

Задача 7.1.

5.1.1

$$A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad C = (1, 0, 2)$$

$$5 \cdot A = 5 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix} \quad A+B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

$$A \cdot C = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times (1, 0, 2) = \begin{pmatrix} 1 \cdot 0 \cdot 2 \\ 2 \cdot 0 \cdot 4 \\ 3 \cdot 0 \cdot 6 \end{pmatrix}$$

5.1.2

$$(5E)^{-1} \quad \det(5E) = 5^5$$

$$M(5E) = \begin{pmatrix} 5^4 & \dots & \dots \\ \dots & 5^4 & \dots \\ \dots & \dots & 5^4 \end{pmatrix} \Rightarrow (5E)^{-1} = \frac{1}{5^5} \begin{pmatrix} 5^4 & \dots & \dots \\ \dots & 5^4 & \dots \\ \dots & \dots & 5^4 \end{pmatrix} = \frac{5^4}{5^5} E = \frac{E}{5}$$

5.2

$$A = \begin{pmatrix} 1 & 12 & 3 \\ 4 & 0 & 6 \\ 7 & 8 & 9 \end{pmatrix} \quad \det(A) = 1(0 \cdot 9 - 6 \cdot 8) - 2(4 \cdot 9 - 6 \cdot 7) + 3(4 \cdot 8 - 7 \cdot 0) =$$

$$= 60$$

5.3

$$A^{-1} = \frac{1}{|A|} A^T$$

$$\det(A) = 60 \text{ (by 5.2)} \quad \det(A) \neq 0$$

$$M = \begin{vmatrix} -48 & -6 & 32 \\ -6 & -12 & -6 \\ 12 & -6 & -8 \end{vmatrix} \quad M_+ = \begin{vmatrix} -48 & 6 & 32 \\ 6 & -12 & 6 \\ 12 & 6 & -8 \end{vmatrix}$$

$$M_+^T = \begin{vmatrix} -48 & 6 & 12 \\ 6 & -12 & 6 \\ 32 & 6 & -8 \end{vmatrix}$$

$$A^{-1} = \frac{1}{60} \begin{pmatrix} -48 & 6 & 12 \\ 6 & -12 & 6 \\ 32 & 6 & -8 \end{pmatrix}$$

$$\textcircled{5.3.2} \quad A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{pmatrix} \quad \text{rang}(A) = 1$$

$$\textcircled{5.4} \quad \begin{aligned} A &= (1, 5) \\ B &= (2, 8) \end{aligned} \quad \bar{A} \cdot \bar{B} = x_1 x_2 + y_1 y_2 = 1 \cdot 2 + 5 \cdot 8 = 42$$

$$\textcircled{5.5} \quad \bar{A} = (1 \ 5 \ 0) ; \bar{B} = (2 \ 8 \ 7) ; \bar{C} = (7 \ 15 \ 3)$$

$$\bar{A} \times [\bar{B} \times \bar{C}] = \begin{vmatrix} 1 & 5 & 0 \\ 2 & 8 & 7 \\ 7 & 15 & 3 \end{vmatrix} = 1(8 \cdot 3 - \frac{3}{2} \cdot 7) - 5(2 \cdot 3 - 7 \cdot 7) = \frac{457}{2}$$