

행렬곱셈의 분할연산

- 다음과 같은 2×2 크기의 A, B 행렬이 있다고 가정하자.
 - 이 때, 행렬의 곱은 다음과 같이 나타낼 수 있음

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- 만약 위 행렬의 각 원소인 a_{ij}, b_{ij} 가 스칼라 값이 아니라 행렬이라면 어떻게 될까?

$$AB = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array} \right] \cdot \left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array} \right] = \left[\begin{array}{c|c} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ \hline A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{array} \right]$$

행렬곱셈 분할연산의 예 (1)

$$A = \left[\begin{array}{cc|cc} 1 & -1 & 3 & 1 \\ 2 & 1 & 2 & 1 \\ \hline 0 & 5 & 3 & 1 \\ -1 & 2 & 1 & 1 \end{array} \right]$$

$$B = \left[\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 2 & -1 & 3 & 3 \\ \hline 5 & 1 & 2 & -1 \\ 0 & 2 & 1 & 0 \end{array} \right]$$

$$A_{11} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \quad B_{12} = \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0 & 5 \\ -1 & 2 \end{bmatrix} \quad A_{22} = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

$$B_{21} = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix} \quad B_{22} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$A_{11}B_{11} + A_{12}B_{21} = \begin{bmatrix} 14 & 6 \\ 14 & 3 \end{bmatrix}$$

$$A_{11}B_{21} + A_{12}B_{22} = \begin{bmatrix} 6 & -5 \\ 12 & 3 \end{bmatrix}$$

$$A_{21}B_{11} + A_{22}B_{21} = \begin{bmatrix} 25 & 0 \\ 8 & 1 \end{bmatrix}$$

$$A_{21}B_{12} + A_{22}B_{22} = \begin{bmatrix} 22 & 12 \\ 7 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 14 & 6 & 6 & -5 \\ 14 & 3 & 12 & 3 \\ 25 & 0 & 22 & 12 \\ 8 & 1 & 7 & 4 \end{bmatrix}$$

행렬곱셈 분할연산의 예 (2)

$$A = \left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & -4 & -2 & 7 & -1 \end{array} \right] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array} \right] = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$AB = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} A_{11}B_1 + A_{12}B_2 \\ A_{21}B_1 + A_{22}B_2 \end{bmatrix}$$

$$A_{11}B_1 = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 5 & -2 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ -2 & 1 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 15 & 12 \\ 2 & -5 \end{bmatrix}$$

$$A_{12}B_2 = \begin{bmatrix} 0 & -4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} -20 & -8 \\ -8 & 7 \end{bmatrix}$$

$$AB = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} A_{11}B_1 + A_{12}B_2 \\ A_{21}B_1 + A_{22}B_2 \end{bmatrix} = \begin{bmatrix} -5 & 4 \\ -6 & 2 \\ \hline 2 & 1 \end{bmatrix}$$

행렬곱셈 분할연산의 예 (3)

$$A = \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 2 & -1 & 4 & 2 & 1 \\ 3 & 1 & -1 & 7 & 5 \end{array} \right] = \begin{bmatrix} I_2 & O_{23} \\ P & Q \end{bmatrix} \quad B = \left[\begin{array}{cc} 4 & -2 \\ 5 & 6 \\ \hline 7 & 3 \\ -1 & 0 \\ 1 & 6 \end{array} \right] = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$AB = \begin{bmatrix} I & O \\ P & Q \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} IX + OY \\ PX + QY \end{bmatrix} = \begin{bmatrix} X \\ PX + QY \end{bmatrix} = \left[\begin{array}{cc} 4 & -2 \\ 5 & 6 \\ \hline 30 & 8 \\ 8 & 27 \end{array} \right]$$