2. **a)** Determine the distance between the given skew lines

L_1: $x = 3 - t$, $y = 4 + 4t$, $z = 1 + 2t$

L_2: $x = t$, $y = 3$, $z = 2t$

$\hat{n}_1 = \langle -1, 4, 2 \rangle$

$\hat{n}_2 = \langle 1, 0, 2 \rangle$

$\hat{n}_1 \times \hat{n}_2 = \langle 8 - 0, -(-2-2), 0 - 4 \rangle$

$= \langle 8, 4, -4 \rangle$

a passing point over L_2 is $P_2(0, 3, 0)$

the eqn of the plane contaning L_2 is as follows

$8(x-0) + 4(y-3) - 4(z-0) = 0$

$\Rightarrow 8x + 4y - 12 - 4z = 0$

$\Rightarrow 8x + 4y - 4z - 12 = 0$

a passing point over L_1 is $P_1(3, 4, 1)$

$\therefore$ the distance between the two lines

$$= \frac{|8 \cdot 3 + 4 \cdot 4 - 4 \cdot 1 - 12|}{\sqrt{8^2 + 4^2 + 4^2}} = \sqrt{6}$$

b) Find the equation of the plane passing through the points $p_1(1, -2, 0)$, $p_2(1, 0, -2)$ and $p_3(-1, 5, 0)$.

$\overrightarrow{P_1P_2} = \langle 0, 2, -2 \rangle$

$\overrightarrow{P_1P_3} = \langle -2, 7, 0 \rangle$

$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \langle 0 + 14, -(0-4), 0 + 4 \rangle$

$= \langle 14, 4, 4 \rangle$

eqn of the plane: $14(x-1) + 4(y+2) + 4(z-0) = 0$

$\Rightarrow 14x - 14 + 4y + 8 + 4z = 0$

$\Rightarrow 14x + 4y + 4z - 6 = 0$

c) Find an equation of a line that is intersection of the planes $x - y + 2z = 0$ and $2x + 3y - z + 1 = 0$.

$\hat{n}_1 = \langle 1, -1, 2 \rangle$

$\hat{n}_2 = \langle 2, 3, -1 \rangle$

$\hat{n}_1 \times \hat{n}_2 = \langle 1 - 6, -(-1-4), 3 + 2 \rangle$

$= \langle -5, 5, 5 \rangle$

for the xy-plane $z = 0$

$\therefore x - y = 0$ and $2x + 3y + 1 = 0$

$x, y = -\frac{1}{5}, -\frac{1}{5}$

$\therefore (-\frac{1}{5}, -\frac{1}{5}, 0)$ is a passing point over that line

and $\langle -5, 5, 5 \rangle$ is the parallel vector

$\therefore$ the eqn of the line:

$x = -\frac{1}{5} - 5t$ $\quad y = -\frac{1}{5} + 5t$ $\quad z = 5t$

3. **a)** Find the area of the triangle with vertices $P_1(1, -1, 0)$ $P_2(-1, 0, 3)$ and $P_3(0, 4, 1)$.

$\overrightarrow{P_1P_2} = \langle -2, 1, 3 \rangle$

$\overrightarrow{P_1P_3} = \langle -1, 5, 1 \rangle$

$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \langle 1 - 15, -(-2+3), -10 + 1 \rangle$

$= \langle -14, -1, -9 \rangle$

area of the triangle $= \frac{1}{2}\sqrt{14^2 + 1^2 + 9^2} = 8.34$

b) Find the vector component (orthogonal projection) of $q = \langle 1, -1, 5 \rangle$ along $p = \langle 4, 0, -1 \rangle$ and orthogonal to p.

$\hat{p} = \langle \frac{4}{\sqrt{17}}, 0, -\frac{1}{\sqrt{17}} \rangle$

$\text{proj}_{\vec{p}}\, \vec{q} = \{ \langle 1, -1, 5 \rangle \cdot \langle \frac{4}{\sqrt{17}}, 0, -\frac{1}{\sqrt{17}} \rangle \} \langle \frac{4}{\sqrt{17}}, 0, -\frac{1}{\sqrt{17}} \rangle$

$= \{ \frac{4}{\sqrt{17}} + 0 - \frac{5}{\sqrt{17}} \} \langle \frac{4}{\sqrt{17}}, 0, -\frac{1}{\sqrt{17}} \rangle$

$= -\frac{1}{\sqrt{17}} \langle \frac{4}{\sqrt{17}}, 0, -\frac{1}{\sqrt{17}} \rangle$

$= \langle -\frac{4}{17}, 0, \frac{1}{17} \rangle$

projection of $\vec{q}$ orthogonal to $\vec{p}$

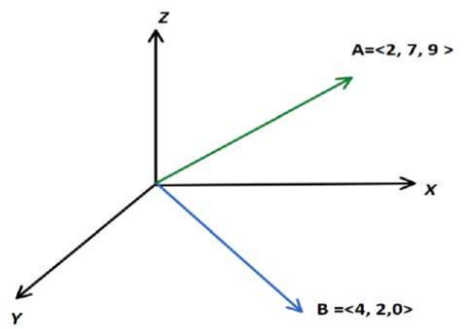
$= \vec{q} - \text{proj}_{\vec{p}}\, \vec{q}$

$= \langle 1, -1, 5 \rangle - \langle -\frac{4}{17}, 0, \frac{1}{17} \rangle$

$= \langle 1 + \frac{4}{17}, -1, 5 - \frac{1}{17} \rangle$

$= \langle \frac{21}{17}, -1, \frac{84}{17} \rangle$

c)



i) Find the angle between vector **A** and **y**-axis.

ii) Find a unit vector that is orthogonal to vector **A** and x-axis.

i) $\vec{A} \cdot \hat{j} = |\vec{A}| \cos\theta$

$\Rightarrow \langle 2, 7, 9 \rangle \cdot \langle 0, 1, 0 \rangle = |\vec{A}| \cos\theta$

$\Rightarrow \cos\theta = \frac{7}{\sqrt{2^2 + 7^2 + 9^2}}$

$\Rightarrow \theta = \cos^{-1}(0.6047) = 0.921$

ii) $\vec{A} \times \hat{i} = \langle 2, 7, 9 \rangle \times \langle 1, 0, 0 \rangle$

$= \langle 0, -(0-9), 0 - 7 \rangle$

$= \langle 0, 9, -7 \rangle$

$\text{unit}(\langle 0, 9, -7 \rangle) = \frac{1}{\sqrt{130}} \langle 0, 9, -7 \rangle$