**17** 以下の問いに答えよ．

(1) $\cos 5\theta$ を $\cos\theta$ で表せ．

(2) $\cos^2 \dfrac{1}{10}\pi$ の値を求めよ．

（信州大 改）

(1)

$\cos 5\theta = \cos(3\theta + 2\theta)$

$= \cos 3\theta \cdot \cos 2\theta - \sin 3\theta \cdot \sin 2\theta$

$= \cos 3\theta (2\cos^2\theta - 1) - \sin 3\theta (2\sin\theta\cos\theta)$ —(※)

ここで，

$\cos 3\theta = \cos(2\theta + \theta)$

$= \cos 2\theta \cdot \cos\theta - \sin 2\theta \cdot \sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta \cdot \sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta \cdot \cos\theta$

$= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$.

$\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta \cdot \cos\theta + \cos 2\theta \cdot \sin\theta$

$= 2\sin\theta\cos\theta \cdot \cos\theta + (2\cos^2\theta - 1) \cdot \sin\theta$

$= 2\sin\theta \cdot \cos^2\theta + 2\sin\theta\cos^2\theta - \sin\theta$

$= 4\sin\theta\cos^2\theta - \sin\theta$

(※)へ代入．

$\cos 5\theta = (4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1) - (4\sin\theta\cos^2\theta - \sin\theta) \cdot 2\sin\theta\cos\theta$

(1項目) $= 8\cos^5\theta - 6\cos^3\theta - 4\cos^3\theta + 3\cos\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta$

(2項目) $= -8\sin^2\theta\cos^3\theta + 2\sin^2\theta\cos\theta$

$= -8(1 - \cos^2\theta)\cos^3\theta + 2(1 - \cos^2\theta)\cos\theta$

$= -8\cos^3\theta + 8\cos^5\theta + 2\cos\theta - 2\cos^3\theta$

$\therefore \cos 5\theta = 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta + 8\cos^5\theta - 10\cos^3\theta + 2\cos\theta$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ //

(2) (1)より

$\cos 5\theta = \cos\theta(16\cos^4\theta - 20\cos^2\theta + 5)$

$\theta = \frac{\pi}{10}$ を代入すると

(左辺) $= \cos\frac{5}{10}\pi = \cos\frac{1}{2}\pi = 0$.

$\therefore\ 0 = \cos\frac{\pi}{10}\left(16\cos^4\frac{\pi}{10} - 20\cos^2\frac{\pi}{10} + 5\right)$

ここで $\cos\frac{\pi}{10} \neq 0$ なので

$16\cos^4\frac{\pi}{10} - 20\cos^2\frac{\pi}{10} + 5 = 0$

$\cos^2\frac{\pi}{10} = x$ とおくと

$16x^2 - 20x + 5 = 0$

$x = \dfrac{20 \pm \sqrt{400 - 4 \cdot 5 \cdot 16}}{2 \cdot 16}$

$= \dfrac{20 \pm \sqrt{80}}{2 \cdot 16}$

$= \dfrac{20 \pm 4\sqrt{5}}{2 \cdot 16}$

$= \dfrac{5 \pm \sqrt{5}}{8}$

よって，$\cos\frac{\pi}{4} < \cos\frac{\pi}{10}$，i.e. $\frac{1}{\sqrt{2}} < \cos\frac{\pi}{10}$

なので，$\frac{1}{2} < \cos^2\frac{\pi}{10}$

$\therefore\ \frac{1}{2} < x$

$\therefore\ x = \dfrac{5 - \sqrt{5}}{8}$

i.e. $\cos^2\frac{\pi}{10} = \dfrac{5 - \sqrt{5}}{8}$ //

$\cos 5\theta$ は，1つ1つ加法定理を丁寧に行うのみ!!

(2)は(1)の仕方を考えて解く．