令和 5 年度第 1 学年 4 組 学年末考査 (準備問題) 数学 1 (その 1)

R6. 2.16

1 以下の値を求めよ.

(1) $(-2)^{-2}$

$= \frac{1}{4}$

(2) $(-2024)^0$

$= 1.$

(3) $8^{\frac{2}{3}}$

$= 4$

(4) $1024^{\frac{1}{10}}$

$= 2$

(5) $\sqrt[3]{27}$

$= 3$

2 以下の式を計算せよ.

(6) $5^2 \times 5^{-2}$

$= 5^{2-2} = 5^0 = 1$

(7) $2^2 \div 4^3 \times 8^2$

$= 2^2 \div 2^6 \times 2^6$

$= \frac{2^2 \cdot 2^6}{2^6} = 2^2 = 4$

(8) $\dfrac{\sqrt[3]{3}}{\sqrt[3]{81}} = \dfrac{(3)^{\frac{1}{3}}}{(3^4)^{\frac{1}{3}}}$

$= \left(\frac{\cancel{3}}{3^{\cancel{4}3}}\right)^{\frac{1}{3}} = \left(\frac{1}{3^3}\right)^{\frac{1}{3}} = \frac{1}{3}$

(9) $\left\{\left(\dfrac{16}{25}\right)^{-\frac{3}{4}}\right\}^{\frac{2}{3}}$

$= \left(\frac{25}{16}\right)^{\frac{\cancel{3}}{\cancel{4}2}\cdot\frac{\cancel{2}}{\cancel{3}}} = \left(\frac{25}{16}\right)^{\frac{1}{2}} = \frac{5}{4}$

(10) $\sqrt[3]{135} - \sqrt[3]{5} + \sqrt[3]{40}$

$= (3^3 \cdot 5)^{\frac{1}{3}} - 5^{\frac{1}{3}} + (2^3 \cdot 5)^{\frac{1}{3}}$

$= 3 \cdot 5^{\frac{1}{3}} - 5^{\frac{1}{3}} + 2 \cdot 5^{\frac{1}{3}} = 4 \cdot 5^{\frac{1}{3}}$

$= 4\sqrt[3]{5}$

3 $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = 3$ のとき, 以下の値を求めよ.

(11) $x^{\frac{3}{2}} + x^{-\frac{3}{2}}$

$\underbrace{(x^{\frac{1}{2}} + x^{-\frac{1}{2}})^3}_{27} = x^{\frac{3}{2}} + 3 \cdot (x^{\frac{1}{2}})^2 \cdot x^{-\frac{1}{2}} + 3 \cdot x^{\frac{1}{2}} \cdot (x^{-\frac{1}{2}})^2 + x^{-\frac{3}{2}}$

$= x^{\frac{3}{2}} + x^{-\frac{3}{2}} + \underbrace{3(x^{\frac{1}{2}} + x^{-\frac{1}{2}})}_{9}$

$\therefore\ x^{\frac{3}{2}} + x^{-\frac{3}{2}} = 18$

(12) $x + x^{-1}$

$\underbrace{(x^{\frac{1}{2}} + x^{-\frac{1}{2}})^2}_{9} = x + 2x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}} + x^{-1}$

$= x + x^{-1} + 2.$

$\therefore\ x + x^{-1} = 7$

(13) $x^2 + x^{-2}$

$\underbrace{(x + x^{-1})^2}_{49} = x^2 + 2 \cdot x \cdot x^{-1} + x^{-2}$

$= x^2 + x^{-2} + 2.$

$\therefore\ x^2 + x^{-2} = 47$

4 以下の方程式, 不等式を解け.

(14) $2^x = 128$

$128 = 2^7$

$\therefore\ x = 7$

(15) $2^x = \dfrac{1}{64}$

$\frac{1}{64} = \left(\frac{1}{2}\right)^6 = 2^{-6}$ $\therefore\ x = -6$

(16) $\left(\dfrac{1}{3}\right)^x = 27$

$27 = 3^3$

$= \left(\frac{1}{3}\right)^{-3}$ $\therefore\ x = -3$

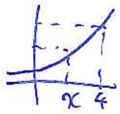
(17) $3^{3x-4} = 243$

$243 = 3^5$

$\therefore\ 3x - 4 = 5$ $x = 3$

(18) $2^x < 16$

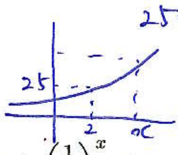
$16 = 2^4$ $\therefore\ 2^x < 2^4$



$x < 4$

(19) $5^x > 25$

$25 = 5^2$

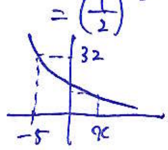


$2 < x$

(20) $\left(\dfrac{1}{2}\right)^x \leqq 32$

$32 = 2^5$

$= \left(\frac{1}{2}\right)^{-5}$



$-5 \leq x$

令和５年度第１学年４組 学年末考査 (準備問題) 数学１ (その２)

R6. 2.16

5 以下の問いに答えよ．

(21) 以下の３つの数の大小を不等式を用いて表せ．

$$\sqrt[3]{4},\quad \sqrt[4]{8},\quad \sqrt[5]{8}$$

$\sqrt[3]{4} = (2^2)^{\frac{1}{3}} = 2^{\frac{2}{3}}$ 中

$\sqrt[4]{8} = (2^3)^{\frac{1}{4}} = 2^{\frac{3}{4}}$ 大

$\sqrt[5]{8} = (2^3)^{\frac{1}{5}} = 2^{\frac{3}{5}}$ 小

$\frac{3}{5} < \frac{2}{3} < \frac{3}{4}$ であるので

$\sqrt[5]{8} < \sqrt[3]{4} < \sqrt[4]{8}$ //

(22) 方程式 $4^x - 3\cdot 2^x - 4 = 0$ を解け．

$2^x = t.$ $t > 0$

$t^2 - 3t - 4 = 0$

$(t-4)(t+1) = 0$

$t > 0$ より $t = 4$

$2^x = 4$

$x = 2$ //

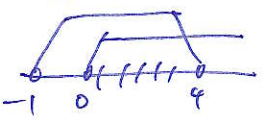
(23) 不等式 $4^x - 3\cdot 2^x - 4 < 0$ を解け．

$2^x = t$ $t > 0$ —①

$t^2 - 3t - 4 < 0$

$(t-4)(t+1) < 0$

$-1 < t < 4$ —②

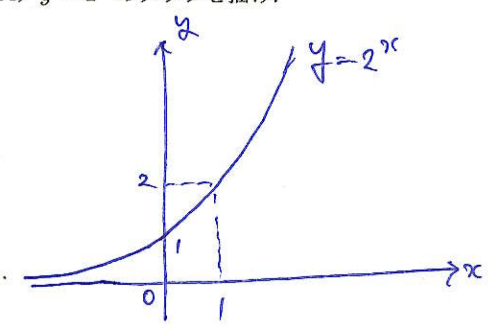


$0 < t < 4$

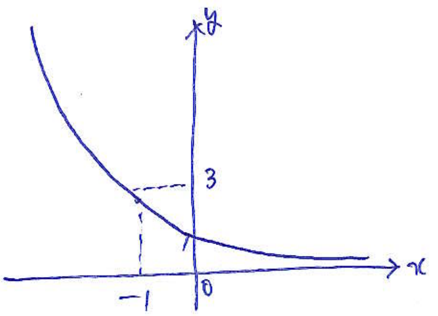
$0 < 2^x < 4$

$x < 2$ //

(24) $y = 2^x$ のグラフを描け．



(25) $y = \left(\frac{1}{3}\right)^x$ のグラフを描け．



令和 5 年度第 1 学年 4 組 学年末考査 (準備問題) 数学 1 (その 3)

R6. 2.16

6 以下の値を求めよ.

(26) $\log_3 27$

$27 = 3^3$

$= 3$

(27) $\log_2 32$

$32 = 2^5$

$= 5$

(28) $\log_2 \frac{1}{64}$

$= -6$

(29) $\log_{\frac{1}{2}} 4$

$= -2$

(30) $2^{\log_2 3}$

$= 3$

7 以下の式を計算せよ.

(31) $\log_8 4 + \log_8 2$

$= \log_8 4 \cdot 2 = \log_8 8 = 1$

(32) $\log_3 18 - \log_3 2$

$= \log_3 \frac{18}{2} = \log_3 9 = 2$

(33) $4\log_5 3 - 2\log_5 15 - \log_5 45$

$= \log_5 3^4 - \log_5 15^2 - \log_5 45$

$= \log_5 \frac{3^4}{15^2 \cdot 45} = \log_5 \frac{1}{5 \cdot 5 \cdot 5} = -3$

(34) $\log_3 5 \cdot \log_5 9$

$= \log_3 9 = 2$

(35) $(\log_3 5 + \log_9 25)(\log_5 9 + \log_{25} 3)$

$2 \quad \frac{1}{2} \quad 2 \quad \frac{1}{2}$

$= 2 + \frac{1}{2} + 2 + \frac{1}{2} = 5$

8 以下の方程式, 不等式を解け.

(36) $\log_2 x = 4$

$2^4 = x \qquad x = 16$

(37) $\log_{\frac{1}{10}} x = 2$

$\left(\frac{1}{10}\right)^2 = x \qquad x = \frac{1}{100}$

(38) $\log_x 9 = 2$

$x > 0$より

$x^2 = 9 \qquad x = 3.$

(39) $\log_4 x = 2$

$4^2 = x. \qquad x = 16$

(40) $\log_2 x + \log_2(x-1) = 1$

真 $x > 0.\ x - 1 > 0 \rightarrow x > 1.$

$\log_2 x(x-1) = \log_2 2$

$x(x-1) = 2$

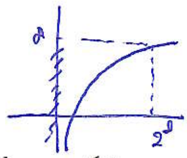
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x > 1$より

$x = 2$

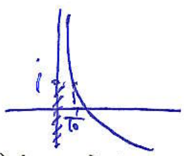
(41) $\log_2 x < 8$



$0 < x < 2^8$

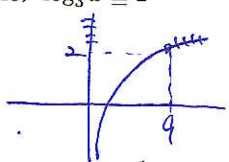
$0 < x < 256$

(42) $\log_{\frac{1}{10}} x \leqq 1$



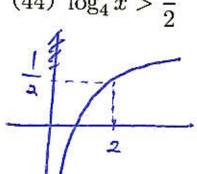
$\frac{1}{10} \leq x$

(43) $\log_3 x \geqq 2$



$9 \leqq x$

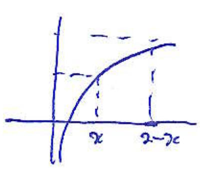
(44) $\log_4 x > \frac{1}{2}$



$2 < x$

(45) $\log_2(2-x) \geqq \log_2 x$

真 $2 - x > 0.\ x > 0 \rightarrow 0 < x < 2.$ —①



$x < 2 - x$

$2x < 2$

$x < 1$ —②

①, ②より

$0 < x < 1$

令和5年度第1学年4組 学年末考査 (準備問題) 数学1 (その4)

R6. 2.16

$\boxed{8}$ 以下の問いに答えよ．

(46) 以下の2つの数の大小関係を不等号を用いて表せ．

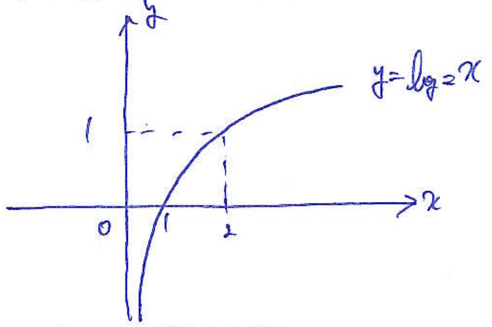
$$2\log_5 3,\quad 3\log_5 2$$

$2\log_5 3 = \log_5 9.$ 大

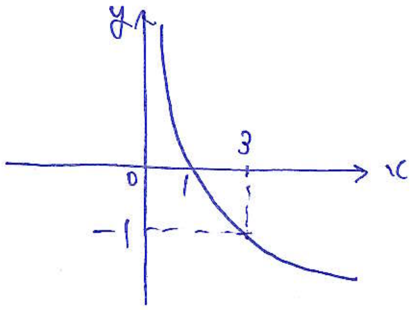
$3\log_5 2 = \log_5 8$ 小

$\therefore\ 3\log_5 2 < 2\log_5 3$

(47) $y = \log_2 x$ のグラフを描け．



(48) $y = \log_{\frac{1}{3}} x$ のグラフを描け．



(49) 3^{10} は何桁か．ただし，$\log_{10} 3 = 0.4771$ とする．

$10^N = 3^{10}$

$\log_{10} 10^N = \log_{10} 3^{10}$

$N = 10 \cdot 0.4771$

$= 4.771.$ $\quad\therefore\ 3^{10} = 10^{4.771}$

i.e. 5桁

$10^4 = 10000$ 5桁

$10^5 = 100000$ 6桁

(50) $\left(\dfrac{1}{2}\right)^{20}$ を小数で表したとき，小数第何位に初めて0でない数が現れるか．ただし，$\log_{10} 2 = 0.3010$ とする．

$10^N = \left(\frac{1}{2}\right)^{20}$

$\log_{10} 10^N = \log_{10}\left(\frac{1}{2}\right)^{20}$

$= \log_{10} 2^{-20}$

$\therefore\ N = -20 \cdot 0.3010$

$= -6.020$

$\therefore\ \left(\frac{1}{2}\right)^{20} = 10^{-6.020}$

$10^{-6} = 0.000001$

$10^{-7} = 0.0000001$

小数第7位