

13 Exercises – Hidden Markov model

1. HMM probabilities

An HMM (hidden Markov model) is a probabilistic graphical model with three types of probabilities.

Transition probabilities:

	L_t	H_t
L_{t-1}	0.2	0.8
H_{t-1}	0.4	0.6

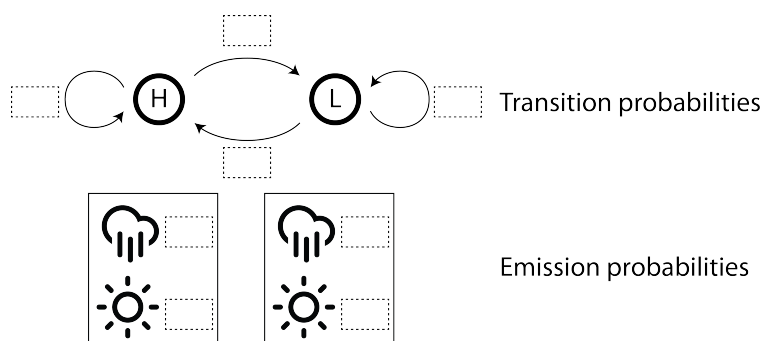
Emission probabilities:

	L	H
Sunny	0.5	0.7
Rain	0.5	0.3

Initial transition probabilities:

$$(L, H) = (0.3, 0.7)$$

- (a) Add the transition and emission probabilities to the graph.

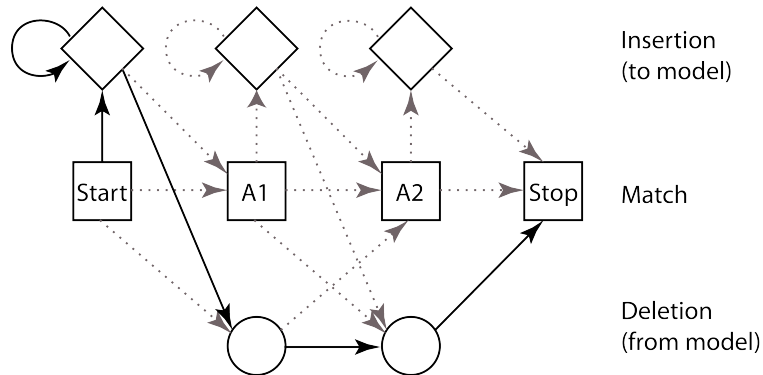


- (b) What are the joint probabilities for (Rain, Rain, Sunny) and (H, L, L)?
- (c) What are the joint probabilities for (Sunny, Rain, Sunny) and (L, H, L)?

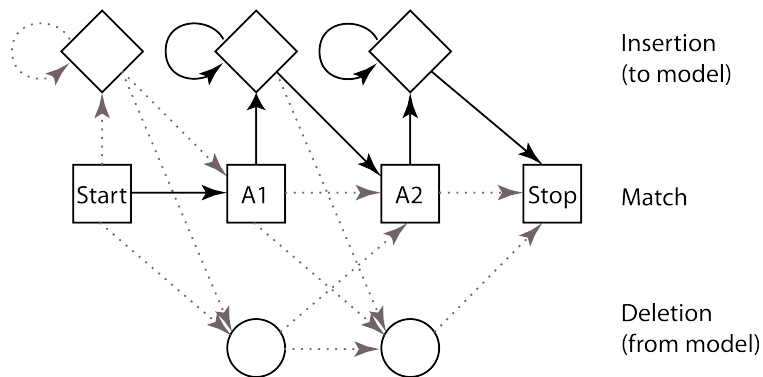
2. HMM profile

A path of an HMM profile represents an alignment between an input sequence and the profile.

- (a) Assume $\text{Seq1} = q_1 q_2$ and its path is indicated with solid lines. Draw the alignment of Seq1 and the profile.

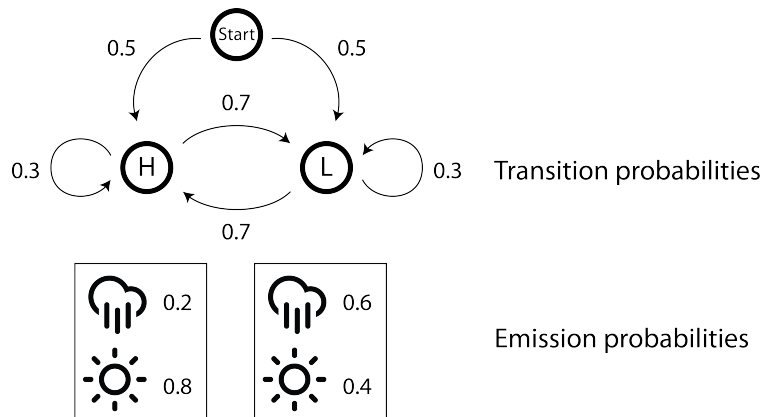


- (b) Assume $\text{Seq2} = q_1 q_2 q_3 q_4 q_5 q_6$ and its path is indicated with solid lines. Draw the alignment of Seq2 and the profile.



3. The Viterbi algorithm

The Viterbi algorithm is a dynamic programming based method to find the optimal path of an HMM with hidden status.



(a) Find the optimal path when observed weather conditions are (Rain, Sunny).

	H	L
Rain		
Sunny		

(b) Find the optimal path when observed weather conditions are (Sunny, Sunny, Rain).

	H	L
Sunny		
Sunny		
Rain		