type

$o ::= int|bool|unit$

$\tau ::= o|\tau \to \tau$

syntax

$e ::= c^o|funcx = e|e + e|e - e|e * e|e/e|$

$e > e|e < e|e <= e|e >= e|e == e|e! = e|$

$func\ x(e) = e|func\ x(e : t) : t = e|e(e)|if\ e\ \{e\}\ else\ \{e\}$

func x = e は変数宣言 x という意味であるので x と略記可能とする以下に型環境と型付け推論規則を示す

$$\Gamma = \{x_1 : \tau_1, x_2 : \tau_2, \cdots x_n : \tau_n\}$$

$$\frac{C_1\ C_2\ \ldots \qquad C_n}{\Gamma \vdash e : \tau}$$

$$\Gamma \vdash C^o : o$$
$$\Gamma\{x : \tau\} \vdash x : \tau$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 + e_2 : int}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 - e_2 : int}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 * e_2 : int}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 / e_2 : int}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 < e_2 : bool}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 > e_2 : bool}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 <= e_2 : bool}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 >= e_2 : bool}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 >= e_2 : bool}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 == e_2 : bool}$$

$$\frac{\Gamma \vdash e_1 : int \qquad \Gamma \vdash e_2 : int}{\Gamma \vdash e_1 != e_2 : bool}$$

$$\frac{\Gamma\{x:\tau_1\} \vdash e:\tau_2 \qquad \Gamma \vdash e_2:\tau_2}{\Gamma \vdash func\ x(e:\tau_1):\tau_2\ :\ \tau_1 \to \tau_2}$$

$$\frac{\Gamma \vdash e_1:\tau_1 \to \tau_2 \qquad \Gamma \vdash e_2:\tau_2}{\Gamma \vdash e_1(e_2):\tau_2}$$

$$\frac{\Gamma \vdash e_1:bool \qquad \Gamma \vdash e_2:\tau_2 \qquad \Gamma \vdash e_3:\tau_3}{\Gamma \vdash if\ e_1\ \{e_2\}else\{e_3\}:\tau}$$

意味論を定義する以下に値を定義する

$$n ::= ...|-2|-1|0|1|2|...$$
$$b ::= true|false$$
$$v ::= n|b|cls(E,x,e)$$
$$tovalue_o c ::= get(c)$$

$$E = \{x_1 \mapsto v_1, x_2 \mapsto v_2, ..., x_n \mapsto v_n\}$$

$$\frac{C_1\ C_2\ \ldots \qquad C_n}{\Gamma \vdash e \Downarrow v}$$

$$E \vdash c^o \Downarrow tovalue_o\ c^o$$
$$E\{x \mapsto v\} \vdash x \Downarrow v$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 + e_2 \Downarrow n_1 + n_2}$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 - e_2 \Downarrow n_1 - n_2}$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 * e_2 \Downarrow n_1 \cdot n_2}$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 / e_2 \Downarrow n_1 / n_2}$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 < e_2 \Downarrow n_1 < n_2}$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 > e_2 \Downarrow n_1 > n_2}$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 <= e_2 \Downarrow n_1 <= n_2}$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 < e_2 \Downarrow n_1 < n_2}$$

$$\frac{E \vdash e_1 \Downarrow n_1 \qquad E \vdash e_2 \Downarrow n_2}{E \vdash e_1 == e_2 \Downarrow n_1 = n_2}$$

$$E \vdash func\ x(e : \tau_1) : \tau_2\ e \Downarrow cls(E, x, e)$$

$$\frac{E \vdash e \Downarrow cls(E, x, e) \qquad E \vdash e_2 \Downarrow v' \qquad E\{x' \mapsto v'\} \vdash e' \Downarrow v}{E \vdash e_1(e_2) \Downarrow v}$$

$$\frac{E \vdash e_1 \Downarrow true \qquad E \vdash e_2 \Downarrow v}{E \vdash if\ e_1\ \{e_2\} else \{e_3\} \Downarrow v}$$

$$\frac{E \vdash e_1 \Downarrow false \qquad E \vdash e_3 \Downarrow v}{E \vdash if\ e_1\ \{e_2\} else \{e_3\} \Downarrow v}$$