1/ Differentiate the following function:

(i). $f(x) = \sec^3\left(\frac{x}{\sqrt{1-x^2}}\right)$

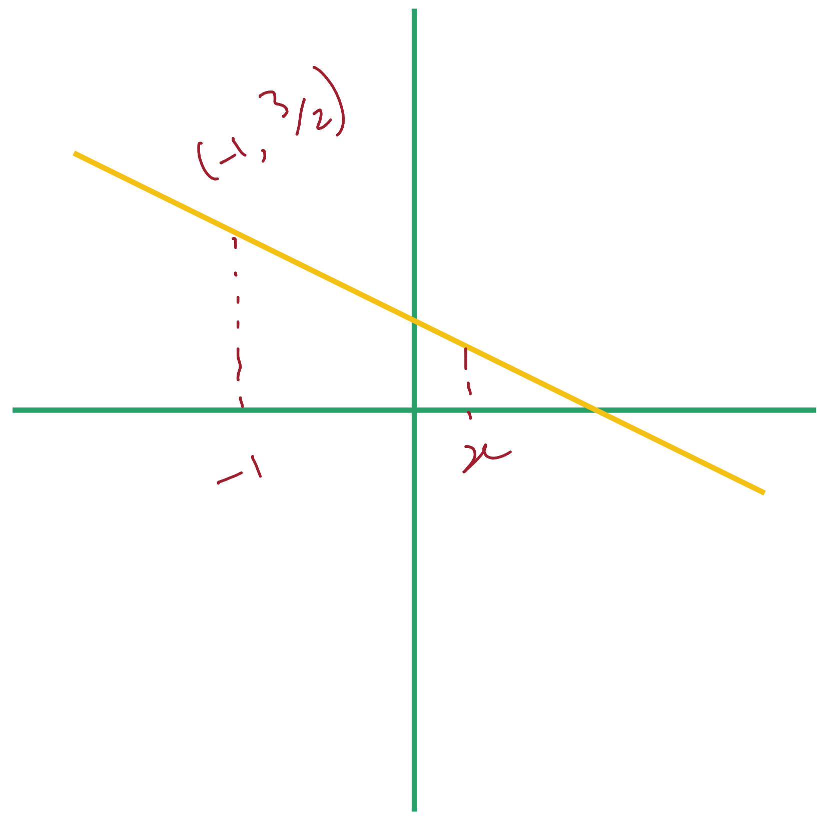
(ii). $g(x) = (x^7 - 3)^{-2}\tan\left(\frac{3}{x}\right)$

i) $3\sec^2\frac{x}{\sqrt{1-x^2}} \cdot \sec\frac{x}{\sqrt{1-x^2}}\tan\frac{x}{\sqrt{1-x^2}} \cdot \frac{\sqrt{1-x^2} - x\frac{-2x}{2\sqrt{1-x^2}}}{1-x^2}$

ii) $-2(x^7-3)^{-3} \cdot 7x^6\tan\frac{3}{x} + (x^7-3)^{-2}\sec^2\frac{3}{x}\left(-3x^{-2}\right)$

2/ (a) Then use simple area formula from geometry to find the area function $A(x)$ that gives the area between the graph of the function $f(x) = 1 - \frac{x}{2}$ and the interval $[-1, x]$. Also, confirm that $A'(x) = f(x)$.

$1 - \frac{-1}{2}$



$\frac{1}{2} \times \left(1 - \frac{x}{2} + \frac{3}{2}\right) \times (x+1)$

$= \frac{1}{2}(x+1)\left(\frac{5-x}{2}\right)$

$= \frac{1}{4}(x+1)(5-x)$

$= \frac{1}{4}(5x+5-x^2-x)$

$= \frac{1}{4}(4x+5-x^2)$

confirm $\frac{1}{4}(4+0-2x)$

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$= 1 - \frac{x}{2}$

b) Evaluate the integral $\int_4^7 f(x)dx$, given that $f(x) = \begin{cases} |3-x|\,; x \le 5 \\ 2\,; x > 5 \end{cases}$. Also verify your result by interpreting the integral geometrically.

$\int_4^5 x - 3\, dx$

$= \left[\frac{x^2}{2} - 3x\right]_4^5$

$= \frac{3}{2}$

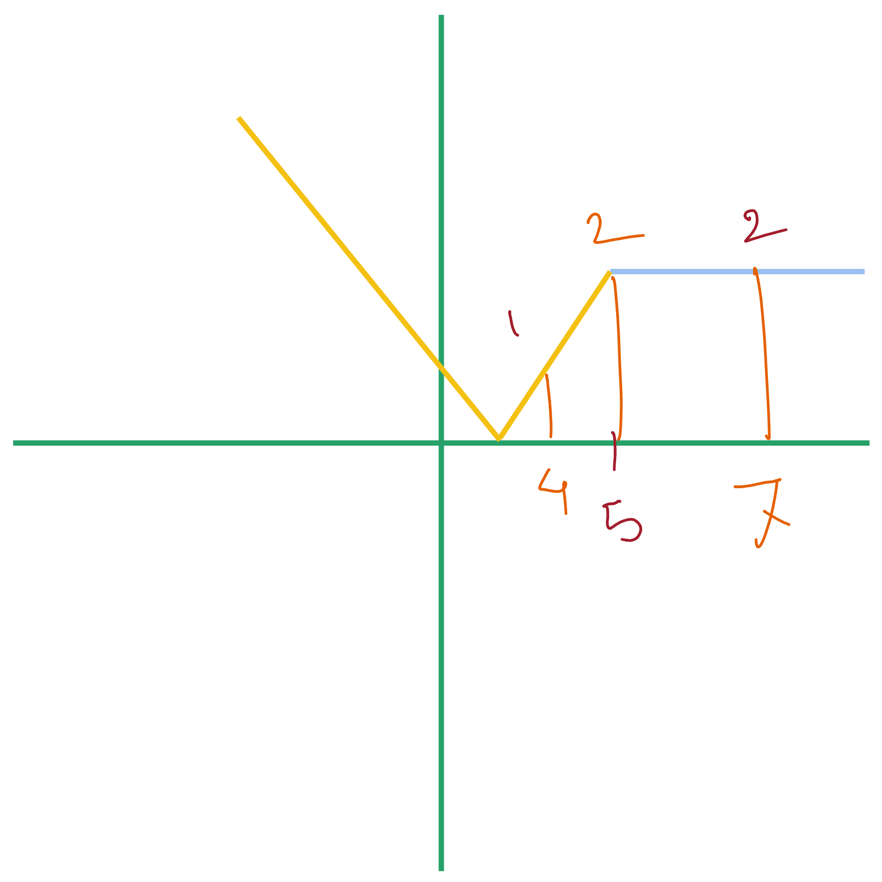
$\int_5^7 2\, dx$

$= [2x]_5^7$

$= 14 - 10$

$= 4$

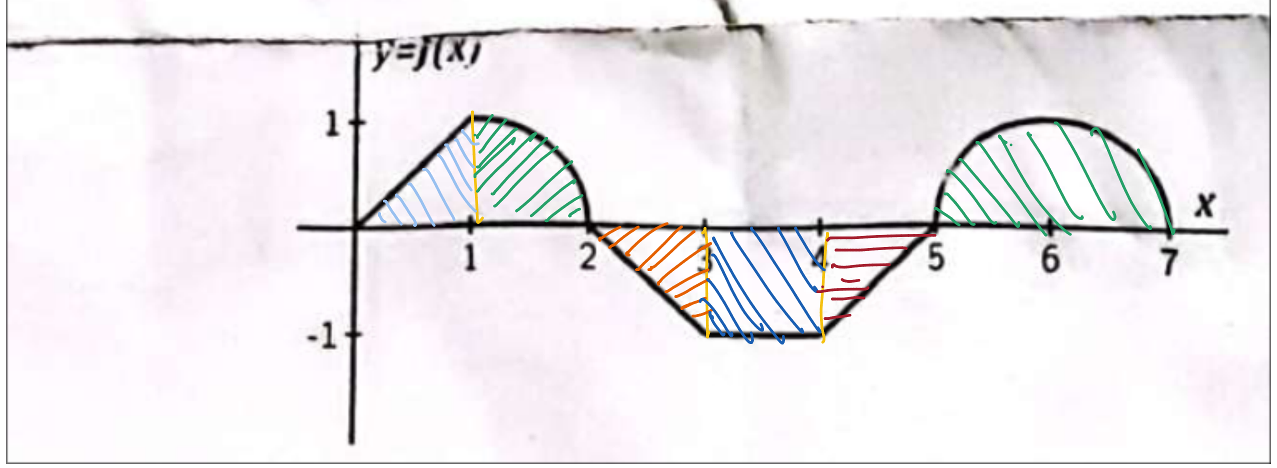
$4 + \frac{3}{2} = \frac{11}{2}$



$$\frac{1}{2}\times(1+2)\times 1 + (2\times 2)$$

$$= \frac{3}{2} + 4 = \frac{11}{2}$$

2. According to the following figure evaluate $\int_0^7 f(x)dx$.

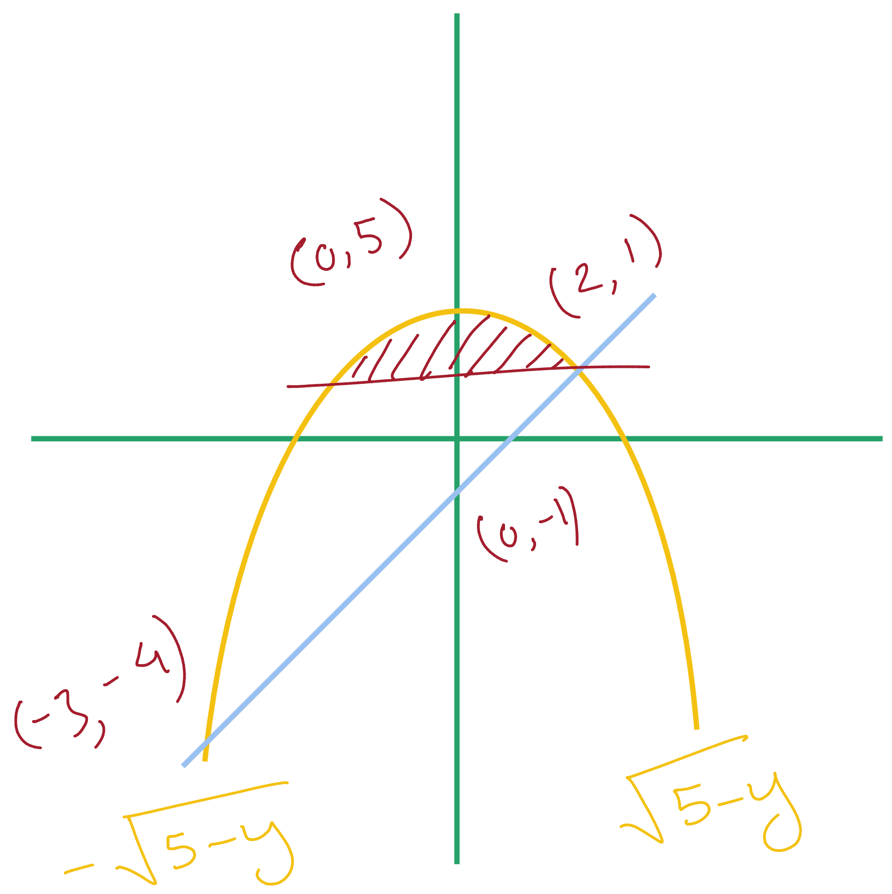


$$\frac{1}{2} + \frac{\pi}{4} + \frac{1}{2} + 1 + \frac{1}{2} + \frac{\pi}{2}$$

$$= 2 + \frac{1}{2} + \frac{3\pi}{4}$$

$$= \frac{8+2+3\pi}{4} = \frac{10+3\pi}{4}$$

3. Find the area between two curves $y = 5 - x^2$ and $x = y + 1$ by (i) integrating with respect to x (ii) integrating with respect to y.



$$5 - x^2 = x - 1$$

$$\Rightarrow x^2 + x - 6 = 6$$

$$\Rightarrow x^2 + 3x - 2x - 6 = 0$$

$$\Rightarrow x(x+3) - 2(x+3) = 0$$

$$\Rightarrow (x-2)(x+3) = 0$$

$$\Rightarrow x = 2, -3$$

i) $\int_{-3}^{2} 5 - x^2 - (x-1)\, dx$

$$= \left[6x - \frac{x^3}{3} - \frac{x^2}{2}\right]_{-3}^{2}$$

$$= \frac{125}{6}$$

ii) $\int_{-4}^{1} y+1-(-\sqrt{5-y})\,dy$

$u=9$

$u=4$

$= \left[\frac{y^2}{2}+y\right]_{-4}^{1} + \int_{-4}^{1}\sqrt{5-y}\,dy$ $\quad u = 5-y$

$\Rightarrow du = -dx$

$= \left[\frac{y^2}{2}+y\right]_{-4}^{1} + \int_{9}^{4}\sqrt{u}\,(-du)$

$= \frac{1}{2}+1-8+4$

$= -\frac{5}{2}$

$= -\frac{2}{3}\left[u^{3/2}\right]_9^4$

$= -\frac{2}{3}(8-27)$

$= \frac{38}{3}$

$= \frac{61}{6}$

$\int_{1}^{5} \sqrt{5-y} - (-\sqrt{5-y})\,dx$

$= 2\int_{1}^{5}\sqrt{5-y}\,dx$

$= 2\int_{4}^{0}\sqrt{u}\,(-du)$ $\quad u=5-y$, $\Rightarrow du=-dx$

$= -2\,\frac{2}{3}\left[u^{3/2}\right]_4^0$ $\quad u=5-1$, $u=5-5$

$= -\frac{4}{3}(0-8)$

$= \frac{32}{3}$

$\frac{61}{6} + \frac{32}{3} = \frac{125}{6}$

4. Evaluate any four of the following integrals.

(i). $\int \frac{2\cos x}{\sec x}dx$

(ii). $\int \frac{dx}{\sqrt{x}(4+x)}$

(iii). $\int \frac{x\,dx}{4^{x^2}}$

(iv). $\int (x^2-1)\sqrt{x+1}\,dx$

(v). $\int \ln(x^2+9)\,dx$

(vi). $\int \frac{dx}{(9x^2-4)^{\frac{3}{2}}}$

i) $\int \frac{2\cos x}{\sec x}\,dx$

$= \int 2\cos^2 x\,dx$

$= \int 1+\cos 2x\,dx$

$= x + \frac{1}{2}\sin 2x + C$

ii) $\int \frac{dx}{\sqrt{x}(4+x)}\,dx$ $\quad u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\int \frac{\sqrt{x}}{\sqrt{x}(4+u^2)}\,du$

$= 2\int \frac{1}{4+u^2}\,du$ $\quad z = \frac{u}{2}$

$\Rightarrow dz = \frac{1}{2}du$

$= \frac{2}{4}\int \frac{1}{1+(\frac{u}{2})^2}\,du$ $\quad \Rightarrow du = 2dz$

$\frac{1}{2}\int \frac{1}{1+z^2}\,2dz$

$= \tan^{-1} z + C$

$= \tan^{-1}\frac{u}{2} + C$

$= \tan^{-1}\frac{\sqrt{x}}{2} + C$

iii) $\displaystyle\int \frac{x}{4^{x^2}}\,dx$ $\qquad u = x^2$, $\quad du = 2x\,dx$

$$= \frac{1}{2}\int \frac{1}{4^u}\,du$$

$$= \frac{1}{2}\int \frac{1}{4^u} \cdot \frac{dz}{4^u \ln 4} \qquad z = 4^u, \quad dz = 4^u \ln 4\,du$$

$$= \frac{1}{2}\int \frac{1}{z^2}\,\frac{dz}{\ln 4}$$

$$= \frac{1}{2\ln 4}\,\frac{z^{-1}}{-1}$$

$$= \frac{-1}{2\ln 4}\,\frac{1}{4^u}$$

$$= -\frac{1}{2\ln 4}\,\frac{1}{4^{x^2}} + C$$

iv) $\displaystyle\int (x^2-1)\sqrt{x+1}\,dx$

$$= \int (x+1)(x-1)\sqrt{x+1}\,du \qquad u = x+1, \quad du = dx$$

$$= \int u\,(u-1-1)\sqrt{u}\,du$$

$$= \int u^{3/2}(u-2)\,du$$

$$= \int u^{5/2} - 2u^{3/2}\,du$$

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v) $\displaystyle\int \ln(x^2+9)\,dx$

$$= \ln(x^2+9)\,x - \int \frac{2x}{x^2+9}\,x\,dx$$

$$= x\ln(x^2+9) - 2\int \frac{x^2}{x^2+9}\,dx$$

$$-2\int \frac{x^2-x^2-9}{x^2+9} + 1\,dx$$

$$= -2\int -9\,\frac{1}{x^2+9} + 1\,dx$$

$$= 18\int \frac{1}{3^2+x^2}\,dx - \int 2\,dx$$

$$= 18\,\frac{1}{3}\cdot\tan^{-1}\frac{x}{3} - 2x$$

$$= 6\tan^{-1}\frac{x}{3} - 2x + C$$