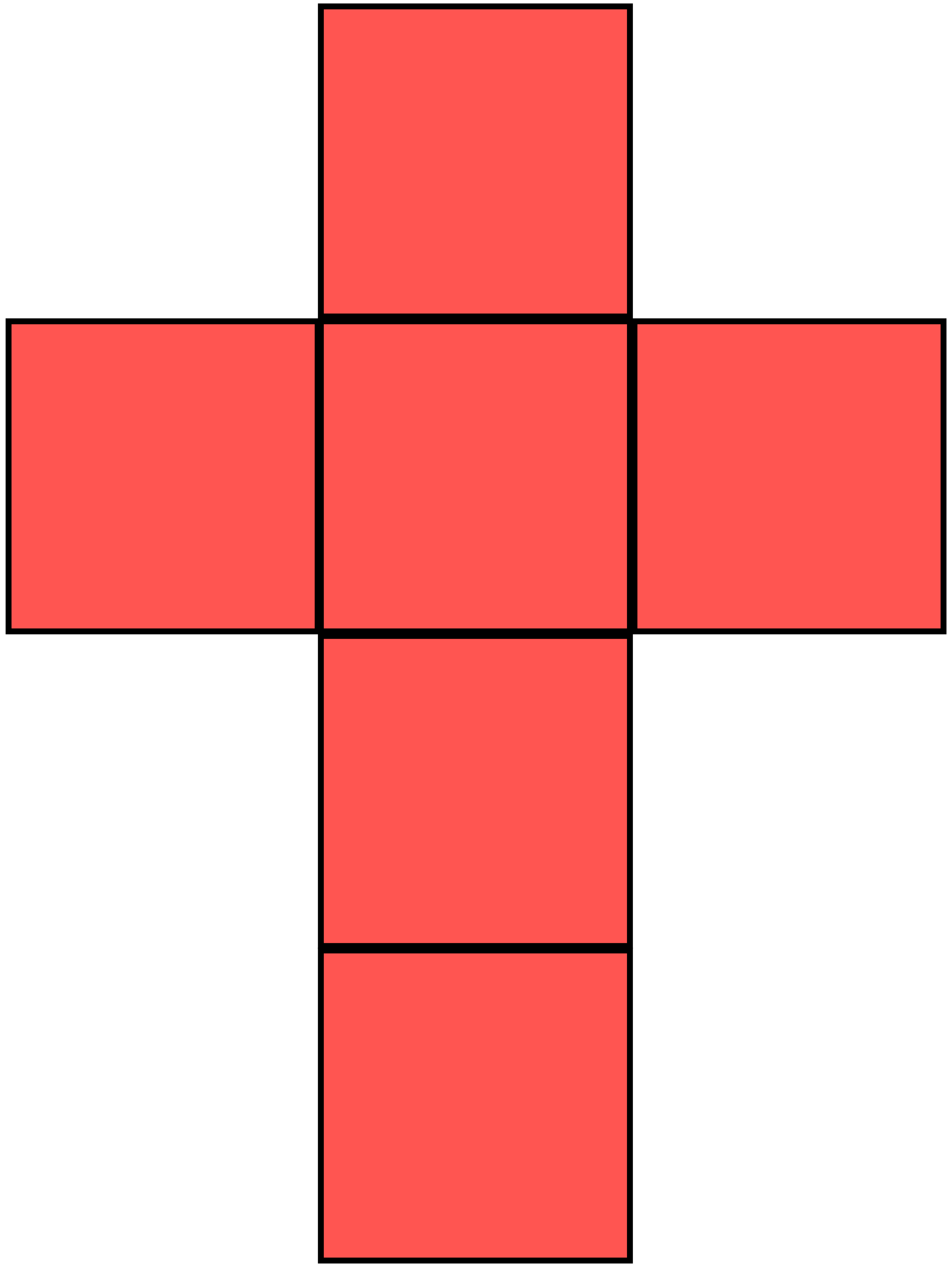
ALEXANDROV PUZZLE

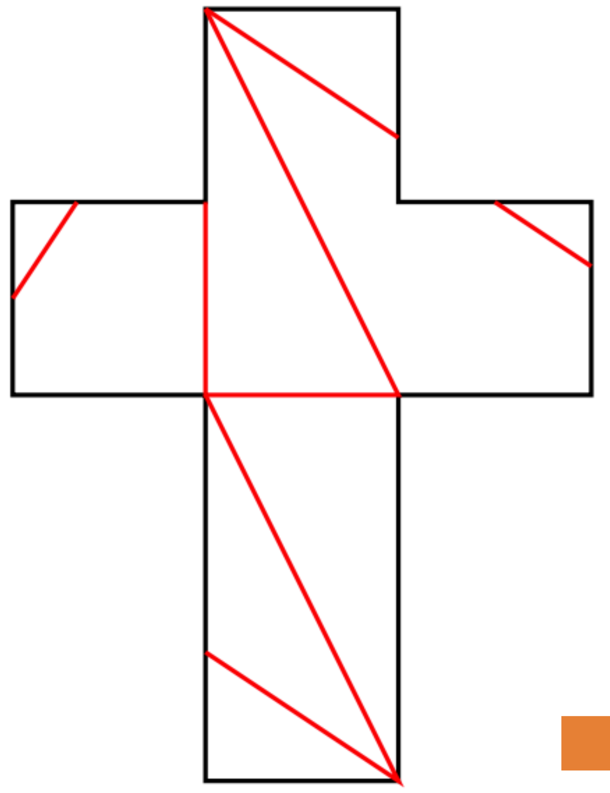
Kodai Takenaga and Shizuo Kaji



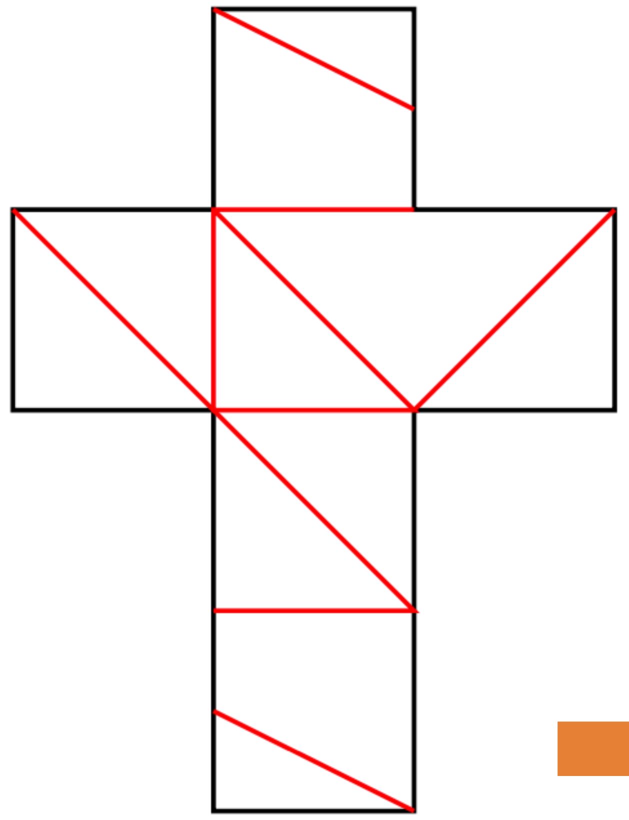
QUESTION

- What polyhedron can you make from this development?

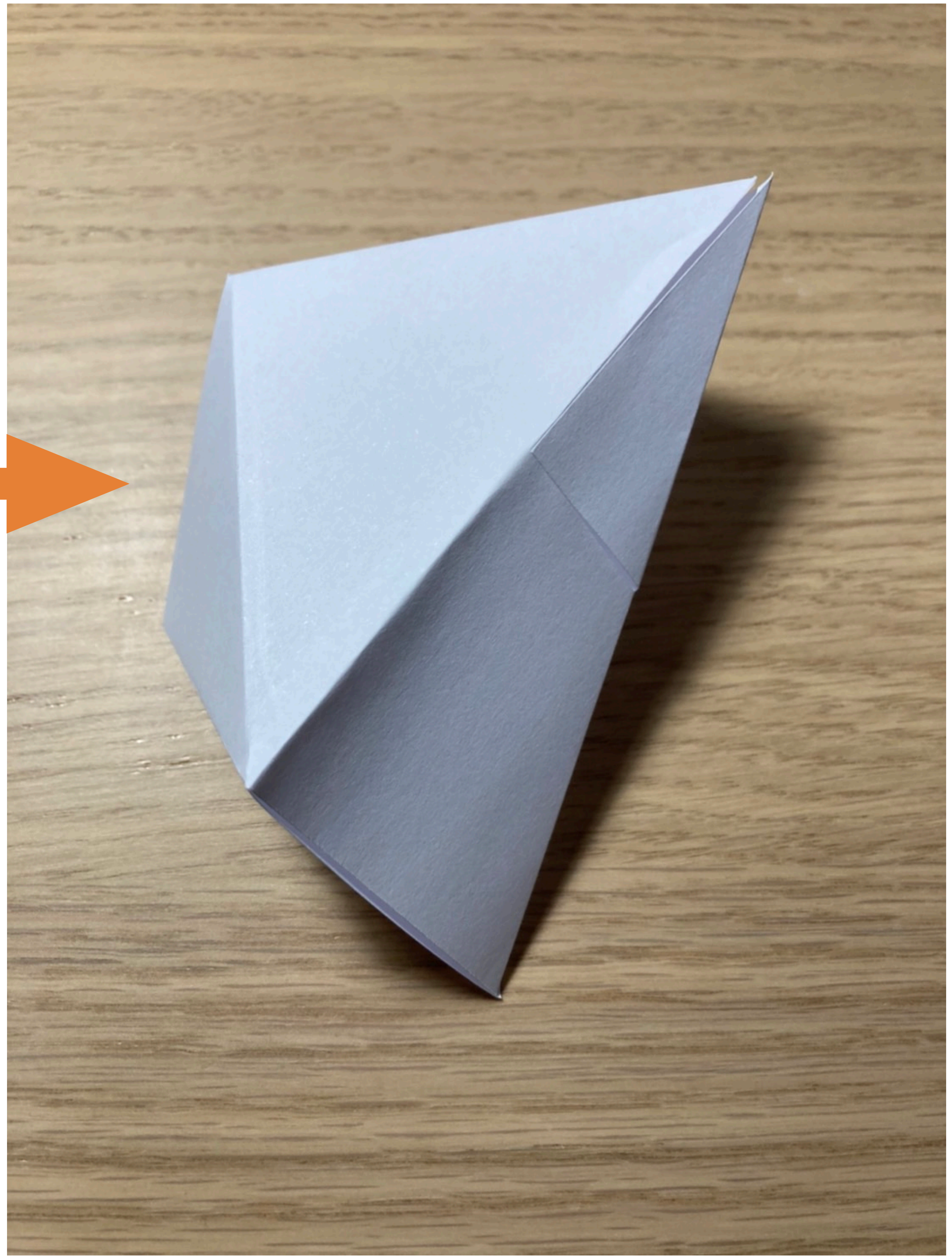
ANSWER!



Tetrahedron

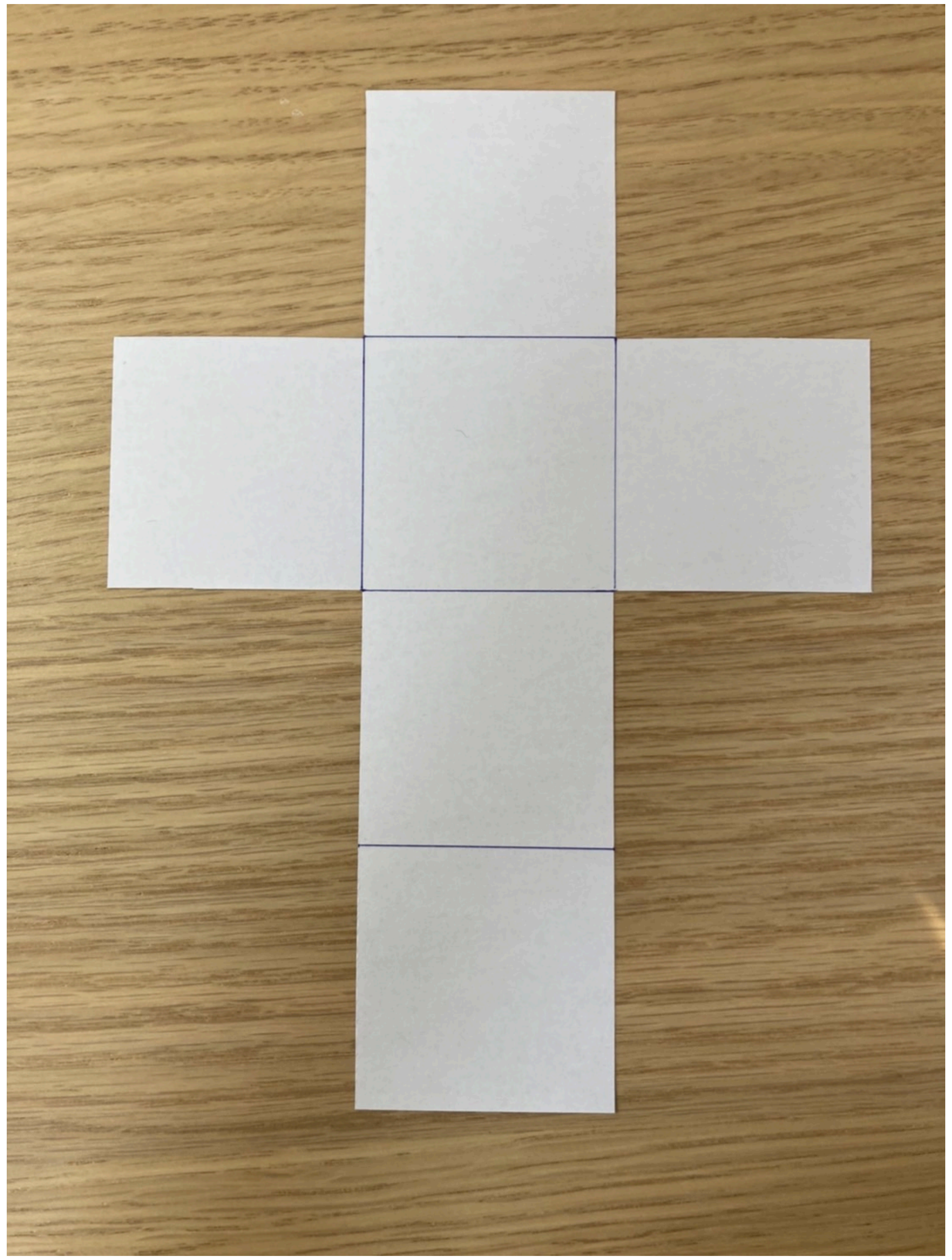


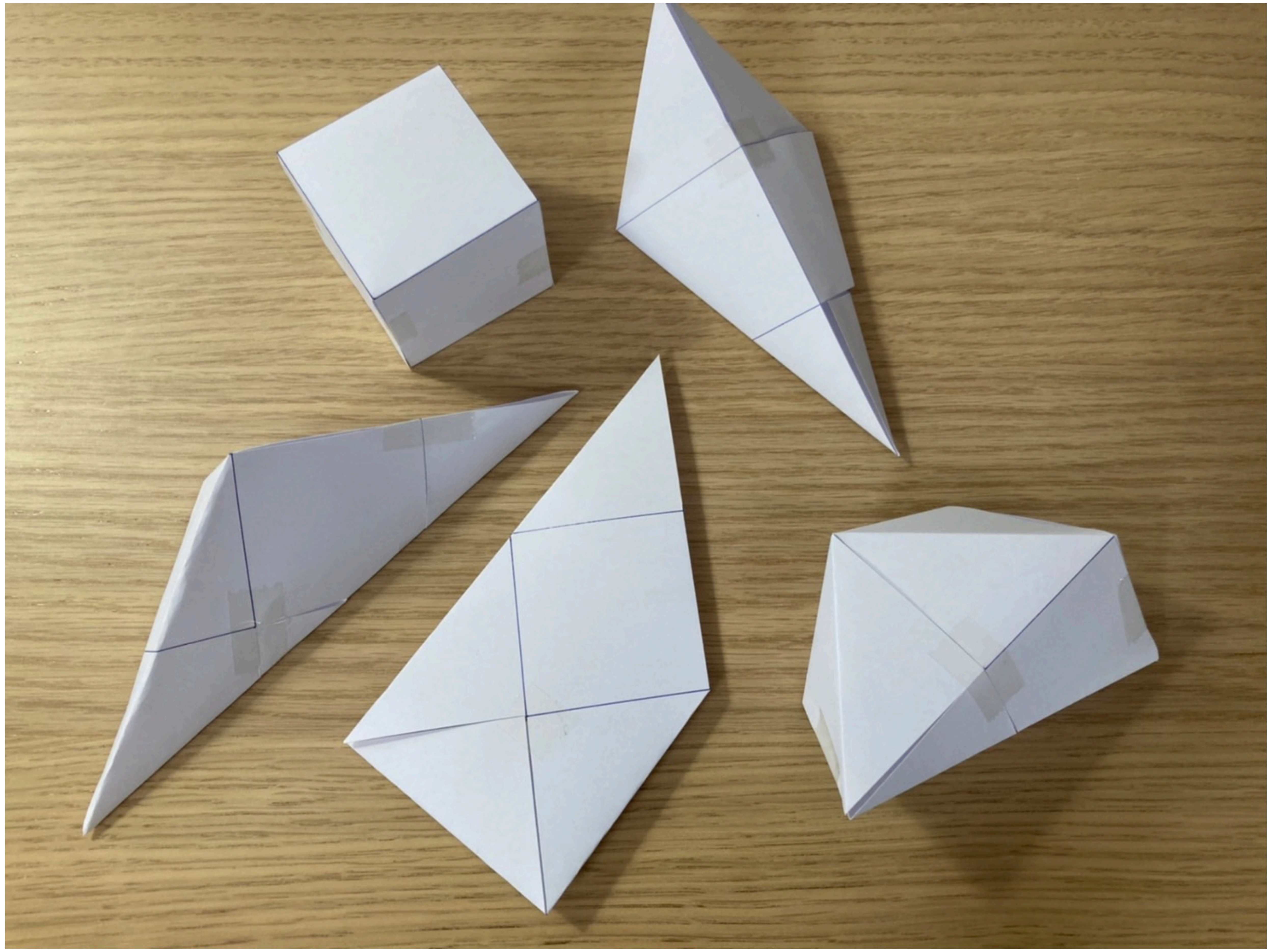
Octahedron



IN FACT…

It is known that there are five kinds of convex polyhedra by glueing each of the fourteen edges





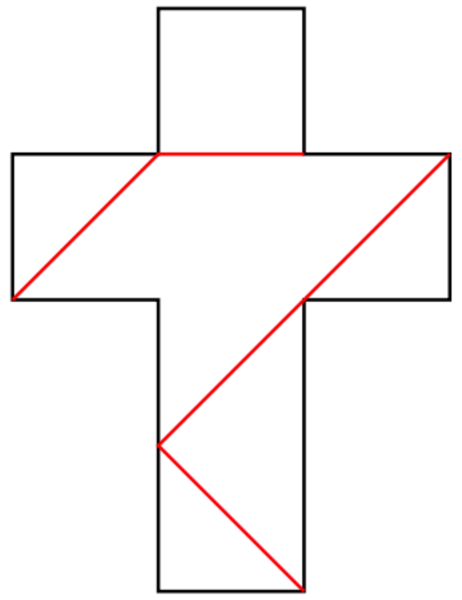
ALEXANDROV'S THEOREM

If edges of a polygon are matched to make the quotient space homeomorphic to the sphere and the sum of angles at most 2π at each vertex, the space has a unique realization as a polyhedron.

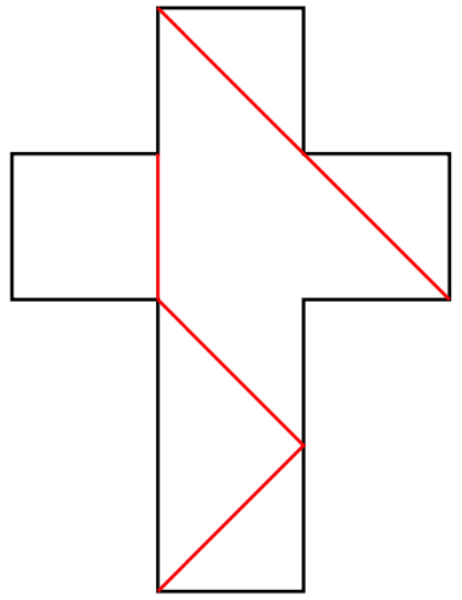
- For every "valid" matching of edges of the Latin cross, we have a polyhedron.
- The polyhedron is unique so that the fold line on the surface is "forced".

We can "feel" the theorem with our puzzle!

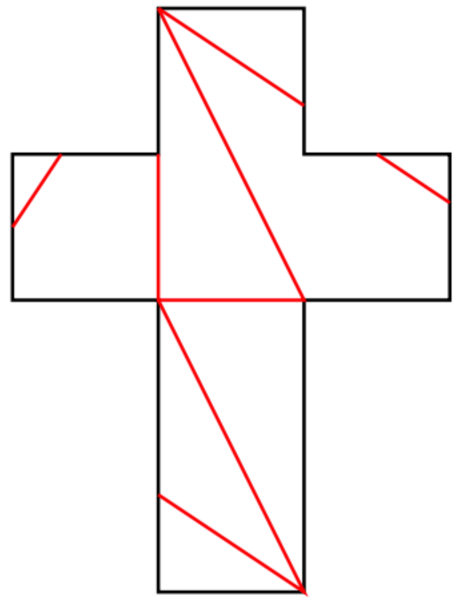
THERE ARE TWO WAYS TO MAKE EACH OF FOUR SHAPES



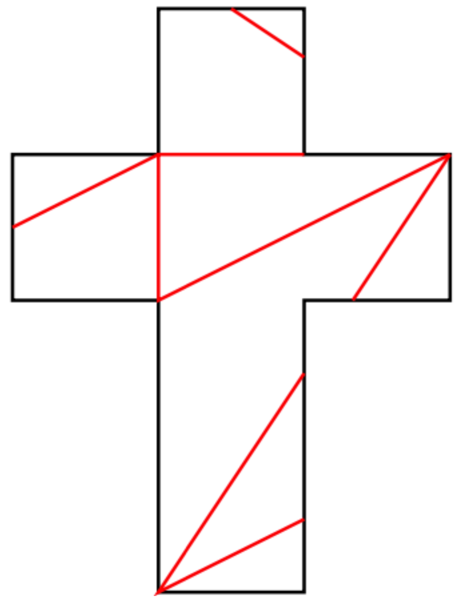
Q_0



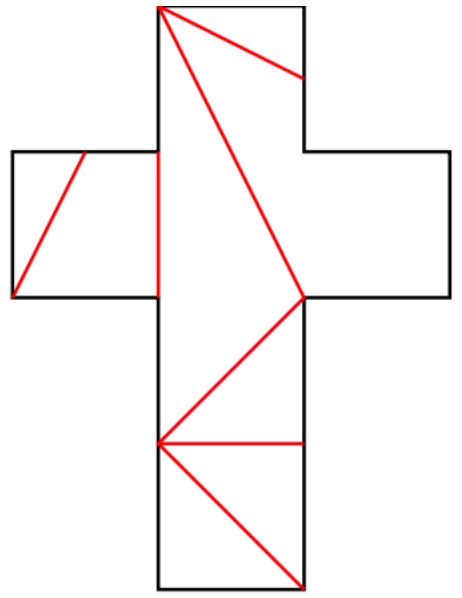
Q_1



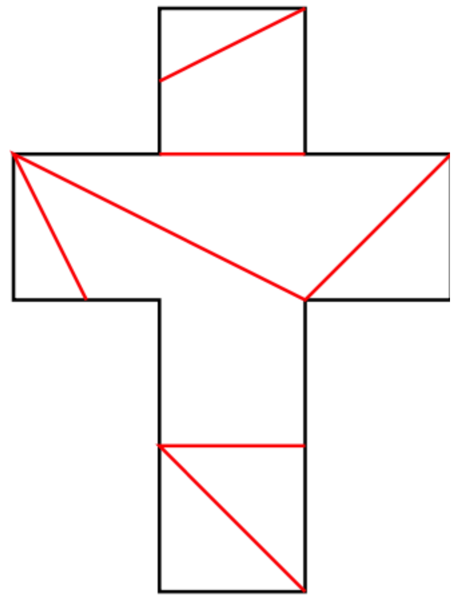
T_0



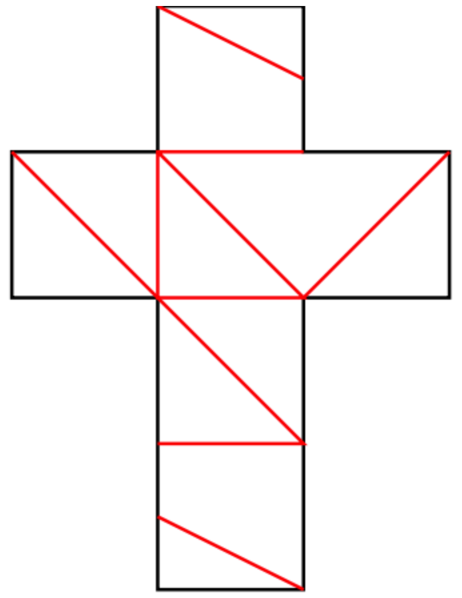
T_1



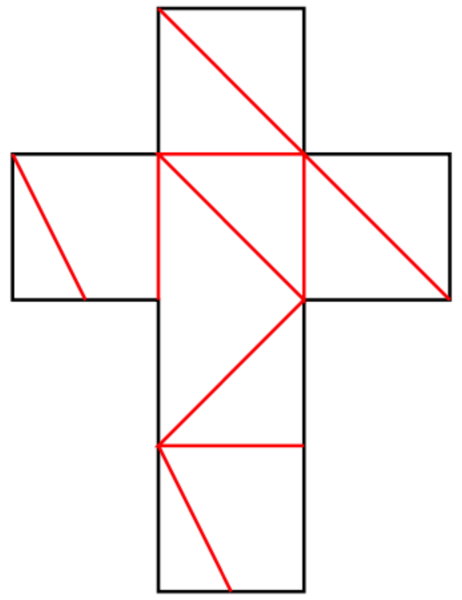
P_0



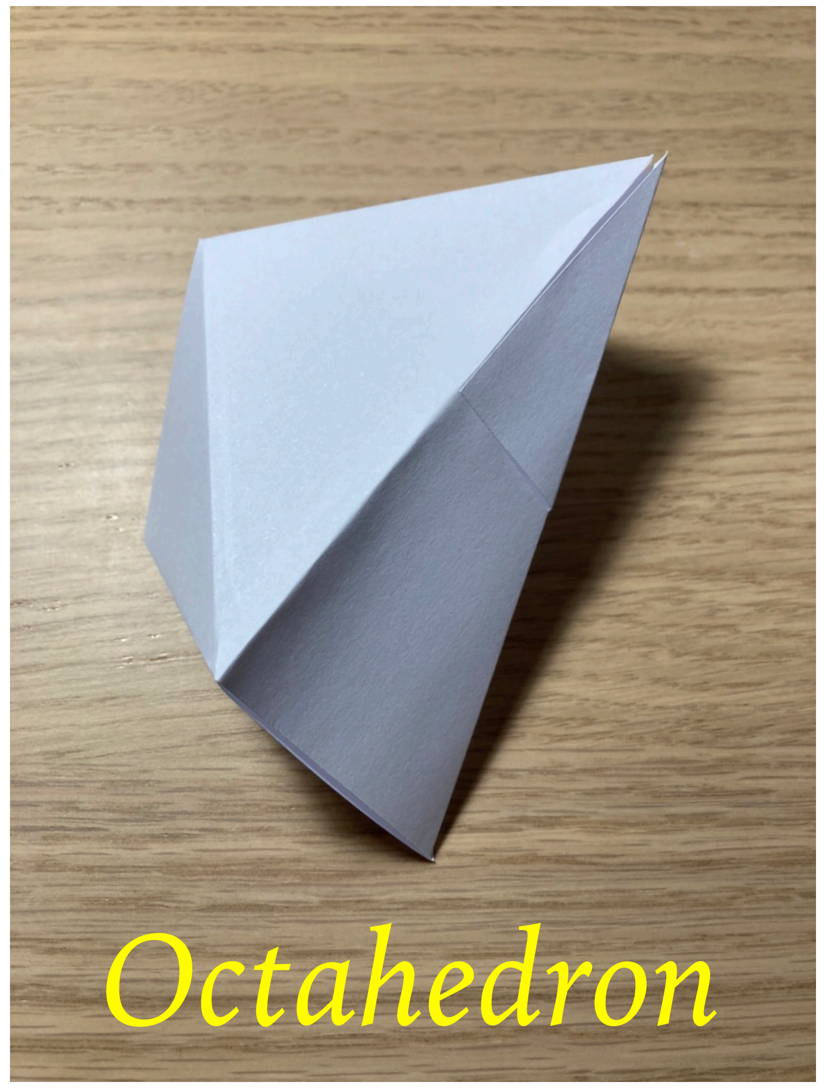
P_1

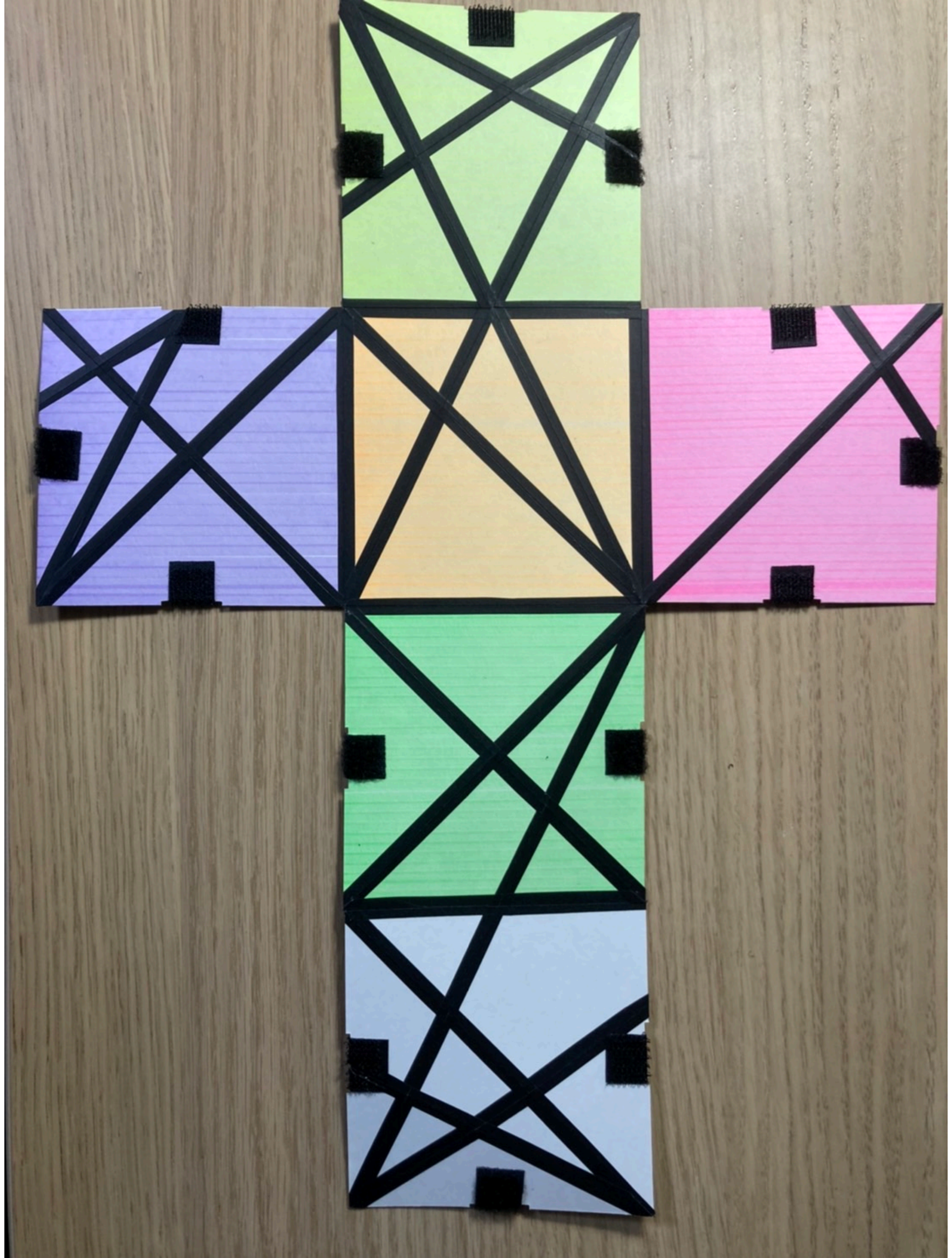


O_0



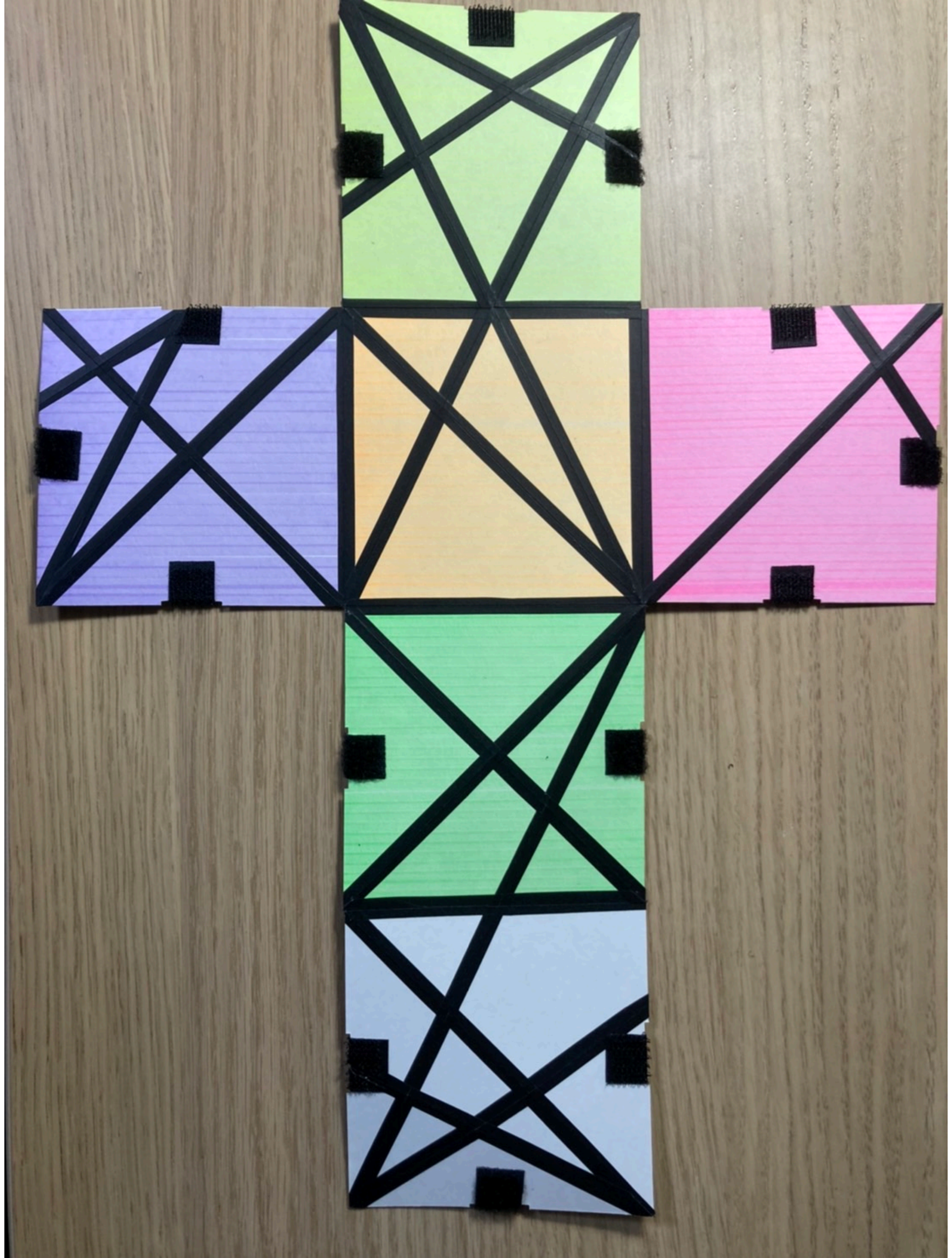
O_1





CREATE A PUZZLE

- Based on these facts, we introduce a physical puzzle consisting of hinged panels that can be folded into five polyhedra.
- Design choice: Combination of fold lines (hinges)

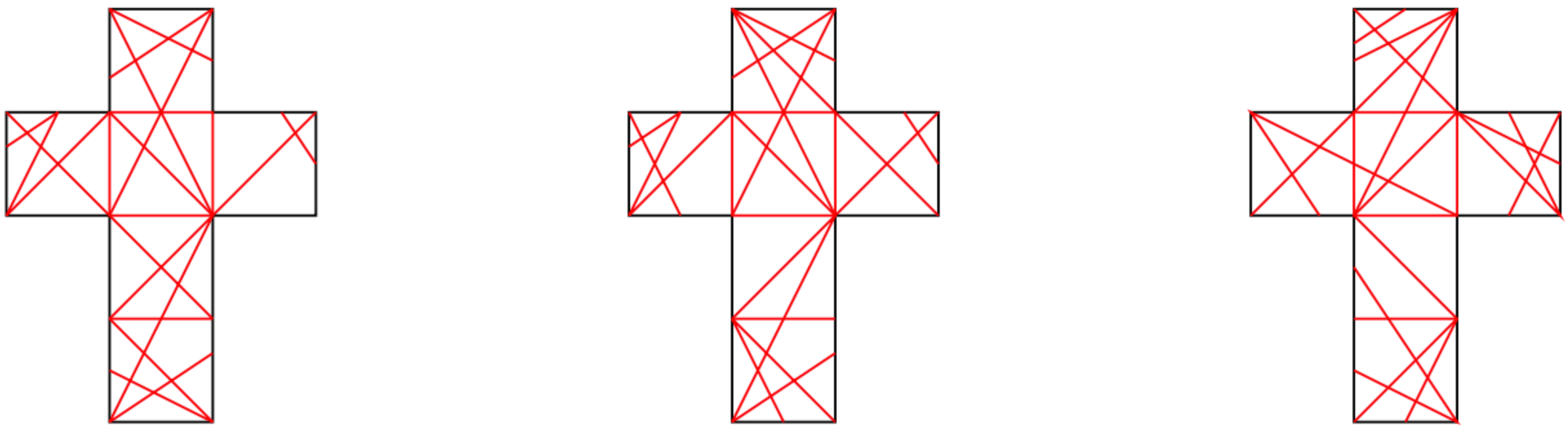


TO CREATE A PUZZLE

➤Six indicators

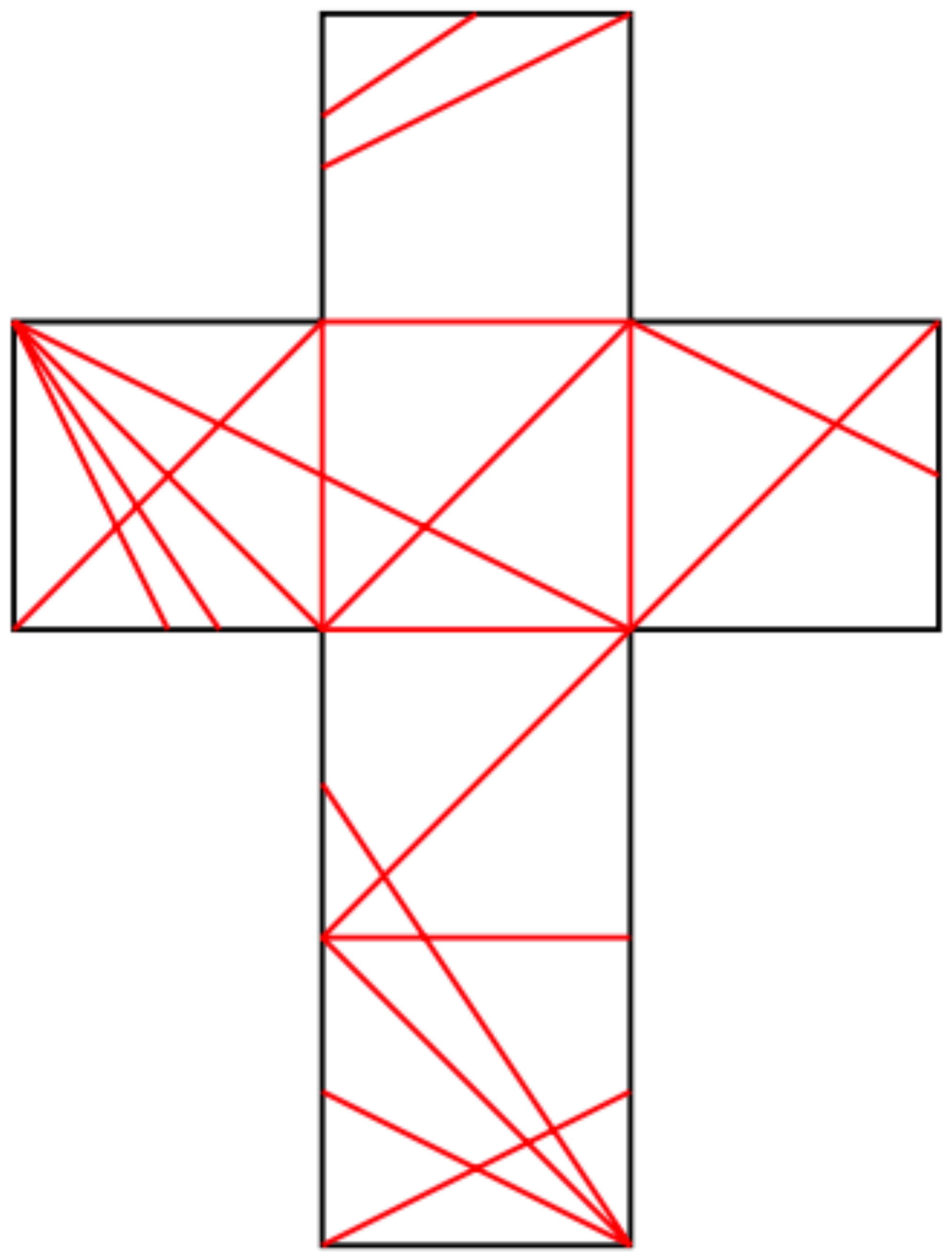
1. *The minimum angle of the panels*
2. *The number of shared edges*
3. *The variance of the number of vertices in each square*
4. *The number of total vertices*
5. *The number of total faces*
6. *The minimum area of faces*

INDICATOR 1 : MIN. ANGLE



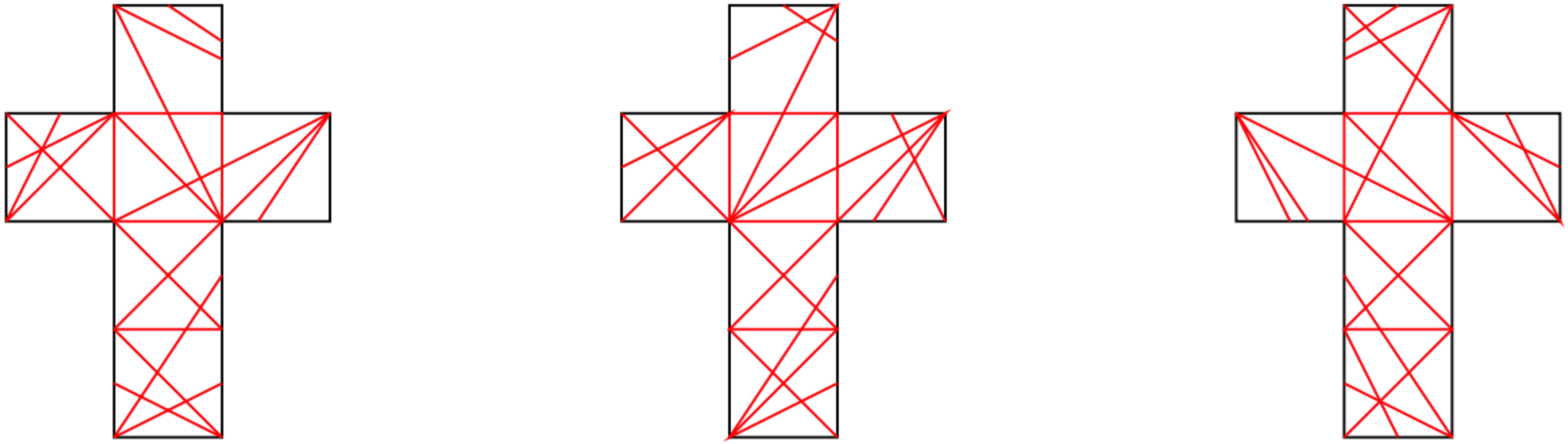
Combinations	num. vert.	num. shared edges	var. of verts.	min. angle	num. faces	min. area
$Q_0 T_0^m P_0 O_0$	16	04	0.556	18.435	40	583
$Q_0 T_0^m P_0 O_1$	17	03	0.917	18.435	42	335
$Q_0^m T_1^m P_0^m O_1^m$	19	03	0.917	18.435	43	133

INDICATOR 2 : NUM. SHARED EDGES



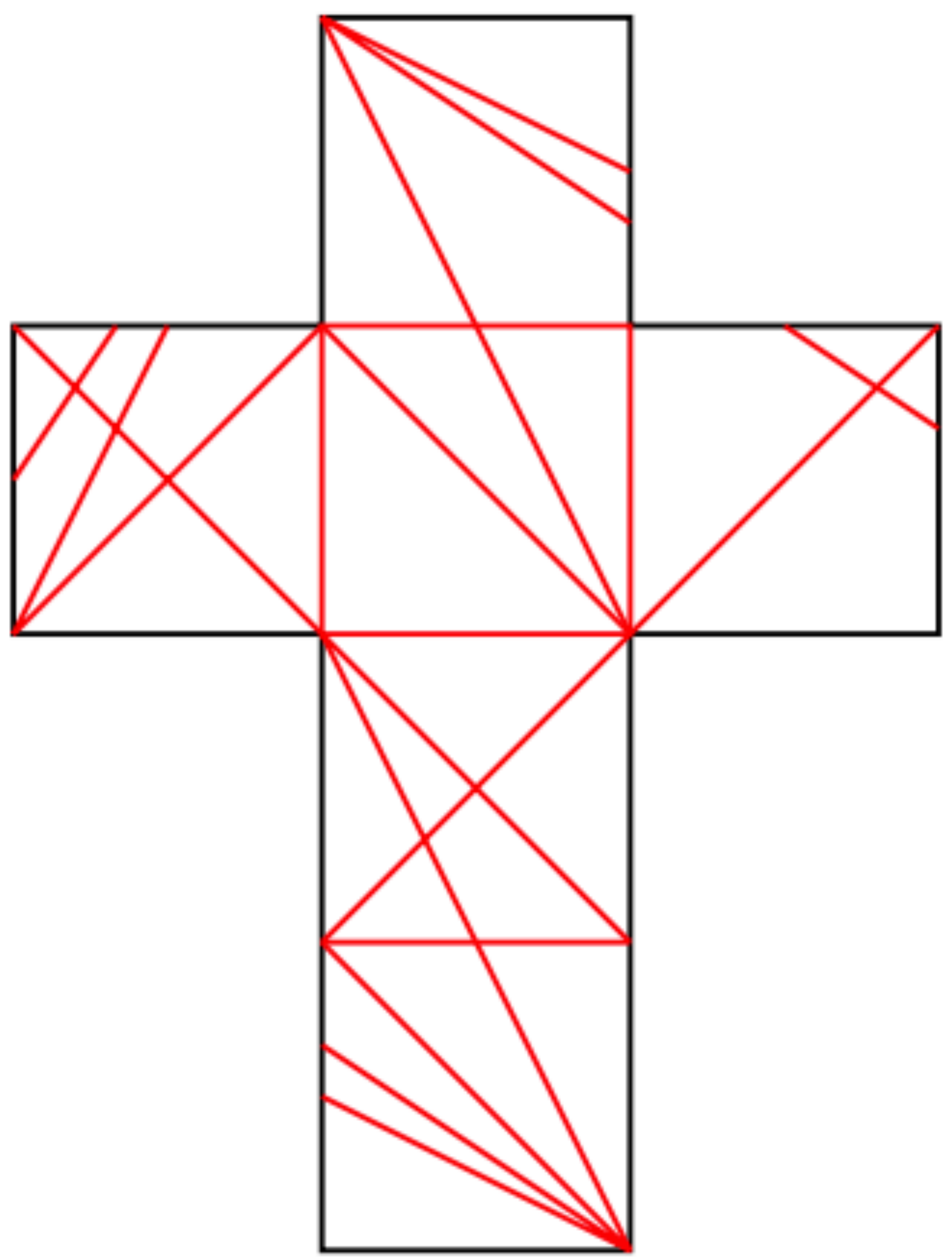
Combinations	num. vert.	num. shared edges	var. of verts.	min. angle	num. faces	min. area
$Q_0T_1^mP_1O_0^m$	12	07	1.472	7.125	33	322

INDICATOR 3 : VAR. OF VERTS.



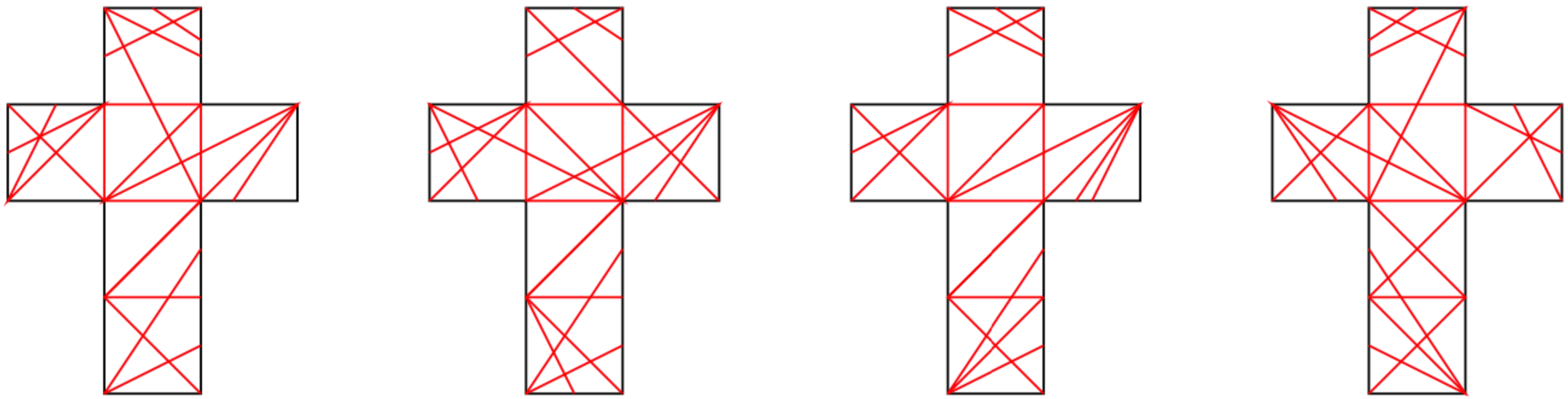
Combinations	num. vert.	num. shared edges	var. of verts.	min. angle	num. faces	min. area
$Q_0 T_1 P_0 O_0$	13	04	0.222	11.310	37	594
$Q_0 T_1 P_0^m O_0^m$	15	04	0.222	11.310	38	341
$Q_0^m T_1^m P_0^m O_1$	13	04	0.222	7.125	37	585

INDICATOR 4 : NUM. VERT. / 5 : NUM. FACES



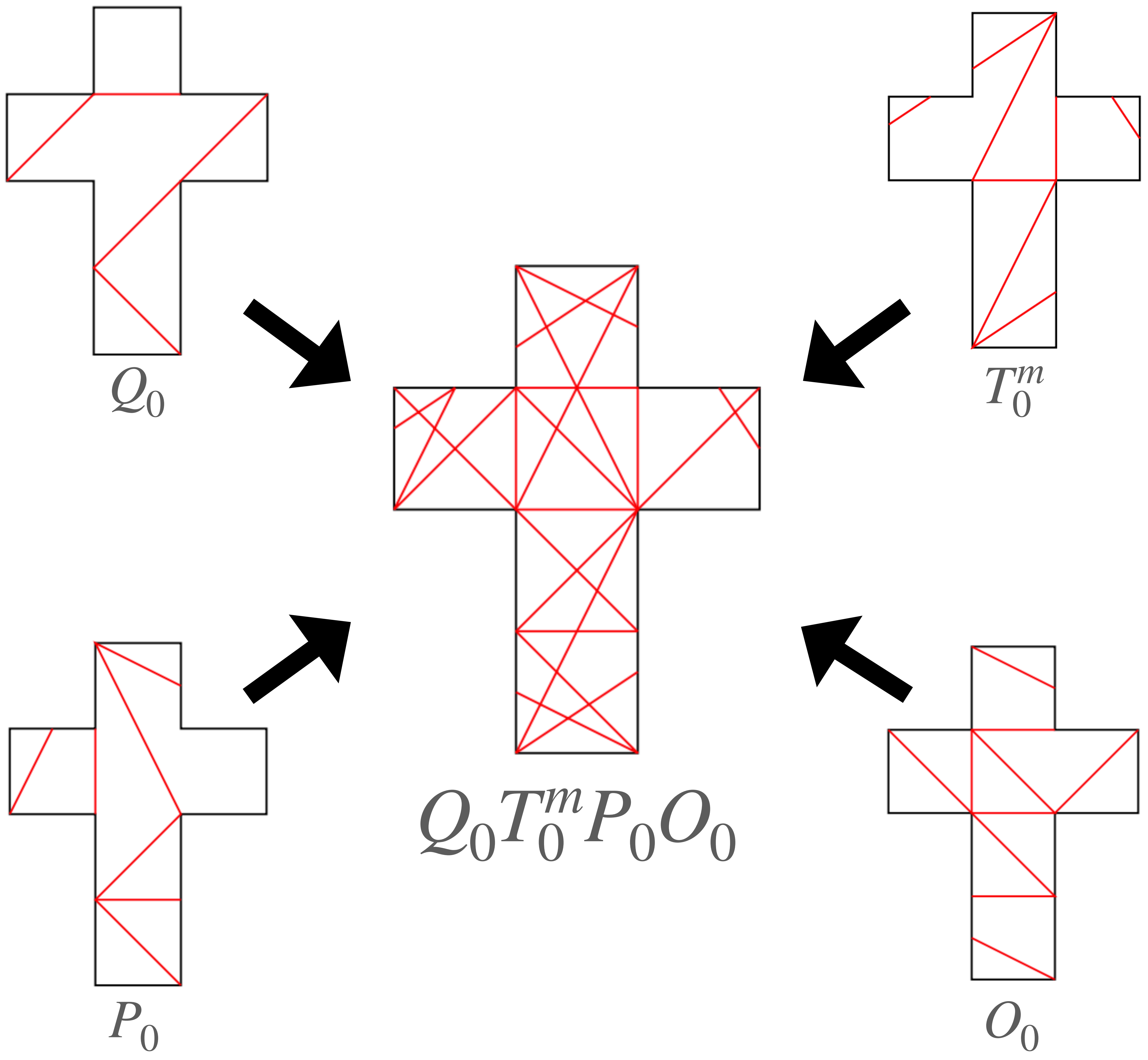
Combinations	num. vert.	num. shared edges	var. of verts.	min. angle	num. faces	min. area
$Q_0T_0P_0O_0$	08	06	0.667	7.125	30	574

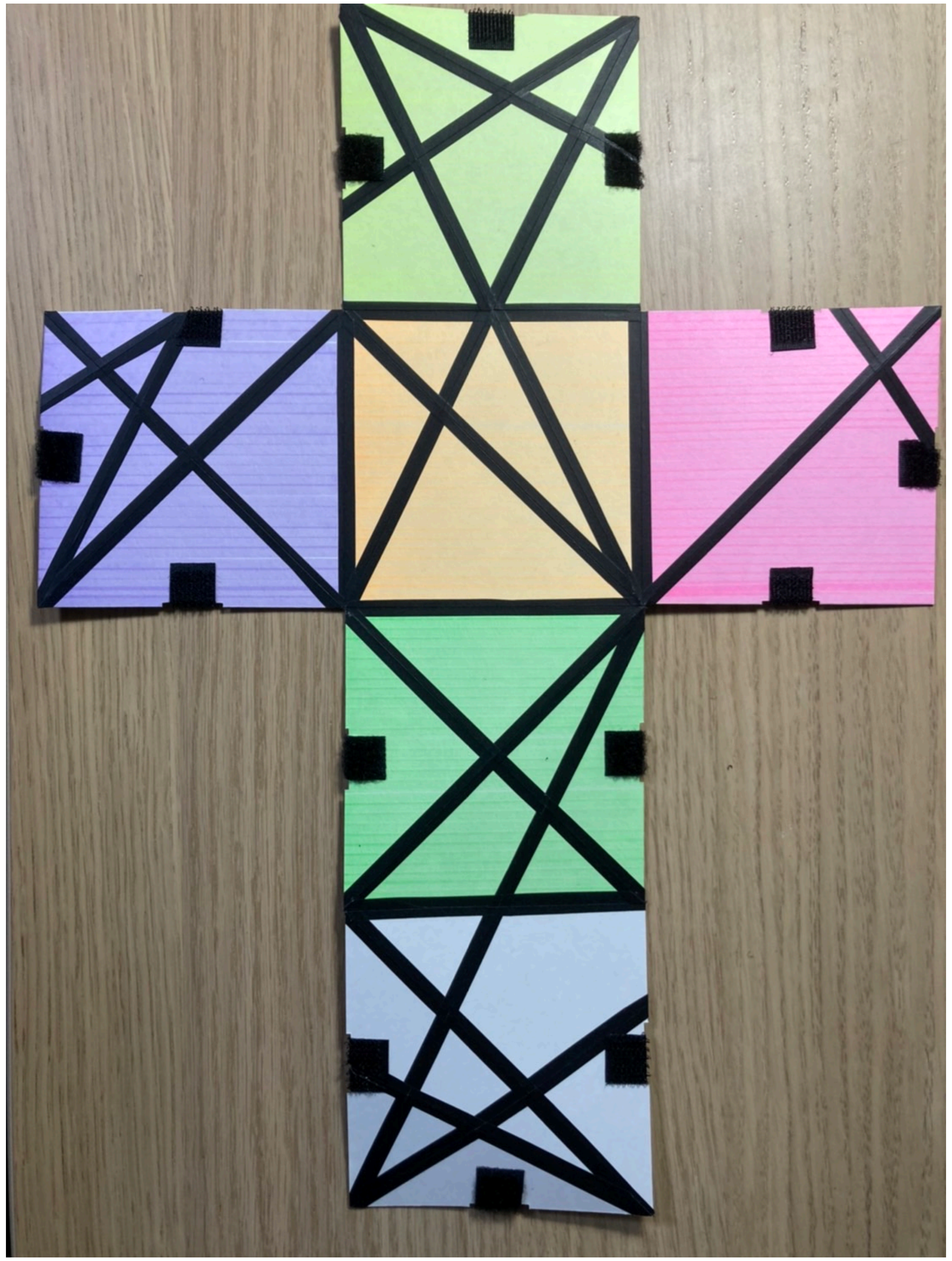
INDICATOR 6 : MIN. AREA



Combinations	num. vert.	num. shared edges	var. of verts.	min. angle	num. faces	min. area
$Q_0T_1P_0O_0^m$	12	05	0.472	11.310	35	608
$Q_0T_1P_1O_1$	18	04	0.556	11.310	41	608
$Q_0T_1P_1^mO_0^m$	09	06	0.667	7.125	31	608
$Q_0T_1^mP_0^mO_0$	17	03	0.250	11.310	42	607

OUR CHOICE IS…



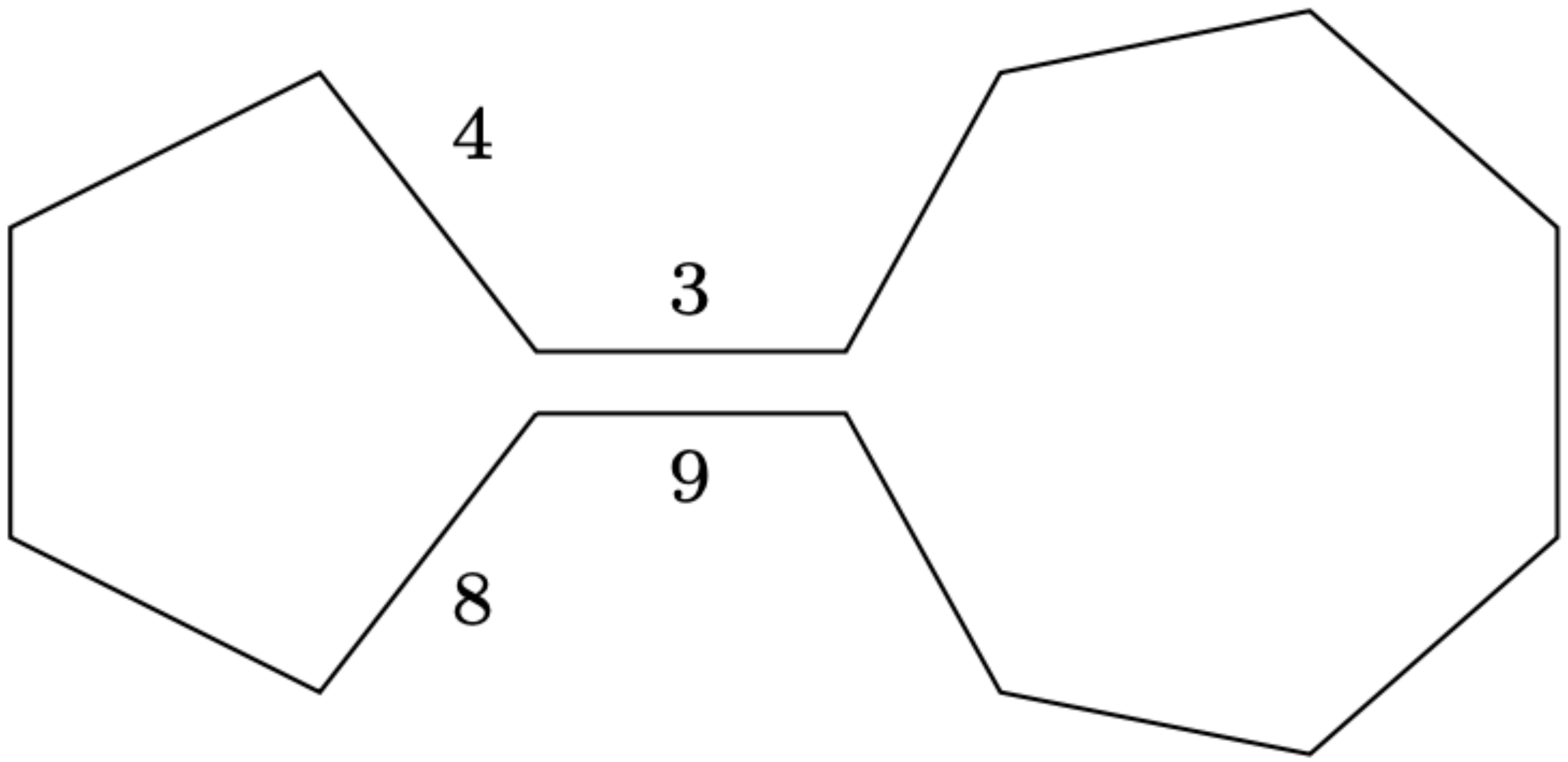


ADHESION

Lemma

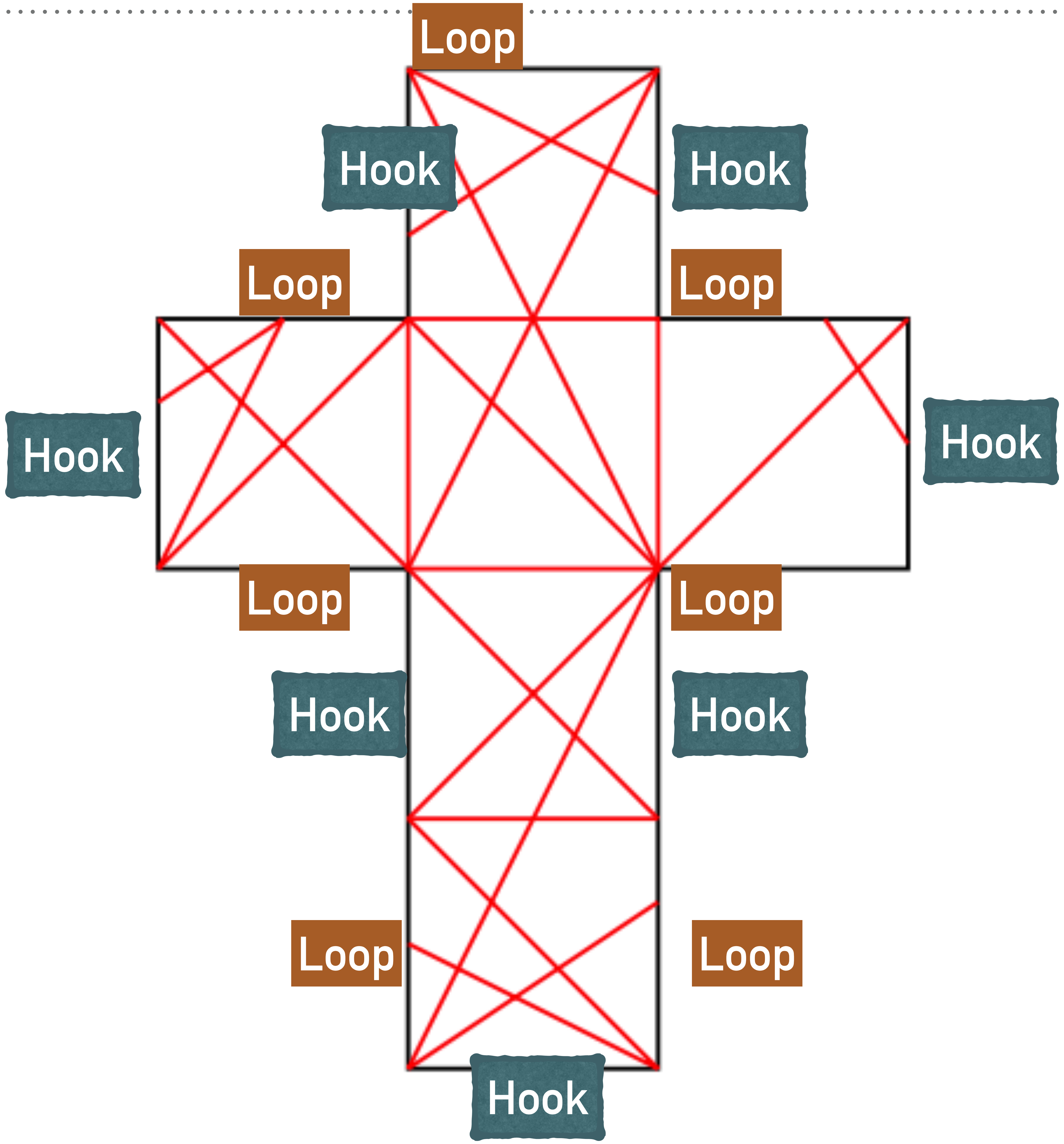
If we number edges of the polygon in a clockwise manner, the parity of glued edges always coincide in a valid matching.

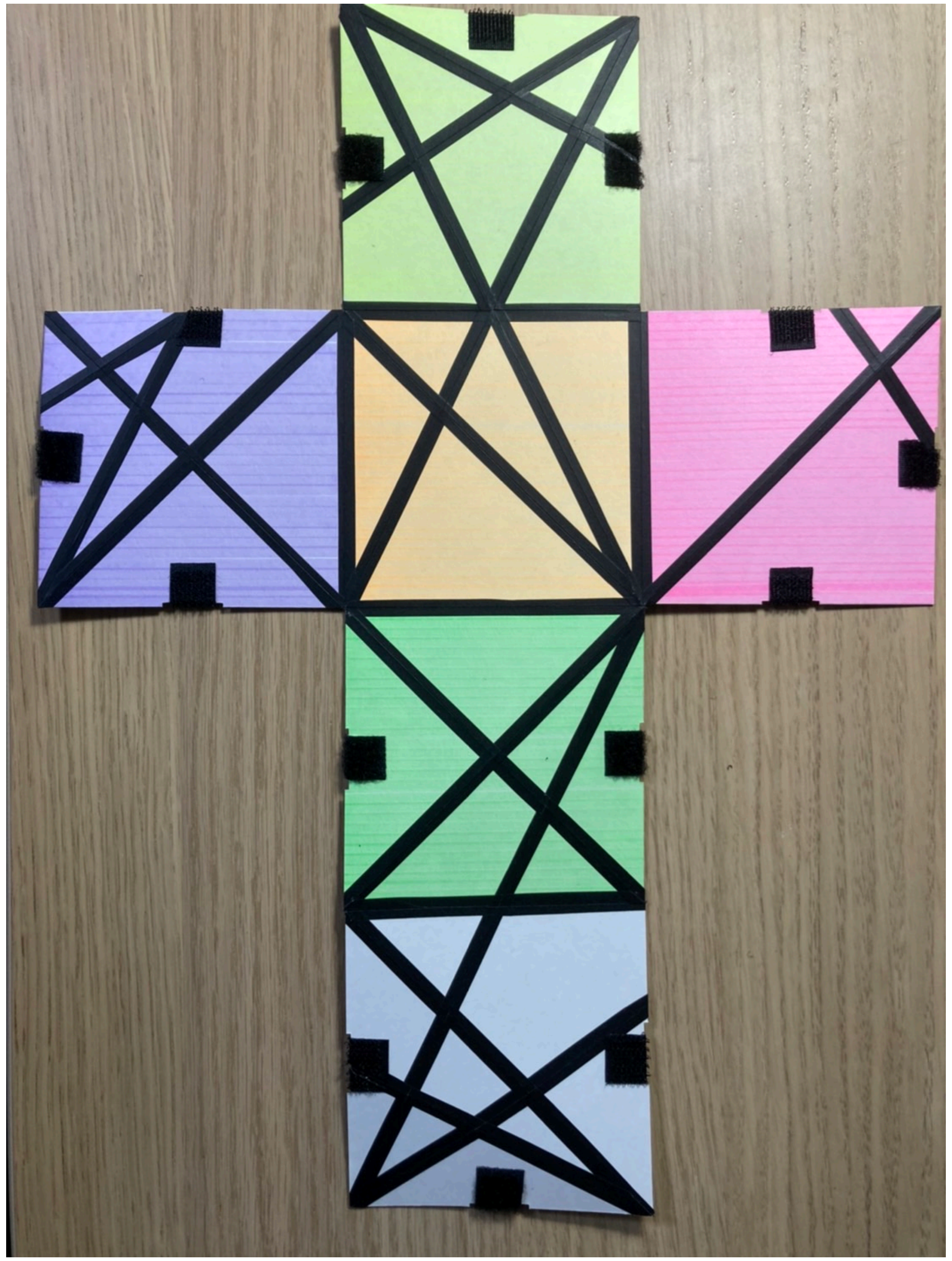
Ex: non-valid matching



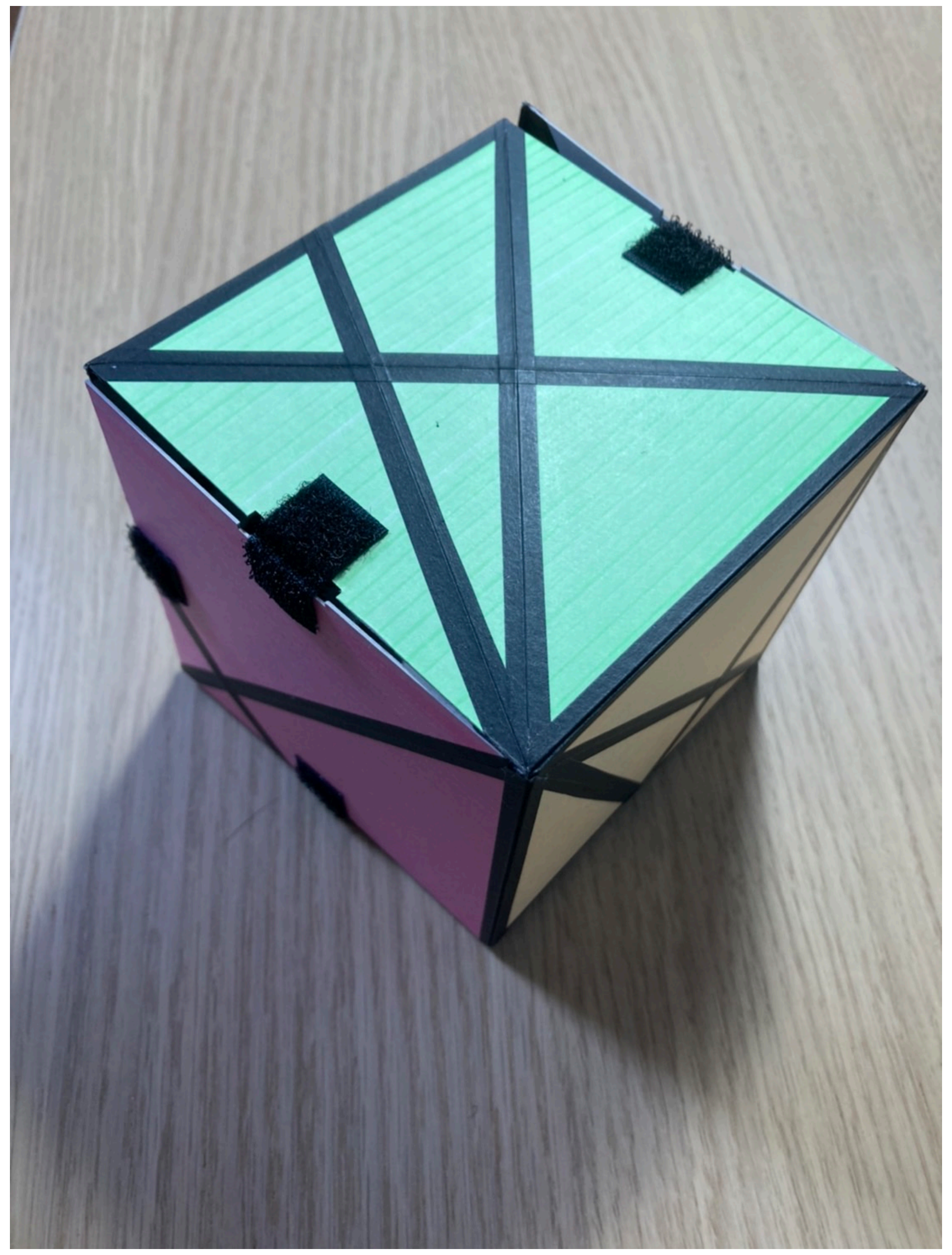
←Cannot be closed to be homeomorphic to the sphere

MAKING PUZZLE…



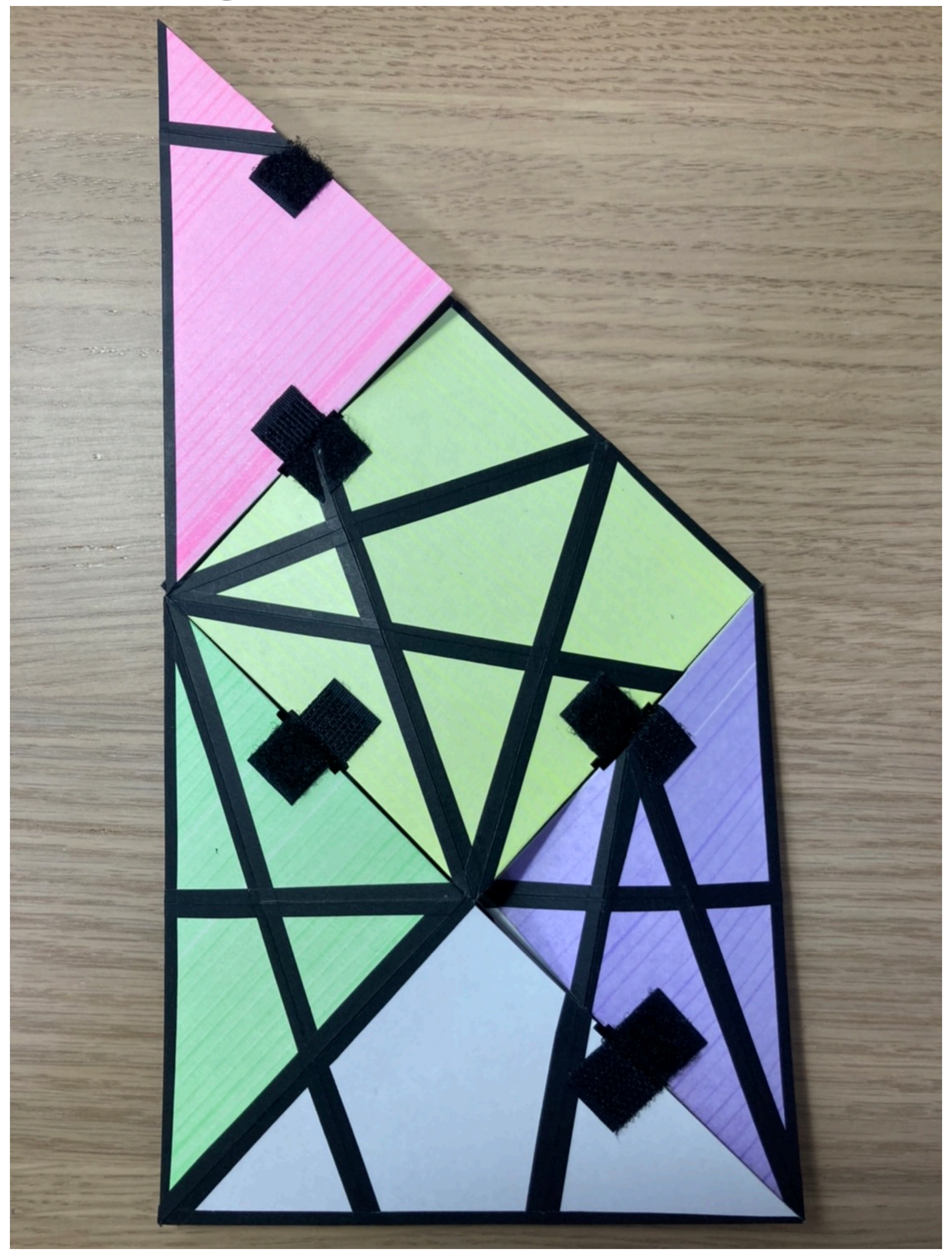


LET'S PLAY



Cube

Doubly-coverd Quadrilateral

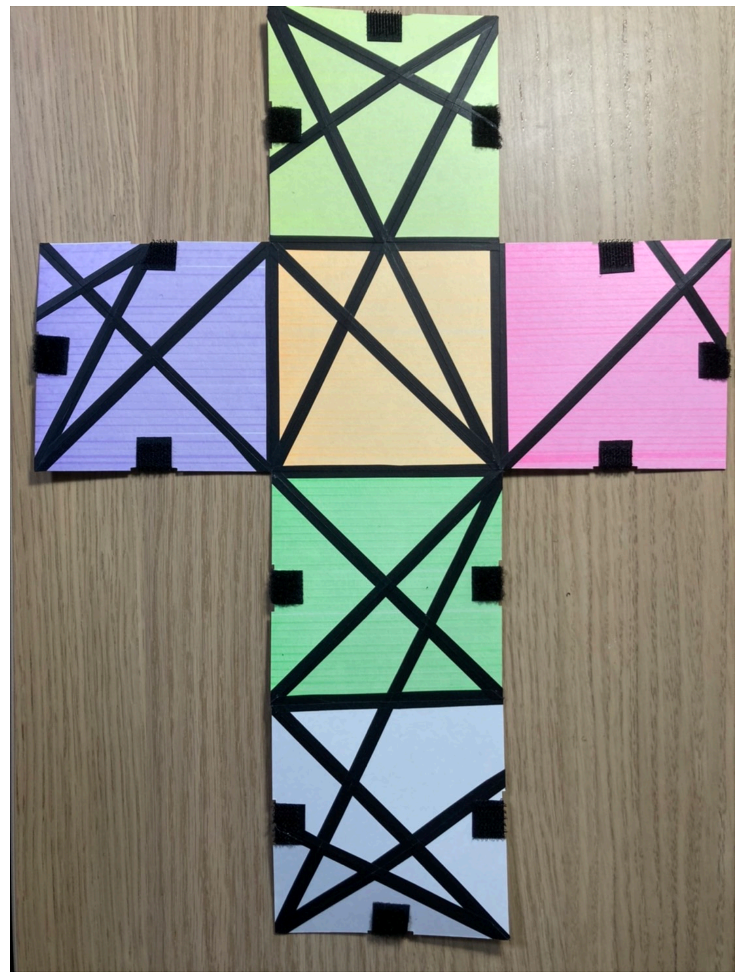


Tetrahedron

Pentahedron

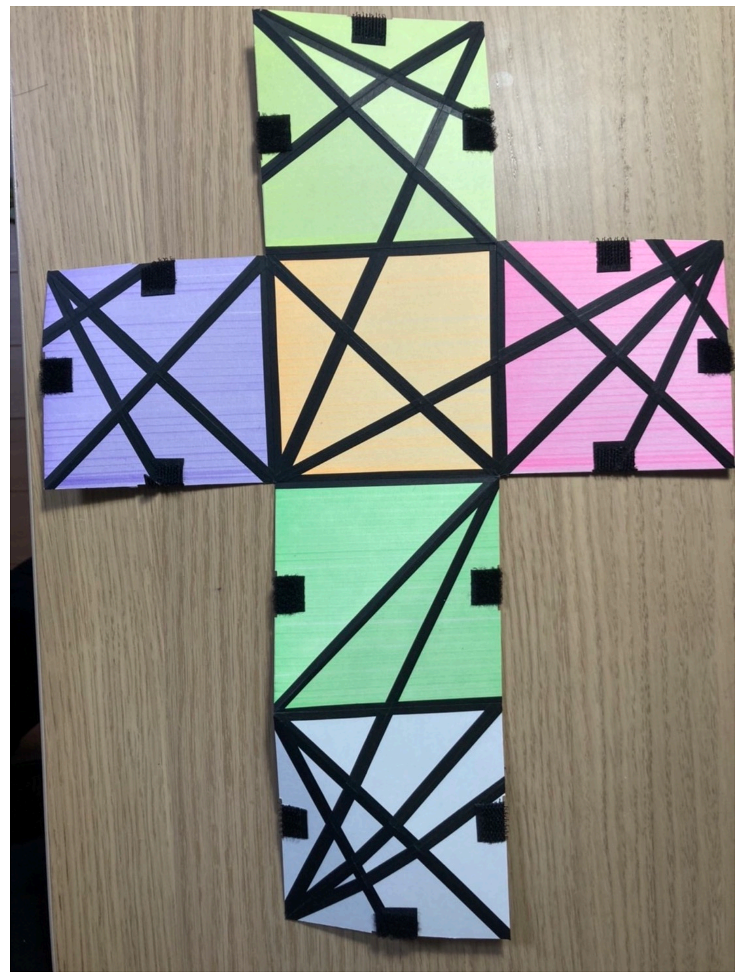
Octahedron

TIME ATTACK FOR TWO COMBINATIONS



Subjects	Affiliation	Order of creation				
Mr. M	Math M1	Cube under 1m	Penta 5m25s	Tetra 21m	Doubly 25m46s	-
Mr. S	Math M1	Cube under 1m	Penta 4m22s	Tetra 20m20s	Octa 28m	-
Mr. K	Math M1	Cube under 1m	Octa 2m20s	Penta 3m30s	Doubly 6m50s	Tetra 11m
Mr. K	Math M1	Cube under 1m	Tetra 7m42s	Octa 30min up	-	-
Ms. I	TOYOTA	Cube under 1m	Octa 3m6s	Penta 5m6s	Doubly 15m22s	Tetra 19m20s

Our choice



Subjects	Affiliation	Order of creation				
Mr. A	Math M1	Cube under 1m	Doubly 3m30	Octa 7m5s	Tetra 19m	Penta 21m50s
Mr. Y	Math M1	Cube under 1m	Octa 3m17s	Penta 3m50s	Doubly 14m50s	-
Mr. M	Math M1	Cube under 1m	Tetra 13m16s	Penta 15m15s	Doubly 21m20s	-
Mr. T	Math M1	Cube under 1m	Doubly 7m20s	Tetra 14m24s	Octa 20m	Penta 27m
Mr. K	SONY	Cube under 1m	Doubly 15m23s	Tetra 19m10s	Octa 24m	Penta 28m35s

FUTURE PROSPECTS

- Other polygons than the Latin cross
- Allowing non edge-to-edge gluing
- Make a commercial product of the puzzle

REFERENCES

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