5 $x=\dfrac{1}{\sqrt{3}+\sqrt{2}}, y=\dfrac{1}{\sqrt{3}-\sqrt{2}}$ のとき，以下の式の値を求めよ．

(1) $x+y$

(2) x^2+y^2

(3) x^3-y^3

(1)

$$x=\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$=\frac{\sqrt{3}-\sqrt{2}}{3-2}=\sqrt{3}-\sqrt{2}.$$

$$y=\sqrt{3}+\sqrt{2}$$

$$\therefore x+y=(\sqrt{3}-\sqrt{2})+(\sqrt{3}+\sqrt{2})$$

$$=2\sqrt{3}$$

(2) x^2+y^2

$$=(\sqrt{3}-\sqrt{2})^2+(\sqrt{3}+\sqrt{2})^2$$

$$=(3-2\sqrt{6}+2)+(3+2\sqrt{6}+2)$$

$$=10$$

(3) x^3-y^3

$$=(x-y)(x^2+xy+y^2)$$

ここで，

$$x-y=(\sqrt{3}-\sqrt{2})-(\sqrt{3}+\sqrt{2})$$

$$=-2\sqrt{2}.$$

$$xy=(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$$

$$=3-2=1$$

$$\therefore (与式)=(-2\sqrt{2})\times(10+1)$$

$$=-22\sqrt{2}$$

3乗の因数分解公式を覚えていなければ
各を3乗して引いてもいいけど… 大変そう。