Chapter 6(1) Some Continuous Probability Distributions

1.Continuous Uniform Distribution

One of the simplest continuous distributions in all of statistics is the **continuous uniform distribution**.

Continuous Uniform Distribution

Uniform Distribution

The density function of the continuous uniform random variable X on the interval $[A, B]$ is

$$f(x; A, B) = \begin{cases} \dfrac{1}{B-A}, & A \leq x \leq B \\ 0, & \text{elsewhere.} \end{cases}$$

Continuous Uniform Distribution

This distribution is characterized by a density function that is 'flat', and thus the probability is uniform in a closed interval, say $[A, B]$.

Continuous Uniform Distribution

The density function for a uniform random variable on the interval $[1, 3]$ is shown in Figure 6.1.

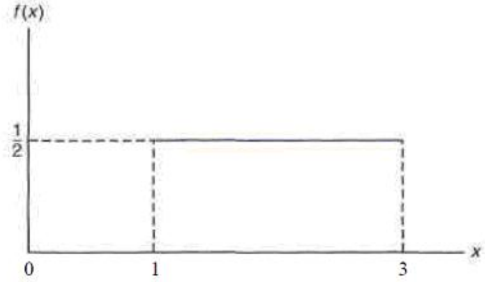

Figure 6.1: The density function for a random variable on the interval [1,3].

Continuous Uniform Distribution

Example 6.1

Suppose that a large conference room for a certain company can be reserved for no more than 4 hours. However, the use of the conference room is such that both long and short conferences occur quite often. In fact, it can be assumed that length X of a conference has a uniform distribution on the interval $[0, 4]$.

(a) What is the probability density function?

(b) What is the probability that any given conference lasts at least 3 hours?

Continuous Uniform Distribution

Theorem 6.1

The mean and variance of the uniform distribution are

$$\mu = \frac{A+B}{2} \quad \text{and} \quad \sigma^2 = \frac{(B-A)^2}{12}.$$

2. Normal Distribution

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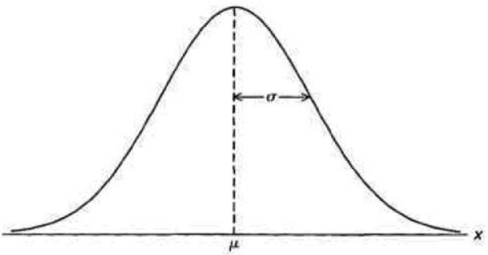

Figure 6.2: The normal curve.

Normal Distribution

The normal distribution is often referred to as the **Gaussian distribution**, in honor of Karl Friedrich Gauss (1777-1855).

Normal Distribution

Normal Distribution

The density function of the normal random variable X, with two parameters μ and $\sigma > 0$, is

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\infty < x < \infty,$$

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$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\infty < x < \infty,$$

where μ and σ are mean and standard deviation of the normal distribution.

Normal Distribution

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2. The curve is symmetric about a vertical axis through the mean μ.

3. The curve has its points of inflection at $x=\mu\pm\sigma$, is concave downward if $\mu-\sigma<X<\mu+\sigma$, and is concave upward otherwise.

4. The normal curve approaches the horizontal axis asymptotically as we proceed in either direction away from the mean.

Normal Distribution

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The two curves are **identical in form** but are **centered at different positions** along the horizontal axis.

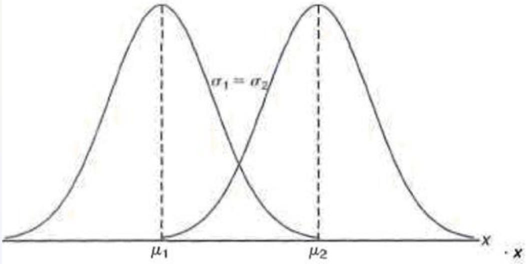

Figure 6.3: Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$.

Normal Distribution

In Figure 6.4 we sketched two normal curves with **the same mean but different standard deviations**.

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The two curves are **centered at exactly the same position** on the horizontal axis, **but** the curve with the larger standard deviation is lower and spreads out farther.

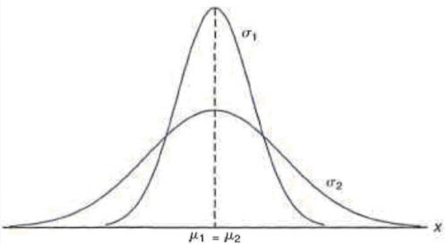

Figure: 6.4: Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$.

Normal Distribution

Remark: The area under a probability curve must be equal to 1, and therefore the more variable the set of observations the lower and wider the corresponding curve will be.

Normal Distribution

Figure 6.5 shows the results of sketching two normal curves having **different means and different standard deviations**.

Clearly, they are **centered at different positions** on the horizontal axis and their shapes reflect the **two different values of σ**.

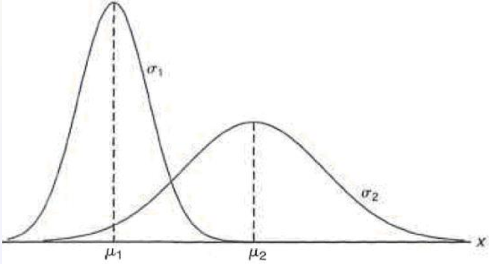

Figure 6.5: Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$.

Normal Distribution

We shall now show that the parameter μ and σ^2 are indeed the **mean** and the **variance** of the normal distribution.