

## 2

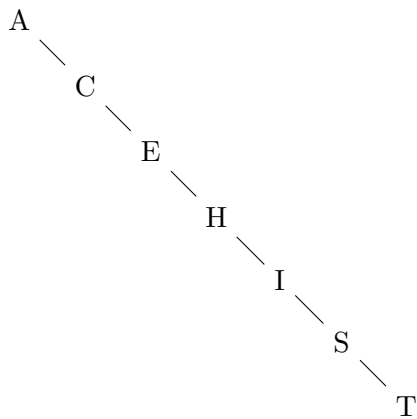
a) A binary tree with 4 childless vertices means it has 4 leaves ( $l = 4$ ).

The number of leaves ( $l$ ) in a full  $m$ -ary tree with  $i$  internal vertices and  $n$  vertices is  $(m - 1) \cdot i + 1$ .

Here,

$$\begin{aligned}(m - 1) \cdot i + 1 &= 4 \\ (2 - 1)(n - 4) + 1 &= 4 \\ n - 4 + 1 &= 4 \\ n &= 4 + 3 \\ n &= 7\end{aligned}$$

b) BST:



## 3

b) Postorder traversal of

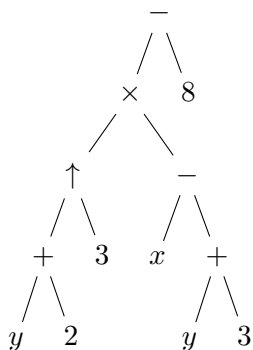
Tree 1: c, d, b, f, g, h, e, a

Tree 2: c, d, b, f, g, h, e, a

c) Prefix notation:

$$(- (\times (\uparrow (+ y 2) 3) (- x (+ y 3))) 8)$$

Binary tree of the expression:



- d) Rewriting the expression in infix format:  $\left(\frac{9}{3} + 5\right) \times (7 - 2)$   
Which evaluates to 40.

4

- a) It will be a weakly connected directed graph.

The direction is implied by one-way/two-way roads and it's weakly connected because vertices must have edges because vertices represent road intersections.

- b) Suppose there are  $o$  number of vertices with odd degree and  $e$  number of vertices with even degree and the number of edges is  $E$ . According to the handshaking theorem  $3o + 4e = 2E$ .

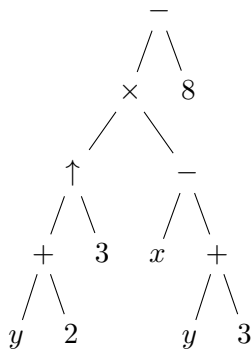
Now, the possible values of  $o$  are 2 and 4 because there should be an even number of vertices with odd degree. So the possible values of  $e$  are 3 and 1.

When  $o = 2$  and  $e = 3$ ,  $E = \frac{6+12}{2} = 9$ .

When  $o = 4$  and  $e = 1$ ,  $E = \frac{12+4}{2} = 8$ .

Therefore the possible number of edges are 8 and 9.

- c) Binary expression tree:



Prefix notation:  $(- (\times (\uparrow (+ y 2) 3) (- x (+ y 3))) 8)$

- d) 40

5

- a) Adjacency matrix:

	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$
$v_1$	0	1	1	0	0	0
$v_2$	1	0	0	0	0	0
$v_3$	0	0	0	1	0	1
$v_4$	0	0	0	0	1	0
$v_5$	0	0	1	1	0	1
$v_6$	0	0	0	0	0	0

- b) Doing the following tests,

- i. number of vertices is the same (7)

- ii. number of edges is the same (9)
- iii. sequence of degrees is not the same  $(3,3,3,3,3,2)$  and  $(3,3,3,4,3,2)$

As the third test fails, the graphs are not isomorphic.

## 6

a)  $C(7, 5) \times C(6, 4) \times 5 \times 7 \times 6 \times 5 \times 4 \times 4 \times 3 \times 2 \times 1$