

(1) for the node with v_1 voltage:

$$\frac{80 - v_1}{20} = \frac{v_1 - v_2}{30} + \frac{v_1 - v_x}{30} = \frac{2v_1 - v_2 - v_x}{30}$$

$$\Rightarrow 2400 - 30v_1 = 40v_1 - 20v_2 - 20v_x$$

$$\Rightarrow -70v_1 - 20v_2 - 20v_x = 2400 \quad \textcircled{1}$$

for the node with v_2 voltage:

$$\frac{v_x - v_2}{30} + \frac{v_1 - v_2}{30} = \frac{v_2 - (-80)}{20}$$

$$\Rightarrow \frac{v_1 - 2v_2 + v_x}{30} = \frac{v_2 + 80}{20}$$

$$\Rightarrow 20v_1 - 40v_2 + 20v_x = 30v_2 + 2400$$

$$\Rightarrow 20v_1 - 70v_2 + 20v_x = 2400 \quad \textcircled{11}$$

for the node x :

$$\frac{v_1 - v_x}{30} = \frac{v_x}{20} + \frac{v_x - v_2}{30} = \frac{3v_x + 2v_x - 2v_2}{60} = \frac{5v_x - 2v_2}{60}$$

$$\Rightarrow 2v_1 - 2v_x = 5v_x - 2v_2$$

$$\Rightarrow 2v_1 + 2v_2 - 7v_x = 0$$

solving $\textcircled{1}$, $\textcircled{10}$ and $\textcircled{11}$

$$v_1 = 26.67$$

$$v_2 = -26.67$$

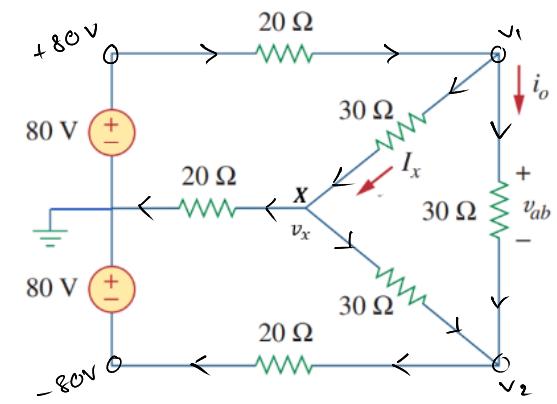
$$v_x = 0$$

$$v_{ab} = v_1 - v_2 = 26.67 + 26.67 = 53.34 \text{ V}$$

$$i_o = \frac{v_{ab}}{30} = 1.78 \text{ A}$$

$$I_x = \frac{v_1 - v_x}{30} = \frac{26.67 - 0}{30} = 0.89 \text{ A}$$

$$v_x = 0$$



(2) a)

Loop 1

$$(i_1 - i_3) + 3(i_1 - i_2) - 12 = 0$$

$$\Rightarrow i_1 - i_3 + 3i_1 - 3i_2 - 12 = 0$$

$$\Rightarrow 4i_1 - 3i_2 - i_3 - 12 = 0 \quad \text{--- (1)}$$

Loop 2

$$3(i_2 - i_1) + 4(i_2 - i_4) = 0$$

$$\Rightarrow 3i_2 - 3i_1 + 4i_2 - 4i_4 = 0$$

$$\Rightarrow -3i_1 + 7i_2 + 12 = 0 \quad \text{--- (2)}$$

Loop 3

$$6i_3 + 8(i_3 - i_4) + (i_3 - i_1) = 0$$

$$\Rightarrow 6i_3 + 8i_3 - 8i_4 + i_3 - i_1 = 0$$

$$\Rightarrow -i_1 + 15i_3 + 24 = 0 \quad \text{--- (3)}$$

Loop 4

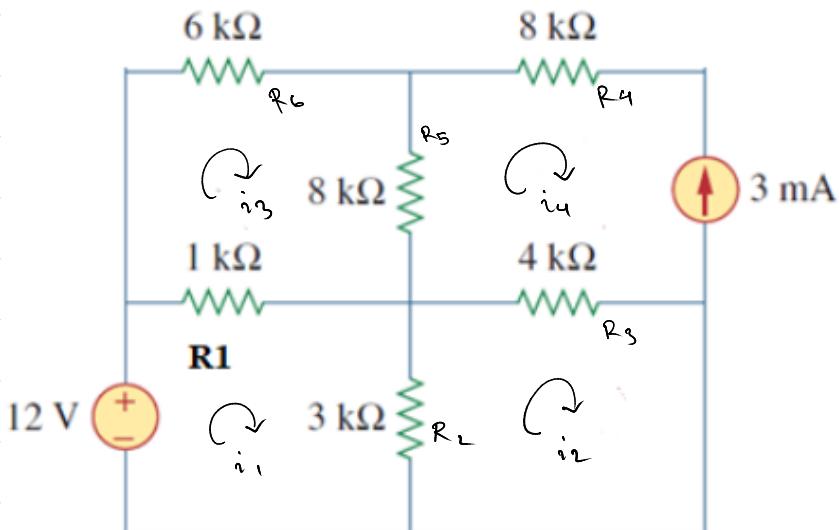
$$i_4 = -3 \text{ mA}$$

solving (1), (2) and (3)

$$i_1 = 1.9856 \text{ mA}$$

$$i_2 = -0.86 \text{ mA}$$

$$i_3 = -1.467 \text{ mA}$$



$$\text{current through } R_1 = i_1 - i_3 = 3.45 \text{ mA}$$

$$\text{--- --- --- } R_2 = i_1 - i_2 = 2.84 \text{ mA}$$

$$\text{--- --- --- } R_3 = i_2 - i_4 = 2.14 \text{ mA}$$

$$\text{--- --- --- } R_4 = 3 \text{ mA}$$

$$\text{--- --- --- } R_5 = i_4 - i_3 = -1.533 \text{ mA}$$

$$\text{--- --- --- } R_6 = i_3 = -1.467 \text{ mA}$$

$$\text{b) } P = i^2 R = (3.45 \text{ mA})^2 \times 1 \text{ k}\Omega \\ = 3.45^2 \times 10^{-6} \times 10^3 \text{ W} \\ = 0.0119 \text{ W}$$

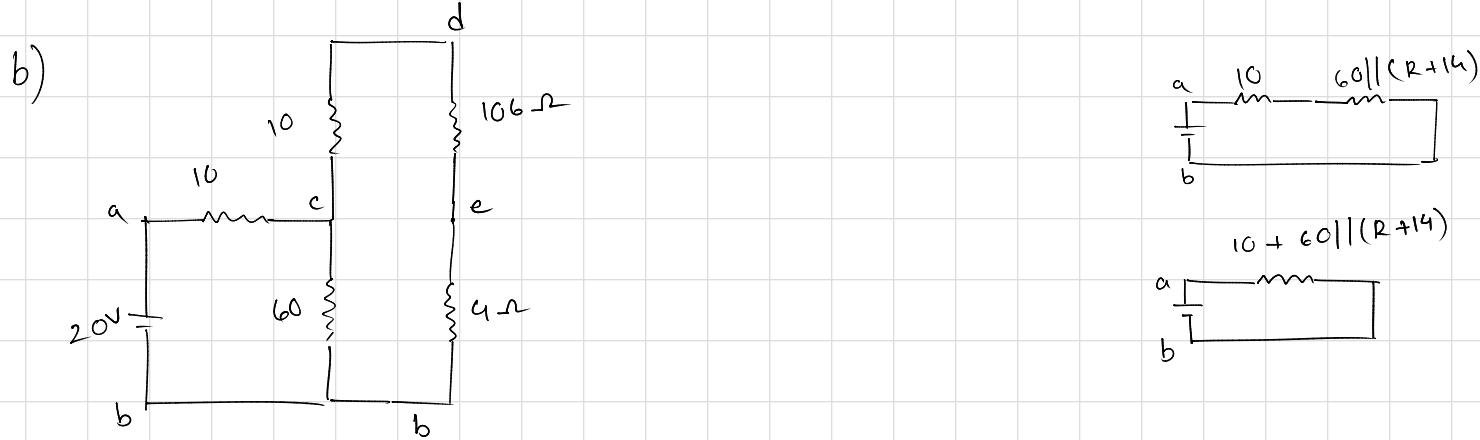
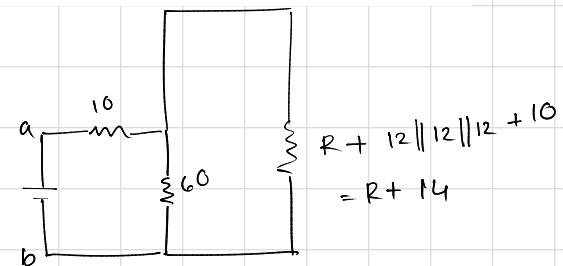
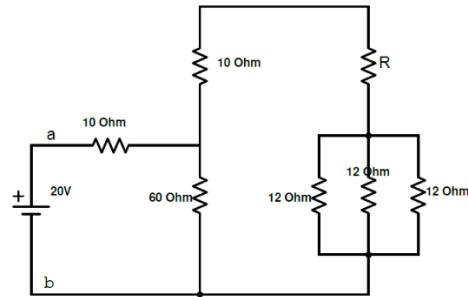
$$(3) \quad a) \quad 10 + 60 \parallel (R+14) = 50$$

$$\Rightarrow \left(60^{-1} + (R+14)^{-1} \right)^{-1} = 40$$

$$\Rightarrow 60^{-1} + (R+14)^{-1} = 40^{-1}$$

$$\Rightarrow R+14 = (40^{-1} - 60^{-1})^{-1}$$

$$\Rightarrow R = (40^{-1} - 60^{-1})^{-1} - 14 = 106 \Omega$$



$$v_{cb} = 20 \times \frac{60 \parallel (106+10+4)}{10 + 60 \parallel (106+10+4)} = 16V$$

$$v_{eb} = 16 \times \frac{4}{10 + 106 + 4} = 0.533V$$

$$i_{ab} = \frac{20}{50} = \frac{2}{5} A = i_{cb}$$

$$i_{cdeb} = \frac{2}{5} \times \frac{60}{60+120} = 0.133A$$

$$i_{eb} = 0.133A$$

$$i(12\Omega) = 0.133 \times \frac{12 \parallel 12}{12 + 12 \parallel 12} = 0.0443A$$