Lazy evaluation in Haskell

exploring some mental models and implementations

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Rev. 0.00.0
WIP

Lazy,... zzz

..., It's fun.

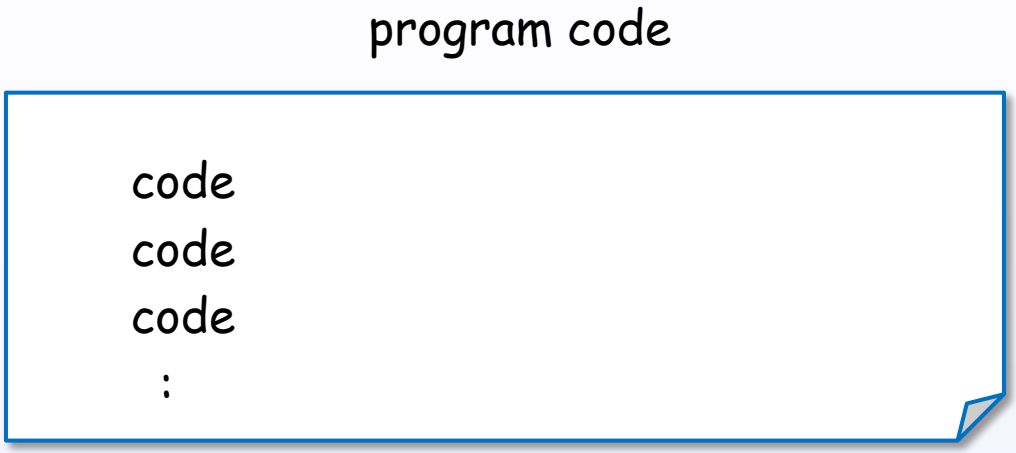
NOTE

- Meaning of terms are different by communities.
- There are a lot of good documents. Please see also references.
- This is written for GHC's Haskell.

Contents

Basic mental models

How to evaluate program code ?



プログラムは、どの順で評価される？

どういうステップ、どういう順で evaluation (exexution, reduction) される？

What are these mental models?

References : [1]

One of the mental model for C program

プログラムは、statement の集まり。Cでは。

C program code

```
main (...) {
  code..
  code..
  code..
  code..
}
```

(1) 文は基本的に、
上から下へ評価
downword

statement order

```
x = func1( func2( a ) );
```

(2) 内側の関数評価が先
（内から外へ。）

```
y = func1( a(x), b(x), c(x) );
```

(3) 同階層では、左側の
（左から右へ。）

```
z = func1( m + n );
```

(4) 引数評価が先
（引数評価から関数評価へ。

要するに、Cのラフメンタルモデル

- 上から下へ
- 内側から外側へ
- 左から右へ
- 引数から関数呼出しへ

References : [1]

One of the mental model for Haskell program

プログラムは、式の集まり。（全体では1つの式）

Haskell program code

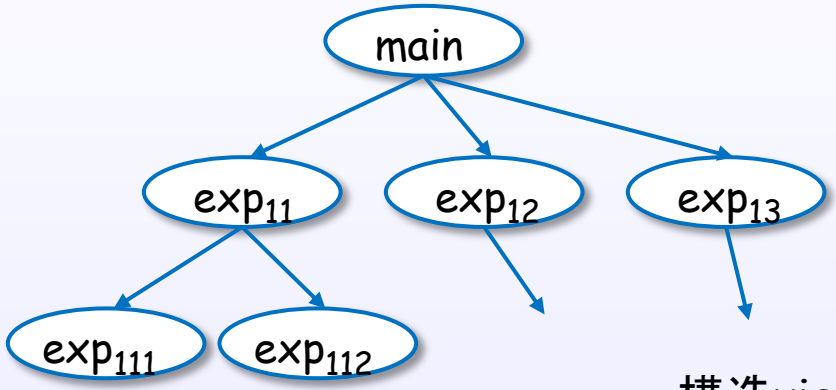
```
main = exp11 exp12 exp13

exp11 = exp111 exp112

exp12 = exp121 exp122 exp123
 :
```

式view

main = ((exp111 exp112) (exp121 (exp1224 ...



構造view

(1) プログラム全体を1つの式と見立てて

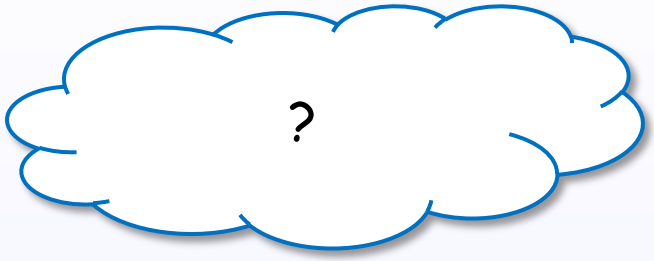
(2) 部分式をある順で評価（簡約）していく

References : [1]

Expression

What is an expression?

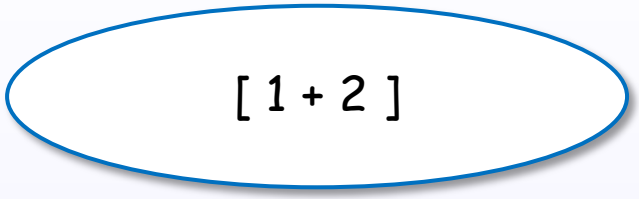
An expression



References : [1]

An expression denotes a value

An expression

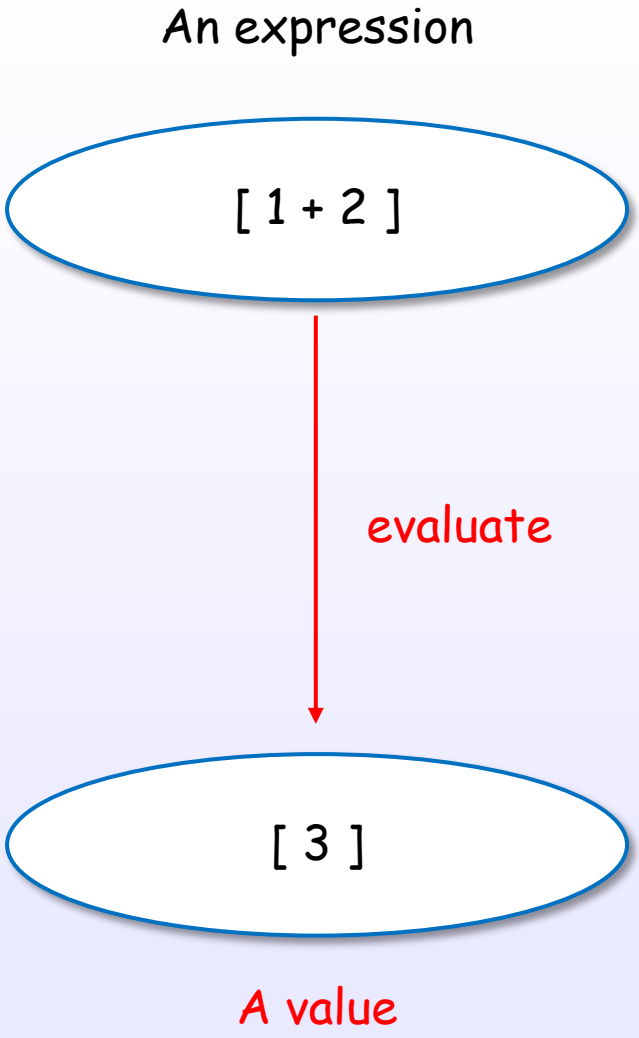


[HR2010]

[Bird, Chapter 2]

References : [1]

An expression evaluates to a value

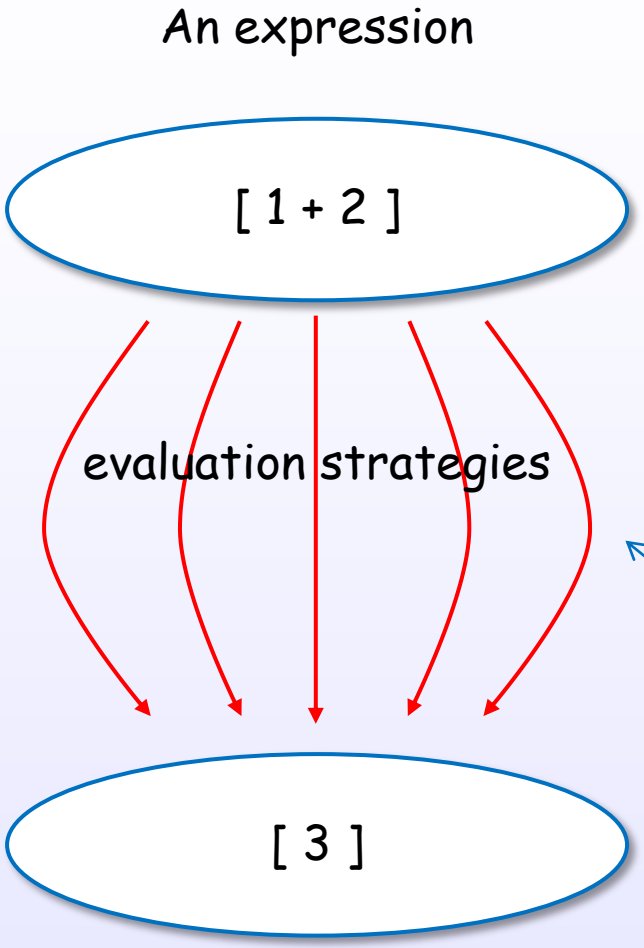


[HR2010]

[Bird, Chapter 2]

References : [1]

There are many evaluation approaches



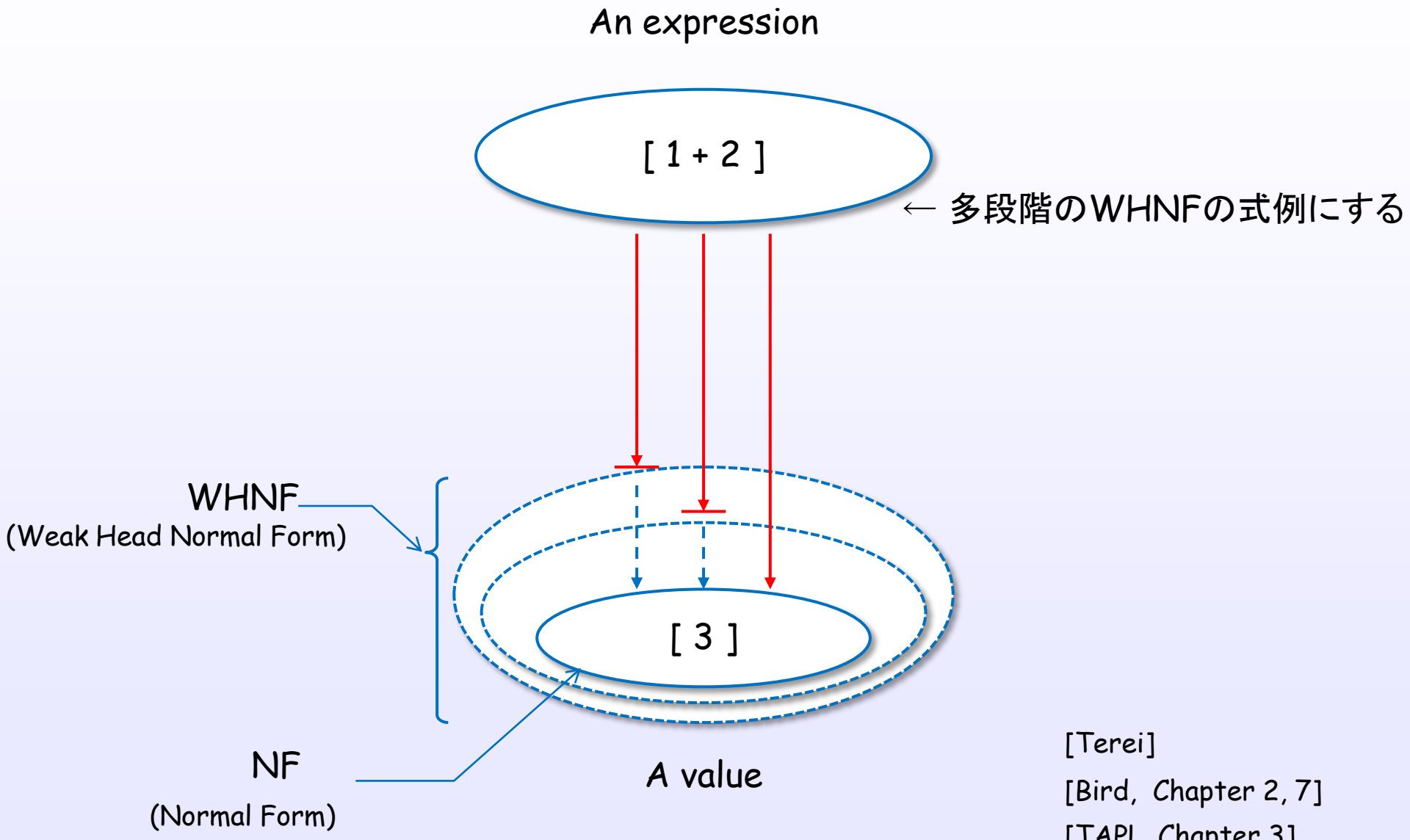
- Strict, Non-strict evaluation
- Eager, Lazy evaluation
- Call-by-value, Call-by-name, Call-by-need, …
- Innermost, Outermost
- Normal order, Application order
- …

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

There are some evaluation levels



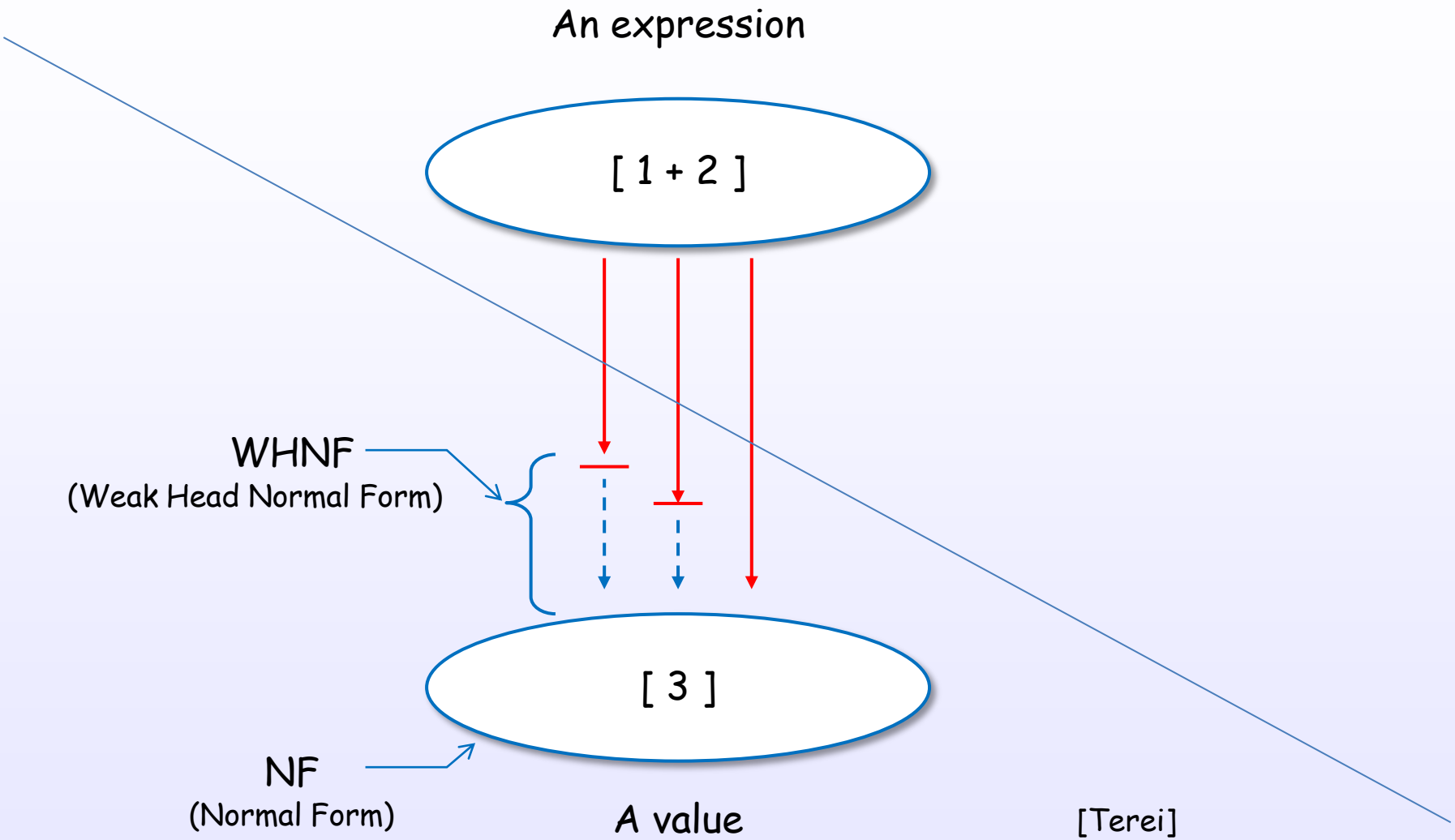
[Terei]

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

There are some evaluation levels



[Terei]

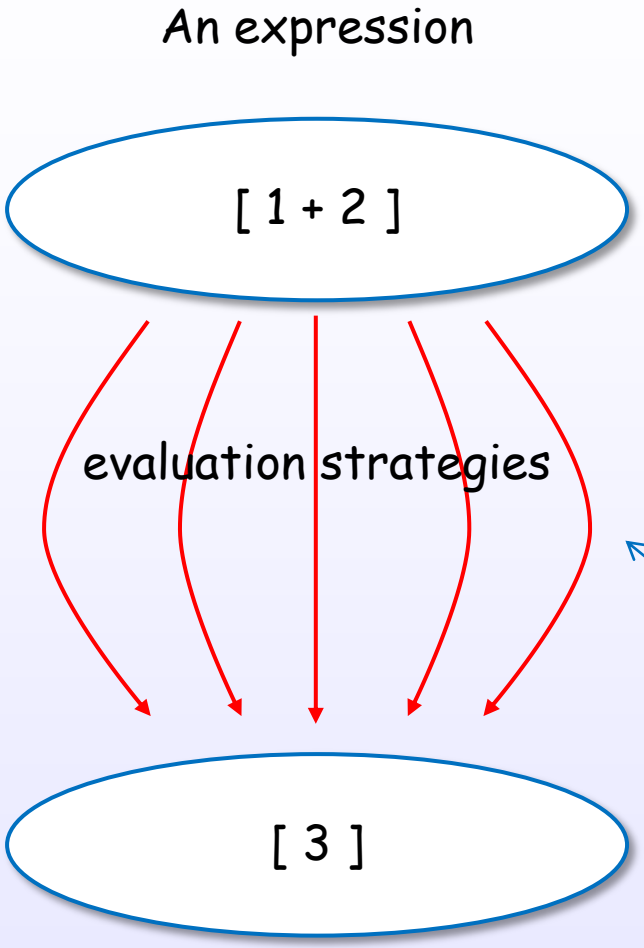
[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

Evaluation strategies

There are many evaluation approaches



- Strict, Non-strict evaluation
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[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

Evaluation layers

denotational semantics

evaluation strategies

implementation techniques

[Bird, Chapter 7]

[Hutton, Chapter 8]

[TAPL, Chapter 3]

References : [1]

Evaluation layers

denotational semantics

- Strict semantics
- Non-strict semantics

---

evaluation strategies

- Eager evaluation (Strict evaluation)
- Nondeterministic evaluation
- Lazy evaluation (Non-strict evaluation)
- ...
- Call-by-Value
- Call-by-Name
- Call-by-Need
- ...

---

implementation techniques

- Lazy graph reduction
- ...

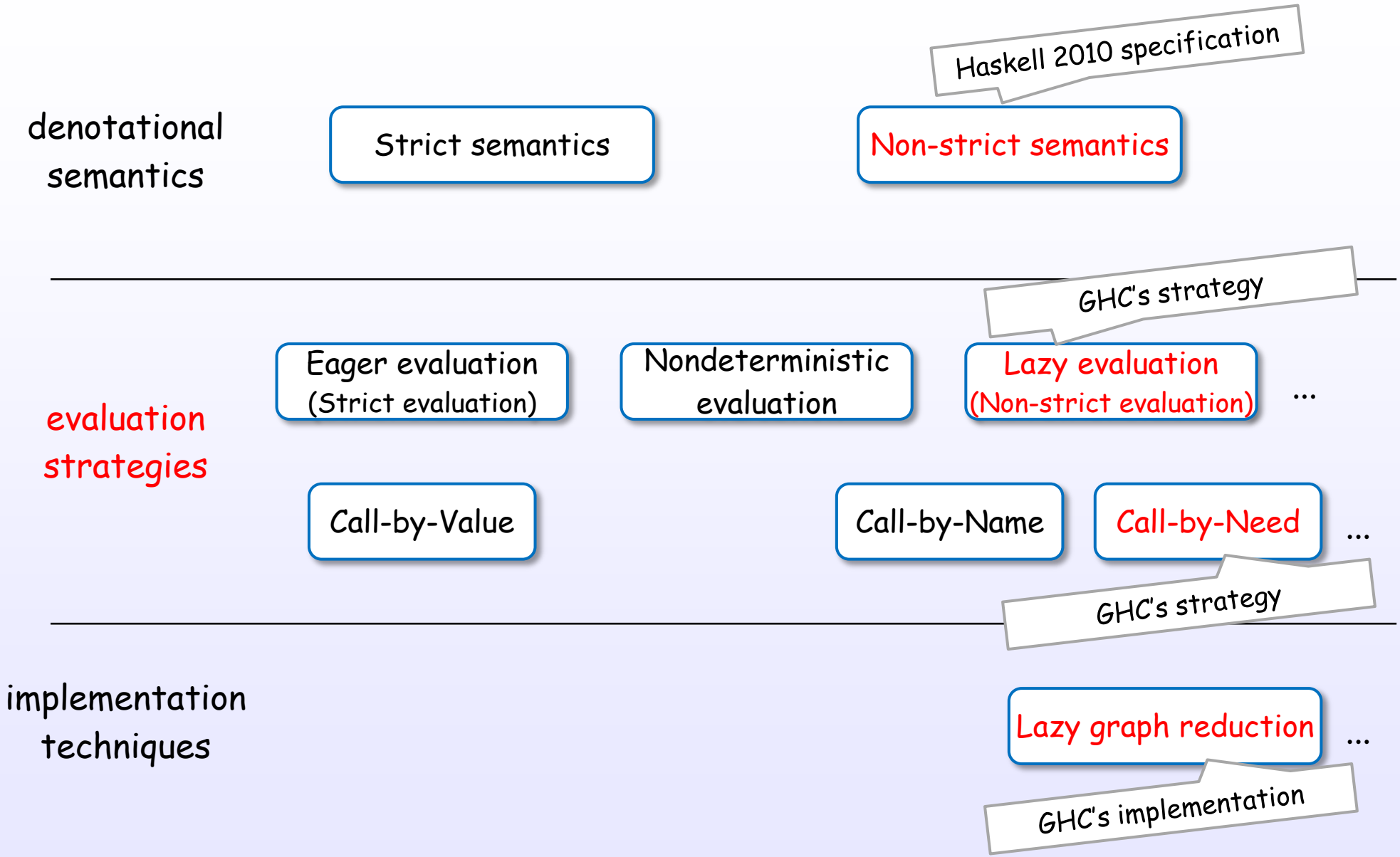
[Bird, Chapter 7]

[Hutton, Chapter 8]

[TAPL, Chapter 3]

References : [1]

Evaluation layers for GHC's Haskell



References : [1]

Evariation strategies and order

a (b c) + d (e (f g))

order

[Bird]
[Hutton]

References : [1]

Simple example of both evaluations

default

C, Java, JavaScript, Python, OCaml, Scheme, ...

Eager evaluation
(Strict evaluation)

square (1 + 2)

argument evaluation first

square (3)

3 * 3

9

default

Haskell (GHC), ...

Lazy evaluation
(Non-strict evaluation)

square (1 + 2)

apply first

(1 + 2) * (1 + 2)

(3) * (3)

9

[Bird]
[Hutton]

References : [1]

Simple example of both evaluations

Eager evaluation
(Strict evaluation)

square (1 + 2)

square (3)

argument evaluated

3 * 3

9

Lazy evaluation
(Non-strict evaluation)

square (1 + 2)

(1 + 2) * (1 + 2)

argument evaluation delayed !

(3) * (3)

9

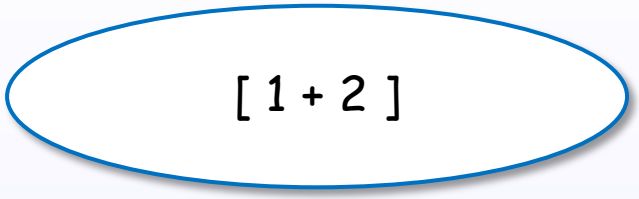
[Bird]
[Hutton]

References : [1]

Expressions in Haskell

An expression denotes a value

An expression

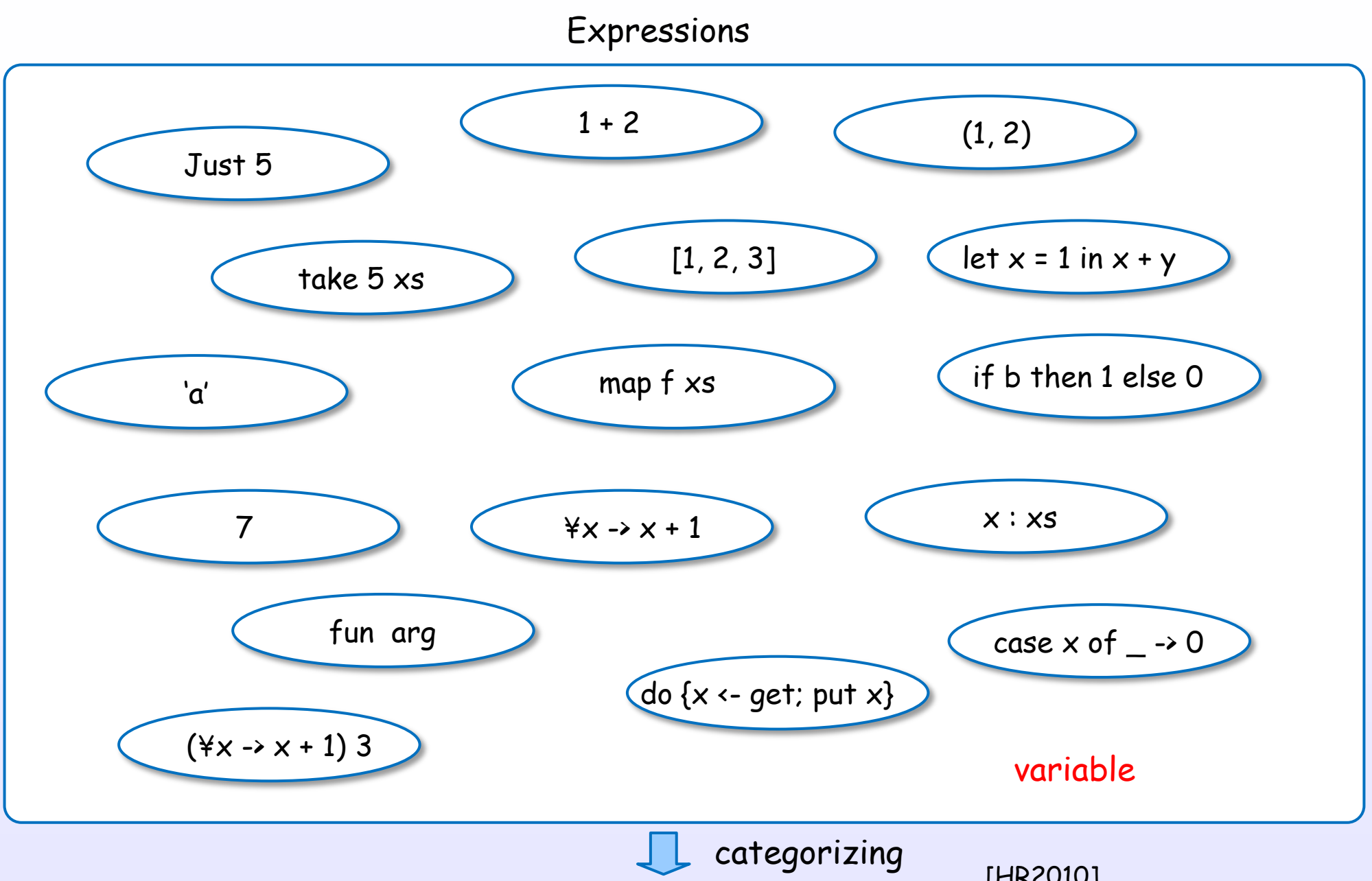


[HR2010]

[Bird, Chapter 2]

References : [1]

There are many expressions in Haskell



[HR2010]

[Bird, Chapter 2]

References : [1]

Expression categories in Haskell

lambda abstraction

¥x -> x + 1

let expression

let x = 1 in x + y

variable

conditional

if b then 1 else 0

case expression

case x of _ -> 0

do expression

do {x <- get; put x}

general constructor, literal and some forms

7

[1, 2, 3]

(1, 2)

'a'

x : xs

Just 5

function application

take 5 xs

(¥x -> x + 1) 3

1 + 2

map f xs

fun arg

[HR2010]

[Bird, Chapter 2]

References : [1]

Specification is defined in Haskell 2010 Language Report

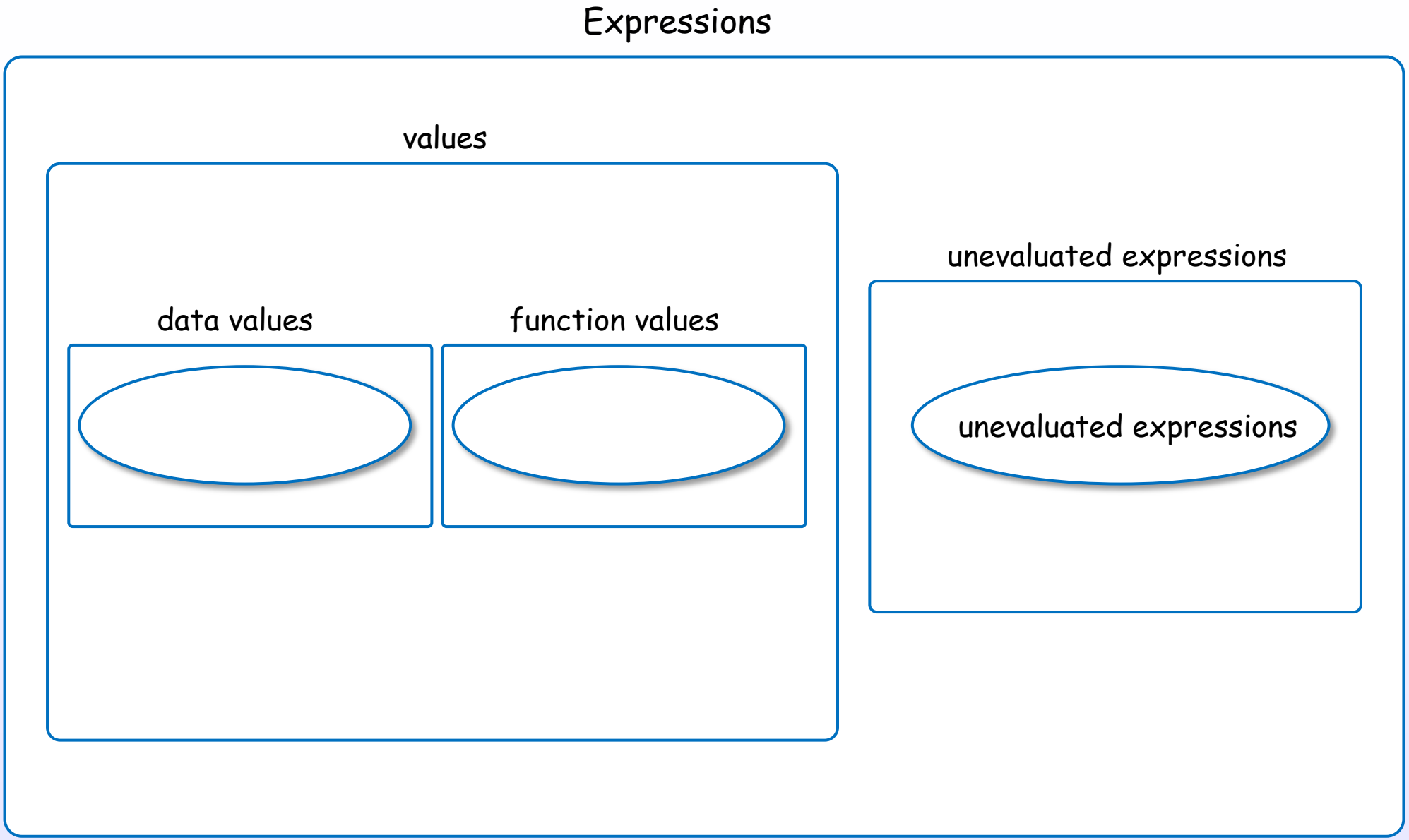
Haskell 2010 Language Report, Chapter 3 Expressions [1]

exp	→	*infixexp* `::` [*context* `=>`] *type*	(expression type signature)
	|	*infixexp*	
infixexp	→	*lexp qop infixexp*	(infix operator application)
	|	`-` *infixexp*	(prefix negation)
	|	*lexp*	
lexp	→	`\` $apat_1$ ... $apat_n$ `->` *exp*	(lambda abstraction, $n \geq 1$)
	|	`let` *decls* `in` *exp*	(let expression)
	|	`if` *exp* [`;`] `then` *exp* [`;`] `else` *exp*	(conditional)
	|	`case` *exp* `of` `{` *alts* `}`	(case expression)
	|	`do` `{` *stmts* `}`	(do expression)
	|	*fexp*	
fexp	→	[*fexp*] *aexp*	(function application)
aexp	→	*qvar*	(variable)
	|	*gcon*	(general constructor)
	|	*literal*	
	|	`(` *exp* `)`	(parenthesized expression)
	|	`(` exp_1 `,` ... `,` exp_k `)`	(tuple, $k \geq 2$)
	|	`[` exp_1 `,` ... `,` exp_k `]`	(list, $k \geq 1$)
	|	`[` exp_1 [`,` exp_2] `..` [exp_3] `]`	(arithmetic sequence)
	|	`[` *exp* `|` $qual_1$ `,` ... `,` $qual_n$ `]`	(list comprehension, $n \geq 1$)
	|	`(` *infixexp qop* `)`	(left section)
	|	`(` $qop_{\langle - \rangle}$ *infixexp* `)`	(right section)
	|	*qcon* `{` $fbind_1$ `,` ... `,` $fbind_n$ `}`	(labeled construction, $n \geq 0$)
	|	$aexp_{\langle qcon \rangle}$ `{` $fbind_1$ `,` ... `,` $fbind_n$ `}`	(labeled update, $n \geq 1$)

References : [1]

Classification of expressions

A value or an unevaluated expression

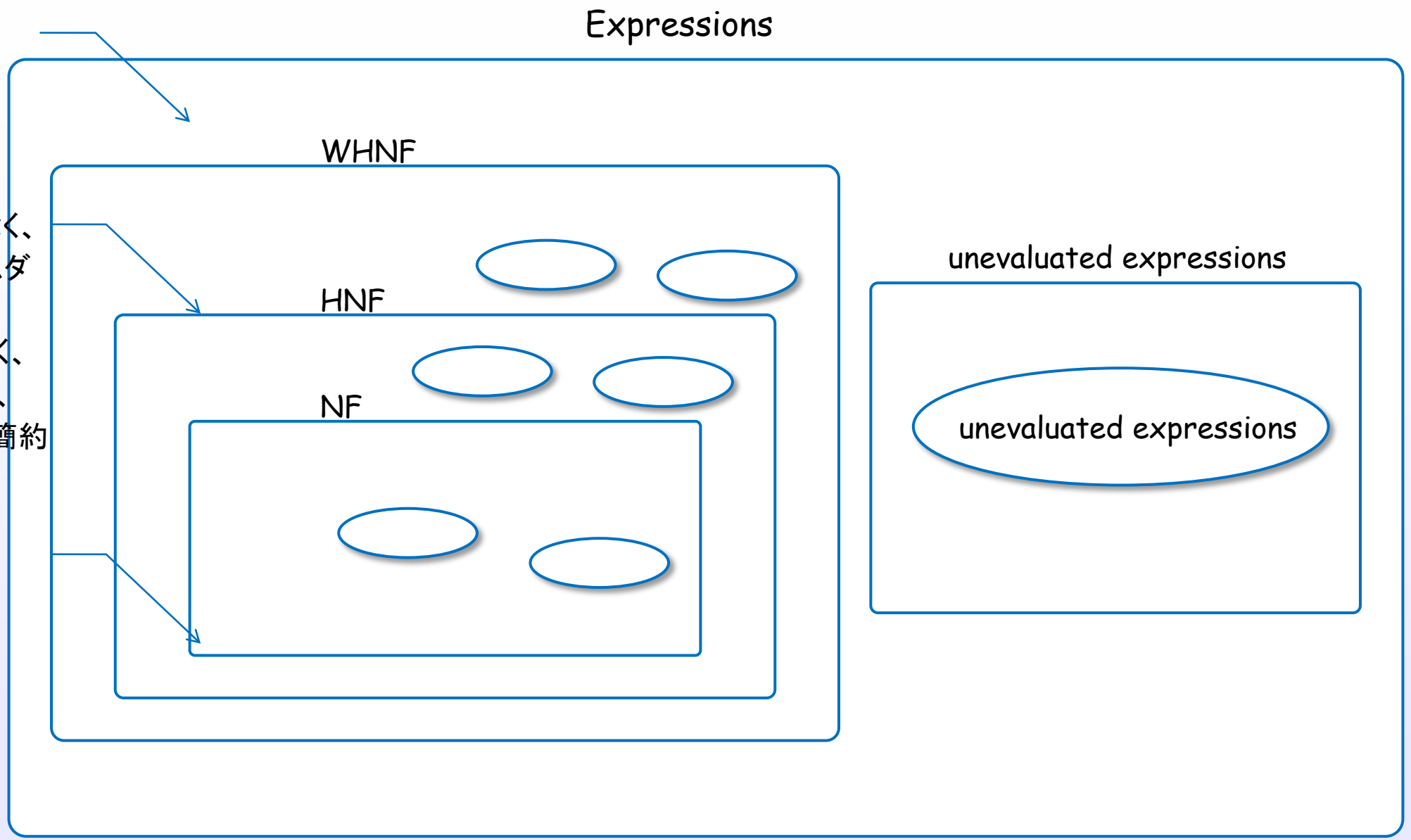


値か否か。値は2種。

[STG]

References : [1]

evaluation level



値には、評価レベルがある。

[STG]

References : [1]

実例との対応付け

[STG]

References : [1]

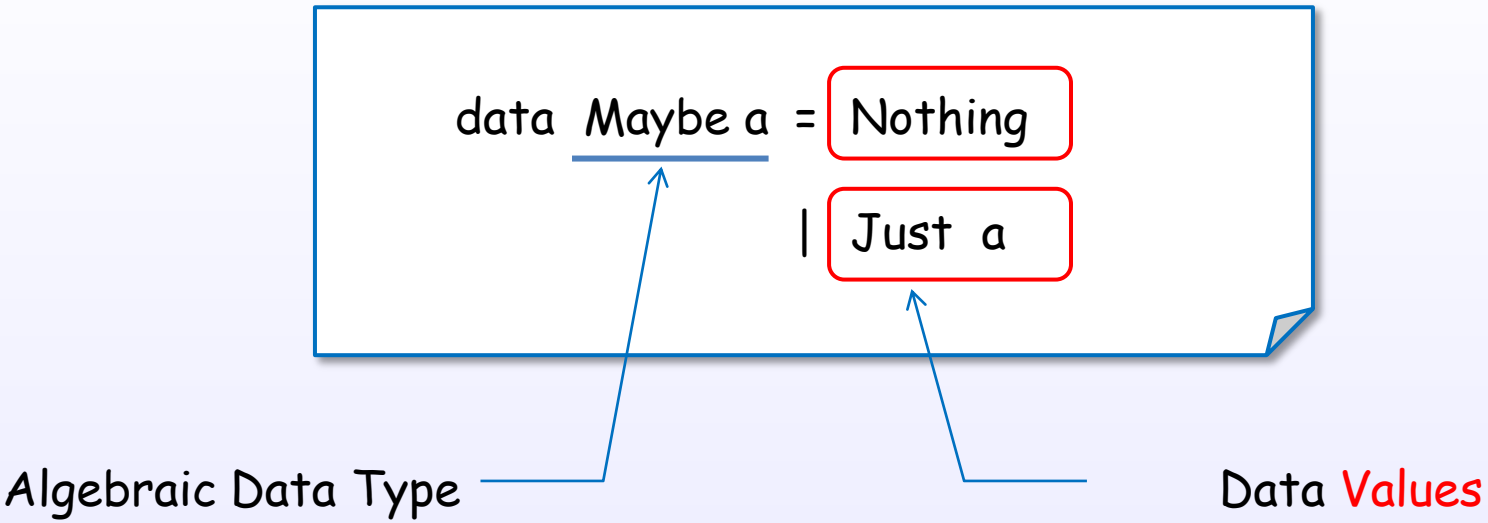
Constructor

Constructor

Constructor is one of the key elements
to understand WHNF and lazy evaluation.

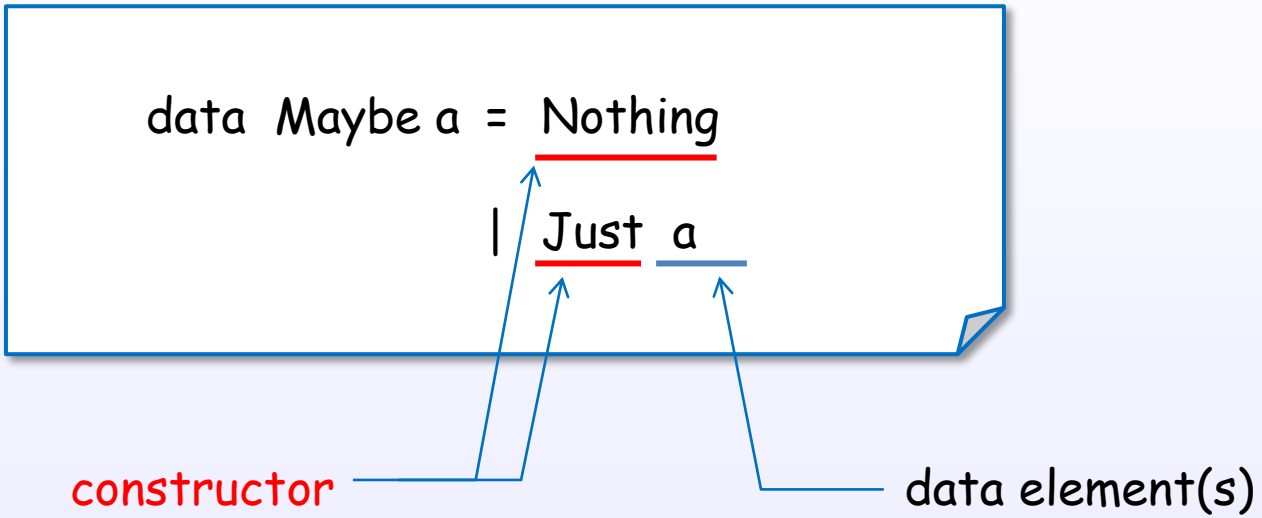
References : [1]

data文で宣言する代数的データ型とその値



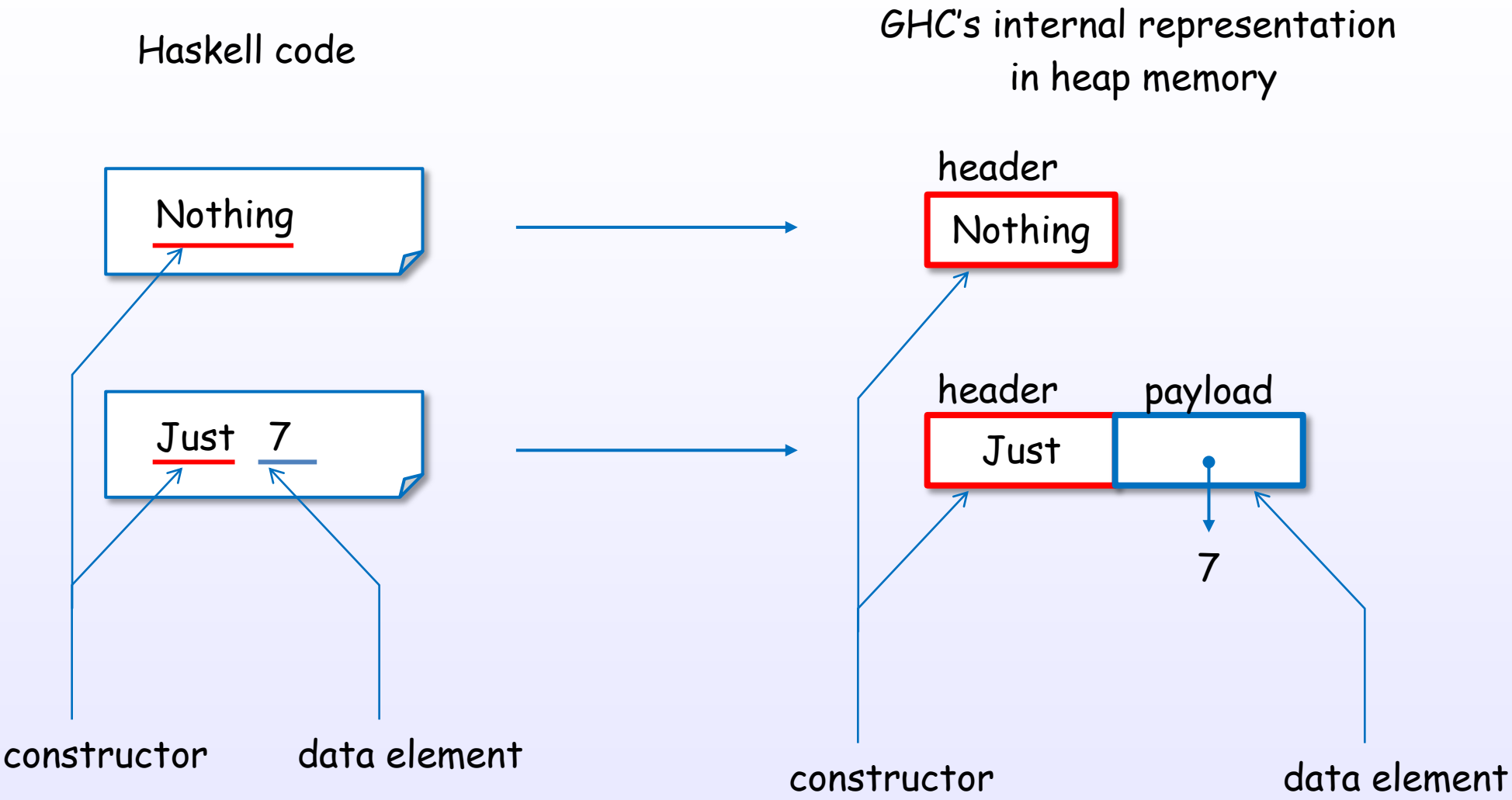
References : [1]

Constructorはdata文で宣言する代数的データ値



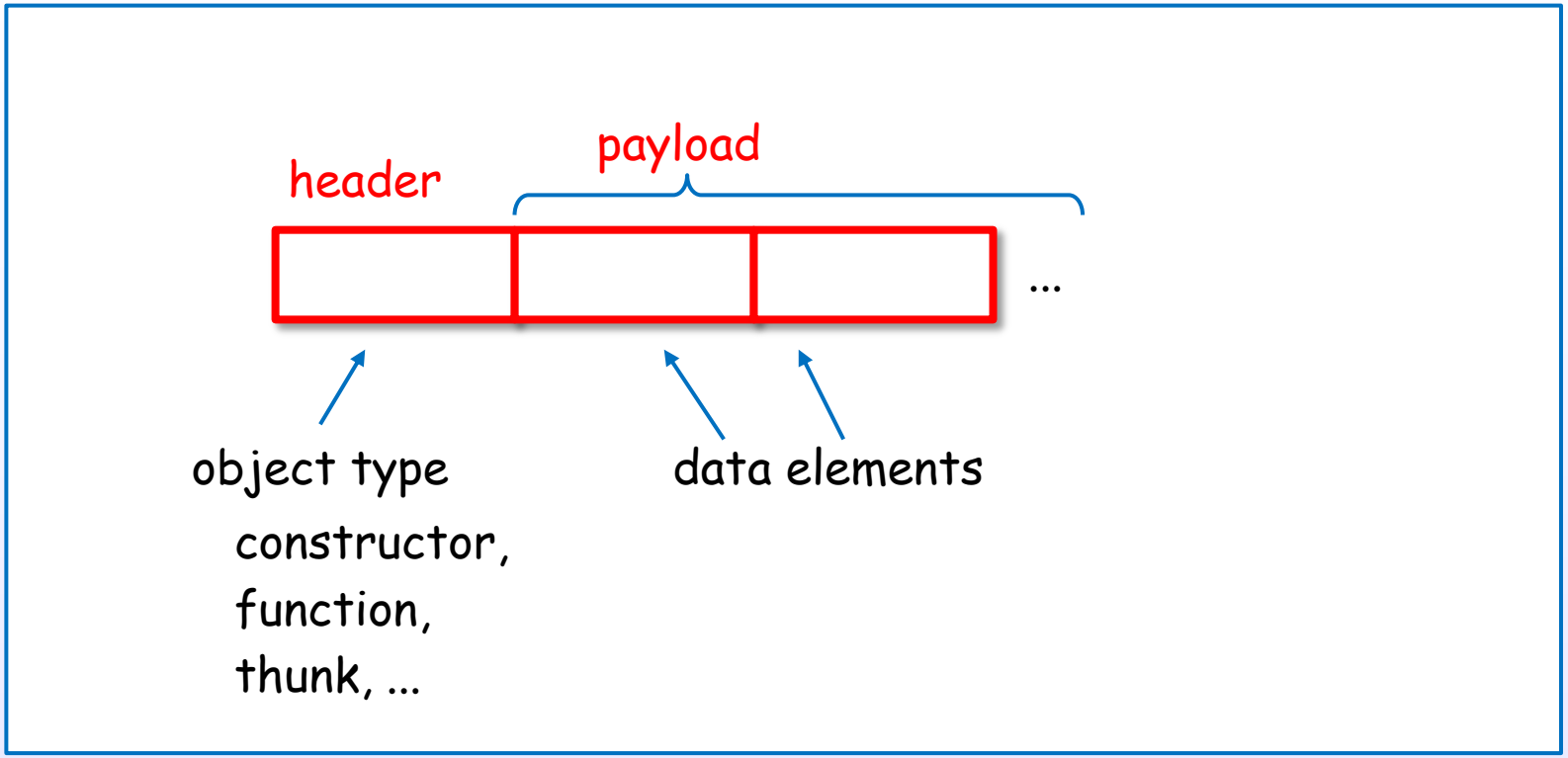
References : [1]

Constructorの内部表現



References : [1]

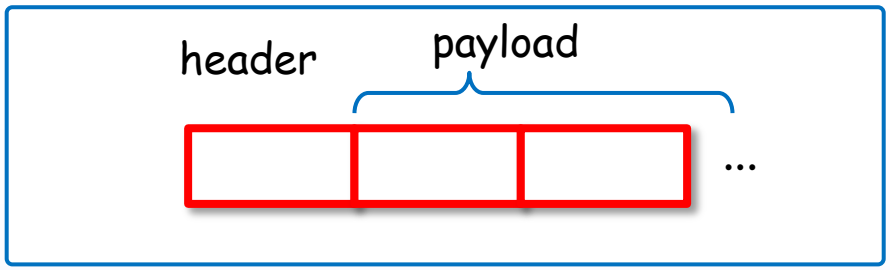
Constructorは統一内部表現で表現される



in heap memory, stack, registers or static memory

References : [1]

統一内部表現



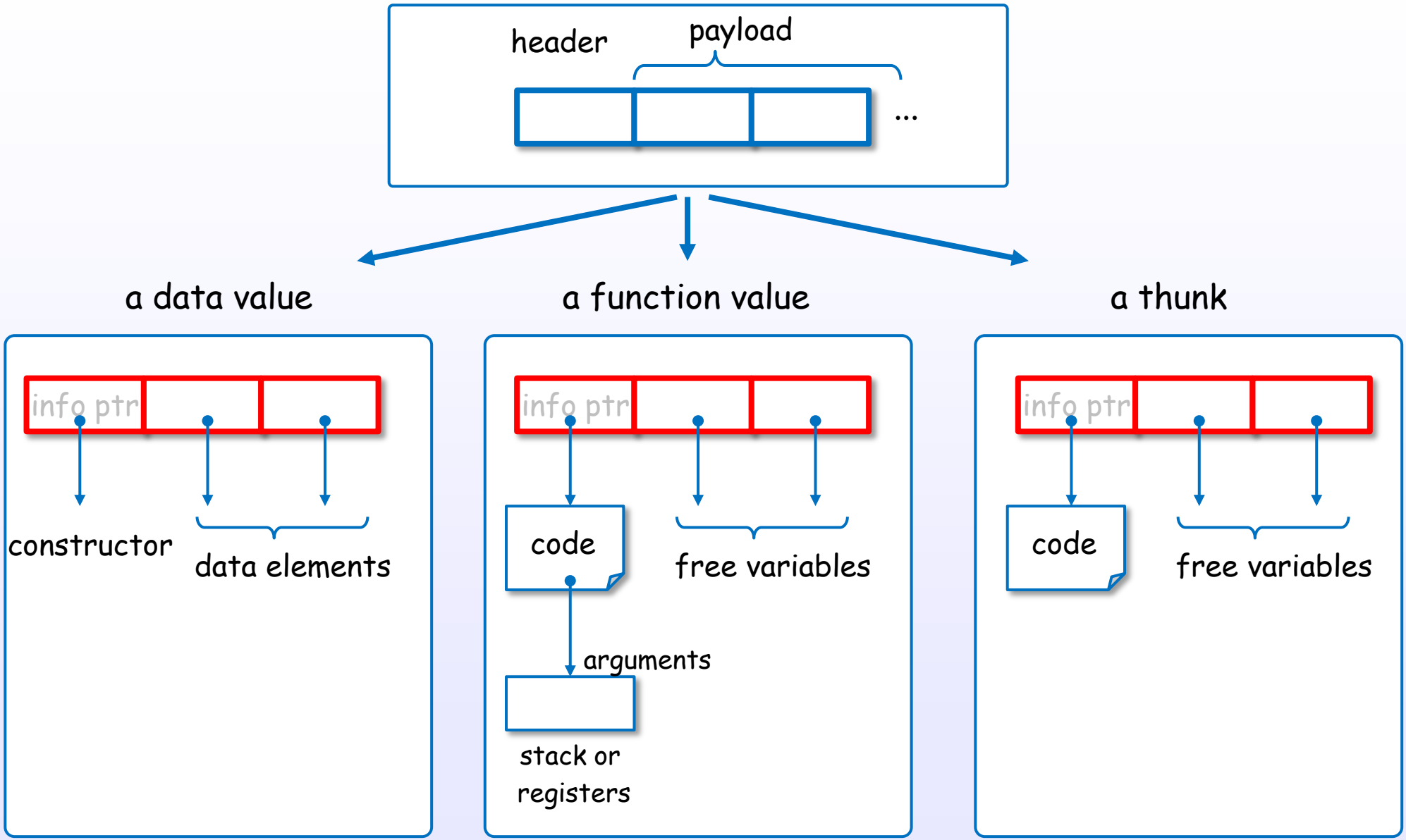
a data value

a function value

a thunk

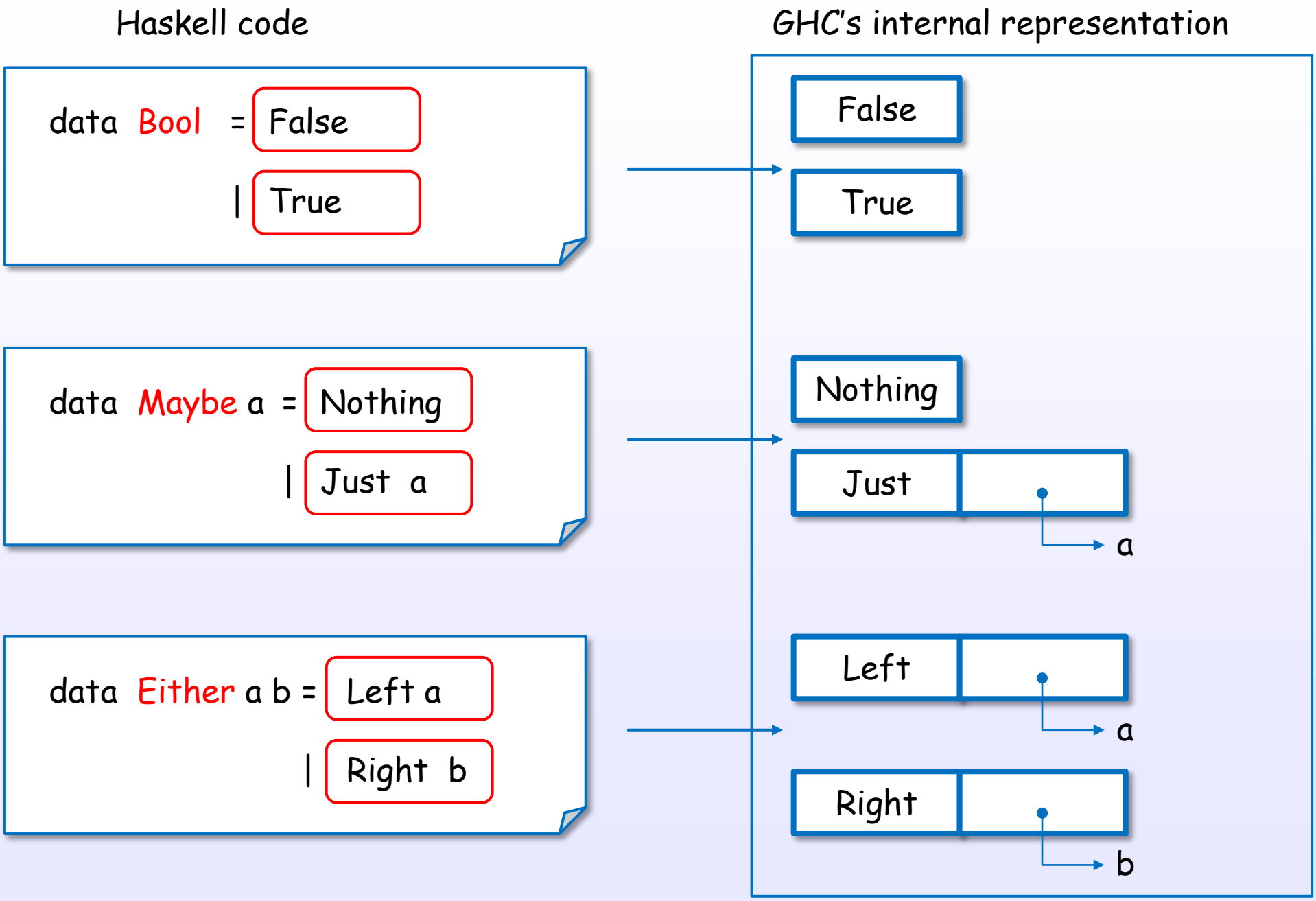
References : [1]

統一内部表現



References : [1]

いろいろなコンストラクタと内部表現



References : [1]

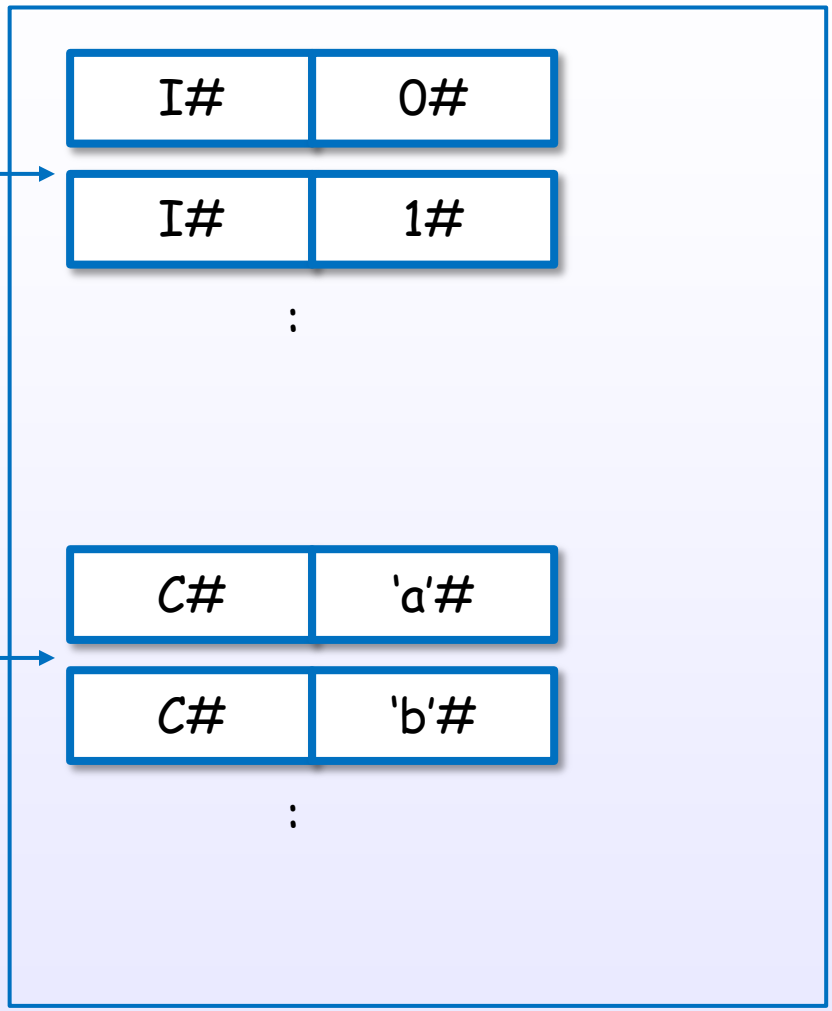
基本データ型も実はコンストラクタで構成されている

Haskell code

```
data Int  = I# 0#
          | I# 1#
          | :
```

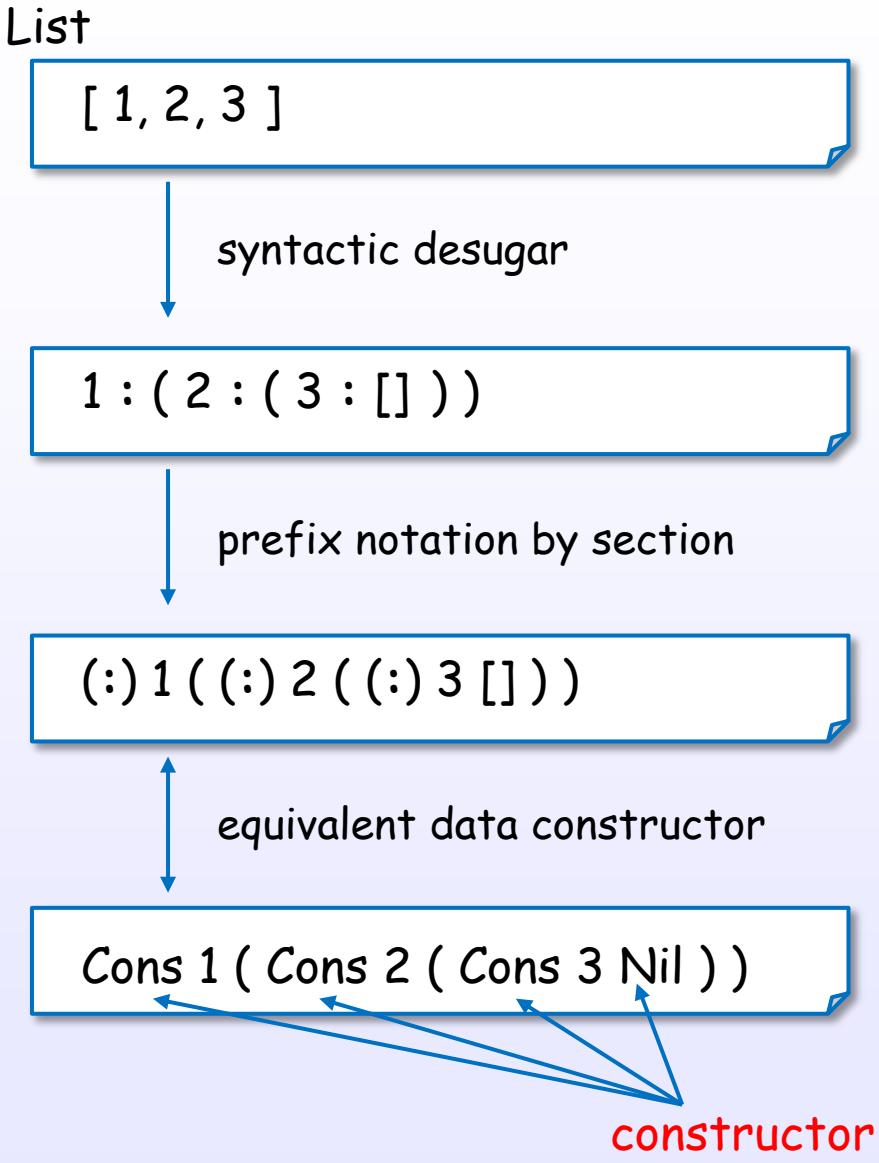
```
data Char = C# 'a'#
          | C# 'b'#
          | :
```

GHC's internal representation



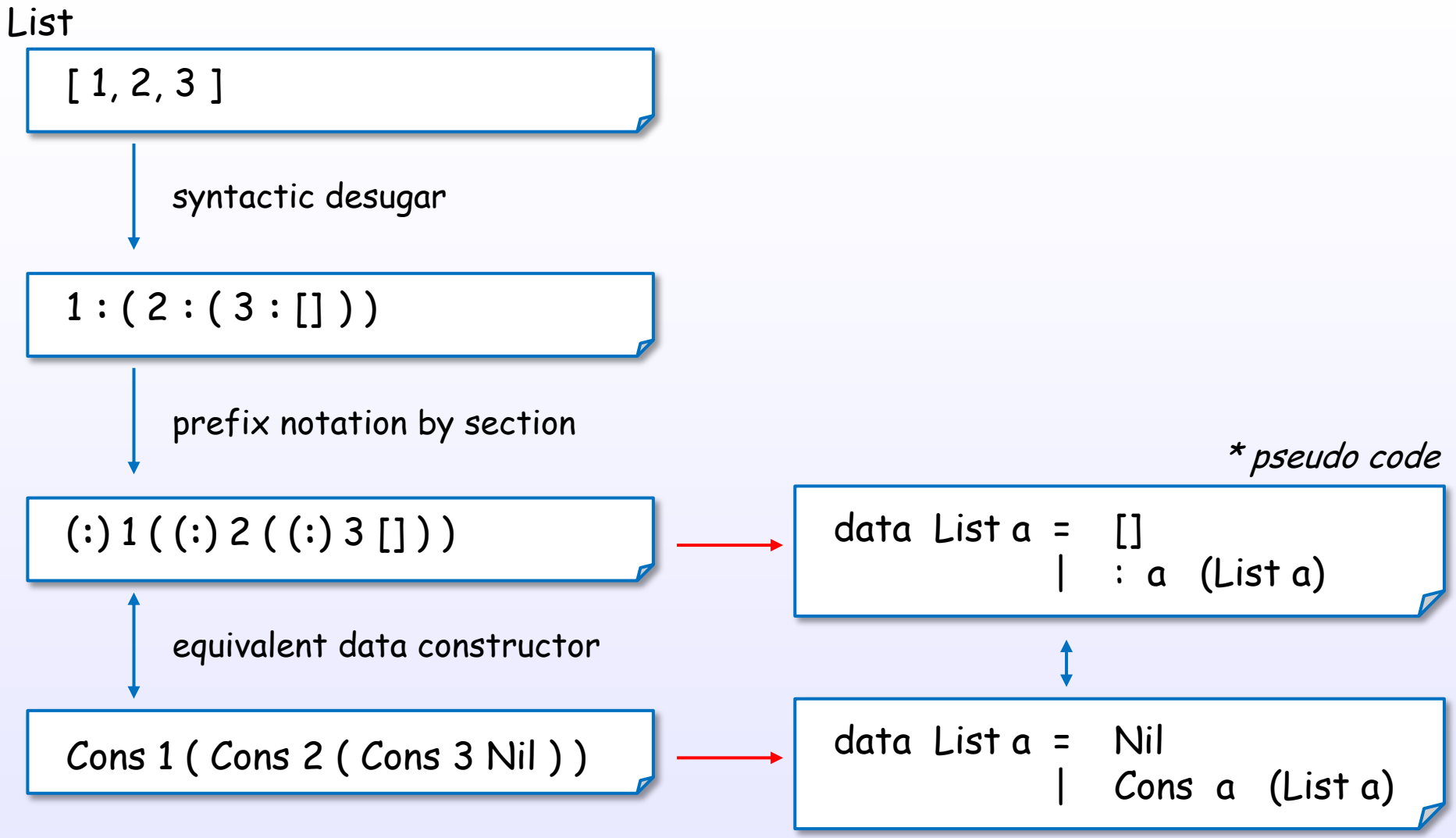
References : [1]

リストも実はコンストラクタで構成されている



References : [1]

リストも実はコンストラクタで構成されている



References : [1]

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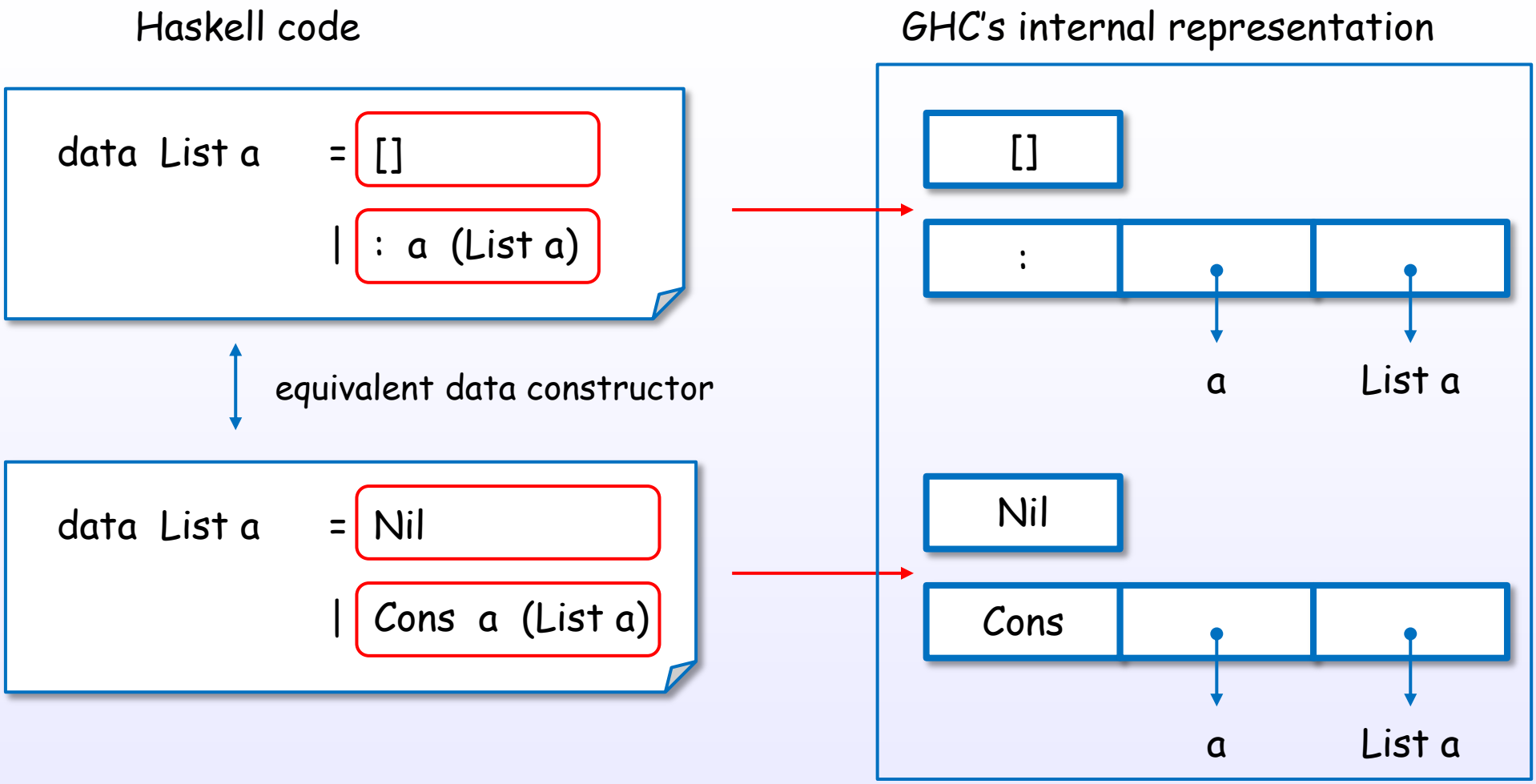


References : [1]

リストも実はコンストラクタで構成されている

Haskell code

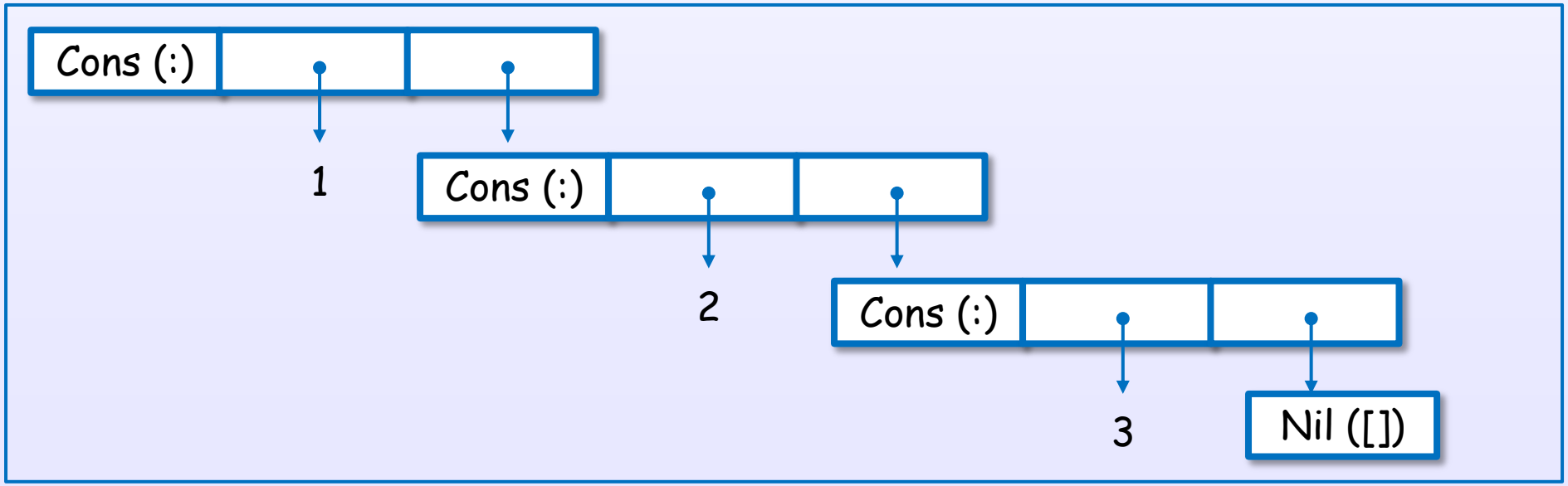
```
[ 1, 2, 3 ]
```

```
1 : ( 2 : ( 3 : [] ) )
```

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

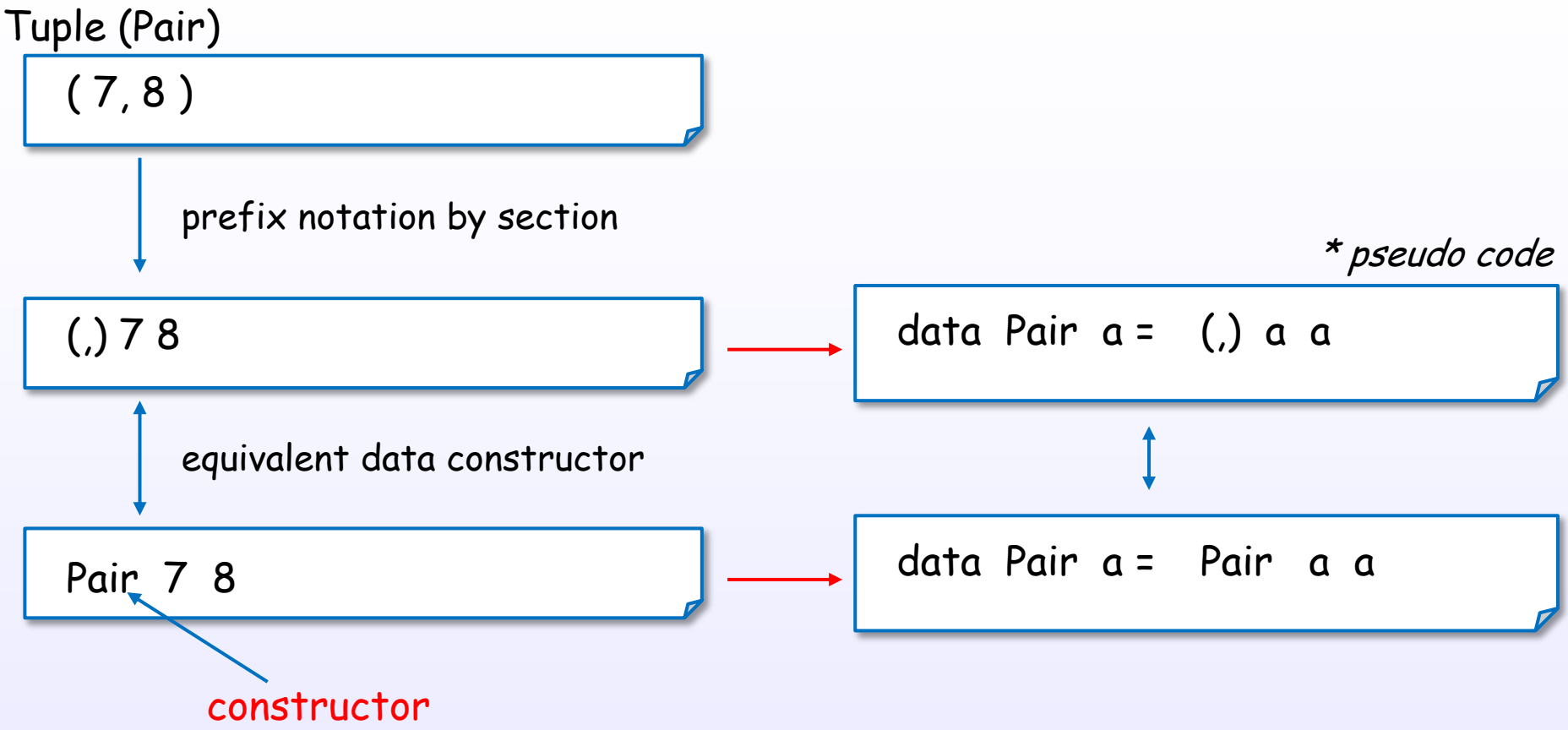
```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

GHC's internal representation



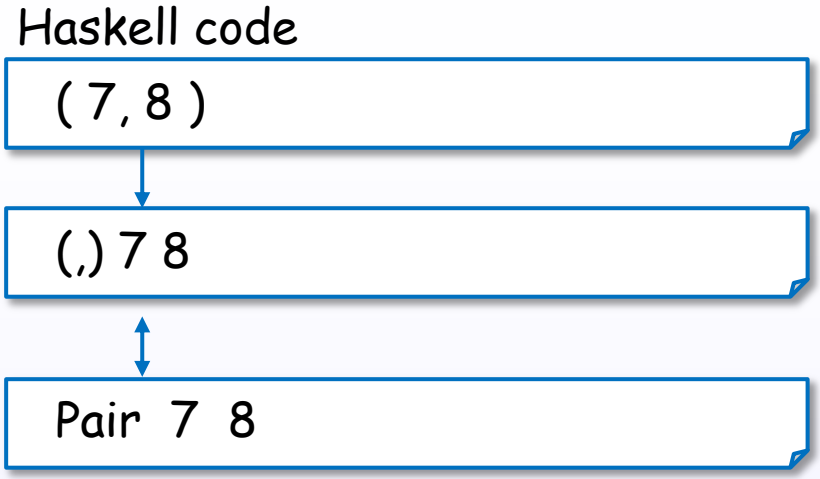
References : [1]

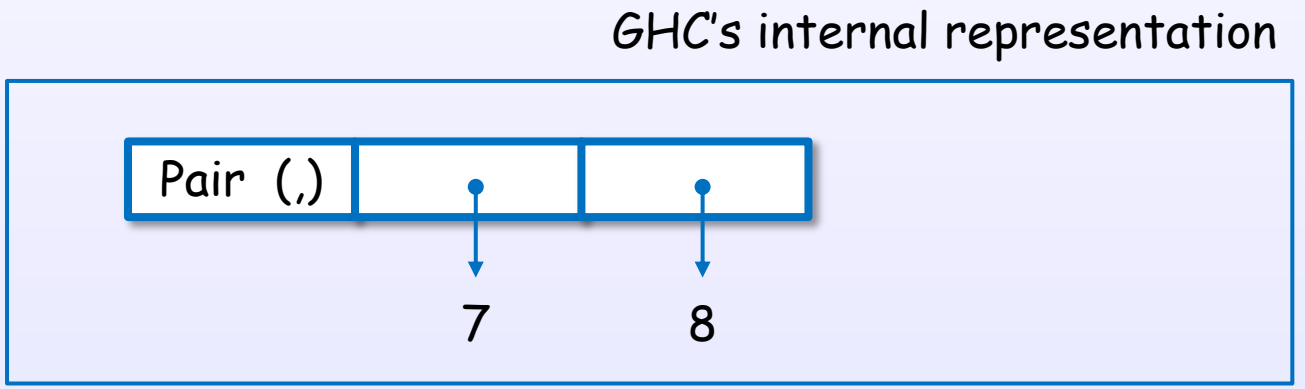
タプルも実はコンストラクタで構成されている



References : [1]

タプルも実はコンストラクタで構成されている





References : [1]

Thunk

Thunk

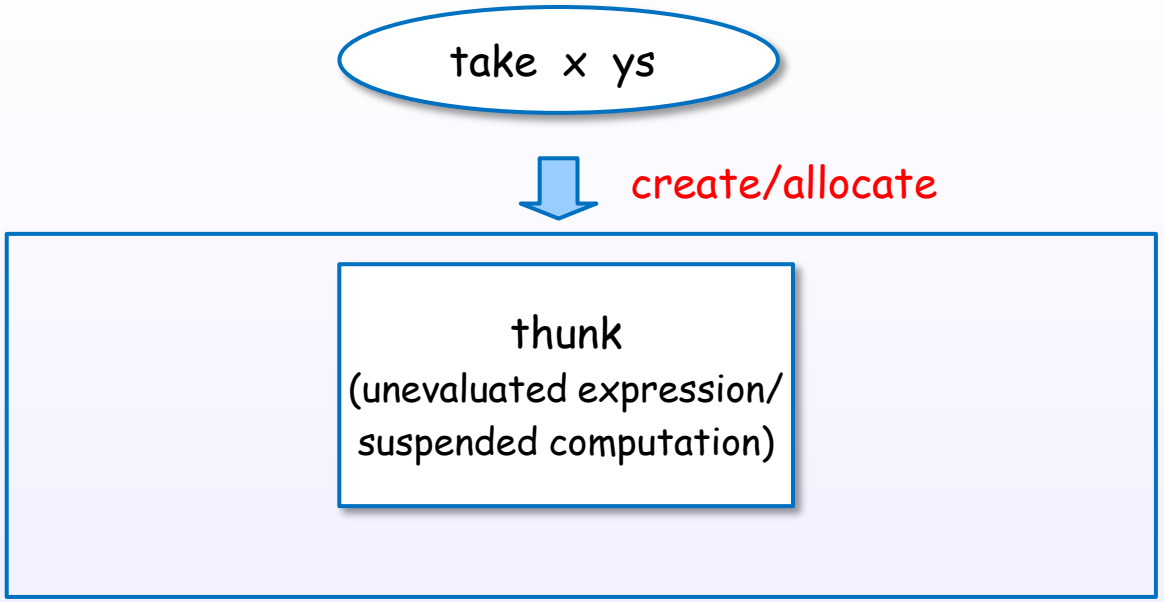
thunk
(unevaluated expression/
suspended computation)

A thunk is an unevaluated expression in heap memory.

References : [1]

Thunk

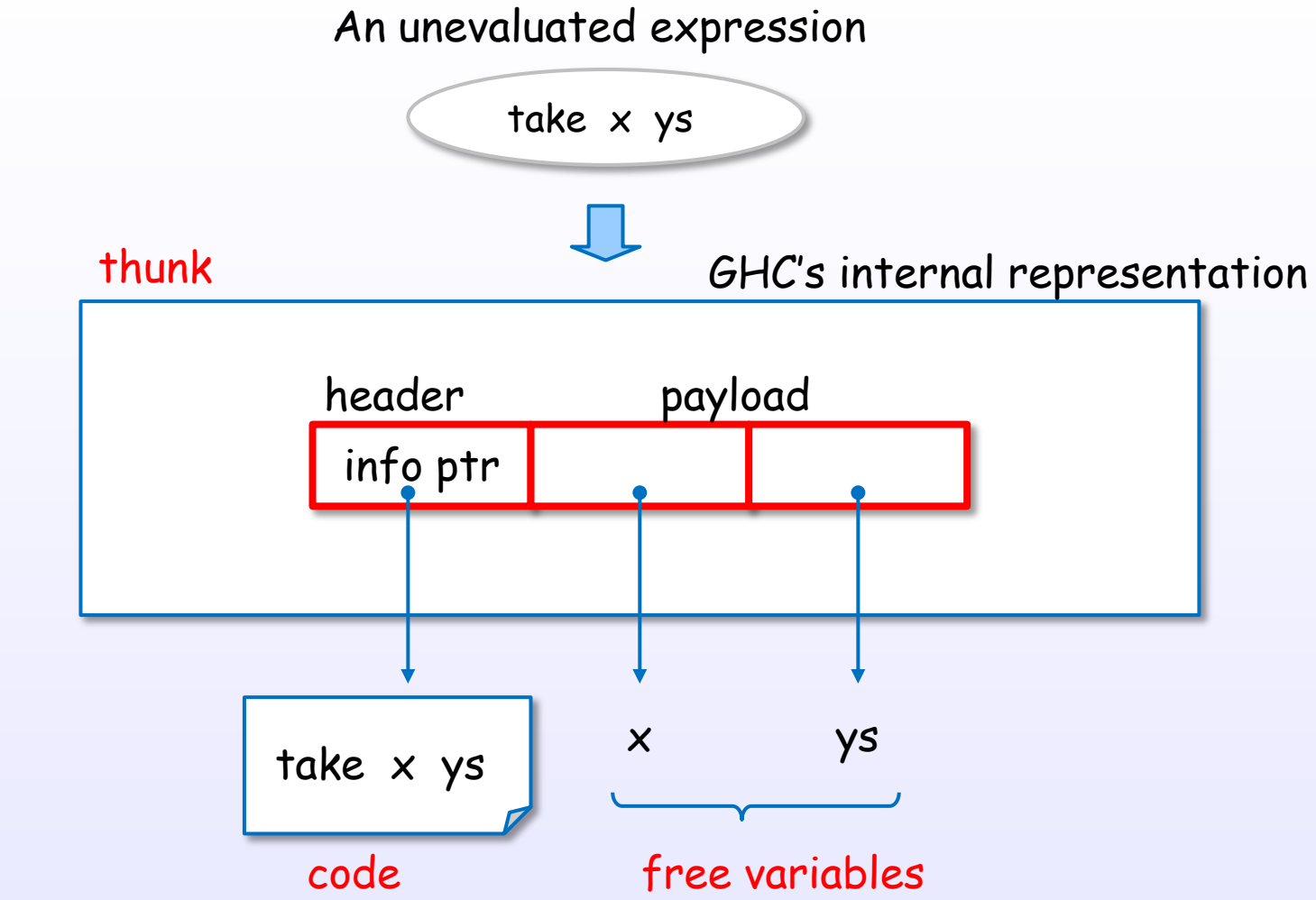
An unevaluated expression



A thunk is created for an unevaluated expression.

References : [1]

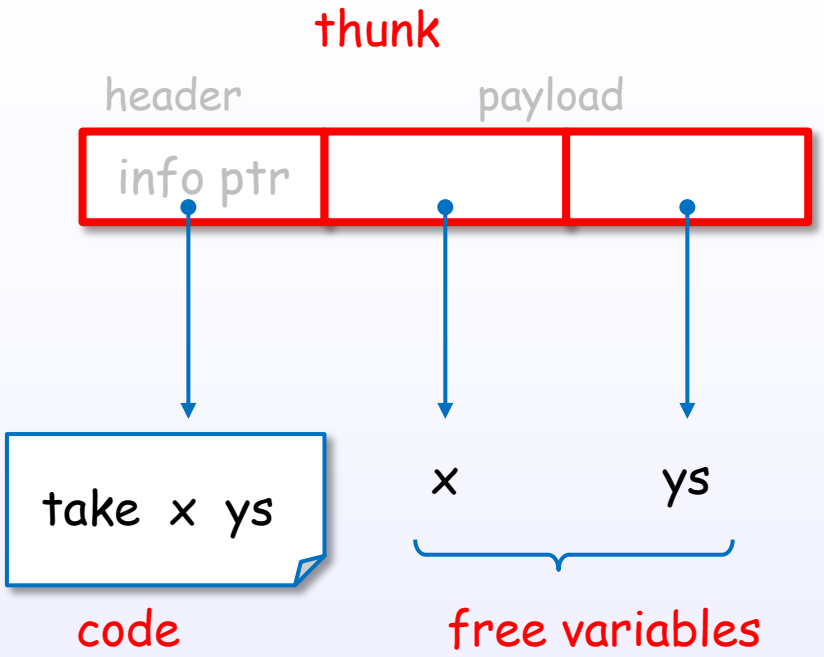
Thunkの内部表現



A thunk is represented with header(code) + payload(free variables).

References : [1]

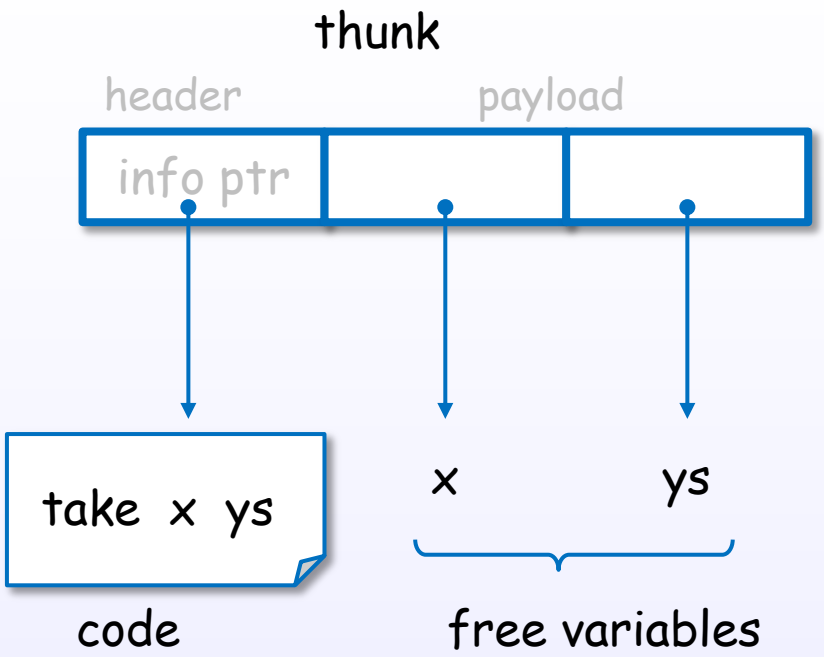
Thunkは、codeとfree variablesをパッケージ化したもの



A thunk is a package of code + free variables.

References : [1]

Thunkは、forcing要求により評価される



forcing

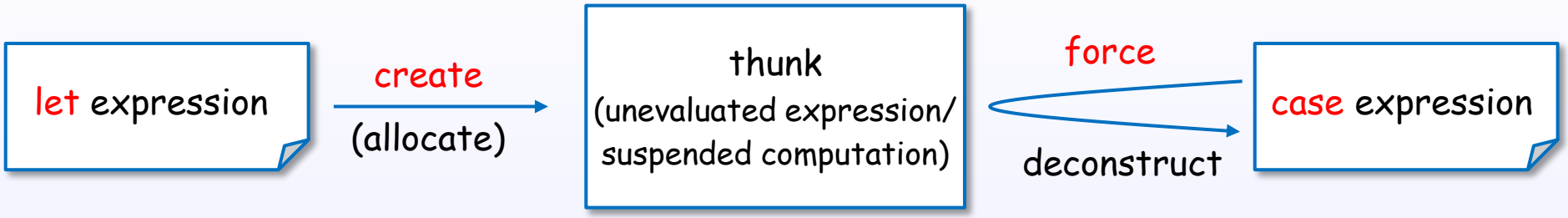
[1, 2, 3]

A evaluated expression

References : [1]

let, case expression

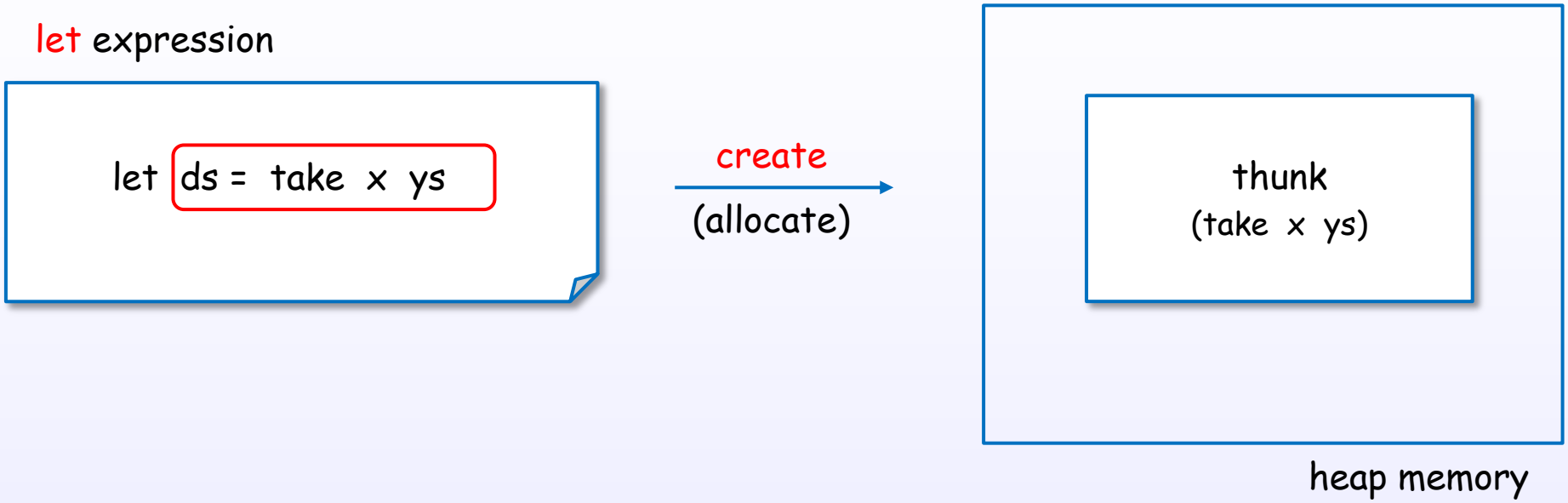
let/case expressions and thunk



A let expression may create a thunk.
A case expression forces and deconstructs the thunk.

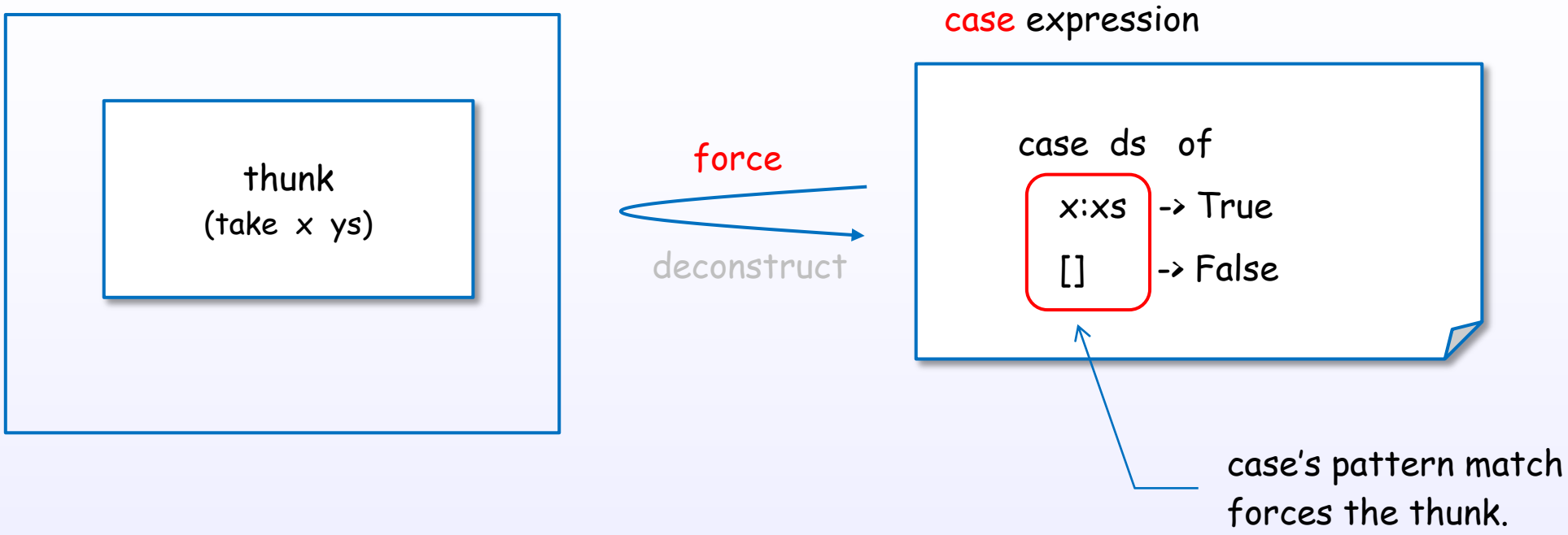
References : [1]

A let expression creates a thunk



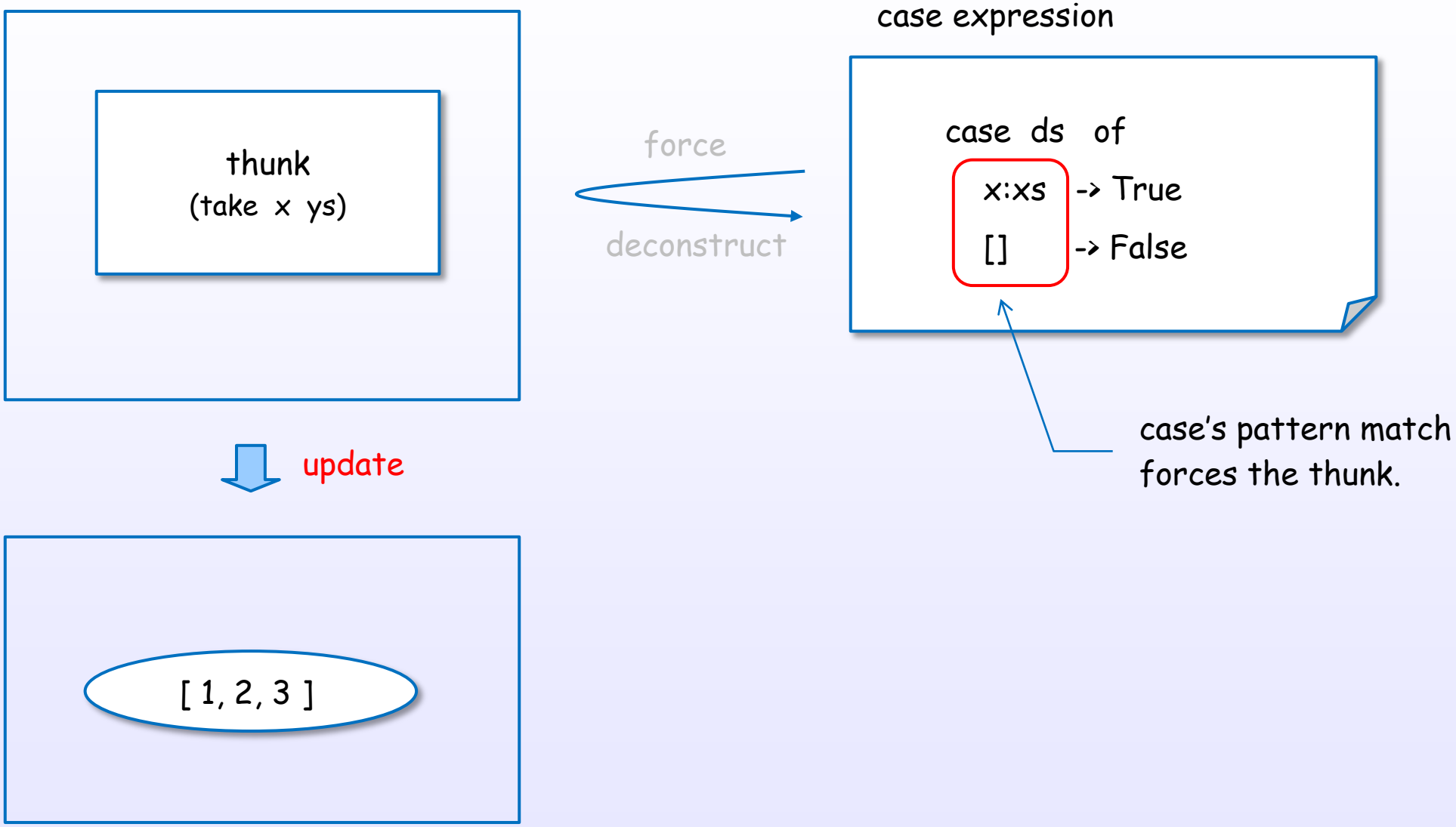
References : [1]

A case expression forces a thunk



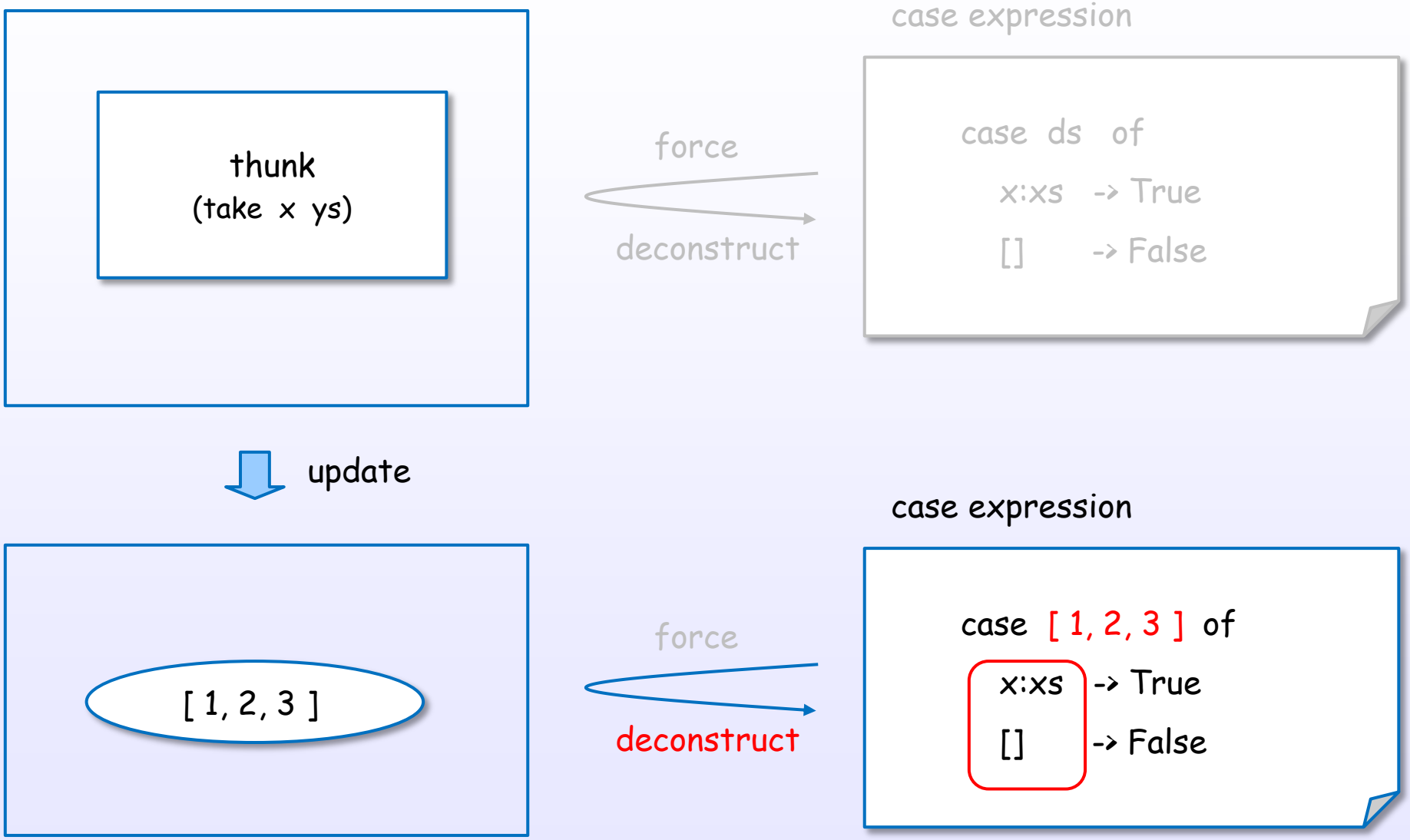
References : [1]

A case expression forces a thunk



References : [1]

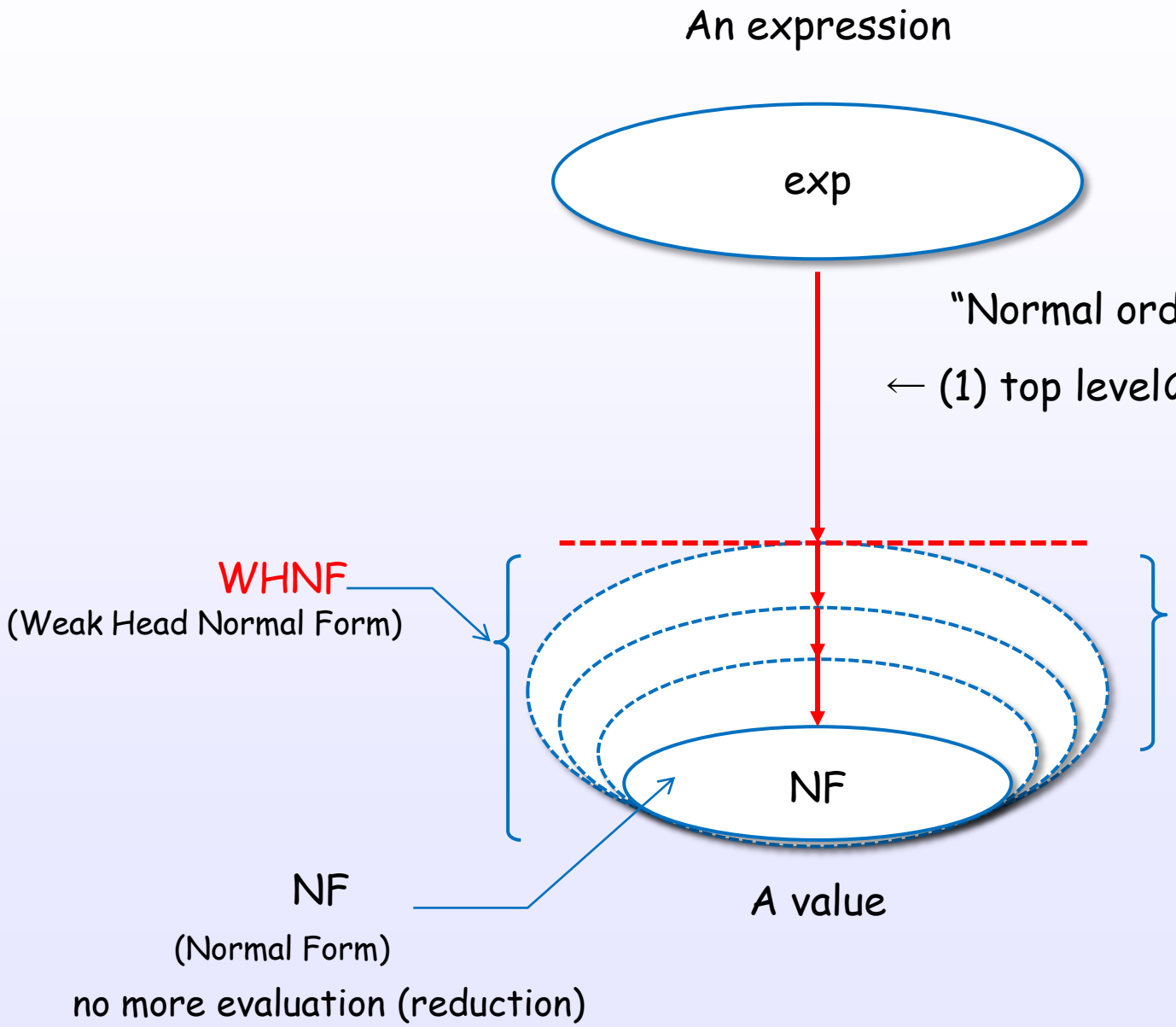
A case expression forces a thunk



References : [1]

WHNF

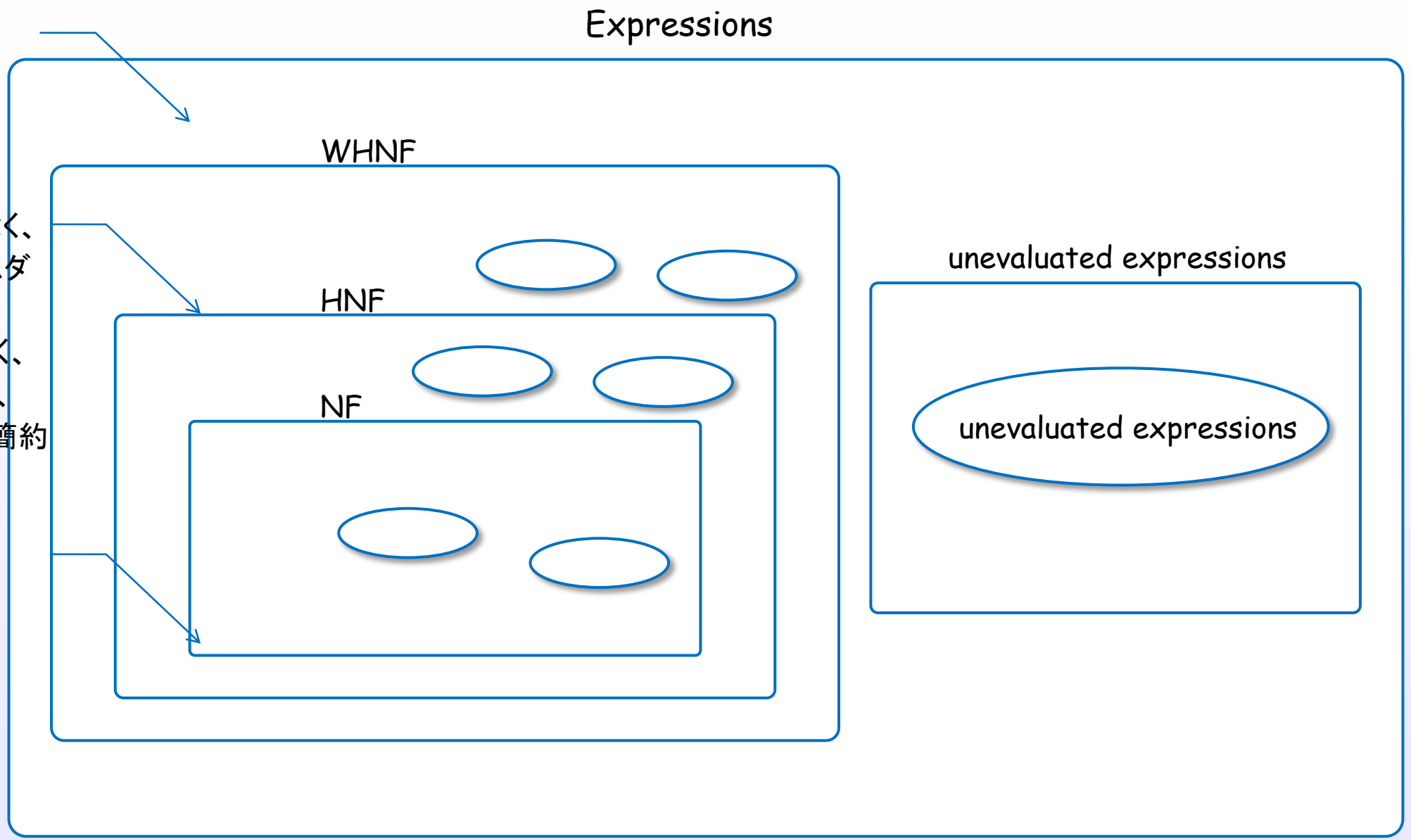
evaluation step (GHC)



References : [1]

再掲

evaluation level

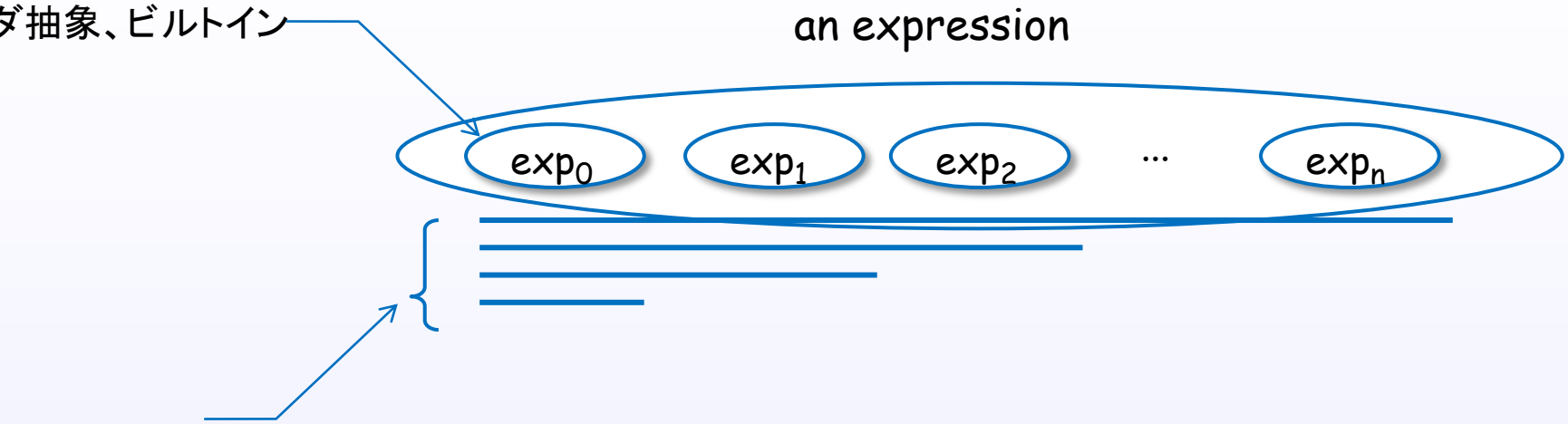


値には、評価レベルがある。

[STG]

References : [1]

WHNF



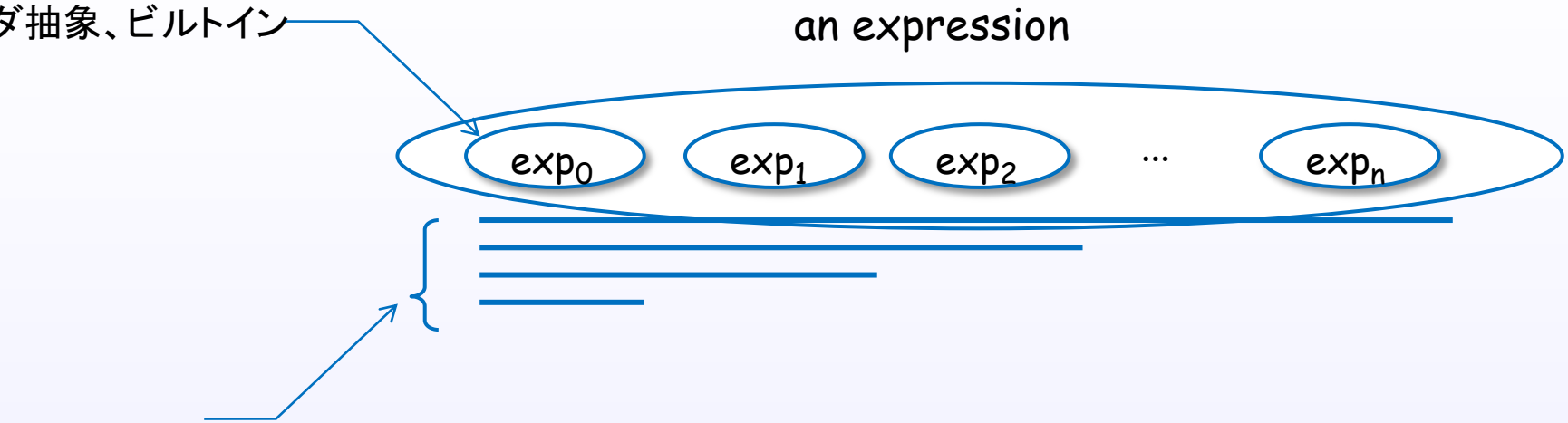
more

An expression has no top level redex, if it is in WHNF.

[slpj-book-1987]

References : [1]

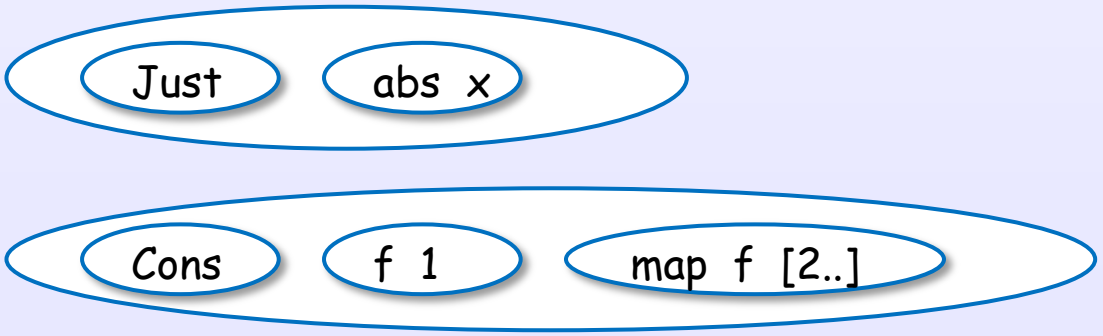
Examples of WHNF



Just (abs x)

Cons (f 1) (map f [2..])

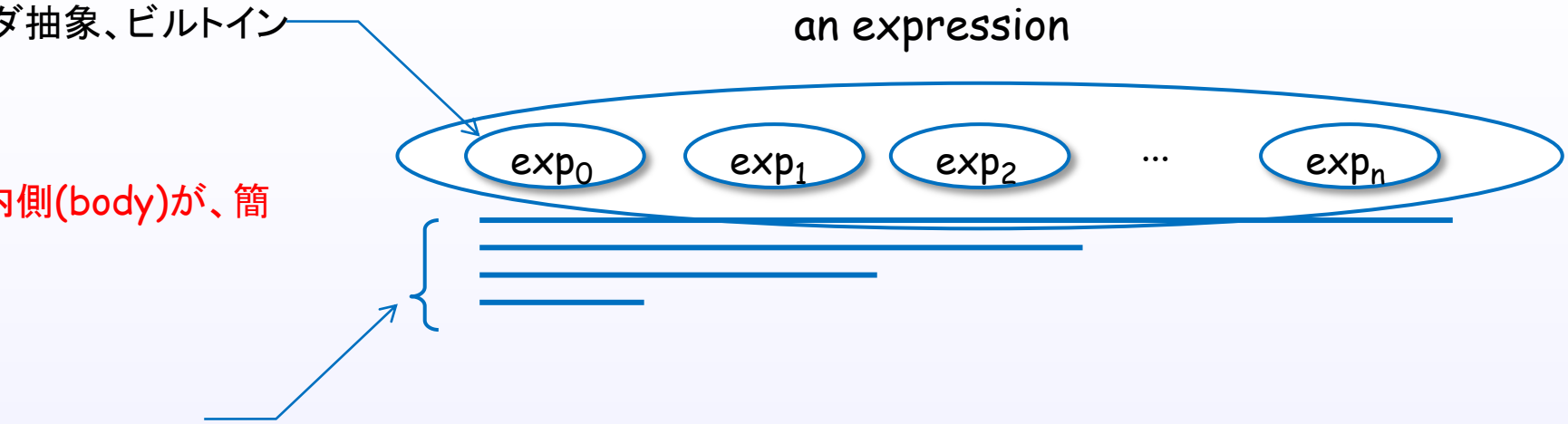
more



[slpj-book-1987]

References : [1]

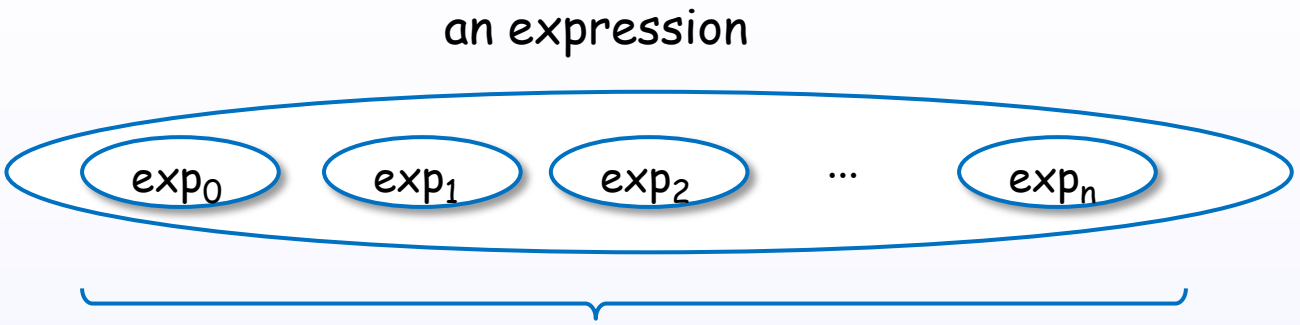
HNF



[slpj-book-1987]

References : [1]

NF

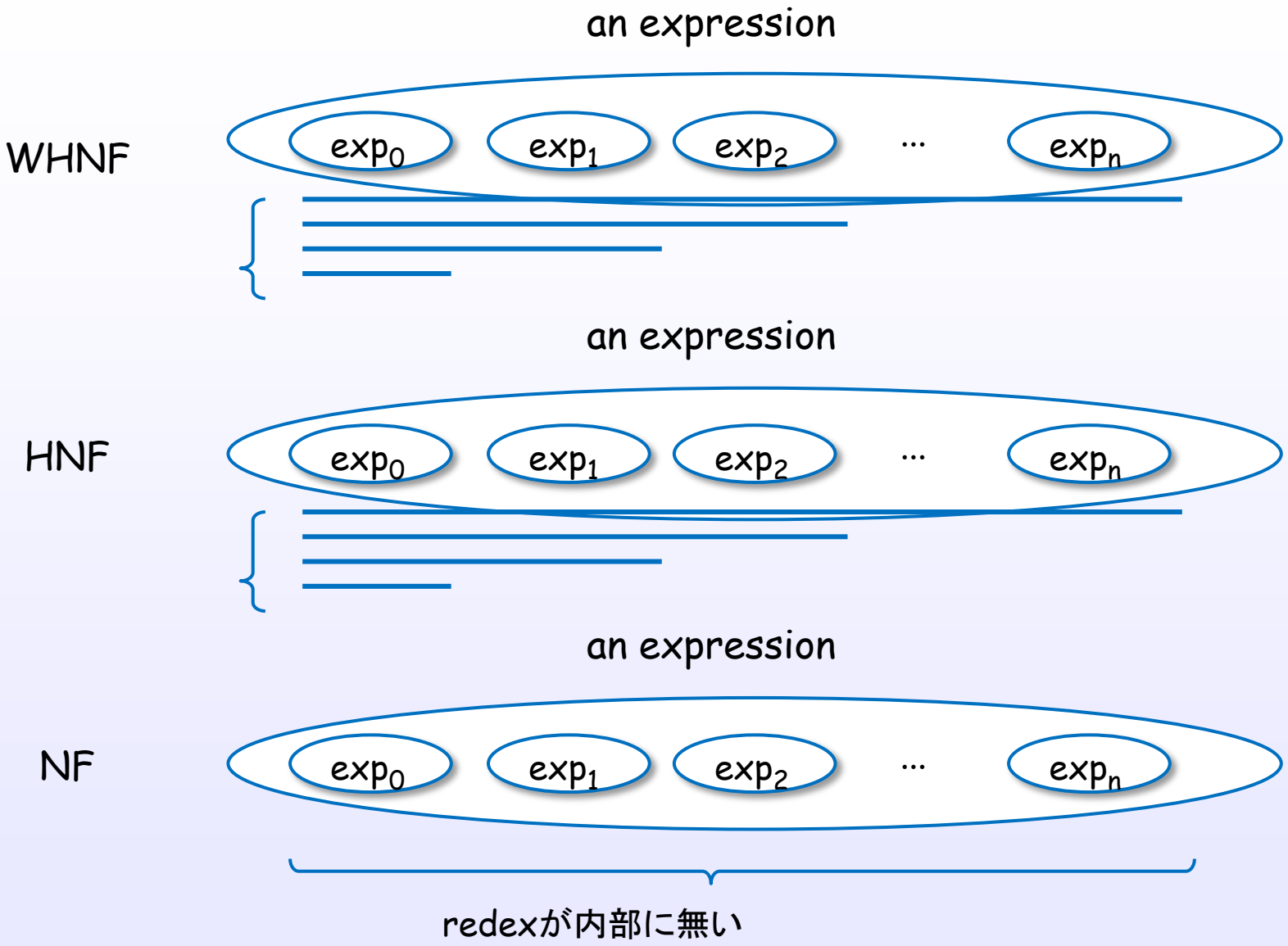


redexが内部に無い

[slpj-book-1987]

References : [1]

WHNF, HNF, NF



[slpj-book-1987]

References : [1]

definition of WHNF and HNF

The implementation of functional programming languages [19]

11.3.1 Weak Head Normal Form

To express this idea precisely we need to introduce a new definition:

DEFINITION

A lambda expression is in *weak head normal form* (WHNF) if and only if it is of the form

$$F\ E_1\ E_2\ \ldots\ E_n$$

where $n \geq 0$;
and either F is a variable or data object
or F is a lambda abstraction or built-in function
and $(F\ E_1\ E_2\ \ldots\ E_m)$ is not a redex for any $m \leq n$.

An expression has no *top-level redex* if and only if it is in weak head normal form.

11.3.3 Head Normal Form

Head normal form is often confus
some discussion. The content of
since for most purposes head nor
form. Nevertheless, we will stick t

DEFINITION

A lambda expression is in *head normal form* (HNF) if and only if it is of the form

$$\lambda x_1.\lambda x_2.\ldots\lambda x_n.(v\ M_1\ M_2\ \ldots\ M_m)$$

where $n, m \geq 0$;
v is a variable (x_i), a data object, or a built-in function;
and $(v\ M_1\ M_2\ \ldots\ M_p)$ is not a redex for any $p \leq m$.

[slpj-book-1987]

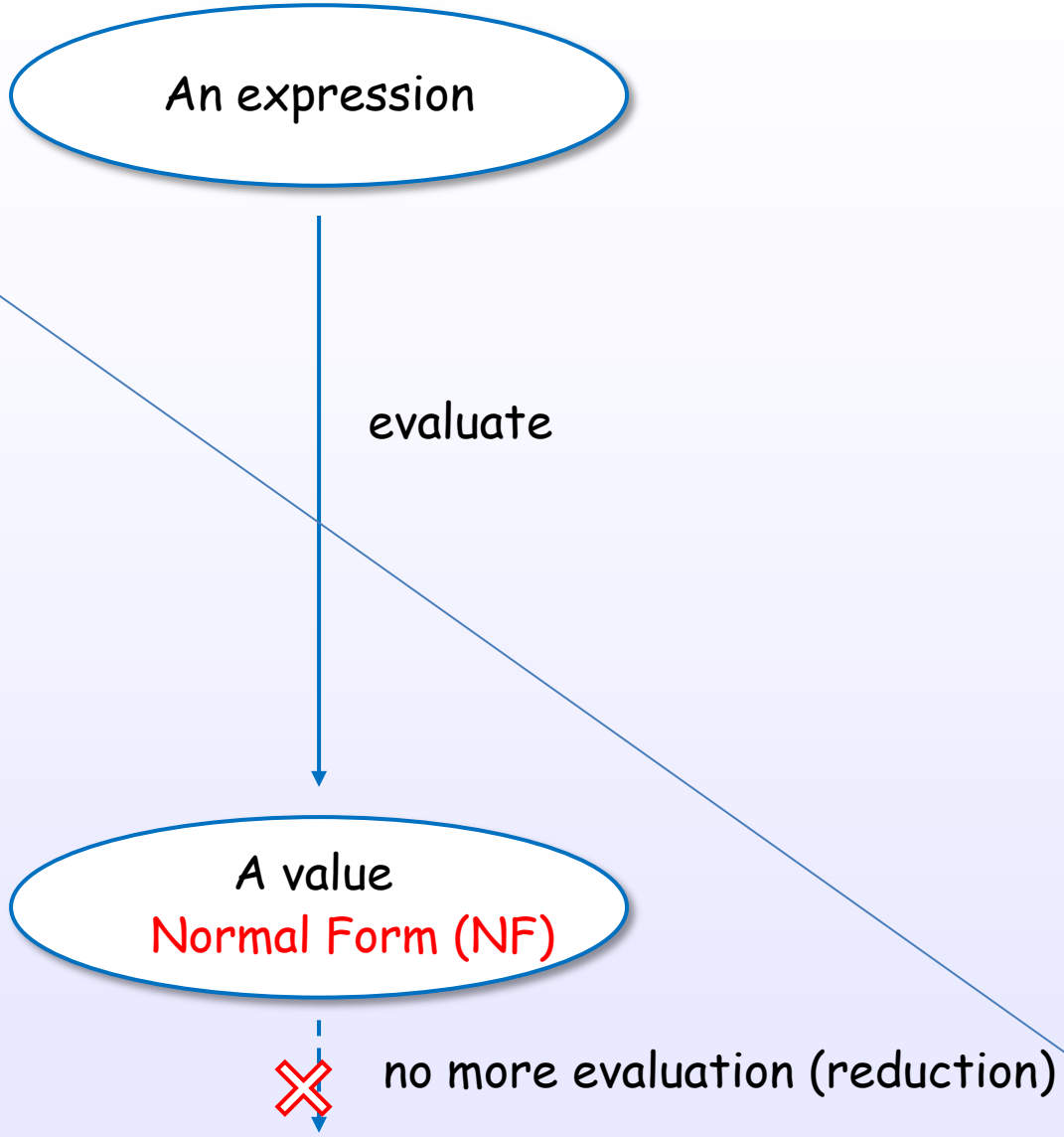
References : [1]

internal representation of WHNF

heap objectイメージ

References : [1]

Normal form は、これ以上評価できない値



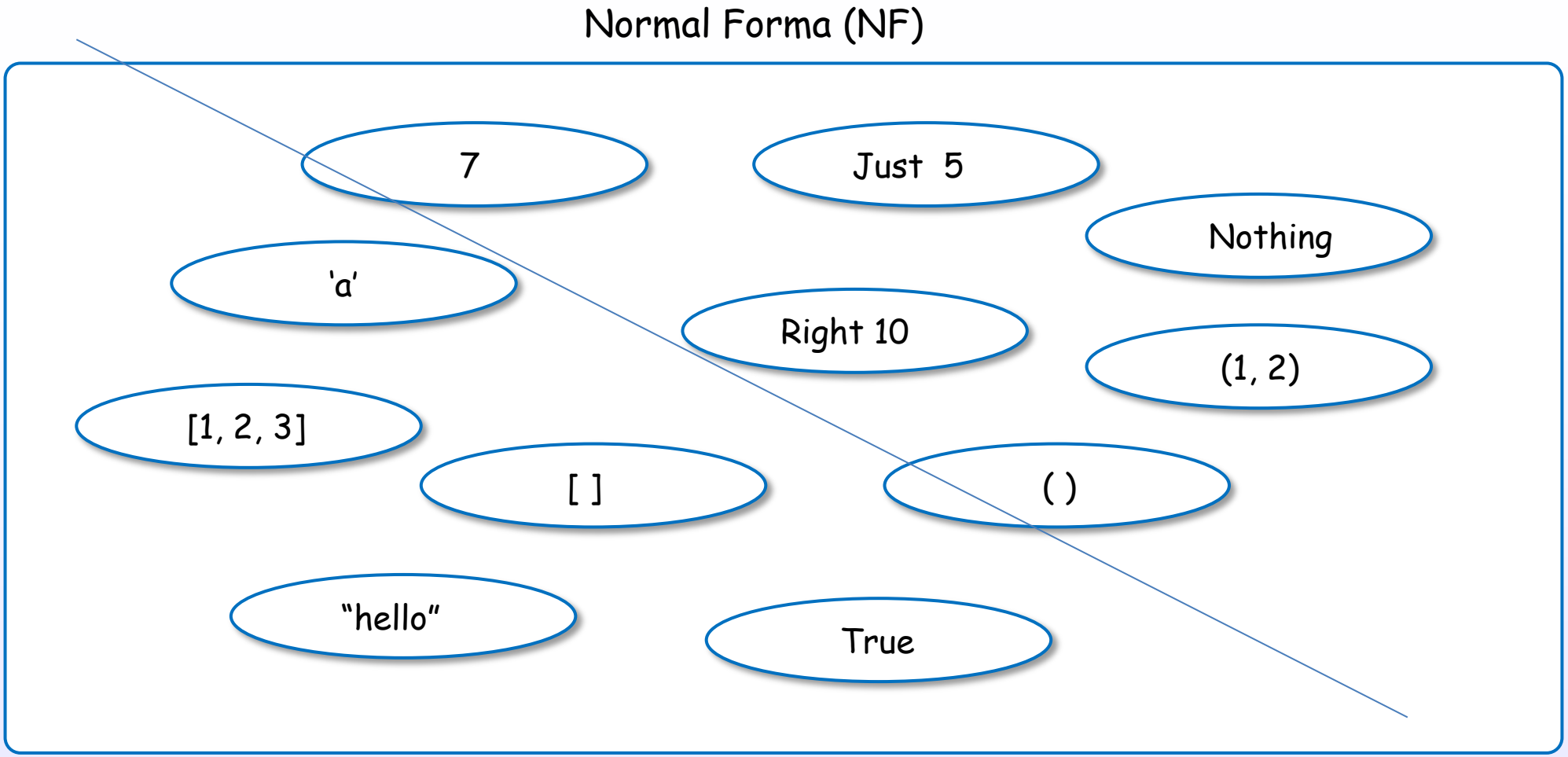
[Terei]

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

Examples of normal form (NF)



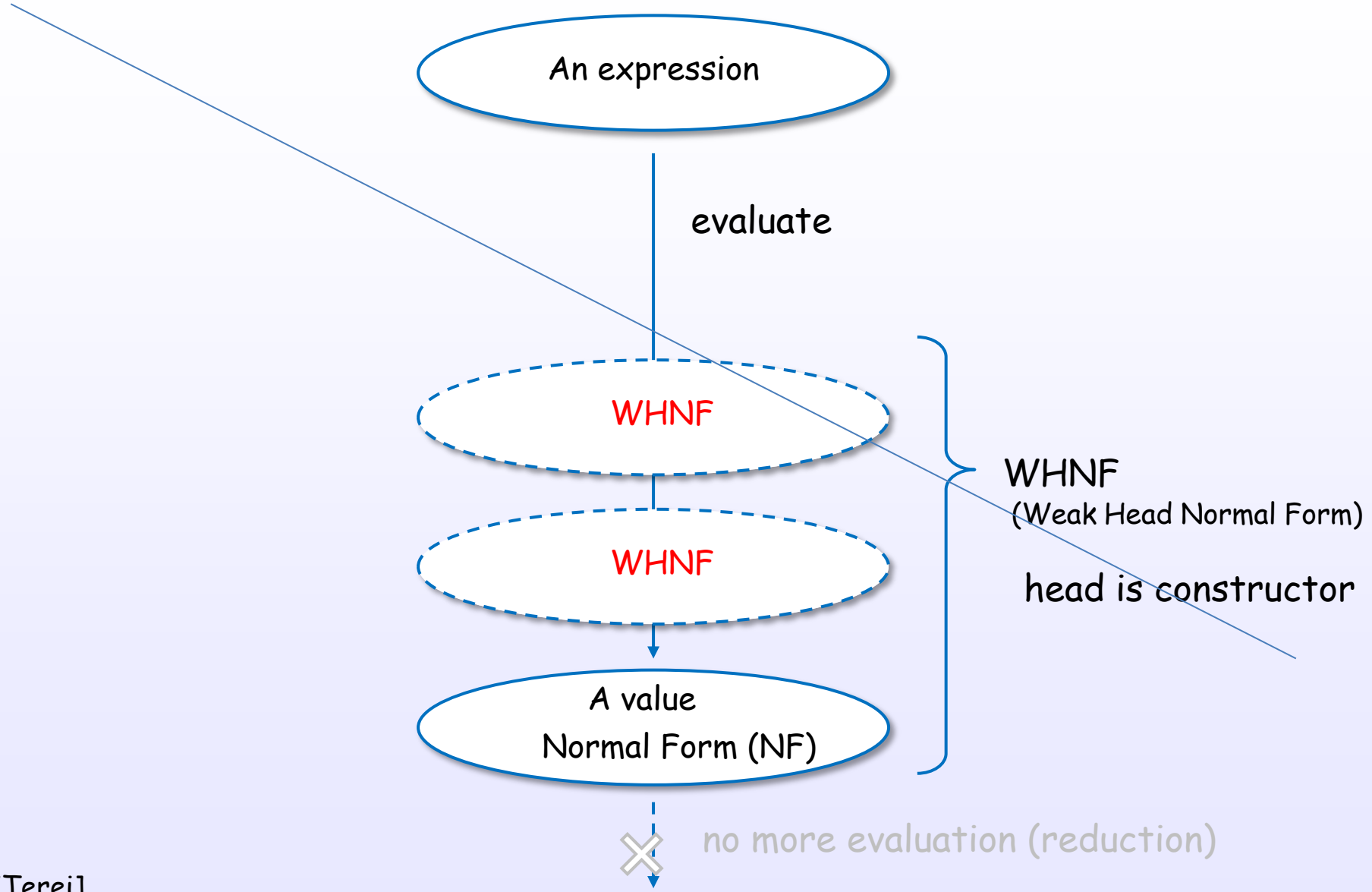
[Terei]

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

Weak Head Normal form は、少なくとも先頭が評価された式



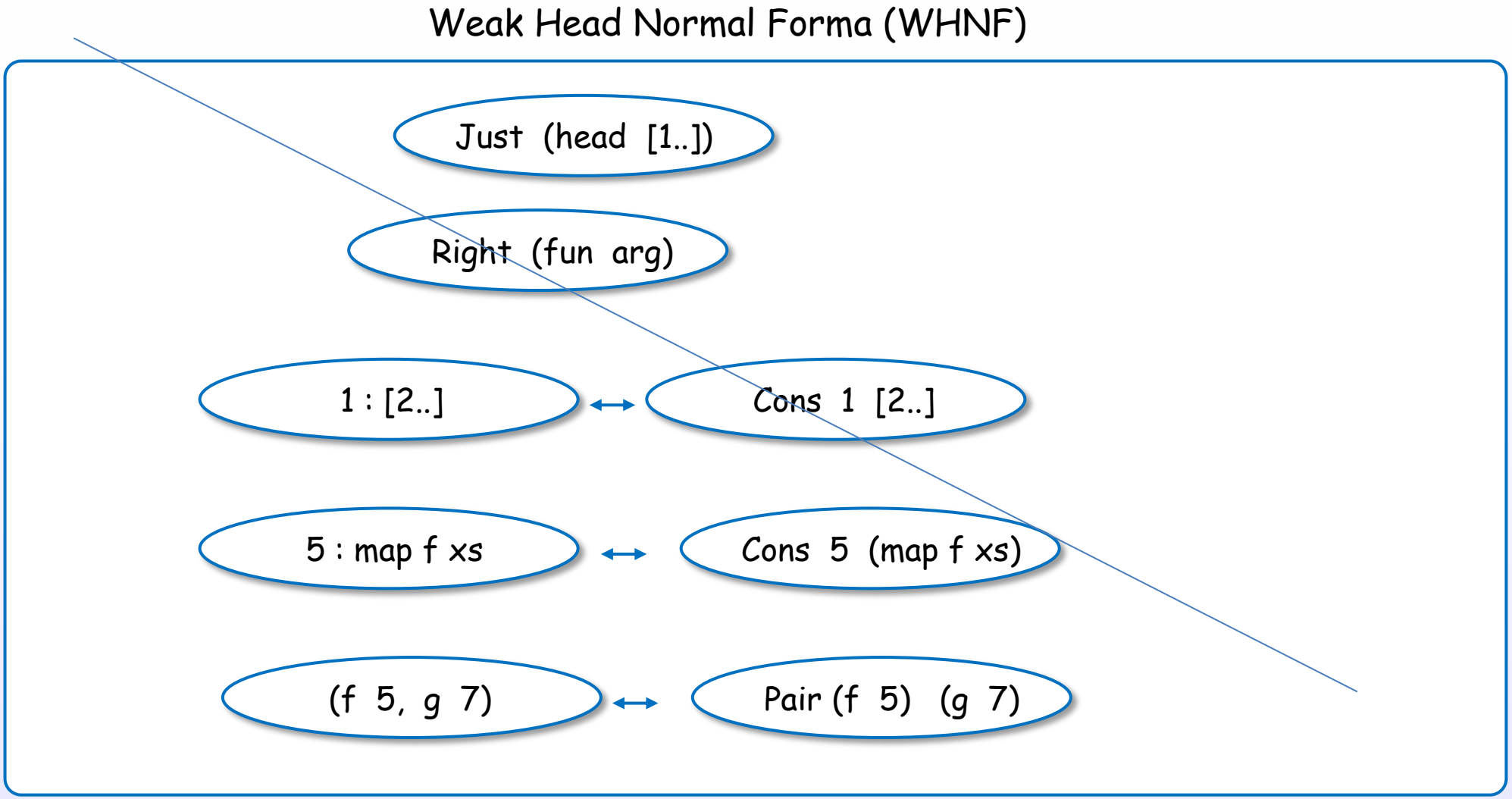
[Terei]

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

Examples of weak head normal form (WHNF)



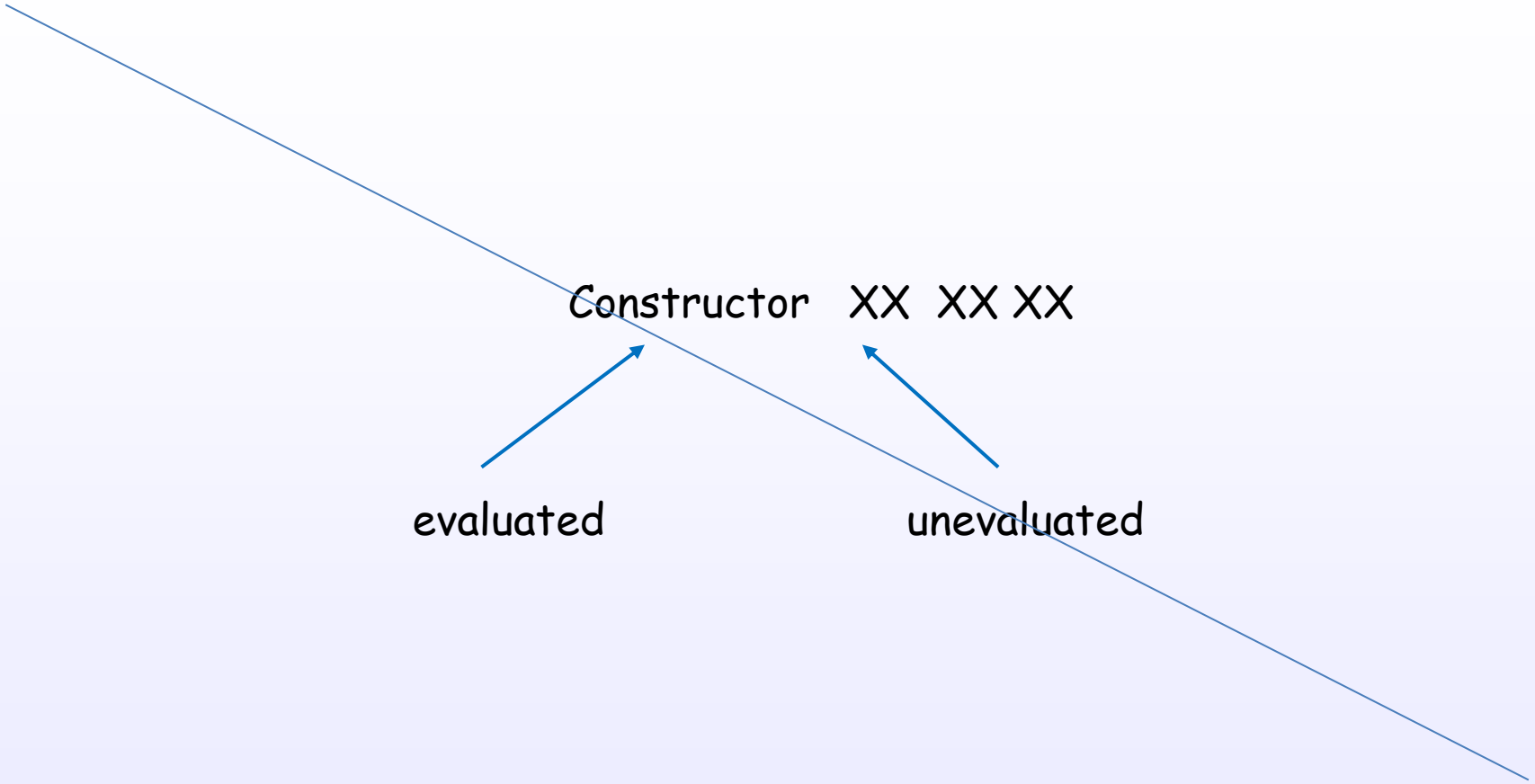
[Terei]

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

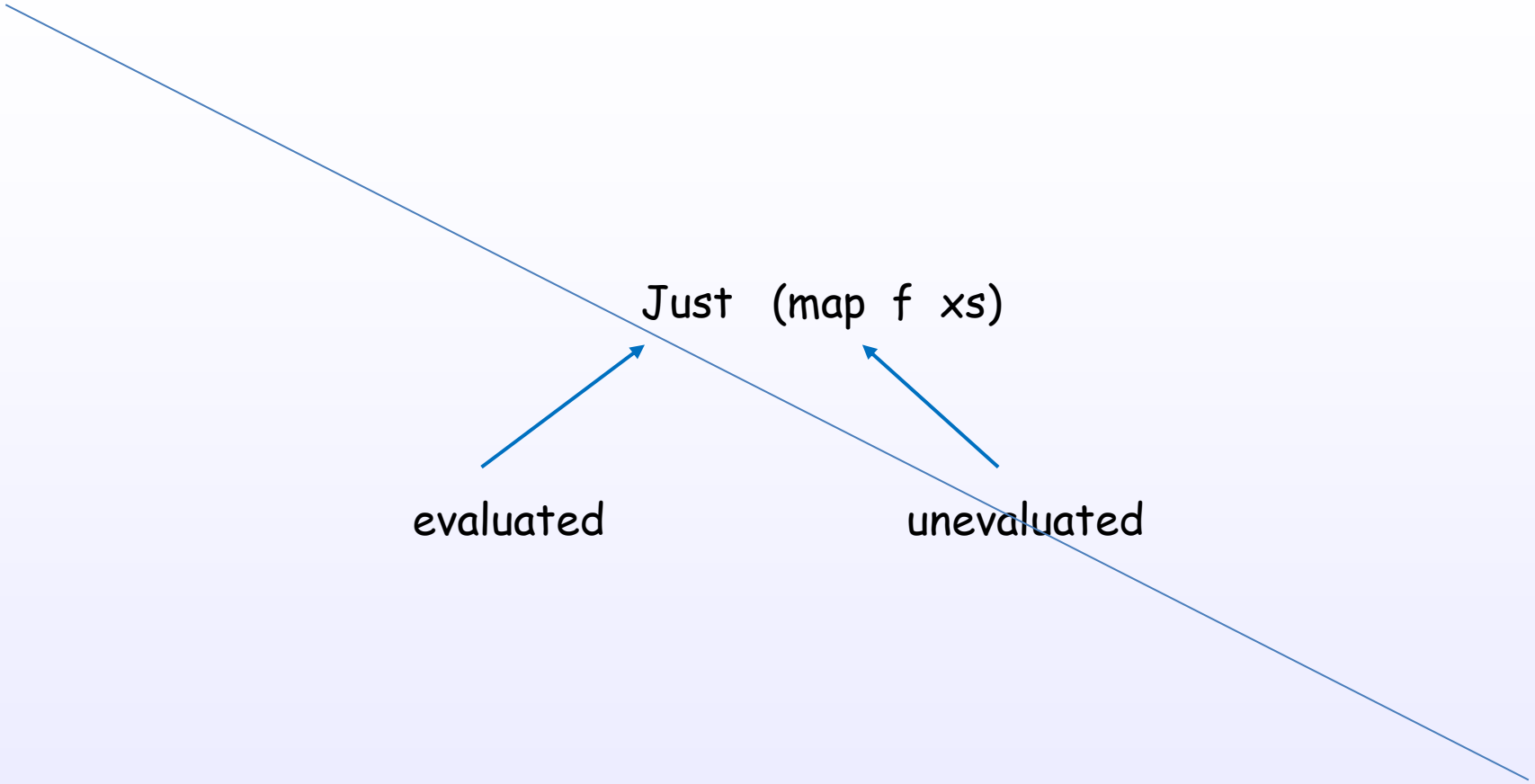
References : [1]

WHNF



References : [1]

WHNF



References : [1]

WHNF

References : [1]

WHNF

[4]

normal form:
an expression without an redexes

head normal form:
an expression where the top level (head) is neither a redex NOR
a lambda abstraction with a reducible body

weak head normal form:
an expression where the top level (head) isn't a redex

[Terei]

References : [1]

WHNF

[4]

evaluation strategies:
call-by-value: arguments evaluated before function entered (copied)
call-by-name: arguments passed unevaluated
call-by-need: arguments passed unevaluated but an expression is only evaluated once (sharing)

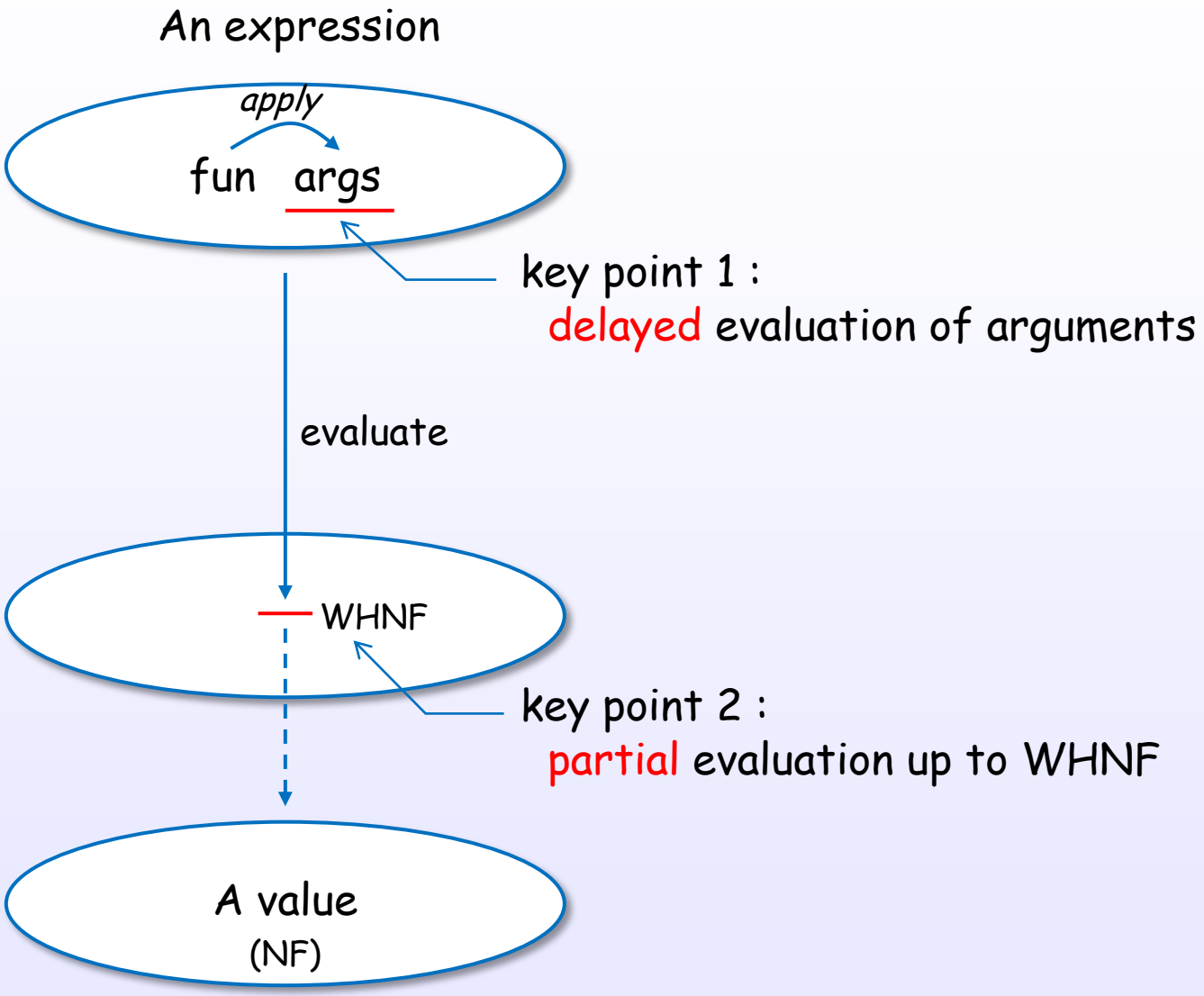
no-strict evaluation Vs. lazy evaluation:
non-strict: Includes both call-by-name and call-by-need, general term for evaluation strategies that don't evaluate arguments before entering a function
lazy evaluation: Specific type of non-strict evaluation. Uses call-by-need (for sharing).

[Terei]

References : [1]

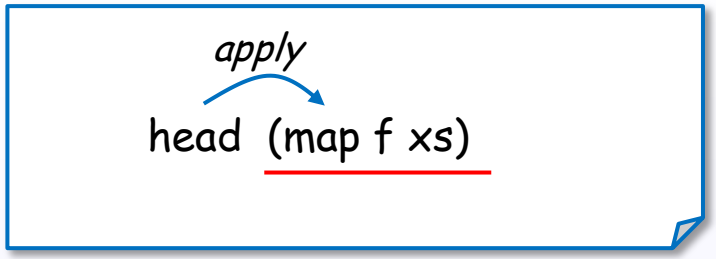
Evaluation in Haskell (GHC)

Key concept of Haskell's lazy evaluation

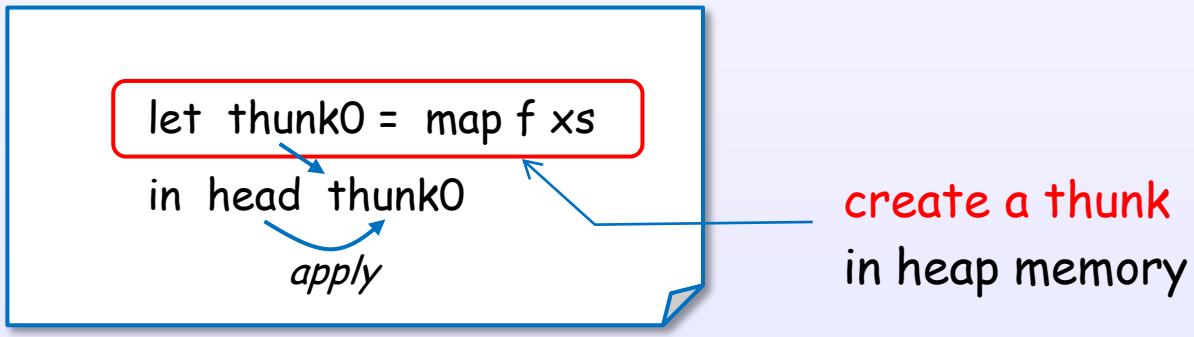


References : [1]

key point 1 : delayed evaluation of arguments



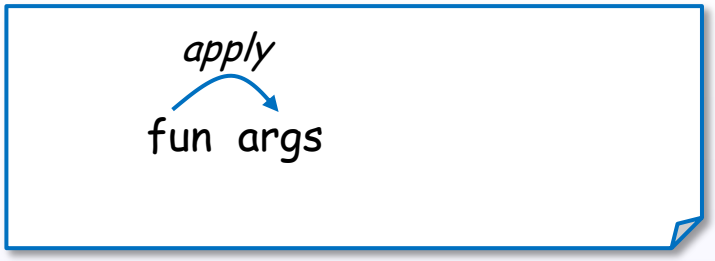
internal transformation by GHC



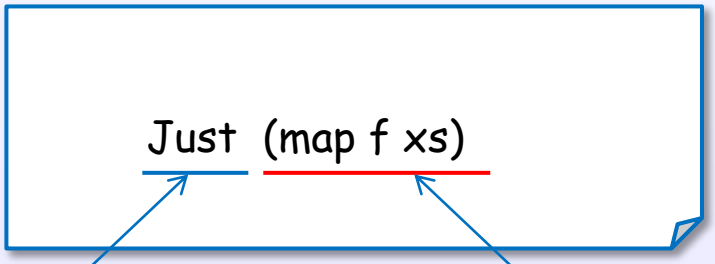
GHC implements lazy evaluation using the thunk.
Evaluation of arguments is delayed with the thunk.

References : [1]

key point 2 : partial evaluation up to WHNF



evaluation up to WHNF



evaluated part
(head constructor)

unevaluated part
(thunk)

GHC can partially evaluate a expression.
Constructor can hold an unevaluated expression (a thunk).

References : [1]

Pattern match

[CIS194]

References : [1]

Examples of evaluation steps

Example of repeat

repeat 1

1 : repeat 1

1 : 1 : repeat 1

1 : 1 : 1 : repeat 1

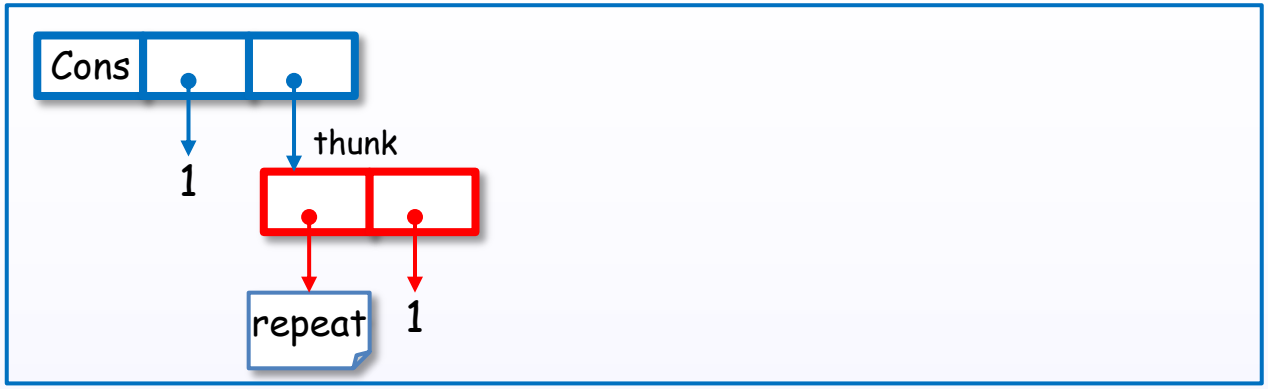
References : [1]

Example of repeat

repeat 1

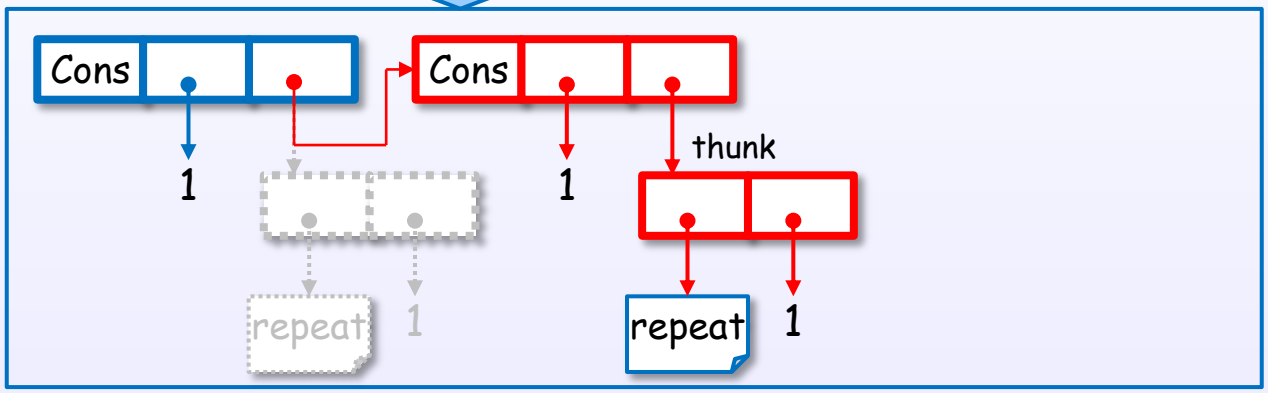
1 : repeat 1

Cons 1 (repeat 1)



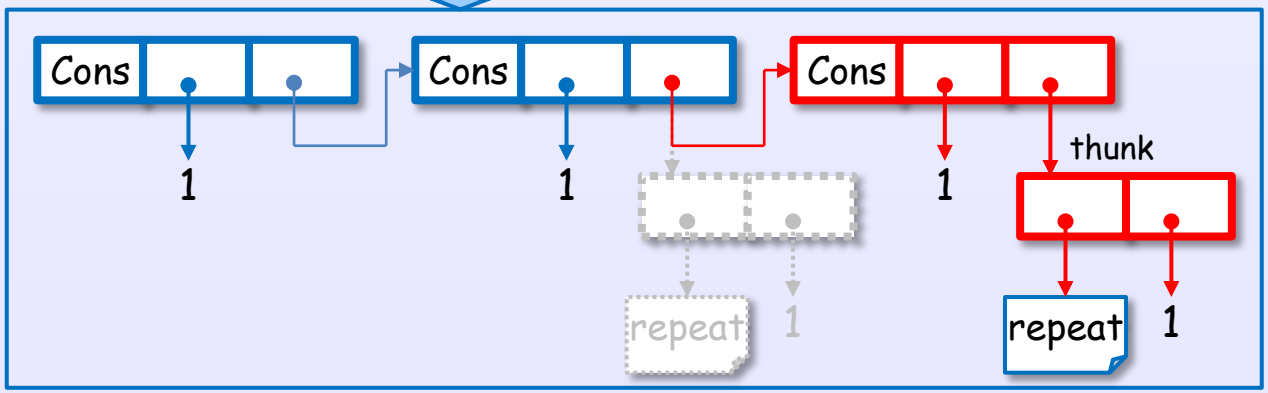
1 : 1 : repeat 1

Cons 1 (Cons 1 (repeat 1))



1 : 1 : 1 : repeat 1

Cons 1 (Cons 1 (Cons 1 (repeat 1)))



References : [1]

Example of map

```
map  f  [1, 2, 3]
```

```
f 1  :  map f [2, 3]
```

```
f 1  :  f 2  :  map f [3]
```

```
f 1  :  f 2  :  f 3
```

...

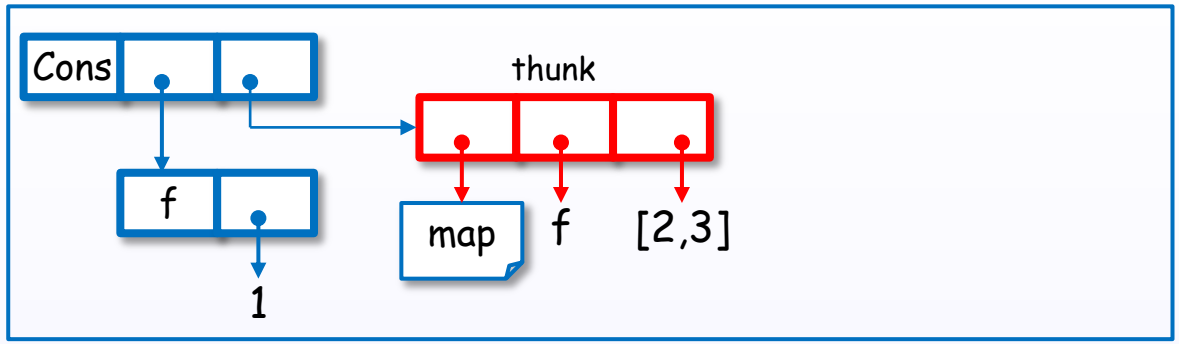
References : [1]

Example of map

map f [1, 2, 3]

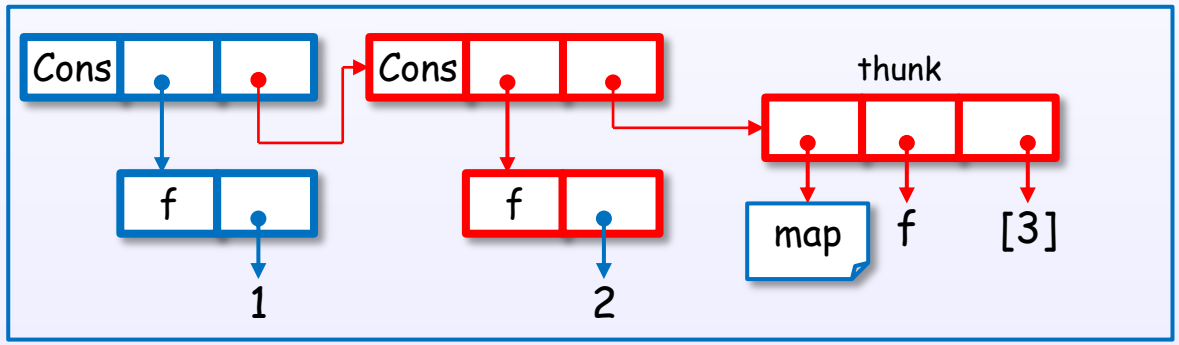
f 1 : map f [2, 3]

Cons (f 1) (map f [2, 3])



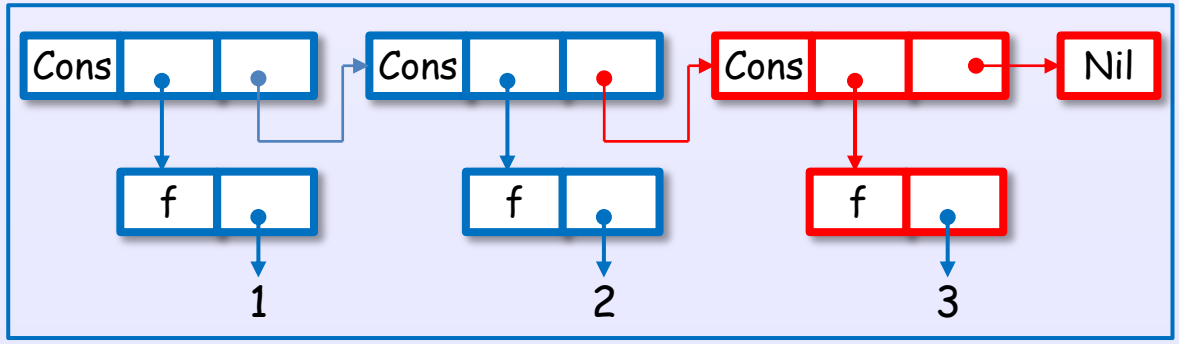
f 1 : f 2 : map f [3]

Cons (f 1) (Cons (f 2) (map f [3]))



f 1 : f 2 : f 3

Cons (f 1) (Cons (f 2) (Cons (f 3) Nil))

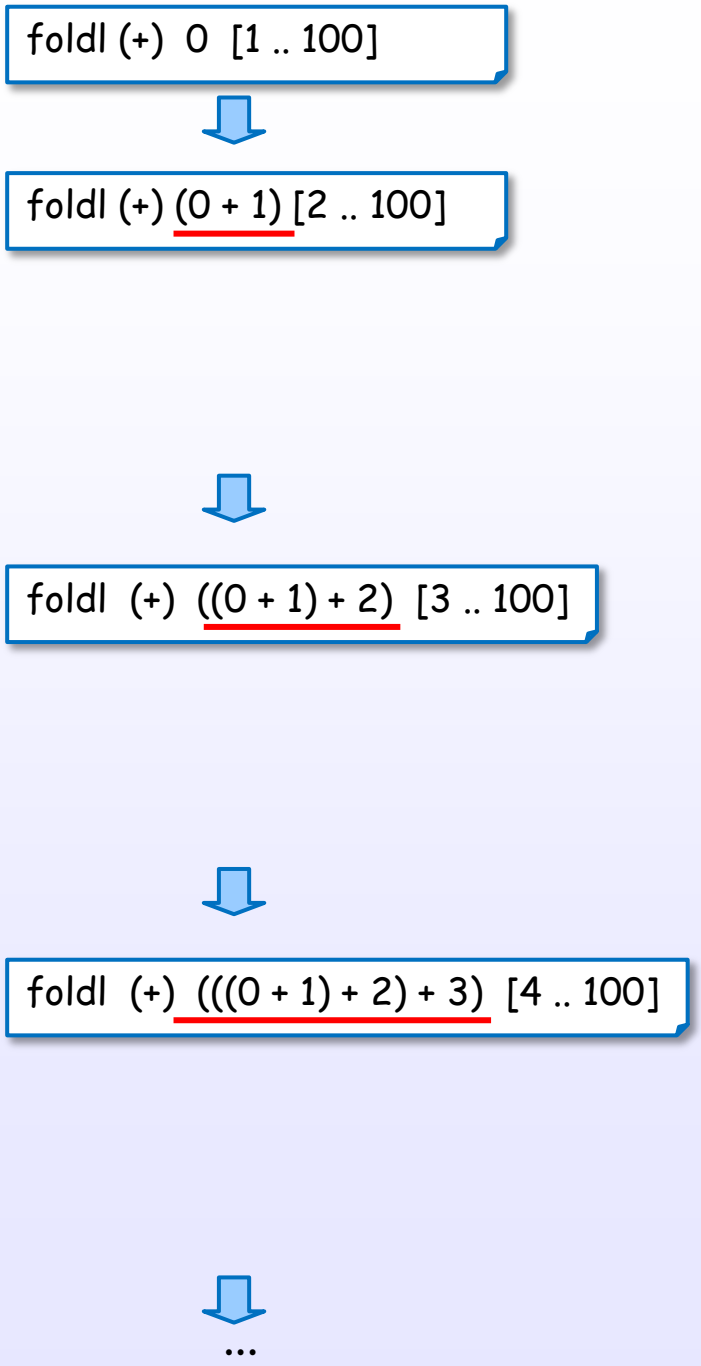


...

...

References : [1]

Example of foldl (non-strict)



References : [1]

Example of foldl (non-strict)

```
foldl (+)  0  [1 .. 100]
```

```
foldl (+) (0 + 1) [2 .. 100]
```

```
let  thunk1 = (0 + 1)
in foldl (+) thunk1 [2 .. 100]
```

```
foldl  (+)  ((0 + 1) + 2)  [3 .. 100]
```

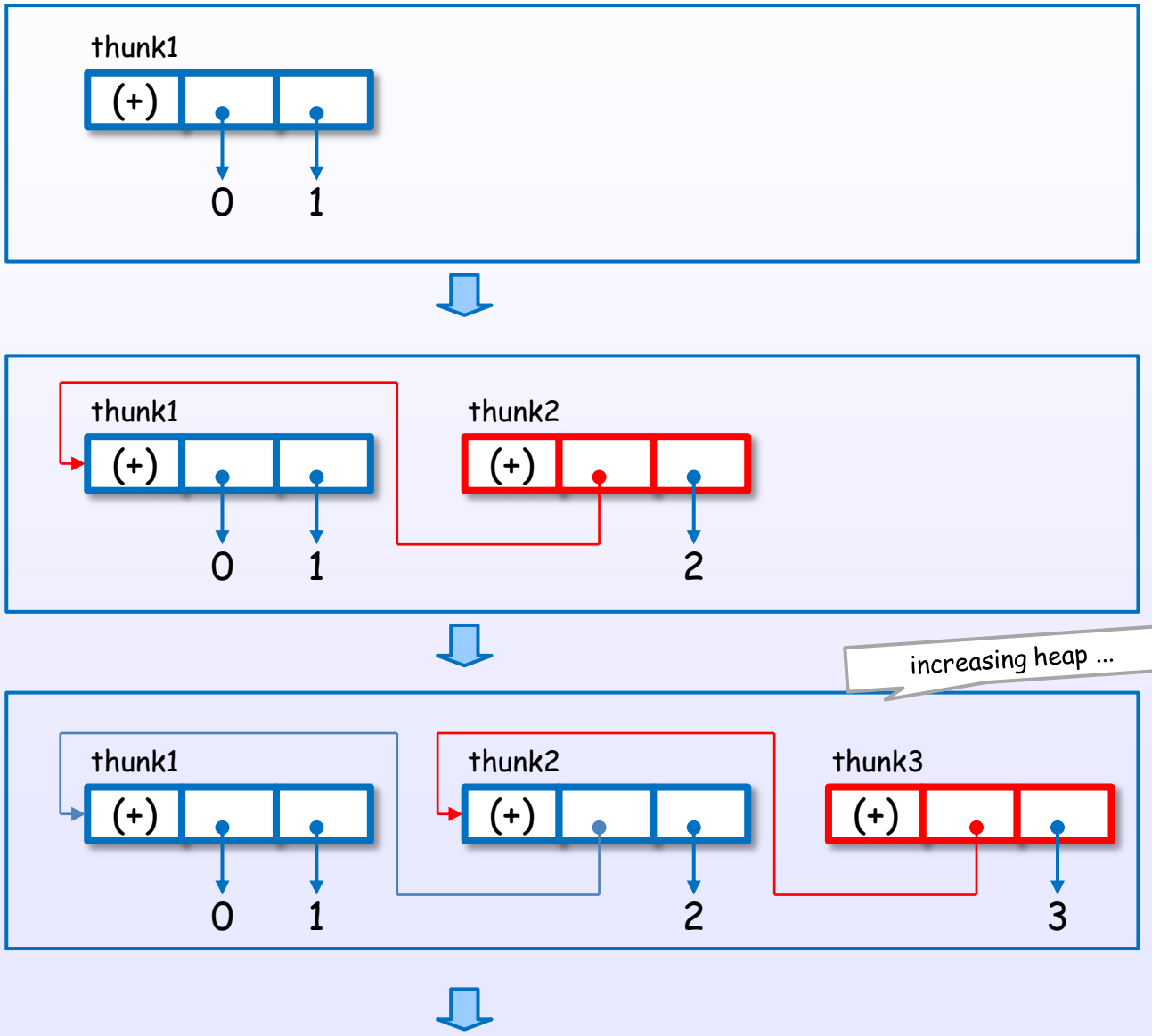
```
let  thunk2 = (thunk1 + 2)
in foldl (+) thunk2 [3 .. 100]
```

```
foldl  (+)  (((0 + 1) + 2) + 3)  [4 .. 100]
```

```
let  thunk3 = (thunk2 + 3)
in foldl (+) thunk3 [4 .. 100]
```

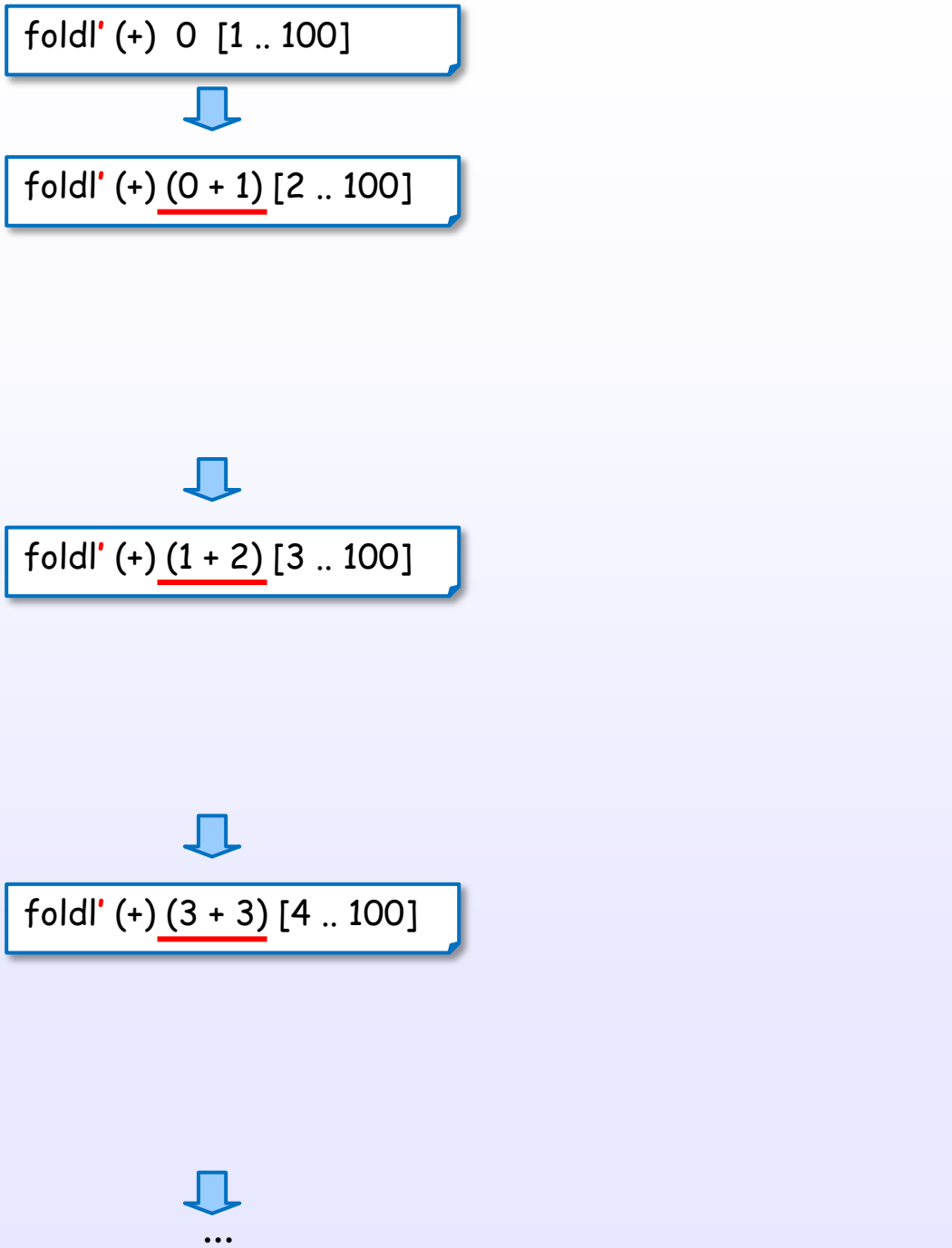
...

heap memory



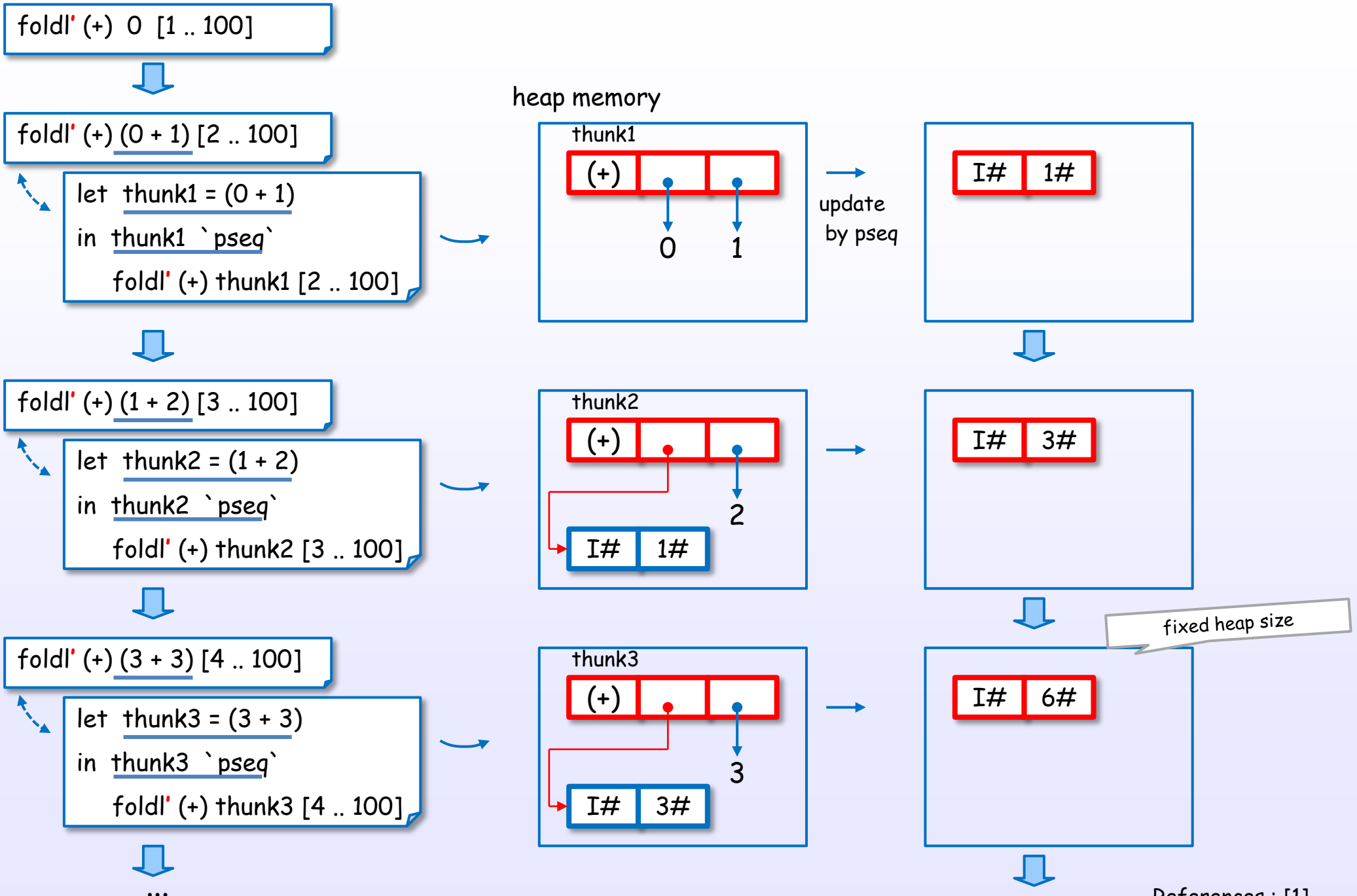
References : [1]

Example of foldl' (strict)



References : [1]

Example of foldl' (strict)



References : [1]

Example of foldl (non-strict) and foldl' (strict)

foldl (+) (0 + 1) [2 .. 100]

foldl (+) ((0 + 1) + 2) [3 .. 100]

foldl (+) (((0 + 1) + 2) + 3) [4 .. 100]

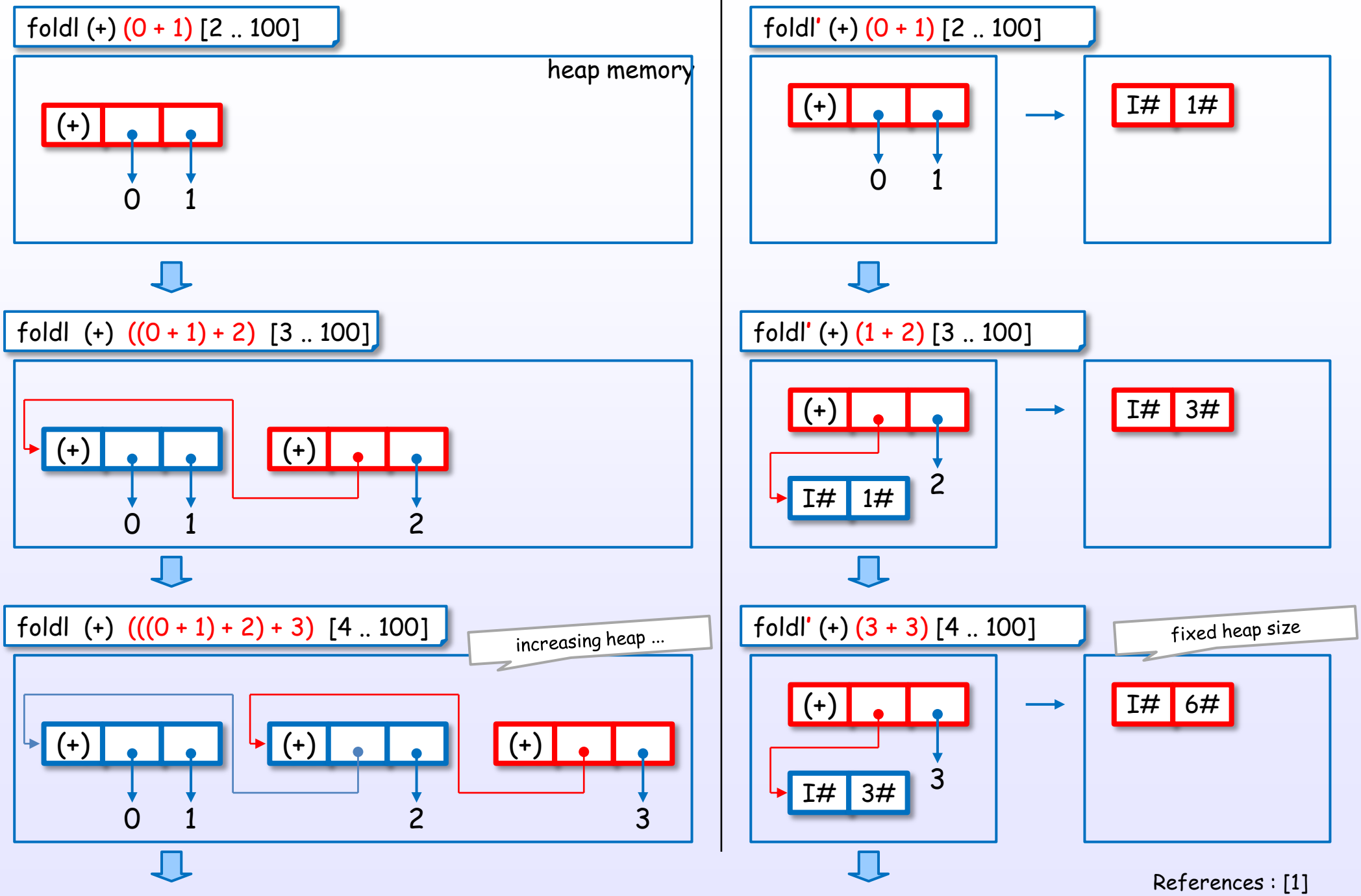
foldl' (+) (0 + 1) [2 .. 100]

foldl' (+) (1 + 2) [3 .. 100]

foldl' (+) (3 + 3) [4 .. 100]

References : [1]

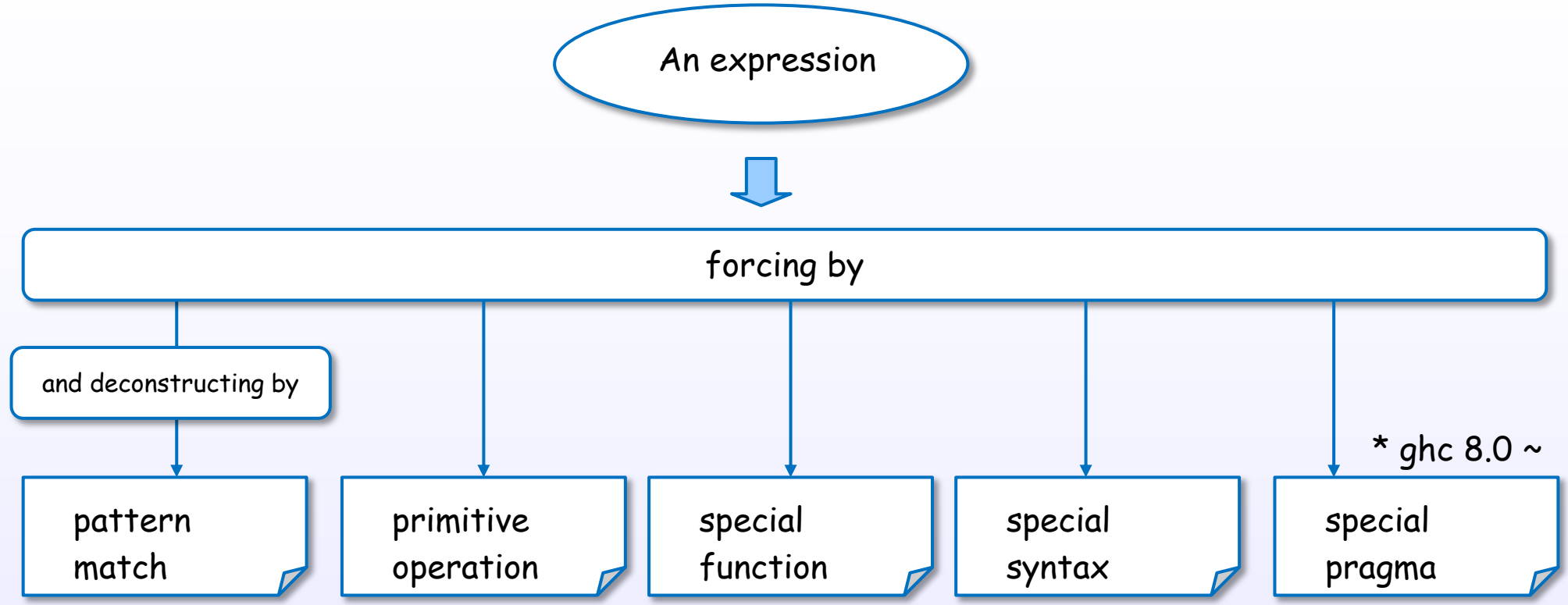
Example of foldl (non-strict) and foldl' (strict)



References : [1]

Control the evaluation in Haskell

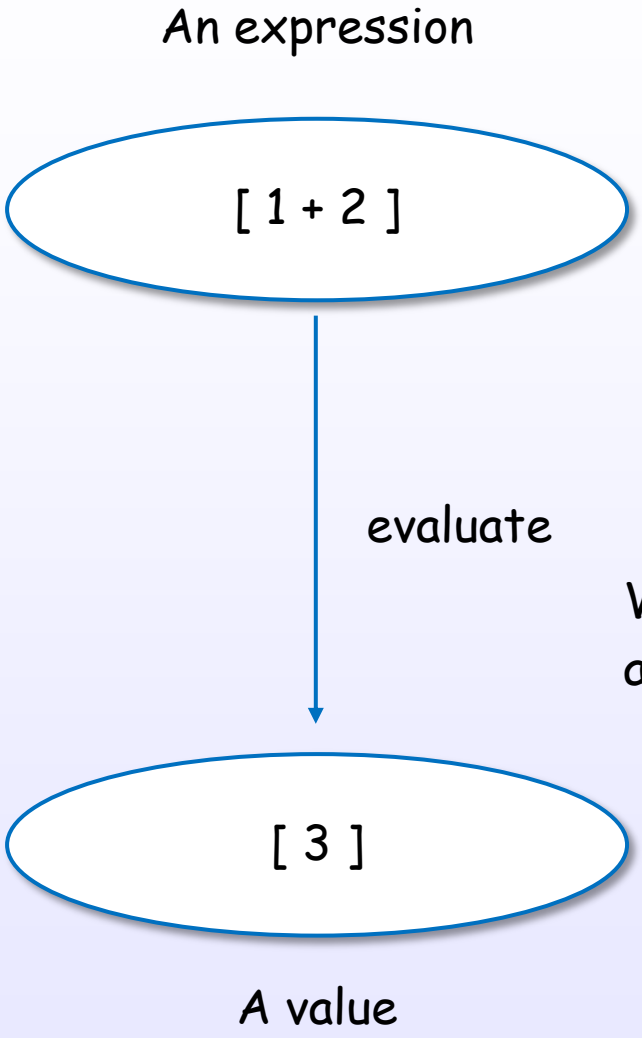
How to drive evaluation



References : [1]

Bottom

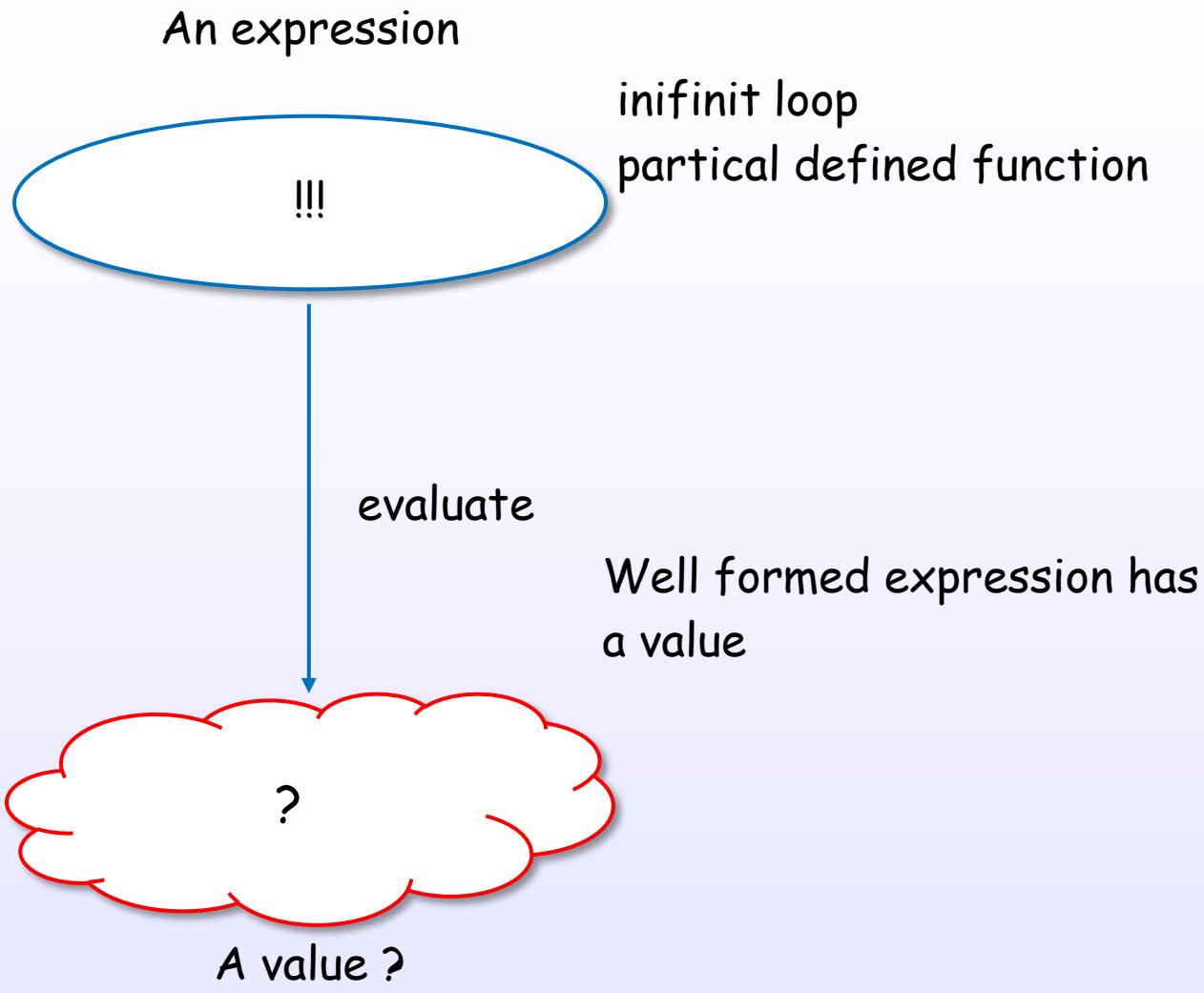
Well formed expression has a value



[Bird, Chapter 2]

References : [1]

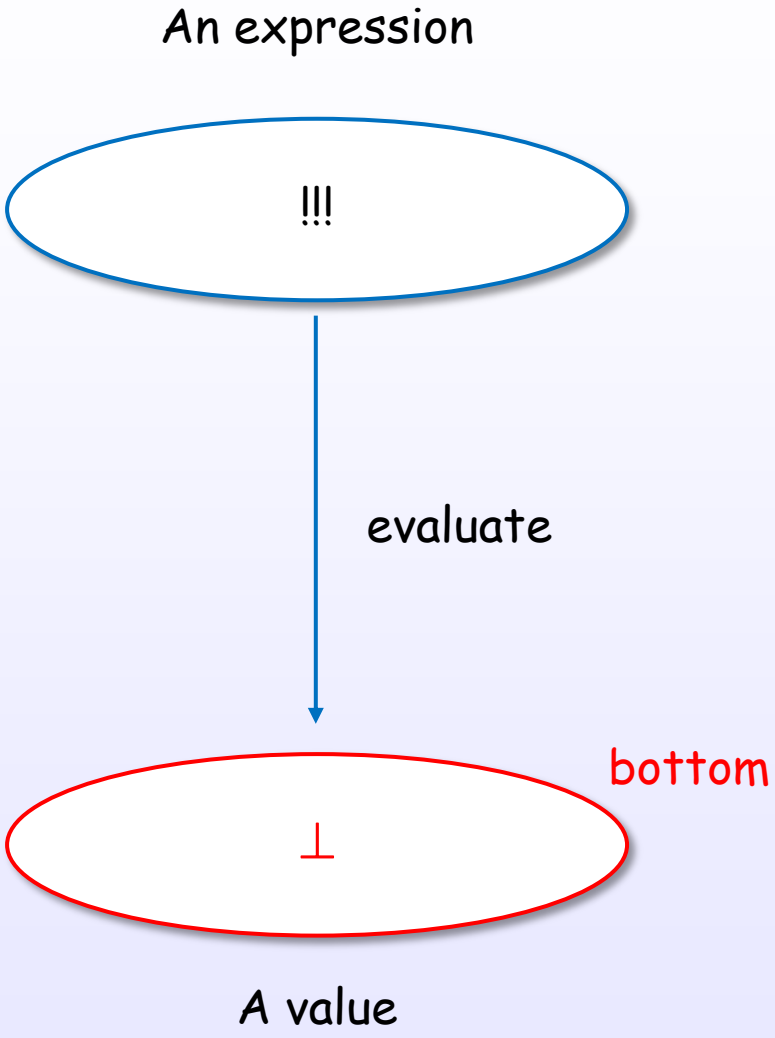
Well formed expression has a value



[Bird, Chapter 2]

References : [1]

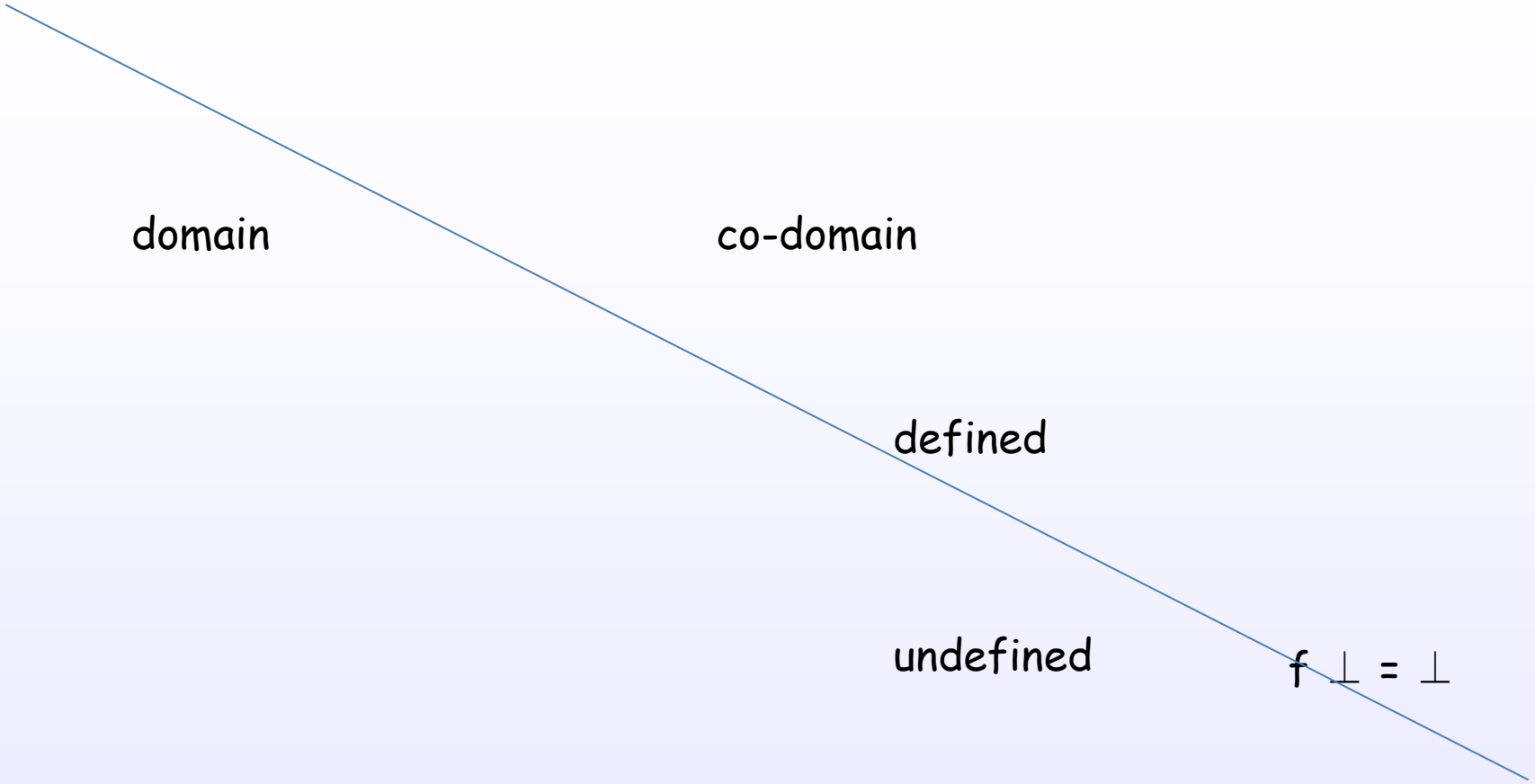
Well formed expression has a value



[Bird, Chapter 2]

References : [1]

Bottom



[Bird, Chapter 2]

References : [1]

Non-strict Semantics

Strictness

$f \perp = \perp$

[Bird, Chapter 2]

References : [1]

Layer

Non-strictness

$f \perp = \perp$

Lazy evaluation

Graph reduction

STG machine

References : [1]

Graph reduction

Tree, Graph

a expression

AST

Tree

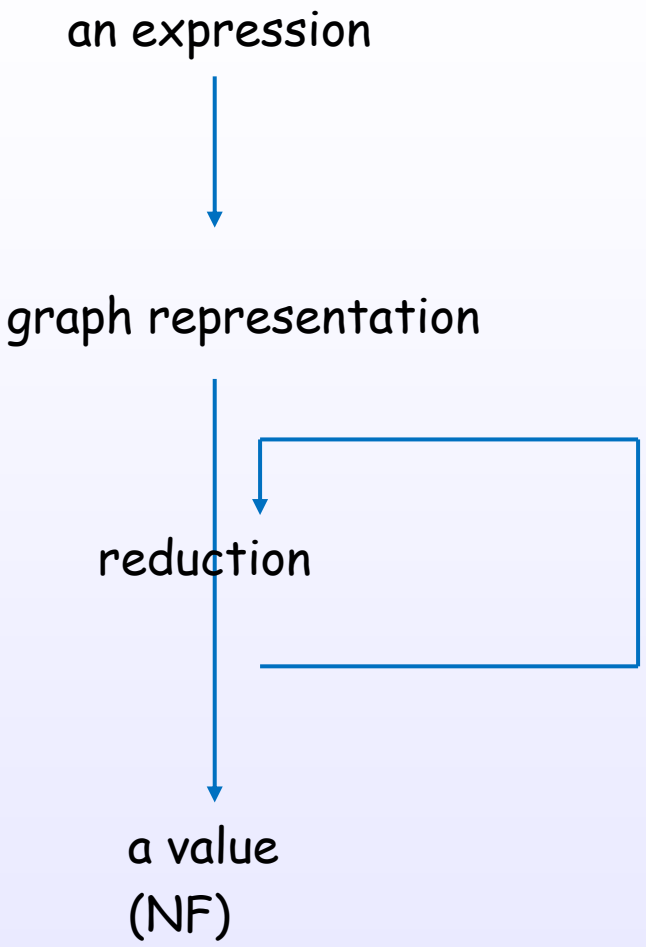
Graph

Shared Term

Lazy

References : [1]

evaluation, reduction



References : [1]

Implementation in GHC

STG heap objects

language	Just 5
implementation	heap object

References : [1]

Layer

Non-strictness

$f \perp = \perp$

Lazy evaluation

Graph reduction

STG machine

References : [1]

Layer

Haskell semantics	take 5 [1..10]
internal representation	graph
STG semantics	heap object
STG machine	

References : [1]

References

References

[1] Haskell 2010 Language Report
https://www.haskell.org/definition/haskell2010.pdf

[2] The Glorious Glasgow Haskell Compilation System (GHC user's guide)
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[4] Types and Programming Languages
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References

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[15] Lazy evaluation
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[17] Haskell/Denotational semantics
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[20] Implementing lazy functional languages on stock hardware: the Spineless Tagless G-machine Version 2.5
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[21] Making a Fast Curry Push/Enter vs Eval/Apply for Higher-order Languages
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[22] I know kung fu: learning STG by example
https://ghc.haskell.org/trac/ghc/wiki/Commentary/Compiler/GeneratedCode

[23] GHC Commentary: The Layout of Heap Objects
https://ghc.haskell.org/trac/ghc/wiki/Commentary/Rts/Storage/HeapObjects

[24] GHC Commentary: Strict & StrictData
https://ghc.haskell.org/trac/ghc/wiki/StrictPragma

[25] GHC illustrated
http://takenobu-hs.github.io/downloads/haskell_ghc_illustrated.pdf

Lazy,... zzz