

$$\begin{array}{cccc}
 & & 1 & \\
 & & 1 & 1 \\
 & 1 & 2 & 1 \\
 1 & 3 & 3 & 1 \\
 1 & 4 & 6 & 4 & 1
 \end{array}$$

58  $x + \frac{1}{x} = 2$  とする. 以下の値を求めよ.

(1)  $x^2 + \frac{1}{x^2}$

(2)  $x^3 + \frac{1}{x^3}$

(3)  $x^4 + \frac{1}{x^4}$

$$\begin{aligned}
 (1) \quad x^2 + \frac{1}{x^2} &= \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} \\
 &= \left(x + \frac{1}{x}\right)^2 - 2 \\
 &= 2^2 - 2 \\
 &= 4 - 2 = \underline{2}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3 \cdot x^2 \cdot \frac{1}{x} - 3 \cdot x \cdot \frac{1}{x^2} \\
 &= \left(x + \frac{1}{x}\right)^3 - 3x - 3 \cdot \frac{1}{x} \\
 &= \left(x + \frac{1}{x}\right)^3 - 3 \cdot \left(x + \frac{1}{x}\right) \\
 &= 2^3 - 2 \cdot 2 \\
 &= 8 - 4 = \underline{4}
 \end{aligned}$$

$$(3) \quad \left(x + \frac{1}{x}\right)^4 = x^4 + 4 \cdot x^3 \cdot \frac{1}{x} + 6 \cdot x^2 \cdot \frac{1}{x^2} + 4x \cdot \frac{1}{x^3} + \frac{1}{x^4}$$

$$= x^4 + \frac{1}{x^4} + 4\left(x^2 + \frac{1}{x^2}\right) + 6$$

$$\begin{aligned}
 \therefore x^4 + \frac{1}{x^4} &= \left(x + \frac{1}{x}\right)^4 - 4\left(x^2 + \frac{1}{x^2}\right) - 6 \\
 &= 2^4 - 4 \cdot 2 - 6 \\
 &= 16 - 8 - 6 \\
 &= \underline{2}
 \end{aligned}$$