1

a) Suppose, $v_1 = 2v_2$ and $e_1 = 5e_2$

A complete graph K_n has $\frac{n(n-1)}{2}$ edges.

Here,

$$e_1 = \frac{v_1(v_1-1)}{2}$$
$$e_2 = \frac{v_2(v_2-1)}{2}$$

Now,

$$\begin{aligned} e_1 &= 5e_2 \\ \frac{v_1(v_1-1)}{2} &= 5\frac{v_2(v_2-1)}{2} \\ v_1(v_1-1) &= 5v_2(v_2-1) \\ 2v_2(2v_2-1) &= 5v_2(v_2-1) \\ 4v_2^2 - 2v_2 &= 5v_2^2 - 5v_2 \\ v_2^2 - 3v_2 &= 0 \\ v_2(v_2-3) &= 0 \\ v_2 &= 3 \end{aligned}$$

So, $v_1 = 6$, $e_1 = 15$ and $e_2 = 3$.

b) According to the handshaking theorem,

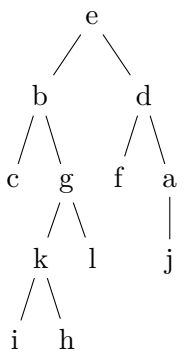
$$\begin{aligned} 9 \times 3 + 3x &= 2|E| \\ 27 + 3x &= 2|E| \end{aligned}$$

x must be odd because 27 is odd and for $27 + 3x$ to be even $3x$ needs to be odd.

The number of edges in a complete graph with 12 vertices $= \frac{12(12-1)}{2} = 66$. Therefore, x can't be greater than 13, otherwise number of edges will exceed 66.

3

a) Binary tree:



b) i. No; a has only one child.

ii. No; c (a leaf) is on height 2.

c)

$$
\begin{aligned}
n &= 271 \\
l &= 226 \\
i &= 271 - 226 \\
&= 45 \\
n &= mi + 1 \\
m &= \frac{n-1}{i} \\
&= \frac{270}{45} \\
&= 6
\end{aligned}
$$

5

a) As Rahat has already build a library and has 5 shelves, the shelves must at least contain 1 book each. And he has 150 books of the same category. So he must have minimum $150 + 4 = 154$ books.

b) $8^{10} + 8^{10} = 2 \times 80\text{B} = 160\text{B}$

c) Remaining letters $= 26 - 12 = 14$. Possible five letter words with 14 letters $= 14^5$.