Lazy evaluation illustrated

for Haskellers

exploring some mental models and implementations

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Rev. 0.00.0
WIP

Lazy,... zzz

..., It's fun!

NOTE

- Meaning of terms are different by communities.
- There are a lot of good documents. Please see also references.
- This is written for GHC's Haskell.

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1. Introduction

1. Introduction

Basic mental models

How to evaluate program in your brain ?

program code

```
code
code
code
  :
```

プログラムは、どの順で評価される？

どういうステップ、どういう順で evaluation (execution, reduction) される？

What are these mental models?

What "mental model" do you have?

References : [1]

One of the mental models for C program

文の並び

```
main (...) {
  code..
  code..
  code..
  code..
}
```

?

入れ子の構造

```
x = func1( func2( a ) );
```

?

引数の並び

```
y = func1( a(x),  b(x),  c(x) );
```

?

関数と引数

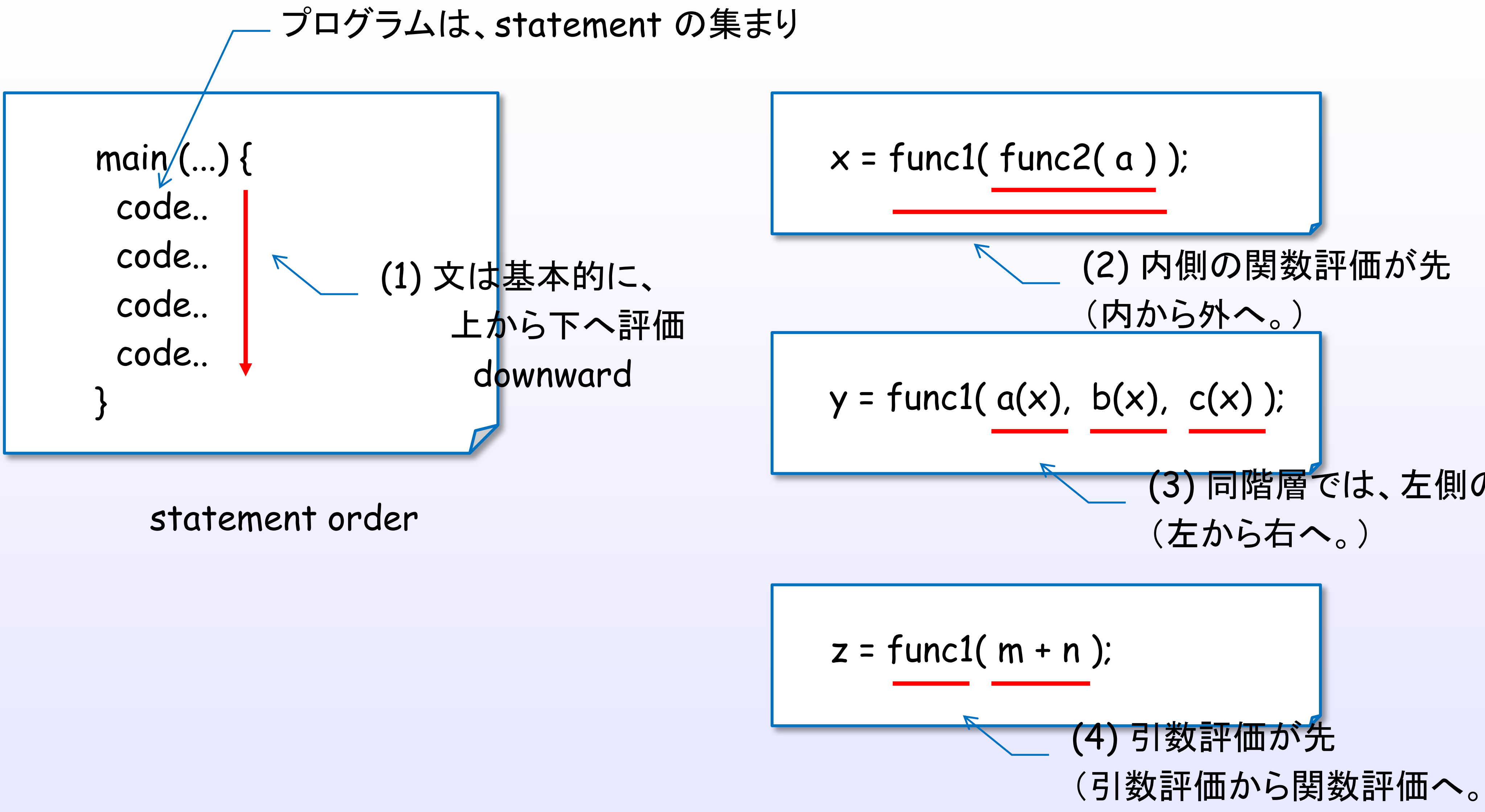
```
z = func1( m + n );
```

?

どのように評価される？
あなたの頭の中の、評価メンタルモデルは？

References : [1]

One of the mental models for C program



プログラムは、statement の集まり

```
main (...) {
  code..
  code..
  code..
  code..
}
```

(1) 文は基本的に、
上から下へ評価
downward

statement order

```
x = func1( func2( a ) );
```

(2) 内側の関数評価が先
（内から外へ。）

```
y = func1( a(x),  b(x),  c(x) );
```

(3) 同階層では、左側の
（左から右へ。）

```
z = func1( m + n );
```

(4) 引数評価が先
（引数評価から関数評価へ。

Each programmers have some mental models in their brain.

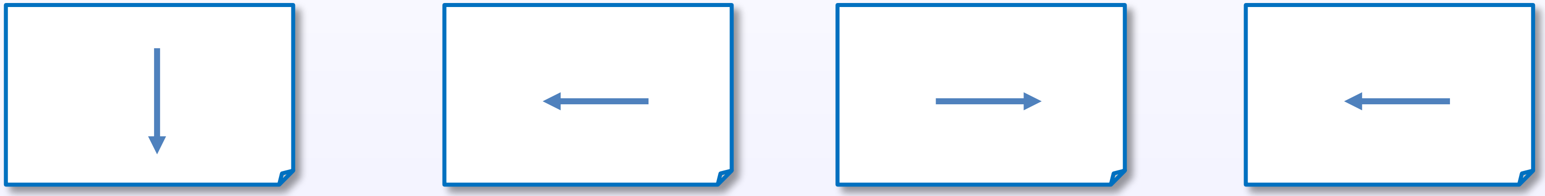
References : [1]

One of the mental models for C program

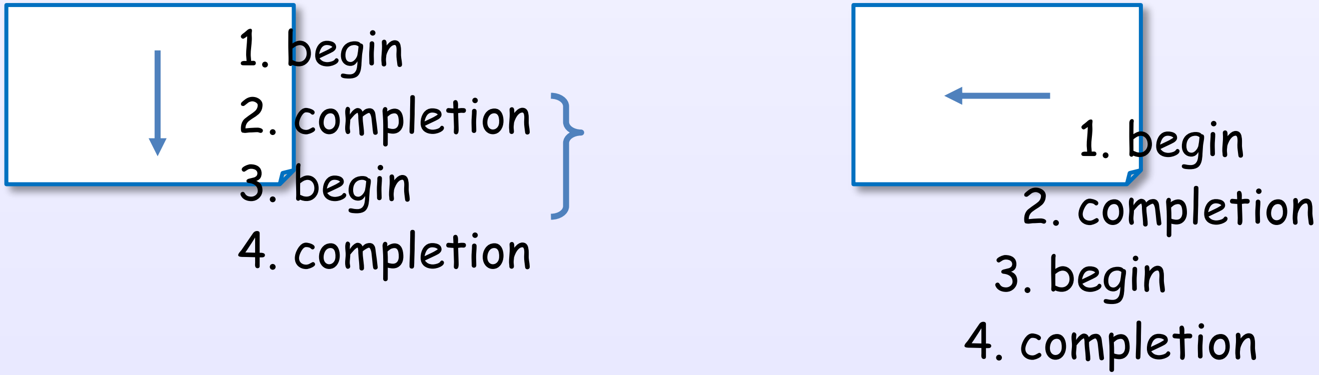
Maybe, You have some implicit mental model in your brain for C program.

(1) program is collection of statements

(2) an order between evaluations of elements



(3) an order between completion and begin of evaluations



1. begin
2. completion
3. begin
4. completion

1. begin
2. completion
3. begin
4. completion

This is an example of an implicit sequential order model for programming languages.

References : [1]

One of the mental models for Haskell program

$\text{main} = \text{exp}_{11}\ (\text{exp}_{12}\ \text{exp}_{13}\ \text{exp}_{14})$

$\text{exp}_{13} = \text{exp}_{131}\ \text{exp}_{132}$

$\text{exp}_{14} = \text{exp}_{141}\ \text{exp}_{142}\ \text{exp}_{143}$

:

どのように評価される？
あなたの頭の中の、評価メンタルモデルは？

References : [1]

One of the mental models for Haskell program

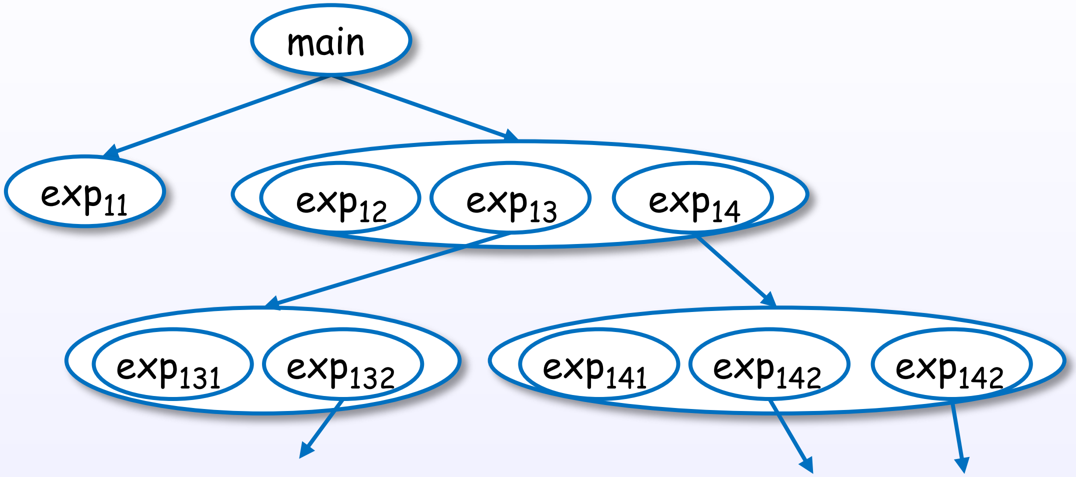
プログラムは、式の集まり

$\text{main} = \text{exp}_{11}\ (\text{exp}_{12}\ \text{exp}_{13}\ \text{exp}_{14})$

$\text{exp}_{13} = \text{exp}_{131}\ \text{exp}_{132}$

$\text{exp}_{14} = \text{exp}_{141}\ \text{exp}_{142}\ \text{exp}_{143}$

:



$\text{main} = \text{exp}_{11}\ (\text{exp}_{12}\ (\text{exp}_{131}\ \text{exp}_{132})\ (\text{exp}_{141}\ \text{exp}_{142}\ \text{exp}_{143}))$

(1) プログラム全体を1つの式と見立てて

(2) 部分式をある順で評価（簡約）していく

(3) 評価は置換により行う

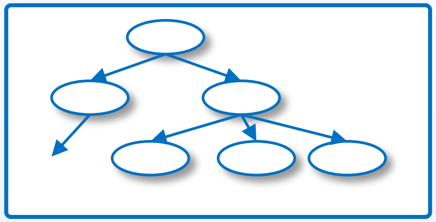
References : [1]

One of the mental models for Haskell program

(1) program is collection of expression's declaration

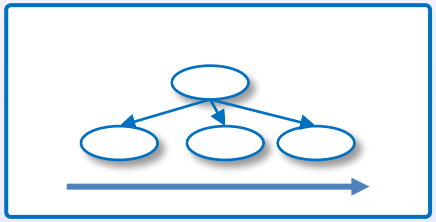
(2) プログラム全体が階層をもった1つの式

```
main = e (e (e (e e) e (e e e) ) )
```

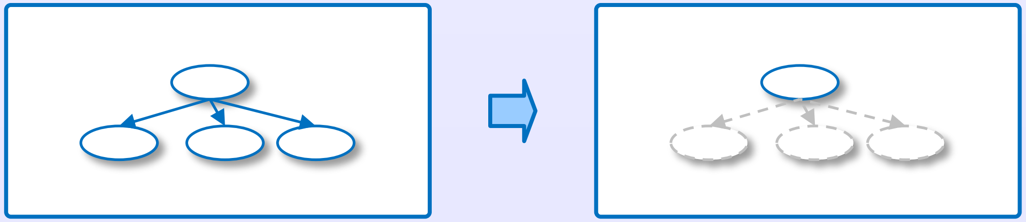


(3) 部分式を、ある順序で評価していく

```
f = e (e (e (e e) e (e e e) ) )
```



(4) 評価は置換により行われる



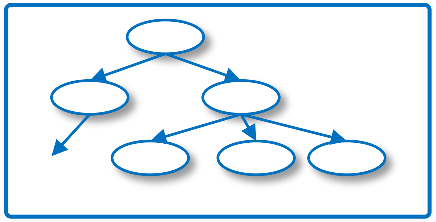
References : [1]

1. Introduction

Lazy evaluation

では、具体的にはどうやって評価される？

f = e (e (e (e e) e (e e e)))



Haskellは purely functional language

↓ no side effect [slpj-book-1987], p.193

order free (so, potentially hi-level optimization and parallelism

↓ call-by-need 可能

GHC chosen lazy evaluation to implement non-strict semantics.

[slpj-book-1987], p.33

References : [1]

GHC chosen lazy evaluation

必要な時に、必要な箇所のみを評価する

(STG p.11)

↓

・引数評価を先送る （case式が来るまで評価しない） call-by-need

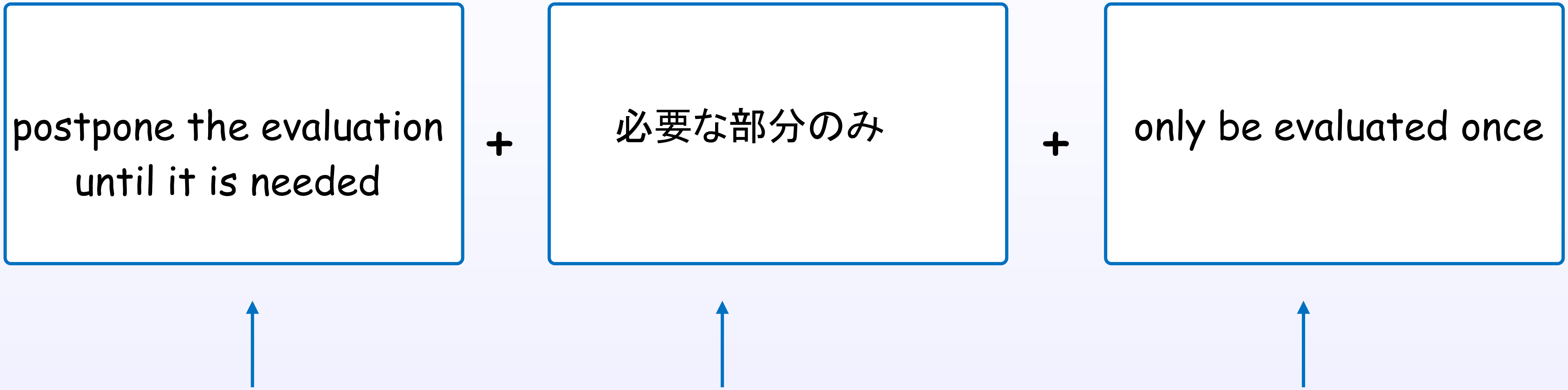
・部分式を完全評価しない （caseのパターンマッチで参照するところのみを評価する）WHNF

これは、計算量を最小化する戦略（メモリ量でなく）

References : [1]

Haskell(GHC) 's lazy evaluation

ingredient of Haskell's “lazy evaluation”



postpone the evaluation until it is needed

+

必要な部分のみ

+

only be evaluated once

call-by-need
normal order reduction
(= leftmost + outermost
reduction)

stop at WHNF

call-by-need (sharing)
self updating model

[slpj-book-1987], 194

[slpj-book-1987], p.198, 23, 194

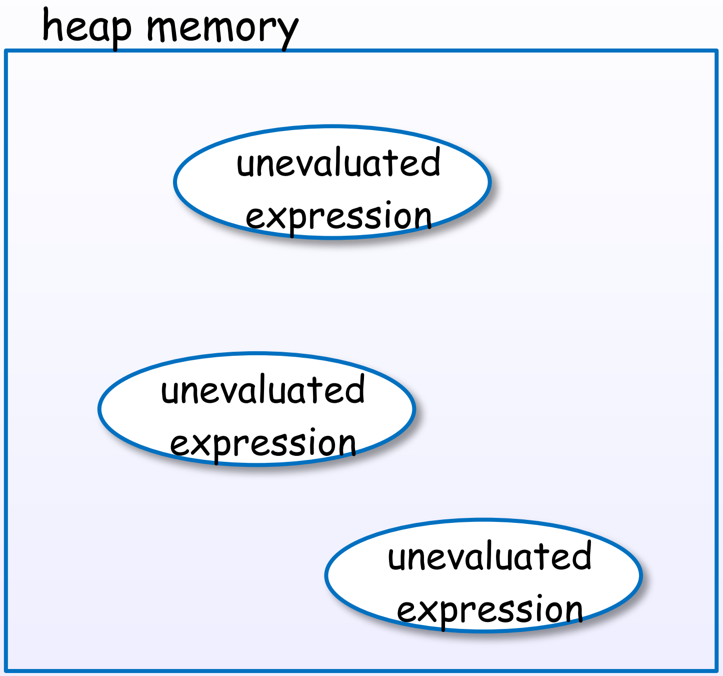
outermost と、call-by-need

call-by-needは、狭義のlazy eval

References : [1]

では、必要な時までどこに置いておく？

postpone →



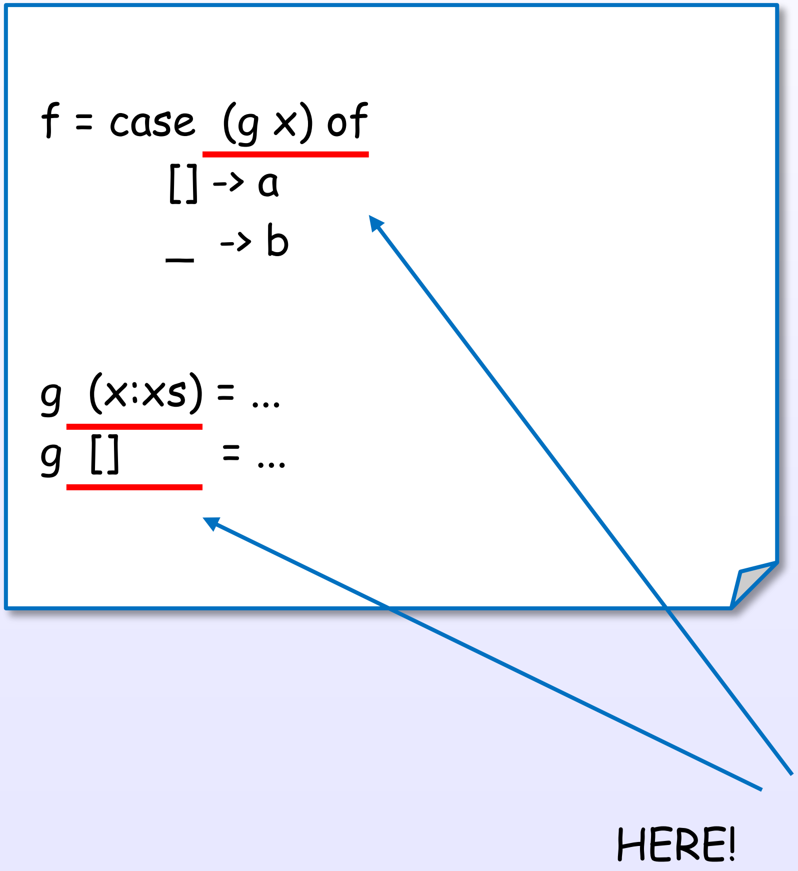
stackでなく、heap。
なので、sequential アクセスでなくて良い。

heapに置いておく

References : [1]

では、必要になるのは、いつ？

case式か、関数定義のパターンマッチで、取りだされるときが、必要なとき



```
f = case  (g x) of
        [] -> a
        _  -> b

g  (x:xs) = ...
g  []       = ...
```

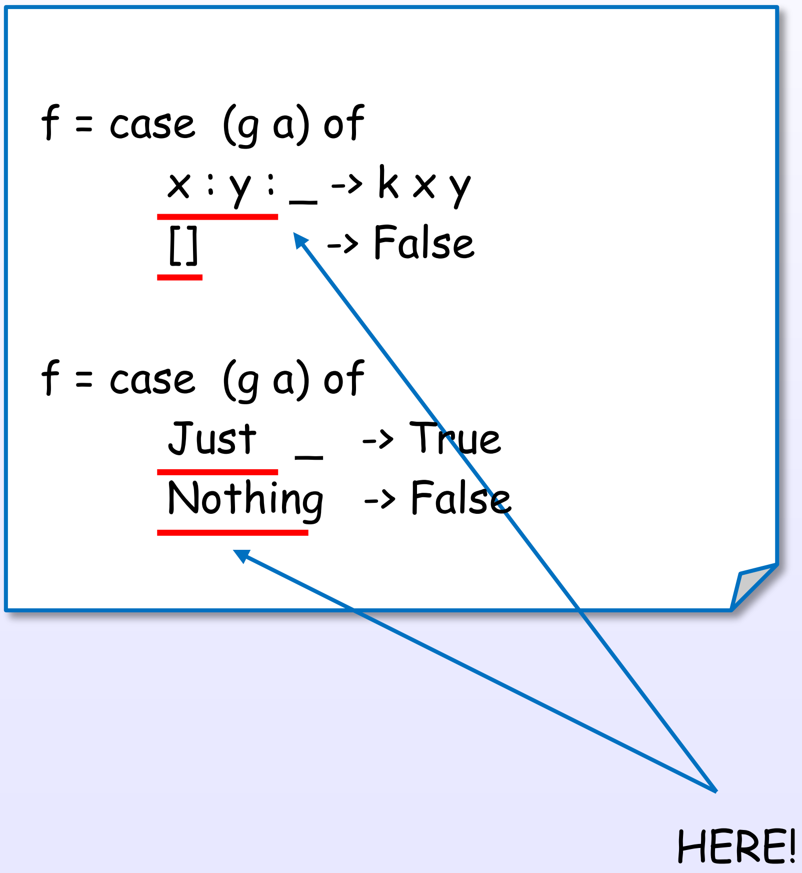
HERE!

pattern match via
case expression and function definition
will {cause, trigger} the evaluation

References : [1]

では、必要な部分とはどこ？

パターンマッチで明示された部分



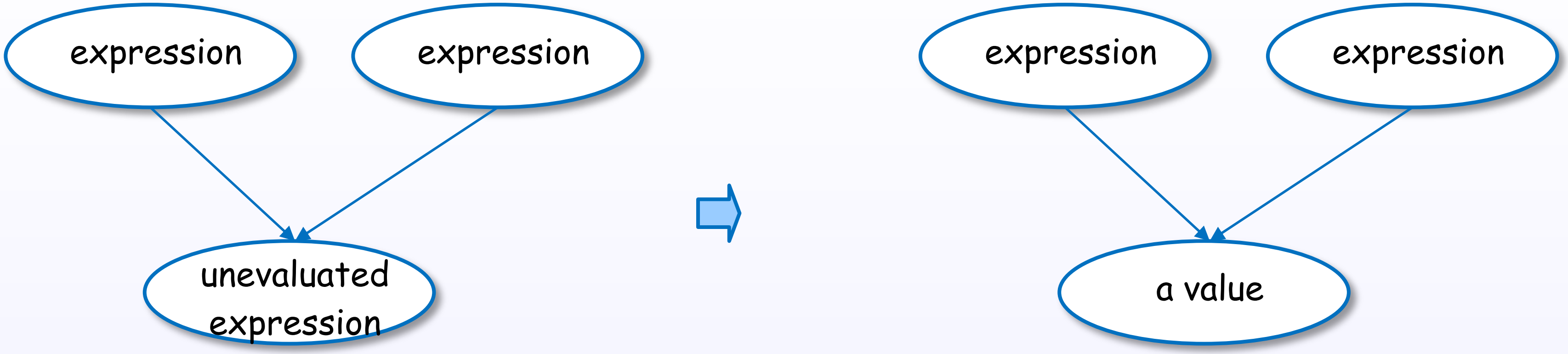
```
f = case  (g a) of
         x : y : _ -> k x y
         []          -> False

f = case  (g a) of
         Just   _    -> True
         Nothing   -> False
```

HERE!

there are components which you need.

References : [1]

どうやって、一度だけ評価する？



self updating

shared term

repeat call

References : [1]

Why lazy evaluation?

(1) normal order reduction guarantees to find a normal form (if one exists)

[slpj-book-1987], p.25

pursue normal order reduction, but stop at WHNF.
This is an essential ingredient of lazy evaluation

(2) lazy evaluation implements non-strict semantics

infinite data structure and stream

[slpj-book-1987], p.194

(3) 不要な評価を避ける

References : [1]

Lazy evaluationの注意点 1

実行タイミングがずれる

code と 実行が同期していない

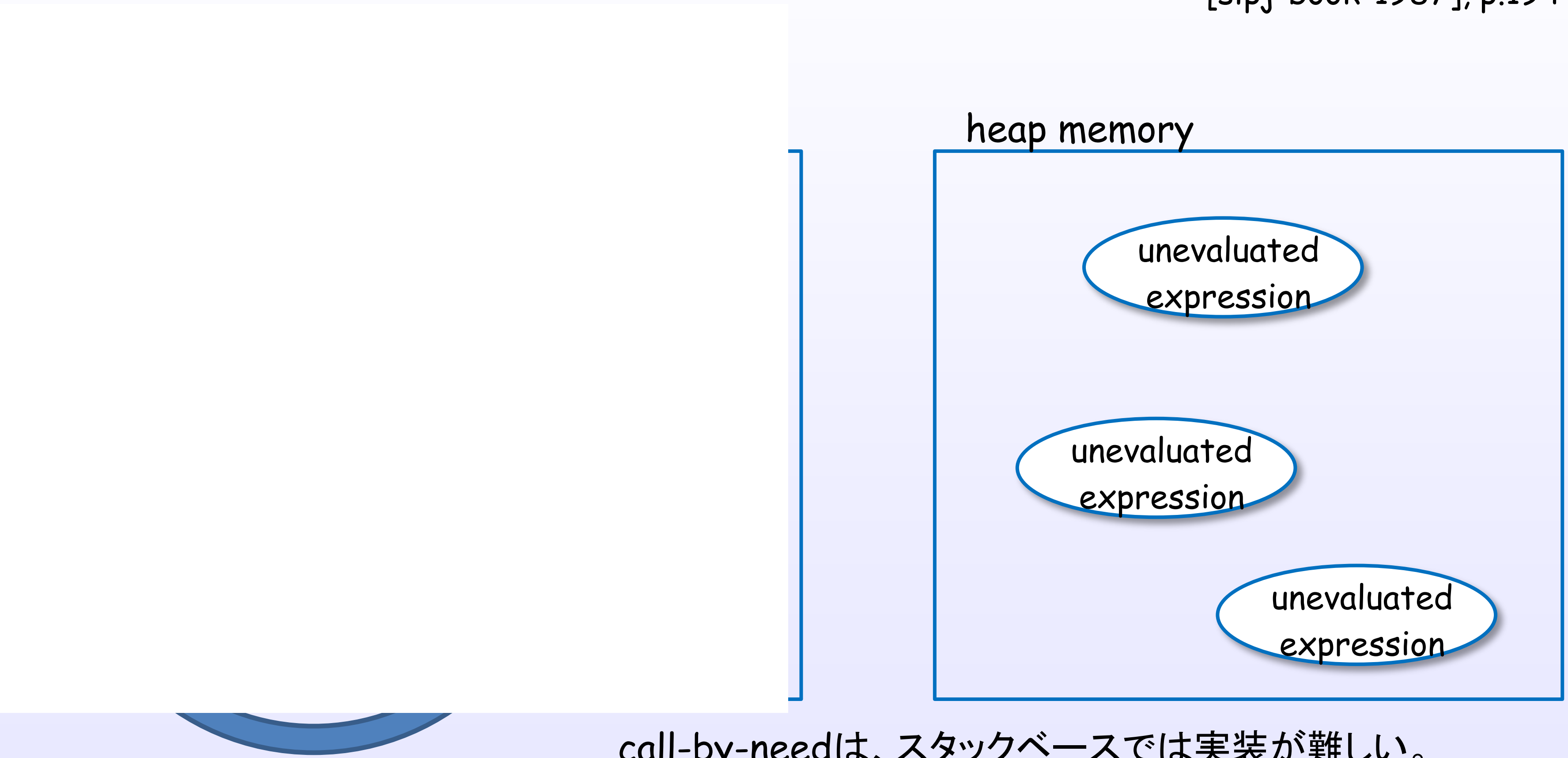
References : [1]

Lazy evaluationの注意点 2

ヒープの使用

ヒープにたまっていく

[slpj-book-1987], p.194



call-by-needは、スタックベースでは実装が難しい。

コントロールが必要

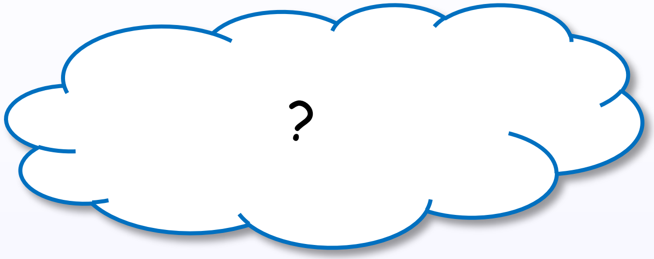
References : [1]

1. Introduction

Expression and value

What is an expression?

An expression



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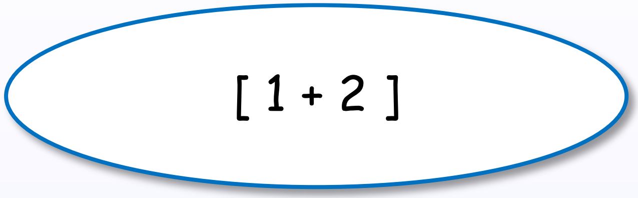
References : [1]

An expression denotes a value

An expression



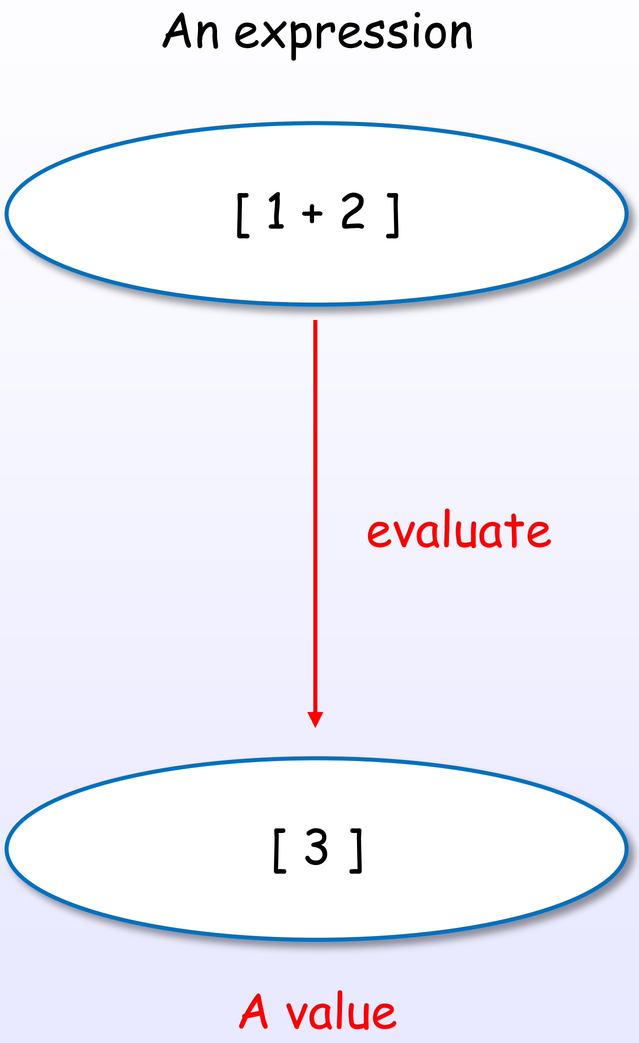
[1 + 2]

[HR2010]

[Bird, Chapter 2]

References : [1]

An expression evaluates to a value



An expression

[1 + 2]

evaluate

[3]

A value

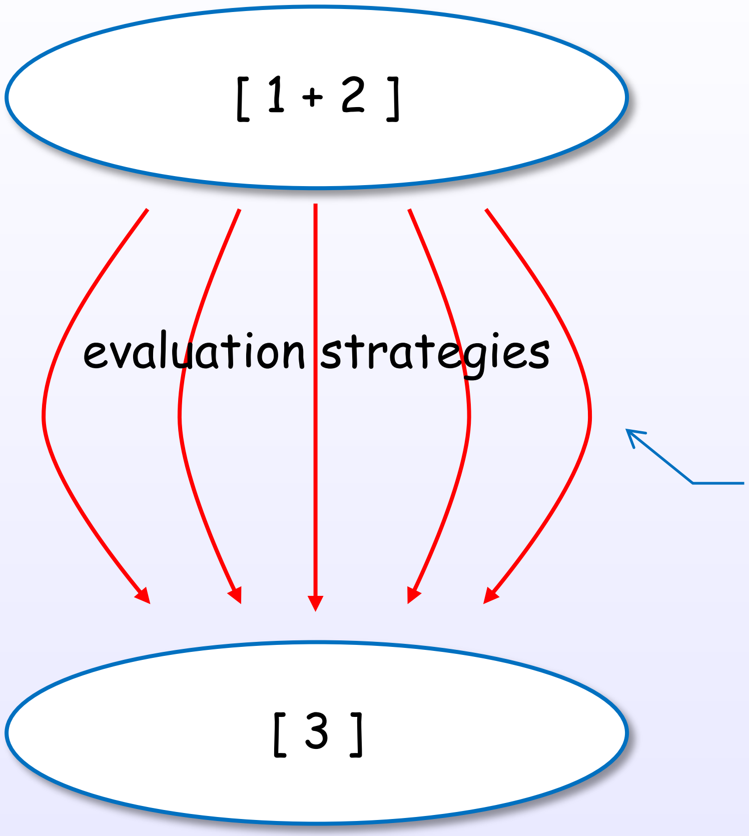
[HR2010]

[Bird, Chapter 2]

References : [1]

There are many evaluation approaches

An expression



[1 + 2]

evaluation strategies

[3]

A value

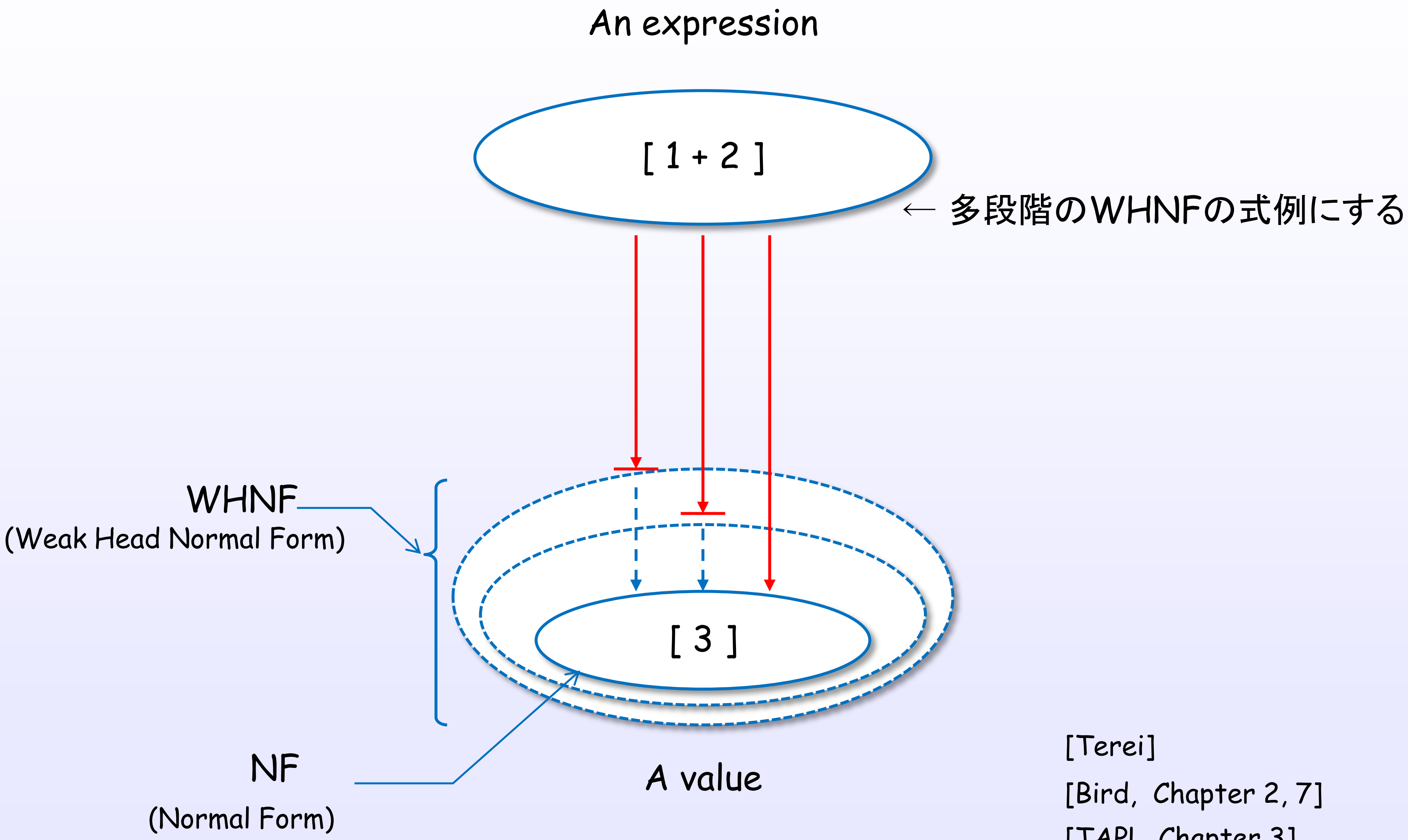
- Strict, Non-strict evaluation
- Eager, Lazy evaluation
- Call-by-value, Call-by-name,
  Call-by-need, …
- Innermost, Outermost
- Normal order, Application order
- …

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

There are some evaluation levels



[Terei]

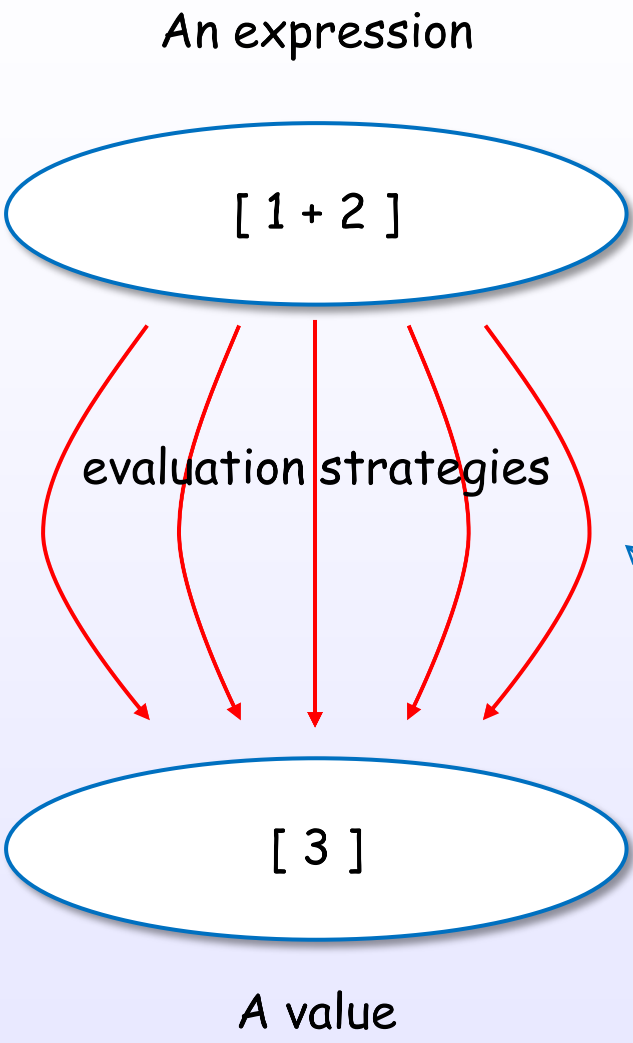
[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

Evaluation strategies

There are many evaluation approaches



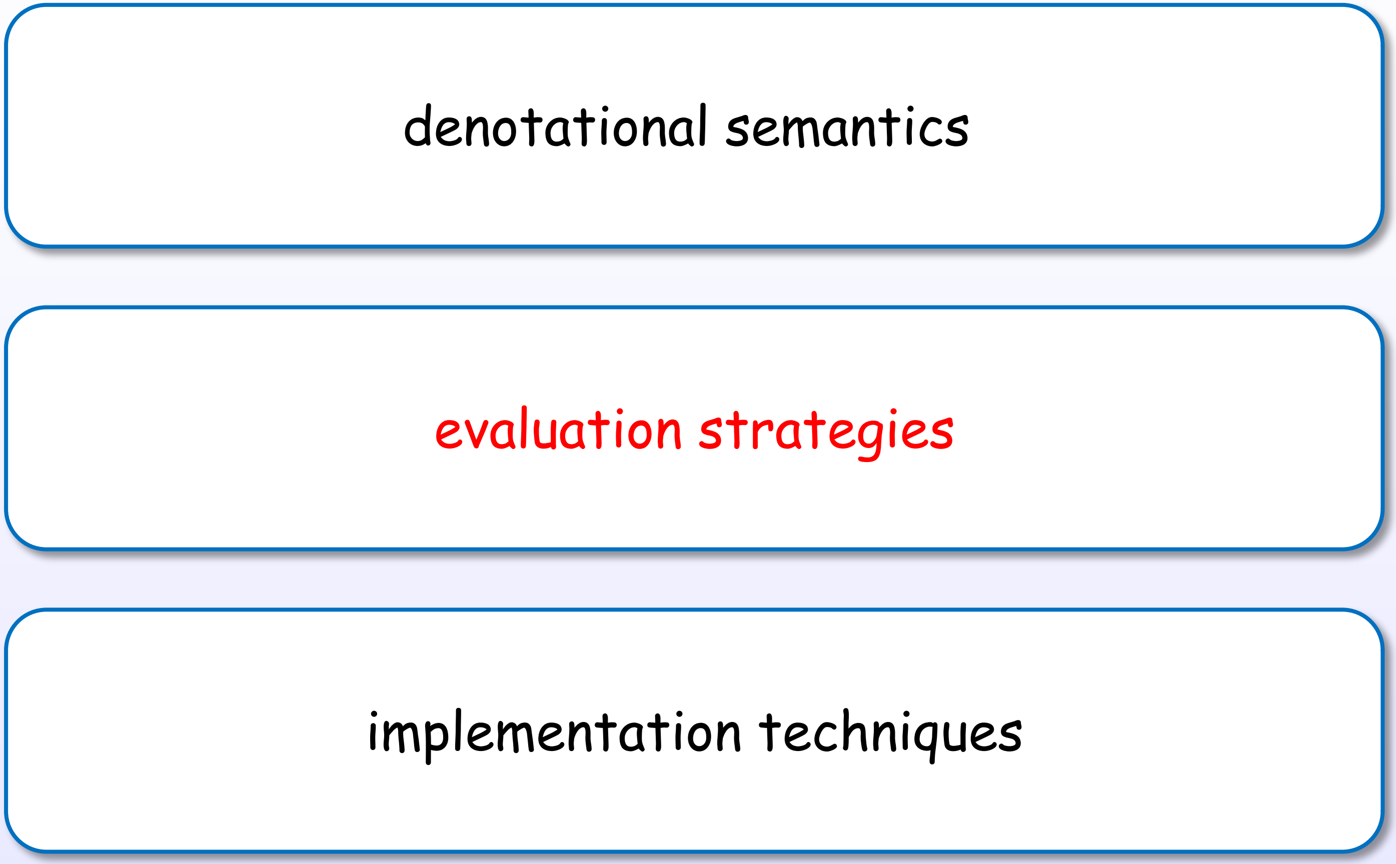
- Strict, Non-strict evaluation
- Eager, Lazy evaluation
- Call-by-value, Call-by-name,
  Call-by-need, …
- Innermost, Outermost
- Normal order, Application order
- …

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

Evaluation layers



denotational semantics

evaluation strategies

implementation techniques

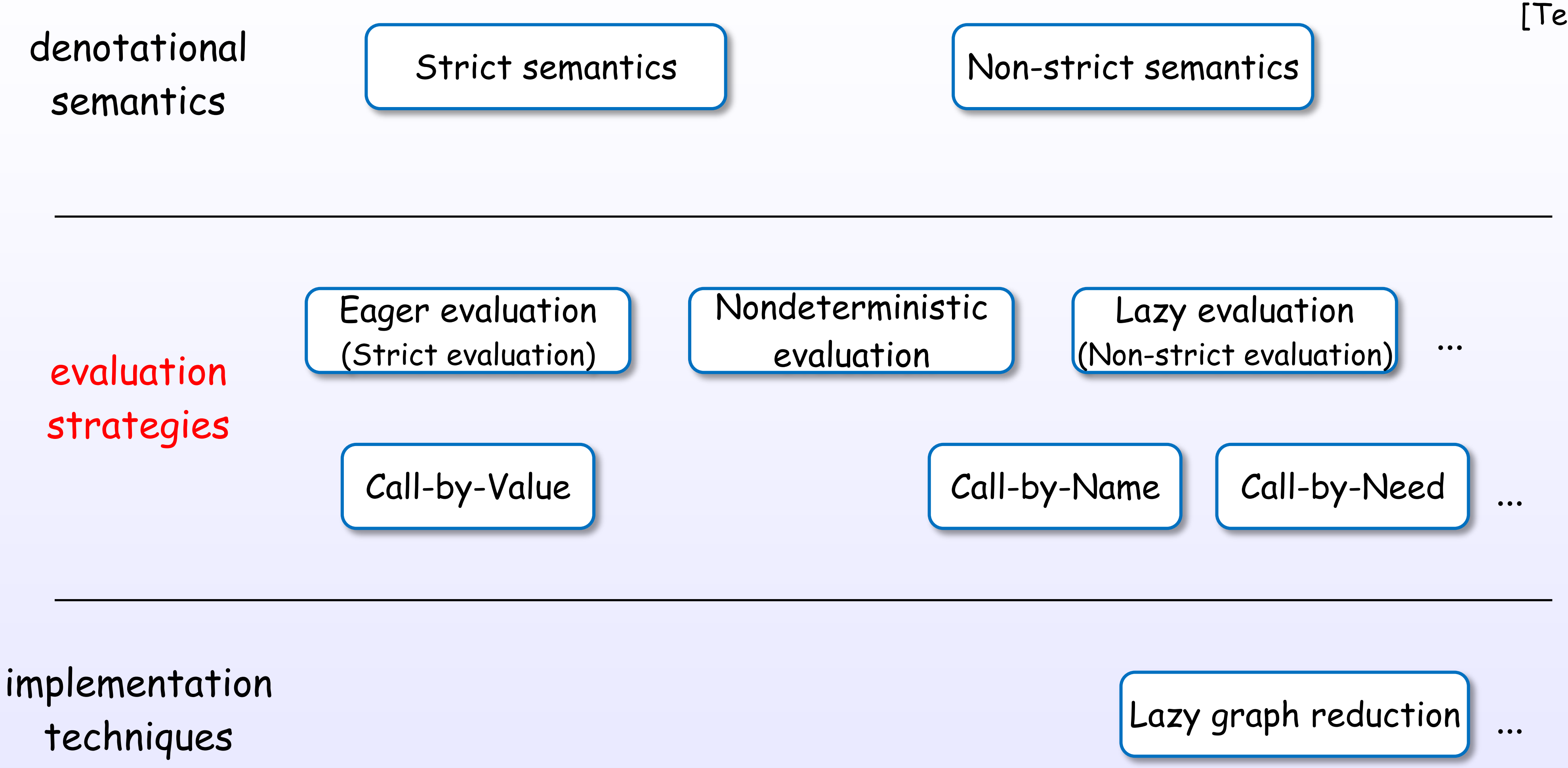
[Bird, Chapter 7]

[Hutton, Chapter 8]

[TAPL, Chapter 3]

References : [1]

Evaluation layers



denotational semantics

Strict semantics

Non-strict semantics

evaluation strategies

Eager evaluation (Strict evaluation)

Nondeterministic evaluation

Lazy evaluation (Non-strict evaluation)

...

Call-by-Value

Call-by-Name

Call-by-Need

...

implementation techniques

Lazy graph reduction

...

[Terei]
eva
call
call
call
eva

no-
non
eva
fun
lazy
sha

[Bir

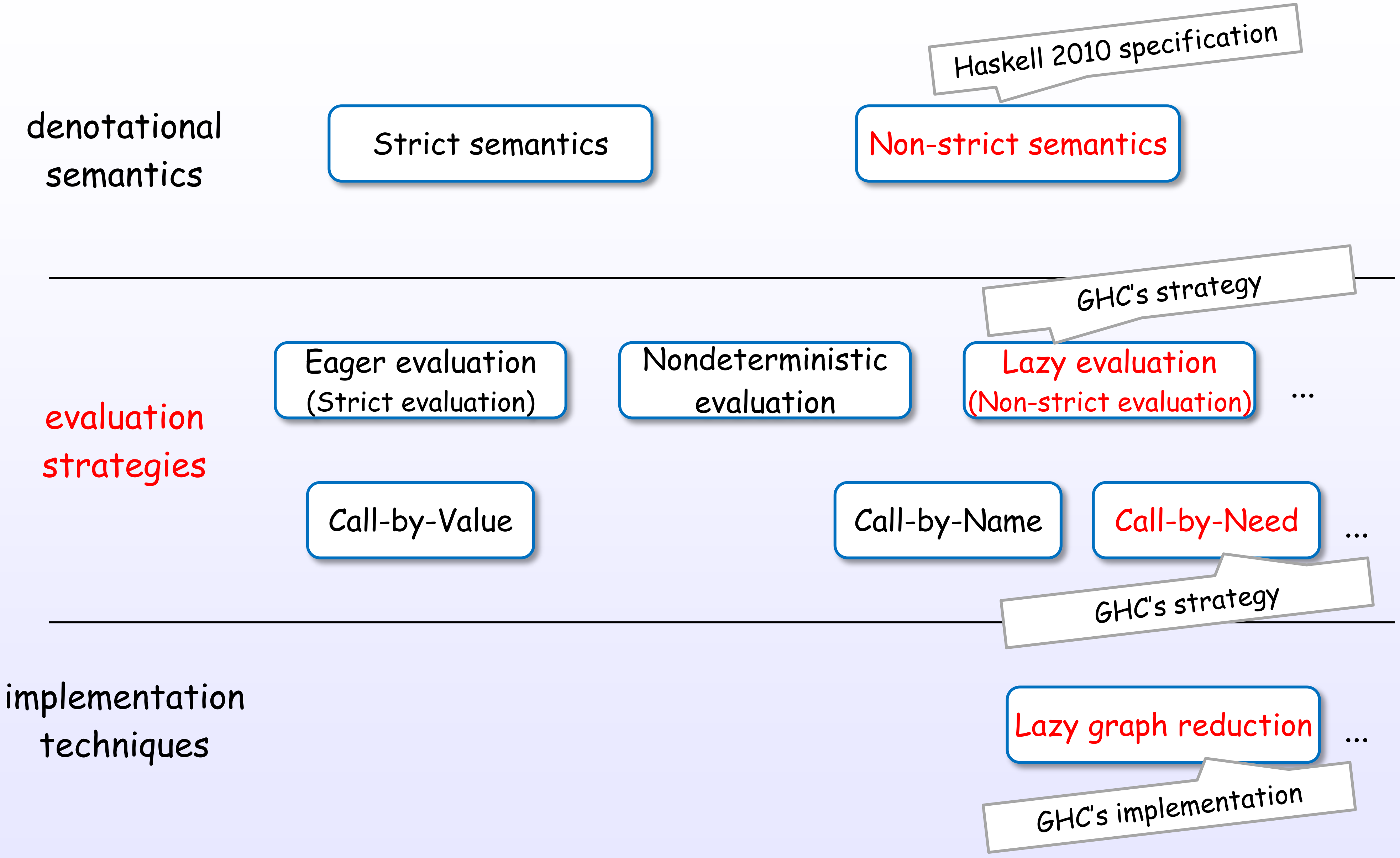
[Bird, Chapter 7]

[Hutton, Chapter 8]

[TAPL, Chapter 3]

References : [1]

Evaluation layers for GHC's Haskell



denotational semantics

Strict semantics

Non-strict semantics

Haskell 2010 specification

evaluation strategies

Eager evaluation (Strict evaluation)

Nondeterministic evaluation

Lazy evaluation (Non-strict evaluation)

GHC's strategy

...

Call-by-Value

Call-by-Name

Call-by-Need

GHC's strategy

...

implementation techniques

Lazy graph reduction

GHC's implementation

...

References : [1]

Evaluation strategies and order

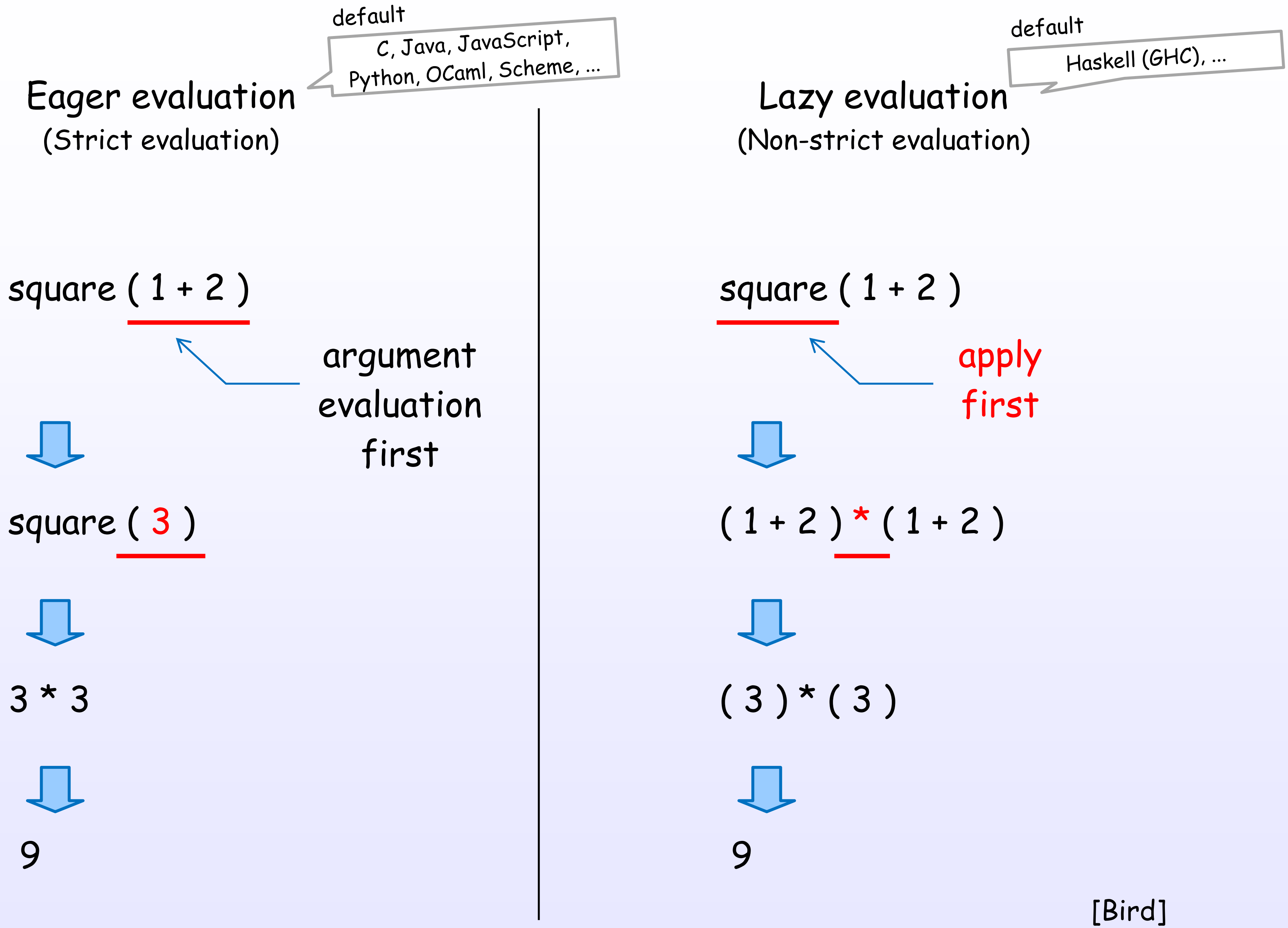
a (b c) + d (e (f g))

order

[Bird]
[Hutton]

References : [1]

Simple example of both evaluations



default

C, Java, JavaScript, Python, OCaml, Scheme, ...

Eager evaluation

(Strict evaluation)

square (1 + 2)

argument evaluation first

square (3)

3 * 3

9

default

Haskell (GHC), ...

Lazy evaluation

(Non-strict evaluation)

square (1 + 2)

apply first

(1 + 2) * (1 + 2)

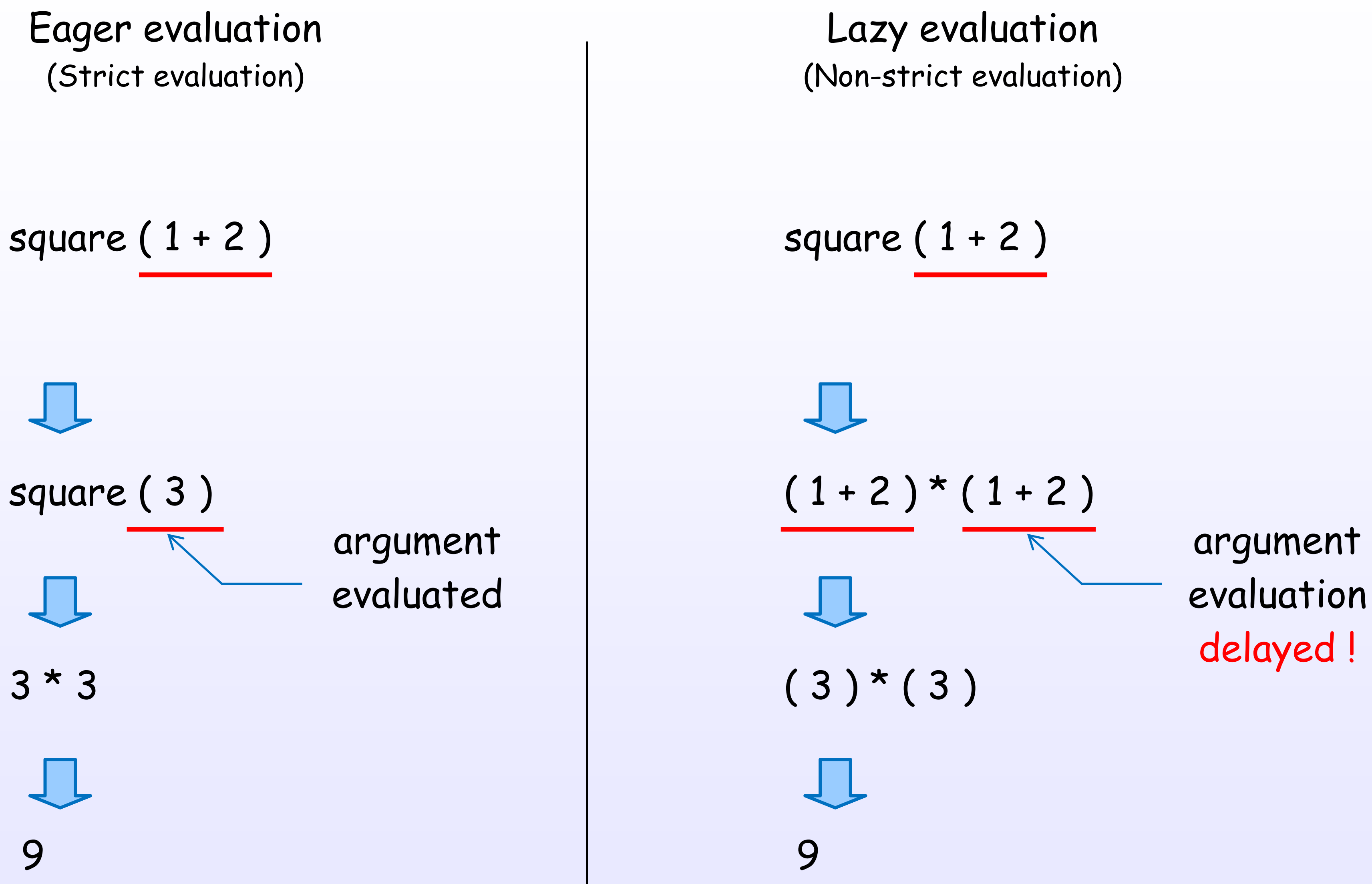
(3) * (3)

9

[Bird]
[Hutton]

References : [1]

Simple example of both evaluations



Eager evaluation

(Strict evaluation)

square (1 + 2)

square (3)

argument evaluated

3 * 3

9

Lazy evaluation

(Non-strict evaluation)

square (1 + 2)

(1 + 2) * (1 + 2)

argument evaluation delayed !

(3) * (3)

9

[Bird]
[Hutton]

References : [1]

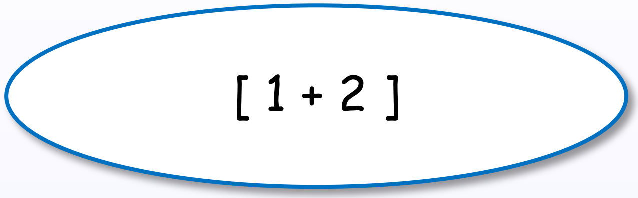
2. Expressions

2. Expressions

Expressions in Haskell

An expression denotes a value

An expression



[1 + 2]

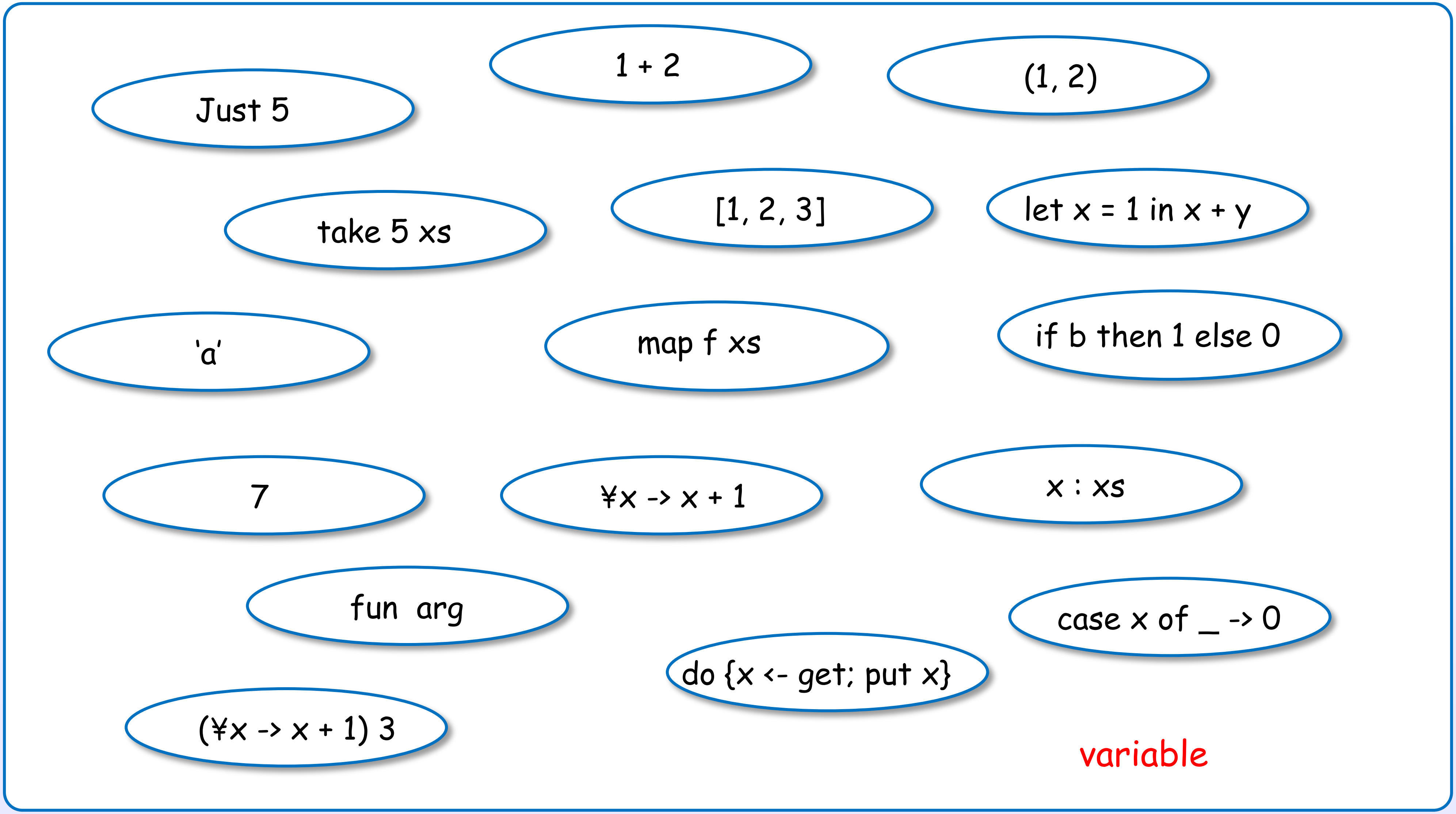
[HR2010]

[Bird, Chapter 2]

References : [1]

There are many expressions in Haskell

Expressions



Just 5

1 + 2

(1, 2)

take 5 xs

[1, 2, 3]

let x = 1 in x + y

'a'

map f xs

if b then 1 else 0

7

¥x -> x + 1

x : xs

fun arg

case x of _ -> 0

do {x <- get; put x}

(¥x -> x + 1) 3

variable

categorizing

[HR2010]

[Bird, Chapter 2]

References : [1]

Expression categories in Haskell

WHNF(a value)、
unevaluated expression
との関連づけを
PAPもWHNFなので注意

lambda abstraction

¥x -> x + 1

let expression

let x = 1 in x + y

variable

conditional

if b then 1 else 0

case expression

case x of _ -> 0

do expression

do {x <- get; put x}

general constructor, literal and some forms

7

[1, 2, 3]

(1, 2)

'a'

x : xs

Just 5

function application

take 5 xs

(¥x -> x + 1) 3

1 + 2

map f xs

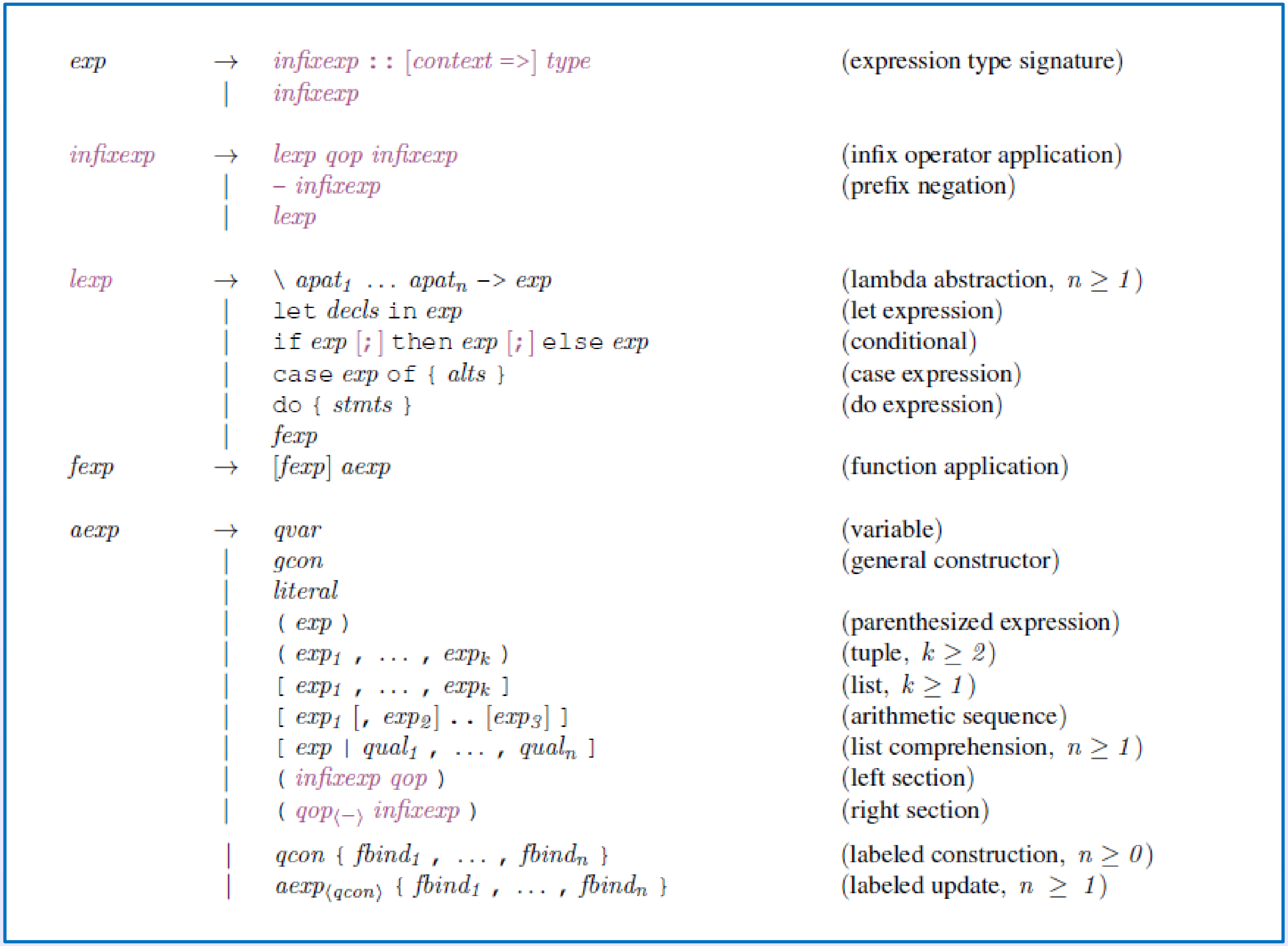
fun arg

[HR2010]
[Bird, Chapter 2

References : [1]

Specification is defined in Haskell 2010 Language Report

Haskell 2010 Language Report, Chapter 3 Expressions [1]

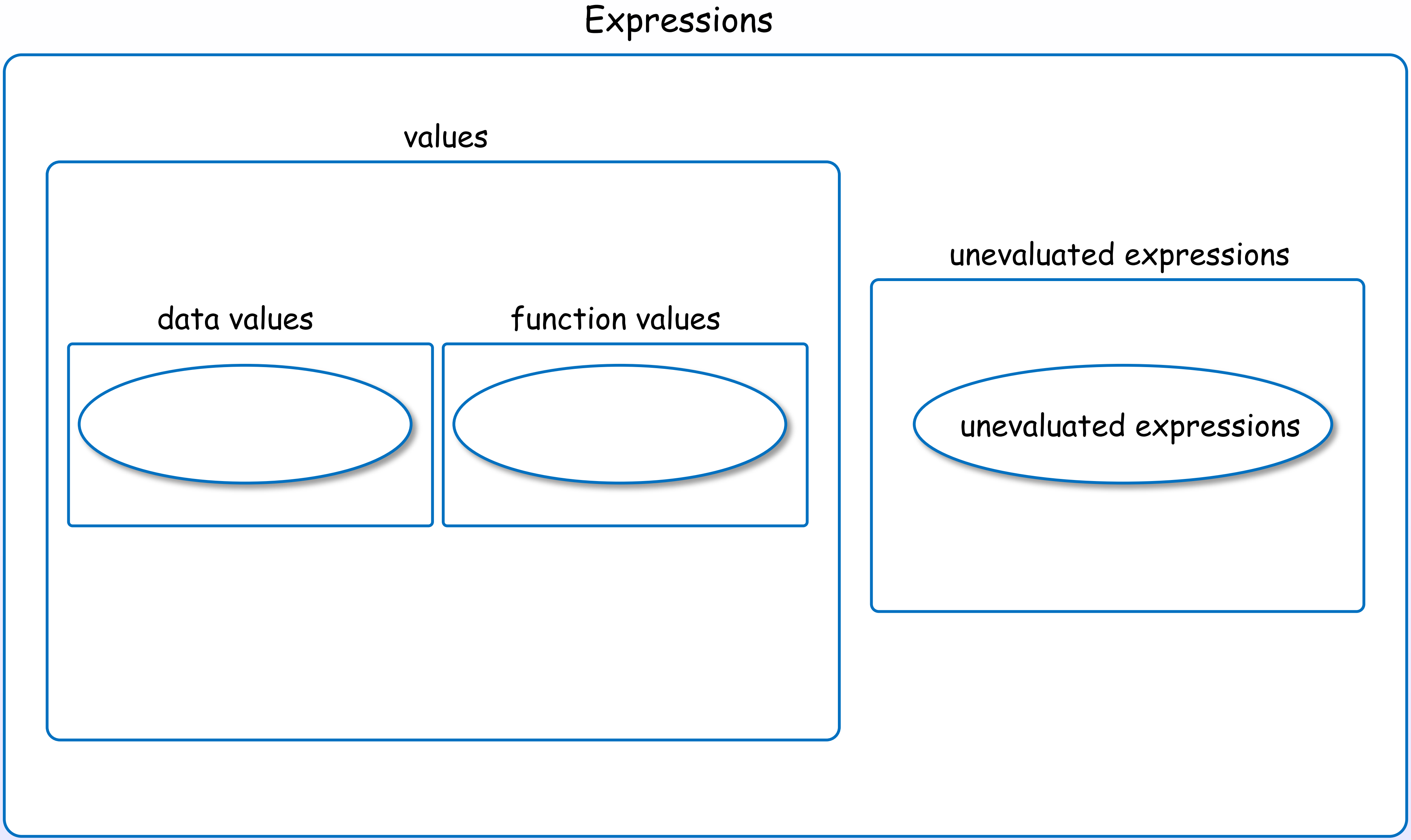


exp	→	*infixexp* :: [*context* =>] *type*	(expression type signature)
	|	*infixexp*	
infixexp	→	*lexp qop infixexp*	(infix operator application)
	|	- *infixexp*	(prefix negation)
	|	*lexp*	
lexp	→	\ $apat_1$ … $apat_n$ -> *exp*	(lambda abstraction, $n \geq 1$)
	|	let *decls* in *exp*	(let expression)
	|	if *exp* [;] then *exp* [;] else *exp*	(conditional)
	|	case *exp* of { *alts* }	(case expression)
	|	do { *stmts* }	(do expression)
	|	*fexp*	
fexp	→	[*fexp*] *aexp*	(function application)
aexp	→	*qvar*	(variable)
	|	*gcon*	(general constructor)
	|	*literal*	
	|	(*exp*)	(parenthesized expression)
	|	(exp_1 , … , exp_k)	(tuple, $k \geq 2$)
	|	[exp_1 , … , exp_k]	(list, $k \geq 1$)
	|	[exp_1 [, exp_2] .. [exp_3]]	(arithmetic sequence)
	|	[*exp* | $qual_1$, … , $qual_n$]	(list comprehension, $n \geq 1$)
	|	(*infixexp qop*)	(left section)
	|	($qop_{\langle - \rangle}$ *infixexp*)	(right section)
	|	*qcon* { $fbind_1$, … , $fbind_n$ }	(labeled construction, $n \geq 0$)
	|	$aexp_{\langle qcon \rangle}$ { $fbind_1$, … , $fbind_n$ }	(labeled update, $n \geq 1$)

References : [1]

Classification of expressions

A value or an unevaluated expression



値か否か。値は2種。

[STG]

References : [1]

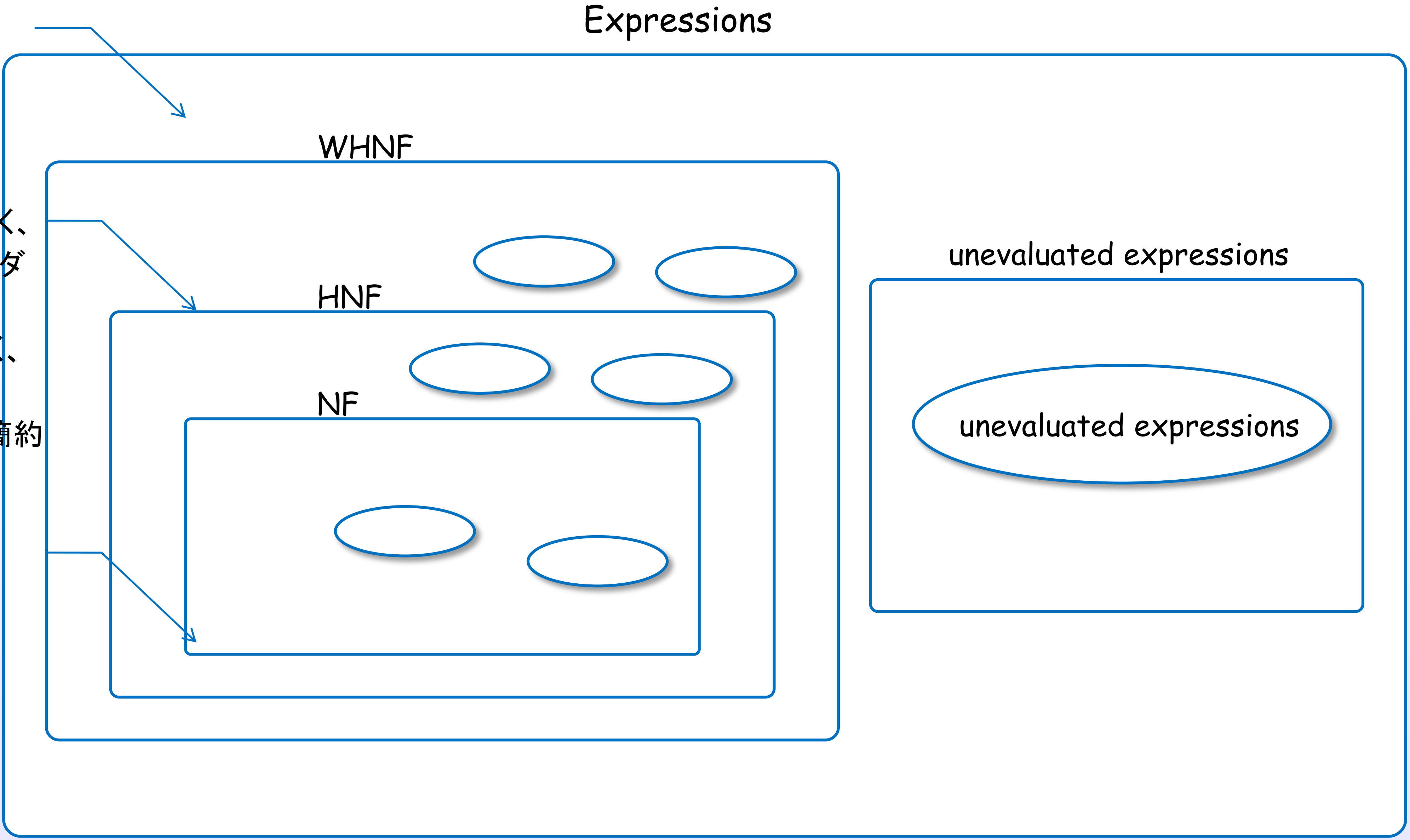
evaluation level

ぐ

でなく、
ラムダ

でなく、
合は、
象も簡約

い。
）



値には、評価レベルがある。

[STG]

References : [1]

実例との対応付け

[STG]

References : [1]

3. Internal representation of expressions

3. Internal representation of expressions

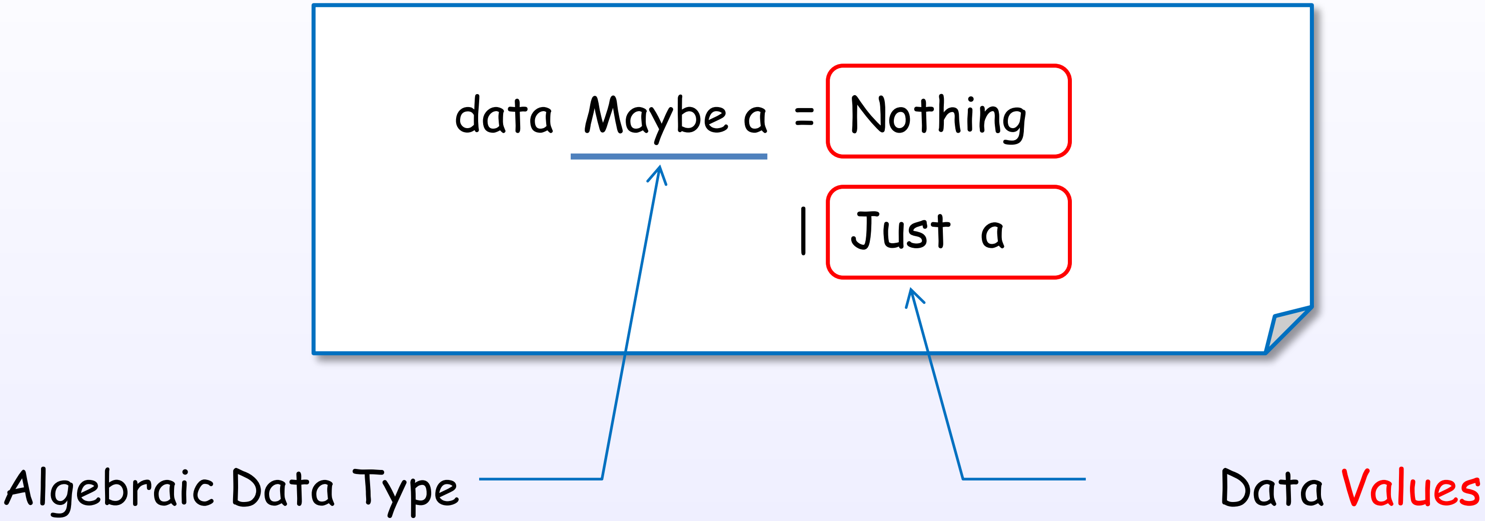
Constructor

Constructor

Constructor is one of the key elements
to understand WHNF and lazy evaluation in Haskell.

References : [1]

data文で宣言する代数的データ型とその値



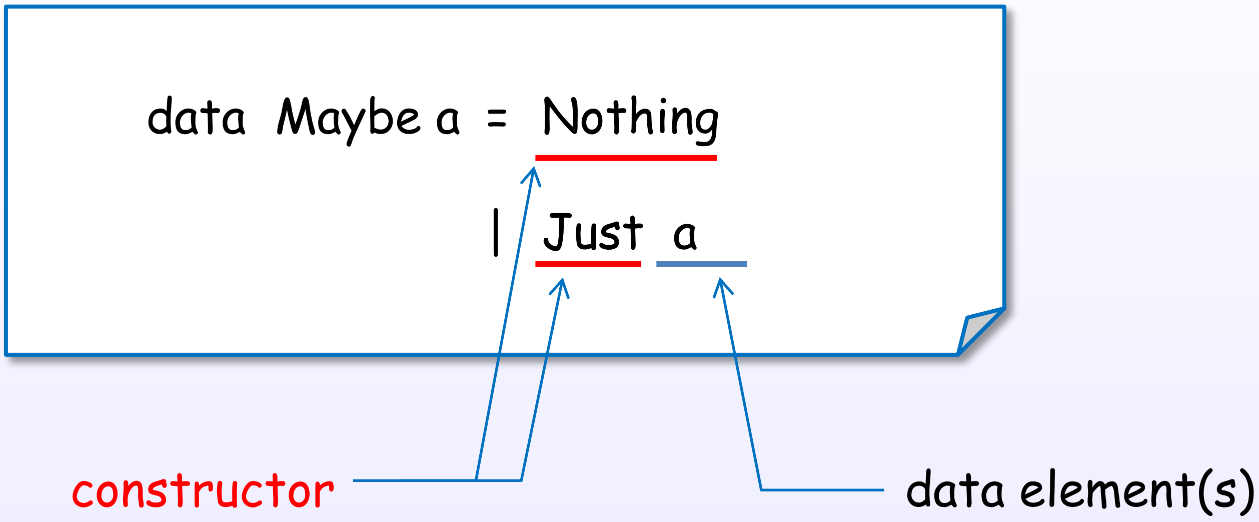
```
data  Maybe a  = Nothing
               | Just  a
```

Algebraic Data Type

Data Values

References : [1]

Constructorはdata文で宣言する代数的データ値



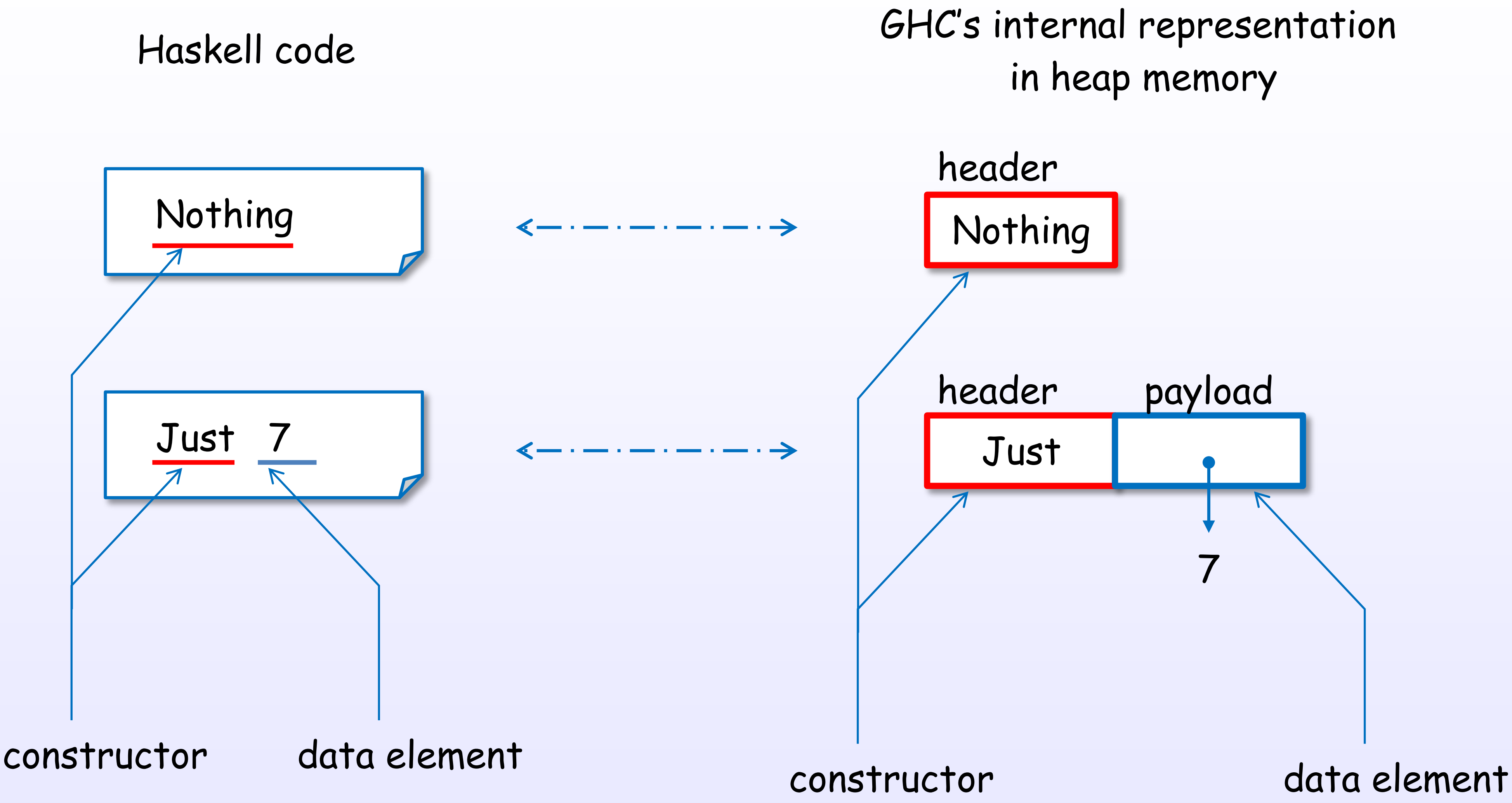
```
data  Maybe a =  Nothing
              |  Just  a
```

constructor

data element(s)

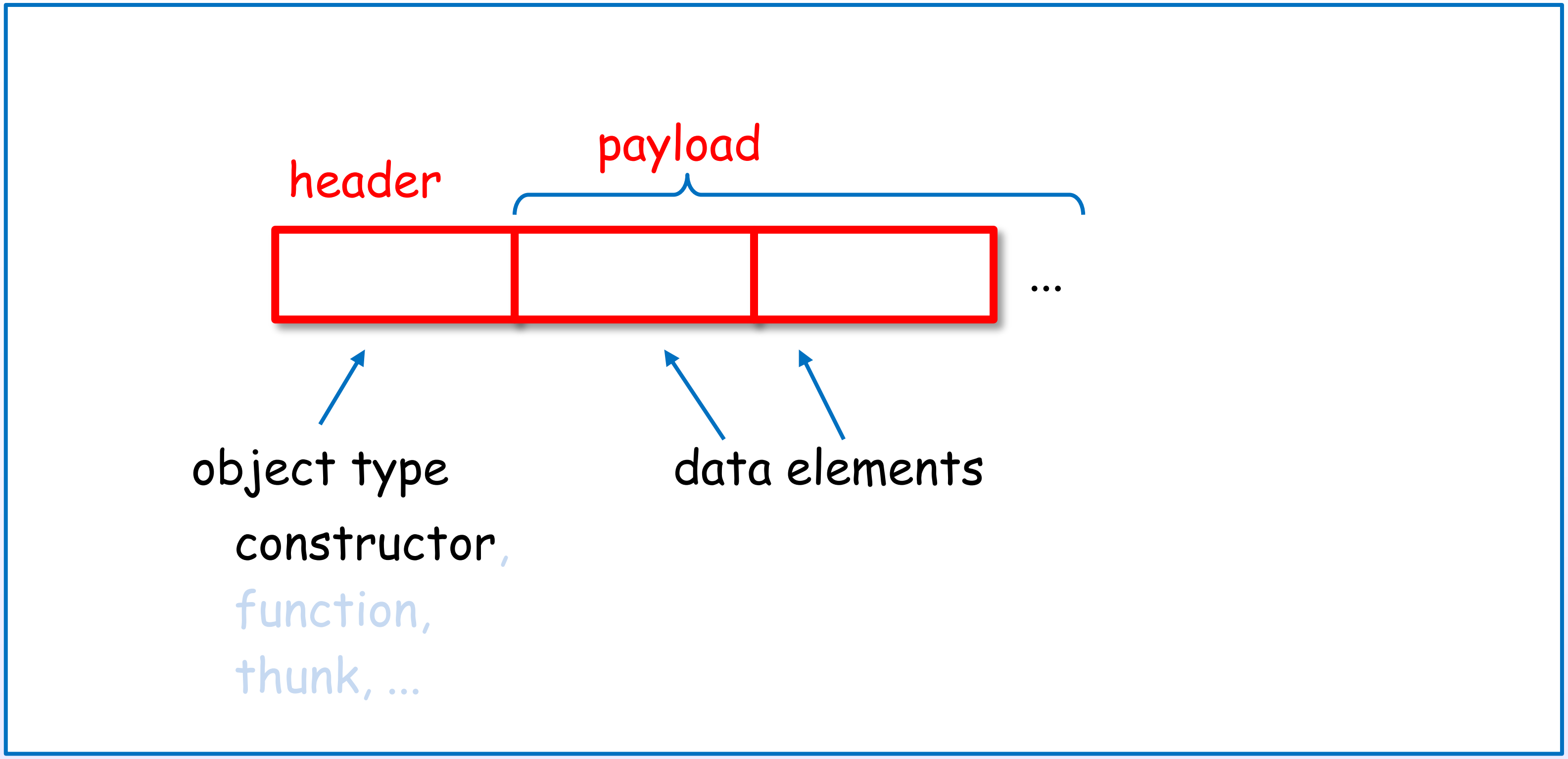
References : [1]

Constructorの内部表現



References : [1]

Constructorは統一内部表現で表現される



header

payload

...

object type

constructor,

function,

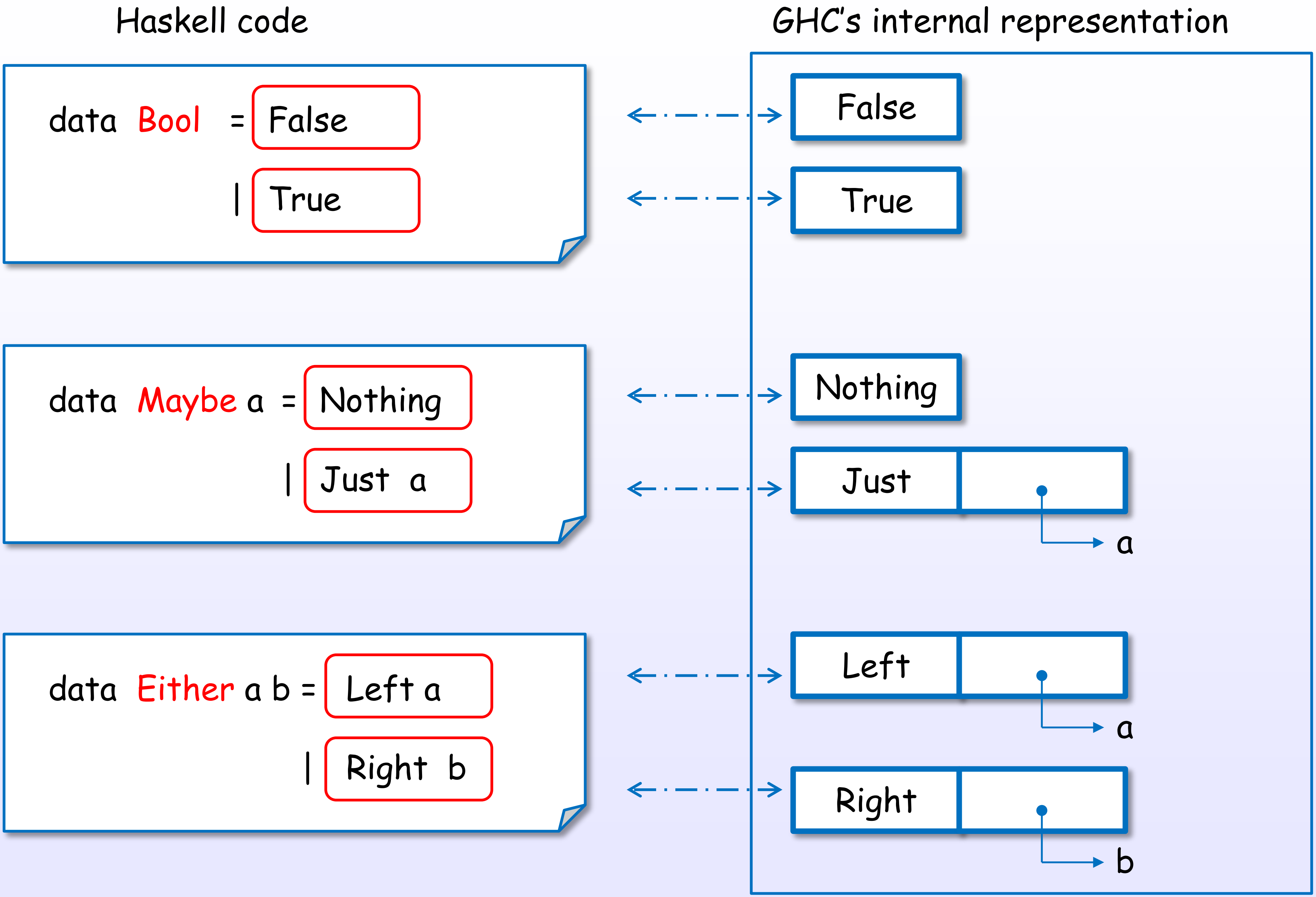
thunk, ...

data elements

in heap memory, stack, registers or static memory

References : [1]

いろいろなコンストラクタと内部表現



Haskell code

```
data  Bool   = False
             | True
```

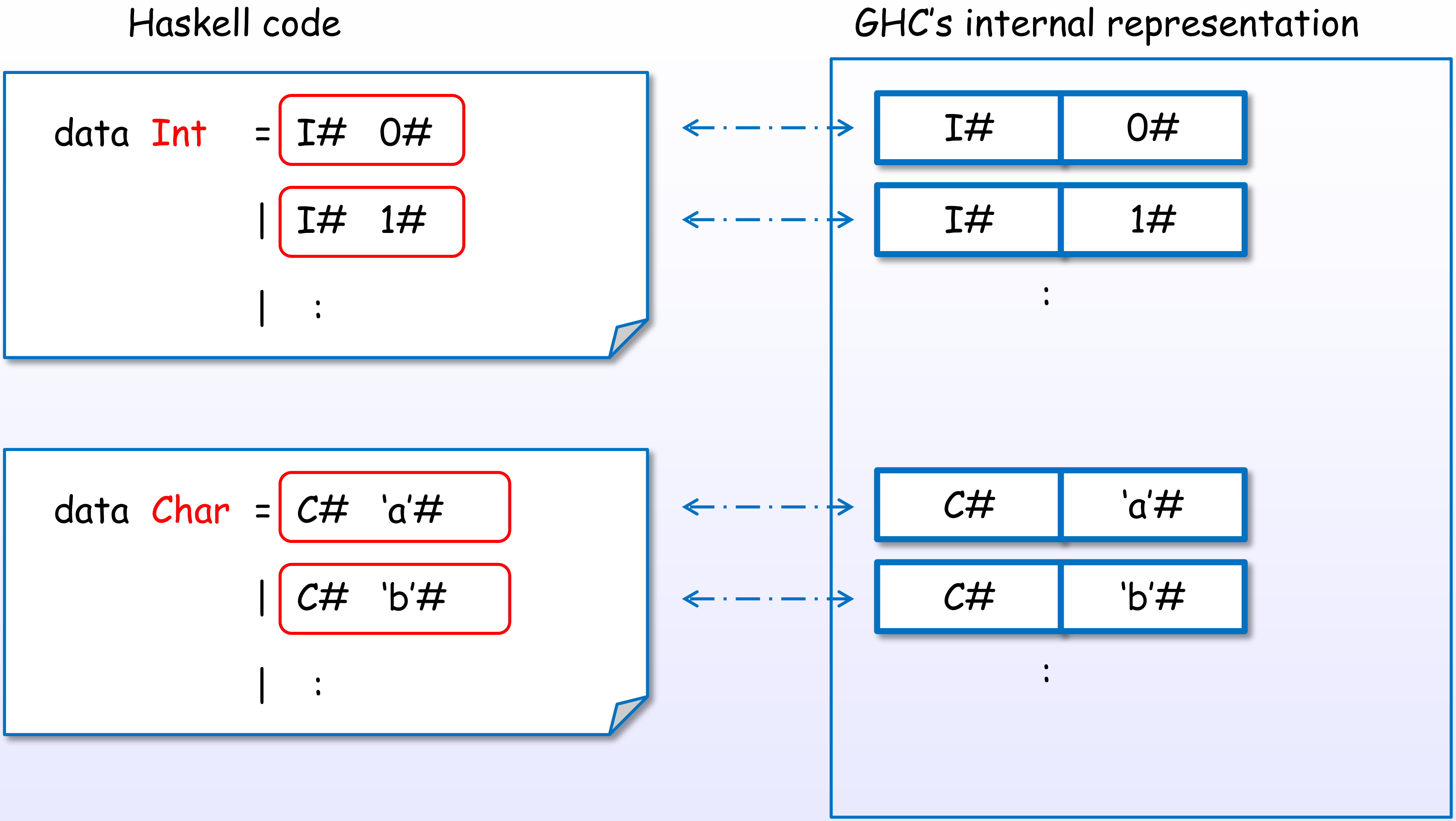
```
data  Maybe a  = Nothing
               | Just  a
```

```
data  Either a b = Left a
                 | Right  b
```

GHC's internal representation

References : [1]

基本データ型も実はコンストラクタで構成されている



Haskell code

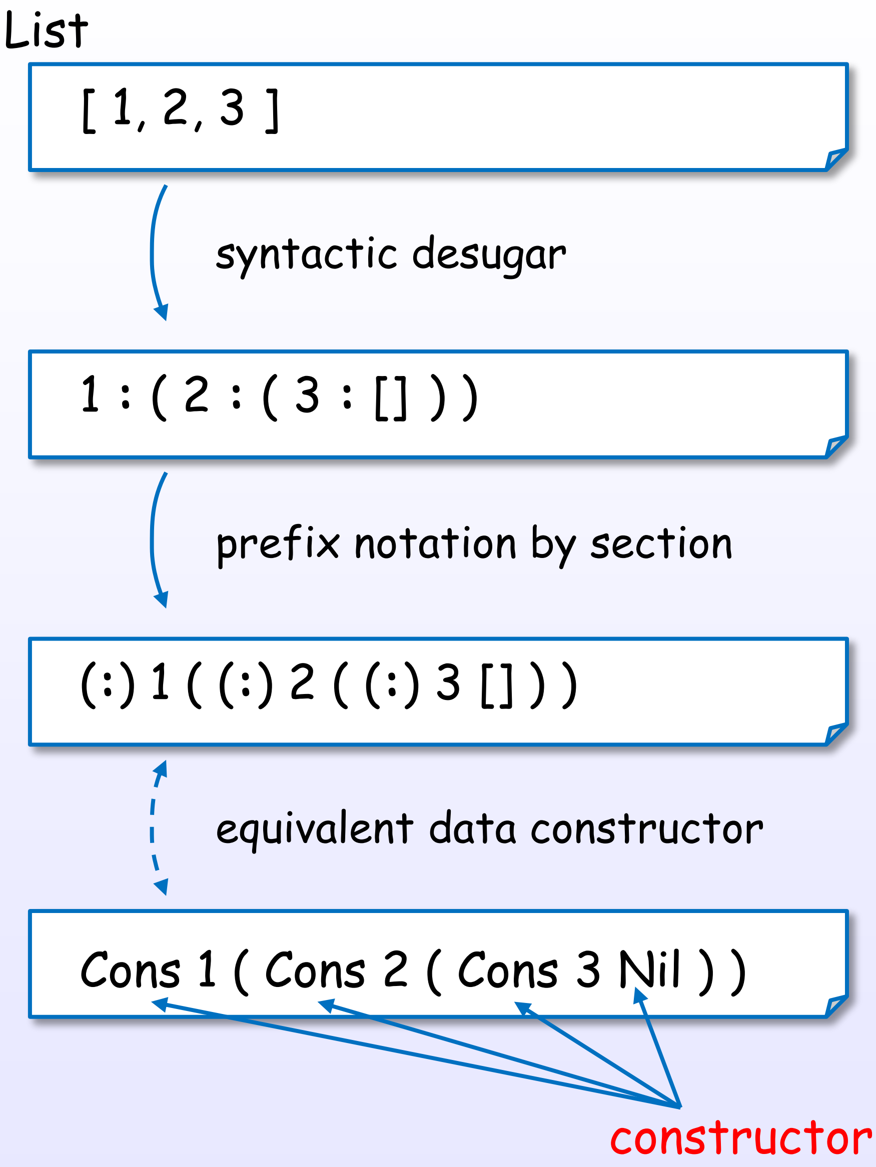
```
data Int  = I# 0#
          | I# 1#
          |  :

data Char = C# 'a'#
          | C# 'b'#
          |  :
```

GHC's internal representation

I#	0#
I#	1#
:	
C#	'a'#
C#	'b'#
:	

References : [1]

リストも実はコンストラクタで構成されている



List

```
[ 1, 2, 3 ]
```

syntactic desugar

```
1 : ( 2 : ( 3 : [] ) )
```

prefix notation by section

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

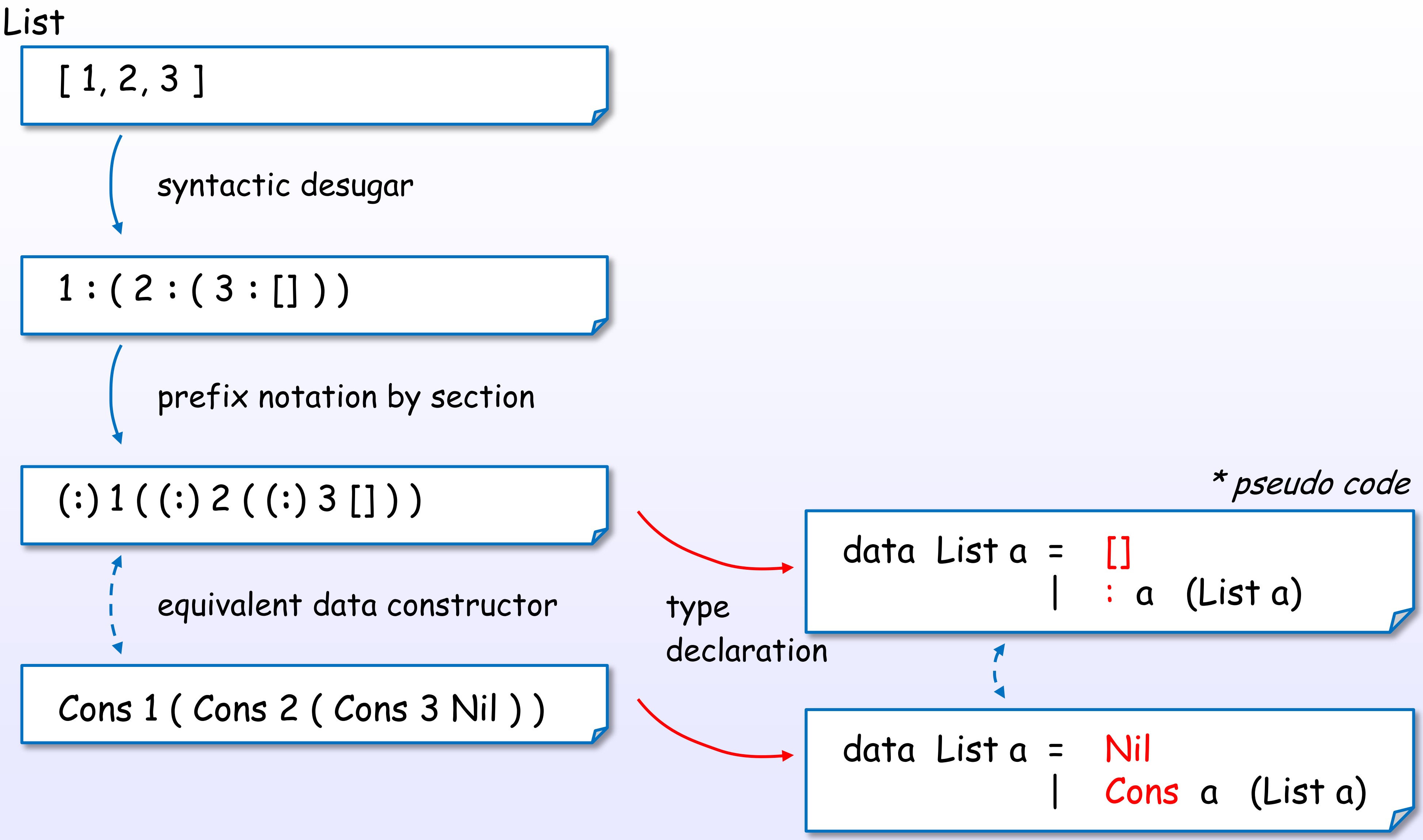
equivalent data constructor

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

constructor

References : [1]

リストも実はコンストラクタで構成されている



List

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prefix notation by section

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

equivalent data constructor

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

type declaration

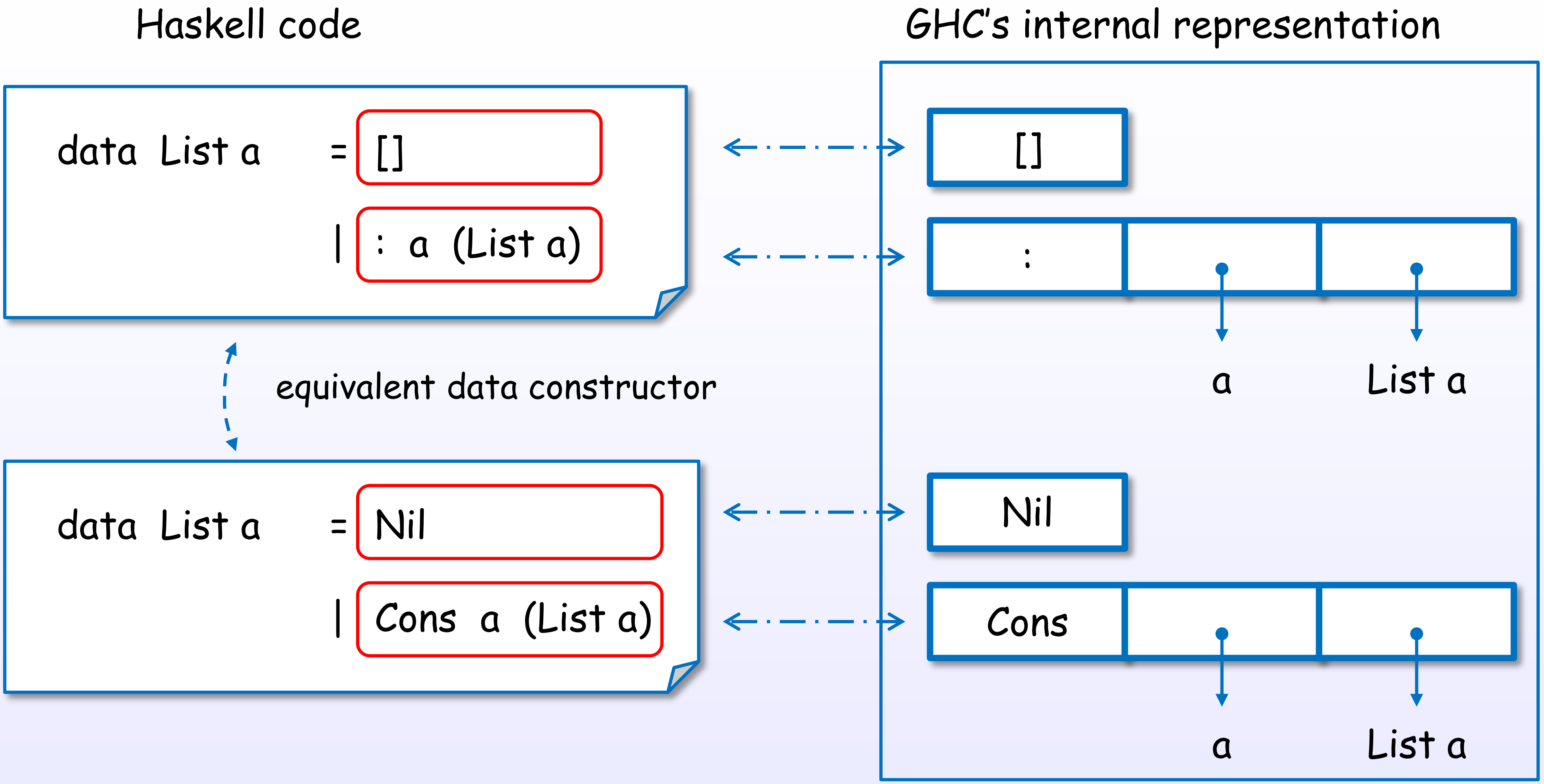
** pseudo code*

```
data  List a  =   []
              |   :  a   (List a)
```

```
data  List a  =   Nil
              |   Cons  a   (List a)
```

References : [1]

リストも実はコンストラクタで構成されている



Haskell code

```
data  List a      = []
                  | :  a  (List a)
```

equivalent data constructor

```
data  List a      = Nil
                  | Cons  a  (List a)
```

GHC's internal representation

[]

:

a

List a

Nil

Cons

a

List a

References : [1]

リストも実はコンストラクタで構成されている

Haskell code

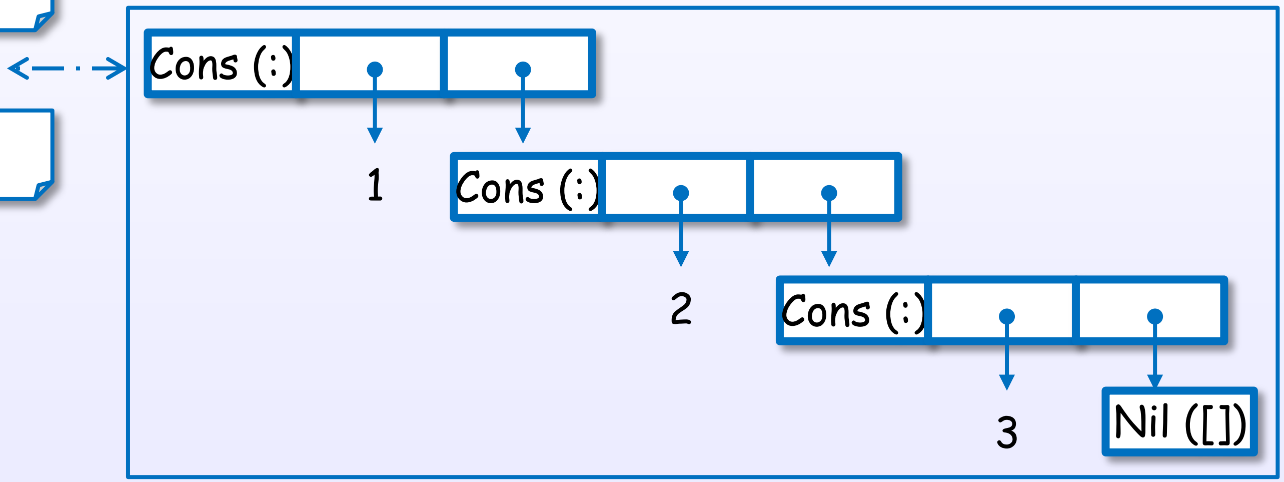
```
[ 1, 2, 3 ]
```

```
1 : ( 2 : ( 3 : [] ) )
```

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

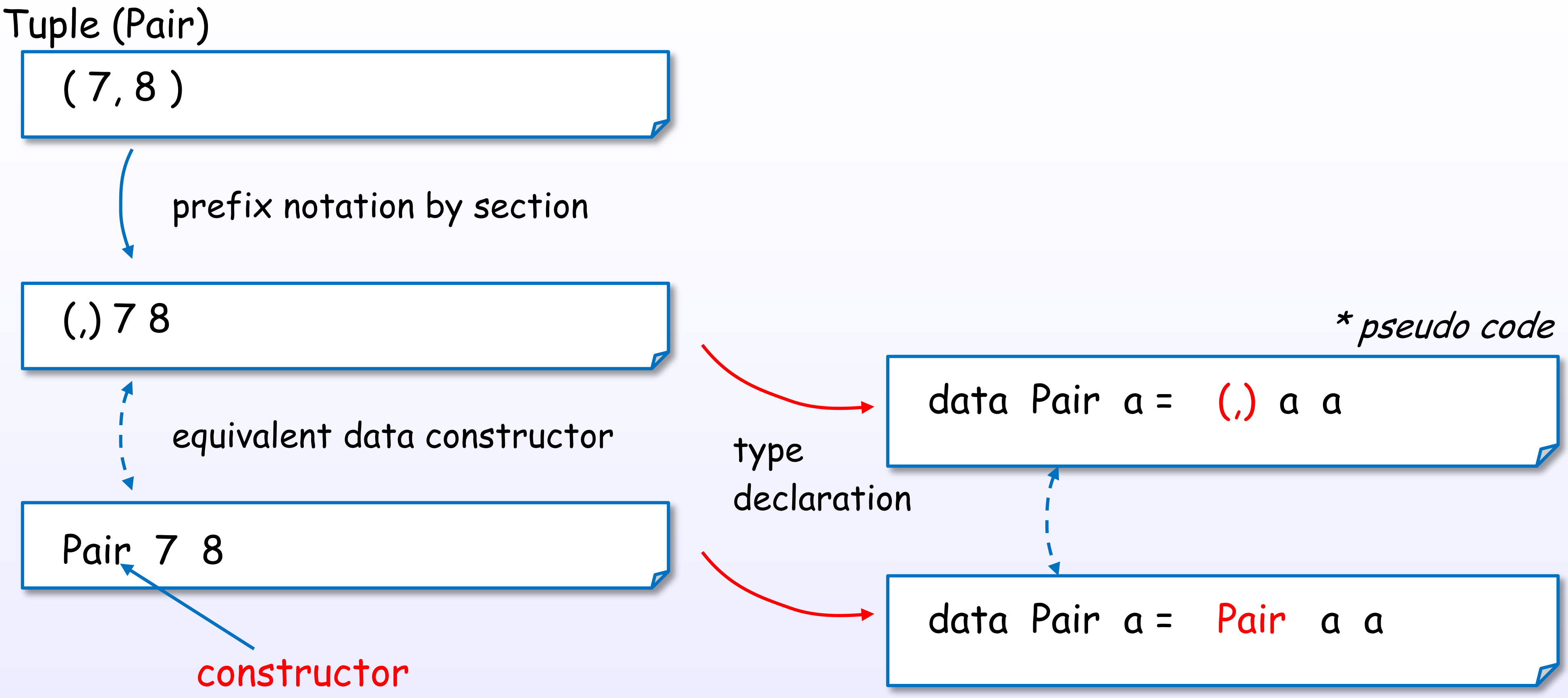
```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

GHC's internal representation



References : [1]

タプルも実はコンストラクタで構成されている



Tuple (Pair)

```
( 7, 8 )
```

prefix notation by section

```
(,) 7 8
```

equivalent data constructor

```
Pair  7  8
```

constructor

type declaration

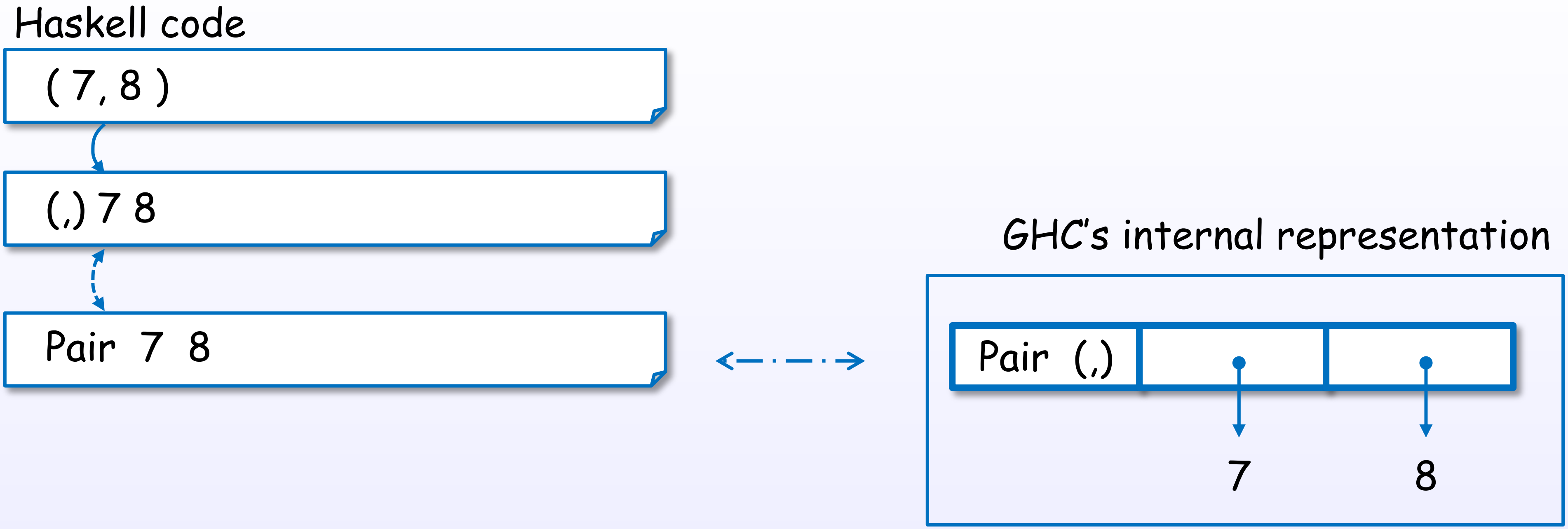
** pseudo code*

```
data  Pair  a =   (,)  a  a
```

```
data  Pair  a =   Pair   a  a
```

References : [1]

タプルも実はコンストラクタで構成されている



Haskell code

```
( 7, 8 )
```

```
(,) 7 8
```

```
Pair  7  8
```

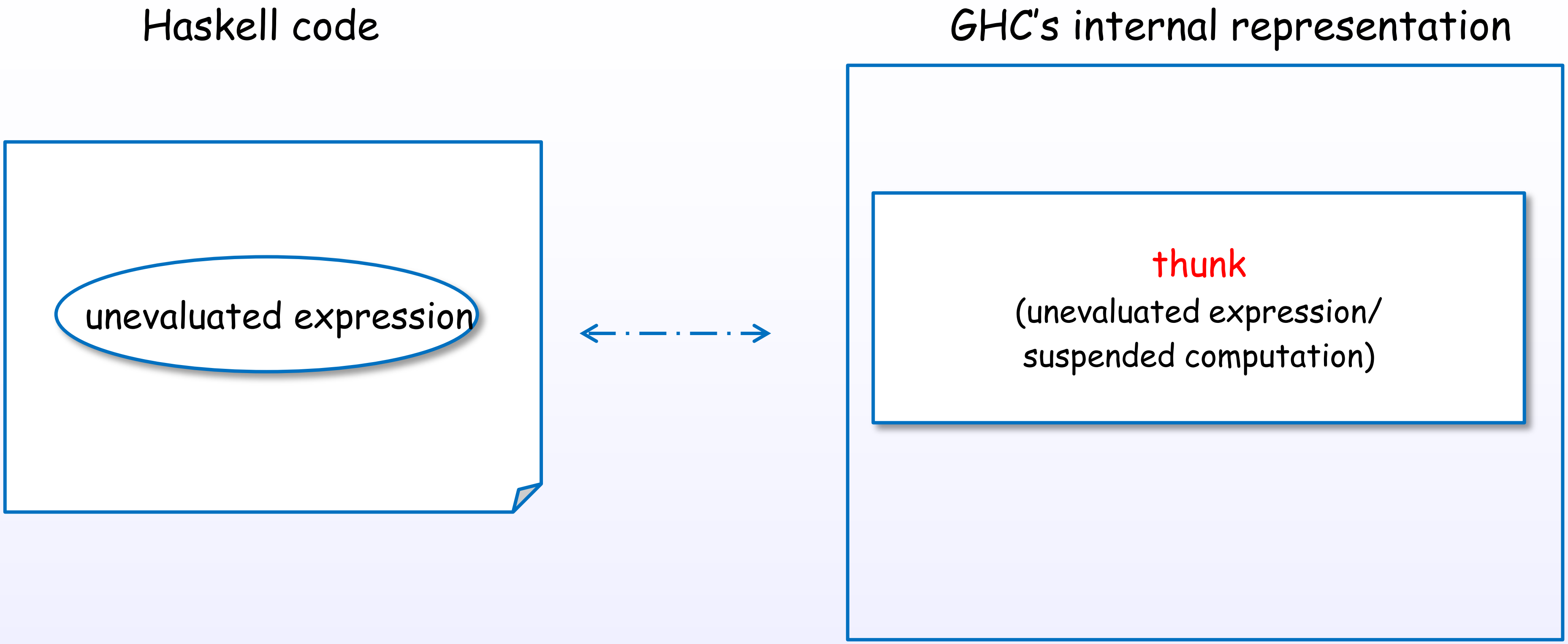
GHC's internal representation

References : [1]

3. Internal representation of expressions

Thunk

Thunk


Haskell code

unevaluated expression

GHC's internal representation

thunk

(unevaluated expression/ suspended computation)

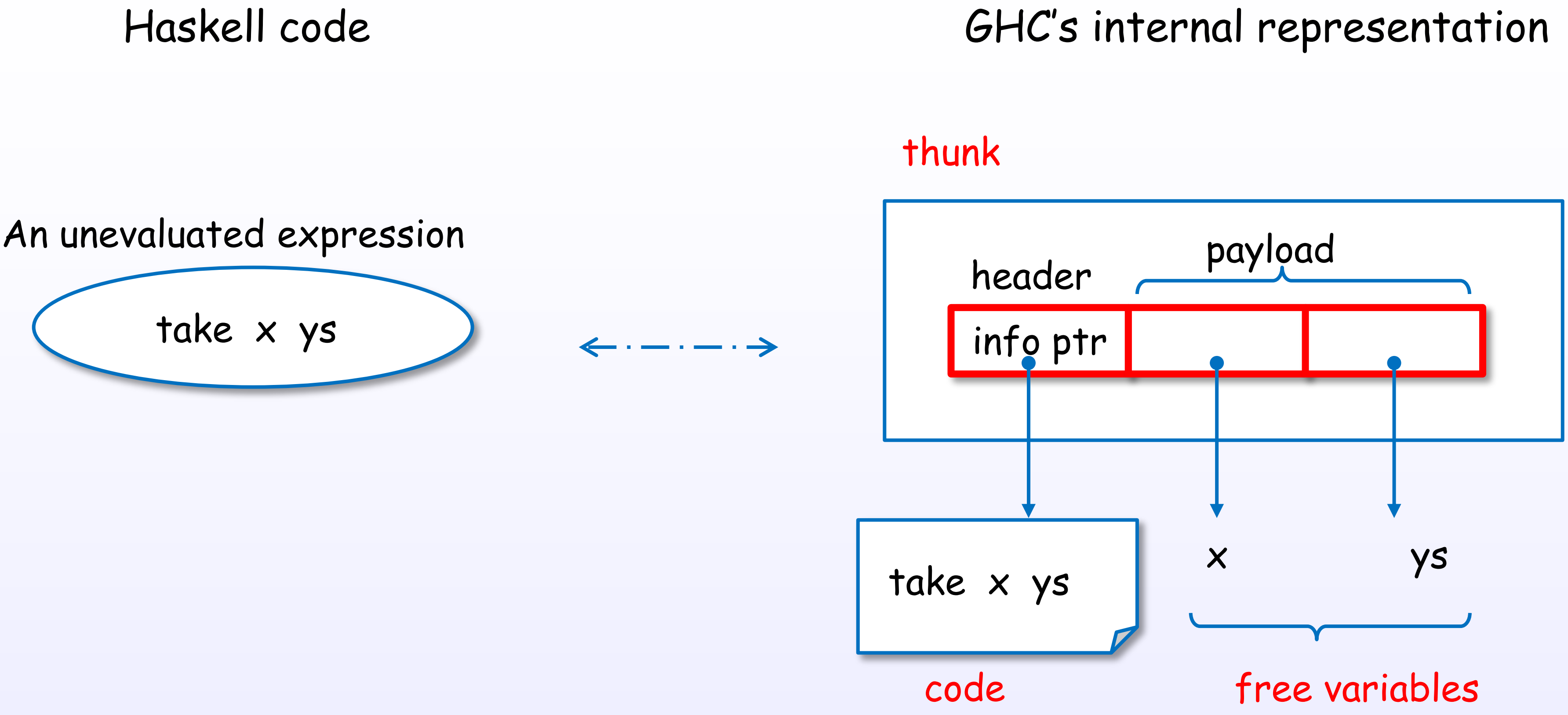
A thunk is an unevaluated expression in heap memory.

A thunk is built to postpone the evaluation.

[parconc, Ch.2]

References : [1]

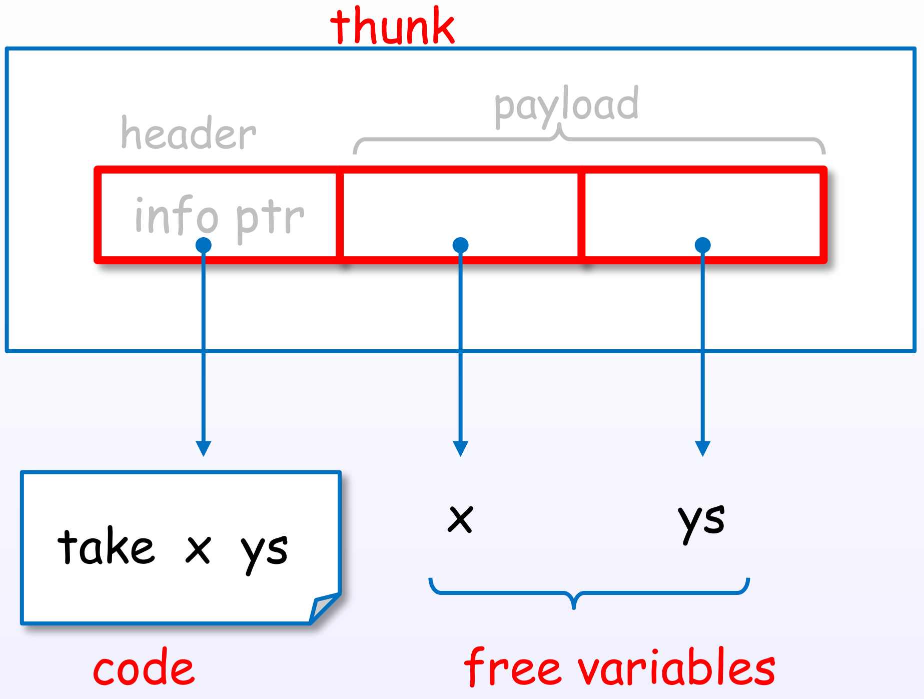
Thunkの内部表現



A thunk is represented with header(code) + payload(free variables).

References : [1]

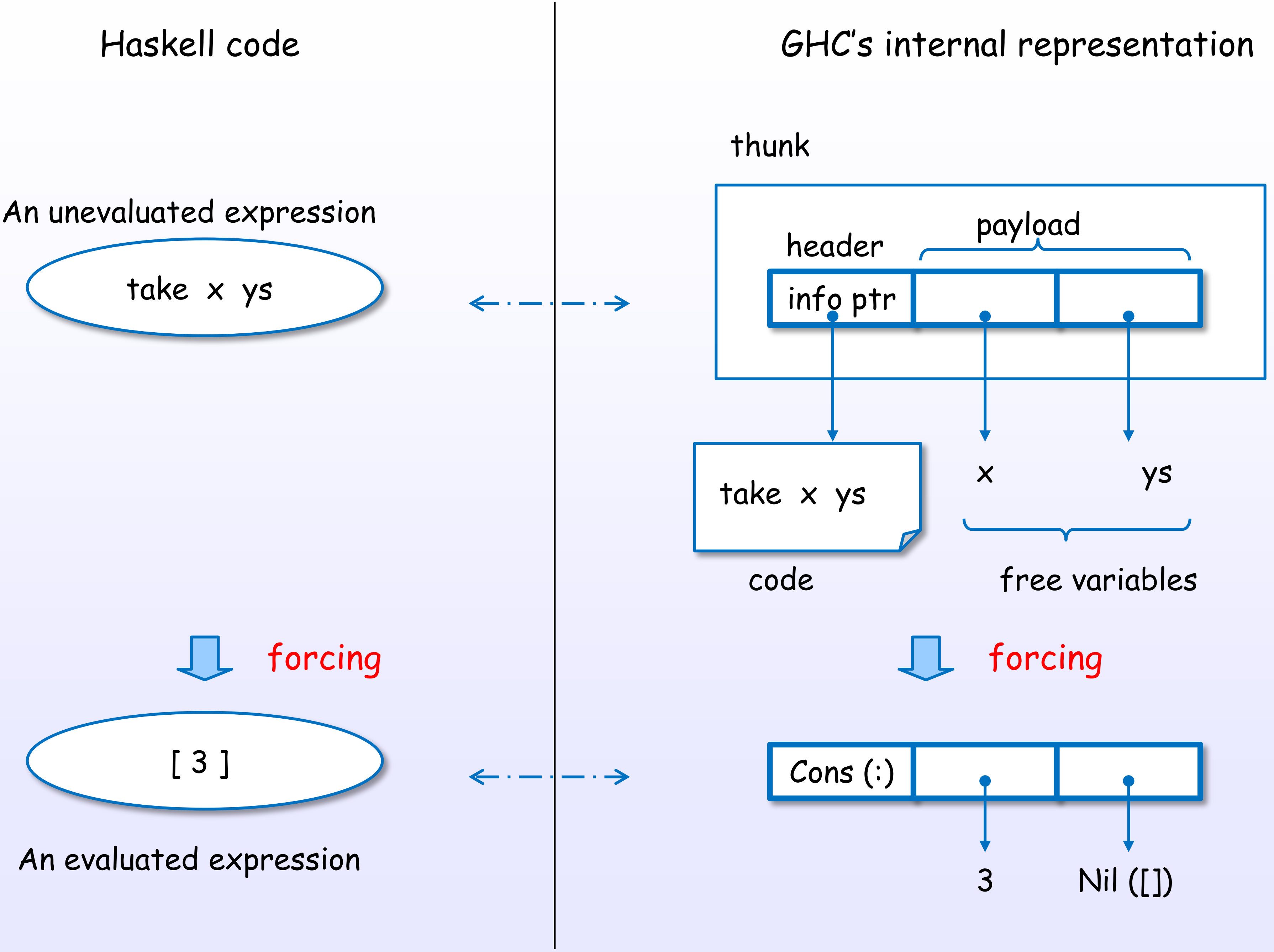
Thunkは、codeとfree variablesをパッケージ化したもの



A thunk is a package of code + free variables.

References : [1]

Thunkは、forcing要求により評価される

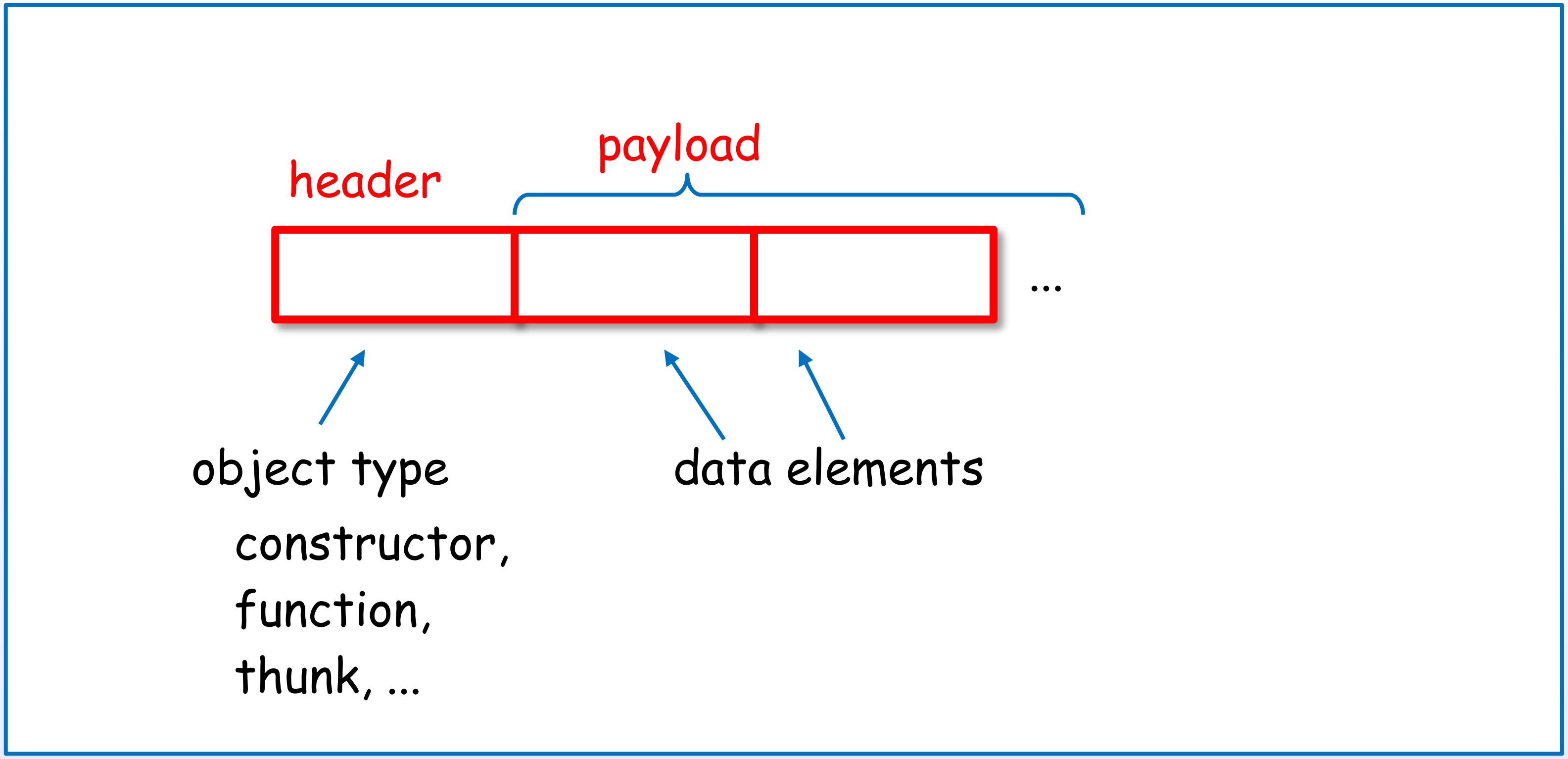


References : [1]

3. Internal representation of expressions

Uniform representation

統一内部表現で表現される



payload

header

...

object type

constructor,

function,

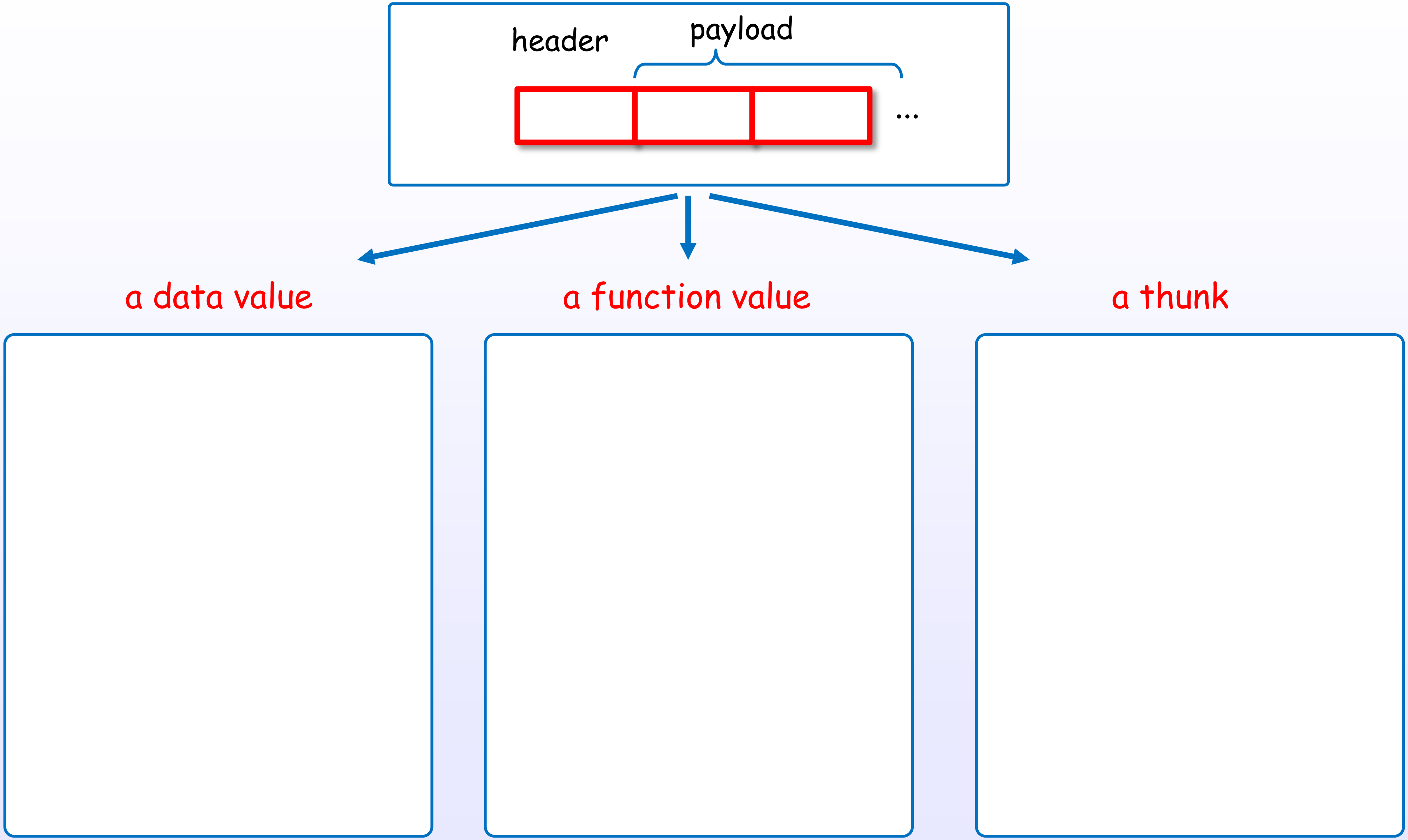
thunk, ...

data elements

in heap memory, stack, registers or static memory

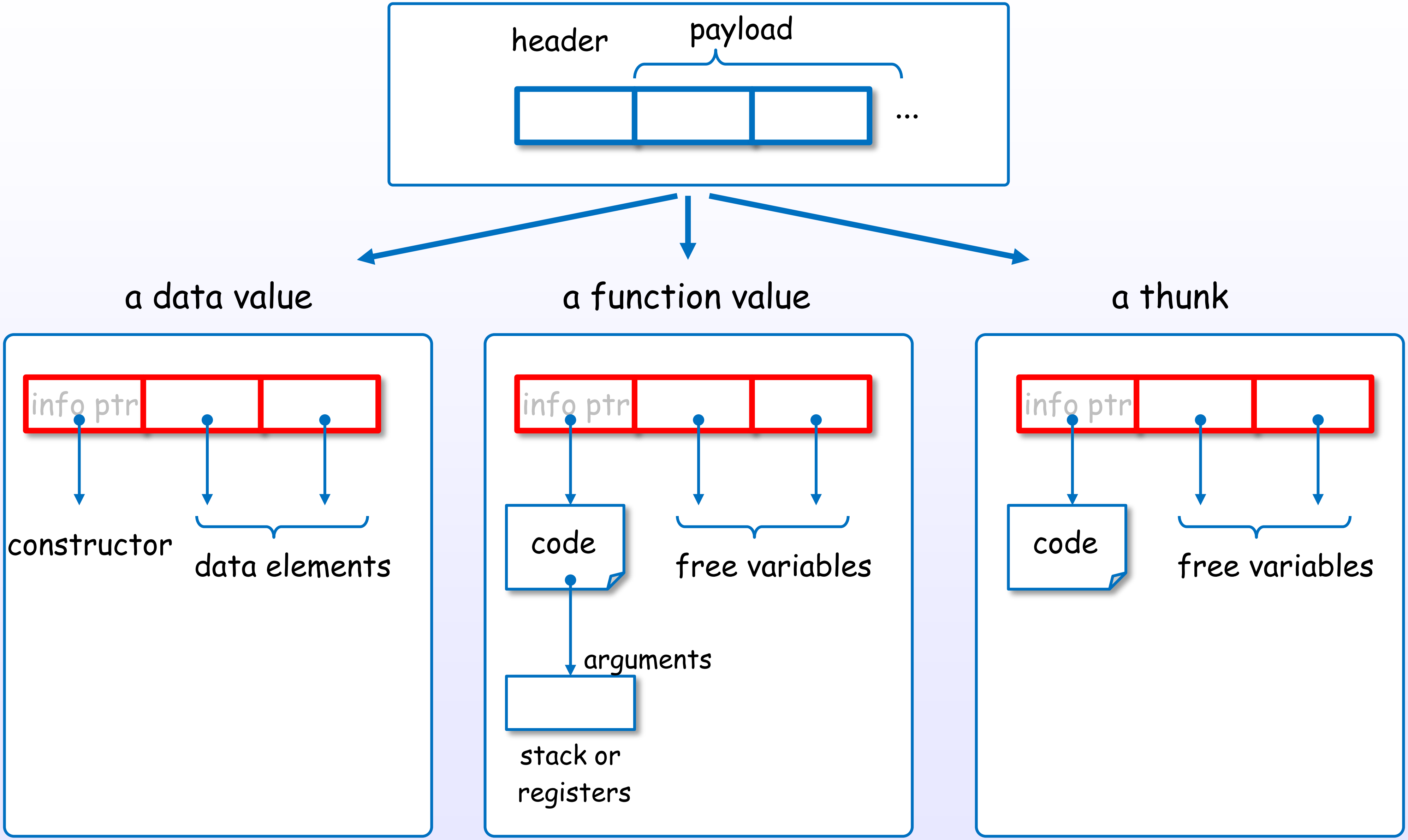
References : [1]

統一内部表現



References : [1]

統一内部表現



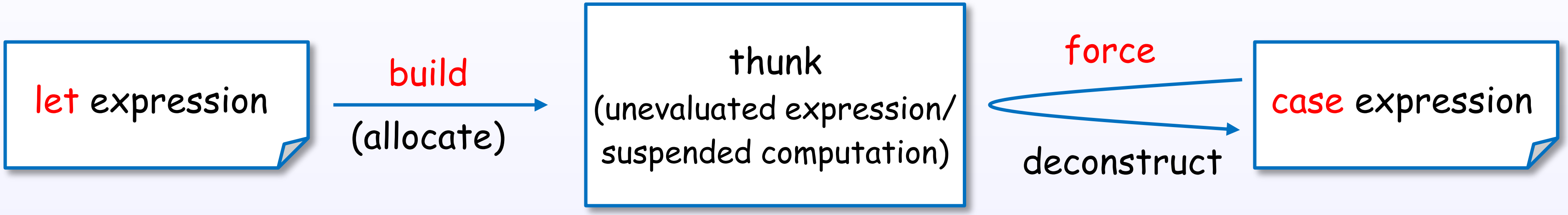
いずれも、広義の、"closure" (= code + environment(free variables))

References : [1]

3. Internal representation of expressions

let, case expression

let/case expressions and thunk



let expression

build

(allocate)

thunk
(unevaluated expression/
suspended computation)

force

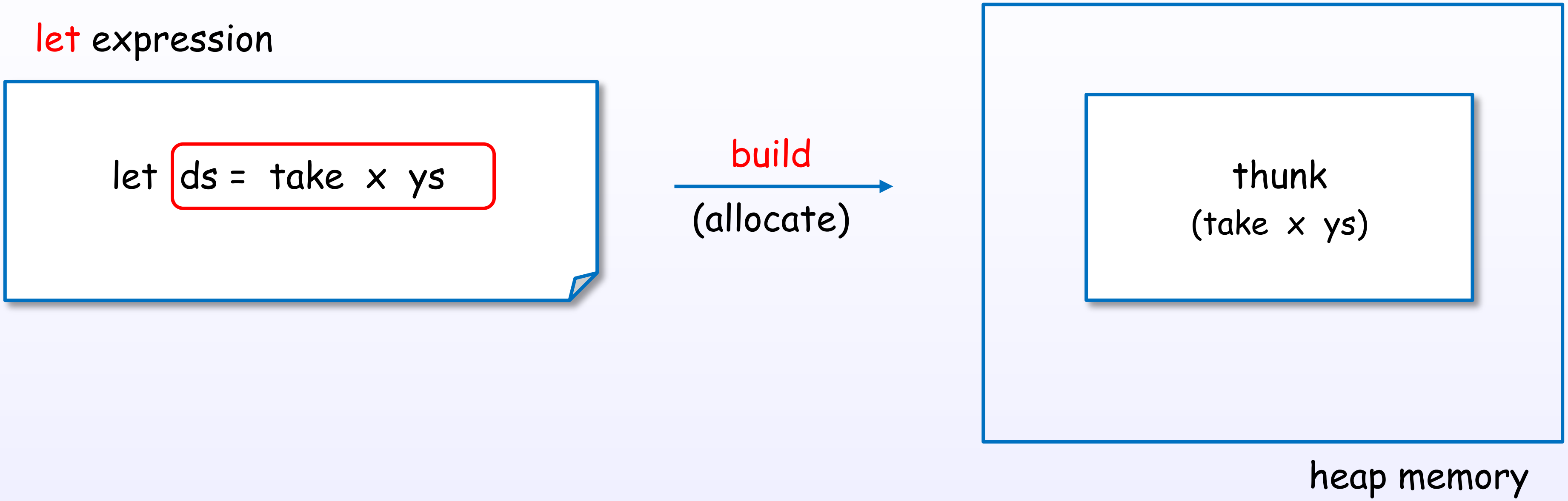
deconstruct

case expression

A let expression may build a thunk.
A case expression forces and deconstructs the thunk.

References : [1]

A let expression builds a thunk



let expression

let ds = take x ys

build

(allocate)

thunk

(take x ys)

heap memory

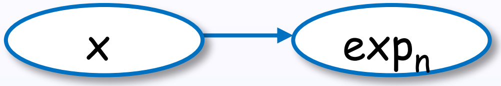
References : [1]

いろいろな表現

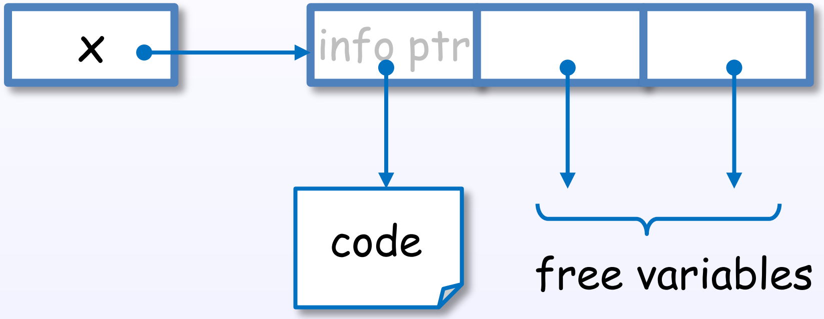
Haskell code

let x = exp_n

Expression

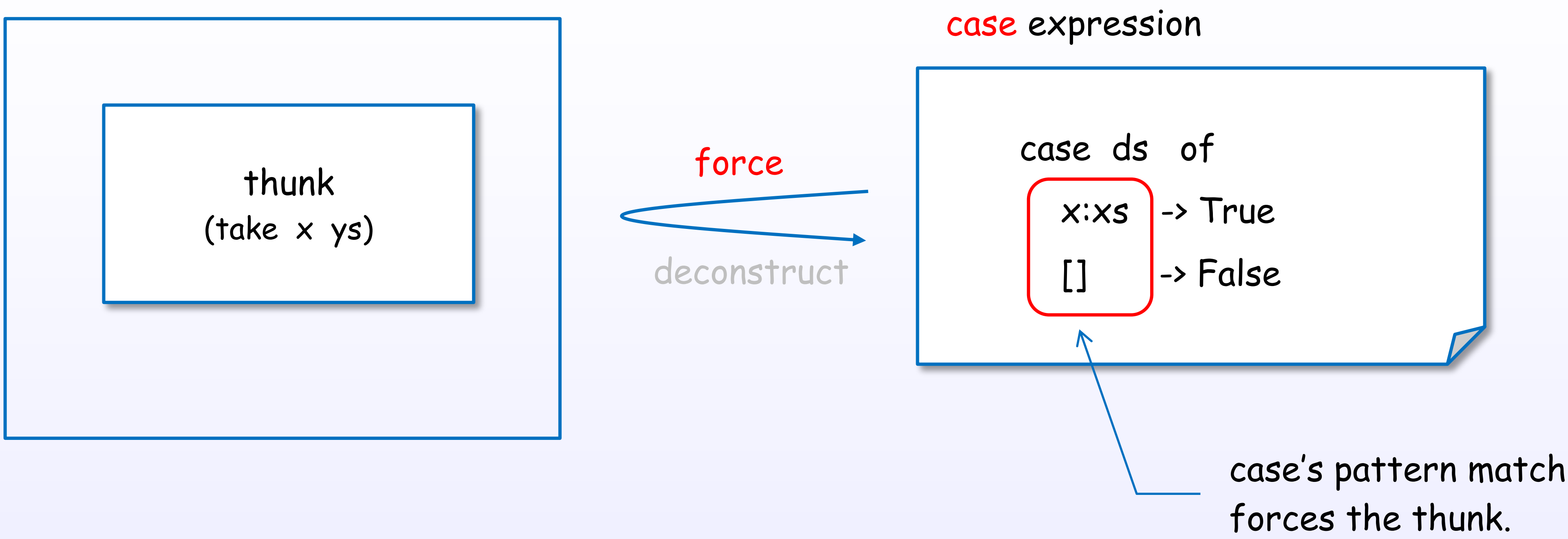


GHC internal representing



References : [1]

A case expression forces a thunk



thunk
(take x ys)

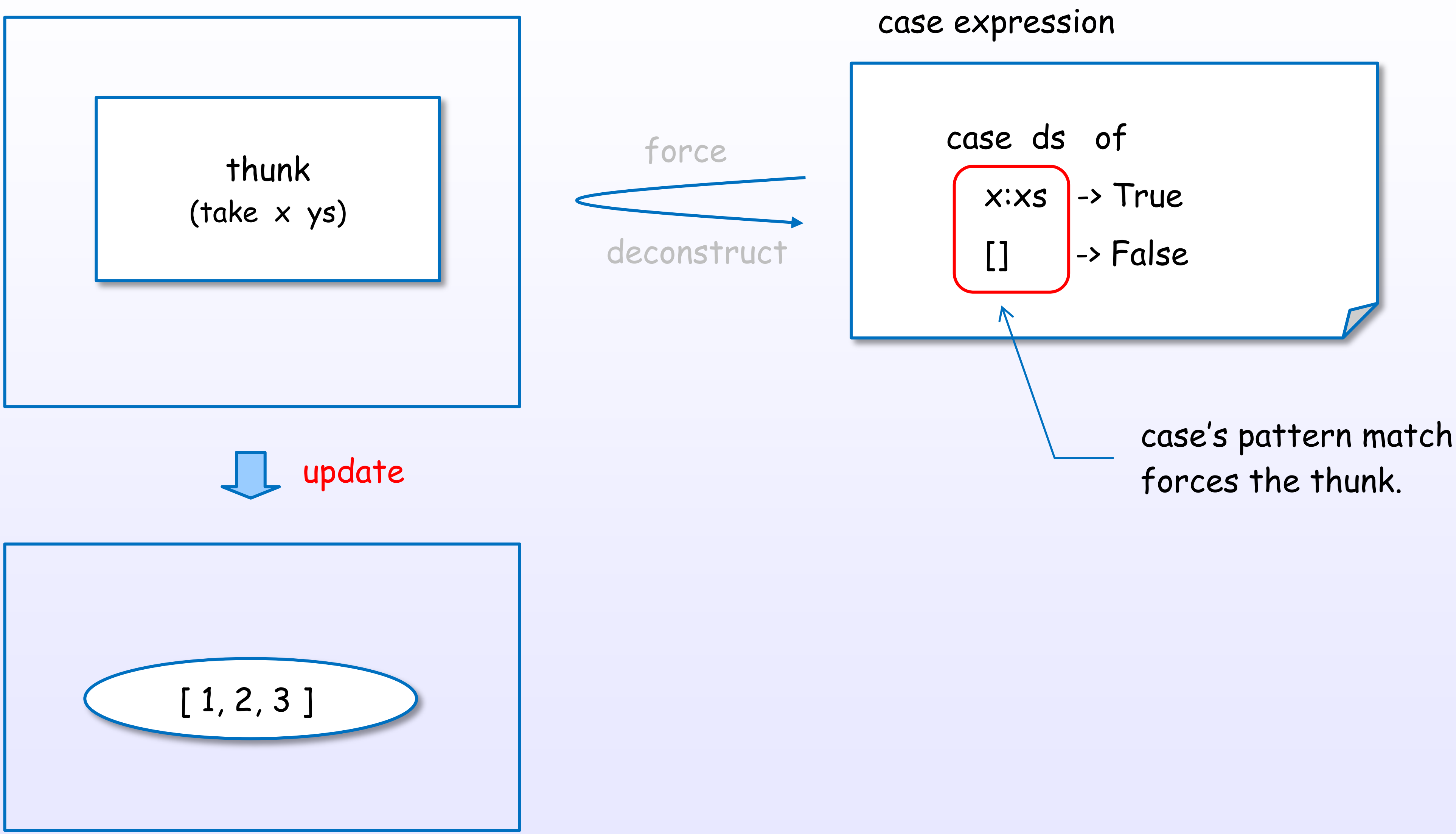
case expression

```
case ds of
  x:xs -> True
  []   -> False
```

case's pattern match forces the thunk.

References : [1]

A case expression forces a thunk



thunk
(take x ys)

force

deconstruct

update

[1, 2, 3]

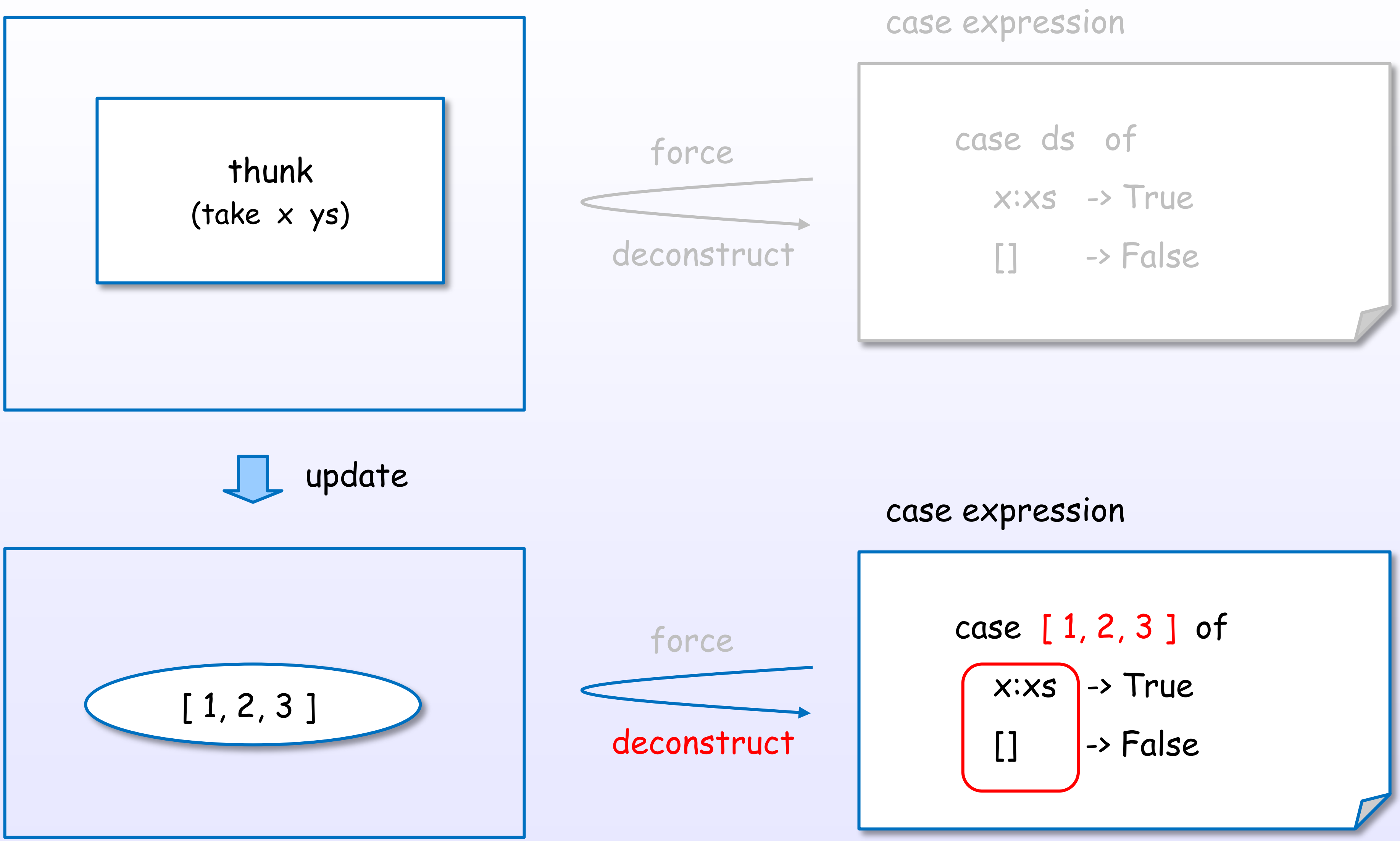
case expression

```
case  ds   of
    x:xs   -> True
    []     -> False
```

case's pattern match
forces the thunk.

References : [1]

A case expression forces a thunk



thunk
(take x ys)

force

deconstruct

case expression

```
case ds of
    x:xs  -> True
    []    -> False
```

update

[1, 2, 3]

force

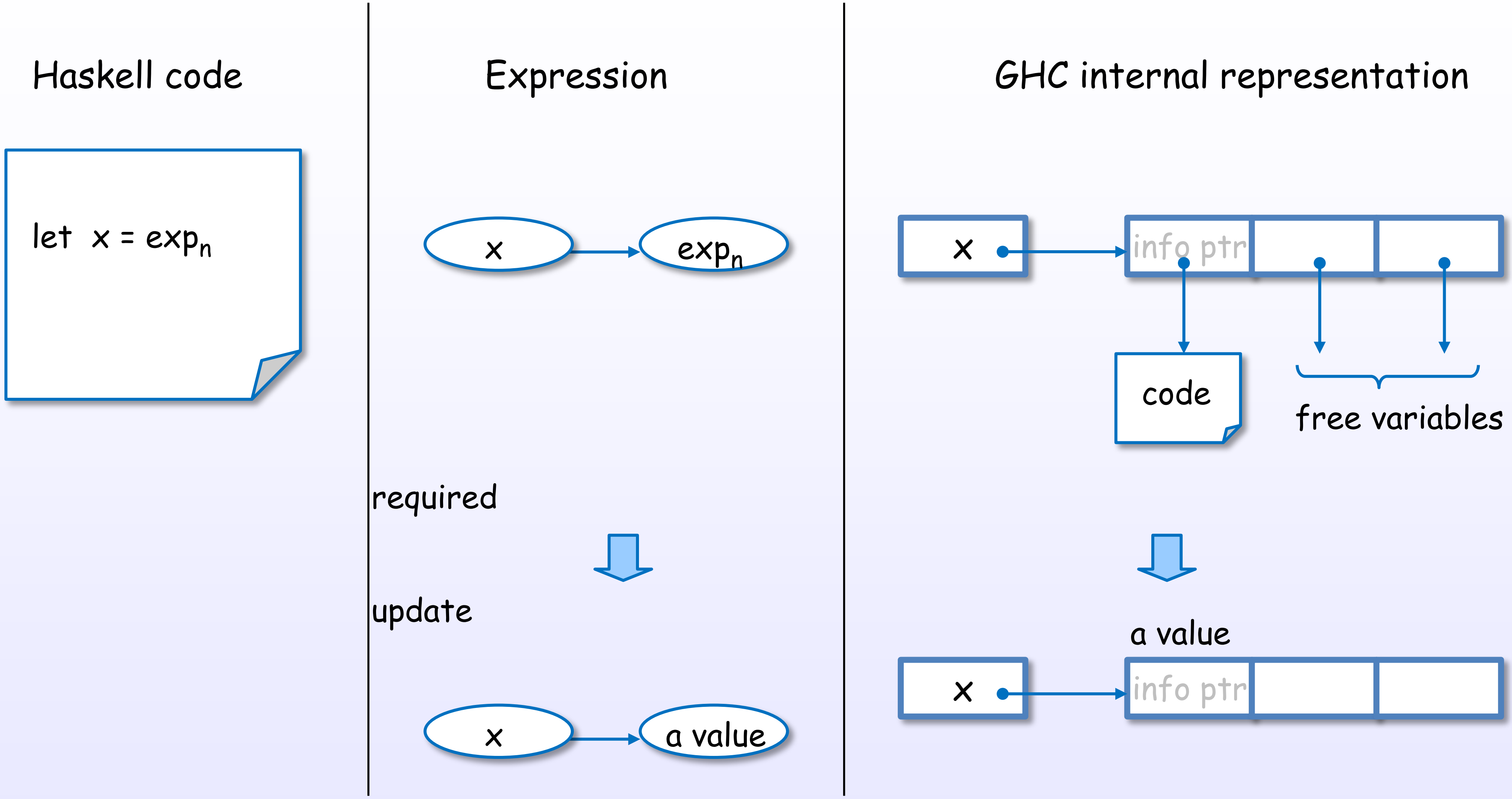
deconstruct

case expression

```
case [ 1, 2, 3 ] of
    x:xs  -> True
    []    -> False
```

References : [1]

forcing and update



Haskell code

let x = exp_n

Expression

x → exp_n

required

update

x → a value

GHC internal representation

x → info ptr

code

free variables

a value

x → info ptr

when a variable is bound
it is generally bound to an unevaluated closure allocated in the heap

このイメージを伝える

References : [1]

3. Internal representation of expressions

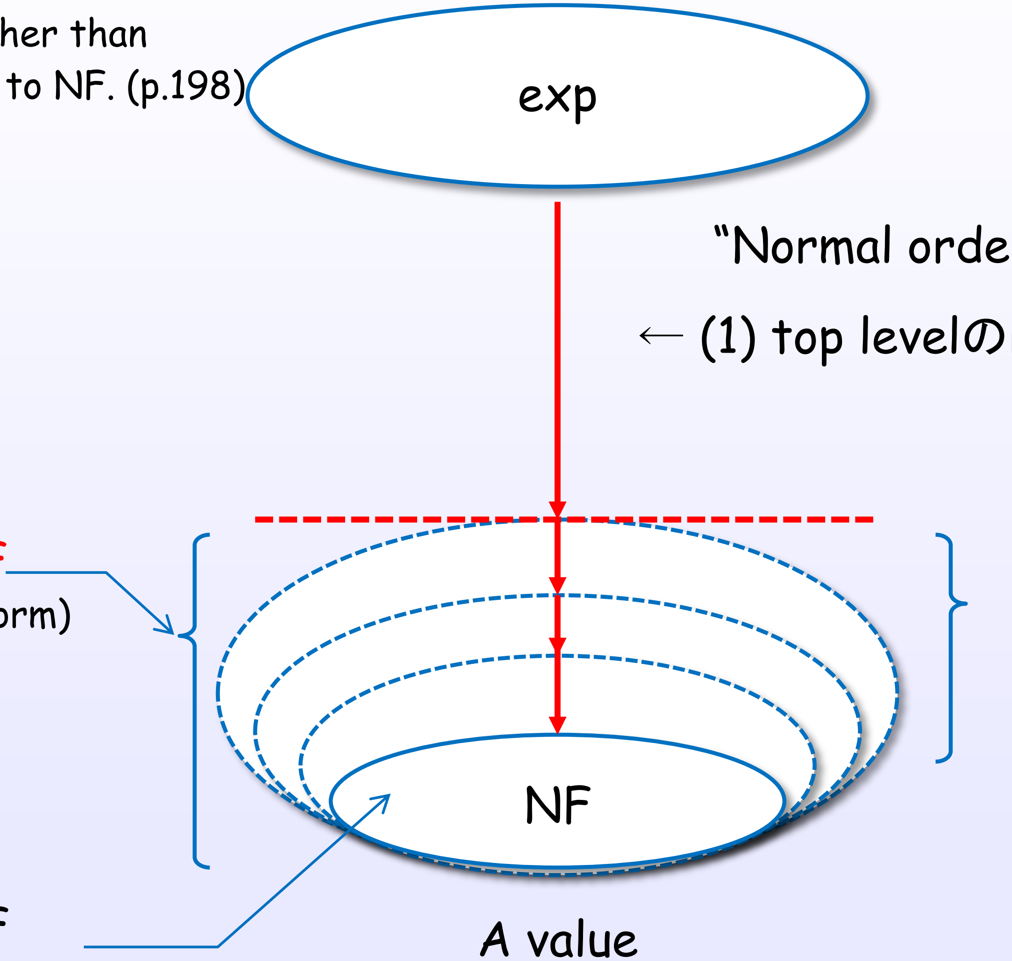
WHNF

evaluation step (GHC)

Our reduction order is therefore to reduce the top-level redex until weak head normal form is reached. (p.198)

We pursue normal order reduction, but stop at WHNF rather than proceeding all the way to NF. (p.198)

An expression



“Normal order reduction of top-level rede

← (1) top levelのreduction

[slpj-book-1987]

← (2) innerのreduction

“Normal order reduction of

A value

no more evaluation (reduction)

[Terei]

[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

References : [1]

再掲

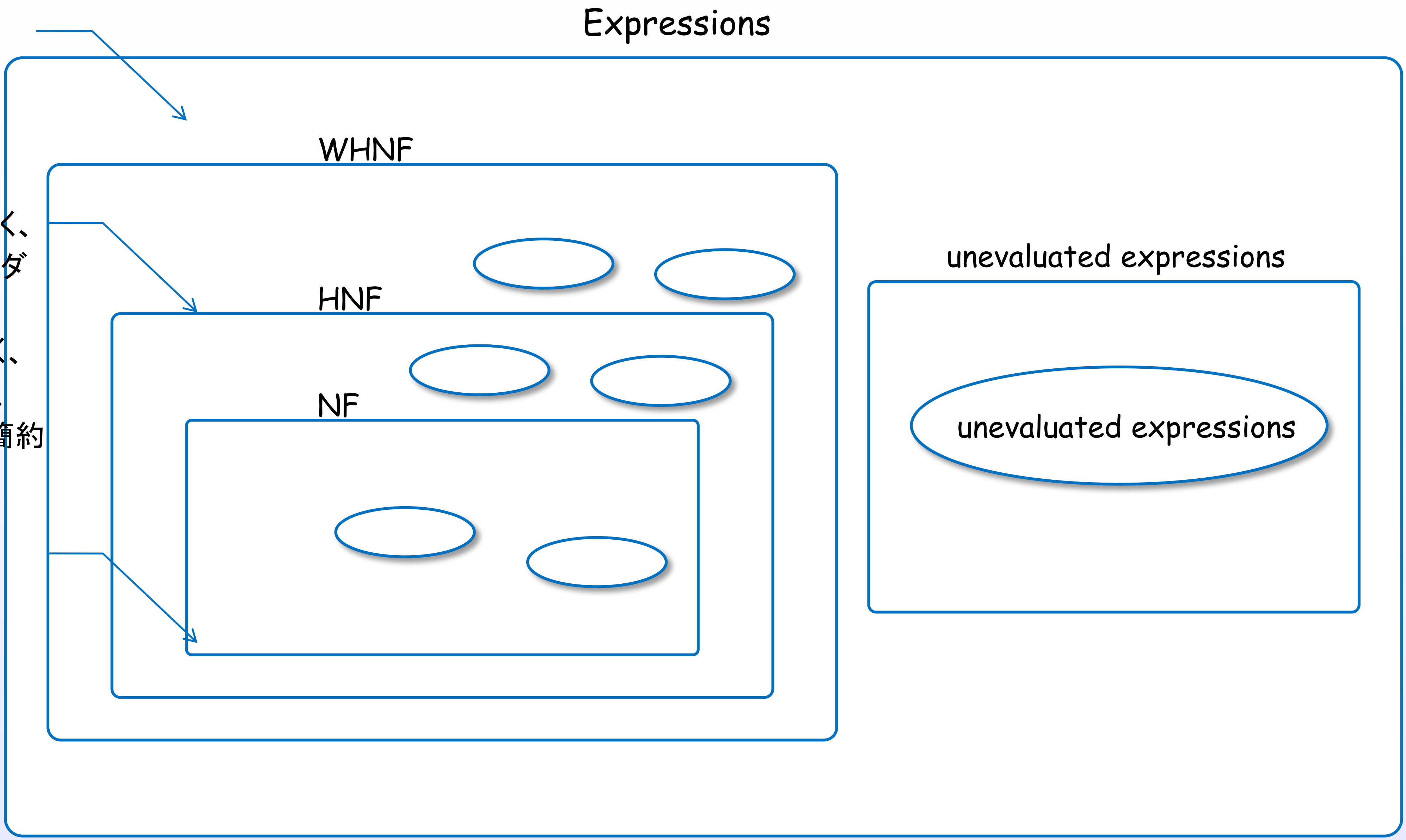
evaluation level

ゞ

でなく、
ラムダ

でなく、
合は、
も簡約

い。
）



[Te

n
a

h
a
a

w
a

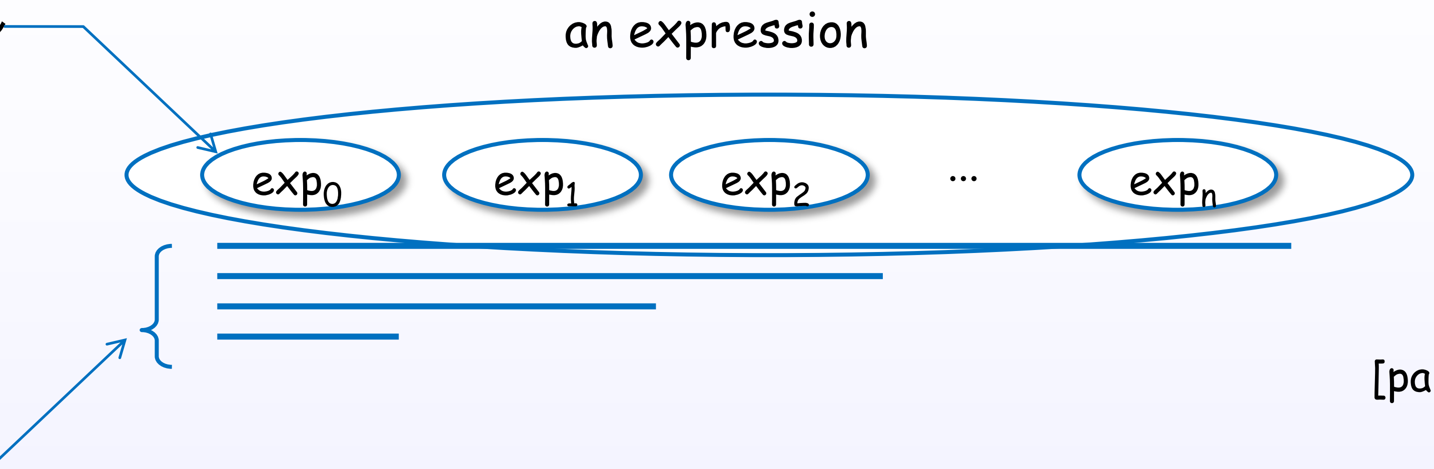
値には、評価レベルがある。

[STG]

References : [1]

WHNF

ダ抽象、ビルトイン



[parconc, Ch.2]

more

An expression has no top level redex, if it is in WHNF.

[slpj-book-1987]

These are in weak head normal form,
but not in normal form, since they contain inner redex. (p.198)

[Terei]

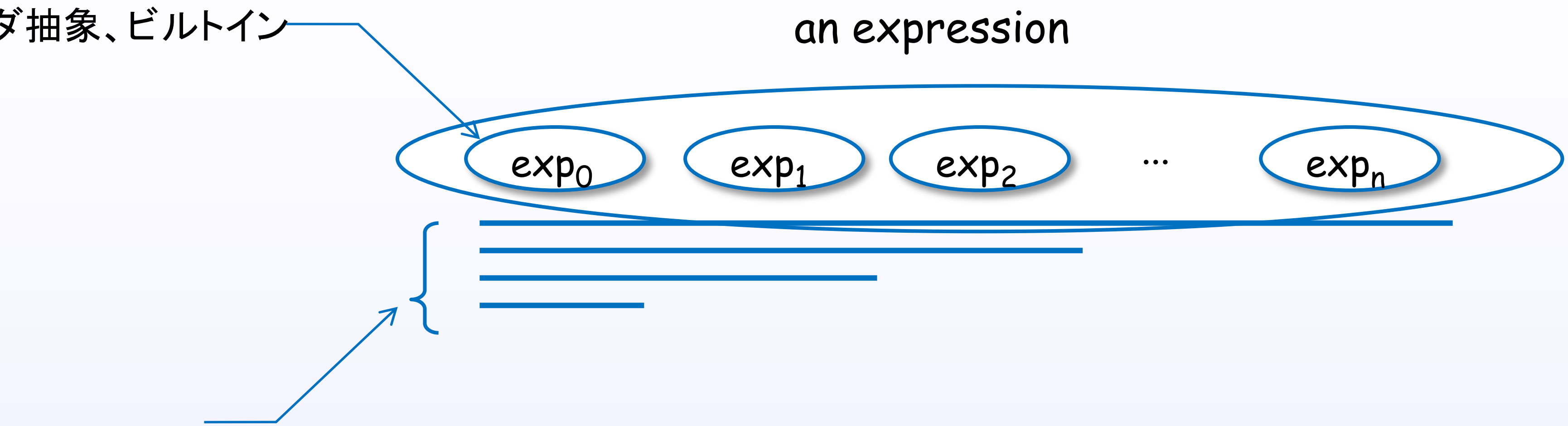
[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

[Terei]

References : [1]

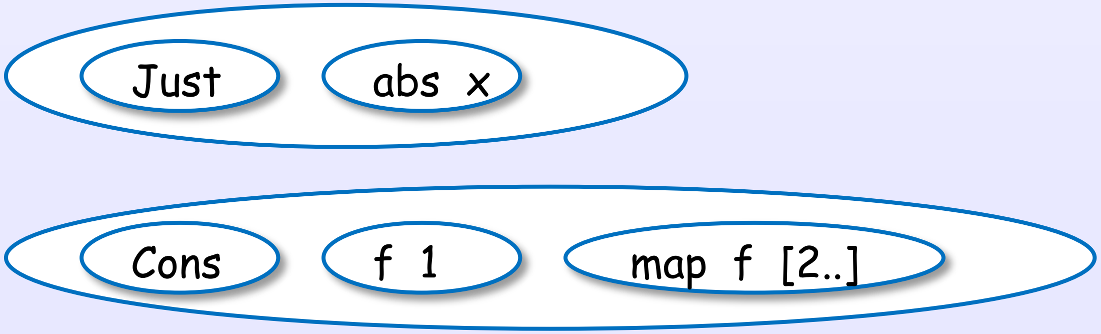
Examples of WHNF



more

Just (abs x)

Cons (f 1) (map f [2..])



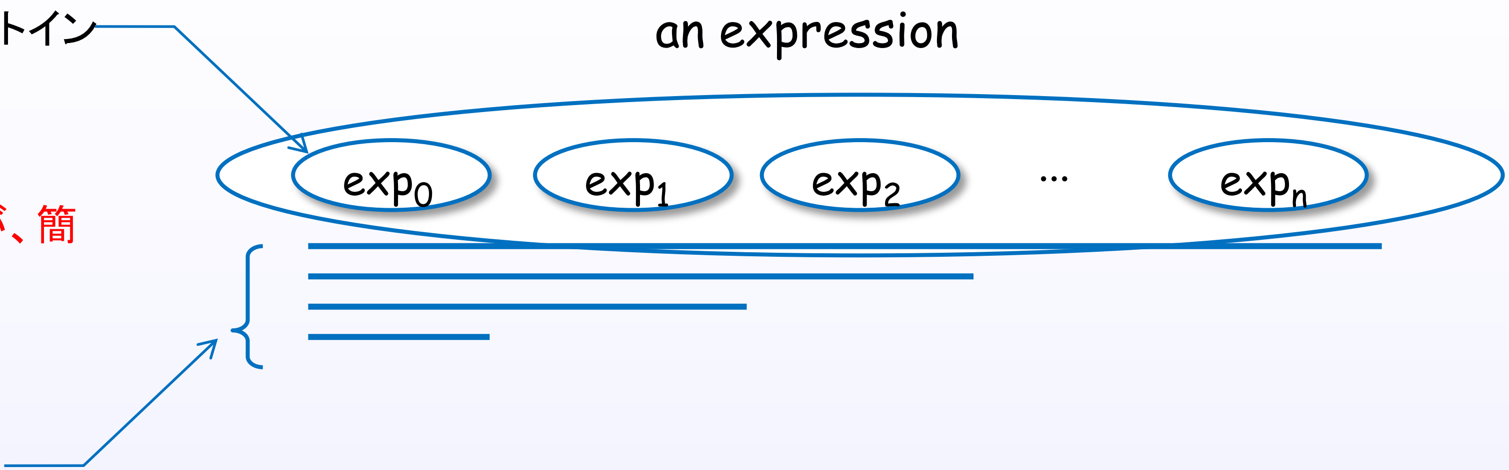
[slpj-book-1987]

References : [1]

HNF

ダ抽象、ビルトイン

内側(body)が、簡



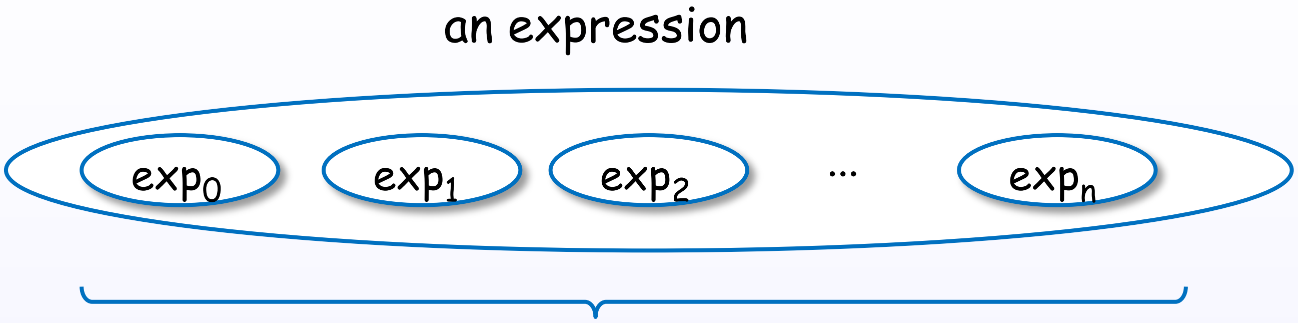
more

[slpj-book-1987]

[Terei]

References : [1]

NF



redexが内部に無い

[slpj-book-1987]

[Terei]

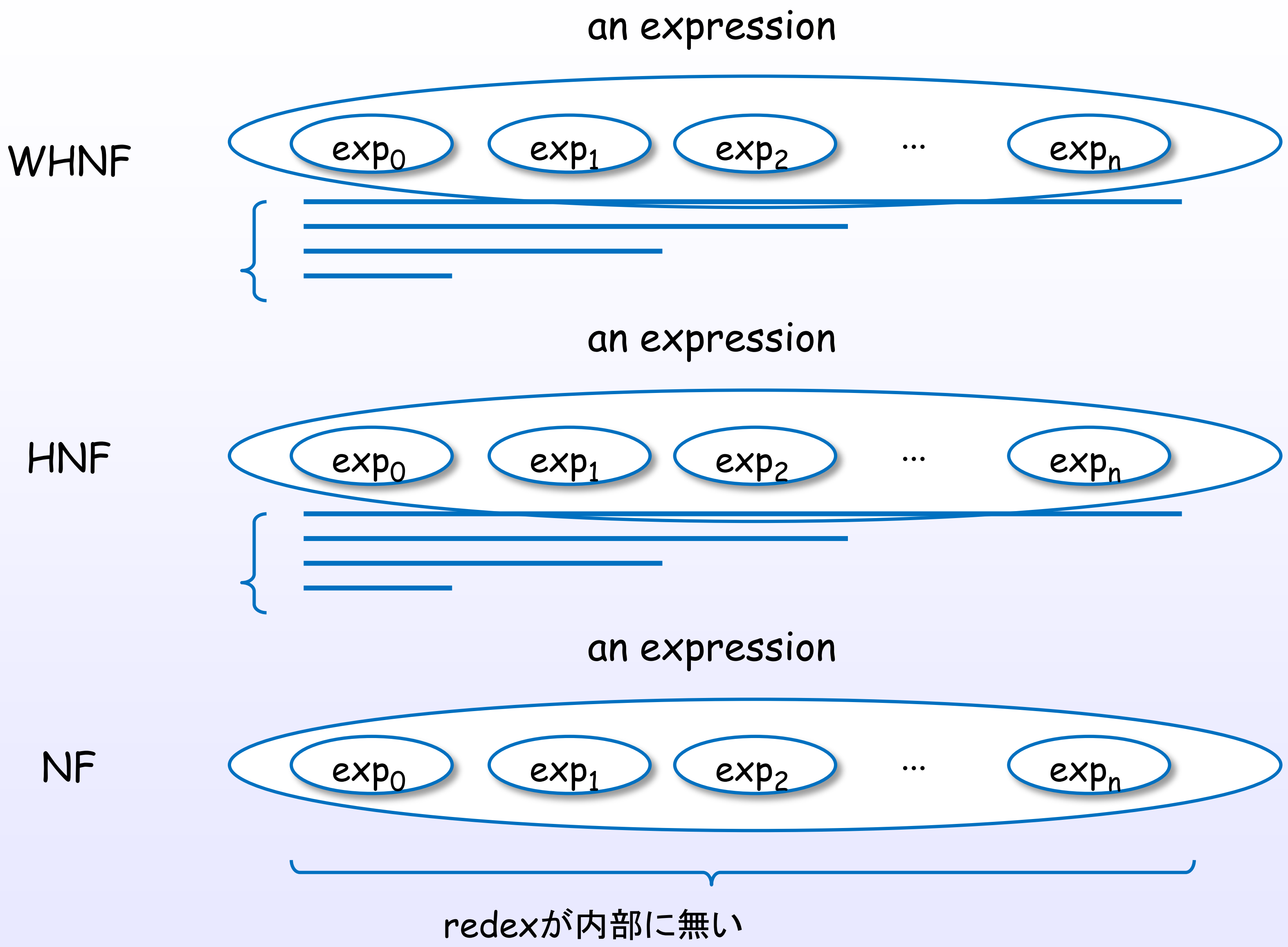
[Bird, Chapter 2, 7]

[TAPL, Chapter 3]

[Terei]

References : [1]

WHNF, HNF, NF



an expression

WHNF

exp_0 exp_1 exp_2 … exp_n

an expression

HNF

exp_0 exp_1 exp_2 … exp_n

an expression

NF

exp_0 exp_1 exp_2 … exp_n

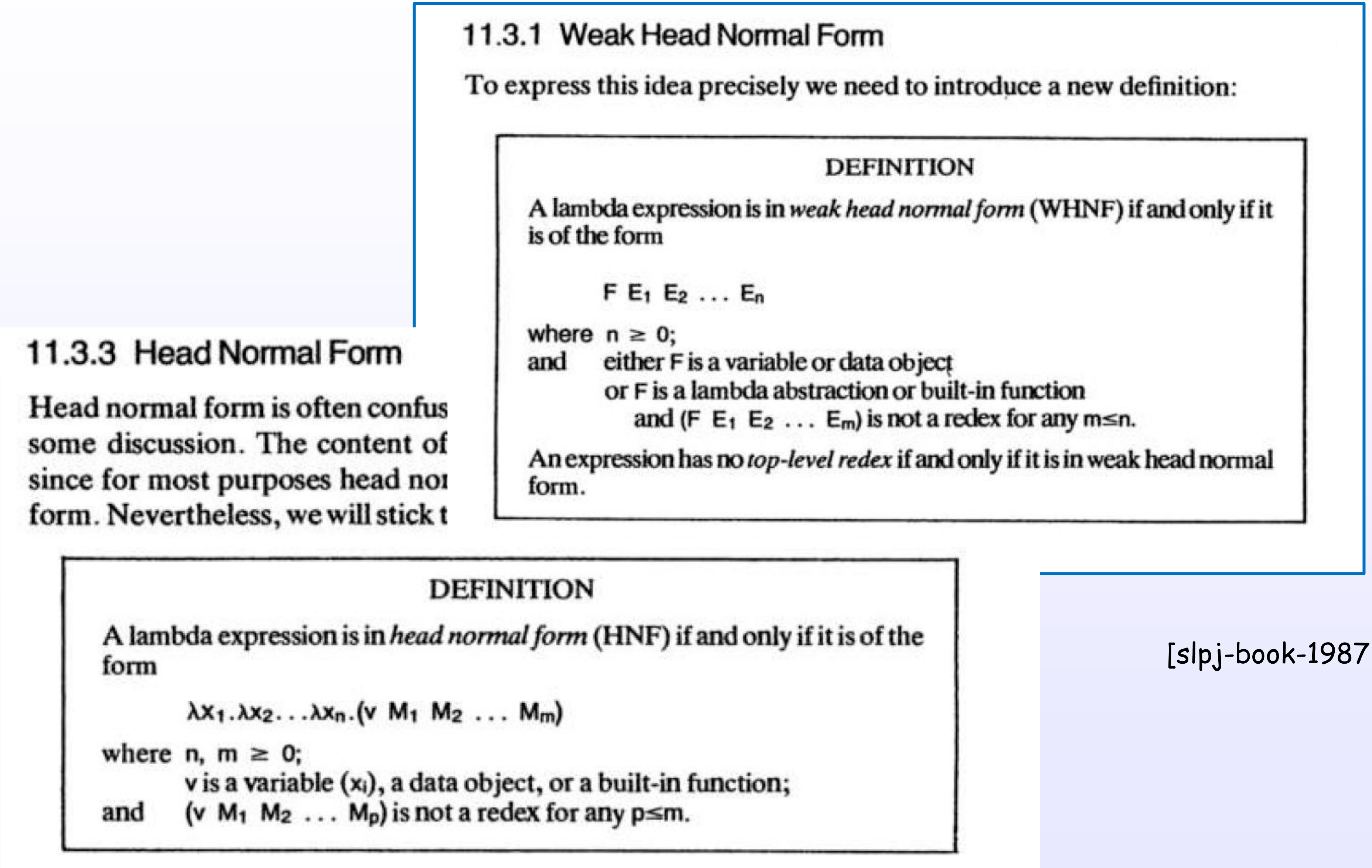
redexが内部に無い

[slpj-book-1987]

References : [1]

definition of WHNF and HNF

The implementation of functional programming languages [19]



11.3.1 Weak Head Normal Form

To express this idea precisely we need to introduce a new definition:

DEFINITION

A lambda expression is in *weak head normal form* (WHNF) if and only if it is of the form

$$F\ E_1\ E_2\ \ldots\ E_n$$

where $n \geq 0$;
and either F is a variable or data object
or F is a lambda abstraction or built-in function
and $(F\ E_1\ E_2\ \ldots\ E_m)$ is not a redex for any $m \leq n$.

An expression has no *top-level redex* if and only if it is in weak head normal form.

11.3.3 Head Normal Form

Head normal form is often confus
some discussion. The content of
since for most purposes head no
form. Nevertheless, we will stick t

DEFINITION

A lambda expression is in *head normal form* (HNF) if and only if it is of the form

$$\lambda x_1.\lambda x_2 \ldots \lambda x_n.(v\ M_1\ M_2\ \ldots\ M_m)$$

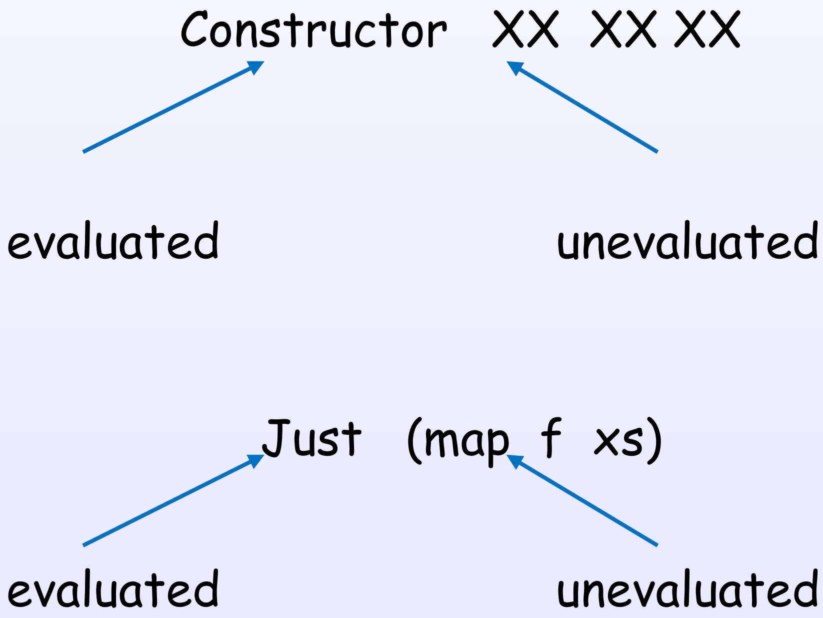
where $n, m \geq 0$;
v is a variable (x_i), a data object, or a built-in function;
and $(v\ M_1\ M_2\ \ldots\ M_p)$ is not a redex for any $p \leq m$.

[slpj-book-1987]

References : [1]

internal representation of WHNF

heap objectイメージ



Constructor XX XX XX

evaluated

unevaluated

Just (map f xs)

evaluated

unevaluated

References : [1]

4. Evaluation

4. Evaluation

Evaluation in Haskell (GHC)

GHC chosen lazy evaluation

必要な時に、必要な箇所のみを評価する

(STG p.11)

↓

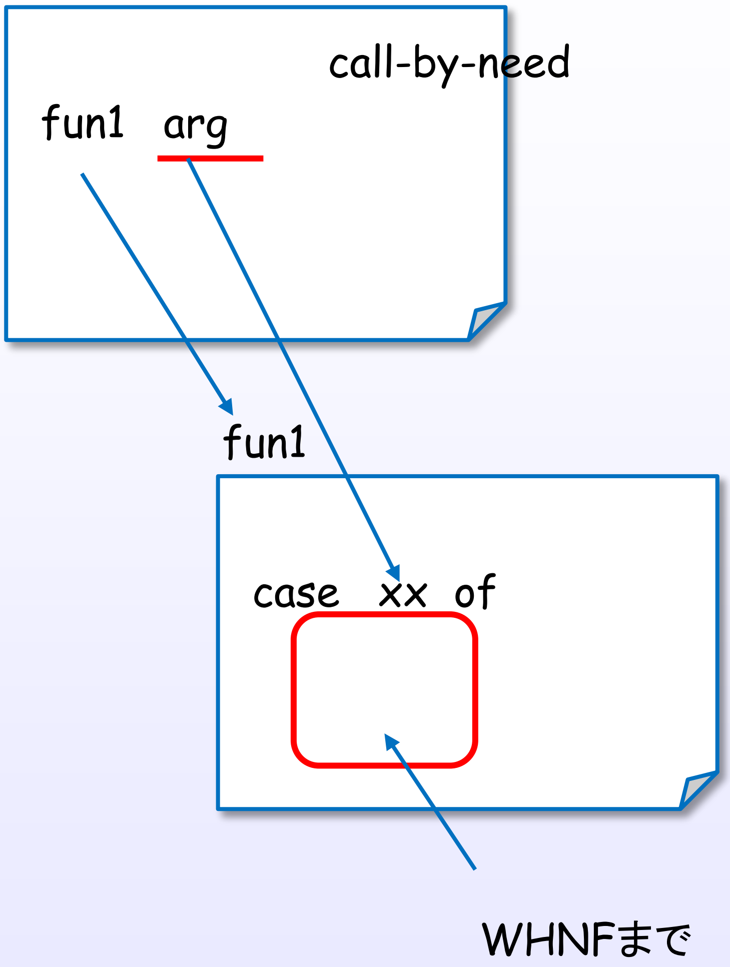
・引数評価を先送る （case式が来るまで評価しない） call-by-need

・部分式を完全評価しない （caseのパターンマッチで参照するところのみを評価する）WHNF

これは、計算量を最小化する戦略（メモリ量でなく）

References : [1]

eval 全体のイメージ



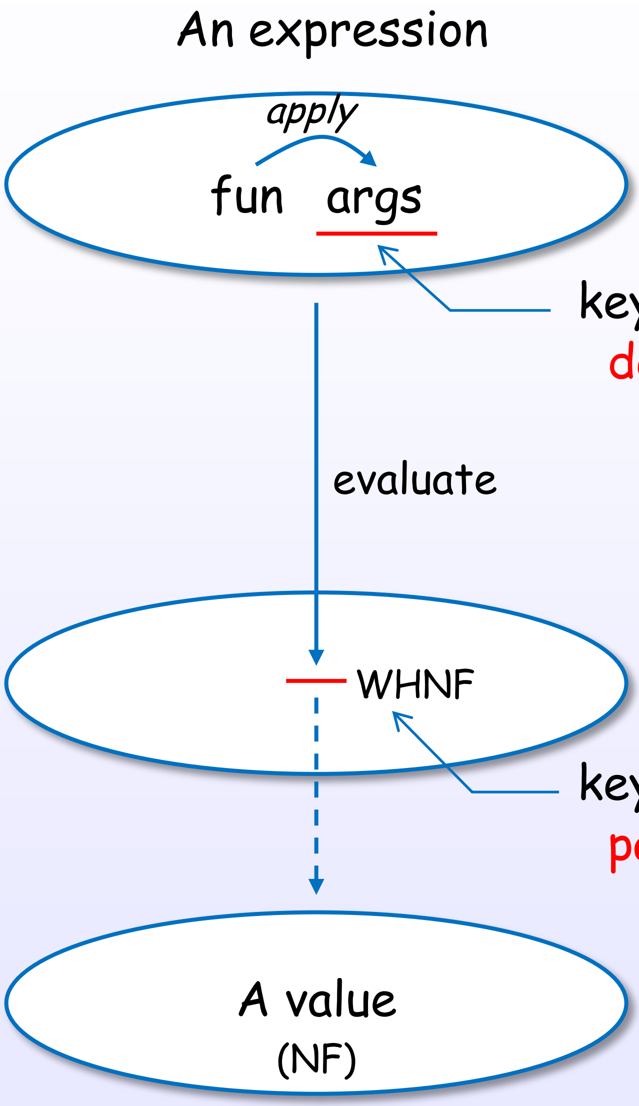
・call-by-need
・WHNFまで
・caseで駆動
・パターンマッチ

を、１枚の絵に

ページ順番の変更を

References : [1]

Key concept of Haskell's lazy evaluation

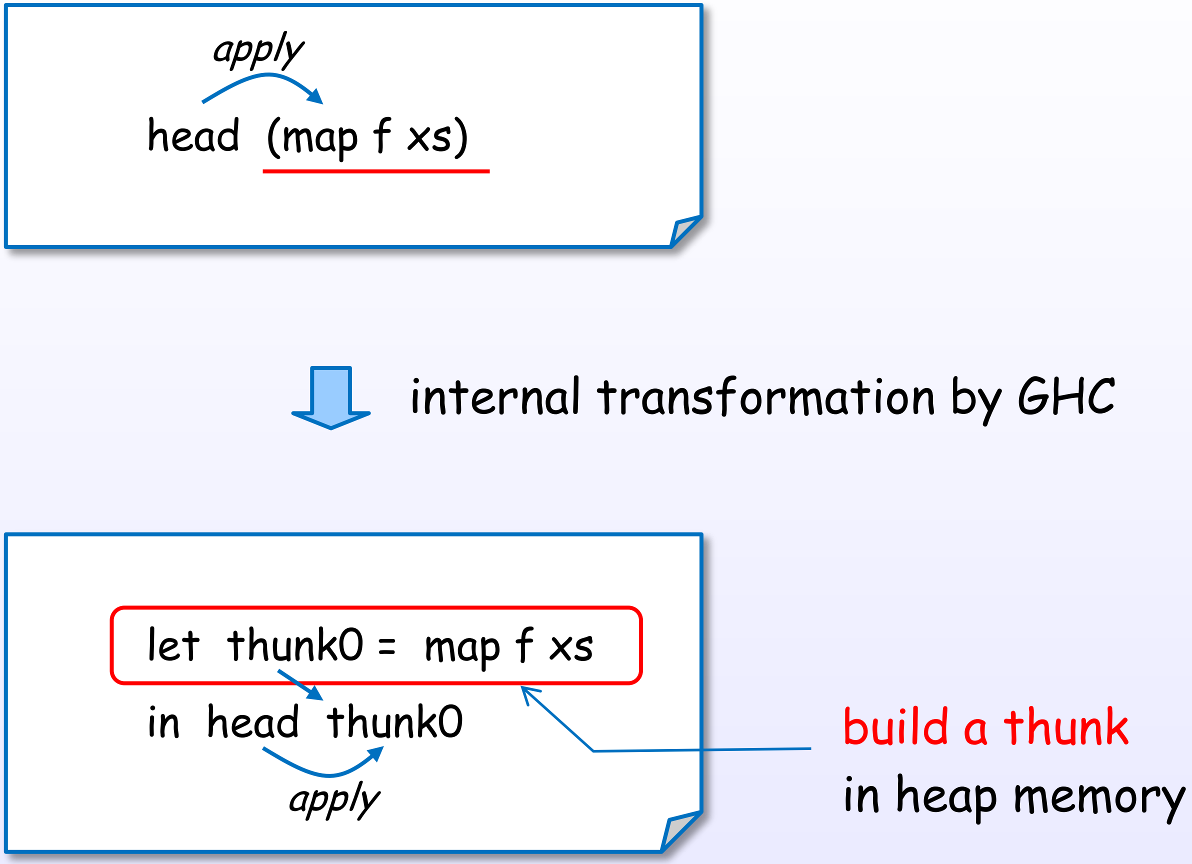


key point 1 :
delayed evaluation of arguments

key point 2 :
partial evaluation up to WHNF

References : [1]

key point 1 : delayed evaluation of arguments



apply

```
head  (map f xs)
```

internal transformation by GHC

```
let  thunk0 =  map f xs
in  head  thunk0
```

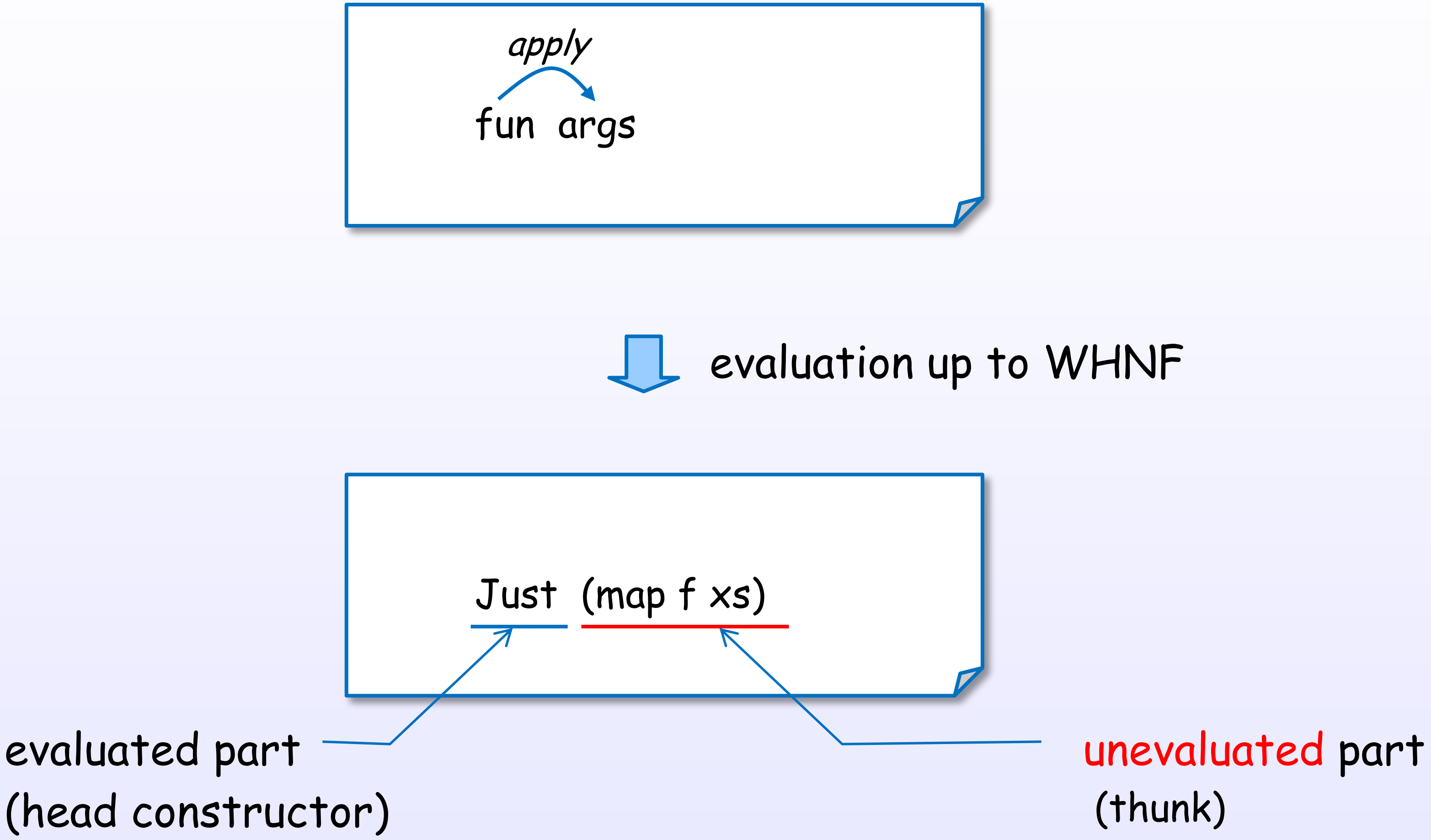
apply

build a thunk
in heap memory

GHC implements lazy evaluation using the thunk.
Evaluation of arguments is delayed with the thunk.

References : [1]

key point 2 : partial evaluation up to WHNF



apply

fun args

evaluation up to WHNF

Just (map f xs)

evaluated part
(head constructor)

unevaluated part
(thunk)

GHC can partially evaluate a expression.
Constructor can hold an unevaluated expression (a thunk).

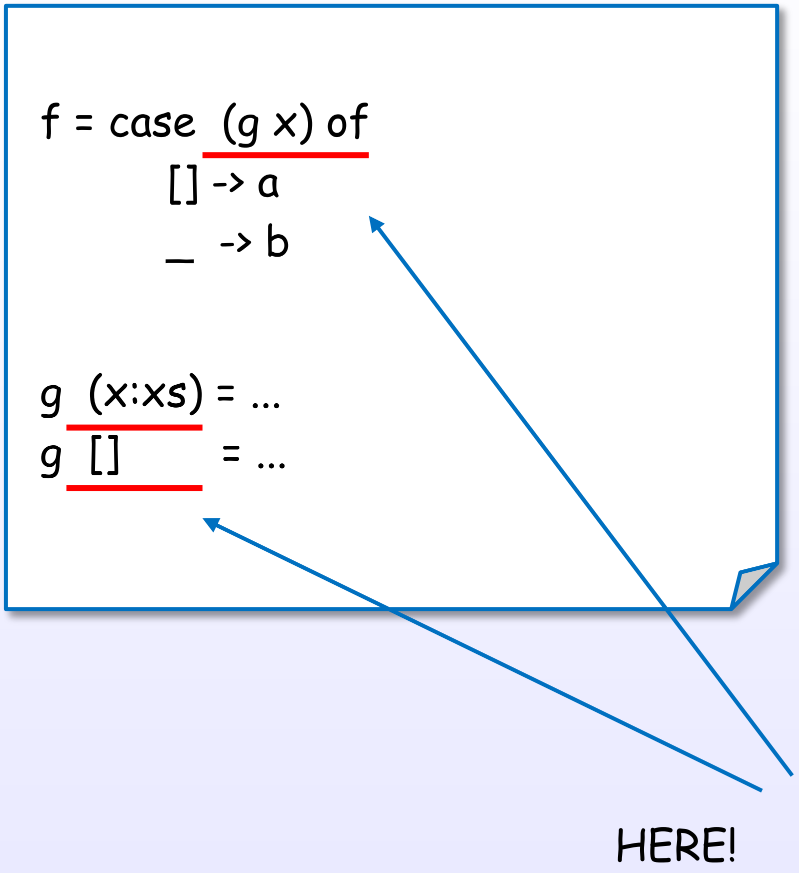
References : [1]

では、必要なときはいつか？

References : [1]

では、必要なときはいつか？

Haskell code



```
f = case  (g x) of
        [] -> a
        _   -> b


g  (x:xs) = ...
g  []       = ...
```

HERE!

pattern match via
case expression and function definition
will {cause, trigger} the evaluation

References : [1]

Pattern match

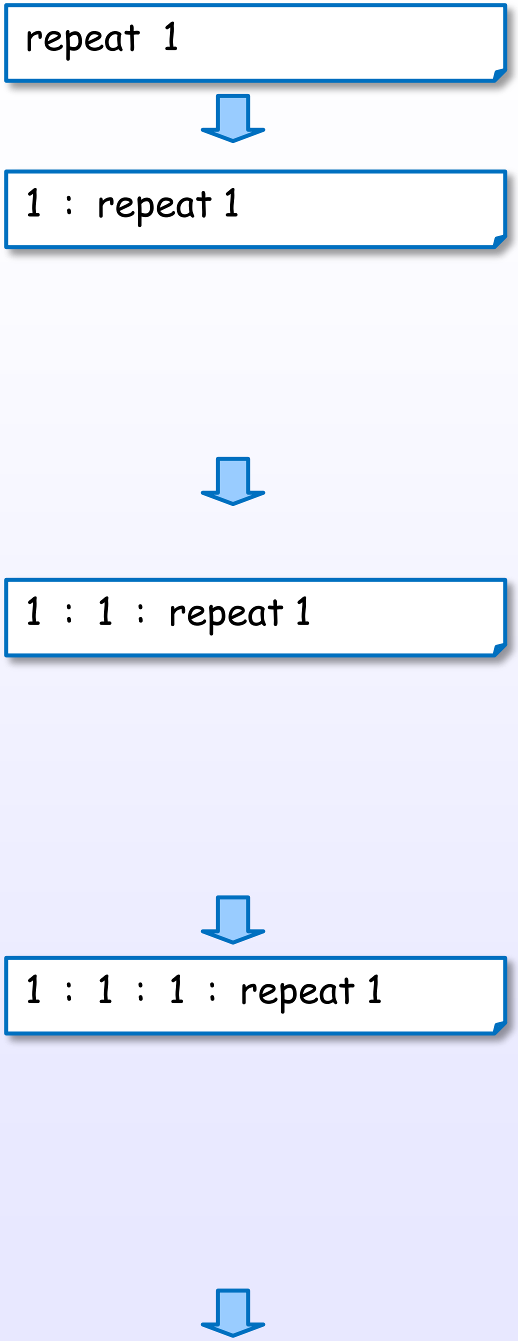
[CIS194]

References : [1]

4. Evaluation

Examples of evaluation steps

Example of repeat



repeat 1

1 : repeat 1

1 : 1 : repeat 1

1 : 1 : 1 : repeat 1

References : [1]

Example of repeat

repeat 1

1 : repeat 1

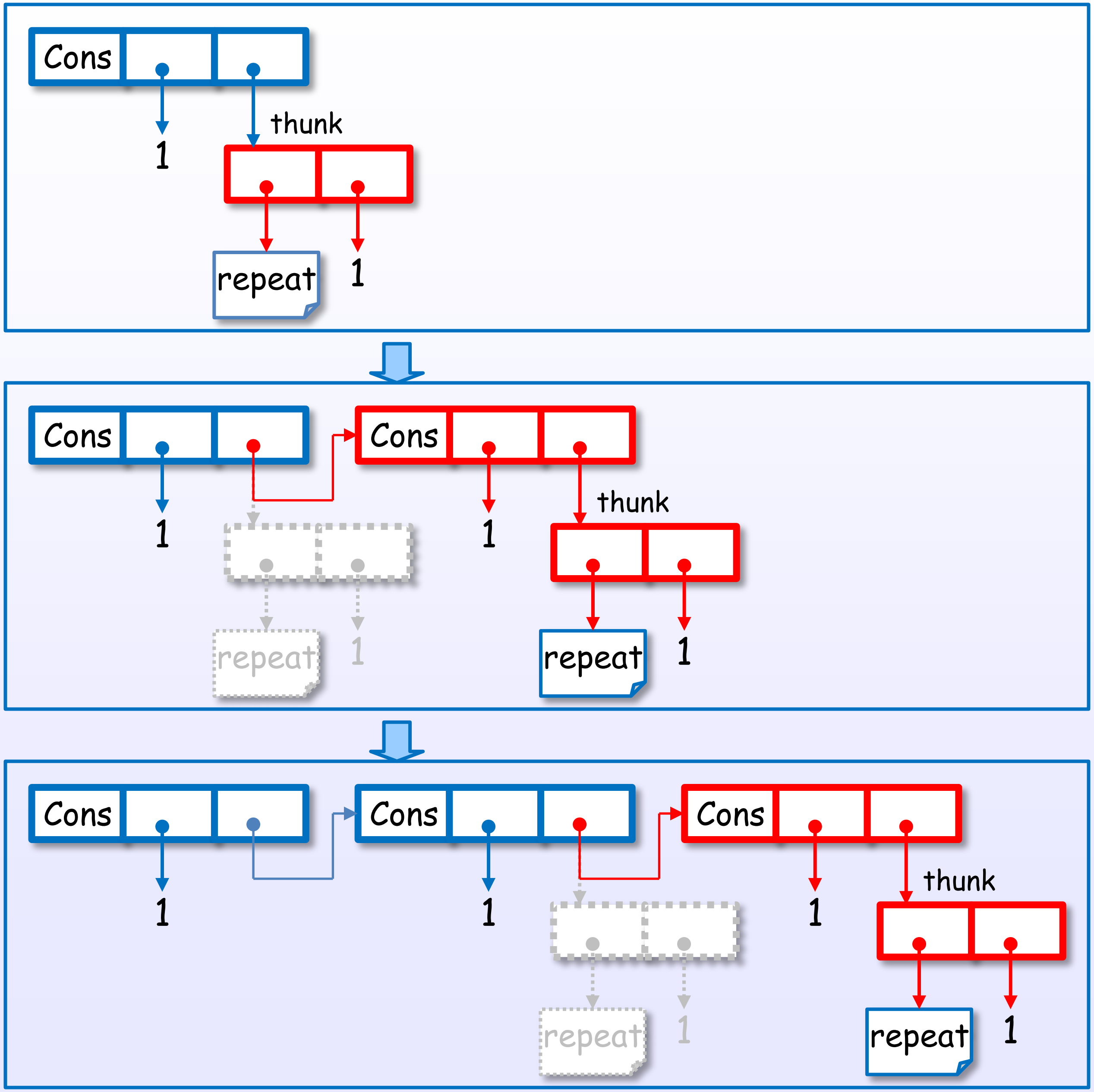
Cons 1 (repeat 1)

1 : 1 : repeat 1

Cons 1 (Cons 1 (repeat 1))

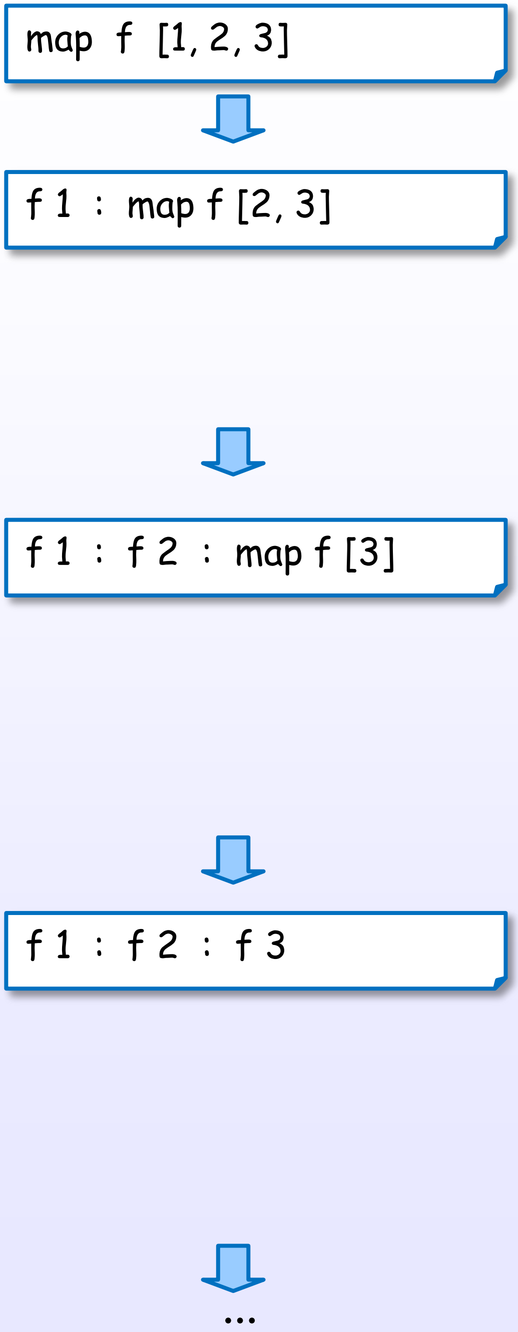
1 : 1 : 1 : repeat 1

Cons 1 (Cons 1 (Cons 1 (repeat 1)))



References : [1]

Example of map



map f [1, 2, 3]

f 1 : map f [2, 3]

f 1 : f 2 : map f [3]

f 1 : f 2 : f 3

...

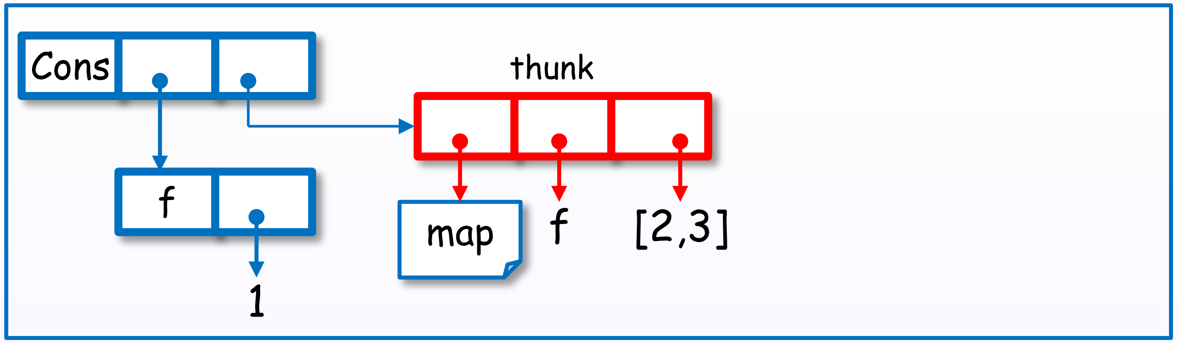
References : [1]

Example of map

map f [1, 2, 3]

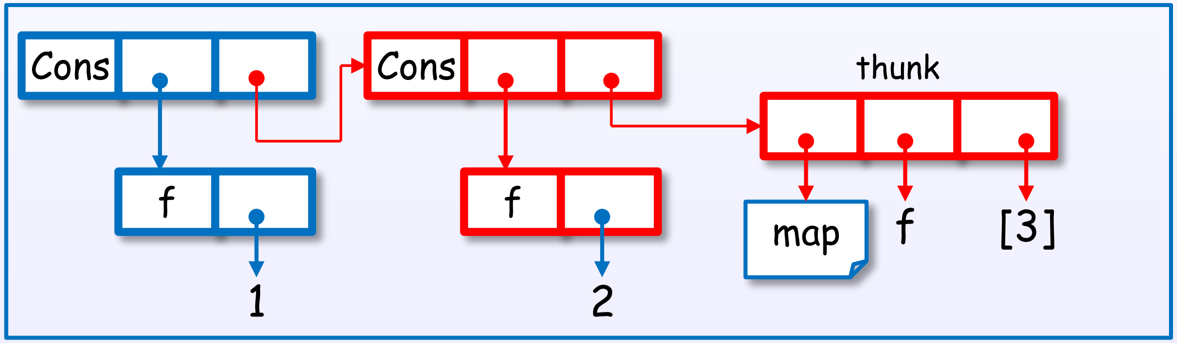
f 1 : map f [2, 3]

Cons (f 1) (map f [2, 3])



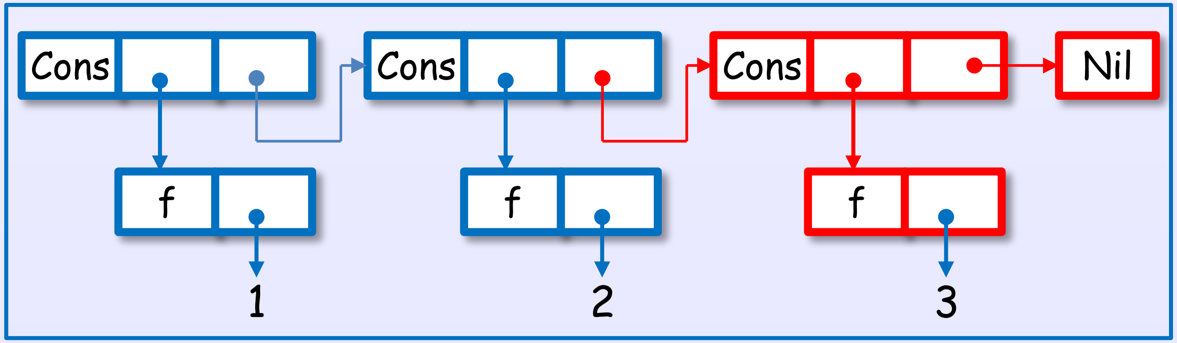
f 1 : f 2 : map f [3]

Cons (f 1) (Cons (f 2) (map f [3]))



f 1 : f 2 : f 3

Cons (f 1) (Cons (f 2) (Cons (f 3) Nil))



...

...

References : [1]

Example of foldl (non-strict)

foldl (+) 0 [1 .. 100]

⇓

foldl (+) $\underline{\texttt{(0 + 1)}}$ [2 .. 100]

⇓

foldl (+) $\underline{\texttt{((0 + 1) + 2)}}$ [3 .. 100]

⇓

foldl (+) $\underline{\texttt{(((0 + 1) + 2) + 3)}}$ [4 .. 100]

⇓

...

References : [1]

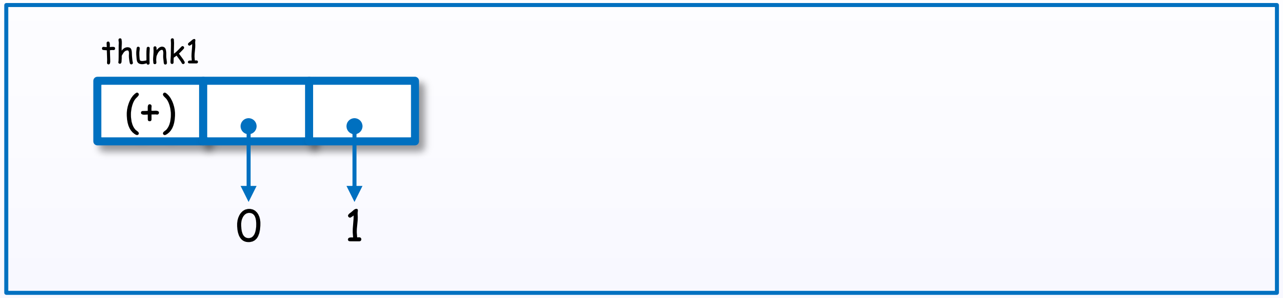
Example of foldl (non-strict)

```
foldl (+)  0  [1 .. 100]
```

```
foldl (+) (0 + 1) [2 .. 100]
```

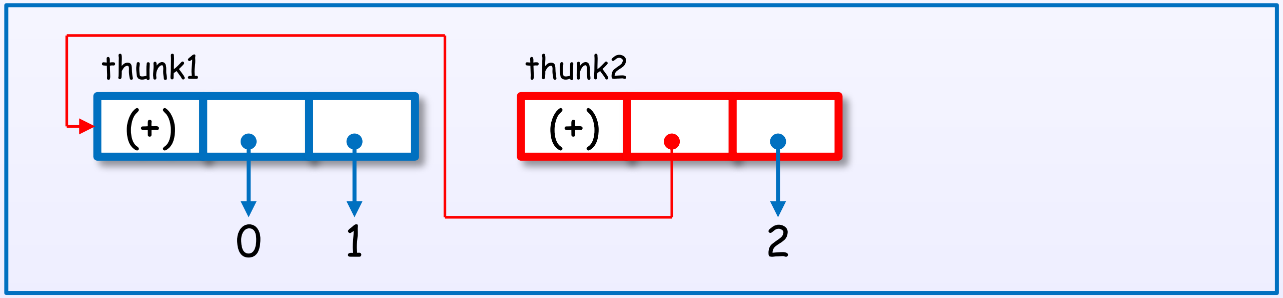
```
let  thunk1 = (0 + 1)
in foldl (+) thunk1 [2 .. 100]
```

heap memory



```
foldl  (+)  ((0 + 1) + 2)  [3 .. 100]
```

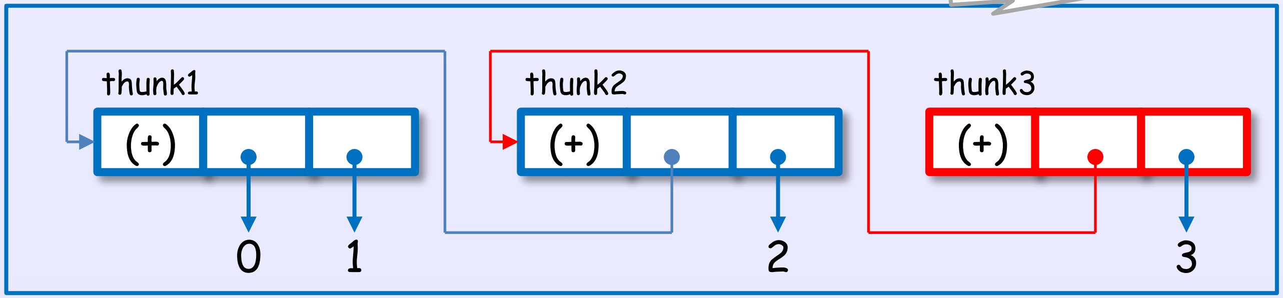
```
let  thunk2 = (thunk1 + 2)
in foldl (+) thunk2 [3 .. 100]
```



```
foldl  (+)  (((0 + 1) + 2) + 3)  [4 .. 100]
```

```
let  thunk3 = (thunk2 + 3)
in foldl (+) thunk3 [4 .. 100]
```

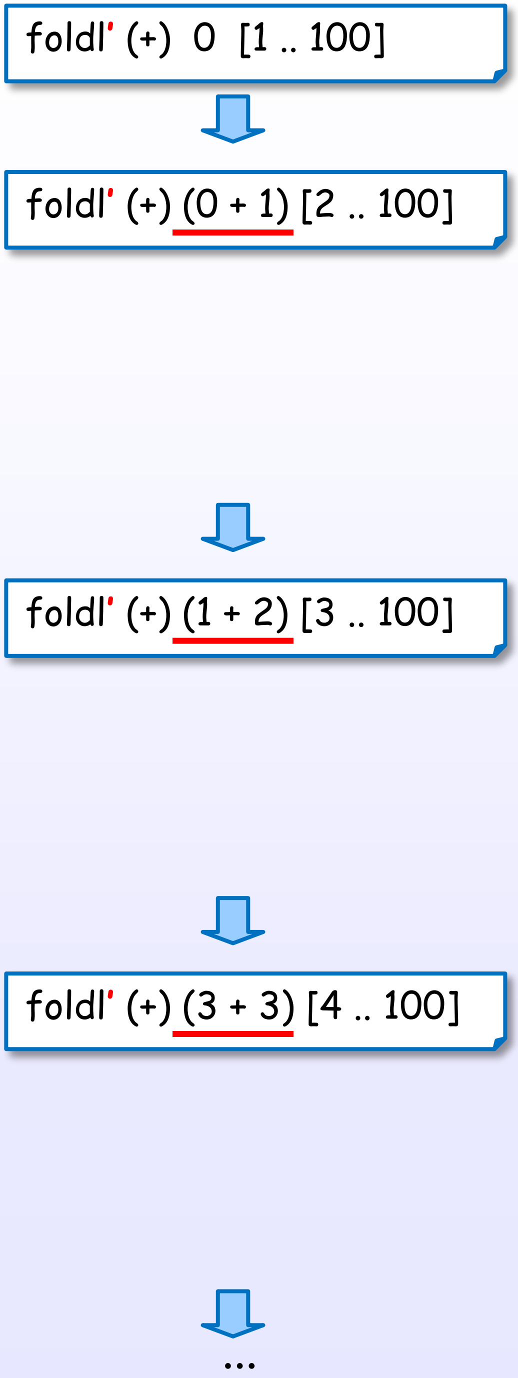
increasing heap ...



...

References : [1]

Example of foldl' (strict)



foldl' (+) 0 [1 .. 100]

foldl' (+) $\underline{(0 + 1)}$ [2 .. 100]

foldl' (+) $\underline{(1 + 2)}$ [3 .. 100]

foldl' (+) $\underline{(3 + 3)}$ [4 .. 100]

…

References : [1]

Example of foldl' (strict)

```
foldl' (+)  0  [1 .. 100]
```

```
foldl' (+) (0 + 1) [2 .. 100]
```

```
let  thunk1 = (0 + 1)
in  thunk1  `pseq`
      foldl' (+) thunk1 [2 .. 100]
```

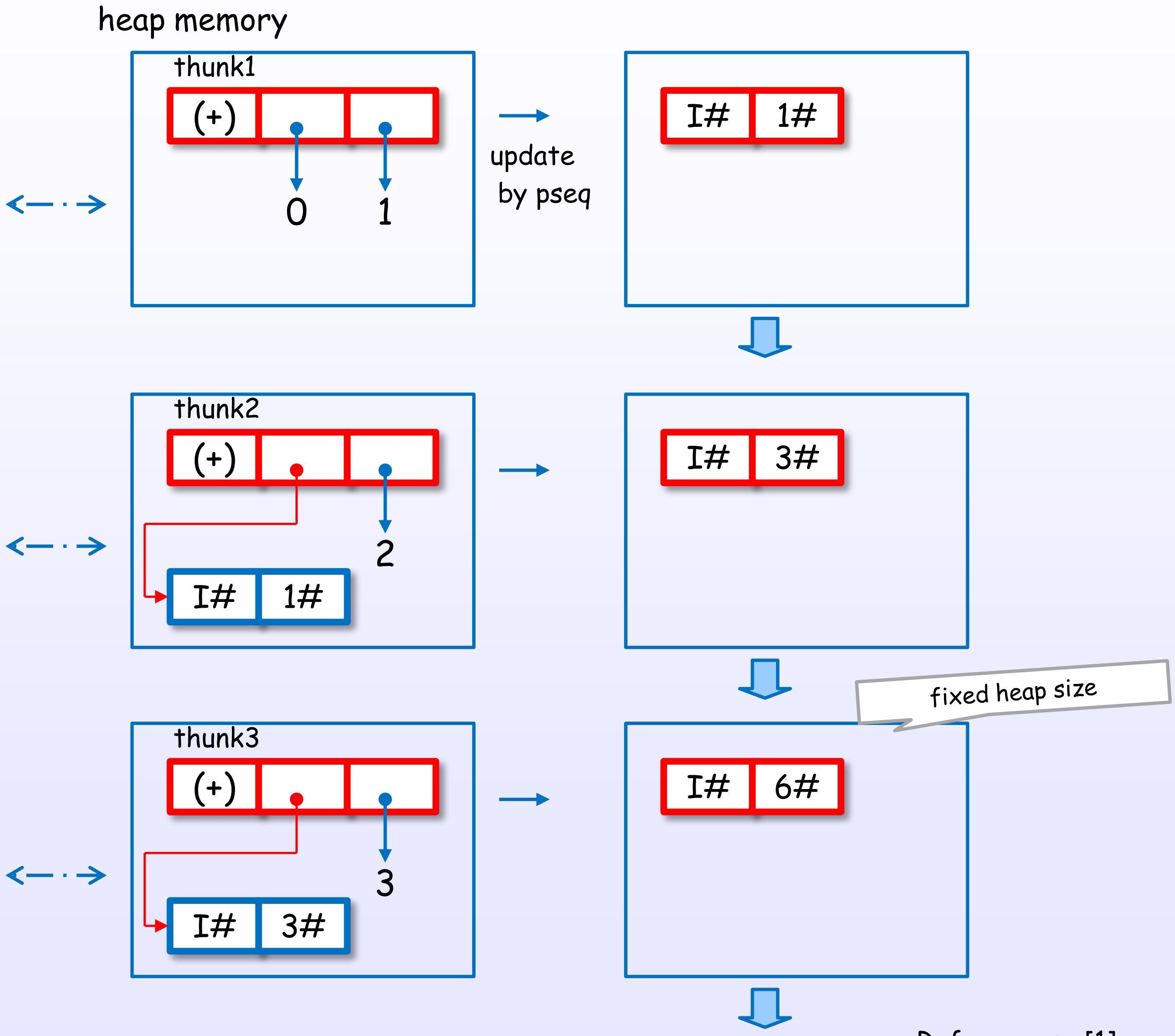
```
foldl' (+) (1 + 2) [3 .. 100]
```

```
let  thunk2 = (1 + 2)
in  thunk2  `pseq`
      foldl' (+) thunk2 [3 .. 100]
```

```
foldl' (+) (3 + 3) [4 .. 100]
```

```
let  thunk3 = (3 + 3)
in  thunk3  `pseq`
      foldl' (+) thunk3 [4 .. 100]
```

...



References : [1]

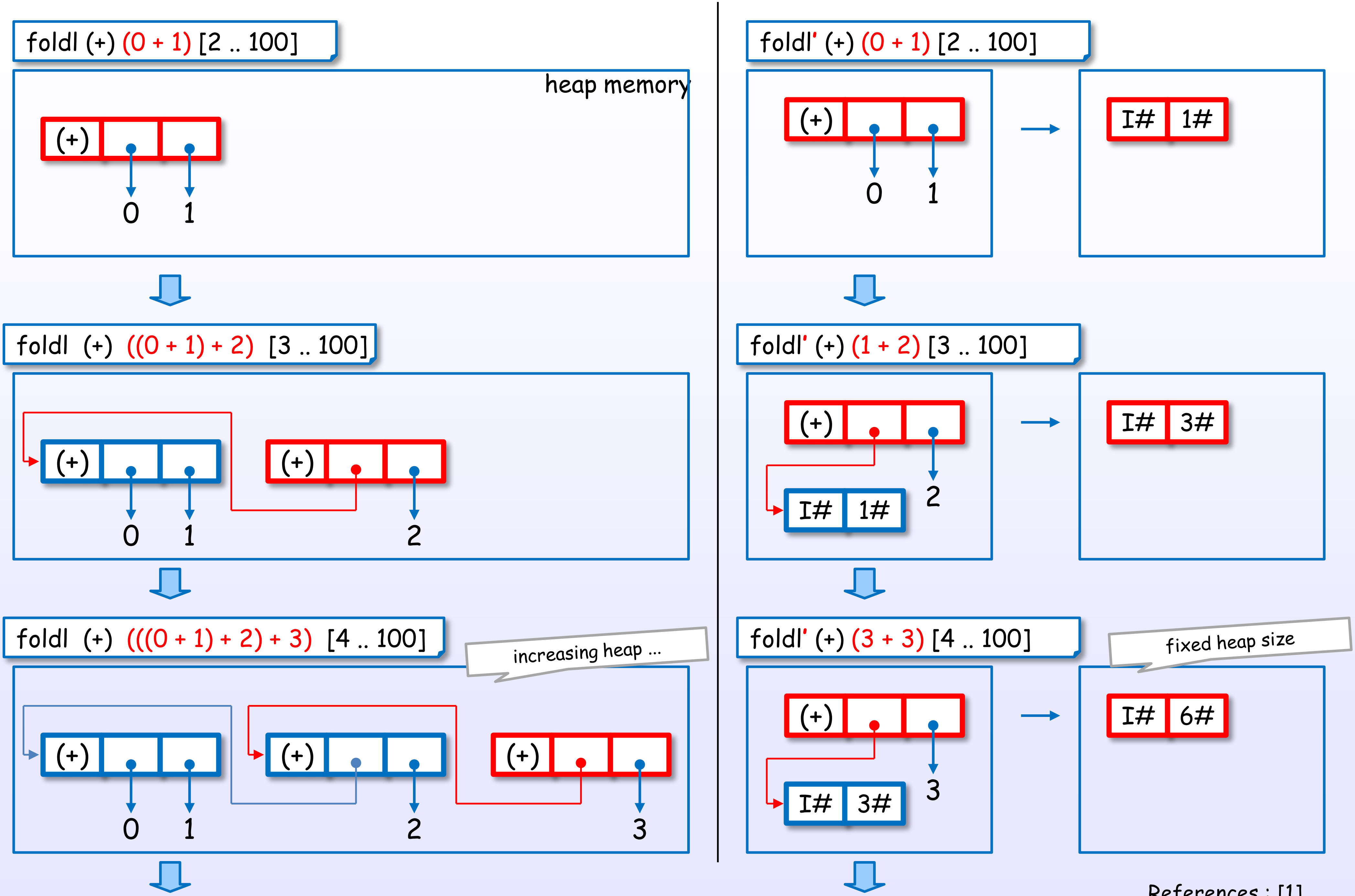
Example of foldl (non-strict) and foldl' (strict)

```
foldl (+) (0 + 1) [2 .. 100]

⇩

foldl  (+)  ((0 + 1) + 2)  [3 .. 100]

⇩

foldl  (+)  (((0 + 1) + 2) + 3)  [4 .. 100]

⇩
```

```
foldl' (+) (0 + 1) [2 .. 100]

⇩

foldl' (+) (1 + 2) [3 .. 100]

⇩

foldl' (+) (3 + 3) [4 .. 100]

⇩
```

References : [1]

Example of foldl (non-strict) and foldl' (strict)



foldl (+) (0 + 1) [2 .. 100]

foldl (+) ((0 + 1) + 2) [3 .. 100]

foldl (+) (((0 + 1) + 2) + 3) [4 .. 100]

increasing heap ...

foldl' (+) (0 + 1) [2 .. 100]

foldl' (+) (1 + 2) [3 .. 100]

foldl' (+) (3 + 3) [4 .. 100]

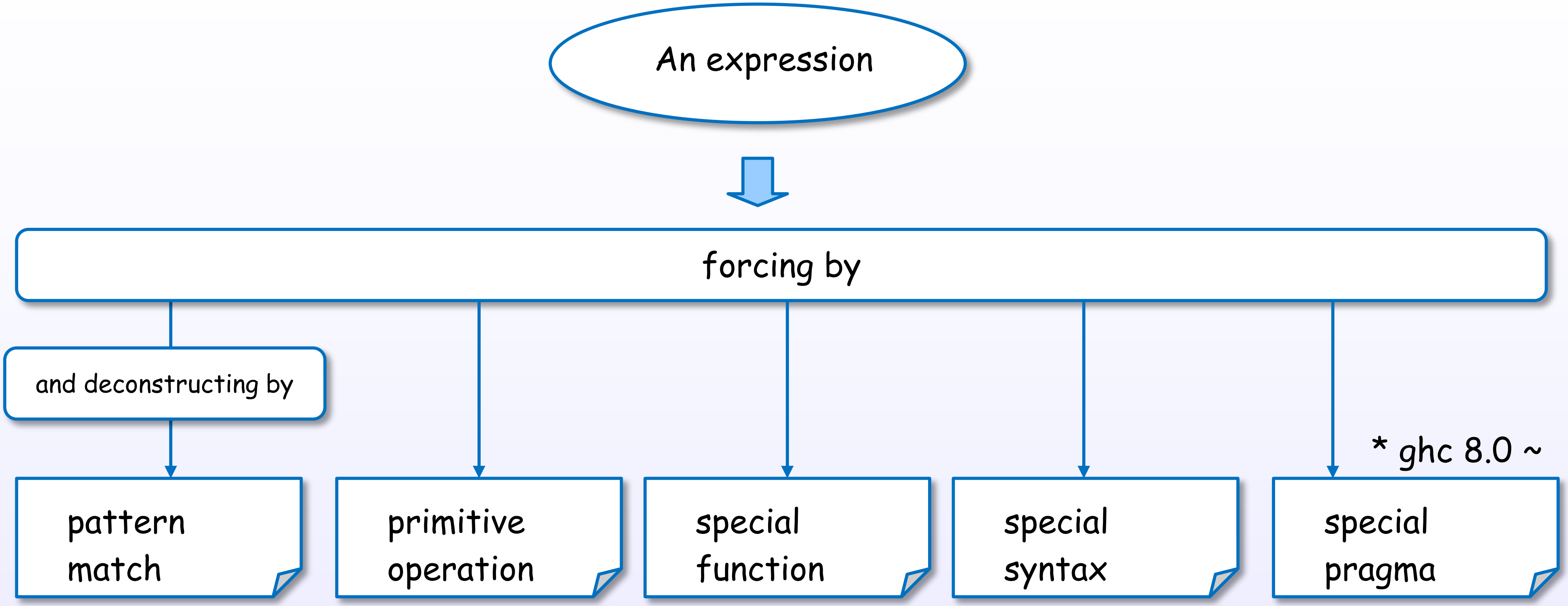
fixed heap size

References : [1]

4. Evaluation

Control the evaluation in Haskell

How to drive evaluation



An expression

forcing by

and deconstructing by

pattern match

case expression
function definition

primitive operation

+, *, ...

special function

seq
$!
rnf
deepseq
force
$!!
pseq
,...

special syntax

!

* ghc 8.0 ~

special pragma

Strict
StrictData

References : [1]

Example of the evaluation by pattern match

case expression

```
case  ds   of
   x:xs  -> f  x  xs
   []    -> False
```

case expression
function definition

References : [1]

Example of the evaluation by primitive operation

primitive operation

$$f\ x\ y = x\ \underline{+}\ y$$

+, *, ...

References : [1]

Example of the evaluation by special function

special function

```
f x y = seq x y
```

seq
$!
rnf
deepseq
force
$!!
pseq
,...

to WHNF
to NF

[parconc, Ch.2]

[RWH, Ch.24-25]

Please refer the document more detail. [xx]
hoogle or hackage

References : [1]

Example of the evaluation by special syntax

special syntax

```
{-# LANGUAGE BangPatterns #-}

f !xs = g xs
```

```
{-# LANGUAGE BangPatterns #-}

data ...
```

BangPattern

[RWH, Ch.25]

Please refer the document more detail. [xx]

[user guide, 7.19]

References : [1]

Example of the evaluation by special pragma

special pragma

```
{-# LANGUAGE Strict #-}

f  xs  =  g  xs
```

* ghc 8.0 ~

```
{-# LANGUAGE StrictData #-}

f  xs  =  g  xs
```

Strict
StrictData

Please refer the document more detail. [xx] [wiki]

References : [1]

Strict analysis

Strict analysis

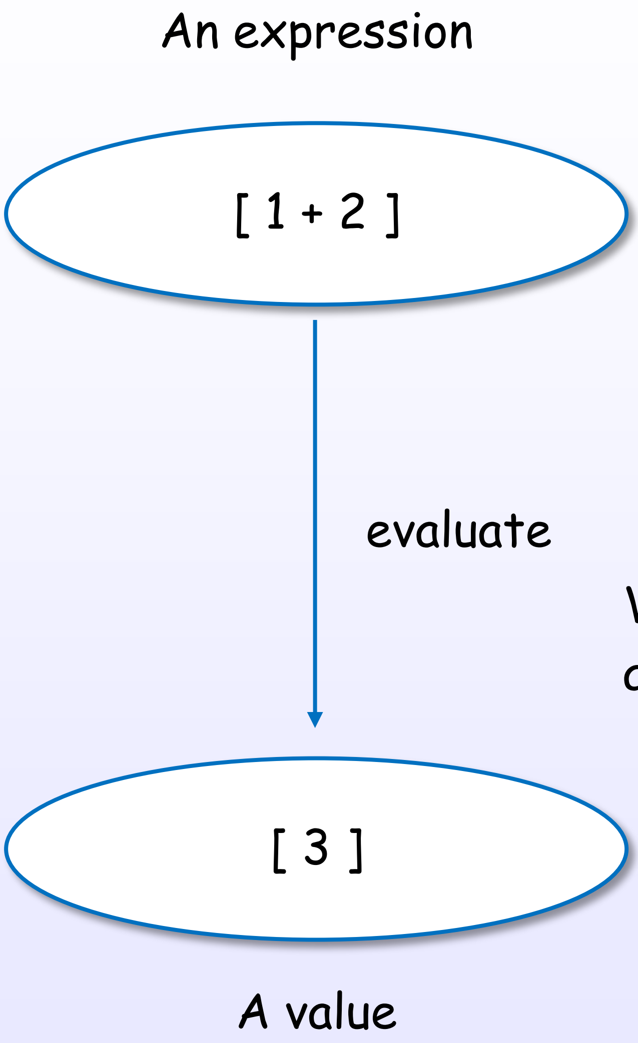
References : [1]

5. Semantics

5. Semantics

Bottom

Well formed expression has a value



An expression

[1 + 2]

evaluate

[3]

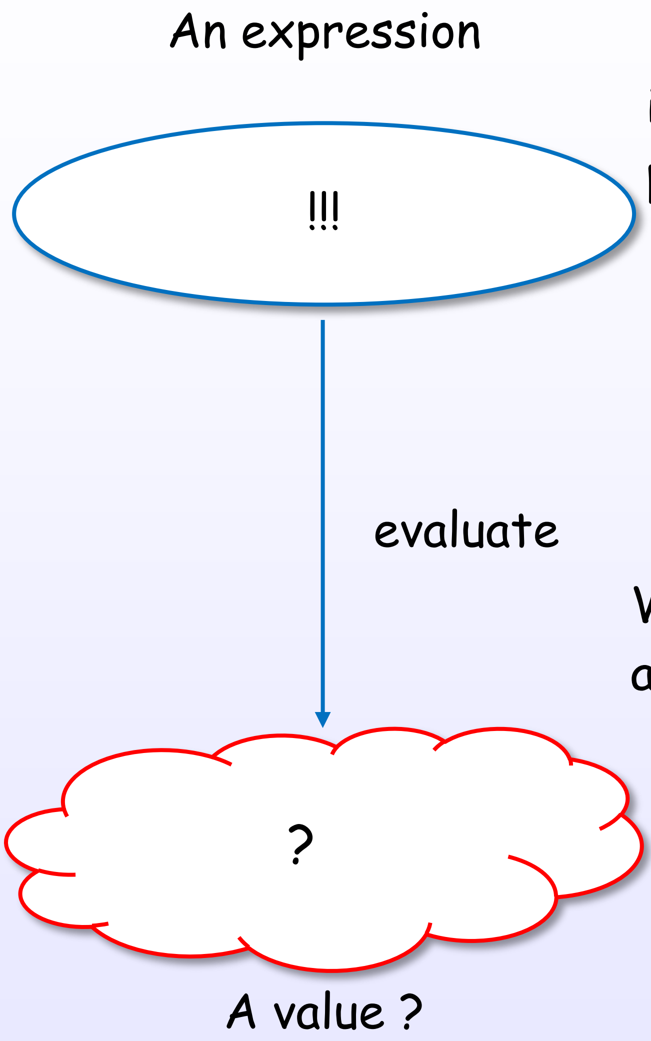
A value

Well formed expression has a value

[Bird, Chapter 2]

References : [1]

Well formed expression has a value



An expression

infinite loop
partial defined function

evaluate

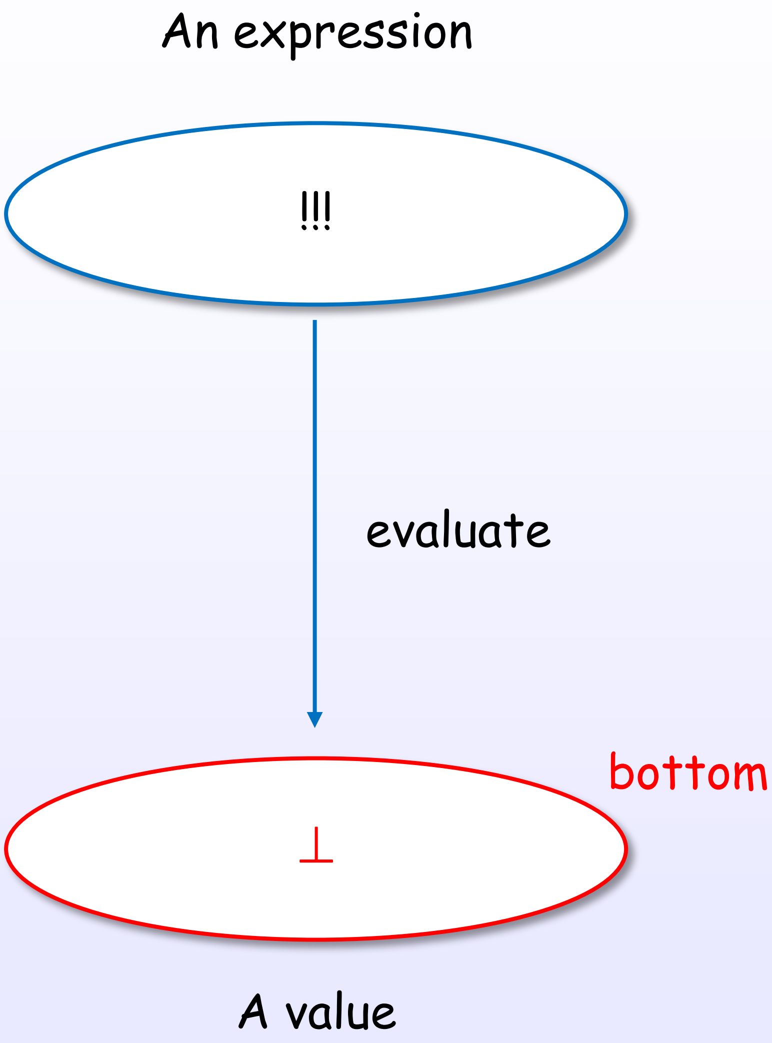
Well formed expression has
a value

A value ?

[Bird, Chapter 2]

References : [1]

Well formed expression has a value



An expression

!!!

evaluate

bottom

$\perp$

A value

[Bird, Chapter 2]

References : [1]

Bottom

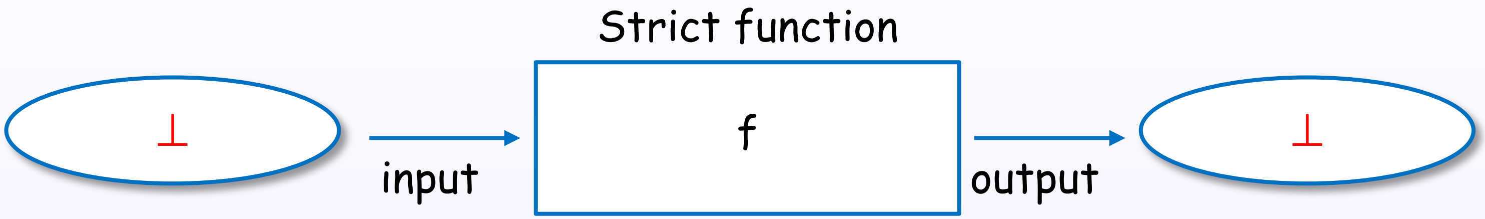
[Bird, Chapter 2]

References : [1]

Non-strict Semantics

Strictness

$$f \perp = \perp$$



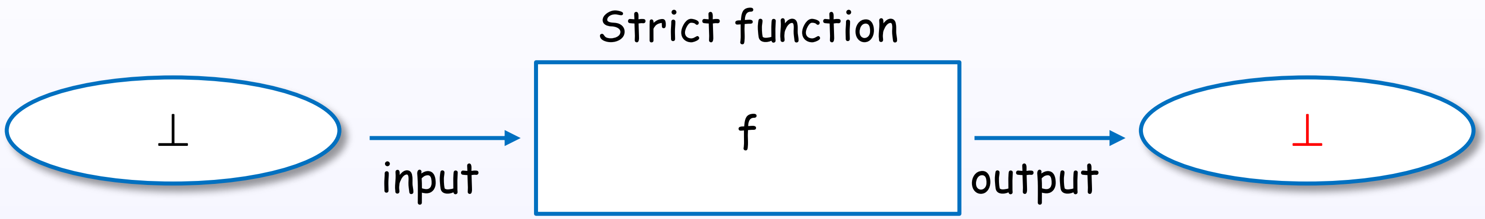
[Bird, Chapter 2]

References : [1]

Strictness and Non-strictness

Strict

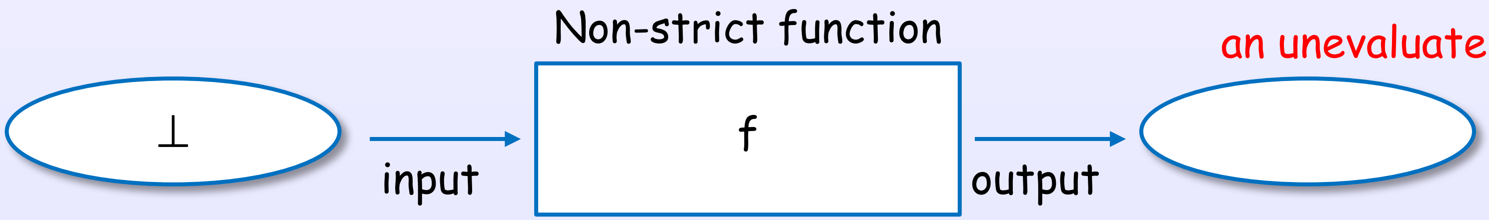
$$f \perp = \perp$$



Non-strict

$$f \perp \neq \perp$$

a value or
an unevaluated expression



[Bird, Chapter 2]

References : [1]

Layer

Non-strictness $f \perp = \perp$

Lazy evaluation GHC chosen lazy evaluation to implement non-strict semantics.

Graph reduction GHC chosen graph reduction to implement lazy evaluation.

STG-machine GHC implements graph reduction by STG-machine.

References : [1]

seq and pseq

[Runtime Support for Multicore Haskell]

$$\begin{aligned} \mathrm{seq}\ a\ \perp &= \perp \\ \mathrm{seq}\ \perp\ a &= \perp \end{aligned}$$

$$\begin{aligned} \mathrm{pseq}\ a\ b &= \perp, && \text{if } a = \perp \\ &= b, && \text{otherwise} \end{aligned}$$

[Snoyman]
Evaluation order and state tokens
https://www.fpcomplete.com/user/snoyberg/general-haskell/advanced/evaluation-order-and-state-tokens

References : [1]

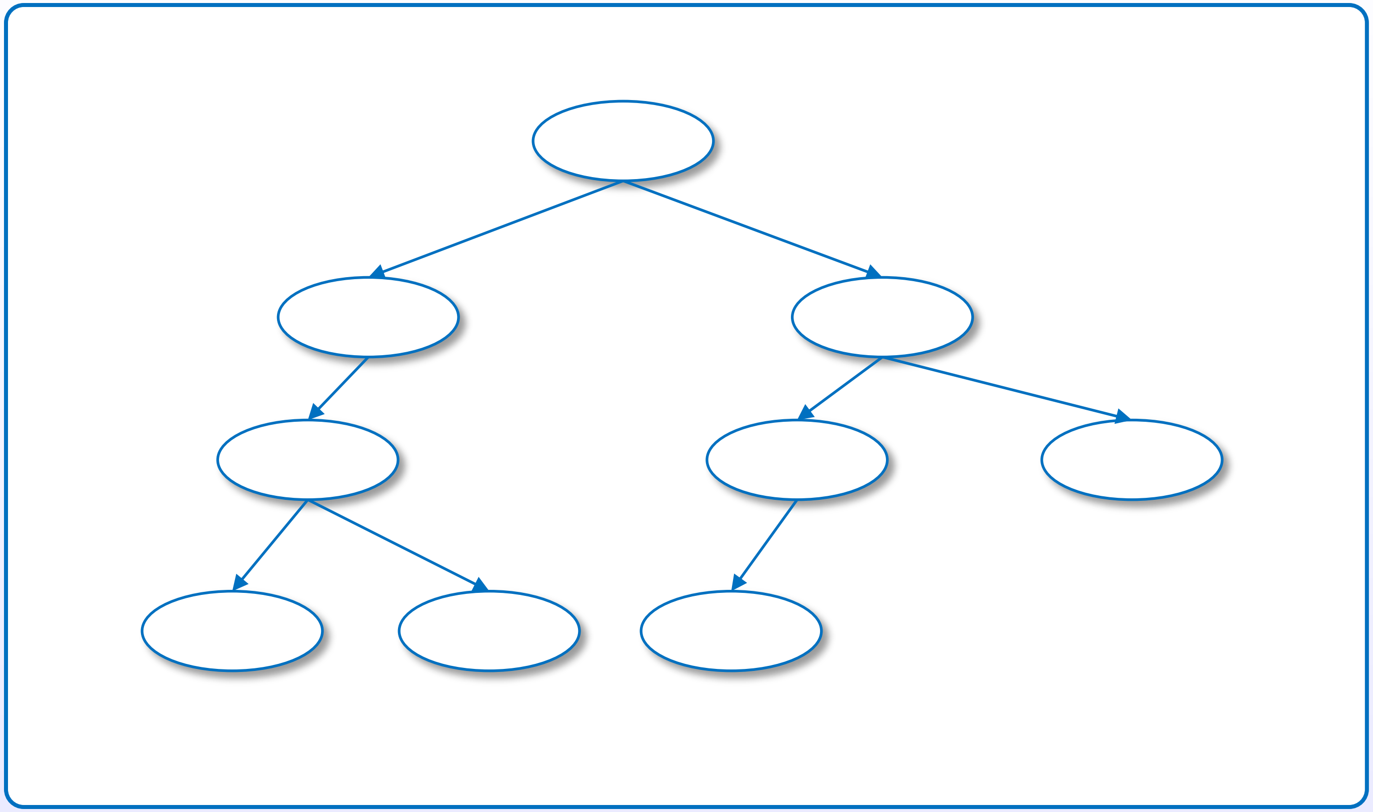
6. Implementation

6. Implementation

Graph reduction

Tree

AST represents an expression

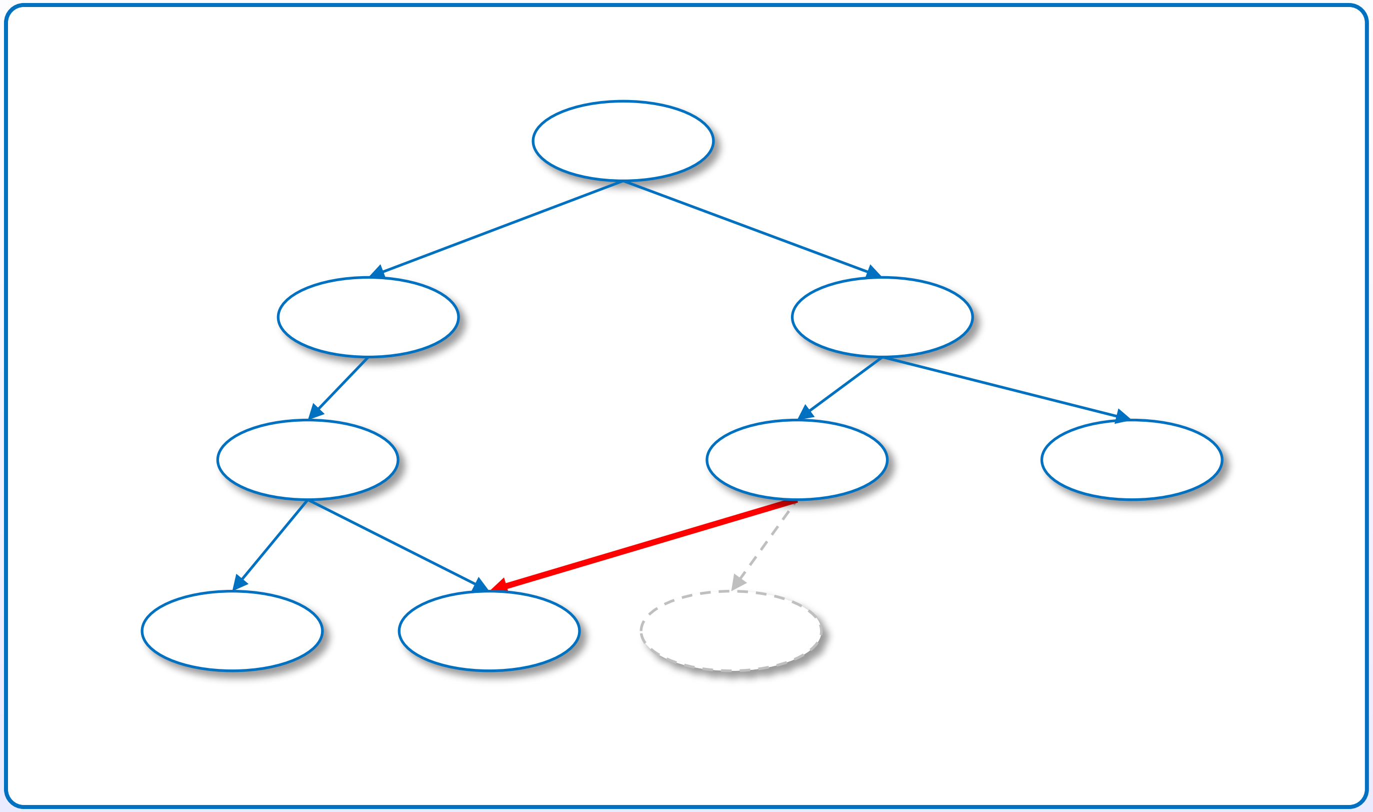


References : [1]

Graph

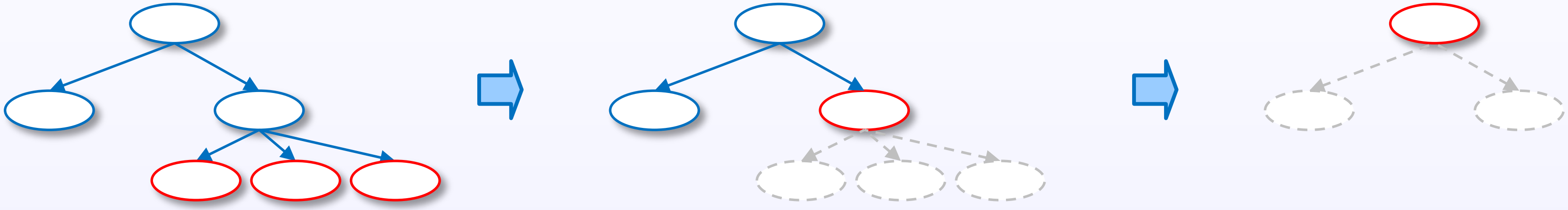
Share the term, looped

not Tree, but Graph



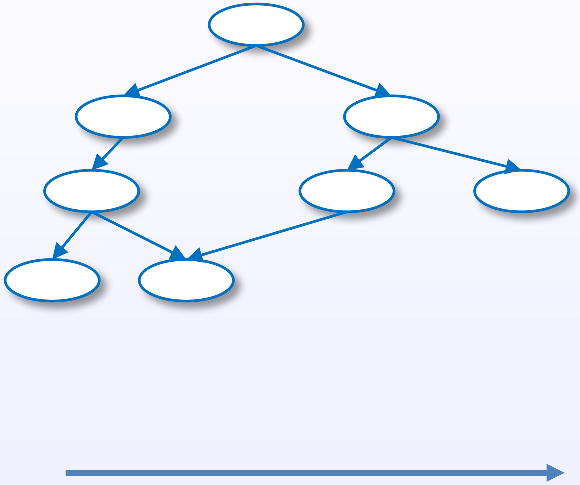
References : [1]

Graph reduction



References : [1]

Graph reduction and lazy



References : [1]

5. Implementation

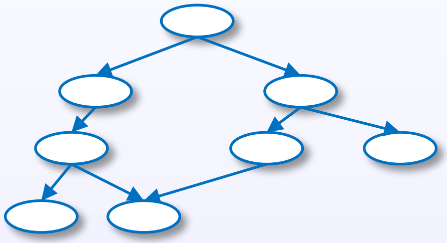
STG-machine

Layer

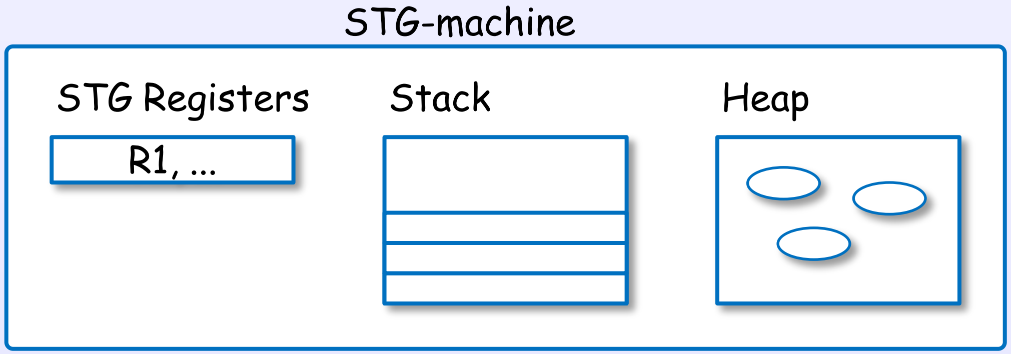
Haskell code

take 5 [1..10]

Internal representation
by graph

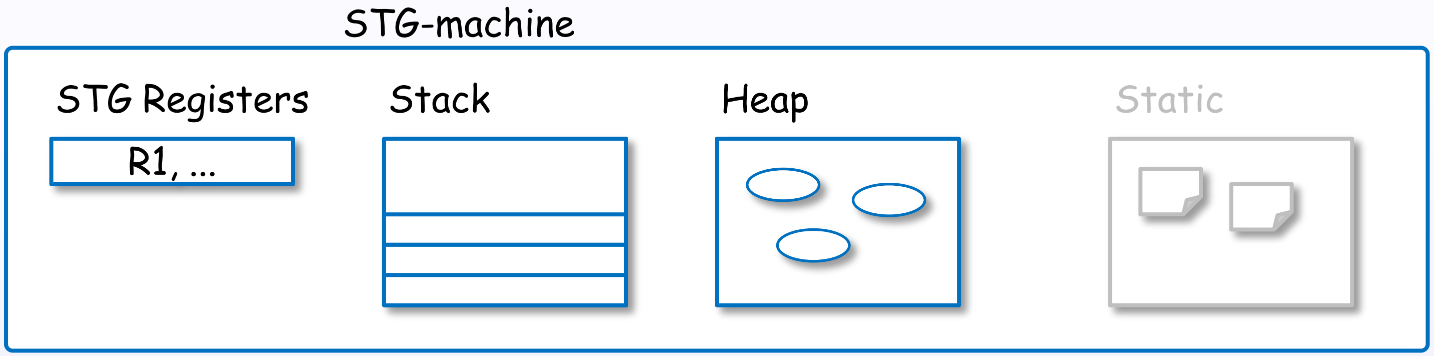


Evaluation (execution, reduction)
by STG-machine



References : [1]

STG-machine

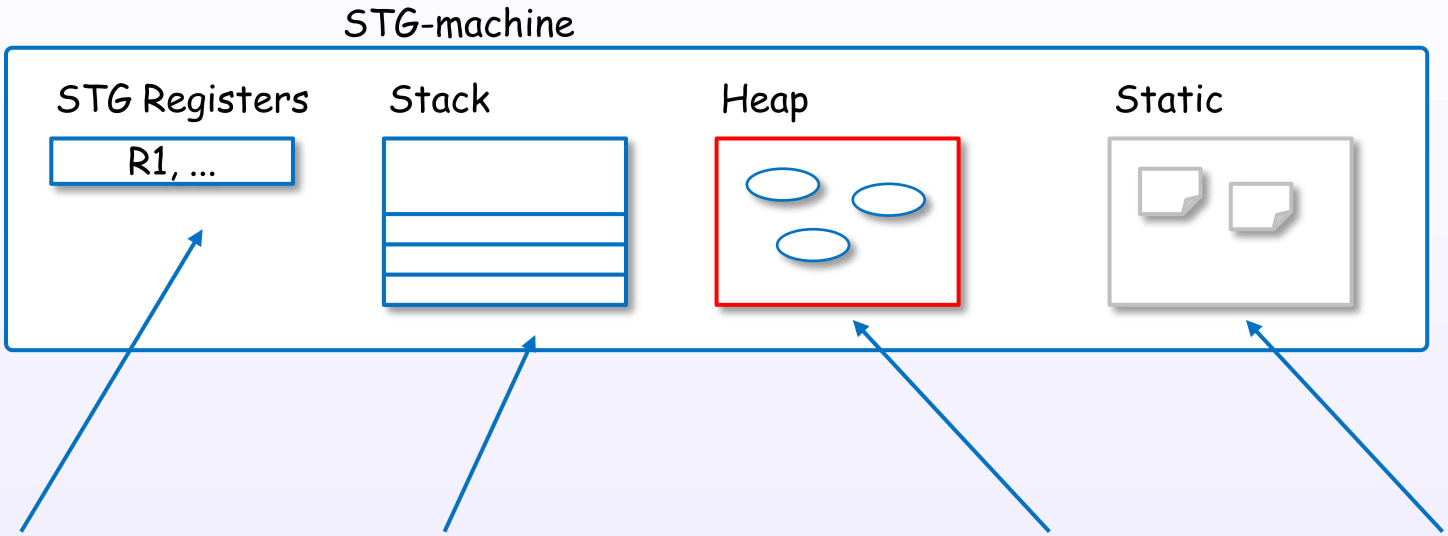


STG-machine is abstraction machine
which is defined by operational semantics.

STG-machine efficiently performs lazy graph reduction.

References : [1]

STG-machine



mainly using for
call/return convention
various control

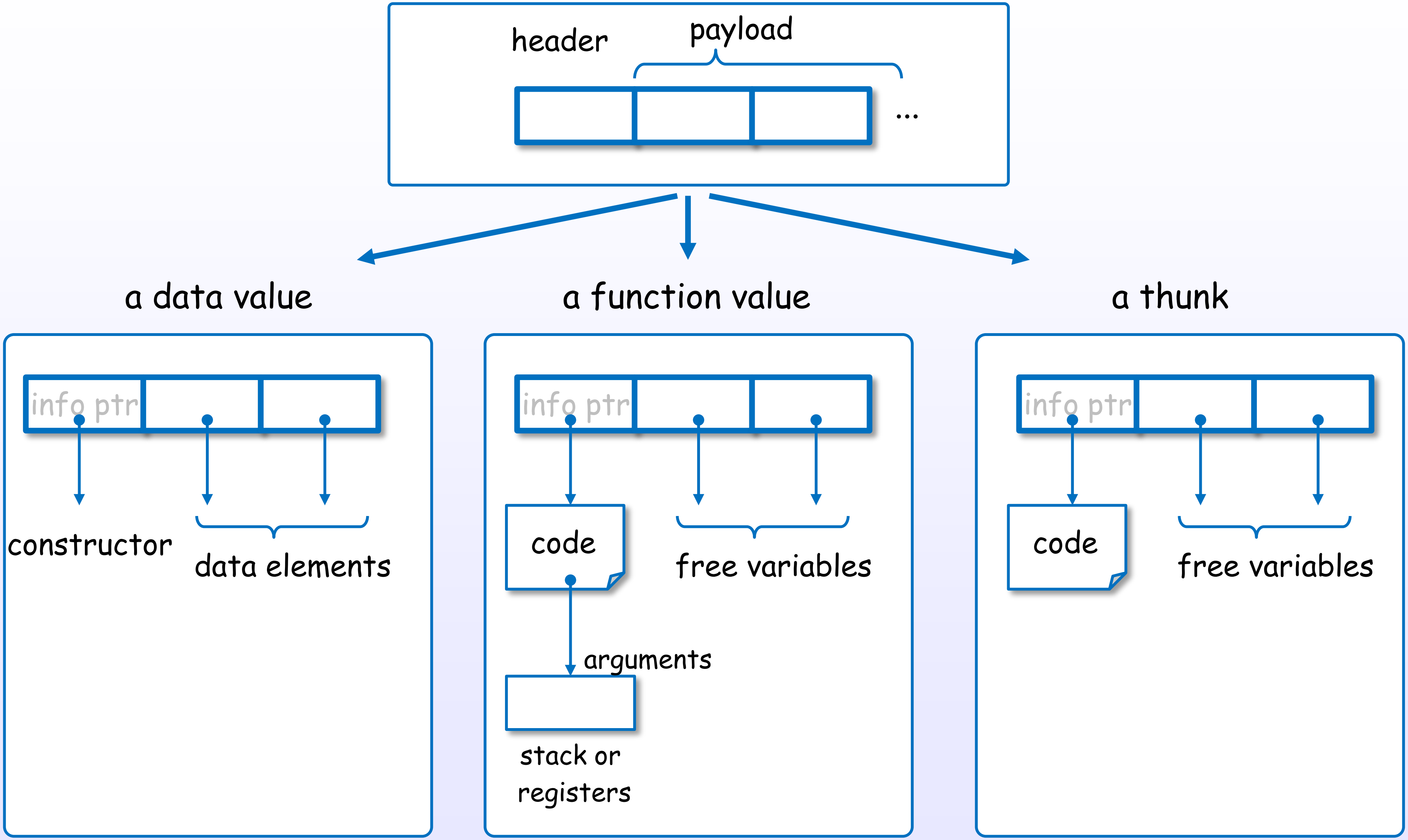
mainly using for
nest continuation
argument passing

mainly using for
allocating thunks

mainly using for
code
static closure

References : [1]

an unified representation in {heap, stack, static} memory



いずれも、広義の、“closure” (= code + environment(free variables))

References : [1]

7. Appendix

References

References

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Lazy,... zzz