1 < \tau < \lambda^{\frac{\sigma}{\sigma-1}} \end{cases}$$

The result for city 1 reflects the fact that the increase in city 1's varieties dominates the decreases in city 2's varieties. Reallocation of labor from the unproductive location to the productive one increases the total mass of available varieties. In the case of city 2's expansion, the net effect is the opposite. However, there are still two positive effects: saving the trade cost on average as the share of varieties without the trade cost rises; and the rising purchasing power. The aggregate effect depends on the size of the relative productivity and the trade cost. When the trade cost is large enough relative to the productivity gap, the negative effect from the unproductive labor reallocation is dominated by the combination of the benefit from saving the trade cost and the purchasing power effect. Therefore, population expansion always increases U_2, regardless of the level of N_2. In the other case, when the trade cost is relatively low, the negative productivity effect catches up with the positive effects at some point ($\bar{N}_2$) and U_2 starts to decline.

Finally, an equilibrium is characterized by the seven conditions obtained above: (5), (8), (11), (12), the national labor clearing condition ($N_1 + N_2 = N$), and the aggregate tradable sector shares

of varieties, other things being equal, cheaper because the average trade cost declines. Fourth, the home market effect raises the purchasing power of the local residents ($\omega \uparrow$ for city 1 residents). The first, third, and fourth effects are positive effects for U_c while the second is negative.

as follows:

$$\sum_{k\in\mathbb{T}} m_{1k} = \left[\frac{\lambda^{-\sigma} - \rho_k\omega^{-\sigma}}{(1-\rho_k^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}$$
$$\sum_{k\in\mathbb{T}} m_{2k} = \left[\frac{1 - \rho_k\lambda^{-\sigma}\omega^{\sigma}}{(1-\rho_k^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}$$

These aggregate shares follow from individual tradable sector's share ((9) and (10)). There are seven unknown variables $\{U_1, U_2, \omega, N_1, N_2, \sum_{k\in\mathbb{T}} m_{1k}, \sum_{k\in\mathbb{T}} m_{2k}\}$ for these seven equations. By making use of the zero-profit conditions for all city-sector pairs, the number of endogenous variables here is fewer than that of the equilibrium definition. Based on these conditions, I now provide some properties of the equilibrium.

The Existence, the Uniqueness, and the Stability of an Equilibrium

First, it can be shown that, given Assumption 1, there exists a unique equilibrium.

Assumption 1. $\gamma > \frac{1-\eta}{(\sigma-1)\min\{\epsilon_k\}_{k\in\mathbb{T}}}$

Proposition 4. *If Assumption 1 holds, there exists a unique equilibrium.*

This corresponds to the sufficient and necessary condition, which Helpman (1998) obtains by numerical simulations, for his model to have a unique stable equilibrium. Compared to his condition ($\gamma > \frac{1}{\sigma-1}$ in this paper's notation), there is an adjusting term, $\frac{1-\eta}{\min\{\epsilon_k\}_{k\in\mathbb{T}}}$. This is sufficient to ensure that whatever the expenditure composition is, the agglomeration force is weaker than the dispersion force.

Next, I analyze whether the unique equilibrium is stable. When U_1 and U_2 can be expressed as functions of only N_1 from (8), (11), and (12), a stable equilibrium is defined as follows.

Definition. A competitive equilibrium is a stable equilibrium if and only if

$$\frac{d\frac{a_1U_1(N_1)}{a_2U_2(N_2)}}{dN_1} < \frac{d\left(\frac{N_1}{N_2}\right)^{\gamma}}{dN_1}$$

[3]This guarantees that, with the migration of agents whose agent-specific utility in city 1 is marginally greater than that of city 2, the expanding city would not experience enough of a relative gain in utility from goods consumption to support the post-migration population allocation. With this condition, the marginal agents in the expanding city find the shrinking one preferable and return to their original location. Consequently, the economy converges back to the original state. For the stability, Proposition 5 is obtained.

Proposition 5. *If Assumption 1 holds, the unique equilibrium is stable.*

When the dispersion force is strong enough, the unique equilibrium is stable.[4]In the rest of the paper, I assume that Assumption 1 holds, and so I focus on the unique and stable equilibrium.

3 Cross-City Analysis

In this section, I illustrate how the two cities differ in the equilibrium depending on their fundamental productivities and amenities. I start with a partial equilibrium analysis to show the directions of the forces that the fundamental differences generate. Then, I lay out general equilibrium results in the case where the cities have the same amenity level and differ only in fundamental productivity ($a_1 = a_2$). The results of different amenities with the same productivity are briefly explained (A detailed explanation is provided in Appendix D).

[3]With V_1 and V_2 s.t. $V_1 = a_1 U_1 N_1^{-\gamma}$ and $V_2 = a_2 U_2 N_2^{-\gamma}$, the condition of the stable equilibrium becomes $\frac{dV_1}{dN_1} < \frac{dV_2}{dN_1}$In the homogeneous agent's interpretation of the model (as discussed in a previous footnote), this guarantees that the expanding city does not offer a higher utility than the shrinking city.

[4]If Assumption 1 does not hold and $\gamma < \frac{1-\eta}{(\sigma-1)\max\{\epsilon_k\}_{k\in\mathbb{T}}}$ and $\mathbb{N} \neq \emptyset$, then all workers living in one city 1 ($N_1 = N$ and $N_2 = N$) become a stable equilibrium. This case corresponds to $\beta\epsilon < 1$ in Helpman (1998)'s prohibitive trade cost. In my model, the existence of a non-tradable sector, rather than a prohibitive trade cost, suffices to generate this result. In the non-tradable sectors, a new marginal city after a deviation from a one-city equilibrium has only a marginal mass of varieties. Therefore, the relative mass of the locally-offered varieties for the preexisting city is unbounded. When $\gamma < \frac{1-\eta}{(\sigma-1)\max\{\epsilon_k\}_{k\in\mathbb{T}}}$, this unbounded gap dominates the dispersion force.

Partial Equilibrium Analysis

For the partial equilibrium analysis, we focus on city 1. Making use of the population allocation with Fréchet taste shock (5), the utility from goods consumption in city 1 in equilibrium (11) and the home market effect (HME) on the wage rate (8) can be written as follows:

$$1 = \left[\frac{1}{\lambda^{\sigma}}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} N_1^{\gamma\epsilon_k \frac{\sigma-1}{\sigma-\eta} - \frac{1-\eta}{\sigma-\eta}} \left(\frac{V}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$
$$+ \left[\frac{1}{\left(\lambda^{\sigma} + \frac{\omega^{\sigma-1}}{\tau^{\sigma-1}}\left(\frac{N_2}{N_1}\right)\frac{\sum_{k\in\mathbb{T}} x_{2k}}{\sum_{k\in\mathbb{T}} x_{1k}}\right)}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in K} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} N_1^{\gamma\epsilon_k \frac{\sigma-1}{\sigma-\eta} - \frac{1-\eta}{\sigma-\eta}} \left(\frac{V}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \tag{13}$$

$$N_1 = \frac{\sum_{k\in\mathbb{T}} m_{2k}}{\sum_{k\in\mathbb{T}} m_{1k}} \omega^{2\sigma-1} \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{\lambda^{\sigma} - \rho\omega^{\sigma}}\right] N_2 \tag{14}$$

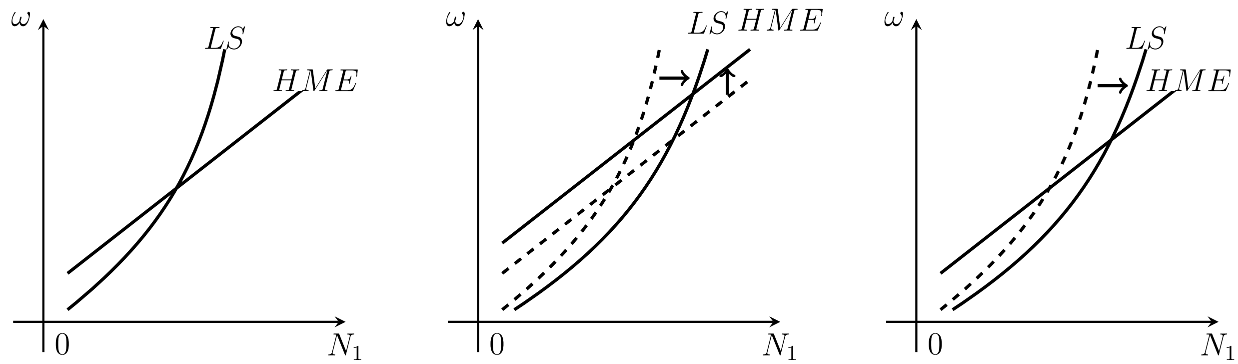
where $V = a_2 U_2 N_2^{\gamma}$ and w_2 is normalized to 1. In this partial equilibrium analysis, I fix $V(= a_2 U_2 N_2^{\gamma})$, and the tradable sector share ratio $(\frac{\sum_{k\in\mathbb{T}} x_{2k}}{\sum_{k\in\mathbb{T}} x_{1k}})$. Then, equation (13) can be interpreted as the labor supply curve in city 1, and, given Assumption 1, N_1 increases in ω and therefore in w_1 . As the wage increases, the city attracts more workers. Similarly, given Assumption 1, N_1 increases in ω and therefore in w_1 from the HME on the wage rate (14). Having a large local market requires a higher input cost to keep the profit at zero. These two curves are depicted in Figure 3a. [5]

When city 1 has a higher productivity ($\lambda \uparrow$), the HME curve shifts up and the labor supply curve shifts to the right, as in Figure (3b). The HME curve shifts for two reasons. First, the local income linearly increases in local productivity as $\omega = \lambda w_1$. Consequently, the local market size expands and the input cost, w_1, needs to rise to keep the zero profit. The shift of the labor supply curve is driven by additional local varieties and reflects the fact that the number of people increases given the income (ω). Higher productivity increases the local labor supply in terms of the efficiency

[5] How the curves intersect is not easy to tell from the equations in this part. The depiction here is based on the theoretical results in the general equilibrium. For cities with asymmetric productivity, the one with higher productivity offers higher local wage and larger population. For cities with asymmetric amenity, the one with higher amenity offers higher local wage and larger population. These are consistent with the labor supply curve intersecting the home market effect curve from below.

Figure 3: Partial Equilibrium Analysis

(a) Original State (b) Higher Productivity ($\lambda \uparrow$) (c) Higher Amenity ($a_1 \uparrow$)



unit and, therefore, the mass of local varieties. Given the prices of goods, which are linear in the wage rate per efficient unit w_1, this reduces the price indices and raises the local utility from goods consumption, and, therefore, it attracts more workers. Because of the shifts of the two curves, the new intersection is located where both the population and the wage are higher than before.

When city 1 has a higher amenity, this shifts the labor supply curve to the right, although it does not affect the HME curve, as in Figure 3c. This reflects the fact that the higher amenity attracts more people, but it does not affect production. As a result, the intersection moves along the HME curve, and both the population and the wage are higher than before. [6]

This analysis, which ignores what is taking place in the other city (N_2, U_2) and the effect through the tradable sector share ratio $(\frac{\sum_{k\in\mathbb{T}} x_{2k}}{\sum_{k\in\mathbb{T}} x_{1k}})$, shows the main forces generated by heterogeneous fundamental productivity and amenity. In the next section, I provide the results in general equilibrium for the case in which cities differ only in terms of productivity. Appendix D provides the results for the case in which there is a different amenity and a common productivity. The city that has better fundamental characteristics (productivity or amenity) becomes larger and offers a higher wage.

[6]While this result is in contrast to what a Rosen-Roback model (Rosen (1979), Roback (1982)) implies, this is not new in the literature. For example, Glaeser and Gottlieb (2009) point out that rising amenities can increase wages because of agglomeration economy.

General Equilibrium with Productivity Difference

In this section, I consider cities that have the same amenity level and differ only in fundamental productivity, and I provide a cross-city analysis for the stable unique equilibrium. The first comparison concerns population allocation and the nominal income ratio. The partial equilibrium analysis (Figure 3b) suggests that city 1 has the larger population and the residents receive the higher income. When we consider a general equilibrium and allow additional variables, including N_2, to move, the movement of N_2 attenuates the agglomeration economy in city 1 because of the decrease in the mass of varieties shipped from city 2, as discussed in Section 2. As a result, this shortens the shift of the labor supply curve in Figure 3b. On the other hand, the shrinking population in city 2 amplifies the home market effect and it shifts further up the HME curve in Figure 3b. After all, it can be proven that the qualitative result does not change and Proposition 6 is obtained (see Appendix C for the proof).

Proposition 6. *In the stable unique equilibrium with Assumption 1 and $a_1 = a_2$, the fundamentally productive city is larger and the nominal income ratio is greater than the fundamental productivity ratio.*

$$N_1 > N_2 \text{ and } \omega > \lambda > 1$$

This result is consistent with a stylized fact that the nominal income level is higher in larger cities, even when observable or unobservable workers' characteristics are taken into account (e.g., Behrens and Robert-Nicoud (2015),Glaeser and Mare (2001)).

The next result concerns expenditure shares.

Proposition 7. *In the stable unique equilibrium, given Assumption 1 and $a_1 = a_2$, the sectoral expenditure share ratio of city 1 to city 2 increases in ϵ_k within the non-tradable sectors and within the tradable sectors, respectively.*

Proof. (9) and (10) provide the expenditure share ratio of a non-tradable sector and a tradable sector, respectively, 8

$$\forall k \in \mathbb{N}, \quad \frac{m_{1k}}{m_{2k}} = \left[\lambda^{-\sigma}\frac{N_2}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \left(\frac{U_1}{U_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$
$$\forall k \in \mathbb{T}, \quad \frac{m_{1k}}{m_{2k}} = \left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{1-\rho\lambda^{-\sigma}\omega^{\sigma}}\frac{N_2}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \left(\frac{U_1}{U_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$

Since $\frac{U_1}{U_2} > 1$, the expenditure share ratio increases in ϵ_k within the non-tradable sectors and within the tradable sectors. □

In the equilibrium, the large and high-income city spends more expenditure in the income-elastic sectors. This difference in the expenditure shares generates the following results in the employment shares.

Proposition 8. *In an equilibrium, for a tradable sector, there is a relationship with the within-tradable employment share ratio and the within-tradable expenditure share ratio such that*

$$\forall k \in \mathbb{T}, \ \frac{\tilde{x}_{1k}}{\tilde{x}_{2k}} = \frac{\frac{\tilde{m}_{1k}}{\tilde{m}_{2k}} - \rho\lambda^{\sigma}\omega^{-\sigma}}{1-\rho\lambda^{\sigma}\omega^{-\sigma}} \frac{1-\rho\omega^{\sigma}\lambda^{-\sigma}}{1-\rho\omega^{\sigma}\lambda^{-\sigma}\frac{\tilde{m}_{1k}}{\tilde{m}_{2k}}}$$

where $\tilde{m}_{ck} = \frac{m_{ck}}{\sum_{k\in\mathbb{T}} m_{ck}}$ *and* $\tilde{x}_{ck} = \frac{x_{ck}}{\sum_{k\in\mathbb{T}} x_{ck}}$. *This shows* $\frac{\tilde{x}_{1k}}{\tilde{x}_{2k}}$ *increases in* $\frac{\tilde{m}_{1k}}{\tilde{m}_{2k}}$. *That is, given an equilibrium, the greater the within-tradable expenditure share difference of a sector, the greater that sector's within-tradable employment share difference. Also, this implies that a city becomes the net exporter in sectors for which the city has a greater relative within-tradable expenditure share as follows:*

$$\forall k \in \mathbb{T}, \ \frac{\tilde{x}_{1k}}{\tilde{x}_{2k}} > \frac{\tilde{m}_{1k}}{\tilde{m}_{2k}} \iff \frac{\tilde{m}_{1k}}{\tilde{m}_{2k}} > 1$$

This is the home market effect in the sectoral specialization. The difference in the expenditure pattern generates comparative advantages and is amplified to that of the employment pattern. Unlike Krugman (1980), who assumes an exogenous taste difference to generate the heterogeneous relative demand, here that difference arises endogenously from the non-homothetic preference. The importance and the endogenous formation of the relative demand are the same as in Matsuyama (2019), but my definition of the relative demand is different. The result shown above demonstrates that when non-tradable sectors exist, the relative size of demand should be measured within the tradable sectors. In Matsuyama (2019), all sectors are tradable and the relative demand size is the same whether it is within the overall economy or within the tradable sectors.

Next, I analyze the aggregate tradable and non-tradable sector expenditure shares, which are equal to their employment shares from Corollary 1. In the equilibrium, it follows from equation (9) and (10) that the ratios of these shares are

$$\frac{\sum_{k\in\mathbb{T}} m_{1k}}{\sum_{k\in\mathbb{N}} m_{1k}} = \left[\frac{1-\rho\lambda^{\sigma}\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}$$

$$\frac{\sum_{k\in\mathbb{T}} m_{2k}}{\sum_{k\in\mathbb{N}} m_{2k}} = \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}$$

This shows that the ratio is determined by two forces. The first is the relative price of goods in the tradable sectors, which shows up in the form of the first factor in each RHS. When two cities start to share the varieties in the tradable sectors through inter-city trade, the relative increase in the mass of varieties is greater in city 2 due to the asymmetric city size. This causes a relatively greater reduction in the the price index in the tradable sectors in city 2. Given the assumption that the sectors are gross complements ($\eta < 1$), this implies that the relative expenditure share in the tradable sectors is lower in city 2, other things being equal. The second force is the income-elasticities of the two groups. Because $U_1 > U_2$ in the equilibrium, the higher the income-elasticities of tradable sectors as a whole are compared to those of the non-tradable sectors, the higher the aggregate tradable sector share in city 1 is compared to that in city 2, other things being equal. This observation becomes important when we think about the effect of sector-specific trade cost reductions. When a non-tradable sector becomes a tradable, it changes the income-elasticities of the two groups and, thereby, the aggregate tradable sector share. Onoda (2022) examines how business services' trade cost reduction has affected cross-city income inequality by this mechanism.

Finally, the price index of a sector differs between locations. The sector price index ratio is given by

$$\frac{P_{1k}}{P_{2k}} = \begin{cases} \frac{w_1}{w_2}\left(\frac{\lambda N_1}{N_2}\frac{x_{1k}}{x_{2k}}\right)^{-\frac{1}{\sigma-1}} & k \in \mathbb{N} \\ \frac{1}{\tau}\left(1 + \frac{\tau^{2(\sigma-1)}-1}{\left(\frac{\tau w_2}{w_1}\right)^{\sigma-1}\lambda\frac{N_1}{N_2}\frac{x_{1k}}{x_{2k}}+1}\right)^{\frac{1}{\sigma-1}} & k \in \mathbb{T} \end{cases}$$

In both groups, $\frac{P_{1k}}{P_{2k}}$ decreases in $\frac{x_{1k}}{x_{2k}}$. As the employment share ratio increases in income-elasticity within a group, the sector price index decreases in income-elasticity within a group. In city 1, the higher expenditure shares on income-elastic goods attract firms in those sectors, and the price indices in those sectors become relatively cheap, reflecting relatively more varieties.

Connection to Stylized Fact

In summary, the fundamentally productive location becomes the large and high-income city, and within each group (tradable and non-tradable), the residents allocate their expenditure relatively more towards income-elastic sectors, which offer relatively richer varieties. On the supply side, the large and high-income city specializes in income-elastic sectors in the sense that workers are employed relatively more in income-elastic sectors within each group, which replicates the sectoral specialization patterns seen in Figure 1 and 2. While what we observe in the real world are the city size, the income level, and the sectoral employment of a city, what generates the relationship between them in this model is the fundamental productivity. Moreover, the model predicts that the demand pattern is amplified to the specialization pattern for tradable sectors; that is, the city with the greater relative within-expenditure share becomes the net exporter of that sector. In Appendix E, I provide an empirical analysis like that of Figure 2 (except that it has both tradable and non-tradable sectors) that shows that the amplification mechanism is consistent with the data. While the income-elasticity estimates in Figure 2 are borrowed from Caron et al. (2020), who estimate them using international trade flow data, the ones in Appendix E are from Comin et al. (2021), who provide estimates for 9 sectors using sectoral employment data from 39 countries. Although the plot has fewer sectors, the sectoral specialization pattern that appears is the same as Figure 2. The fact that the pattern becomes clearer in the tradable sectors is consistent with the theory's amplification effect.

4 Robustness Check

In this section, I address an alternative explanation of the cities' specialization patterns in Figure 1 and 2. The positive relationships are possibly driven by an omitted variable--that is, skilled-labor supply in cities. This is a reasonable concern given that skilled workers, who are at the same time high income earners, tend to reside in large cities, and those cities tend to host skill-intensive sectors (Davis and Dingel (2020)). Also, and as is well known, there is a positive correlation between the skill intensity and the income-elasticity of a sector (Caron et al. (2014) and Caron et al. (2020)). I address this concern in two ways. First, I test if the positive relationship in Figure 1 is robust to controlling the skill intensities of sectors. Second, I use the elasticities of employment share that are conditioned on local skilled-labor supply for Figure 1 and 2.

In Appendix F, I provide an empirical exercise with an alternative specification as an additional robustness check. The specification follows Nunn (2007), and it tests the association of the income level and the income elasticities of the local industries. This test also shows that there is a positive relationship even when we control the association of the skilled-labor supply and the skill intensities.

Controlling Skill Intensities in Figure 1

In order to test if the positive relationship in Figure 1 is robust to controlling the skill intensities of sectors, I regress the elasticities of employment share on the income-elasticities of demand and the skill intensities. The regression model is given by

$$\xi_k = \alpha + \beta\epsilon_k + \gamma\theta_k + e_k$$

where

ξ_k : elasticity of employment share of industry k with respect to MSA's population

ϵ_k : income elasticity of demand for industry k output

θ_k : skill intensity of industry k

e_k : error term for industry k

ξ_k is the variable of the y-axis in Figure 1. Each ξ_k is obtained by regressing the log of employment shares of sector k in MSAs on the log of MSAs' population sizes. The population data are from the Bureau of Economic Analysis and the employment data are from the Country Business Patterns. In theses log regressions, the region of a MSA is controlled. ϵ_k is the variable of the x-axis in Figure 1 and θ_k is the control. For these two variables, I use the estimates by Caron et al. (2020), who obtain the estimates through a structural estimation with international trade data that have both heterogeneous skill-intensities and income-elasticities. If the cities' specialization pattern in Figure 1 only reflects that large cities specialize in skill-intensive sectors by the supply side mechanism (Davis and Dingel (2020)), $\beta = 0$ and $\gamma > 0$ are expected. I implement the cross-sectional regression for three years and the results are shown in Table 1.

In all three years, β is significantly positive whereas γ is not. A caveat is that all of ξ_k, ϵ_k, and θ_k that are used in the regressions are estimated values, and, therefore, the standard errors here are not precise for the hypothesis test. Nevertheless, these results suggest that the positive relationship in Figure 1 is robust to the supply-side explanation.

Table 1: Results with Controlling Skill Intensity

	Population elasticity of employment share		
	2006	2011	2016
	(1)	(2)	(3)
Income elasticity (β)	1.288***	1.201***	0.937***
	(0.337)	(0.318)	(0.285)
Skill intensity (γ)	−0.145	0.067	0.132
	(0.690)	(0.653)	(0.583)
Observations	33	33	33
R^2	0.464	0.486	0.433
Adjusted R^2	0.429	0.452	0.395
F Statistic (df = 2; 30)	13.010***	14.183***	11.440***

Notes: ***,**, and * denote significance at the 1 %, 5%, and 10% levels respectively.

Controlling Skill Supply in Figure 1 and 2

The second robustness check is done by controlling skill supply when the elasticities of employment share with respect to population and that of MSA's income are obtained. Specifically, the college graduate share in the labor force in a MSA is controlled[7]. Figure 4 and 5 use these conditional elasticities in the y-axes. As the regression line shows, the positive relationships still clearly exist. This shows that the alternative explanation cannot deny the mechanism that is proposed by my model.

[7] The labor force data are from Census via IPUMS.

Figure 4: With Population Elasticity of Employment Share Conditioned on Skill Supply

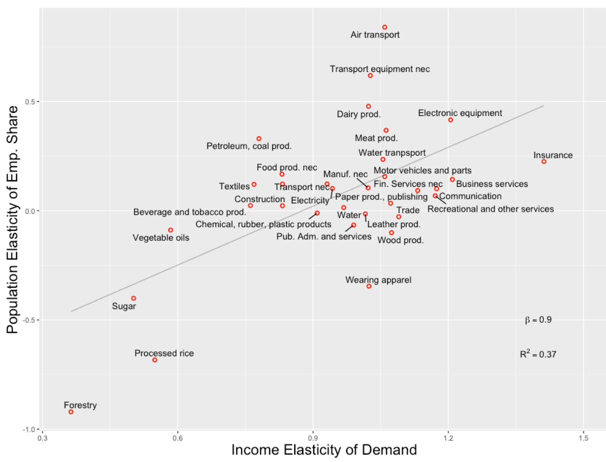
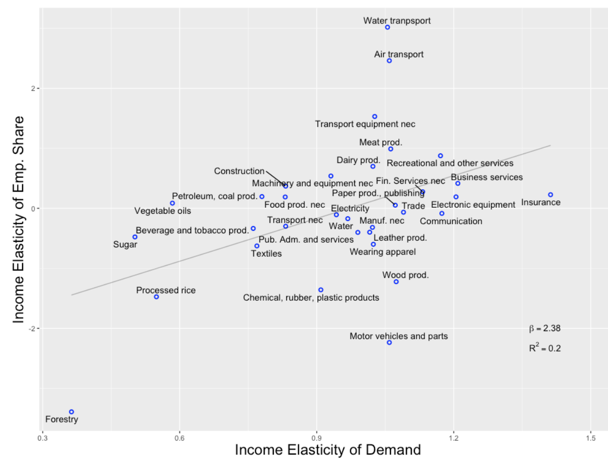


Figure 5: With income-elasticity of Employment Share Conditioned on Skill Supply



MSA population are 2016 data from the Bureau of Economic Analysis (BEA). The income of a MSA is per capita personal income in 2016; from the BEA. Employment shares are calculated from 2016 CBP data. When obtaining the income-elasticity of employment share, the college graduate share in the labor force and the region $\in \{Northeast, Midwest, South, West\}$ are controlled. Income-elasticity estimates are fromCaron et al. (2020). The unweighted regression lines are shown.

5 Conclusion

In this paper, I first report a new stylized fact: large cities specialize in income-elastic sectors. To explain this, I present a two-city model and demonstrate how a fundamental difference in productivity and in amenity generates the patterns of a city's size, income level, and industrial composition, which is consistent with the production patterns. In addition, the model reveals the importance of the aggregate share of tradable sectors in local wages. This provides an additional reason why we should understand the mechanism that drives the industrial composition. In Onoda (2022), I examine the impact of the aggregate tradable share in a city-size wage premium and how a sectoral trade cost reduction affects wage inequality across locations by changing the pattern of the aggregate tradable share. The endogenous and heterogeneous shares of tradable and non-tradable sectors deserve more investigations, and my model can be a useful framework for such studies both in international trade and in urban economics.

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Appendix

A Data

To create Figure 2, I borrow estimates of income-elasticities from Caron et al. (2020). Using 2007 international trade data for 109 countries, these scholars estimate the elasticities for 49 sectors. The elasticity varies from 0.137 for "Processed rice" to 1.311 for "Financial services nec". For employment data, I use datasets from Country Business Patterns (CBP). CBP provides employment data for sectors classified annually according to the North American Industry Classification System

(NAICS) in metropolitan areas. The classification in Caron et al. (2020), which is different from NAICS, is called GTAP. In most cases, one GTAP code corresponds to multiple 3-digit or 4-digit NAICS codes. Following Carrico et al. (2012), and mapping NAICS data to GTAP, I create employment data by GTAP. For income level of MSAs, I use per capita personal income available in the Bureau of Economic Analysis. Table 2 displays the distribution of income levels and population in 2006. The largest MSA in 2006 in this sample is New York-Newark-Jersey City, (NY-NJ-PA), which had a population of 19,200,372, while the smallest is Ocean City, NJ, which had a population of 98,695. Income level varies substantially across MSAs: \$18,720 in McAllen-Edinburg-Mission, TX, is the lowest and \$83,059 in Bridgeport-Stamford-Norwalk, CT, is the highest.

Table 2: Distribution of Income and Population across MSAs in 2006

	Min	Q1	Q2	Q3	Max
Income	\$18,720	\$30,190	\$33,383	\$37,878	\$83,059
Population	98,695	166,923	352,204	782,757	19,200,372

B Derivation of Demand Function and Indirect Utility

Derivation of Demand Function

The FOCs are

$$\begin{aligned}
U :& 1+\mu\frac{\eta}{\eta-1}\left[\sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}}\left[\sum_k \frac{\epsilon_k}{\eta}\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}-1} Q_k^{\frac{\eta-1}{\eta}}\right]=0\\
Q_k :& \mu\left[\sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}}\left[\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{-1}{\eta}}\right]=\xi\\
q(\nu) :& \xi\left[\int_{\Omega_k} q(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{1}{\sigma-1}} q^{-\frac{1}{\sigma}}=\zeta p(\nu)
\end{aligned}$$

where μ, ξ, and ζ are lagrange multipliers. Note $\left[\sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}}$ can be removed because it is 1.

First, derive the usual result with CES.

$$P_k Q_k^{\frac{1}{\sigma}} = p(\nu) q(\nu)^{\frac{1}{\sigma}}$$

With this result, the FOC w.r.t Q_k, and that w.r.t $q(\nu)$,

$$\begin{aligned} \mu \left[\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}} \right] &= \zeta P_k Q_k \\ \implies \frac{\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}}{\beta_l^{\frac{1}{\eta}} U^{\frac{\epsilon_l}{\eta}} Q_l^{\frac{\eta-1}{\eta}}} &= \frac{P_k Q_k}{P_l Q_l} \end{aligned} \tag{15}$$

Next, using (15),

$$\begin{aligned} E &= \sum_k P_l Q_l \frac{\beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}}}{\beta_l^{\frac{1}{\eta}} U^{\frac{\epsilon_l}{\eta}} Q_l^{\frac{\eta-1}{\eta}}} \\ &= \frac{P_l Q_l}{\beta_l^{\frac{1}{\eta}} U^{\frac{\epsilon_l}{\eta}} Q_l^{\frac{\eta-1}{\eta}}} \sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} Q_k^{\frac{\eta-1}{\eta}} \\ &= \frac{P_l Q_l}{\beta_l^{\frac{1}{\eta}} U^{\frac{\epsilon_l}{\eta}} Q_l^{\frac{\eta-1}{\eta}}} \\ \implies Q_l &= E^{\eta} \beta_l U^{\epsilon_l} P_l^{-\eta} \end{aligned} \tag{16}$$

$$E^{\eta} \beta_l U^{\epsilon_l} P_l^{1-\eta}$$

Derivation of Indirect Utility

I can implicitly express the indirect utility by plugging (16) into the definition of C,

$$\begin{aligned} 1 =& \left[\sum_k \beta_k^{\frac{1}{\eta}} U^{\frac{\epsilon_k}{\eta}} \left[E^{\eta} \beta_k U^{\epsilon_k} P_k^{-\eta} \right]^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ =& E^{\eta} \left[\sum_k \beta_k U^{\epsilon_k} P_k^{1-\eta} \right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

C Proof of Propositions and Lemma

Proof of Proposition 1

I begin by substituting the optimized production into the zero-profit condition. As for the optimized production, the optimized prices are as follows.

$$\forall k \in K,\ p_{cck} = p_{ck} = \frac{\sigma}{\sigma - 1}\psi_k w_c$$
$$\forall k \in \mathbb{T},\ p_{cc'k} = \tau p_{ck} = \frac{\sigma}{\sigma - 1}\tau\psi_k w_c$$

Then, the zero-profit condition implies $\pi_{ck} = 0$ for all k in K and $c \in \{1, 2\}$. It follows for all (c, c') in $\{(1, 2), (2, 1)\}$,

$$\forall k \in \mathbb{N},\ q_{ck}\psi_k w_c \left[\frac{1}{\sigma - 1}\right] - \phi_k w_c = 0$$
$$\forall k \in \mathbb{T},\ \left(q_{ck} + q_{cc'k}\tau\right)\psi_k w_c \left[\frac{1}{\sigma - 1}\right] - \phi_k w_c = 0$$

The total labor demand by a firm in sector k in city c, L_{ck}, is pinned down as

$$L_{ck} = \begin{cases} q_{ck}\psi_k + \phi_k = \sigma\phi_k & k \in \mathbb{N} \\ \left(q_{ck} + q_{cc'k}\tau\right)\psi_k + \phi_k = \sigma\phi_k & k \in \mathbb{T} \end{cases} \tag{17}$$

The labor demand is determined solely by the fixed cost. Now, I use a normalization. It can be shown that β_k, ϕ_k and ψ_k affect the equilibrium values of $N_1, N_2, w_1, w_2, U_1, U_2$ only through $\beta_k^{\frac{1}{1-\eta}}\phi_k^{\frac{1}{\sigma-1}}\psi_k$. Therefore, given any set of parameters, replacing $\{\beta_k, \phi_k, \psi_k\}_{k\in K}$ by $\left\{\tilde{\beta}_k, \frac{1}{\sigma}, \frac{\sigma-1}{\sigma}\right\}_{k\in K}$ where $\tilde{\beta}_k = \left(\frac{\beta_k^{\frac{1}{1-\eta}}\phi_k^{\frac{1}{\sigma-1}}\psi_k}{(\frac{1}{\sigma})^{\frac{1}{\sigma-1}}(\frac{\sigma-1}{\sigma})}\right)^{1-\eta}$ does not affect the equilibrium values of those variables. One caveat is that the price index is affected by this change. Let $\tilde{P}_k$ be the new price index given $\left\{\tilde{\beta}_k, \frac{1}{\sigma}, \frac{\sigma-1}{\sigma}\right\}_{k\in K}$. Then, $P_k = (\frac{1}{\sigma\phi_k})^{\frac{1}{\sigma-1}}(\frac{\sigma}{\sigma-1}\psi_k)\tilde{P}_k$. Following Matsuyama (2019), I set $\psi_k = \frac{\sigma-1}{\sigma}$ and $\phi_k = \frac{1}{\sigma}$ so that $p_{ck} = w_c$ for all $k \in K$ and $L_{ck} = 1$, which requires that β_k is replaced by

$\tilde{\beta}_k = \left(\frac{\beta_k^{\frac{1}{1-\eta}} \phi_k^{\frac{1}{\sigma-1}} \psi_k}{(\frac{1}{\sigma})^{\frac{1}{\sigma-1}} (\frac{\sigma-1}{\sigma})} \right)^{1-\eta}$. Then, it follows from equation (17) and the normalization that the aggregate supply of goods by a firm is given by

$$\forall k \in \mathbb{N},\ q_{ck} = 1$$

$$\forall k \in \mathbb{T},\ q_{ck} + q_{cc'k}\tau = 1$$

Next, to equate demand to supply, I derive the aggregate demand for a variety in sector k in city c. I let D_{ck} denote the aggregate demand, and it is as follows:

$$D_{ck} = p_{ck}^{-\sigma} A_{ck}$$

where

$$A_{ck} = N_c \tilde{P}_{ck}^{\sigma} Q_{ck} + \rho_k N_{c'} \tilde{P}_{c'k}^{\sigma} Q_{c'k}$$

$$\rho_k = \begin{cases} 0 & k \in \mathbb{N} \\ \rho = \tau^{1-\sigma} & k \in \mathbb{T} \end{cases}$$

Equating demand (D_{ck}) and supply (q_{ck} for $k \in \mathbb{N}$ and $q_{ck} + q_{cc'k}\tau$ for $k \in \mathbb{T}$) with $p_{ck} = w_c$ gives, $\forall c \in \{1, 2\}, \forall k \in K$

$$1 = w_c^{-\sigma} A_{ck} \tag{18}$$

This equates supply and demand, and it reflects the zero-profit condition. This illustrates that, given a sector, the city with greater aggregate demand has a higher wage and, given a city, this equates the aggregate demands across sectors because the wage is common. To make use of equation (18), I use two different expressions of the demand function to substitute for the Q_{ck} that is contained in A_{ck}.

$$Q_{ck} = \begin{cases} \tilde{P}_{ck}^{-1} E_c m_{ck} \\ \tilde{\beta}_k \tilde{P}_{ck}^{-\eta} E_c^{\eta} U_c^{\epsilon_k} \end{cases} \tag{19}$$

The first follows from equation (3) and the second from equation (2). With the first expression, equation (18) becomes

$$w_c^{\sigma} = N_c \tilde{P}_{ck}^{\sigma-1} E_c m_{ck} + \rho_k N_{c'} \tilde{P}_{c'k}^{\sigma-1} E_{c'} m_{c'k} \tag{20}$$

Proposition 1 follows from (20) for a non-tradable sector $k \in \mathbb{N}$

$$\begin{aligned} w_1^{\sigma} &= \tilde{P}_{1k}^{\sigma-1} N_1 E_1 m_{1k} \\ &= \frac{N_1 E_1 m_{1k}}{x_{1k} \lambda N_1 w_1^{1-\sigma}} \\ \Longrightarrow\ x_{1k} &= m_{1k} \end{aligned} \tag{21}$$

where I use the price index $\tilde{P}_{1k}^{1-\sigma} = x_{1k} \lambda N_1 \left(w_1\right)^{1-\sigma}$ for $k \in \mathbb{N}$ and x_{ck} is the employment share in sector k in city c

Proof of Corollary (1)

This follows from (21) as follows:

$$\begin{aligned} \sum_{k \in \mathbb{N}} x_{1k} &= \sum_{k \in \mathbb{N}} m_{1k} \\ \Longrightarrow\ 1 - \sum_{k \in \mathbb{T}} x_{1k} &= 1 - \sum_{k \in \mathbb{T}} m_{1k} \\ \Longrightarrow\ \sum_{k \in \mathbb{T}} x_{1k} &= \sum_{k \in \mathbb{T}} m_{1k} \end{aligned} \tag{22}$$

Proof of Proposition 2

This follows from zero-profit conditions for a tradable sector $k \in \mathbb{T}$ ((20)) for the two cities that

$$\begin{aligned} \frac{w_1^\sigma - \rho w_2^\sigma}{1-\rho^2} &= \tilde{P}_{1k}^{\sigma-1} N_1 E_1 m_{1k} \\ &= \frac{N_1 \lambda w_1 m_{1k}}{x_{1k}\lambda N_1 w_1^{1-\sigma} + \rho x_{2k} N_2 w_2^{1-\sigma}} \end{aligned} \tag{23}$$

where I use the price index $\tilde{P}_{1k}^{1-\sigma} = x_{1k}\lambda N_1 \left(w_1\right)^{1-\sigma} + \rho x_{2k} N_2 w_2^{1-\sigma}$ and x_{ck} is the employment share in sector k in city c. It follows that

$$x_{1k}\lambda N_1 w_1^{1-\sigma} + \rho x_{2k} N_2 w_2^{1-\sigma} = (1-\rho^2)\frac{N_1 \lambda w_1 m_{1k}}{w_1^\sigma - \rho w_2^\sigma} \tag{24}$$

Aggregate over tradable sectors and use the income ratio $\omega = \frac{E_1}{E_2} = \frac{\lambda w_1}{w_2}$,

$$\sum_{k\in\mathbb{T}} x_{1k}\lambda^\sigma N_1 \omega^{1-\sigma} + \rho \sum_{k\in\mathbb{T}} x_{2k} N_2 = (1-\rho^2)\frac{N_1 \omega \sum_{k\in\mathbb{T}} m_{1k}}{\lambda^{-\sigma}\omega^\sigma - \rho}$$

Transforming this making use of(22), the following city size ratio is obtained.

$$\frac{N_1}{N_2} = \frac{\sum_{k\in\mathbb{T}} m_{2k}}{\sum_{k\in\mathbb{T}} m_{1k}} \omega^{2\sigma-1}\left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{\lambda^\sigma - \rho\omega^\sigma}\right]$$

Proof of Lemma 1

I use the second form of the demand function (19). (18) becomes

$$w_c^\sigma = \tilde{\beta}_k \tilde{P}_{ck}^{\sigma-\eta} N_c E_c^\eta U_c^{\epsilon_k} + \rho_k \tilde{\beta}_k \tilde{P}_{c'k}^{\sigma-\eta} N_{c'} E_{c'}^\eta U_{c'}^{\epsilon_k} \tag{25}$$

It follows from 25 of sector $k \in \mathbb{T} \cup \mathbb{N}$ for the two cities ((25)),

$$\frac{w_1^\sigma - \rho_k w_2^\sigma}{1-\rho_k^2} = \tilde{\beta}_k \tilde{P}_{1k}^{\sigma-\eta} N_1 E_1^\eta U_1 \tag{26}$$

I eliminate $\tilde{P}_{nk}$ using (3). From (3) and (4),

$$\begin{aligned} \tilde{P}_{1k} &= \left[\frac{m_{1k} \left(\sum_l \tilde{\beta}_l U_1 \tilde{P}_{1l}^{1-\eta} \right)}{\tilde{\beta}_k U_1} \right]^{\frac{1}{1-\eta}} \\ &= \left[\frac{m_{1k} E_1^{1-\eta}}{\tilde{\beta}_k U_1} \right]^{\frac{1}{1-\eta}} \end{aligned}$$

Plug this into (26),

$$\begin{aligned} \frac{w_1^\sigma - \rho_k w_2^\sigma}{1-\rho_k^2} &= \tilde{\beta}_k \left[\frac{m_{1k} E_1^{1-\eta}}{\tilde{\beta}_k U_1} \right]^{\frac{\sigma-\eta}{1-\eta}} N_1 E_1^\eta U_1 \\ m_{1k}^{\frac{\sigma-\eta}{1-\eta}} &= \frac{w_1^\sigma - \rho_k w_2^\sigma}{1-\rho_k^2} \frac{\tilde{\beta}_k^{\frac{\sigma-1}{1-\eta}} U_1}{N_1 (\lambda w_1)^\sigma} \\ &= \frac{\lambda^{-\sigma} - \rho_k \omega^{-\sigma}}{1-\rho_k^2} \frac{\tilde{\beta}_k^{\frac{\sigma-1}{1-\eta}} U_1}{N_1} \\ m_{1k} &= \left[\frac{\lambda^{-\sigma} - \rho_k \omega^{-\sigma}}{(1-\rho_k^2) N_1} \right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \end{aligned}$$

Proof of Proposition 8

For a tradable sector in city 1, from (24),

$$x_{1k} \lambda^\sigma N_1 + \rho x_{2k} N_2 \omega^{\sigma-1} = (1-\rho^2) \frac{N_1 m_{1k}}{\lambda^{-\sigma} - \rho \omega^{-\sigma}} \tag{27}$$

The counterpart of this with m_{2k} can be obtained as

$$\rho x_{1k}\lambda^{\sigma}N_1 + x_{2k}N_2\omega^{\sigma-1} = (1-\rho^2)\frac{N_2 m_{2k}}{\omega^{-\sigma}-\rho\lambda^{-\sigma}}\omega^{-1} \tag{28}$$

Solve (27) and (28) for x_{1k} and x_{2k} using 8.

$$\begin{aligned}
\frac{x_{1k}}{\sum_{k\in\mathbb{T}} x_{1k}} &= \frac{\frac{m_{1k}}{\sum_{k\in\mathbb{T}} m_{1k}} - \rho\lambda^{\sigma}\omega^{-\sigma}\frac{m_{2k}}{\sum_{k\in\mathbb{T}} m_{2k}}}{1-\rho\lambda^{\sigma}\omega^{-\sigma}} \\
\frac{x_{2k}}{\sum_{k\in\mathbb{T}} x_{2k}} &= \frac{\frac{m_{2k}}{\sum_{k\in\mathbb{T}} m_{2k}} - \rho\omega^{\sigma}\lambda^{-\sigma}\frac{m_{1k}}{\sum_{k\in\mathbb{T}} m_{1k}}}{1-\rho\omega^{\sigma}\lambda^{-\sigma}}
\end{aligned}$$

The employment share ratio immediately follows from this.

Proof of Proposition 4 and Proposition 5

I introduce new variables, V_1 and V_2, defined as

$$V_1 = a_1 U_1 N_1^{-\gamma}$$

$$V_2 = a_2 U_2 N_2^{-\gamma}$$

Using them, the equilibrium conditions can be rewritten as follows:

$$N_1^{\frac{1-\eta}{\sigma-\eta}} = \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}}\sum_{k\in\mathbb{N}}\tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}}\left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} + \left[\frac{\lambda^{-\sigma}-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}}\sum_{k\in\mathbb{T}}\tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}}\left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} \tag{29}$$

$$N_2^{\frac{1-\eta}{\sigma-\eta}} = \sum_{k\in\mathbb{N}}\tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}}\left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}}\sum_{k\in\mathbb{T}}\tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}}\left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} \tag{30}$$

$$\left(\frac{N_2}{N_1}\right)^{\frac{\sigma-1}{\sigma-\eta}} = \frac{\sum_{k\in\mathbb{T}}\tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}}\left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}}\tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}}\left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}\lambda^{\sigma}\left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}}\omega^{1-2\sigma} \tag{31}$$

$$V_1 = V_2$$

$$N = N_1 + N_2$$

I prove the existence and uniqueness of the equilibrium by showing that V_1 and V_2 can be expressed as monotone continuous functions of N_1 and that they have a unique intersection by the intermediate value theorem.

(i) V_1 and V_2 can be expressed as functions of N_1 First, I show that V_1 decreases in N_1 and that V_2 increases in N_1. It follows from(29) and (30) that

$$\frac{\partial V_1(N_1,\omega)}{\partial\omega} < 0, \frac{\partial V_2(N_2,\omega)}{\partial\omega} > 0$$

Then, notice that, given N_1, N_2, the RHS of (31) decreases in ω, taking into account $\frac{\partial V_1(\omega,N_1)}{\partial\omega}$ and $\frac{\partial V_2(\omega,N_2)}{\partial\omega}$. Also, given N_1, N_2,

$$\lim_{\omega\to\rho^{\frac{1}{\sigma}}\lambda} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \lambda^{\sigma} \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}} \omega^{1-2\sigma} \to +\infty \tag{32}$$

$$\lim_{\omega\to\rho^{-\frac{1}{\sigma}}\lambda} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}N_1^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}N_2^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \lambda^{\sigma} \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}} \omega^{1-2\sigma} \to 0 \tag{33}$$

Therefore, given N_1, N_2, it uniquely pins down ω_1 and, consequently, V_1 and V_2 from (29) and (30), respectively. With the workers clearing condition, V_1 and V_2 are functions of N_1.

(ii) ω increases in N_1 Suppose ω weakly decreases in N_1. Then, $V_1N_1^{\gamma}$ must strictly increase and $V_2N_2^{\gamma}$ strictly decreases in N_1 to satisfy (29) and (30), respectively. Then, ω needs to strictly increase in N_1 to satisfy (31). This contradicts that ω weakly decreases in N_1. Therefore, ω strictly increases in N_1. Assuming the differentiability,

$$\frac{d\omega}{dN_1} > 0$$

(iii) V_1 and V_2 are continuous in N_1 Given N_1 (and therefore N_2), (29) and (30) imply that $V_1 N_1^{\gamma}$ and $V_2 N_2^{\gamma}$ are continuous in ω on $(\rho^{\frac{1}{\sigma}}\lambda, \rho^{-\frac{1}{\sigma}})$, decrease and increase in ω, respectively, and are nonzero. Then, given N_1, the RHS of (31) is a continuous decreasing function of ω. Combined with (32) and (33), it follows that for all $N_1 \in (0, N)$ there exists $\omega \in (\rho^{\frac{1}{\sigma}}\lambda, \rho^{-\frac{1}{\sigma}}\lambda)$. Since ω increases in N_1, this implies ω is continuous on $N_1 \in (0, N)$. It immediately follows from (29) and (30) that V_1 and V_2 are continuous in $N_1 \in (0, N)$.

(iv) V_1 decreases and V_2 increases in N_1 Transform (29) and (30),

$$
\begin{aligned}
1 = {} & \lambda^{-\sigma\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \\
& + \left[\frac{\lambda^{-\sigma} - \rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
1 = {} & \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \\
& + \left[\frac{1 - \rho\lambda^{-\sigma}\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}
\end{aligned} \tag{35}
$$

Given Assumption 1, it follows that

$$
\frac{\partial V_1(N_1,\omega)}{\partial N_1} < 0, \ \frac{\partial V_2(N_2,\omega)}{\partial N_2} < 0
$$

Combining the results so far, the signs of the total derivatives can be obtained.

$$\frac{dV_1(N_1,\omega)}{dN_1} = \frac{\partial V_1(N_1,\omega)}{\partial N_1} + \frac{\partial V_1(N_1,\omega)}{\partial \omega_1}\frac{d\omega}{dN_1} < 0$$
$$\frac{dV_2(N_2,\omega)}{dN_1} = \frac{\partial V_2(N_2,\omega)}{\partial N_2}\frac{dN_2}{dN_1} + \frac{\partial V_2(N_2,\omega)}{\partial \omega}\frac{d\omega}{dN_1} > 0$$

(iv) $V_1 > V_2$ when $N_1 \to 0$ and $V_1 < V_2$ when $N_1 \to N$ and unique intersection When $N_1 \to 0$, $\omega \to \rho^{\frac{1}{\sigma}}\lambda$. Suppose $V_1 \not\to \infty$. This contradicts (34). Therefore, $V_1 \to \infty$ ($N_1 \to 0$). Meanwhile, V_2 is bounded following from (35). The same can be done when $N_1 \to 0$ or $N_2 \to N$. Thus,

$$\lim_{N_1\to 0} V_1 > \lim_{N_1\to 0} V_2$$

$$\lim_{N_1\to N} V_1 < \lim_{N_1\to N} V_2$$

Since V_1 and V_2 are a continuous decreasing function and a continuous increasing function of N_1, respectively, there is a unique intersection at $N_1 \in (0, N)$ by the intermediate value theorem. In addition, this shows that it is unstable for all workers to live in one location because after marginal migration, the utility of the small new city is greater than that of the large original city, and additional migration follows.

Proof of Proposition of 6

First, I prove that the productive city becomes larger by showing that the intersection of V_1 and V_2 in the proof of 4 is located in $N_1 \in (\frac{N}{2}, N)$. When $N_1 = N_2 = \frac{N}{2}$, suppose $V_1 \leq V_2$. Then, (29) and (30) implies that it is necessary that

$$\omega_{|\frac{N}{2}} > \lambda$$

Substitute $\frac{1-\rho\lambda^{-\sigma}\omega^{-\sigma}}{\lambda^{-\sigma}-\rho\omega^{\sigma}}$ in (31) with (29) and (30).

$$\begin{aligned}
\left(\frac{N_2}{N_1}\right)^{\frac{\sigma-1}{\sigma-\eta}} &= \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 N_1^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 N_2^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \lambda^{\sigma} \left[\frac{1-\rho\lambda^{-\sigma}\omega^{\sigma}}{\lambda^{-\sigma}-\rho\omega^{-\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}} \omega^{1-2\sigma} \\
&= \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 N_1^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 N_2^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \left(\lambda\omega^{-1}\right)^{2\sigma-1} \lambda^{1-\sigma} \\
&\cdot \left[\frac{1-\left[\frac{1}{N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 N_2^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2 N_2^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \left(\frac{N_2}{N_1}\right)^{\frac{1-\eta}{\sigma-\eta}}\right]^{\frac{\sigma-1}{1-\eta}} \\
&\cdot \left[\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 N_1^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{1-\left[\frac{\lambda^{-\sigma}}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1 N_1^{\gamma}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \left(\frac{N_2}{N_1}\right)^{\frac{1-\eta}{\sigma-\eta}}\right]^{\frac{\sigma-1}{1-\eta}}
\end{aligned}$$

Evaluate this equation at $N_1 = N_2 = \frac{N}{2}$.

$$
\begin{aligned}
1 = &\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \left(\lambda\omega_{|\frac{N}{2}}^{-1}\right)^{2\sigma-1} \\
&\cdot \lambda^{1-\sigma}\left[\frac{1-\left[\frac{2}{N}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}} \\
&\cdot \left[\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{1-\left[\frac{2}{N}\lambda^{-\sigma}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}} \\
< &\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \\
&\cdot \left[\frac{1-\left[\frac{2}{N}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}} \frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{1-\left[\frac{2}{N}\lambda^{-\sigma}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}} \\
= &\left[\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}+1} \left[\frac{1-\left[\frac{2}{N}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_2\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}{1-\left[\frac{2}{N}\lambda^{-\sigma}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(V_1\left(\frac{N}{2}\right)^{\gamma}\right)^{\epsilon_k\frac{\sigma-1}{\sigma-\eta}}}\right]^{\frac{\sigma-1}{1-\eta}}
\end{aligned}
$$

where the inequality follows from $\lambda < \omega_{|\frac{N}{2}}$. When $V_1 \leq V_2$, the value of the RHS is always smaller than 1. This contradicts the inequality. Therefore, $V_1 > V_2$ at $N_1 = \frac{N}{2}$, which implies that the unique intersection is located in $N_1 \in (\frac{N}{2}, N)$ because V_1 decreases and V_2 increases in N_1.

As for the wage level, think about the equilibrium conditions when $N_1 = N_2$. Suppose $\omega \leq \lambda$.

$$1 \leq \sum_{k \in \mathbb{N}} \lambda^{-\sigma \frac{1-\eta}{\sigma-\eta}} \left[\frac{1}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1 + \sum_{k \in \mathbb{T}} \lambda^{-\sigma \frac{1-\eta}{\sigma-\eta}} \left[\frac{1}{(1+\rho)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1$$

$$1 \geq \sum_{k \in \mathbb{N}} \left[\frac{1}{N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \sum_{k \in \mathbb{T}} \left[\frac{1}{(1+\rho)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}$$

$$\begin{aligned} 1 &\geq \frac{\lambda^{-\sigma \frac{1-\eta}{\sigma-\eta}} \left[\frac{1-\rho}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k \in \mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\left[\frac{1-\rho}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k \in \mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \lambda \\ &= \frac{\lambda^{\frac{\sigma-1}{\sigma-\eta}\eta} \sum_{k \in \mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}}{\sum_{k \in \mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}}} \end{aligned}$$

Since the first and the second imply $U_1 > U_2$, the RHS of the third is greater than 1. This contradicts the inequality in the third. Therefore, $\omega(N_1 = N_2 = \frac{N}{2}) > \lambda$. Because $N_1 > \frac{N}{2}$, and ω increases in N_1, $\omega > \lambda$.

D Equilibrium with Asymmetric Amenity

In this appendix, I impose symmetric fundamental productivity ($\lambda = 1$) and show how the asymmetric amenity ($a_1 \neq a_2$) generates a cross-city difference. I let city 1 have a higher amenity ($a_1 > a_2$) without loss of generality. The system of equations characterizing an equilibrium is given by

$$
\begin{aligned}
1 &= \left[\frac{1}{N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\omega^{-\sigma}}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \\
1 &= \left[\frac{1}{N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} + \left[\frac{1-\rho\omega^{\sigma}}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \\
\frac{\sum_{k\in\mathbb{T}} m_{1k}}{\sum_{k\in\mathbb{T}} m_{2k}} \frac{N_1}{N_2} &= \omega^{2\sigma-1} \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho\omega^{\sigma}}\right] \\
\sum_{k\in\mathbb{T}} m_{1k} &= \left[\frac{1-\rho\omega^{-\sigma}}{(1-\rho^2)N_1}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_1^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \\
\sum_{k\in\mathbb{T}} m_{2k} &= \left[\frac{1-\rho\omega^{\sigma}}{(1-\rho^2)N_2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} U_2^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \\
\frac{a_1 U_1}{a_2 U_2} &= \left(\frac{N_1}{N_2}\right)^{\gamma}
\end{aligned}
$$

Let V_1 and V_2 be

$$
V_1 = a_1 U_1 N_1^{-\gamma}
$$

$$
V_2 = a_2 U_2 N_2^{-\gamma}
$$

The system becomes

$$
\begin{aligned}
&\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \\
&\quad + \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} = 1 \\
&\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \\
&\quad + \left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} = 1 \\
&\frac{\left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_1^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}}{\left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} N_2^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}} \frac{N_1}{N_2} = \omega^{2\sigma-1}\left[\frac{1-\rho\omega^{-\sigma}}{1-\rho\omega^{\sigma}}\right] \\
&V_1 = V_2
\end{aligned}
$$

Consider the relative level of V_1 and V_2 at $N_1 = N_2 = \frac{N}{2}$ from the first three equations.

$$
\begin{aligned}
&\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \\
&\quad + \left[\frac{1-\rho\omega^{\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} = 1 \\
&\sum_{k\in\mathbb{N}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} \\
&\quad + \left[\frac{1-\rho\omega^{-\sigma}}{1-\rho^2}\right]^{\frac{1-\eta}{\sigma-\eta}} \sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}} = 1 \\
&\frac{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_1}{a_1}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}}{\sum_{k\in\mathbb{T}} \tilde{\beta}_k^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{V_2}{a_2}\right)^{\epsilon_k \frac{\sigma-1}{\sigma-\eta}} \left(\frac{N}{2}\right)^{\left(\gamma\epsilon_k - \frac{1-\eta}{\sigma-1}\right)\frac{\sigma-1}{\sigma-\eta}}} = \omega^{2\sigma-1}\left[\frac{1-\rho\omega^{-\sigma}}{1-\rho\omega^{\sigma}}\right]^{\frac{\sigma-1}{\sigma-\eta}}
\end{aligned}
$$

Suppose $\omega > 1$. The first and the second equations imply $\frac{V_1}{a_1} > \frac{V_2}{a_1}$. However, the third implies $\frac{V_1}{a_1} < \frac{V_2}{a_1}$. These contradict each other. So, when $N_1 = N_2 = \frac{N}{2}$, the first three equations imply

$\omega \leq 1$. Then, it follows from the third

$$\frac{V_1(N_1 = \frac{N}{2})}{a_1} \geq \frac{V_2(N_1 = \frac{N}{2})}{a_1} \implies V_1(N_1 = \frac{N}{2}) \geq \frac{a_1}{a_2} V_2(N_1 = \frac{N}{2}) > V_2(N_1 = \frac{N}{2})$$

Therefore, it follows from the intermediate value theorem that the the equilibrium is located in $N_1 \in (\frac{N}{2}, N)$ when Assumption 1 holds. In addition, since ω increases in N_1, the equilibrium wage level is higher in city 1 than in city 2 ($\omega = \frac{w_1}{w_2} > 1$).

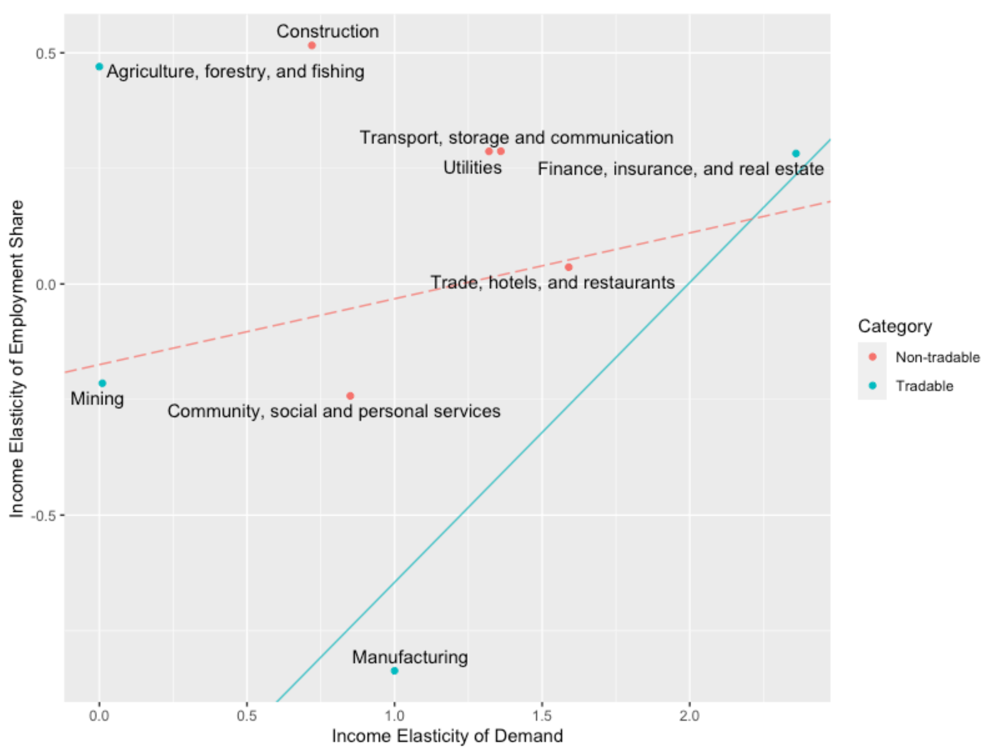
E Amplification Effect Test with Non-tradable Sectors

Figure 1 and 2 use income-elasticity estimates obtained from international trade data. Therefore, the scope of these estimates is limited to tradable sectors. To determine the degree to which the model's prediction with non-tradable sectors is consistent with the actual production pattern, I use the income-elasticity estimates (which include non-tradables) calculated by Comin et al. (2021) for 9 sectors: (1) agriculture, forestry and fishing; (2) mining and quarrying; (3) manufacturing; (4) public utilities; (5) construction; (6) wholesale and retail trade, hotels and restaurants; (7) transport, storage and communication; (8) finance, insurance, and real estate; and (9) community, social and personal services. The procedure is largely the same as before except for two differences. First, because the Comin et al. (2021) sector classification is based on Groningen's 10-Sector Database , I follow the US Census concordance from 2002 NAICS to ISIC Rev3.1., and so I map the MSA's sectoral employment data in 4 digit NAICS to the 9 sectors [8]. Second, the employment shares are defined within the non-tradable sectors and the tradable sectors following the model's implication: (1) agriculture, (2) mining, (3) manufacturing, and (8) FIR are classified as tradable and the rest are classified as as non-tradable. The result is shown in Figure 6. The dashed (red) line is the weighted regression line of the elasticity of the within-employment share of the non-tradable sectors on the income-elasticities, with the U.S. aggregate sectoral employment as the weights. The solid

[8]When one NAICS code corresponds to multiple codes in ISIC, I allocate the value evenly.

(blue) line is that of the tradable sectors. Although the small sample size should not be ignored, it is noteworthy that this graph is consistent with the model predictions in two respects. First, whether or not the sectors are tradable, Figure 6 shows the same positive relationship as in Figure 2, which the model successfully generates. Second, the positive relationship is stronger for tradable sectors, which is consistent with the model's prediction that the home market effect amplifies the expenditure share difference.

Figure 6: Elasticity of the Within-Employment Share with respect to MSA's Income Level Conditioned on Skill Supply and Elasticity of Demand with respect to Income with Comin et al. (2021) estimates.



Income is per capita personal income in 2006 from the Bureau of Economic Analysis. The within-employment share is derived as the share of employment of a sector in the corresponding group (tradable or non-tradable) in each 2006 MSA in CBP. When obtaining the income-elasticity of the within employment share, the college graduate share in the labor force and the region $\in \{Northeast, Midwest, South, West\}$ are controlled. income-elasticity estimates are from Comin et al. (2021) and are for OECD countries.

F Alternative Specification

I test the association of the income level and the income-elasticities of the local industries with alternative identification. This test follows Nunn (2007), and the regression model is given by

$$\begin{aligned} y_{mk} = & \alpha \cdot \exp(\beta_{income} \cdot \epsilon_k \cdot \log(Income_m) + \beta_{skill} \cdot \theta_k \cdot \log(College_m) \\ & + \sum_m \gamma_m D_m + \sum_{region} \gamma_{k,region} D_{k,region})) \cdot e_{mk} \end{aligned} \tag{36}$$

where

y_{mk} : employment level of industry k in MSA m

ϵ_k : income elasticity of demand in industry k

$Income_m$: per capita personal income in MSA m

θ_k : skill intensity of goods in industry$k$

$College_m$: college graduate share in labor force in MSA m

D_m : MSA dummy variable

$D_{k,region}$: (industry k×region$\in \{Northeast, Midwest, South, West\}$) dummy variable

e_{mk} : error term for MSA m × industry k

The coefficient of interest is β_{income}. When β_{income} is positive, the employment level rises more for high ϵ_k as the income level rises, which is consistent with the stylized fact and the model prediction. Two points are noteworthy. First, as the regression model shows, I implement level regressions by the Poisson Pseudo Maximum Likelihood (PPML) estimation. As discussed in Silva and Tenreyro (2006), log-linear regressions require a very specific condition on error terms to obtain consistent estimators. Moreover, in log-linear estimations, it is problematic when zeros are contained in the data. On the other hand, PPML provides consistent estimators that do not require this condition, and it is efficient with various error term patterns. For this reason, PPML is very common in gravity equation estimations in international trade where zeros are prevalent and error terms show heteroskedasiticy. In my dataset, 17% of the sample is zero. To address these zeros, I use PPML, and I set employment levels (instead of employment shares) as the dependent variable so that the error terms show a heteroskedasticity pattern that is suitable for PPML in terms of efficiency. Second, I control the supply side effect using the skill-intensity of sectors, as in the main empirical work. This time, the interaction term, $\theta_k \cdot \log(College_m)$, is used to implement the control, and the estimates of skill intensities are again borrowed from Caron et al. (2020), who obtained them through a structural estimation that had both heterogeneous skill-intensities and income-elasticities.

Table 3: Regression Results of Alternative Specification

	employment (PPML)			log(employment+1)		
	2006	2011	2016	2006	2011	2016
	(1)	(2)	(3)	(4)	(5)	(6)
β_{income}	2.331***	2.310***	1.970***	3.708***	3.443***	2.773***
	(0.214)	(0.211)	(0.207)	(0.564)	(0.520)	(0.481)
β_{skill}	2.519***	3.299***	3.390***	2.329***	3.113***	3.502***
	(0.228)	(0.222)	(0.246)	(0.610)	(0.583)	(0.588)
Observations	8,382	8,415	7,986	8,382	8,415	7,986
R^2				0.852	0.855	0.857
Adjusted R^2				0.845	0.848	0.851

Notes: ***,**, and * denote significance at the 1 %, 5%, and 10% levels, respectively. MSA and industry×region are controlled in all of the regressions.

Because this regression is cross-sectional, I implement it for three different years separately to make use of the annual data. The results are shown in Table 3. The results with PPML are shown in the left three columns, while those in the right three columns show log-linear regressions after

1 is added to every observation to take care of zeros. In all of the results, β_{income} is significantly positive, which is consistent with the stylized fact and the model prediction. The size of β_{income} is between 1.9 and 2.3 in PPML. For example, to see the impact of this β_{income} using 2006 data, suppose, first, that City A has a per capita income that is higher than the sample mean ($34,765) by one standard deviation ($7,381) and, second, that City B has a per capita income that is lower than the sample mean by one standard deviation. Then, the employment ratio of a sector with $\epsilon_k = 1.2$ over that with $\epsilon_k = 0.8$ ($\frac{N_{\epsilon_k=1.2}}{N_{\epsilon_k=0.8}}$) is 1.48 times greater in City A than in City B.

It is not straightforward to measure the explanatory power of the PPML estimators because technically they are obtained by a maximum likelihood estimation. To construct a measurement for the PPML estimators, I calculate the estimated employment shares implied by the fitted values for the employment levels and obtain the residual sum of squares (RSS) from the difference between the estimated shares and the actual data. Using this RSS, a measurement which analogous to R^2 in linear regressions is constructed and summarized in Table 4. In the first row, RSS_{FE} is the RSS when the regression has only the FE effects of MSA and industry $\times$ region and TSS is the residual sum of squares by the unconditional mean of the employment shares. Table 4 shows that the FE effects explain 93% of TSS in all of the three years. In the second row, $RSS_{FE+skill}$ is obtained by the regression that has the skill interaction term as an additional control. Given an industry and a region, the skill supply effect explains 9%-12% of the variation. Similarly, in the third row, $RSS_{FE+income}$ is obtained by the regression, here with the income interaction term instead of the skill interaction, and the income effect explains 13% of the variation in all the three years. Finally, in the last row, $RSS_{FE+skill+income}$ is by the regression that has the same controls as (36), and the skill and income effects jointly capture 19%-22% of the variation. These results suggest that, given an industry and a region, the income effect can explain a significant portion of the variation in the employment share, even after controlling the skill supply. Moreover, the explanatory power is not smaller than that of the skill effect.

Table 4: Explanatory Power

	2006	2011	2016
$1 - \frac{RSS_{FE}}{TSS}$	0.93	0.93	0.93
$1 - \frac{RSS_{FE+skill}}{RSS_{FE}}$	0.09	0.12	0.09
$1 - \frac{RSS_{FE+income}}{RSS_{FE}}$	0.13	0.13	0.13
$1 - \frac{RSS_{FE+skill+income}}{RSS_{FE}}$	0.19	0.22	0.19