

57  $x = \frac{2}{3-\sqrt{5}}, y = \frac{2}{3+\sqrt{5}}$  とする.

(1)  $x, y$  を有理化せよ.

(2)  $x+y, xy$  を求めよ.

(3)  $x^2+y^2$  を求めよ.

(4)  $x^4-y^4$  を求めよ.

$$\begin{aligned} (1) \quad x &= \frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} \\ &= \frac{2(3+\sqrt{5})}{9-5} = \frac{1}{2}(3+\sqrt{5}) \end{aligned}$$

$$\begin{aligned} y &= \frac{2}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} \\ &= \frac{2(3-\sqrt{5})}{9-5} = \frac{1}{2}(3-\sqrt{5}) \end{aligned}$$

$$\begin{aligned} (2) \quad x+y &= \frac{1}{2}(3+\sqrt{5}) + \frac{1}{2}(3-\sqrt{5}) \\ &= \frac{1}{2} \cdot 3 + \frac{1}{2} \cdot 3 = 3 \end{aligned}$$

$$\begin{aligned} xy &= \frac{1}{2}(3+\sqrt{5}) \cdot \frac{1}{2}(3-\sqrt{5}) \\ &= \frac{1}{4}(9-5) = 1 \end{aligned}$$

$$\begin{aligned} (3) \quad x^2+y^2 &= (x+y)^2 - 2xy \\ &= 3^2 - 2 \cdot 1 \\ &= 9 - 2 = 7 \end{aligned}$$

$$\begin{aligned} (4) \quad x^4-y^4 &= (x^2+y^2)(x^2-y^2) \\ &= (x^2+y^2)(x+y)(x-y) \end{aligned}$$

$$\begin{aligned} &= 7 \cdot 3 \\ &= 21 \end{aligned}$$

$$\begin{aligned} x-y &= \frac{1}{2}(3+\sqrt{5}) - \frac{1}{2}(3-\sqrt{5}) \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} \therefore x^4-y^4 &= 7 \cdot 3 \cdot \sqrt{5} \\ &= 21\sqrt{5} \end{aligned}$$