Course Title :
Population Dynamics [Environmental Sciences]
Environmental Management and Policy III

(Advanced course of)

The Theory in Bio-Demography

Kinya Nishimura & Takenori Takada

Background

Oct. 10 and 17

Date	Lecturer	Class hr.
Sept. 26	KN	2 – 5
Oct. 3	KN	2 - 4
Oct. 10	Tak	2 - 4
Oct. 17	Tak	2 - 4

TK

- ✓ Application to complex life history
- ✓ Basic population metrics
- ✓ Application to conservation ecology

Concept: From age-structure to stage-structure
(From human demography to bio-demography)

After revision

Class 2 hr.	10:25 - 11:55	90 min.
Class 3 hr.	13:15 - 14:45	90 min.
Class 4 hr.	15:10 - 16:40	90 min.

Before revision

Class 2 hr.	10:25 - 12:10	105 min.
Class 3 hr.	13:00 - 14:45	105 min.
Class 4 hr.	14:55 - 16:40	105 min.

高田　壮則　Takenori Takada (Tak)

- Born in Hakodate, the southern part of Hokkaido
- Speciality: Mathematical ecology
- https://taktakada.github.io

Hobbies: Hiking, Skiing, River-fishing

What I am planning this year

The 3rd Meeting of Power Demography

Date: Nov. 20 (Fri), 2020 13:00–17:30.
Place：Online meeting using zoom. https://taktakada.github.io

Speakers and titles:
Tak Takada (Hokkaido Univ.) Introductory talk
Hiroshi Tomimatsu (Yamagata Univ.) Population biology of a forest herb Trillium camschatcense in anthropogenically fragmented and geographically peripheral habitats
Fumiko Ishihama (NIES) Estimating the extinction risk of "immortal" tree - simulation analysis using a shoot-dynamics matrix model -
Hiroyuki Yokomizo (NIES) Inter-stage flow matrices: a new population statistic derived from life history matrices
Richard Shefferson (Univ. of Tokyo) Addressing the importance of individual history on population dynamics with R package Lefko3

Questions and registration：Tak Takada e-mail: takada@ees.hokudai.ac.jp

Your turn !!!

- Name
- Coming from?
- B.M.D course
- Laboratory? What do you study?
- Good at math?

Purpose

- From age-structure to stage-structure

  (From human demography to bio-demography)
- How to apply matrix algebra to bio-demography?
- How to obtain the basic population metrics using the matrix?
- How to apply them in conservation ecology?

A little bit math

Why should(?) we use
linear algebra (matrix)
in demographic study?

Answer: History & Theoretical thought
(4 pages later) (Next file)

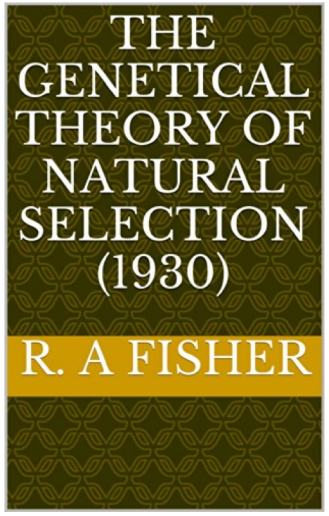
Demography?

Demography is the statistical study of populations, especially human beings. As a very general science, it can analyze any kind of dynamic living population (population dynamics). (From Wikipedia)

Human demography

Nishimura-sensei

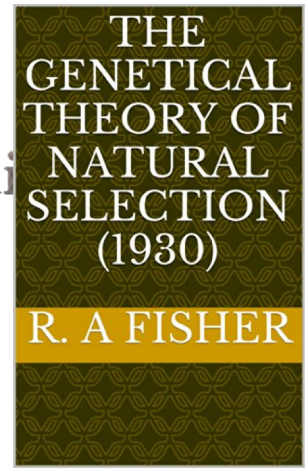
Bio-demography
(Plants and Animals)



One of the most famous textbooks in ecology

He did not use matrix framework

Fundamental theories of population ecology in 1930s



CONTENTS

(logarithm of λ)

Euler–Lotka Equation

$$1 = \sum_{i=1}^{n} b_i l_i \lambda^{-i}$$

reproductive value

$$\frac{v_j}{v_1} = \frac{\lambda^j}{l_j} \sum_{i=j} b_i l_i \lambda^{-i-1}$$

Who gave birth to this baby?

Nishimura Part

$$\underbrace{1}_{\text{All sum}} = b_1 l_1 \lambda^{-1} + \cdots + b_{x-1} l_{x-1} \lambda^{-(x-1)} + \underbrace{b_x l_x \lambda^{-x}}_{\text{Probability that the newborn's parent is age-}x} + b_{x+1} l_{x+1} \lambda^{-(x+1)} + \cdots$$

The name is "Euler–Lotka Equation", which allows us to obtain λ for given b_x and l_x .

Chronological table

1826 Babbage Life table

A Comparative View of the various Institutions for the Assurance of Lives.

Nishimura Part

1910 Sharpe & Lotka Euler-Lotka equation

1912 Frobenius Peron–Frobenius theorem

1930 Fisher Basic theory, Defining "Reproductive value"

1941 Bernardelli Age-structured "Leslie model"

1945 Leslie Leslie matrix

1963, 1965 Lefkovitch Stage-structured "Matrix population model"

1978 Caswell Sensitivity analysis (New metrics)

1986 De Kroon et al. Elasticity analysis (New metrics)

2015 Salguero-Gomez et al. COMPADRE Plant Database

History of Matrix Population Model

- transition matrix model or projection matrix model

1) Bernardelli (1941), Lewis (1942)

2) Leslie (1945, 1948,1959,1966)

Age-structured model " Leslie matrix model"

Drastic change from age-structure to stage-structure

3) Lefkovitch(1963, 1965)

Stage-structured model in a beetle.

4) Theoretical and applied studies have been developed since 1970s.

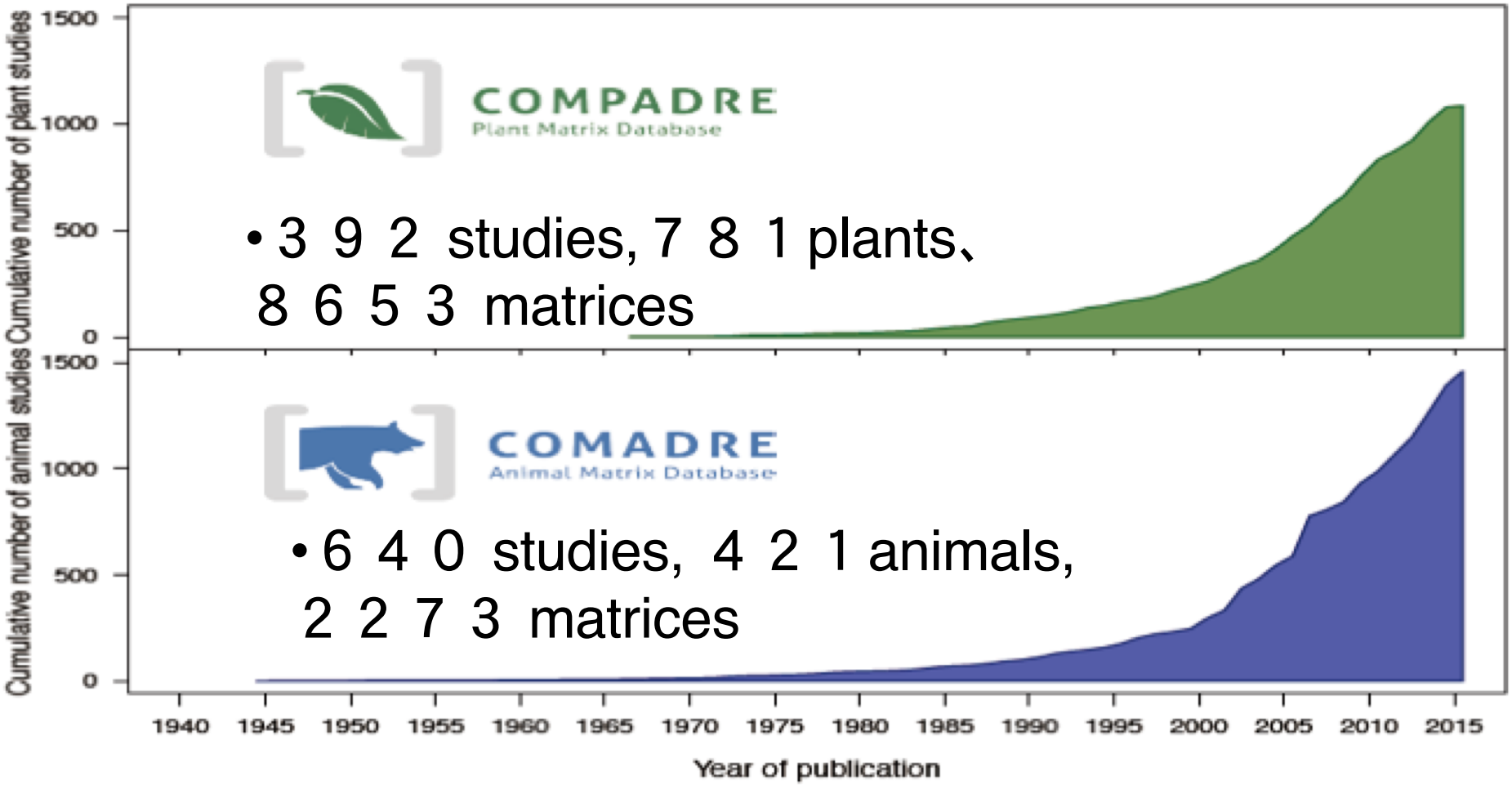
Schaffer & Samson(1985) Grizzly Lande(1988) Owl

Damman & Cain(1998) Wild ginger Lindborg & Ehrlen(2000) Wild maze

+ Many plant & animal species → **Big database**

Database of Matrix Population Model

COMPADRE Salguero-Gómez et al. (2015)
COMADRE Salguero-Gómez, Owen Jones et al. (2016)



Matrix Population model in mathematical ecology?

Mathematical models in ecology

1) Dynamical model:

Malthus model, Logistic model, Lotka-Volterra model, matrix population model, diffusion equation model, lattice model, etc.

(To grasp the characteristics of the dynamics)

2) Evolutionary model:

Egg-size model, Leaf expansion and shedding model, Reproductive schedule model, Foraging strategy model, Hawk-Dove game, Sex ratio model

(To obtain optimal strategy or evolutionarily stable strategy)

Classification of dynamical model

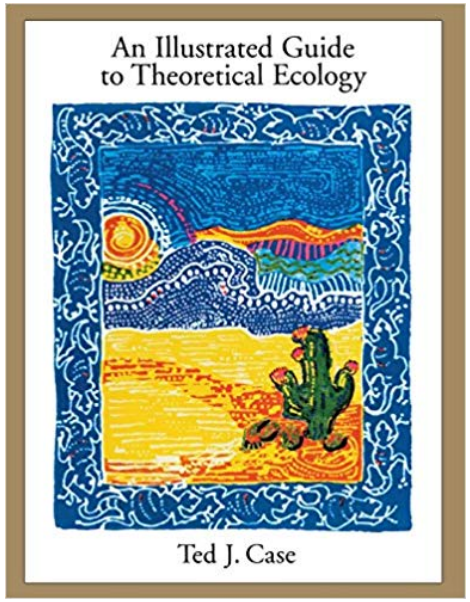
Model	Time	Space	No. of variables	Memo
Malthus model	Conti.	None	1	One species
Logistic model	Conti.	None	1	Density effect
Competition equation	Conti.	None	>1	Multi-species, Competition
Lotka-Volterra model	Conti.	None	>1	Multi-species, Predation
Continuity equation	Conti.	None	1	Size growth and transition
Diffusion equation model	Conti.	Available	1	Diffusion of individuals
Population matrix model	Disc.	None	>1	Transition of life stage
Lattice model	Disc.	Available	1,>1	Spatial dispersion

Classification of evolutionary model

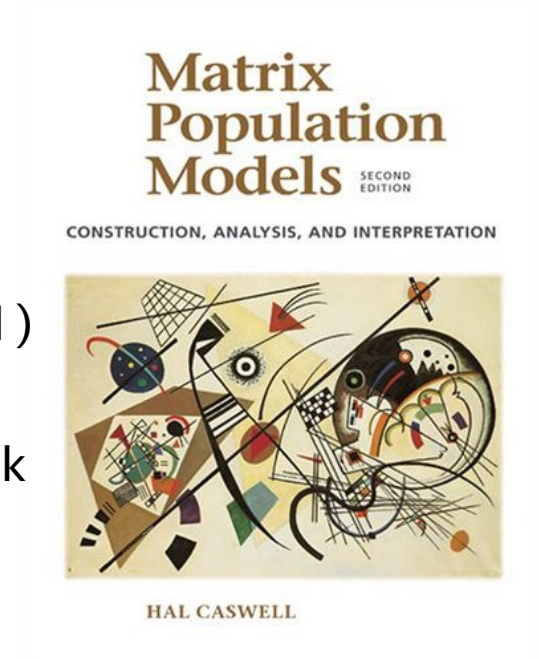
model	strategy	No. of species	memo
Egg-size model	1 var., conti.	1	Optimal
Leaf expansion and shedding model	>1 var., conti.	1	Optimal
Reproductive schedule mode	function, conti.	1	Optimal
Hawk-Dove model	two strategies, discrete	1	E S S
Sex ratio model	1 var., conti.	1	E S S

Textbook

Case (2000) in 3rd & 4th chapters.

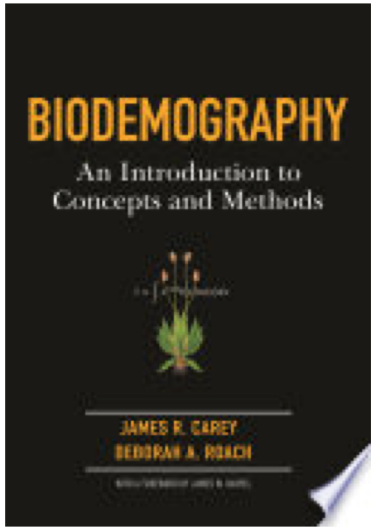


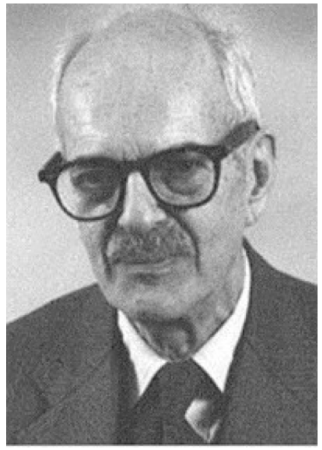
Hal Caswell (2001)
720pp.
Standard textbook



Carey & Roach (2020))

500pp.
Standard textbook





Introduction to the Mathematics of Populations

Nathan Keyfitz (1968)

Keyfitz (1968)

Questions

(1) There are n numbers, $x_1, x_2, \ldots, x_n$. The probability of getting x_i is given by $p(x_i)$. Then, answer the formula of the expectation (average).

(2) Answer the formula of the determinant of following matrix.

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

(3) For a given survivorship curve, $l(x)$, answer the formula of remaining lifetime of individuals with age y.

(4) For a given survivorship ($l(x)$) and age-dependent fertility ($b(x)$), how many children can be expected in the lifetime?

Break time

The 1st Japanese textbook

Introduction to Mathematical biology (1950)
数理生物学概論（by 小松勇作）
by Yusaku Komatsu (Utokyo, Math.）

- Chap. 1　Population growth
- Chap. 2　Genetics
- Chap. 3　Diffusion in cells
- Chap. 4　neuron model
- Chap. 5　Models on white blood cells and on circulation of blood in heart

He did not know population matrix model.

Break time

Japanese Society for Mathematical Biology (JSMB)

- Members About 500
- Starting as an informal organization (1989)

  Organizing the symposium every year.
- Founding the academic society (2004)

  Annual meeting, several awards

  Publication of the newsletter