2.

a) Determine whether the given lines are skew.

L_1: $x = 2 - t$, $y = -1 + 2t$, $z = -1 - 5t$

L_2: $x = -t$, $y = 5 - 5t$, $z = 3 + 2t$

$\vec{v_1} = \langle -1, 2, -5 \rangle$

$\vec{v_2} = \langle -1, -5, 2 \rangle$

there is no such k such that $\vec{v}_1 = k\,\vec{v_2}$ thus the lines are not parallel

let the intersection point of the lines be (x_0, y_0, z_0)

$\therefore \quad x_0 = 2 - t_1 \quad y_0 = -1 + 2t_1 \quad z_0 = -1 - 5t_1$

and $\quad x_0 = -t_2 \quad y_0 = 5 - 5t_2 \quad z_0 = 3 + 2t_2$

$2 - t_1 = -t_2 \qquad -1 + 2t_1 = 5 - 5t_2 \qquad -1 - 5t_1 = 3 + 2t_2$

solving the first two eqns

$t_1 = 16/7 \quad t_2 = 2/7$

now $\quad -1 - 5 \cdot \frac{16}{7} = 3 + 2\,\frac{2}{7}$

$\Rightarrow \; -\frac{87}{7} = \frac{25}{7}$

which is a contradiction

therefore the lines don't intersect

and the lines are skew.

b) Find the equation of line of intersection of the planes.

$x + y + z - 5 = 0$ and $5x - 2y + 4z = 7$.

$\vec{n_1} = \langle 1, 1, 1 \rangle$

$\vec{n_2} = \langle 5, -2, 4 \rangle$

$\vec{v} = \vec{n_1} \times \vec{n_2} = \langle 4+2, -(4-5), -2-5 \rangle$

$= \langle 6, 1, -7 \rangle$

on the xy-plane $z = 0$

$\therefore \quad x + y = 5$ and $5x - 2y = 7$

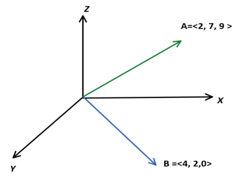
$x = 17/7 \quad y = 18/7$

$\therefore \; (17/7, 18/7, 0)$ is a common passing point of the planes

$\therefore$ the eqn of the intersecting line

$x = 17/7 + 6t \; ; \; y = 18/7 + t \; ; \; z = -7t$

3.



a) Find the orthogonal projection of ***B*** along ***A***.

b) Find the angle between vector ***A*** and **y**-axis.

c) Find a unit vector that is orthogonal to vector ***A*** and ***x*** −axis.

a) $\vec{A} = \langle 2, 7, 9 \rangle$

$\hat{a} = \frac{1}{\sqrt{134}} \langle 2, 7, 9 \rangle$

$\text{proj}_{\vec{A}} \vec{B} = (\vec{B} \cdot \hat{a})\, \hat{a}$

$= \frac{1}{\sqrt{134}} (8 + 14 + 0) \cdot \hat{a}$

$= \frac{22}{\sqrt{134}} \cdot \frac{1}{\sqrt{134}} \langle 2, 7, 9 \rangle$

$= \frac{11}{67} \langle 2, 7, 9 \rangle$

b) $A \cdot \hat{j} = |A| \cdot 1 \cdot \cos\theta$

$\Rightarrow \langle 2, 7, 9 \rangle \cdot \langle 0, 1, 0 \rangle = \sqrt{2^2 + 7^2 + 9^2}\, \cos\theta$

$\Rightarrow \cos\theta = \frac{7}{\sqrt{134}}$

$\Rightarrow \theta = 52.79°$

c) $\vec{v} = \vec{A} \times \hat{i} = \langle 2, 7, 9 \rangle \times \langle 1, 0, 0 \rangle$

$= \langle 0, -(0 - 9), 0 - 7 \rangle$

$= \langle 0, 9, -7 \rangle$

unit $\vec{v} = \frac{1}{\sqrt{130}} \langle 0, 9, -7 \rangle$