CAL - Lista 3: Vinícius Takeo

$$\sum_{i=0}^{h} (h-i)2^{i} \qquad h = \lfloor \log_2 n \rfloor$$

$$\sum_{i=0}^{h} 2^{i}h - 2^{i}.i$$

$$h\sum_{i=0}^{h} 2^{i} - \sum_{i=0}^{h} 2^{i}\, i$$

$$\sum_{i=0}^{h} 2^{i} = 2^{0} + 2^{i} + \dots + 2^{h} + 2^{h+1}$$

$$\sum_{i=0}^{h} 2^{i} + 2^{h+1} = 2^{0} + \sum_{i=0}^{h} 2^{i+1}$$

$$\cancel{\sum_{i=0}^{h} 2^{i}} + 2^{h+1} = 1 + \cancel{2}\sum_{i=0}^{h} 2^{i}$$

$$2^{h+1} - 1 = \sum_{i=0}^{h} 2^{i} \Rightarrow \sum_{i=0}^{h} 2^{i} = 2^{h+1} - 1$$

$$\sum_{i=0}^{h} 2^i i = \cancel{2^0 \cdot 0} + 2^1 \cdot 1 + \dots + h \cdot 2^i + (h+1)2^{i+1}$$

$$\sum_{i=0}^{h} 2^i i + (h+1)2^{h+1} = \sum_{i=0}^{h} (i+1) * 2^{i+1}$$

$$\cancel{\sum_{i=0}^{h} 2^i} i + (h+1)2^{h+1} = 2\sum_{i=0}^{h} i 2^i + 2\sum_{i=0}^{h} 2^i$$

$$\sum_{i=0}^{h} i 2^i = (h+1)2^{h+1} - 2\sum_{i=0}^{h} 2^i$$

$$\sum_{i=0}^{h} i 2^i = 2 \cdot 2^h h + 2^h - 2(2 \cdot 2^h - 1)$$

$$\sum^{h} i 2^i = 2 \cdot 2^h h + 2^h - 4 \cdot 2^h + 2$$

Voltando lá em cima...

$$h\sum_{i=0}^{h} 2^i - \sum_{i=0}^{h} 2^i i$$

$$h(2^{h+1} - 1) - (2.2^h.h + 2^h - 4.2^h + 2)$$

$$h(2^{h+1} - 1) - 2.2^h.h - 2^h + 4.2^h - 2$$

$$\cancel{2.2^h h} - h - \cancel{2.2^h.h} - 2^h + 4.2^h - 2$$

$$3.2^h - h - 2$$

$$3.2^{\log_2 n} - \log_2 n - 2 \qquad h = \log_2 n$$

$$3n - \log_2 n - 2$$

$$a^{\log_a x} = x$$

$\therefore\ O(n)$