

INF281 Exercise 05

1. Non-parametric test

A non-parametric test is used to determine a p-value for the optimal score of a global alignment. Assume we randomly generated 9 sequences and calculated the alignment scores as follows.

q: AACG

Seq No.	1	2	3	4	5	6	7	8	9
Score	0.2	0.4	0.5	1.2	1.2	1.5	1.9	2.2	2.1

- (a) What are H_0 (null hypothesis) and H_1 (alternative hypothesis) if you want to use a statistical hypothesis test to evaluate a global pairwise alignment in terms of finding homologues?

Solution: H_0 : Sequences are not homologous, H_1 : Sequences are homologous

- (b) Calculate the p-value for the alignment below.

q: AACG

d: AGTG

Score: 2

The p-value can be calculated as:

$$p = (b + 1)/(n + 1)$$

where b is the number of randomly generated scores above the score of the original alignment, and n is the sample size.

Solution: p-value: 0.3

- (c) Is the test result statistically significant when $\alpha = 0.05$?

Solution: No.

- (d) What is the conclusion of the test in terms of finding homologues?

Solution: q and d are not homologous.

2. Gumbel distribution

Use the alignment scores between q and 5 randomly generated sequences below to answer the following questions.

q: ACGTA

Randomly generated sequence	Alignment score
ACGTA	4
TTACG	5
CGCGA	6
ATTAT	4
CGATC	6

- (a) What is the mean of the scores?

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Solution: 5

- (b) What is the standard deviation of the scores?

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

Solution: 1

- (c) The parameters μ and λ of the Gumbel distribution can be estimated from the mean and the variance of a sample. Calculate lambda and mu.

$$\begin{aligned}\text{lamda} &\approx \frac{1.282}{s} \\ \text{mu} &\approx \bar{x} - \frac{0.577}{\text{lamda}}\end{aligned}$$

Solution: lamda: 1.282, mu: 4.55

(d) What is the p-value when the score of an alignment is 4.55?

x	exp(x)
0	1
-1	0.3679
-2	0.1353
-3	0.0497

$$P[Y > 4.55] = 1 - F_Y(4.55) = 1 - \exp(-e^{-\lambda(4.55-\mu)})$$

Solution: $1 - 0.3679 = 0.6321$

(e) What is the conclusion of the test in terms of finding homologues?

Solution: The sequences of the alignment with score 4.55 are not homologous.

3. Bit score

BLAST reports bit scores, which are equivalent to the information content that are calculated from raw scores.

- Bit score: $S' = \frac{(\lambda S - \ln K)}{\ln 2}$
- S: raw score
- K and λ : Karlin-Altschul statistics

x	$\ln x$
1	0
2	0.693
3	1.099

Assume K: 3 and λ : 0.1 and use the table above to answer the following questions.

(a) What is the bit score when the raw score is 80.29?

Solution: $\frac{0.1 \times 80.29 - 1.099}{0.693} = \frac{6.93}{0.693} = 10$

(b) $2^{S'}$ indicates the expected search space size that one can find an alignment with score at least S by chance alone. What is the $2^{S'}$ value when the raw score is 17.92?

Solution: $S' = \frac{0.1 \times 17.92 - 1.099}{0.693} = \frac{0.693}{0.693} = 1; \quad 2^1 = 2$

4. Bit score to e-value

The e-value represents the expected number of hits when homologous sequence are searched on a database of a particular size. It can be calculated from a bit score as follows.

$$E = \frac{\text{Size of search space}}{2^{\text{bit-score}}}$$

- (a) What is the e-value when the size of search space is 3200 and the bit score is 4?

Solution: $\frac{3200}{2^4} = \frac{3200}{16} = 200$

- (b) Assume we can calculate a search space size as $m \times n$ where m is the query sequence size and n is the total character size of a database. What is the e-value when m is 20, n is 4000, and the bit score is 3?

Solution: $\frac{20 \times 4000}{2^3} = \frac{80000}{8} = 10000$

5. **Raw score to e-value** The e-value can be directly calculated from a raw score as $E(S) = Kmne^{-\lambda S}$. Use the values below to answer the following questions.

- K: 2
- λ : 0.1

x	exp(x)
0	1
-1	0.3679
-10	0.000045

- (a) What is $E(10)$ when m is 10 and n is 100?

Solution: $E(10) = 210100e^{-1} = 7358$

- (b) What is $E(100)$ when m is 10 and n is 100?

Solution: $E(100) = 210100e^{-10} = 0.09$

- (c) What is $E(100)$ when m is 100 and n is 1000?

Solution: $E(100) = 21001000e^{-10} = 9$