

Optimization Algorithms on Riemannian Manifold

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Riemannian Manifold

Definition (Riemannian Manifold)

M を可微分多様体とする. 任意の接空間 $T_x M$ に内積 $g : T_x M \times T_x M \rightarrow \mathbb{R}$ が定まっているとき, 組 (M, g) をリーマン多様体 (Riemannian Manifold) といい, g をリーマン計量と呼ぶ.

Definition (Gradient)

(M, g) をリーマン多様体とし, $f : M \rightarrow \mathbb{R}$ を可微分写像とする.
 $x \in M$ について

$$\forall \xi \in T_x M, g(\operatorname{grad} f(x), \xi) = Df(x)[\xi]$$

を満たす一意な $\operatorname{grad} f(x) \in T_x M$ を f の x での勾配 (Gradient) と呼ぶ.

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Sphere

Example (Sphere)

自然数 $n \geq 2$ に対して, 球面 $S^{n-1} := \{x \in \mathbb{R}^n \mid x^\top x = 1\}$ は可微分多様体となる. また, $x \in S^{n-1}$ での接空間 $T_x S^{n-1}$ は

$$T_x S^{n-1} = \{z \in \mathbb{R}^n \mid x^\top z = 0\}$$

となる. ここで, $g_x : T_x S^{n-1} \times T_x S^{n-1} \rightarrow \mathbb{R}$ を

$$g_x(\xi, \eta) = \xi^\top \eta$$

と定めれば, g_x は $T_x S^{n-1}$ の内積となるので, S^{n-1} はリーマン多様体である.

Stiefel Manifold

Example (Stiefel Manifold)

$X = [x_1 x_2 \cdots x_p] \in \mathbb{R}^{n \times p} (n \geq p)$ で, $\{x_i\}_{i=1}^p$ が正規直交系であるような $n \times p$ 行列全体は可微分多様体となる. この多様体をシュティーフェル多様体 (*Stiefel Manifold*) といい, $\text{St}(p, n)$ と表す. すなわち

$$\text{St}(p, n) = \{X \in \mathbb{R}^{n \times p} \mid X^\top X = \mathbb{I}_p\}.$$

である.

$$X \in \text{St}(p, n)$$

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