05124265: Reinforcement Learning
Exercise 3

Tal Grossman, 201512282 , Moshe Yelisevitch, 207423104

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1 Theory

for theory sections please see handwritten solution.

RL - Ex 3.

Q1. $\boxed{\text{Yes.}}$ Proof:

$$\pi^*(s) \overset{\text{by definition}}{=} \arg\max_a Q^*_M(s,a) = \arg\max_a Q^*_M(s,a) + \underbrace{f(s)}_{\text{const. w.r.t. a}} = \arg\max_a Q^*_{M'}(s,a)$$

$\pi^{*'} = \arg\max_a Q^*_{M'}(s,a)$ is defined as the optimal policy for $Q^*_{M'}(s,a)$, so we proved that $= \pi^*(s)$ is the optimal policy for $Q^*_{M'}(s,a)$ also!

Q3. 1. Each iteration we run π_t, the opt. policy for iter. t

Q 2:

- Bellman eq for m:

$$Q^*_M = \sum_{s'} P(s'|s,a)\left[R(s,a,s') + \gamma \max_{a'} Q^*_M(s',a')\right]$$

- Bellman eq for m':

$$Q^*_{M'} = \sum_{s'} P(s'|s,a)\left[R'(s,a,s') + \gamma \max_{a'} Q^*_{M'}(s',a')\right]$$

Plug in R' as R

$$= \sum_{s'} P(s'|s,a)\left[R(s,a,s') + \phi(s) - \gamma\,\phi(s') + \gamma \max_{a'} Q^*_{M'}(s',a')\right]$$

$$= \sum_{s'} P(s|s,a) R(s,a,s') + \sum_{s'} P(s'|s,a)\,\phi(s) - \gamma \sum_{s'} P(s'|s,a)\,\phi(s')$$
$$+ \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q^*_{M'}(s',a')$$

$\phi(s)$ is independent of s'

$$= \sum_{s'} P(s|s,a) R(s,a,s') + \phi(s)\sum_{s'} P(s'|s,a) - \gamma \sum_{s'} P(s'|s,a)\,\phi(s')$$
$$+ \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q^*_{M'}(s',a')$$

$\sum_{s'} p(s'|s,a) = 1$

$$= \sum_{s'} P(s|s,a) R(s,a,s') + \phi(s)\cdot 1 - \gamma \sum_{s'} P(s'|s,a)\,\phi(s')$$
$$+ \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q^*_{M'}(s',a')$$

$$\Rightarrow \quad Q^*_{M'}(s,a) = Q^*_M(s,a) + \underbrace{\phi(s) - \gamma \sum_{s'} P(s'|s,a)\,\phi(s')}$$

Since this term is a state depenent and independent of a, it does not affect on the slection of the optimal policy.

$\Rightarrow$ The optimal policy for M and M' are identical //

Q3. 1. Each iteration we run π_t, the opt. policy for iter. t,

until a new state-action is found. Let's assume by

contradiction that exists a pair (s',a') such that

$R'_t(s,a') = 1$ we didn't discover at iteration t. In other words, the

optimal policy π_t stuck on (s,a): $R'_t(s,a) = 0$, So $V_t^{\pi} = 0$.

Given a strongly connected MDP, we know (s',a') can be visited

So a policy π' that does so will get a reward of 1 at (s',a'),

thus π is not optimal, in contradiction!

Each iteration must end!

2. $\boxed{T_{iteration} = O(|S|)}$. For example, in the case that we should go over all the states that we have already visited in order to go from S_0 to (s', a')

3. every state s in the set S has a set of possible action $A(s)$, So in order to go over the entire S,a space we need $\sum_{s \in S} |A(s)| = \boxed{|S| \cdot |A| \text{ iterations}}$

4. $T_{exploitation} = T_{iteration} \cdot num\_iterations = O(|S|) \cdot |S| \cdot |A| =$

$= \boxed{O(|S|^2 \cdot |A|)}$

Q 4 :

1)
- from the starting position $(b,1)$ the legal moves are:

$(b,1) \to (a,3)$

$(c,3)$

$(d,2)$

- to return to $(b,1)$ after 2 steps the knigt must move to position which he can return to $(b,1)$, possible moves are:

$(a,3) \to (b,1)$

$(c,3) \to (b,1)$

$(d,2) \to (b,1)$

- So, initially the knight has 3 legal moves
  - from each it can return to $(b,1)$

$\Rightarrow P = \frac{1}{3} \cdot \frac{1}{8} \cdot 3 = \frac{1}{8}$ //

2)- the markov chain is irreducible since its possible to get from any state to any other,
for the knigt it can evetually get to any square from any starting position.

- the markov chain is periodic since the knigt can return to any state every "pair" steps (מספר זוגי של צעדים), thus the biggest divisor is 2, thus periodic with $T=2$

3) the mean recurrence time T_i to a state i is the reciprocal of the stationay distribution Π_i for that state

Since the knight spends an equal amount of time on each square:

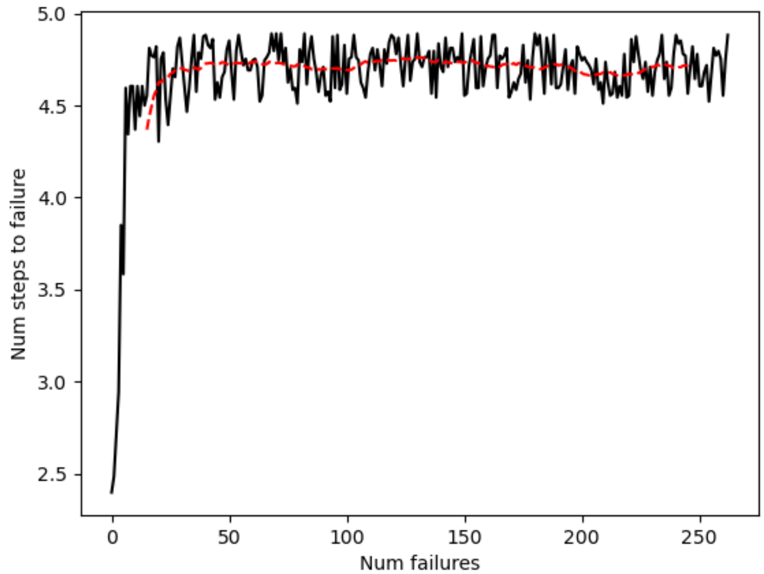
$$\Pi_i = 1/64 \quad \forall i$$

$$\Rightarrow \quad T_{corner} = \frac{1}{\Pi_{corner}} = 64$$

2 Programming

2.1 Question 1: Off-Policy Model-Based

completed in python in the attached file **control.py**
In that specific run, it took 263 iterations to converge. The plot of the failure rate is as follows:



2.2 Question 2: Q-Learning

2.2.1 Item 1 - tabular_Q.py

The percentage of successful episodes is roughly 56.6% . The Q-table is as follows:

```
Score over time: 0.566
Final Q-Table Values
[[1.21427361e-01 1.83254485e-02 1.48874363e-02 1.92531811e-02]
 [1.82363151e-03 1.64267572e-03 5.90394023e-04 1.63101858e-01]
 [0.00000000e+00 3.74026090e-03 2.63652325e-03 2.57552930e-01]
 [5.99671903e-04 2.84269525e-04 2.21128911e-04 1.03078260e-01]
 [2.72628630e-01 2.31807525e-03 6.13800917e-04 1.96965210e-03]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [1.72937261e-04 8.75123858e-08 1.84143980e-01 3.24574625e-05]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [2.86188773e-03 4.22361064e-04 5.35911767e-04 3.60103396e-01]
 [0.00000000e+00 3.54639242e-01 0.00000000e+00 2.76921632e-03]
 [1.05852451e-01 1.79812038e-03 2.10456247e-04 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 2.85918529e-01 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 8.55830849e-01 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 0.00000000e+00]]
```

2.2.2 Item 2 - network_Q.py

The percentage of successful episodes is roughly 34.5%. This result is worse than what we achieved with tabular Q-learning, probably due to the fact that the network is not deep and complicated enough to capture the complexity of the environment. We believe that with a deeper network with some activation functions, we could achieve better results.

```
Score over time: 0.345
```