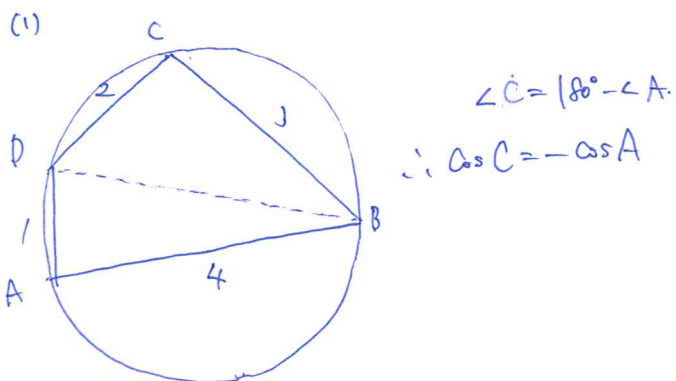


28 円に内接する四角形 ABCD について, $AB=4$, $BC=3$, $CD=2$, $DA=1$ である. 以下の問いに答えよ.

(1) $\cos A$ の値を求めよ.

(2) 四角形 ABCD の面積 S を求めよ.



$\triangle ABD$ で余弦定理.

$$\begin{aligned} BD^2 &= 1 + 16 - 2 \cdot 1 \cdot 4 \cdot \cos A \\ &= 17 - 8 \cos A. \quad \text{--- ①} \end{aligned}$$

$\triangle BCD$ で余弦定理.

$$\begin{aligned} BD^2 &= 4 + 9 - 2 \cdot 2 \cdot 3 \cdot \cos C \\ &= 13 + 12 \cos A \quad \text{--- ②} \end{aligned}$$

① - ②

$$0 = 4 - 20 \cos A$$

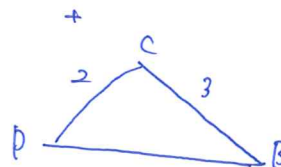
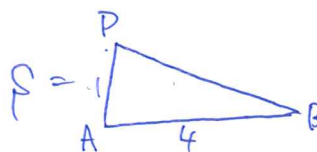
$$\cos A = \frac{1}{5}$$

(2) $\sin^2 A + \cos^2 A = 1$

$$\sin A = \frac{2\sqrt{6}}{5}$$

$$C = 180^\circ - A$$

$$\begin{aligned} \sin C &= \sin(180^\circ - A) \\ &= \sin A = \frac{2\sqrt{6}}{5} \end{aligned}$$



$$= \frac{1}{2} \cdot 4 \cdot \frac{2\sqrt{6}}{5} + \frac{1}{2} \cdot 3 \cdot \frac{2\sqrt{6}}{5}$$

$$= \frac{\sqrt{6}}{5} (4 + 3) = \frac{7\sqrt{6}}{5}$$