

6 加法定理

6.1 計算練習

「75° の三角比を求めたい。」 (← 目標)

加法定理

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \\ &\quad (\text{符号同順})\end{aligned}$$

加法定理を使って 75° の三角比を求めてみよう。

$$75^\circ = 30^\circ + 45^\circ \text{ となる}$$

$$\begin{aligned}\sin 75^\circ &= \sin(30^\circ + 45^\circ) \\ &= \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ \\ &= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{1 + \sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

$$\begin{aligned}\cos 75^\circ &= \cos(30^\circ + 45^\circ) \\ &= \cos 30^\circ \cdot \cos 45^\circ - \sin 30^\circ \cdot \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\tan 75^\circ &= \tan(30^\circ + 45^\circ) \\ &= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \cdot \tan 45^\circ} \\ &= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1} \\ &= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} = \frac{1 + \sqrt{3}}{\sqrt{3} - 1} = 2 + \sqrt{3}\end{aligned}$$

計算練習

(1) 15° の三角比を求めよ。

$$\begin{aligned}15^\circ &= 45^\circ - 30^\circ \text{ となる} \\ \sin 15^\circ &= \sin(45^\circ - 30^\circ) \\ &= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\cos 15^\circ &= \cos(45^\circ - 30^\circ) \\ &= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\tan 15^\circ &= \frac{\sin 15^\circ}{\cos 15^\circ} \\ &= \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = 2 - \sqrt{3}\end{aligned}$$

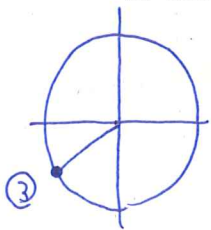
(2) $\frac{11}{12}\pi$ の三角比を求めよ。

$$\begin{aligned}\frac{11}{12}\pi &= \frac{3}{4}\pi + \frac{1}{6}\pi \text{ となる} \\ \sin \frac{11}{12}\pi &= \sin\left(\frac{3}{4}\pi + \frac{1}{6}\pi\right) \\ &= \sin \frac{3}{4}\pi \cdot \cos \frac{1}{6}\pi + \cos \frac{3}{4}\pi \cdot \sin \frac{1}{6}\pi \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{1}{\sqrt{2}}\right) \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\cos \frac{11}{12}\pi &= \cos\left(\frac{3}{4}\pi + \frac{1}{6}\pi\right) \\ &= \cos \frac{3}{4}\pi \cdot \cos \frac{1}{6}\pi - \sin \frac{3}{4}\pi \cdot \sin \frac{1}{6}\pi \\ &= -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\ &= -\frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

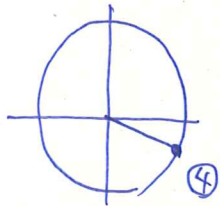
$$\begin{aligned}\tan \frac{11}{12}\pi &= \frac{\sin \frac{11}{12}\pi}{\cos \frac{11}{12}\pi} \\ &= -\frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = -(2 - \sqrt{3})\end{aligned}$$

- (3) α の動径が第3象限, β の動径が第4象限にあり,
 $\sin \alpha = -\frac{3}{5}, \cos \beta = \frac{4}{5}$ のとき, 以下の問いに答えよ.
 (a) $\cos \alpha$ の値を求めよ.



$$\begin{aligned} R^2 \alpha + \cos^2 \alpha &= |r|^2 \\ \cos^2 \alpha &= \frac{16}{25} \\ \text{左図より, } \cos \alpha &= \text{㉑} \\ \therefore \cos \alpha &= -\frac{4}{5} \end{aligned}$$

- (b) $\sin \beta$ の値を求めよ.



$$\begin{aligned} R^2 \beta + \cos^2 \beta &= |r|^2 \\ R^2 \beta &= \frac{9}{25} \\ \text{左図より, } R \beta &= \text{㉒} \\ \therefore R \beta &= -\frac{3}{5} \end{aligned}$$

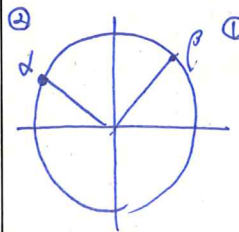
- (c) $\sin(\alpha + \beta)$ の値を求めよ.

$$\begin{aligned} R(\alpha + \beta) &= R \alpha \cos \beta + \cos \alpha R \beta \\ &= -\frac{3}{5} \cdot \frac{4}{5} + \left(-\frac{4}{5}\right) \cdot \left(-\frac{3}{5}\right) \\ &= 0 \end{aligned}$$

- (d) $\cos(\alpha - \beta)$ の値を求めよ.

$$\begin{aligned} \cos(\alpha - \beta) &= \cos \alpha \cos \beta + R \alpha R \beta \\ &= \left(-\frac{4}{5}\right) \cdot \frac{4}{5} + \left(-\frac{3}{5}\right) \cdot \left(-\frac{3}{5}\right) \\ &= \frac{-16}{5} + \frac{9}{5} = -\frac{7}{5} \end{aligned}$$

- (4) α の動径が第2象限, β の動径が第1象限にあり,
 $\sin \alpha = \frac{2}{3}, \cos \beta = \frac{3}{5}$ のとき, $\sin(\alpha - \beta), \cos(\alpha + \beta)$ の値を求めよ.



$$\begin{aligned} R^2 \alpha + \cos^2 \beta &= |r|^2 \\ \cos^2 \alpha &= \frac{5}{9} \\ R^2 \beta &= \frac{16}{25} \end{aligned}$$

∴ 左図より, $\cos \alpha = \text{㉑}$, $R \beta = \text{㉒}$

$$\therefore \cos \alpha = -\frac{\sqrt{5}}{3}, R \beta = \frac{4}{5}$$

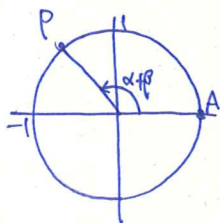
$$\begin{aligned} R(\alpha - \beta) &= R \alpha \cos \beta - \cos \alpha R \beta \\ &= \frac{2}{3} \cdot \frac{3}{5} - \left(-\frac{\sqrt{5}}{3}\right) \cdot \frac{4}{5} \\ &= \frac{6}{15} + \frac{4\sqrt{5}}{15} \\ &= \frac{6 + 4\sqrt{5}}{15} \end{aligned}$$

$$\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - R \alpha R \beta \\ &= \left(-\frac{\sqrt{5}}{3}\right) \cdot \frac{3}{5} - \frac{2}{3} \cdot \frac{4}{5} \\ &= \frac{-3\sqrt{5} - 8}{15} \end{aligned}$$

6.2 証明

加法定理

<証明>



左図に示す。

 $A(1, 0)$ $P(\cos(\alpha+\beta), \sin(\alpha+\beta))$

すると

$$\begin{aligned} AP^2 &= \cos^2(\alpha+\beta) + (1 - \sin(\alpha+\beta))^2 \\ &= \cos^2(\alpha+\beta) + 1 - 2\sin(\alpha+\beta) + \sin^2(\alpha+\beta) \\ &= 2 - 2\sin(\alpha+\beta) \end{aligned}$$

同様に $Q(\cos \alpha, \sin \alpha)$ $R(\cos \beta, \sin \beta)$ また、 $\cos(-\beta) = \cos \beta$ $\sin(-\beta) = -\sin \beta$

すると

$$\begin{aligned} QR^2 &= (\cos \alpha - \cos \beta)^2 + (\sin \alpha - (-\sin \beta))^2 \\ &= \cos^2 \alpha - 2\cos \alpha \cos \beta + \cos^2 \beta \\ &\quad + (\sin^2 \alpha + 2\sin \alpha \sin \beta + \sin^2 \beta) \\ &= 2 - 2\cos \alpha \cos \beta + 2\sin \alpha \sin \beta \end{aligned}$$

中心角が同じだから $AP = QR$

$$\therefore AP^2 = QR^2$$

$$2 - 2\cos(\alpha+\beta) = 2 - 2\cos \alpha \cos \beta + 2\sin \alpha \sin \beta$$

$$\therefore \cos(\alpha+\beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

また、 β を $-\beta$ とおきかえ

$$\begin{aligned} \cos(\alpha-\beta) &= \cos \alpha \cdot \cos(-\beta) - \sin \alpha \sin(-\beta) \\ &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad (2) \end{aligned}$$

$$\text{すなわち } \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \quad (1)$$

(2) の α を $\frac{\pi}{2} - \alpha$ とおきかえ

$$\begin{aligned} \sin(\alpha+\beta) &= \cos\left(\frac{\pi}{2} - (\alpha+\beta)\right) \\ &= \cos\left(\frac{\pi}{2} - \alpha - \beta\right) \\ &= \sin(\alpha+\beta) \end{aligned}$$

$$\begin{aligned} \sin(\alpha+\beta) &= \cos\left(\frac{\pi}{2} - \alpha\right) \cos \beta + \sin\left(\frac{\pi}{2} - \alpha\right) \sin \beta \\ &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \end{aligned}$$

$$\therefore \sin(\alpha+\beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad (3)$$

また、 β を $-\beta$ とおきかえ

$$\begin{aligned} \sin(\alpha-\beta) &= \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta) \\ &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \quad (4) \end{aligned}$$

$$\text{すなわち } \tan \theta = \frac{\sin \theta}{\cos \theta} \quad (5)$$

$$\begin{aligned} \tan(\alpha+\beta) &= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \\ &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \end{aligned}$$

分子・分母を $\cos \alpha \cos \beta$ で割る

$$\begin{aligned} \tan(\alpha+\beta) &= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\ &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \end{aligned}$$

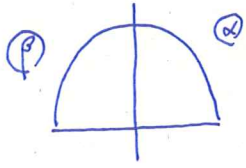
$$\therefore \tan(\alpha+\beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \quad (6)$$

また、 β を $-\beta$ とおきかえ

$$\begin{aligned} \tan(\alpha-\beta) &= \frac{\tan \alpha + \tan(-\beta)}{1 - \tan \alpha \tan(-\beta)} \\ &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \quad (7) \end{aligned}$$

6.3 演習

(1) $\sin \alpha = \frac{3}{5}$ ($0 < \alpha < \frac{\pi}{2}$), $\cos \beta = -\frac{4}{5}$ ($\frac{\pi}{2} < \beta < \pi$) のとき, $\sin(\alpha + \beta)$, $\cos(\alpha - \beta)$, $\tan(\alpha - \beta)$ の値を求めよ.



$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad (*)$$

$$\cos^2 \alpha = \frac{16}{25}$$

$$\cos \alpha = \frac{4}{5}$$

上図より $\cos \alpha > 0$, $\sin \beta > 0$

$$\therefore \cos \alpha = \frac{4}{5}, \sin \beta = \frac{3}{5}$$

$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \frac{3}{5} \cdot \left(-\frac{4}{5}\right) + \frac{4}{5} \cdot \frac{3}{5} \end{aligned}$$

$$= 0$$

$$\begin{aligned} \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \frac{4}{5} \cdot \left(-\frac{4}{5}\right) + \frac{3}{5} \cdot \frac{3}{5} \end{aligned}$$

$$= -\frac{7}{25}$$

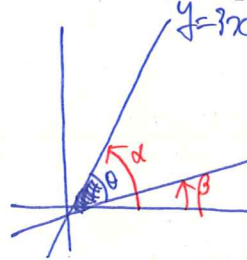
$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\therefore \tan \alpha = \frac{3}{4}, \tan \beta = -\frac{3}{4} \quad (*)$$

$$\begin{aligned} \tan(\alpha - \beta) &= \frac{\frac{3}{4} - \left(-\frac{3}{4}\right)}{1 + \frac{3}{4} \cdot \left(-\frac{3}{4}\right)} \\ &= \frac{\frac{6}{4}}{1 - \frac{9}{16}} = \frac{\frac{6}{4}}{\frac{7}{16}} \end{aligned}$$

$$= \frac{6 \cdot 16}{7 \cdot 4} = \frac{24}{7}$$

(2) 2 直線 $y = 3x$, $y = \frac{1}{2}x$ のなす鋭角を求めよ.



左図より α, β は鋭角.

求める角 θ は

$$\theta = \alpha - \beta$$

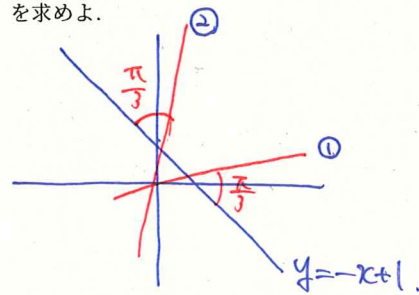
$$\therefore \tan \alpha = 3, \tan \beta = \frac{1}{2}$$

$$\begin{aligned} \tan \theta &= \tan(\alpha - \beta) \\ &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ &= \frac{3 - \frac{1}{2}}{1 + 3 \cdot \frac{1}{2}} = 1 \end{aligned}$$

$$\tan \theta = 1 \quad \theta = \text{鋭角} \tan \theta$$

$$\theta = \frac{\pi}{4}$$

(3) 原点を通り, 直線 $y = -x + 1$ と $\frac{1}{3}\pi$ の角をなす直線の方程式を求めよ.



$$\textcircled{1} \text{ は } x \text{ 軸と } \frac{\pi}{3} \text{ の角をなす直線} \quad \frac{\pi}{3} - \frac{\pi}{4} = \frac{1}{12}\pi$$

$$\textcircled{2} \text{ は, } \frac{3}{4}\pi - \frac{1}{3}\pi = \frac{5}{12}\pi$$

\therefore 求める直線の傾きは

$$\textcircled{1} \tan \frac{1}{12}\pi = \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{4}}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} = 2 - \sqrt{3}$$

$$\textcircled{2} \tan \frac{5}{12}\pi = \frac{\tan \frac{3}{4}\pi - \tan \frac{1}{3}\pi}{1 + \tan \frac{3}{4}\pi \tan \frac{1}{3}\pi}$$

$$= \frac{-1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = 2 + \sqrt{3}$$

\therefore 求める直線は

$$y = (2 - \sqrt{3})x, \quad y = (2 + \sqrt{3})x$$

7 加法定理の応用

7.1 復習

加法定理を思い出す。

$$(1) \sin(\alpha + \beta)$$

$$= \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$(2) \sin(\alpha - \beta)$$

$$= \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$(3) \cos(\alpha + \beta)$$

$$= \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$(4) \cos(\alpha - \beta)$$

$$= \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$(5) \tan(\alpha + \beta)$$

$$= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$(6) \tan(\alpha - \beta)$$

$$= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

計算練習

$$(1) \sin\left(\frac{1}{3}\pi + \frac{1}{4}\pi\right)$$

$$= \sin \frac{1}{3}\pi \cdot \cos \frac{1}{4}\pi + \cos \frac{1}{3}\pi \cdot \sin \frac{1}{4}\pi$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$(2) \cos\left(\frac{1}{12}\pi\right)$$

$$\frac{1}{12}\pi = \frac{1}{3}\pi - \frac{1}{4}\pi$$

$$= \cos\left(\frac{1}{3}\pi - \frac{1}{4}\pi\right)$$

$$= \cos \frac{1}{3}\pi \cdot \cos \frac{1}{4}\pi + \sin \frac{1}{3}\pi \cdot \sin \frac{1}{4}\pi$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

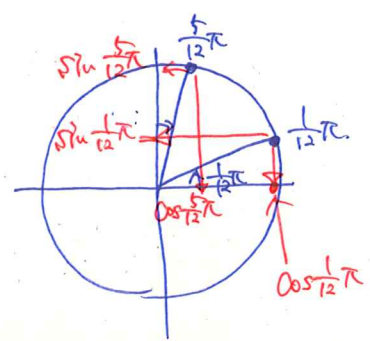
$$(3) \sin\left(\frac{1}{12}\pi\right)$$

$$= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \sin \frac{\pi}{3} \cdot \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \cdot \sin \frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$



$$(4) \sin\left(\frac{5}{12}\pi\right)$$

$$= \cos \frac{1}{12}\pi$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$(5) \cos\left(\frac{5}{12}\pi\right)$$

$$= \sin \frac{1}{12}\pi$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$(6) \tan\left(\frac{5}{12}\pi\right)$$

$$= \frac{\sin \frac{5}{12}\pi}{\cos \frac{5}{12}\pi}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = \frac{\sqrt{2}(\sqrt{3} + 1)}{\sqrt{2}(\sqrt{3} - 1)}$$

$$= \frac{2 + \sqrt{3}}{1}$$

$$\begin{aligned}\textcircled{7.2} \quad \sin(\alpha+\beta) &= \sin\alpha \cos\beta + \cos\alpha \sin\beta \\ \cos(\alpha+\beta) &= \cos\alpha \cos\beta - \sin\alpha \sin\beta \\ \tan(\alpha+\beta) &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}\end{aligned}$$

7.2 2倍角

考える

$2\alpha = \alpha + \alpha$ と考えることで、 2α の三角比を考える。

(1) $\sin(\alpha + \alpha)$ を α の三角比で表そう。

$$\begin{aligned}&= \sin\alpha \cdot \cos\alpha + \cos\alpha \cdot \sin\alpha \\ &= \underline{2 \sin\alpha \cos\alpha} \quad \text{4}\end{aligned}$$

(2) $\cos(\alpha + \alpha)$ を α の三角比で表そう。

$$\begin{aligned}&= \cos\alpha \cdot \cos\alpha - \sin\alpha \cdot \sin\alpha \\ &= \underline{\cos^2\alpha - \sin^2\alpha} \quad \text{4}\end{aligned}$$

(3) $\cos(\alpha + \alpha)$ を $\sin\alpha$ で表そう。

$$\begin{aligned}&= \cos^2\alpha - \sin^2\alpha \\ &= (1 - \sin^2\alpha) - \sin^2\alpha \\ &= \underline{1 - 2\sin^2\alpha} \quad \text{4}\end{aligned}$$

$$\begin{aligned}\sin^2\alpha + \cos^2\alpha &= 1 \\ \cos^2\alpha &= 1 - \sin^2\alpha\end{aligned}$$

(4) $\cos(\alpha + \alpha)$ を $\cos\alpha$ で表そう。

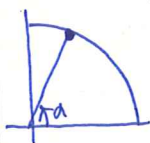
$$\begin{aligned}&= \cos^2\alpha - \sin^2\alpha \\ &= \cos^2\alpha - (1 - \cos^2\alpha) \\ &= \underline{2\cos^2\alpha - 1} \quad \text{4}\end{aligned}$$

(5) $\tan(\alpha + \alpha)$ を $\tan\alpha$ で表そう。

$$\begin{aligned}&= \frac{\tan\alpha + \tan\alpha}{1 - \tan\alpha \tan\alpha} \\ &= \underline{\frac{2\tan\alpha}{1 - \tan^2\alpha}} \quad \text{4}\end{aligned}$$

練習問題

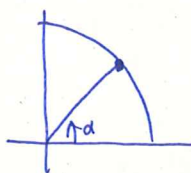
(1) $0 < \alpha < \frac{\pi}{2}$ で、 $\sin\alpha = \frac{4}{5}$ のとき、 $\sin 2\alpha$ の値を求めよ。



$$\begin{aligned}\sin^2\alpha + \cos^2\alpha &= 1 \\ \cos\alpha &= \frac{3}{5}\end{aligned}$$

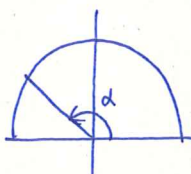
$$\begin{aligned}\sin 2\alpha &= 2\sin\alpha \cos\alpha \\ &= 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \underline{\frac{24}{25}} \quad \text{4}\end{aligned}$$

(2) $0 < \alpha < \frac{\pi}{2}$ で、 $\sin\alpha = \frac{3}{5}$ のとき、 $\cos 2\alpha$ の値を求めよ。



$$\begin{aligned}\cos 2\alpha &= 1 - 2\sin^2\alpha \\ &= 1 - 2 \cdot \left(\frac{3}{5}\right)^2 \\ &= 1 - \frac{18}{25} = \underline{\frac{7}{25}} \quad \text{4}\end{aligned}$$

(3) $\frac{\pi}{2} < \alpha < \pi$ で、 $\cos\alpha = -\frac{\sqrt{5}}{3}$ のとき、 $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$ の値を求めよ。



$$\begin{aligned}\sin^2\alpha + \cos^2\alpha &= 1 \\ \sin\alpha &= \frac{2}{3}\end{aligned}$$

$$\begin{aligned}\sin 2\alpha &= 2\sin\alpha \cos\alpha \\ &= 2 \cdot \frac{2}{3} \cdot \left(-\frac{\sqrt{5}}{3}\right) = \underline{\frac{-4\sqrt{5}}{9}} \quad \text{4}\end{aligned}$$

$$\begin{aligned}\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= \frac{5}{9} - \frac{4}{9} = \underline{\frac{1}{9}} \quad \text{4}\end{aligned}$$

$$\begin{aligned}\tan 2\alpha &= \frac{\sin 2\alpha}{\cos 2\alpha} \\ &= \frac{\frac{-4\sqrt{5}}{9}}{\frac{1}{9}} = \underline{-4\sqrt{5}} \quad \text{4}\end{aligned}$$