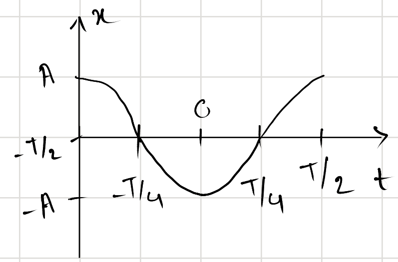
1. (a) Why we observe damped harmonic motion in RLC circuit? 2 CO1
(b) The equation of displacement of a simple harmonic oscillator is $x = A\cos(\omega t + \pi)$. Plot displacement vs. time and acceleration vs. time graphically. What is the phase difference between displacement and acceleration? 2 CO1
(c) Draw a transverse wave and show the wavelength on the wave. 2 CO1

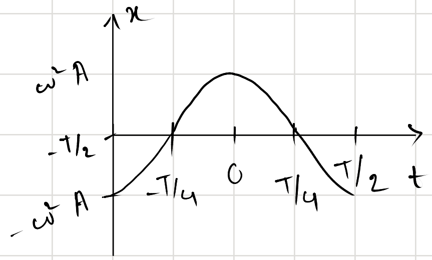
a) Because of the Resistor the flow of current is impeded and thus the damping is observed.

b)



$$\omega t + \pi = 0$$

$$\Rightarrow t = -\pi \times \frac{1}{\omega} = -\pi \times \frac{T}{2\pi} = -\frac{T}{2}$$



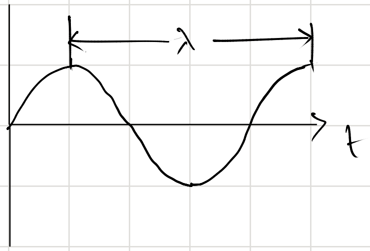
$$x = A\cos(\omega t + \pi)$$

$$\Rightarrow a = -\omega^2 A\cos(\omega t + \pi)$$

$$= \omega^2 A \cos(\pi + \omega t + \pi) = \omega^2 A \cos(\omega t + 2\pi)$$

phase difference $(2\pi - \pi) = \pi$

c)



wavelength λ

2. (a) Consider a mass-spring system oscillating in SHM and where the equation of displacement is
$$y = 7\sin(8t - \tfrac{\pi}{4}).$$
If the block has mass m = 2 kg, calculate:
(i) time period of the oscillation
(ii) the velocity at t = 0.3 sec
Consider all the units in S.I. unit system.
(b) A block attached to a spring is suspended vertically. If the block is pushed 7 cm upward from the equilibrium position and released at t = 0. The mass of the block is 5 kg and the spring constant is k = 22 N/m.
(i) Calculate the potential energy at x = 3 cm.
(ii) Calculate the kinetic energy at the same position.

a) $y = 7\sin(8t - \pi/4)$

i) $\omega = 8 \text{ rad s}^{-1}$ $\quad T = \frac{2\pi}{\omega} = \frac{\pi}{4}$ s

ii) $v = 7 \cdot 8 \cos(8t - \pi/4)$

$= 56 \cos(8 \times 0.3 - \pi/4)$

$= -2.45 \text{ m s}^{-1}$

b) $A = 7 \times 10^{-2}$ m $\quad m = 5$ kg $\quad K = 22$ N/m

i) $E_p = \frac{1}{2} k x^2 = \frac{1}{2} 22 \times (3 \times 10^{-2})^2$

$= 9.9 \times 10^{-3}$ J

ii) $E_K = \frac{1}{2} K A^2 - E_p$

$= \frac{1}{2} 22 \times (7 \times 10^{-2})^2 - 9.9 \times 10^{-3}$

$= 0.044$ J

3. (a) John constructed an RLC circuit with the value, C = 0.009 μF, L = 0.5 mH, and R= 200Ω respectively. 4
(i) Whether the circuit is oscillatory, calculate the frequency of oscillation of the RLC circuit.
(ii) What will be the value of resistance R if he wants to produce critical damping?
(b) For a damped oscillator, m = 0.30 kg, k = 19.6 N/m, and b = 0.00086 kg/s. The oscillator is released at t = 0 and the amplitude is 10 cm. 4
(i) Calculate the frequency of oscillations of the oscillator.
(ii) How long does it take for the amplitude of the damped oscillator to drop to one half of its initial value?

a) i) $\omega = \sqrt{\frac{1}{LC}} = \sqrt{(0.5\times10^{-3} \times 0.009\times10^{-6})^{-1}}$

$= 471404.52$ rad/s

$\gamma = \frac{R}{L} = \frac{200}{0.5\times10^{-3}} = 400000\ s^{-1}$

$\omega^2 = 2.22\times10^{11}$

$\frac{\gamma^2}{4} = 4\times10^{10}$

since $\omega^2 > \frac{\gamma^2}{4}$, the circuit is oscillatory

$\omega_1 = \sqrt{\omega^2 - \frac{\gamma^2}{4}} = 426874.9492$ rad/s

$f = \frac{1}{T} = \frac{\omega_1}{2\pi} = 67939.258$ Hz

ii) $\omega^2 = \frac{\gamma^2}{4} = \frac{R^2}{4L^2}$

$\Rightarrow R = \sqrt{4\omega^2L^2} = 2\omega L = 471.405\ \Omega$

b) i) $\omega = \sqrt{\frac{k}{m}} = 8.08$ rad/s

$\gamma = \frac{b}{m} = 0.002867\ s^{-1}$

$\omega_1 = \sqrt{\omega^2 - \frac{\gamma^2}{4}} = 8.079$ rad/s

$f = \frac{1}{T} = \frac{\omega}{2\pi} = 1.286$ Hz

ii) $x = Ae^{-\gamma/2\, t}\cos(\omega_1 t)$

$\Rightarrow \frac{A}{2} = Ae^{-\gamma/2\, t}$

$\Rightarrow -\gamma/2\ t = \ln 1/2$

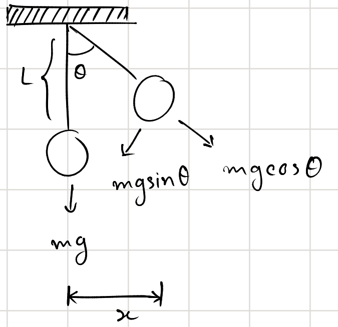
$\Rightarrow t = -\frac{2}{\gamma}\ln 1/2 = 483.535$ s

initial value?

4. (a) Show that, for a simple pendulum in SHM, $\frac{d^2\theta}{dt^2} + g\frac{\theta}{L} = 0$.
(b) For a mass-spring system oscillating in SHM, the equation of displacement is, $x = A\sin\omega t$
Show that the total energy of the oscillator is, $E = \frac{1}{2}KA^2$. Plot energy vs. displacement graph.

a) restoring force $F = -mg\sin\theta$

$= -mg\theta$ [for small θ]



now, $x = L\theta$

$\Rightarrow \frac{d^2x}{dt^2} = L\frac{d^2\theta}{dt^2}$

$\Rightarrow a = \frac{F}{m} = \frac{-mg\theta}{m} = L\frac{d^2\theta}{dt^2}$

$\Rightarrow -g\theta = L\frac{d^2\theta}{dt^2}$

$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$

b) $E_p = \int F\,dx = \int ma\,dx$

$= m\int \omega^2 x\,dx$

$= k\,\frac{1}{2}x^2$

$= \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t)$

$x = A\sin(\omega t)$

$v = \omega A\cos(\omega t)$

$a = -\omega^2 A\sin(\omega t)$

$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t)$

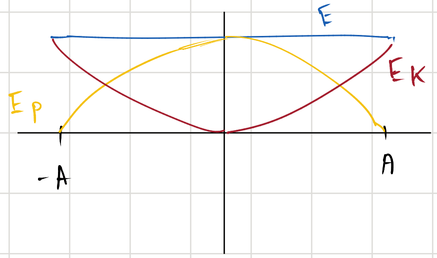
$= \frac{1}{2}kA^2\cos^2(\omega t)$

$E = E_p + E_K$

$= \frac{1}{2}kA^2(\sin^2(\omega t) + \cos^2(\omega t))$

$= \frac{1}{2}kA^2$

$E_p = \frac{1}{2}kx^2 \qquad E_K = \frac{1}{2}k(A^2 - x^2)$



5. (a) Show that, the equation of displacement of the particles of a medium for a progressive wave is $y = A \sin\frac{2\pi}{\lambda}(vt-x)$. Calculate the value of $\frac{d^2y}{dt^2}$. Here the symbols have their usual meanings.
(b) Derive the differential equation of a mass-spring system oscillating in DHM. Write the conditions of three types of damped harmonic motion and graphically represent them by plotting displacement vs. time graphs.

a) eqn of a wave is as follows

$$y = A \sin(\omega t)$$

since the wave is progressive there will be a variable path difference in the eqn; let it be x

for a path diff of λ, the phase diff $\phi = 2\pi$ rad

$\therefore$ for x, $\phi = \frac{2\pi}{\lambda} x$

now the eqn, $y = A \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)$

$$= A \sin\left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right)$$

$$= A \sin \frac{2\pi}{\lambda}\left(\frac{\lambda}{T} t - x\right)$$

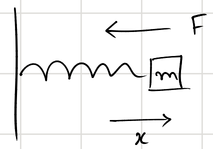
$$= A \sin \frac{2\pi}{\lambda}(vt - x)$$

$$\frac{dy}{dt} = A\omega \cos\left(\omega t - \frac{2\pi}{\lambda} x\right)$$

$$\frac{d^2y}{dt^2} = -A\omega^2 \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)$$

$$= -\frac{4\pi^2}{\lambda^2} v^2 A \sin \frac{2\pi}{\lambda}(vt - x) \quad \left[\omega = \frac{2\pi}{T} = \frac{2\pi}{\lambda} v\right]$$

b)



let b be the friction constant

$$F = -kx - bv$$

$$\Rightarrow ma = -kx - bv$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{b}{m}v = -\omega^2 x - \gamma v$$

$$\Rightarrow \frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega^2 x = 0 \quad \text{——— (1)}$$

Let $x = Ae^{i(\omega t + \phi)} = Be^{pt}$

$$\Rightarrow \frac{dx}{dt} = Be^{pt} \cdot p$$

$$\Rightarrow \frac{d^2x}{dt^2} = Be^{pt} \cdot p^2$$

now eqn (1),

$$Be^{pt} \cdot p^2 + \gamma \cdot Be^{pt} \cdot p + \omega^2 Be^{pt} = 0$$

$$\Rightarrow p^2 + \gamma p + \omega^2 = 0$$

$$\Rightarrow p = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\omega^2}}{2}$$

$$= -\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - \omega^2}$$

if $\omega^2 > \gamma^2/4$, it's oscillatory DHM

$\omega^2 = \gamma^2/4$, " critical "

$\omega^2 < \gamma^2/4$, " overdamped HM