13 Exercise solutions – Hidden Markov model

1. **HMM probabilities**

   An HMM (hidden Markov model) is a probabilistic graphical model with three types of probabilities.

   Transition probabilities:

	L_t	H_t
L_{t-1}	0.2	0.8
H_{t-1}	0.4	0.6

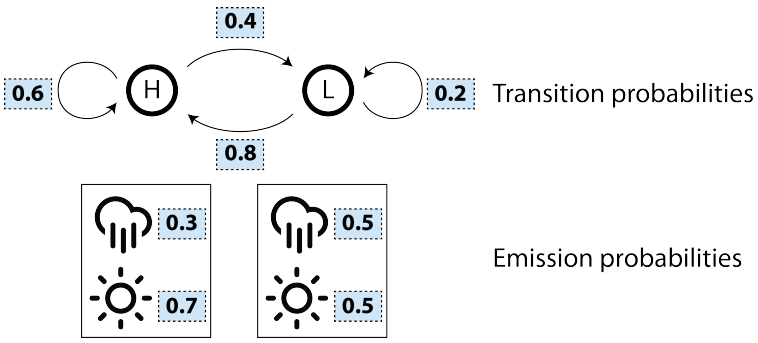
   Emission probabilities:

	L	H
Sunny	0.5	0.7
Rain	0.5	0.3

   Initial transition probabilities:

$$(\text{L, H}) = (0.3, 0.7)$$

   (a) Add the transition and emission probabilities to the graph.



   (b) What are the joint probabilities for (Rain, Rain, Sunny) and (H, L, L)?

   Solution: $\text{p(H)p(Rain|H)} \times \text{p(L|H)p(Rain|L)} \times \text{p(L|L)p(Sunny|L)}$
   $= 0.7 \times 0.3 \times 0.4 \times 0.5 \times 0.2 \times 0.5 = 0.0042$

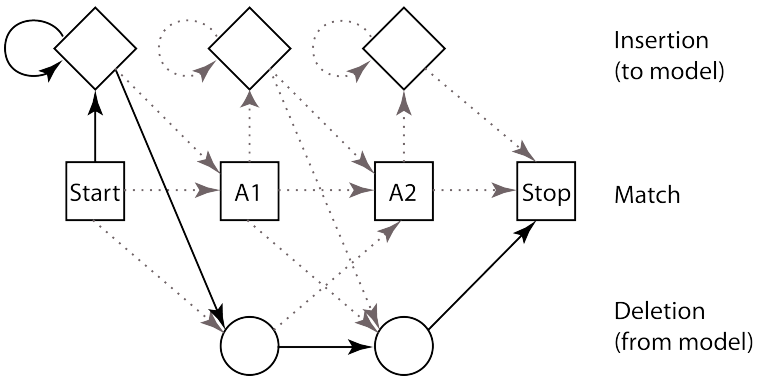
   (c) What are the joint probabilities for (Sunny, Rain, Sunny) and (L, H, L)?

   Solution: $\text{p(L)p(Sunny|L)} \times \text{p(H|L)p(Rain|H)} \times \text{p(L|H)p(Sunny|L)}$
   $= 0.3 \times 0.5 \times 0.8 \times 0.3 \times 0.4 \times 0.5 = 0.0072$

2. **HMM profile**

A path of an HMM profile represents an alignment between an input sequence and the profile.

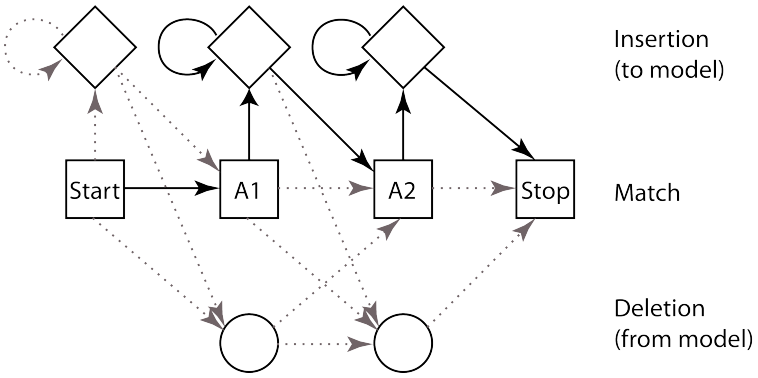
(a) Assume Seq1 = q1 q2 and its path is indicated with solid lines. Draw the alignment of Seq1 and the profile.



Solution:

```
   Seq1: q1 q2 -  -
Profile: -  -  A1 A2
```

(b) Assume Seq2 = q1 q2 q3 q4 q5 q6 and its path is indicated with solid lines. Draw the alignment of Seq2 and the profile.

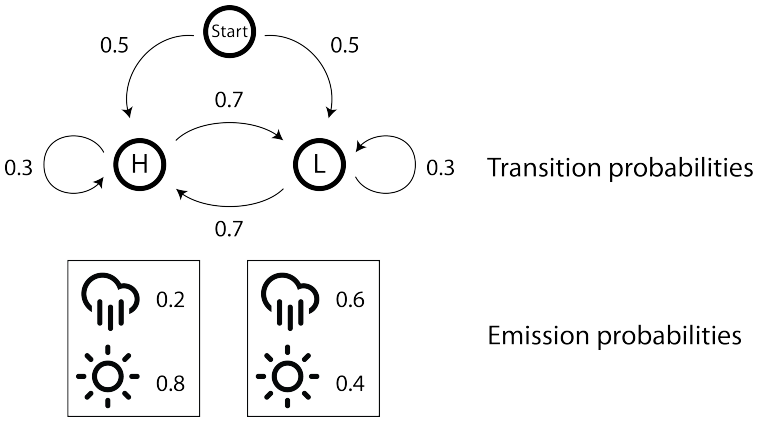


Solution:

```
   Seq2: q1 q2 q3 q4 q5 q6
Profile: A1 -  -  A2 -  -
```

3. **The Viterbi algorithm**

The Viterbi algorithm is a dynamic programming based method to find the optimal path of an HMM with hidden status.



(a) Find the optimal path when observed weather conditions are (Rain, Sunny).

	H	L
Rain	$0.5 \times 0.2 = 0.1$	$0.5 \times 0.6 = 0.3$
Sunny	(H) $0.1 \times 0.3 \times 0.8 = 0.024$ (L) $0.3 \times 0.7 \times 0.8 = 0.168$	(H) $0.1 \times 0.7 \times 0.4 = 0.028$ (L) $0.3 \times 0.3 \times 0.4 = 0.036$

Solution: (L, H)

(b) Find the optimal path when observed weather conditions are (Sunny, Sunny, Rain).

	H	L
Sunny	$0.5 \times 0.8 = 0.4$	$0.5 \times 0.4 = 0.2$
Sunny	(H) $0.4 \times 0.3 \times 0.8 = 0.096$ (L) $0.2 \times 0.7 \times 0.8 = 0.112$	(H) $0.4 \times 0.7 \times 0.4 = 0.112$ (L) $0.2 \times 0.3 \times 0.4 = 0.024$
Rain	(H) $0.112 \times 0.3 \times 0.2 = 0.007$ (L) $0.112 \times 0.7 \times 0.2 = 0.016$	(H) $0.112 \times 0.7 \times 0.6 = 0.047$ (L) $0.112 \times 0.3 \times 0.6 = 0.02$

Solution: (L, H, L)