106 【三角関数】

θ の関数 $y=\sin 2\theta+\sin\theta+\cos\theta$ について，最大値，最小値を求めよ．

$\sin 2\theta = 2\sin\theta\cos\theta$ より

$y = 2\sin\theta\cos\theta + \sin\theta + \cos\theta$

$t = \sin\theta + \cos\theta$ とおく．

$t = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right)$

$= \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

$\therefore -\sqrt{2} \leq t \leq \sqrt{2}$．

また

$t^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$\therefore 2\sin\theta\cos\theta = t^2 - 1$．

よって

$y = t^2 - 1 + t$

$= t^2 + t - 1 \qquad (-\sqrt{2} \leq t \leq \sqrt{2})$

$= \left(t + \frac{1}{2}\right)^2 - \frac{5}{4}$

軸 $t = -\frac{1}{2}$

図より $t = -\frac{1}{2}$ で Min $-\frac{5}{4}$

$t = \sqrt{2}$ で Max. $1+\sqrt{2}$