1. a) Consider the following system of linear equation

$$x + 2y - 2z = 3$$
$$3x - y + z = 1$$
$$-x + 5y - 5z = 5$$

Solve the above system by elementary row operations.

$$\begin{array}{cccc} 1 & 2 & -2 & 3 \\ 3 & -1 & 1 & 1 \\ -1 & 5 & -5 & 5 \end{array} \rightarrow \begin{array}{cccc} 1 & 2 & -2 & 3 \\ 0 & -7 & 7 & -8 \\ 0 & 7 & -7 & 8 \end{array} \rightarrow \begin{array}{cccc} 1 & 2 & -2 & 3 \\ 0 & 7 & -7 & 8 \\ 0 & 7 & -7 & 8 \end{array} \rightarrow \begin{array}{cccc} 1 & 2 & -2 & 3 \\ 0 & 1 & -1 & 8/7 \\ 0 & 0 & 0 & 0 \end{array} \rightarrow \begin{array}{cccc} 1 & 0 & 0 & 5/7 \\ 0 & 1 & -1 & 8/7 \\ 0 & 0 & 0 & 0 \end{array}$$

$r_2 = -3r_1 + r_2$
$r_3 = r_1 + r_3$

$r_2 = -r_2$

$r_2 = r_2/7$
$r_3 = r_2 - r_3$

$r_1 = r_1 - 2r_2$

$x = \frac{5}{7}$

$y - z = 8/7 \Rightarrow y = \frac{8}{7} + p$

$z = p$

b) Make a statement about the number of solutions of the following system. Confirm your conclusion algebraically. (3)

$$x + y = 4$$
$$3x + 3y = 6$$

no solutions

$x + y = 4 \Rightarrow x = 4 - y$

$x + y = 3 \Rightarrow 4 - y + y = 3 \Rightarrow 4 = 3$ math error

c) Determine the value(s) of "$k$" for which the system has no solutions, unique solution, or infinitely many solutions. (2)

$$x + ky = 6$$
$$2x + 6y = 10$$

$x + ky = 6$

$x + 3y = 10$

no solutions for $K = 3$

unique solutions for any other value of K

there is no such value of K for which there are infinite solutions

2. Consider the following matrices

$$A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 1 & 2 \\ 0 & -2 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \quad P = \begin{bmatrix} 4 & 1 & 0 & 3 \end{bmatrix} \quad Q = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \end{bmatrix}$$

a) Find $A^{-1}$ by using matrix inversion algorithm. Also, find $X$ such that $AX = b$. (5)

$$\begin{array}{cccccc} 0 & 1 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & -2 & -5 & 0 & 0 & 1 \end{array} \rightarrow \begin{array}{cccccc} 0 & 1 & 3 & 1 & 0 & 0 \\ 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \rightarrow \begin{array}{cccccc} 0 & 1 & 0 & -5 & 0 & -3 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}$$

$r_2 = r_2 - r_1$
$r_3 = 2r_1 + r_3$

$r_1 = r_1 + 3r_2$
$r_2 = r_2 + r_3$

$$\begin{array}{cccccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -5 & 0 & -3 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}$$

swap(1,2)

$A x = b$

$\Rightarrow X = A^{-1} b$

$$= \begin{bmatrix} 1 & 1 & 1 \\ -5 & 0 & -3 \\ 2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1+2+4 \\ 5+0-12 \\ -2+0+4 \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 2 \end{bmatrix}$$

b) Find $P(B)$ for $P(x) = x^2 - x + 1$. $\boxed{\text{Is } P(B) \text{ a solution of } P(x)}$? $\rightarrow$ বুঝি না T_T

$$B^2 = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 9+5 & 3+2 \\ 15+10 & 5+4 \end{bmatrix} = \begin{bmatrix} 14 & 5 \\ 25 & 9 \end{bmatrix}$$

$$P(B) = \begin{bmatrix} 14 & 5 \\ 25 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 4 \\ 20 & 8 \end{bmatrix}$$

c) Find $tr(P+Q)$ and also evaluate $\det(PQ)$.

$P + Q$ is not possible ×

$PQ = [4 + 2 + 0 - 3] = [3]$

$\det[3] = 3$

3. Find the eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$. Also verify that $AX = \lambda X$. (5)

$A - \lambda I = 0$

$\Rightarrow \begin{bmatrix} 4-\lambda & -1 \\ 2 & 1-\lambda \end{bmatrix} = 0$

$\Rightarrow (4-\lambda)(1-\lambda) + 2 = 0$

$\Rightarrow 4 - \lambda - 4\lambda + \lambda^2 + 2 = 0$

$\Rightarrow \lambda^2 - 5\lambda + 6 = 0$

$\Rightarrow \lambda = 2, 3$

for $\lambda = 2$

$\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x - y = 0$

let $y = p$

$x = p/2$

$p \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$

for $\lambda = 3$

$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

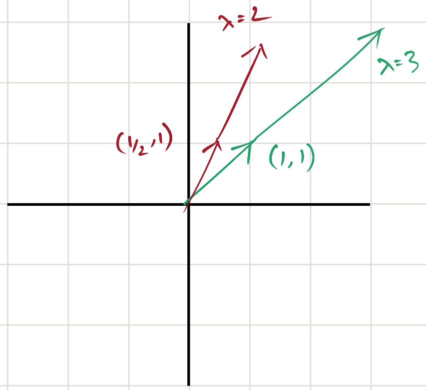
$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x - y = 0$

let $y = p$

$x = p$

$p \begin{bmatrix} 1 \\ 1 \end{bmatrix}$




verify

$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2-1 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4-1 \\ 2+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

a) Solve the following higher order differential equations:

i) $y'' + 9y = 0$ ; $y(\frac{\pi}{3}) = 2$, $y'(\frac{\pi}{3}) = 6$

ii) $y'' - 6y' + 9y = 2e^{3x} + 5e^{-2x}\cos x - 3$

i) $m^2 + 9 = 0$

$\Rightarrow m = \pm 3i$

$y = e^0 [c_1 \cos 3x + c_2 \sin 3x]$ $\quad y' = -3c_1 \sin 3x + 3c_2 \cos 3x$

$y(\pi/3) = 2$ $\quad y'(\pi/3) = 6$

$\Rightarrow c_1 \cos \pi + c_2 \sin \pi = 2$ $\quad \Rightarrow -3c_1 \sin \pi + 3c_2 \cos \pi = 6$

$\Rightarrow -c_1 = 2$ $\quad \Rightarrow -3c_2 = 6$

$\Rightarrow c_1 = -2$ $\quad \Rightarrow c_2 = -2$

$y = -2\cos 3x - 2\sin 3x$

ii) $m^2 - 6m + 9 = 0$

$\Rightarrow m^2 - 2.m.3 + 3^2 = 0$

$\Rightarrow (m-3)^2 = 0$

$\Rightarrow m = 3$

$y_c = c_1 e^{3x} + c_2 x e^{3x}$

$y = \frac{1}{D^2 - 6D + 9} 2e^{3x} = 2x \frac{1}{2D - 6} e^{3x} = 2x^2 \frac{1}{2} e^{3x} = x^2 e^{3x}$

$y = \frac{1}{D^2 - 6D + 9} 5e^{-2x} \cos x = 5e^{-2x} \frac{1}{(D-2)^2 - 6(D-2) + 9} \cos x = 5e^{-2x} \frac{1}{D^2 - 4D + 4 - 6D + 12 + 9} \cos x$

$= 5e^{-2x} \frac{1}{D^2 - 10D + 25} \cos x$

$= 5e^{-2x} \frac{1}{-1 - 10D + 25} \cos x$

$= -5e^{-2x} \frac{1}{10D - 24} \cos x$

$= -\frac{5}{2} e^{-2x} \frac{5D + 12}{25D^2 - 144} \cos x$

$= -\frac{5}{2} e^{-2x} \frac{1}{-25 - 144} (5D + 12) \cos x$

$= \frac{5}{338} e^{-2x} (-5\sin x + 12\cos x)$

$y = \frac{1}{D^2 - 6D - 9} (-3) = -3 \frac{1}{0 - 0 - 9} = \frac{1}{3}$

b) Solve: $y(x+y)dx + x^2 dy = 0$

Bernoulli out of syllabus