# 11 Exercise solutions – Construction of scoring matrix

#### 1. Scoring schemes for protein alignments

Calculate the score of the alignment by using different scoring schemes.

Seq1 R-HIC Seq2 RDDCC

(a) Use the identity with a simple scoring scheme as match: 1, mismatch: 0, and gap penalty: 0.

Solution: 2

(b) Use the genetic code.

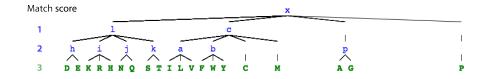
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|----------|--------------------|-----------------|-----------------|--------------|----------|
| First    | $Second\ position$ |                 |                 |              | Third    |
| position | T                  | $^{\mathrm{C}}$ | Α               | G            | position |
| Т        | F                  | S               | Y               | С            | Т        |
|          | F                  | $\mathbf{S}$    | Y               | $\mathbf{C}$ | С        |
| 1        | ${ m L}$           | $\mathbf{S}$    | Stop            | Stop         | A        |
|          | ${ m L}$           | $\mathbf{S}$    | Stop            | W            | G        |
|          | L                  | Р               | Н               | R            | Т        |
| С        | L                  | Р               | $_{\mathrm{H}}$ | $\mathbf{R}$ | С        |
|          | ${ m L}$           | Р               | Q               | $\mathbf{R}$ | A        |
|          | ${ m L}$           | Р               | Q               | $\mathbf{R}$ | G        |
|          | I                  | Τ               | N               | S            | Т        |
|          | I                  | ${ m T}$        | N               | $\mathbf{S}$ | С        |
| A        | I                  | ${ m T}$        | K               | $\mathbf{R}$ | A        |
|          | M                  | ${ m T}$        | K               | $\mathbf{R}$ | G        |
| G        | V                  | A               | D               | G            | Т        |
|          | V                  | A               | D               | G            | С        |
|          | V                  | A               | $\mathbf{E}$    | G            | A        |
|          | V                  | A               | E               | G            | G        |

| A            | Ala                  | Alanine       |
|--------------|----------------------|---------------|
| $\mathbf{C}$ | Cys                  | Cysteine      |
| D            | Asp                  | Aspartic acid |
| $\mathbf{E}$ | $\operatorname{Glu}$ | Glutamic acid |
| $\mathbf{F}$ | Phe                  | Phenylalanine |
| G            | Gly                  | Glycine       |
| Η            | His                  | Histidine     |
| I            | Ile                  | Isoleucine    |
| K            | Lys                  | Lysine        |
| L            | Leu                  | Leucine       |
| $\mathbf{M}$ | Met                  | Methionine    |
| Ν            | $\operatorname{Asn}$ | Asparagine    |
| Ρ            | $\operatorname{Pro}$ | Proline       |
| Q            | Gln                  | Glutamine     |
| $\mathbf{R}$ | $\operatorname{Arg}$ | Arginine      |
| $\mathbf{S}$ | $\operatorname{Ser}$ | Serine        |
| $\mathbf{T}$ | $\operatorname{Thr}$ | Threonine     |
| V            | Val                  | Valine        |
| W            | Trp                  | Tryptophan    |
| Y            | Tyr                  | Tyrosine      |
|              |                      |               |

Solution: 9

CGU --- CAU AUU UGU CGU GAU GAU UGU UGU

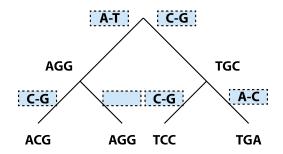
(c) Use the AACH.



**Solution:** 3 + 0 + 1 + 1 + 3 = 8

# 2. Probabilities of accepted mutations

Use a phylogenetic tree below to calculate the probabilities of accepted mutations. The tree contains sequences of four OTUs.



- (a) Estimate the mutations and fill them in the boxes next to the edges.
- (b) Count the occurrences of mutations and fill them in the matrix. Note that a mutation  $A \to B$  is equivalent with a mutation  $B \to A$ .

|   | A | С | G | Т |
|---|---|---|---|---|
| A |   | 1 |   | 1 |
| С | 1 |   | 3 |   |
| G |   | 3 |   |   |
| Т | 1 |   |   |   |

(c) Use the following definitions and calculate  $f_{CG}$ ,  $f_C$  and f.

 $f_{ab}$ : The number of mutations from a to b or from b to a

 $f_a$ : Total number of mutations in which a takes part

f: Twice the total number of mutations

 $f_{CG}: 3$ 

 $f_C: 1+3=4$ 

f: 2(1+1+3) = 10

(d) Use the following definition and calculate  $p_C$ .

 $p_a$ : The relative occurrence of a in the observed sequences

 $p_C: \ 3/12$ 

### 3. Relative mutability of PAM

Relative mutability is calculated from frequencies of estimated mutilation and background probabilities.

$$m_a: \frac{1}{100p_a} \times \frac{f_a}{f}$$

 $f_a$ : Total number of point mutations in which a takes part

f: Twice the total number of point mutations

 $p_a$ : Relative occurrence of a in the observed sequences

Assume that the frequencies are pre-calculated as follows.

• Frequencies of estimated mutations

$$f_A: 2, \quad f_G: 3, \quad f_C: 3, \quad f_T: 2$$
  
 $f: 10$ 

• Background probabilities

$$p_A: 3/10, \quad p_G: 2/10, \quad p_C: 4/10, \quad p_T: 1/10$$

(a) Calculate the probabilities of point mutations by  $\frac{f_a}{f}$ .

$$\frac{f_A}{f}: \ 2/10$$
  $\frac{f_G}{f}: \ 3/10$   $\frac{f_C}{f}: \ 3/10$   $\frac{f_T}{f}: \ 2/10$ 

(b) Calculate  $100p_a$ .

$$100p_A: 30$$
  $100p_G: 20$   $100p_C: 40$   $100p_T: 10$ 

(c) Calculate the relative mutability  $m_a$ .

$$m_A: \ 2/300 \ m_G: \ 3/200 \ m_C: \ 3/400 \ m_T: \ 2/100$$

# 4. Mutation probabilities of PAM

Mutation probabilities are calculated from relative mutability.

$$m_{ab}: m_a \times \frac{f_{ab}}{f_a}, \qquad m_{aa}: 1 - m_a$$

 $f_{ab}$ : Total number of point mutations in which a takes part

 $f_a$ : Twice the total number of point mutations

 $m_a$ : Relative mutability of a

Assume that the frequencies are pre-calculated as follows.

• Frequencies of estimated mutations

 $f_{AC}: 8, \quad f_A: 32$ 

• Relative mutability

 $m_A: 0.004$ 

(a) Calculate  $M_{AC}$ .

**Solution:**  $0.004 \times \frac{8}{32} = 0.001$ 

(b) Calculate  $M_{AA}$ .

**Solution:** 1 - 0.004 = 0.996

# 5. Odds ratios of PAM

Odds ratios are calculated from mutation probabilities and background probabilities.

$$O_{ab} = \frac{M_{ab}}{p_b} = m_a \times \frac{f_{ab}}{f_a} \times \frac{1}{f_b} = \frac{1}{100} \times \frac{f_{ab}}{f} \times \frac{1}{p_a p_b}$$

Assume that the frequencies are pre-calculated as follows.

$$f_{AC}: 16, \quad f: 400, \quad p_A: 0.2, \quad p_c: 0.4$$

(a) Calculate  $O_{AC}$ .

**Solution:**  $(1/100) \times (16/400) \times (1/(0.2 \times 0.4)) = 0.005$ 

(b) Calculate  $O_{CA}$ .

Solution: 0.005

#### 6. BLOSUM

BLOSUM uses several thousand blocks to calculate the probabilities of accepted mutation. Use the following definitions and Block1 & Block2 to solve the problems.

 $f_{ab}$ : Frequencies of an observed pair a and b.

T: Total number of pairs from all blocks.

The number of pairs can be calculated as 1/2wm(m-1).

$$p_a: p_a = f_{aa} + \sum_{e \neq a} f_{ae}/2$$

 $e_{aa}:p_ap_a$ 

$$e_{ab}: p_a p_b + p_b p_a = 2p_a p_b$$

Block1 Block2 CAGC GGA GTAC GTA CAGC

(a) Count the occurrences of all pairs.

|   | Α | G | С | Т |
|---|---|---|---|---|
| A | 2 | 2 | 0 | 2 |
| G | 2 | 2 | 2 | 1 |
| С | 0 | 2 | 4 | 0 |
| Т | 2 | 1 | 0 | 0 |

(b) Calculate T.

**Solution:** 
$$(1/2 \times 4 \times 3 \times 2) + (1/2 \times 3 \times 2 \times 1) = 12 + 3 = 15$$

(c) Calculate  $f_{AA}$  and  $f_{AG}$ .

**Solution:** 
$$f_{AA}: 2/15, \quad f_{AG}: 2/15$$

(d) Calculate  $p_A$  and  $p_G$ .

**Solution:** 
$$p_A: 8/30, p_G: 9/30$$

(e) Calculate  $e_{AA}$  and  $e_{AG}$ .

**Solution:** 
$$e_{AA}: 64/900, \quad e_{AG}: 144/900$$

(f) Calculate  $f_{AA}/e_{AA}$  and  $f_{AG}/e_{AG}$ .

**Solution:** 
$$f_{AA}/e_{AA}: 1.875, \quad f_{AG}/e_{AG}: 0.833$$