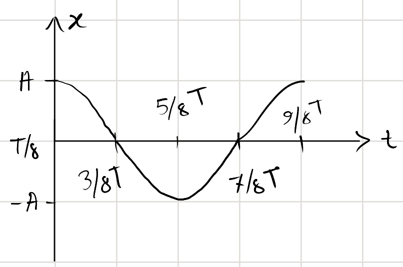
(a) Why do we observe Damped Harmonic Motion (DHM) in a mass-spring system in our real life instead of Simple Harmonic Motion (SHM)? 2 CO1
(b) What type of change in current flow will be observed if we replace the resistor of an RLC circuit with a conducting wire? Explain briefly. (Consider the wire that has approximate no resistance) 2 CO1
(c) The equation of displacement of a simple harmonic oscillator is $x = A\cos\left(\omega t - \frac{\pi}{4}\right)$. Graphically represent the displacement and velocity of the oscillator. 2 CO1

a) Systems internal friction and external forces like air pressure and gravity.

b) We will observe SHM instead of DHM since the current will not be impeded.

c) $x = A\cos\left(\omega t - \frac{\pi}{4}\right)$



$\omega t - \frac{\pi}{4} = 0$

$\Rightarrow t = \frac{\pi}{4} \cdot \frac{1}{\omega} = \frac{\pi}{4} \cdot \frac{T}{2\pi} = \frac{T}{8}$

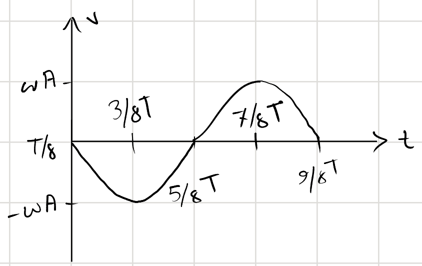
$T/8 + T/4 = \frac{3}{8} T$

$3/8\, T + T/4 = 5/8\, T$

$5/8\, T + T/4 = 7/8\, T$

$7/8\, T + T/4 = 9/8\, T$

$v = -A\omega \sin(\omega t - \pi/4)$



(a) Labid wants to construct an RLC circuit that produces critical damping. He has a capacitor and inductor with value, C = 0.05 μF, L = 0.2 mH respectively. 2 CO3
(i) What is the value of resistance he must connect to make his desired circuit?
(ii) If R = 500 Ω, is the circuit oscillatory? If oscillatory, find the frequency of oscillation.

i) $\omega^2 = \frac{r^2}{4}$

$\Rightarrow \frac{1}{LC} = \frac{R^2}{4L^2}$

$\Rightarrow R^2 = \frac{4L^2}{LC} = 4\frac{L}{C}$

$\Rightarrow R = 2\sqrt{L/C} = 2\sqrt{(0.2\times10^{-3})/(0.05\times10^{-6})}$

$= 126.49\ \Omega$

ii) $R = 500\ \Omega$

$\omega^2 = \frac{1}{LC} = \frac{1}{0.05\times10^{-6}\times 0.2\times10^{-3}} = 10^{11}$

$\frac{r^2}{4} = \frac{R^2}{4L^2} = \frac{500^2}{4\times(0.2\times10^{-3})^2} = 1.56\times10^{12}$

$\omega^2 < \frac{r^2}{4}$; the circuit is not oscillatory.

(a) Suppose a spring block-system starts moving from the equilibrium as we apply force on it. The block has mass m = 6.4 kg and is designed to oscillate with a angular frequency ω = 56 rads^{-1} with amplitude 15 cm. Calculate: 3 CO3
(i) the kinetic energy at x = 14 cm from the equilibrium point,
(ii) mathematically calculate the position where the kinetic energy is 0.

i) $E_K = \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}m\omega^2(A^2 - x^2)$

$= \frac{1}{2}\times 6.4\times 56^2\,(0.15^2 - 0.14^2)$

$= 29.102\ J$

ii) $\frac{1}{2}k(A^2 - x^2) = p$ [qs is illegible]

$\Rightarrow A^2 - x^2 = \frac{2p}{k}$

$\Rightarrow x = \sqrt{A^2 - 2p/k}$

(b) A 3 kg block is attached to a spring and the spring constant is k = 19.6 N/m. The block is held 6 cm from equilibrium and released at t = 0. 2 CO3
(i) Write an equation for x vs. time.
(ii) Calculate the velocity at t= 3 s and acceleration at t = 0.5 s.

i) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{19.6}{3}} = \frac{7\sqrt{30}}{15}$

$x = 0.06\cos\left(\frac{7\sqrt{30}}{15}t\right)$

ii) $v = -\frac{7\sqrt{30}}{15}\cdot 0.06\cdot\sin\left(\frac{7\sqrt{30}}{15}\cdot 3\right)$

$= -0.15\ m/s$

$a = -\left(\frac{7\sqrt{30}}{15}\right)^2\cdot 0.06\cdot\cos\left(\frac{7\sqrt{30}}{15}\times 0.5\right)$

$= -0.113\ m/s^2$

(c) A particle with mass 50 g executes simple harmonic motion given by the equation y = $\sin(10t - \frac{\pi}{4})$ .Calculate the (i) velocity and acceleration at t = 5 s (ii) total energy at t = 3s. 3

i) $v = y' = 10\cos(10t - \pi/4)$

$v(5) = 10\cos(50 - \pi/4) = 4.968\ m/s$

$a = y'' = -100\sin(10t - \pi/4)$

$a(5) = -100\sin(50 - \pi/4) = 86.786\ m/s^2$

ii) $E = \frac{1}{2}kA^2 = \frac{1}{2}\times m\omega^2 A^2$

$= \frac{1}{2}\times 0.05\times 10^2\times 1^2$

$= 2.5\ J$

(b) For a damped oscillator m = 380 gm, k = 19.6 N/m and b = 82 gm/s. The oscillator is released at t = 0 and the amplitude is 5 cm. 3 CO3
(i) How long does it take for the amplitude of the damped oscillations to drop to one fourth of its initial value?
(ii) How may complete cycle of oscillations be found after t = 6 s?

1) $r = \frac{b}{m} = \frac{82}{380} = 0.216\ s^{-1}$

$x = A e^{-r/2\ t}$

$\Rightarrow \frac{A}{4} = A e^{-r/2\ t}$

$\Rightarrow -\frac{r}{2} t = \ln 1/4$

$\Rightarrow t = -\frac{2}{r} \ln 1/4 = 12.85$ sec

ii) $\omega_1 = \sqrt{\omega^2 - r^2/4} = \sqrt{\frac{K}{m} - \frac{r^2}{4}} = 7.181$ rad/s

$T = \frac{2\pi}{\omega_1} = 0.875$ s

$\frac{6}{T} = 6.857$

$\therefore$ 6 complete cycles will be formd after 6 seconds.

(c) When a simple harmonic motion is propagated through a medium, the displacement of the particle at any instant of time is given by $y = 15sin\ (5t - 0.066x)$. 3 CO3
Calculate the (i) wavelength, (ii) wave velocity, (iii) amplitude and (iv) frequency.

$y = 15 \sin (5t - 0.066x)$

$A \sin \frac{2\pi}{\lambda} (vt - x)$

$\frac{2\pi}{\lambda} = 0.066$

$\Rightarrow \lambda = \frac{2\pi}{0.066} = 95.2$ m

$\frac{2\pi}{\lambda} v = 5$

$\Rightarrow v = 5 \cdot \frac{\lambda}{2\pi} = 75.76$ m/s

$A = 15$ m

$v = f\lambda \Rightarrow f = \frac{v}{\lambda} = 0.796$ Hz

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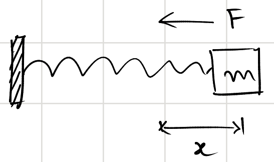
(a) Drive the differential equation of a mass spring system oscillating in simple harmonic motion. Write down the possible solution of the differential equation and graphically plot it.



$F = -kx$

$\Rightarrow a = -\frac{k}{m} x = -\omega^2 x$

$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$

$\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{d}{dx} v$

$\Rightarrow v \frac{d}{dx} v + \omega^2 x = 0$

$\Rightarrow \int v\ dv = -\omega^2 \int x\, dx$

$\Rightarrow \frac{1}{2} v^2 = -\frac{1}{2} \omega^2 x^2 + c$

$\Rightarrow v^2 = -\omega^2 x^2 + c$

when $v = 0$ $x = A$

$\therefore c = \omega^2 A^2$

$v^2 = -\omega^2 x^2 + \omega^2 A^2$

$\Rightarrow v = \omega \sqrt{A^2 - x^2}$

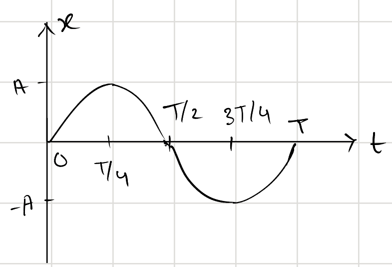
$\Rightarrow \frac{dx}{dt} = \omega \sqrt{A^2 - x^2}$

$\Rightarrow \int \frac{1}{\sqrt{A^2 - x^2}} dx = \int \omega\, dt$

$\Rightarrow \frac{1}{A} \int \frac{1}{\sqrt{1 - (\frac{x}{A})^2}} dx$

$\Rightarrow \sin^{-1} \frac{x}{A} = \omega t + \phi$

$\Rightarrow x = A \sin(\omega t + \phi)$



(b) For a body oscillating in simple harmonic motion, the equation of displacement is, $y = Acos\left(\omega t + \frac{\pi}{3}\right)$. Calculate the equations of velocity, acceleration, potential energy and kinetic energy. Graphically plot potential energy vs. time and kinetic energy vs. time graph. 4 CO2

$y = A\cos\left(\omega t + \frac{\pi}{3}\right)$

$v = y' = -A\omega \sin\left(\omega t + \frac{\pi}{3}\right)$

$a = y'' = -A\omega^2 \cos\left(\omega t + \pi/3\right)$

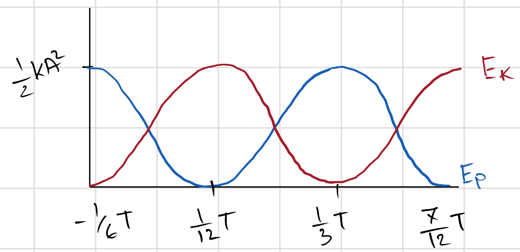
$E_p = \int_0^y F\, dy = \int_0^y -ky\, dy = -\frac{1}{2} k y^2$

$E_K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(-A\omega \sin(\omega t + \pi/3)\right)^2$

$= \frac{1}{2} m \left(A^2 \omega^2 \{1 - \cos^2(\omega t + \pi/3)\}\right)$

$= \frac{1}{2} K A^2 \left(1 - \cos^2(\omega t + \pi/3)\right)$

$= \frac{1}{2} k (A^2 - x^2)$



$\omega t + \frac{\pi}{3} = 0$

$\Rightarrow t = -\frac{\pi}{3} \cdot \frac{1}{\omega}$

$= -\frac{\pi}{3} \times \frac{T}{2\pi}$

$= -\frac{T}{6}$

5. (a) For a progressive wave show that, y = $A\sin\frac{2\pi}{\lambda}(vt - x)$ where the symbols have their usual meaning. 2 CO2

Drive the equation of velocity of the particle from the above equation. 2

honk honk

$$y = A\sin\frac{2\pi}{\lambda}(v_w t - x) = A\sin\left(\frac{2\pi}{\lambda}v_w t - \frac{2\pi}{\lambda}x\right)$$

$$\Rightarrow \frac{dy}{dt} = A\cos\left(\frac{2\pi}{\lambda}v_w t - \frac{2\pi}{\lambda}x\right)\cdot\frac{2\pi}{\lambda}v_w$$

$$\Rightarrow v = \frac{2\pi}{\lambda}v_w A\cos\left(\frac{2\pi}{\lambda}v_w t - \frac{2\pi}{\lambda}x\right)$$

$$\frac{dy}{dx} = -A\cos\left(\frac{2\pi}{\lambda}v_w t - \frac{2\pi}{\lambda}x\right)$$

$$\frac{\frac{dy}{dt}}{\frac{dy}{dx}} = -v_w$$

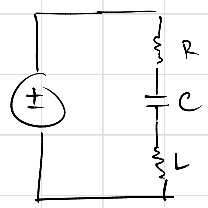
$$\frac{dy}{dt} = -v_w\frac{dy}{dx}$$

$$\boxed{v_p = -v_w\frac{dy}{dx}}$$

$$\frac{d^2y}{dt^2} = -v_w\frac{dy}{dt^2}$$

(b) An inductor, a resistor and a charged capacitor are connected to a circuit. Derive differential equation for the circuits and write down the solution of the equation for oscillatory damping. 3 CO2

$$\frac{Q}{C} + iR + L\frac{di}{dt} = 0$$



$$C = \frac{Q}{V}$$

$$\Rightarrow \frac{1}{C}i + R\frac{di}{dt} + L\frac{d^2i}{dt^2} = 0$$

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} = 0$$

$$\omega^2 = \frac{1}{LC} \qquad \gamma = \frac{R}{L}$$

$$Q(t) = Q_0 e^{-\gamma/2\, t}\cos(\omega_1 t)$$

$$= Q_0 e^{-\frac{R}{2L}t}\cos(\omega_1 t)$$