AM 147: Computational Methods and Applications: Winter 2021

Homework #3

Instructor: Abhishek Halder

Due: January 26, 2021

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW3.zip` via CANVAS. For example, `HalderAbhishekHW3.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m files (MATLAB scripts) for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

Problem 1

Bisection method **($15 + 15 = 30$ points)**

(a) Write a MATLAB .m function named `bisection.m` that computes a real root for any nonlinear equation of the form $f(x) = 0$ within numerical tolerance ε, where f is continuous in the interval $[a, b]$. Your MATLAB function should take the inputs: a, b, f, ε and return an approximation for the root x_{approx}. It is a good practice (but not mandatory) to pass an additional input for maximum number of iterations.

(b) For any positive integer n, the degree n Laguerre polynomial $L_n(x)$ has the general form

$$L_n(x) = \frac{\exp(x)}{n!}\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(\exp(-x)x^n\right).$$

For example, when $n = 5$, we get the degree 5 (quintic) polynomial

$$L_5(x) = \frac{1}{120}\left(-x^5 + 25x^4 - 200x^3 + 600x^2 - 600x + 120\right).$$

It is well-known that all n roots of $L_n(x)$ are positive real, and that all of them are located within the interval $(0, n + (n-1)\sqrt{n}\,]$.

Write a MATLAB code `YourlastnameYourfirstnameHW3p1.m` that plots a graph of the function $L_5(x)$. By visually inspecting this plot, call the function `bisection.m` from part (a) in your code `YourlastnameYourfirstnameHW3p1.m` to numerically compute all 5 roots of $L_5(x)$ within

tolerance $\varepsilon = 10^{-4}$. In other words, executing your code `YourlastnameYourfirstnameHW3p1.m` should generate a plot of $L_5(x)$, AND print its all 5 roots approximated via bisection method in MATLAB command window.

Problem 2

Newton's method **(20 points)**

Write a MATLAB script `YourlastnameYourfirstnameHW3p2.m` that performs 6 Newton iterations to compute $5^{1/7}$ starting with $x_0 = 2$, that is, the script should compute $x_1, x_2, x_3, x_4, x_5, x_6$. In your MATLAB script, declare `power(5,1/7)` as x_{true}, and in the same code, also compute the relative error $|x_6 - x_{\text{true}}|/|x_{\text{true}}|$.