1.	a) Solve the following system by Gauss-Jordan elimination method $x - y - z + 2p = 2$ $2x + y - 2z + p = 0$ $-x + 2y - z - 4p = -1$	[6]
	b) State (do not solve) how many solutions does the following set of equations have? $x + y = 1$ $3x + 3y = -5$	[2]
	c) Solve the homogeneous system of linear equations $3x - 2y - z + 3w = 0$ $x - y + 2z - 2w = 0$	[2]

a)

$$\begin{matrix} 1 & -1 & -1 & 2 & 2 \\ 2 & 1 & -2 & 1 & 0 \\ -1 & 2 & -1 & -4 & -1 \end{matrix} \rightarrow \begin{matrix} 1 & -1 & -1 & 2 & 2 \\ 0 & 3 & 0 & -3 & -4 \\ 0 & 1 & -2 & -2 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & -3 & 0 & 3 \\ 0 & 0 & 6 & 3 & -7 \\ 0 & 1 & -2 & -2 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & -3 & 0 & 3 \\ 0 & 0 & 1 & 1/2 & -7/6 \\ 0 & 1 & -2 & -2 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 0 & 3/2 & -1/2 \\ 0 & 0 & 1 & 1/2 & -7/6 \\ 0 & 1 & 0 & -1 & -4/3 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 0 & 3/2 & -1/2 \\ 0 & 1 & 0 & -1 & -4/3 \\ 0 & 0 & 1 & 1/2 & -7/6 \end{matrix}$$

$r_2 = -2r_1 + r_2$
$r_3 = r_1 + r_3$

$r_1 = r_1 + r_3$
$r_2 = r_2 - 3r_3$

$r_2 = r_2/6$

$r_1 = r_1 + 3r_2$
$r_3 = 2r_1 + r_3$

swap(2,3)

$x + \frac{3}{2}p = -\frac{1}{2}$

$\Rightarrow x = -\frac{1}{2} - \frac{3}{2}p$

$y - p = -\frac{4}{3}$

$\Rightarrow y = -\frac{4}{3} + p$

$z + \frac{1}{2}p = -\frac{7}{6}$

$\Rightarrow z = -\frac{7}{6} - \frac{1}{2}p$

b)

$$\begin{matrix} 1 & 1 & 1 \\ 3 & 3 & -5 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 1 \\ 0 & 0 & -8 \end{matrix}$$

no solutions

c)

$$\begin{matrix} 3 & -2 & -1 & 3 & 0 \\ 1 & -1 & 2 & -2 & 0 \end{matrix} \rightarrow \begin{matrix} 0 & 1 & -7 & 9 & 0 \\ 1 & -1 & 2 & -2 & 0 \end{matrix} \rightarrow \begin{matrix} 0 & 1 & -7 & 9 & 0 \\ 1 & 0 & -5 & 7 & 0 \end{matrix}$$

$y - 7z + 9w = 0$ $\quad$ $x - 5z + 7w = 0$

$\Rightarrow y = 7z - 9w$ $\quad$ $\Rightarrow x = 5z - 7w$

2.	a) Given that, $x - 2y + z = 1$ $x - 3y - z = 0$ $2x + y - 2z = 2$	[5]
	i. Write the above system of linear equations in the form $AX = B$, where A, X and B are matrices. ii. Find the inverse of A and hence solve the above system of linear equations.	
	b) Find eigenvalues and eigenvectors of the Matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 6 \end{bmatrix}$. Also sketch the eigenspace in xy −coordinates.	[5]

a) i)

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & -3 & -1 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

ii)

$$\begin{matrix} 1 & -2 & 1 & 1 & 1 & 0 & 0 \\ 1 & -3 & -1 & 0 & 0 & 1 & 0 \\ 2 & 1 & -2 & 2 & 0 & 0 & 1 \end{matrix} \rightarrow \begin{matrix} 0 & 1 & 2 & 1 & 1 & -1 & 0 \\ 1 & -3 & -1 & 0 & 0 & 1 & 0 \\ 0 & 5 & -4 & 0 & -2 & 0 & 1 \end{matrix} \rightarrow \begin{matrix} 0 & 1 & 2 & 1 & 1 & -1 & 0 \\ 1 & 0 & 5 & 3 & 3 & -2 & 0 \\ 0 & 0 & -14 & -5 & -7 & 5 & 1 \end{matrix} \rightarrow \begin{matrix} 0 & 1 & 2 & 1 & 1 & -1 & 0 \\ 1 & 0 & 5 & 3 & 3 & -2 & 0 \\ 0 & 0 & 1 & 5/14 & 1/2 & -5/14 & -1/14 \end{matrix}$$

$r_1 = r_1 - r_2$
$r_3 = -2r_1 + r_3$

$r_2 = 3r_1 + r_2$
$r_3 = -5r_1 + r_3$

$r_3 = r_3/-14$

$$\begin{matrix} 1 & 0 & 0 & 17/14 & 1/2 & -3/14 & 5/14 \\ 0 & 1 & 0 & 2/7 & 0 & -2/7 & 1/7 \\ 0 & 0 & 1 & 5/14 & 1/2 & -5/14 & -1/14 \end{matrix} \leftarrow \begin{matrix} 0 & 1 & 0 & 2/7 & 0 & -2/7 & 1/7 \\ 1 & 0 & 0 & 17/14 & 1/2 & -3/14 & 5/14 \\ 0 & 0 & 1 & 5/14 & 1/2 & -5/14 & -1/14 \end{matrix}$$

swap(1,2)

$r_1 = r_1 - 2r_3$
$r_2 = r_2 - 5r_3$

$x = 17/14$

$y = 2/7$

$z = 5/14$

b) $A = \begin{bmatrix} 1 & 2 \\ 0 & 6 \end{bmatrix}$ $\quad (A - \lambda I)x = 0$

$A - \lambda I = 0$

$\Rightarrow \begin{bmatrix} 1-\lambda & 2 \\ 0 & 6-\lambda \end{bmatrix} = 0$

$\Rightarrow (1-\lambda)(6-\lambda) = 0$

$\Rightarrow \lambda = 1, 6$

for $\lambda = 1$

$\begin{bmatrix} 0 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

let $x = p$ $\quad \begin{bmatrix} p \\ 0 \end{bmatrix} = p \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

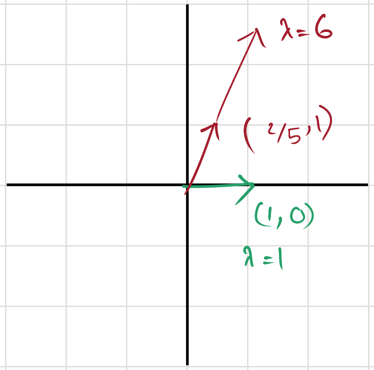
$y = 0$

for $\lambda = 6$

$\begin{bmatrix} -5 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

let $y = p$ $\quad \begin{bmatrix} \frac{2}{5}p \\ p \end{bmatrix} = p \begin{bmatrix} 2/5 \\ 1 \end{bmatrix}$

$x = \frac{2}{5}p$



3. a) Given $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 1 & 1 & 2 \\ 3 & 0 & 0 & 0 \end{bmatrix}$, $B = [1 \quad 2 \quad 1 \quad 3]$ [5]

i. Find $\det(A)$ and $\det(B)$

ii. Evaluate AB and BA.

iii. Find $3A - 3$.

iv. Find x, Such that $tr(A) = x^2 + 3$

i) $(-1)^{4+1} \, 3 \times (2 \times (3 - 0)) = -18$

ii) $\begin{bmatrix} 1 & 2 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & 1 & 1 & 2 \\ 3 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1+0+2+9 & 1+0+1+0 & 1+6+1+0 & 0+0+2+0 \end{bmatrix}$

$= \begin{bmatrix} 12 & 2 & 8 & 2 \end{bmatrix}$

AB not possible

iii) $3A - 3 = \begin{bmatrix} 3 & 3 & 3 & 0 \\ 0 & 0 & 9 & 0 \\ 6 & 3 & 3 & 6 \\ 9 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 3 & 0 \\ 0 & -3 & 9 & 0 \\ 6 & 3 & 0 & 6 \\ 9 & 0 & 0 & -3 \end{bmatrix}$

iv) $tr(A) = 2 = x^2 + 3$

$\Rightarrow x = \sqrt{-1} = \pm i$

b) Solve $(3x^2y + 2x)dx + (x^3 + 2y)dy = 0$

$\quad\quad M \quad\quad\quad\quad N$

$M_y = 3x^2 \quad N_x = 3x^2$ exact ✓

$F_x = 3x^2y + 2x$

$\Rightarrow F = x^3y + x^2 + g(y) = x^3y + x^2 + y^2$

$\Rightarrow F_y = x^3 + g'(y)$

$\Rightarrow x^3 + 2y = x^3 + g'(y)$

$\Rightarrow y^2 = g(y)$

4. a) Solve the following second order ordinary differential equations

i) $\frac{d^2y}{dx^2} + 16\frac{dy}{dx} + 64y = 0$; $y(0) = 1$, $y'(0) = 0$.

ii) $y'' + y' + 16y = 0$

a) i) $m^2 + 16m + 64 = 0$

$$\Rightarrow m = \frac{-16 \pm \sqrt{16^2 - 4\times 64}}{2} = -8$$

$$y = c_1 e^{-8x} + c_2 x e^{-8x}$$

$y(0) = 1$

$\Rightarrow \quad c_1 = 1$

$$y = e^{-8x} + 8x e^{-8x}$$

$$y' = -8 c_1 e^{-8x} + c_2 \left[e^{-8x} - 8x e^{-8x} \right]$$

$y'(0) = 0$

$\Rightarrow \quad -8c_1 + c_2[1 - 0] = 0$

$\Rightarrow \quad -8 + c_2 = 0$

$\Rightarrow \quad c_2 = 8$

ii) $m^2 + m + 16 = 0$

$$\Rightarrow m = \frac{-1 \pm \sqrt{1^2 - 64}}{2} = -\frac{1}{2} \pm \frac{1}{2} i \sqrt{7\times 9} = -\frac{1}{2} \pm \frac{3\sqrt{7}}{2} i$$

$$y = e^{-t/2}\left[c_1 \cos \frac{3\sqrt{7}}{2} t + c_2 \sin \frac{3\sqrt{7}}{2} t \right]$$

b) Solve $y'' + y' + y = 2e^x - \cos 2x - \ln 5 + e^{2x}\sin x$.

$$m^2+m+1=0$$

$$\Rightarrow m = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$y_c = e^{-x/2}\left[c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right]$$

$$y = \frac{1}{D^2+D+1} 2e^x = 2\frac{1}{1+1+1}e^x = \frac{2}{3}e^x$$

$$y = -\frac{1}{D^2+D+1}\cos 2x = -\frac{1}{-4+D+1}\cos 2x = -\frac{D+3}{D^2-3^2}\cos 2x = -\frac{D+3}{-4-9}\cos 2x$$

$$= \frac{1}{13}\left(-2\sin 2x + 3\cos 2x\right)$$

$$y = -\frac{1}{D^2+D+1}\ln 5 = -\ln 5 \frac{1}{0+0+1} = -\ln 5$$

$$y = \frac{1}{D^2+D+1} e^{2x}\sin x$$

$$= e^{2x}\frac{1}{(D+2)^2+D+3}\sin x$$

$$= e^{2x}\frac{1}{D^2+4D+4+D+3}\sin x$$

$$= e^{2x}\frac{1}{D^2+5D+7}\sin x$$

$$= e^{2x}\frac{1}{5D+6}\sin x$$

$$= e^{2x}\frac{5D-6}{25D^2-36}\sin x$$

$$= e^{2x}\frac{1}{-25-36}(5\cos x - 6\sin x) = -\frac{1}{61}e^{2x}(5\cos x - 6\sin x)$$