1

a) There are total 10 (7+3) choices for each part of the journey. Since both subtasks must be completed for the journey to be complete, this scenario will follow the product rule.

Therefore, the number of ways to complete this journey is $10 \times 10 = 100$.

b) There are 52 letters (case-sensitive) and 10 digits.

i. $2 \times (52 \times 51 \times 50 \times 49) \times (10 \times 9 \times 8 \times 7) = 65,493,792,000$
ii. $2 \times (52 \times 52 \times 52 \times 52) \times (10 \times 9 \times 8 \times 7) = 73,701,089,280$

c) We will categorize the friends according the type of chocolate they get. So, there will be three category or boxes to put them in. At least one category or box must have 10 people in it.

Here,

$$
\begin{aligned}
k &= 3 \\
n &= 10 \\
N &= ?
\end{aligned}
$$

Now,

$$
\begin{aligned}
\left\lceil \frac{N}{k} \right\rceil &= n \\
\left\lceil \frac{N}{3} \right\rceil &= 10 \\
N &= 28
\end{aligned}
$$

Therefore, at least 28 friends must be invited.

d) If we don't want to seat CS students next to each other, we have 16 (15+1) places between the BBA students where we can seat them. (one after every BBA student and one at the front)

The number of possible ways to seat 15 BBA students in 15 places is $\mathrm{P}(15, 15)$
And the number of possible ways to seat 12 CS students in 16 places is $\mathrm{P}(16, 12)$

Therefore, the total number of possible ways is $\mathrm{P}(15, 15) \times \mathrm{P}(16, 12)$

2

a) The sum of degrees for this graph is $4 \times 5 + 9 \times 2 = 38$.

According to the Handshaking Theorem,

$$
\begin{aligned}
\text{sum of degrees} &= 2 \times \text{number of vertices} \\
\text{number of vertices} &= \frac{\text{sum of degrees}}{2} \\
&= \frac{38}{2} \\
&= 19
\end{aligned}
$$

b) In a wheel graph with $n + 1$ (denoted W_n) vertices, there are $2n$ edges.

So, the total number of edges in the graph W_{110} is $2 \times 110 = 220$.

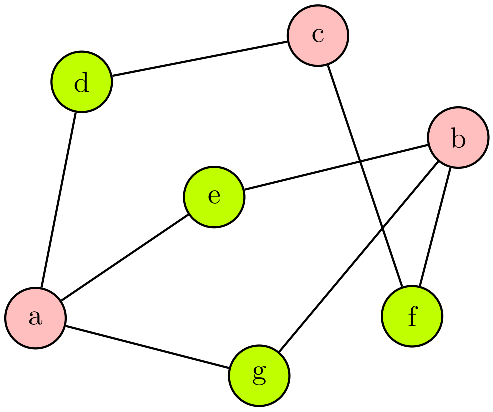
c) The following table shows the degree of each vertex.

Vertex	degree
0	4
1	2
2	3
3	1
4	4
5	3
6	1
Sum	18

The number of edges in the graph is 9.

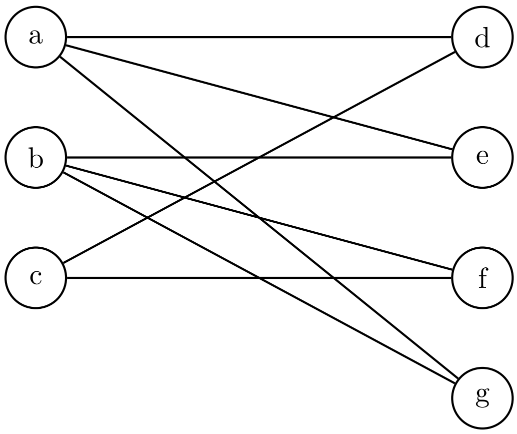
According to the Handshaking Theorem the sum of degrees must be equal to twice of the number of edges, which is true for the given graph since $2 \times 9 = 18$.

d) Coloring the given graph using the two-coloring algorithm.



As we can see, every edge connects two differently colored vertex. Therefore, the given graph is bipartite.

Redrawing the graph in bipartite form.

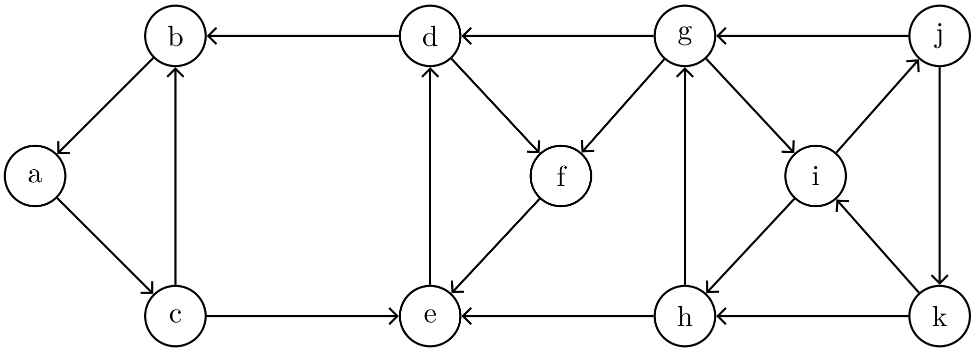


3.c

i. Traversing the graph and creating a stack according to the finishing time of each vertex. The following is an illustration of the traverse path (from left to right e.g. a to b).

a	→	b	→	c								
			→	d	→	g	→	j	→	i	→	k
							→	h				
					→	e	→	f				

The stack sorted by finishing time: a b d e f g h j i k c

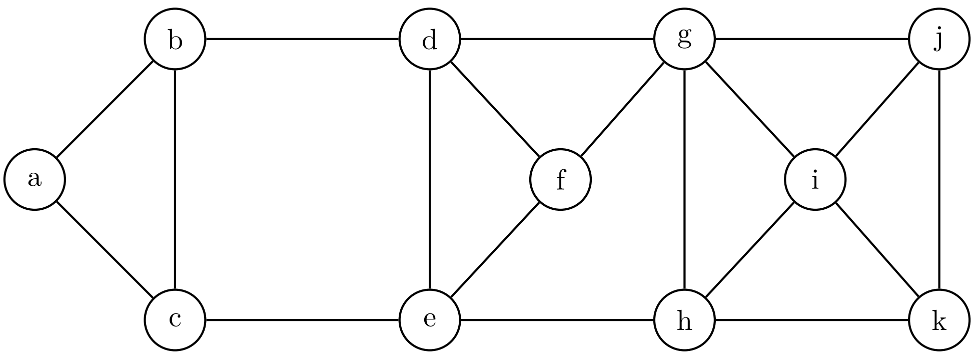


Now after reversing the direction of the edges (the graph above), let's traverse again starting from the vertex at the top of the stack.

a	→	c	→	b				
			→	e	→	d	→	f
g	→	i	→	h				
			→	j	→	k		

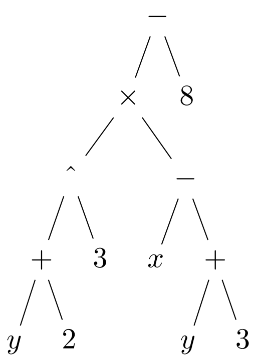
So, we can reach c,b,e,d,f from a and i,h,j,k from g. Therefore, there are two strongly connected components in this graph which are {a, b, c, d, e, f} and {g, h, i, j, k}.

ii. If we look at the underlying non-directed graph (the graph below) of the given graph, it is a complete graph meaning there is a path from one vertex to any other vertex. Therefore, the given graph is weakly connected.



4

b) Expression tree:



c) The given scenario can be represented as a full 4-ary tree where the persons are the vertices and the recipients are the children of the sender. Therefore, there are 400 leaves in this tree ($l = 400$).

The number of leaves (l) in a full m-ary tree with i internal vertices and n vertices is $(m - 1) \cdot i + 1$.

Here,

$$\begin{aligned}(4-1)\cdot i + 1 &= 400\\ i &= \frac{400-1}{3}\\ i &= \frac{399}{3}\\ i &= 133\end{aligned}$$

133 people sent out the letter.