PEARSON

For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education A lvanced Level

MATHEMATICS 9164/1

PAPER 1

JUNE 2006 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in th spaces provided on the a paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accura cy required is not specified in t question, then in the case of an angle it should be given to th nearest degree, and in other should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of eac 1 question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations ar 1 candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where approp iate.
You are reminded of the need for clear presentation in your a iswers.

This question paper consists of 7 printed p ges and 1 blank page.

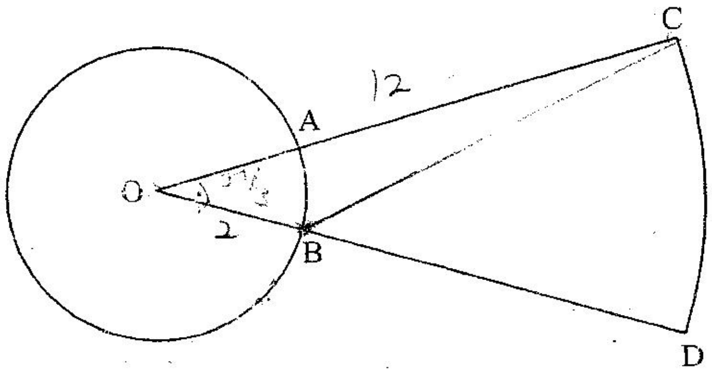
[Turn over

1 Express $\dfrac{5}{(x+1)(x^2+4)}$ in partial fractions . [4]

2 Solve the equation $|4x+3| = |x-5|$. [4]

3 Find the particular solution of the differential equation $\dfrac{dy}{dx} = -\dfrac{x}{y}$ that passes through the point (5; 12) and sketch it. [5]

4



The figure shows a circle centre O and radius 2 cm. The radii OA and OB are produced to C and D so that AC = BD = 10 cm. CD is part of an arc of a circle centre O and radius 12 cm.

Given that the area of the sector OCD is six times the area of the circle with radius OA, show that the angle AOB $= \dfrac{\pi}{3}$ radians. [2]

Hence find (i) the exact perimeter of the figure ACDB bounded by the straight lines AC, BD and the arcs CD and BA, [2]

(ii) the exact length of the line joining B to C. [2]

$(x-5)(x-5)$
$x^2 - 5x - 5x + 25$
$(4x+3)(4x+3)$
$16x^2 + 12x + 12$

$-(-b + C)$
$-b + C$

$\dfrac{-(bx + c)}{x^2 + 1}$

5 Given that $y = \sec x$, show that $\dfrac{dy}{dx} = \sec x \tan x$. [2]

Express $\dfrac{d^2y}{dx^2}$ in terms of $\sec x$ and $\tan x$. [2]

Hence find the Maclaurin expansion for $\sec x$ up to and including the term in x^2. [2]

6 **(a)** Express the complex number $z = \dfrac{6 + 4i}{1 + 5i}$ in the form $a + bi$.

Hence find $|z|$ and $\arg(z)$. [4]

(b) Show by substitution that $w = 2 - 3i$ is a root of the equation

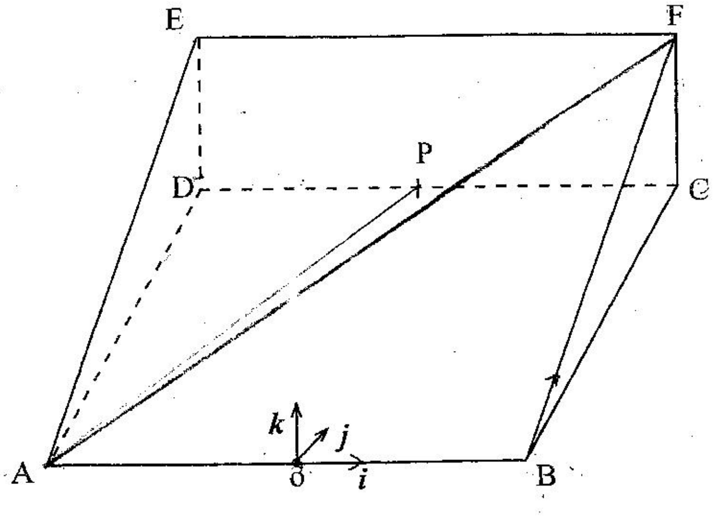
$$w^2 - 4w + 13 = 0.$$

7 **(a)** Write down the first five terms of the sequence U_n, where $U_{n+2} = U_{n+1} + 2U_n$, $U_1 = 2$ and $U_2 = 3$.

State whether the sequence oscillates, converges or diverges. [1]

(b) Given that the sum of the first two terms of a geometric progression is 90 and the sum to infinity is $91\frac{3}{7}$, find the two possible values of the common ratio.

8



The diagram shows a solid triangular prism standing on a horizontal rectangular base ABCD. The rectangular face CDEF is vertical. The edge AB has mid-point O, and the edge DC has mid-point P. Unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are taken parallel to edges AB, BC and CF respectively. The rectangular base has length AB = 10 units and width BC = 8 units. $\overrightarrow{CF} = 6\boldsymbol{k}$. Calculate

(i) $\overrightarrow{OC}$, [1]

(ii) $\overrightarrow{EP}$, [1]

(iii) angle FAP. [5]

9 **(a)** Express $ln\left(\dfrac{2x^2 - 3x - 2}{\sqrt{1 - x}}\right)$ in the form

$ln(ax + b) + ln(cx + d) + g\,ln(1 - x)$ where a, b, c, d and g are constants to be found. [3]

(b) Solve the equation $\dfrac{e^{3x} - e^{-3x}}{2} = 4$, giving your solution correct to 3 significant figures. [5]

10 A, B and C are points $(-5; 3)$, $(7; -5)$ and $(5; 5)$ respectively .

(i) Find the equation of the line which passes through A and B, giving your answer in the form $ax + by + c = 0$. [2]

(ii) Given that D is the mid-point of AB, show that CD is perpendicular to AB. [3]

(iii) Hence or otherwise show that the area of ΔABC is 52 units^2. [3]

11 The functions f and g are defined by:

$f: x \mapsto ln(1 + x)$, for $x > -1$

$g: x \mapsto x - 6$, $x \in \mathbb{R}$

By graphical considerations, show that the equation $f(x) = g(x)$ has exactly two real roots.

Use the iteration $X_{n+1} = 6 + ln(x_n + 1)$ with $x_1 = 6$ to find one root of the equation correct to 3 significant figures.

Show that the equation $f(x) = g(x)$ may be written in the form $e^{x-6} - x - 1 = 0$,

Use the Newton–Raphson method twice starting with $x_1 = 0$ to obtain an approximation to the smaller root of the equation giving your answer correct to 4 significant figures.

12 Given that $x = 3 + 2\cos^2\theta$,
$y = -1 + 5\sin^2\theta$

for $0 \leq \theta \leq 2\pi$,

(i) obtain expressions in terms of θ for

$\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$. Hence find $\frac{dy}{dx}$, [4]

(ii) describe fully the graph of y against x by considering maximum and minimum values of $(x; y)$ and $\frac{dy}{dx}$ from **(ii)** above. [5]

$2x^2-4x+x-2$
$2x(x-2)+1(x-2)$
$(2x+1)(x-2)$

[Turn over

13 **(a)** Show that the line $y = x - 2m$ always meets the curve $xy = 8$, for all the values of m, where $m \in \mathbb{IR}$. [3]

(b) Given that the function $f(x) = 2x^3 + ax^2 - 11x + 6$ is divisible by $(x + 2)$, find the value of a. [2]

With this value of a, show that $f(x)$ is also divisible by $(2x - 1)$ and $(x - 3)$. [2]

Hence, or otherwise find $f(x - 2)$ in factor form. [2]

14 **(a)** It is given that $\tan\alpha = \dfrac{1}{4}$ and $\tan\beta = \dfrac{3}{5}$, where α and β are acute. Use this information to prove that $(\alpha + \beta) = 45°$. [3]

(b) Express $7\cos\theta - 24\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and α is acute, giving the value of α correct to the nearest one tenth of a degree. [2]

Hence find

(i) values of θ, correct to one tenth of a degree, for $0 \leq \theta \leq 360°$ which satisfy $7\cos\theta - 24\sin\theta = 16$, [3]

(ii) the greatest and least values of $\dfrac{1}{27 + 7\cos\theta - 24\sin\theta}$. [2]

(a) The graph of $y = x^2$ is first stretched in the positive x-direction by a scale factor of 4. It is then translated by 3 units in the negative x-direction. Write the equation of the resulting graph. [3]

(b) Given the function $y = f(x)$, where $f(x) = \dfrac{1}{x + 1}$, $x > -1$

(i) find $y = f^{-1}(x)$, [3]

(ii) state how the graph of $y = f^{-1}(x)$ is related to the graph of $y = f(x)$, [1]

(iii) determine the range of $y = f(x)$. [1]

(iv) sketch the graph of $y = f(x) + 2$. [2]

16 **(a)** **(i)** Use a suitable substitution to show that

$$\int_{-1}^{0} x(2x+1)^7\,dx = \frac{1}{18}.$$ [6]

(ii) The graph of $y = x(2x+1)^7$ cuts the x-axis at $(0; 0)$ and $\left(-\frac{1}{2};\ 0\right)$. Give a reason why the area between the curve $y = x(2x+1)^7$, the x-axis and the lines $x = -1$ and $x = 0$ is not $\frac{1}{18}$. [1]

(b) Find $\int \ln 3x\,dx$. [3]

$(x-2)(x-2)(x-2)$

$(x^2-4x+4)(x-2)$