Kernel method

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1 Functional Analysis

Definition 1.1. (vector space)
Let K, $\mathbb{R}$ be a field and a set with two operations, addition and scalar multiplication. If V statisfies

1. V becomes commutative group by addition,
2. $\forall \alpha \in \mathbb{K}, \forall u, v \in V, e\alpha(u+v) = \alpha u + \alpha v$,
3. $\forall \alpha, \beta \in \mathbb{K}, \forall v \in V, (\alpha + \beta)v = \alpha v + \beta v$,
4. $\forall \alpha, \beta \in \mathbb{K}, \forall v \in V, \alpha(\beta v) = (\alpha\beta)v$ and
5. $\exists 1_{\mathbb{K}} \in \mathbb{K}$ s.t. $\forall v \in V, 1_{\mathbb{K}} v = v$.

Then, V is called **vector space** over $\mathbb{K}$.

We consider only real verctor space or complex vector space in this article. So, $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$.

Definition 1.2. (normed vector space)
Let V, $\|\cdot\|$ be a vector space over $\mathbb{K}$ and a map from $V \times V$ to $\mathbb{K}$. $\|\cdot\|$ is called **norm** on $\mathbb{K}$ when $\|\|$ statisfies

1. $\forall v \in V, \|v\| \geq 0$,
2. $\forall v \in V, \|v\| = 0 \iff v = 0$,
3. $\forall \alpha \in \mathbb{K}, \forall v \in V, \|\alpha v\| = |\alpha|\|v\|$ and
4. $\forall v, w \in V, \|v + w\| \leq \|v\| + \|w\|$.

A pair $(V, \|\|)$ is called **normed vector space** or **normed space**

Proposition 1.3. Suppose that $(V, \|\|)$ is normed space. Then, norm space is metric space by a distance generated $d : V \times V \to \mathbb{K}$ by norm,

$$d(x, y) = \|x - y\|.$$