$\boxed{57}$ $x = \dfrac{2}{3-\sqrt{5}}, y = \dfrac{2}{3+\sqrt{5}}$ とする.

(1) x, y を有理化せよ.

(2) $x+y, xy$ を求めよ.

(3) x^2+y^2 を求めよ.

(4) x^4-y^4 を求めよ.

(1)

$$x = \frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{2(3+\sqrt{5})}{9-5} = \underline{\frac{1}{2}(3+\sqrt{5})}\ //$$

$$y = \frac{2}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$$

$$= \frac{2(3-\sqrt{5})}{9-5} = \underline{\frac{1}{2}(3-\sqrt{5})}\ //$$

(2)

$$x+y = \frac{1}{2}(3+\sqrt{5}) + \frac{1}{2}(3-\sqrt{5})$$

$$= \frac{1}{2}\cdot 3 + \frac{1}{2}\cdot 3 = \underline{3}\ //$$

$$xy = \frac{1}{2}(3+\sqrt{5})\cdot\frac{1}{2}(3-\sqrt{5})$$

$$= \frac{1}{4}(9-5) = \underline{1}\ //$$

(3)

$$x^2+y^2 = (x+y)^2 - 2xy$$

$$= 3^2 - 2\cdot 1$$

$$= 9 - 2 = \underline{7}\ //$$

(4)

$$x^4-y^4 = (x^2+y^2)(x^2-y^2)$$

$$= (x^2+y^2)(x+y)(x-y)$$

ここで、(2),(3)より

$$x^2+y^2 = 7$$

$$x+y = 3.$$

また、

$$x-y = \frac{1}{2}(3+\sqrt{5}) - \frac{1}{2}(3-\sqrt{5})$$

$$= \sqrt{5}$$

$$\therefore x^4-y^4 = 7\cdot 3\cdot\sqrt{5}$$

$$= \underline{21\sqrt{5}}\ //$$