Lazy evaluation illustrated

for Haskell divers

exploring some mental models and implementations

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Rev. 0.00.0
WIP

Lazy,... zzz

..., It's fun!

NOTE

- Meaning of terms are different by communities.
- There are a lot of good documents. Please see also references.
- This is written for GHC's Haskell.

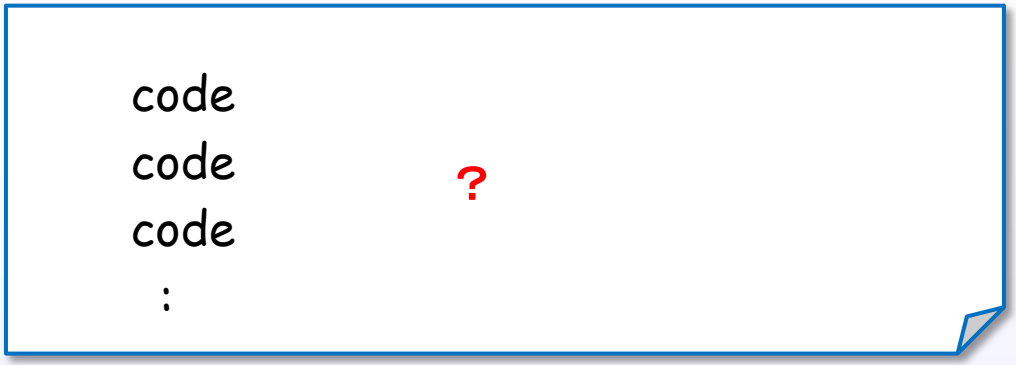
Contents

1. Introduction

Basic mental models

How to evaluate a program in your brain ?

a program



How to evaluate (execute, reduce) the program in your brain?

What "mental model" do you have?

One of the mental models for C program

C program

A sequence of statements

```
main (...) {
  code..
  code..
  code..
  code..
}
```

?

A nested structure

```
x = func1( func2( a ) );
```

?

A sequence of arguments

```
y = func1( a(x), b(x), c(x) );
```

?

A function and arguments

```
z = func1( m + n );
```

?

How to evaluate (execute, reduce) the program in your brain?

What step, what order, ... ?

One of the mental models for C program

C program — A program is a collection of statements.

A sequence of statements

```
main (...) {
  code..
  code..
  code..
  code..
}
```

Statements are executed downward.

A nested structure

```
x = func1( func2( a ) );
```

from inner to outer

A sequence of arguments

```
y = func1( a(x),  b(x),  c(x) );
```

from left to right

A function and arguments

```
z = func1( m + n );
```

arguments first
apply second

Each programmer has some mental models in their brain.

One of the mental models for C program

Maybe, You have some implicit mental model in your brain for C program.

(1) A program is a collection of statements.

(2) There is the order between evaluations of elements.

(3) There is the order between termination and start of evaluations.

This is a syntactically straightforward model for programming languages.

(an implicit sequential order model)

One of the mental models for Haskell program

Haskell program

```
main = exp11 (exp12 exp13 exp14 )

exp13 = exp131 exp132

exp14 = exp141 exp142 exp143
   :
```

?

How to evaluate (execute, reduce) the program in your brain?

What step, what order, ... ?

One of the mental models for Haskell program

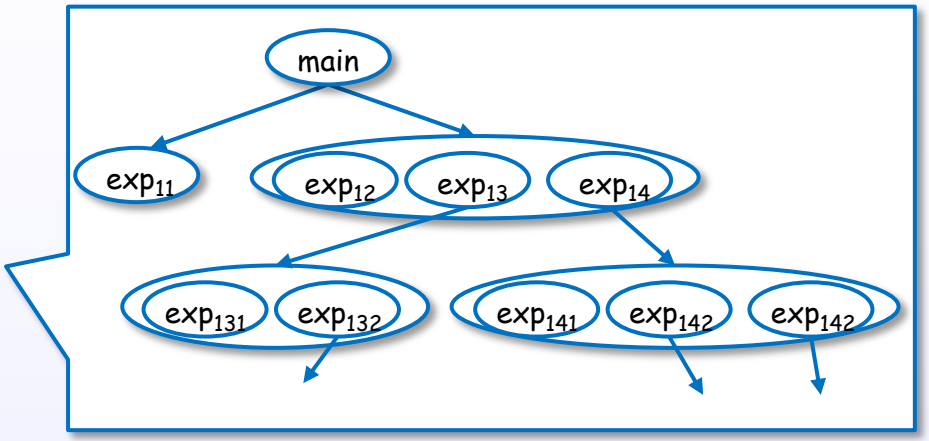
Haskell program

A program is a collection of expressions.

```
main = exp11 (exp12 exp13 exp14 )

exp13 = exp131 exp132

exp14 = exp141 exp142 exp143
   :
```




```
main = exp11 (exp12 (exp131 exp132 ) (exp141 exp142 exp143) )
```

A entire program is regarded as a single expression.

The subexpression is evaluated (reduced) in some order.

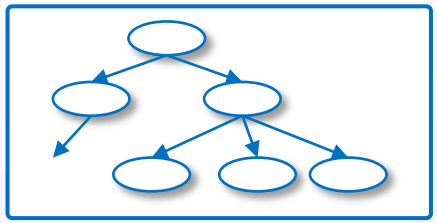
The evaluation is performed by replacement.

One of the mental models for Haskell program

(1) A program is a collection of expressions.

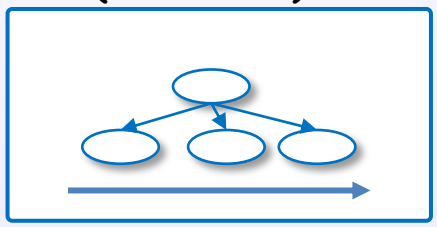
(2) A entire program is regarded as a single expression.

```
main = e (e (e (e e) e (e e e) ) )
```

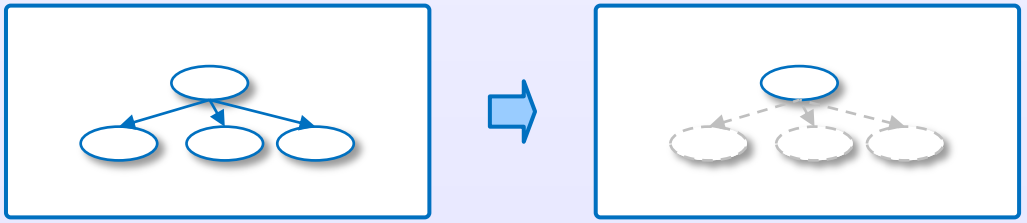


(3) The subexpressions are evaluated (reduced) in some order.

```
f = e (e (e (e e) e (e e e) ) )
```



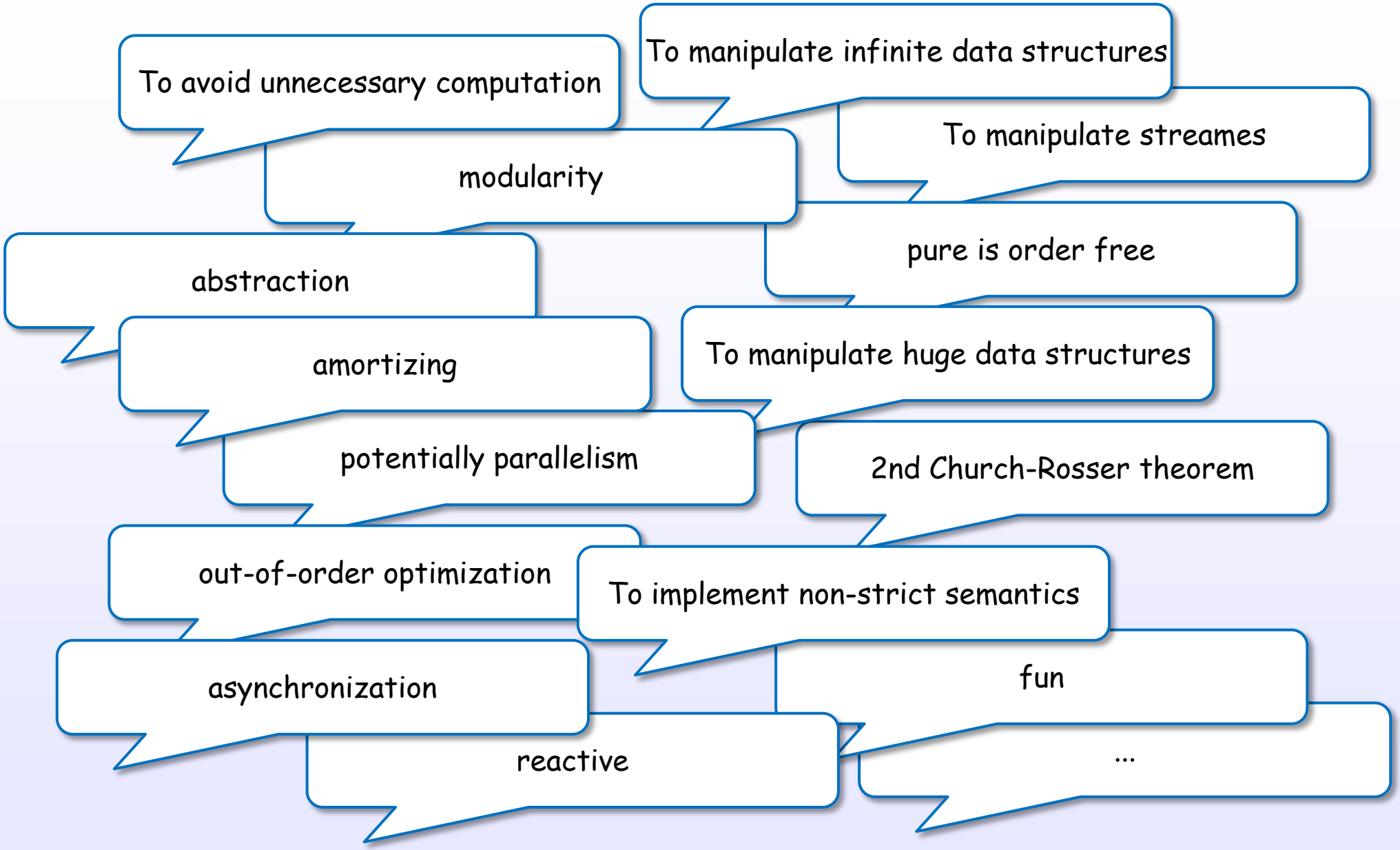
(4) The evaluation is performed by replacement.



This is an example of an expression reduction model for Haskell.

Lazy evaluation

Why lazy evaluation?



There are various reasons ☺

References : [H4], [H3], [B2], [B7], [B8], [D2], [D12], [D13], [D14]

Haskell(GHC) 's lazy evaluation

Lazy evaluation

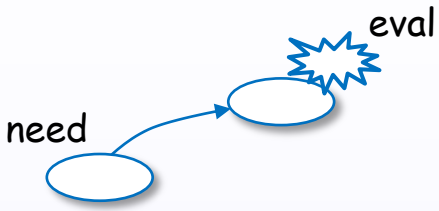
evaluate only when needed

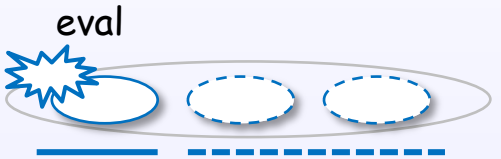
+

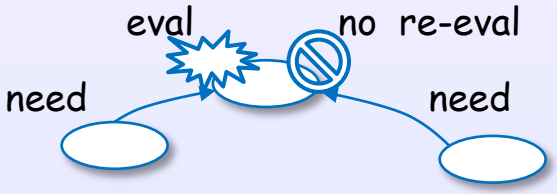
evaluate only enough

+

evaluate at most once



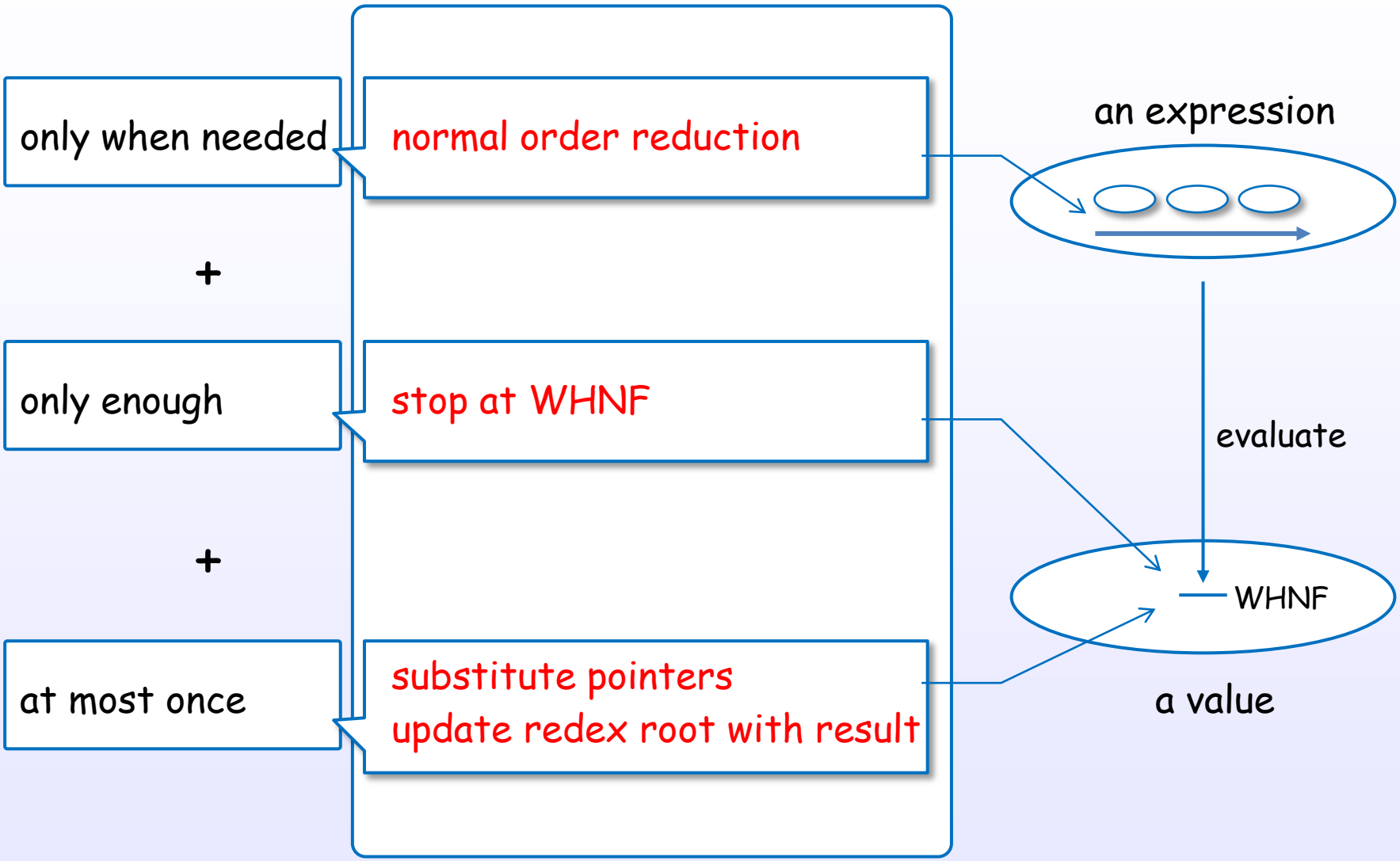




"Lazy" is "delay and avoidance" rather than "delay".

References : [B2] Ch.7, [H4] Ch.11, 12, [D2]

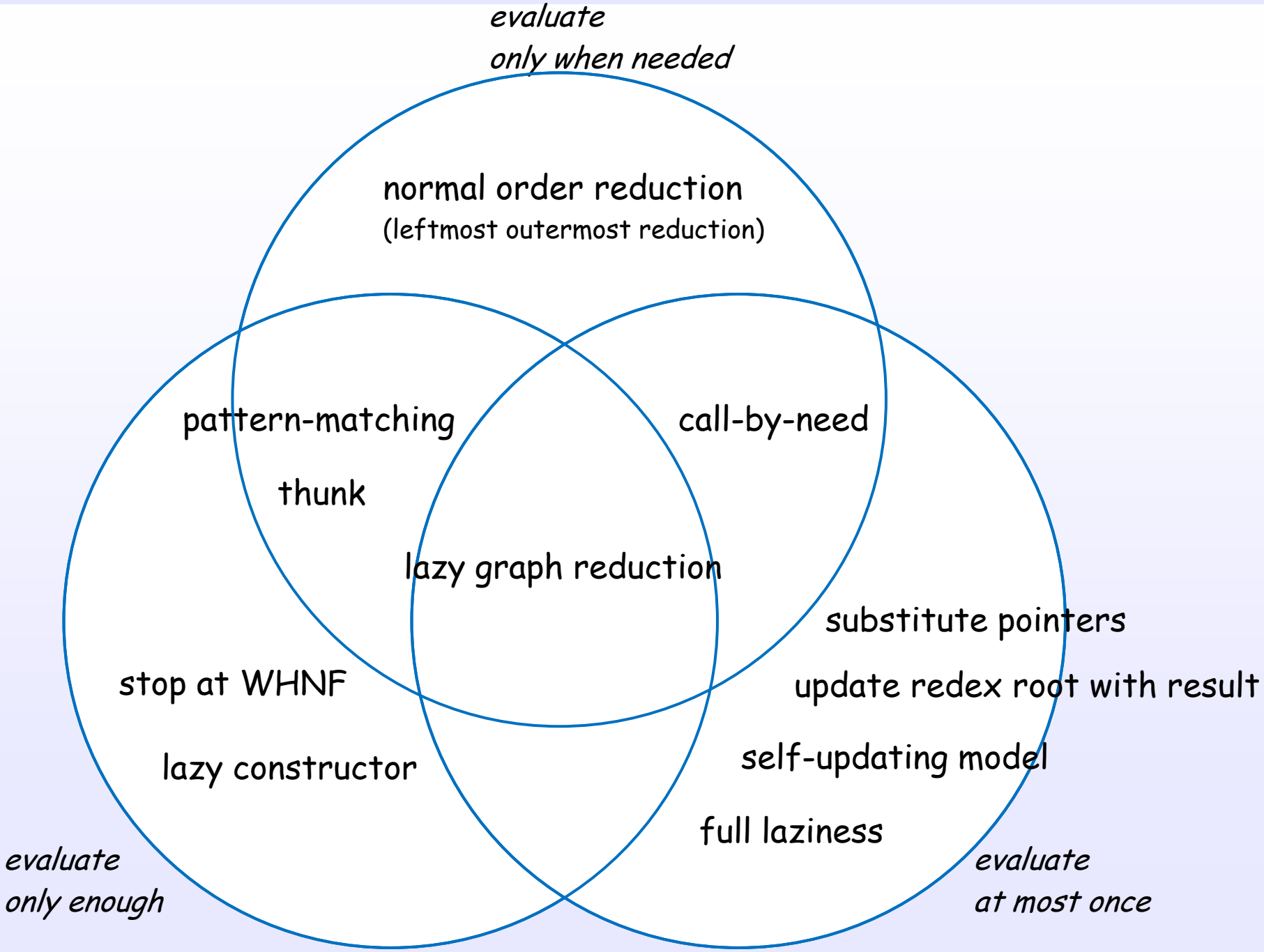
Ingredient of Haskell(GHC) 's lazy evaluation



This strategy is implemented by lazy graph reduction.

References : [B2] Ch.7, [H4] Ch.11, 12, [D2]

Techniques of Haskell(GHC) 's lazy evaluation

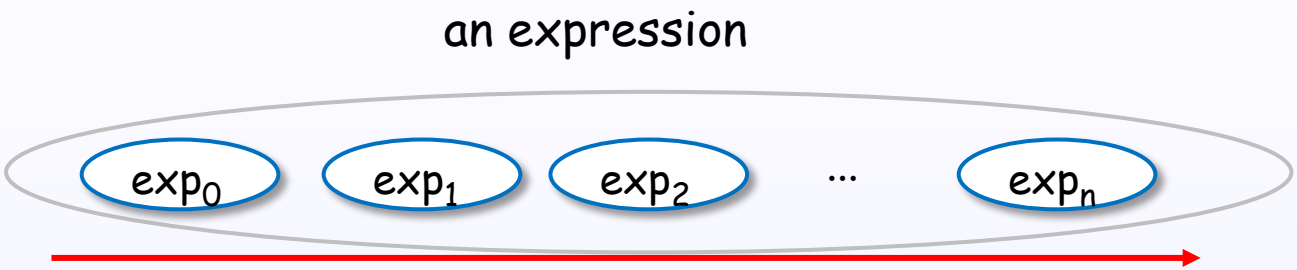


References : [B2] Ch.7, [H4] Ch.2, 11, 12, 15, [H5], [D2]

1. Introduction

Simple questions

What order?



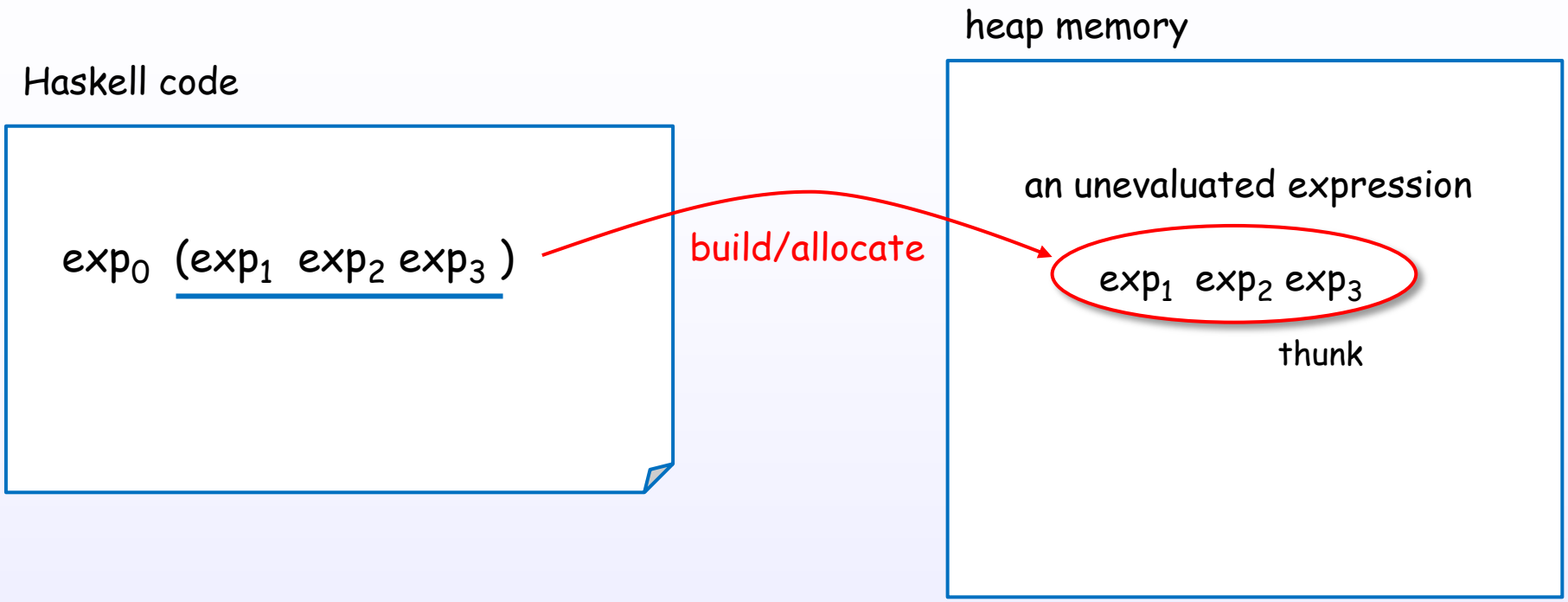
An expression is evaluated by normal order (leftmost outermost redex first).

Normal order reduction guarantees to find a normal form (if one exists).

To avoid unnecessary computation, normal order reduction chooses to apply the function rather than first evaluating the argument.

References : [H4] Ch.2, 11, [B6] Ch.5

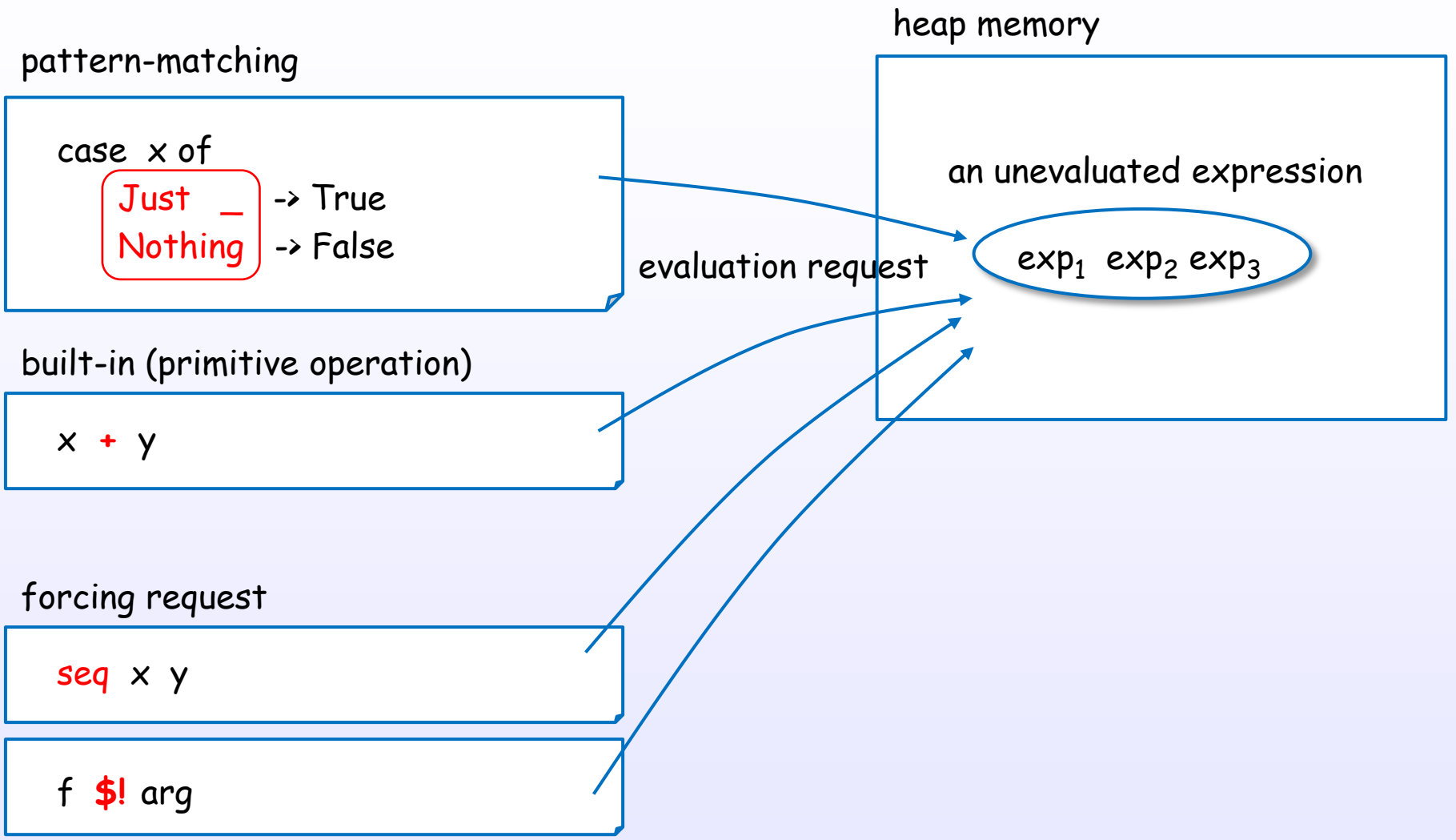
How to postpone?



To postpone the evaluation, an unevaluated expression is built in the heap memory.

References : [H4], [H5]

When needed?

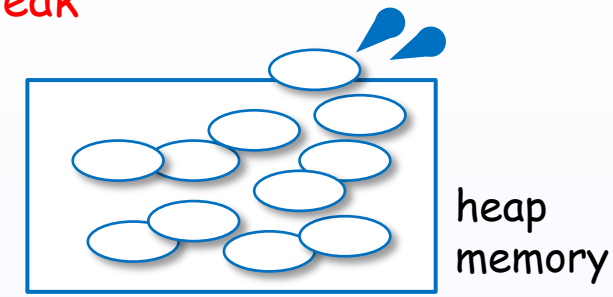


Pattern-matching or forcing request drive the evaluation.

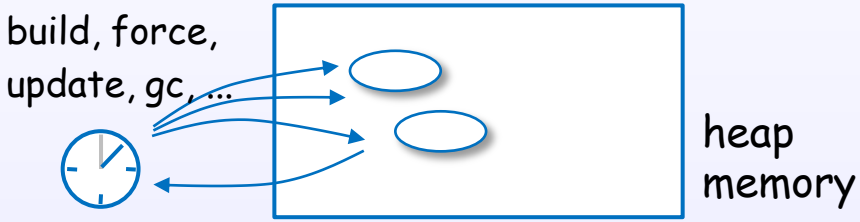
References : [H4], [D2], [D5]

What to be careful about?

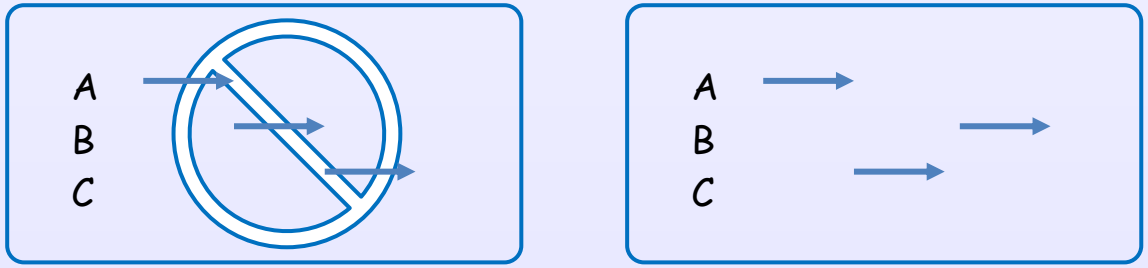
To consider hidden space leak



To consider performance cost to postpone unevaluated expressions



To consider evaluation (execution) order and timing in real world



You can avoid the pitfalls by controlling the evaluation.

References : [H4], [D2], [D5]

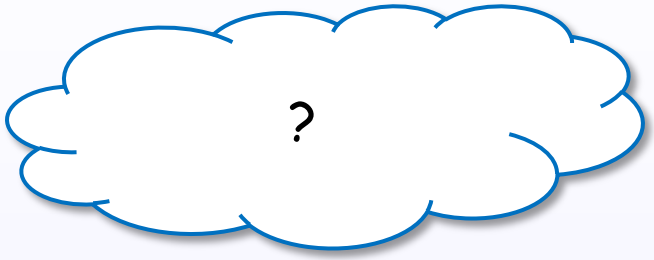
2. Expressions

2. Expressions

Expression and value

What is an expression?

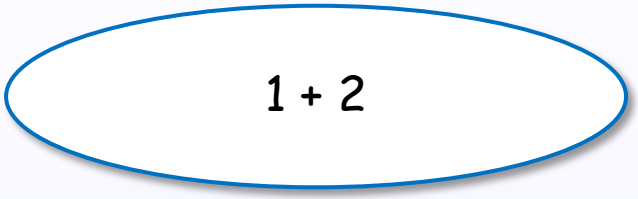
An expression



References : [B1] Ch.1, [B2] Ch.2, [B6] Ch.3

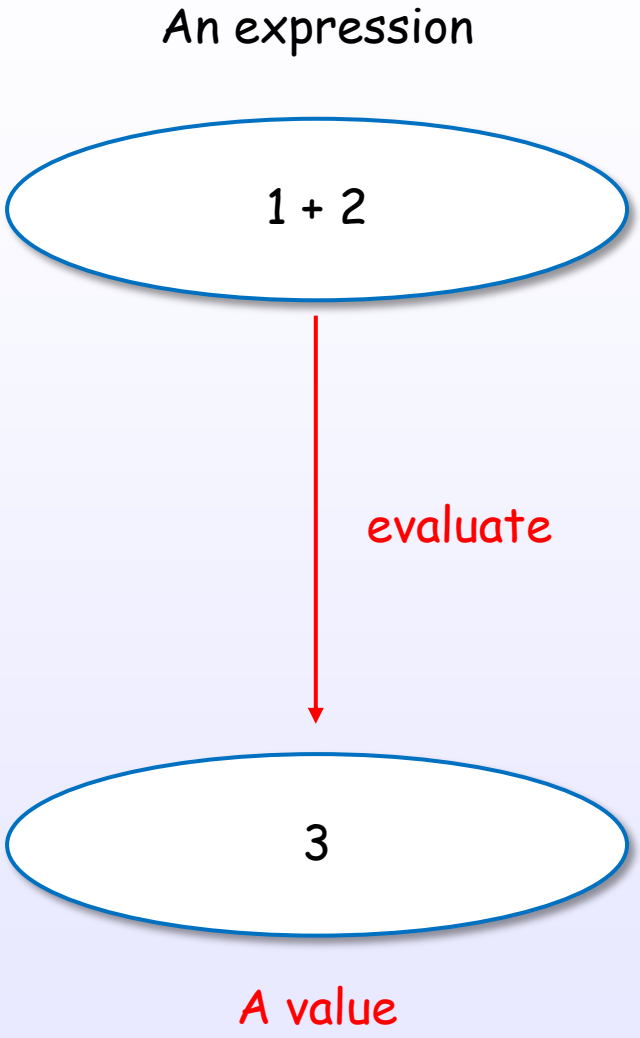
An expression denotes a value

An expression



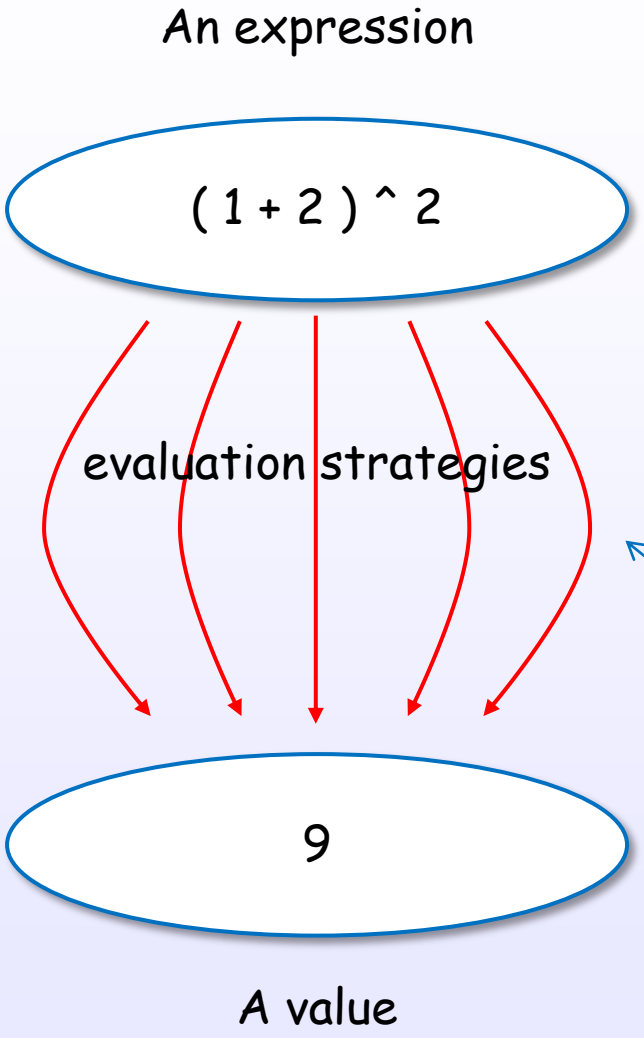
References : [B1] Ch.1, [B2] Ch.2, [H1] Ch.1, [B6] Ch.3

An expression evaluates to a value



References : [B1] Ch.1, [B2] Ch.2, [H1] Ch.1, [B6] Ch.3

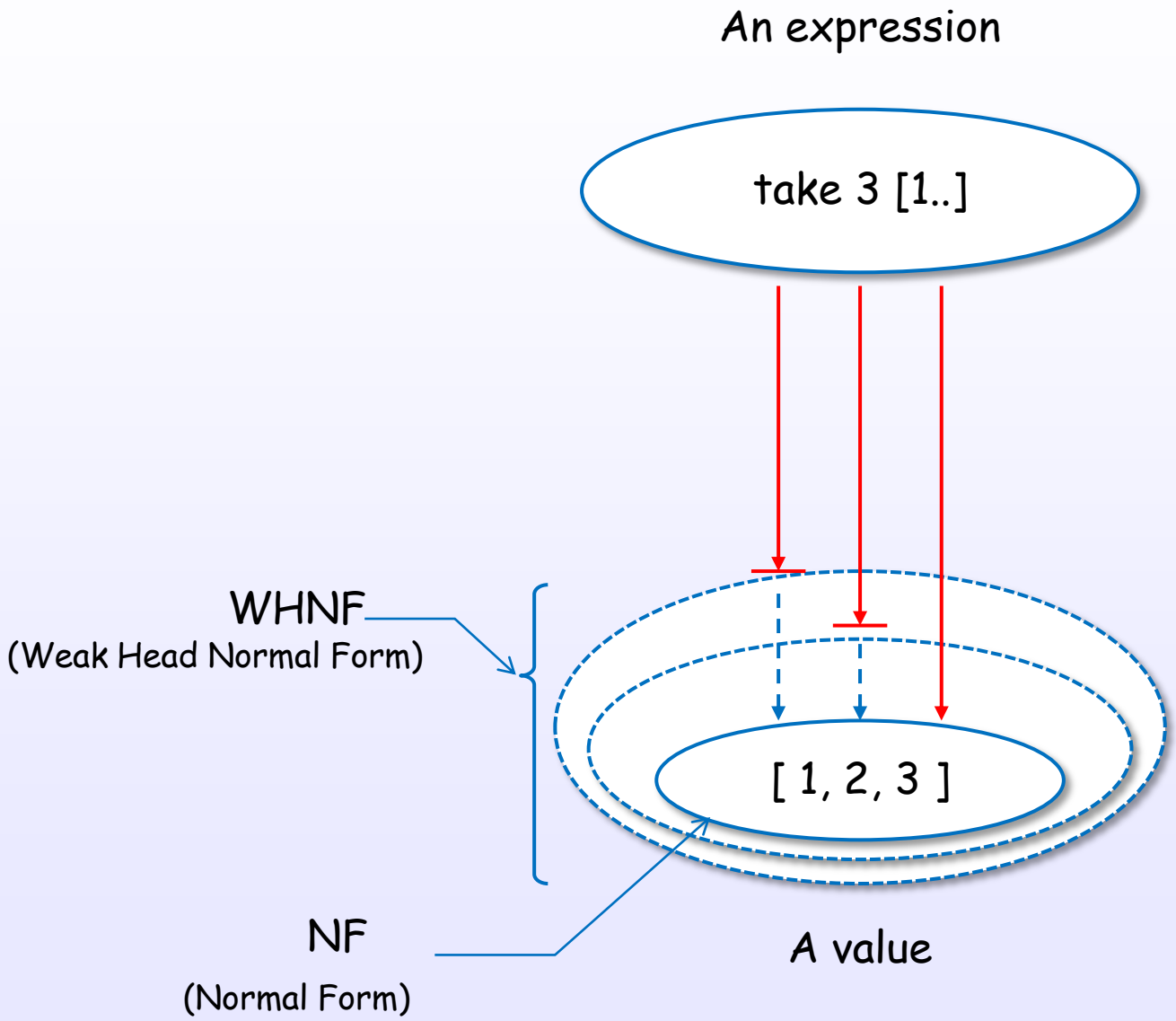
There are many evaluation approaches



- Strict, Non-strict evaluation
- Eager, Lazy evaluation
- Call-by-value, Call-by-name, Call-by-need, …
- Innermost, Outermost
- Normal order, Applicative order
- …

References : [B2] Ch.2, 7, [B6] Ch.3, [D1]

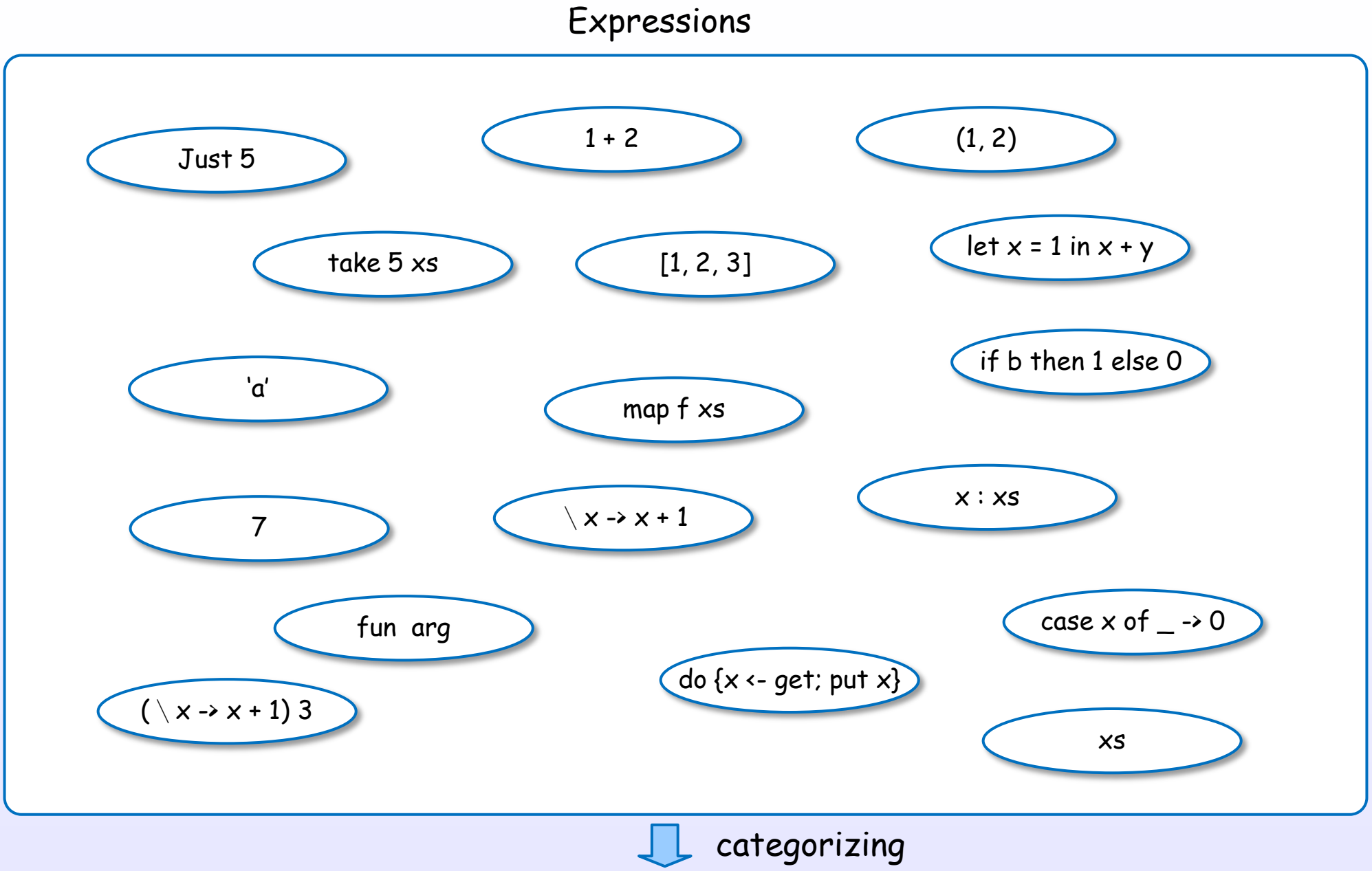
There are some evaluation levels



References : [D3], [B2] Ch.2, 7, [B6] Ch.3, [D1]

Expressions in Haskell

There are many expressions in Haskell



References : [B2] Ch.2, [H1] Ch.3

Expression categories in Haskell

lambda abstraction

```
\ x -> x + 1
```

let expression

```
let x = 1 in x + y
```

conditional

```
if b then 1 else 0
```

case expression

```
case x of _ -> 0
```

do expression

```
do {x <- get; put x}
```

function application

```
take 5 xs
```

```
( \ x -> x + 1) 3
```

```
1 + 2
```

```
map f xs
```

```
fun  arg
```

general constructor, literal and some forms

```
7
```

```
[1, 2, 3]
```

```
(1, 2)
```

```
'a'
```

```
x : xs
```

```
Just 5
```

variable

```
xs
```

References : [H1] Ch.3, [B2] Ch.2

Specification is defined in Haskell 2010 Language Report

"Haskell 2010 Language Report, Chapter 3 Expressions" [H1]

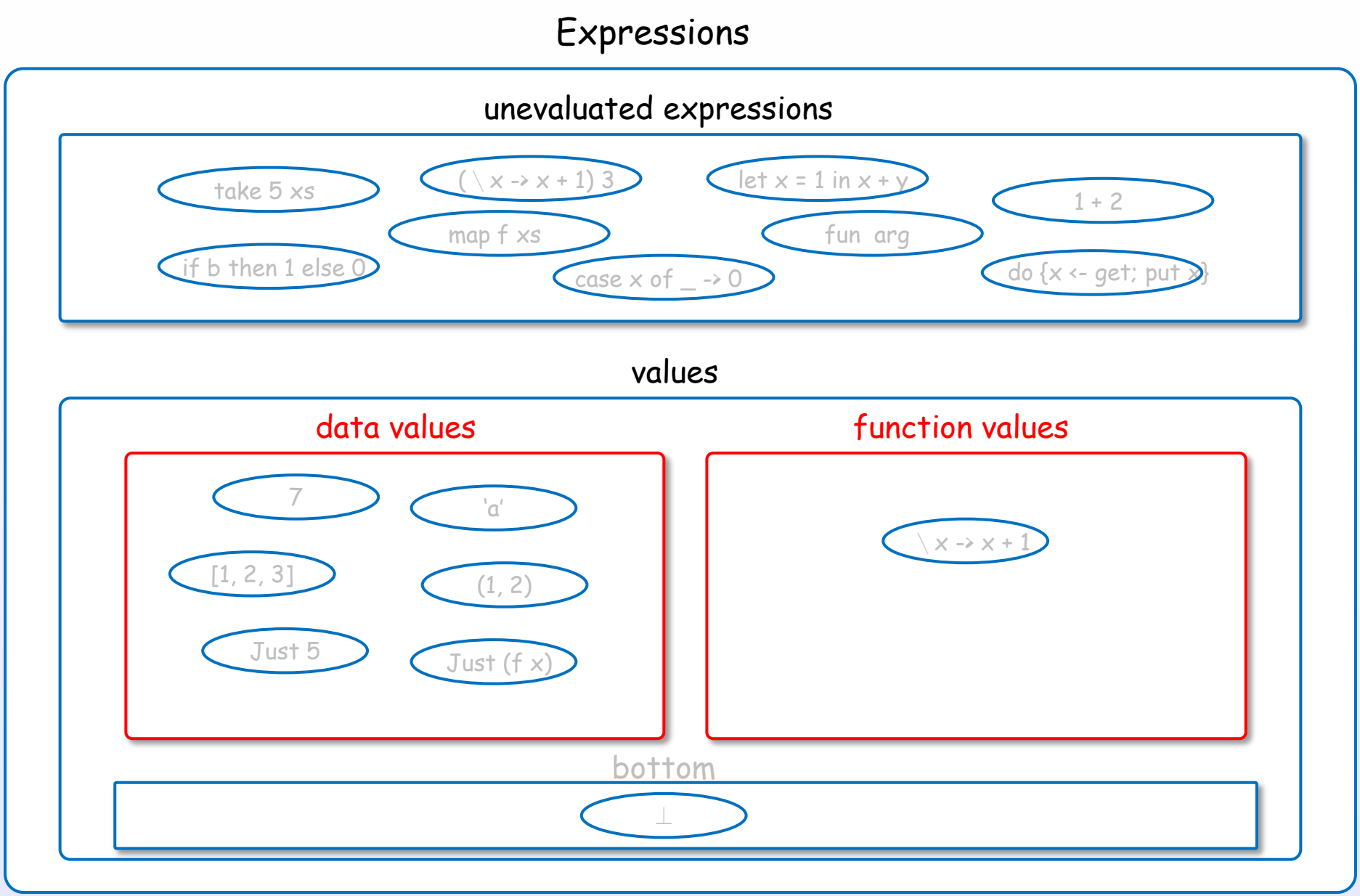
exp	→	*infixexp* `::` [*context* `=>`] *type*	(expression type signature)
	|	*infixexp*	
infixexp	→	*lexp qop infixexp*	(infix operator application)
	|	`-` *infixexp*	(prefix negation)
	|	*lexp*	
lexp	→	`\` $apat_1$ ... $apat_n$ `->` *exp*	(lambda abstraction, $n \geq 1$)
	|	`let` *decls* `in` *exp*	(let expression)
	|	`if` *exp* [`;`] `then` *exp* [`;`] `else` *exp*	(conditional)
	|	`case` *exp* `of` `{` *alts* `}`	(case expression)
	|	`do` `{` *stmts* `}`	(do expression)
	|	*fexp*	
fexp	→	[*fexp*] *aexp*	(function application)
aexp	→	*qvar*	(variable)
	|	*gcon*	(general constructor)
	|	*literal*	
	|	`(` *exp* `)`	(parenthesized expression)
	|	`(` exp_1 `,` ... `,` exp_k `)`	(tuple, $k \geq 2$)
	|	`[` exp_1 `,` ... `,` exp_k `]`	(list, $k \geq 1$)
	|	`[` exp_1 [`,` exp_2] `..` [exp_3] `]`	(arithmetic sequence)
	|	`[` *exp* `|` $qual_1$ `,` ... `,` $qual_n$ `]`	(list comprehension, $n \geq 1$)
	|	`(` *infixexp qop* `)`	(left section)
	|	`(` $qop_{\langle - \rangle}$ *infixexp* `)`	(right section)
	|	*qcon* `{` $fbind_1$ `,` ... `,` $fbind_n$ `}`	(labeled construction, $n \geq 0$)
	|	$aexp_{\langle qcon \rangle}$ `{` $fbind_1$ `,` ... `,` $fbind_n$ `}`	(labeled update, $n \geq 1$)

References : [H1] Ch.3, [B2] Ch.2

2. Expressions

Classification by values and forms

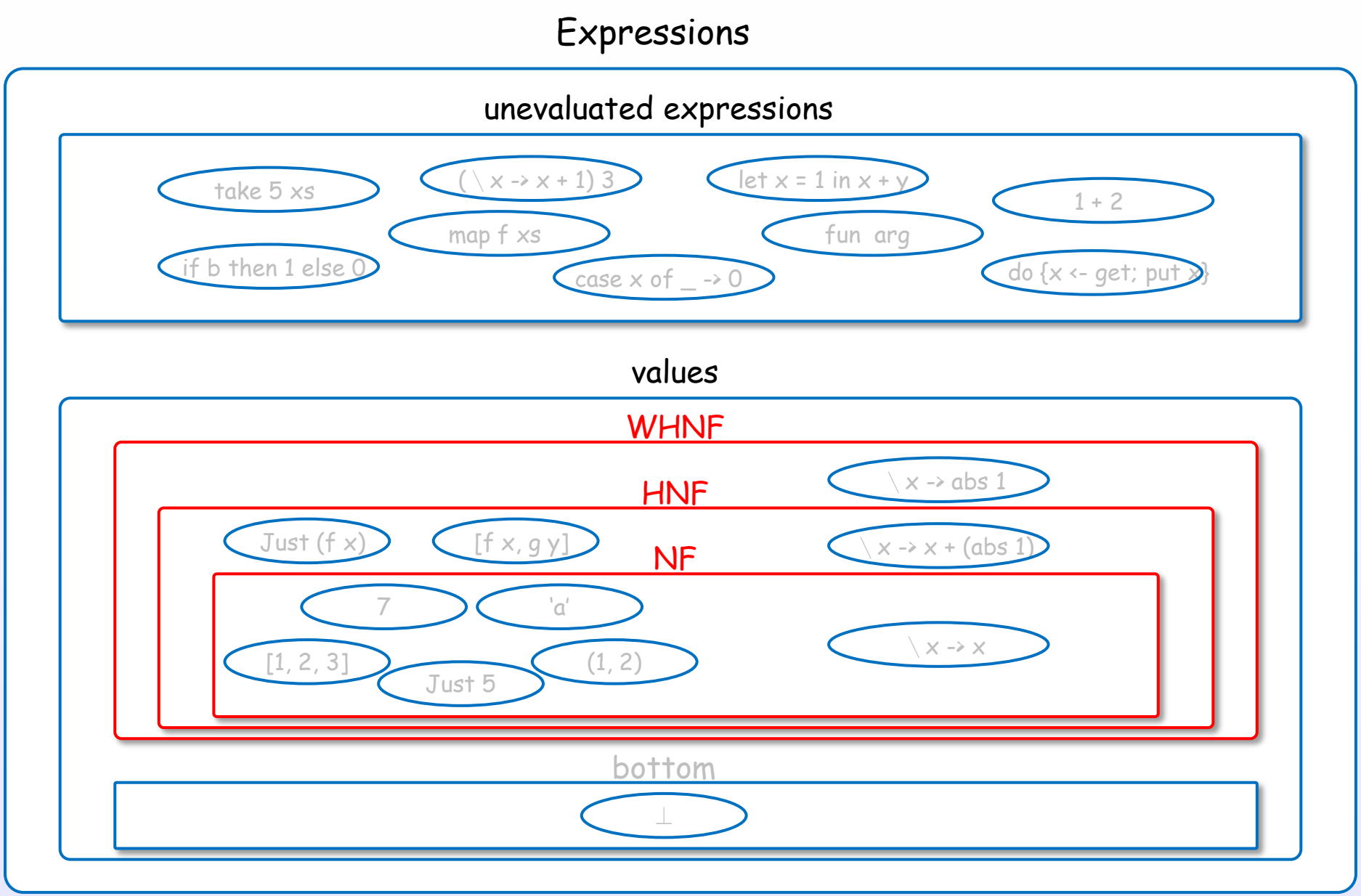
Classification by values



Values are data values or function values.

References : [H5]

Classification by forms



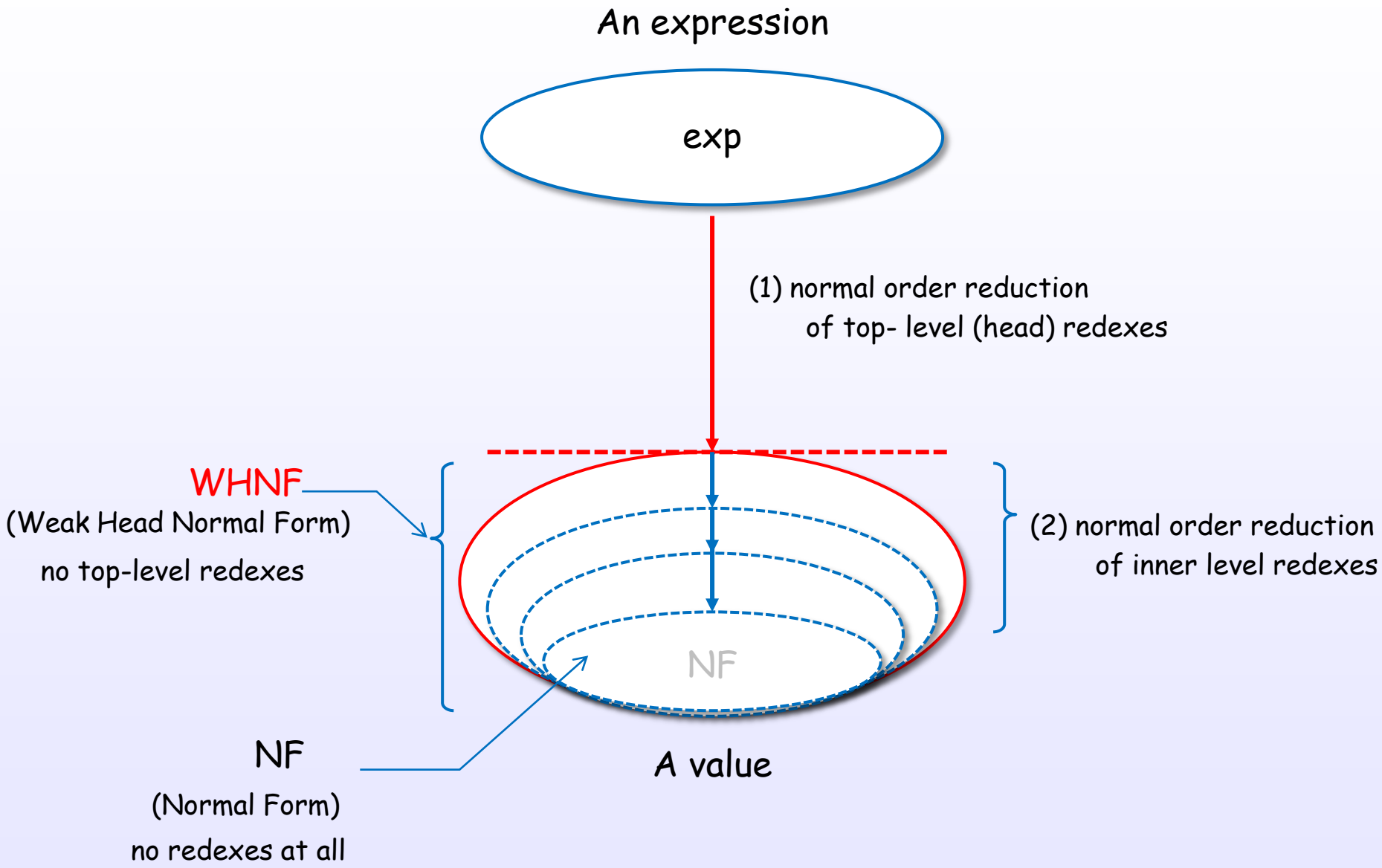
Values are WHNF, HNF or NF.

References : [H4] Ch.11, [D3], [B6] Ch.3, [B2] Ch.2, 7, [D1], [W1]

2. Expressions

WHNF

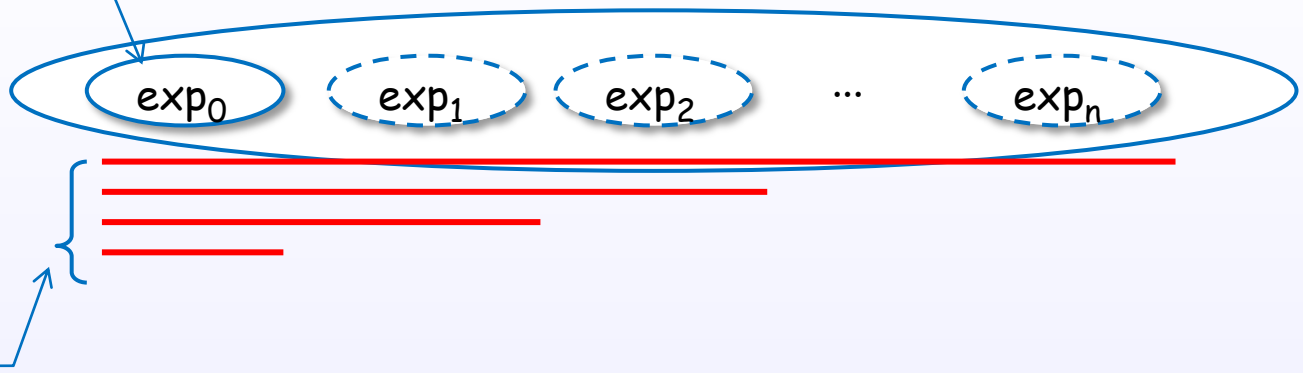
WHNF is one of the form for the evaluated values



References : [H4] Ch.11, [D3], [B6] Ch.3, [B2] Ch.2, 7, [D1], [W1]

WHNF

top-level (head) is
a constructor or
a lambda abstraction



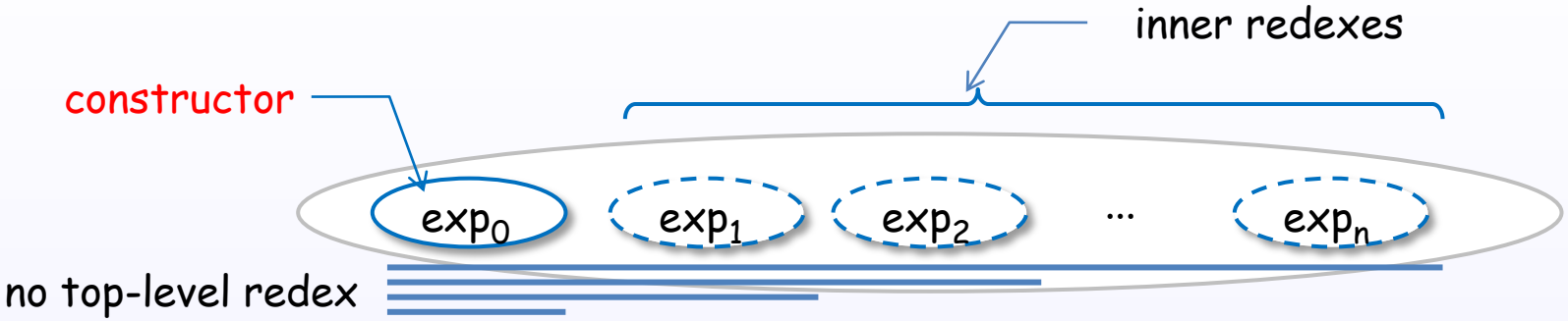
no top-level redex

WHNF is a value which has evaluated top-level

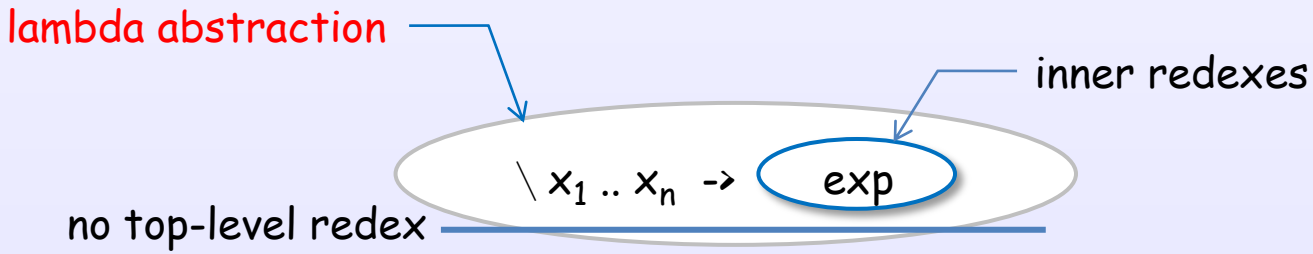
References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

WHNF for a data value and a function value

a data value in WHNF



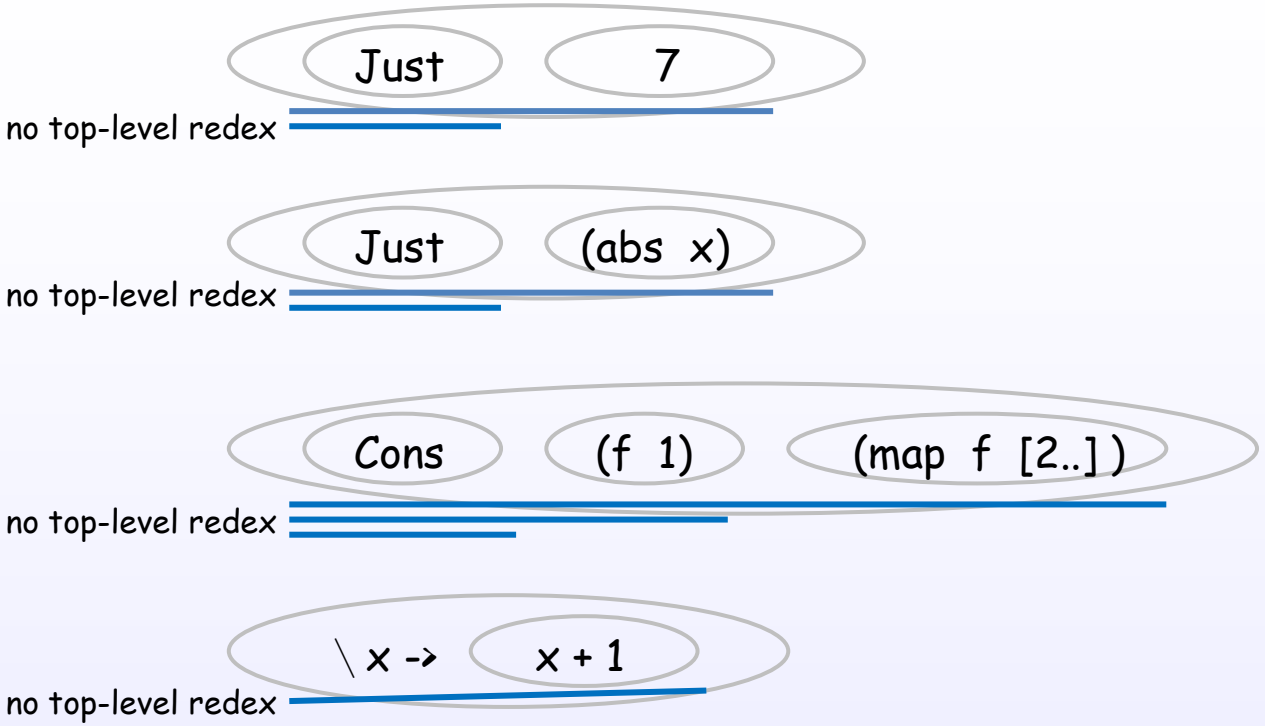
a function value in WHNF



References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

Examples of WHNF

WHNF



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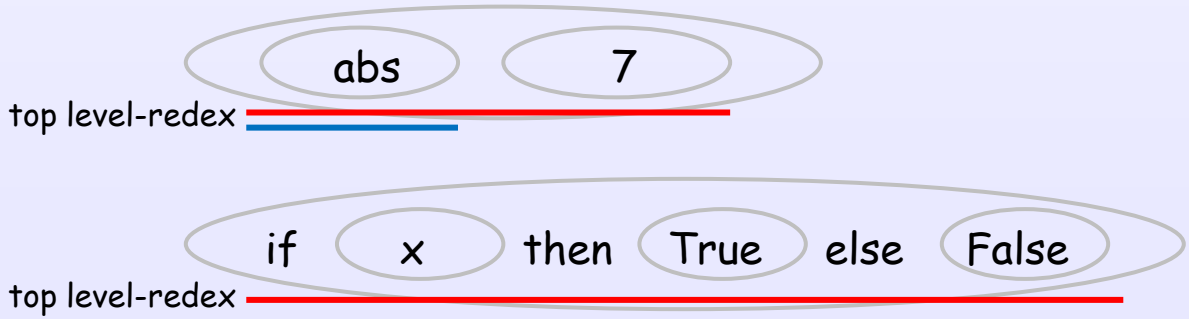
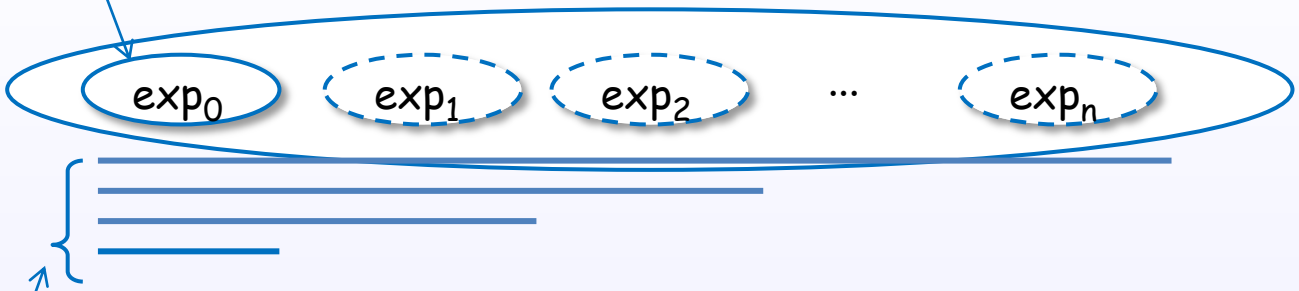
no WHNF



References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

HNF

top-level (head) is
a constructor or
a lambda abstraction with no top-level redex



no top-level redex

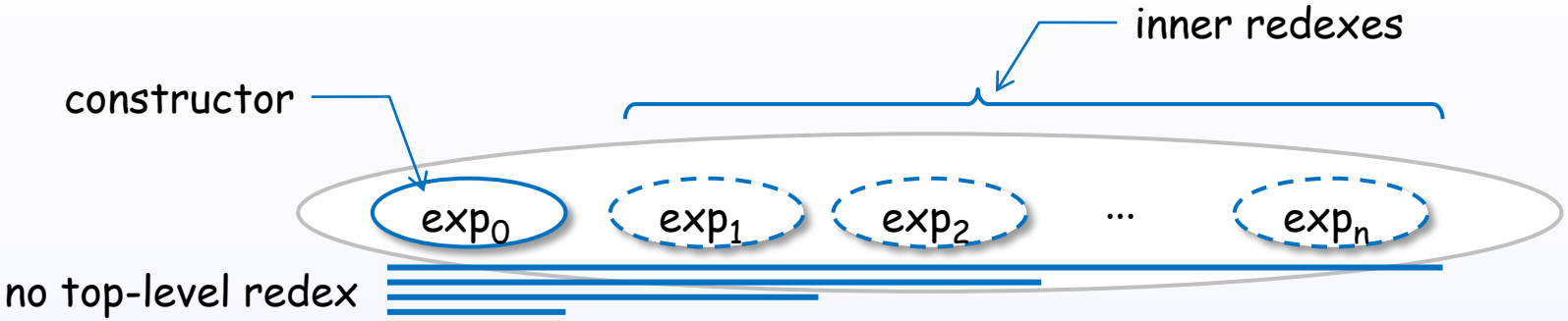
HNF is a value which has evaluated top-level

* GHC uses WHNF rather than HNF.

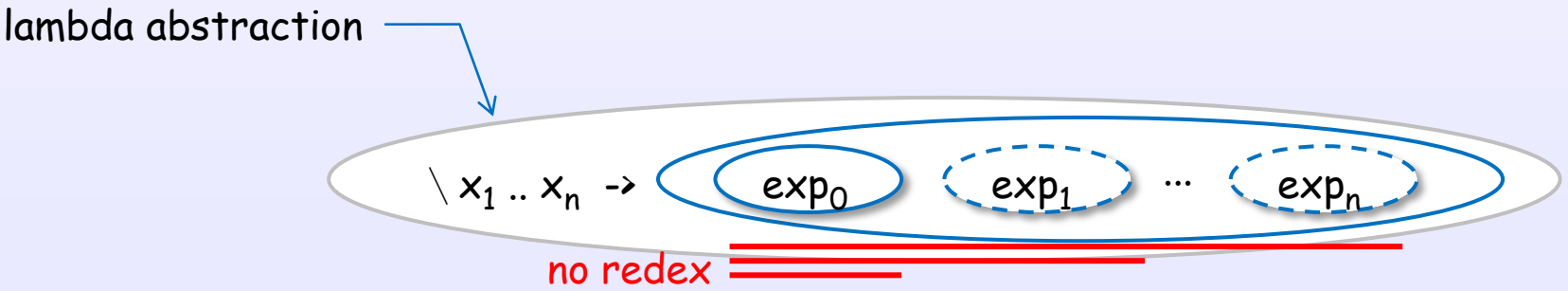
References : [H4] Ch.11, [D3], [B3]

HNF for a data value and a function value

a data value in HNF (same as WHNF)



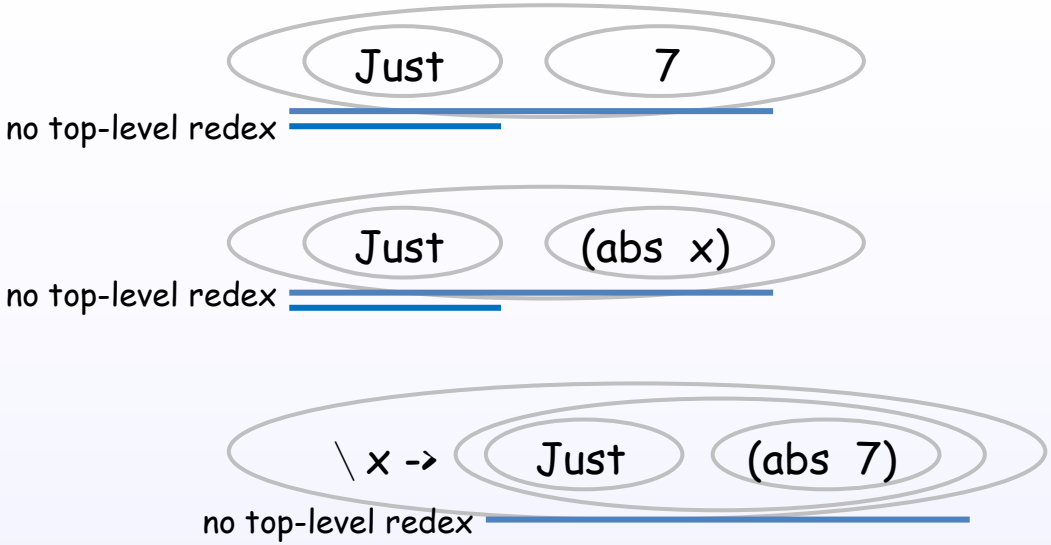
a function value in HNF



References : [H4] Ch.11, [D3], [B3]

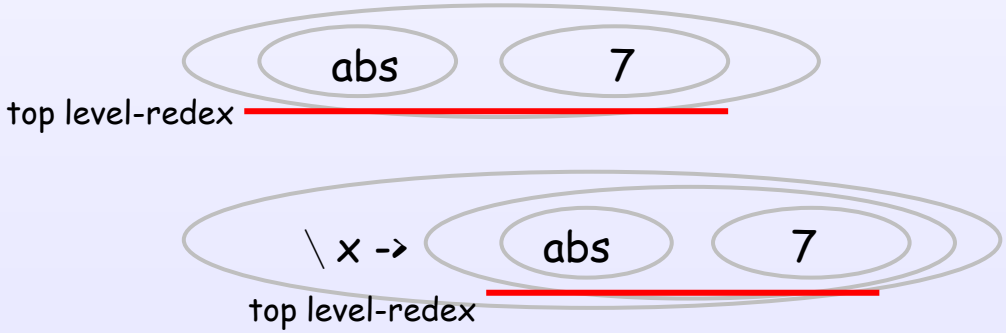
Examples of HNF

HNF



---

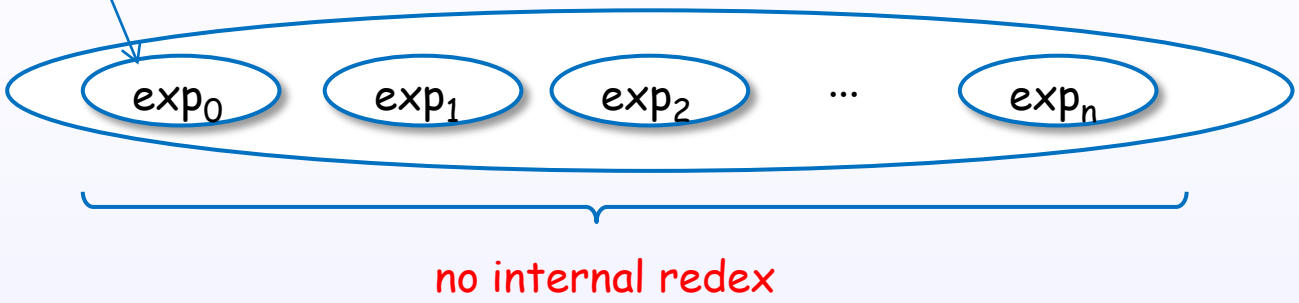
no HNF



References : [H4] Ch.11, [D3], [B3]

NF

top-level (head) is
a constructor or
a lambda abstraction

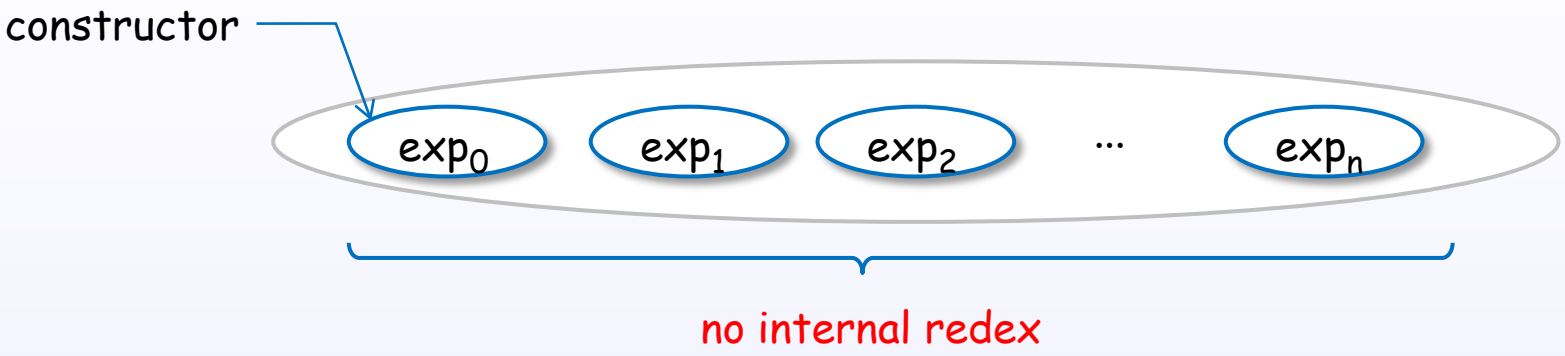


NF is a value which has no redex.

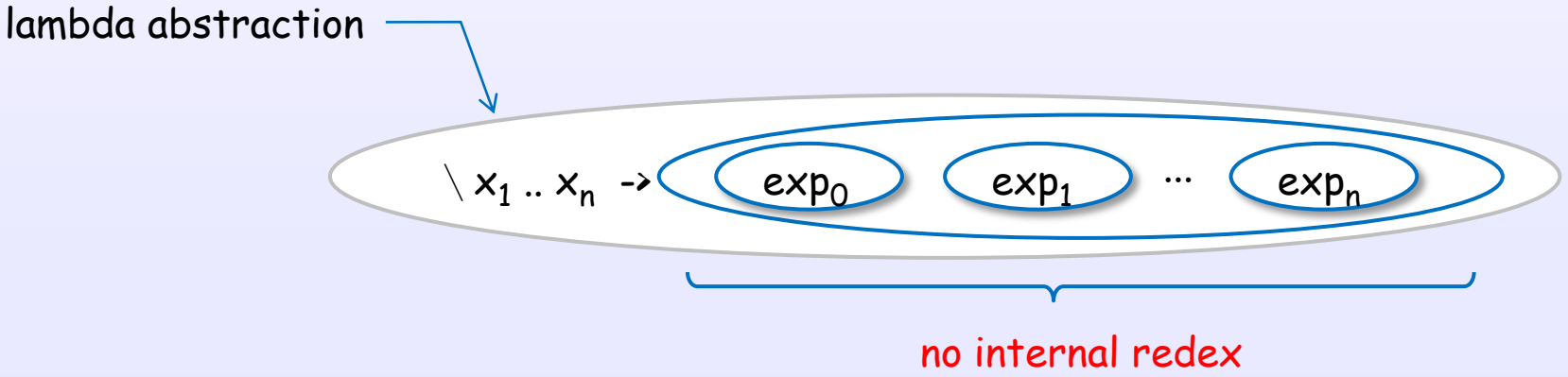
References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

NF for a data value and a function value

a data value in NF



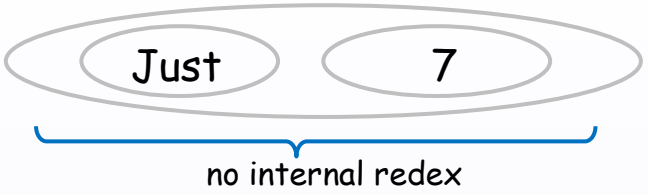
a function value in NF

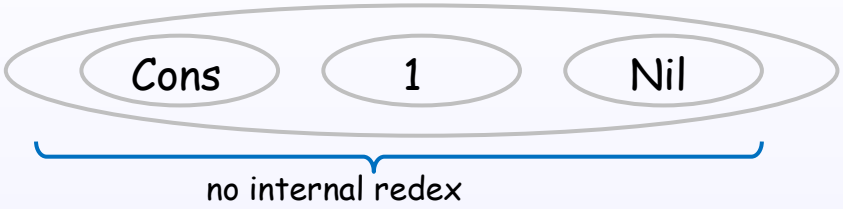


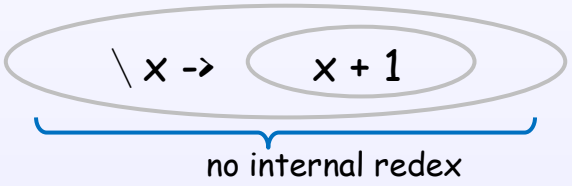
References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

Examples of NF

NF

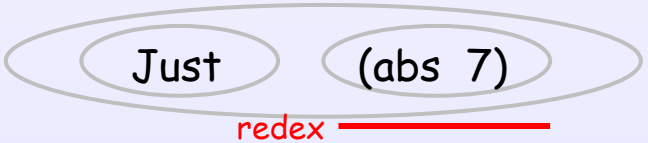


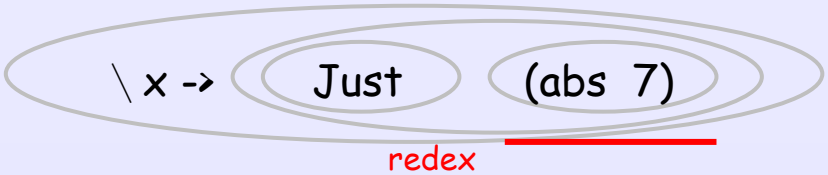




---

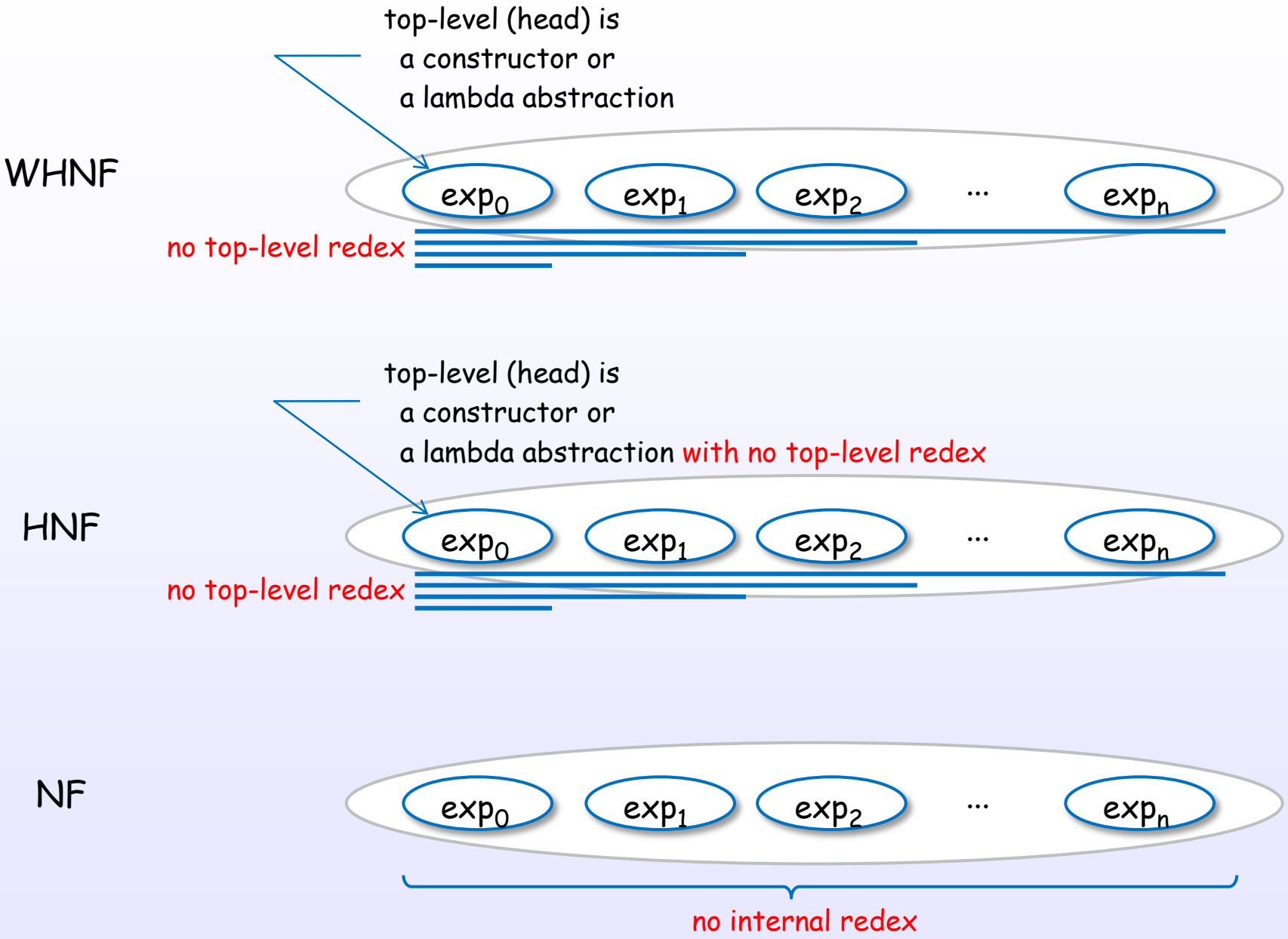
no NF





References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

WHNF, HNF, NF



References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

Definition of WHNF and HNF

"The implementation of functional programming languages" [H4]

11.3.1 Weak Head Normal Form

To express this idea precisely we need to introduce a new definition:

DEFINITION

A lambda expression is in *weak head normal form* (WHNF) if and only if it is of the form

$$F\ E_1\ E_2\ \ldots\ E_n$$

where $n \geq 0$;
and either F is a variable or data object
or F is a lambda abstraction or built-in function
and $(F\ E_1\ E_2\ \ldots\ E_m)$ is not a redex for any $m \leq n$.

An expression has no *top-level redex* if and only if it is in weak head normal form.

11.3.3 Head Normal Form

Head normal form is often confused
some discussion. The content of th
since for most purposes head norma
form. Nevertheless, we will stick to t

DEFINITION

A lambda expression is in *head normal form* (HNF) if and only if it is of the form

$$\lambda x_1.\lambda x_2 \ldots \lambda x_n.(v\ M_1\ M_2\ \ldots\ M_m)$$

where $n, m \geq 0$;
v is a variable (x_i), a data object, or a built-in function;
and $(v\ M_1\ M_2\ \ldots\ M_p)$ is not a redex for any $p \leq m$.

References : [H4] Ch.11

3. Internal representation of expressions

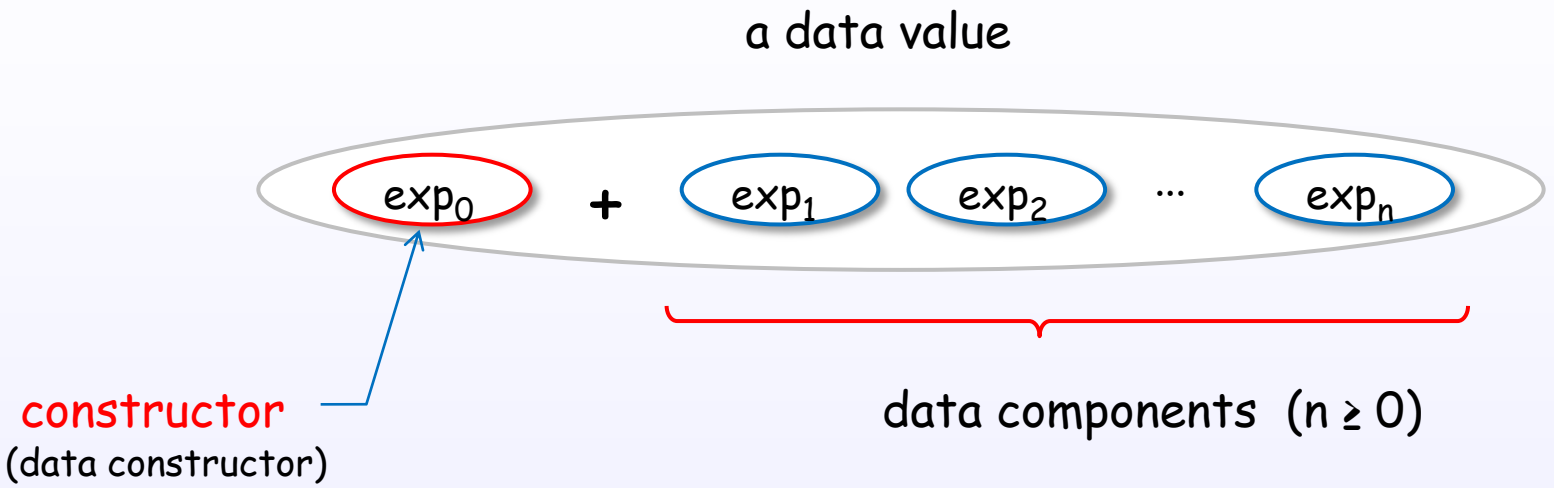
3. Internal representation of expressions

Constructor

Constructor

Constructor is one of the key elements
to understand WHNF and lazy evaluation in Haskell.

Constructor

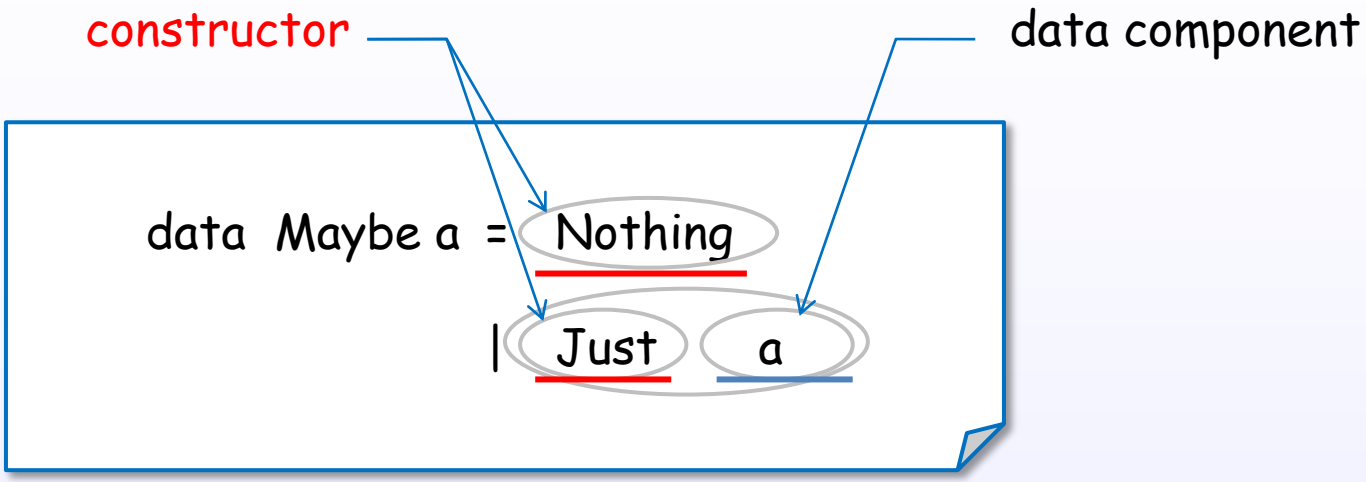


A constructor builds a structured data value.

A constructor distinguishes the data value in expressions.

References : [B2], [H1], [H4] Ch.2, 10, [B6] Ch.11

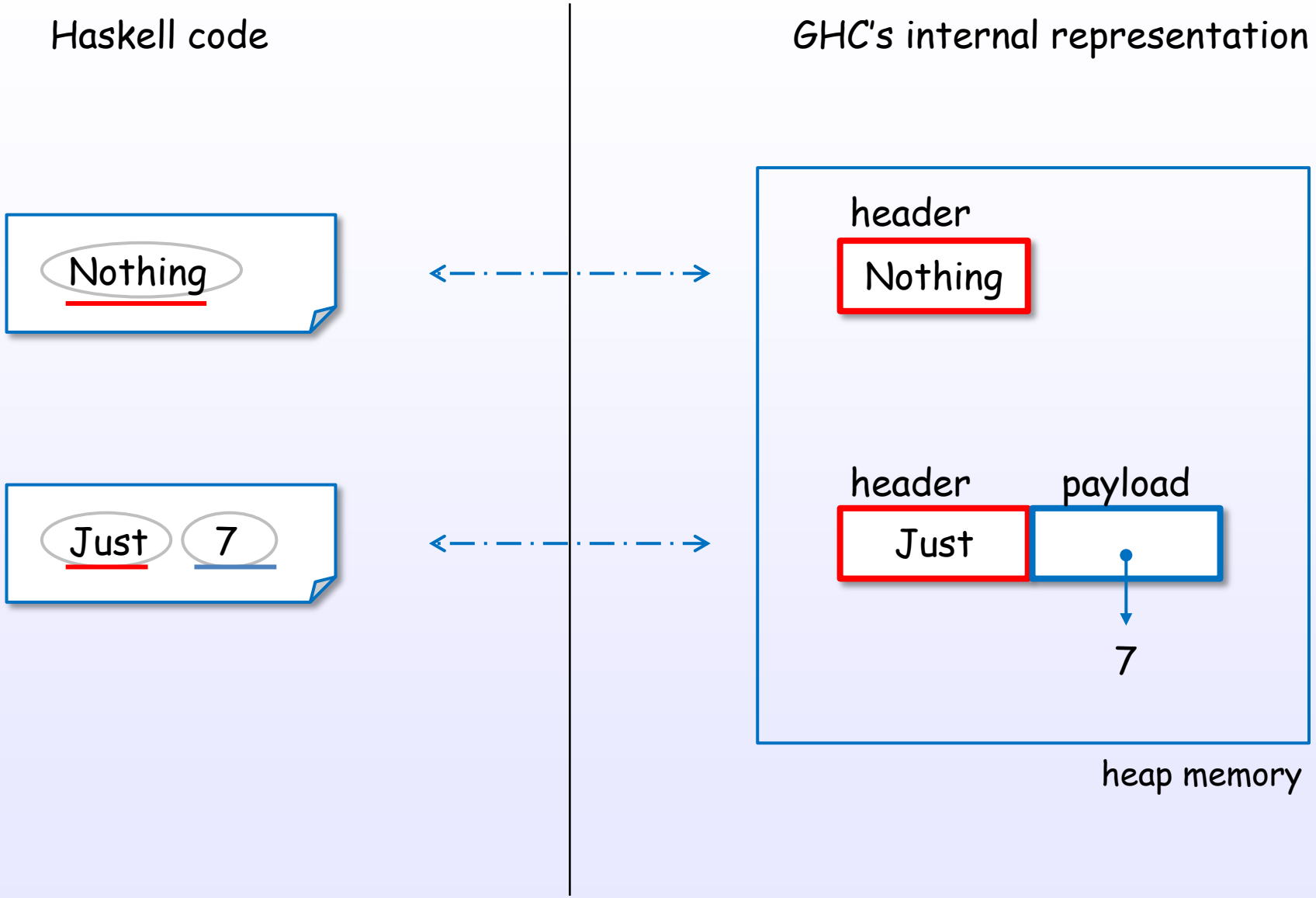
Constructors and data declaration



Constructors are defined by data declaration.

References : [B2], [H1]

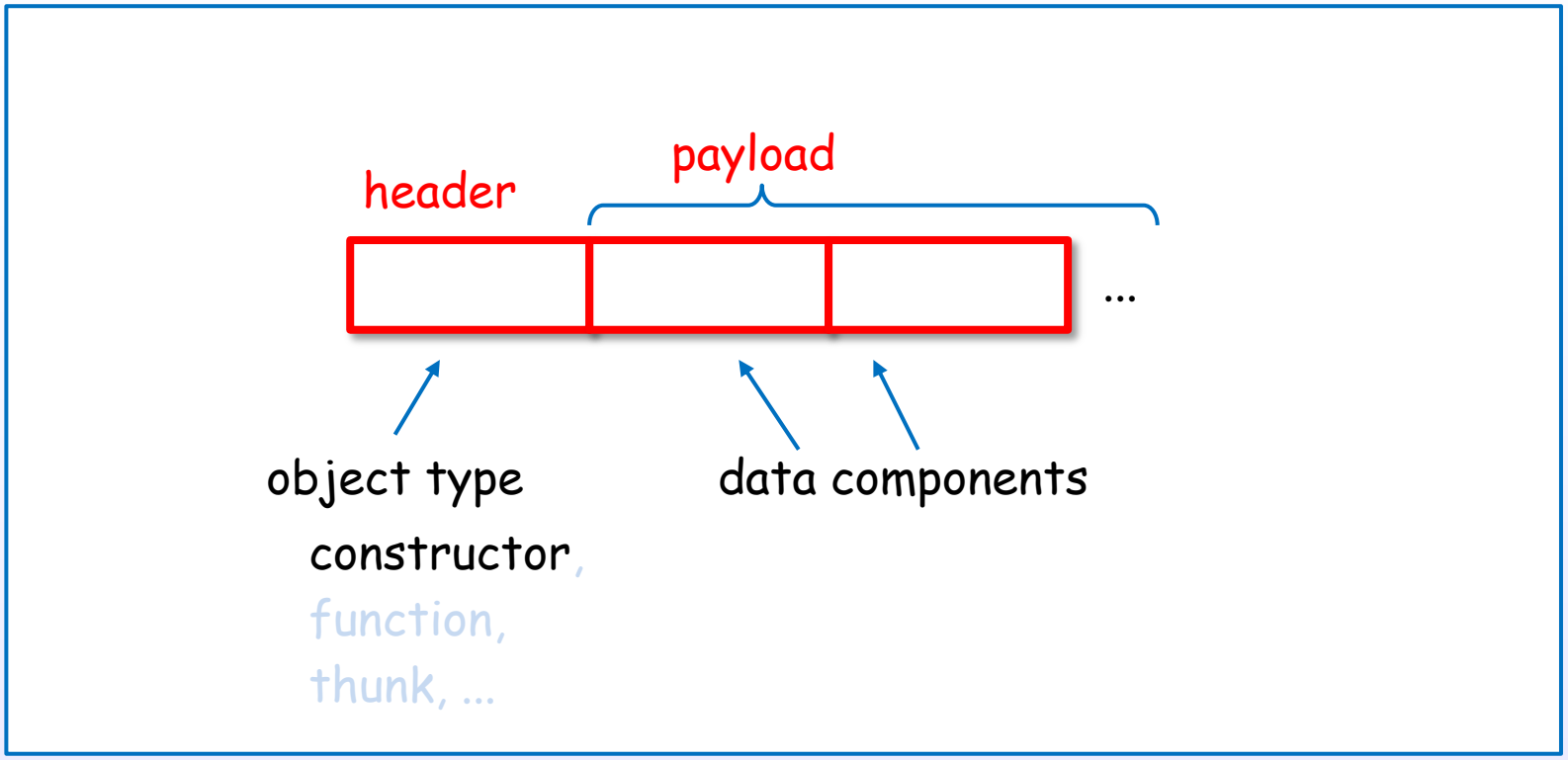
Internal representation of Constructors for data values



References : [H11], [H10], [H5], [H6], [H7]

Constructors are represented uniformly

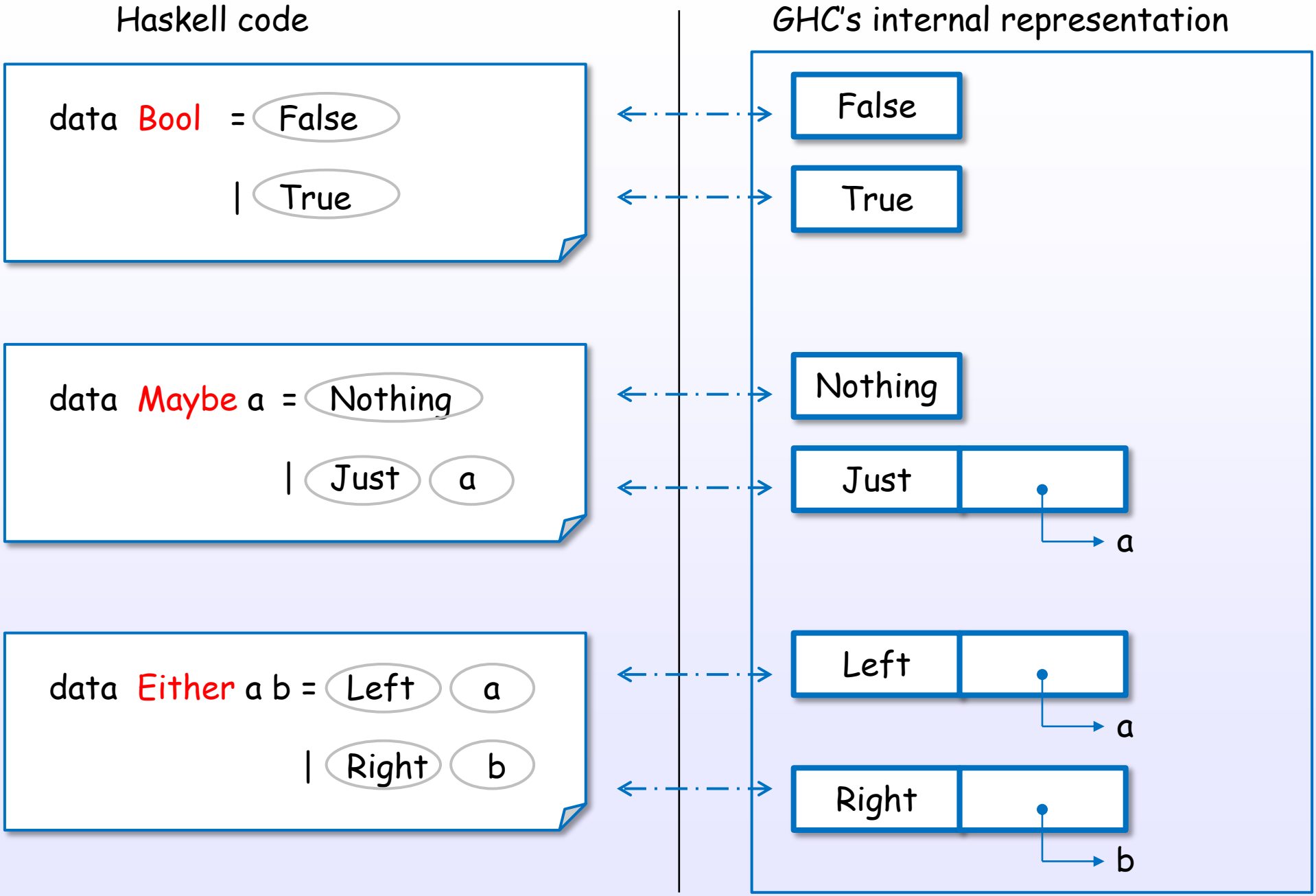
GHC's internal representation



A data value is represented with header(constructor) + payload(components).

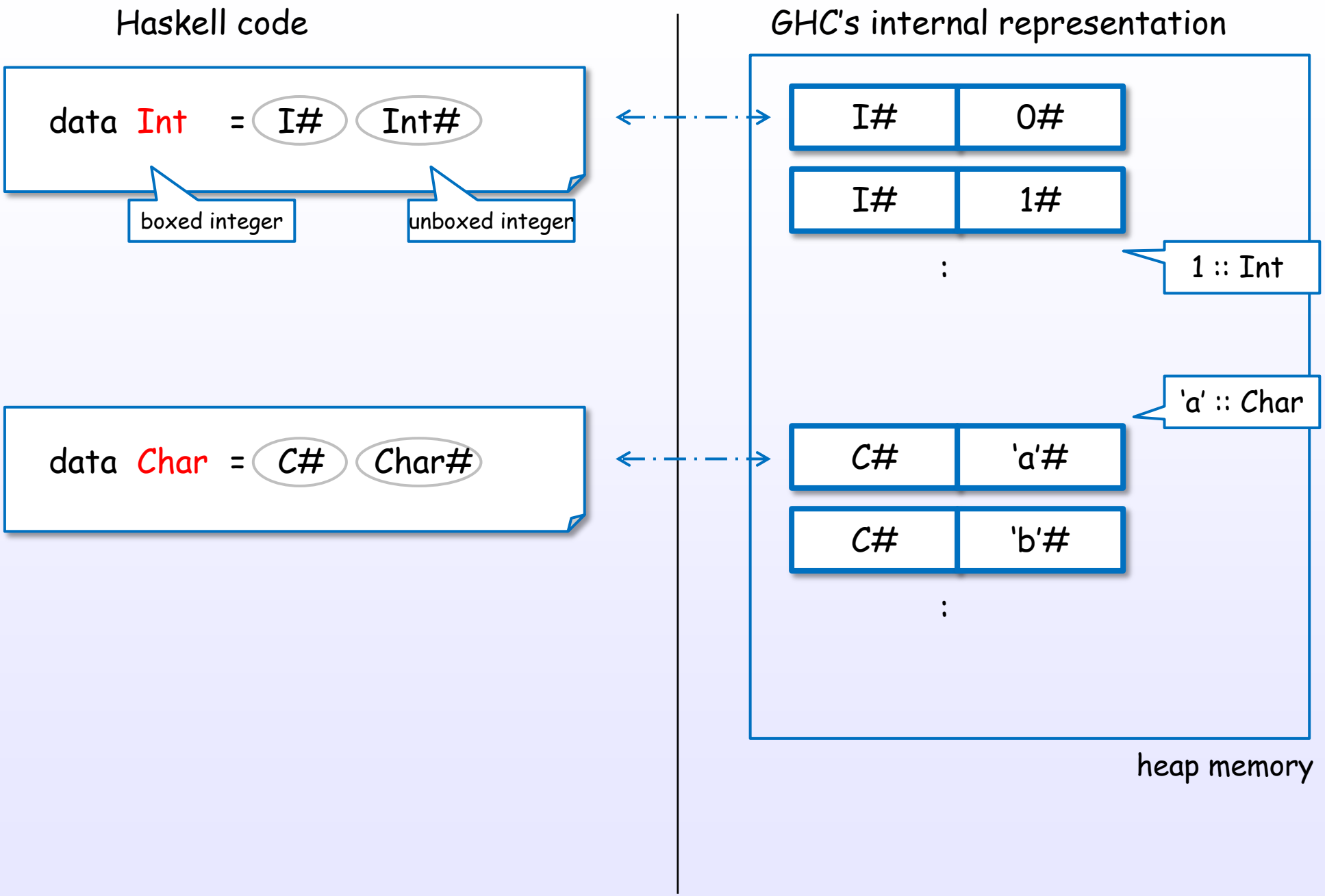
References : [H11], [H10], [H5], [H6], [H7], [D15]

Representation of various constructors



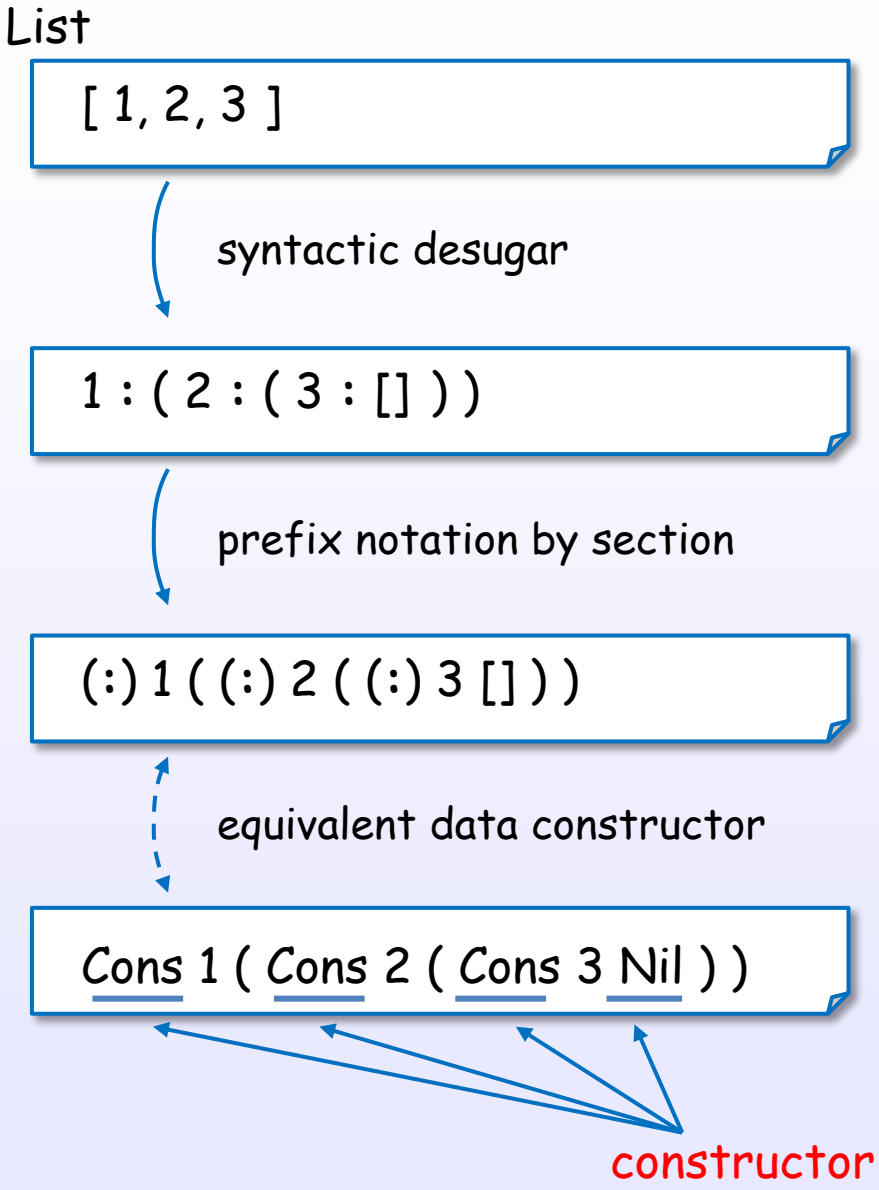
References : [H11], [H10], [H5], [H6], [H7]

Primitive data types are also represented with constructors



References : [H11], [H10], [H5], [H6], [H7]

List is also represented with constructors



References : [H11], [H10], [H5], [H6], [H7]

List is also represented with constructors

List

[1, 2, 3]

syntactic desugar

1 : (2 : (3 : []))

prefix notation by section

(:) 1 ((:) 2 ((:) 3 []))

equivalent data constructor

Cons 1 (Cons 2 (Cons 3 Nil))

type declaration

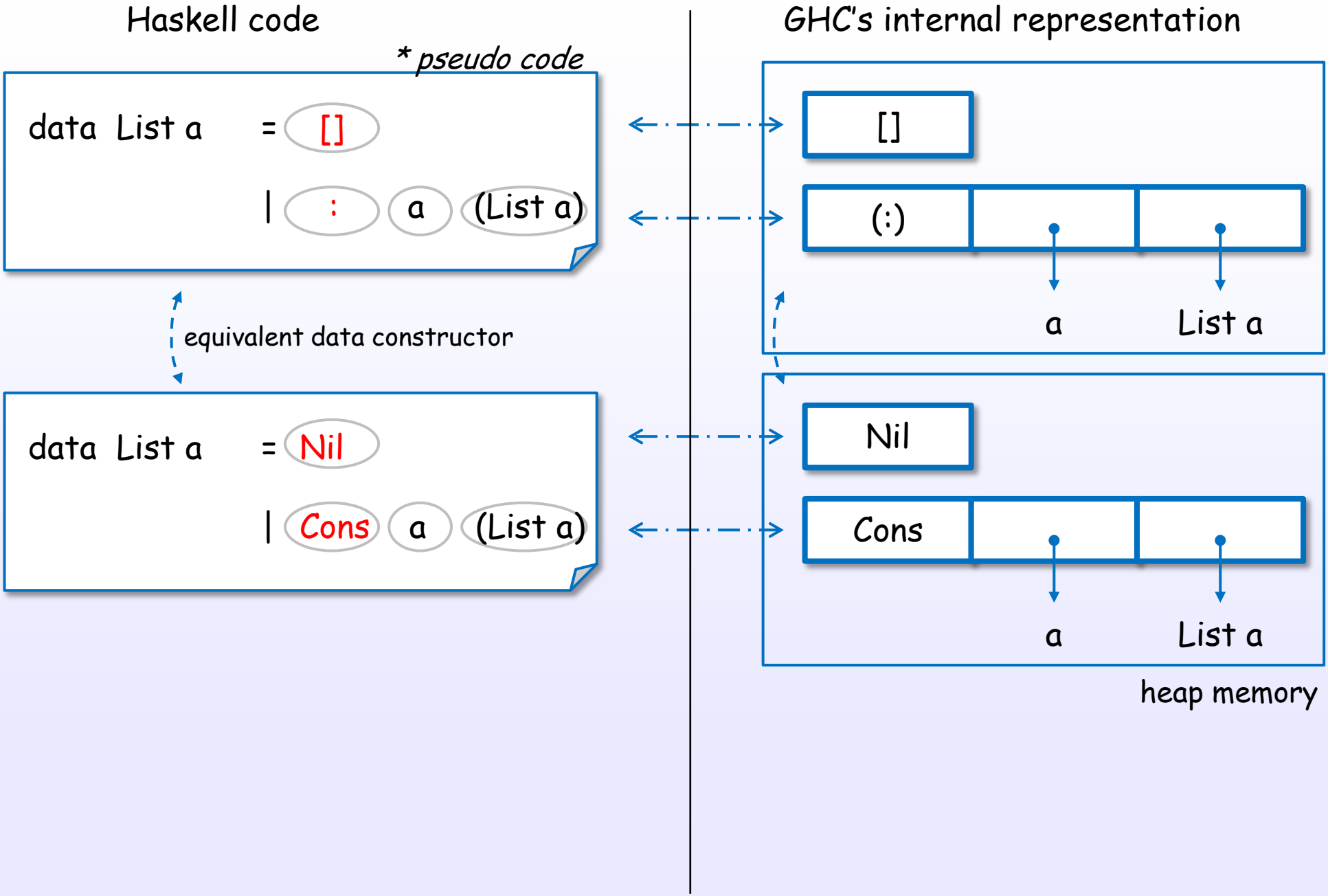
* pseudo code

```
data List a = []
            | :  a (List a)
```

```
data List a = Nil
            | Cons a (List a)
```

References : [H11], [H10], [H5], [H6], [H7]

List is also represented with constructors



References : [H11], [H10], [H5], [H6], [H7]

List is also represented with constructors

Haskell code

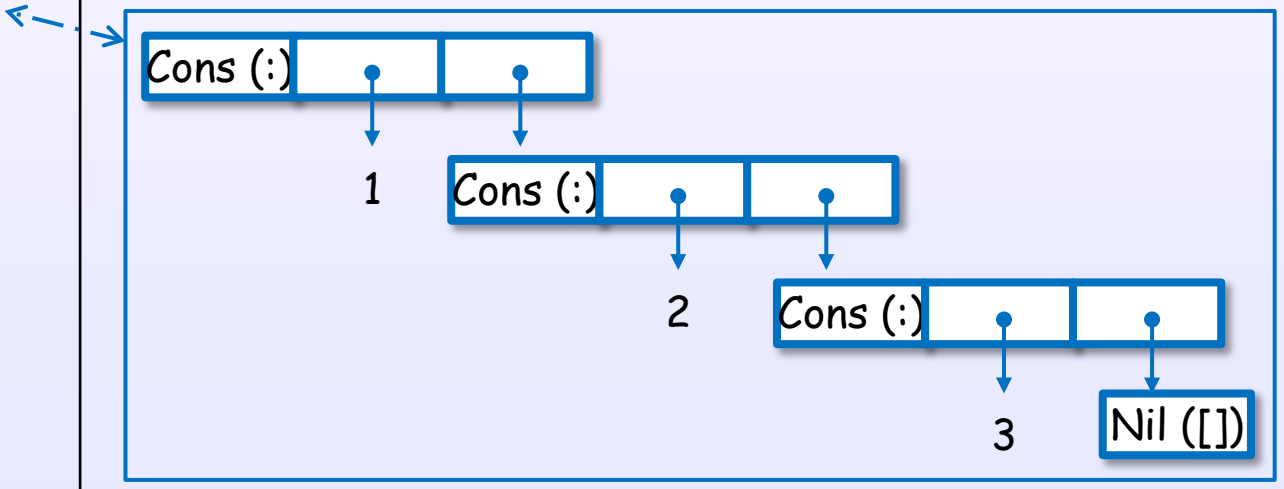
```
[ 1, 2, 3 ]
```

```
1 : ( 2 : ( 3 : [] ) )
```

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

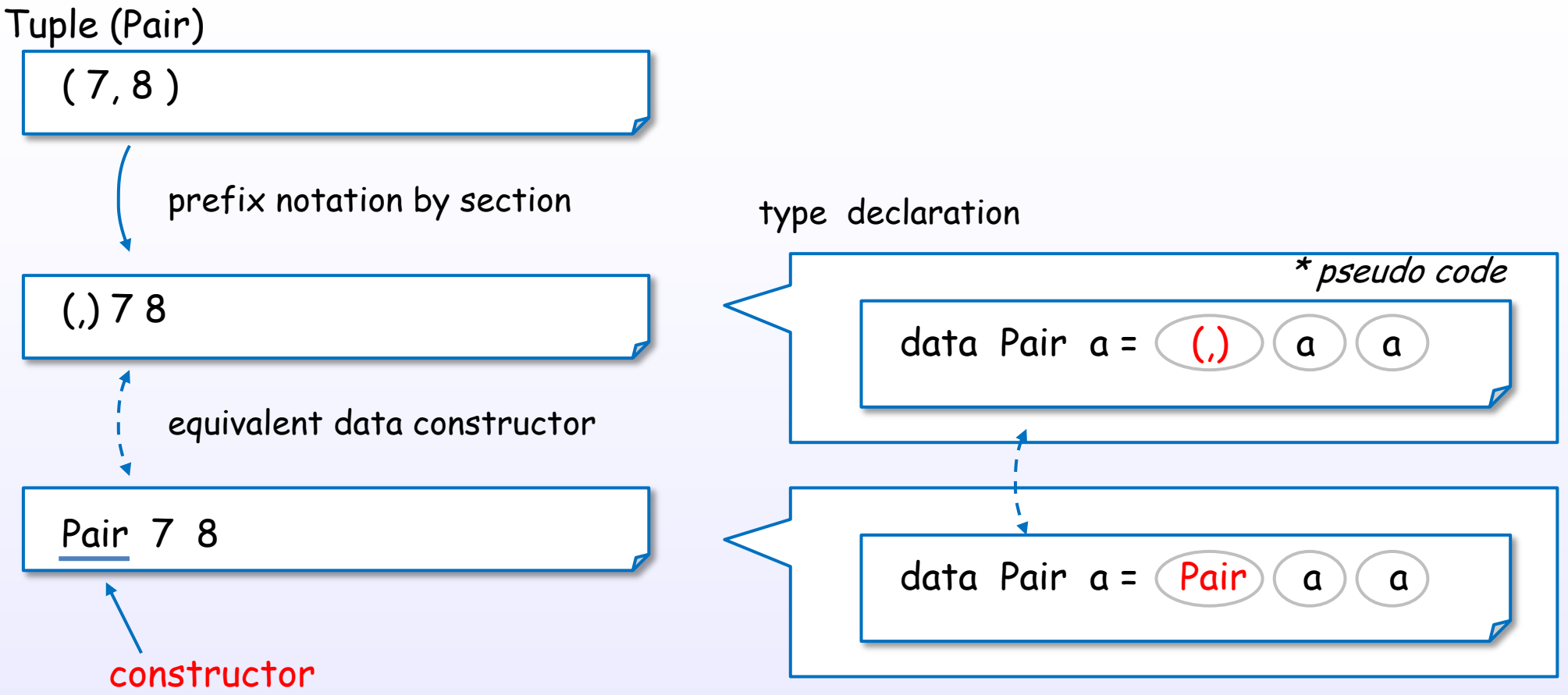
```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

GHC's internal representation



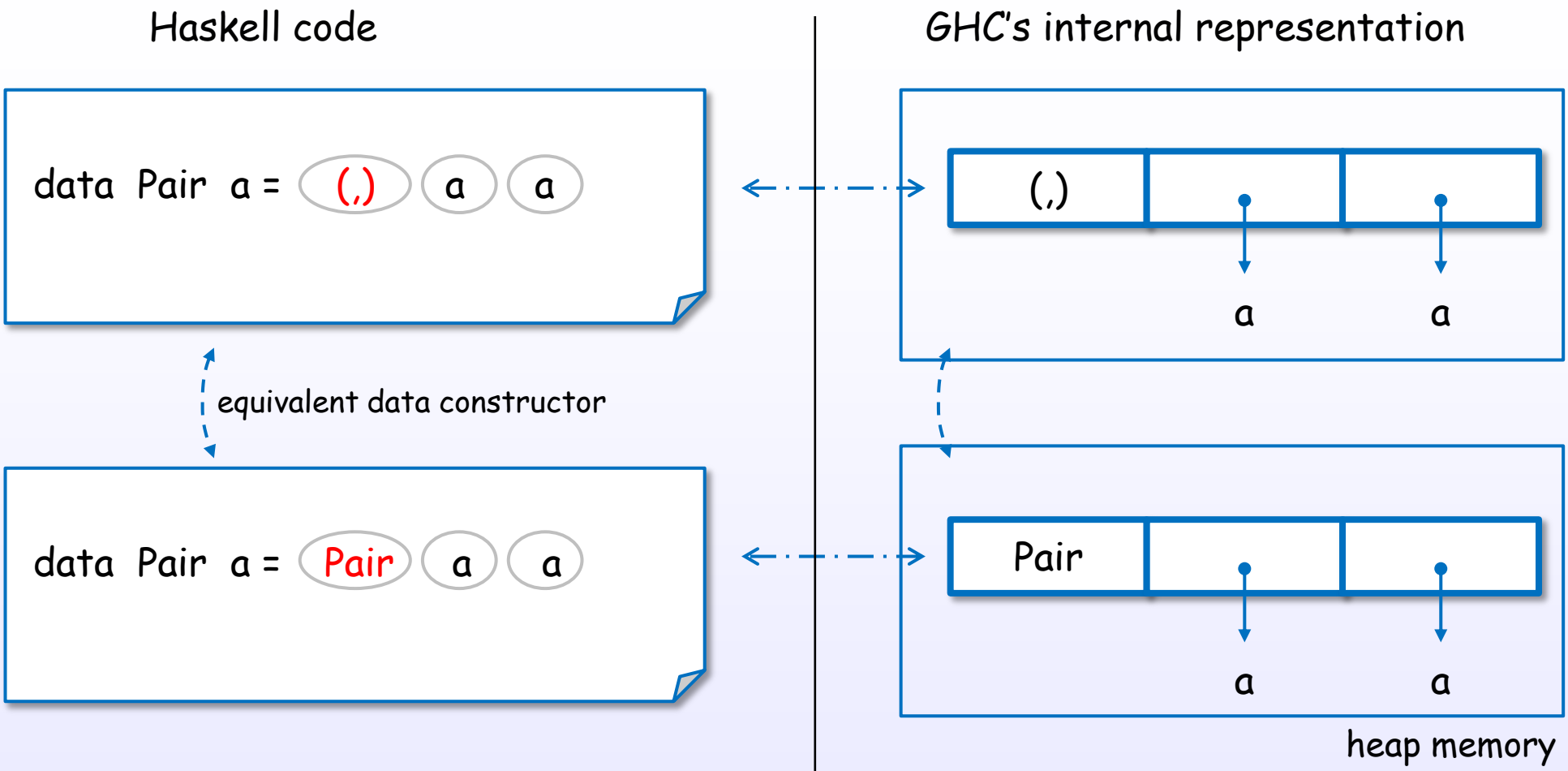
References : [H11], [H10], [H5], [H6], [H7]

Tuple is also represented with constructor



References : [H11], [H10], [H5], [H6], [H7]

Tuple is also represented with constructor



References : [H11], [H10], [H5], [H6], [H7]

Tuple is also represented with constructor

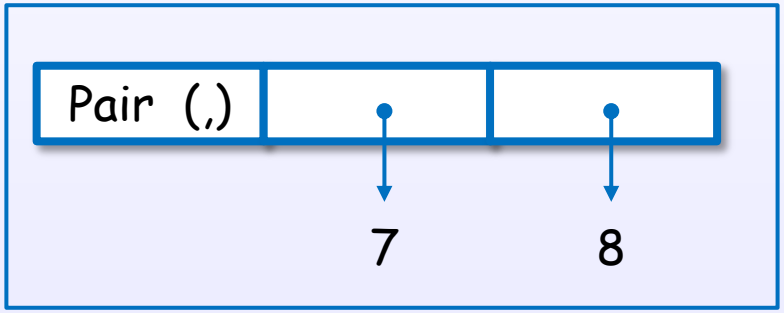
Haskell code

(7, 8)

(,) 7 8

Pair 7 8

GHC's internal representation

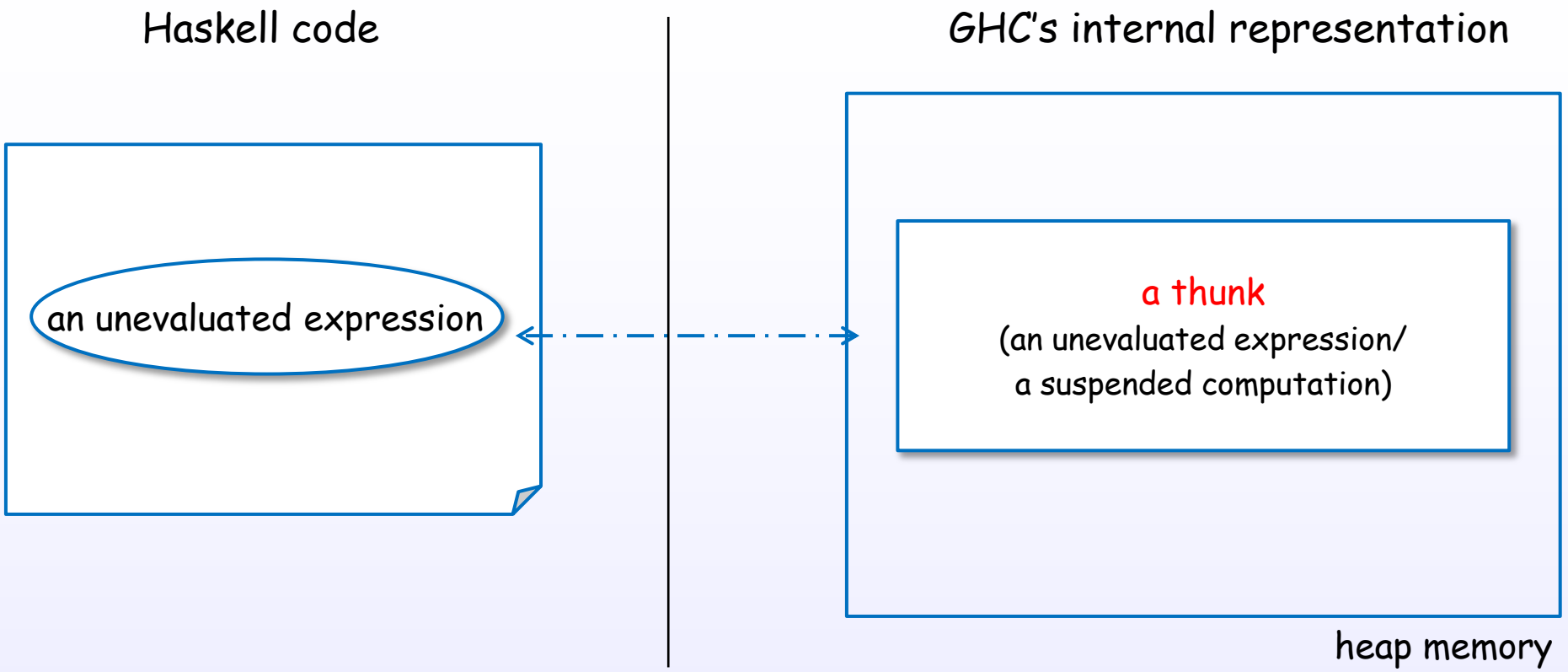


References : [H11], [H10], [H5], [H6], [H7]

3. Internal representation of expressions

Thunk

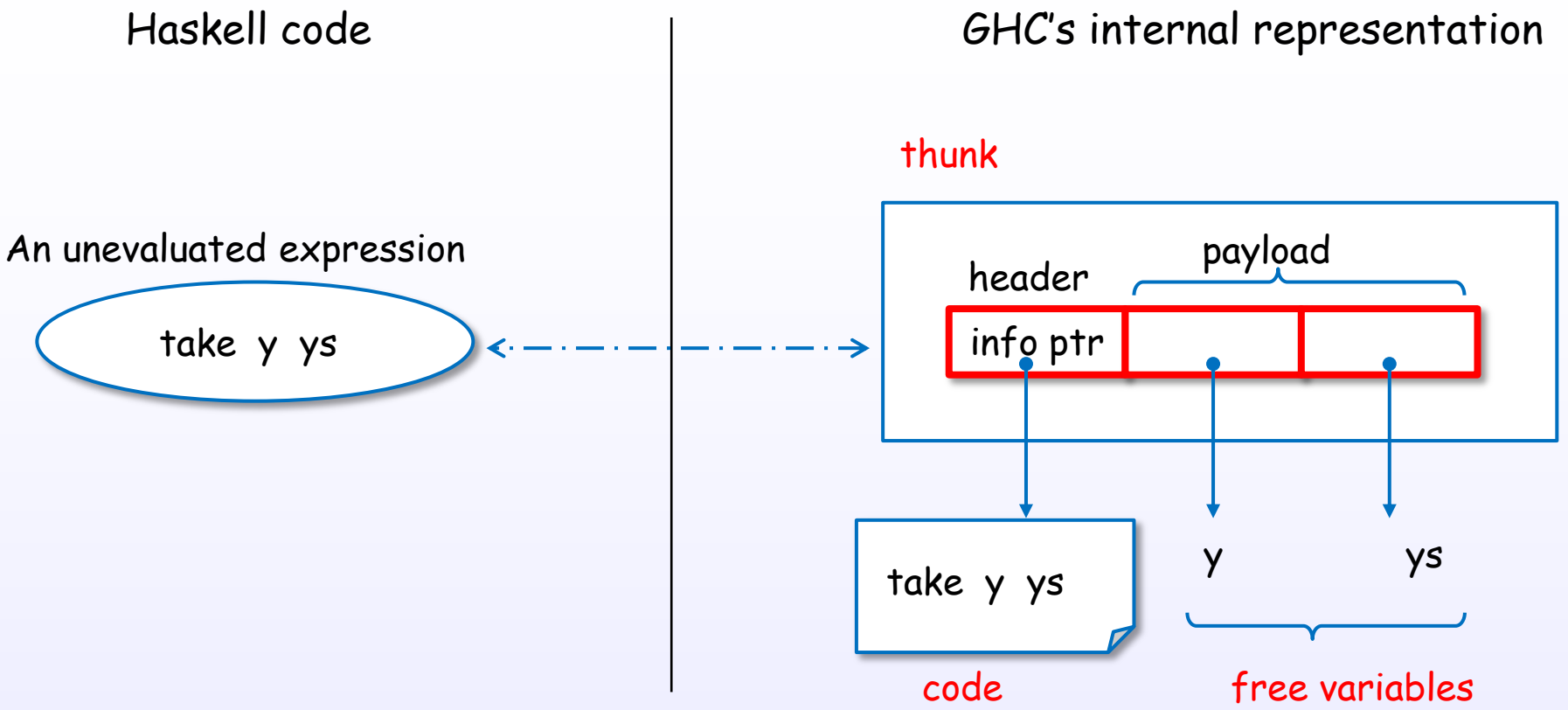
Thunk



A thunk is an unevaluated expression in heap memory.
A thunk is built to postpone the evaluation.

References : [B5] Ch.2, [D5], [W1], [H10], [H5], [D7]

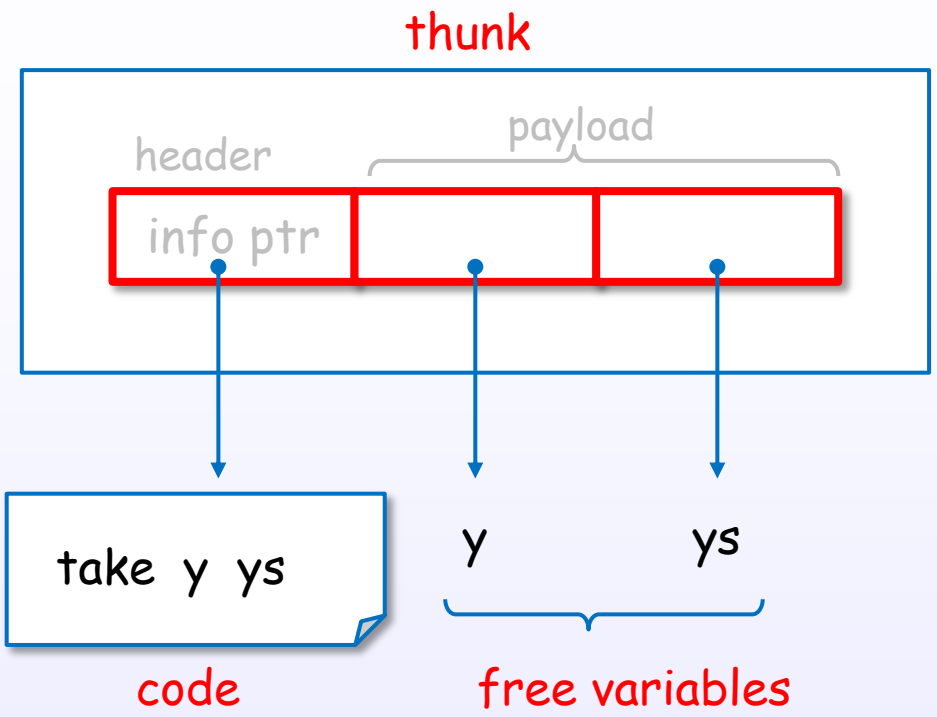
Internal representation of thunk



A thunk is represented with header(code) + payload(free variables).

References : [H11], [H10], [D2], [H5], [H6], [H7], [B5] Ch.2, [D5], [W1]

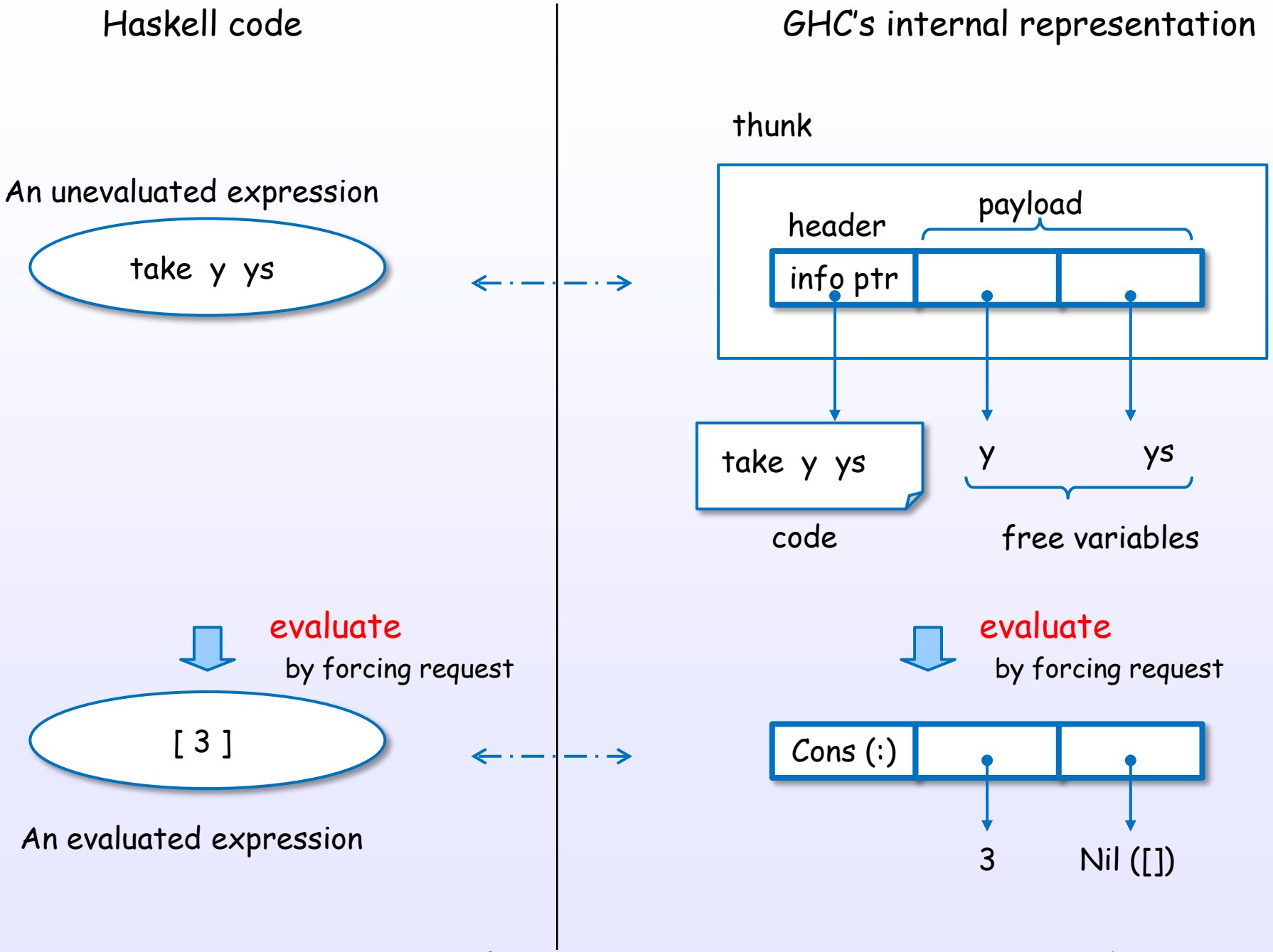
A thunk is a package



A thunk is a package of code + free variables.

References : [D2], [H11], [H10], [H5], [H6], [H7], [B5] Ch.2, [D5], [W1]

A thunk is evaluated by forcing request

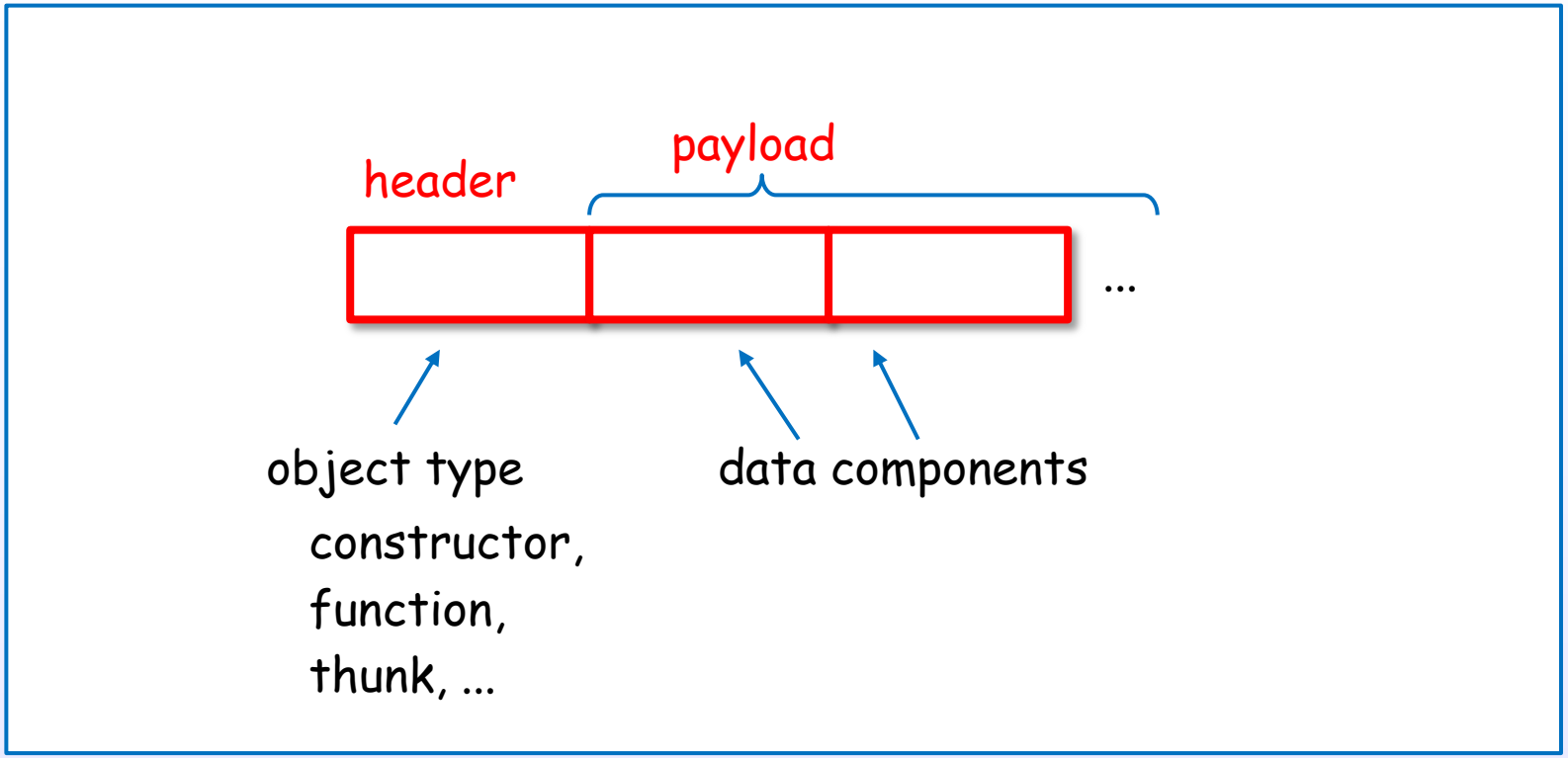


References : [D7], [D2], [H11], [H10], [H5], [H6], [H7], [B5] Ch.2, [D5], [W1], [D15]

3. Internal representation of expressions

Uniform representation

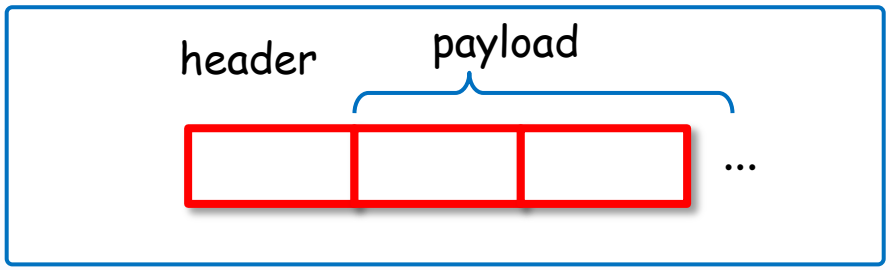
Every object is uniformly represented in memory



in heap memory, stack or static memory

References : [H11], [H10], [H5], [H6], [H7], [D15]

Every object is uniformly represented in memory



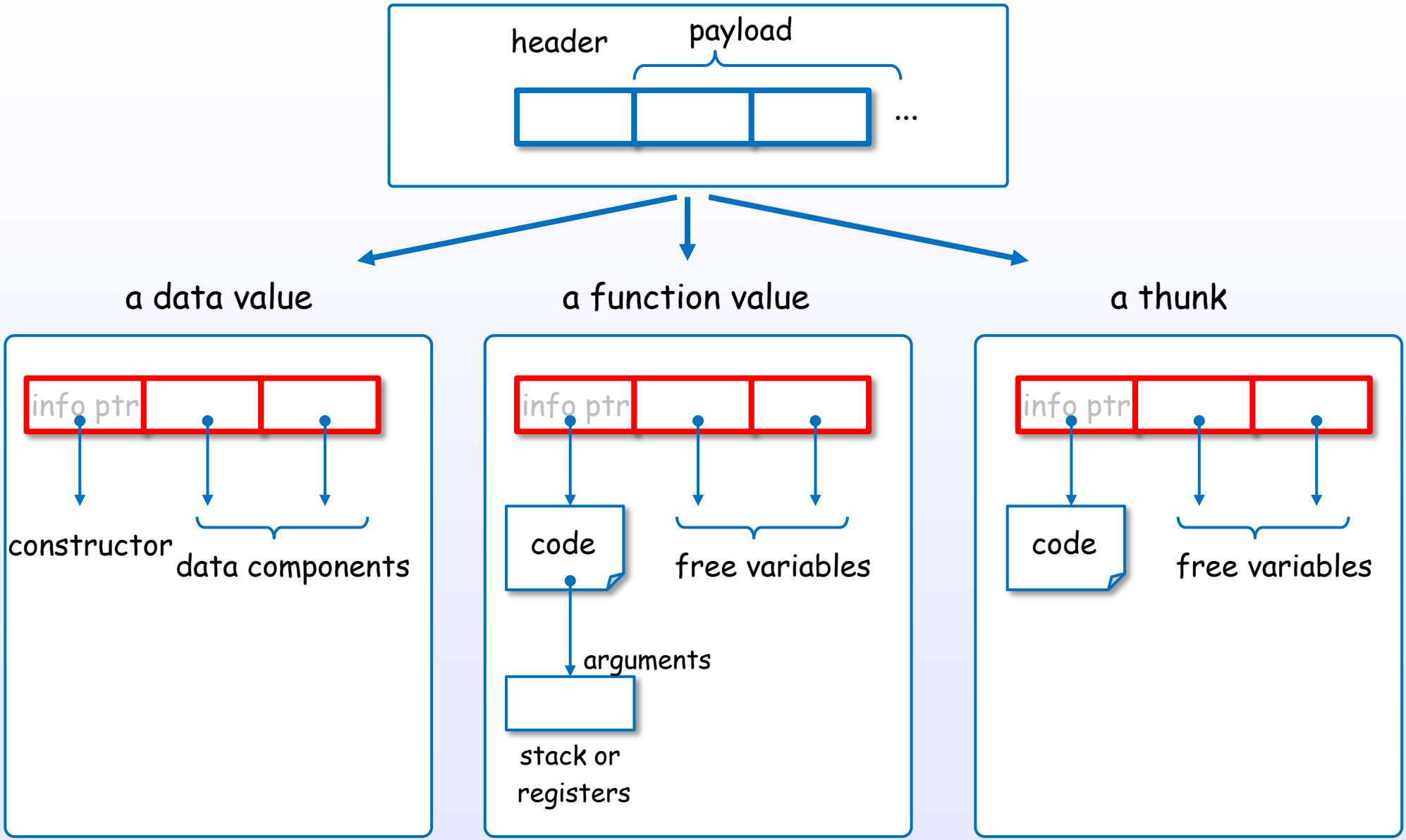
a data value

a function value

a thunk

References : [H11], [H10], [H5], [H6], [H7], [D15]

Every object is uniformly represented



References : [H11], [H10], [H5], [H6], [H7], [D15]

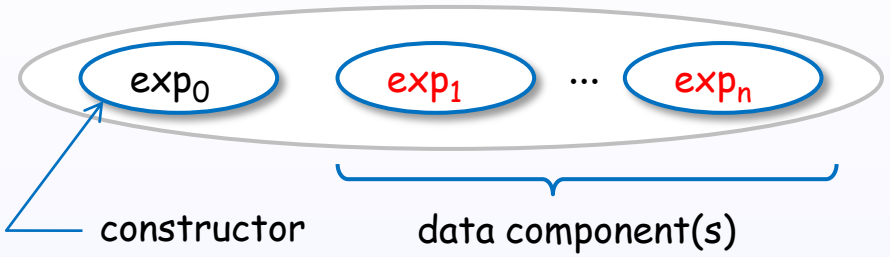
3. Internal representation of expressions

WHNF

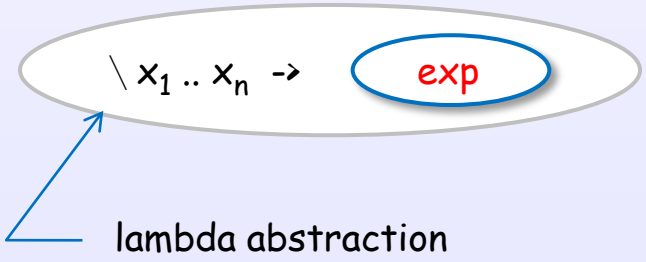
Internal representation of WHNF

Haskell code

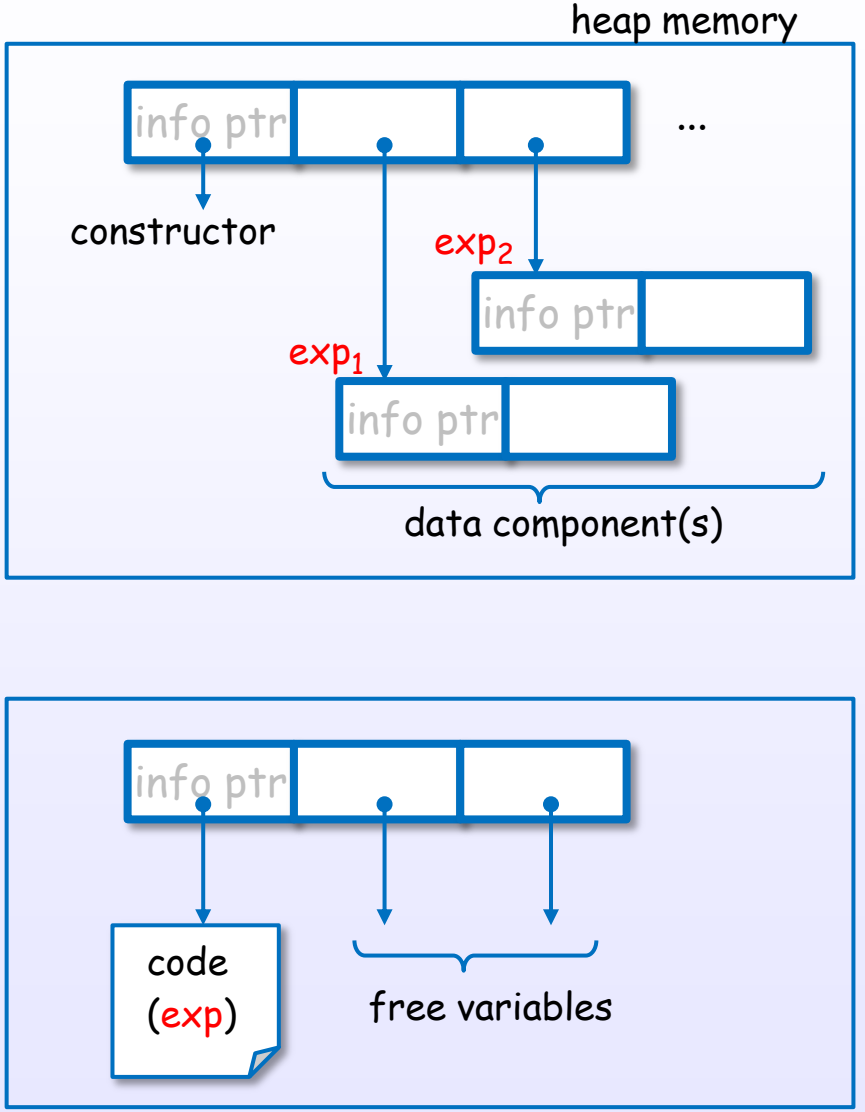
a data value in WHNF



a function value in WHNF

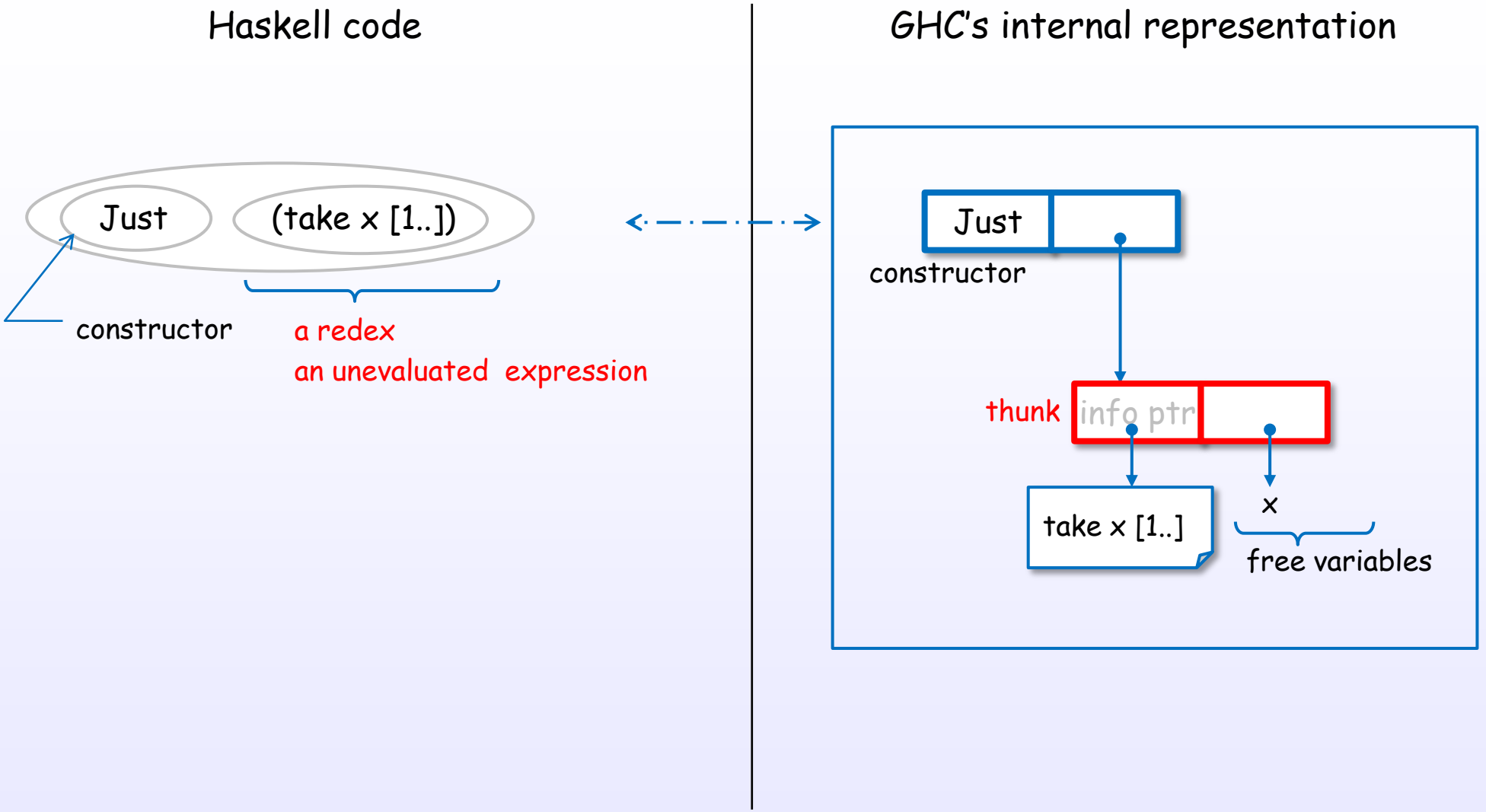


GHC's internal representation



References : [H11], [H5], [H6], [H7], [H10]

Example of WHNF for a data value

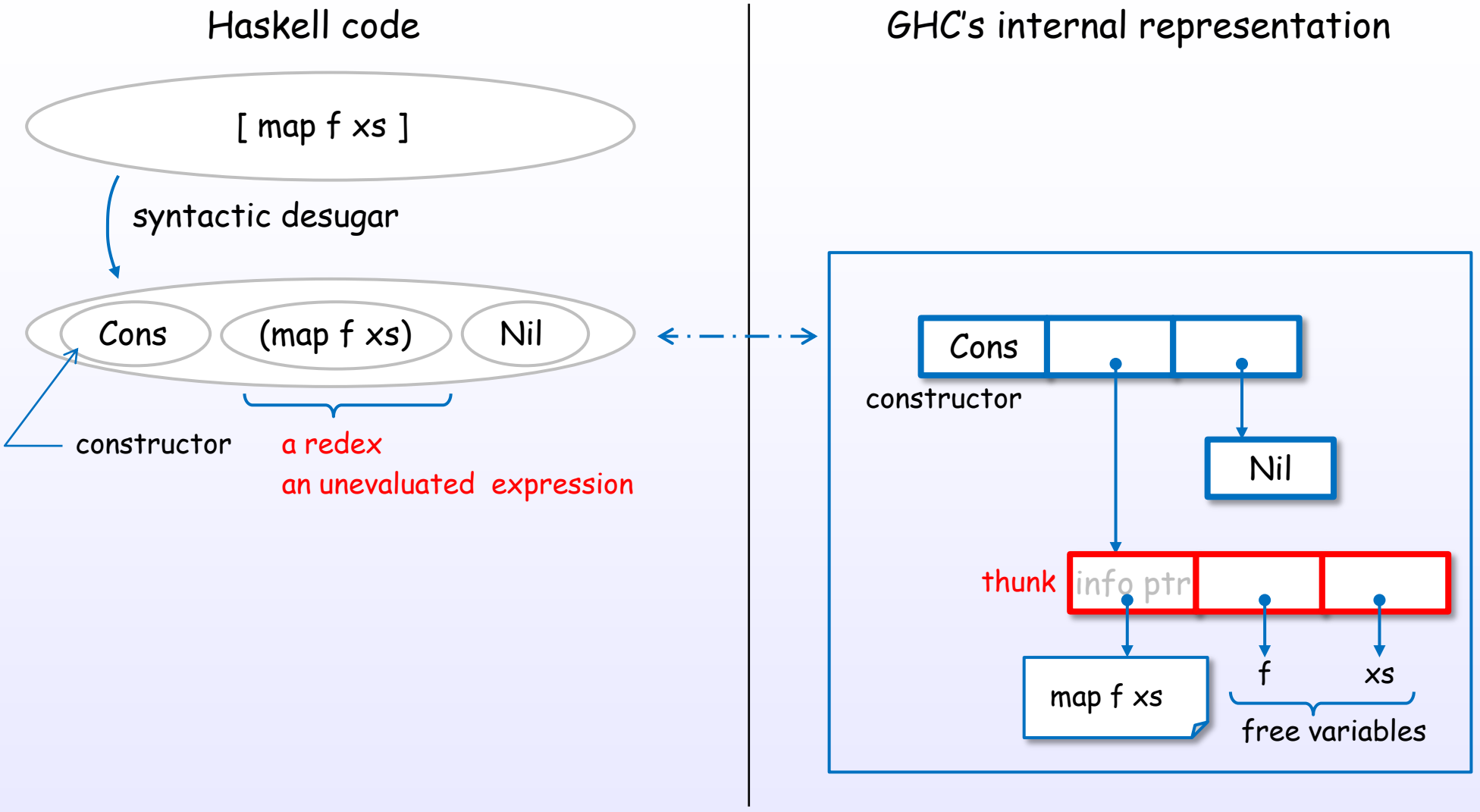


Constructors can contain unevaluated expressions by thunks.

Haskell's constructors are lazy constructors.

References : [H11], [H5], [H6], [H7], [H10]

Example of WHNF for a data value



References : [H11], [H5], [H6], [H7], [H10]

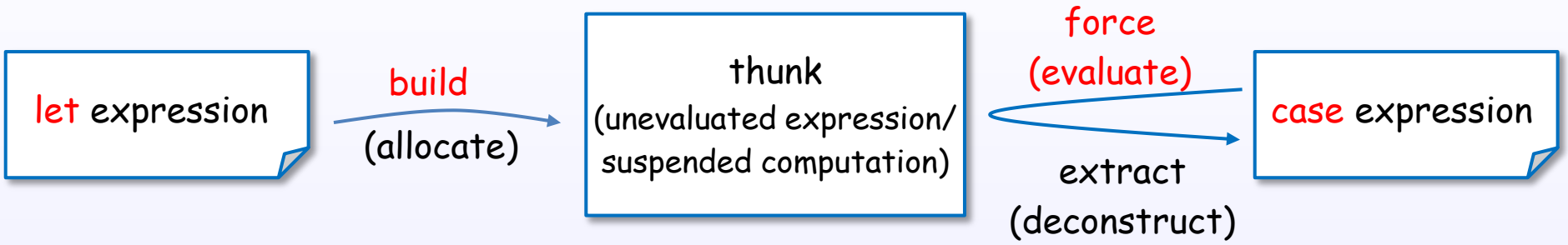
3. Internal representation of expressions

let, case expression

let, case expression

let and case expressions are special role in the evaluation

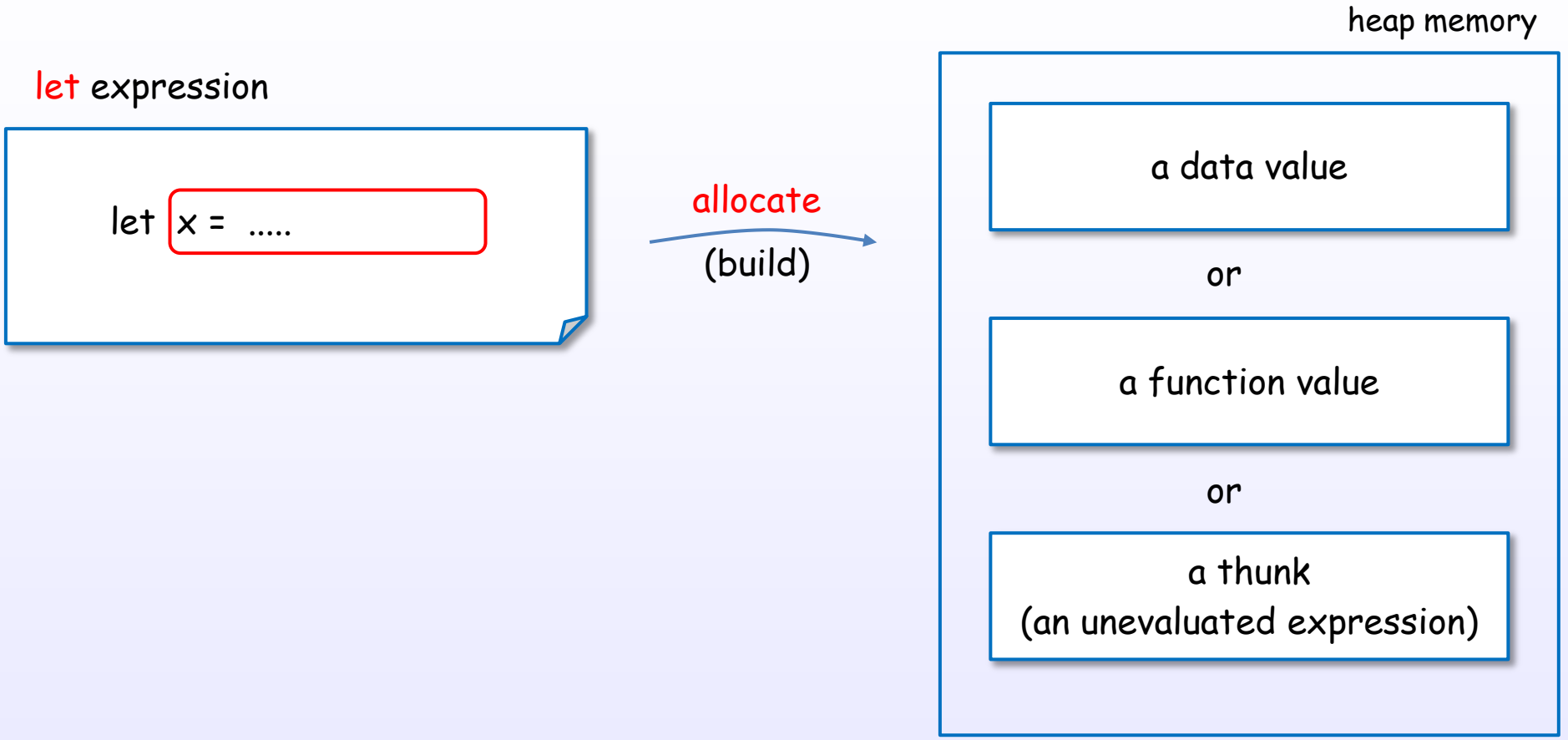
let/case expressions and thunk



A let expression may build a thunk.
A case expression evaluates (forces) and deconstructs the thunk.

References : [H5], [H6], [H7], [H10]

A let expression may allocates a heap object



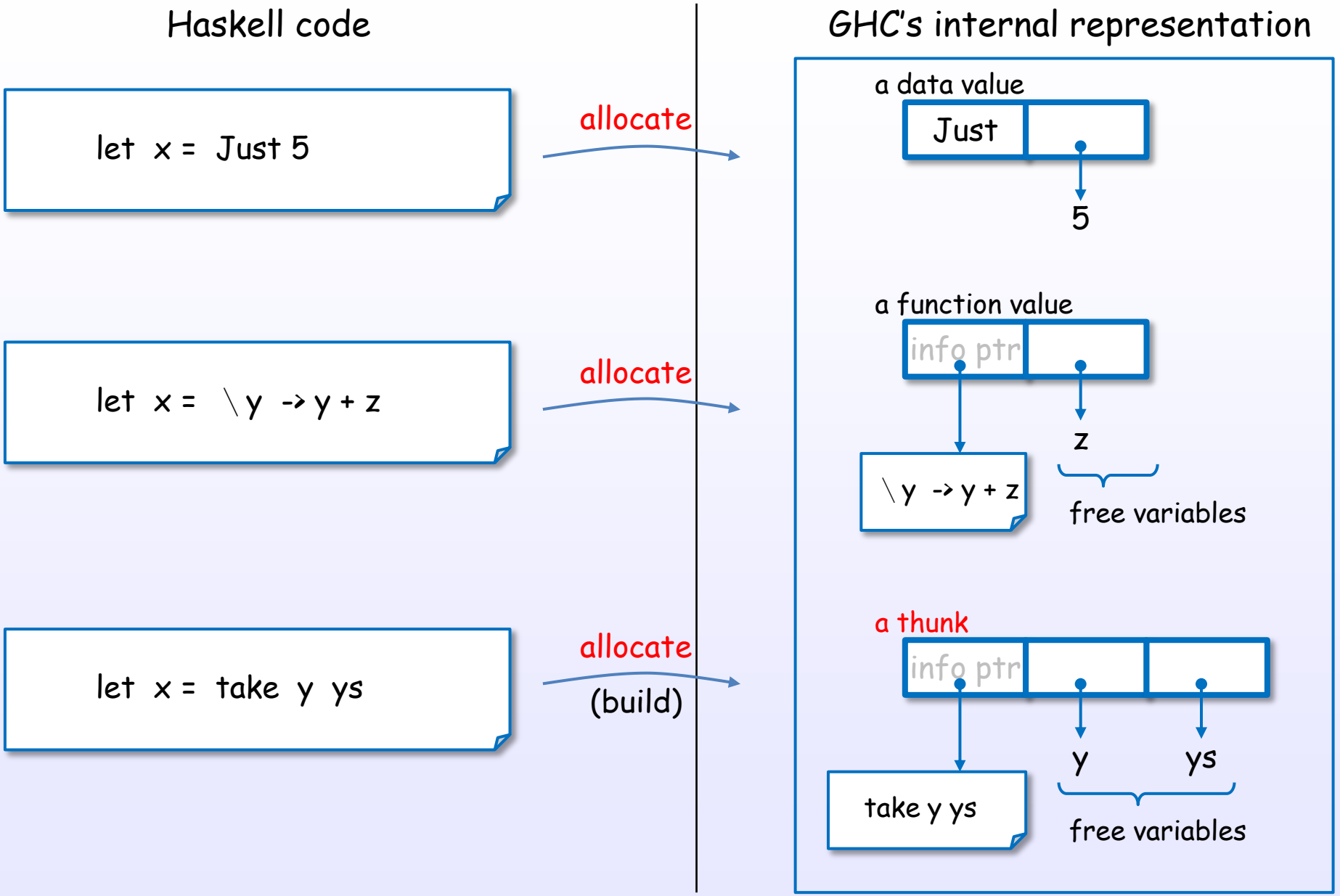
A let expression may allocates an object in the heap.

(If GHC can optimize it, the let expression may not allocate.)

* At exactly, STG language's let expression rather than Haskell's let expression

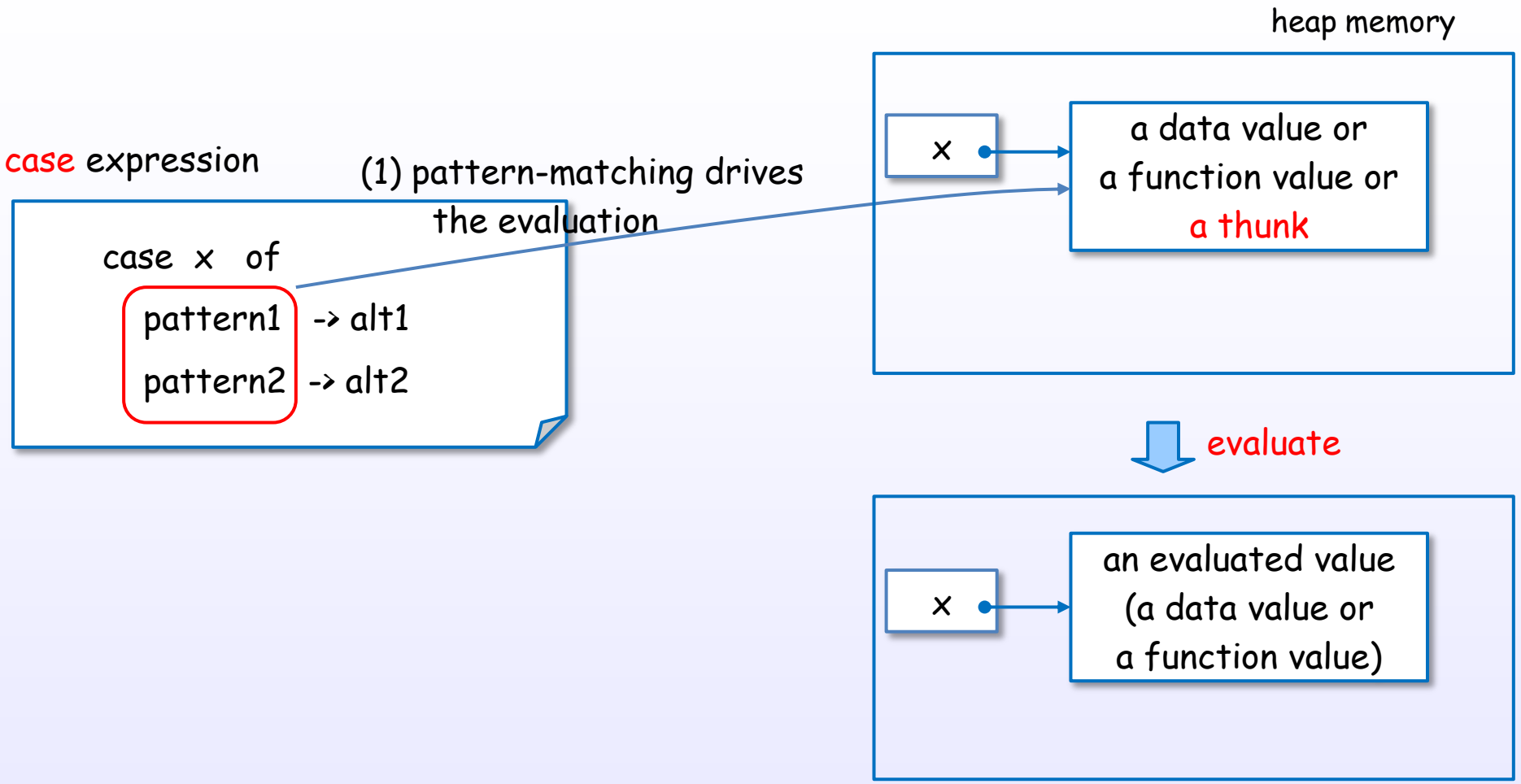
References : [H5], [H6], [H7], [H10]

Example of let expressions



References : [H5], [H6], [H7], [H10]

A case expression evaluates a subexpression

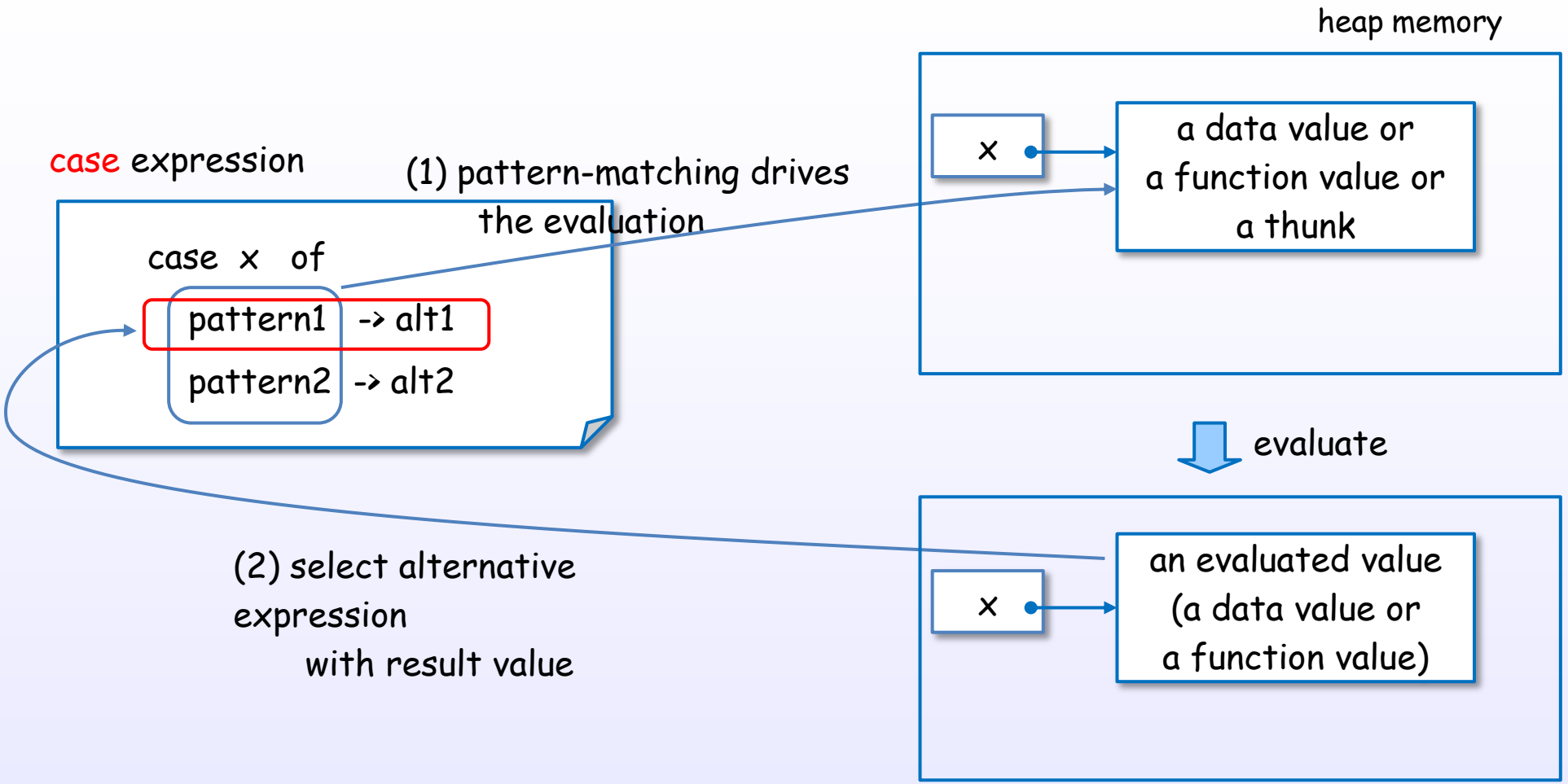


Pattern-matching drives the evaluation.

* At exactly, STG language's case expression rather than Haskell's case expression

References : [H5], [H6], [H7], [H10], [D2]

A case expression also perform case analysis

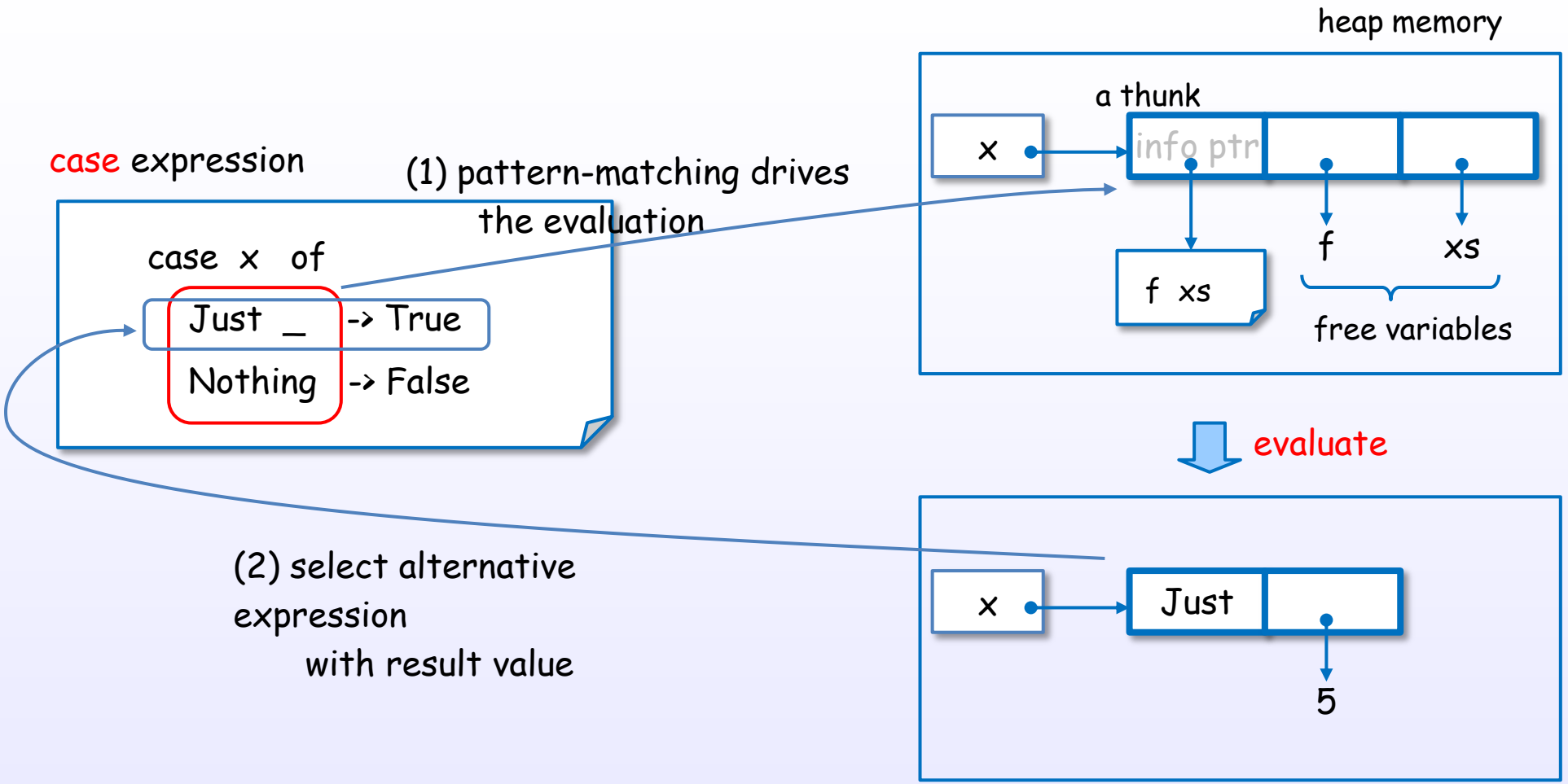


A case expression evaluates a subexpression
and optionally performs case analysis on its value.

* At exactly, STG language's case expression rather than Haskell's case expression

References : [H5], [H6], [H7], [H10], [D2]

Example of a case expression



A case expression's pattern-matching says "I need the value".

References : [H5], [H6], [H7], [H10], [D2]

Pattern-matching in function definition

pattern-matching in function definition

```
f Just _   = True
f Nothing  = False
```

syntactic desugar

pattern-matching in case expression

```
f x = case x of
        Just _   -> True
        Nothing  -> False
```

A function's pattern-matching is syntactic sugar of case expression.

A function's pattern-matching also drives the evaluation.

References : [H1]

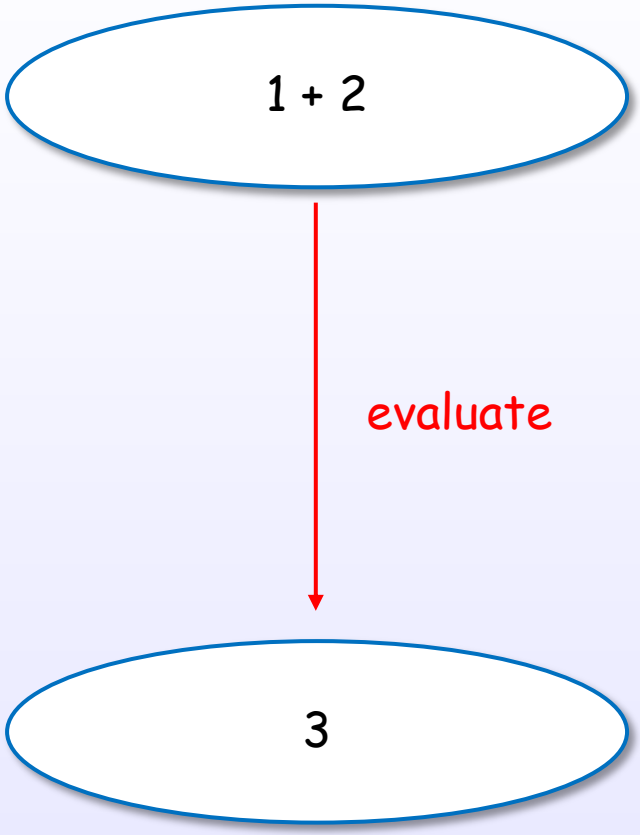
4. Evaluation

4. Evaluation

Evaluation strategies

Evaluation

An expression

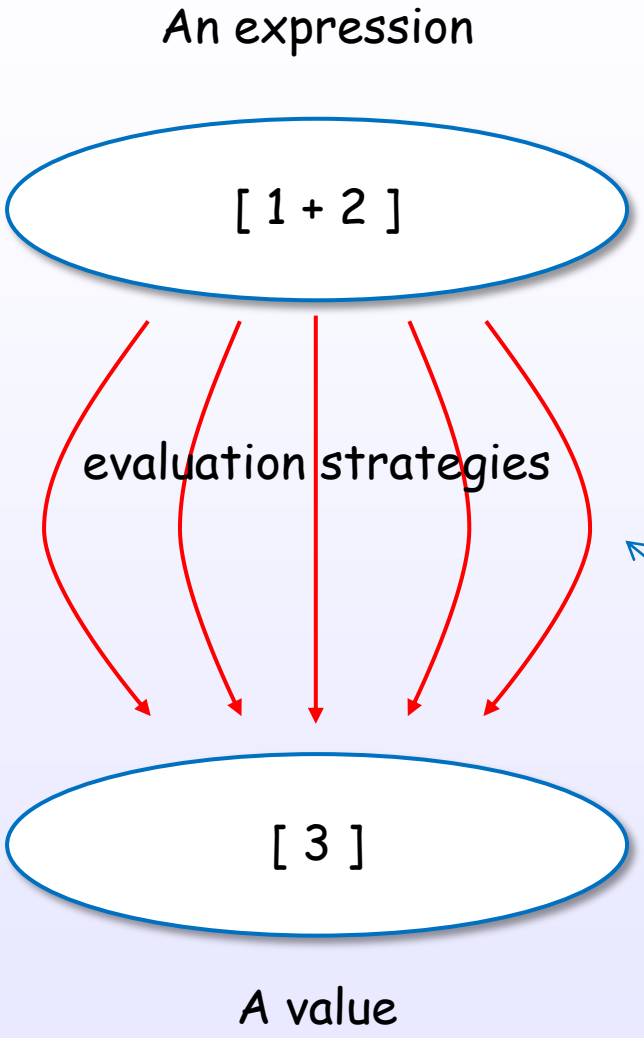


A value

The evaluation produces a value from an expression.

References : [B1] Ch.1, [B2] Ch.2, [H1] Ch.1, [B6] Ch.3

There are many evaluation approaches



- Strict, Non-strict evaluation
- Eager, Lazy evaluation
- Call-by-value, Call-by-name, Call-by-need, …
- Innermost, Outermost
- Normal order, Application order
- …

References : [B3] Ch.2, 7, [B6] Ch.3, [D1]

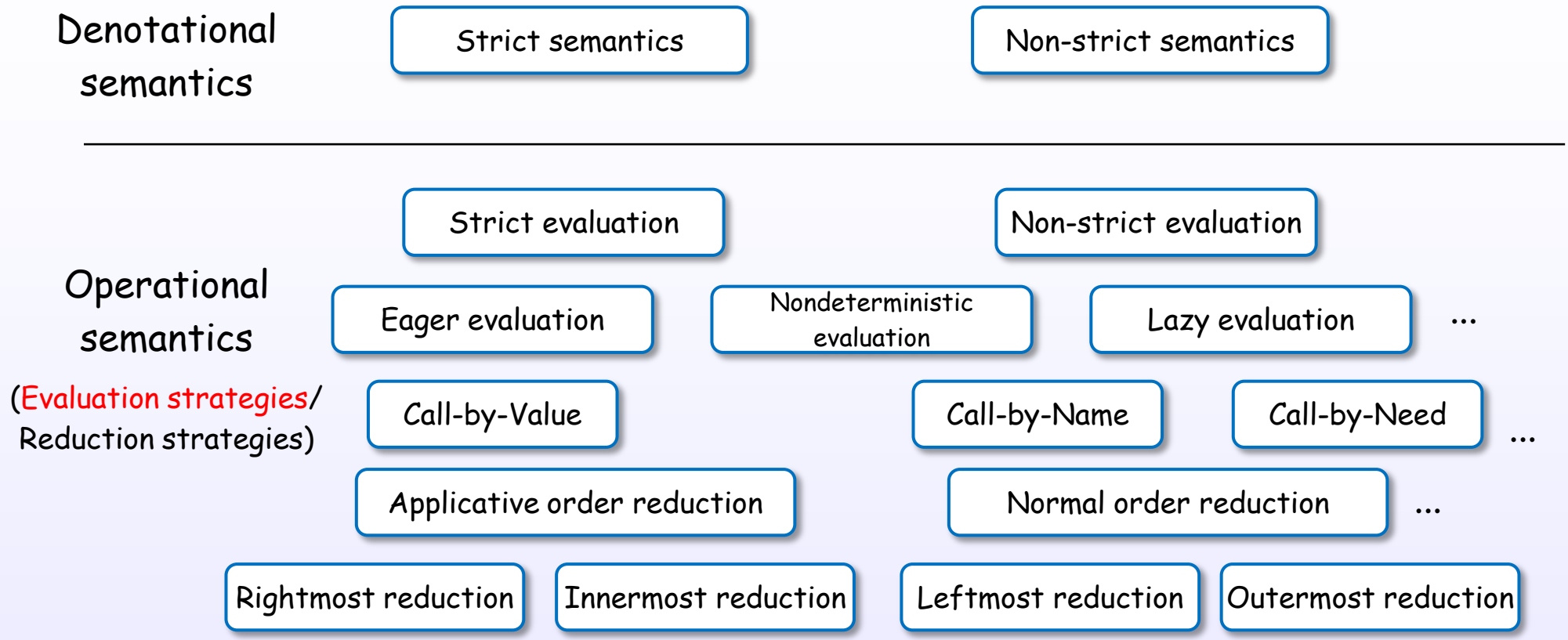
Evaluation concept layer

Denotational semantics

Operational semantics
(Evaluation strategies / Reduction strategies)

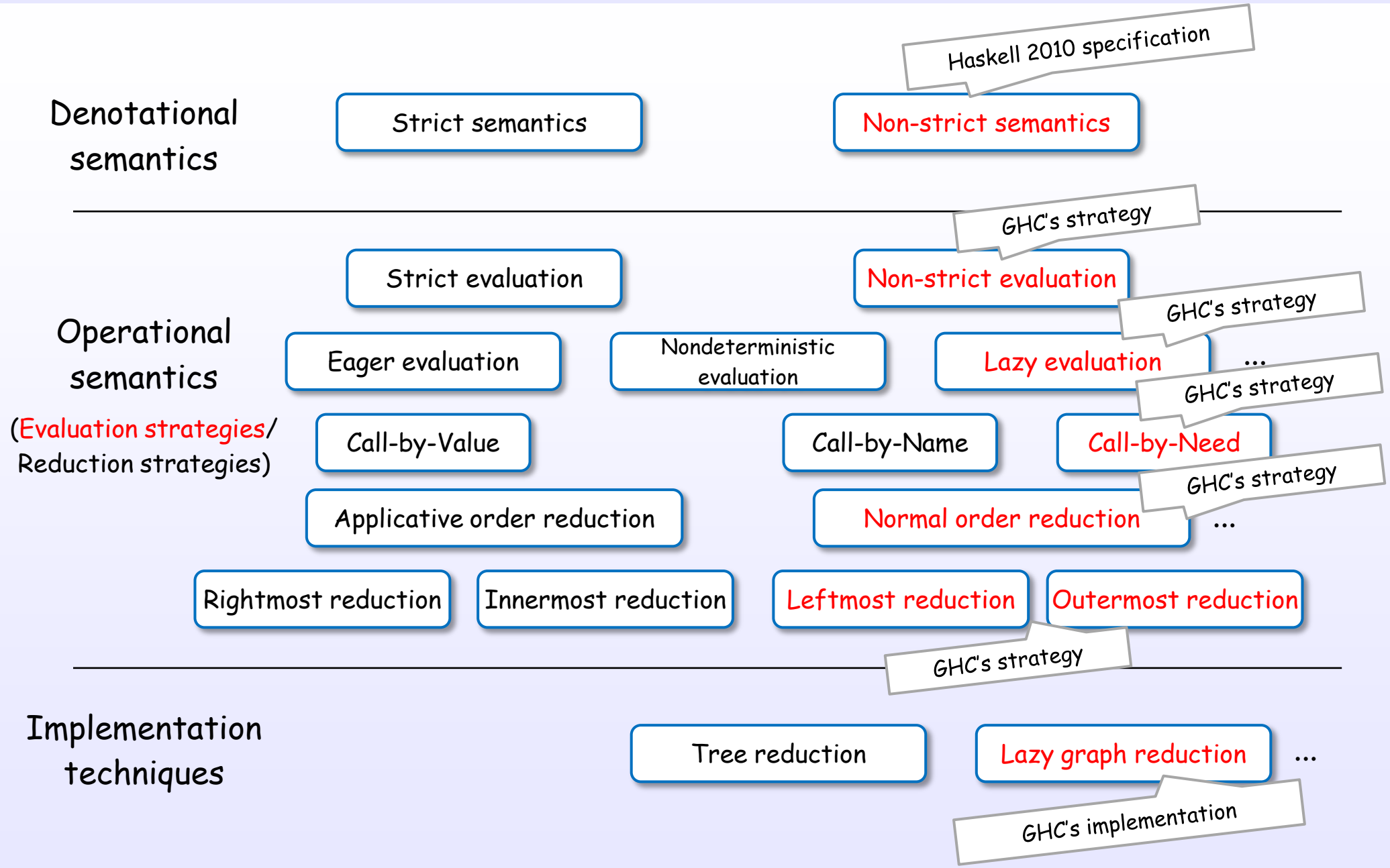
Implementation techniques

Evaluation layer for GHC's Haskell



References : [D3], [D1], [D2], [D5], [D4], [B2] Ch.7, [B3] Ch.8, [B6] Ch.5, [W1], [W2], [W3], [B7], [B8]

Evaluation layer for GHC's Haskell



References : [D3], [D1], [D2], [D5], [D4], [B2] Ch.7, [B3] Ch.8, [B6] Ch.5, [W1], [W2], [W3], [B7], [B8]

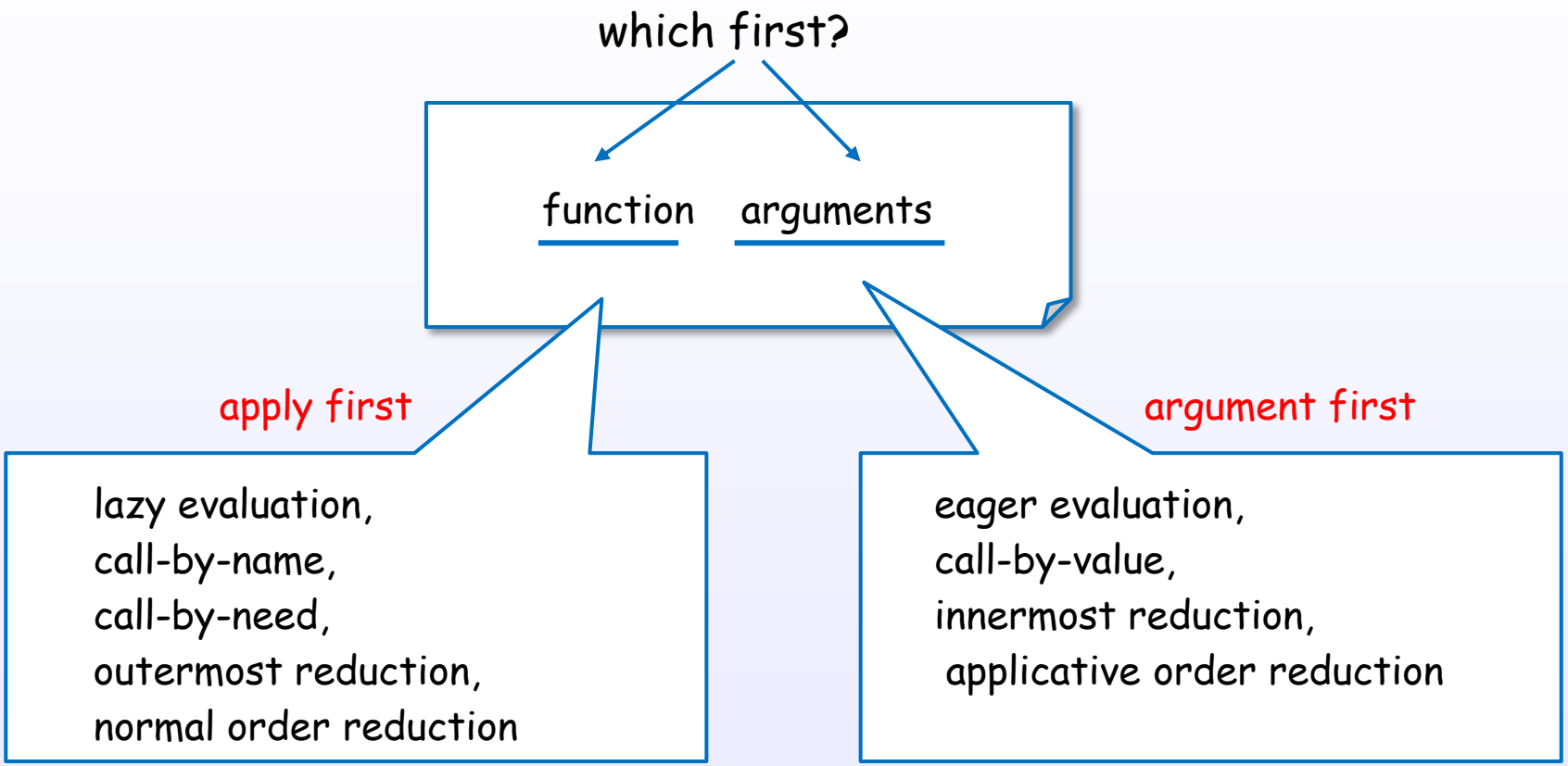
Evaluation strategies

Each evaluation strategy decides how to operate the evaluation about

ordering,
region,
trigger condition,
termination condition,
re-evaluation, ...

References : [D3], [D1], [D2], [D5], [D4], [B2] Ch.7, [B3] Ch.8, [B6] Ch.5, [W1], [W2], [W3], [B7], [B8]

One of the important points is the order



References : [D3], [D1], [D2], [D5], [D4], [B2] Ch.7, [B3] Ch.8, [B6] Ch.5, [W1], [W2], [W3], [B7], [B8]

Simple example of typical evaluations

call-by-value

default

C, Java, JavaScript, Python, OCaml, Scheme, ...

square (1 + 2)

argument evaluation first

call-by-need

default

Haskell (GHC), ...

square (1 + 2)

apply first

References : [B2] Ch.7, [B3] Ch.8, [D4], [B6] Ch.5

Simple example of typical evaluations

call-by-value

square (1 + 2)

square (3)

evaluation is performed

3 * 3

9

call-by-need

square (1 + 2)

(1 + 2) * (1 + 2)

evaluation is delayed !

(3) * (3)

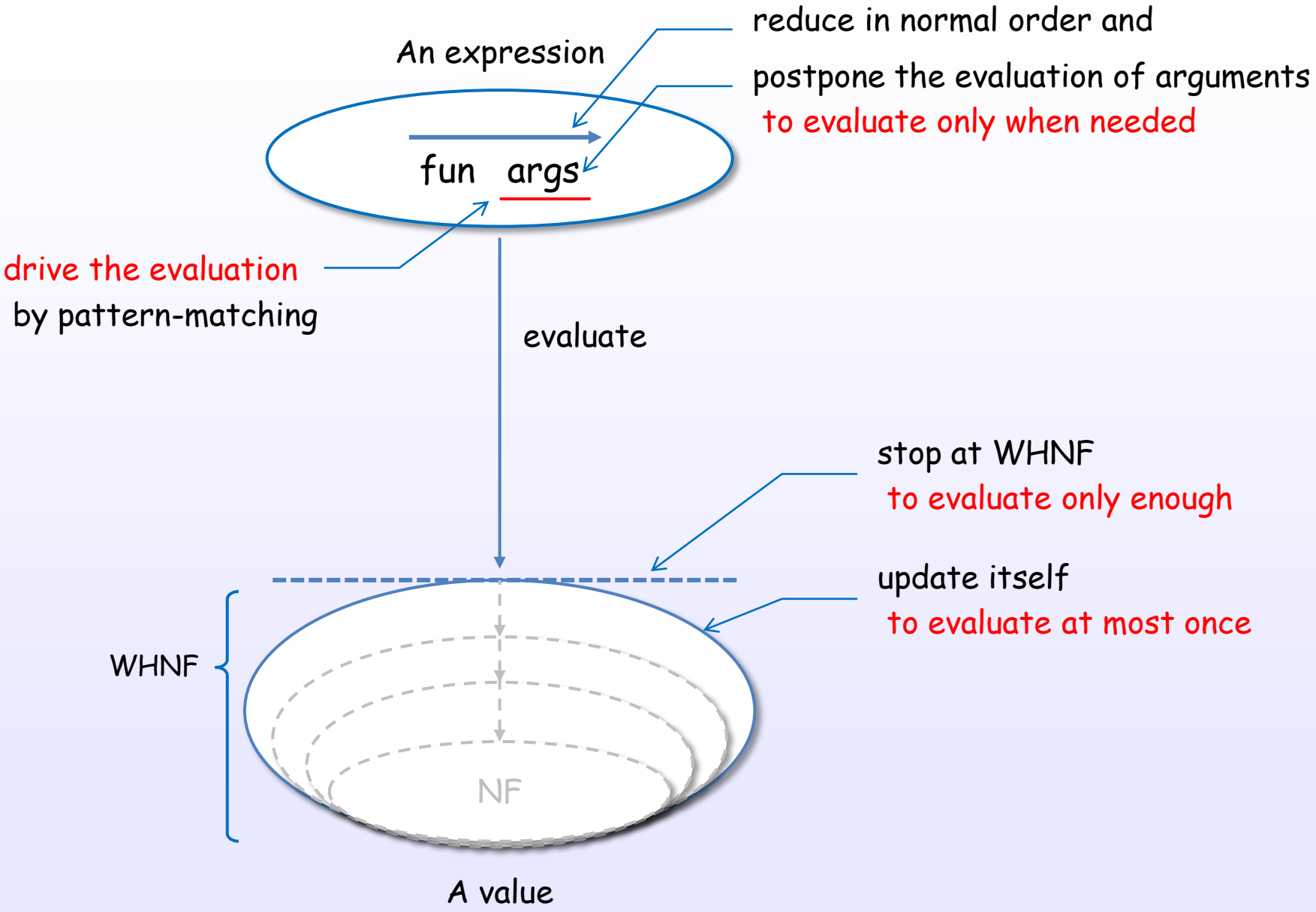
9

References : [B2] Ch.7, [B3] Ch.8, [D4], [B6] Ch.5

4. Evaluation

Evaluation in Haskell (GHC)

Key concepts of Haskell's lazy evaluation



References : [H4] Ch.11, 12, [H5], [H6], [D2]

Postpone the evaluation of arguments

Haskell code

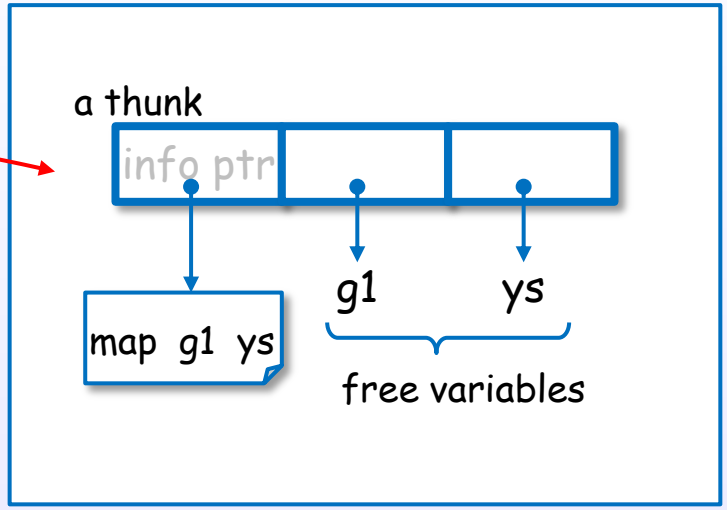
```
fun  (map  g1  ys)
```

internal translation

```
let  thunk0 =  map  g1  ys
in  fun  thunk0
```

postpone

(build)

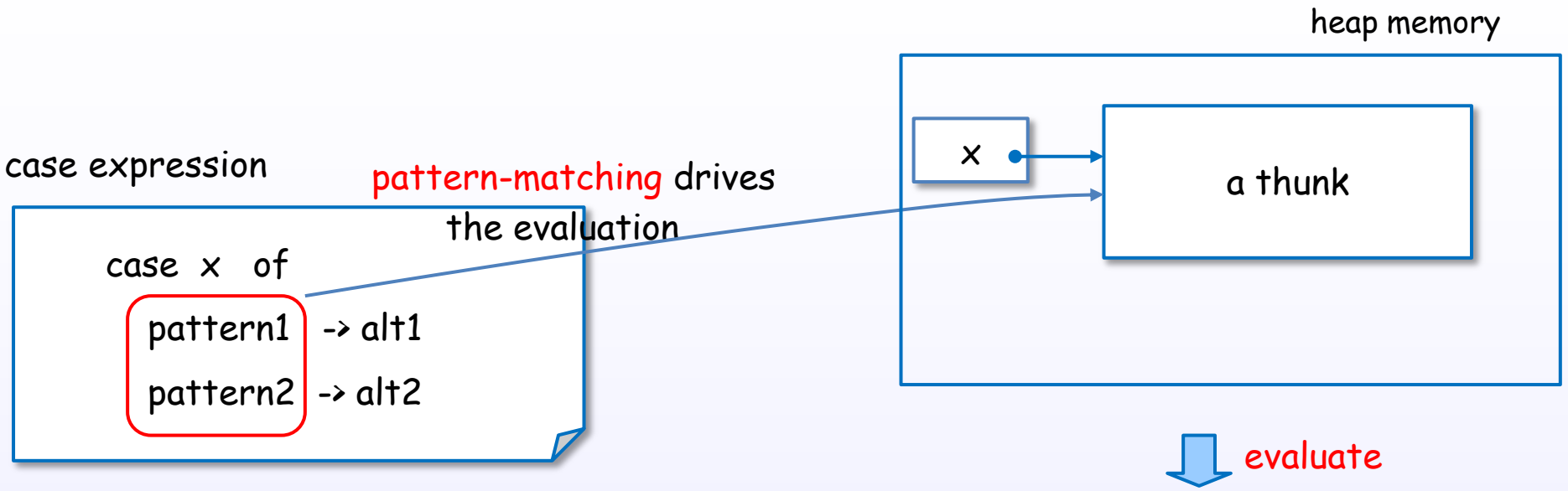


postpone the evaluation by a thunk which build with let expression

(When GHC can optimize it by analysis, the thunk may not be build.)

References : [H5], [H6], [H10]

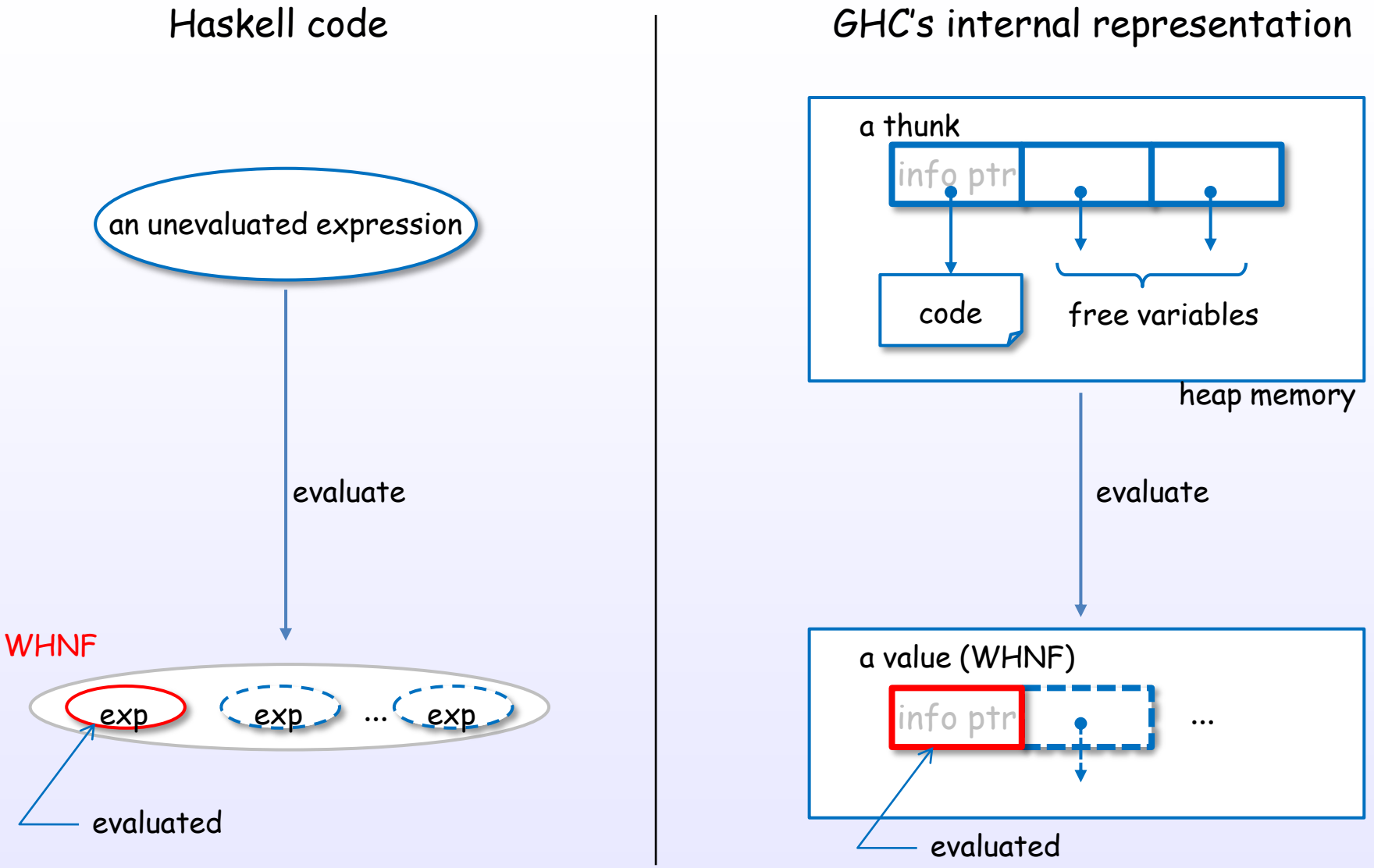
Pattern-matching drives the evaluation



drive the evaluation by pattern-matching

References : [H5], [H6], [H7], [H10], [D2]

Stop at WHNF



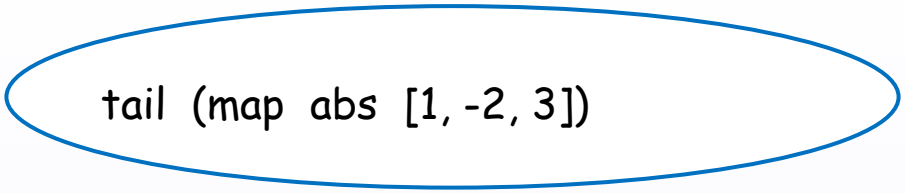
stop the evaluation at WHNF

References : [H5], [H6], [H10]

4. Evaluation

Examples of evaluation steps

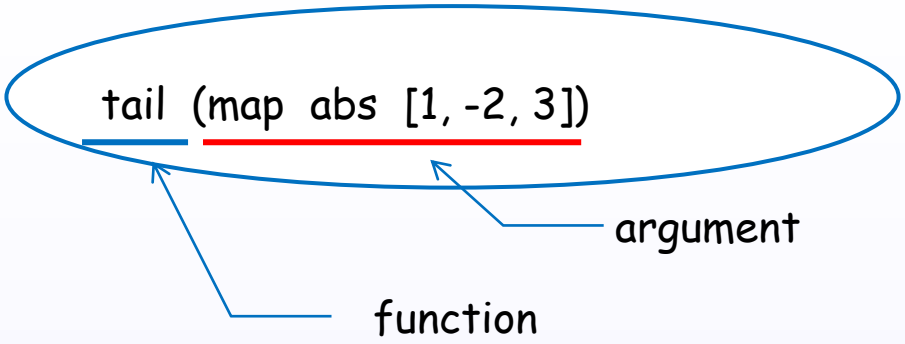
(1) Example of GHC's evaluation



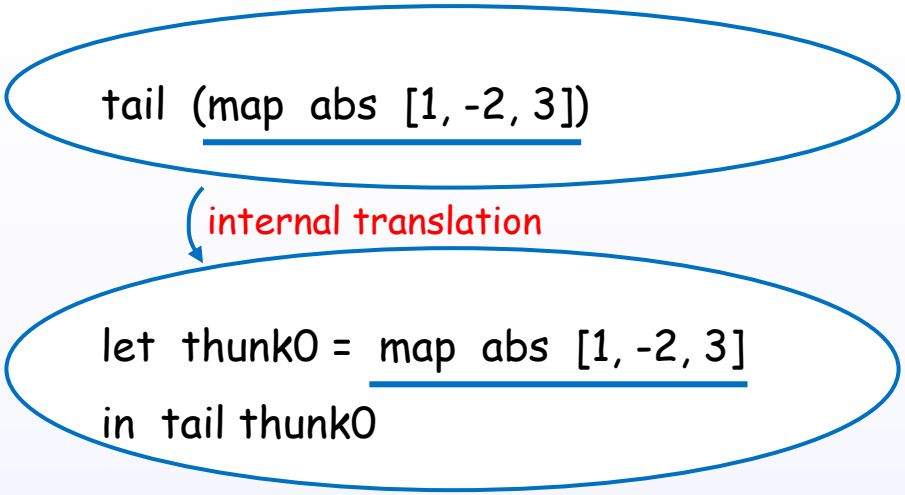
Let's evaluate. It's time to magic!

* no optimizing case (without –O)

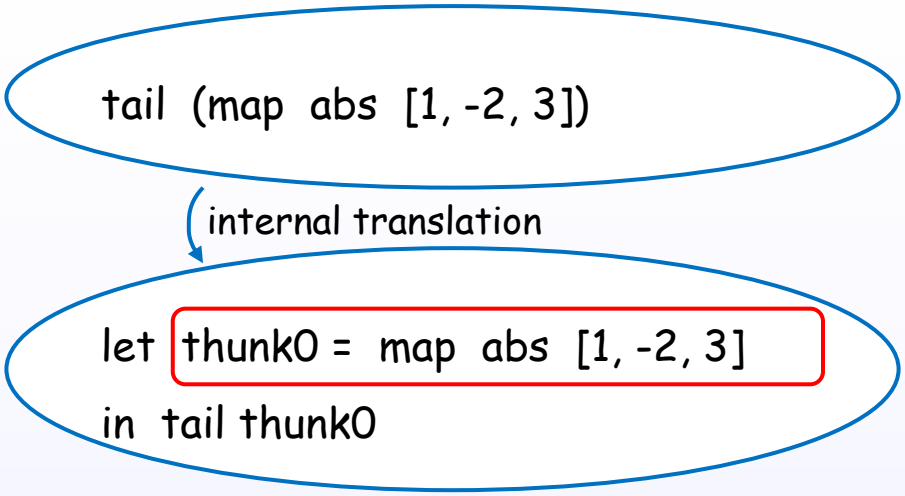
(2) How to postpone the evaluation of arguments?

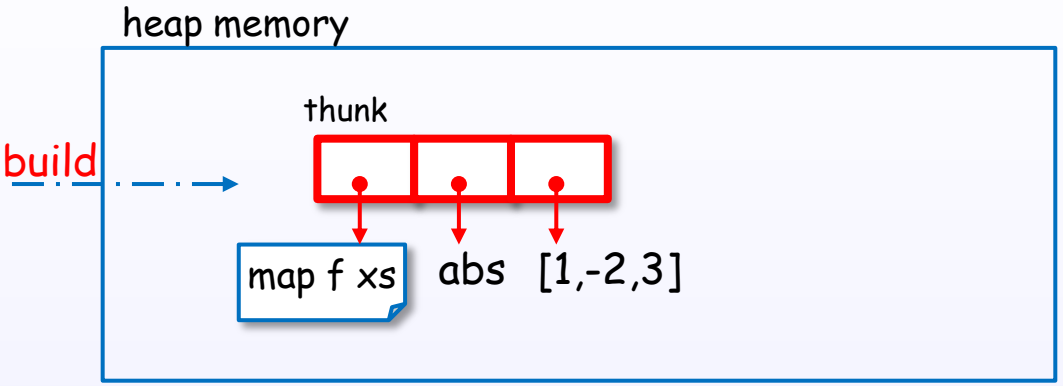


(3) GHC internally translates the expression

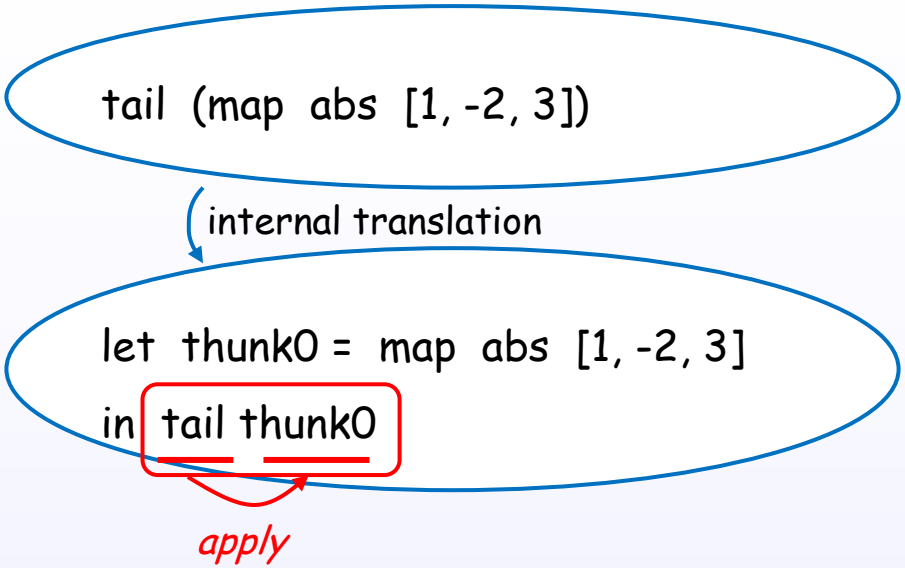


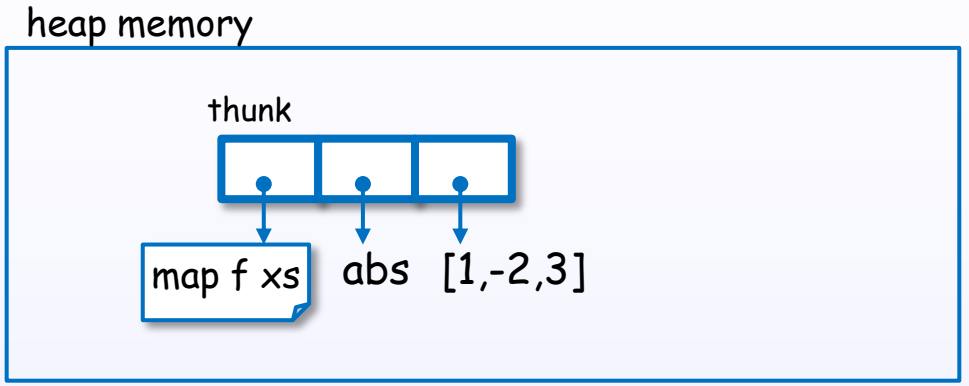
(4) a let expression builds a thunk



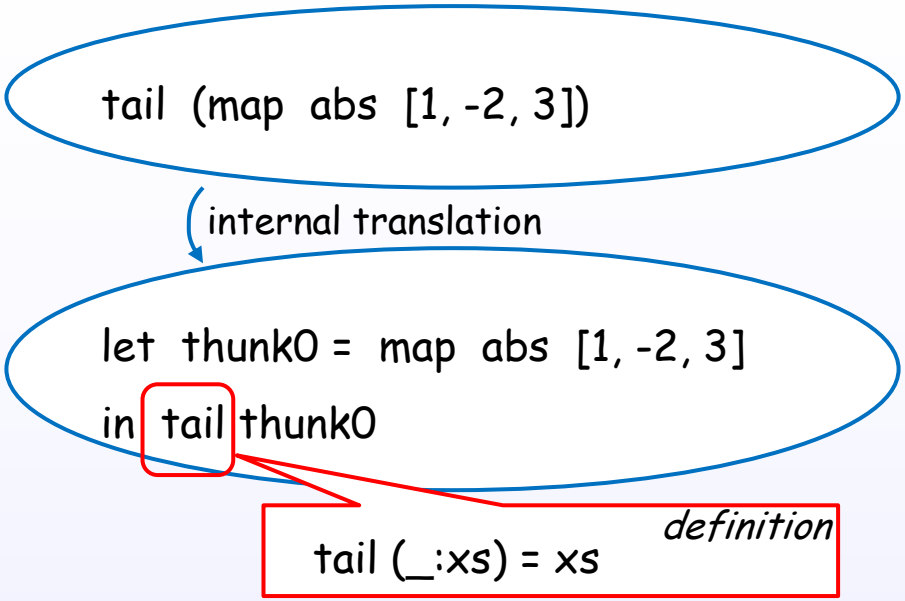


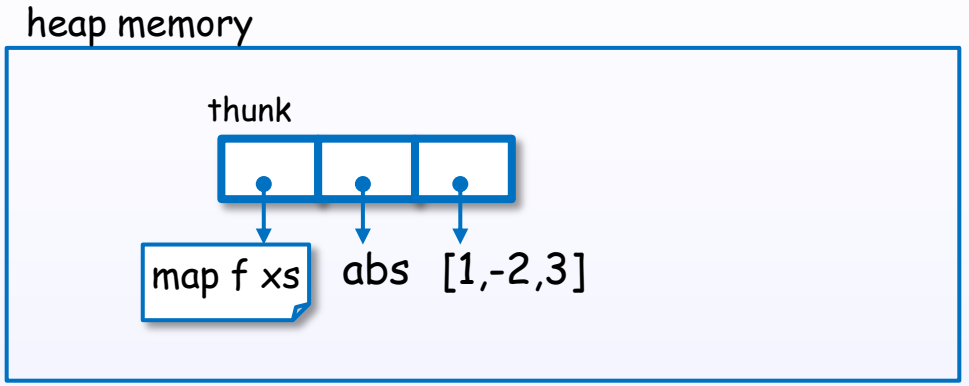
(5) function apply to argument



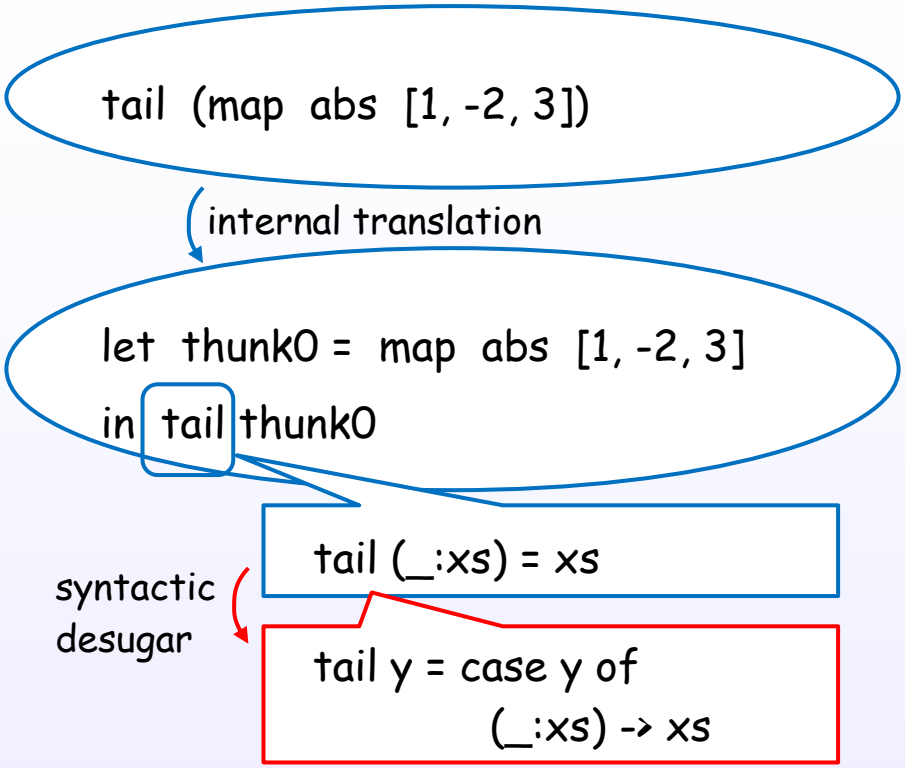


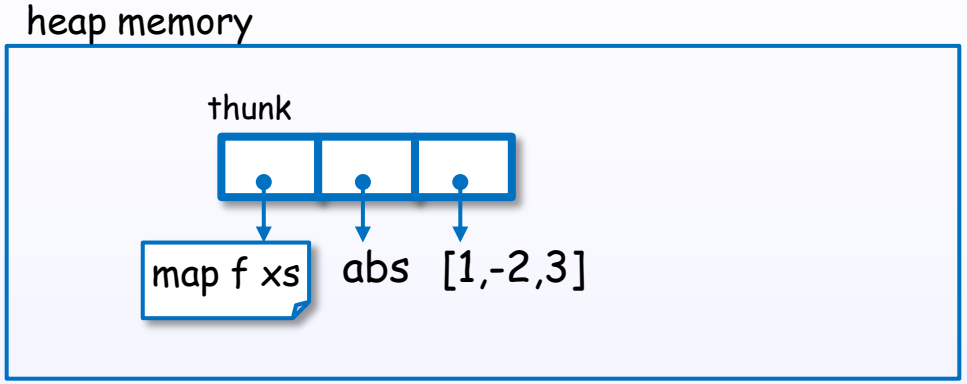
(6) tail is defined here



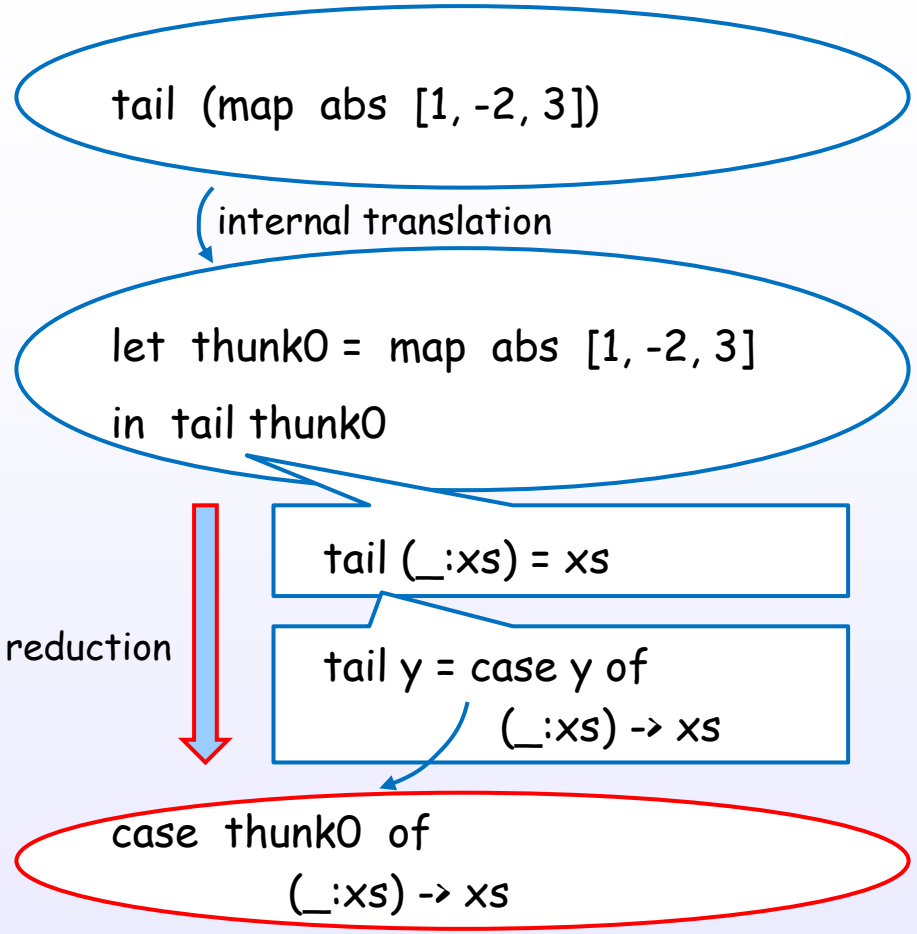


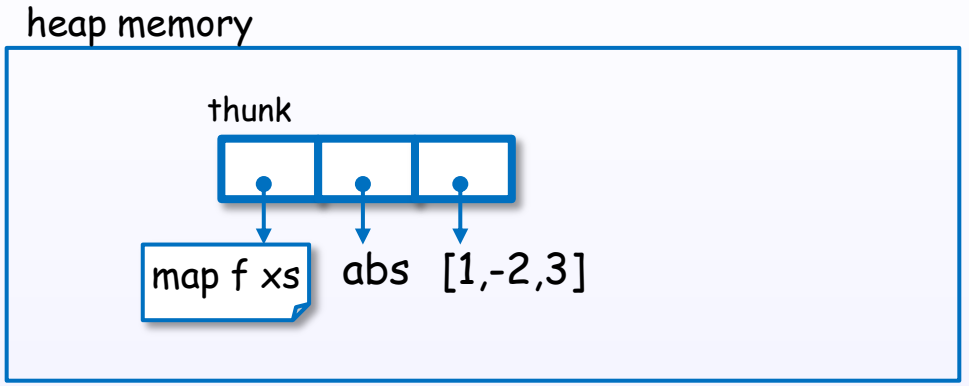
(7) function is syntactic sugar



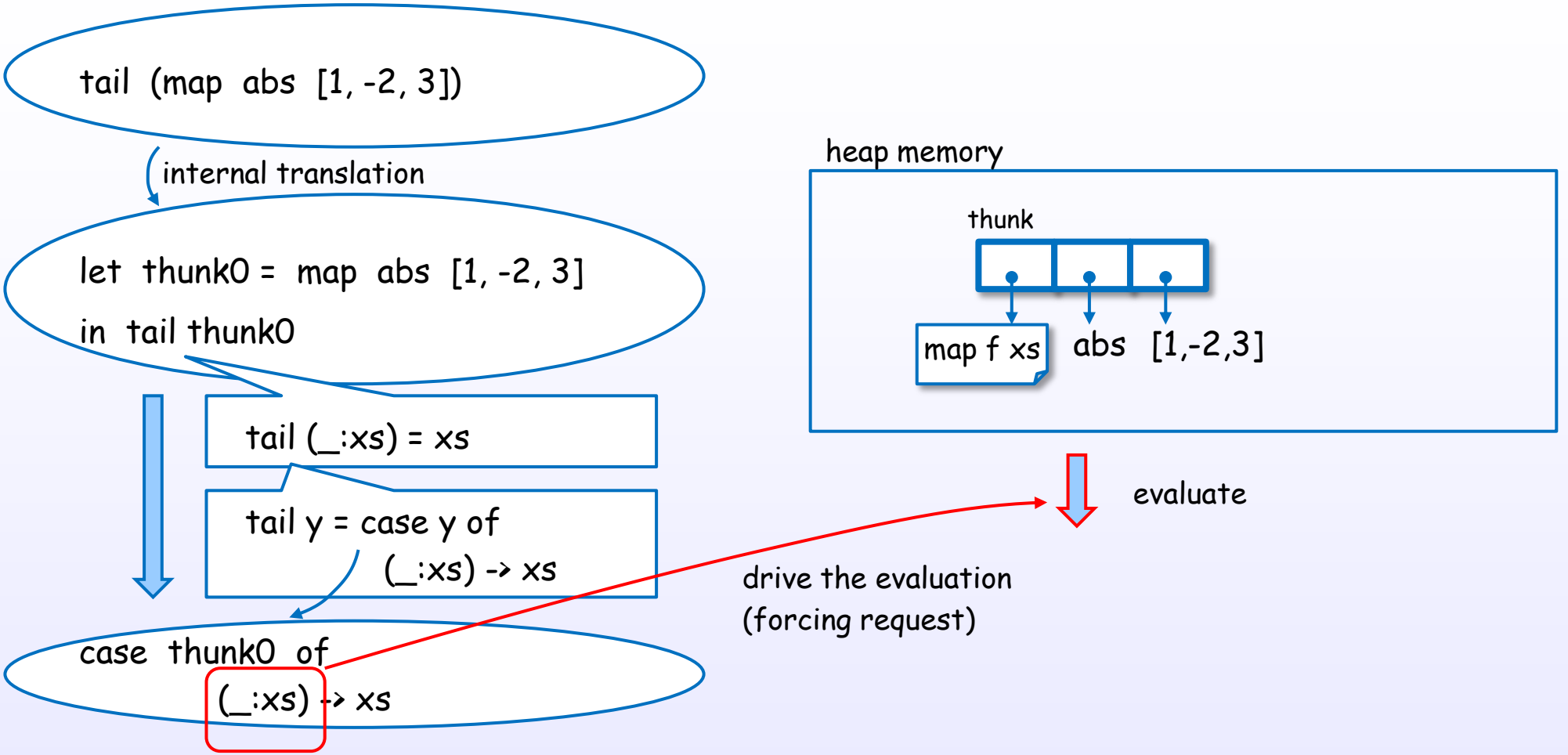


(8) substitute function body (beta reduction)

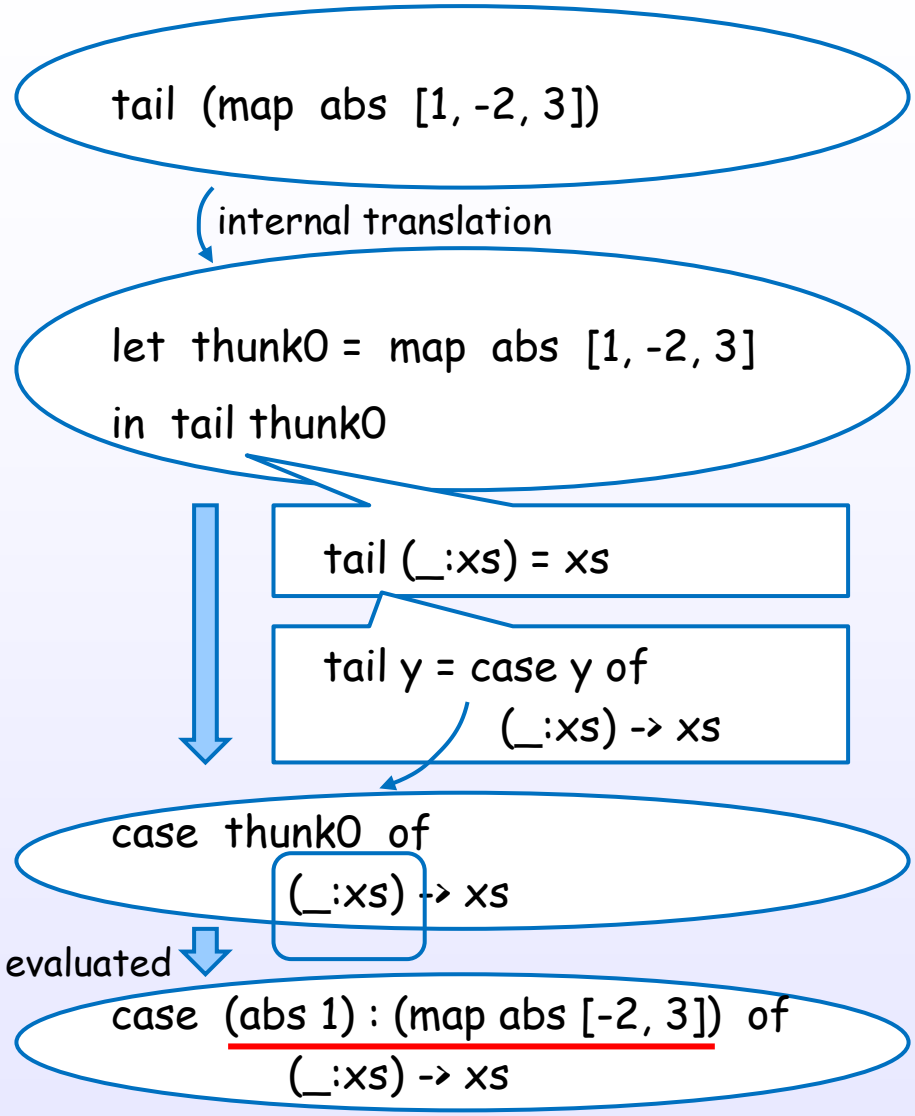


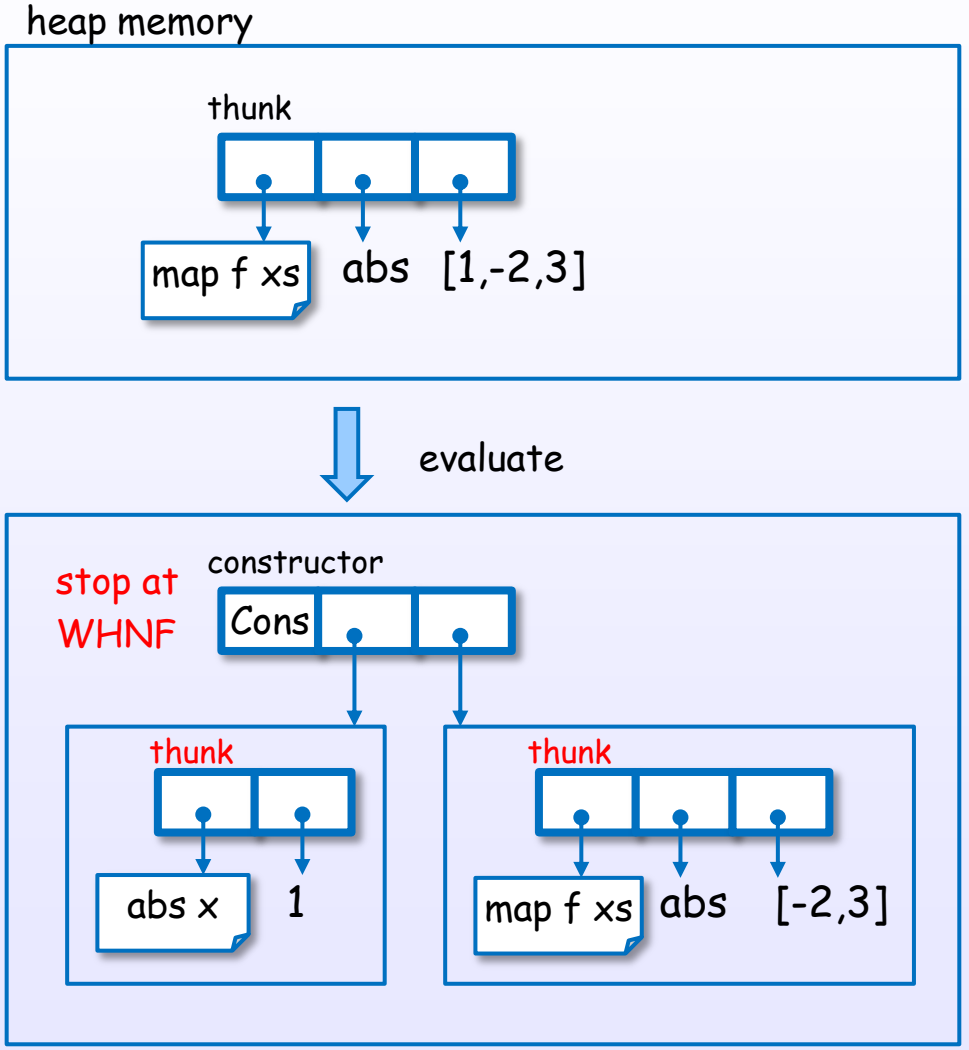


(9) case pattern-matching drives the evaluation

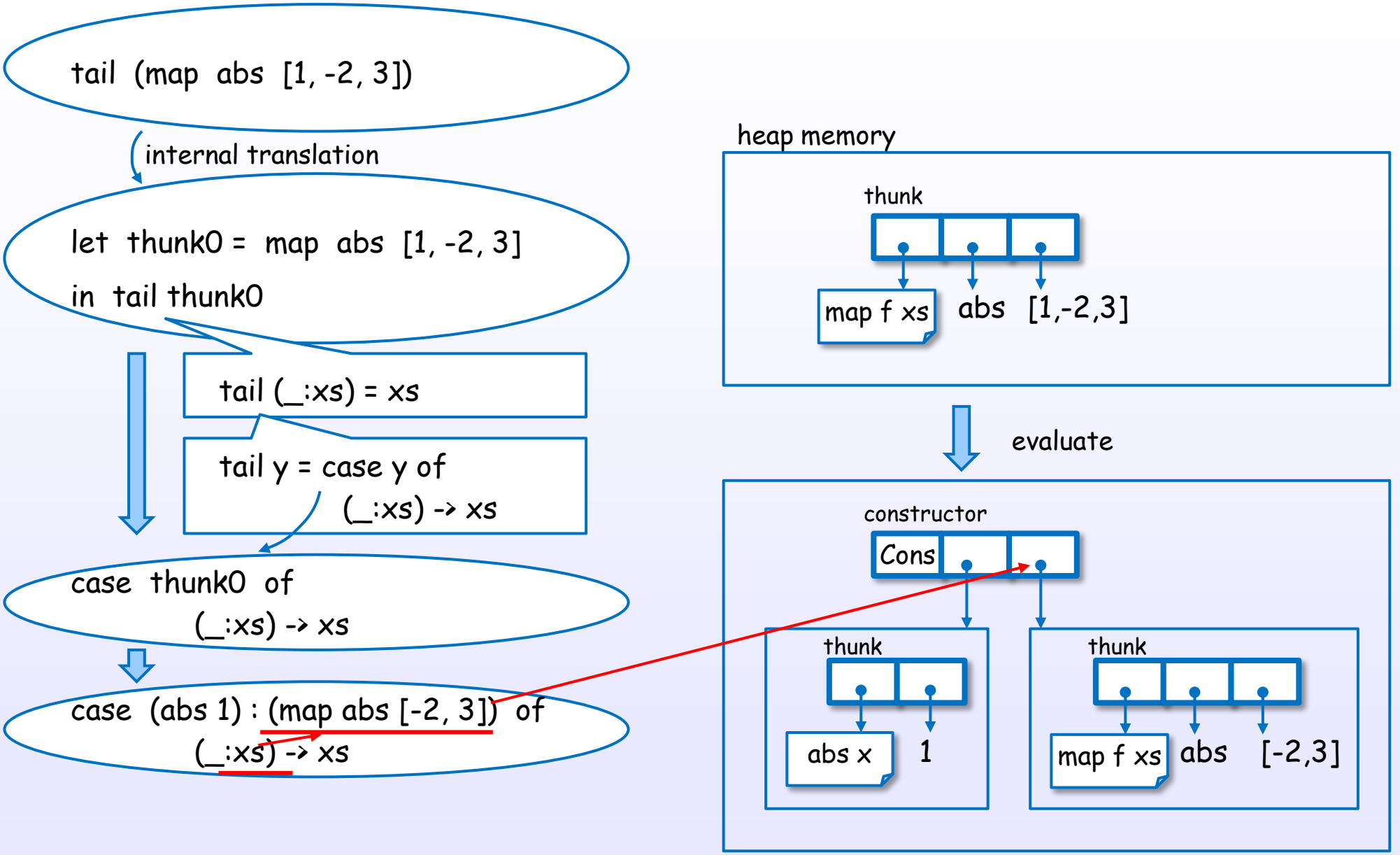


(10) but, stop at WHNF

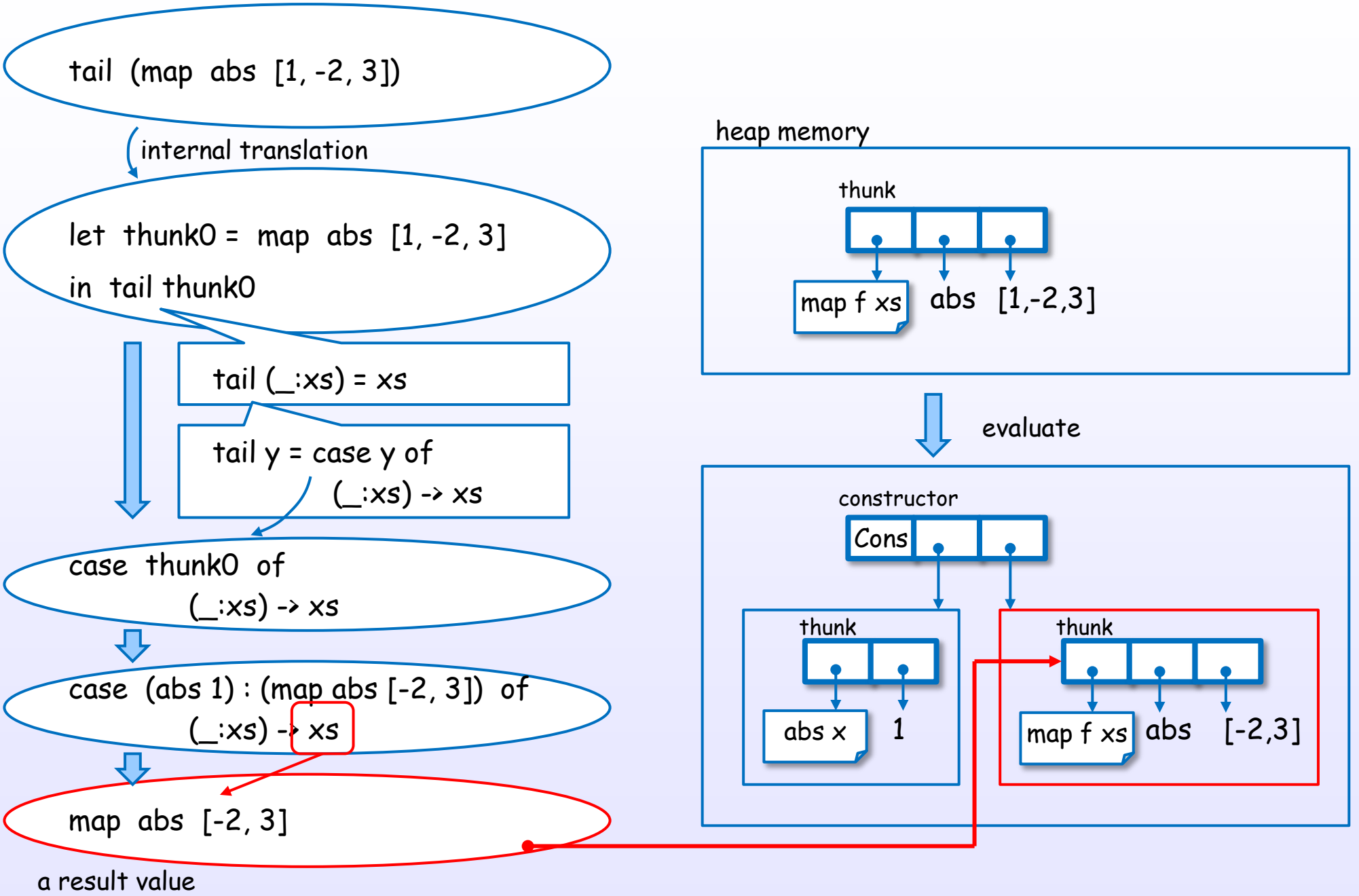




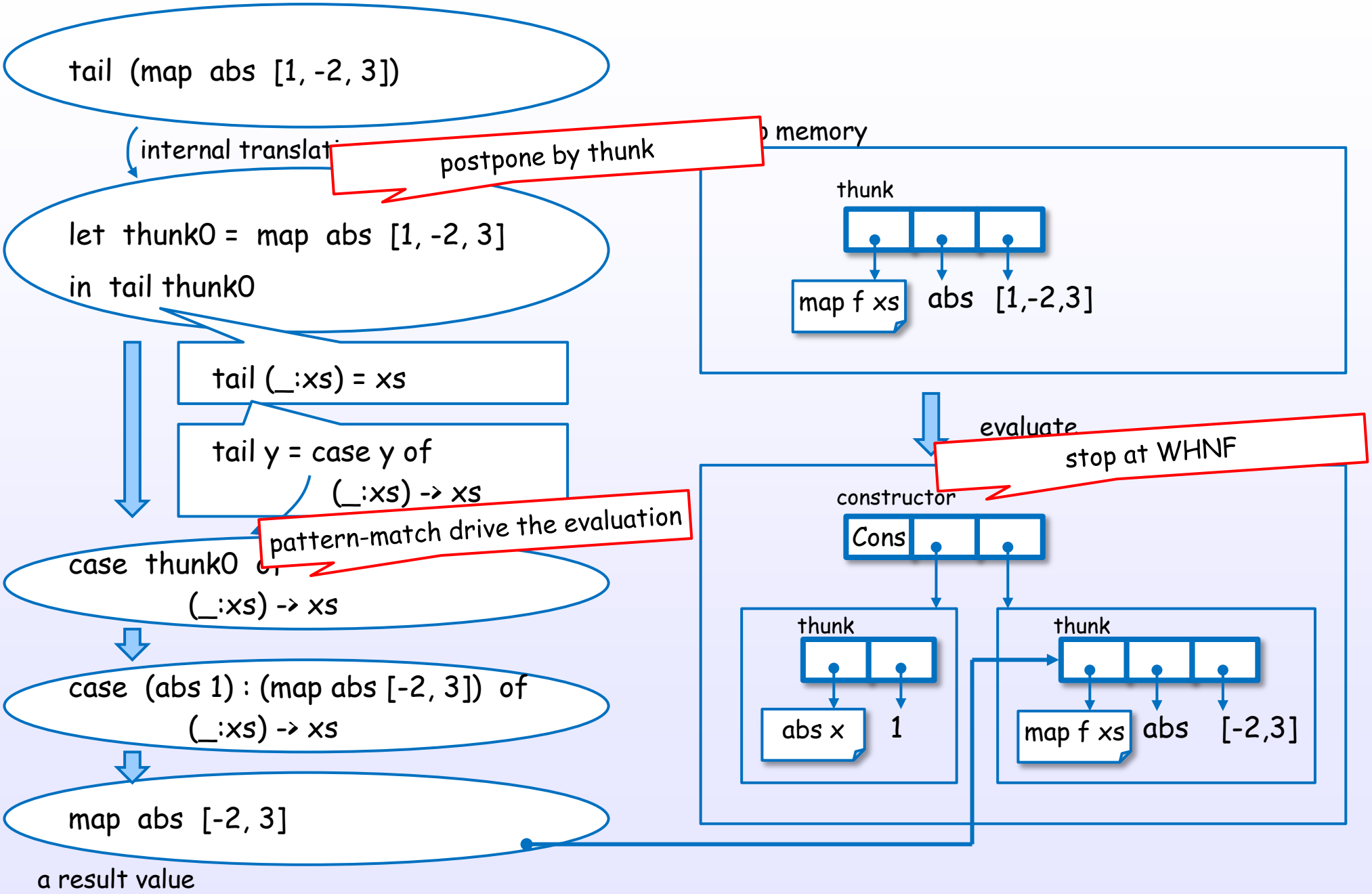
(11) bind variables to result



(12) return the value



Key points


tail (map abs [1, -2, 3])
internal translat
postpone by thunk
let thunk0 = map abs [1, -2, 3]
in tail thunk0
memory
thunk
map f xs
abs
[1,-2,3]
tail (_:xs) = xs
tail y = case y of
(_:xs) -> xs
evaluate
stop at WHNF
pattern-match drive the evaluation
constructor
Cons
case thunk0 of
(_:xs) -> xs
thunk
abs x
1
thunk
map f xs
abs
[-2,3]
case (abs 1) : (map abs [-2, 3]) of
(_:xs) -> xs
map abs [-2, 3]
a result value

Examples of evaluations

Example of repeat

repeat 1

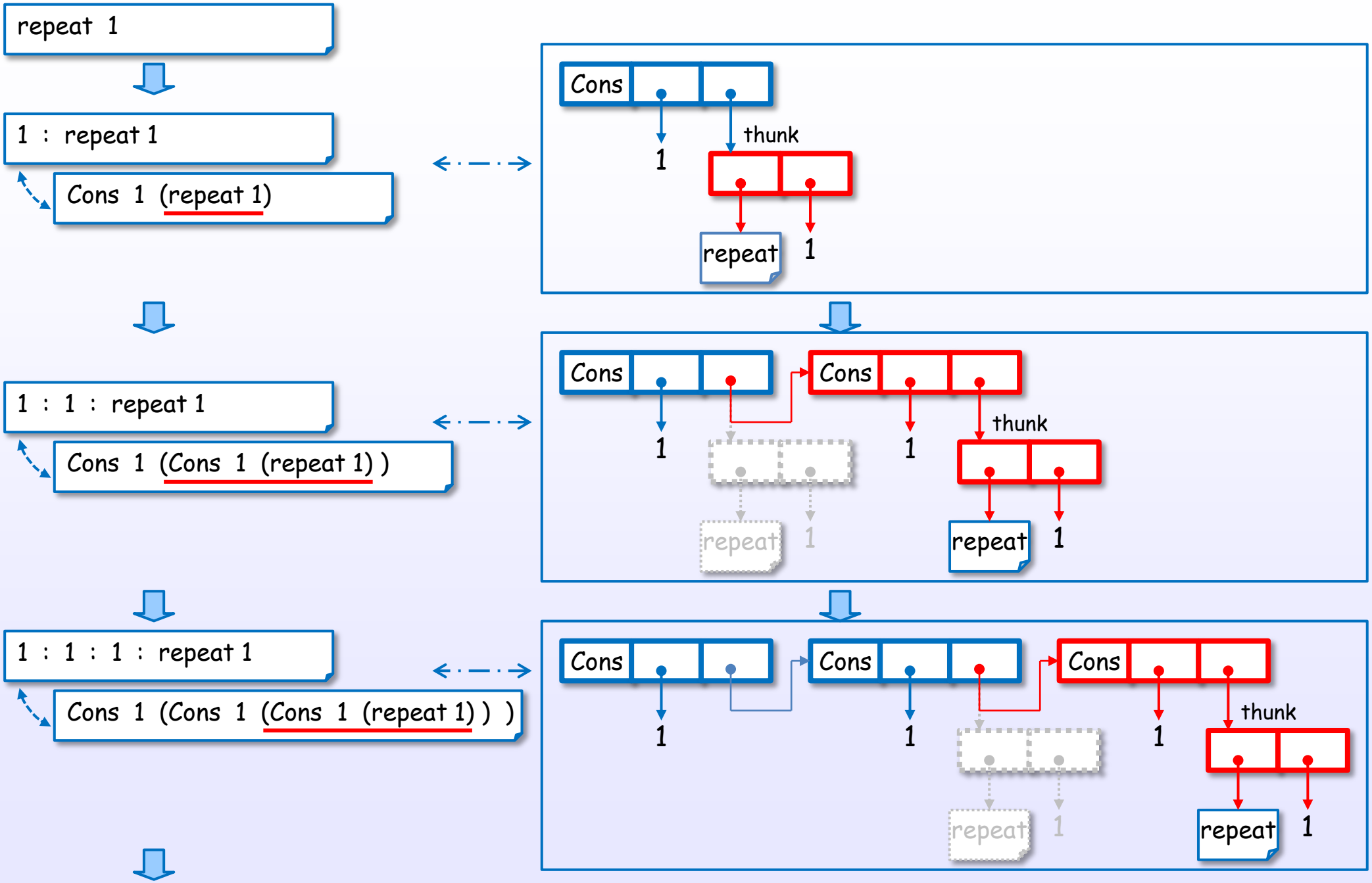
1 : repeat 1

1 : 1 : repeat 1

1 : 1 : 1 : repeat 1

References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of repeat



References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of map

```
map  f  [1, 2, 3]
```

```
f 1  :  map f [2, 3]
```

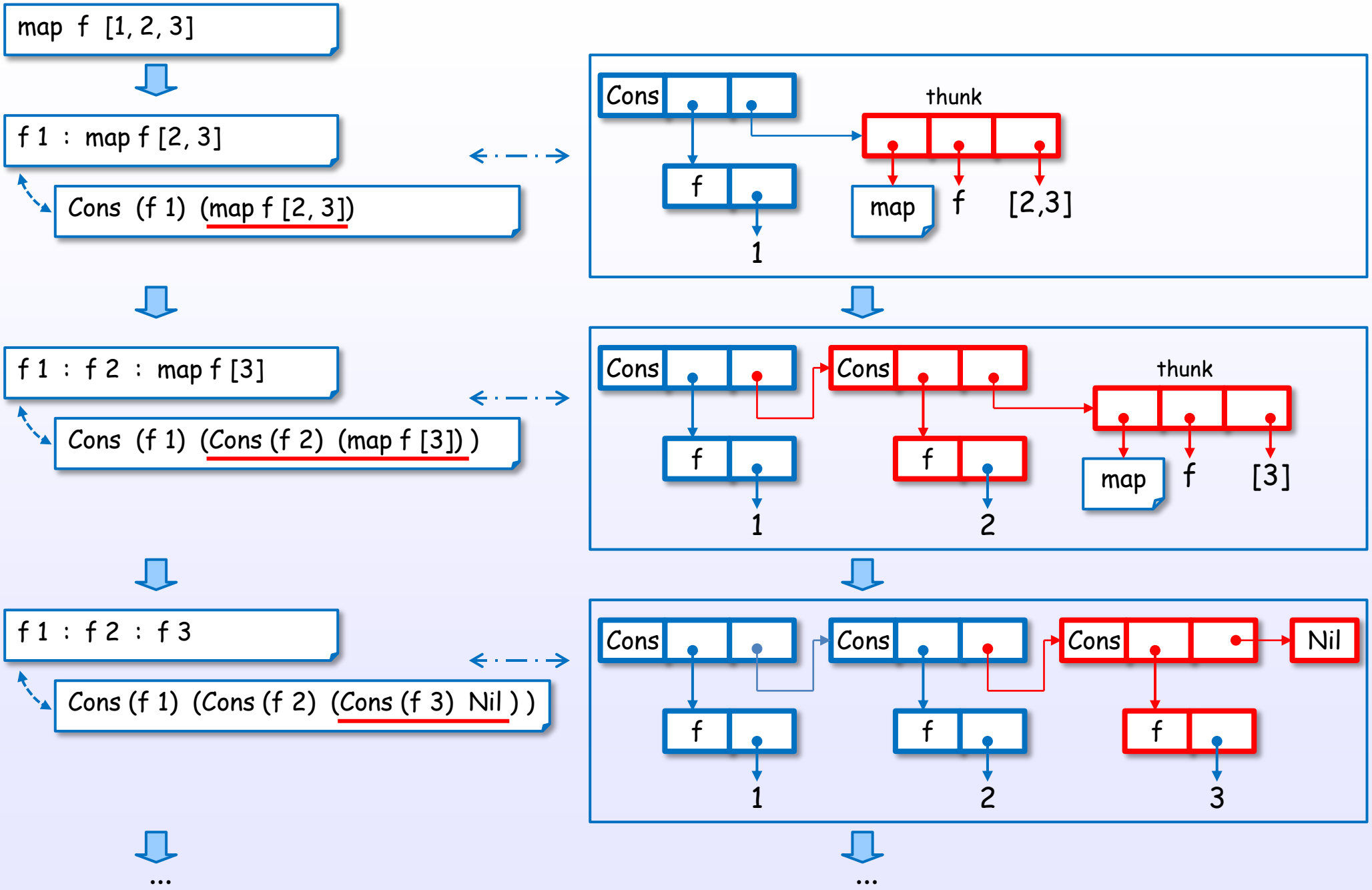
```
f 1  :  f 2  :  map f [3]
```

```
f 1  :  f 2  :  f 3
```

...

References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of map



References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl (non-strict)

```
foldl (+)  0  [1 .. 100]
```

```
foldl (+) (0 + 1) [2 .. 100]
```

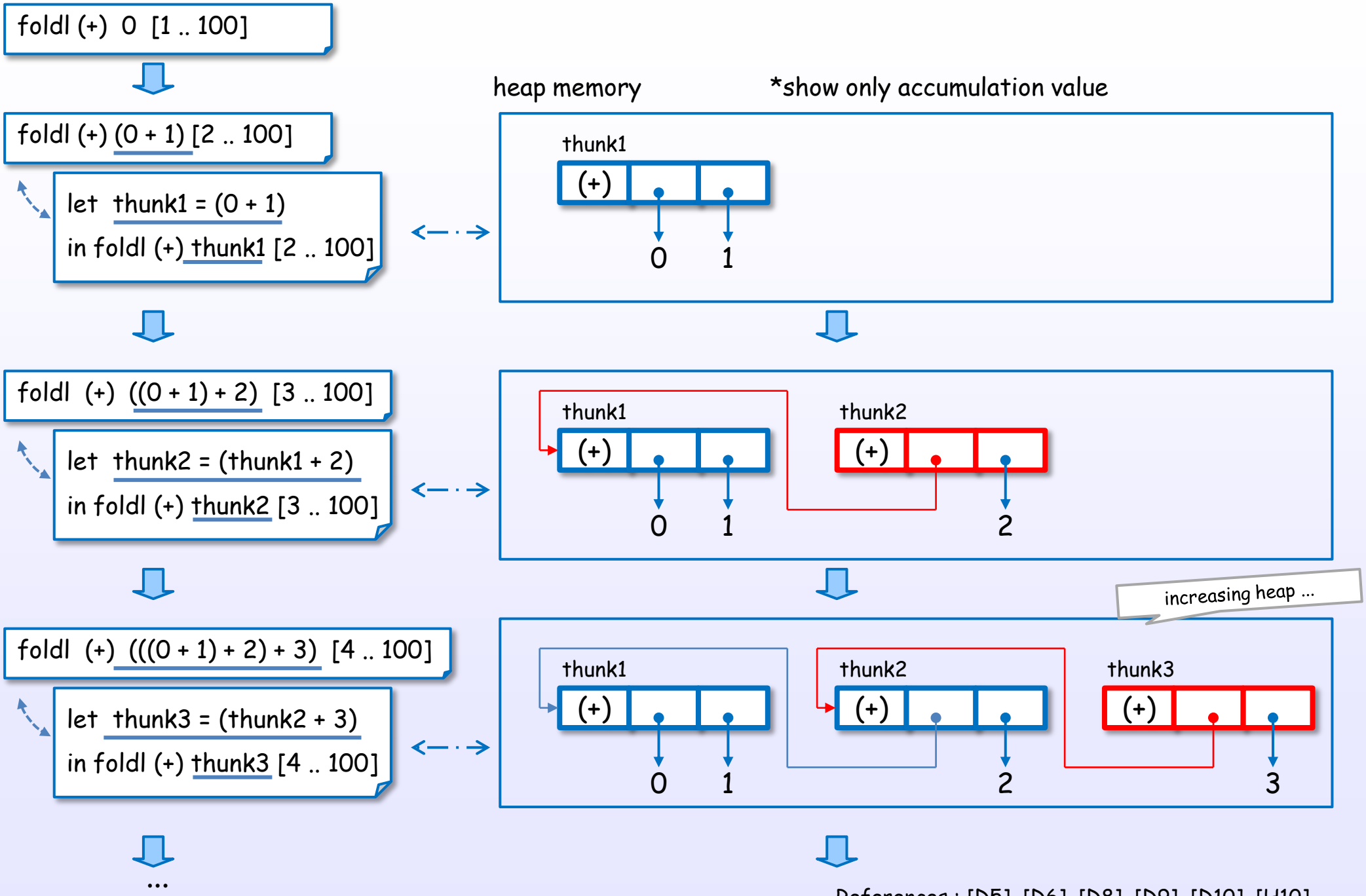
```
foldl  (+)  ((0 + 1) + 2)  [3 .. 100]
```

```
foldl  (+)  (((0 + 1) + 2) + 3)  [4 .. 100]
```

...

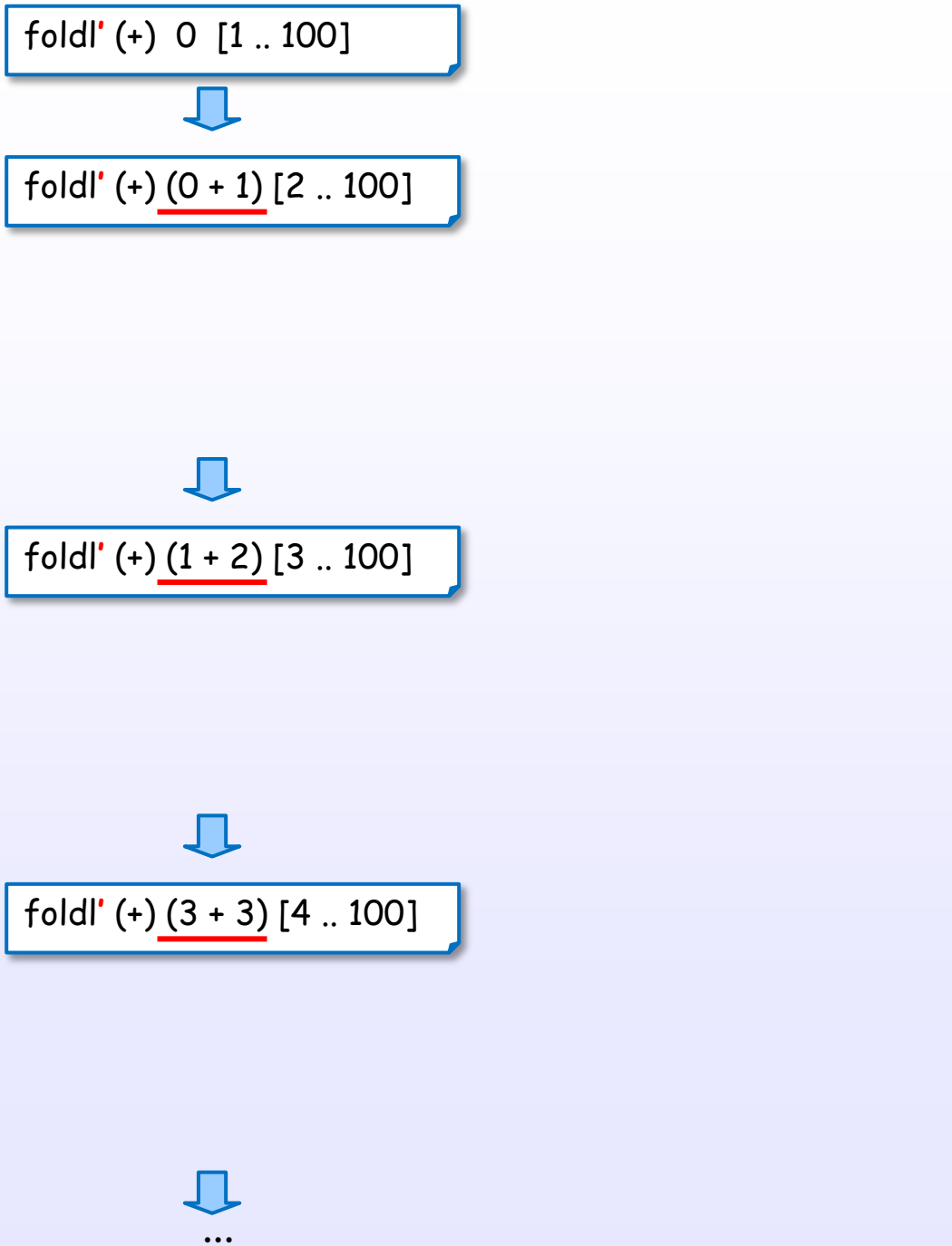
References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl (non-strict)



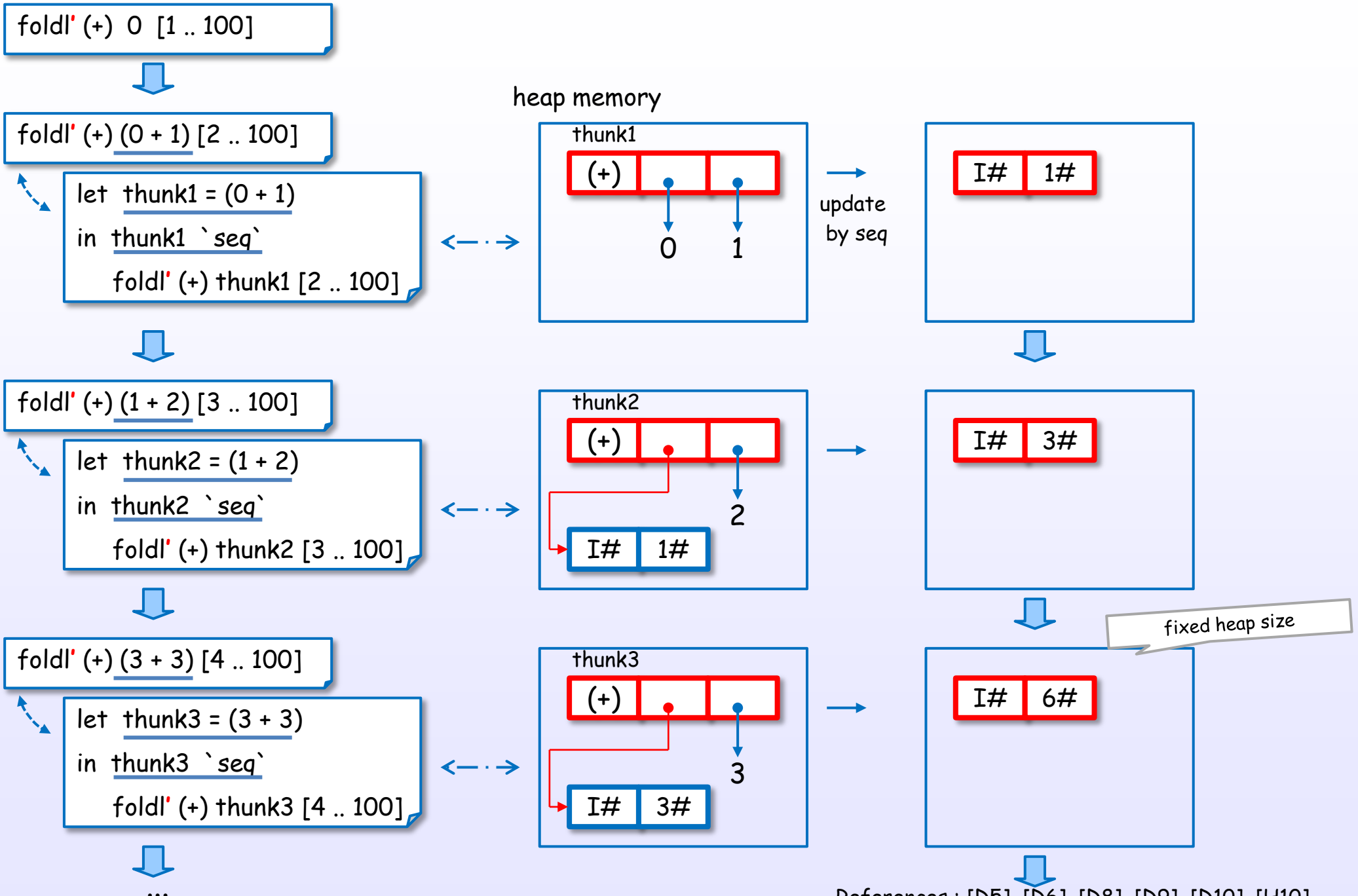
References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl' (strict)



References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl' (strict)



Example of foldl (non-strict) and foldl' (strict)

foldl (+) (0 + 1) [2 .. 100]

foldl (+) ((0 + 1) + 2) [3 .. 100]

foldl (+) (((0 + 1) + 2) + 3) [4 .. 100]

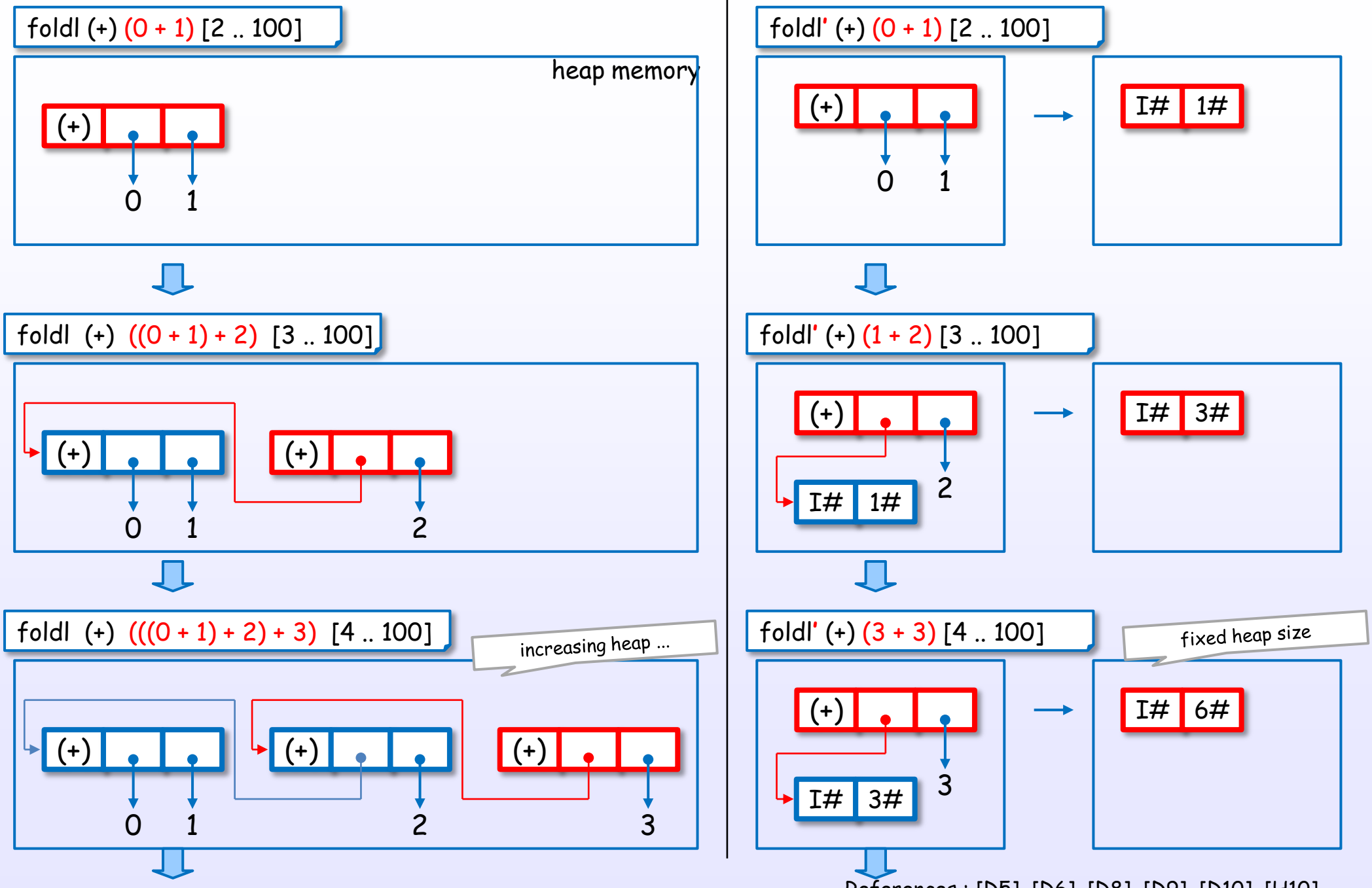
foldl' (+) (0 + 1) [2 .. 100]

foldl' (+) (1 + 2) [3 .. 100]

foldl' (+) (3 + 3) [4 .. 100]

References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl (non-strict) and foldl' (strict)

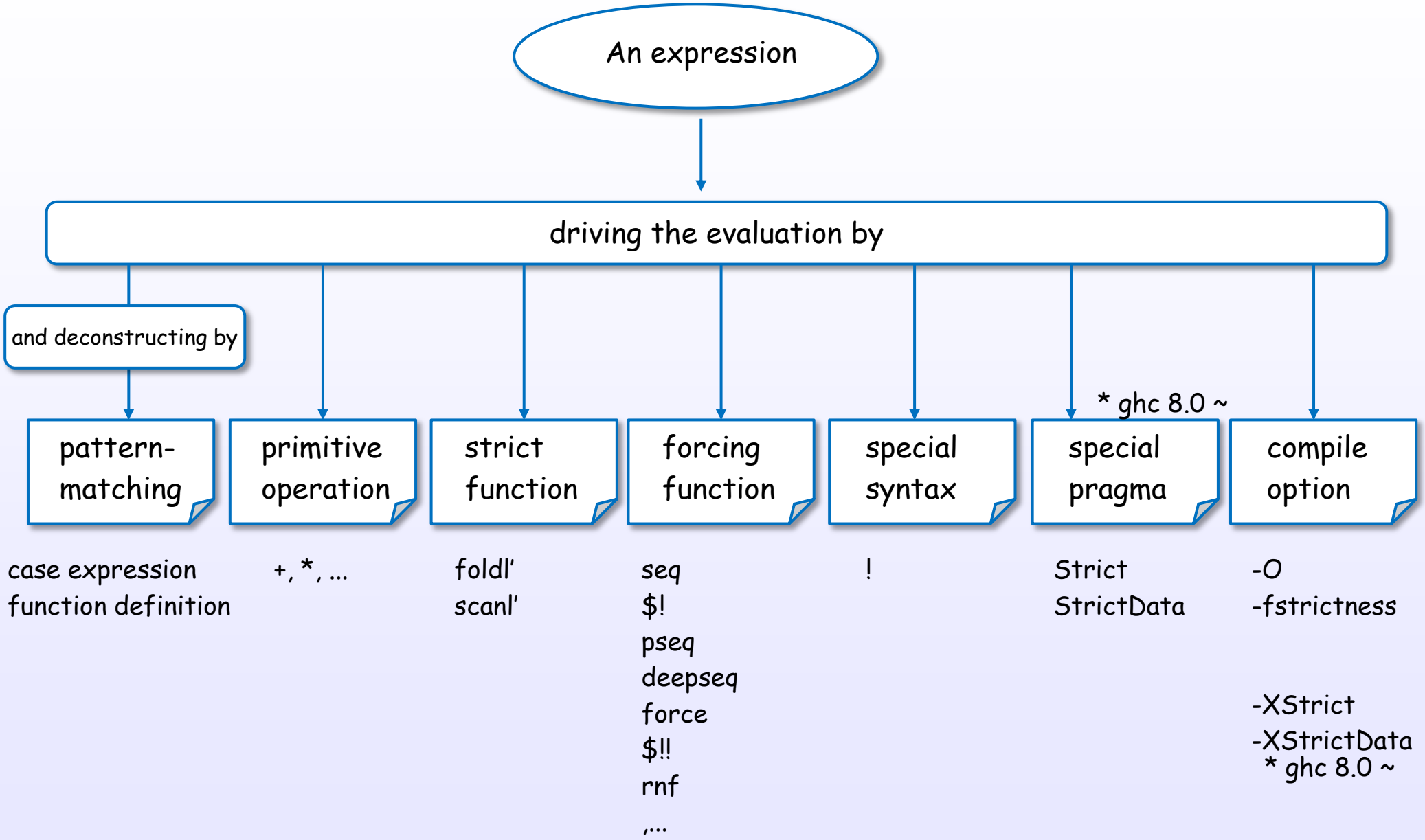


References : [D5], [D6], [D8], [D9], [D10], [H10]

4. Evaluation

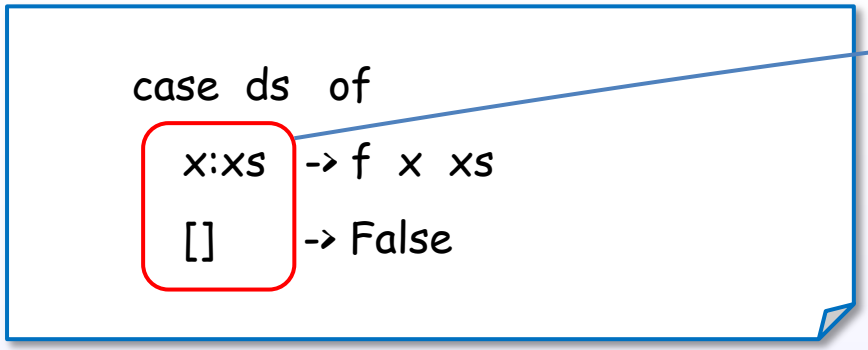
Controlling the evaluation

How to drive the evaluation



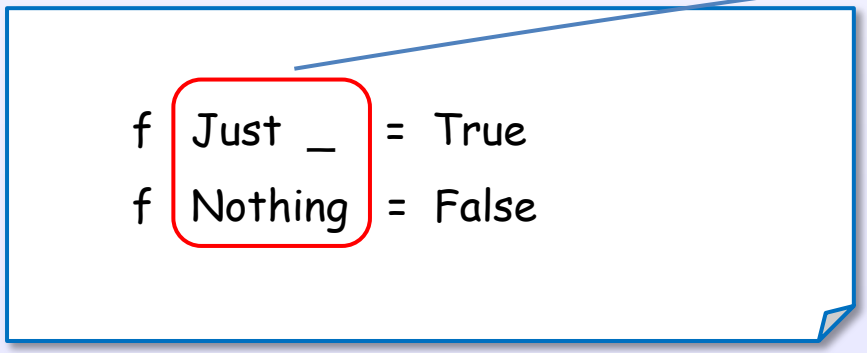
(1) Evaluation by pattern-matching

pattern-matching in case expression



forcing
(drive the evaluation of the thunk)

pattern-matching in function definition



forcing
(drive the evaluation of the thunk)

References : [H1] Ch.3, [D2], [D1], [H5], [W1]

(1) Evaluation by pattern-matching

Strict patterns drive the evaluation

case expression

```
case ds of
  x:xs -> f x xs
  []   -> False
```

function definition

```
f Just _  = True
f Nothing = False
```

Lazy patterns postpone the evaluation.

let binding pattern

```
let (x:xs) = fun args
```

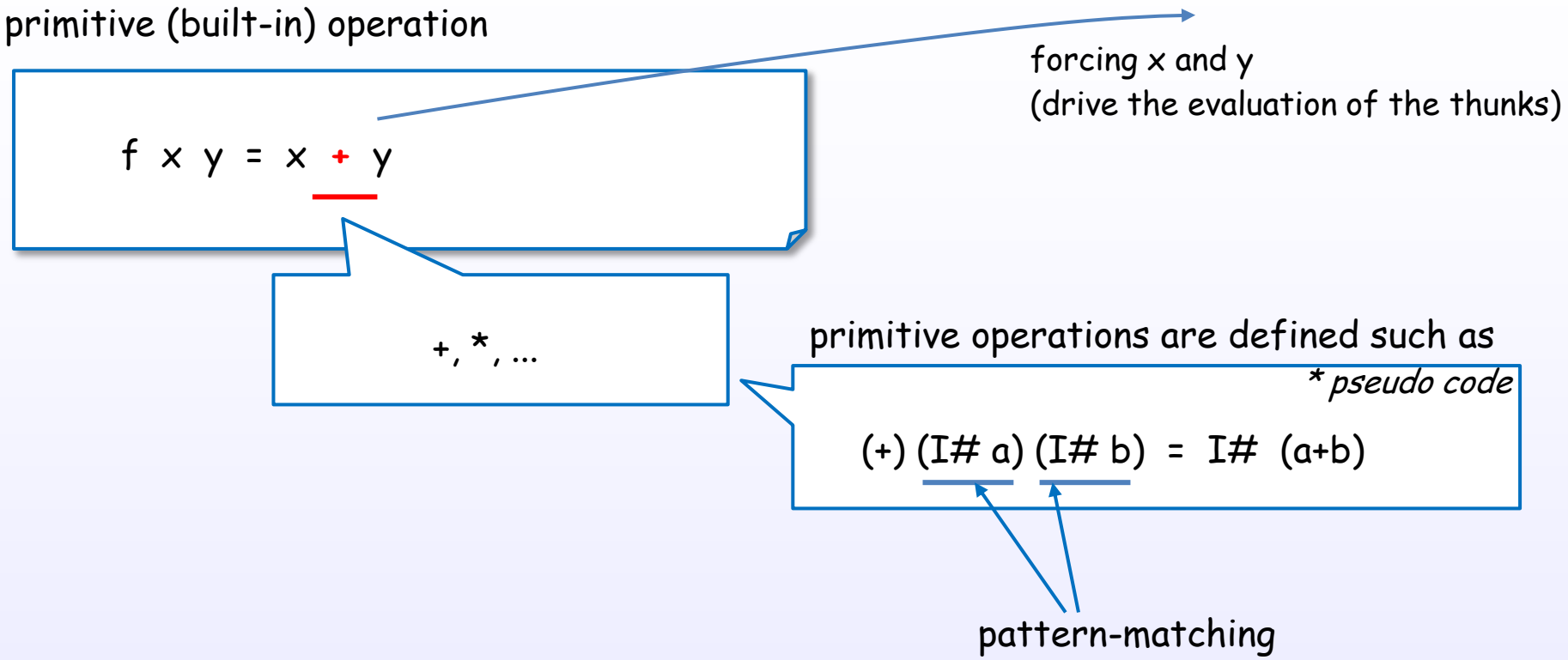
irrefutable patterns [H1] 3.17

```
f ~(Just _ ) = True
f ~(Nothing) = False
```

There are two kinds of pattern-matching.

References : [H1] Ch.3, [D2], [D1], [H5], [W1]

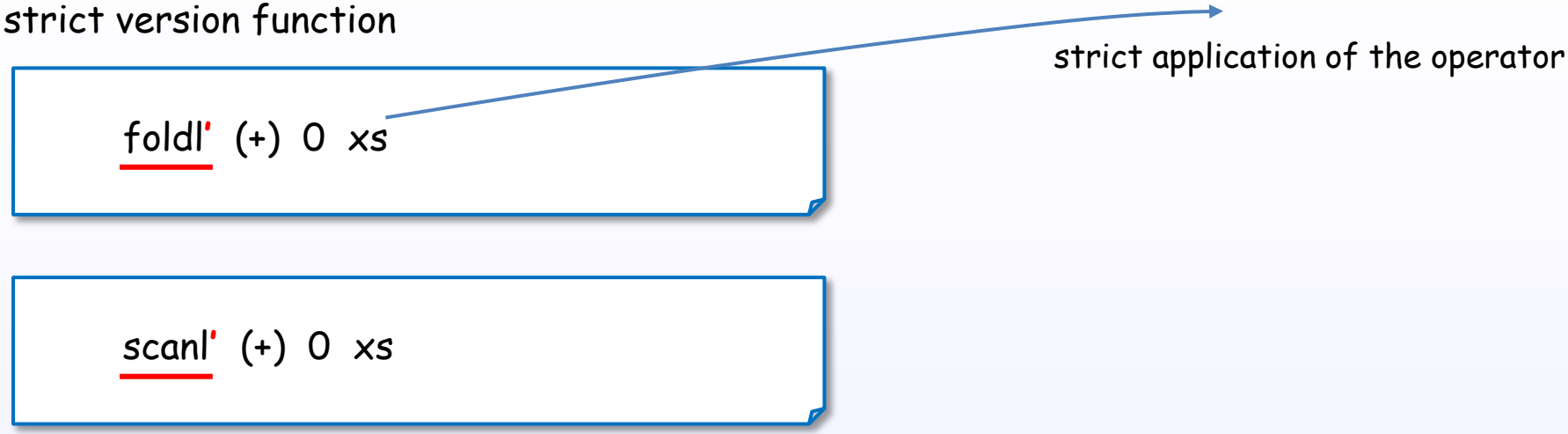
(2) Evaluation by primitive operation



References : [D3], [H5], [H12]

(3) Evaluation by strict version function

strict version function



References : [B4] Ch.25, [D9], [B6] Ch.7, [B2], [S1], [S2]

(4) Evaluation by forcing function

forcing functions to WHNF

```
seq x y
```

```
f $! x
```

```
pseq x y
```

forcing functions to NF

```
deepseq x y
```

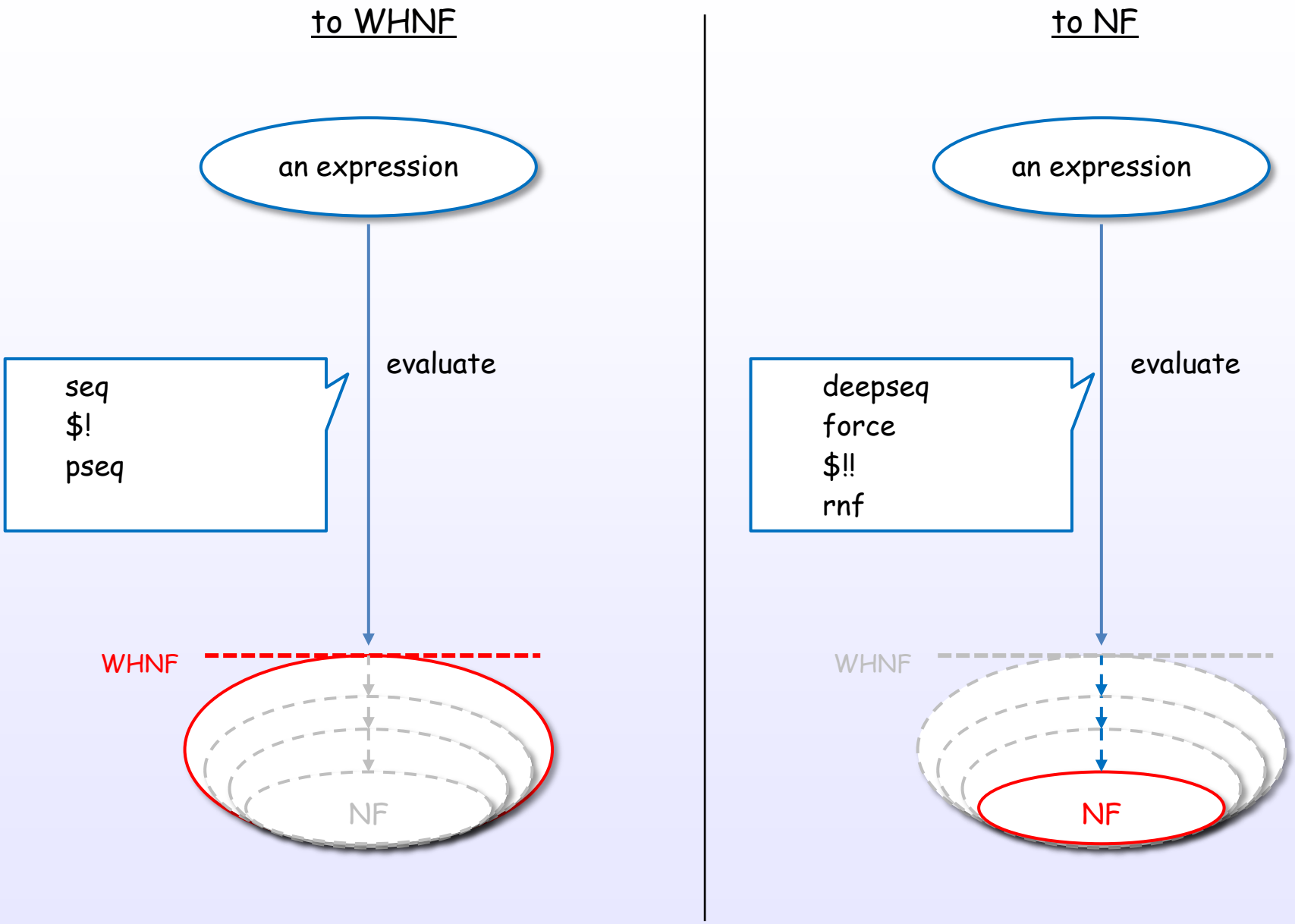
```
f $!! x
```

```
force x
```

```
rnf x
```

References : [B5] Ch.2, [B4] Ch.24, 25, [B6] Ch.7, [H1] Ch.6, [D2], [B2], [D1], [D2]. [S1], [S2]

(4) Evaluation by forcing function



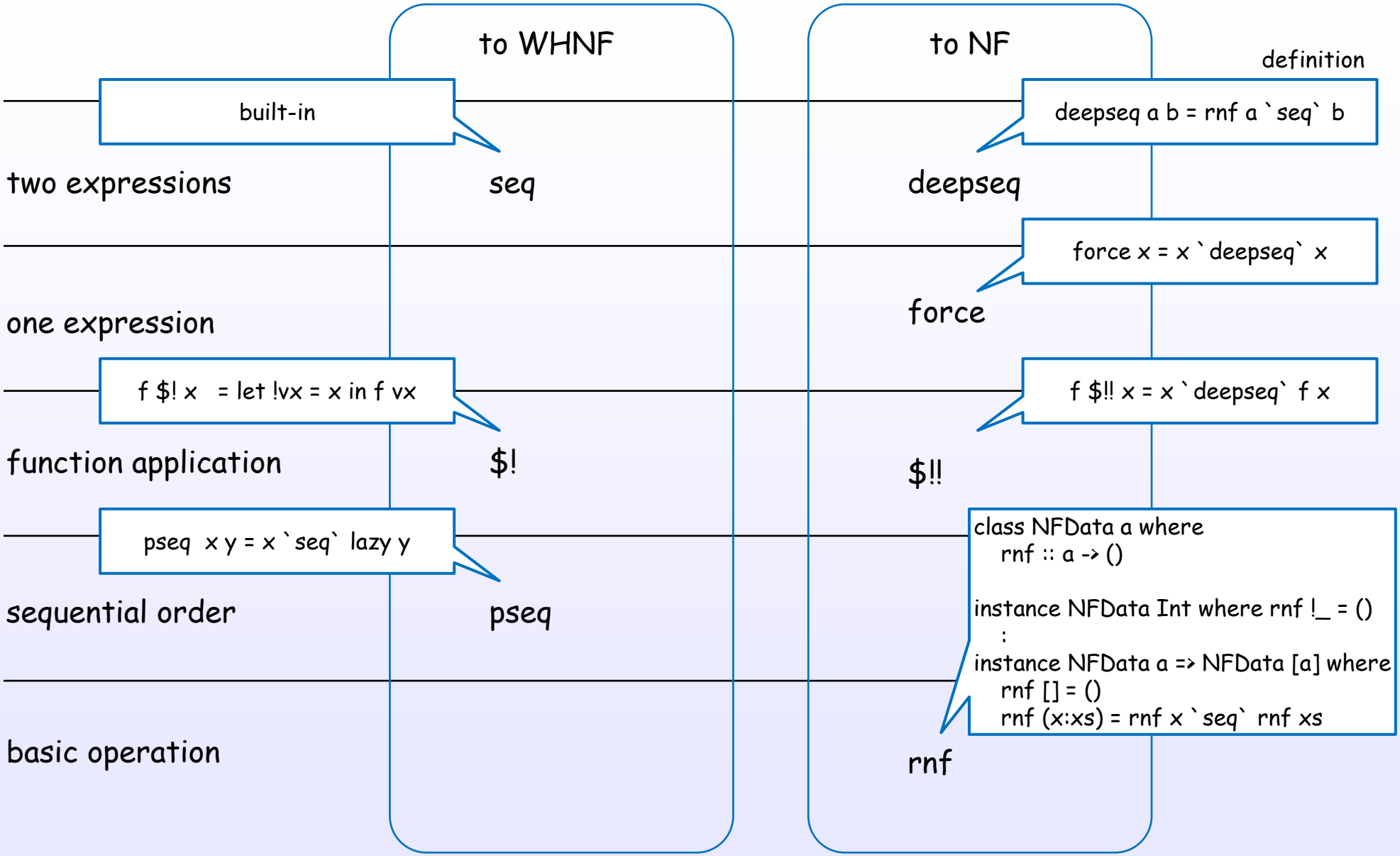
References : [B5] Ch.2, [B4] Ch.24, 25, [H1] Ch.6, [D2], [B2], [D1], [D2]. [S1], [S2]

(4) Evaluation by forcing function

	to WHNF	to NF
two arguments	seq	deepseq
one argument		force
function application	$!	$!!
sequential order	pseq	
basic operation		rnf

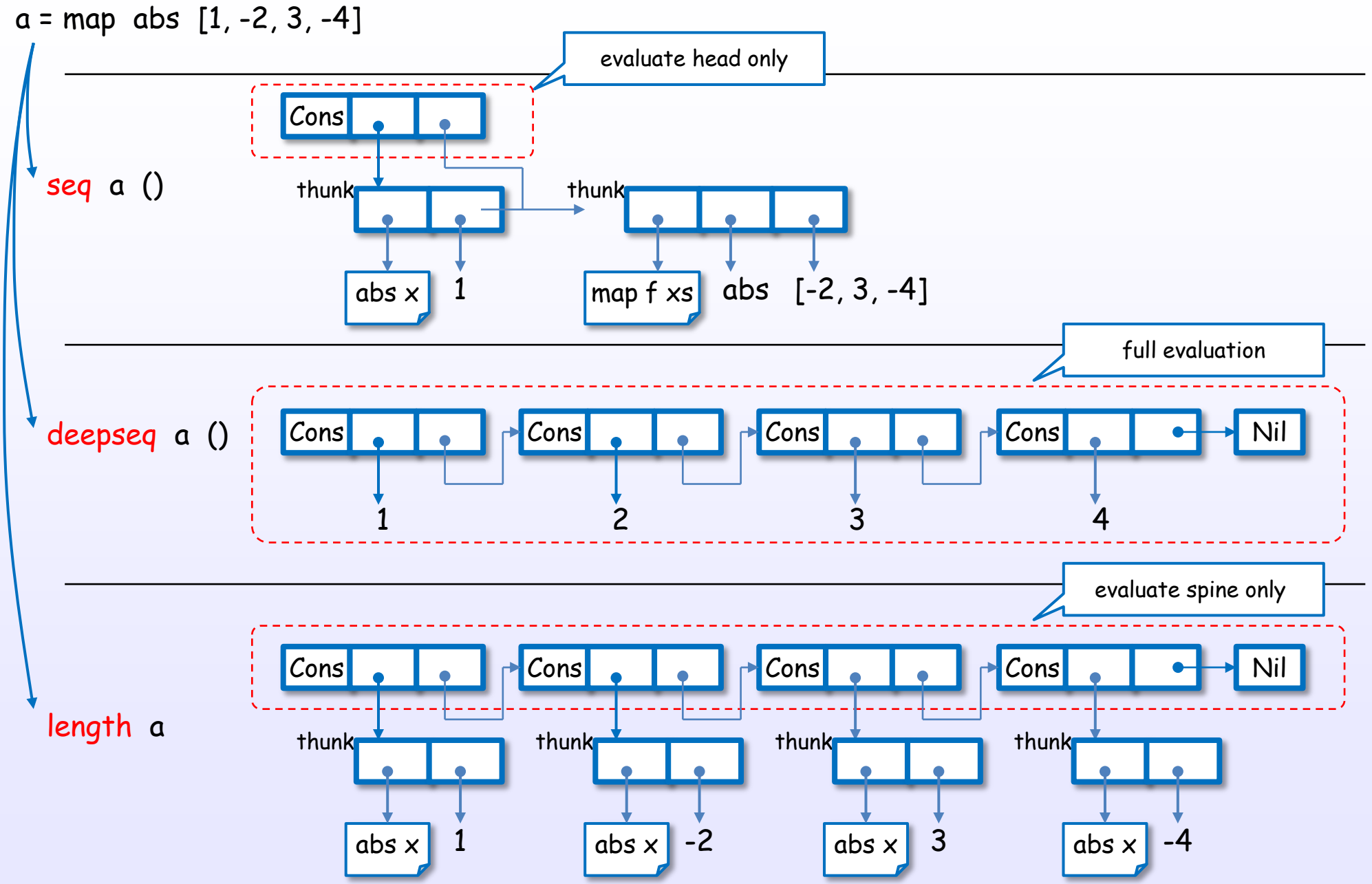
References : [B5] Ch.2, [B4] Ch.24, 25, [H1] Ch.6, [D2], [B2], [D1], [D2]. [S1], [S2]

(4) Evaluation by forcing function



References : [B5] Ch.2, [B4] Ch.24, 25, [H1] Ch.6, [D2], [B2], [D1], [D2]. [S1], [S2]

(4) Evaluation by forcing function



References : [B5] Ch.2, [B4] Ch.24, 25, [H1] Ch.6, [D2], [B2], [D1], [D10]. [S1], [S2]

(5) Evaluation by special syntax

Strictness annotation

Bang pattern [H2] 7.19

see also Strict pragma

```
{-# LANGUAGE BangPatterns #-}

f !xs = g xs
```

arguments are evaluated
before function application

Strictness flag [H1] 4.2.1

see also StrictData and Strict pragma

```
data Pair = Pair !a !b
```

arguments are evaluated
before constructor application

Strictness annotations assist strictness analysis.

References : [D1], [H2] Ch.7, [H1] Ch.4, [B4] Ch.25, [B2] Ch.7, [W6], [H4] Ch.22, [W4]

(6) Evaluation by special pragma

Special pragma for strictness language extension

Strict pragma * ghc 8.0 ~

see also bang pattern and strictness flag

```
{-# LANGUAGE Strict #-}

let f xs = g xs in f ys
data Pair = Pair a b
```

arguments are evaluated before application

StrictData pragma * ghc 8.0 ~

see also strictness flag

```
{-# LANGUAGE StrictData #-}

data Pair = Pair a b
```

Strict and StrictData pragmas are module level control.
These can use in ghc 8.0 or later.

References : [H13]

(7) Evaluation by compile option

Compile option

strictness analysis

```
$ ghc  -O
```

Turn on optimization.
Imply -O.

```
$ ghc  -fstrictness
```

Turn on strictness analysis.
Implied by -O.

strictness language extension * ghc 8.0 ~

```
$ ghc  -XStrict
```

apply Strict pragma

```
$ ghc  -XStrictData
```

apply StrictData pragma

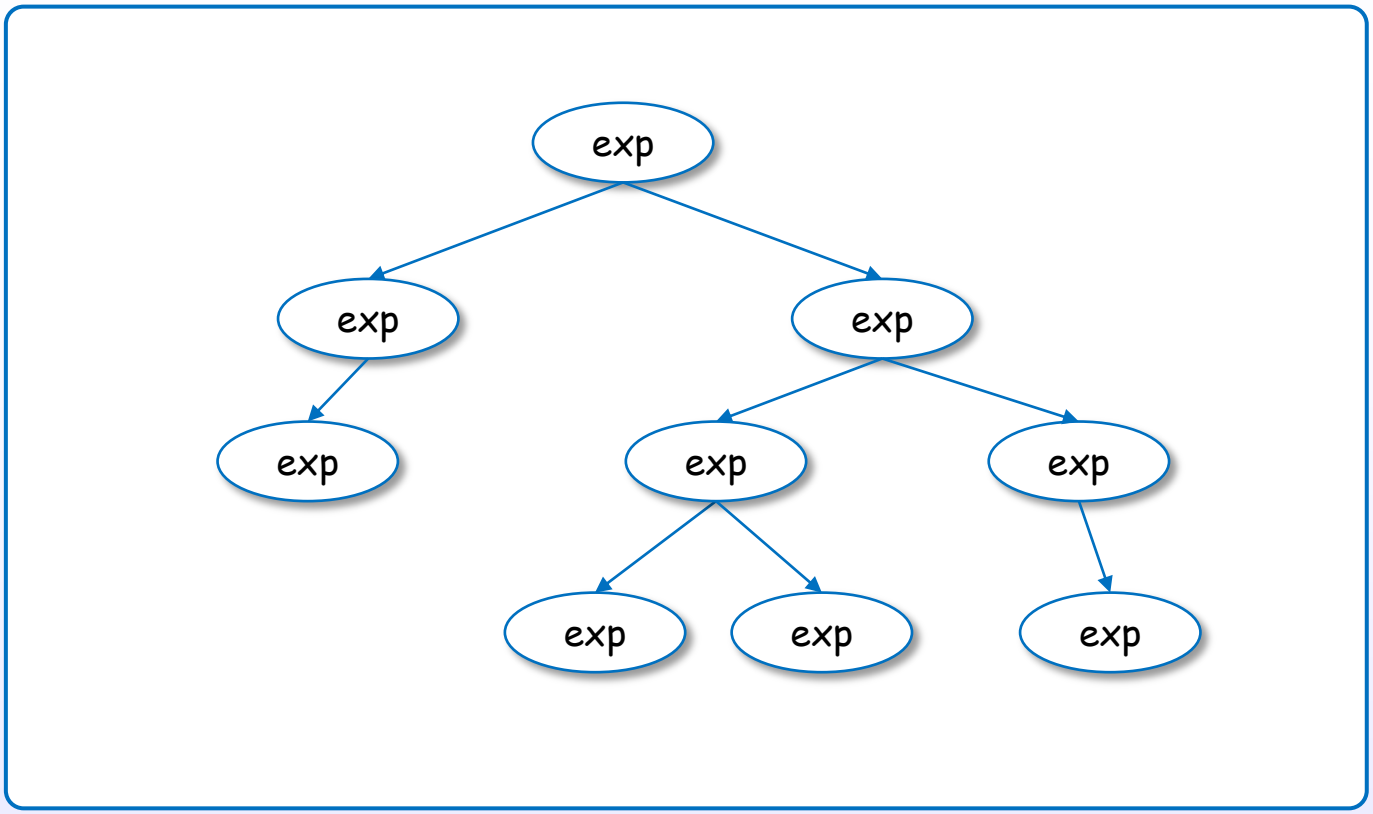
References : [H2], [H13]

5. Implementation of evaluator

5. Implementation of evaluator

Lazy graph reduction

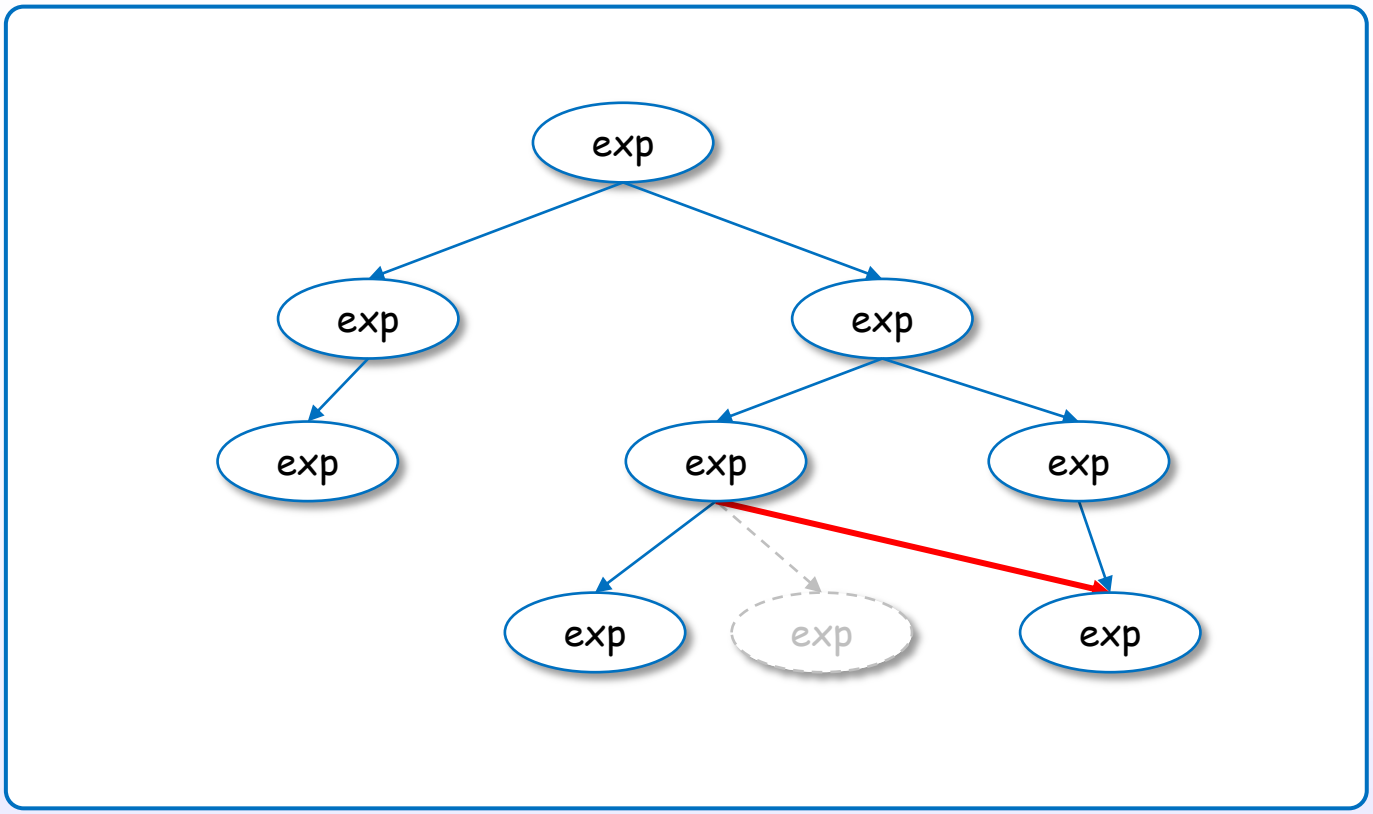
Tree



An expression can be represented in the form of Abstract Syntax Tree (AST).
AST is reduced using stack (sequential access memory).

References : [D3], [D2], [D5], [W5], [H4] Ch.12, [D8] Ch.3

Graph



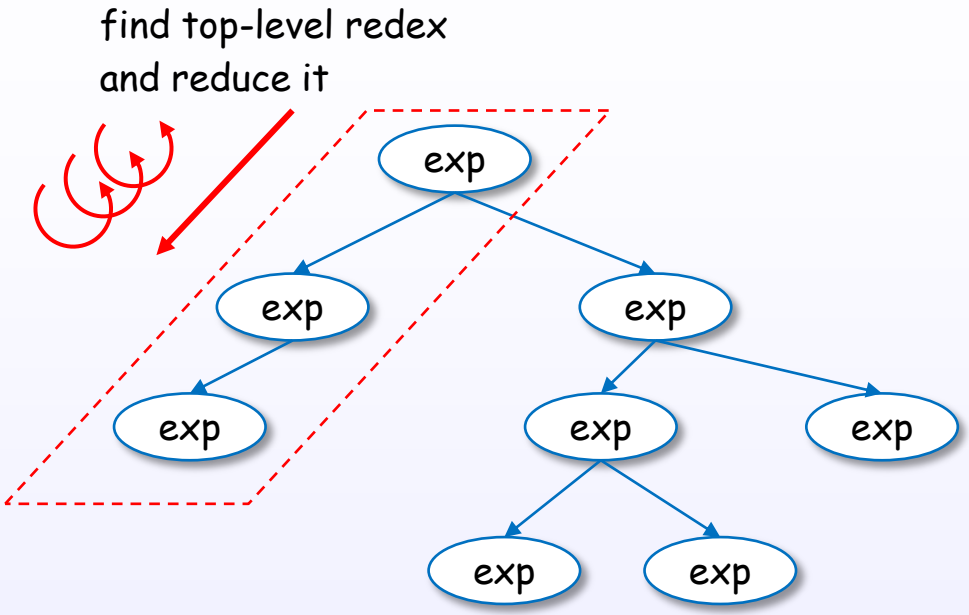
An expression can be also represented in the form of Graph.

Graph can share subexpressions to evaluate at once.

So, graph is reduced using heap (random access memory) rather than stack.

References : [D3], [D2], [D5], [W5], [H4] Ch.12, [D8] Ch.3

Normal order graph reduction



Normal order reduction specifies reducing the leftmost outermost redex (top-level redex). Given an application of a function, the outermost redex is the function application itself.

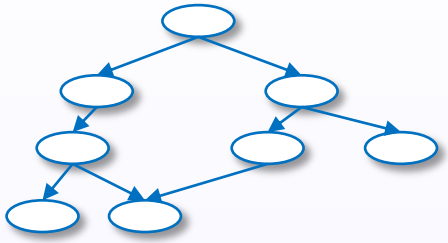
References : [D3], [D2], [D5], [W5], [H4] Ch.11, 12, [D8] Ch.3

5. Implementation of evaluator

STG-machine

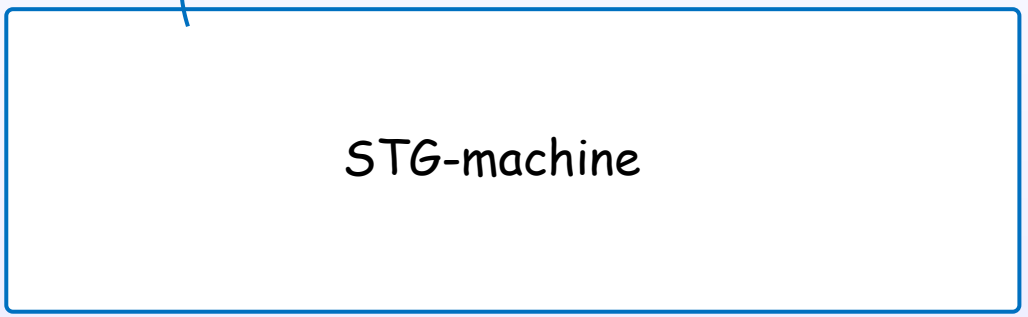
Abstract machine

Graph
(expression)



evaluate
(reduce / execute)

Evaluator
(abstract machine)



GHC uses abstract machine to reduce the expression.
It's called "STG-machine".

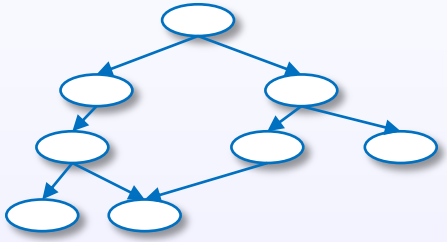
References : [H5], [H6], [H7], [D15]

Concept layer

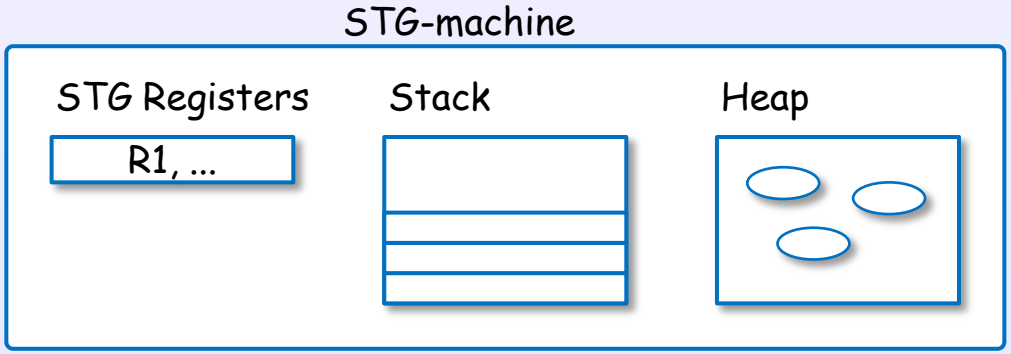
Haskell code

```
take 5 [1..10]
:
```

Graph
(internal representation
of the expression)

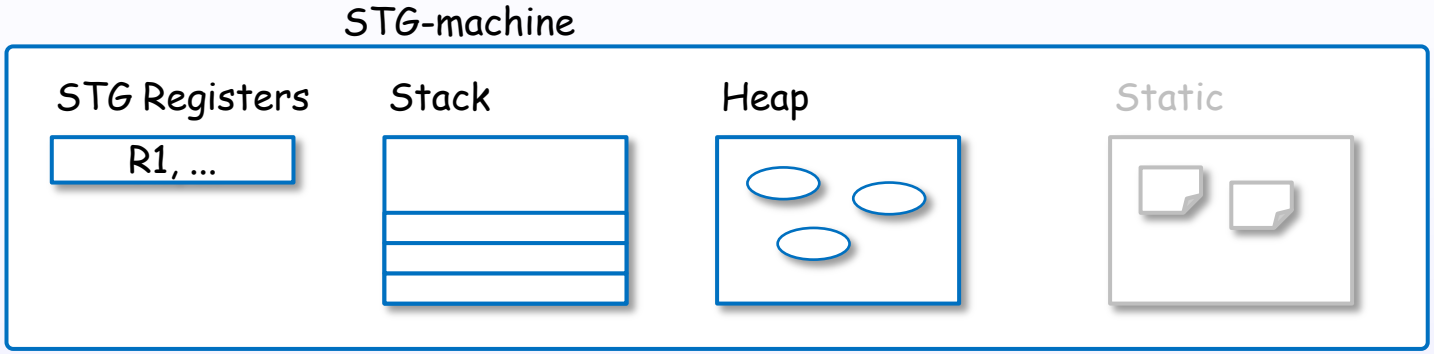


Evaluator (reducer, executer)
(abstract machine)



References : [H5], [H6], [H7], [D15]

STG-machine

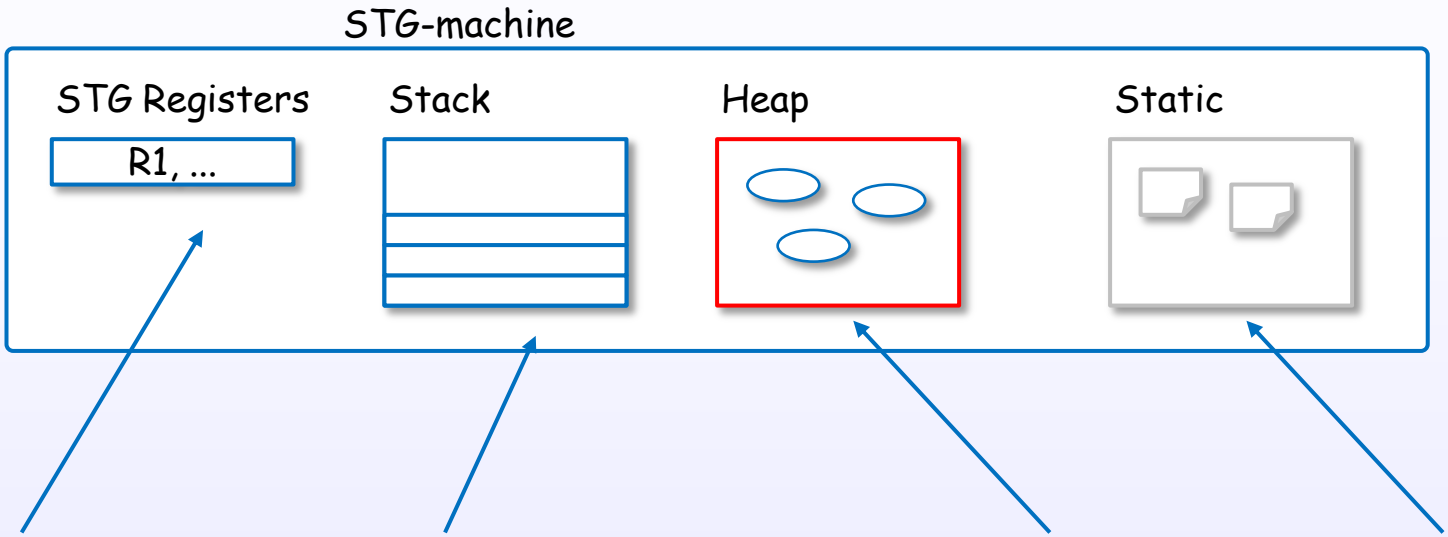


STG-machine is abstraction machine
which is defined by operational semantics.

STG-machine efficiently performs lazy graph reduction.

References : [H5], [H6], [H7], [D15]

STG-machine



mainly used for
call/return convention
various control

mainly used for
nest continuation
argument passing

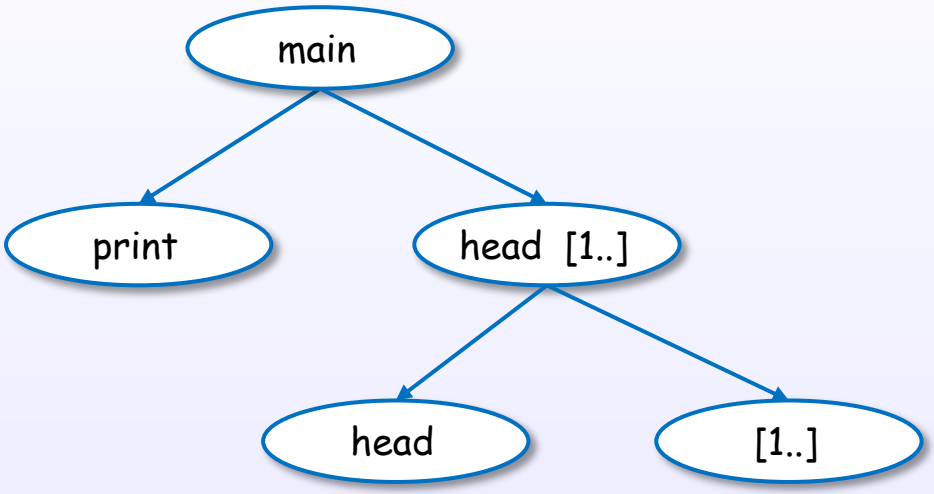
mainly used for
allocating objects
(thunks, datas,
functions)

mainly used for
code
static objects

References : [H5], [H6], [H7], [D15]

Example of mapping a code to a graph

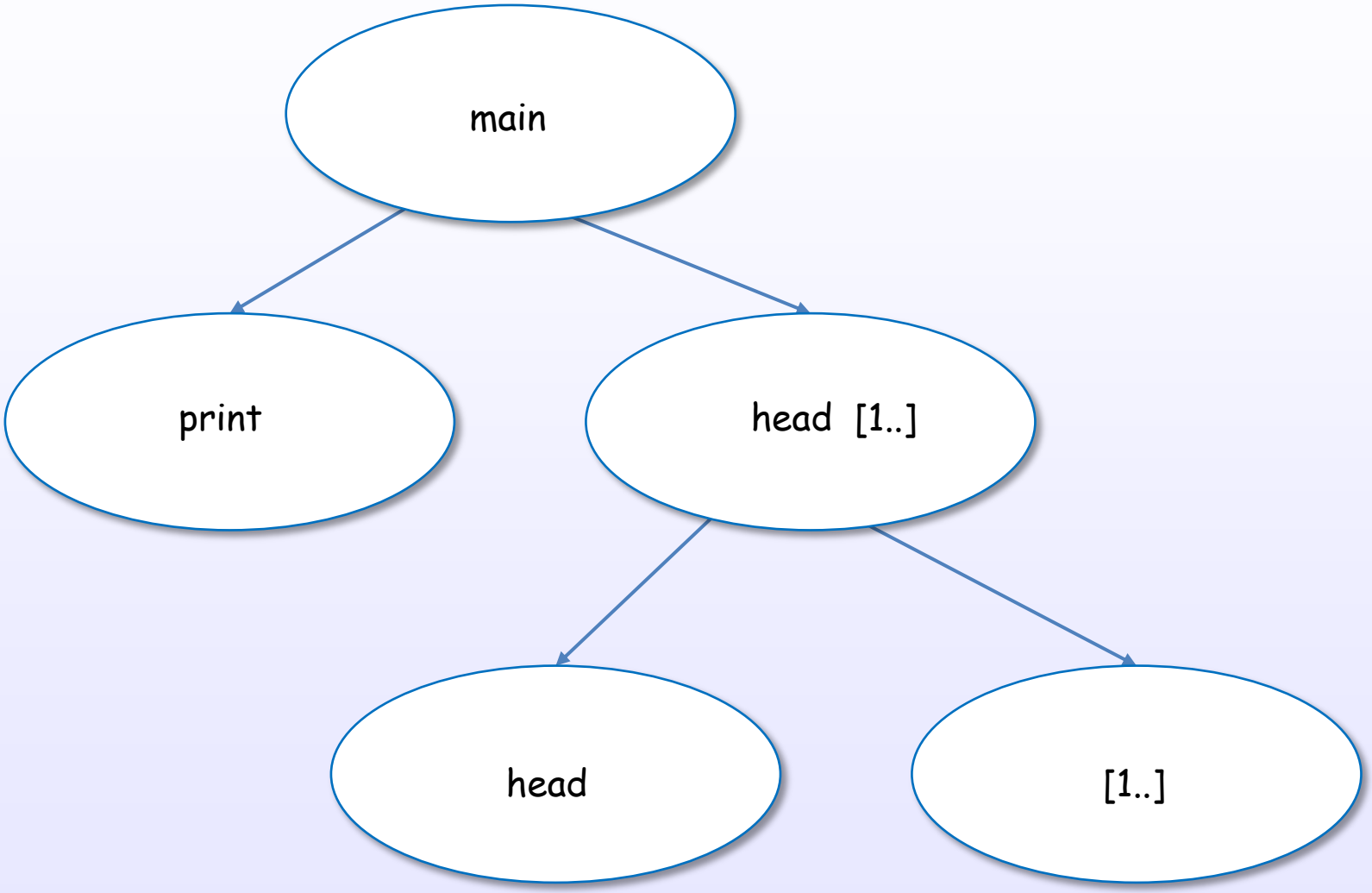
main = print (head [1..])



References : [H5], [H10]

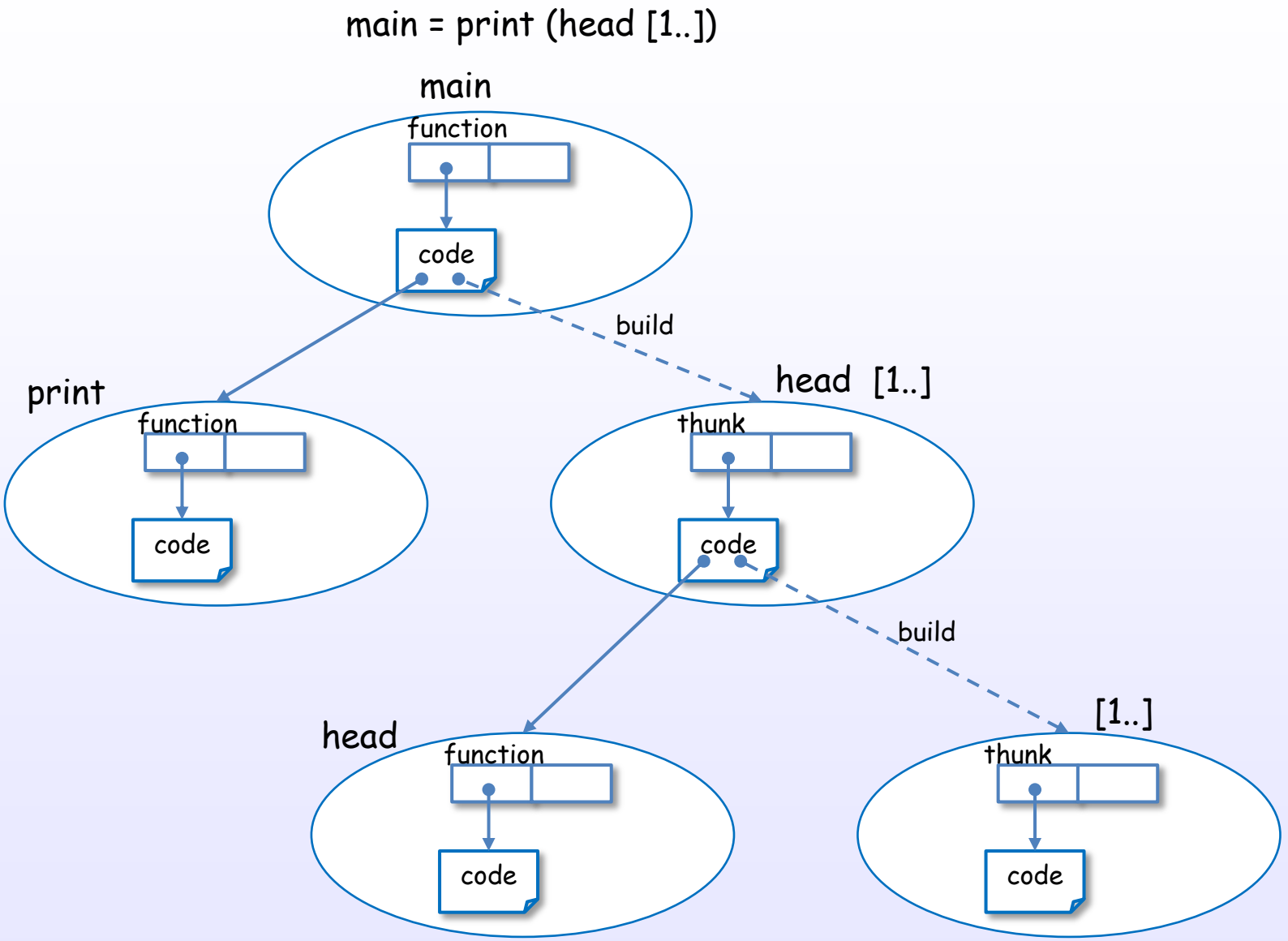
Example of mapping a code to a graph

main = print (head [1..])



References : [H5], [H10]

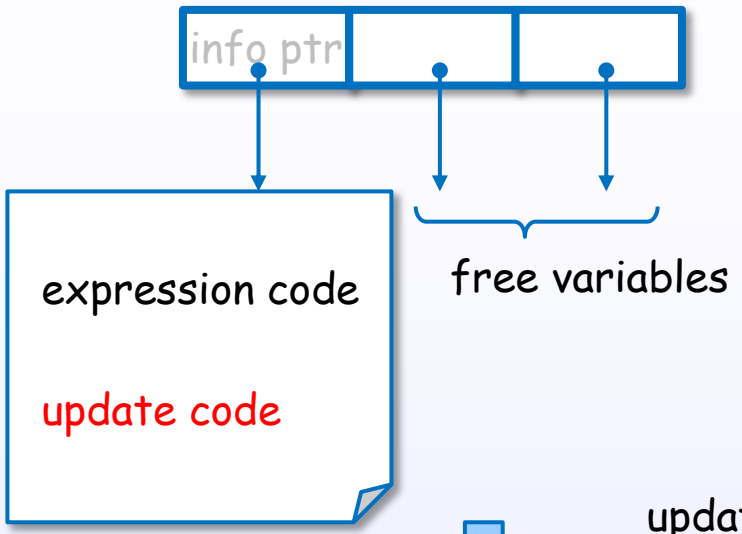
Example of mapping a code to a graph



References : [H5], [H10]

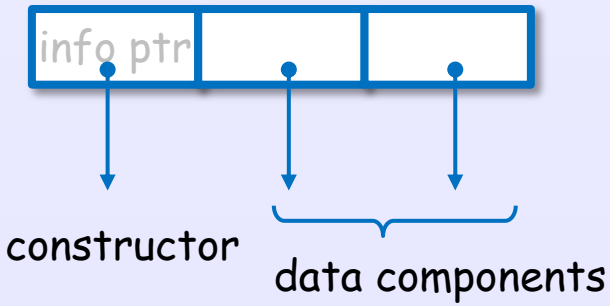
Self-updating model

a thunk



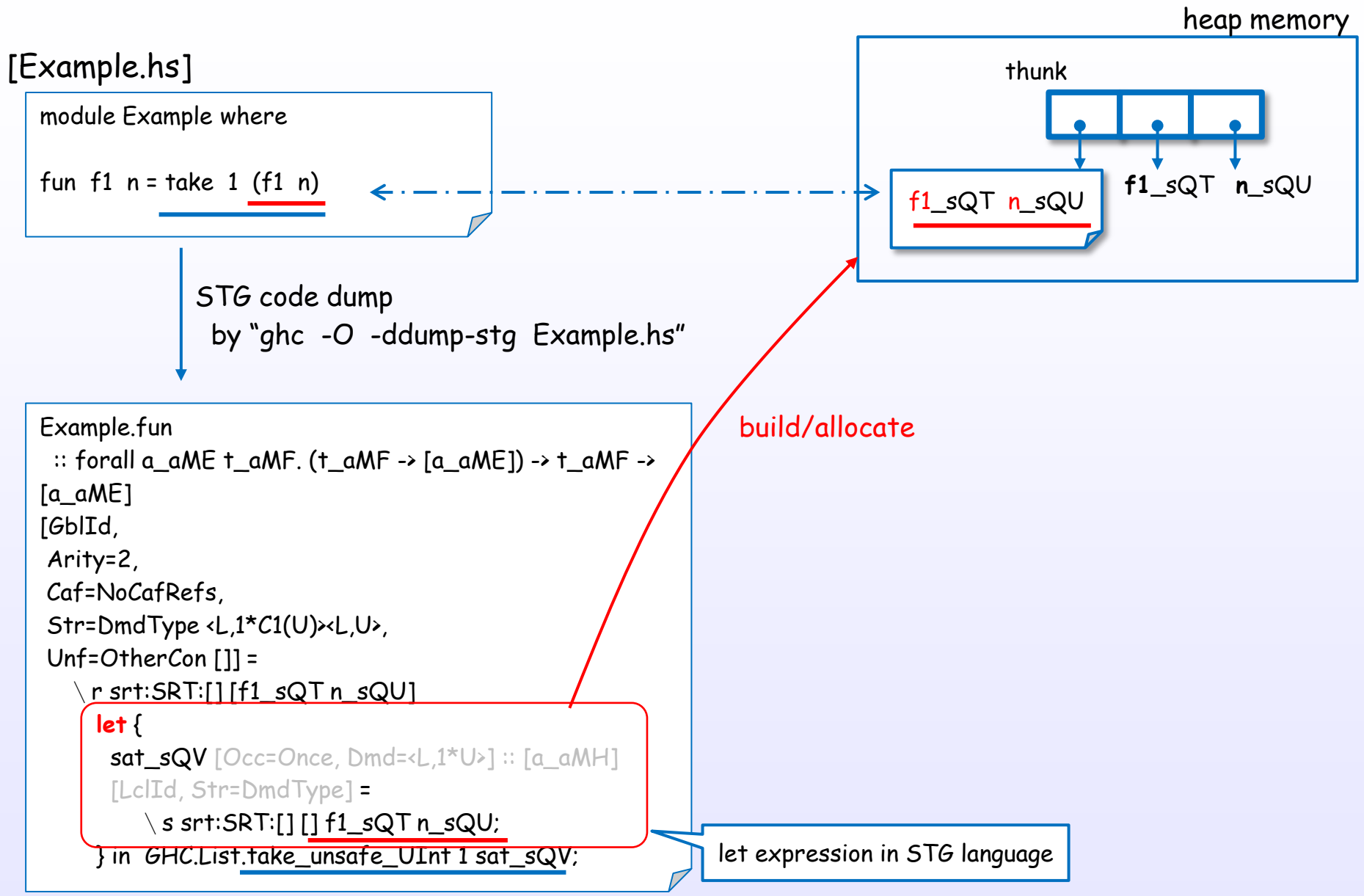
update
(replace myself to result value)

a data value



References : [H5], [H6], [H7], [D15]

STG-dump shows which expression is build as thunks

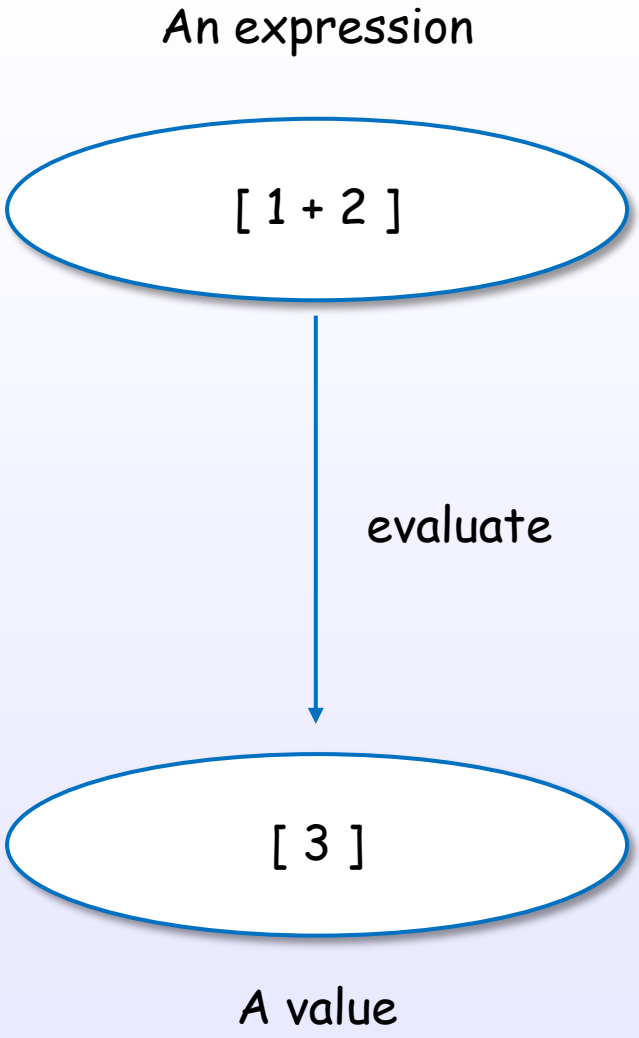


References : [H5], [H6], [H7], [D15]

6. Semantics

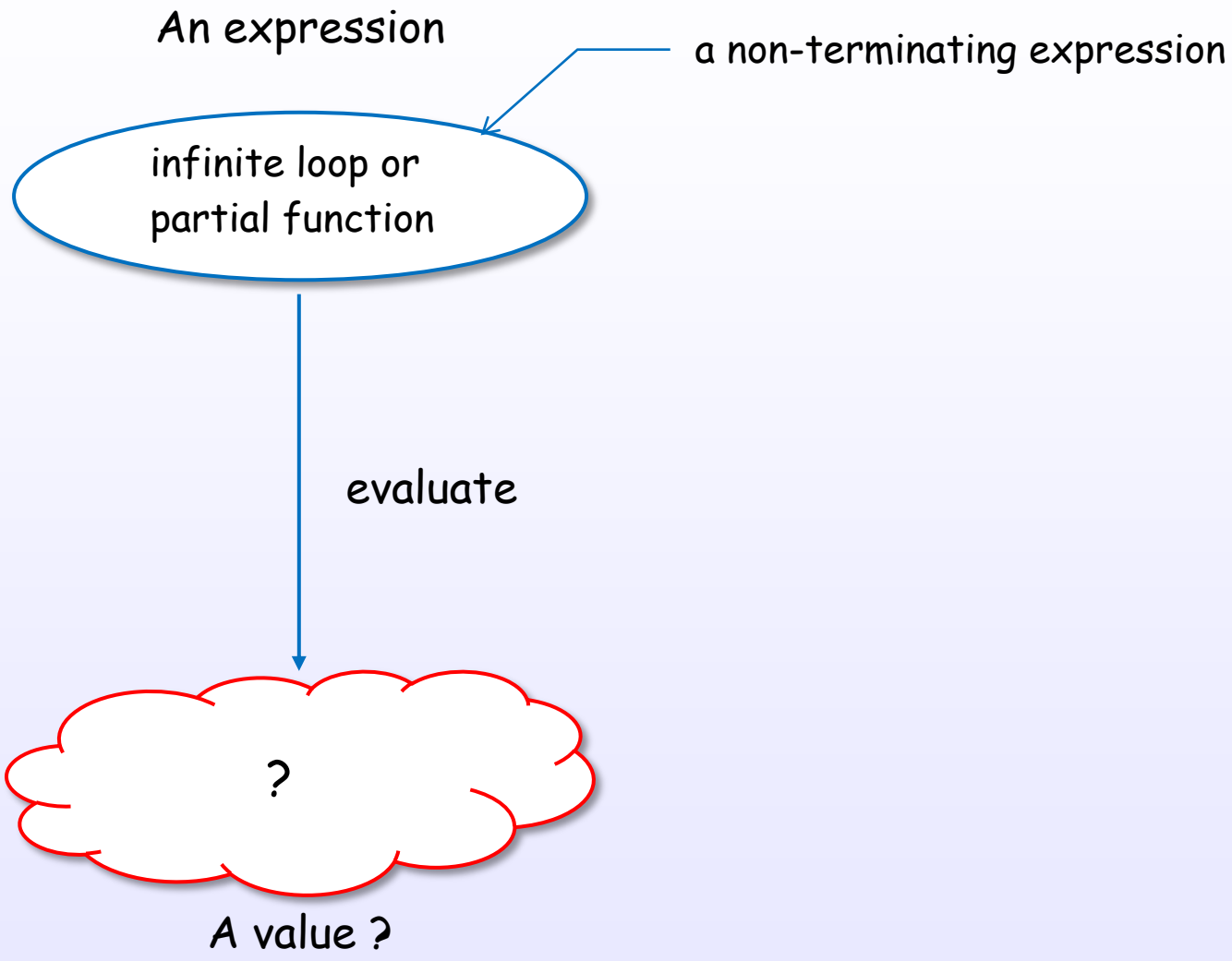
Bottom

Well formed expression should have a value



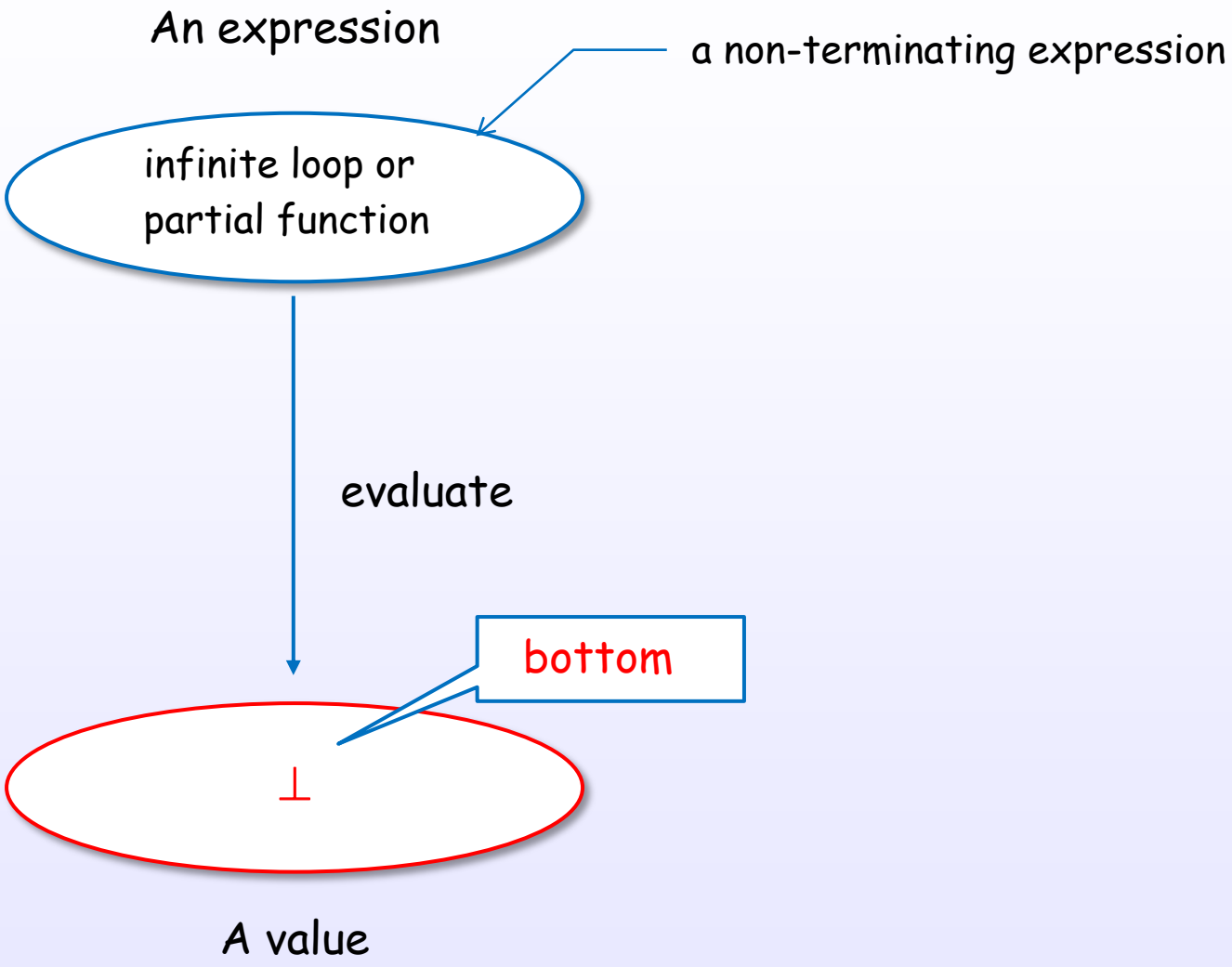
References : [B2] Ch.2, [W4]

What is a value in this case?



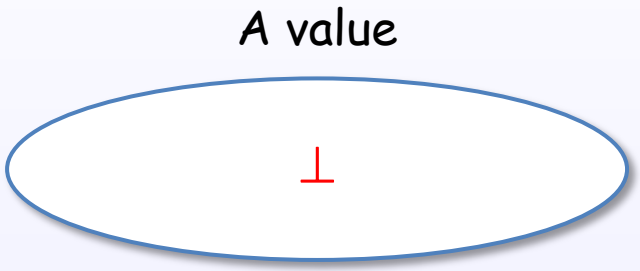
References : [B2] Ch.2, [H1] Ch.3, [W4], [H4] Ch.2, 22

A value "bottom" is introduced



References : [B2] Ch.2, [H1] Ch.3, [W4], [H4] Ch.2, 22

Bottom



Bottom (⊥) is "an undefined value".

Bottom (⊥) is "a non-terminating value".

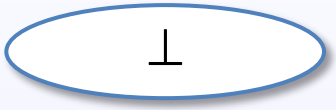
References : [B2] Ch.2, 9, [H1] Ch.3, [W4], [H4] Ch.2, 22

"undefined" represents bottom in Haskell

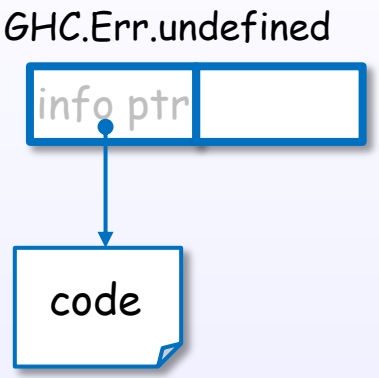
Haskell code

```
undefined :: a
```

Expression



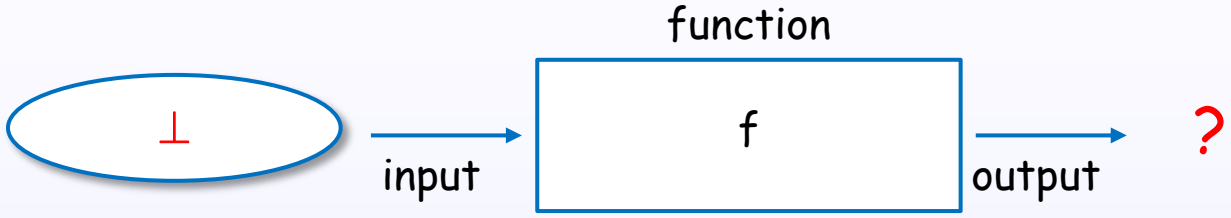
GHC's internal representation



References : [B2] Ch.2, 9, [H1] Ch.3, [W4], [H4] Ch.2, 22

Non-strict Semantics

Strictness

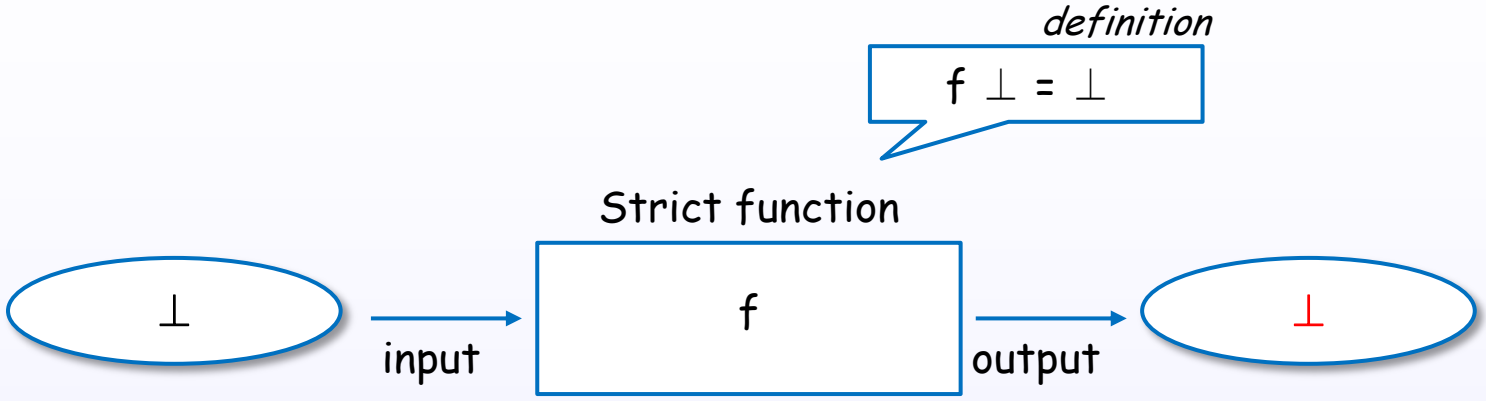


Strictness is property of the function.

"given a non-terminating arguments, the function will terminate?"

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Strict function

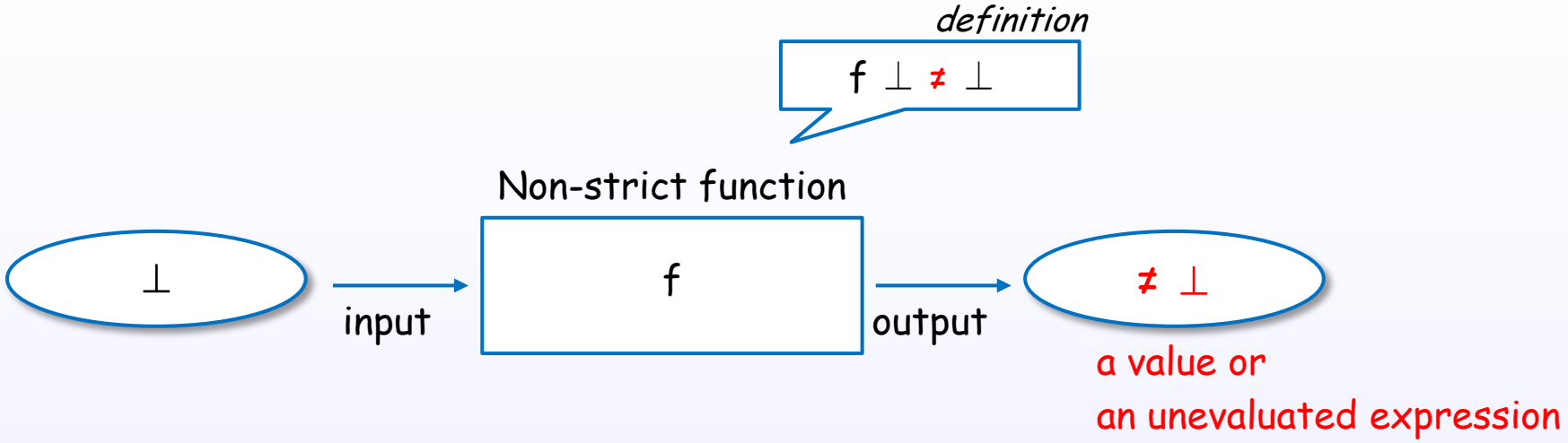


Strict function's output is bottom when input is bottom.

given a non-terminating arguments, strict function will not terminate.

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Non-strict function



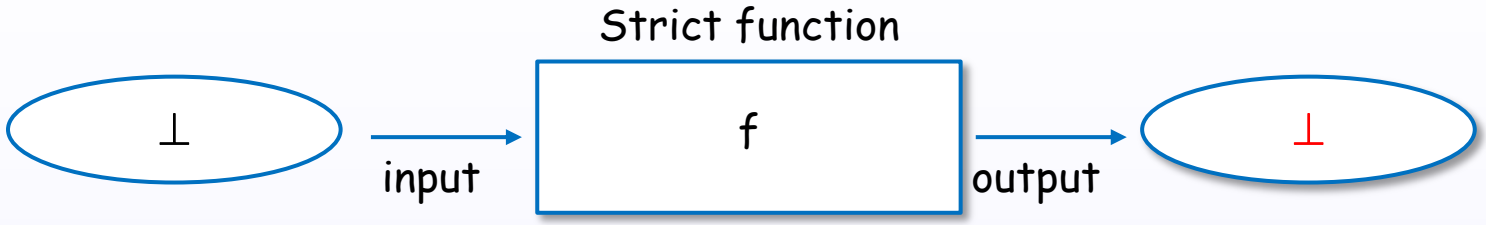
Non-strict function's output is not bottom when input is bottom.

given a non-terminating arguments, non-strict function will terminate.

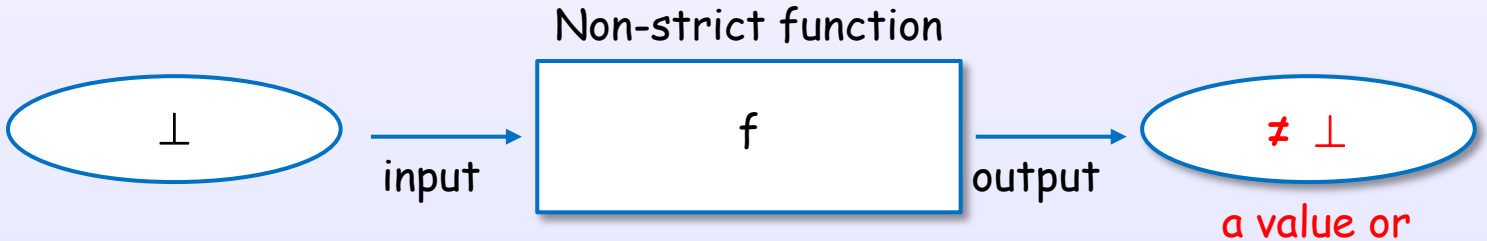
References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Strict and Non-strict functions

Strict



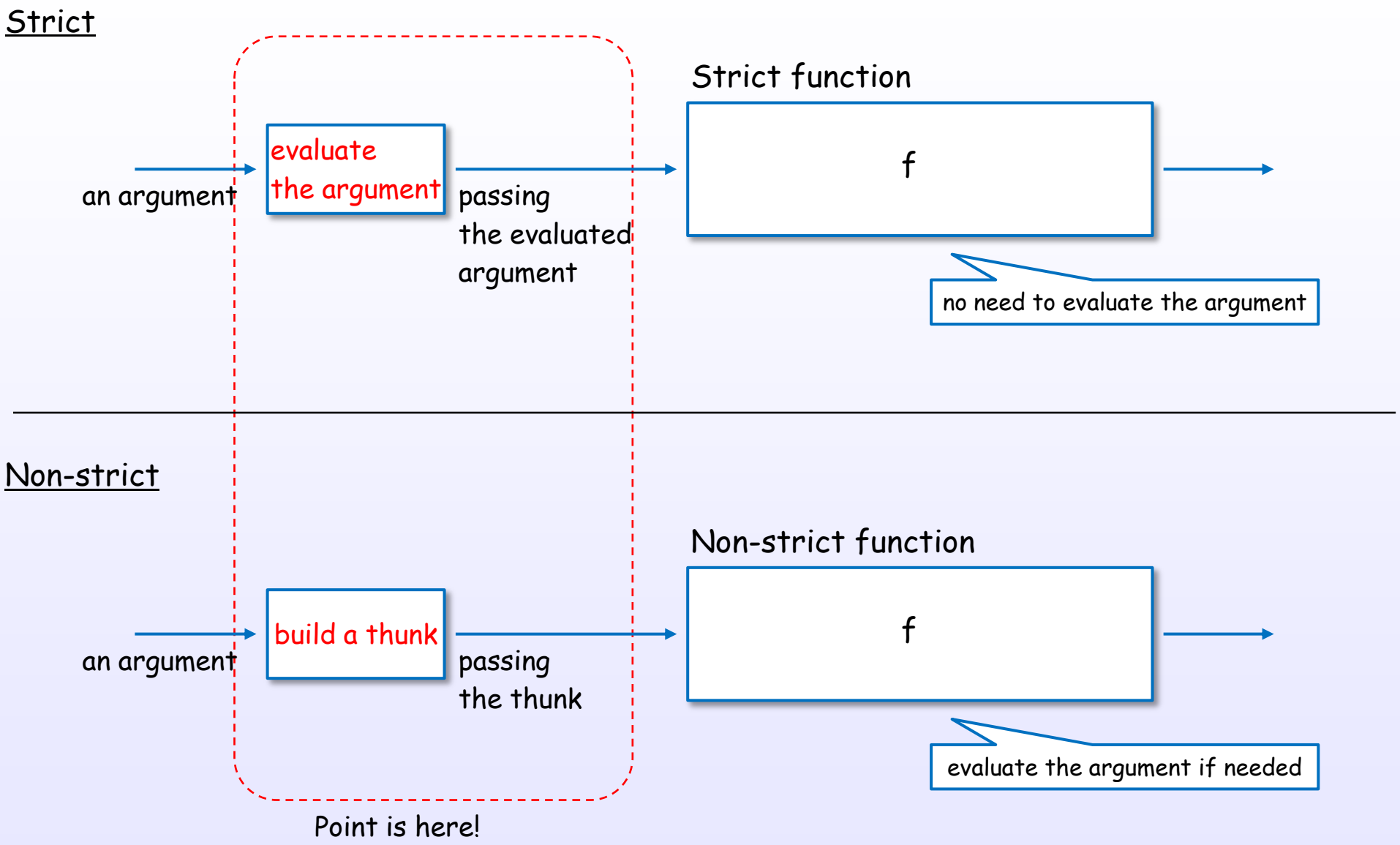
Non-strict



a value or
an unevaluated expression

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

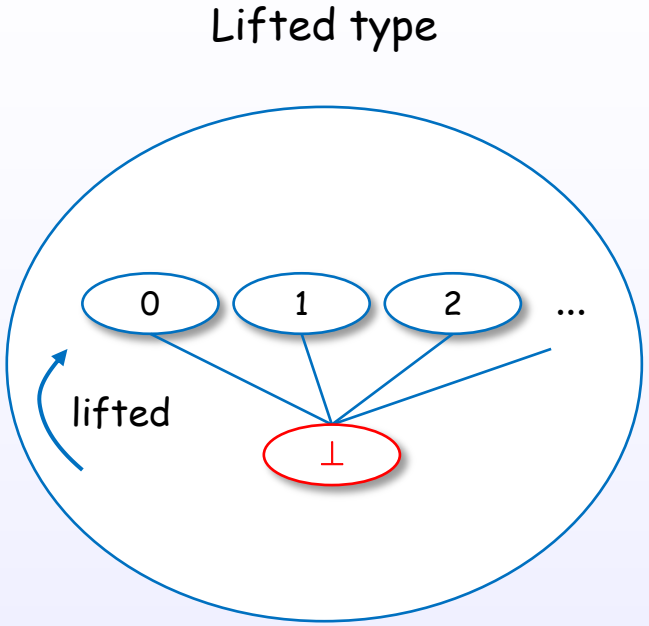
Function application and strictness



References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Lifted and boxed types

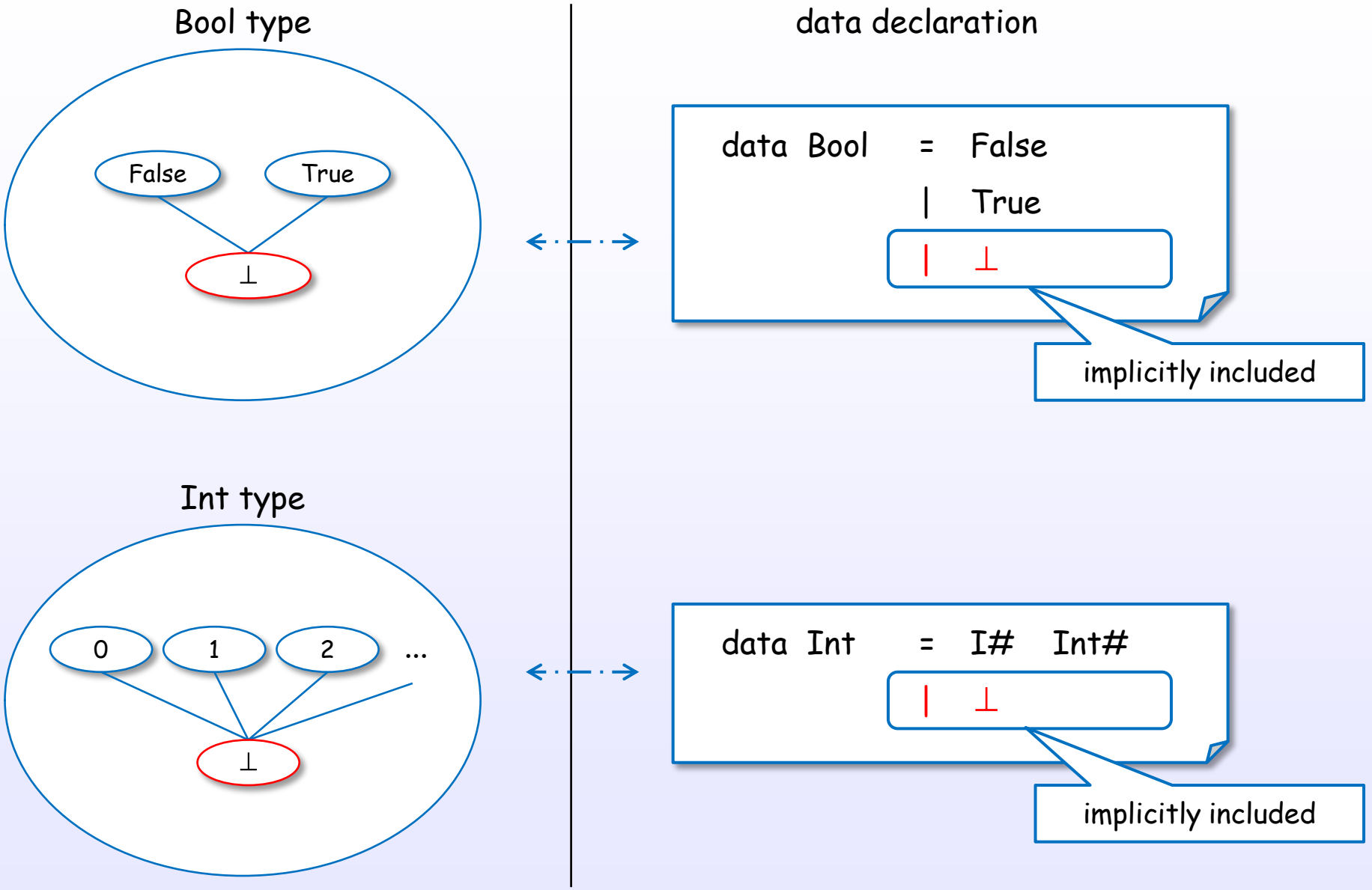
Lifted types



Lifted types include bottom as an element.

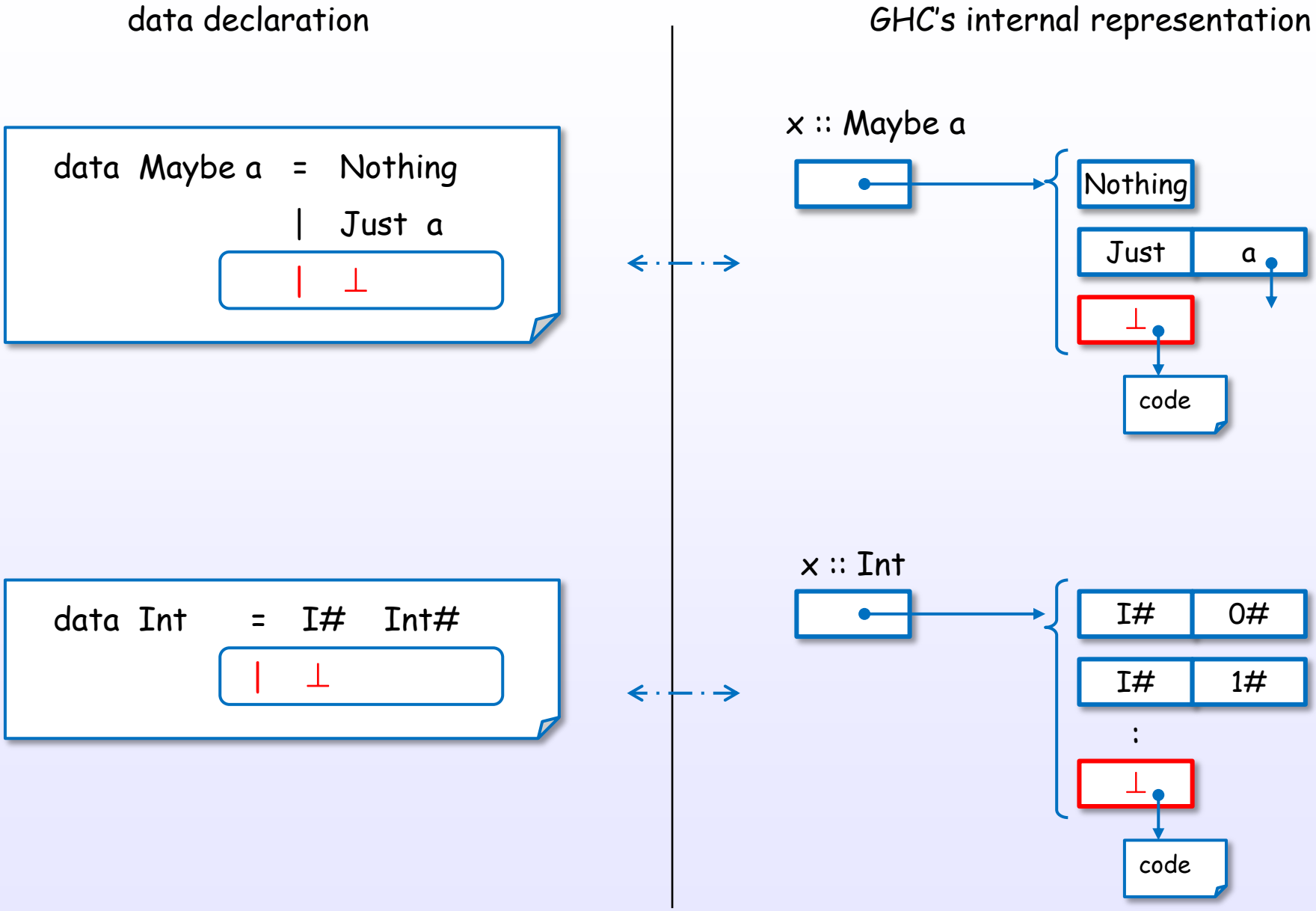
References : [W4], [B2], [H14]

Lifted type's declaration implicitly include bottom



References : [W4], [B2], [H14]

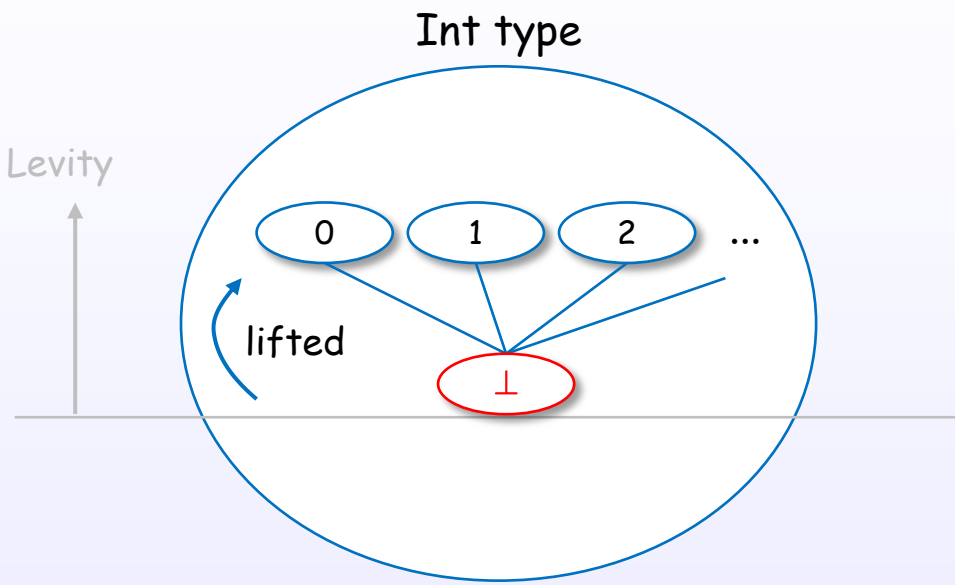
Lifted type are also implemented by uniform representation



References : [W4], [B2], [H14]

Lifted and unlifted types

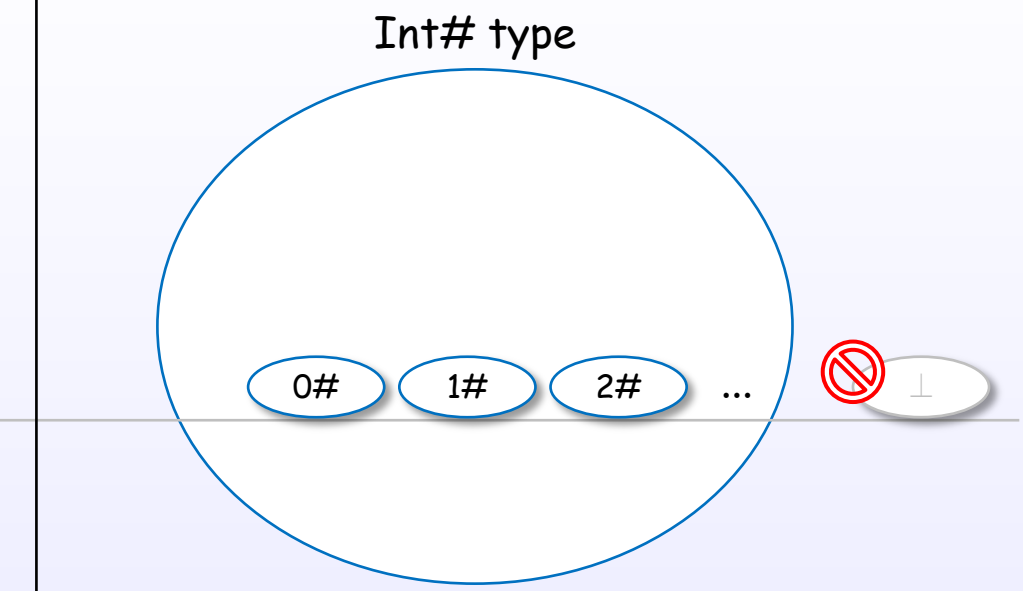
Lifted types



Lifted types include bottom.

(Bool, Int, Char, Maybe, List, ...)

Unlifted types



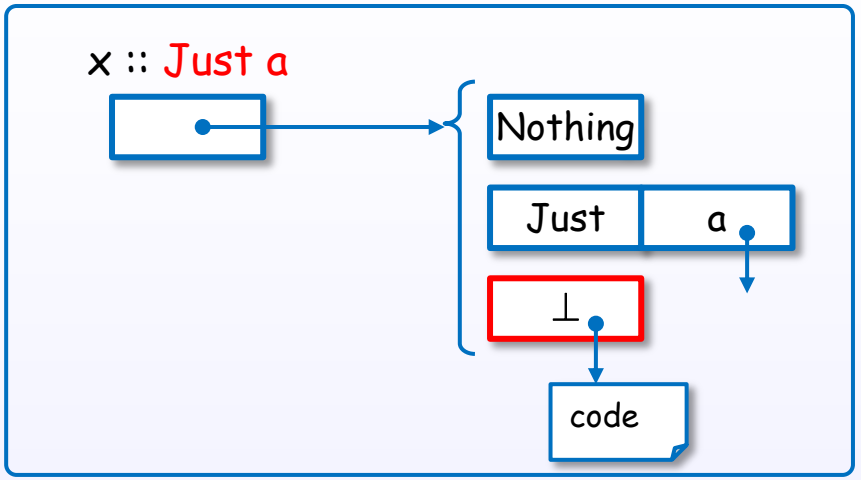
Unlifted types do not include bottom.

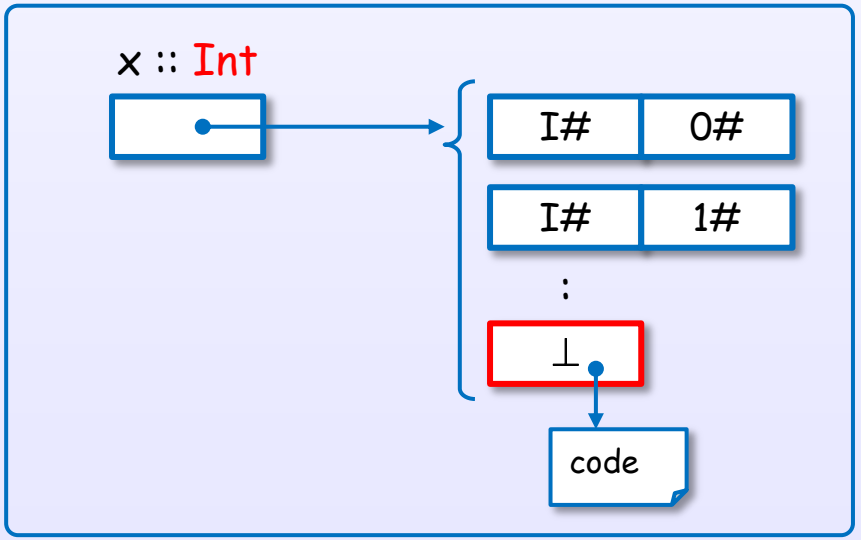
(Int#, Char#, Addr#, Array#, ByteArray#, ...)

References : [W4], [B2], [H14]

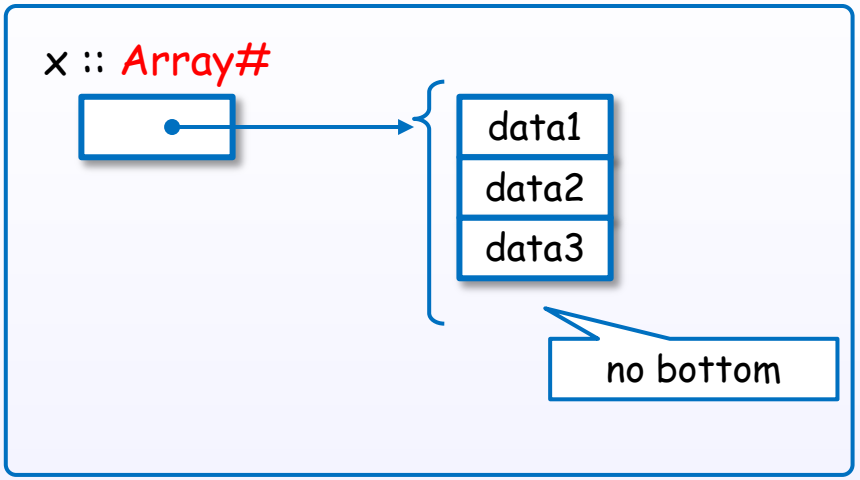
Example of lifted and unlifted types

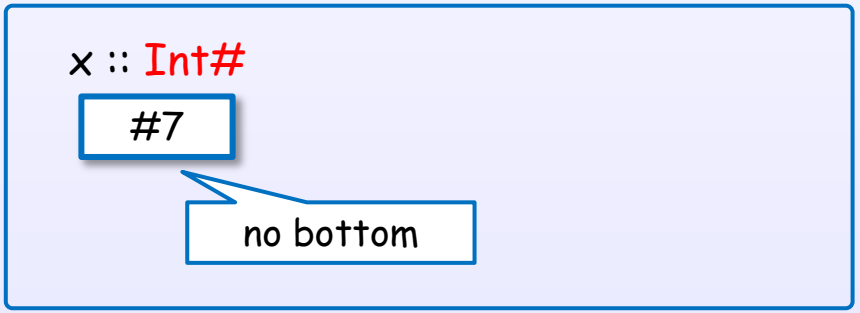
Lifted types





Unlifted types





References : [W4], [B2], [H14]

Boxed and unboxed type

Boxed types

Boxed types are represented as a pointer.

Unboxed types

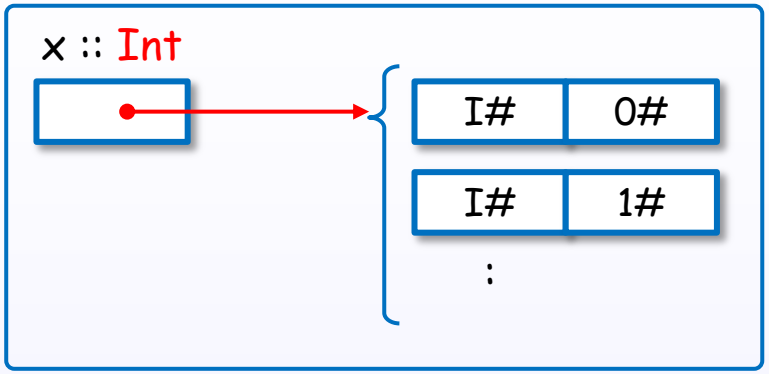
Unboxed types are represented other than a pointer.

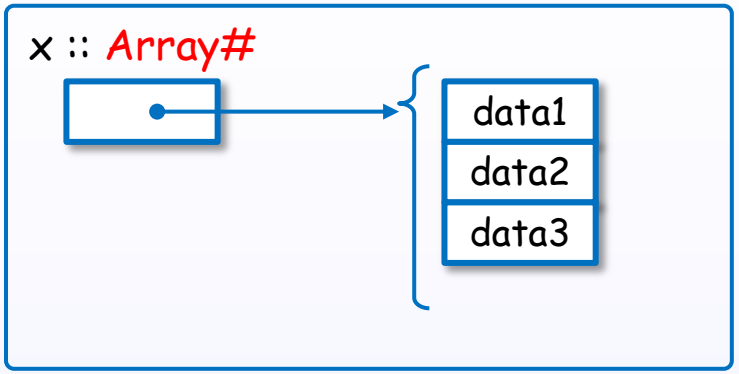
- no bottom (can't be lifted)
- no thunk (can't be postponed)
- no polymorphism (non-uniform size)
+ low cost memory size (no pointer)
+ high performance (no wrap/unwrap)

References : [W4], [B2], [H14]

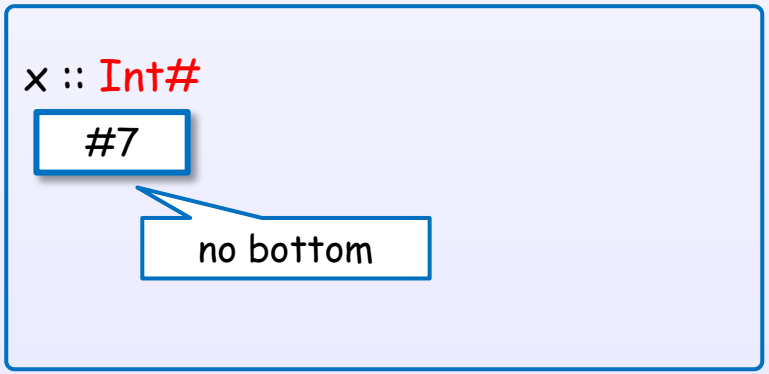
Example of boxed and unboxed types

Boxed types





Unboxed types



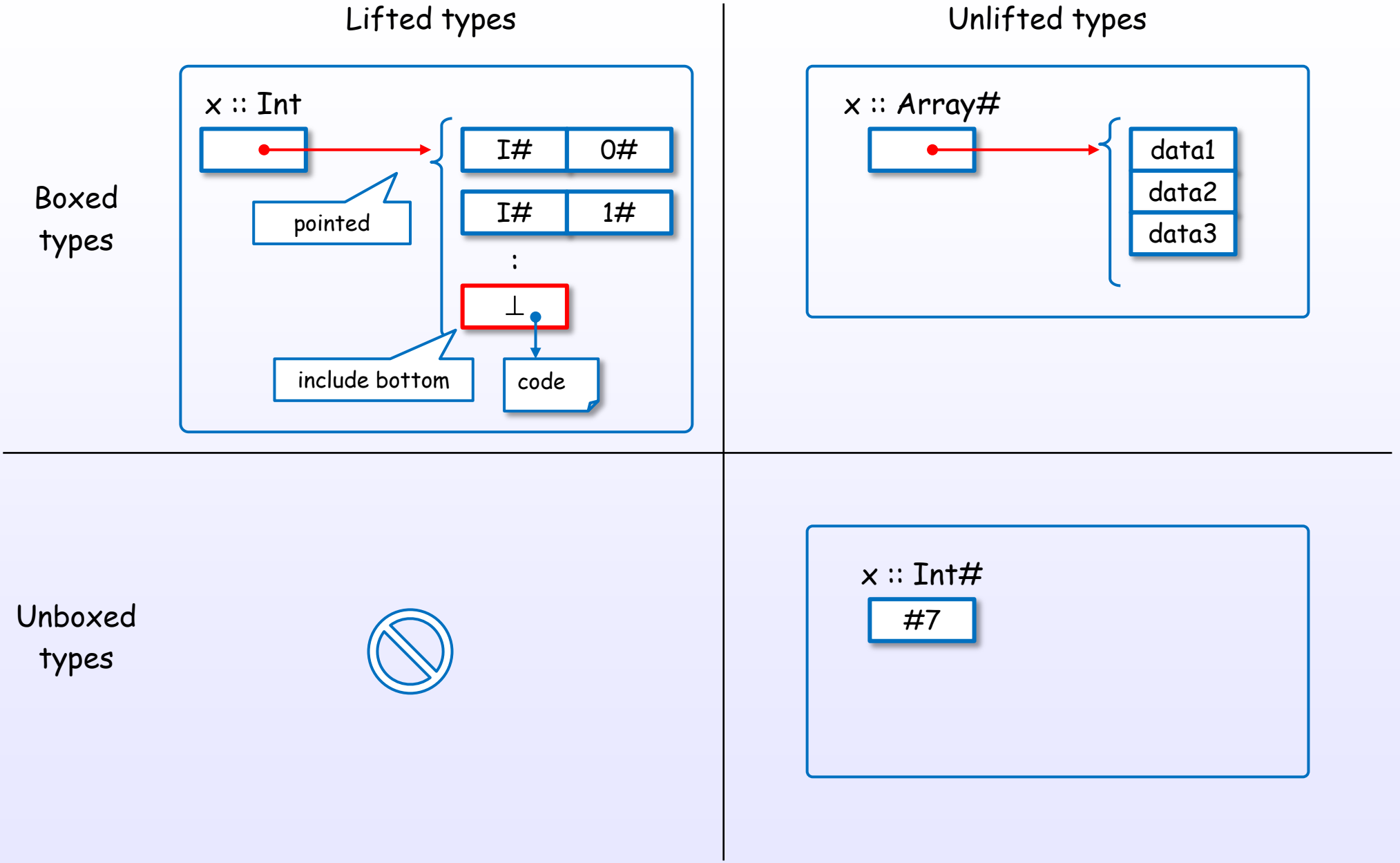
References : [W4], [B2], [H14]

Lifted and boxed type

	Lifted types	Unlifted types
Boxed types	Int Char Float Maybe :	Array# ByteArray# : no bottom
Unboxed types	 unboxed can't be lifted	Int# Char# Float# : no bottom no thunk

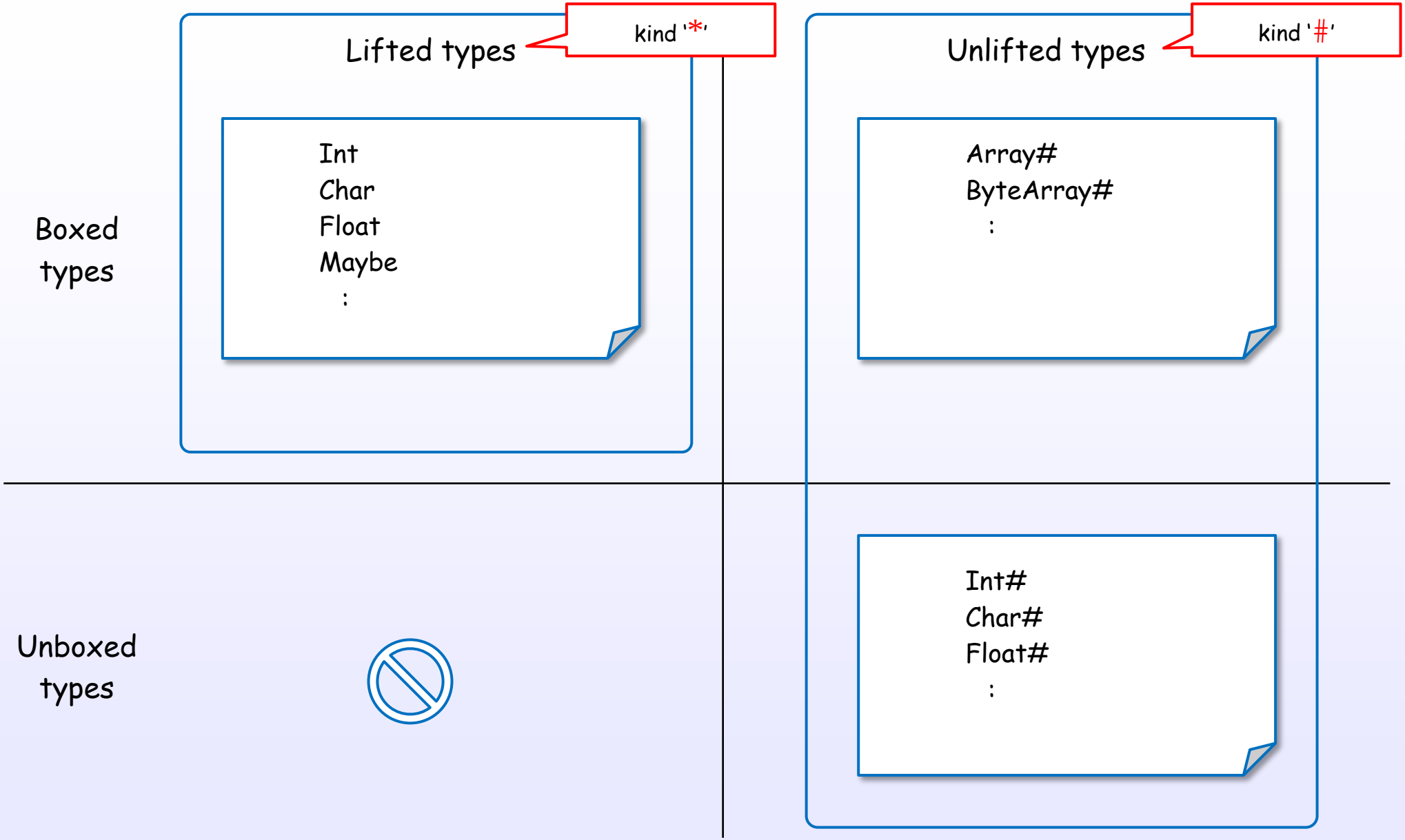
References : [W4], [B2], [H14]

Example of lifted and boxed type



References : [W4], [B2], [H14]

Kinds and types



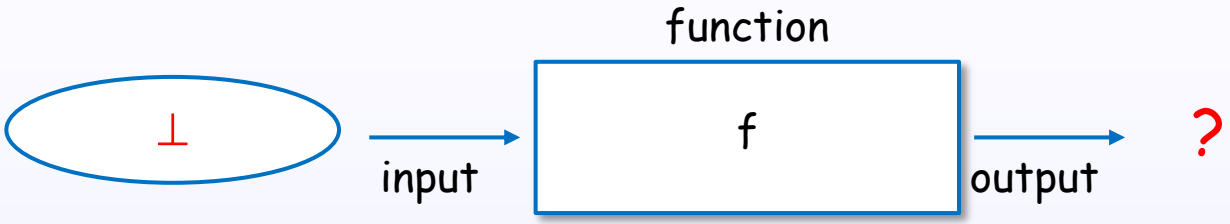
Note:
Identifier's '#' customarily means "primitive" rather than "unboxed" or "unlifted".
Kind's '#' means "unlifted".

References : [B6] Ch.29, [W4], [B2], [H14]

6. Semantics

Strict analysis

Strict analysis

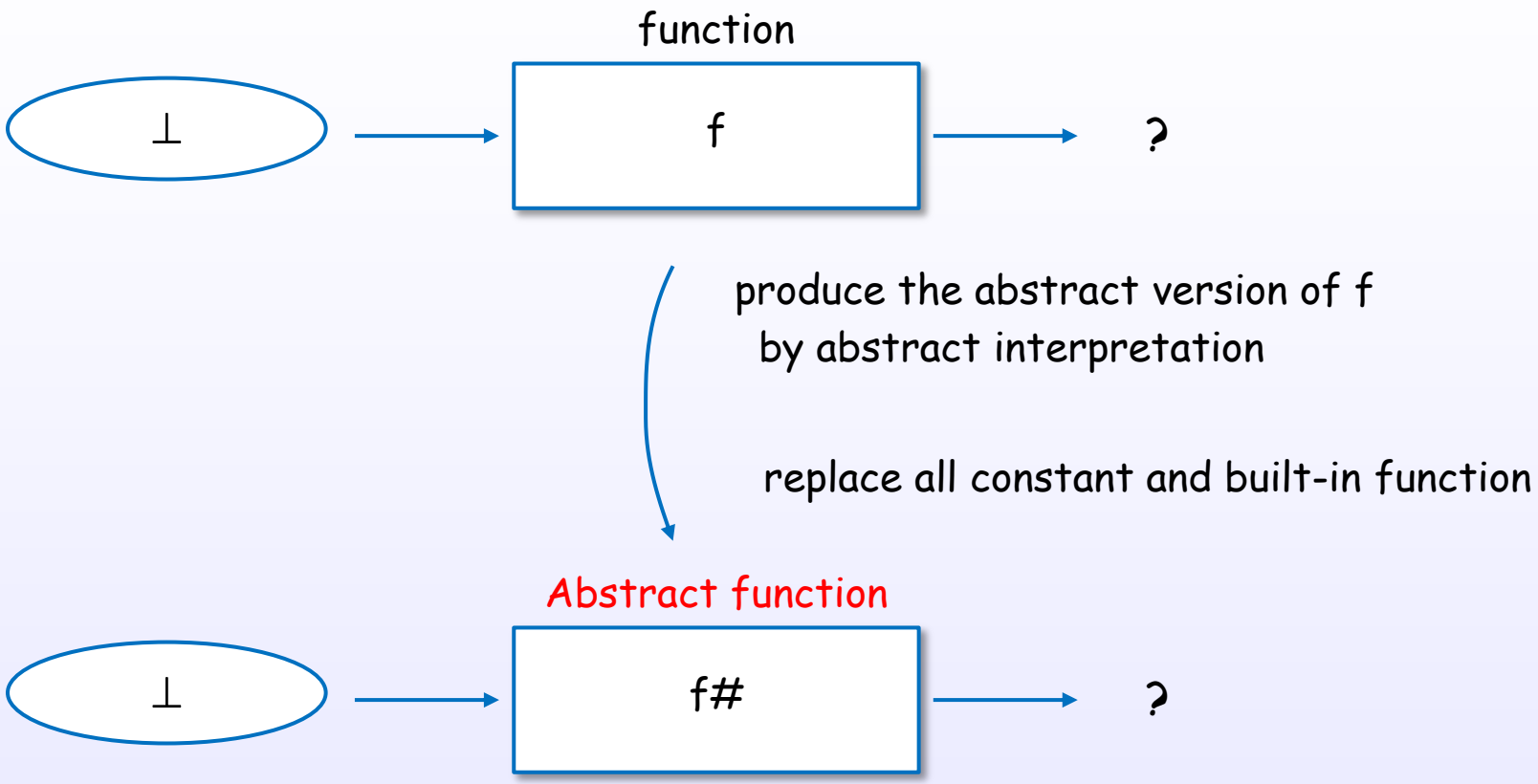


Strictness analysis analyzes whether a function is sure to evaluate its argument.

"-ddump-stranal" and "-ddump-strsigs" show strictness analysis information.

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H16], [H13], [H2]

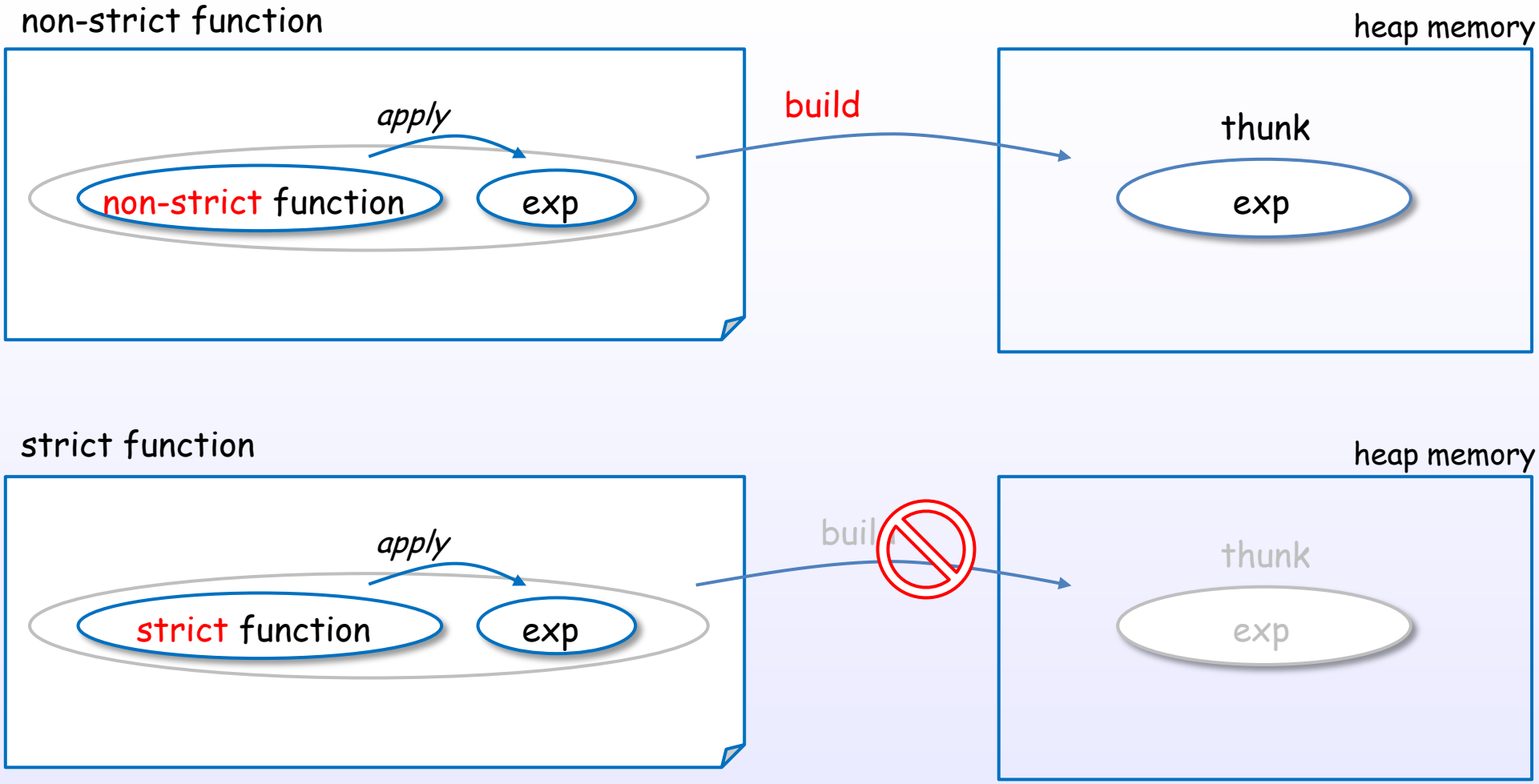
Strict analysis using abstract function



Abstract function can decide strictness without going via full evaluation.

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H16], [H13]

Strictness analysis are used to avoid the thunk

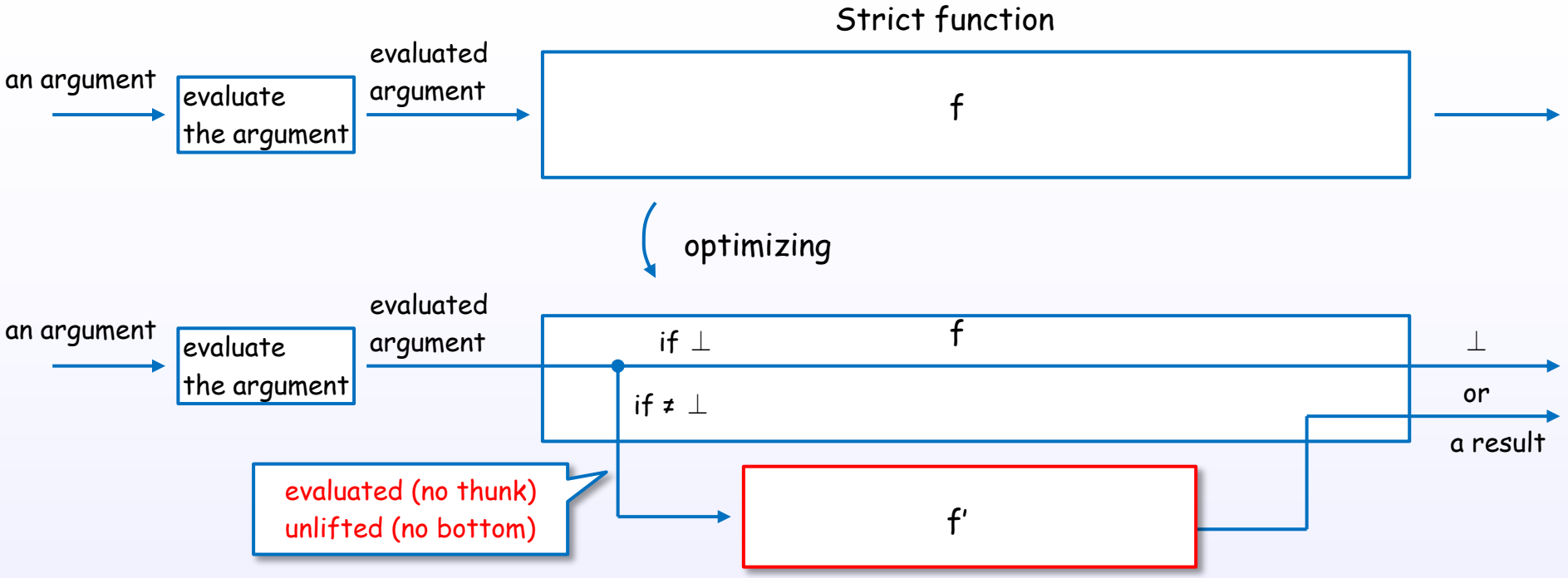


If GHC knows that a function is strict, arguments is evaluated before application.

GHC finds strict functions by "strictness analysis (demand analysis)".

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H16], [H13]

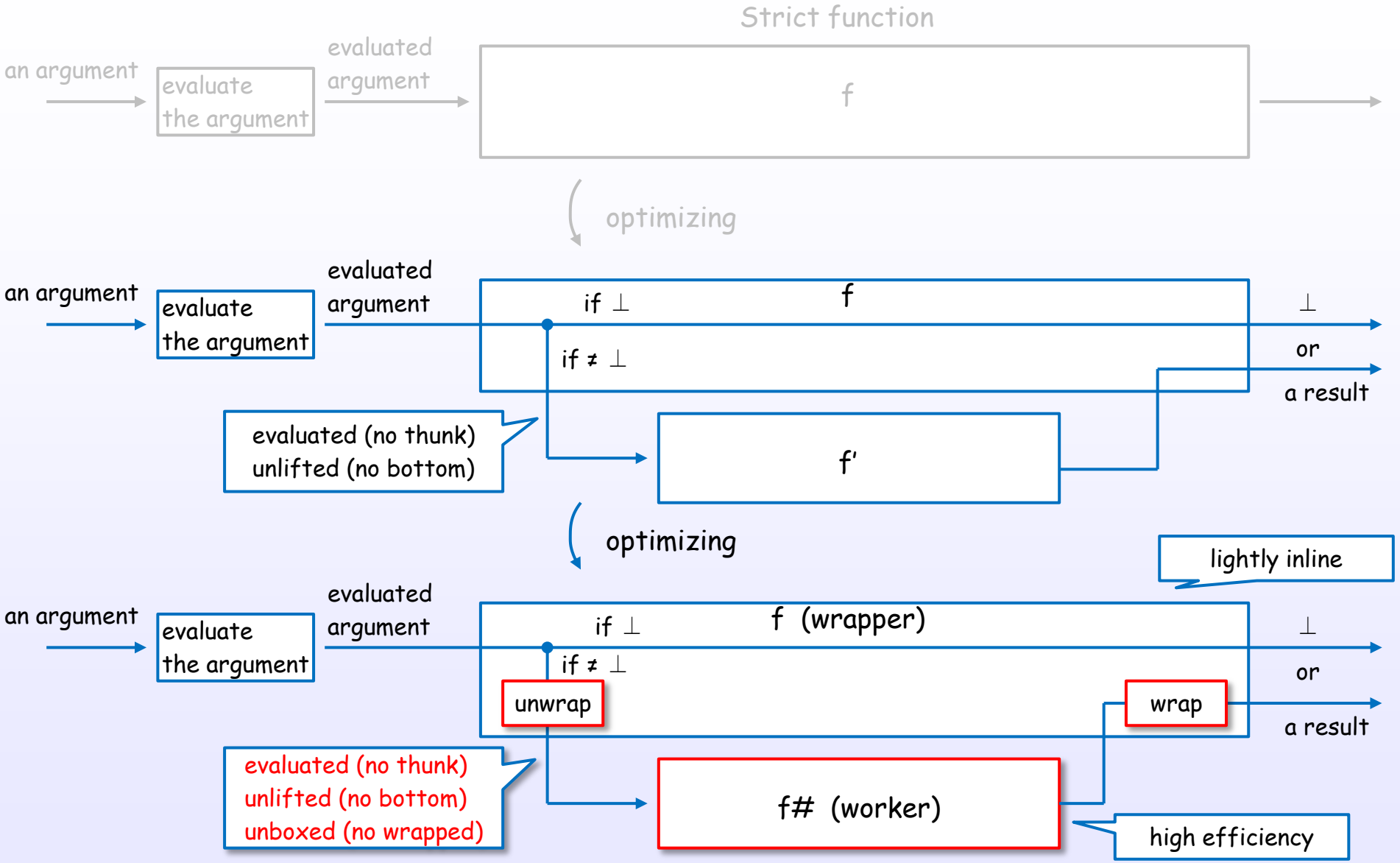
Strictness analysis are also used to optimize



Strictness function can be optimized to assume no thunk, no bottom.

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H16], [H13]

Strictness analysis are also used to optimize



Strictness function can be optimized to assume no thunk, no bottom, no wrap.

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H16], [H13]

6. Semantics

Sequential order

"seq" doesn't guarantee the evaluation order

specification

```
seq a b  =  ⊥,     if a = ⊥
         =  b,     otherwise
```

strictness for each arguments

```
seq ⊥ b  =  ⊥      // a is strict
seq a ⊥  =  ⊥      // b is strict
```

"seq" function only guarantee that it is strict in both arguments.
This semantics property makes no operational guarantee about order of evaluation.

References : [H9], [D11], [H1] Ch.6, [S1]

"seq" and "pseq"

specification

```
seq a b  =  ⊥,     if a = ⊥
         =  b,     otherwise
```

```
seq ⊥ b  =  ⊥      // a is strict
seq a ⊥  =  ⊥      // b is strict
```

specification

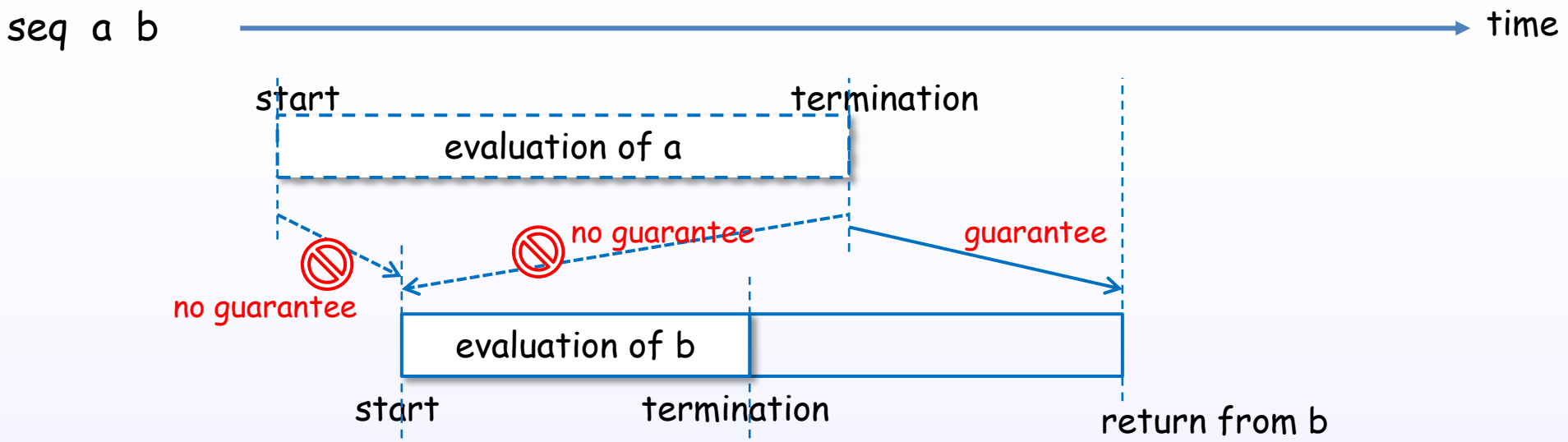
```
pseq a b  =  ⊥,     if a = ⊥
          =  b,     otherwise
```

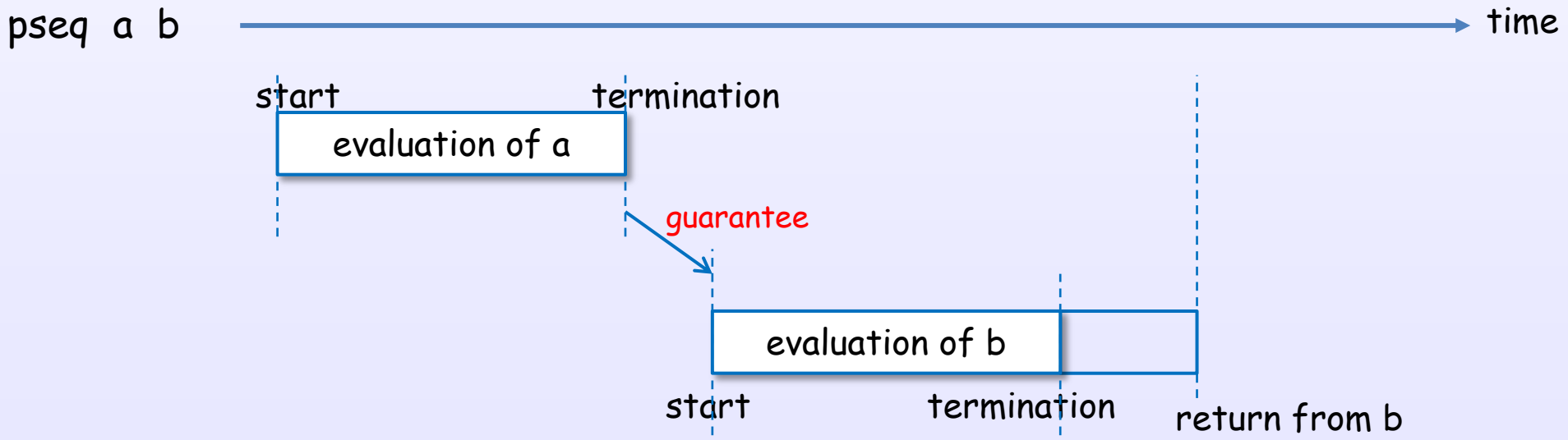
```
pseq ⊥ b  =  ⊥      // a is strict
pseq a ⊥  =  ⊥      // b is strict
```

Both of denotational semantics are the same.
But "pseq" makes operational guarantee about order of evaluation.

References : [H9], [D11], [H1] Ch.6, [S1]

Evaluation order of "seq" and "pseq"





References : [H9], [D11], [H1] Ch.6, [S1]

Implementation of "seq" and "pseq"

specification

```
seq a b  =  ⊥,     if a = ⊥
         =  b,     otherwise
```

Haskell's built-in

specification

```
pseq a b  =  ⊥,     if a = ⊥
          =  b,     otherwise
```

```
pseq x y = x `seq` lazy y
```

GHC's "lazy" function restrains the strictness analysis.

"seq" is built-in function.
"pseq" is implemented by built-in functions ("seq" and "lazy").

References : [H9], [D11], [H1] Ch.6, [H2] Ch.7, [S1]

7. Appendix

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Lazy,... zzz

to be as lazy as possible...