NOV 2009

7 (a) Express $4\left(\sqrt{3} - i\right)$ in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [3]

(b) Given that $x_1 = 1 + 2i$ is a root of the equation
$x^4 - 4x^3 - 6x^2 + 20x - 75 = 0$, find the other three roots. [5]

(c) Express $\sin 5\theta$ in terms of powers of $\sin\theta$ and hence show that
$\sin 5\theta - 5\sin\theta = 16\sin^5\theta - 20\sin^3\theta$.

Find $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(16\sin^5\theta - 20\sin^3\right) d\theta$, giving your answer in exact form. [9]

JUNE 2004

7a). Use de Moivre's theorem to express $\sin 5\theta$ in terms of powers of $\sin\theta$. (5)
b). Given Z4 = $8 - 8\sqrt{3}i$, find all possible values of Z, giving the answers in the form a + bi with a and b correct to two decimal places. (7)
c). Sketch on an Argand diagram the locus of Z, where Z + 4 = Z – 4i (2)
Hence or otherwise state the Cartesian equation of the locus. (1)

JUE 2007

3 **(a)** Illustrate on an Argand diagram the set of points representing the complex number z satisfying both

$$|z-1-2i| \le 3 \text{ and } \arg(z-2-i) = \frac{3\pi}{4}.$$ [3]

(b) Given that $z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ and $w = \sqrt{3}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$,

find the modulus and argument of

(i) zw, [2]

(ii) $\frac{z}{w}$. [2]

(c) Given that $z = 1 + i\sqrt{3}$, prove that $z^{11} = 2^{10}(1 - i\sqrt{3})$. [3]

NOV 2003

Sketch the following locus on an Argand diagram:

Arg (z – 1) = Π [4]

(z – 4l) 3

(b) Express cos5θ in terms of cosines of multiple angles. [7]

(c) Show that 2 + 3l is a root of the equation z' – 3z2 + 9z + 13 = 0.

Hence find the other two roots. [6]

NOV 2010

6. (a) The complex number z = a + bi where a and b are positive real numbers.

(i) Given that w = iz, write down w in terms of a and b and

explain the geometrical relationship between z and w.[2]
(ii) Another complex number v = ½ z + w.

Represent clearly on the same Argand diagram the complex numbers z, w and v.
Find v if the complex number z = 3 + 2i. [6]
(b) Use De Moivre’s theorem to find the 4 roots of unity giving your answers in exponential form. [5]

NOV 2004

9 A complex number Z has modulus 8 and argument $\frac{\pi}{4}$. Another complex number W has modulus $\frac{1}{2}$ and argument $\frac{\pi}{8}$.

(a) Write each of the following complex numbers in the form a + *i*b.

(i) ZW^4 [6]

(ii) $\frac{Z^2}{W^2}$ [6]

(b) Find the smallest value of n such that $|W^n| < 0.01$. [3]

JUNE 2012

6 **(a)** It is given that the complex number a whose conjugate is $\bar{a}$.
satisfies the equation $4a\bar{a} + 12i = 8a + 16$.

Find the two possible values of a, giving your answer in the form $p + \mathrm{i}q$ where p and q are real. [6]

NOV 2008

1. A complex number z has modulus 8 and argument $3\pi/4$.
State the modulus and argument of z^2. [2]
Using these values show the number z^2 on an Argand diagram, and hence express z^2 in the form a + bi. [2]

NOV 2013

6 (a) (i) Show by using de Moivre's theorem that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$. 16

(ii) Deduce that $\cos\frac{\pi}{8}$ is a root of the equation $8x^4 - 8x^2 + 1 = 0$.

(iii) Write down the other three roots in a similar form. [10] X

(b) Express in the form $a + bi$

$$\frac{e^{\frac{3}{4}\pi i}}{e^{\frac{\pi}{2}i}}.$$ 8 [3] 7

(c) (i) On the same axes, draw a diagram showing the locus of z in each of the following

1. $|z| < 2$

2. $\frac{\pi}{6} < \arg(z) < \frac{\pi}{3}$

Shade the region which is common to both loci in **(i)**. [3]

SPECIMEN 2003

7 (i) Use De Moivre's Theorem to express $\cos 4\theta$ in terms of $\cos\theta$. [4]

(ii) Find all the roots of $z^4 = -16$ in the form $a + ib$, where a and b are real.

REPRESENT THESE ROOTS ON AN ARGAND DIAGRAM [8]

NOV 2008

5 (a) Find the modulus and argument of $\dfrac{(1+i)^5}{(1-i)^7}$ for $-\pi < \arg \le \pi$. [4]

(b) Sketch in an Argand diagram the set of points representing all complex numbers z satisfying both of the inequalities

$$|z-2i| \le 2 \text{ and } |z-2i| \le |z|.$$ [3]

(c) Use de Moivre's theorem to express $\sin 5\theta$ in terms of $\sin\theta$. [5]

k $k^2-2k+k+k+2$

NOV 2007

4 Given that $f(x) = x^5 - 3x^4 - 16x + 48$,

(i) show that $2i$ is a root of $f(x) = 0$, [2]

(ii) state another complex root of $f(x) = 0$, [2]

(iii) find the quadratic factor of $f(x)$, [2]

(iv) factorise $f(x)$ completely and hence solve the equation $f(x) = 0$. [1]

NOV 2011

8 (a) Express in exponential form $\left(\frac{3}{5}+\frac{4i}{5}\right)^{20}-\left(\frac{3}{5}-\frac{4i}{5}\right)^{20}$. [5]

(b) (i) Prove that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$ based on de Moivres theorem.

(ii) Hence find the first four exact values of θ for which $\tan^4\theta - 4\tan^3\theta - 6\tan^2\theta - 4\tan\theta + 1 = 0$. [10]

NOV 2009

5 The complex numbers z and w are given by $z = -3 + 2i$ and $w = 5 + 4i$.

Find

(i) $|z|$, [1]

(ii) $\arg z$, [2]

(iii) $\dfrac{z}{w}$ in the form $a + ib$ where a and b are exact.

Hence represent $\dfrac{z}{w}$ in an Argand diagram. [3]

NOV 2006

7 **(a)** The equation $3z^3 - 10z^2 + 20z - 16 = 0$ has $1 - \sqrt{3}i$ as one of its roots.

(i) Find the other roots. [5]

(ii) Sketch the roots in an Argand diagram. [2]

(b) Express $3\sqrt{3} - 3i$ in the form $re^{\theta i}$. [3]

Hence find the 4th root of $3\sqrt{3} - 3i$ giving your answers correct to 2 decimal places. [5]

NOV 2006

6 **(a)** By using the substitution $z = x + y$, show that the Cartesian equation of the circle representing the complex number z, where

$|z+1| = 2|z-1|$, can be expressed in the form $Ax^2 + Bx + Cy^2 + D = 0$, where A, B, C and D are integers. [3]

Sketch this circle on an Argand diagram. [3]

(b) Use De Moivre's theorem to express $\cos 6\theta$ in terms of powers of $\cos\theta$. [6]

(c) Solve the equation $z^4 + 8 + i8\sqrt{3} = 0$ giving your roots in the form $r(\cos\theta + i\sin\theta)$. [8]