Section4. Applications of the Normal Distribution

Example 6.7

A certain type of storage battery lasts, on average, 3.0 years, with a standard deviation of 0.5 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years.

Solution

X: life of the battery, $X \sim N(3, 0.5^2)$. We have

$$P(X < 2.3) = P(\frac{X-3}{0.5} < \frac{2.3-3}{0.5}) = P(Z < -1.4) = 0.0808.$$

Example 6.8

An electronic firm manufacturers light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Solution

X: life of the light bulbs, $X \sim N(800, 40^2)$. We have

$$
\begin{aligned}
P(778 < X < 834) &= P(\frac{778-800}{40} < \frac{X-800}{40} < \frac{834-800}{40}) \\
&= P(-0.55 < Z < 0.85) \\
&= P(Z < 0.85) - P(Z < -0.55) \\
&= 0.8023 - 0.2912 = 0.5111
\end{aligned}
$$

Example 6.9

In an industrial process the diameter of a ball bearing is an important part. The buyers set specifications on the diameter to be 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean 3.0 and standard deviation $\sigma = 0.005$. On the average, how many manufactured ball bearings will be scraped?

Solution

$$P(2.99 < X < 3.01) = P(-2.0 < Z < 2.0)$$

From Table A.3, $P(Z < -2.0) = 0.0228$. Due to symmetry of the normal distribution, we find that

$$P(-2.0 < Z < 2.0) = 1 - 2 \times 0.0228 = 0.9544.$$

As a result, it is anticipated that on the average, 4.56% of manufactured ball bearing will be scarapped.

Example 6.10

Gauges are used to reject all components where a certain dimension is not within the specification $1.50 \pm d$. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 0.2. Determine the value d such that the specifications 'cover' 95% of the measurements.

Solution

From Table A.3 we know that

$$P(-1.96 < Z < 1.96) = 0.95$$

Therefore,

$$1.96 = \frac{(1.50 + d) - 1.50}{0.2}$$

from which we obtain

$$d = 0.2 \times 1.96 = 0.392$$

An illustration of the specification is shown in Figure 6.17

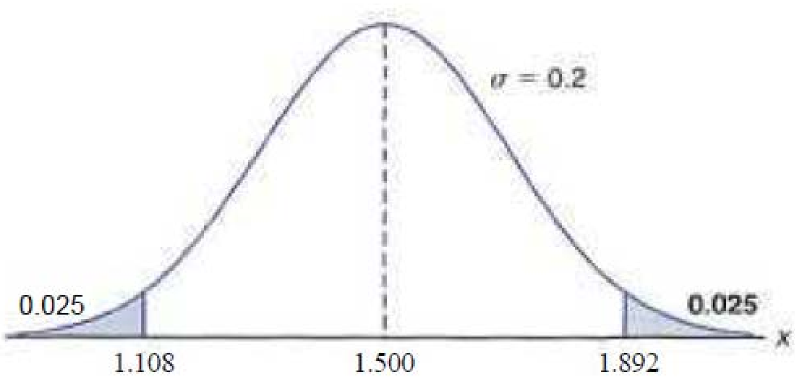

Figure 6.17: Specifications for Example 6.10.

Example 6.11

A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms?

Solution

A percentage is found by multiplying the relative frequency by 100%. Since the relative frequency for an interval is equal to the probability of falling in the interval, we must find the area to the right of $x = 43$ to the corresponding z-value, obtaining the area to the left of z from Table A.3, and then subtracting this area from 1. We find

$$z = \frac{43 - 40}{2} = 1.5.$$

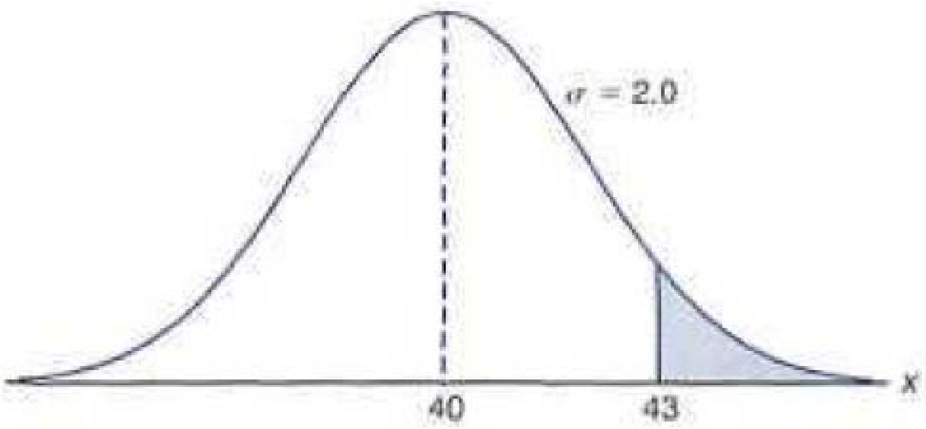

Figure 6.18: Area for Example 6.11.

Hence

$P(X > 43) = P(Z > 1.5) = 1 - P(Z < 1.5) = 1 - 0.9332 = 0.0668$

Therefore, 6.68% of the resistors will have a resistance exceeding 43 ohms.