Complexidade de Algoritmos
Lista de Exercícios 5

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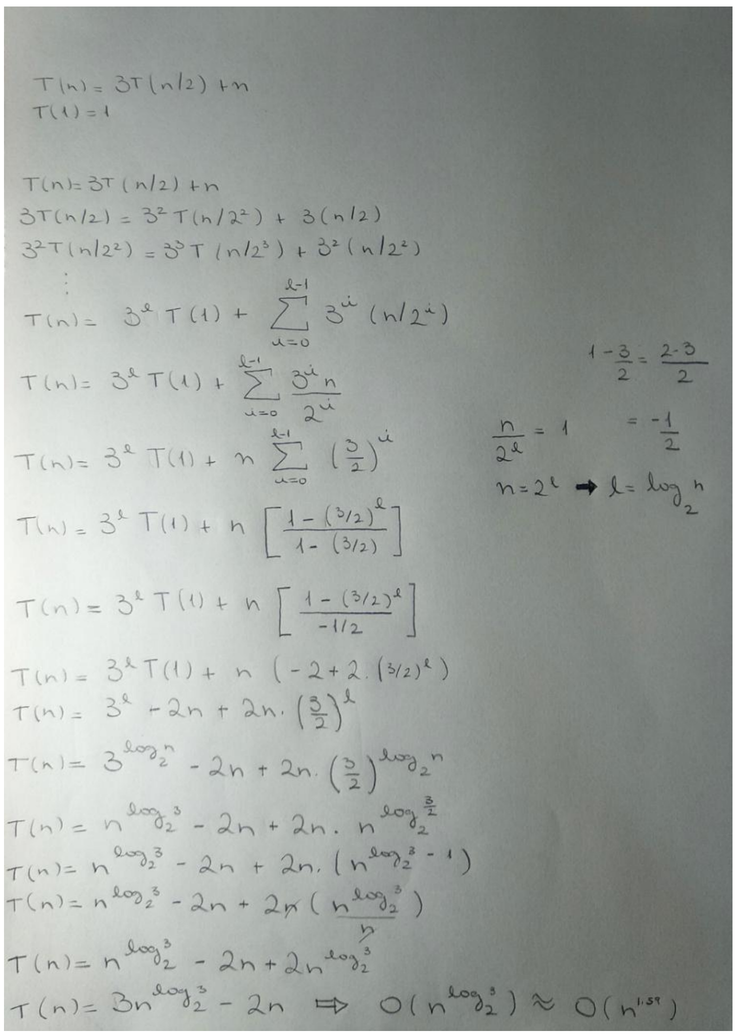
1 Recorrência



$T(n) = 3T(n/2) + n$
$T(1) = 1$

$T(n) = 3T(n/2) + n$
$3T(n/2) = 3^2 T(n/2^2) + 3(n/2)$
$3^2 T(n/2^2) = 3^3 T(n/2^3) + 3^2 (n/2^2)$
$\vdots$
$T(n) = 3^l T(1) + \sum_{i=0}^{l-1} 3^i (n/2^i)$
$T(n) = 3^l T(1) + \sum_{i=0}^{l-1} \frac{3^i n}{2^i}$
$T(n) = 3^l T(1) + n \sum_{i=0}^{l-1} \left(\frac{3}{2}\right)^i$
$T(n) = 3^l T(1) + n \left[\frac{1-(3/2)^l}{1-(3/2)}\right]$
$T(n) = 3^l T(1) + n \left[\frac{1-(3/2)^l}{-1/2}\right]$
$T(n) = 3^l T(1) + n\,(-2 + 2.(3/2)^l)$
$T(n) = 3^l - 2n + 2n.\left(\frac{3}{2}\right)^l$
$T(n) = 3^{\log_2 n} - 2n + 2n.\left(\frac{3}{2}\right)^{\log_2 n}$
$T(n) = n^{\log_2 3} - 2n + 2n.\ n^{\log_2 \frac{3}{2}}$
$T(n) = n^{\log_2 3} - 2n + 2n.\,(n^{\log_2 3 - 1})$
$T(n) = n^{\log_2 3} - 2n + 2\cancel{n}\left(\frac{n^{\log_2 3}}{\cancel{n}}\right)$
$T(n) = n^{\log_2 3} - 2n + 2n^{\log_2 3}$
$T(n) = 3n^{\log_2 3} - 2n \Rightarrow O(n^{\log_2 3}) \approx O(n^{1.59})$

$1 - \frac{3}{2} = \frac{2-3}{2} = -\frac{1}{2}$

$\frac{n}{2^l} = 1$
$n = 2^l \Rightarrow l = \log_2 n$

Figura 1: Resolução da Recorrência

2 Multiplicação de 15x10

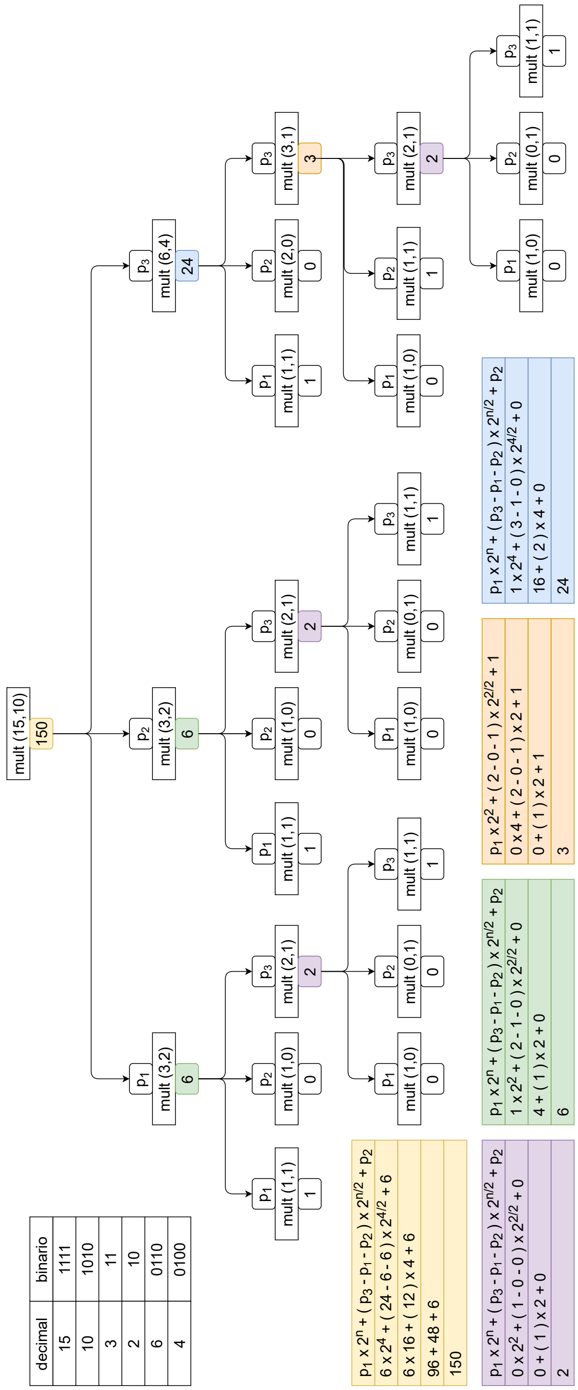


Figura 2: Multiplicação de 15 por 10.