Chapter 2. Probability

1. Sample Space

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• classify items coming off an assembly line as "defective" or "nondefective";

• the volume of gas released in a chemical reaction when the concentration of an acid is varied.

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We shall refer to any recording of information, whether it be numerical or categorical, as an **observation**.

- The number $2, 0, 1$ and 2, representing the number of accidents that occurred for each month from January through April during the past year at the intersection of Driftwood Lane and Royal Oak Drive, constitute a set of **observations**.

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- The categorical data N, D, N, D and D,representing the items found to be defective or nondefective when five items are inspected, are recorded as **observations**.

experiment

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• the launching of a missile and observing its velocity at specified times.

• The opinions of voters concerning a new sales tax can also be considered as observations of an experiment.

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Example: a coin is tossed repeatedly

- cannot predict the result of a given toss
(i.e. the outcome depend on chance)
- know the entire set of possibilities for each toss.

sample space

Definition 2.1 The set of all possible outcomes of a statistical experiment is called the **sample space** and is represented by S.

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Each outcome in a sample space is called an **element** or a **sample point**.

sample space

What's the possible outcomes when a coin is tossed?

sample space

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The sample space may be written

$$S = \{H, T\},$$

where H and T corresponds to 'heads' and 'tails', respectively.

sample space

Example 2.1 Consider the experiment of tossing a die.

interested in the number that shows on the top face

$$S_1 = \{1, 2, 3, 4, 5, 6\}$$

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sample space

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S_1 provides **more information** than S_2.

sample space

Remarks:

• more than one sample space can be used to describe the outcomes of an experiment.

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- It is desirable to use a sample space that gives the most information.

sample space

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The sample space is

$$S = \{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}.$$

sample space

sample space with a large or infinite number of sample points

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sample space

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How to describe?

by a statement or rule

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- all points (x, y) on the boundary or the interior of a circle of radius 2 with center at the origin

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- all points (x, y) on the boundary or the interior of a circle of radius 2 with center at the origin

$$S = \{(x, y)|x^2 + y^2 \leq 4\}$$

2. Events

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event A: the outcome is divisible by 3.

Event A will occur if the outcome is an element of the set $A = \{3, 6\}$.

$A = \{3, 6\}$ is the subset of the sample space $S_1 = \{1, 2, 3, 4, 5, 6\}$ in Example 2.1.

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Definition 2.2 An **event** is a subset of a sample space.

event

Example 2.4. Given the sample space $S = \{t|t \geq 0\}$, where t is the life in years of a certain electronic component. Event A is that the component fails before the end of the fifth year.

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$$\text{subset} \quad A = \{t | 0 \leq t < 5\}$$

event

Two special subset

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a subset that includes the entire sample space S

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- a subset that includes the entire sample space S
- a subset contains no elements at all, denoted by $\emptyset$, called **null set.**

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event

Two special subset

a subset that includes the entire sample space S

a subset contains no elements at all, denoted by $\emptyset$, called **null set.**

Example

$B = \{x | x \quad is \quad an \quad even \quad factor \quad of \quad 7\}$,

then $B = \emptyset$, since the only possible factors of 7 are odd numbers 1 and 7.

complement

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the complement of A: A'

complement

Example 2.5 Let R be the event that a red card is selected from an ordinary deck of 52 playing cards and let S be the entire deck. What is R'?

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R' is the event that the card selected from the deck is not red but a black card.

intersection

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Example 2.7

Let P be the event that a person selected at random while dining at a popular cafeteria is a taxpayer, and let Q be the event that the person is over 65 years of age. Then the event $P \cap Q$ is the set of all taxpayers in the cafeteria who are over 65 years of age.

intersection

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That is, M and N have no elements in common and, therefore, cannot both occur simultaneously.

intersection

Definition 2.5 Two events A and B are **mutually exclusive**, or **disjoint** if $A \cap B = \emptyset$, that is, if A and B have no elements in common.

union

Definition 2.6 The **union** of the two events A and B, denote by symbol $A \cup B$, is the event containing all the elements that belong to A or B or both.

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Example 2.10 Let $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$; then

$$A \cup B = \{a, b, c, d, e\}.$$

union

Example 2.11 Let P be the event that an employee selected at random from an oil drilling company smokes cigarettes. Let Q be the event that the employee selected drinks alcoholic beverages. Then the event $P \cup Q$ is the set of all employees who either drink or smoke, or do both.

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Example 2.12 If $M = \{x|3 < x < 9\}$, $N = \{x|5 < x < 12\}$,

$$\text{then } M \cup N = \{x|3 < x < 12\}.$$

The relationship between events and the corresponding sample space can be illustrated graphically by means of **Venn diagrams**.

several results

The following results may easily be verified by means of **Venn diagrams**

$A \cap \emptyset = \emptyset \qquad A \cup \emptyset = A$

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$$A \cap A' = \emptyset \qquad A \cup A' = S$$

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$$A \cap A' = \emptyset \qquad A \cup A' = S$$
$$S' = \emptyset \qquad \emptyset' = S$$

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The following results may easily be verified by means of **Venn diagrams**

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$A \cap A' = \emptyset \qquad A \cup A' = S$

$S' = \emptyset \qquad \emptyset' = S$

$(A')' = A$

several results

The following results may easily be verified by means of **Venn diagrams**

$$A \cap \emptyset = \emptyset \qquad A \cup \emptyset = A$$
$$A \cap A' = \emptyset \qquad A \cup A' = S$$
$$S' = \emptyset \qquad \emptyset' = S$$
$$(A')' = A$$

$$(A \cap B)' = A' \cup B'$$
$$(A \cup B)' = A' \cap B'$$

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The following results may easily be verified by means of **Venn diagrams**

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