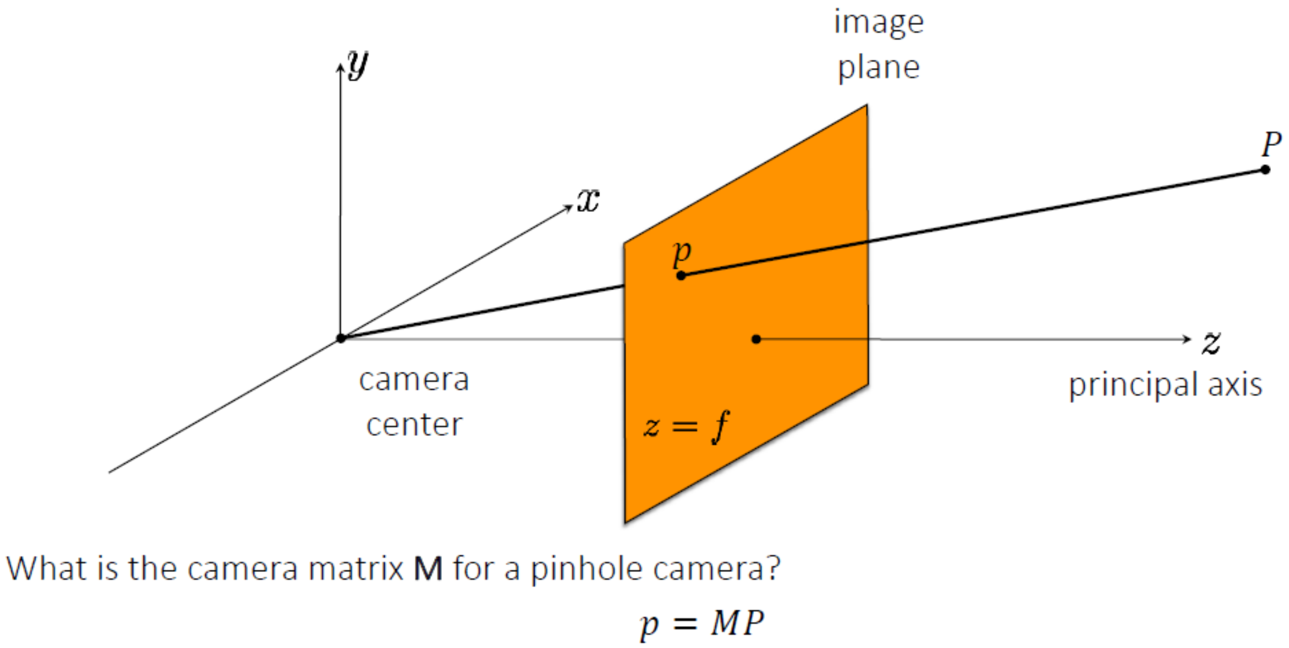
EE 046746 - Technion - Computer Vision

Tutorial 10 - Camera Calibration and Epipolar Geometry

Agenda

Camera Model

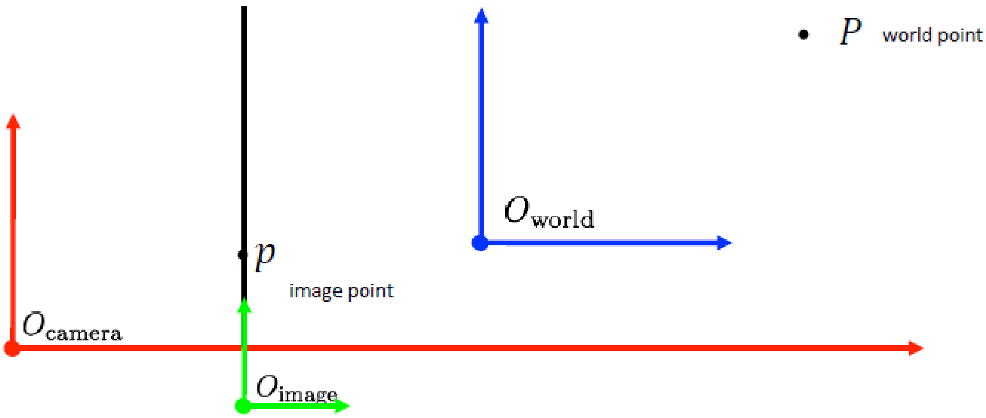
The (rearranged) pinhole camera



- A 3D world point P is projected by the camera matrix M to the 2D image point p

The Camera Matrix

In general, there are *three*, generally different, coordinate systems.



We need to know the transformations between them.

- M is a 3×4 matrix comprised of two sets of parameters: **Intrinsic** and **Extrinsic**.
- What is the decomposed structure of M?

The Camera Matrix

$$\mathrm{M} = K\,[R|t]$$

$$\mathrm{M} = \underbrace{\begin{bmatrix} f & 0 & m_x \\ 0 & f & m_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsic parameters}} \underbrace{\left[\begin{array}{ccc:c} r_1 & r_2 & r_3 & t_1 \\ r_4 & r_5 & r_6 & t_2 \\ r_7 & r_8 & r_9 & t_3 \end{array}\right]}_{\text{extrinsic parameters}}$$

$$\underbrace{\mathbf{R} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}}_{\text{3D rotation}} \qquad \underbrace{\mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}}_{\text{3D translation}}$$

The Camera Matrix

- How many degress of freedom so far?
- And after switching f with f_x and f_y and adding skew s?

Camera Calibration

- Estimation of M
- Separating Extrinsic and Intrinsic Parameters

Geometric Camera Calibration: Estimating M

- Given a set of matched points $\{P_i, p_i\}$, we want to estimate M
  - Use the camera model: $p_i = MP_i$
  - Where did we get such matched points?
- Same trick as in the Homography tutorial $\rightarrow$ switch to row-wise representation of the unknowns:

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} - & m_1^T & - \\ - & m_2^T & - \\ - & m_3^T & - \end{bmatrix} P$$

Estimating M

- Resulting equation for x and y in heterogeneous coordinates:

$$\tilde{x} = \frac{m_1^T P}{m_3^T P}, \tilde{y} = \frac{m_2^T P}{m_3^T P}$$

- Rearranging to solve for m_i:

$$m_1^T P - \tilde{x} m_3^T P = 0$$
$$m_2^T P - \tilde{y} m_3^T P = 0$$

- What is the dimension of $m_i^T P$?

Estimating M

- Rearrange into a matrix for N_p points:

$$\begin{bmatrix} P_i^T & 0^T & -\tilde{x}_i P_i^T \\ 0^T & P_i^T & -\tilde{y}_i P_i^T \\ . & . & . \\ . & . & . \\ P_{N_p}^T & 0^T & -\tilde{x}_{N_p} P_{N_p}^T \\ 0^T & P_{N_p}^T & -\tilde{y}_{N_p} P_{N_p}^T \end{bmatrix} \begin{bmatrix} | \\ m_1 \\ | \\ m_2 \\ | \\ m_3 \\ | \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \\ 0 \end{bmatrix} \leftrightarrow Am = 0$$

- What are the dimensions? How much points N_p do we need?

Estimating M

- boils down to the problem:

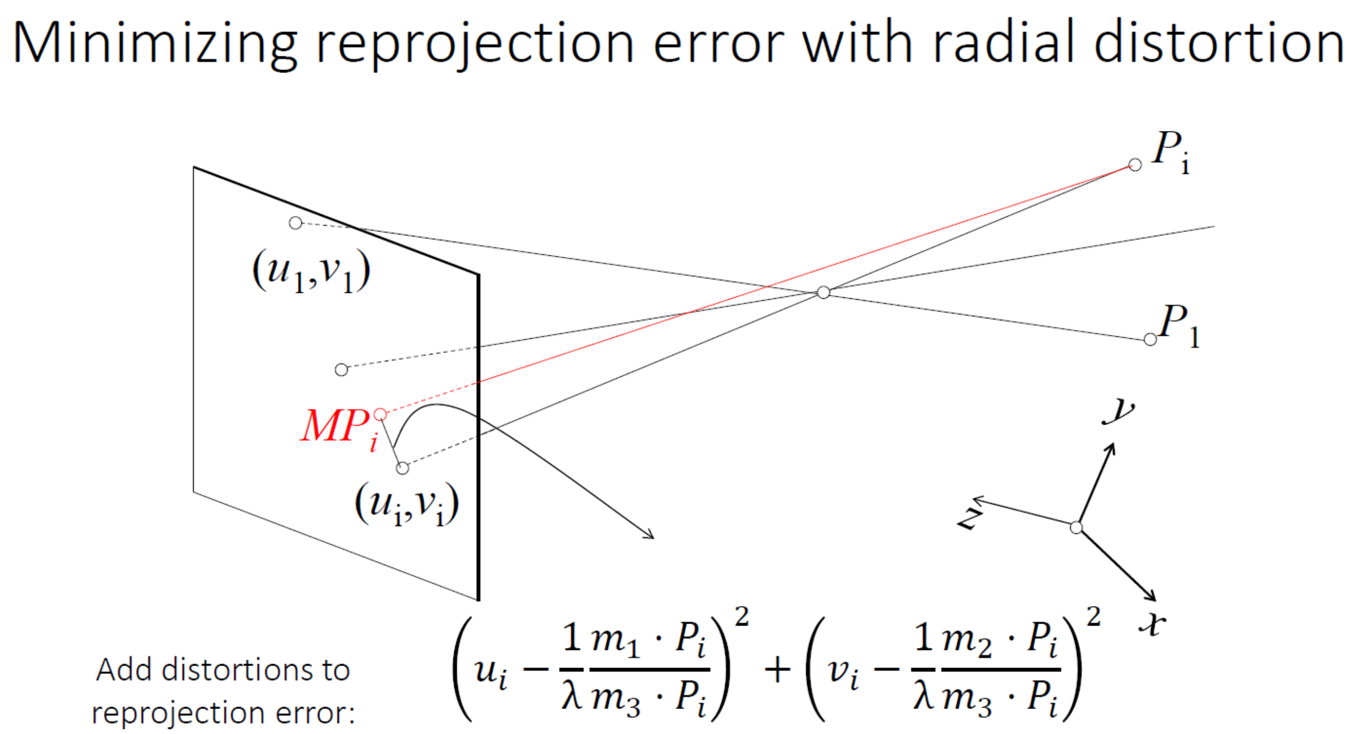
$$\hat{m} = \arg\min_m \|Am\|^2 \ s.t. \ \|m\|^2 = 1$$

- Solution via SVD of $A = U \Sigma V^T$:
  - $\hat{m}$ is the column of V corresponding to the smallest eigen-value.
- How about separating M to $K\,[R|t]$?

Decomposition of M to K, R & t

- rewrite M:
$$M = K\,[R|t] = K\,[R|-Rc] = [N|-Nc]$$
- c can be found via SVD of M due to the relation:
$$Mc = 0$$
- Then N can be further decomposed into $N = KR$:
  - How? using [QR (https://en.wikipedia.org/wiki/QR_decomposition)](https://en.wikipedia.org/wiki/QR_decomposition) decomposition because K is upper triangular and R is orthogonal
- However..
  - Does not take into account noise, radial distortions, hard to impose prior knowledge (e.g. f), etc.
  - Solution?

Minimize reprojection error



- Where the radial distortion model is: $\lambda = 1 + k_1 r^2 + k_2 r^4 + k_3 r^6$

Minimize reprojection error

- Radial distortion is multiplicative:

$$x_{rad} = x\left[1 + k_1 r^2 + k_2 r^4 + k_3 r^6\right]$$
$$y_{rad} = y\left[1 + k_1 r^2 + k_2 r^4 + k_3 r^6\right]$$

- Usually we also include tangential distortion:

$$x_{tan} = x + \left[2p_1 xy + p_2\left(r^2 + 2x^2\right)\right]$$
$$y_{tan} = y + \left[p_1\left(r^2 + 2y^2\right) + 2p_2 xy\right]$$

- We end up with 5 parameters to estimate:

$$\text{distortion coefficients} = [k_1, k_2, k_3, p_1, p_2]^T$$

Chessboard Calibration in OpenCV

- Take a notebook and paste a chesspattern
- Capture this pattern from several angles and positions
- Calibrate using OpenCV

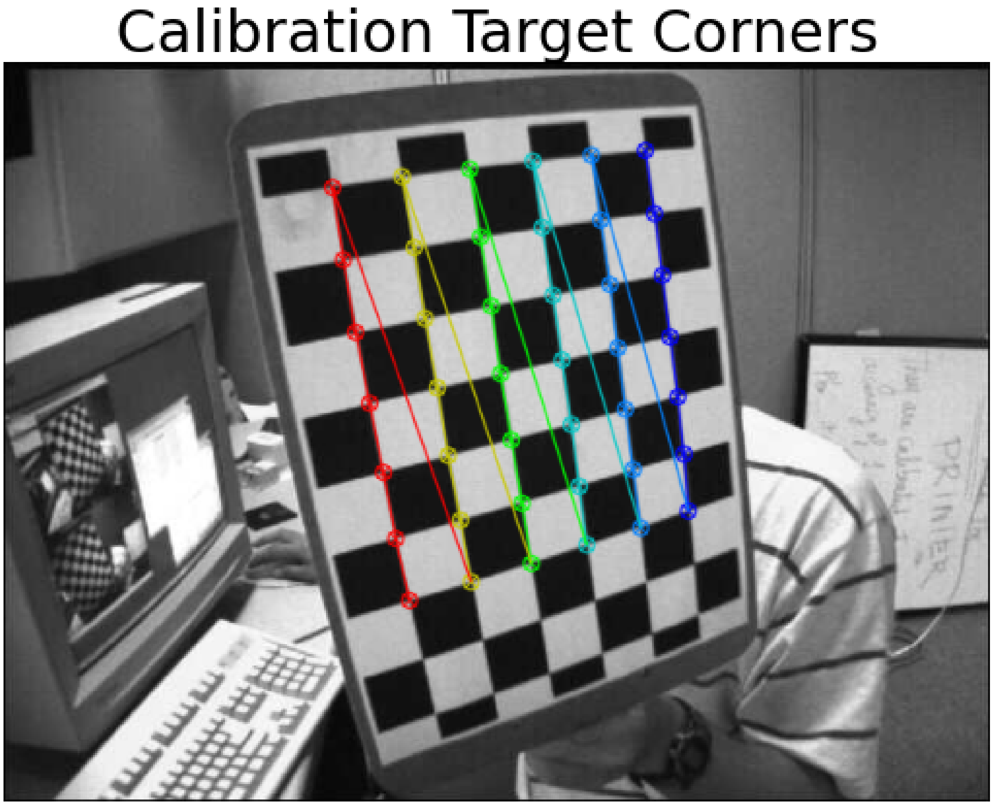
Chessboard Calibration in OpenCV

- Getting the 3D to 2D points correspondences from a known planar object
- Chessboard has fixed distances between squares known apriori
- Camera static and chessboard moves $\leftrightarrow$ chessboard static and camera moves
- Camera moves $\leftrightarrow$ Extrinsic parameters in each frame change
- Therefore we got the matches of real world points and camera points $\{P_i, p_i\}_{i=1}^{N}$!
  - $P_i = [X_i, Y_i, Z_i = 0]$, where, X_i, Y_i set by periodicity of the chessbaord
  - $p_i = [x_i, y_i]$, detected corners in the image

In [1]:

```
import numpy as np
import cv2
import glob
# termination criteria
criteria = (cv2.TERM_CRITERIA_EPS + cv2.TERM_CRITERIA_MAX_ITER, 30, 0.001)
# prepare object points, like (0,0,0), (1,0,0), (2,0,0) ....,(6,5,0)
objp = np.zeros((6*7,3), np.float32)
objp[:,:2] = np.mgrid[0:7,0:6].T.reshape(-1,2)
# Arrays to store object points and image points from all the images.
objpoints = [] # 3d point in real world space
imgpoints = [] # 2d points in image plane.
images = glob.glob('./assets/left*.jpg')
for fname in images:
    img = cv2.imread(fname)
    gray = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
    # Find the chess board corners
    ret, corners = cv2.findChessboardCorners(gray, (7,6), None)
    # If found, add object points, image points (after refining them)
    if ret == True:
        objpoints.append(objp)
        corners2 = cv2.cornerSubPix(gray,corners, (11,11), (-1,-1), criteria)
        imgpoints.append(corners)
        # Draw and display the corners
        imlast = cv2.drawChessboardCorners(img, (7,6), corners2, ret)
        cv2.imshow('img', img)
        cv2.waitKey(500)
cv2.destroyAllWindows()
```

In [2]:
```python
# show the last image and the detected corners
import matplotlib.pyplot as plt
plt.figure(figsize=(11,7))
plt.imshow(imlast)
plt.axis('off')
plt.title('Calibration Target Corners', fontsize=30)
plt.show()
```



In [3]:
```python
# now that we have object points and and image points, we just apply OpenCV bu
iltin function
ret, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(objpoints, imgpoints, gray.
shape[::-1], None, None)

# resulting camera matrix
print("M = ")
print(repr(mtx))

# distortion coeff.
print("distortion coeff = ")
print(repr(dist))
```

```
M =
array([[534.07088364,   0.        , 341.53407554],
       [  0.        , 534.11914595, 232.94565259],
       [  0.        ,   0.        ,   1.        ]])
distortion coeff =
array([[-2.92971637e-01,  1.07706962e-01,  1.31038376e-03,
        -3.11018780e-05,  4.34798110e-02]])
```

In [6]:
```
# Rodriguez rotation vectors and translation vectors
print("Rotation vector 1 = ")
print(rvecs[0])
print("Translation vector 1 = ")
print(tvecs[0])

print("\n . \n . \n . \n")
```

```
Rotation vector 1 =
[[-0.43239599]
 [ 0.25603401]
 [-3.08832021]]
Translation vector 1 =
[[ 3.79739146]
 [ 0.89895018]
 [14.8593055 ]]

 .
 .
 .
```

In [7]:
```
print("Rotation vector N = ")
print(rvecs[-1])
print("Translation vector N = ")
print(tvecs[-1])
```

```
Rotation vector N =
[[-0.17288944]
 [-0.46764681]
 [ 1.34745198]]
Translation vector N =
[[ 1.81888151]
 [-4.2642919 ]
 [12.45728517]]
```

In [8]:
```
# let us examine the distortion on a given image
img = cv2.imread('./assets/left12.jpg')
h,  w = img.shape[:2]
newcameramtx, roi = cv2.getOptimalNewCameraMatrix(mtx, dist, (w,h), 1, (w,h))

# undistort
dst = cv2.undistort(img, mtx, dist, None, newcameramtx)
# crop the image
x, y, w, h = roi
dst = dst[y:y+h, x:x+w]
```

In [9]:

```python
# plot the image ebfore and after fixing distortion
plt.figure(figsize=(18,13))
plt.subplot(1,2,1)
plt.imshow(img)
plt.axis('off')
plt.title('Distorted', fontsize=30)
plt.subplot(1,2,2)
plt.imshow(dst)
plt.axis('off')
plt.title('Undistorted', fontsize=30)
plt.show()
```

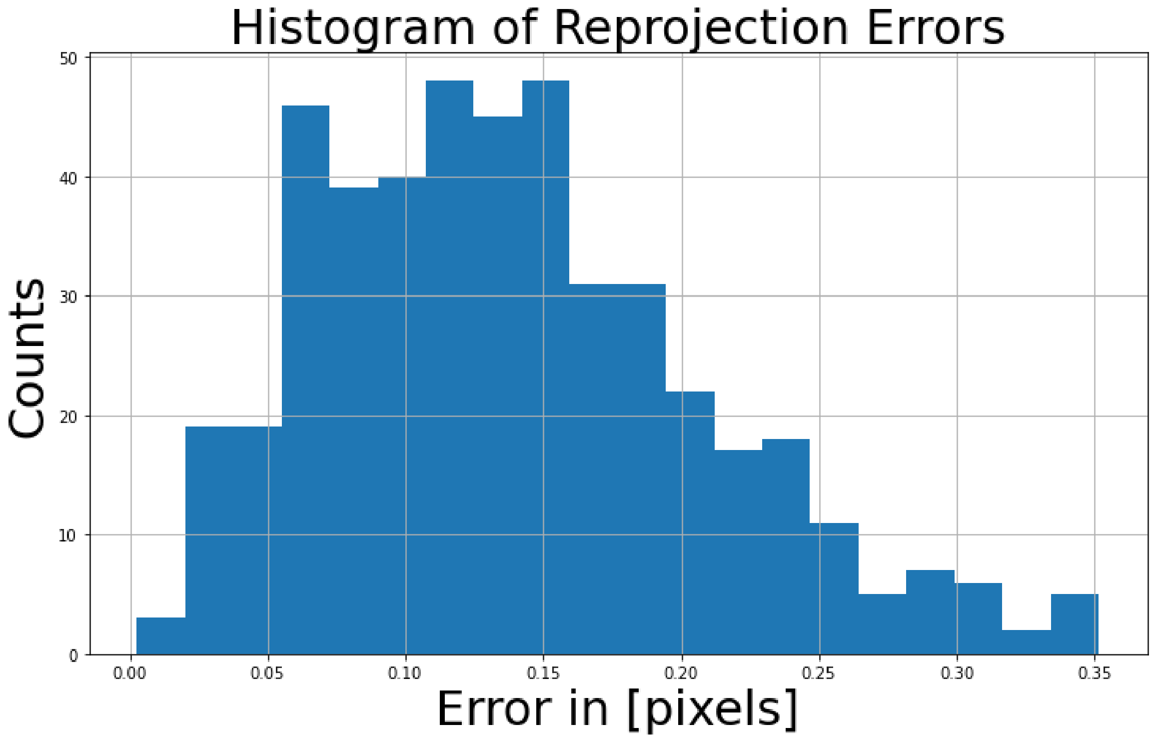
In [10]:

```python
# checking reprojection errors for validation - ideally we should get ~0
mean_error = 0
errs_all = []
for i in range(len(objpoints)):
    imgpoints2, _ = cv2.projectPoints(objpoints[i], rvecs[i], tvecs[i], mtx, d
ist)
    errs_all.append(np.squeeze(np.sqrt(np.sum((imgpoints[i] - imgpoints2)**2,a
xis=2))))

# all reprojection errors in absolute pixel values
errs_all = np.hstack(errs_all)
print("Mean error: {:.4f} px".format(errs_all.mean()))
```

```
Mean error: 0.1387 px
```

In [11]:
```
fig = plt.figure(figsize=(12, 7))
ax = fig.add_subplot(1, 1 ,1)
ax.hist(errs_all, 20)
ax.set_title("Histogram of Reprojection Errors", fontsize=30)
ax.set_xlabel('Error in [pixels]', fontsize=30)
ax.set_ylabel('Counts', fontsize=30)
ax.grid()
```



Homography Quiz

Homography Quiz

- **Quiz1**: Prove that a 3×3 homography transform H is sufficient to describe the mapping between a planar 3D object and a camera, i.e. point matches of the form $\{p_i, P_i\}$, where $p_i = [x_i, y_i, w_i]^T$ and $P_i = [X_i, Y_i, Z_i, 1]^T$ satisfying $aX_i + bY_i + cZ_i + d = 0$.

Homography Quiz

In this special case of a **planar scene**, we do not need the full 3×4 camera matrix M, and we can make due with a 3×3 homography matrix H. The proof is relatively straight forward, and rely on the following observation:

Since the points in 3D lie on a plane:

$$aX + bY + cZ + d = 0$$

we can switch sides and write down the plane equation for Z, such that

$$P_i = \left[X_i, Y_i, -\frac{a}{c}X_i - \frac{b}{c}Y_i - \frac{d}{c}, 1\right]^T$$

.

Homography Quiz

- This leads to the main conclusion that the 4D homogeneous coordinates are redundant and can be written down by a 3D homogeneous coordinates:

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{a}{c} & -\frac{b}{c} & -\frac{d}{c} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix}$$

- Now coming back to the general camera matrix M how can we conclude it can be reduced to a homography?
  - The answer lies in simplifying the matrix-vector product.

Homography Quiz

- A general M satisfy the relation:

$$\begin{bmatrix} x_i \\ y_i \\ w_i \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

- Assuming the scene is planar, we plug in the plane equation for $Z_i = -\frac{a}{c}X_i - \frac{b}{c}Y_i - \frac{d}{c}$:

$$\begin{bmatrix} x_i \\ y_i \\ w_i \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ -\frac{a}{c}X_i - \frac{b}{c}Y_i - \frac{d}{c} \\ 1 \end{bmatrix}$$

Homography Quiz

- Now, let us look more closely at the formula for x_i:

$$x_i = m_{11}X_i + m_{12}Y_i + m_{13}\left(-\frac{a}{c}X_i - \frac{b}{c}Y_i - \frac{d}{c}\right) + m_{14}$$

- The terms can be rearranged and written as:

$$x_i = \left(m_{11} - \frac{a}{c}m_{13}\right)X_i + \left(m_{12} - \frac{b}{c}m_{13}\right)Y_i + \left(m_{14} - m_{13}\frac{d}{c}\right)$$

Homography Quiz

- Similarly, this can be done for y_i and w_i:

$$\begin{aligned} y_i &= \left(m_{21} - \frac{a}{c}m_{23}\right)X_i + \left(m_{22} - \frac{b}{c}m_{23}\right)Y_i + \left(m_{24} - m_{23}\frac{d}{c}\right) \\ w_i &= \left(m_{31} - \frac{a}{c}m_{33}\right)X_i + \left(m_{32} - \frac{b}{c}m_{33}\right)Y_i + \left(m_{34} - m_{33}\frac{d}{c}\right) \end{aligned}$$

Homography Quiz

- Therefore, rewriting this in matrix form we get the following relation:

$$\begin{bmatrix} x_i \\ y_i \\ w_i \end{bmatrix} = \begin{bmatrix} m_{11} - \frac{a}{c} m_{13} & m_{12} - \frac{b}{c} m_{13} & m_{14} - \frac{d}{c} m_{13} \\ m_{21} - \frac{a}{c} m_{23} & m_{22} - \frac{b}{c} m_{23} & m_{24} - \frac{d}{c} m_{23} \\ m_{31} - \frac{a}{c} m_{33} & m_{32} - \frac{b}{c} m_{33} & m_{34} - \frac{d}{c} m_{33} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix}$$

- Meaning, the 3D point P_i can be indeed reduced to a 3D homogeneous vector $[X_i, Y_i, 1]^T$, and is related to the 2D point in the image $p_i = [x_i, y_i, w_i]$ through a 3×3 homography H that is a function of the entries in the general 3×4 camera matrix M, and the normal to the plane $[a, b, c, d]^T$.

Homography Quiz

- **Quiz2**: Prove that there exists a homography H that satisfies:
$$p_1 \equiv H p_2$$
between the 2D points (in homogeneous coordinates) p_1 and p_2 in the images of a plane Π captured by two 3×4 camera projection matrices M_1 and M_2, respectively. The symbol $\equiv$ stands for equality $up\ to\ scale$.

(Note: A degenerate case happens when the plan Π contains both cameras' centers, in which case there are infinite choices of H satisfying the equation. You can ignore this special case in your answer.)

Homography Quiz

- Plane in 3D using homogeneous coordinates is given by:
$$n^T P = 0$$
Where n, P are homogeneous vectors (4 numbers each) and n is the normal to the plane.
- Therefore, we can find a basis of 3 vectors u_1, u_2, u_3 in R^4, such that each point on the plane is given by:
$$P = \sum_{i=1}^{3} \alpha_i u_i$$
- The projection of 3D point P to the j^{th} image point p_j is given by:
$$p_j = M_j P = \sum_{i=1}^{3} \alpha_i M_j u_i$$

Homography Quiz

- If we denote $v_j^i = M_j u_i$ we get:

$$p_1 = \sum_{i=1}^{3} \alpha_i v_1^i$$
$$p_2 = \sum_{i=1}^{3} \alpha_i v_2^i$$

- Hence, the relation between the two points is a 3×3 matrix satisfying:

$$\begin{bmatrix} | & | & | \\ v_1^1 & v_1^2 & v_1^3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} | & | & | \\ v_2^1 & v_2^2 & v_2^3 \\ | & | & | \end{bmatrix}$$

Homography Quiz

- Relation to camera calibration:
  - Recall that we have 11 degrees of freedom in M.
  - If all the calibration points are on a plane, we get at most 8 independent equations out of 4 pts.
  - Any 5^{th} point will result in constraints that are linearly dependent on the constraints from the previous 4 pts on the plane.
  - Therefore, in estimating M we can't rely on a single image of the chessboard.

Homography Quiz

- **Quiz3**: Prove that there exists a homography H that satisfies the equation $p_1 = Hp_2$, given two cameras separated by a pure rotation. That is, for camera 1, $p_1 = K_1 \left[I|0 \right] P$, and for camera 2, $p_2 = K_2 \left[R|0 \right] P$. Note that K_1 and K_2 are the 3×3 intrinsic matrices of the cameras and are different. I is 3×3 identity matrix, 0 is a 3×1 zero vector and P is a point in 3D space. R is the 3×3 rotation matrix of the camera.

Homography Quiz

- Since the last column is zero, we can see that:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R^{-1}K_2^{-1}p_2$$

- Substituting this in the second equation we get:

$$p_1 = K_1R^{-1}K_2^{-1}p_2$$

- Therefore, the resulting homography is given by:

$$H = K_1R^{-1}K_2^{-1}$$

Homography Quiz

- Take away is 2 cameras differing only in rotation can't triangulate!
- Remember where this was useful?
  - Panorama stitching! (There we did not care about recovering depth)

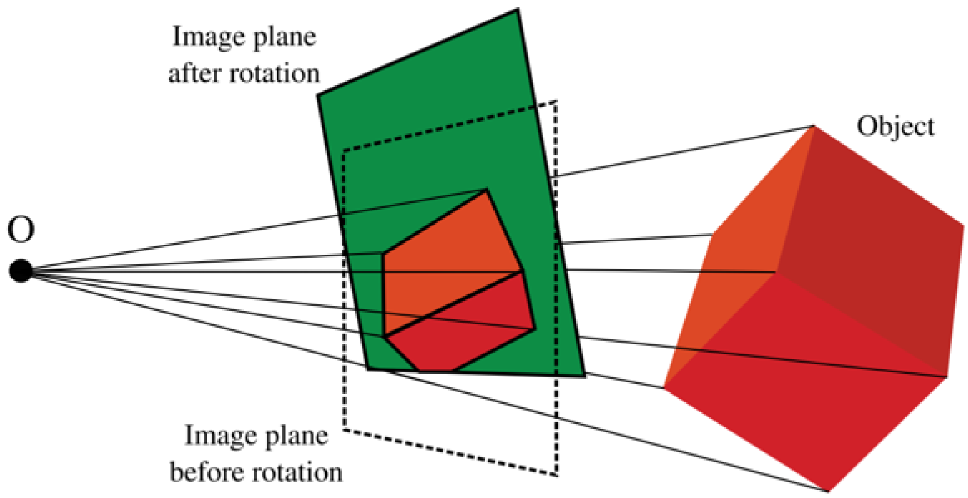


Figure 15.14 Images under pure camera rotation. When the camera rotates but does not translate, the bundle of rays remains the same, but is cut by a different plane. It follows that the two images are related by a homography.

- Image Source - Prince.

Homography Quiz

- **Quiz4**: Suppose that a camera is rotating about its center C, keeping the intrinsic parameters K constant. Let H be the homography that maps the view from one camera orientation to the view at a second orientation. Let θ be the angle of rotation between the two. Show that H^2 is the homography corresponding to a rotation of 2θ.

Homography Quiz

- We have just shown that for such a scenario:
$$H_{2\to 1} = K_1 R_\theta^{-1} K_2^{-1}$$
$$H_{1\to 2} = K_2 R_\theta K_1^{-1}$$
- Applying the constraint $K_1 = K_2 \equiv K$ gets us:
$$H_{1\to 2} = K R_\theta K^{-1}$$
- Applying $H_{1\to 2}$ twice gets us:
$$H_{1\to 2}^2 = K R_\theta K^{-1} K R_\theta K^{-1} = K R_\theta R_\theta K^{-1}$$
- Since $R_\theta R_\theta = R_{2\theta}$, we indeed get:
$$H_{1\to 2}^2 = K R_{2\theta} K^{-1}$$

Which is a homography that corresponds to a rotation of 2θ.

Homography Quiz

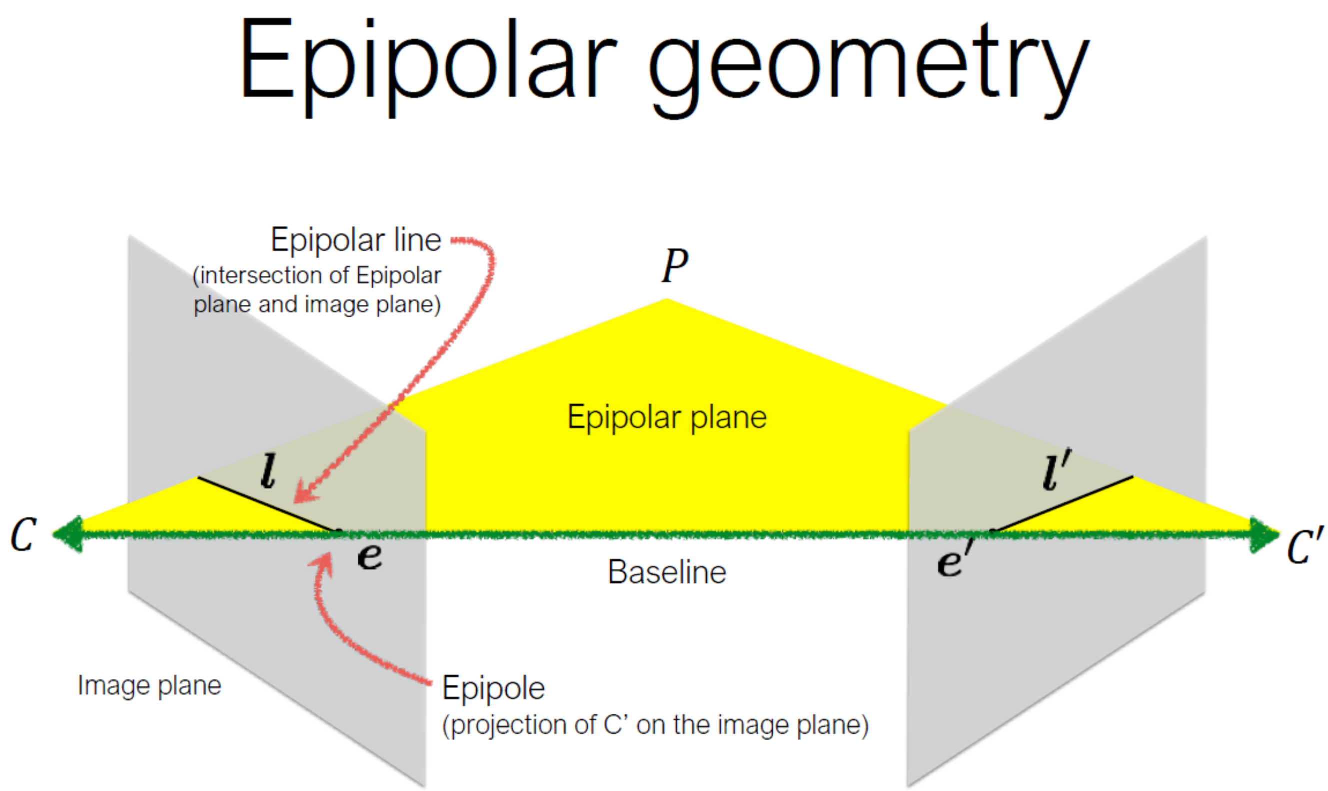
- **Quiz5**: Prove that points on a single line do not uniquely constrain the homography H. In other words, prove that we need points on at least 2 different directions within the plane to estimate a homography reliably. (Quiz 7)

Homography Quiz

- If all points lie on a line there is a 3×1 vector l such the $l^T p = 0$ for all points p.
- Now suppose you found a homography matrix H such that $p'_j = Hp_j$, and yet all your points satisfy $l^T p_j = 0$.
- Then it is easy to see that for every 3×1 vector v the matrix $H' = H + vl^T$ will also satisfy $p'_j = H' p_j$
- This implies that there is no unique H which explains points on the same line.

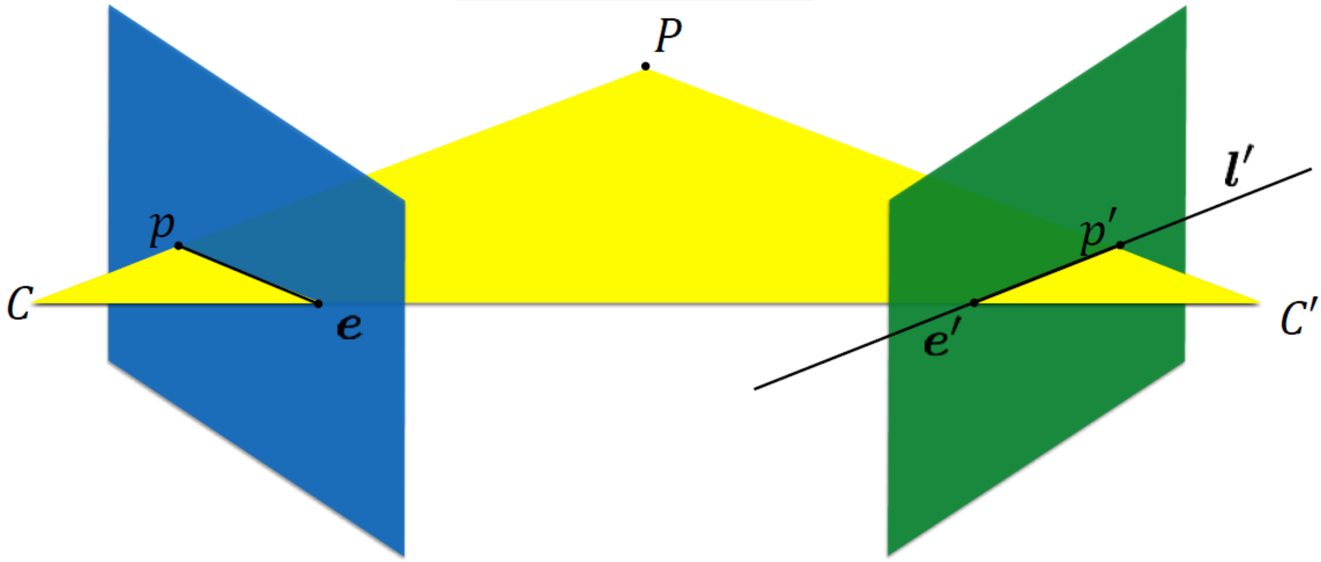
Epipolar Geometry

Epipolar Lingo



Given a point in one image,
multiplying by the **essential matrix** will tell us
the **epipolar line** in the second view.

$$\boxed{Ep = l'}$$



Essential Matrix vs Homography

What's the difference between the essential matrix and a homography?

They are both 3 x 3 matrices but …

$$Ep = l'$$

Essential matrix maps a **point** to a **line**

$$Hp = p'$$

Homography maps a **point** to a **point**

When can we use a homography? And when only an essential matrix?

Homography applies only for planer scenes

- Given by the equation $p'^T Ep = 0$
  - and also equals $E = R[dC]_\times, \quad E = [t]_\times R$, where $t = -RdC$
- **Properties:**
  - $p'^T Ep = 0$
  - $p^T l = 0, \quad p'^T l' = 0$
  - $l' = Ep, \quad l = E^T p'$
  - $e'^T E = 0, \quad Ee = 0$

Fundamental Matrix

$$\widehat{p'}^T E \hat{p} = 0$$

The Essential matrix operates on image points expressed in
normalized coordinates
(points have been aligned (normalized) to camera coordinates)

$$\widehat{p'} = K'^{-1} p' \qquad \hat{p} = K^{-1} p$$

camera point   image point

Writing out the epipolar constraint in terms of image coordinates

$$p'^T K'^{-T} E K^{-1} p = 0$$

$$p'^T (K'^{-T} E K^{-1}) p = 0$$

$$p'^T F p = 0$$

The Fundamental matrix (From Spring 2019 Q4)

1. Explain how you can calculate F from the known K, R, and t of the camera.

Solution

1.
$$F \triangleq K^{-T} E K'^{-1}$$
$$E = [t]_\times R$$
$$F = K^{-T} [t]_\times R K'^{-1}$$

where, $[t]_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$

Fundamental Matrix Properties

- **Properties:**
  - $p'^T F p = 0$
  - $p^T l = 0, p'^T l' = 0$
  - $l' = Fp, l = F^T p'$
  - $e'^T F = 0, Fe = 0$

The Fundamental matrix (From Spring 2019 Q4)

1. Explain shortly the steps of the algorithm for estimating F (when K, R, t are unknown). How many DOF does F have? How many sets of points are needed to find it?

Fundamental Matrix Estimation

Solution

1. Assume we are given 2D to 2D M matched image points:
$$\{p_i, p_i'\}_{i=1}^{M}$$

- Each cosspondence should satisfy:
$$p_i^T F p_i' = 0 \leftrightarrow \begin{bmatrix} x_i & y_i & 1 \end{bmatrix}^T \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = 0$$
- How to solve?
  - The 8-point algorithm $\leftrightarrow$ arrange into homogeneous linear equations and SVD..

Fundamental Matrix Estimation

$$\begin{bmatrix} x_1x'_1 & x_1y'_1 & x_1 & y_1x'_1 & y_1y'_1 & y_1 & x'_1 & y'_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_Mx'_M & x_My'_M & x_M & y_Mx'_M & y_My'_M & y_M & x'_M & y'_M & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix} = \mathbf{0}$$

- 8 DOF, solve **SVD** under the constraint $||f|| = 1$

The Fundamental matrix (From Spring 2019 Q4)

1. Under what conditions does $F = E$?

Solution

1. Since $F \triangleq K'^{-T}EK^{-1}$, when $K' = K = I$ we'll get $F = I^{-1}EI^{-1} = E$

note that $F = K'EK$, that's a hint that we want to constraint K and K'

The Fundamental matrix (From Spring 2019 Q4)

1. Suppose we know that the camera planes are parallel (after rectification - i.e. they are only translated on the x axis). Does that information allow us to use less sets of matching points to find F?

Solution

1. The camera matrices are given by $M = K[I|0]$ and $M' = K'[R|t]$, where

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}, K' = \begin{bmatrix} f' & 0 & 0 \\ 0 & f' & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = I, t = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix}$$

We know that $F = K'^{-T}[t]_\times RK^{-1}$, so by plugging in the relevant parameters:

$$F = \begin{bmatrix} 1/f' & 0 & 0 \\ 0 & 1/f' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x/f' \\ 0 & -t_x/f & 0 \end{bmatrix}$$

F now has only 2 DoFs, so we only need 2 points.

- Note that Even if we look at more complex intrinsic matrices, and 3 degrees of translation, we can still spare 3 DoFs (and 3 points) by not having a rotation matrix!

Fundamental Matrix Demo

In [12]:

```
# start by detecting features and matching them with SIFT
img1 = cv2.imread('./assets/left.jpg',0)  #queryimage # left image
img2 = cv2.imread('./assets/right.jpg',0) #trainimage # right image
sift = cv2.SIFT_create()
# find the keypoints and descriptors with SIFT
kp1, des1 = sift.detectAndCompute(img1,None)
kp2, des2 = sift.detectAndCompute(img2,None)
# FLANN parameters
FLANN_INDEX_KDTREE = 1
index_params = dict(algorithm = FLANN_INDEX_KDTREE, trees = 5)
search_params = dict(checks=50)
flann = cv2.FlannBasedMatcher(index_params,search_params)
matches = flann.knnMatch(des1,des2,k=2)
```

In [13]:

```
# ratio test as per Lowe's paper
pts1 = []
pts2 = []
for i,(m,n) in enumerate(matches):
    if m.distance < 0.8*n.distance:
        pts2.append(kp2[m.trainIdx].pt)
        pts1.append(kp1[m.queryIdx].pt)
```

In [14]:

```
# estimating the fundamental matrix
pts1 = np.int32(pts1)
pts2 = np.int32(pts2)
F, mask = cv2.findFundamentalMat(pts1,pts2,cv2.FM_LMEDS)
# We select only inlier points
pts1 = pts1[mask.ravel()==1]
pts2 = pts2[mask.ravel()==1]
```

In [15]:

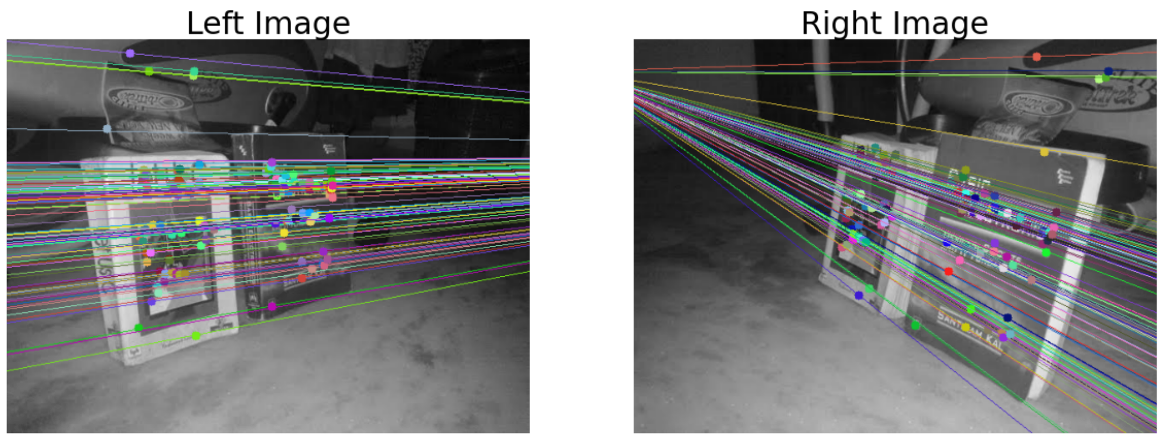
```
# drawing epilines
def drawlines(img1,img2,lines,pts1,pts2):
    ''' img1 - image on which we draw the epilines for the points in img2
        lines - corresponding epilines '''
    r,c = img1.shape
    img1 = cv2.cvtColor(img1,cv2.COLOR_GRAY2BGR)
    img2 = cv2.cvtColor(img2,cv2.COLOR_GRAY2BGR)
    for r,pt1,pt2 in zip(lines,pts1,pts2):
        color = tuple(np.random.randint(0,255,3).tolist())
        x0,y0 = map(int, [0, -r[2]/r[1] ])
        x1,y1 = map(int, [c, -(r[2]+r[0]*c)/r[1] ])
        img1 = cv2.line(img1, (x0,y0), (x1,y1), color,1)
        img1 = cv2.circle(img1,tuple(pt1),5,color,-1)
        img2 = cv2.circle(img2,tuple(pt2),5,color,-1)
    return img1,img2
```

In [16]:

```
# Find epilines corresponding to points in right image (second image) and
# drawing its lines on left image
lines1 = cv2.computeCorrespondEpilines(pts2.reshape(-1,1,2), 2,F)
lines1 = lines1.reshape(-1,3)
img5,img6 = drawlines(img1,img2,lines1,pts1,pts2)
# Find epilines corresponding to points in left image (first image) and
# drawing its lines on right image
lines2 = cv2.computeCorrespondEpilines(pts1.reshape(-1,1,2), 1,F)
lines2 = lines2.reshape(-1,3)
img3,img4 = drawlines(img2,img1,lines2,pts2,pts1)
```

In [17]:

```
plt.figure(figsize=(20,10))
plt.subplot(121)
plt.imshow(img5)
plt.axis('off')
plt.title('Left Image', fontsize=30)
plt.subplot(122)
plt.imshow(img3)
plt.axis('off')
plt.title('Right Image', fontsize=30)
plt.show()
```



Recommended Videos

Warning!

- These videos do not replace the lectures and tutorials.
- Please use these to get a better understanding of the material, and not as an alternative to the written material.

Video By Subject

- QR decomposition for square matrices - The Bright Side of Mathematics (https://www.youtube.com/watch?v=FAnNBw7d0vg&ab_channel=TheBrightSideofMathematics)
- Epipolar and Essential matrix - William Hoff (https://www.youtube.com/watch?v=Opy8xMGCDrE)
- Fundamental Matrix - William Hoff (https://www.youtube.com/watch?v=wb9245ZAoaE)
- The Fundamental Matrix Song - Daniel Wedge (https://www.youtube.com/watch?v=DgGV3l82NTk)

Credits

- EE 046746 - Moshe Kimhi (https://www.linkedin.com/in/moshekimhi/),Elias Nehme (https://eliasnehme.github.io/), Dahlia Urbach (https://il.linkedin.com/in/dahlia-urbach-97a816123)
- Simon Lucey (CMU)
- Multiple View Geometry in Computer Vision - Hartley and Zisserman - Chapter 6
- Least–squares Solution of Homogeneous Equations (cmp.felk.cvut.cz/cmp/courses/XE33PVR/WS20072008/Lectures/Supporting/constrained_lsq.pdf) - Center for Machine Perception - Tomas Svoboda
- Computer vision: models, learning and inference (http://www.computervisionmodels.com/) , Simon J.D. Prince - Chapter 15
- Computer Vision: Algorithms and Applications (https://www.springer.com/gp/book/9781848829343) - Richard Szeliski - Sections 2.1.5, 6.2. , 7.1, 7.2, 11.1
- Icons from Icon8.com (https://icons8.com/) - https://icons8.com (https://icons8.com)