Lazy evaluation in Haskell

exploring some mental models and implementations

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Rev. 0.00.0
WIP

Lazy,... zzz

..., It's fun.

NOTE

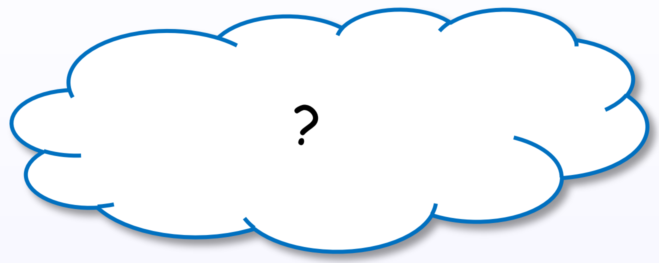
- Meaning of terms are different by communities.
- There are a lot of good documents. Please see also references.
- This is written for GHC's Haskell.

Contents

Introduction

What is an expression?

An expression



References : [1]

An expression denotes a value

An expression

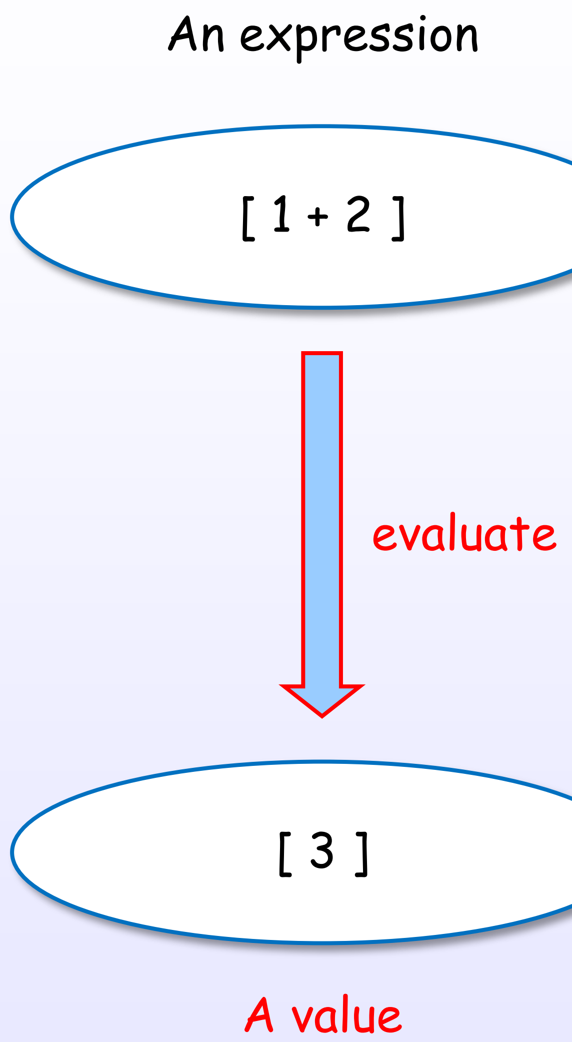
[1 + 2]

[HR2010]

[Bird, Chapter 2]

References : [1]

An expression evaluates to a value

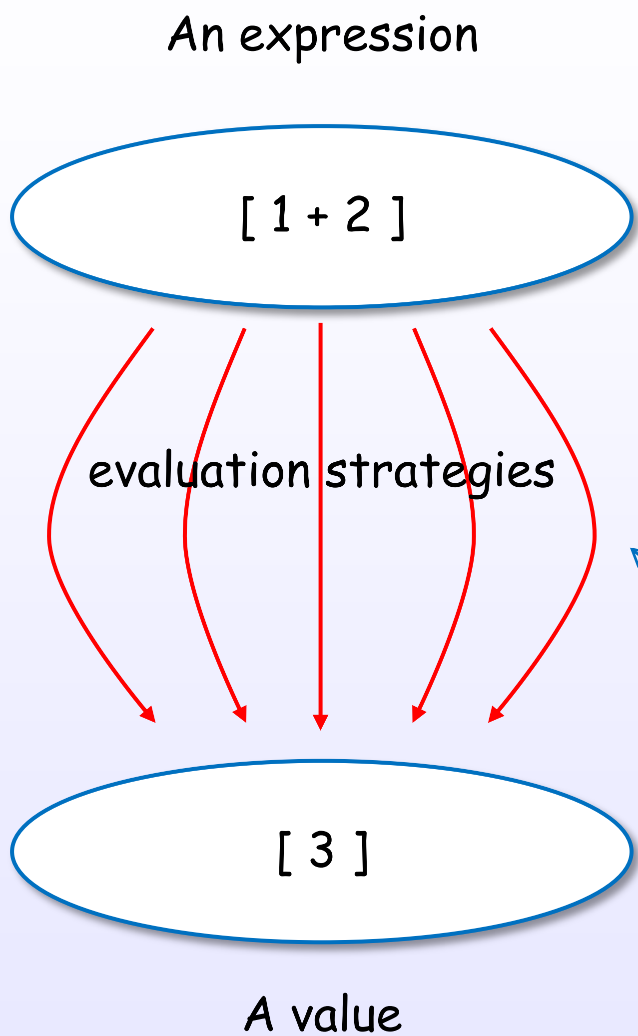


[HR2010]

[Bird, Chapter 2]

References : [1]

There are many evaluation approaches



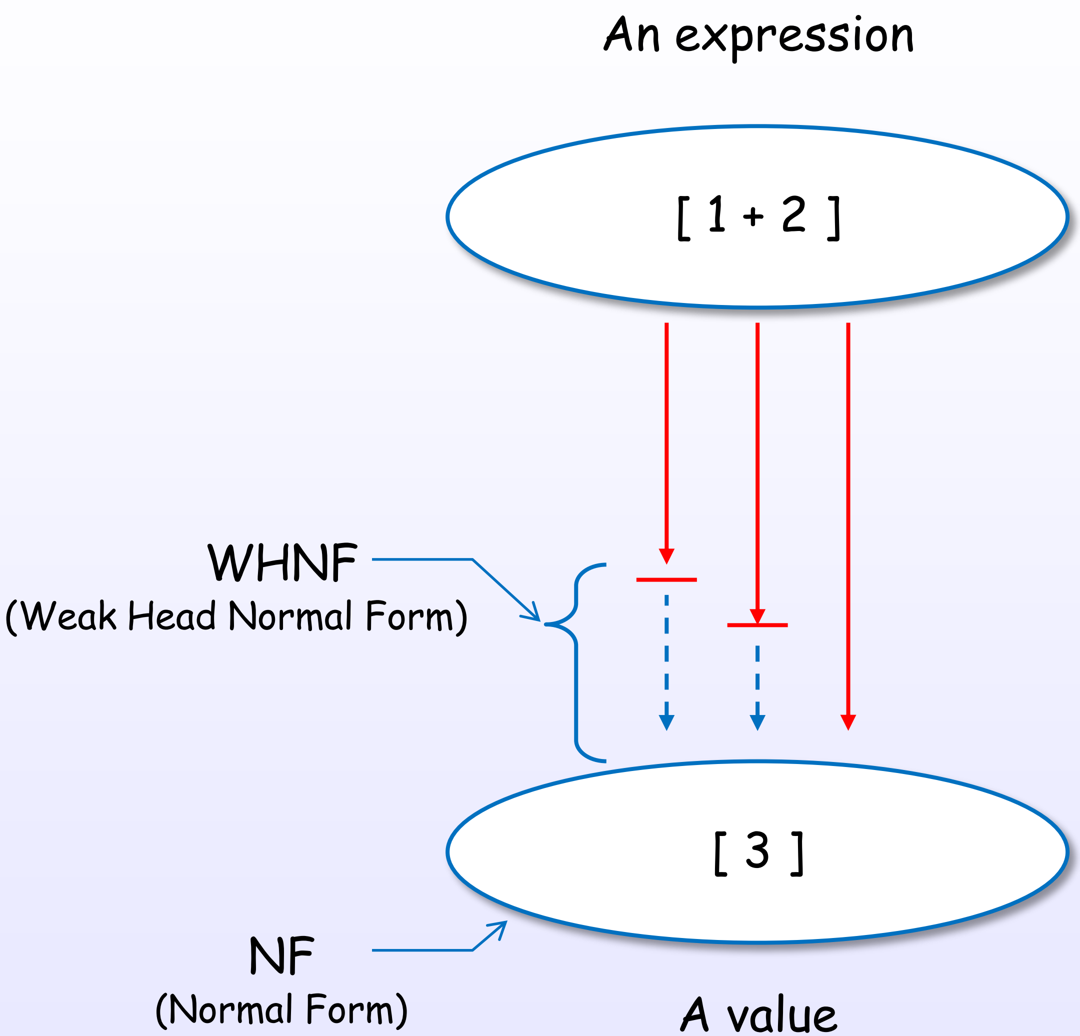
- Strict, Non-strict evaluation
- Eager, Lazy evaluation
- Call-by-value, Call-by-name, Call-by-need, …
- Innermost, Outermost
- Normal order, Application order
- …

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

There are some evaluation levels

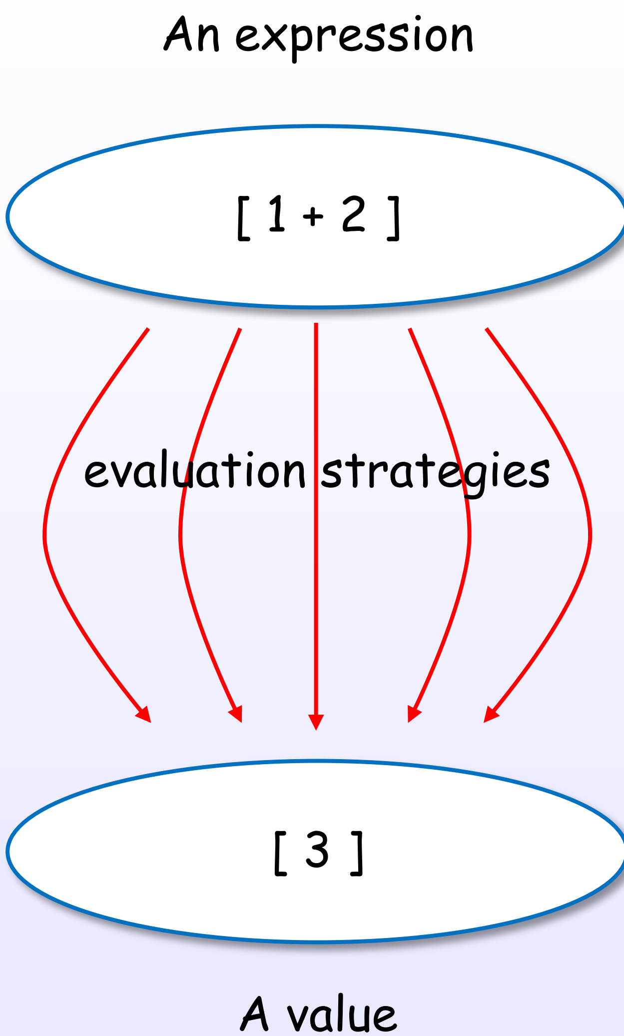


[Terei]
[Bird, Chapter 2, 7]
[TAPL, Chapter 3]

References : [1]

Evaluations

There are many evaluation approaches



- Strict, Non-strict evaluation
- Eager, Lazy evaluation
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  Call-by-need, …
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- …

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

Evaluation layers

denotational semantics

evaluation strategy

evaluation implementation

[Bird, Chapter 7]

[Hutton, Chapter 8]

[TAPL, Chapter 3]

References : [1]

Evaluation layers

denotational semantics	Strict semantics		Non-strict semantics	
evaluation strategy	Eager evaluation (Strict evaluation)	Nondeterministic evaluation	Lazy evaluation (Non-strict evaluation)	…
	Call-by-Value		Call-by-Name, Call-by-Need	…
evaluation implementation			Graph reduction	…

[Bird, Chapter 7]
[Hutton, Chapter 8]
[TAPL, Chapter 3]

References : [1]

Evaluation layers for GHC's Haskell

denotational semantics	Strict semantics		Non-strict semantics		
evaluation strategy	Eager evaluation (Strict evaluation)	Nondeterministic evaluation	Lazy evaluation (Non-strict evaluation)		…
	Call-by-Value		Call-by-Name	Call-by-Need	…
evaluation implementation				Graph reduction	…

Haskell 2010 specification

GHC's strategy

GHC's strategy

GHC's implementation

References : [1]

Simple example of both evaluations

default
C, Java, JavaScript, Python, OCaml, Scheme, ...

Eager evaluation
(Strict evaluation)

square (1 + 2)

argument evaluation first

⇩

square (3)

⇩

3 * 3

⇩

9

default
Haskell (GHC), ...

Lazy evaluation
(Non-strict evaluation)

square (1 + 2)

apply first

⇩

(1 + 2) * (1 + 2)

⇩

(3) * (3)

⇩

9

[Bird]
[Hutton]

References : [1]

Simple example of both evaluations

Eager evaluation (Strict evaluation)	Lazy evaluation (Non-strict evaluation)
square (1 + 2)	square (1 + 2)
square (3)	(1 + 2) * (1 + 2)
3 * 3	(3) * (3)
9	9

argument evaluated

argument evaluation delayed !

[Bird]
[Hutton]

References : [1]

Expressions in Haskell

An expression denotes a value

An expression

[1 + 2]

[HR2010]

[Bird, Chapter 2]

References : [1]

There are many expressions in Haskell

Expressions

- 1 + 2
- (1, 2)
- Just 5
- take 5 xs
- [1, 2, 3]
- let x = 1 in x + y
- 'a'
- map f xs
- if b then 1 else 0
- 7
- ¥x -> x + 1
- x : xs
- fun arg
- case x of _ -> 0
- do {x <- get; put x}
- (¥x -> x + 1) 3

categorizing

[HR2010]

[Bird, Chapter 2]

References : [1]

Expression categories in Haskell

lambda abstraction

- `¥x -> x + 1`

let expression

- `let x = 1 in x + y`

conditional

- `if b then 1 else 0`

case expression

- `case x of _ -> 0`

do expression

- `do {x <- get; put x}`

general constructor, literal and some forms

- `7`
- `[1, 2, 3]`
- `(1, 2)`
- `'a'`
- `x : xs`
- `Just 5`

function application

- `take 5 xs`
- `(¥x -> x + 1) 3`
- `1 + 2`
- `map f xs`
- `fun arg`

[HR2010]
[Bird, Chapter 2

References : [1]

Specification is defined in Haskell 2010 Language Report

Haskell 2010 Language Report, Chapter 3 Expressions [1]

exp	→	*infixexp* `::` [*context* `=>`] *type*	(expression type signature)
	|	*infixexp*	
infixexp	→	*lexp qop infixexp*	(infix operator application)
	|	`-` *infixexp*	(prefix negation)
	|	*lexp*	
lexp	→	`\` $apat_1$ … $apat_n$ `->` *exp*	(lambda abstraction, $n \geq 1$)
	|	`let` *decls* `in` *exp*	(let expression)
	|	`if` *exp* [`;`] `then` *exp* [`;`] `else` *exp*	(conditional)
	|	`case` *exp* `of` `{` *alts* `}`	(case expression)
	|	`do` `{` *stmts* `}`	(do expression)
	|	*fexp*	
fexp	→	[*fexp*] *aexp*	(function application)
aexp	→	*qvar*	(variable)
	|	*gcon*	(general constructor)
	|	*literal*	
	|	`(` *exp* `)`	(parenthesized expression)
	|	`(` exp_1 `,` … `,` exp_k `)`	(tuple, $k \geq 2$)
	|	`[` exp_1 `,` … `,` exp_k `]`	(list, $k \geq 1$)
	|	`[` exp_1 [`,` exp_2] `..` [exp_3] `]`	(arithmetic sequence)
	|	`[` *exp* `|` $qual_1$ `,` … `,` $qual_n$ `]`	(list comprehension, $n \geq 1$)
	|	`(` *infixexp qop* `)`	(left section)
	|	`(` $qop_{\langle - \rangle}$ *infixexp* `)`	(right section)
	|	*qcon* `{` $fbind_1$ `,` … `,` $fbind_n$ `}`	(labeled construction, $n \geq 0$)
	|	$aexp_{\langle qcon \rangle}$ `{` $fbind_1$ `,` … `,` $fbind_n$ `}`	(labeled update, $n \geq 1$)

References : [1]

Constructor

Constructor

Constructor is one of the key elements
to understand WHNF and lazy evaluation.

References : [1]

data文で宣言する代数的データ型とその値

```
data  Maybe a  =  Nothing
               |  Just  a
```

Algebraic Data Type

Algebraic Data Value

References : [1]

Constructorはdata文で宣言する代数的データ値

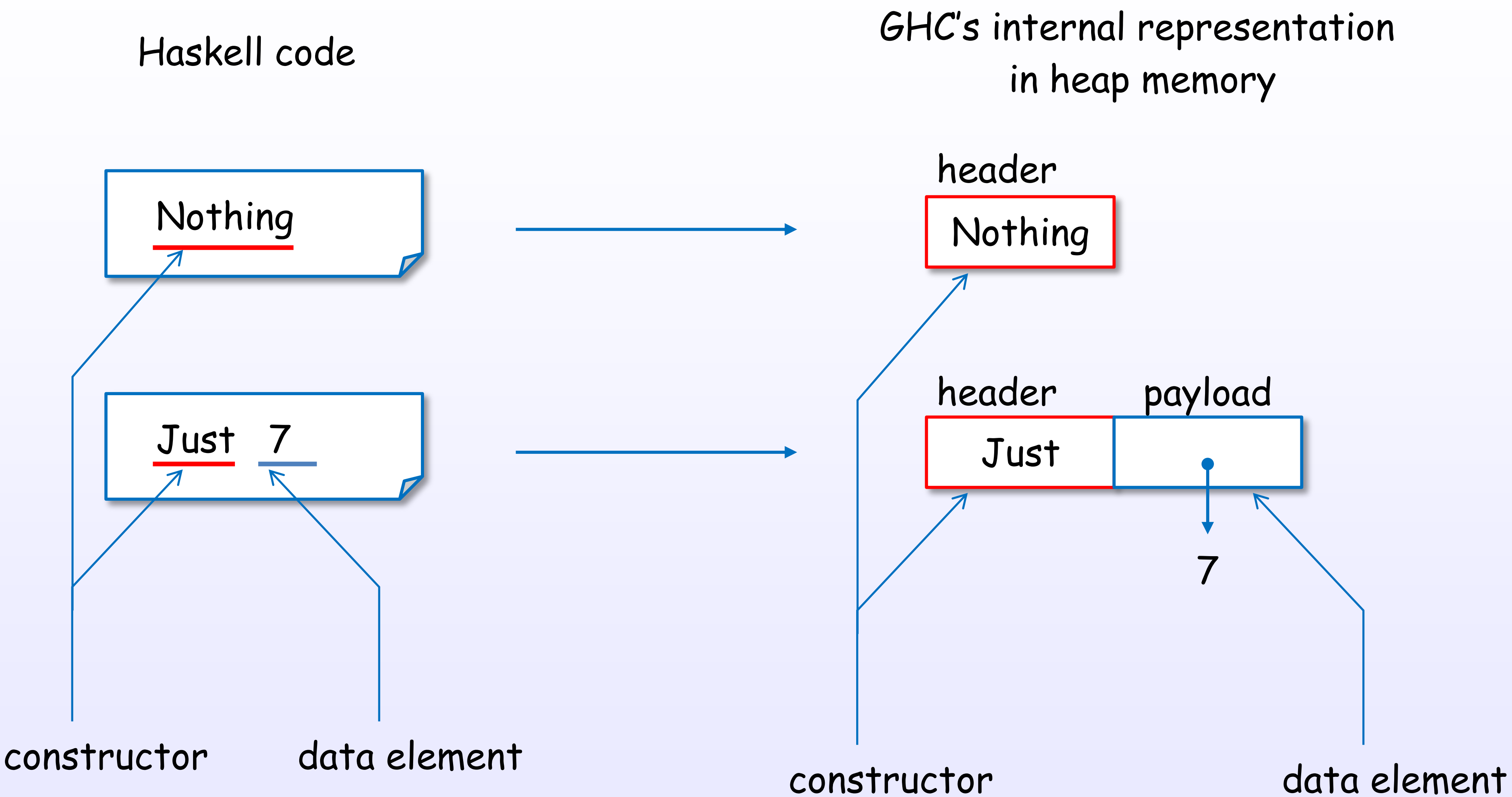
```
data  Maybe a  =  Nothing
                | Just  a
```

constructor

data element(s)

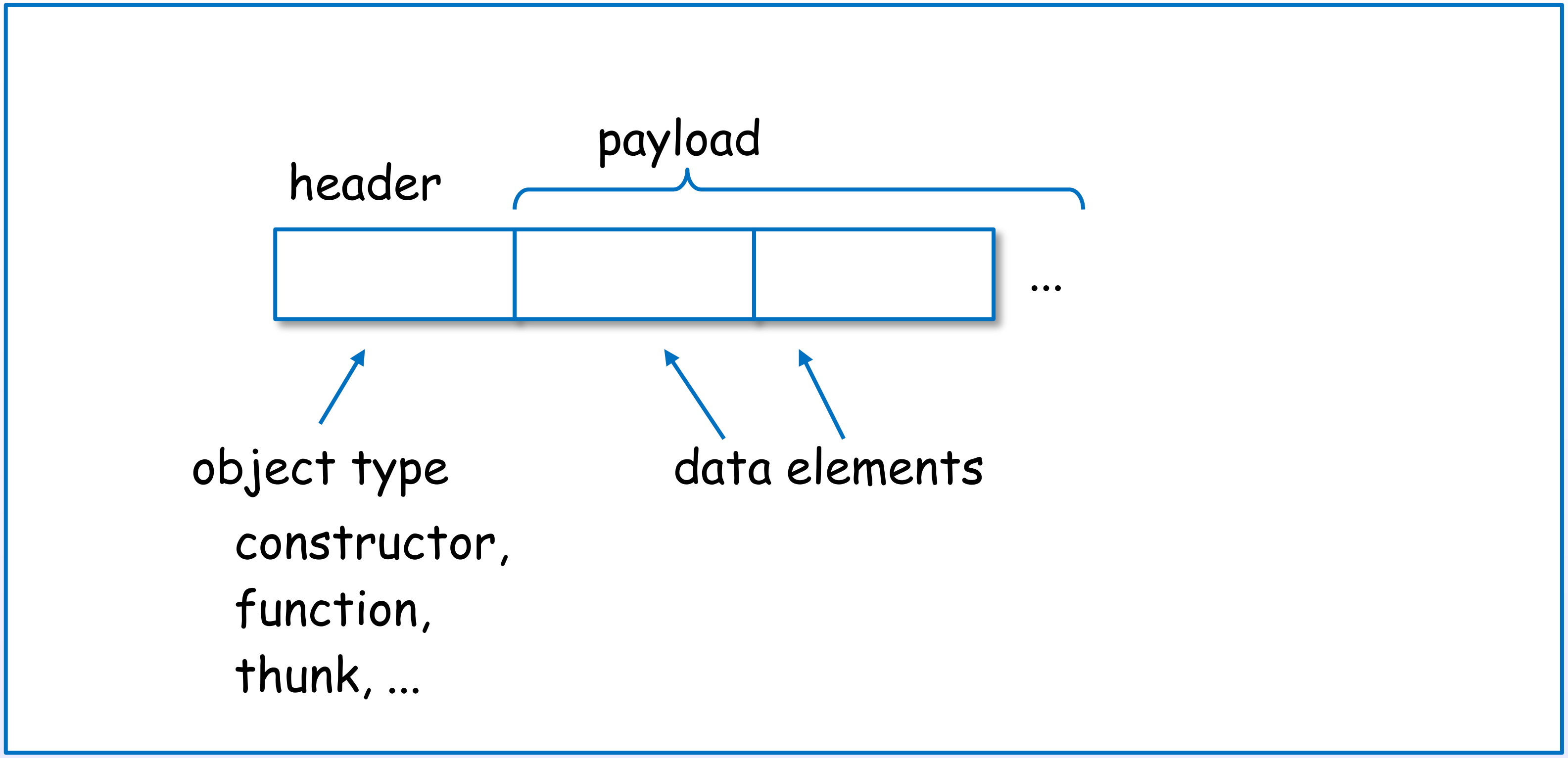
References : [1]

Constructorの内部表現



References : [1]

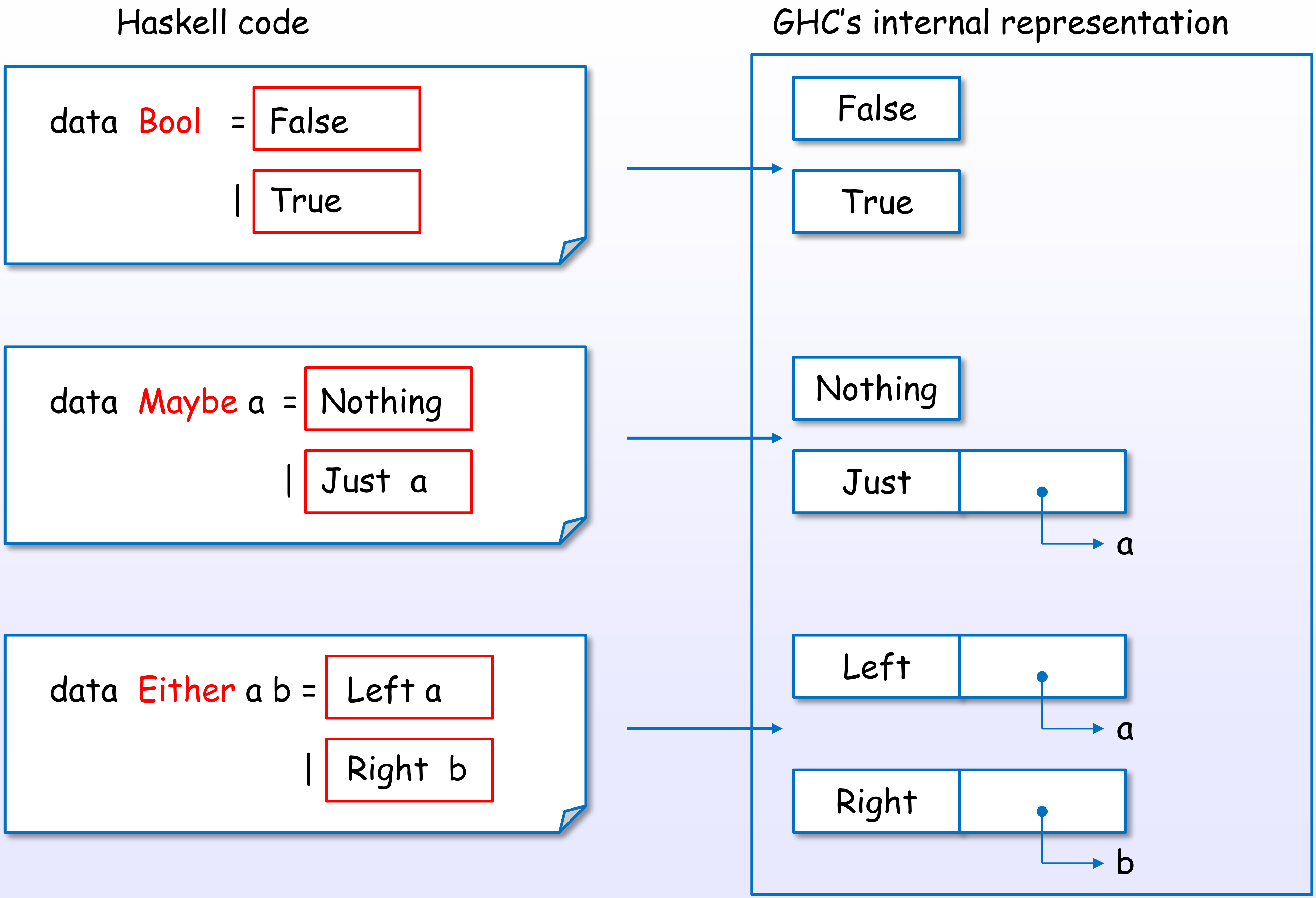
Constructorは統一内部表現で表現される



in heap memory, stack, registers or static memory

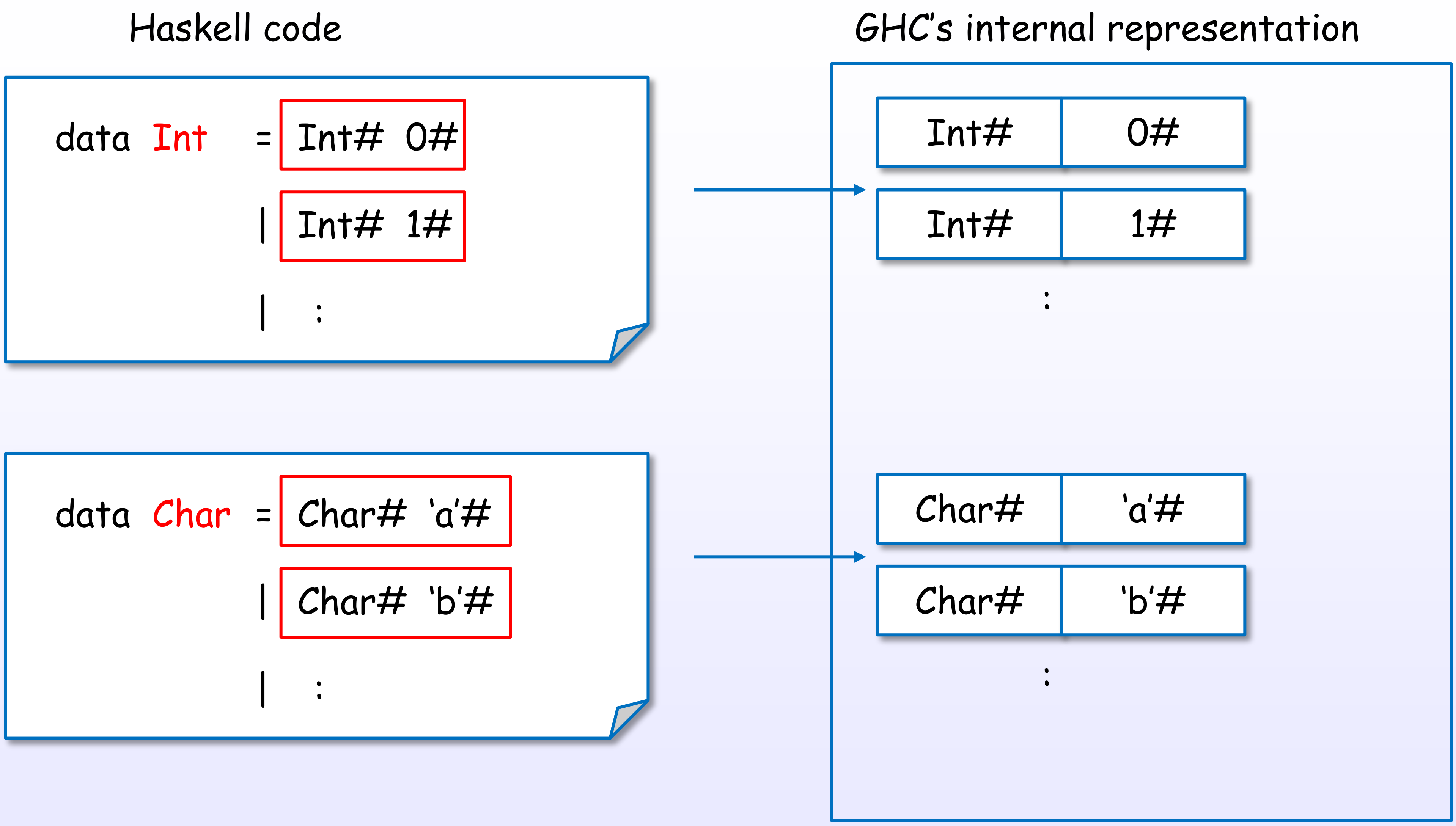
References : [1]

いろいろなコンストラクタと内部表現



References : [1]

基本データ型も実はコンストラクタで構成されている



References : [1]

リストも実はコンストラクタで構成されている

List

```
[ 1, 2, 3 ]
```

↓ syntactics desugar

```
1 : ( 2 : ( 3 : [] ) )
```

↓ prefix notation by section

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

↕ equivalent data constructor

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

constructor

References : [1]

リストも実はコンストラクタで構成されている

List

```
[ 1, 2, 3 ]
```

syntactics desugar

```
1 : ( 2 : ( 3 : [] ) )
```

prefix notation by section

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

equivalent data constructor

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

** pseudo code*

```
data  List a  =   []
              |   :  a   (List a)
```

```
data  List a  =   Nil
              |   Cons  a   (List a)
```

References : [1]

リストも実はコンストラクタで構成されている

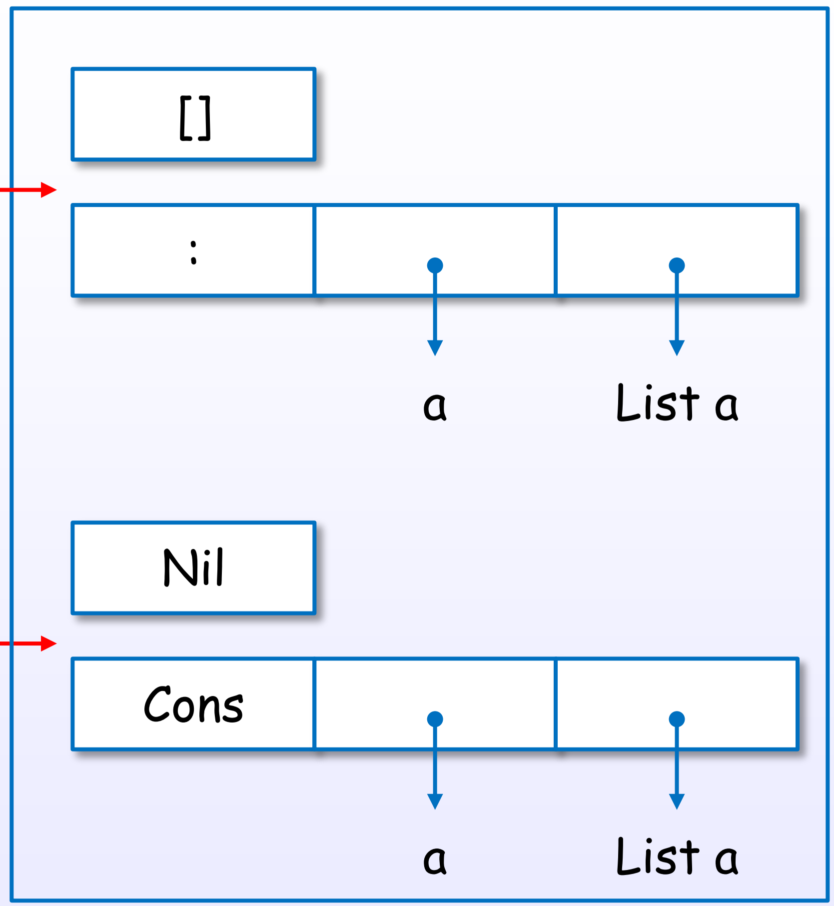
Haskell code

```
data  List a    = []
                | : a (List a)
```

equivalent data constructor

```
data  List a    = Nil
                | Cons  a  (List a)
```

GHC's internal representation



References : [1]

リストも実はコンストラクタで構成されている

Haskell code

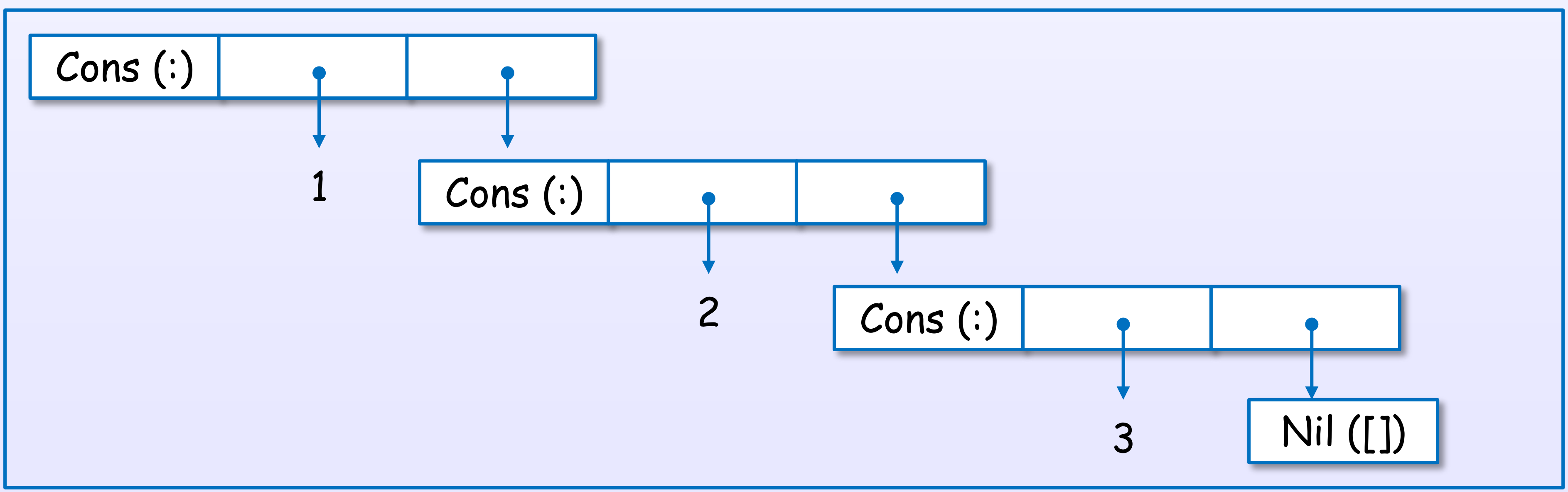
```
[ 1, 2, 3 ]
```

```
1 : ( 2 : ( 3 : [] ) )
```

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

GHC's internal representation



References : [1]

タプルも実はコンストラクタで構成されている

Tuple (Pair)

(7, 8)

prefix notation by section

(,) 7 8

equivalent data constructor

Pair2 7 8

constructor

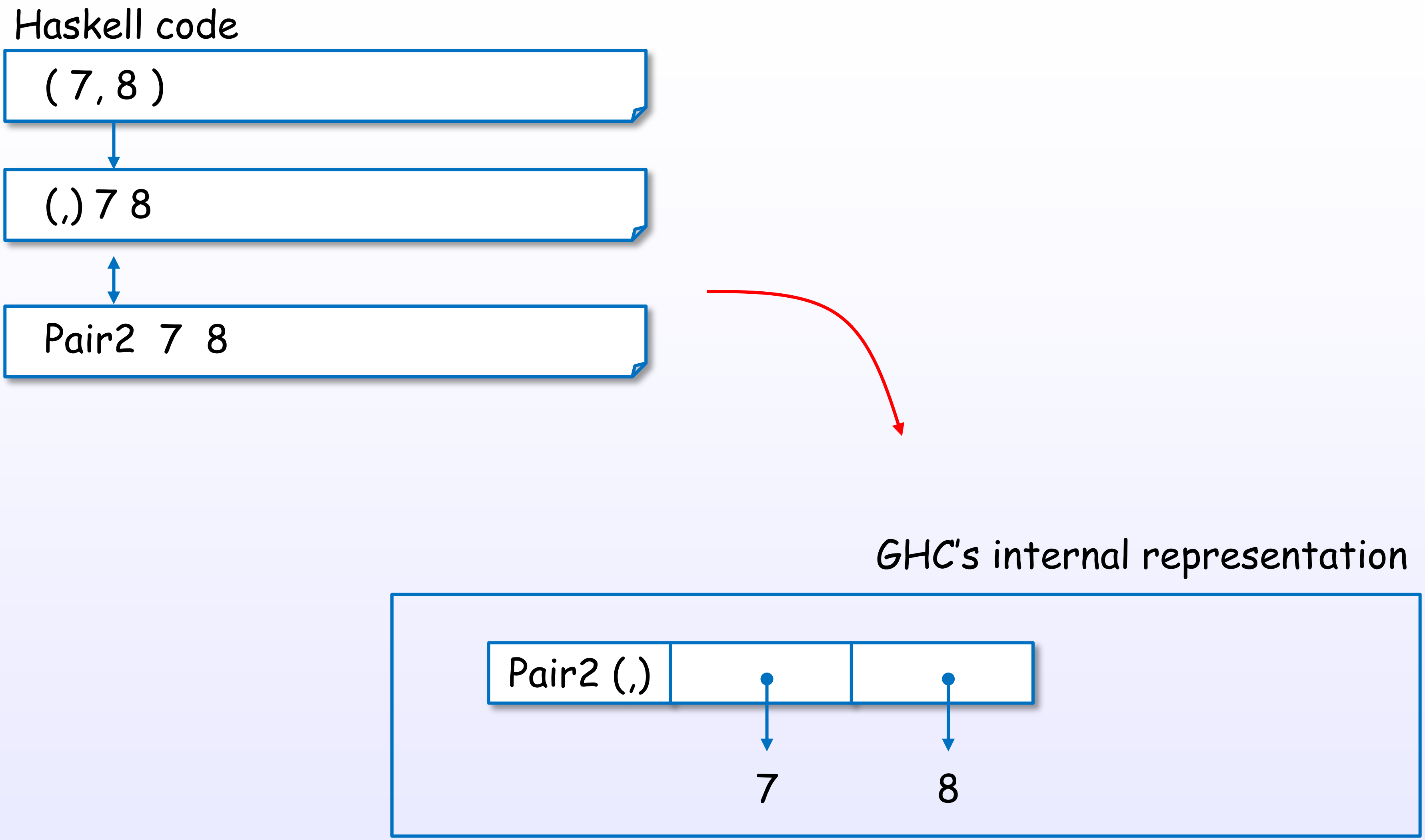
* *pseudo code*

data Pair2 a = (,) a a

data Pair2 a = Pair2 a a

References : [1]

タプルも実はコンストラクタで構成されている



References : [1]

Thunk

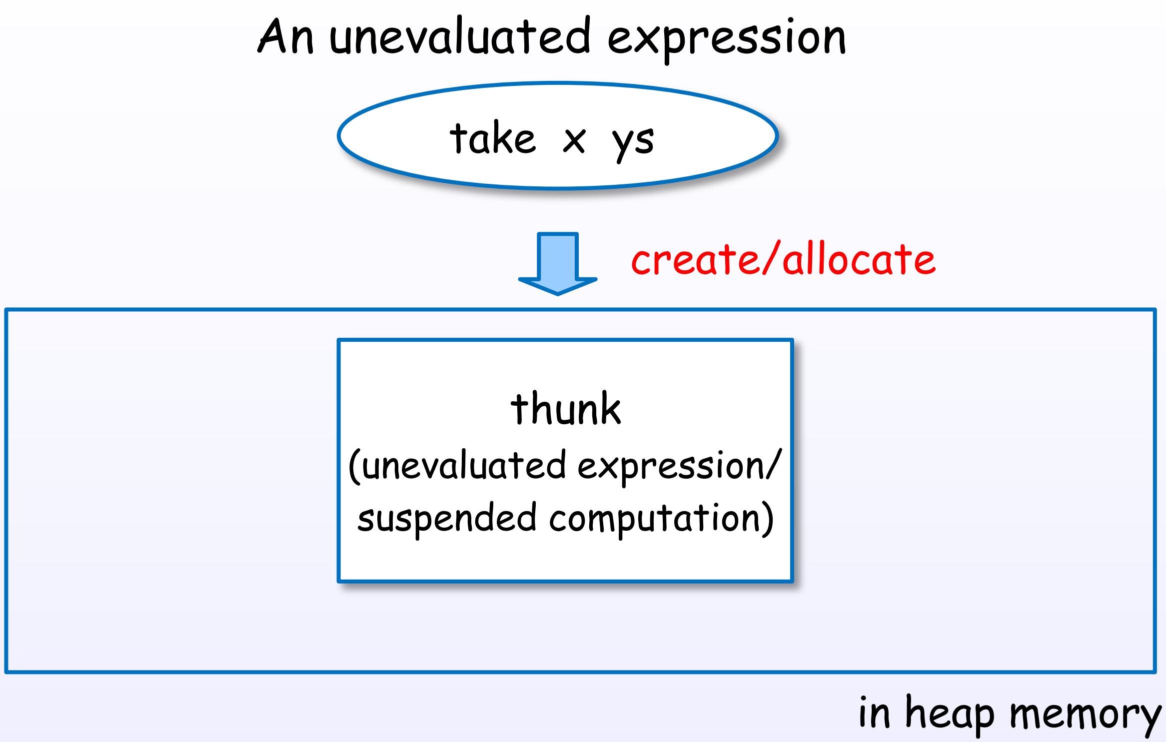
Thunk

thunk
(unevaluated expression/
suspended computation)

A thunk is an unevaluated expression in heap memory.

References : [1]

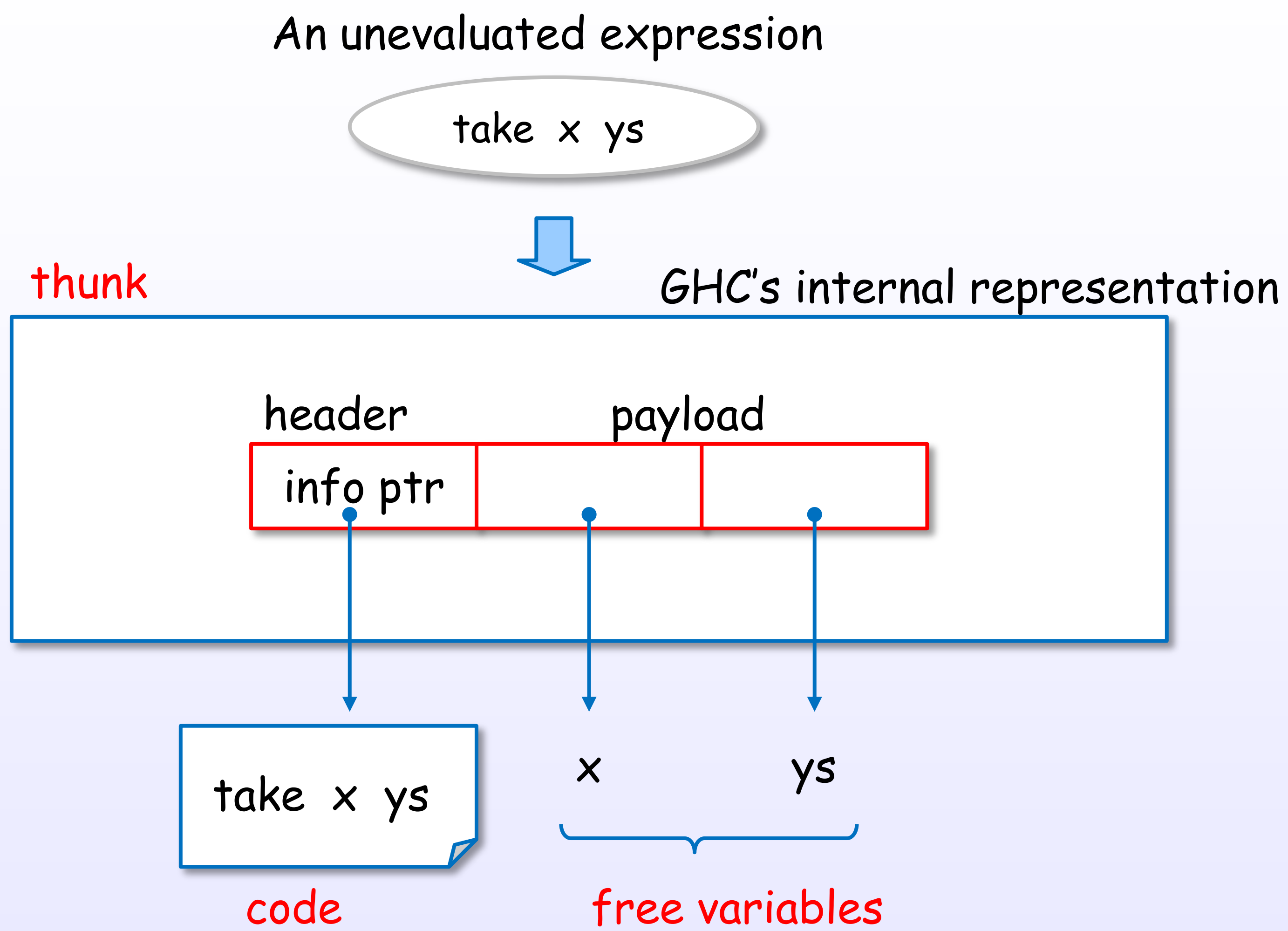
Thunk



A thunk is created for an unevaluated expression.

References : [1]

Thunkの内部表現

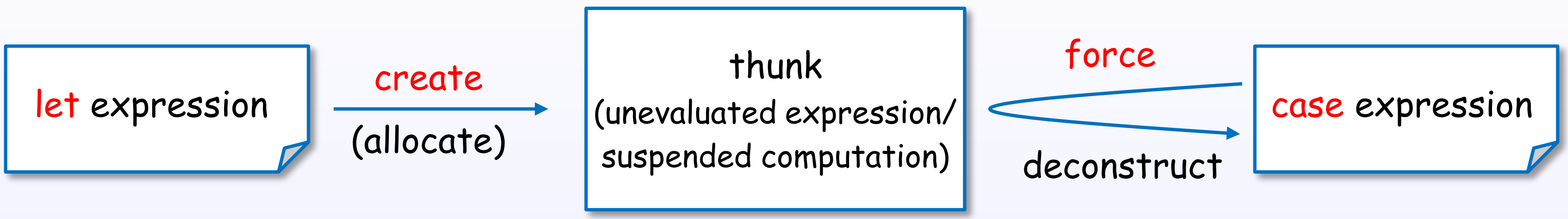


A thunk is represented with header(code) + payload(free variables).

References : [1]

let, case expression

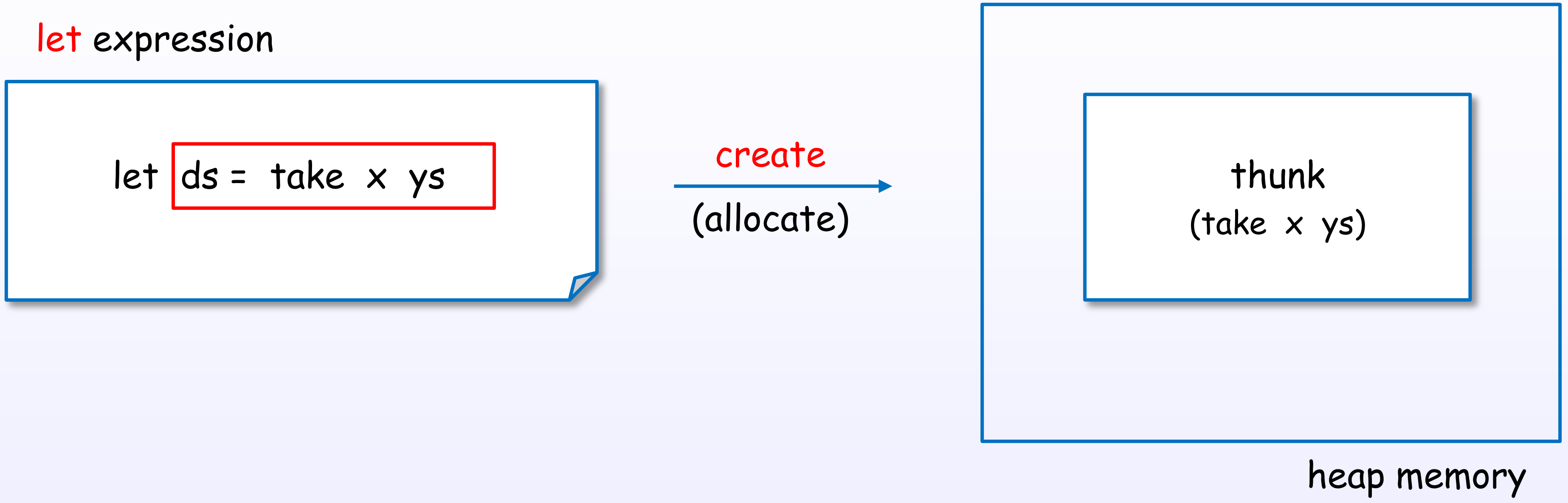
let/case expressions and thunk



A let expression may create a thunk.
A case expression forces and deconstructs the thunk.

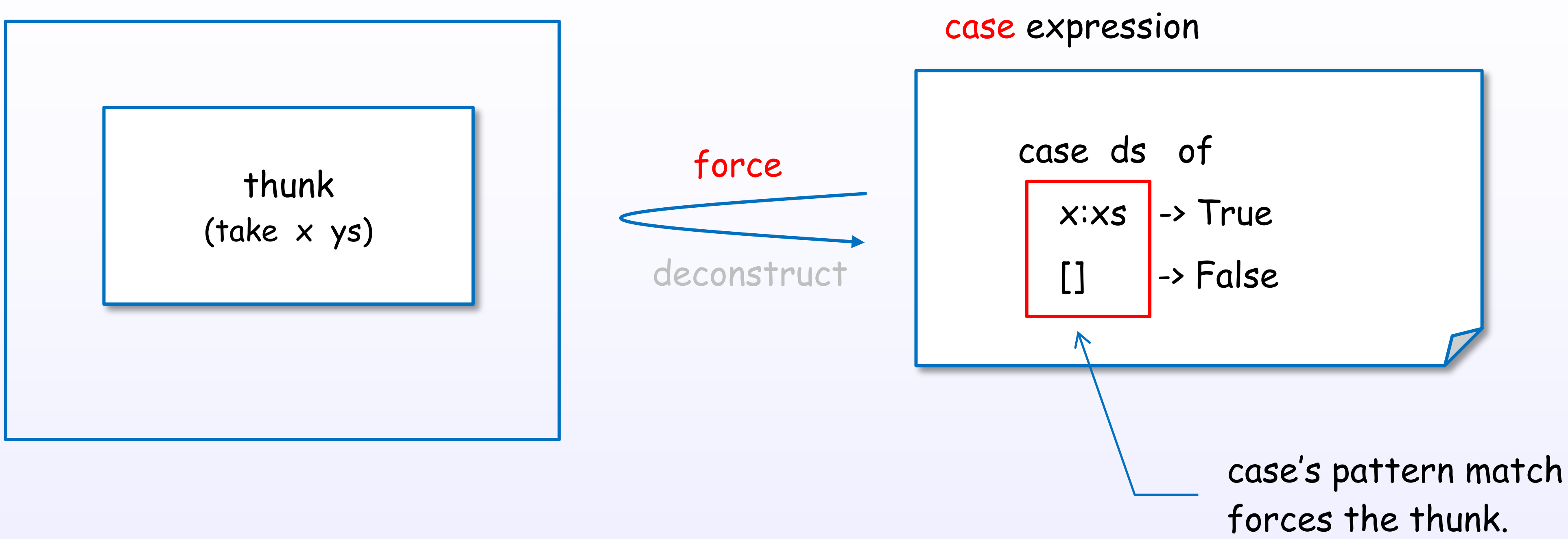
References : [1]

A let expression creates a thunk



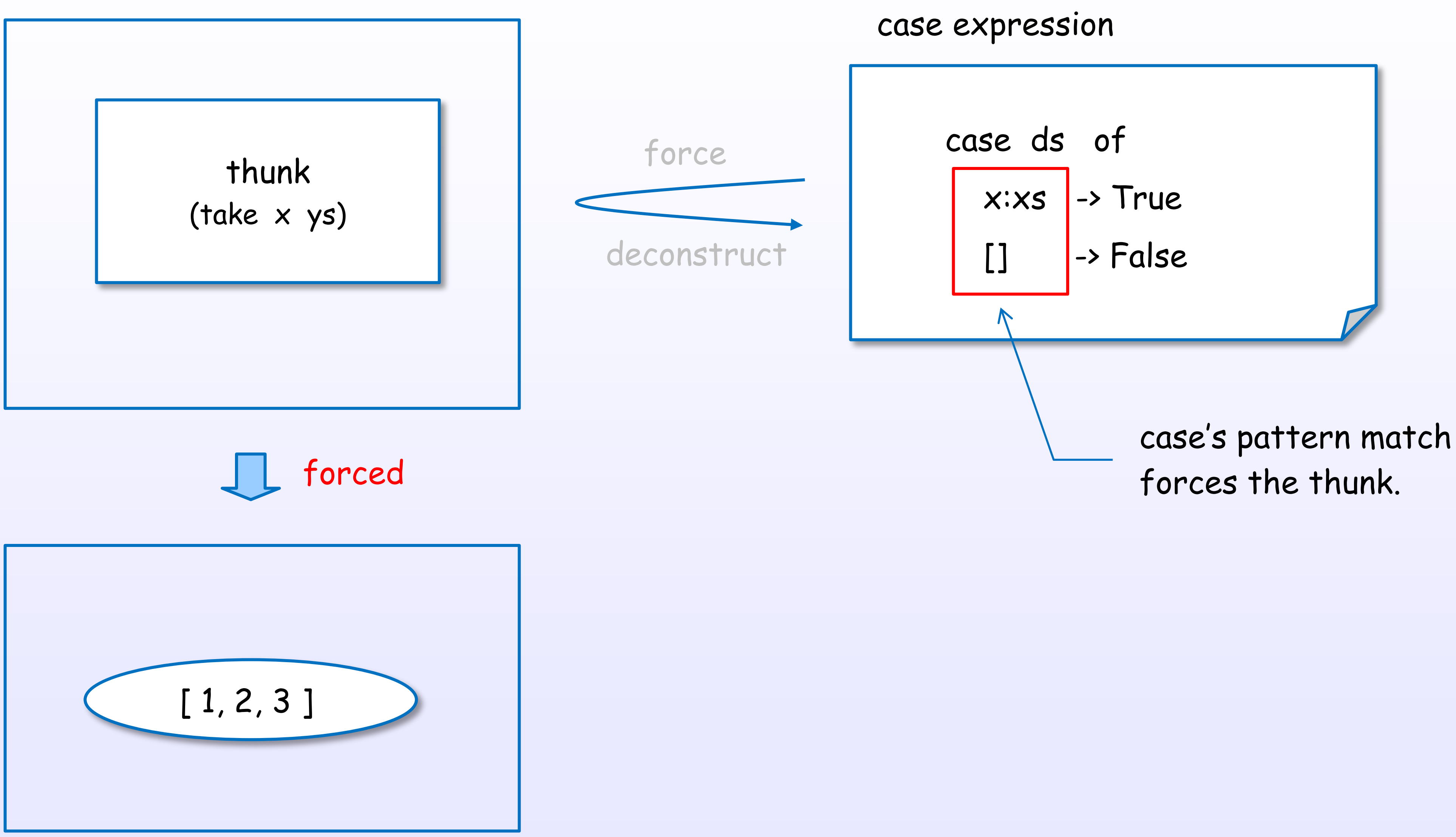
References : [1]

A case expression forces a thunk



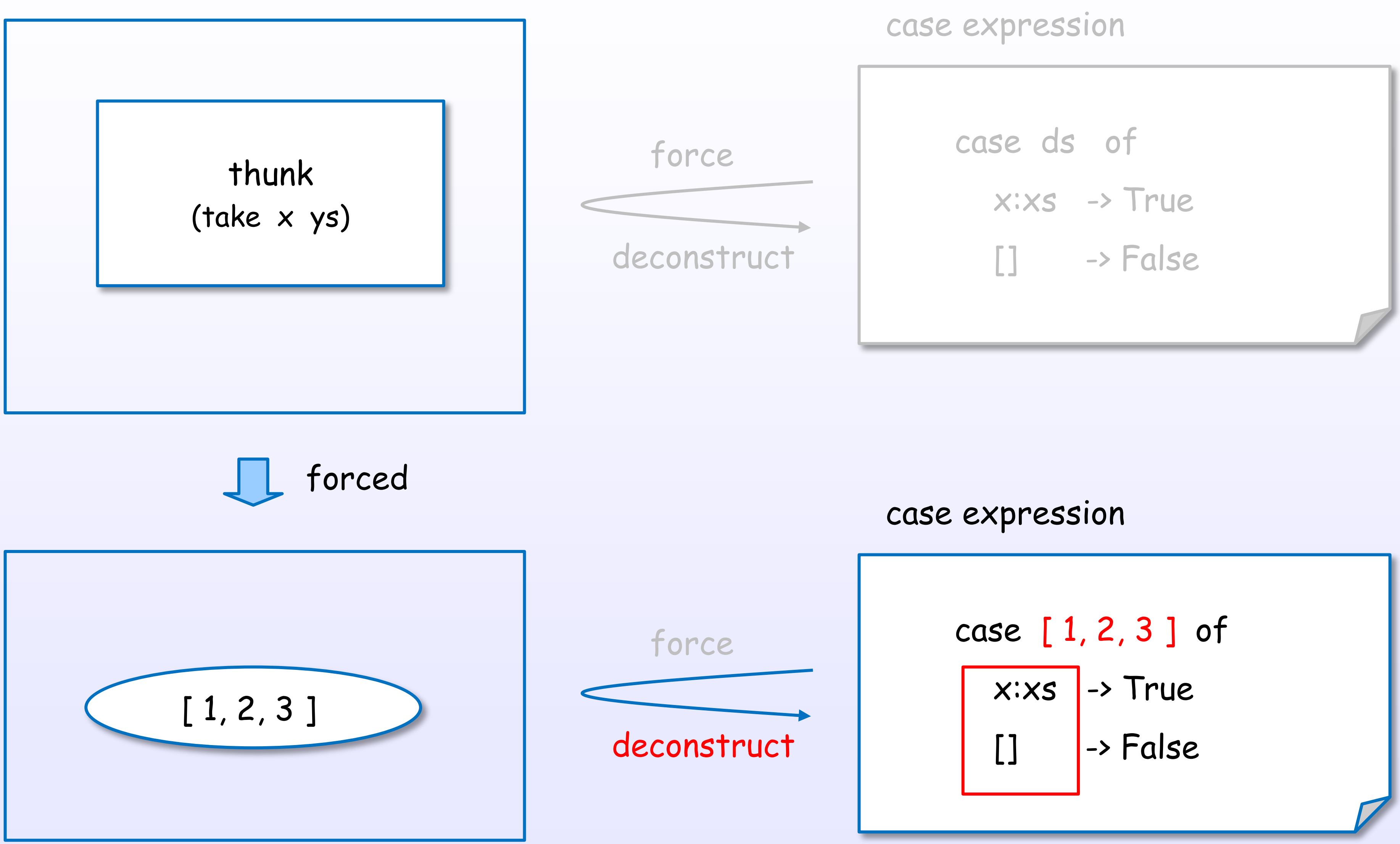
References : [1]

A case expression forces a thunk



References : [1]

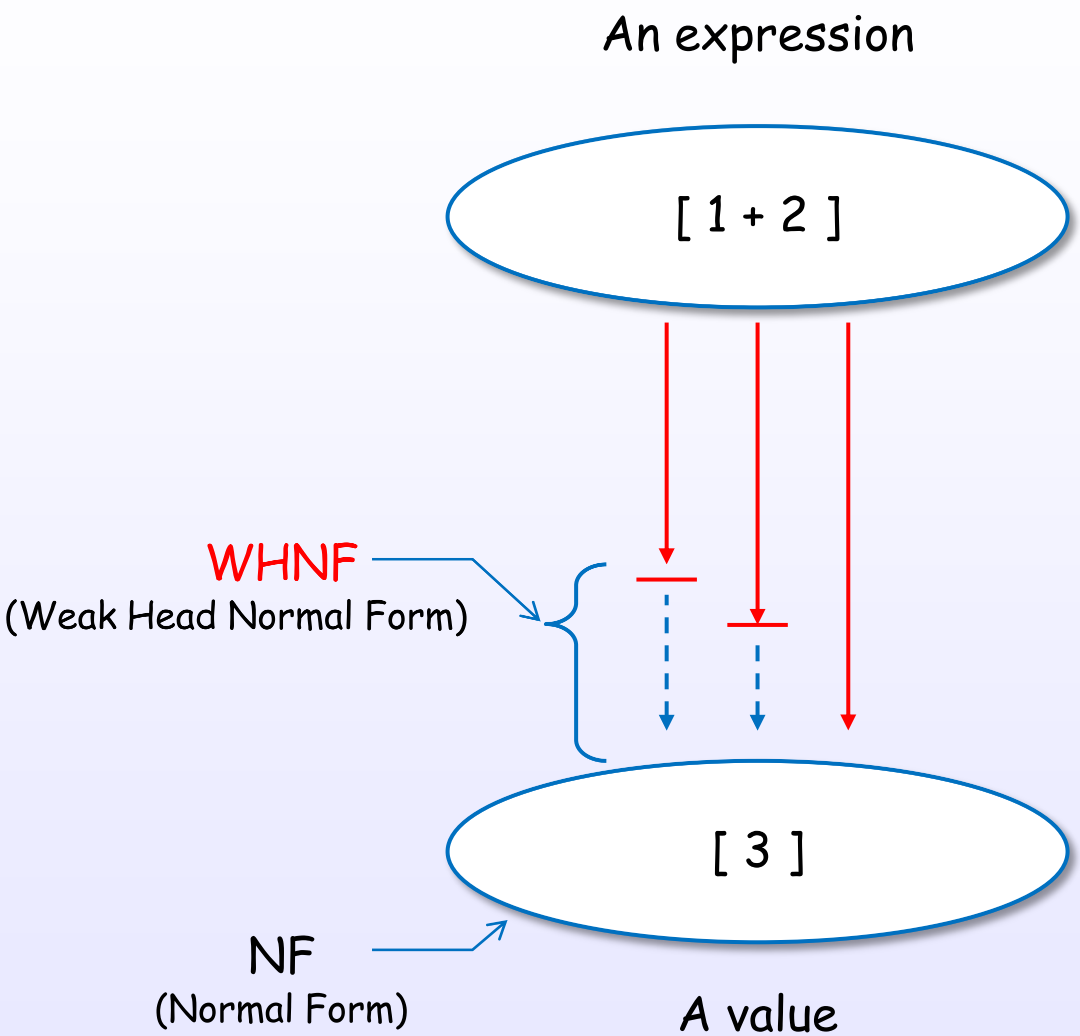
A case expression forces a thunk



References : [1]

WHNF

There are some evaluation levels



[Terei]

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

WHNF

[4]

normal form:
an expression without an redexes

head normal form:
an expression where the top level (head) is neither a redex NOR
a lambda abstraction with a reducible body

weak head normal form:
an expression where the top level (head) isn't a redex

[Terei]

References : [1]

WHNF

[4]

evaluation strategies:
call-by-value: arguments evaluated before function entered (copied)
call-by-name: arguments passed unevaluated
call-by-need: arguments passed unevaluated but an expression is only evaluated once (sharing)

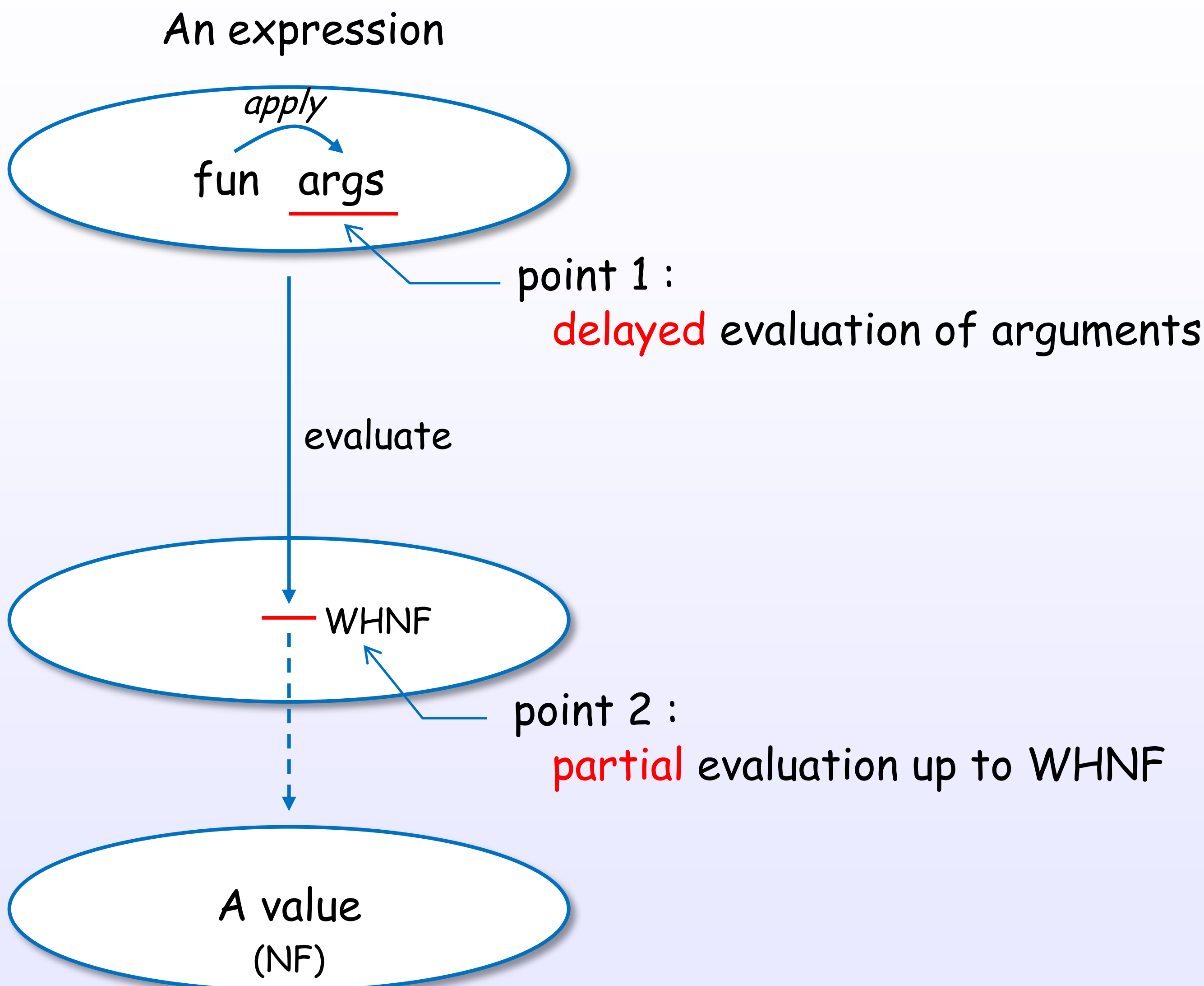
no-strict evaluation Vs. lazy evaluation:
non-strict: Includes both call-by-name and call-by-need, general term for evaluation strategies that don't evaluate arguments before entering a function
lazy evaluation: Specific type of non-strict evaluation. Uses call-by-need (for sharing).

[Terei]

References : [1]

Evaluation in Haskell (GHC)

Key concept of Haskell's lazy evaluation



References : [1]

point 1 : delayed evaluation of arguments

apply

```
head (map f xs)
```

internal transformation by GHC

```
let thunk0 = map f xs
in head thunk0
```

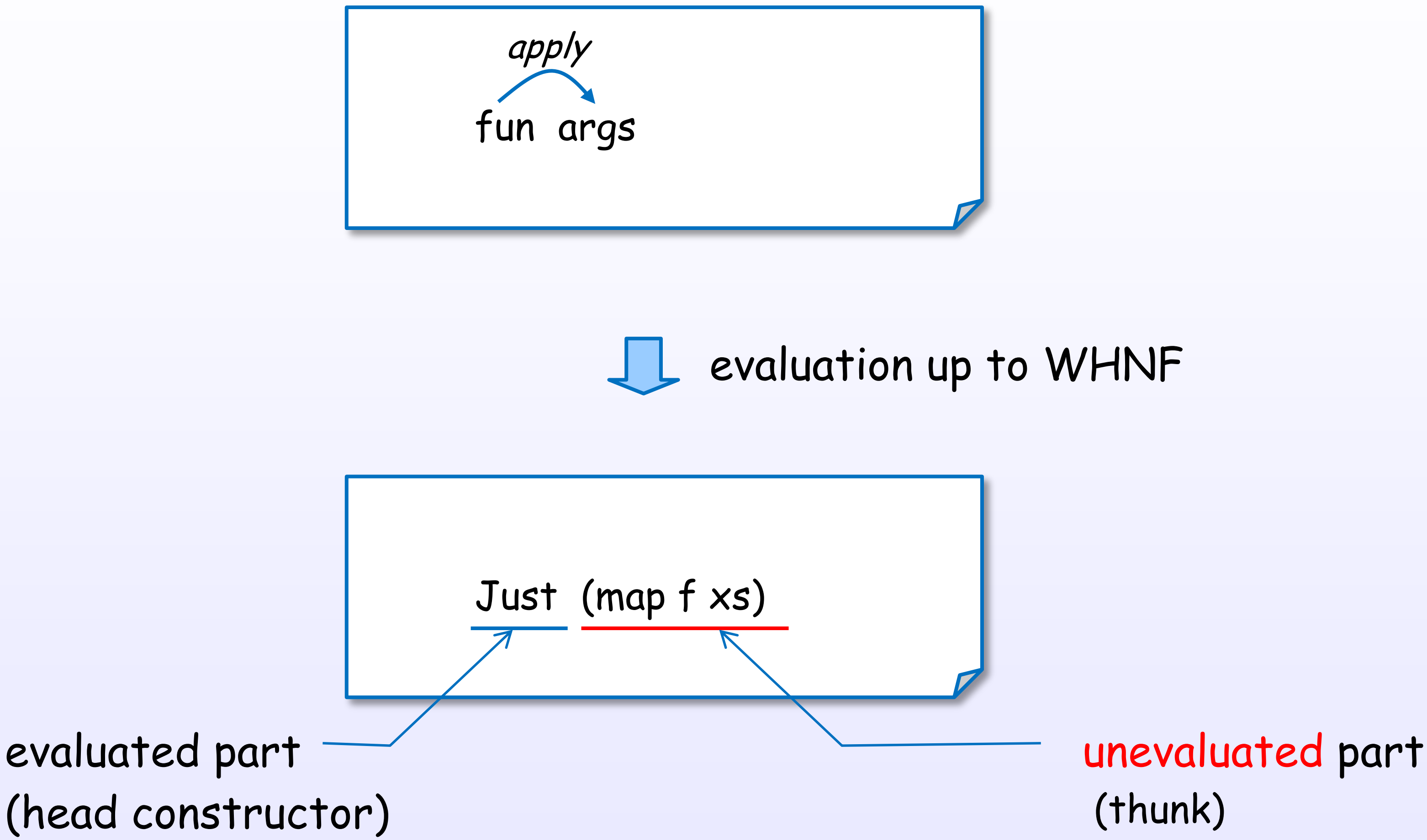
apply

create a thunk
in heap memory

GHC implements lazy evaluation using the thunk.
Evaluation of arguments is delayed with the thunk.

References : [1]

point 2 : partial evaluation up to WHNF



GHC can partially evaluate a expression.
Constructor can hold an unevaluated expression (a thunk).

References : [1]

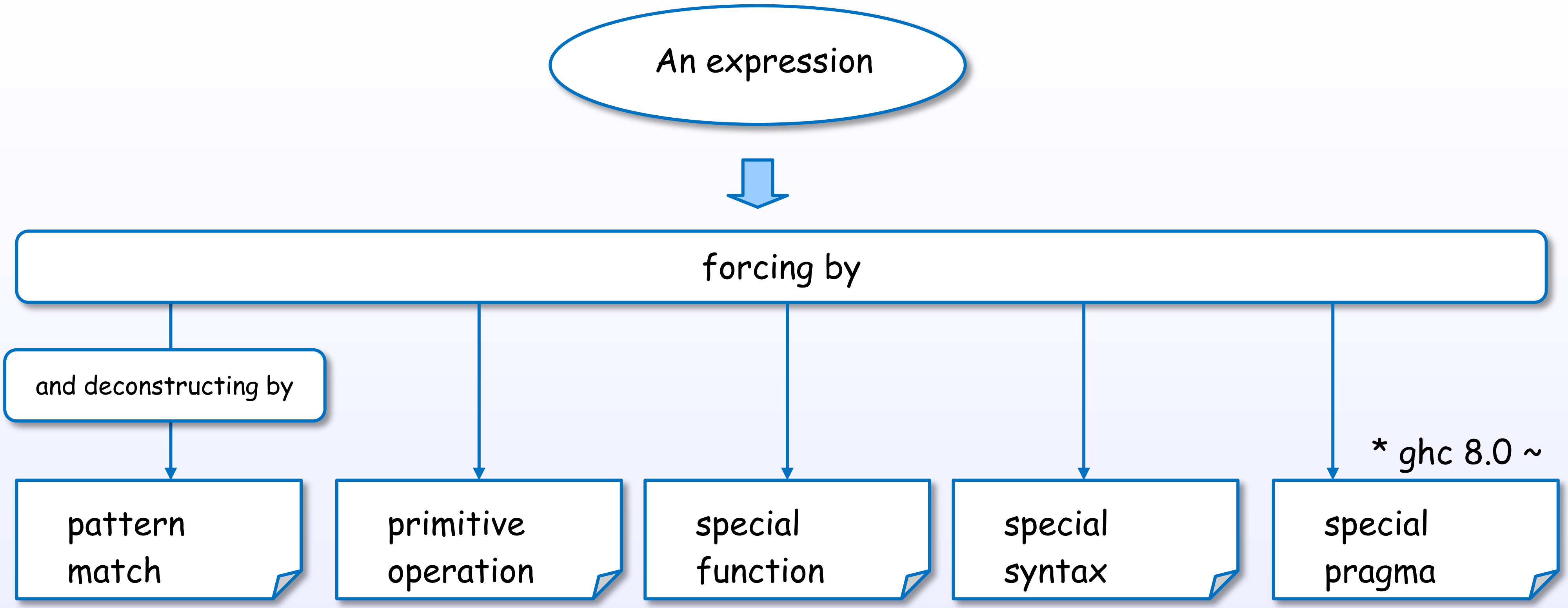
Pattern match

[CIS194]

References : [1]

Control the evaluation in Haskell

How to drive evaluation



case expression
function definition

+, *, ...

seq
$!
rnf
deepseq
force
$!!
pseq
,...

!

Strict
StrictData

IO モナドも入れる事

References : [1]

Implementation in GHC

Tree, Graph

a expression

AST

Tree

Graph

Shared Term

Lazy

References : [1]

evaluation, reduction

an expression

↓

graph representation

↓

reduction

↓

a value
(NF)

References : [1]

Expressions examples

References : [1]

STG heap objects

language	Just 5
implementation	heap object

References : [1]

Layer

Non-strictness $f \perp = \perp$

Lazy evaluation

Graph reduction

STG machine

References : [1]

Layer

Haskell semantics	take 5 [1..10]
internal representation	graph
STG semantics	heap object
STG machine	

References : [1]

Semantics

Bottom

domain

co-domain

defined

undefined

$f \perp = \perp$

[Bird, Chapter 2]

References : [1]

Strictness, Bottom

[Bird, Chapter 2]

References : [1]

References

References

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References

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Lazy,... zzz