

# Lazy evaluation illustrated

## for Haskellers

*exploring some mental models and implementations*

Takenobu T.

Lazy,... zzz

..., It's fun!

## NOTE

- Meaning of terms are different by communities.
- There are a lot of good documents. Please see also references.
- This is written for GHC's Haskell.

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- Basic mental models
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- WHNF

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# 1. Introduction

# 1. Introduction

Basic mental models

# How to evaluate program in your brain ?

program code



プログラムは、どの順で評価される？

どういうステップ、どういう順で evaluation (execution, reduction) される？

What are these mental models?

What "mental model" do you have?

# One of the mental models for C program

## 文の並び

```
main (...) {  
  code..  
  code..  
  code..  
  code..  
}
```

A red curly brace groups the four `code..` lines, with a red question mark to its right.

## 入れ子の構造

```
x = func1( func2( a ) );
```

A red question mark is positioned below the underlined `func2( a )` argument.

## 引数の並び

```
y = func1( a(x), b(x), c(x) );
```

A red question mark is positioned below the underlined arguments `a(x)`, `b(x)`, and `c(x)`.

## 関数と引数

```
z = func1( m + n );
```

A red question mark is positioned below the underlined expression `m + n`.

どのように評価される？

あなたの頭の中の、評価メンタルモデルは？



# One of the mental models for C program

プログラムは、statement の集まり

```
main(...) {  
  code..  
  code..  
  code..  
  code..  
}
```

(1) 文は基本的に、  
上から下へ評価  
downward

statement order

```
x = func1( func2( a ) );
```

(2) 内側の関数評価が先  
(内から外へ。)

```
y = func1( a(x), b(x), c(x) );
```

(3) 同階層では、左側の  
(左から右へ。)

```
z = func1( m + n );
```

(4) 引数評価が先  
(引数評価から関数評価へ。)

Each programmers have some mental models in their brain.

# One of the mental models for C program

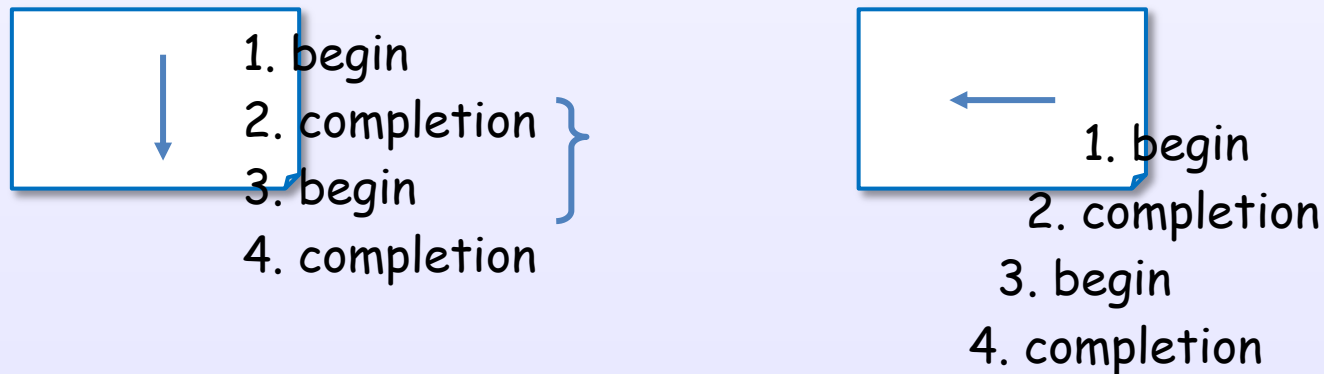
Maybe, You have some implicit mental model in your brain for C program.

(1) program is collection of statements

(2) an order between evaluations of elements



(3) an order between completion and begin of evaluations



This is an example of an implicit sequential order model for programming languages.

# One of the mental models for Haskell program

```
main = exp11 (exp12 exp13 exp14 )
```

```
exp13 = exp131 exp132
```

```
exp14 = exp141 exp142 exp143
```

```
:
```

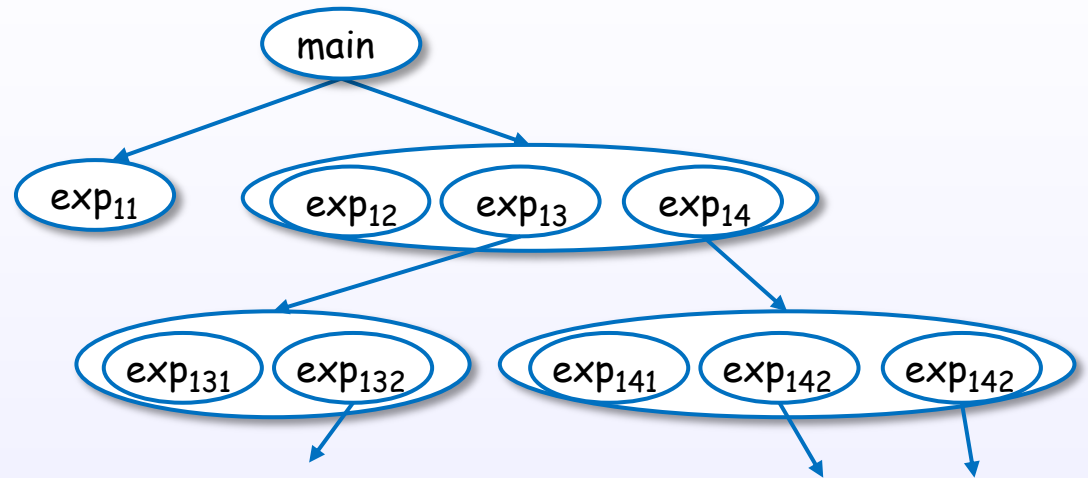
どのように評価される？

あなたの頭の中の、評価メンタルモデルは？

# One of the mental models for Haskell program

プログラムは、式の集まり

```
main = exp11 (exp12 exp13 exp14)  
exp13 = exp131 exp132  
exp14 = exp141 exp142 exp143  
:
```



$\text{main} = \text{exp}_{11} (\text{exp}_{12} (\text{exp}_{131} \text{exp}_{132}) (\text{exp}_{141} \text{exp}_{142} \text{exp}_{143}))$

(1) プログラム全体を1つの式と見立てて

(2) 部分式をある順で評価(簡約)していく

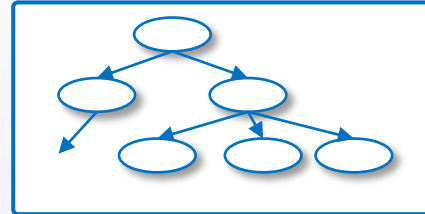
(3) 評価は置換により行う

# One of the mental models for Haskell program

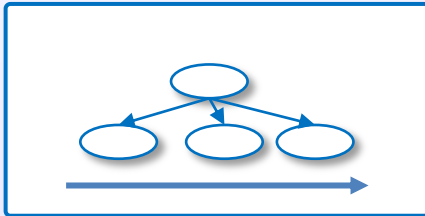
(1) program is collection of expression's declaration

(2) プログラム全体が階層をもった1つの式

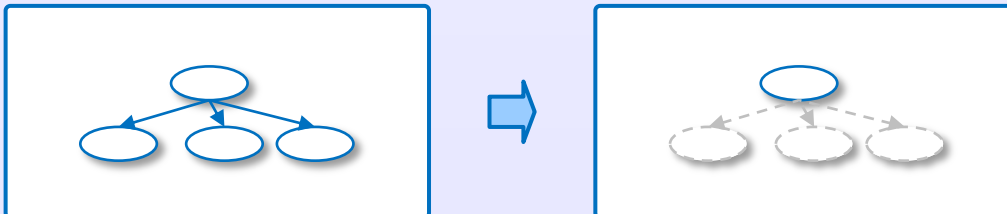
```
main = e (e (e (e e) e (e e e) ) )
```



(3) 部分式を、ある順序で評価していく



(4) 評価は置換により行われる

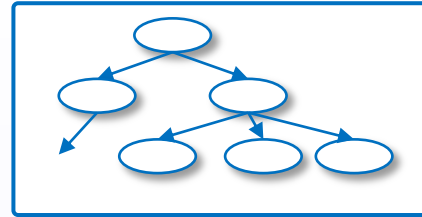


# 1. Introduction

Lazy evaluation

## では、具体的にはどうやって評価される？

```
main = e (e (e (e e) e (e e e) ) )
```



Haskellは purely functional language

↓ no side effect [slpj-book-1987], p.193

order free (so, potentially hi-level optimization and parallelism)

↓ call-by-need 可能

GHC chosen lazy evaluation to implement non-strict semantics.

[slpj-book-1987], p.33

# GHC chosen lazy evaluation

必要な時に、必要な箇所のみを評価する

(STG p.11)



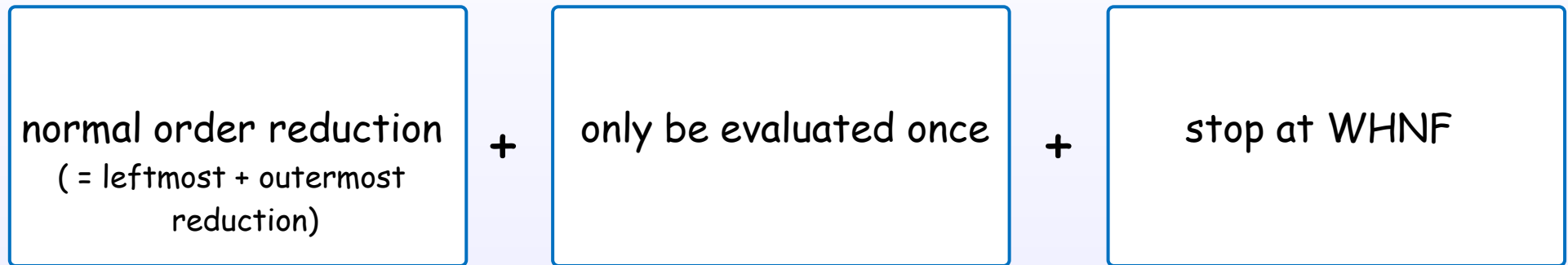
- ・引数評価を先送る (case式が来るまで評価しない) call-by-need
- ・部分式を完全評価しない (caseのパターンマッチで参照するところのみを評価する) WHNF

これは、計算量を最小化する戦略(メモリ量でなく)



# Haskell(GHC) 's lazy evaluation

ingredient of Haskell's "lazy evaluation"



[slpj-book-1987], 194

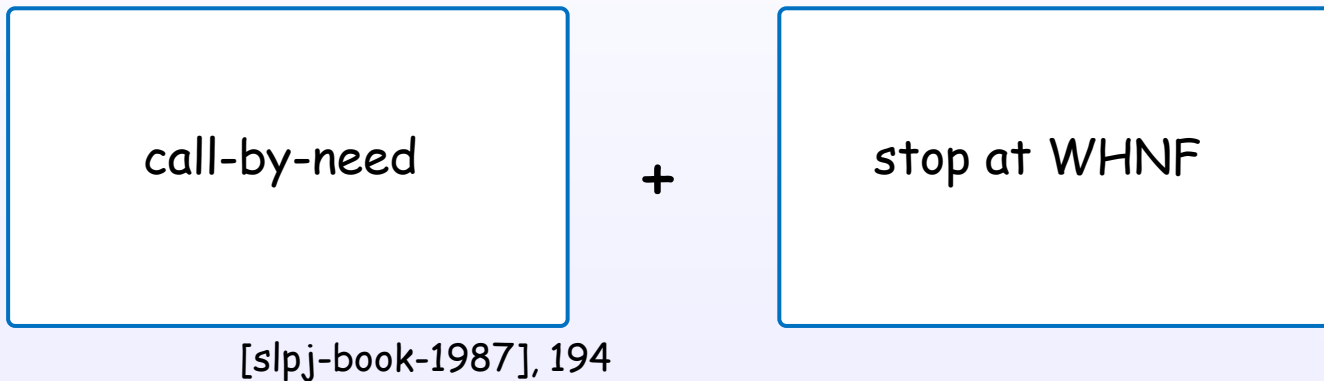
outermost と、call-by-need

call-by-needは、狭義のlazy eval

[slpj-book-1987], p.198, 23, 194

# Haskell(GHC) 's lazy evaluation

ingredient of Haskell's "lazy evaluation"



outermost と、call-by-need

call-by-needは、狭義のlazy eval

[slpj-book-1987], p.198, 23, 194

# Why lazy evaluation?

(1) normal order reduction guarantees to find a normal form (if one exists)

[slpj-book-1987], p.25

pursue normal order reduction, but stop at WHNF.

This is an essential ingredient of lazy evaluation

(2) lazy evaluation implements non-strict semantics

infinite data structure and stream

[slpj-book-1987], p.194

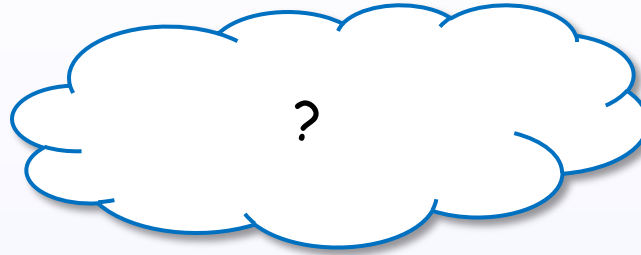
(3) 不要な評価を避ける

# 1. Introduction

Expression and value

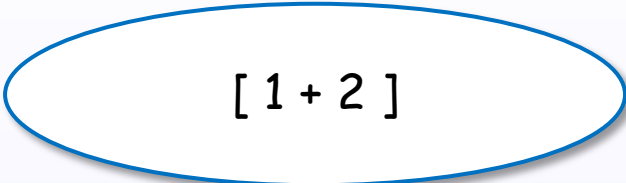
# What is an expression?

An expression



# An expression denotes a value

An expression



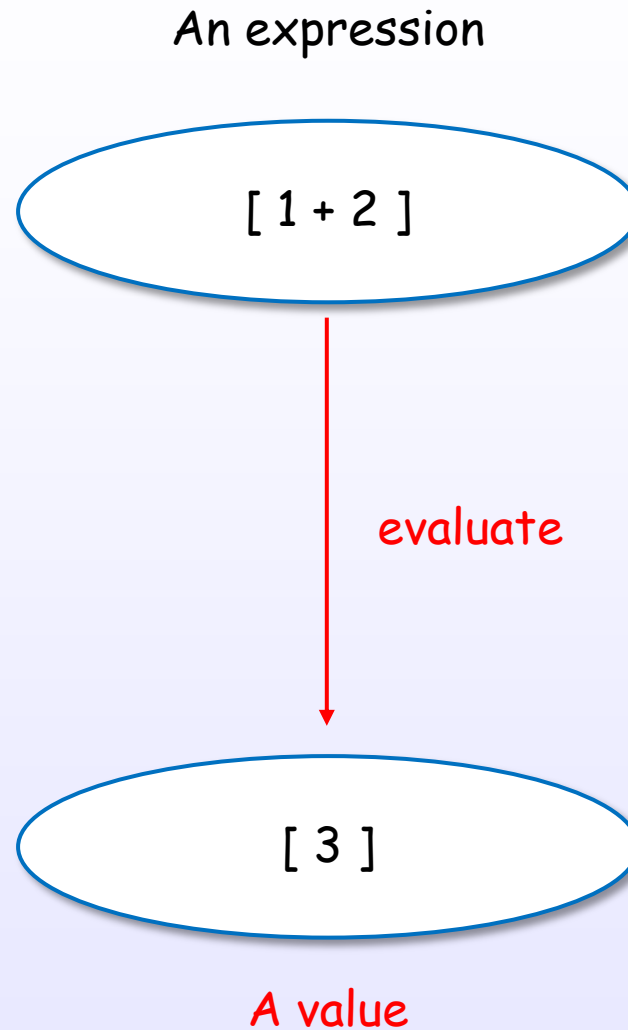
[ 1 + 2 ]

[HR2010]

[Bird, Chapter 2]

References : [1]

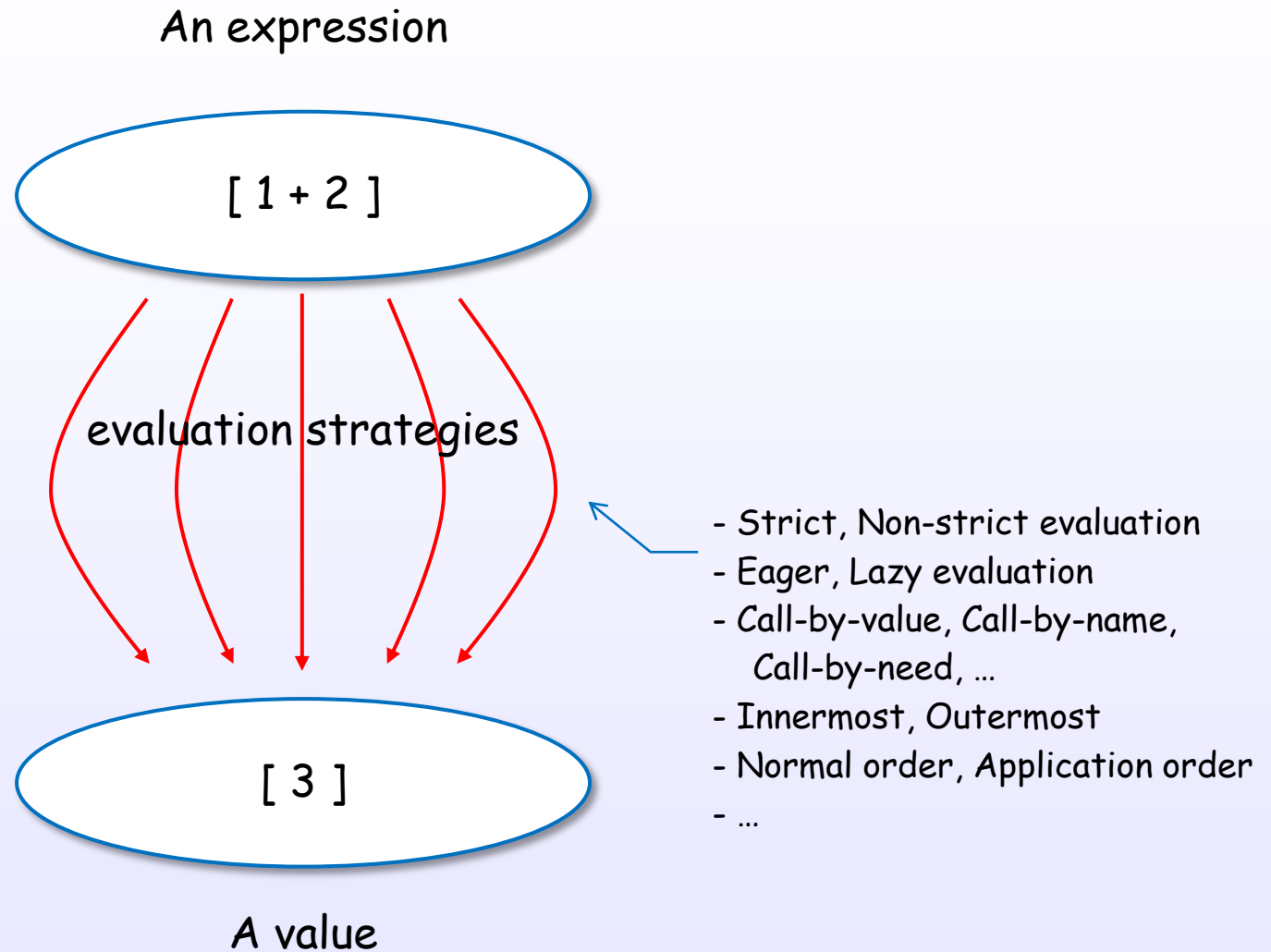
# An expression evaluates to a value



[HR2010]

[Bird, Chapter 2]

# There are many evaluation approaches

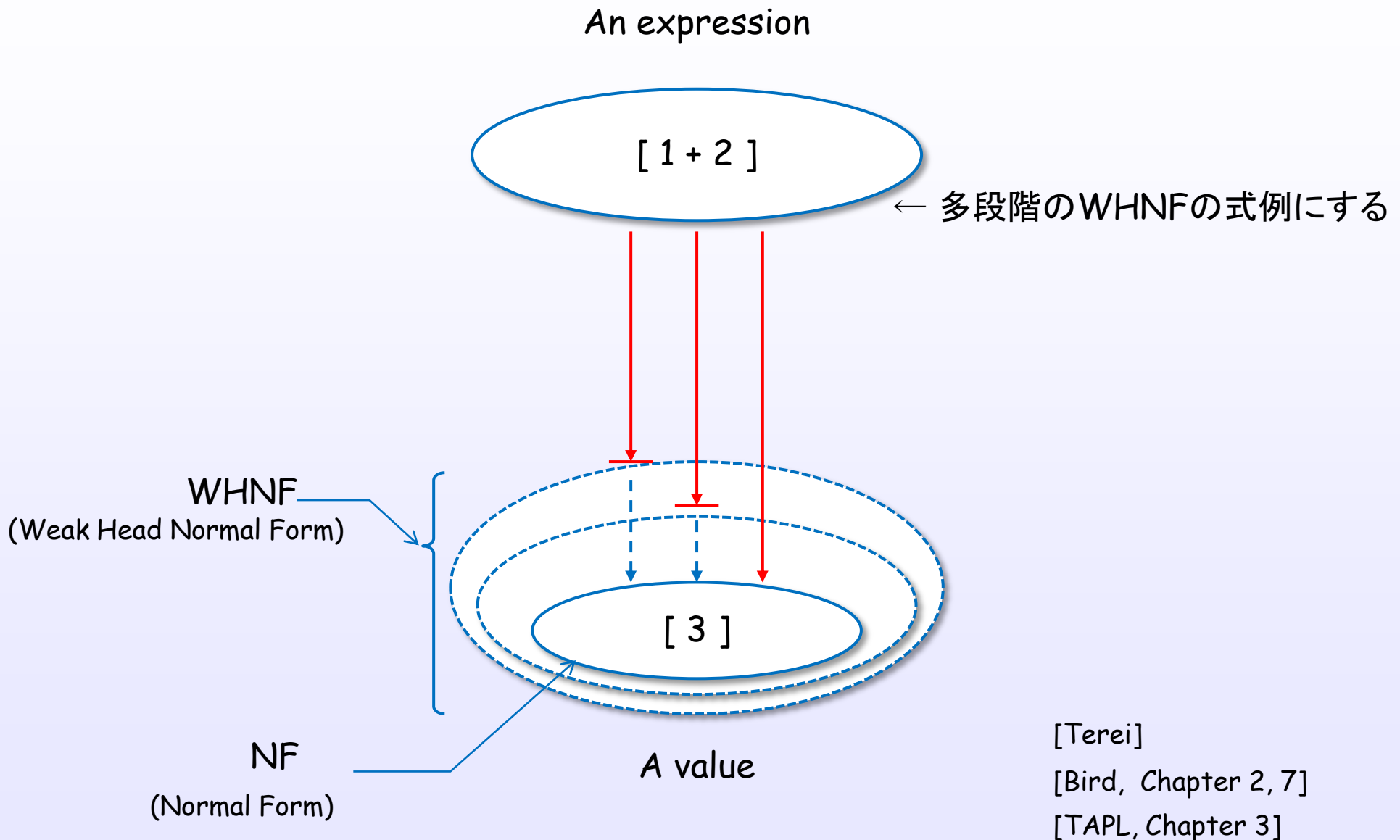


[Bird, Chapter 2, 7]

[TAPL, Chapter 3]



# There are some evaluation levels



[Terei]

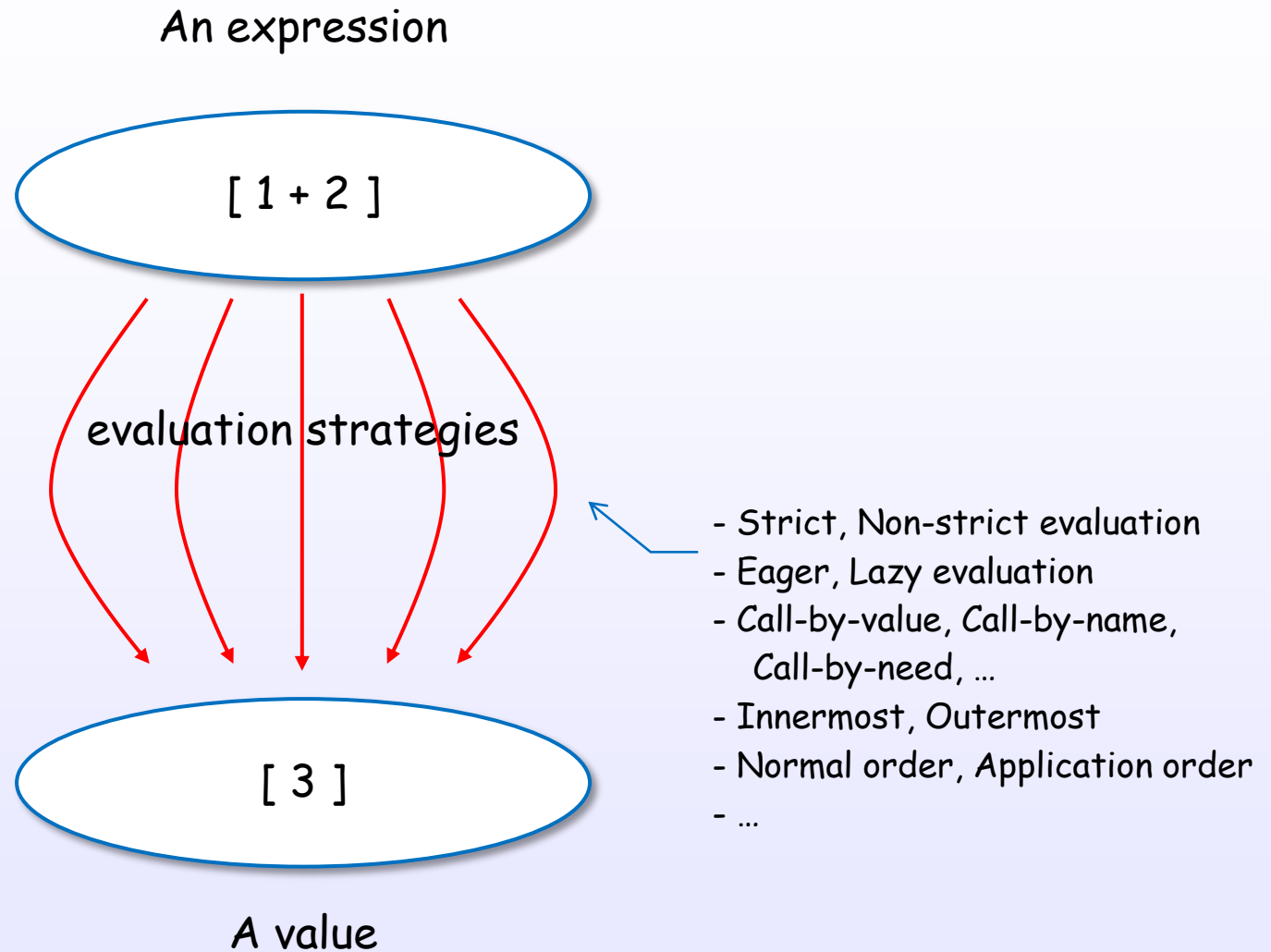
[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

# 1. Introduction

Evaluation strategies

# There are many evaluation approaches



[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

# Evaluation layers

denotational semantics

evaluation strategies

implementation techniques

[Bird, Chapter 7]

[Hutton, Chapter 8]

[TAPL, Chapter 3]

References : [1]

# Evaluation layers

denotational  
semantics

Strict semantics

Non-strict semantics

[Terej]  
eva  
call  
call  
call  
eva

evaluation  
strategies

Eager evaluation  
(Strict evaluation)

Nondeterministic  
evaluation

Lazy evaluation  
(Non-strict evaluation)

...

no-  
non  
eva  
fun  
lazy  
sha

Call-by-Value

Call-by-Name

Call-by-Need

...

implementation  
techniques

Lazy graph reduction

...

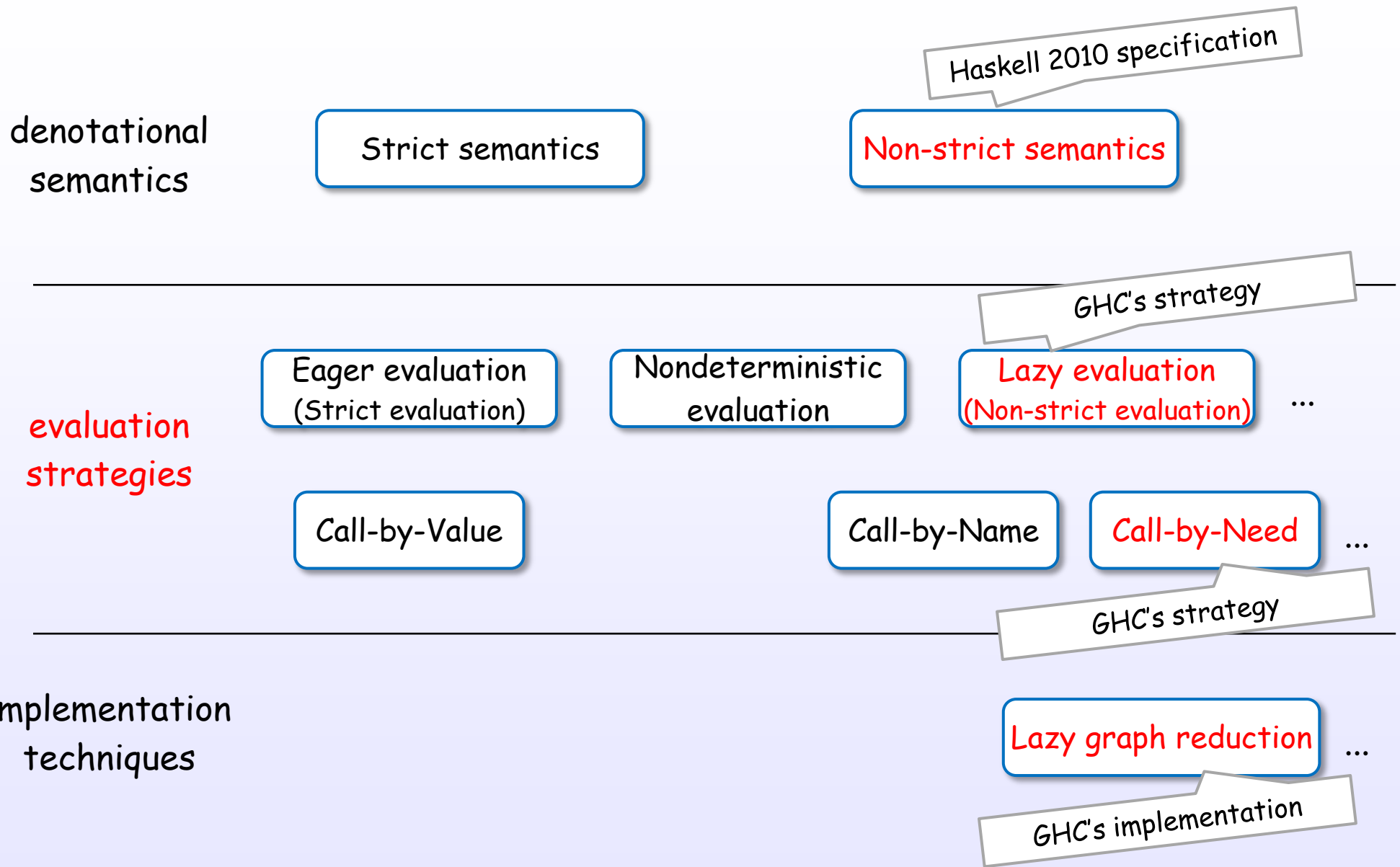
[Bird, Chapter 7]

[Hutton, Chapter 8]

[TAPL, Chapter 3]

References : [1]

# Evaluation layers for GHC's Haskell



# Evaluation strategies and order

$a(b\ c) + d(e\ (f\ g))$

order

[Bird]  
[Hutton]

References : [1]

# Simple example of both evaluations

## Eager evaluation (Strict evaluation)

default  
C, Java, JavaScript,  
Python, OCaml, Scheme, ...

square ( 1 + 2 )



argument  
evaluation  
first

square ( 3 )



3 \* 3



9

## Lazy evaluation (Non-strict evaluation)

default  
Haskell (GHC), ...

square ( 1 + 2 )



apply  
first

( 1 + 2 ) \* ( 1 + 2 )



( 3 ) \* ( 3 )



9

[Bird]  
[Hutton]



# Simple example of both evaluations

Eager evaluation  
(Strict evaluation)

square ( 1 + 2 )



square ( 3 )



3 \* 3



9

argument  
evaluated

Lazy evaluation  
(Non-strict evaluation)

square ( 1 + 2 )



( 1 + 2 ) \* ( 1 + 2 )



( 3 ) \* ( 3 )



9

argument  
evaluation  
**delayed !**

[Bird]  
[Hutton]

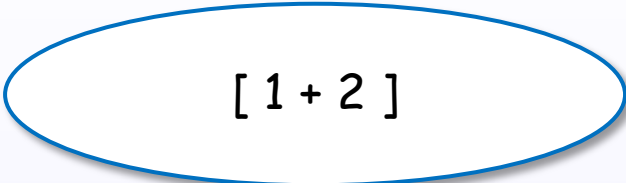
## 2. Expressions

## 2. Expressions

### Expressions in Haskell

# An expression denotes a value

An expression



[ 1 + 2 ]

[HR2010]

[Bird, Chapter 2]

References : [1]

# There are many expressions in Haskell

## Expressions

Just 5

$1 + 2$

$(1, 2)$

take 5 xs

$[1, 2, 3]$

let  $x = 1$  in  $x + y$

'a'

map f xs

if b then 1 else 0

7

$\forall x \rightarrow x + 1$

$x : xs$

fun arg

case x of  $\_ \rightarrow 0$

$(\forall x \rightarrow x + 1) 3$

do { $x \leftarrow$  get; put x}

variable



categorizing

[HR2010]

[Bird, Chapter 2]

References : [1]

# Expression categories in Haskell

WHNF(a value)、  
unevaluated expression  
との関連づけを  
PAPもWHNFなので注意

## lambda abstraction

$\forall x \rightarrow x + 1$

## let expression

let  $x = 1$  in  $x + y$

## variable

## conditional

if  $b$  then 1 else 0

## case expression

case  $x$  of  $\_ \rightarrow 0$

## do expression

do { $x \leftarrow \text{get}$ ; put  $x$ }

## general constructor, literal and some forms

7

[1, 2, 3]

(1, 2)

'a'

$x : xs$

Just 5

## function application

take 5  $xs$

$(\forall x \rightarrow x + 1)$  3

1 + 2

map  $f$   $xs$

fun arg

[HR2010]  
[Bird, Chapter 2]

# Specification is defined in Haskell 2010 Language Report

## Haskell 2010 Language Report, Chapter 3 Expressions [1]

<i>exp</i>	→	<i>infixexp</i> :: [context =>] type   <i>infixexp</i>	(expression type signature)
<i>infixexp</i>	→	<i>lexp</i> <i>qop</i> <i>infixexp</i>   - <i>infixexp</i>   <i>lexp</i>	(infix operator application) (prefix negation)
<i>lexp</i>	→	\ <i>apat</i> <sub>1</sub> ... <i>apat</i> <sub><i>n</i></sub> -> <i>exp</i>   let <i>decls</i> in <i>exp</i>   if <i>exp</i> [ <i>i</i> ] then <i>exp</i> [ <i>i</i> ] else <i>exp</i>   case <i>exp</i> of { <i>alts</i> }   do { <i>stmts</i> }   <i>fexp</i>	(lambda abstraction, $n \geq 1$ ) (let expression) (conditional) (case expression) (do expression)
<i>fexp</i>	→	[ <i>fexp</i> ] <i>aexp</i>	(function application)
<i>aexp</i>	→	<i>qvar</i>   <i>gcon</i>   <i>literal</i>   ( <i>exp</i> )   ( <i>exp</i> <sub>1</sub> , ... , <i>exp</i> <sub><i>k</i></sub> )   [ <i>exp</i> <sub>1</sub> , ... , <i>exp</i> <sub><i>k</i></sub> ]   [ <i>exp</i> <sub>1</sub> [, <i>exp</i> <sub>2</sub> ] .. [ <i>exp</i> <sub>3</sub> ] ]   [ <i>exp</i>   <i>qual</i> <sub>1</sub> , ... , <i>qual</i> <sub><i>n</i></sub> ]   ( <i>infixexp</i> <i>qop</i> )   ( <i>qop</i> { - } <i>infixexp</i> )   <i>qcon</i> { <i>fbind</i> <sub>1</sub> , ... , <i>fbind</i> <sub><i>n</i></sub> }   <i>aexp</i> <sub>{<i>qcon</i>}</sub> { <i>fbind</i> <sub>1</sub> , ... , <i>fbind</i> <sub><i>n</i></sub> }	(variable) (general constructor)  (parenthesized expression) (tuple, $k \geq 2$ ) (list, $k \geq 1$ ) (arithmetic sequence) (list comprehension, $n \geq 1$ ) (left section) (right section)  (labeled construction, $n \geq 0$ ) (labeled update, $n \geq 1$ )

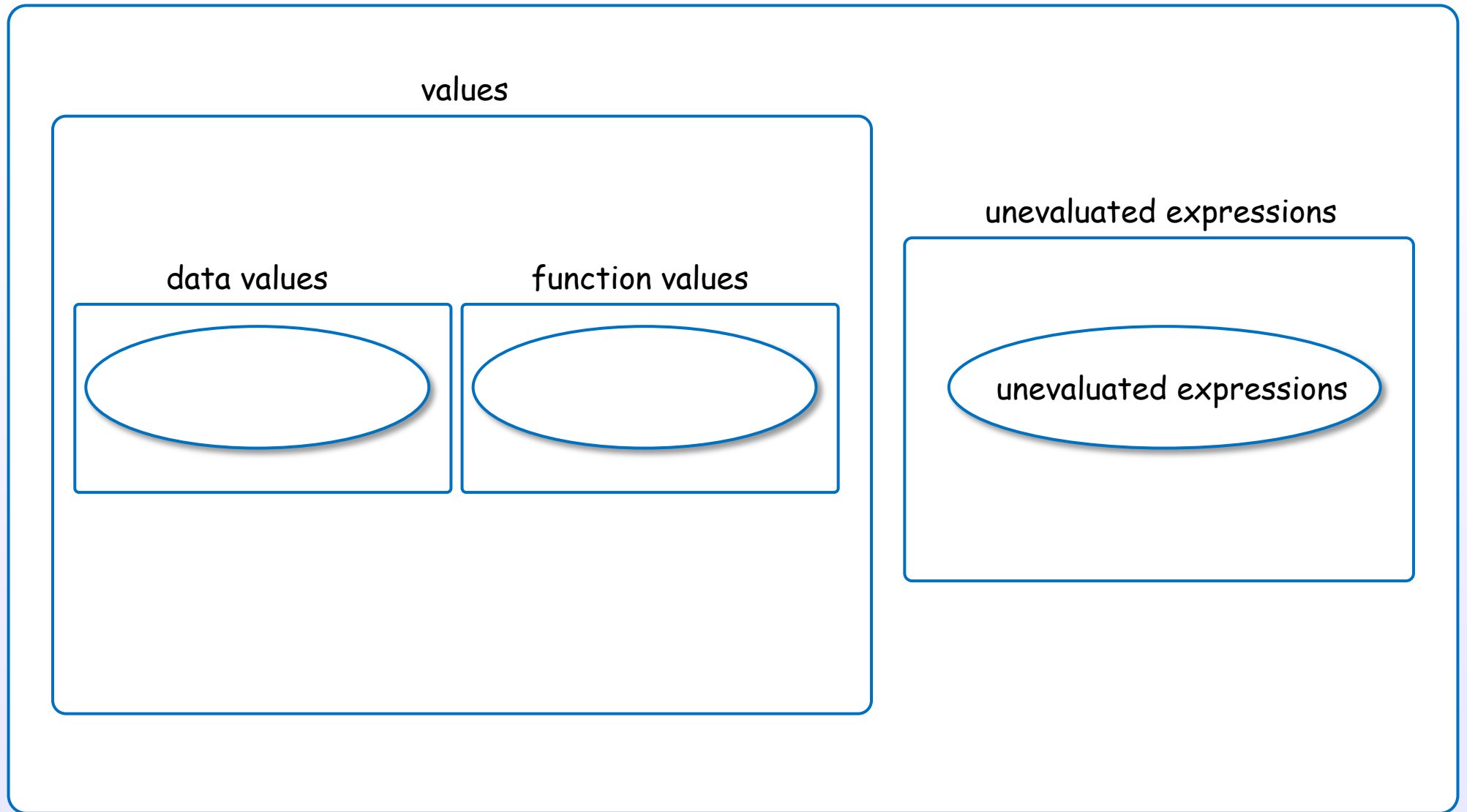
## 2. Expressions

### Classification of expressions



# A value or an unevaluated expression

## Expressions

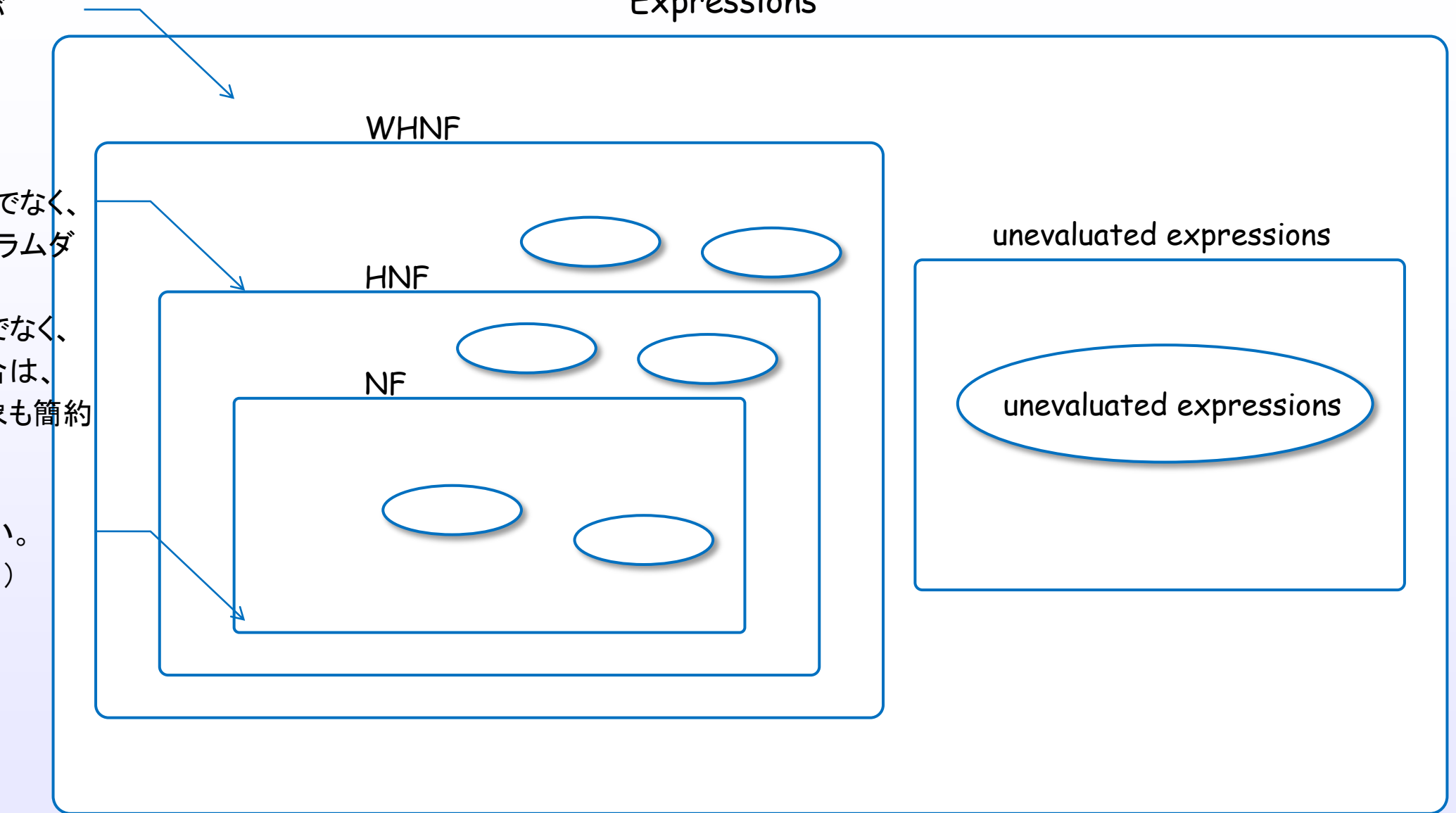


値か否か。値は2種。

[STG]

# evaluation level

## Expressions



値には、評価レベルがある。

[STG]

# 実例との対応付け

[STG]

References : [1]

### 3. Internal representation of expressions

### 3. Internal representation of expressions

Constructor

# Constructor

Constructor is one of the key elements to understand WHNF and lazy evaluation in Haskell.

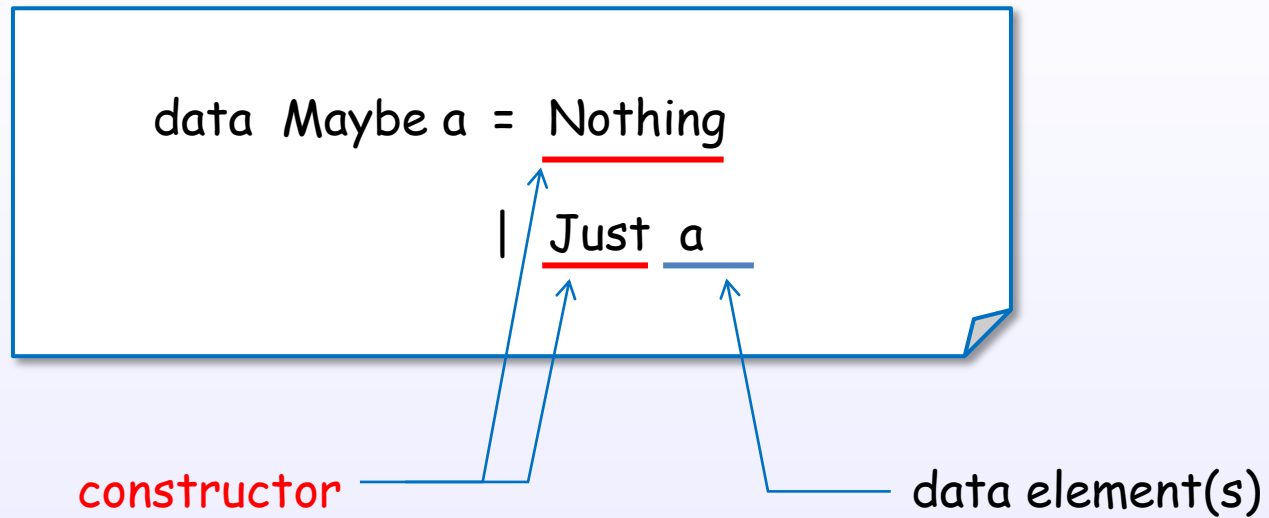
# data文で宣言する代数的データ型とその値

```
data Maybe a = Nothing  
              | Just a
```

Algebraic Data Type

Data **Values**

# Constructorはdata文で宣言する代数的データ値

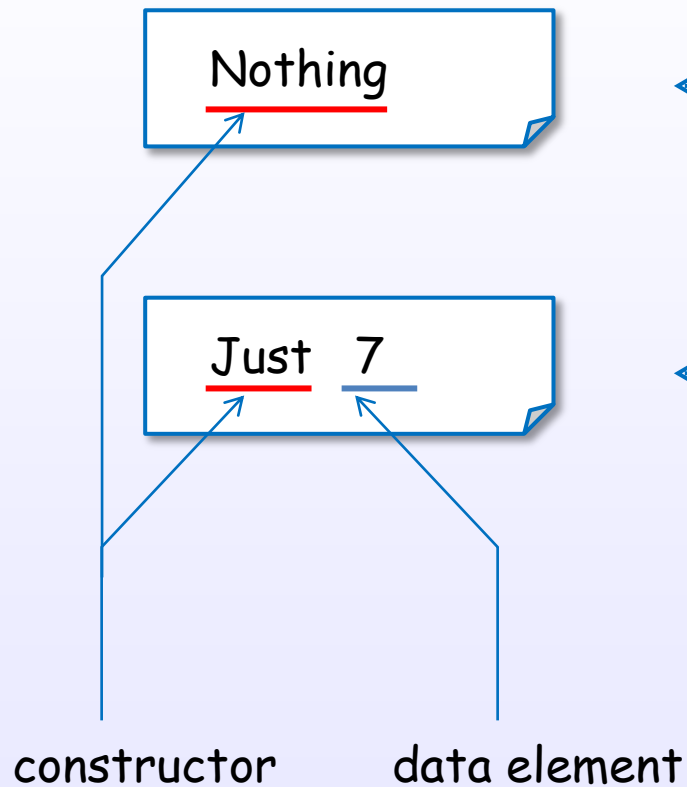




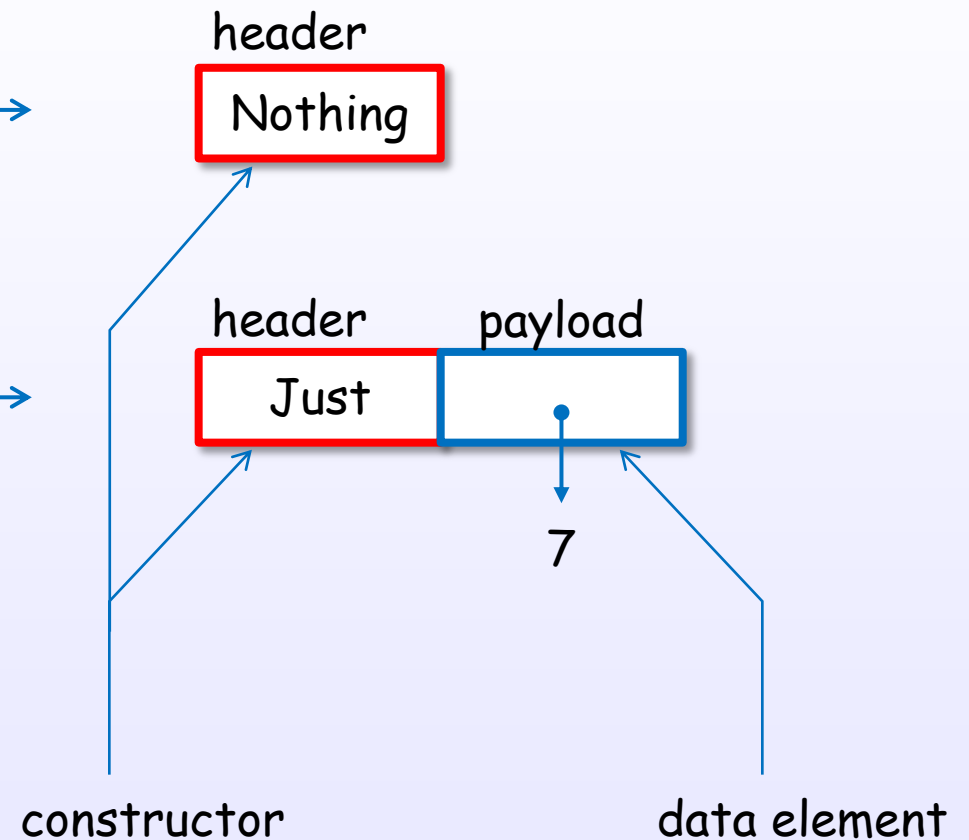
# Constructorの内部表現

↑ data values

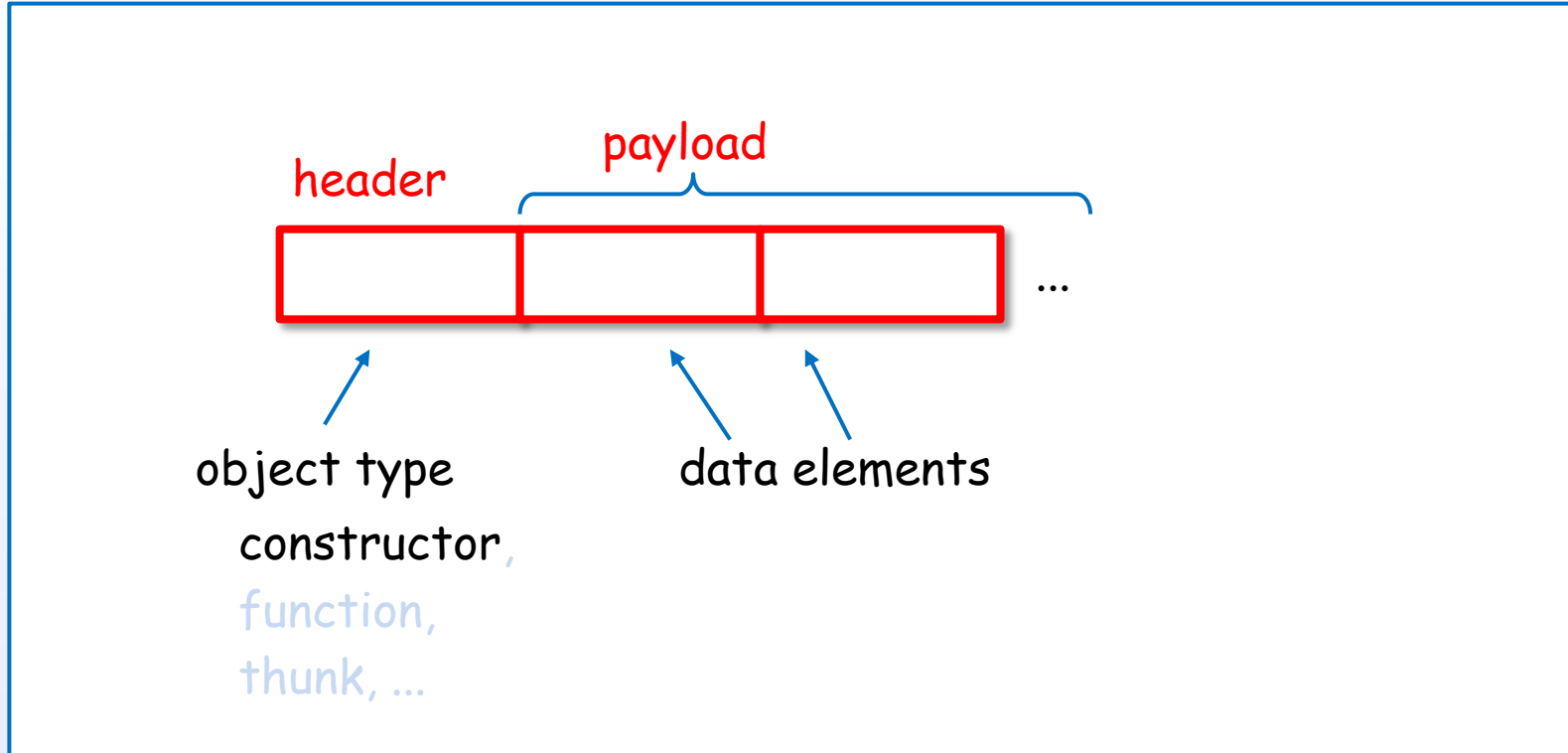
Haskell code



GHC's internal representation  
in heap memory



# Constructorは統一内部表現で表現される



in heap memory, stack, registers or static memory

# いろいろなコンストラクタと内部表現

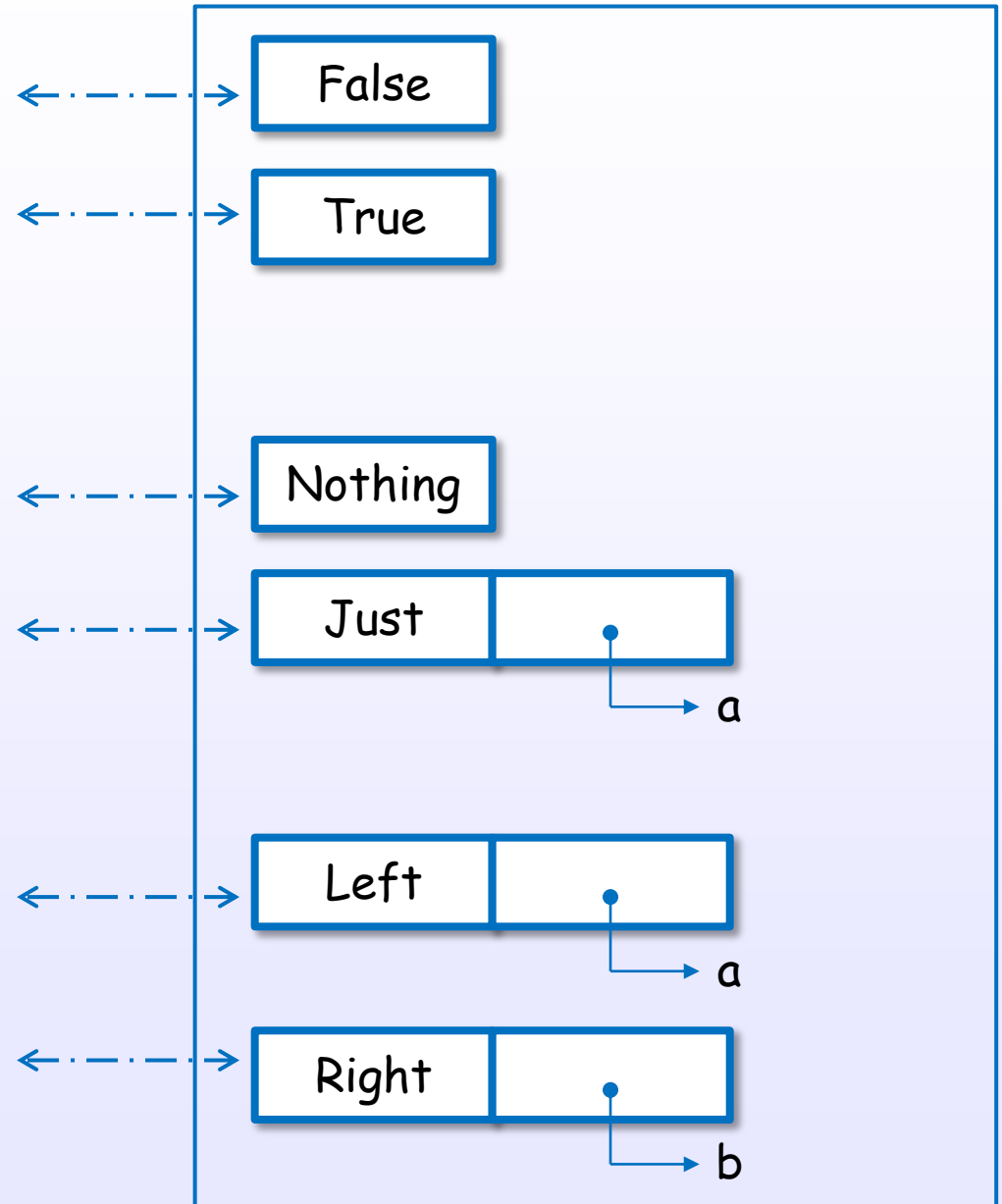
Haskell code

```
data Bool = False  
          | True
```

```
data Maybe a = Nothing  
              | Just a
```

```
data Either a b = Left a  
                 | Right b
```

GHC's internal representation



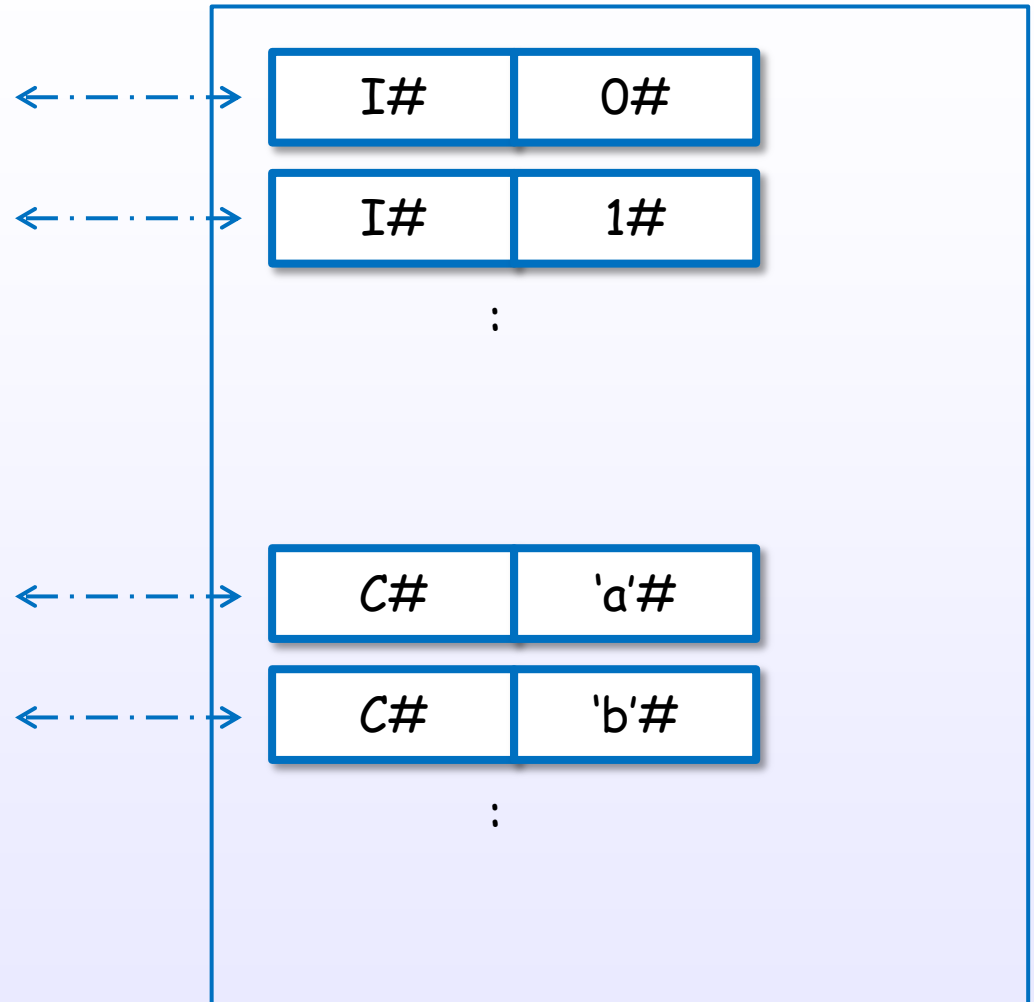
# 基本データ型も実はコンストラクタで構成されている

Haskell code

```
data Int = I# 0#  
        | I# 1#  
        | :  
        | :
```

```
data Char = C# 'a'#  
          | C# 'b'#  
          | :  
          | :
```

GHC's internal representation



# リストも実はコンストラクタで構成されている

List

```
[ 1, 2, 3 ]
```

syntactic desugar

```
1 : ( 2 : ( 3 : [] ) )
```

prefix notation by section

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

equivalent data constructor

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

constructor

# リストも実はコンストラクタで構成されている

List

```
[ 1, 2, 3 ]
```

syntactic desugar

```
1 : ( 2 : ( 3 : [] ) )
```

prefix notation by section

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

equivalent data constructor

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

type  
declaration

*\* pseudo code*

```
data List a = []  
            | : a (List a)
```

```
data List a = Nil  
            | Cons a (List a)
```

# リストも実はコンストラクタで構成されている

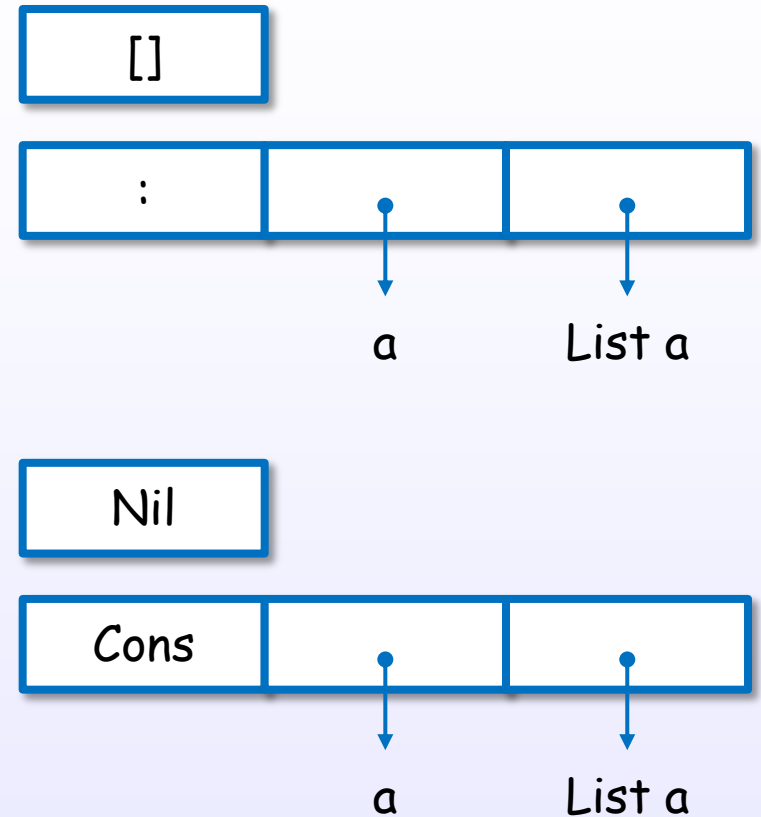
Haskell code

```
data List a = []  
            | : a (List a)
```

equivalent data constructor

```
data List a = Nil  
            | Cons a (List a)
```

GHC's internal representation



# リストも実はコンストラクタで構成されている

Haskell code

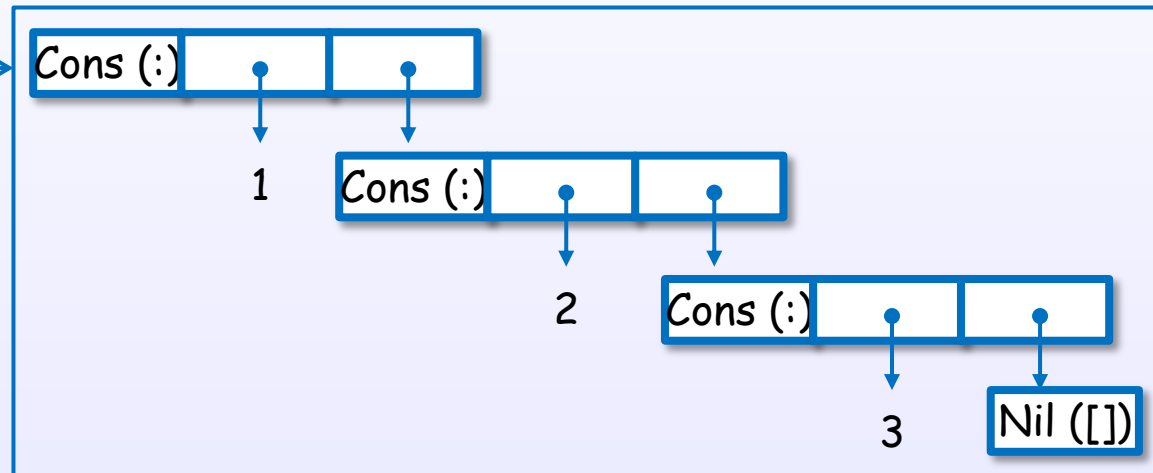
```
[ 1, 2, 3 ]
```

```
1 : ( 2 : ( 3 : [] ) )
```

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

GHC's internal representation





# タプルも実はコンストラクタで構成されている

Tuple (Pair)

( 7 , 8 )

prefix notation by section

(,) 7 8

equivalent data constructor

Pair 7 8

constructor

type  
declaration

*\* pseudo code*

data Pair a = (,) a a

data Pair a = Pair a a

# タプルも実はコンストラクタで構成されている

Haskell code

```
(7, 8)
```

```
(,) 7 8
```

```
Pair 7 8
```

GHC's internal representation

```
Pair (,)
```

7

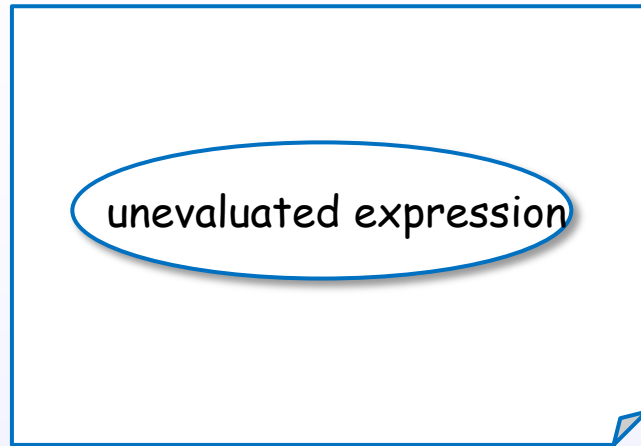
8

### 3. Internal representation of expressions

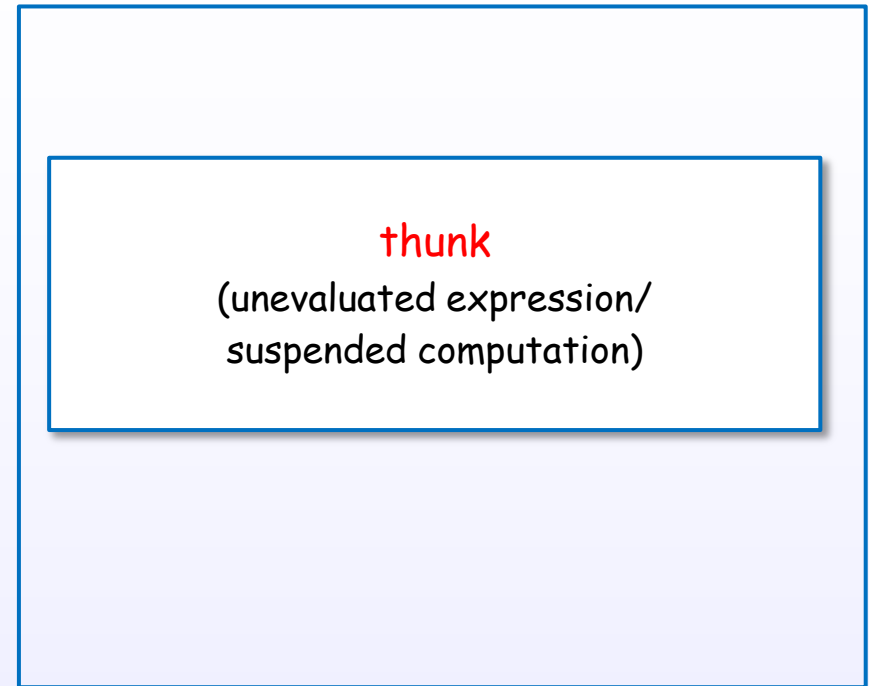
Thunk

# Thunk

Haskell code



GHC's internal representation



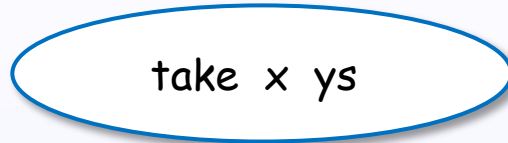
A thunk is an **unevaluated** expression in heap memory.  
A thunk is built to **postpone** the evaluation.

# Thunkの内部表現

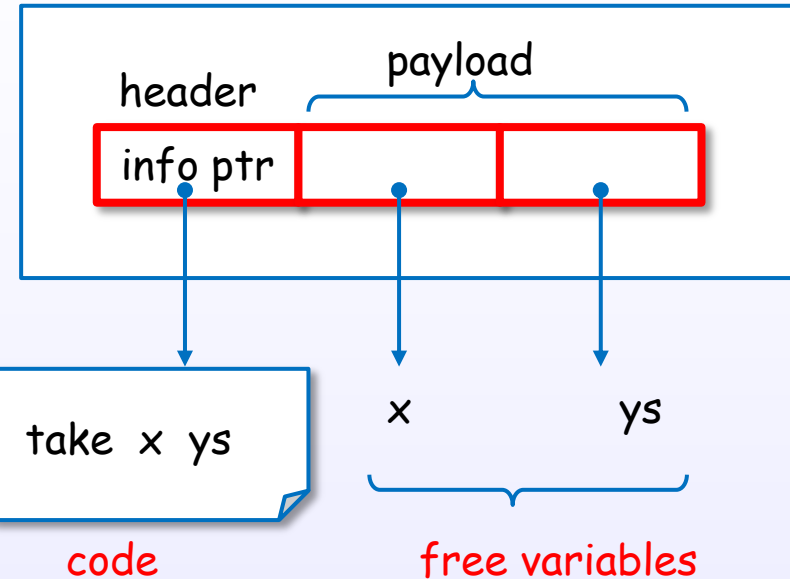
Haskell code

GHC's internal representation

An unevaluated expression

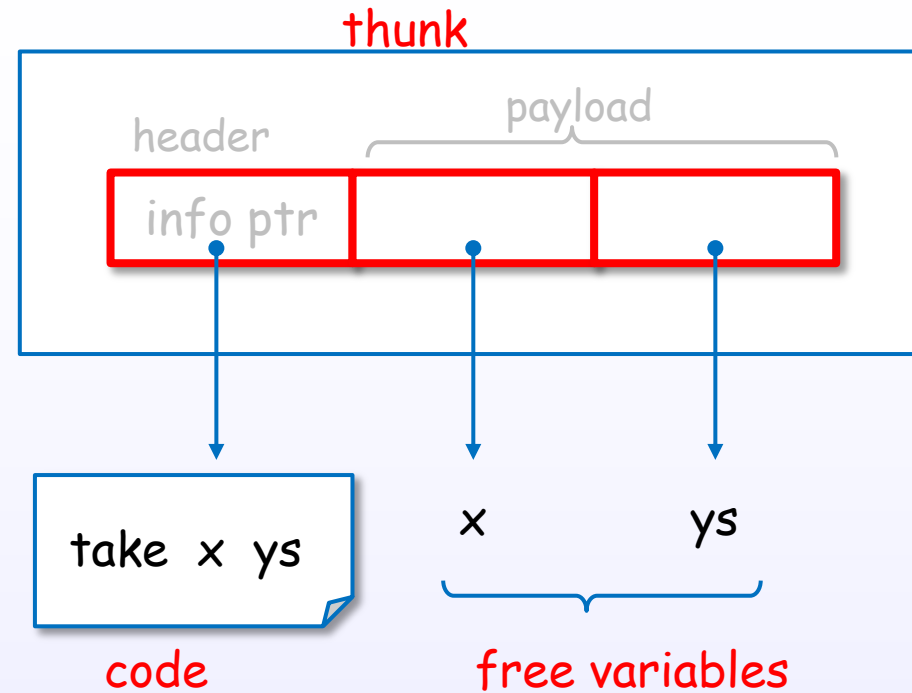


thunk



A thunk is represented with header(code) + payload(free variables).

# Thunkは、codeとfree variablesをパッケージ化したもの



A thunk is a package of code + free variables.

# Thunkは、forcing要求により評価される

Haskell code

An unevaluated expression

take x ys



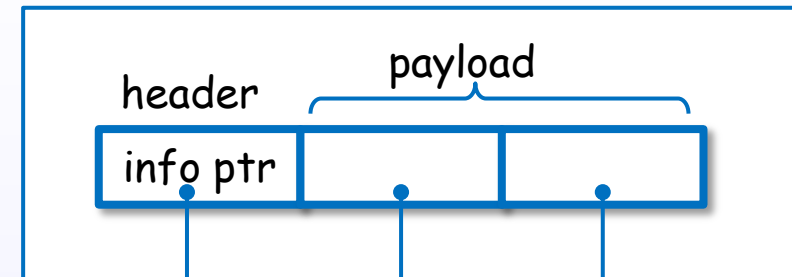
forcing

[ 3 ]

An evaluated expression

GHC's internal representation

thunk



take x ys

code

x

ys

free variables



forcing



3

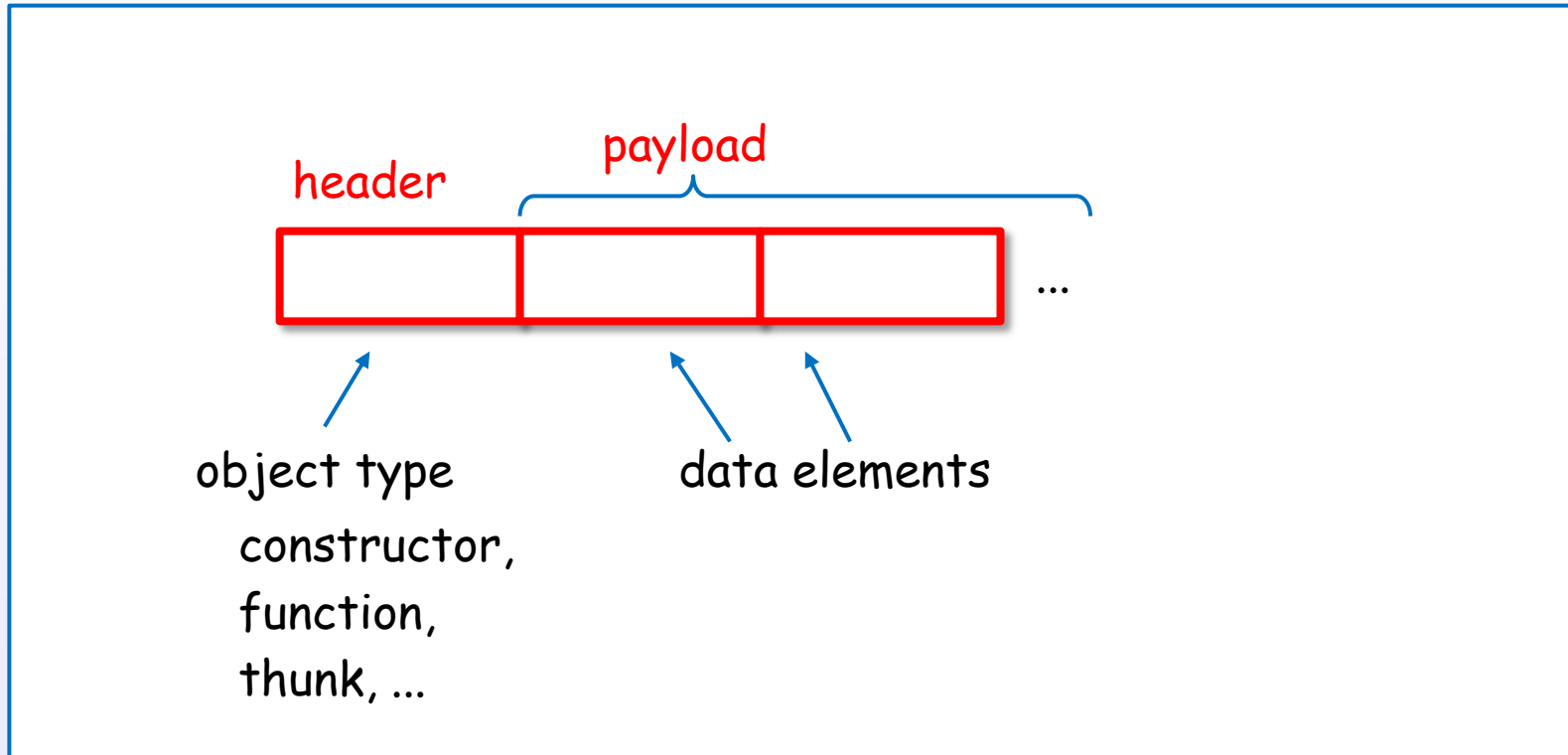
Nil ([)]

### 3. Internal representation of expressions

Uniform representation

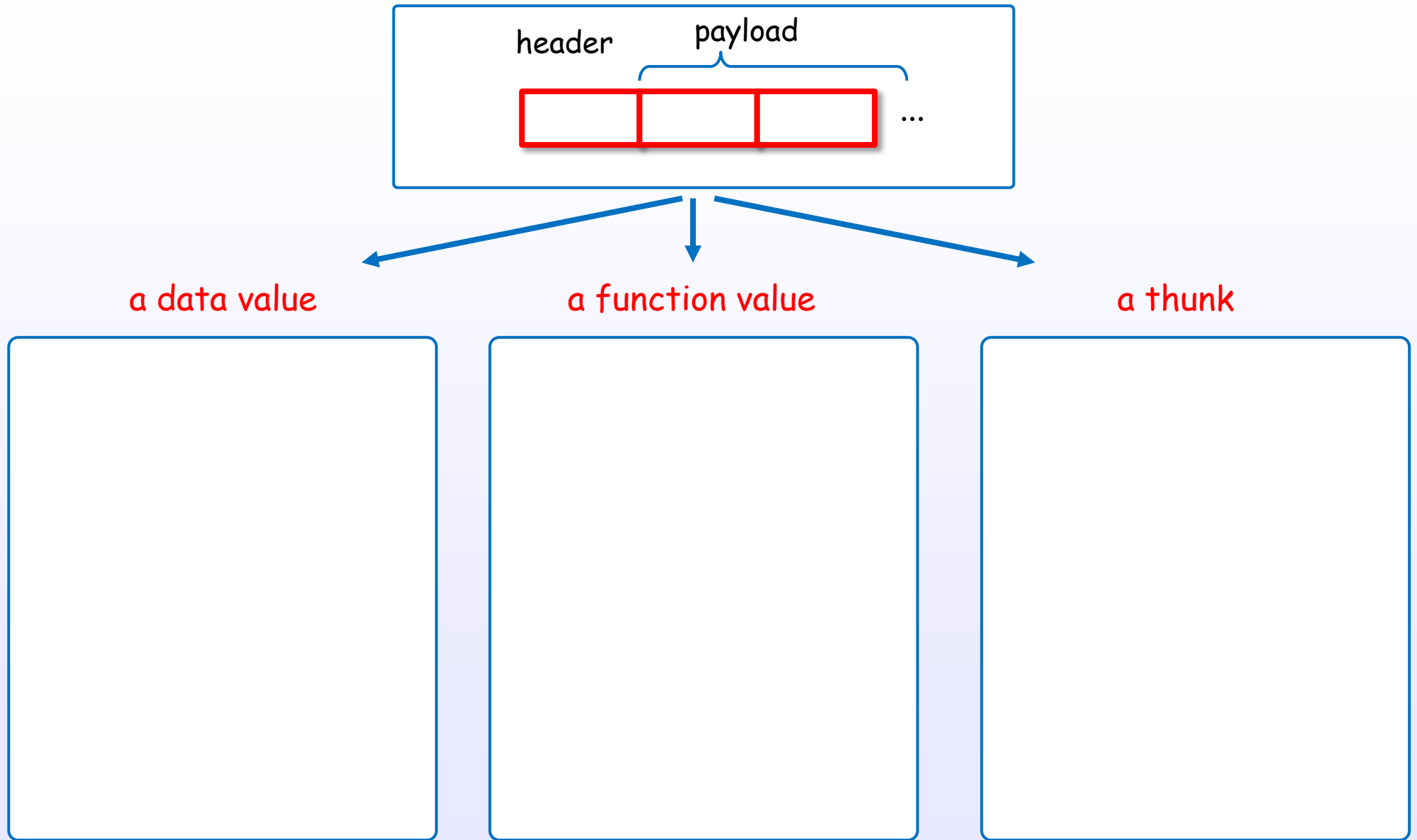


# 統一内部表現で表現される

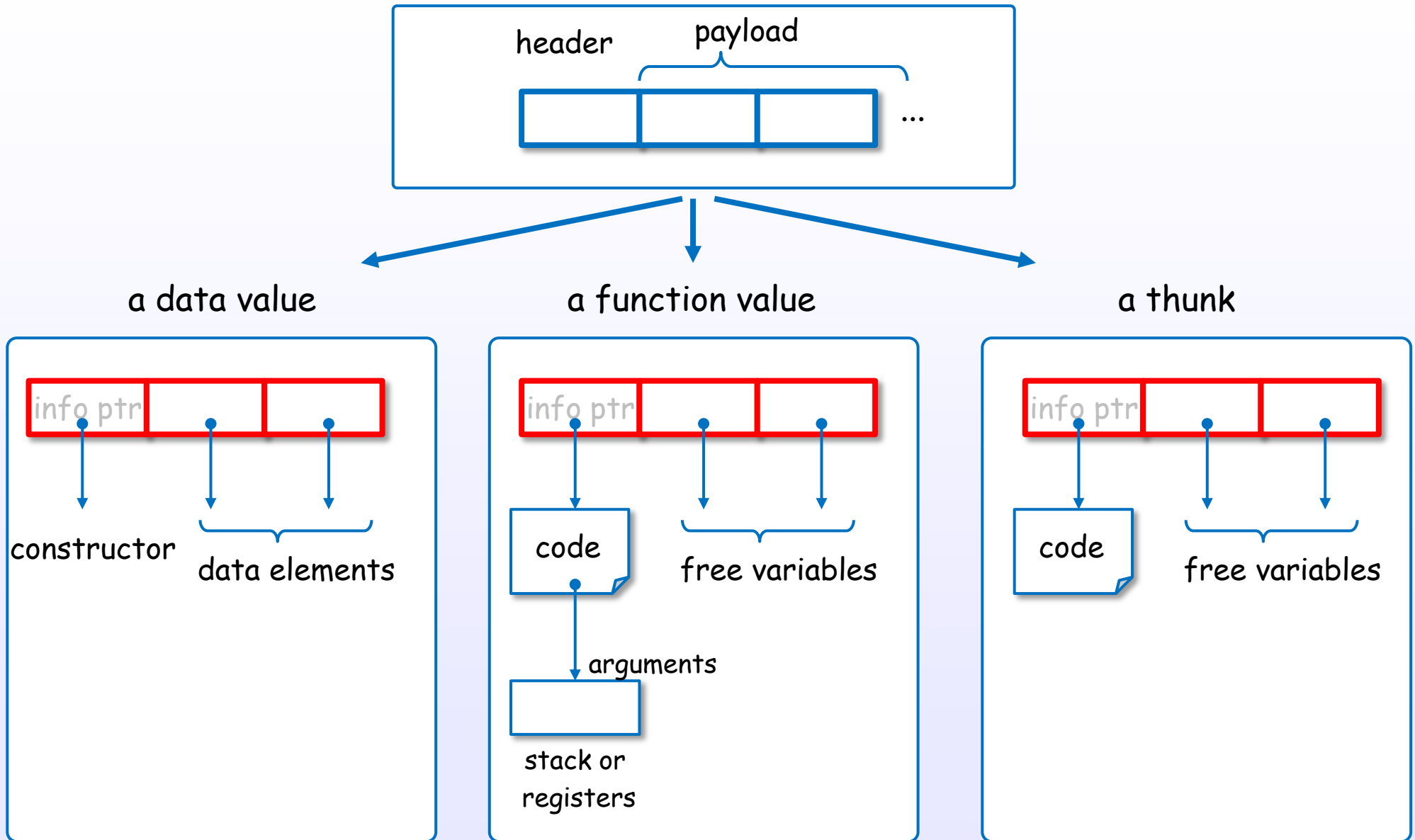


in heap memory, stack, registers or static memory

# 統一内部表現



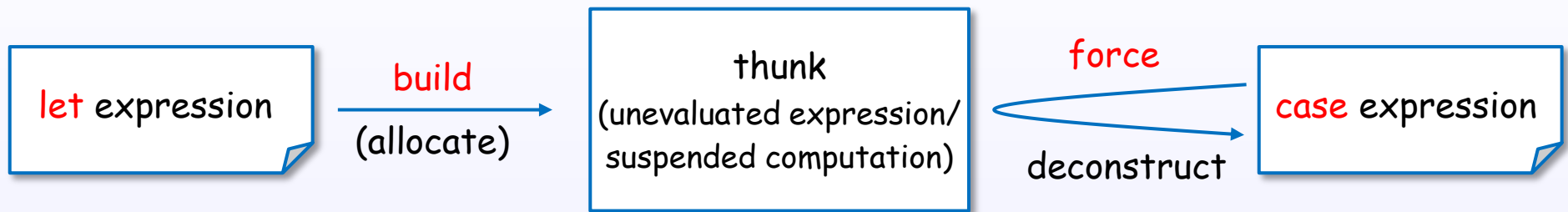
# 統一内部表現



### 3. Internal representation of expressions

let, case expression

# let/case expressions and thunk



A let expression may build a thunk.

A case expression forces and deconstructs the thunk.

# A let expression builds a thunk

let expression

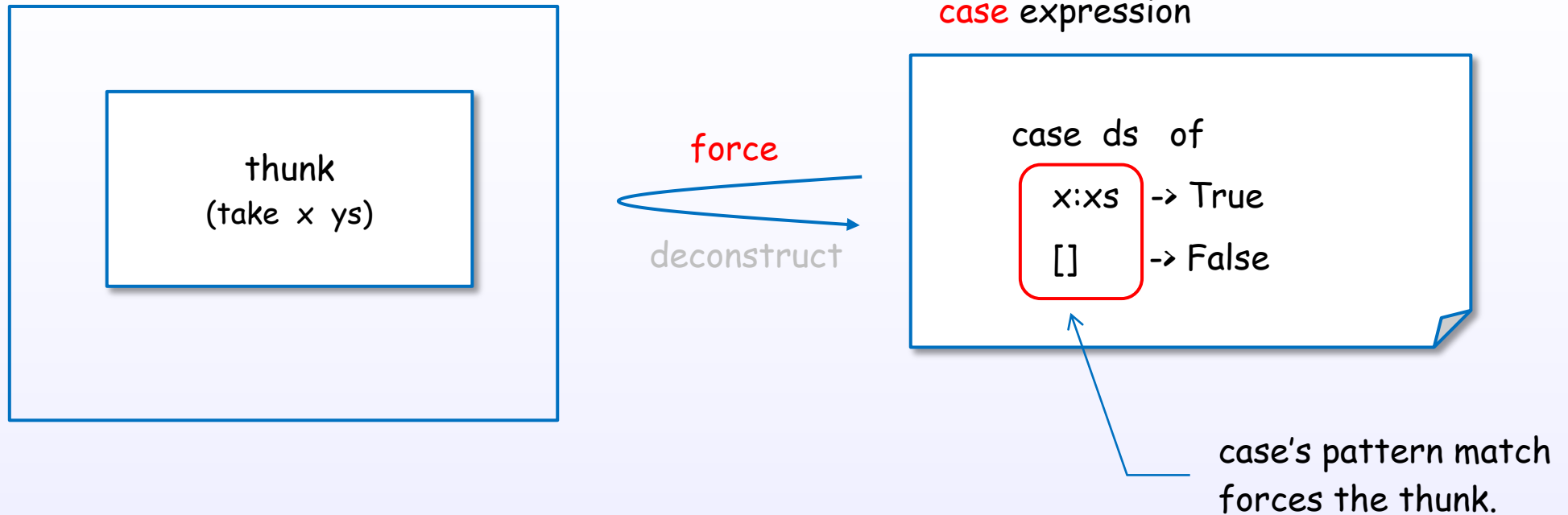
let ds = take x ys

build  
(allocate)

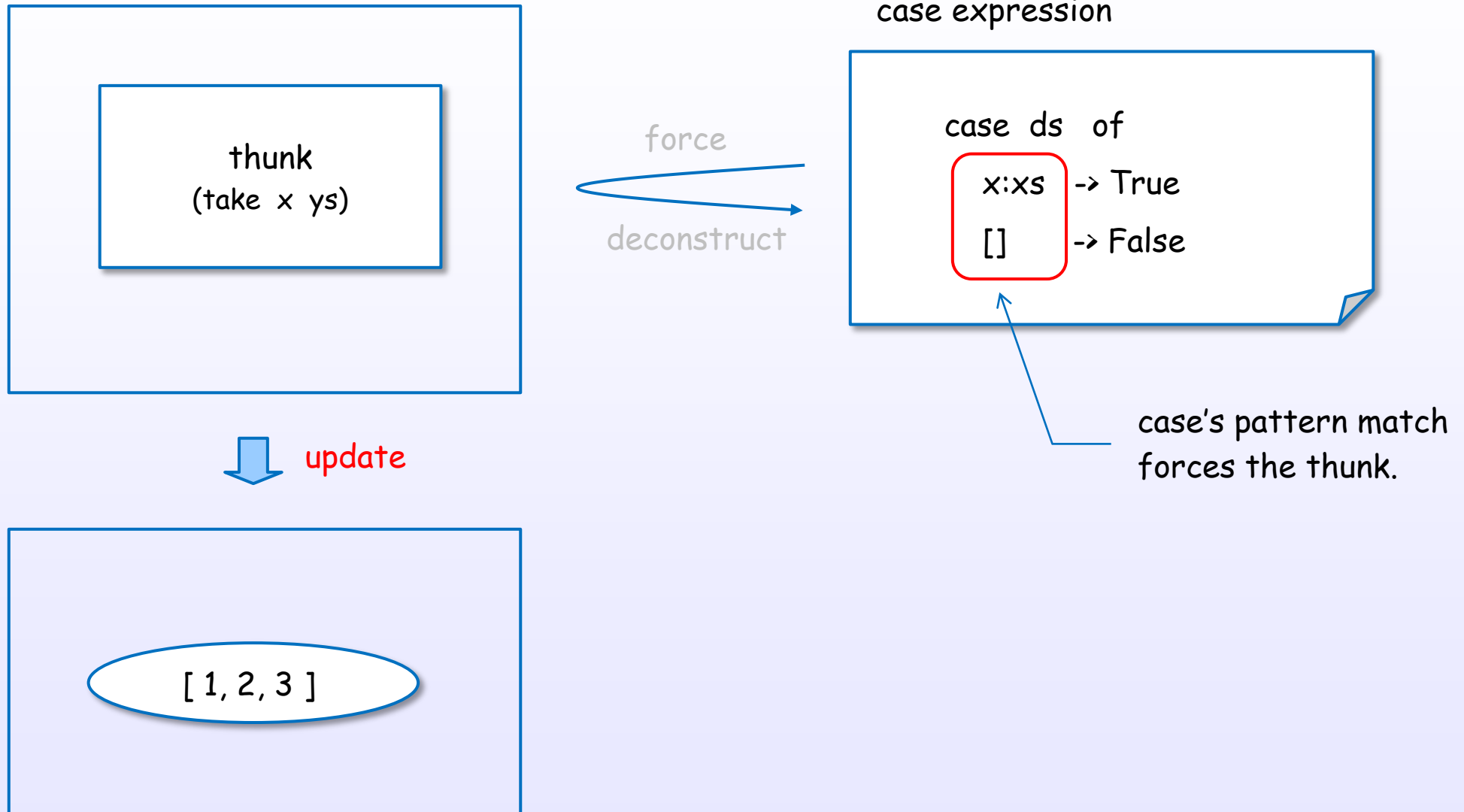
thunk  
(take x ys)

heap memory

# A case expression forces a thunk

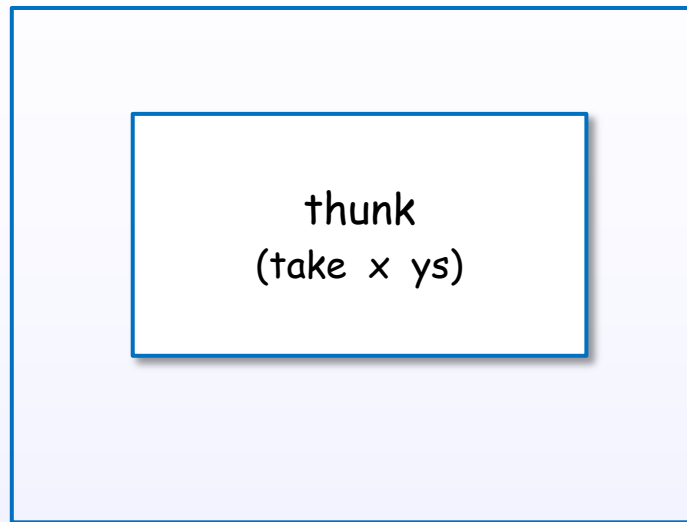


# A case expression forces a thunk

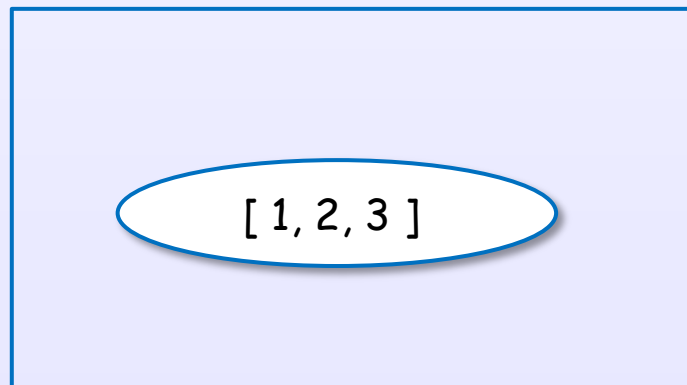




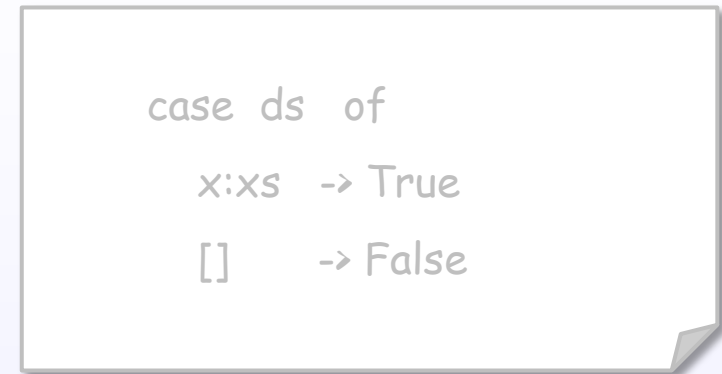
# A case expression forces a thunk



↓ update

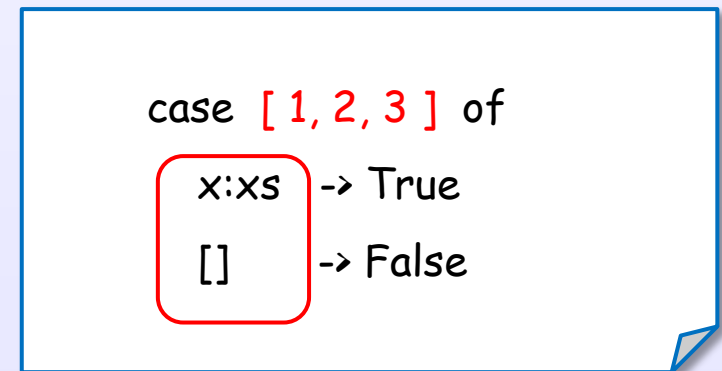


case expression



force  
→  
deconstruct

case expression



force  
→  
deconstruct

### 3. Internal representation of expressions

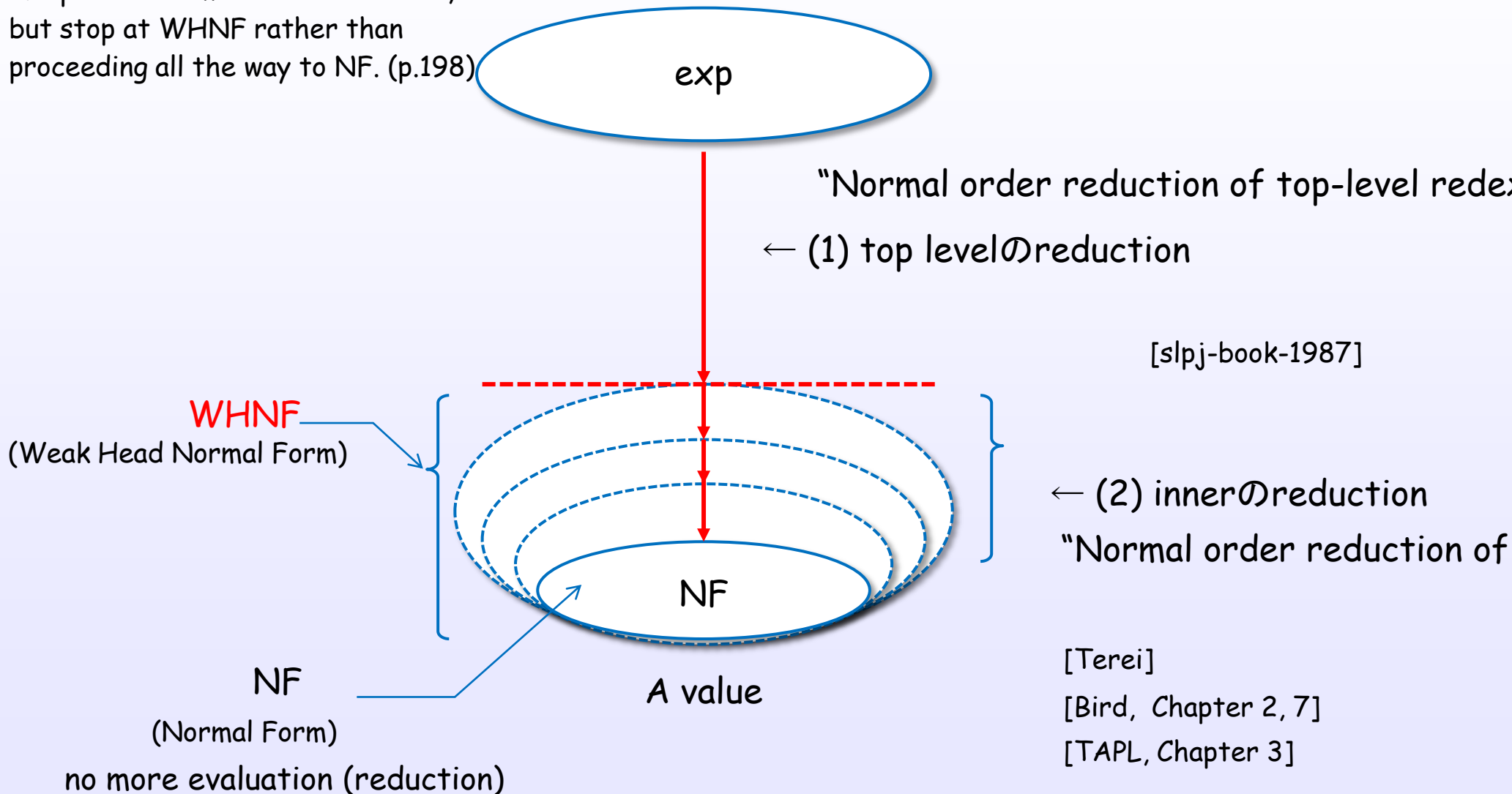
WHNF

# evaluation step (GHC)

Our reduction order is therefore to reduce the top-level redex until weak head normal form is reached. (p.198)

An expression

We pursue normal order reduction, but stop at WHNF rather than proceeding all the way to NF. (p.198)



[slpj-book-1987]

[Terei]

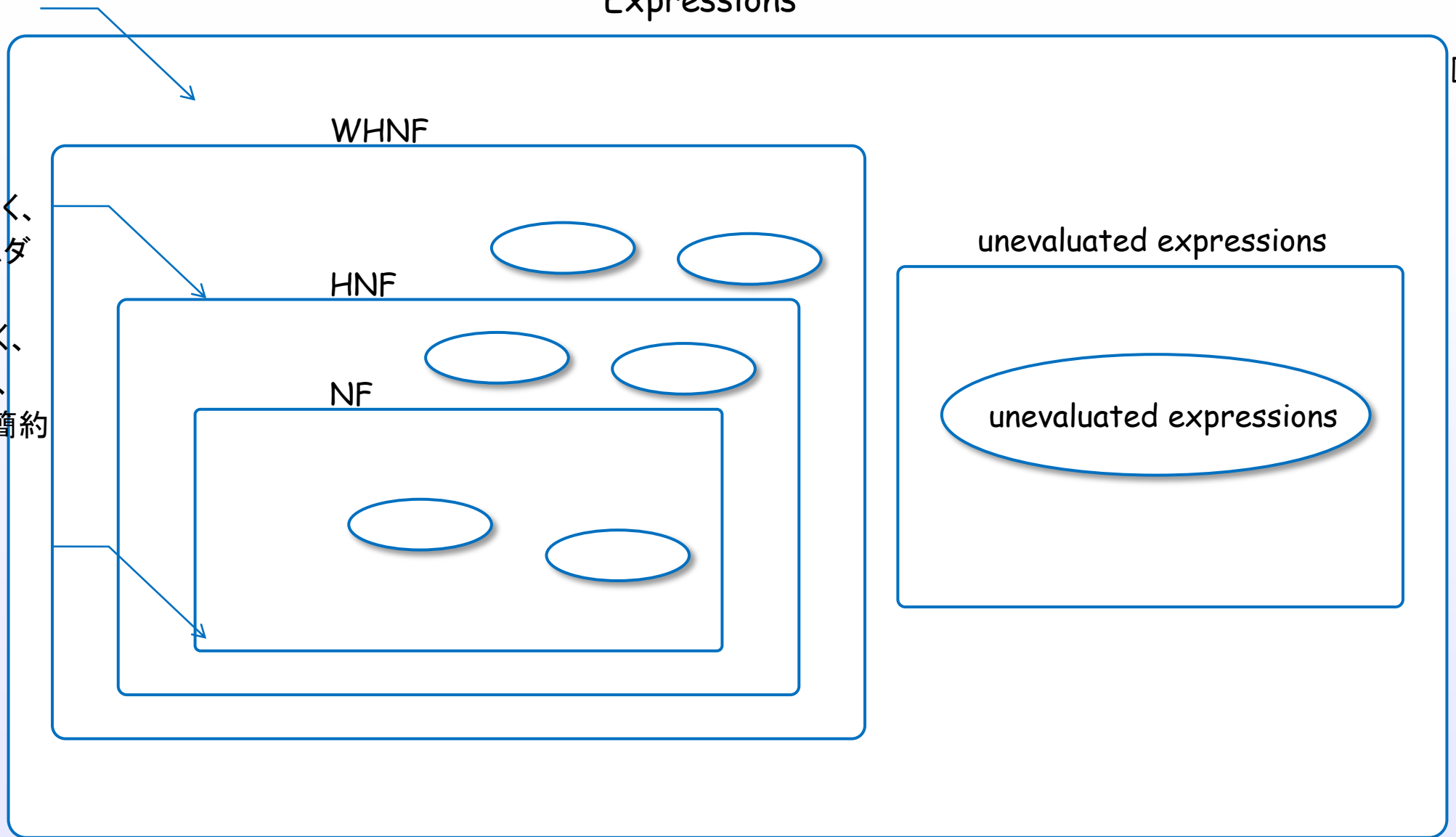
[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

再掲

# evaluation level

## Expressions

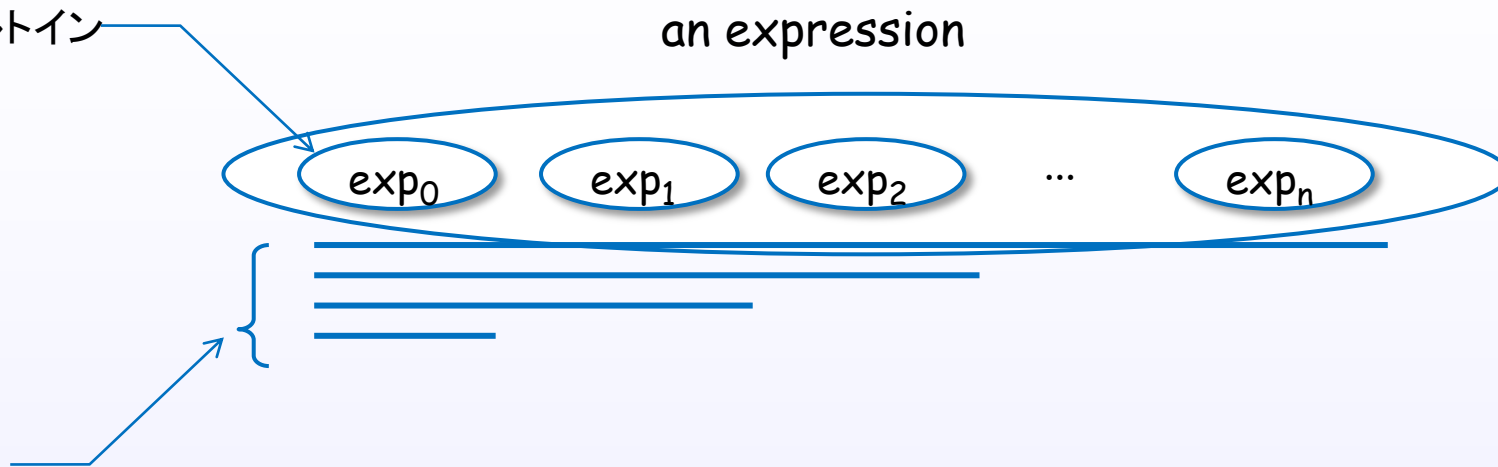


値には、評価レベルがある。

[STG]

# WHNF

データ抽象、ビルトイン



more

An expression has no top level redex, if it is in WHNF.

[slpj-book-1987]

These are in weak head normal form,  
but not in normal form, since they contain inner redex. (p.198)

[Terei]

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

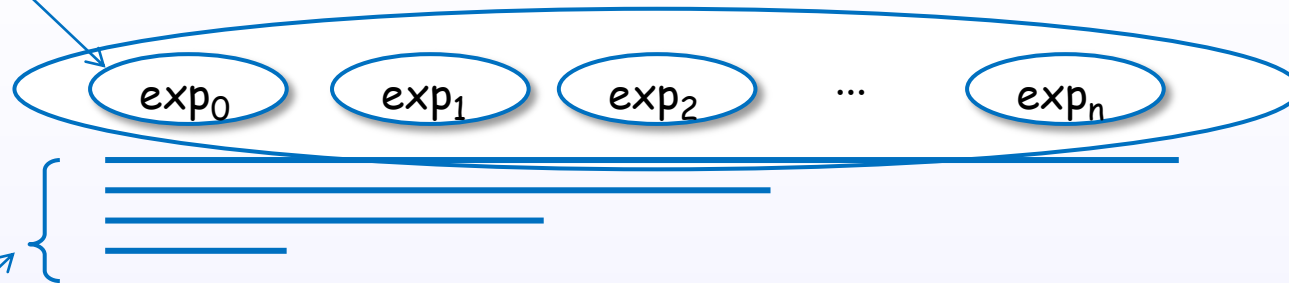
[Terei]

References : [1]

# Examples of WHNF

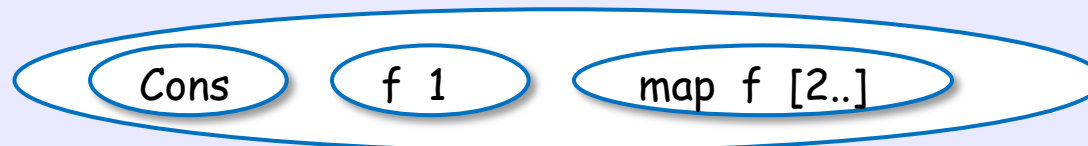
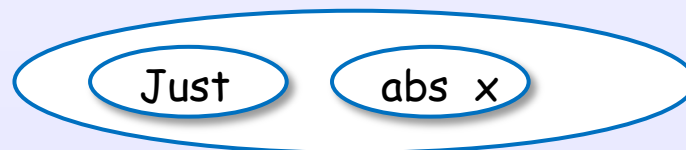
データ抽象、ビルトイン

an expression



Just (abs x)

Cons (f 1) (map f [2..])



[slpj-book-1987]

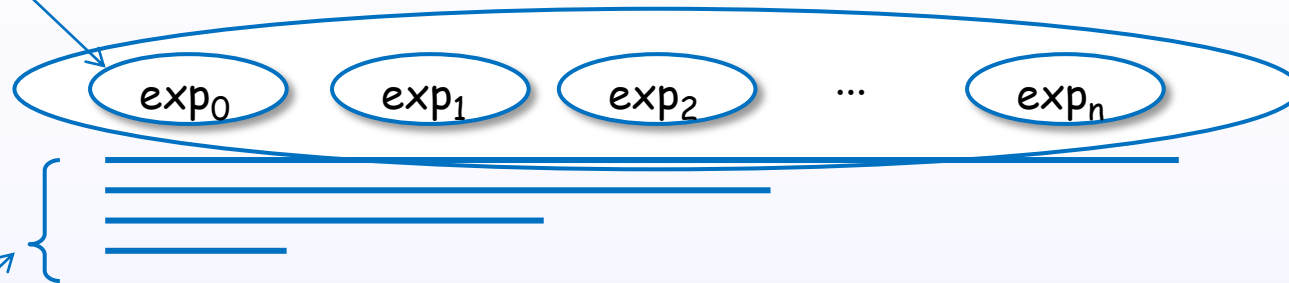
# HNF

データ抽象、ビルトイン

内側(body)が、簡

more

an expression



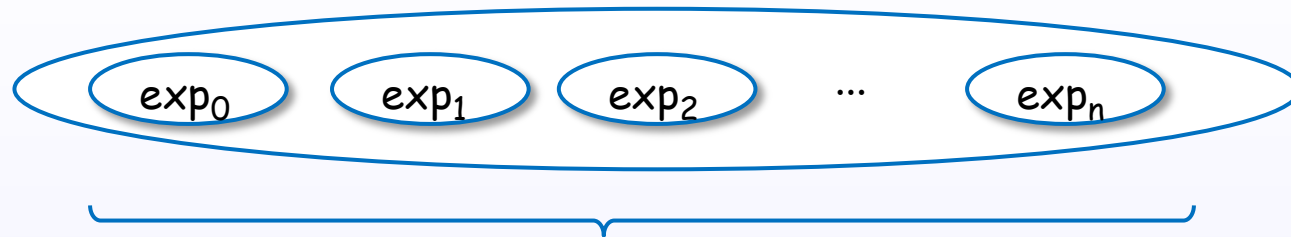
[slpj-book-1987]

[Terei]

References : [1]

# NF

an expression



redexが内部に無い

[slpj-book-1987]

[Terei]

[Bird, Chapter 2, 7]

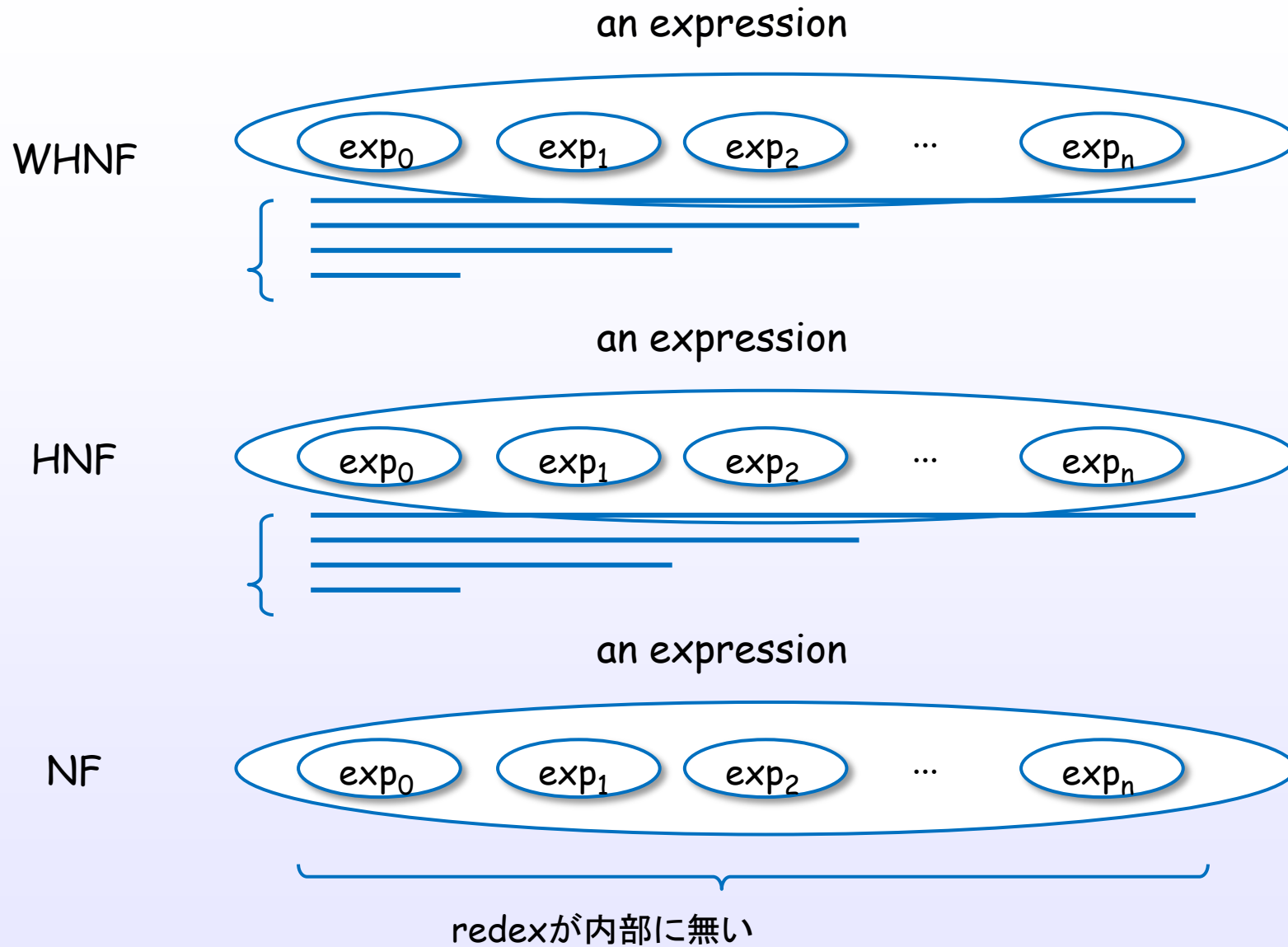
[TAPL, Chapter 3]

[Terei]

References : [1]



# WHNF, HNF, NF



[slpj-book-1987]

References : [1]

## definition of WHNF and HNF

## The implementation of functional programming languages [19]

### 11.3.1 Weak Head Normal Form

To express this idea precisely we need to introduce a new definition:

### DEFINITION

A lambda expression is in *weak head normal form* (WHNF) if and only if it is of the form

$$F \ E_1 \ E_2 \ \dots \ E_n$$

where  $n \geq 0$ ;

and either F is a variable or data object  
or F is a lambda abstraction or built-in function  
and  $(F \ E_1 \ E_2 \ \dots \ E_m)$  is not a redex for any  $m \leq n$ .

An expression has no *top-level redex* if and only if it is in weak head normal form.

### 11.3.3 Head Normal Form

Head normal form is often confusing and requires some discussion. The content of the `head` field is since for most purposes head normal form is the same as head normal form. Nevertheless, we will stick to the head normal form.

### DEFINITION

A lambda expression is in *head normal form* (HNF) if and only if it is of the form

$$\lambda x_1. \lambda x_2. \dots \lambda x_n. (v \ M_1 \ M_2 \ \dots \ M_m)$$

where  $n, m \geq 0$ ;

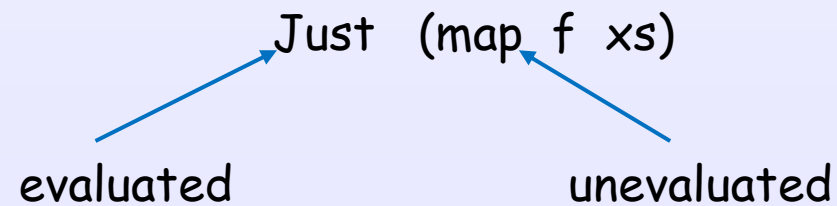
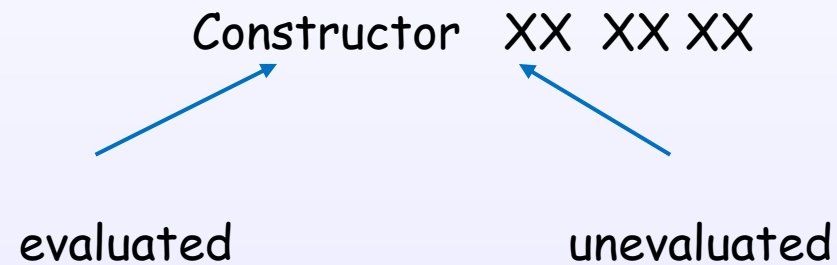
**v** is a variable ( $x_i$ ), a data object, or a built-in function;

and  $(v \ M_1 \ M_2 \ \dots \ M_p)$  is not a redex for any  $p \leq m$ .

[slpj-book-1987]

# internal representation of WHNF

heap objectイメージ



## 4. Evaluation

## 4. Evaluation

Evaluation in Haskell (GHC)

# GHC chosen lazy evaluation

必要な時に、必要な箇所のみを評価する

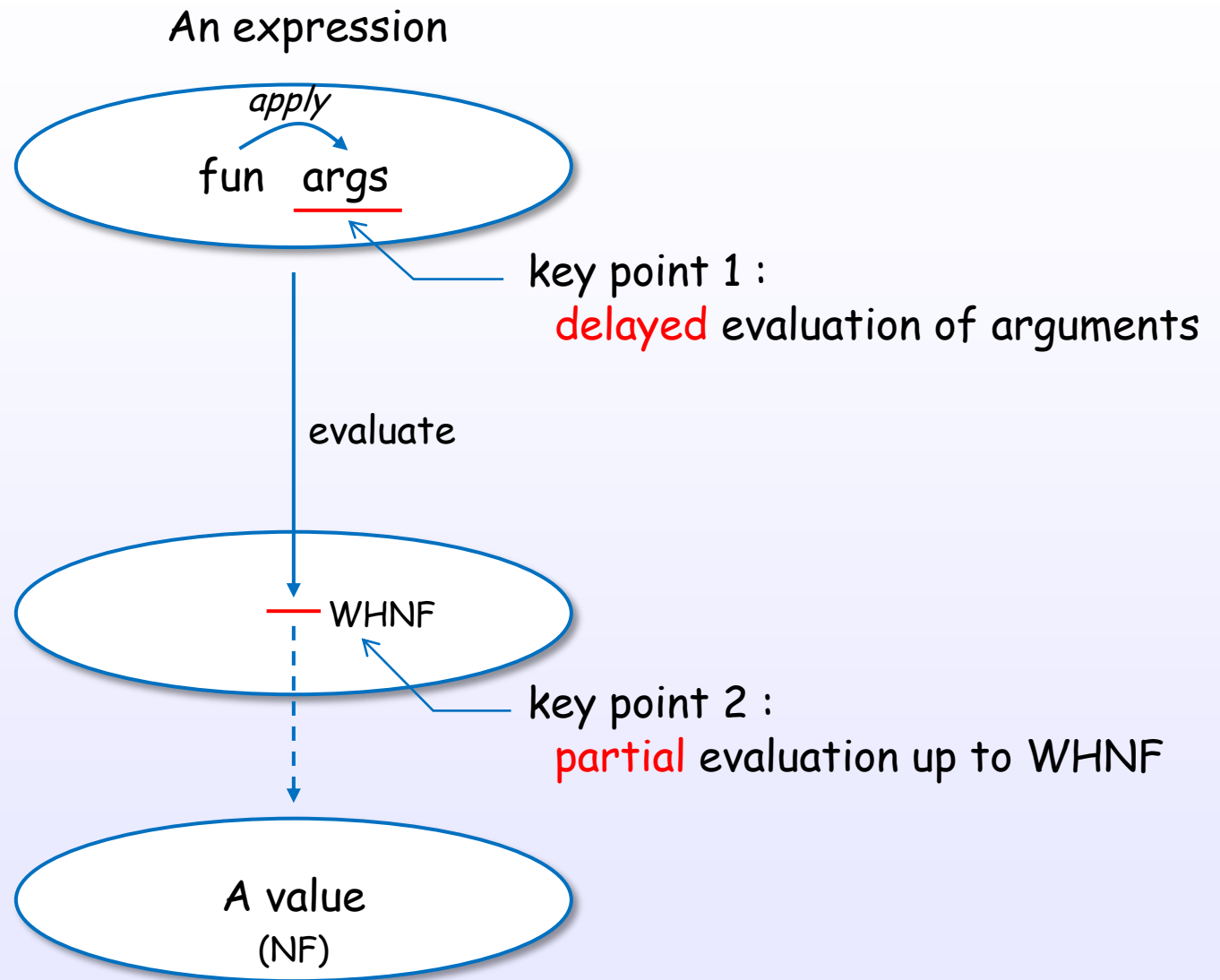
(STG p.11)



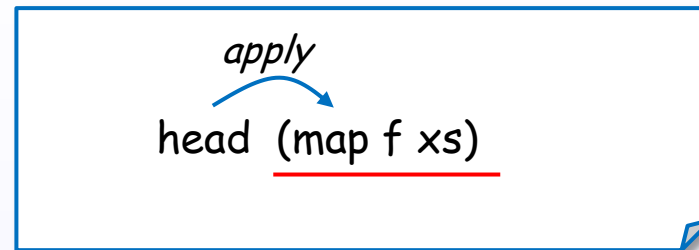
- ・引数評価を先送る (case式が来るまで評価しない) call-by-need
- ・部分式を完全評価しない (caseのパターンマッチで参照するところのみを評価する) WHNF

これは、計算量を最小化する戦略(メモリ量でなく)

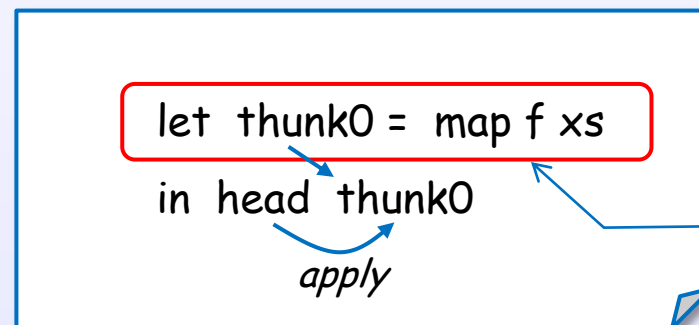
# Key concept of Haskell's lazy evaluation



# key point 1 : delayed evaluation of arguments



↓ internal transformation by *GHC*

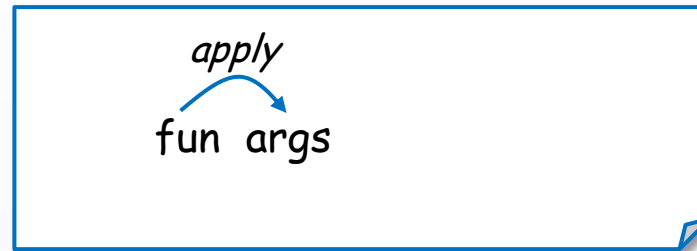


build a thunk  
in heap memory

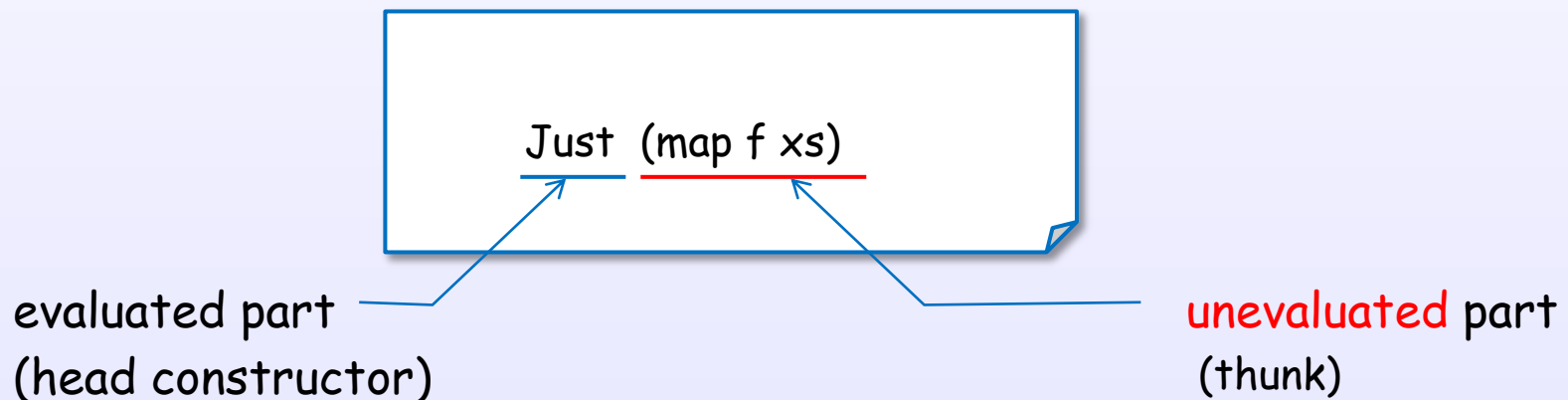
*GHC* implements lazy evaluation using the thunk.  
Evaluation of arguments is delayed with the thunk.



## key point 2 : partial evaluation up to WHNF



↓ evaluation up to WHNF



GHC can partially evaluate an expression.  
Constructor can hold an unevaluated expression (a thunk).

では、必要なときはいつか？

# では、必要なときはいつか？

Haskell code

```
f = case (g x) of  
  [] -> a  
  _  -> b
```

```
g (x:xs) = ...  
g []    = ...
```

pattern match via  
case expression and function definition  
will {cause, trigger} the evaluation

HERE!

# Pattern match

[CIS194]

## 4. Evaluation

Examples of evaluation steps

# Example of repeat

repeat 1



1 : repeat 1



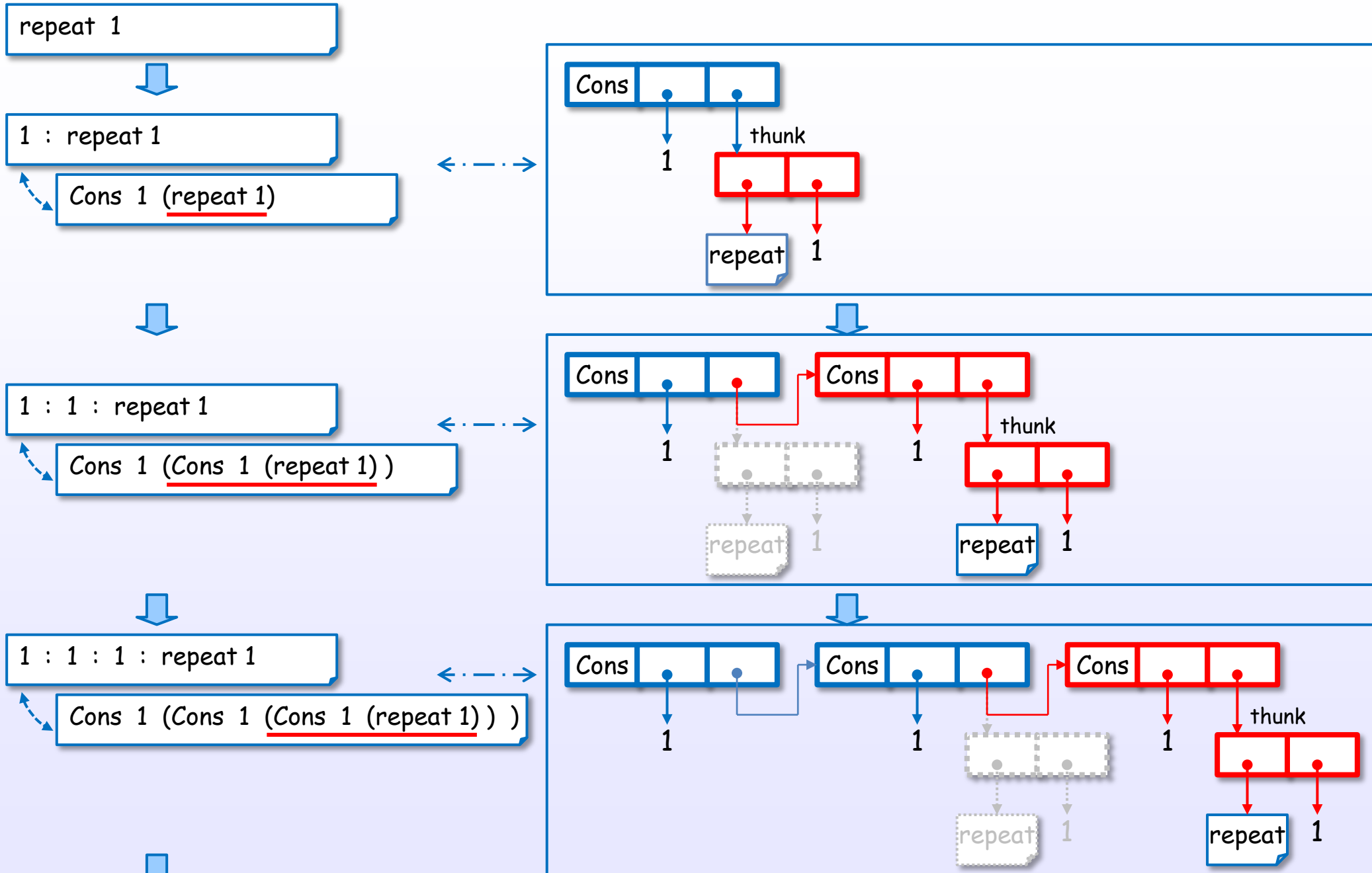
1 : 1 : repeat 1



1 : 1 : 1 : repeat 1



# Example of repeat



# Example of map

```
map f [1, 2, 3]
```



```
f 1 : map f [2, 3]
```



```
f 1 : f 2 : map f [3]
```



```
f 1 : f 2 : f 3
```



...



# Example of map

map f [1, 2, 3]



f 1 : map f [2, 3]



Cons (f 1) (map f [2, 3])



f 1 : f 2 : map f [3]



Cons (f 1) (Cons (f 2) (map f [3]))



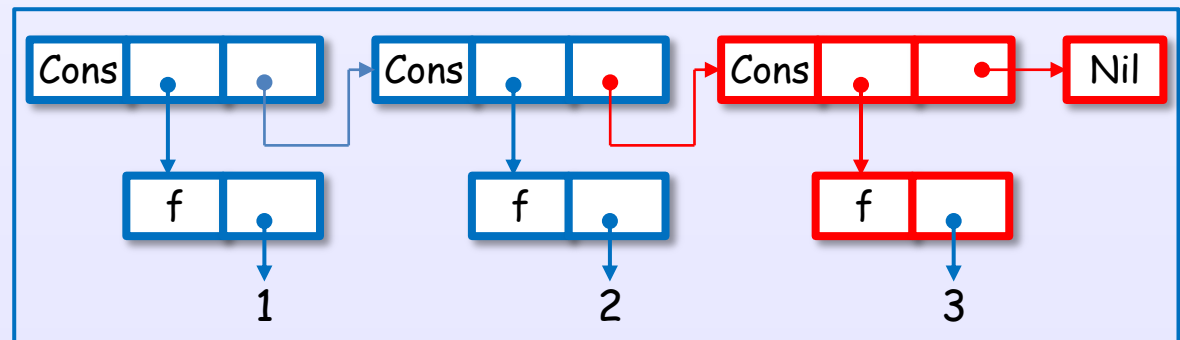
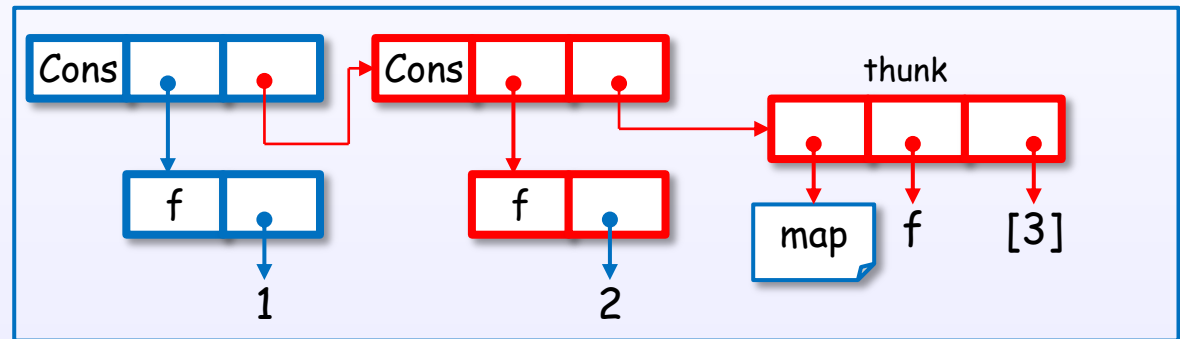
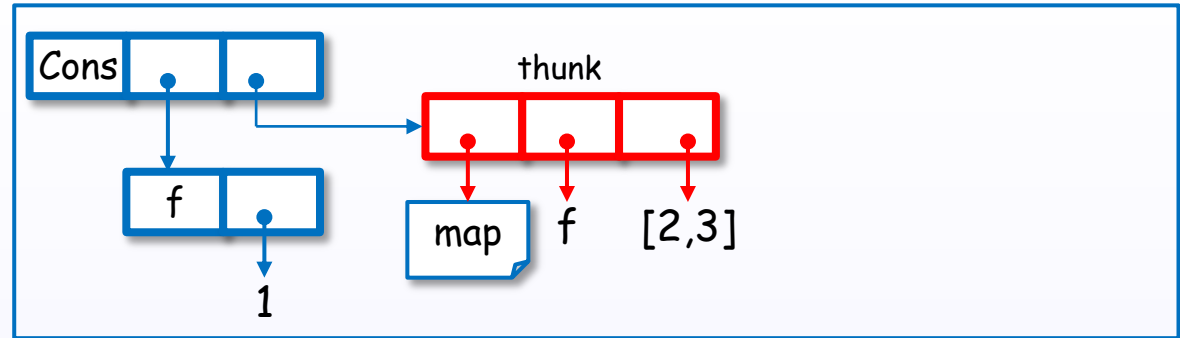
f 1 : f 2 : f 3



Cons (f 1) (Cons (f 2) (Cons (f 3) Nil))



...



...

# Example of foldl (non-strict)

`foldl (+) 0 [1 .. 100]`



`foldl (+) (0 + 1) [2 .. 100]`



`foldl (+) ((0 + 1) + 2) [3 .. 100]`



`foldl (+) ((((0 + 1) + 2) + 3) [4 .. 100]`



...

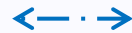
# Example of foldl (non-strict)

```
foldl (+) 0 [1 .. 100]
```

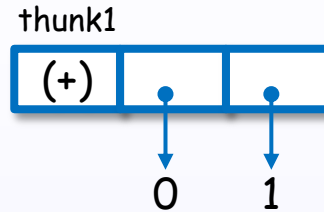


```
foldl (+) (0 + 1) [2 .. 100]
```

```
let thunk1 = (0 + 1)  
in foldl (+) thunk1 [2 .. 100]
```

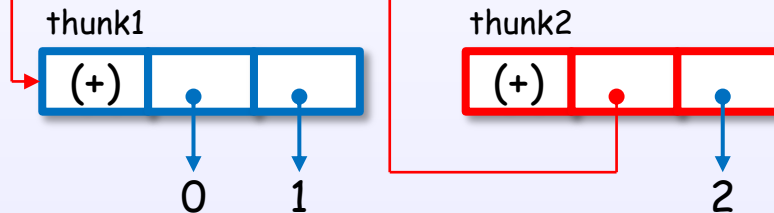


heap memory



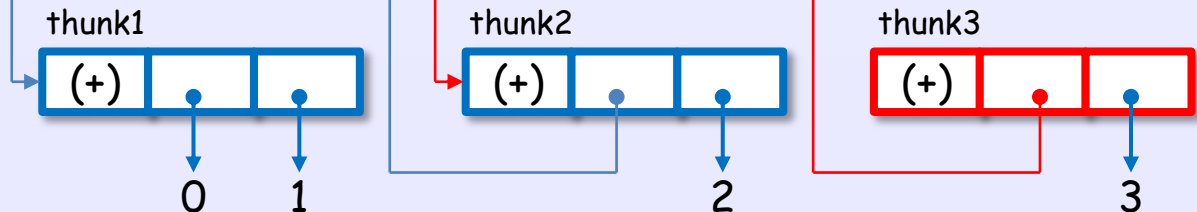
```
foldl (+) ((0 + 1) + 2) [3 .. 100]
```

```
let thunk2 = (thunk1 + 2)  
in foldl (+) thunk2 [3 .. 100]
```



```
foldl (+) (((0 + 1) + 2) + 3) [4 .. 100]
```

```
let thunk3 = (thunk2 + 3)  
in foldl (+) thunk3 [4 .. 100]
```



increasing heap ...



...

# Example of foldl' (strict)

foldl' (+) 0 [1 .. 100]



foldl' (+) (0 + 1) [2 .. 100]



foldl' (+) (1 + 2) [3 .. 100]



foldl' (+) (3 + 3) [4 .. 100]



...

# Example of foldl' (strict)

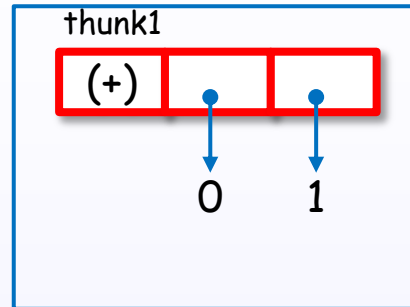
foldl' (+) 0 [1 .. 100]



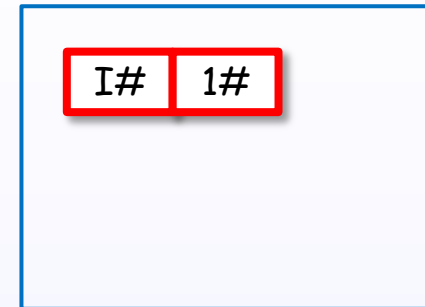
foldl' (+) (0 + 1) [2 .. 100]

let thunk1 = (0 + 1)  
in thunk1 `pseq`  
foldl' (+) thunk1 [2 .. 100]

heap memory

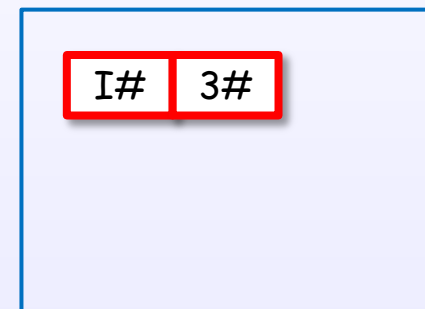
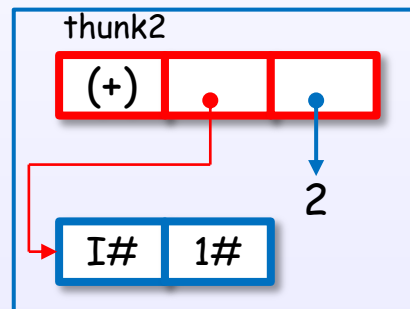


update  
by pseq



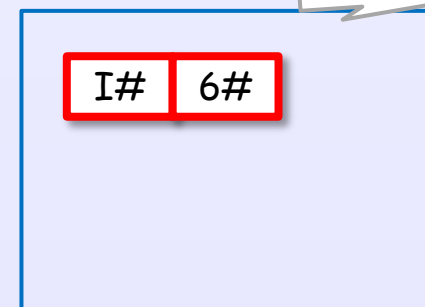
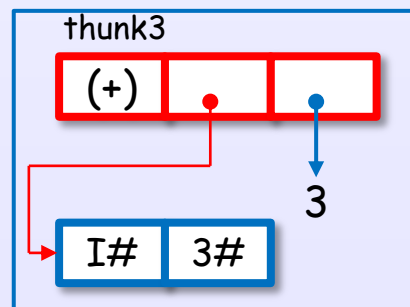
foldl' (+) (1 + 2) [3 .. 100]

let thunk2 = (1 + 2)  
in thunk2 `pseq`  
foldl' (+) thunk2 [3 .. 100]



foldl' (+) (3 + 3) [4 .. 100]

let thunk3 = (3 + 3)  
in thunk3 `pseq`  
foldl' (+) thunk3 [4 .. 100]



fixed heap size



...

References : [1]

# Example of foldl (non-strict) and foldl' (strict)

foldl (+) (0 + 1) [2 .. 100]



foldl (+) ((0 + 1) + 2) [3 .. 100]



foldl (+) (((0 + 1) + 2) + 3) [4 .. 100]



foldl' (+) (0 + 1) [2 .. 100]



foldl' (+) (1 + 2) [3 .. 100]



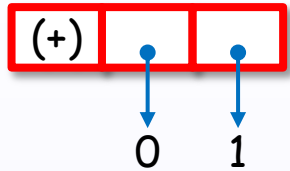
foldl' (+) (3 + 3) [4 .. 100]



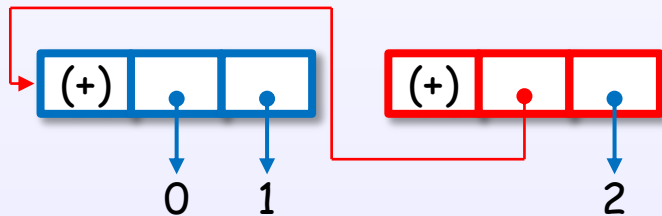
# Example of foldl (non-strict) and foldl' (strict)

foldl (+) (0 + 1) [2 .. 100]

heap memory

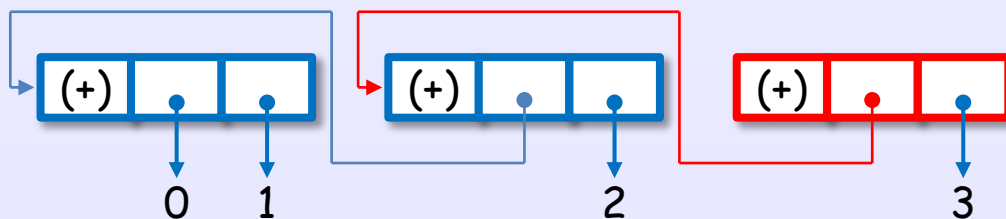


foldl (+) ((0 + 1) + 2) [3 .. 100]

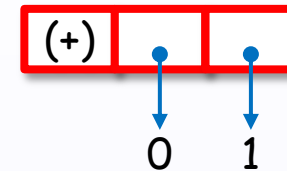


foldl (+) (((0 + 1) + 2) + 3) [4 .. 100]

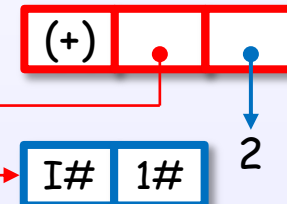
increasing heap ...



foldl' (+) (0 + 1) [2 .. 100]

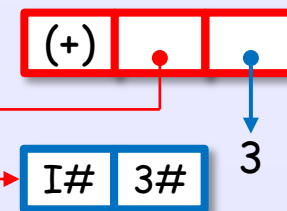


foldl' (+) (1 + 2) [3 .. 100]



foldl' (+) (3 + 3) [4 .. 100]

fixed heap size



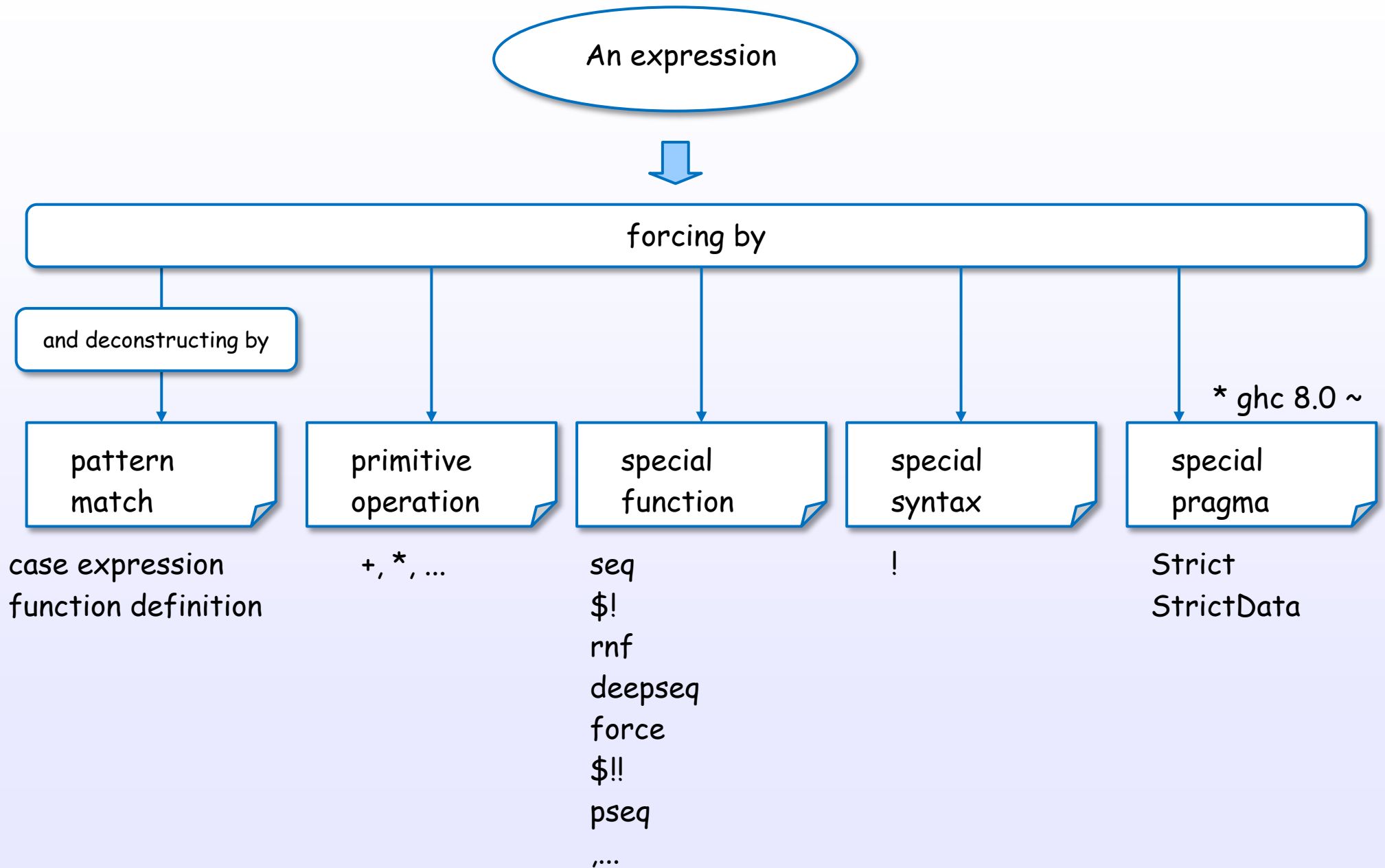
References : [1]

## 4. Evaluation

Control the evaluation in Haskell



# How to drive evaluation



# Example of the evaluation by pattern match

case expression

```
case ds of
  x:xs -> f x xs
  []   -> False
```

case expression  
function definition

# Example of the evaluation by primitive operation

primitive operation

$$f \ x \ y = x \ \underline{+} \ y$$

$+, *, \dots$

# Example of the evaluation by special function

special function

$f \times y = \text{seq} \times y$

seq  
\$!  
rnf  
deepseq  
force  
\$!!  
pseq  
...

Please refer the document more detail. [xx]  
hoogle or hackage

# Example of the evaluation by special syntax

special syntax

```
{-# LANGUAGE BangPatterns #-}
```

```
f !xs = g xs
```

BangPattern

!

Please refer the document more detail. [xx]

[user guide, 7.19]

# Example of the evaluation by special pragma

special pragma

```
{-# LANGUAGE Strict #-}
```

```
f xs = g xs
```

\* ghc 8.0 ~

```
{-# LANGUAGE StrictData #-}
```

```
f xs = g xs
```

Strict  
StrictData

Please refer the document more detail. [xx]

[wiki]

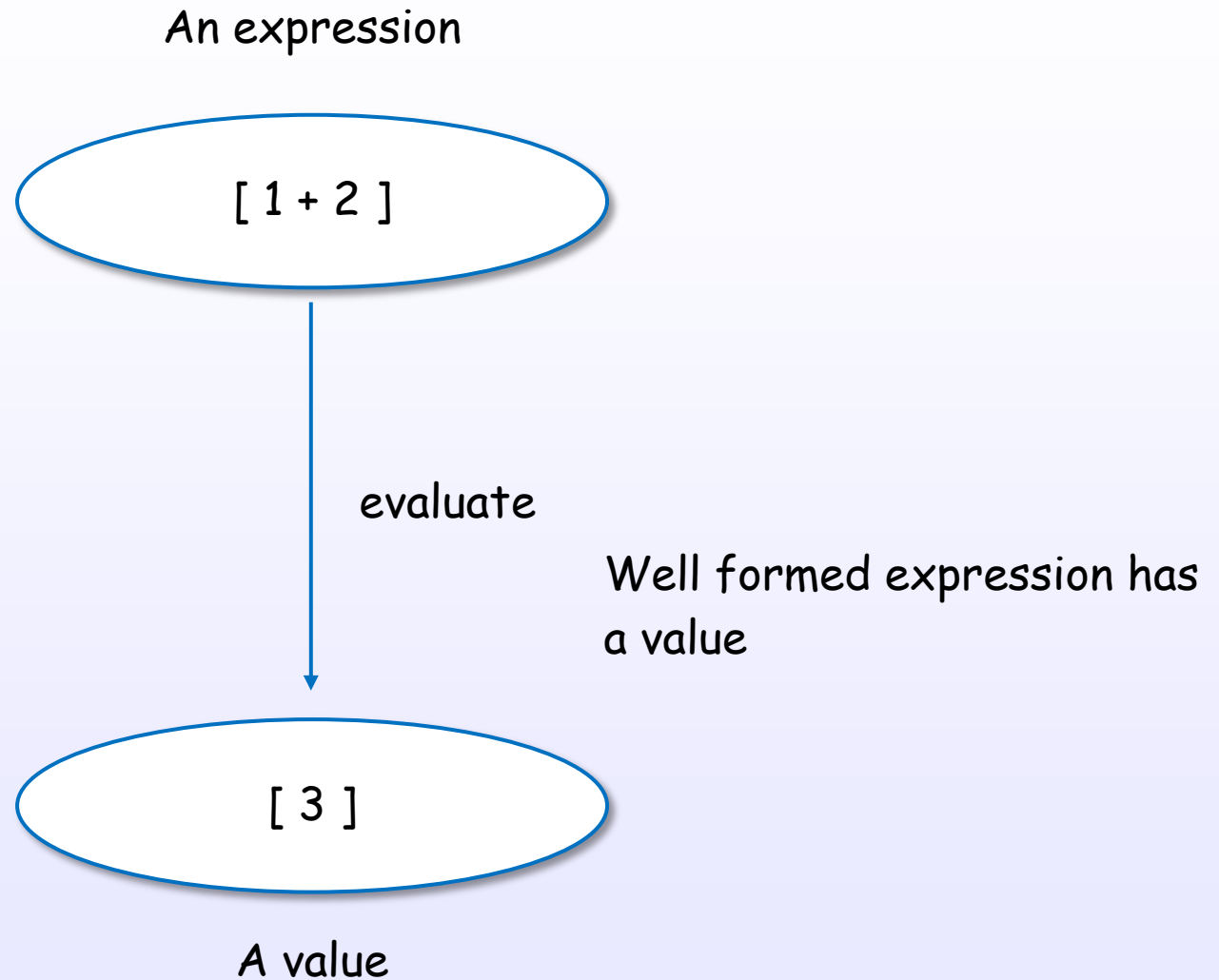
## 5. Semantics

## 5. Semantics

Bottom



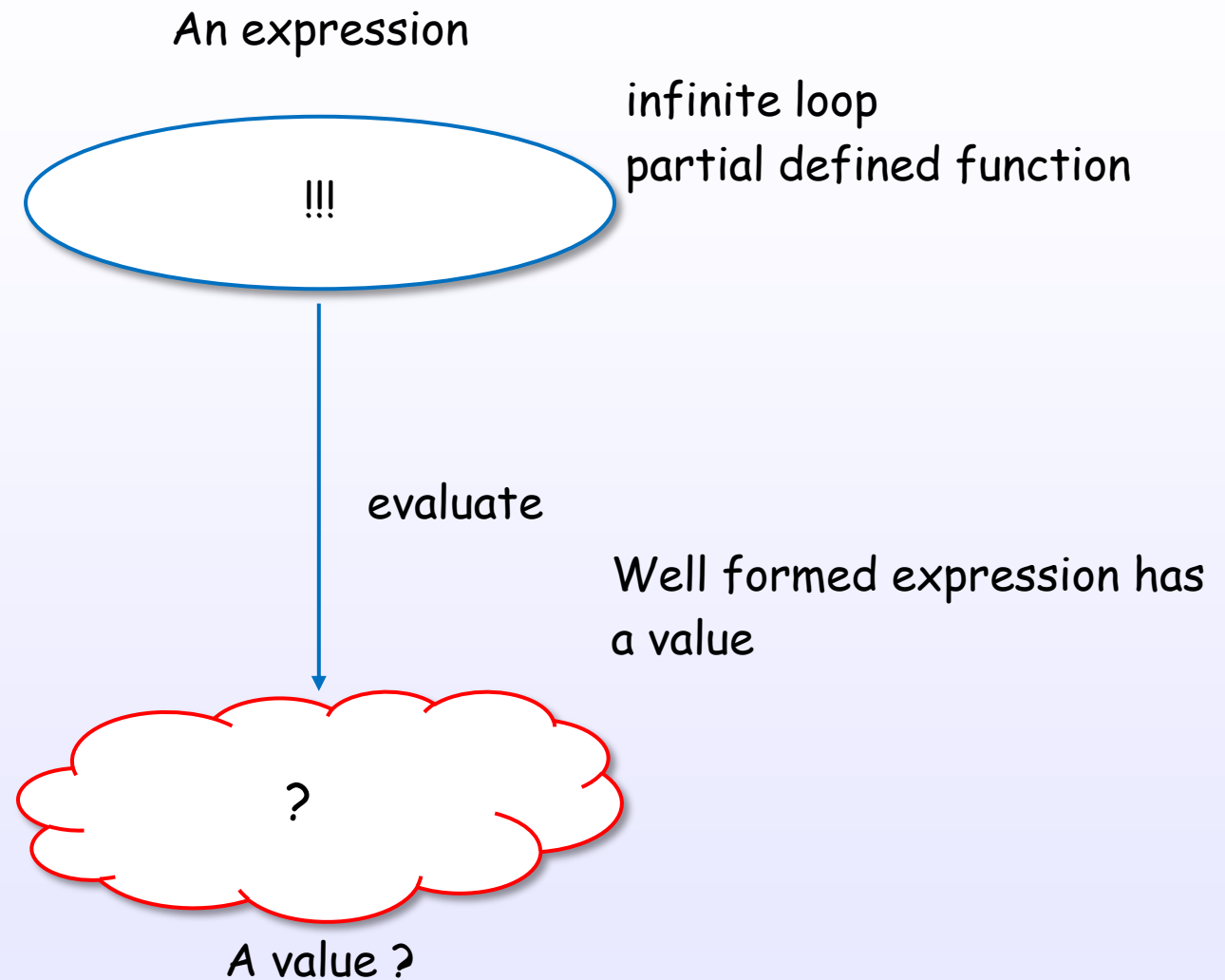
# Well formed expression has a value



[Bird, Chapter 2]

References : [1]

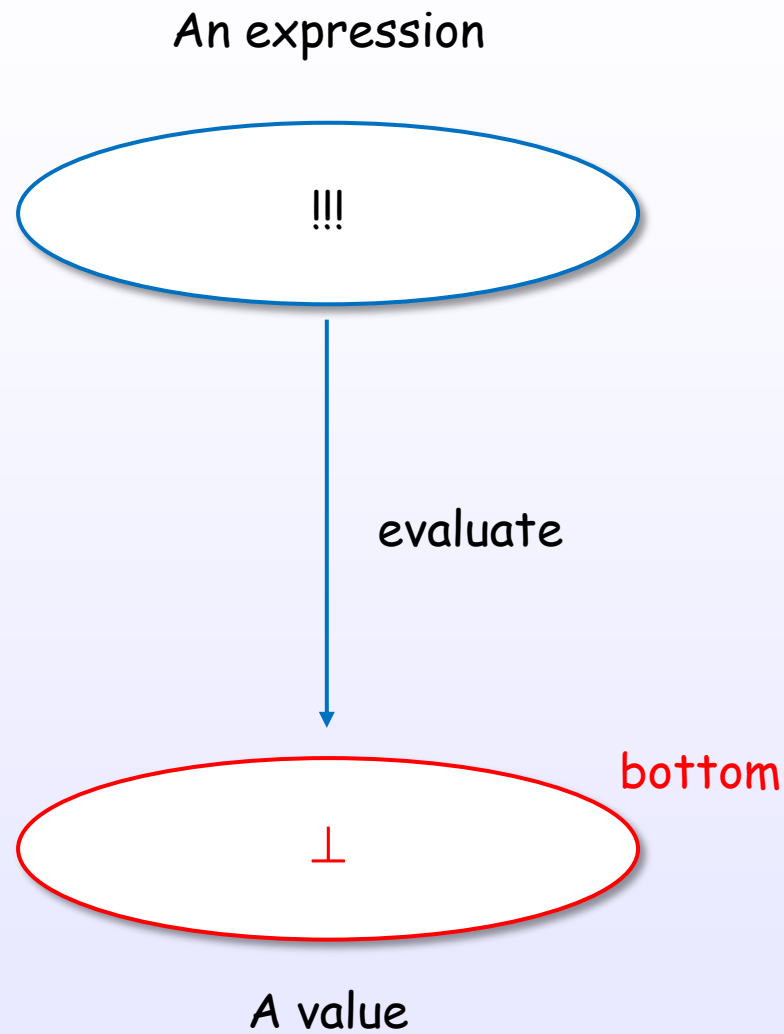
# Well formed expression has a value



[Bird, Chapter 2]

References : [1]

# Well formed expression has a value



[Bird, Chapter 2]

References : [1]

# Bottom

[Bird, Chapter 2]

References : [1]

## 5. Semantics

### Non-strict Semantics

# Strictness

$$f \perp = \perp$$

[Bird, Chapter 2]

# Layer

Non-strictness

$$f \perp = \perp$$

Lazy evaluation

GHC chosen lazy evaluation to implement non-strict semantics.

Graph reduction

GHC chosen graph reduction to implement lazy evaluation.

STG-machine

GHC implements graph reduction by STG-machine.

## 6. Implementation

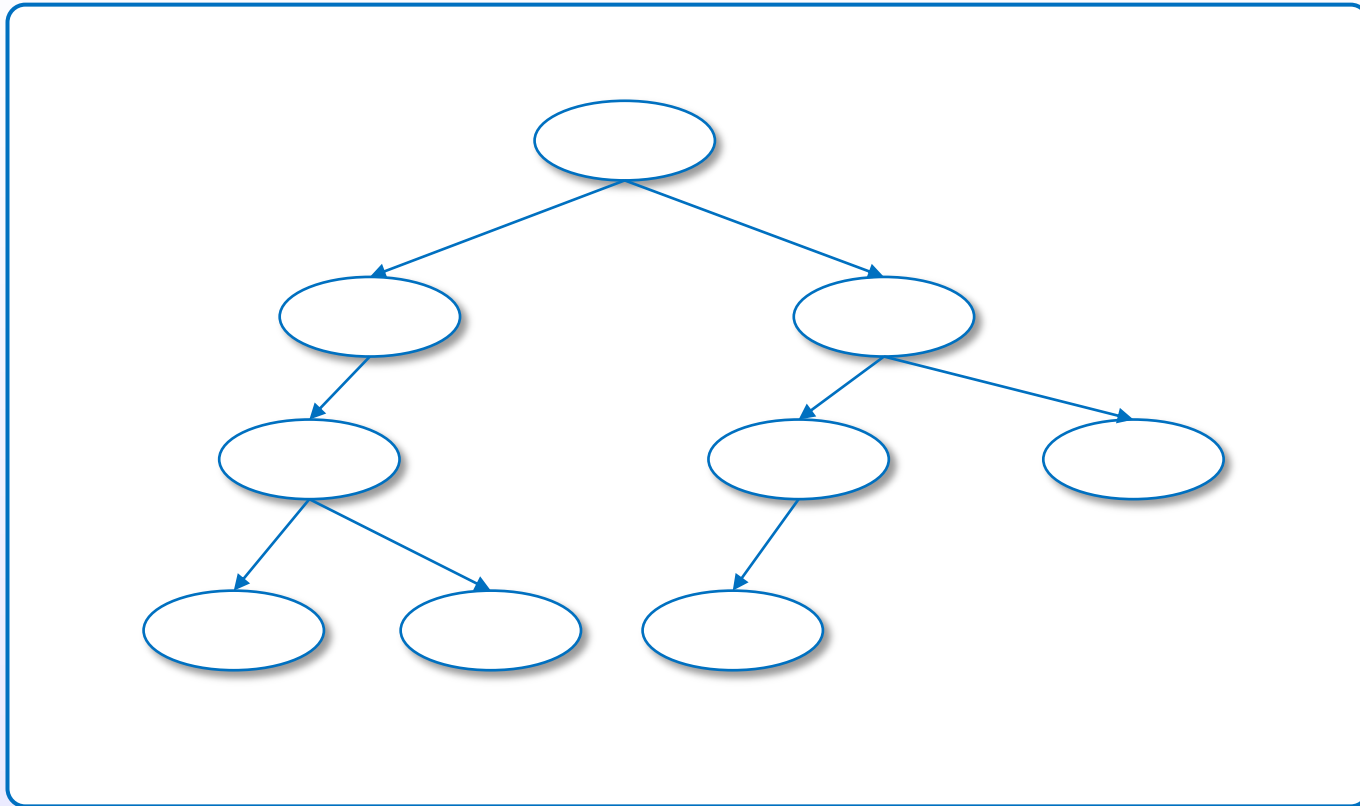


## 6. Implementation

Graph reduction

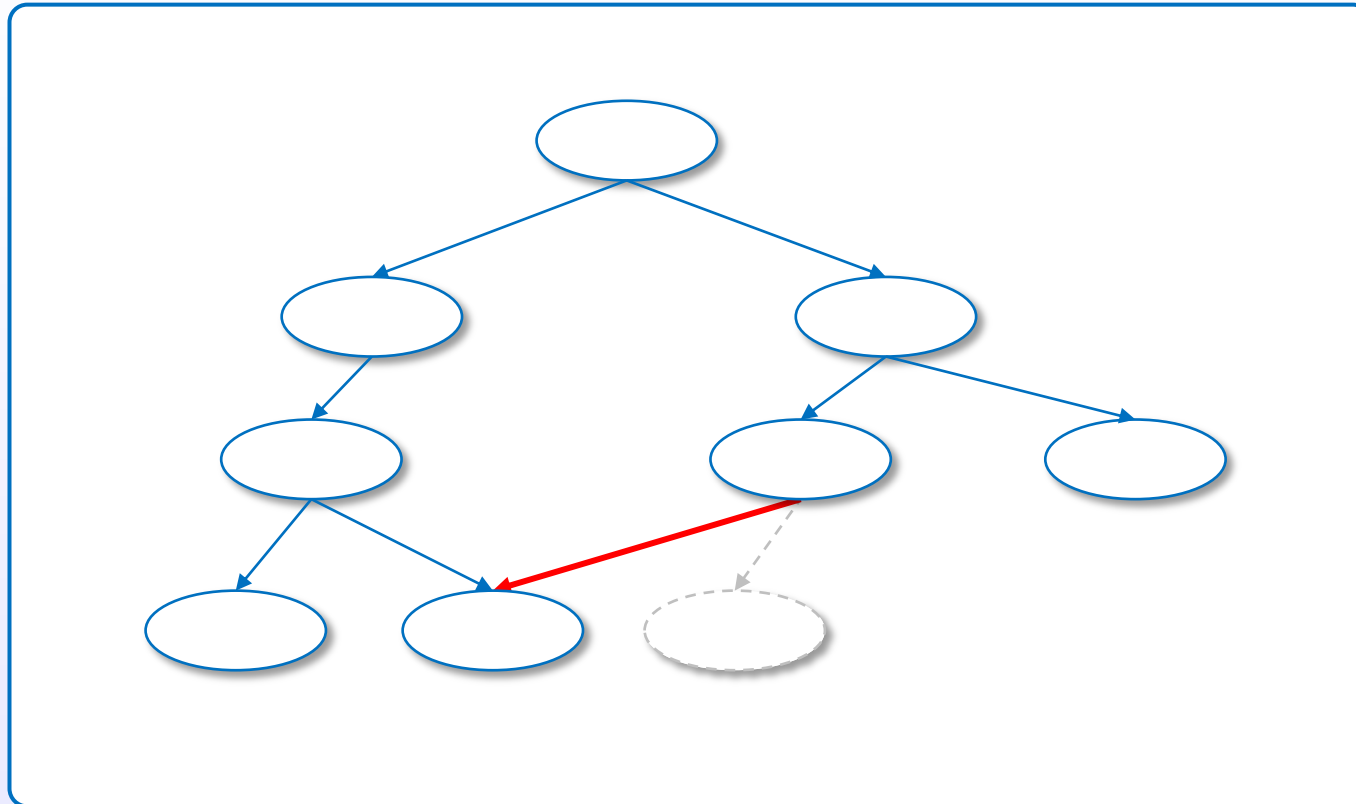
# Tree

AST represents an expression

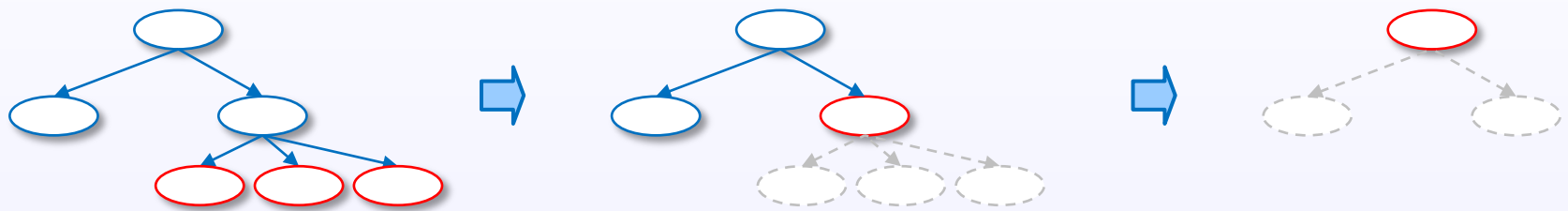


# Graph

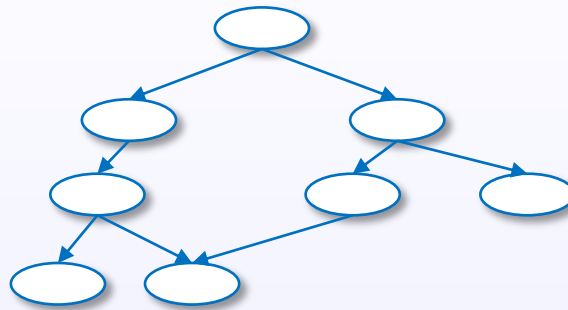
Share the term, looped  
not Tree, but Graph



# Graph reduction



# Graph reduction and lazy



## 5. Implementation

STG-machine

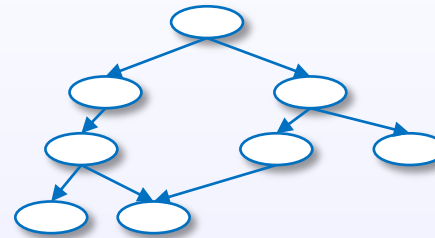
# Layer

Haskell code

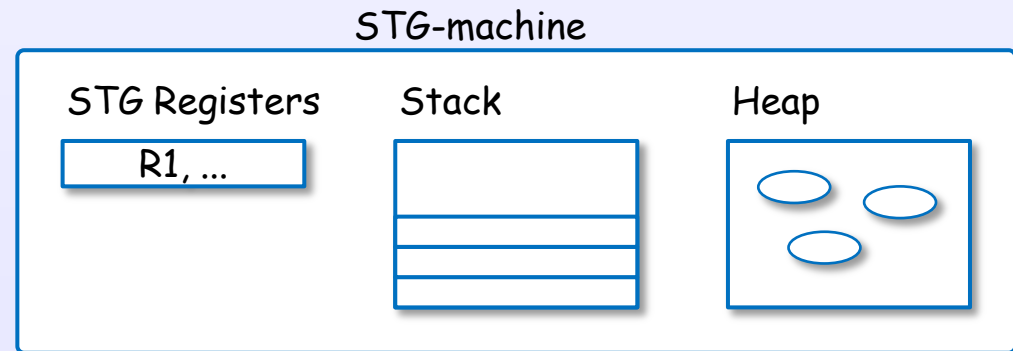
take 5 [1..10]

---

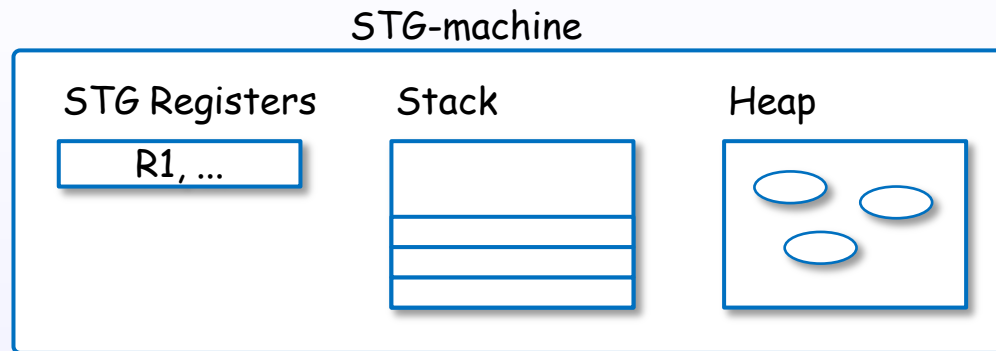
Internal representation  
by graph



Evaluation (execution, reduction)  
by STG-machine



# STG-machine



STG-machine is abstraction machine  
which is defined by operational semantics.

STG-machine efficiently performs lazy graph reduction.



## 7. Appendix

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Lazy,... <sup>!!!</sup>