1

Answer all questions.

a) Solve the following system by Gauss-Jordan elimination method **[5]**

$$x - y + 2z + 3p = 3$$
$$3x + y - 2z + p = 0$$
$$-x + 2y - z - p = -1$$

$$\begin{matrix} 1 & -1 & 2 & 3 & 3 \\ 3 & 1 & -2 & 1 & 0 \\ -1 & 2 & -1 & -1 & -1 \end{matrix} \rightarrow \begin{matrix} 1 & -1 & 2 & 3 & 3 \\ 0 & 4 & -8 & -8 & -9 \\ 0 & 1 & 1 & 2 & 2 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 3 & 5 & 5 \\ 0 & 0 & -12 & -16 & -17 \\ 0 & 1 & 1 & 2 & 2 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 3 & 5 & 5 \\ 0 & 1 & 1 & 2 & 2 \\ 0 & 0 & 1 & 4/3 & 17/12 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 0 & 1 & 3/4 \\ 0 & 1 & 0 & 2/3 & 7/12 \\ 0 & 0 & 1 & 4/3 & 17/12 \end{matrix}$$

$$x + p = 3/4 \quad \Rightarrow \quad x = 3/4 - p$$
$$y + 2/3\,p = 7/12 \qquad y = 7/12 - 2/3\,p$$
$$z + 4/3\,p = 17/12 \qquad z = 17/12 - 4/3\,p$$

b) Solve the homogeneous system of linear equations **[5]**

$$x - 2y + z - w = 0$$
$$2x - 3y + 2z + w = 0$$
$$3x - 2y + z - w = 0$$

$$\begin{matrix} 1 & -2 & 1 & -1 & 0 \\ 2 & -3 & 2 & 1 & 0 \\ 3 & -2 & 1 & -1 & 0 \end{matrix} \rightarrow \begin{matrix} 1 & -2 & 1 & -1 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 4 & -2 & 2 & 0 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 1 & 5 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & -2 & -10 & 0 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 1 & 5 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 0 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 5 & 0 \end{matrix}$$

$$x = 0$$
$$y = -3p$$
$$z = -5p$$
$$w = p$$

2

a) Find the Eigenvalues and corresponding Eigenvector of Matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$. Also draw the Eigen space in xy –plane. [5]

$$AX - \lambda X = 0$$

$$\Rightarrow (A - \lambda I)X = 0$$

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \det \begin{bmatrix} 1-\lambda & 0 \\ 0 & -2-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)(2+\lambda) = 0$$

$$\Rightarrow \lambda = 1, -2$$

for $\lambda = 1$

$$\begin{bmatrix} 1-\lambda & 0 \\ 0 & -2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ -3y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

let $x = p$

$$-3y = 0 \Rightarrow y = 0$$

$$X = \begin{bmatrix} p \\ 0 \end{bmatrix} = p \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for $\lambda = -2$

$$\begin{bmatrix} 1-\lambda & 0 \\ 0 & -2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

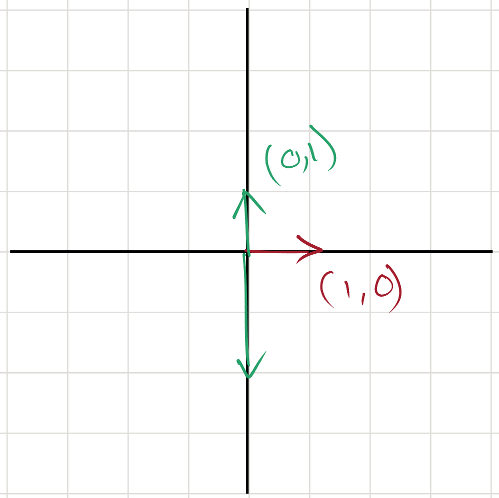
$$\Rightarrow \begin{bmatrix} -3 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3x \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x = 0 \Rightarrow x = 0$$

let $y = q$,

$$X = \begin{bmatrix} 0 \\ q \end{bmatrix} = q \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



b) Find the inverse of $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ by applying inversion [5]

algorithm.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & -5 & 6 & -3 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -6/5 & 3/5 & -1/5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/5 & 3/5 & -1/5 \\ 0 & 1 & 0 & 2/5 & -1/5 & 2/5 \\ 0 & 0 & 1 & -6/5 & 3/5 & -1/5 \end{array}\right]$$

$r_2 = r_2 - 2r_1$ $\qquad$ $r_3 = r_3 - 3r_2$ $\qquad$ $r_3 = r_3/5$ $\qquad$ $r_1 = r_1 + r_3$; $r_2 = r_2 - 2r_3$

③

Consider a Matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 4 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ [4]

i) Find the Cofactor Matrix of A [2]

ii) Find $\det(A)$. [4]

iii) Find $P(A)$, where $P(x) = -A^2 + 5 + 2A + A^T$

(i) $\begin{bmatrix} 4 & -2 & -2 \\ -4 & 1 & 1 \\ -12 & 6 & 2 \end{bmatrix}$

(ii) $\det(A) = 4 - 2 - 6$

$= -4$

(4)

a) Solve $y'' + y' + y = e^{-2x} + \sin 3x + \ln 5 - 4^x + e^x \cos 2x$ [6]

$$m^2 + m + 1 = 0$$

$$\Rightarrow m = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$y_c = e^{-1/2}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right]$$

$$y = \frac{1}{D^2+D+1} e^{-2x} = \frac{1}{4-2+1} e^{-2x} = \frac{1}{3} e^{-2x}$$

$$y = \frac{1}{D^2+D+1} \sin 3x = \frac{1}{-9+D+1} \sin 3x = \frac{D+8}{(D-8)(D+8)} \sin 3x$$

$$= \frac{D+8}{D^2-8^2} \sin 3x = \frac{1}{-9-64}(D+8)\sin 3x = -\frac{1}{73}(3\cos 3x + 8\sin 3x)$$

$$y = \frac{1}{D^2+D+1} \ln 5 = \ln 5$$

$$y = \frac{1}{D^2+D+1}(-4^x) = -\frac{1}{D^2+D+1} e^{\ln 4^x} = -\frac{1}{D^2+D+1} e^{x\ln 4} = -\frac{4^x}{(\ln 4)^2 + \ln 4 + 1}$$

$$y = \frac{1}{D^2+D+1} e^x \cos 2x = e^x \frac{1}{D^2+2D+1+D+1+1}\cos 2x = e^x \frac{1}{D^2+3D+3}\cos 2x = e^x \frac{1}{-4+3D+3}\cos 2x$$

$$= e^x \frac{1}{3D-1}\cos 2x = e^x \frac{3D+1}{9D^2-1}\cos 2x = e^x \frac{3D+1}{-36-1}\cos 2x = -\frac{1}{37}(-6\sin 2x + \cos 2x)$$

$$= \frac{1}{37}(6\sin 2x - \cos 2x)$$

b) Solve the following second order ordinary differential equation

$$y'' - 4y' + 4y = 0 \qquad y(0) = -1 \qquad y'(0) = 1 \qquad [4]$$

$m^2 - 4m + 4 = 0$

$\Rightarrow \quad m = 2, 2$

$y = c_1 e^{2x} + c_2 x e^{2x}$ $\qquad$ $y' = 2c_1 e^{2x} + 2c_2 x e^{2x} + c_2 e^{2x}$

$y(0) = -1$ $\qquad$ $y'(0) = 1$

$\Rightarrow \quad c_1 = -1$ $\qquad$ $\Rightarrow \quad 2c_1 + c_2 = 1$

$\Rightarrow \quad c_2 = 1 + 2 = 3$

$y = -e^{2x} + 3xe^{2x}$