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b) Suppose that the the bipartite sets of the complete bipartite graph $K_{m,n}$ are V_1 and V_2, where $|V_1| = m$ and $|V_2| = n$.

Degree of every vertex in V_1 is n (being a complete graph) and degree of every vertex in V_2 is m. Therefore the sum of degree of this graph $= mn + nm = 2mn$. So, according to the handshaking theorem, the number of edges $= mn$.

c) The sum of degrees in the given graph $= 4x + 5y$. According to the handshaking theorem, $4x + 5y = 2e$ where e is the number edges. So the value of $4x + 5y$ must be even as $2e$ is even. For that to be true, $4x$ and $5y$ both must be even.

Here 4 being even, $4x$ is definitely even. Now, for $5y$ to be even, y must be even as 5 is odd.

Therefore, among x and y, y must be even.