2. a) Find the distance between the given skew lines

L_1: $x = 1 + 7t$, $y = 3 + t$, $z = 5 - 3t$

L_2: $x = 4 - t$, $y = 6$, $z = 7 + 2t$

$\vec{v_1} = \langle 7, 1, -3 \rangle$

$\vec{v_2} = \langle -1, 0, 2 \rangle$

$\vec{n} = \langle 2, -(14-3), 0+1 \rangle$

$= \langle 2, -11, 1 \rangle$

passing point on L_2 $P_2(4, 6, 7)$

eqn of the plane containing L_2

$2(x-4) - 11(y-6) + 1(z-7) = 0$

$\Rightarrow 2x - 8 - 11y + 66 + z - 7 = 0$

$\Rightarrow 2x - 11y + z + 51 = 0$

passing point on L_1: $P_1(1, 3, 5)$

distance between L_1 and $L_2 = \dfrac{|2 - 33 + 5 + 51|}{\sqrt{2^2 + 11^2 + 1^2}}$

$= 2.227$

b) Find the equation of the plane passing through the points $\boldsymbol{p_1(0,-2,3)}$, $\boldsymbol{p_2(3,0,-2)}$ and $\boldsymbol{p_3(-2,\ 3,0)}$.

$\overrightarrow{P_1P_2} = \langle 3, 2, -5 \rangle$

$\overrightarrow{P_1P_3} = \langle -2, 5, -3 \rangle$

$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \langle -6 + 25, -(-9-10), 15 + 4 \rangle$

$= \langle 19, 19, 19 \rangle \rightarrow$ normal

passing point $(0, -2, 3)$

eqn of the plane

$19(x-0) + 19(y+2) + 19(z-3) = 0$

$\Rightarrow 19x + 19y + 38 + 19z - 57 = 0$

$\Rightarrow 19x + 19y + 19z - 19 = 0$

c) Let $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$ be the lines

$\boldsymbol{L_1}$: $x = 1 + 2t$, $y = 3 - t$, $z = 3t$

$\boldsymbol{L_2}$: $x = 4 + 3t$, $y = 2 - 5t$, $z = -1 + 2t$

i) Are the lines parallel?

ii) Do the lines intersect?

i) $\vec{v_1} = \langle 2, -1, 3 \rangle$

$\vec{v_2} = \langle 3, -5, 2 \rangle$

no such scalar multiple t for which $\vec{v_1} = t\,\vec{v_2}$

$\therefore$ not parallel

ii) let (x_0, y_0, z_0) be the intersecting point

for L_1: $x_0 = 1 + 2t_1$, $y_0 = 3 - t_1$, $z_0 = 3t_1$

for L_2: $x_0 = 4 + 3t_2$, $y_0 = 2 - 5t_2$, $z_0 = -1 + 2t_2$

$1 + 2t_1 = 4 + 3t_2$ $\qquad$ $3 - t_1 = 2 - 5t_2$

$\Rightarrow 2t_1 - 3t_2 = 3$ $\qquad$ $\Rightarrow -t_1 + 5t_2 = -1$

$t_1 = 12/7$

$t_2 = 1/7$

now $3t_1 = -1 + 2t_2$

$\Rightarrow 3\,\dfrac{12}{7} = -1 + 2\,\dfrac{1}{7}$

$\Rightarrow \dfrac{36}{7} = -\dfrac{5}{7}$

which is a contradiction

$\therefore$ the lines don't intersect

3. a) Find the area of the triangle with vertices $\boldsymbol{P_1(1,3,0)}$ $\boldsymbol{P_2(-2,0,1)}$ and $\boldsymbol{P_3(0,5,-6)}$.

a) $\overrightarrow{P_1P_2} = \langle -3, -3, 1 \rangle$

$\overrightarrow{P_1P_3} = \langle -1, 2, -6 \rangle$

$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \langle 18 - 2, -(18+1), -6 - 3 \rangle$

$= \langle 16, -19, -9 \rangle$

area $= \dfrac{1}{2}\sqrt{16^2 + 19^2 + 9^2}$

$= 19.209$

b) Find the vector component (orthogonal projection) of $\boldsymbol{p} =< -\mathbf{1}, -\mathbf{2}, \mathbf{0} >$ along $\boldsymbol{q} =< \mathbf{0}, \mathbf{0}, \mathbf{1} >$ and orthogonal to $\boldsymbol{q}$.

c) Determine the angle that is made by $< -\sqrt{\mathbf{3}},\ \mathbf{4},\ \mathbf{5} >$ with the positive $\boldsymbol{x}$-axis and also with the positive $\boldsymbol{z}$-axis.

b) $|\vec{q}| = \sqrt{1^2} = 1$ thus $\vec{q}$ is a unit vector

$\mathrm{proj}_{\vec{q}}\, \vec{p} = (\vec{p} \cdot \vec{q})\, \vec{q}$

$= (\langle -1, -2, 0 \rangle \cdot \langle 0, 0, 1 \rangle)\vec{q}$

$= (0 + 0 + 0)\, \vec{q}$

$= \vec{0}$ (null vector)

component orthogonal to $\vec{q}$ $= \vec{p} - \mathrm{proj}_{\vec{q}}\, \vec{p}$

$= \langle -1, -2, 0 \rangle - \langle 0, 0, 0 \rangle$

$= \langle -1, -2, 0 \rangle$

$\langle -\sqrt{3}, 4, 5 \rangle \cdot \langle 1, 0, 0 \rangle = -\sqrt{3}$

$\Rightarrow \sqrt{3 + 4^2 + 5^2} \times 1 \times \cos\theta_1 = -\sqrt{3}$

$\Rightarrow \cos\theta_1 = -\dfrac{\sqrt{3}}{2\sqrt{11}}$

$\Rightarrow \theta_1 = 1.835$ rad

$\cos\theta_2 = \dfrac{5}{2\sqrt{11}} \Rightarrow \theta_2 = 0.717$ rad

$\left\langle -\dfrac{\sqrt{3}}{2\sqrt{11}}, \dfrac{4}{2\sqrt{11}}, \dfrac{5}{2\sqrt{11}} \right\rangle$