

1 指数

1.1 復習

以下の計算をせよ。(7)以降は推測せよ。

$$(1) 2^6 = 64$$

↑ $\times 2$

$$(2) 2^5 = 32$$

↑ $\times 2$

$$(3) 2^4 = 16$$

↑ $\times 2$

$$(4) 2^3 = 8$$

↑ $\times 2$

$$(5) 2^2 = 4$$

↑ $\times 2$

$$(6) 2^1 = 2$$

↑ $\times 2$

$$(7) 2^0 = 1$$

↑ $\times 2$

$$(8) 2^{-1} = \frac{1}{2}$$

↑ $\times 2$

$$(9) 2^{-2} = \frac{1}{4}$$

1.2 復習, 推測

左の結果も参考にしつつ, 以下の計算をせよ。

$$(1) (-5)^3 = -125$$

↑ $\times (-5)$

$$(2) (-5)^2 = 25$$

↑ $\times (-5)$

$$(3) (-5)^1 = -5$$

↑ $\times (-5)$

$$(4) (-5)^0 = 1$$

↑ $\times (-5)$

$$(5) (-5)^{-1} = -\frac{1}{5}$$

↑ $\times (-5)$

$$(6) (-5)^{-2} = \frac{1}{25}$$

↑ $\times (-5)$

$$(7) (-5)^{-3} = -\frac{1}{125}$$

1.3 復習

以下の計算をせよ。

$$(1) 3^2 2^2 = 9 \times 4 \\ = 36$$

$$(2) (2^2)^3 = 4^3 \\ = 64$$

$$(3) (2 \times 3)^3 = 6^3 \\ = 216$$

$$(4) \frac{2^{10}}{2^5} = \frac{\cancel{2 \times 2 \times 2 \times 2 \times 2} \times 2^5}{\cancel{2 \times 2 \times 2 \times 2 \times 2}} \\ = 2^5 \\ = 32$$

$$(5) \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} \\ = \frac{8}{27}$$

1.4 一般化

以下の計算をせよ。

$$(1) a^3 a^4 = a^7$$

$$(2) (a^2)^3 = a^6$$

$$(3) (a \times b)^3 = a^3 b^3$$

$$(4) \frac{a^9}{a^5} = a^4$$

$$(5) \left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

1.5 拡張

石倉記

指数法則

$a \neq 0, b \neq 0, m, n \in \mathbb{Z}$

$$1. a^m a^n = a^{m+n}$$

$$2. (a^m)^n = a^{mn}$$

$$3. (ab)^m = a^m b^m$$

$$4. \frac{a^m}{a^n} = a^{m-n}$$

$$5. \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

この性質は、 m, n : 整数以外でも成立するに拡張可能。

例

$$2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^{\frac{1}{2} + \frac{1}{2}}$$

$$= 2^1 = 2$$

$$\text{つまり } 2^{\frac{1}{2}} = \sqrt{2}$$

(2回かける2に1/2ずつ)

規則的に...

$$2^2 = 4$$

$\uparrow \times 2$

$$\begin{array}{c} 2^1 = 2 \\ \uparrow + \frac{1}{2} \\ 2^{\frac{1}{2}} = \sqrt{2} \\ \uparrow + \frac{1}{2} \\ 2^0 = 1 \end{array}$$

$2^{\frac{1}{2}}$ は 2回かける2に1/2ずつ
 $\rightarrow \sqrt{2}$!!

1.6 問題

以下の値を求めよ。

(1) $9^{\frac{1}{2}}$

$$= (3^2)^{\frac{1}{2}} = 3^{2 \cdot \frac{1}{2}} = 3$$

(2) $8^{\frac{2}{3}}$

$$= 2^{3 \cdot \frac{2}{3}} = 2^2 = 4$$

(3) $81^{-\frac{1}{4}} = (3^4)^{-\frac{1}{4}}$

$$= 3^{-1} = \frac{1}{3}$$

(4) $125^{\frac{4}{3}} = (5^3)^{\frac{4}{3}}$

$$= 5^4 = 25^2 = 625$$

(5) $3^{\frac{3}{2}} \times 9^{\frac{1}{4}} \times 81^{-\frac{3}{8}}$

$$= 3^{\frac{3}{2}} \times 3^{2 \cdot \frac{1}{4}} \times 3^{4 \cdot \frac{-3}{8}}$$

$$= 3^{\frac{3}{2} + \frac{1}{2} - \frac{3}{2}} = 3^{\frac{1}{2}}$$

(6) $2^{\frac{5}{2}} \times 8^{\frac{3}{4}} \div 4^{-\frac{1}{4}}$

$$= 2^{\frac{5}{2}} \times 2^{3 \cdot \frac{3}{4}} \times 4^{\frac{1}{4}}$$

$$= 2^{\frac{5}{2}} \times 2^{\frac{9}{4}} \times 2^{\frac{1}{2}}$$

$$= 2^{\frac{5}{2} + \frac{9}{4} + \frac{1}{2}}$$

$$= 2^{3 + 2 + \frac{1}{4}} = 2^{\frac{21}{4}}$$

1.7 根号拡張

$$2^{\frac{1}{2}} = \sqrt{2} \quad \leftarrow \text{指数の拡張が10.}$$

2回拡張2に73分の1の正の値。

同じように...

$$2^{\frac{1}{3}} \dots 3回拡張2に73分の1の正の値。$$

$$= \sqrt[3]{2} \quad \text{と} \quad \frac{1}{3} < \dots$$

一般化

$$\boxed{2^{\frac{1}{n}} = \sqrt[n]{2}}$$

計算可能と見る。指数の拡張に可及的 easy?

1.8 問題

以下の値を求めよ。

$$(1) \sqrt[3]{8} = 8^{\frac{1}{3}} = 2^{3 \cdot \frac{1}{3}} = 2$$

$$(2) \sqrt[4]{\frac{1}{16}} = (2^{-4})^{\frac{1}{4}} = 2^{-1} = \frac{1}{2}$$

$$(3) \sqrt[4]{81} = (3^4)^{\frac{1}{4}} = 3$$

$$(4) \sqrt[4]{4} \sqrt[3]{2} = 4^{\frac{1}{4}} \times 2^{\frac{1}{3}} = (2^2)^{\frac{1}{4}} \times 2^{\frac{1}{3}} = 2^{\frac{2}{4}} \times 2^{\frac{1}{3}} = 2^{\frac{1}{2}} \times 2^{\frac{1}{3}} = 2^{\frac{5}{6}} = 2^{\frac{5}{6}}$$

$$(5) (\sqrt[3]{5})^2 = (5^{\frac{1}{3}})^2 = 5^{\frac{2}{3}}$$

$$(6) \frac{\sqrt[4]{2}}{\sqrt[5]{32}} = \frac{2^{\frac{1}{4}}}{(2^5)^{\frac{1}{5}}} = \left(\frac{2}{2^5} \right)^{\frac{1}{4}} = (2^{-4})^{\frac{1}{4}} = 2^{-1} = \frac{1}{2}$$

$$(7) \sqrt[3]{\sqrt[5]{64}} = \left((2^6)^{\frac{1}{5}} \right)^{\frac{1}{3}} = 2^{6 \cdot \frac{1}{5} \cdot \frac{1}{3}} = 2^{\frac{2}{5}}$$

$$(8) \sqrt[4]{5} \div \sqrt{5} \times \sqrt[3]{5} = 5^{\frac{1}{4}} \times 5^{-\frac{1}{2}} \times 5^{\frac{1}{3}} = 5^{\frac{2}{4} - \frac{2}{4} + \frac{1}{3}} = 5^{\frac{1}{3}}$$