

17 以下の問いに答えよ。

(1) $\cos 5\theta$ を $\cos \theta$ で表せ。

(2) $\cos^2 \frac{1}{10}\pi$ の値を求めよ。

$$\begin{aligned} (1) \quad \cos 5\theta &= \cos(3\theta + 2\theta) \\ &= \cos 3\theta \cdot \cos 2\theta - \sin 3\theta \cdot \sin 2\theta \\ &= \cos 3\theta (2\cos^2 \theta - 1) - \sin 3\theta (2\sin \theta \cos \theta) \quad \text{--- (A)} \end{aligned}$$

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$$\begin{aligned} \cos 3\theta &= \cos(2\theta + \theta) \\ &= \cos 2\theta \cdot \cos \theta - \sin 2\theta \cdot \sin \theta \\ &= (2\cos^2 \theta - 1)\cos \theta - 2\sin \theta \cos \theta \cdot \sin \theta \\ &= 2\cos^3 \theta - \cos \theta - 2\sin^2 \theta \cdot \cos \theta \\ &= 2\cos^3 \theta - \cos \theta - 2(1 - \cos^2 \theta)\cos \theta \\ &= 2\cos^3 \theta - \cos \theta - 2\cos \theta + 2\cos^3 \theta \\ &= 4\cos^3 \theta - 3\cos \theta. \end{aligned}$$

$$\begin{aligned} \sin 3\theta &= \sin(2\theta + \theta) \\ &= \sin 2\theta \cdot \cos \theta + \cos 2\theta \cdot \sin \theta \\ &= 2\sin \theta \cos \theta \cdot \cos \theta + (2\cos^2 \theta - 1)\sin \theta \\ &= 2\sin \theta \cos^2 \theta + 2\sin \theta \cos^2 \theta - \sin \theta \\ &= 4\sin \theta \cos^2 \theta - \sin \theta \end{aligned}$$

※ 1/17 入.

$$\begin{aligned} \cos 5\theta &= (4\cos^3 \theta - 3\cos \theta)(2\cos^2 \theta - 1) \\ &\quad - (4\sin \theta \cos^2 \theta - \sin \theta) \cdot 2\sin \theta \cos \theta \end{aligned}$$

$$\begin{aligned} (1) \text{項目} &= 8\cos^5 \theta - 6\cos^3 \theta - 4\cos^3 \theta + 3\cos \theta \\ &= 8\cos^5 \theta - 10\cos^3 \theta + 3\cos \theta \end{aligned}$$

$$\begin{aligned} (2) \text{項目} &= -8\sin^2 \theta \cos^3 \theta + 2\sin^2 \theta \cos \theta \\ &= -8(1 - \cos^2 \theta)\cos^3 \theta + 2(1 - \cos^2 \theta)\cos \theta \\ &= -8\cos^3 \theta + 8\cos^5 \theta + 2\cos \theta - 2\cos^3 \theta \end{aligned}$$

$$\begin{aligned} \therefore \cos 5\theta &= 8\cos^5 \theta - 10\cos^3 \theta + 3\cos \theta \\ &\quad + 8\cos^5 \theta - 10\cos^3 \theta + 2\cos \theta \\ &= 16\cos^5 \theta - 20\cos^3 \theta + 5\cos \theta \end{aligned}$$

(1/10 4.11 7.28)

(2) (1) 3'

$$\cos \frac{1}{5}\theta = \cos \theta (16\cos^4 \theta - 20\cos^2 \theta + 5)$$

$$\theta = \frac{\pi}{10} \text{ 2/12/13/14}$$

$$\left(\frac{1}{5}\pi\right) = \cos \frac{1}{5}\pi = \cos \frac{1}{2}\pi = 0.$$

$$\therefore 0 = \cos \frac{\pi}{10} (16\cos^4 \frac{\pi}{10} - 20\cos^2 \frac{\pi}{10} + 5)$$

$$\therefore \cos \frac{\pi}{10} \neq 0 \text{ 7/12/13/14}$$

$$16\cos^4 \frac{\pi}{10} - 20\cos^2 \frac{\pi}{10} + 5 = 0$$

$$\cos^2 \frac{\pi}{10} = x \text{ 2/12/13/14}$$

$$16x^2 - 20x + 5 = 0$$

$$x = \frac{20 \pm \sqrt{400 - 4 \cdot 5 \cdot 16}}{2 \cdot 16}$$

$$= \frac{20 \pm \sqrt{80}}{2 \cdot 16}$$

$$= \frac{20 \pm 4\sqrt{5}}{2 \cdot 16}$$

$$= \frac{5 \pm \sqrt{5}}{8}$$

$$\text{I 2, } \cos \frac{\pi}{4} < \cos \frac{\pi}{10}, \text{ i.e. } \frac{1}{\sqrt{2}} < \cos \frac{\pi}{10}$$

$$\text{7/12/13/14, } \frac{1}{2} < \cos^2 \frac{\pi}{10}$$

$$\therefore \frac{1}{2} < x$$

$$\therefore x = \frac{5 - \sqrt{5}}{8}$$

$$\text{i.e. } \cos^2 \frac{\pi}{10} = \frac{5 - \sqrt{5}}{8}$$

$\cos 5\theta$ は、10/10 加法定理を丁寧に使う!!

(2) は (1) の仮定を参考で解く。