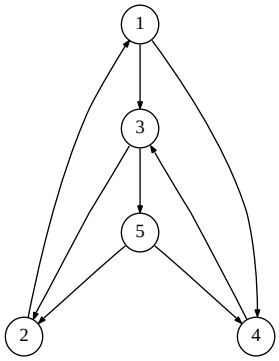
Description of the problem (noname)



arc	c_{ij}	u_{ij}
(1, 3)	8	∞
(1, 4)	-2	5
(2, 1)	3	∞
(3, 2)	1	1
(3, 5)	1	4
(4, 3)	-11	∞
(5, 2)	2	5
(5, 4)	9	∞

node	b_i
1	2
2	-4
3	2
4	-3
5	3

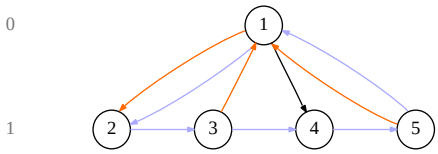
Legend

Arc color	Description
	Natural arc
	Artificial arc
	Swapping arc
	Natural arc in $\mathcal{L}$ set
	Natural arc in $\mathcal{U}$ set
	Thread array

Dashed arcs enter the Tree
Dotted arcs leave the Tree

***1

Description of the initial iteration (F1-noname)



$z = 9$			
arc	x_{ij}^k	$\mathbb{E}$	c_{ij}
$(1, 2)_{\mathrm{a}}$	4	$\mathcal{T}$	1
$(1, 3)$	0	$\mathcal{L}$	0
$(1, 4)$	3	$\mathcal{T}$	0
$(2, 1)$	0	$\mathcal{L}$	0
$(3, 1)_{\mathrm{a}}$	2	$\mathcal{T}$	1
$(3, 2)$	0	$\mathcal{L}$	0
$(3, 5)$	0	$\mathcal{L}$	0
$(4, 3)$	0	$\mathcal{L}$	0
$(5, 1)_{\mathrm{a}}$	3	$\mathcal{T}$	1
$(5, 2)$	0	$\mathcal{L}$	0
$(5, 4)$	0	$\mathcal{L}$	0

node	thread	pred	depth	π
1	2	0	0	0
2	3	1	1	-1
3	4	1	1	1
4	5	1	1	0
5	1	1	1	1

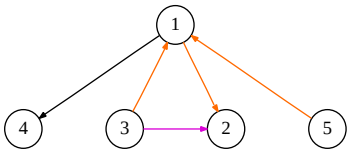
***2

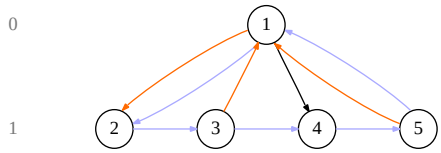
Iteration 1 (F1-noname)

Reduced Costs Table

Set $\mathcal{L}$ $(c_{ij}^{\pi} \geq 0)$													
π_3	-	π_1	+	c_{13}	=	(1)	−	(0)	+	(0)	=	1	
π_1	-	π_2	+	c_{21}	=	(0)	−	(-1)	+	(0)	=	1	
π_2	-	π_3	+	c_{32}	=	(-1)	−	(1)	+	(0)	=	-2	$*$
π_5	-	π_3	+	c_{35}	=	(1)	−	(1)	+	(0)	=	0	
π_3	-	π_4	+	c_{43}	=	(1)	−	(0)	+	(0)	=	1	
π_2	-	π_5	+	c_{52}	=	(-1)	−	(1)	+	(0)	=	-2	$*$
π_4	-	π_5	+	c_{54}	=	(0)	−	(1)	+	(0)	=	-1	$*$

The arc (3, 2) enters the tree.





The arc (3, 2) reaches its upper bound and is moved to the $\mathcal{U}$ set, the tree does not change.

$z = 7$, $\delta = 1$			
arc	x_{ij}^{k-1}	x_{ij}^{k}	$\mathbb{E}$
$(1, 2)_{\mathrm{a}}$	4	3	$\mathcal{T}$
(1, 3)	0	0	$\mathcal{L}$
(1, 4)	3	3	$\mathcal{T}$
(2, 1)	0	0	$\mathcal{L}$
$(3, 1)_{\mathrm{a}}$	2	1	$\mathcal{T}$
(3, 2)	0	1	$\mathcal{U}$
(3, 5)	0	0	$\mathcal{L}$
(4, 3)	0	0	$\mathcal{L}$
$(5, 1)_{\mathrm{a}}$	3	3	$\mathcal{T}$
(5, 2)	0	0	$\mathcal{L}$
(5, 4)	0	0	$\mathcal{L}$

node	thread	pred	depth	π
1	2	0	0	0
2	3	1	1	-1
3	4	1	1	1
4	5	1	1	0
5	1	1	1	1

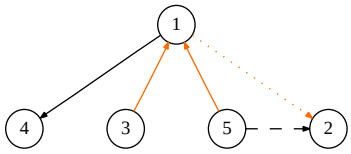
Iteration 1: ***3

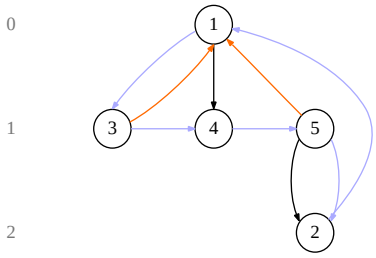
Iteration 2 (F1-noname)

Reduced Costs Table

Set $\mathcal{L}$ $(c_{ij}^{\pi} \geq 0)$													
π_3	-	π_1	+	c_{13}	=	(1)	−	(0)	+	(0)	=	1	
π_1	-	π_2	+	c_{21}	=	(0)	−	(−1)	+	(0)	=	1	
π_5	-	π_3	+	c_{35}	=	(1)	−	(1)	+	(0)	=	0	
π_3	-	π_4	+	c_{43}	=	(1)	−	(0)	+	(0)	=	1	
π_2	-	π_5	+	c_{52}	=	(−1)	−	(1)	+	(0)	=	−2	*
π_4	-	π_5	+	c_{54}	=	(0)	−	(1)	+	(0)	=	−1	*
Set $\mathcal{U}$ $(c_{ij}^{\pi} \leq 0)$													
π_2	-	π_3	+	c_{32}	=	(−1)	−	(1)	+	(0)	=	−2	

The arc (5, 2) enters the tree.





The artificial arc (1, 2) reaches its lower bound and is removed from the problem.

$z = 1$, $\delta = 3$			
arc	x_{ij}^{k-1}	x_{ij}^{k}	$\mathbb{E}$
$(1, 2)_{\mathrm{a}}$	3	0	
(1, 3)	0	0	$\mathcal{L}$
(1, 4)	3	3	$\mathcal{T}$
(2, 1)	0	0	$\mathcal{L}$
$(3, 1)_{\mathrm{a}}$	1	1	$\mathcal{T}$
(3, 2)	1	1	$\mathcal{U}$
(3, 5)	0	0	$\mathcal{L}$
(4, 3)	0	0	$\mathcal{L}$
$(5, 1)_{\mathrm{a}}$	3	0	$\mathcal{T}$
(5, 2)	0	3	$\mathcal{T}$
(5, 4)	0	0	$\mathcal{L}$

node	thread	pred	depth	π
1	3	0	0	0
2	1	5	2	1
3	4	1	1	1
4	5	1	1	0
5	2	1	1	1

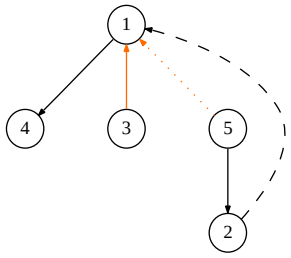
Iteration 2: ***3

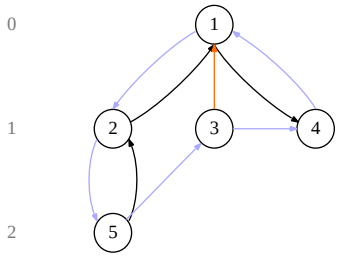
Iteration 3 (F1-noname)

Reduced Costs Table

Set $\mathcal{L}$ $(c_{ij}^{\pi} \geq 0)$													
π_3	-	π_1	+	c_{13}	=	(1)	−	(0)	+	(0)	=	1	
π_1	-	π_2	+	c_{21}	=	(0)	−	(1)	+	(0)	=	−1	*
π_5	-	π_3	+	c_{35}	=	(1)	−	(1)	+	(0)	=	0	
π_3	-	π_4	+	c_{43}	=	(1)	−	(0)	+	(0)	=	1	
π_4	-	π_5	+	c_{54}	=	(0)	−	(1)	+	(0)	=	−1	*
Set $\mathcal{U}$ $(c_{ij}^{\pi} \leq 0)$													
π_2	-	π_3	+	c_{32}	=	(1)	−	(1)	+	(0)	=	0	

The arc (2, 1) enters the tree.





The artificial arc (5, 1) reaches its lower bound and is removed from the problem.

$z = 1$, $\delta = 0$			
arc	x_{ij}^{k-1}	x_{ij}^{k}	$\mathbb{E}$
(1, 3)	0	0	$\mathcal{L}$
(1, 4)	3	3	$\mathcal{T}$
(2, 1)	0	0	$\mathcal{T}$
$(3, 1)_{\mathrm{a}}$	1	1	$\mathcal{T}$
(3, 2)	1	1	$\mathcal{U}$
(3, 5)	0	0	$\mathcal{L}$
(4, 3)	0	0	$\mathcal{L}$
$(5, 1)_{\mathrm{a}}$	0	0	
(5, 2)	3	3	$\mathcal{T}$
(5, 4)	0	0	$\mathcal{L}$

node	thread	pred	depth	π
1	2	0	0	0
2	5	1	1	0
3	4	1	1	1
4	1	1	1	0
5	3	2	2	0

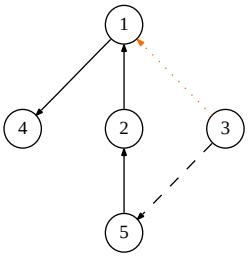
Iteration 3: ***3

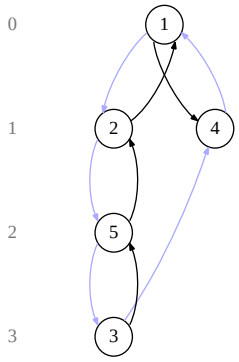
Iteration 4 (F1-noname)

Reduced Costs Table

Set $\mathcal{L}$ $(c_{ij}^{\pi} \geq 0)$												
π_3	-	π_1	+	c_{13}	=	(1)	−	(0)	+	(0)	=	1	
π_5	-	π_3	+	c_{35}	=	(0)	−	(1)	+	(0)	=	−1	*
π_3	-	π_4	+	c_{43}	=	(1)	−	(0)	+	(0)	=	1	
π_4	-	π_5	+	c_{54}	=	(0)	−	(0)	+	(0)	=	0	
Set $\mathcal{U}$ $(c_{ij}^{\pi} \leq 0)$												
π_2	-	π_3	+	c_{32}	=	(0)	−	(1)	+	(0)	=	−1	

The arc (3, 5) enters the tree.





The artificial arc (3, 1) reaches its lower bound and is removed from the problem.

$z = 0$, $\delta = 1$			
arc	x_{ij}^{k-1}	x_{ij}^{k}	$\mathbb{E}$
(1, 3)	0	0	$\mathcal{L}$
(1, 4)	3	3	$\mathcal{T}$
(2, 1)	0	1	$\mathcal{T}$
$(3, 1)_{\mathrm{a}}$	1	0	
(3, 2)	1	1	$\mathcal{U}$
(3, 5)	0	1	$\mathcal{T}$
(4, 3)	0	0	$\mathcal{L}$
(5, 2)	3	4	$\mathcal{T}$
(5, 4)	0	0	$\mathcal{L}$

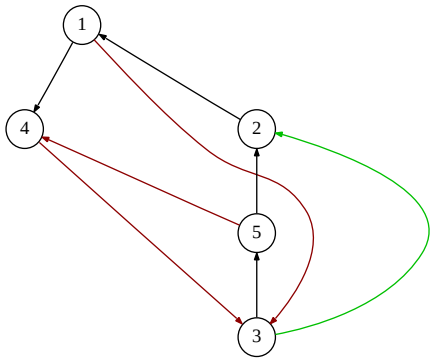
node	thread	pred	depth	π
1	2	0	0	0
2	5	1	1	0
3	4	5	3	0
4	1	1	1	0
5	3	2	2	0

Iteration 4: ***3

Solution of the problem (F1-noname)

Reduced Costs Table

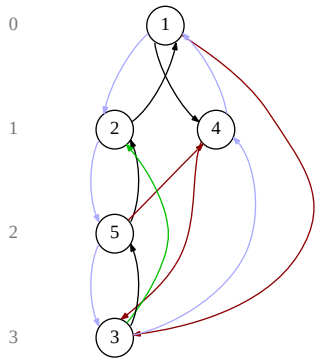
Set $\mathcal{L}$ ($c_{ij}^{\pi} \geq 0$)											
π_3	-	π_1	+	c_{13}	=	(0)	−	(0)	+	(0)	= 0
π_3	-	π_4	+	c_{43}	=	(0)	−	(0)	+	(0)	= 0
π_4	-	π_5	+	c_{54}	=	(0)	−	(0)	+	(0)	= 0
Set $\mathcal{U}$ ($c_{ij}^{\pi} \leq 0$)											
π_2	-	π_3	+	c_{32}	=	(0)	−	(0)	+	(0)	= 0



$z = 0$		
arc	x_{ij}^{k}	$\mathbb{E}$
(1, 3)	0	$\mathcal{L}$
(1, 4)	3	$\mathcal{T}$
(2, 1)	1	$\mathcal{T}$
(3, 2)	1	$\mathcal{U}$
(3, 5)	1	$\mathcal{T}$
(4, 3)	0	$\mathcal{L}$
(5, 2)	4	$\mathcal{T}$
(5, 4)	0	$\mathcal{L}$

***4

Description of the initial iteration (noname)



$z = 7$			
arc	x_{ij}^k	$\mathbb{E}$	c_{ij}
(1, 3)	0	$\mathcal{L}$	8
(1, 4)	3	$\mathcal{T}$	−2
(2, 1)	1	$\mathcal{T}$	3
(3, 2)	1	$\mathcal{U}$	1
(3, 5)	1	$\mathcal{T}$	1
(4, 3)	0	$\mathcal{L}$	−11
(5, 2)	4	$\mathcal{T}$	2
(5, 4)	0	$\mathcal{L}$	9

node	thread	pred	depth	π
1	2	0	0	0
2	5	1	1	3
3	4	5	3	6
4	1	1	1	2
5	3	2	2	5

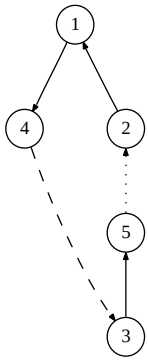
***5

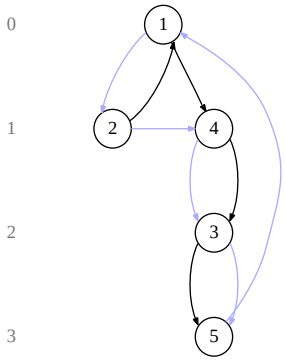
Iteration 1 (noname)

Reduced Costs Table

Set $\mathcal{L}$ $(c_{ij}^{\pi} \geq 0)$													
π_3	-	π_1	+	c_{13}	=	(6)	−	(0)	+	(8)	=	14	
π_3	-	π_4	+	c_{43}	=	(6)	−	(2)	+	(−11)	=	−7	*
π_4	-	π_5	+	c_{54}	=	(2)	−	(5)	+	(9)	=	6	
Set $\mathcal{U}$ $(c_{ij}^{\pi} \leq 0)$													
π_2	-	π_3	+	c_{32}	=	(3)	−	(6)	+	(1)	=	−2	

The arc (4, 3) enters the tree.





The arc (5, 2) reaches its upper bound and is moved to the $\mathcal{U}$ set.

$z = 0$, $\delta = 1$			
arc	x_{ij}^{k-1}	x_{ij}^{k}	$\mathbb{E}$
(1, 3)	0	0	$\mathcal{L}$
(1, 4)	3	4	$\mathcal{T}$
(2, 1)	1	2	$\mathcal{T}$
(3, 2)	1	1	$\mathcal{U}$
(3, 5)	1	2	$\mathcal{T}$
(4, 3)	0	1	$\mathcal{T}$
(5, 2)	4	5	$\mathcal{U}$
(5, 4)	0	0	$\mathcal{L}$

node	thread	pred	depth	π
1	2	0	0	0
2	4	1	1	3
3	5	4	2	13
4	3	1	1	2
5	1	3	3	12

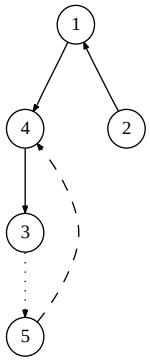
Iteration 1: ***6

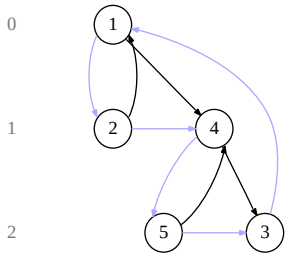
Iteration 2 (noname)

Reduced Costs Table

Set $\mathcal{L}$ ($c_{ij}^{\pi} \geq 0$)												
π_3	-	π_1	+	c_{13}	=	(13)	−	(0)	+	(8)	=	21	
π_4	-	π_5	+	c_{54}	=	(2)	−	(12)	+	(9)	=	−1	$*$
Set $\mathcal{U}$ ($c_{ij}^{\pi} \leq 0$)												
π_2	-	π_3	+	c_{32}	=	(3)	−	(13)	+	(1)	=	−9	
π_2	-	π_5	+	c_{52}	=	(3)	−	(12)	+	(2)	=	−7	

The arc (5, 4) enters the tree.





The arc (3, 5) reaches its upper bound and is moved to the $\mathcal{U}$ set.

$z = -2$, $\delta = 2$			
arc	x_{ij}^{k-1}	x_{ij}^{k}	$\mathbb{E}$
(1, 3)	0	0	$\mathcal{L}$
(1, 4)	4	4	$\mathcal{T}$
(2, 1)	2	2	$\mathcal{T}$
(3, 2)	1	1	$\mathcal{U}$
(3, 5)	2	4	$\mathcal{U}$
(4, 3)	1	3	$\mathcal{T}$
(5, 2)	5	5	$\mathcal{U}$
(5, 4)	0	2	$\mathcal{T}$

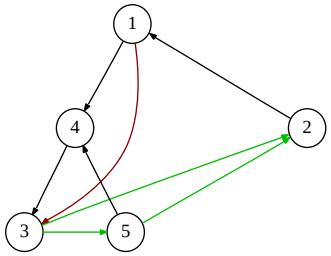
node	thread	pred	depth	π
1	2	0	0	0
2	4	1	1	3
3	1	4	2	13
4	5	1	1	2
5	3	4	2	11

Iteration 2: ***6

Solution of the problem (noname)

Reduced Costs Table

Set $\mathcal{L}$ ($c_{ij}^{\pi} \geq 0$)												
π_3	-	π_1	+	c_{13}	=	(13)	−	(0)	+	(8)	=	21
Set $\mathcal{U}$ ($c_{ij}^{\pi} \leq 0$)												
π_2	-	π_3	+	c_{32}	=	(3)	−	(13)	+	(1)	=	−9
π_5	-	π_3	+	c_{35}	=	(11)	−	(13)	+	(1)	=	−1
π_2	-	π_5	+	c_{52}	=	(3)	−	(11)	+	(2)	=	−6



$z = -2$		
arc	x_{ij}^{k}	$\mathbb{E}$
(1, 3)	0	$\mathcal{L}$
(1, 4)	4	$\mathcal{T}$
(2, 1)	2	$\mathcal{T}$
(3, 2)	1	$\mathcal{U}$
(3, 5)	4	$\mathcal{U}$
(4, 3)	3	$\mathcal{T}$
(5, 2)	5	$\mathcal{U}$
(5, 4)	2	$\mathcal{T}$

***7