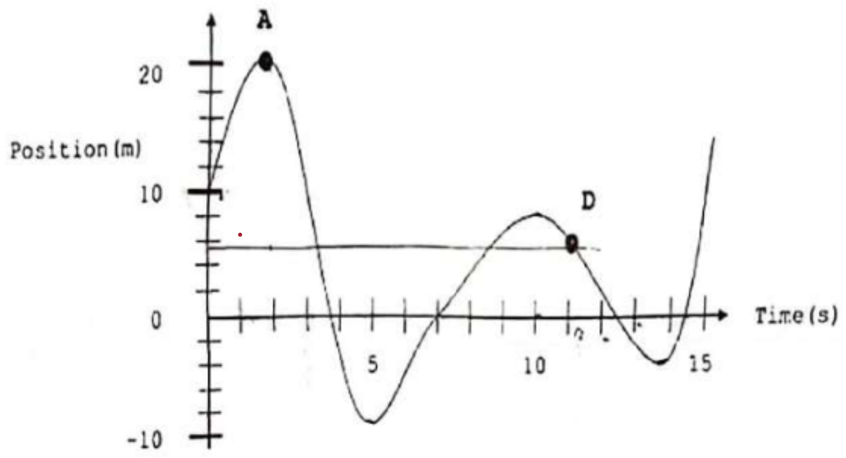
1. (a) The following figure represents a position function of a particle at time t.

(i) Find the average velocity over the time from A to D.

(ii) Find the value(s) of t at which the instantaneous velocity is zero.

(iii) Roughly sketch the velocity graph of the particle.



(b) Consider the function

$$f(x) = \begin{cases} x^2 - 1, & x \le 2 \\ 2x - 1, & x > 2 \end{cases}$$

(i) Sketch the graph of $f(x)$.

(ii) Determine whether the function $f(x)$ is continuous and differentiable at $x = 2$.

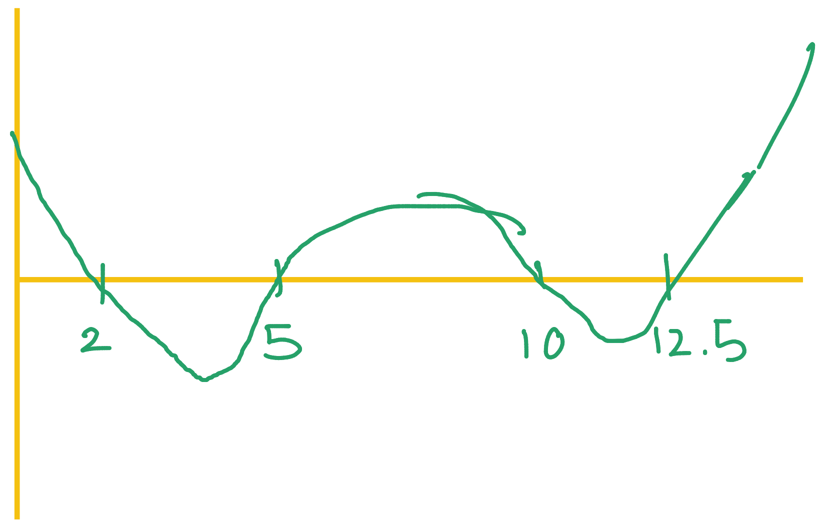
a) i. $\text{avg} = \dfrac{f(11) - f(2)}{11 - 2}$

$= \dfrac{5 - 20}{9}$

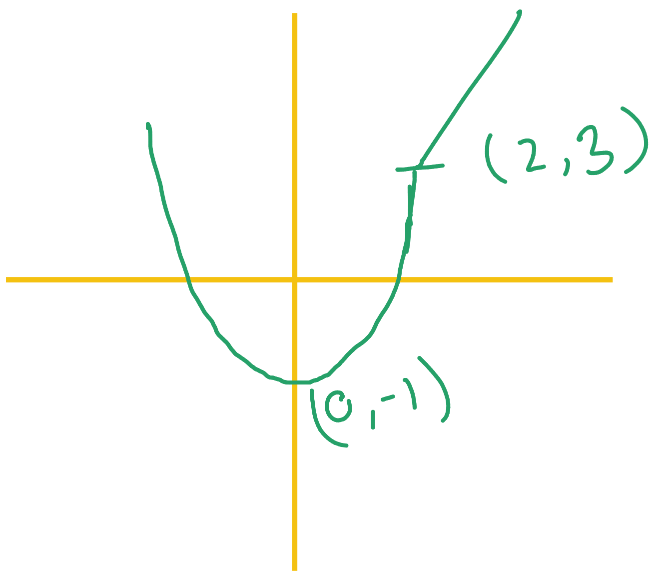
$= -\dfrac{5}{3}$

ii. $x = 2, 5, 10, 12.5$

iii.



b) i.



ii. LHL: $\lim\limits_{x \to 2^-} x^2 - 1 = 3$ RHL: $\lim\limits_{x \to 2^+} 2x - 1 = 3$

$f(2) = 3$ continuous at $x = 2$

LHD: $\lim\limits_{x \to 2^-} \dfrac{f(x) - f(2)}{x - 2}$

$= \lim\limits_{x \to 2^-} \dfrac{x^2 - 1 - (2^2 - 1)}{x - 2}$

$= \lim\limits_{x \to 2^-} \dfrac{x^2 - 4}{x - 2}$

$= \lim\limits_{x \to 2^-} x + 2$

$= 4$

RHD: $\lim\limits_{x \to 2^+} \dfrac{f(x) - f(2)}{x - 2}$

$= \lim\limits_{x \to 2^+} \dfrac{2x - 1 - (4 - 1)}{x - 2}$

$= \lim\limits_{x \to 2^+} \dfrac{2x - 4}{x - 2}$

$= 2$

not diff at $x = 2$

2. (a) Find the derivative of $f(x) = 5 + 2x - x^2$ with respect to x by using the formula $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, and use it to find the equation of tangent line to $f(x)$ at $x = -1$. [3]

(b) The following table defines the values of $f(x), g(x), f'(x)$ and $g'(x)$ at x. [3]

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	4	-2	7	-3
0	7	3	-1	-2

If $u(x) = f(x)g(x)$, and $v(x) = \frac{f(x)}{g(x)}$, then find $u'(-1)$, and $v'(0)$.

b) $u'(-1) = f'(-1)g(-1) + f(-1)g'(-1)$

$= (7)(-2) + (4)(-3)$

$= -14 - 12$

$= -26$

$v'(0) = \dfrac{f'(0)g(0) - f(0)g'(0)}{g(0)^2}$

$= \dfrac{(-1)(3) - (7)(-2)}{3^2}$

$= \dfrac{11}{9}$

a) $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \dfrac{\cancel{5} + \cancel{2x} + 2h - \cancel{x^2} - 2xh - h^2 - \cancel{5} - \cancel{2x} + \cancel{x^2}}{h}$

$= \lim_{h\to 0} 2 - 2x - h$

$= 2 - 2x$

$x = -1 \quad 5 + (2)(-1) - (-1)^2$

$= 5 - 2 - 1$

$= 2$

$(-1, 2)$ slope $= 2 - 2(-1) = 2 + 2 = 4$

$y - 2 = 4(x + 1)$

$y = 4x + 6$

(c) Use chain rule to evaluate the following derivatives:

(i) If $x = \tan u$ and $u = t^3 - 2t\cos t + 5$, then find $\frac{dx}{dt}$.

(ii) If $y = \cot^3\sqrt{2 - 3\sin x}$, then find $\frac{dy}{dx}$.

i) $x = \tan(t^3 - 2t\cos t + 5)$

$\Rightarrow \dfrac{dx}{dt} = \sec^2(t^3 - 2t\cos t + 5)\left[3t^2 - 2\{\cos t - t\sin t\} + 0\right]$

$= (3t^2 - 2\cos t + 2t\sin t)\sec^2(t^3 - 2t\cos t + 5)$

ii) $y = \cot^3\sqrt{2 - 3\sin x}$

$\dfrac{dy}{dx} = 3\cot^2\sqrt{2-3\sin x} \times \left(-\csc^2\sqrt{2-3\sin x}\right) \times \dfrac{0 - 3\cos x}{2\sqrt{2-3\sin x}}$

3. (a) Evaluate the following integrals:

(i) $\int \frac{x^4-x^2+x^3-1}{x^3}dx$ (ii) $\int \frac{x^2}{\sqrt{1-4x^6}}dx$

(b) Use integration by parts to evaluate the following integrals:

(i) $\int e^{-x}\sin 2x\,dx$ (ii) $\int x\tan^{-1}2x\,dx$

a) i. $\int \frac{x^4-x^2+x^3-1}{x^3}dx$

$= \int x - \frac{1}{x} + 1 - x^{-3}\,dx$

$= \frac{x^2}{2} - \ln x + x - \frac{x^{-2}}{-2} + c$

ii. $\int \frac{x^2}{\sqrt{1-4x^6}}dx$

$(x^3)^2$

$\frac{1}{3}\int \frac{1}{\sqrt{1-4u^2}}du$ $\quad u = x^3$, $du = 3x^2\,dx$

$\frac{1}{6}\int \frac{1}{\sqrt{1-z^2}}dz$ $\quad 2u = z$, $dz = 2\,du$

$\frac{1}{6}\sin^{-1} z = \frac{1}{6}\sin^{-1}(2u)$

$= \frac{1}{6}\sin^{-1}(2x^3)$

b) i. $\int \underset{v}{e^{-x}}\,\underset{u}{\sin 2x}\,dx = I$

$= \sin 2x\int e^{-x}dx - \int\left[\frac{d}{dx}\sin 2x\int e^{-x}dx\right]dx$

$= -\sin 2x\, e^{-x} - \int -2\cos 2x\, e^{-x}\,dx$

$= -\sin 2x\, e^{-x} + 2\underline{\int e^{-x}\cos 2x\,dx}$

$\cos 2x\int e^{-x}dx - \int \frac{d}{dx}\cos 2x\int e^{-x}dx\,dx$

$= -e^{-x}\cos 2x - \int(-2\sin 2x)(-e^{-x})\,dx$

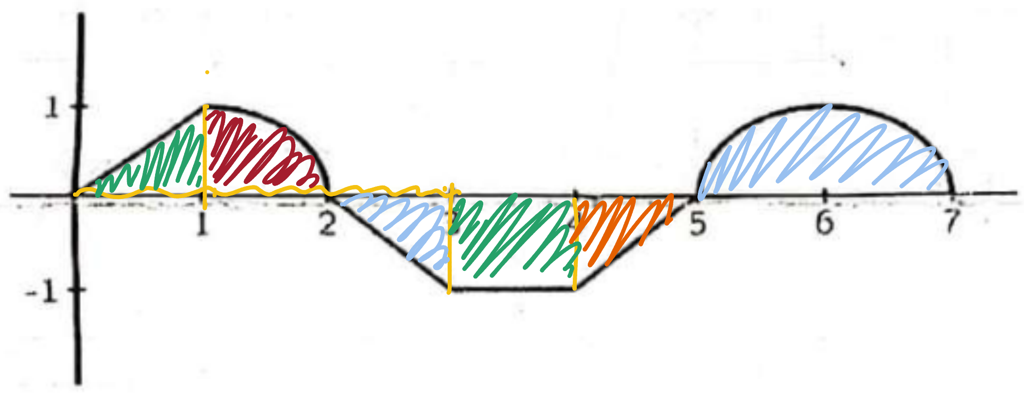
$= -e^{-x}\cos 2x - 2\int e^{-x}\sin 2x\,dx$

$I = -e^{-x}\cos 2x - 2I$

$\Rightarrow I = -\frac{1}{3}e^{-x}\cos 2x$

4. (a) The graph of $f(x)$ is shown. Use the graph to **evaluate** the following integrals:

(i) $\int_0^3 f(x)dx$ (ii) $\int_3^7 f(x)dx$



ii) $1\times 1 + \frac{1}{2}\times 1\times 1 + \frac{1}{2}\pi 1^2$

$= 1 + \frac{1}{2} + \frac{\pi}{2}$

$= \frac{3+\pi}{2}$

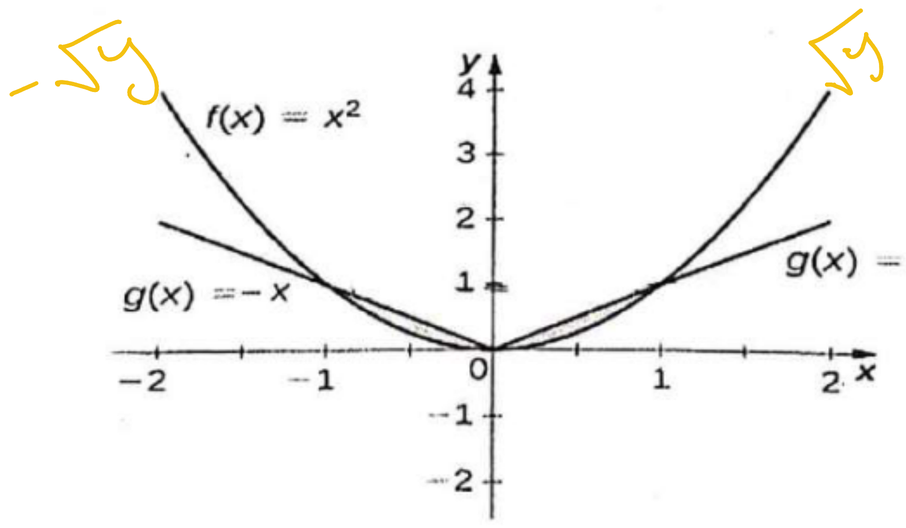
i) $\frac{1}{2}\times 1\times 1 + \frac{1}{4}\pi 1^2 + \frac{1}{2}\times 1\times 1$

$= 1 + \frac{\pi}{4}$

(b) **Find the area of the shaded region by**

(i) integrating with respect to x.

(ii) integrating with respect to y.



i) $\int_{-1}^{0} -x - x^2 \, dx$

$= -\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-1}^{0}$

$= -\left[0 - \left(\frac{1}{2} - \frac{1}{3}\right)\right]$

$= -\left[-\frac{3-2}{6}\right]$

$= \frac{1}{6}$

$\int_0^1 x - x^2 \, dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \left(\frac{1}{2} - \frac{1}{3} - 0\right)$

$= \frac{3-2}{6} = \frac{1}{6}$

$\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

ii) $\int_0^1 -y - (-\sqrt{y}) \, dy$

$= \left[\frac{2}{3}y^{3/2} - \frac{y^2}{2}\right]_0^1$

$= \frac{2}{3} - \frac{1}{2} - 0$

$= \frac{4-3}{6} = \frac{1}{6}$

$\int_0^1 \sqrt{y} - y \, dy$

$= \left[\frac{2}{3}y^{3/2} - \frac{y^2}{2}\right]_0^1$

$= \frac{2}{3} - \frac{1}{2}$

$= \frac{1}{6}$