1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

58 $x+\dfrac{1}{x}=2$ とする．以下の値を求めよ．

(1) $x^2+\dfrac{1}{x^2}$

(2) $x^3+\dfrac{1}{x^3}$

(3) $x^4+\dfrac{1}{x^4}$

(1) $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2\cdot x\cdot\frac{1}{x}$

$=\left(x+\frac{1}{x}\right)^2-2$

$=2^2-2$

$=4-2=2$

(2) $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\cdot x^2\cdot\frac{1}{x}-3x\cdot\frac{1}{x^2}$

$=\left(x+\frac{1}{x}\right)^3-3x-3\cdot\frac{1}{x}$

$=\left(x+\frac{1}{x}\right)^3-3\cdot\left(x+\frac{1}{x}\right)$

$=2^3-3\cdot 2$

$=8-6=2$

(3) $\left(x+\frac{1}{x}\right)^4=x^4+4\cdot x^3\cdot\frac{1}{x}+6\cdot x^2\cdot\frac{1}{x^2}+4x\cdot\frac{1}{x^3}+\frac{1}{x^4}$

$=x^4+\frac{1}{x^4}+4\left(x^2+\frac{1}{x^2}\right)+6$

$\therefore\ x^4+\frac{1}{x^4}=\left(x+\frac{1}{x}\right)^4-4\left(x^2+\frac{1}{x^2}\right)-6$

$=2^4-4\cdot 2-6$

$=16-8-6$

$=2$