

Matrix Computations*

Course Title:	Matrix Computations*
Course prerequisites:	Optional course
Class hour:	32
Department:	Department of Mathematics
Students:	14 级信息与计算科学(全英文)

Course description

Matrix Computation is more exciting now than at almost any time in the past. Its applications continue to spread to more and more fields. Largely due to the computer revolution of the last half century, matrix Computation has risen to a role of prominence in the mathematical curriculum rivaling that of calculus. Modern software has also made it possible to dramatically improve the way the course is taught. This course is an introduction to Linear Algebra. It also introduces the basics of abstract mathematics as well as a software package that is valuable both for learning and using linear algebra. The main development of the course will follow the text

Course goals.

- To provide students with a good understanding of the concepts and methods of linear algebra, described in detail in the syllabus.
- To help the students develop the ability to solve problems using linear algebra.
- To connect linear algebra to other fields both within and without mathematics.
- To develop abstract and critical reasoning by studying logical proofs and the axiomatic method as applied to linear algebra.

Textbook

Numerical Linear Algebra with Applications, First Edition 2015, William Ford, University of the Pacific.

Grades

Your course grade will be determined using the following formula:

30% Class Presentation

30% Programming

40% Final Manuscript

Topics

This is a 1-semester course in matrix computation for students who have completed two semesters of calculus. It covers Matrices, Linear Equations, Subspaces, Determinants, Eigenvalues and Eigenvectors, Orthogonal Vectors and Matrices. We will follow the textbook and aim to cover in full or in part the following chapters:

1. Matrices
 - 1.1 Matrix Arithmetic
 - 1.1.1 Matrix Product
 - 1.1.2 The Trace
 - 1.1.3 MATLAB Examples
 - 1.2 Linear Transformations
 - 1.2.1 Rotations
 - 1.3 Powers of Matrices
 - 1.4 Nonsingular Matrices
 - 1.5 The Matrices Transpose and Symmetric Matrices
2. Systems of Equations
 - 2.1 Introduction to Linear Equations
 - 2.2 Solving square linear systems
 - 2.3 Gaussian Elimination
 - 2.3.1 Upper-Triangular Form
 - 2.4 Systematic Solution of Linear Systems
 - 2.5 Computing the inverse
 - 2.6 Homogeneous systems
 - 2.7 Application: A Truss
3. Subspaces
 - 3.1 Introduction
 - 3.2 Subspaces
 - 3.3 Linear independence
 - 3.4 Basis of a Subspace
 - 3.5 The Rank of a Matrix
4. Determinants
 - 4.1 Developing the Determinant of a 2×2 and a 3×3 Matrix
 - 4.2 Expansion by Minors
 - 4.3 Computing a Determinant Using Row Operations
 - 4.4 Application: Encryption
5. Eigenvalues and Eigenvectors
 - 5.1 Definitions and Examples
 - 5.2 Selected Properties of Eigenvalues
 - 5.3 Diagonalization
 - 5.3.1 Powers of Matrices
 - 5.4 Applications
 - 5.5 Computing Eigenvalues and Eigenvectors using MATLAB
6. Orthogonal Vectors and Matrices
 - 6.1 Introduction

- 6.2 The Inner Product
- 6.3 Orthogonal Matrices
- 6.4 Symmetric Matrices and Orthogonality
- 6.5 The L^2 Inner Product
- 6.6 The Cauchy-Schwarz Inequality

- 7. Gaussian Elimination and the LU Decomposition
 - 7.1 LU Decomposition
 - 7.2 Using LU to Solve Equations
 - 7.3 Elementary Row Matrices
 - 7.4 Derivation of the LU Decomposition
 - 7.4.1 Colon Notation
 - 7.4.2 The LU Decomposition Algorithm
 - 7.4.3 LU Decomposition Flop Count
 - 7.5 Gaussian Elimination with Partial Pivoting
 - 7.5.1 Derivation of $PA=LU$
 - 7.5.2 Algorithm for Gaussian Elimination with Partial Pivoting
 - 7.6 Using the LU Decomposition to solve $Ax=b$
 - 7.7 Find inverse of A
 - 7.8 Stability and Efficiency of Gaussian Elimination
 - 7.9 Iterative Refinement

- 8. Gram-Schmidt Orthonormalization
 - 8.1 The Gram-Schmidt Process
 - 8.2 Numerical Stability of the Gram-Schmidt Process
 - 8.3 The QR Decomposition
 - 8.3.1 Efficiency
 - 8.3.2 Stability
 - 8.4 Applications of the QR Decomposition
 - 8.4.1 Computing the Determinant
 - 8.4.2 Finding an Orthonormal Basis for the Range of a Matrix

- 9. The Singular Value Decomposition
 - 9.1 The SVD Theorem
 - 9.2 Using the SVD to Determine Properties of a Matrix
 - 9.2.1 The Four Fundamental Subspaces of a Matrix
 - 9.3 SVD and Matrix Norms
 - 9.4 Geometric Interpretation of the SVD
 - 9.5 Computing the SVD Using MATLAB

- 10. Least-Squares Problems
 - 10.1 Existence and Uniqueness of Least-Squares Solutions
 - 10.1.1 Existence and Uniqueness Theorem
 - 10.1.2 Normal Equations and Least-squares Solutions

10.1.3 The Pseudoinverse

11. Basic Iterative Methods

11.1 Jacobi Method

11.2 The Gauss-Seidel Iterative Method

11.3 The SOR Iteration

11.4 Convergence of the Basic Iterative Methods

11.4.1 Matrix Form of the Jacobi Iteration

11.4.2 Matrix Form of the Gauss-Seidel Iteration

11.4.3 Matrix Form for SOR

11.4.4 Conditions Guaranteeing Convergence

11.4.5 The Spectral Radius and Rate of Convergence

11.4.6 Convergence of the Jacobi and Gauss-Seidel Methods for Diagonally Dominant

Matrices

11.4.7 Choosing w for SOR

11.5 Applications: Poisson's Equation

12. Krylov Subspace Methods

12.1 Large, Sparse Matrices

12.1.1 Storage of Sparse Matrices

12.2 The CG method

12.2.1 The Method of Steepest Descent

12.2.2 From Steepest Descent to CG

12.2.3 Convergence

12.3 Preconditioning

12.4 Preconditioning for CG

12.4.1 Incomplete Cholesky Decomposition

12.4.2 SSOR Preconditioner

12.5 Krylov Subspaces

12.6 The Arnoldi Method

12.6.1 An Alternative Formulation of the Arnoldi Decomposition

12.7 GMRES

12.7.1 Preconditioned GMRES

12.8 The Symmetric Lanczos Method

12.9 The MINRES Methods

13. Large Sparse Eigenvalue Problems

13.1 The Power Method

13.2 Eigenvalue Computation Using the Arnoldi Process

13.2.1 Estimating Eigenvalues Without Restart or Deflation

13.2.2 Estimating Eigenvalues Using Restart

13.2.3 A Restart Method Using Deflation

13.2.4 Restart Strategies

13.3 The Implicitly Restarted Arnoldi Method

13.3.1 Convergence of the Arnoldi Iteration

13.4 Eigenvalue Computation Using the Lanczos Process