



# EE 046746 - Technion - Computer Vision

## Tutorial 07 - Homography, Alignment & Panoramas

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### Agenda

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- Matching Local Features
- Parametric Transformations
- Computing Parametric Transformations
  - Affine
  - Projective
- RANSAC
- Panorama
  - Warping
  - Image Blending (Feathering)
- Kornia & Transformations in Deep Learning
- Recommended Videos
- Credits

The largest panorama in the world (2014): Mont Blanc

[In2WHITE Video](#)

[In2WHITE Full Image](#)

Homographic usage examples:

<http://blog.flickr.net/en/2010/01/27/a-look-into-the-past/>

<https://www.instagram.com/albumplusart/>

In [1]:

```
%%html
<iframe src="http://www.in2white.com/" width="700" height="600"></iframe>
```



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In [2]:

```
# imports for the tutorial
import numpy as np,sys
import matplotlib.pyplot as plt
from PIL import Image
import cv2
from scipy import signal
%matplotlib inline
```

In [3]:

```
# plot images function
def plot_images(image_list, title_list, subplot_shape=(1,1), axis='off', fontsize=30, figsize=(4,4), cmap=[],
    plt.figure(figsize=figsize)
    for ii, im in enumerate(image_list):
        c_title = title_list[ii]
        if len(cmap) > 1:
            c_cmap = cmap[ii]
        else:
            c_cmap = cmap[0]
        plt.subplot(subplot_shape[0], subplot_shape[1],ii+1)
        plt.imshow(im, cmap=c_cmap)
        plt.title(c_title, fontsize=fontsize)
        plt.axis(axis)
```



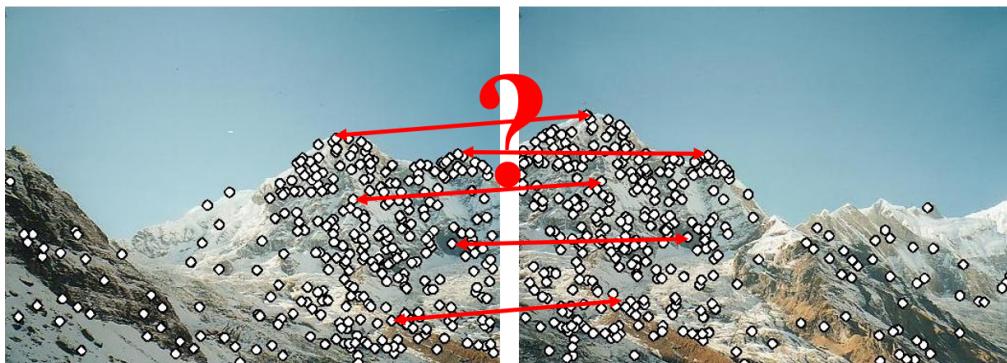
## Matching Local Features

---

Feature matching

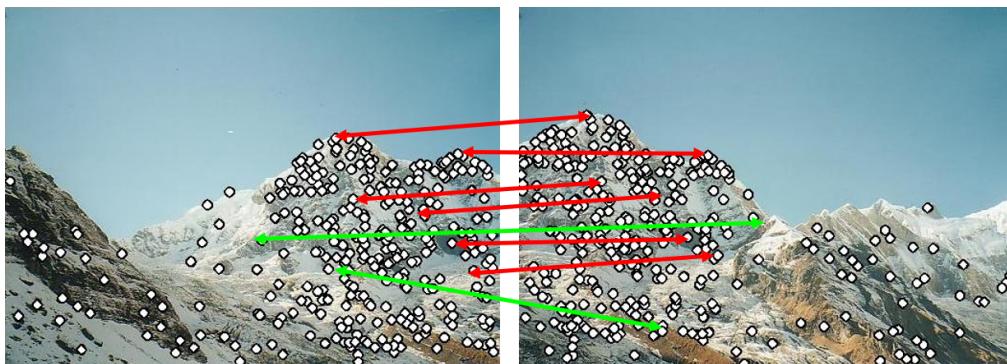
---

- We know how to detect and describe good points
- Next question: How to match them?



Typical feature matching results

- Some matches are correct
- Some matches are incorrect



- Solution: search for a set of geometrically consistent matches



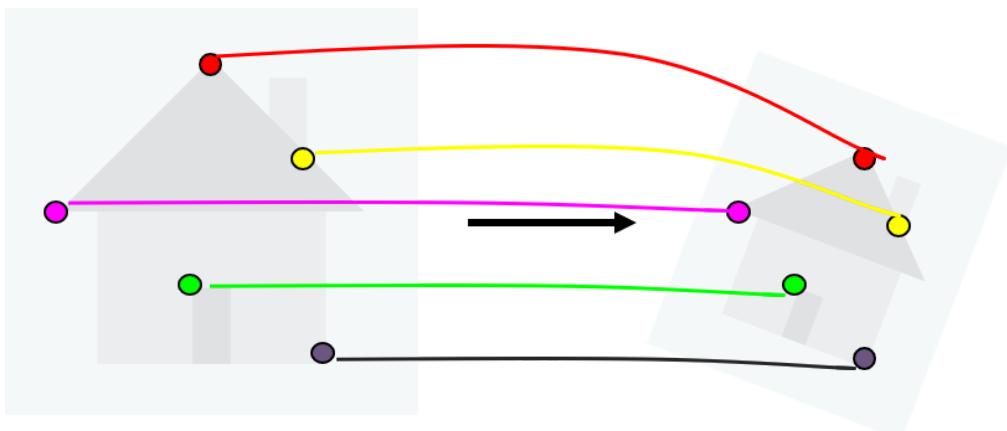
## Parametric Transformations

---

### Image Alignment

---

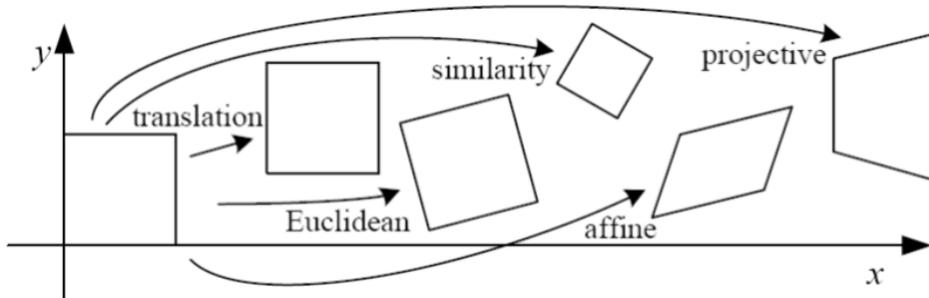
Given a set of matches, what parametric model describes a geometrically consistent transformation?



## Basic 2D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I & t \end{bmatrix}_{2 \times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} R & t \end{bmatrix}_{2 \times 3}$	3	lengths	
similarity	$\begin{bmatrix} sR & t \end{bmatrix}_{2 \times 3}$	4	angles	
affine	$\begin{bmatrix} A \end{bmatrix}_{2 \times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3 \times 3}$	8	straight lines	

( $x$  is in homogeneous coordinates)



## Parametric (Global) Warping

Examples of parametric warps:

```
In [4]: im = cv2.imread('./assets/tut_8_exm.jpg')
im = cv2.cvtColor(im, cv2.COLOR_BGR2RGB)
rows, cols, d = im.shape
print(im.shape)

# Translation:
tx,ty = [10,20]
h_T = np.float32([[1,0,tx],[0,1,ty],[0,0,1]])

# Rotation
theta = np.deg2rad(20)
h_R = np.float32([[np.cos(theta),-np.sin(theta),0],[np.sin(theta),np.cos(theta),0],[0,0,1]])

# Affine
h_AFF = np.array([[ 1.26666667,-0.5,-60],[-0.33333333,1,66.66666667],[0,0,1]])

H = [h_T,h_R,h_AFF]

img = [im]
for h in H:
    img.append(cv2.warpPerspective(im, h,(cols,rows)))
Titles = ['Original','Translation','Rotation','Affine']

# Perspective
pts1 = np.float32([[100,77],[320,105],[100,150],[385,170]])
```

```

pts2 = np.float32([[0,0],[300,0],[0,100],[300,50]])
M = cv2.getPerspectiveTransform(pts1,pts2)
im_perspective = cv2.warpPerspective(im,M,(400,300))

# Cylindrical (non linear)
def cylindricalWarp(img, K):
    """This function returns the cylindrical warp for a given image and intrinsics matrix K"""
    h_,w_ = img.shape[:2]
    # pixel coordinates
    y_i, x_i = np.indices((h_,w_))
    X = np.stack([x_i,y_i,np.ones_like(x_i)],axis=-1).reshape(h_*w_,3) # to homog
    Kinv = np.linalg.inv(K)
    X = Kinv.dot(X.T).T # normalized coords
    # calculate cylindrical coords (sin\theta, h, cos\theta)
    A = np.stack([np.sin(X[:,0]),X[:,1],np.cos(X[:,0])],axis=-1).reshape(w_*h_,3)
    B = K.dot(A.T).T # project back to image-pixels plane
    # back from homog coords
    B = B[:, :-1] / B[:, [-1]]
    # make sure warp coords only within image bounds
    B[(B[:,0] < 0) | (B[:,0] >= w_) | (B[:,1] < 0) | (B[:,1] >= h_)] = -1
    B = B.reshape(h_,w_,-1)

    img_rgba = cv2.cvtColor(img, cv2.COLOR_RGB2RGBA) #BGR2BGRA for transparent borders...
    # warp the image according to cylindrical coords
    return cv2.remap(img_rgba, B[:, :, 0].astype(np.float32), B[:, :, 1].astype(np.float32),
                     cv2.INTER_AREA, borderMode=cv2.BORDER_TRANSPARENT)

# Camera parameters:
K = np.array([[200,0,cols/2],[0,400,rows/2],[0,0,1]]) # mock intrinsics
img_cyl = cylindricalWarp(im, K)

```

(362, 484, 3)

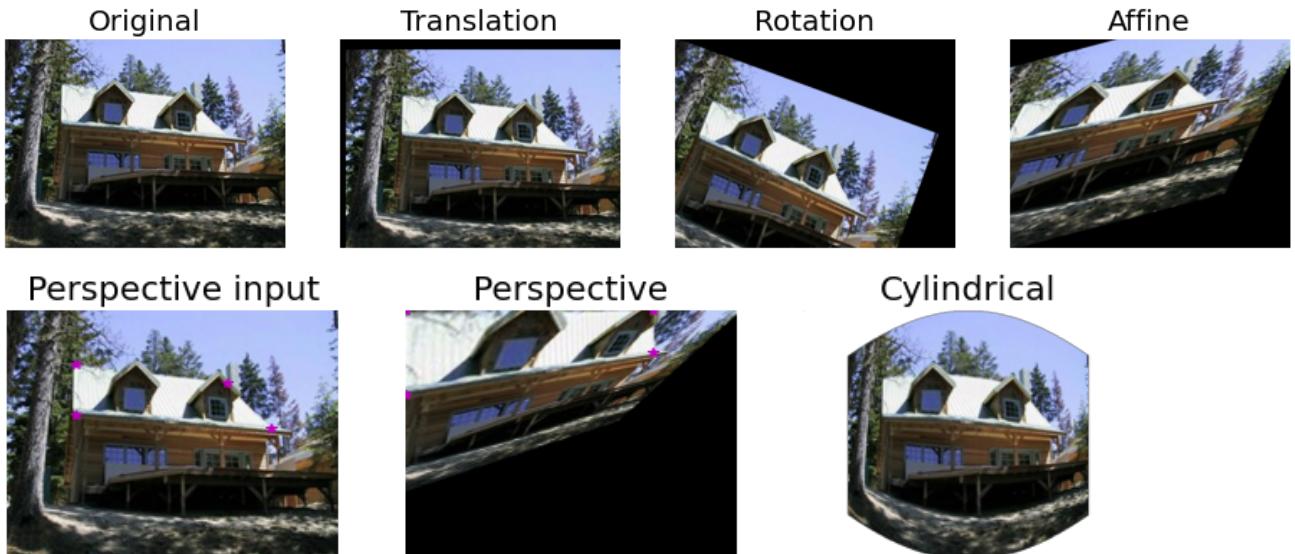
In [5]:

```

plot_images(img,Titles,(1,4),figsize=(16,4),fontsize=20)

plt.figure(figsize=(12,4))
plt.subplot(131)
plt.imshow(im,plt.title('Perspective input',fontsize=20))
plt.plot(pts1[:,0],pts1[:,1],'m*')
plt.axis('off')
plt.subplot(132),plt.imshow(im_perspective,plt.title('Perspective',fontsize=20))
plt.plot(pts2[:,0],pts2[:,1],'m*')
plt.axis('off')
plt.subplot(133),plt.imshow(img_cyl)
plt.title('Cylindrical',fontsize=20)
_ = plt.axis('off')

```



- [Code source](#) - OpenCV

- [Cylinder code source](#) - More Than Technical
- [Cylinder coordinates](#)



## Basic 2D Transformations

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$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

$$\begin{bmatrix} x' \\ y' \\ w \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective  
(Homography)

### Basic 2D Transformations - Translation

---

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

In [6]:

```
# Translation:
tx,ty = [10,20]
h_T = np.float32([[1,0,tx],[0,1,ty],[0,0,1]])
im_T = cv2.warpPerspective(im, h_T,(cols,rows))

plot_images([img[0],img[1]],[Titles[0],Titles[1]],(1,2),figsize=(12,12),fontsize=20)
```

Original



Translation



### Basic 2D Transformation - Rotation

---

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Around which point do we rotate the image?

In [7]:

```
# Rotation:
theta = np.deg2rad(20)
h_R = np.float32([[np.cos(theta), -np.sin(theta), 0], [np.sin(theta), np.cos(theta), 0], [0, 0, 1]])
im_R = cv2.warpPerspective(im, h_R, (cols, rows))

plot_images([img[0], img[2]], [Titles[0], Titles[2]], (1,2), figsize=(12,12), fontsize=20)
```

Original



Rotation



### Basic 2D Transformations – Translation and Rotation (2D rigid body motion)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & t_x \\ \sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Euclidean distances are preserved
- Combination of rotation and translation, which one applied first?

In [8]:

```
H = h_T @ h_R
im_temp = cv2.warpPerspective(im, H, (cols, rows))
plot_images([img[0], im_temp], [Titles[0], 'Rigid'], (1,2), figsize=(12,12), fontsize=20)
```

Original



Rigid



### Basic 2D Transformations – Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

In [9]:

```
# scaling
```

```

s = 1.5
h_s = np.array([[s, 0, 0], [0, s, 0], [0, 0, 1]], np.float32)
im_s = cv2.warpPerspective(im, h_s, (cols, rows))

plot_images([img[0], im_s], [Titles[0], 'Scale'], (1, 2), figsize=(12,12), fontsize=20)

```

Original



Scale



## Basic 2D Transformations – Similarity

Similarity transform (4 DoF) = translation + rotation + scale

```

In [10]: h_sim = h_s @ h_T @ h_R
im_sim = cv2.warpPerspective(im, h_sim, (cols, rows))

plot_images([img[0], im_sim], [Titles[0], 'Similarity'], (1,2), figsize=(12,12), fontsize=20)

```

Original



Similarity



## Basic 2D Transformation - Aspect Ratio

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

```

In [11]: # Aspect Ratio
a = 1 / 2
h_ar = np.array([[a, 0, 0], [0, 1 / a, 0], [0, 0, 1]], np.float32)
im_ar = cv2.warpPerspective(im, h_ar, (cols,rows))

plot_images([img[0], im_ar], [Titles[0], 'Aspect Ratio'], (1,2), figsize=(12,12), fontsize=20)

```

Original



Aspect Ratio



### Basic 2D Transformations – Shear

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

In [12]:

```
# shear
a,b = (0.5, 0.1)
h_sh = np.array([[1, a, 0],[b, 1, 0], [0, 0, 1]], np.float32)
im_sh = cv2.warpPerspective(im, h_sh, (cols, rows))

plot_images([img[0], im_sh], [Titles[0], 'Shear'], (1,2), figsize=(12,12), fontsize=20)
```

Original



Shear



### Basic 2D Transformations – Affine

Affine transform (6 DoF) = translation + rotation + scale + aspect ratio +shear

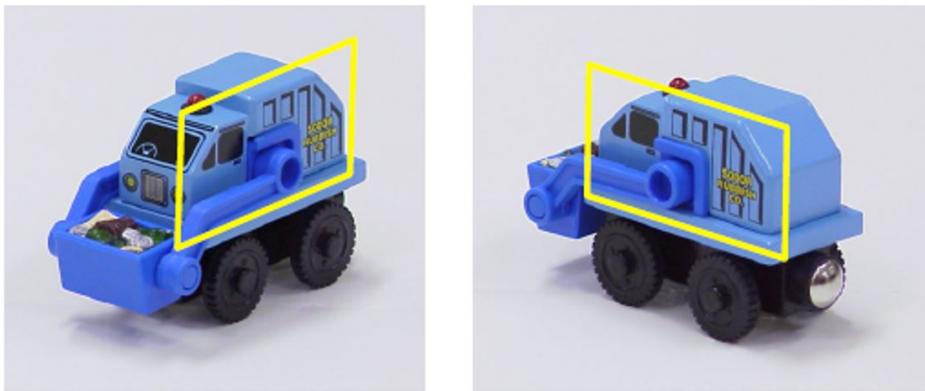
In [13]:

```
# affine transform
h_aff = h_ar @ h_sh @ h_s @ h_T @ h_R
im_aff = cv2.warpPerspective(im, h_aff, (cols * 4, rows * 4))

plot_images([img[0], im_aff], [Titles[0], 'Affine'], (1,2), figsize=(12,12), fontsize=20)
```



- Simple fitting procedure (linear least squares)
- Approximates viewpoint change for roughly planar objects and roughly orthographic camera
- Can be used to initialize fitting for more complex models



### Basic 2D Transformations – Projective - a.k.a - Homographic

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ \textcolor{red}{h_7} & \textcolor{red}{h_8} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = u/w$$

$$y' = v/w$$

**Non-linear!**

```
In [14]: # Perspective
pts1 = np.float32([[100,77],[320,105],[100,150],[385,170]])
pts2 = np.float32([[0,0],[300,0],[0,100],[300,50]])
h_per = cv2.getPerspectiveTransform(pts1,pts2)
im_perspective = cv2.warpPerspective(im,h_per,(400,300))

plot_images([img[0], im_perspective], [Titles[0], 'Projective'], (1, 2), figsize=(12,12), fontsize=20)
```



## When do we get Homography?

Homography maps between:

- points on a plane in the world and their positions in an image
- points in two different images of the **same** plane
- two images of a 3D object where the camera has rotated but not translated

For far away objects:

- works fine for small viewpoint changes



## Computing Parametric Transformations

---

- [Affine](#)
- [Projective](#)

### Computing Affine Transformation

---

- Assuming we know correspondences, how do we get transformation?

$$\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \dots \\ x'_i \\ y'_i \\ \dots \end{bmatrix} = \begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i & y_i & 1 \\ \dots & & & & & \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{bmatrix}$$

$$b = Ah$$

- Solve with Least-squares  $\|Ah - b\|^2$

$$h = (A^T A)^{-1} A^T b$$

In Python:

```
A_inv = pinv(A)
```

```
h = np.linalg.pinv(A)@b
```

- How many matches (correspondence pairs) do we need to solve?
- Once we have solved for the parameters, how do we compute the coordinates of the corresponding point for any pixel  $(x_{new}, y_{new})$ ?

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x'_{new} = Hx_{new}$$

## Computing Projective Transformation

---

- Recall working with homogenous coordinates

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$x'_i = u_i/w_i$$

$$y'_i = v_i/w_i$$

- We get the following non-linear equation:

$$x'_i = \frac{h_1x_i + h_2y_i + h_3}{h_7x_i + h_8y_i + h_9}$$

$$y'_i = \frac{h_4x_i + h_5y_i + h_6}{h_7x_i + h_8y_i + h_9}$$

- We can re-arrange the equation

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & \dots & -x_i x'_i & -y_i x'_i & -x'_i \\ 0 & 0 & 0 & x_i & y_i & 1 & \dots & -x_i y'_i & -y_i y'_i & -y'_i \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} \dots \\ 0 \\ 0 \\ \dots \end{bmatrix}$$

- We want to find a vector  $h$  satisfying

$$Ah = 0$$

where A is full rank. We are obviously not interested in the trivial solution  $h = 0$  hence we add the constraint

$$\|h\| = 1$$

- Thus, we get the homogeneous Least square equation:

$$\arg \min_h \|Ah\|_2^2, s.t. \|h\|_2^2 = 1$$

Compute Projective transformation using SVD:

$$\arg \min_h \|Ah\|_2^2, \text{ s.t. } \|h\|_2^2 = 1$$

- Let decompose  $A$  using SVD:  $A = UDV^T$ , where  $U$  and  $V$  are orthonormal matrix, and  $D$  is a diagonal matrix.
  - Need a reminder on SVD? [Click Here](#)
- From orthonormality of  $U$  and  $V$ :

$$\|UDV^T h\| = \|DV^T h\|$$

$$\|V^T h\| = \|h\|$$

Hence, we get the following minimization problem:

$$\arg \min_h \|DV^T h\| \text{ s.t. } \|h\| = 1$$

- Substitute  $y = V^T h$ :

$$\arg \min_h \|Dy\| \text{ s.t. } \|y\| = 1$$

- $D$  is a diagonal matrix with decreasing values. Then, it is clear that  $y = [0, 0, \dots, 1]^T$ .
- Therefore, choosing  $h$  to be the last column in  $V$  will minimize the equation.

In Python:

```
(U,D,Vh) = np.linalg.svd(A,False)
h = Vh.T[:, -1]
```

**Some more options to find  $h$ :**

- Lagrange multipliers - [Least-squares Solution of Homogeneous Equations](#)
- Using EVD (eigenvalue decomposition) on  $A^T A$ .
- If we know our transformation is nearly Affine we can get an approximate solution using linear least squares



## RANSAC

---

- [Tutorial 2](#)
- [Lecture 6](#)

The RANSAC algorithm is extremely simple, **but it often**

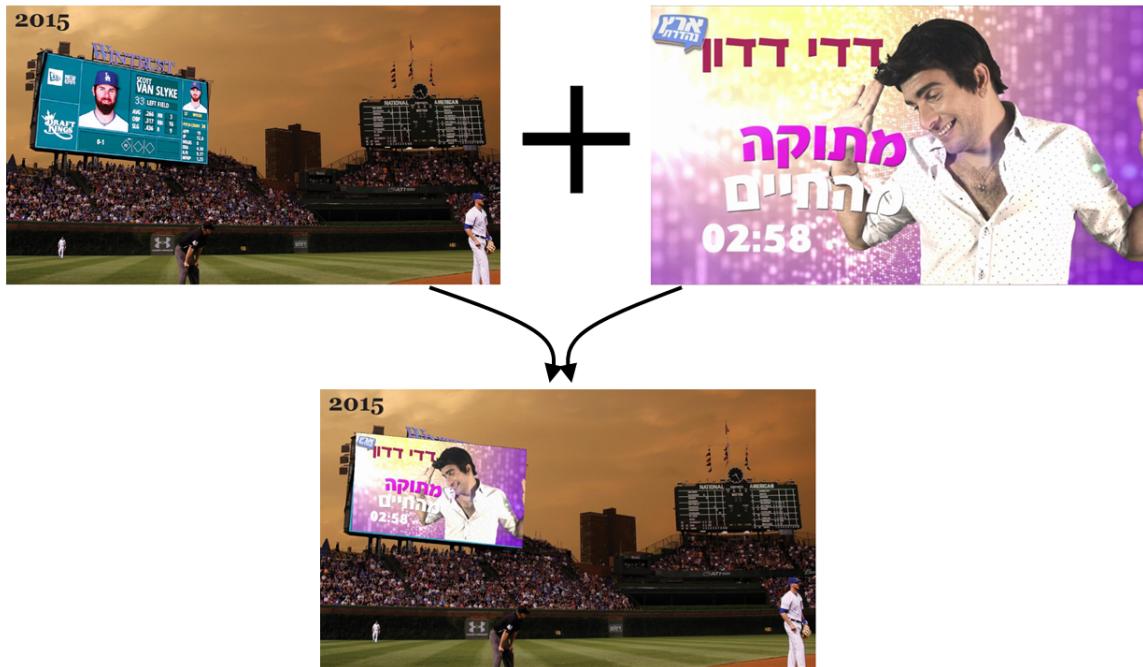
- Does not produce correct model with user-defined probability
- Outputs an inaccurate model
- Does not handle degeneracies
- Can be sped up (by orders of magnitude)
- Does not guarantee minimum running time
- Needs information about scale of the noise
- Does not handle multiple models efficiently

Many improved algorithms:

- [PROSAC](#)
  - Key idea is to assume that the similarity measure predicts correctness of a match

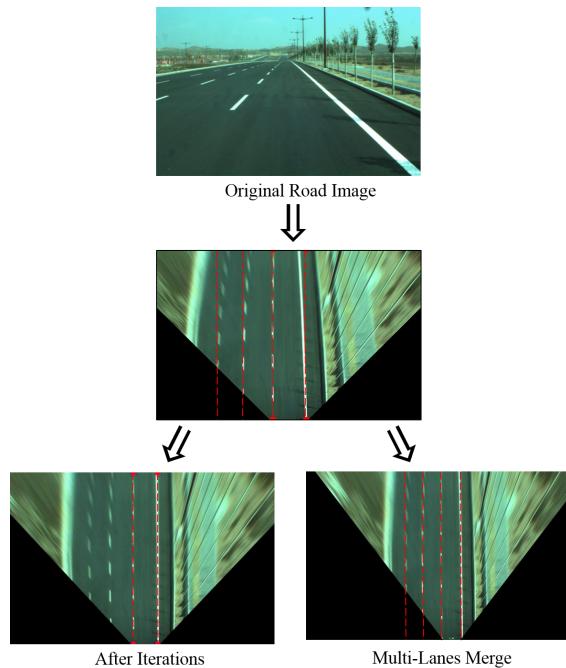
- Randomized RANSAC
  - Each step take a random subset of the query points and perform RANSAC
- KALMANSAC
- and more...
- Estimating homography with RANSAC in OpenCV: `cv2.findHomography(src_pts, dst_pts, cv2.RANSAC)`

### Cool application 1: Planting images into other images



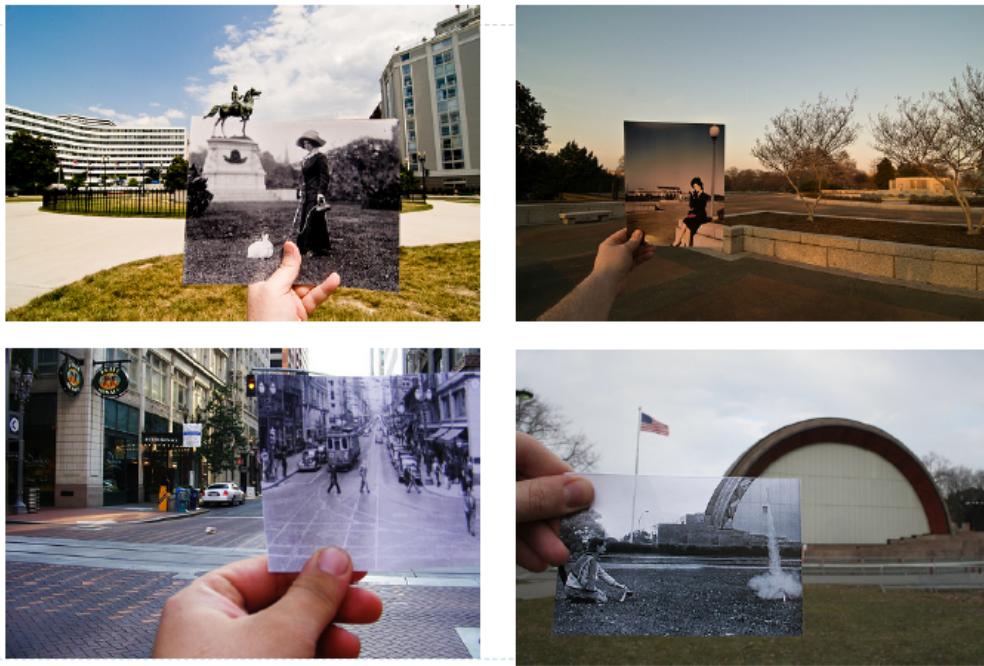
- result achieved with inverse warping and an affine transform

### Cool application 2: BirdEye



- [Image Source](#)

### Cool application 3: Looking into the past



- result achieved with inverse warping and a homography
- [Image Source from Flickr](#)

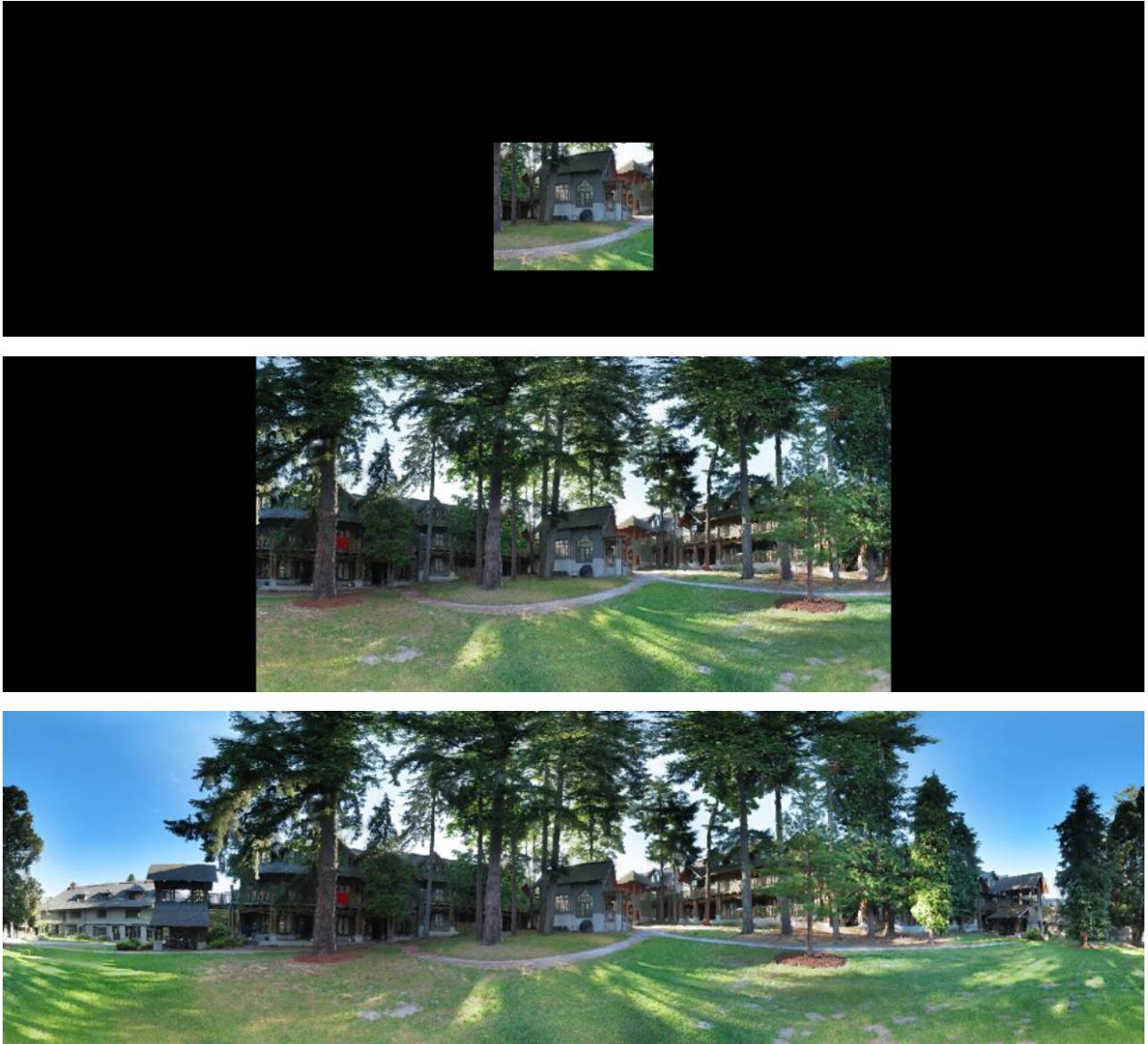


## Panorama

- 
- Warping
  - [Image Blending \(Feathering\)](#)

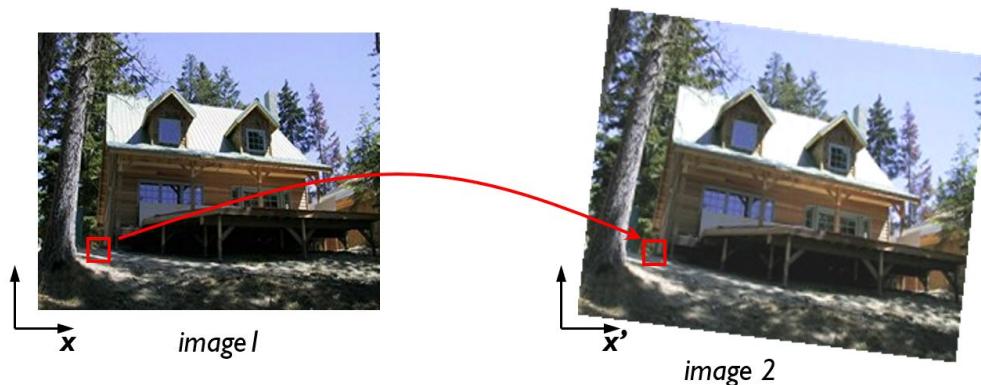


Obtain a wider angle view by combining multiple images



## Warp - What we need to solve?

- Given source and target images, and the transformation between them, how do we align them?
- Send each pixel  $x$  in image1 to its corresponding location  $x'$  in image 2

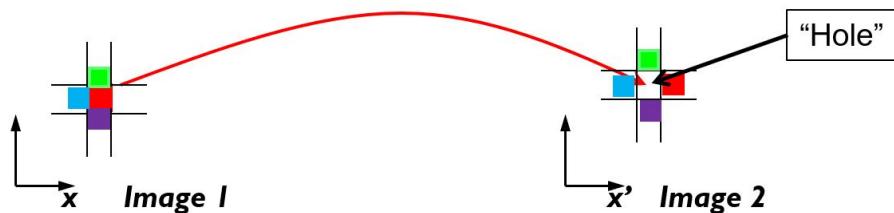


## Forward Warping

- What if pixel lands “between” two pixels?
- Answer: add “contribution” to several pixels and normalize (splatting)

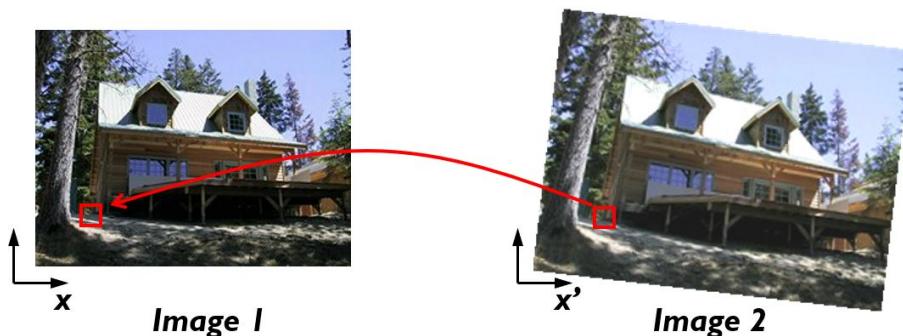


- Limitation: Holes (some pixels are never visited)



## Inverse Warping

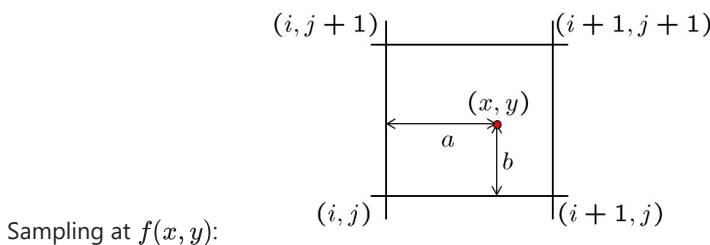
- For each pixel  $x'$  in image 2 find its origin  $x$  in image 1
- Problem: What if pixel comes from “between” two pixels?



- Answer: interpolate color value from neighbors



### Bilinear Interpolation



$$\begin{aligned} f(x, y) = & (1 - a)(1 - b)f[i, j] \\ & + a(1 - b)f[i + 1, j] \\ & + abf[i + 1, j + 1] \\ & + (1 - a)bf[i, j + 1] \end{aligned}$$

Python:

- `interp2d()` - <https://docs.scipy.org/doc/scipy/reference/generated/scipy.interpolate.interp2d.html>
- Inverse warping in OpenCV: `cv2.warpPerspective(im, *, cv2.WARP_INVERSE_MAP)`

In [15]:

```
import numpy as np
from scipy import interpolate
import matplotlib.pyplot as plt

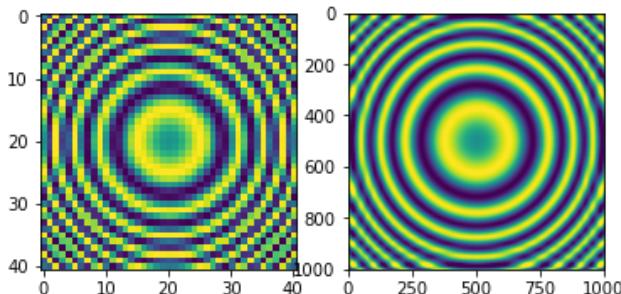
# original samples
x = np.arange(-5.01, 5.01, 0.25)
y = np.arange(-5.01, 5.01, 0.25)
xx, yy = np.meshgrid(x, y)
z = np.sin(xx**2+yy**2)

# interpolated samples
xnew = np.arange(-5.01, 5.01, 1e-2)
ynew = np.arange(-5.01, 5.01, 1e-2)
```

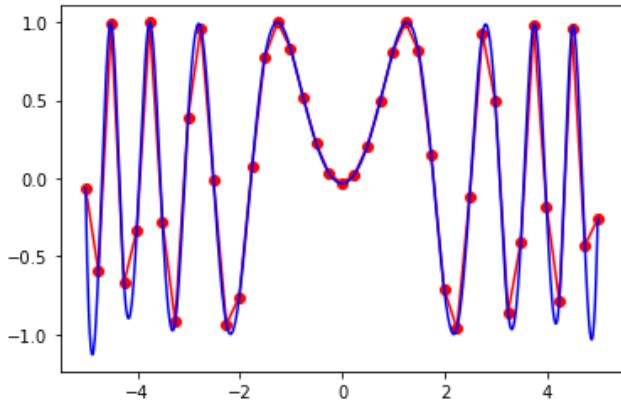
In [16]:

```
## Cubic
f = interpolate.interp2d(x, y, z, kind='cubic')
znew = f(xnew, ynew)

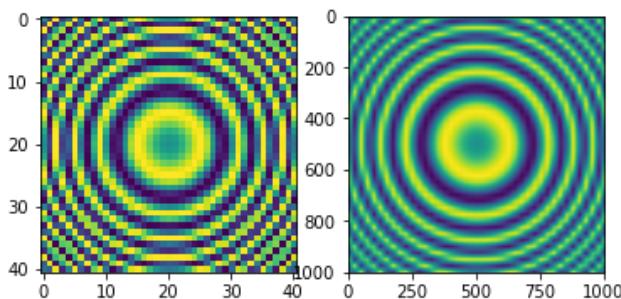
plt.figure()
plt.subplot(121)
plt.imshow(z, vmin=-1, vmax=1)
plt.subplot(122)
plt.imshow(znew, vmin=-1, vmax=1)
plt.show()
```



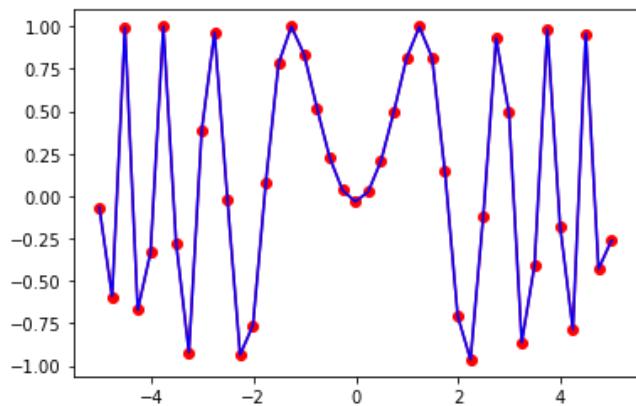
```
In [17]:  
plt.figure()  
plt.plot(x, z[0, :], 'ro-', xnew, znew[0, :], 'b-')  
plt.show()
```



```
In [18]:  
##linear  
f2 = interpolate.interp2d(x, y, z, kind='linear')  
znew2 = f2(xnew, ynew)  
  
plt.figure()  
plt.subplot(121)  
plt.imshow(z, vmin=-1, vmax=1)  
plt.subplot(122)  
plt.imshow(znew2, vmin=-1, vmax=1)  
plt.show()
```

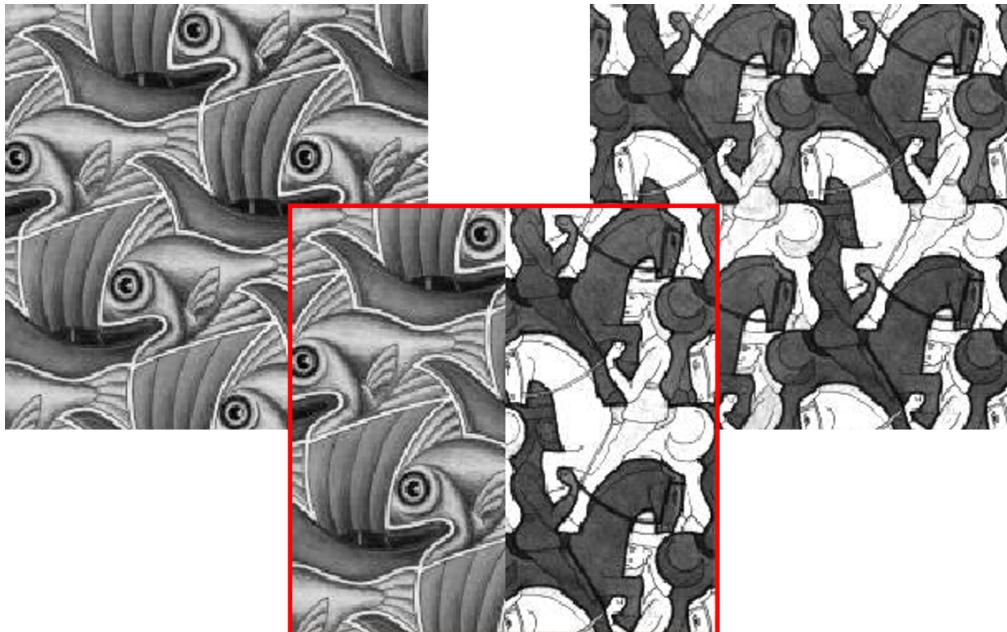


```
In [19]:  
plt.figure()  
plt.plot(x, z[0, :], 'ro-', xnew, znew2[0, :], 'b-')  
plt.show()
```



## Image Blending

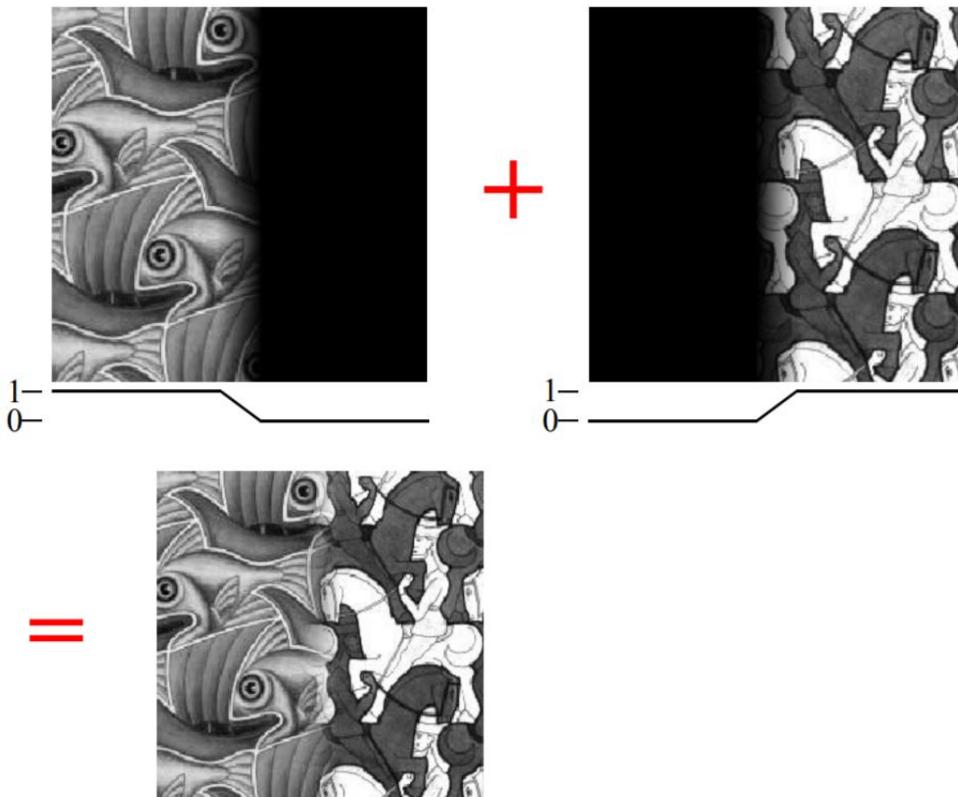
---



- Alpha blending
- Pyramid blending

## Alpha Blending

---



### Pyramid Blending:

1. Build a Gaussian pyramid for each image
2. Build the Laplacian pyramid for each image
3. Decide/find the blending border (in the example: left half belongs to image 1, and right half to image 2 -> the blending border is `cols/2`)
  - Split by index, or
  - Split using a 2 masks (can be weighted masks)
4. Construct a new mixed pyramid - mix each level separately according to (3)
5. Reconstruct a blend image from the mixed pyramid

In [20]:

```
def create_pyrs(A,B):
    # generate Gaussian pyramid for A
    G = A.copy()
    gpA = [G]
    for i in range(6):
        G = cv2.pyrDown(G)
        gpA.append(G)
    # generate Gaussian pyramid for B
    G = B.copy()
    gpB = [G]
    for i in range(6):
        G = cv2.pyrDown(G)
        gpB.append(G)
    # generate Laplacian Pyramid for A
    lpA = [gpA[5]]
    for i in range(5,0,-1):
        GE = cv2.pyrUp(gpA[i])
        L = cv2.subtract(gpA[i-1],GE)
        lpA.append(L)
    # generate Laplacian Pyramid for B
    lpB = [gpB[5]]
    for i in range(5,0,-1):
        GE = cv2.pyrUp(gpB[i])
        L = cv2.subtract(gpB[i-1],GE)
```

```
        lpB.append(L)
    return lpA,lpB
```

```
In [21]: def blend_images(A,B):
    lpA,lpB = create_pyrs(A,B)
    # Now add left and right halves of images in each Level
    LS = []
    for la,lb in zip(lpA,lpB):
        rows,cols,dpt = la.shape
        ls = np.hstack((la[:,0:int(cols/2)], lb[:,int(cols/2):])) #mixing can also be done with a mask
        LS.append(ls)
    # now reconstruct
    ls_ = LS[0]
    for i in range(1,6):
        ls_ = cv2.pyrUp(ls_)
        ls_ = cv2.add(ls_, LS[i])
    # image with direct connecting each half
    real = np.hstack((A[:,0:int(cols/2)],B[:,int(cols/2):]))
    return real, ls_
```

```
In [22]: def switch_texture(A,B):
    lpA,lpB = create_pyrs(A,B)
    # Now add left and right halves of images in each Level
    LS = []
    # for la,lb in zip(lpA,lpB):
    #     rows,cols,dpt = la.shape
    #     ls = np.hstack((la[:,0:int(cols/2)], lb[:,int(cols/2):])) #mixing can also be done with a mask
    #     LS.append(ls)
    # now reconstruct
    ls_ = lpA[0]
    for i in range(1,6):
        ls_ = cv2.pyrUp(ls_)
        ls_ = cv2.add(ls_, lpB[i])
    # image with direct connecting each half
    real = np.hstack((A[:,0:int(cols/2)],B[:,int(cols/2):]))
    return real, ls_
```

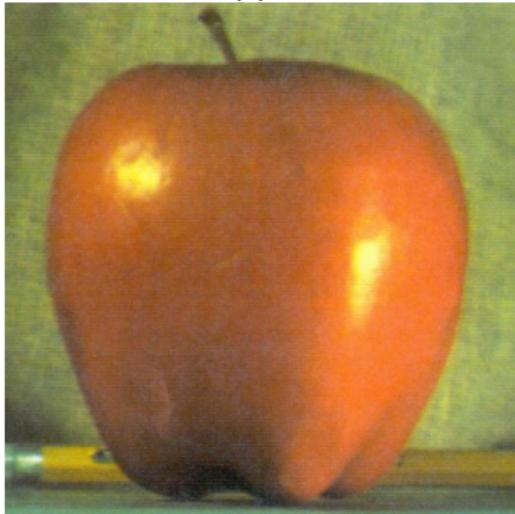
```
In [23]: def alpha_blend(A,B,MASK):
    A = np.float32(A)
    B = np.float32(B)
    return np.uint8(A*MASK), np.uint8(A*MASK+B*(1-MASK))
```

```
In [24]: A = cv2.imread('../assets/apple.jpg')
B = cv2.imread('../assets/orange.jpg')
real,ls_ = blend_images(A,B)

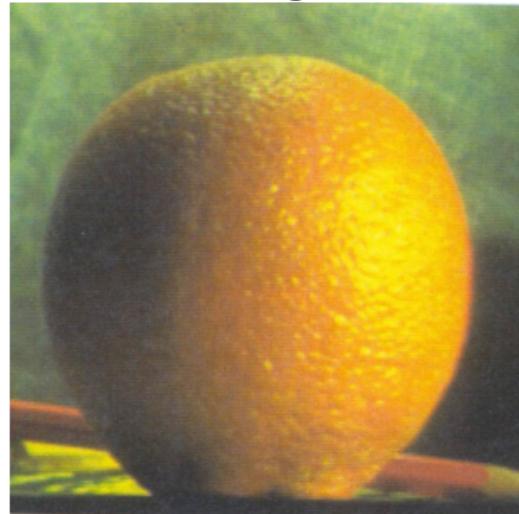
A_ = cv2.cvtColor(A,cv2.COLOR_BGR2RGB)
B_ = cv2.cvtColor(B,cv2.COLOR_BGR2RGB)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

```
In [25]: plot_images([A_,B_],[ 'Apple' , 'Orange' ],(1,2),figsize=(12,12),fontsize=20)
```

Apple

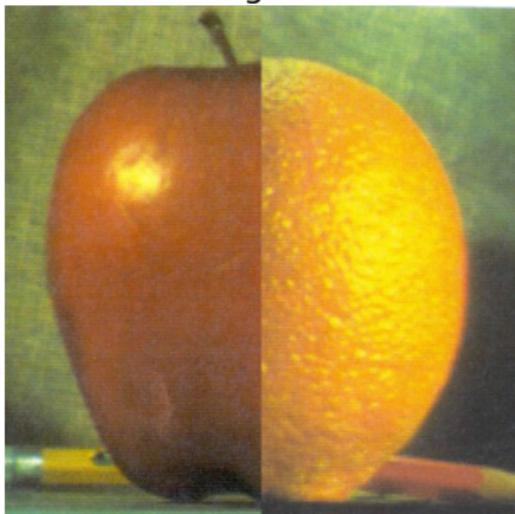


Orange



```
In [26]: plot_images([real,ls_],['Original','Pyramid Blend'],(1,2),figsize=(12,12),fontsize=20)
```

Original



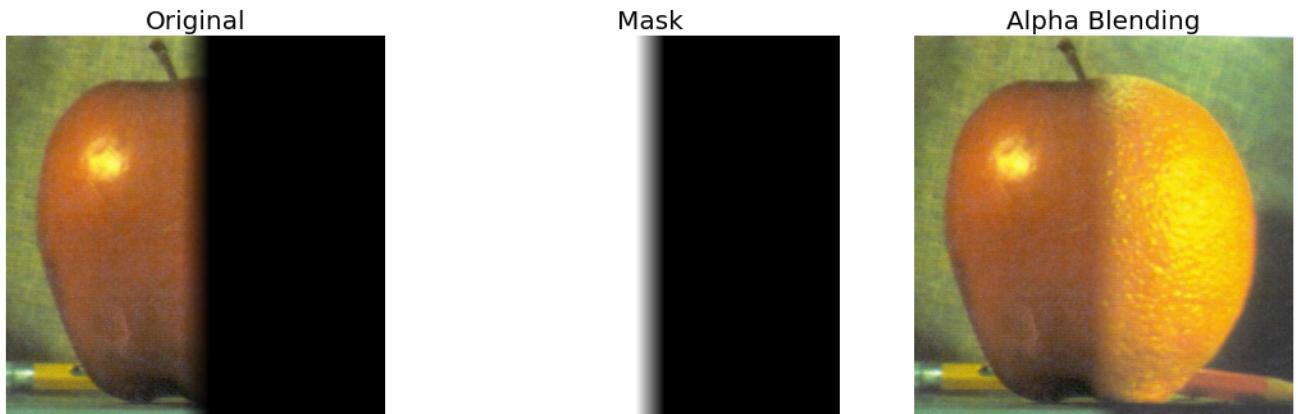
Pyramid Blend



- Alpha blending example:

```
In [27]: MASK = np.ones_like(A,np.float32)
rows,cols,dpt = MASK.shape
w=20
v_dec = np.linspace(1,0,2*w)
MASK[:,int(cols/2):]=0
MASK[:,(int(cols/2)-w):(int(cols/2)+w)]=np.tile(np.reshape(v_dec,[1,-1,1]),[rows,1,3])
real,ls_ = alpha_blend(A,B,MASK)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

```
In [28]: plot_images([real,MASK,ls_],['Original','Mask','Alpha Blending'],(1,3),figsize=(18,12),fontsize=20)
```



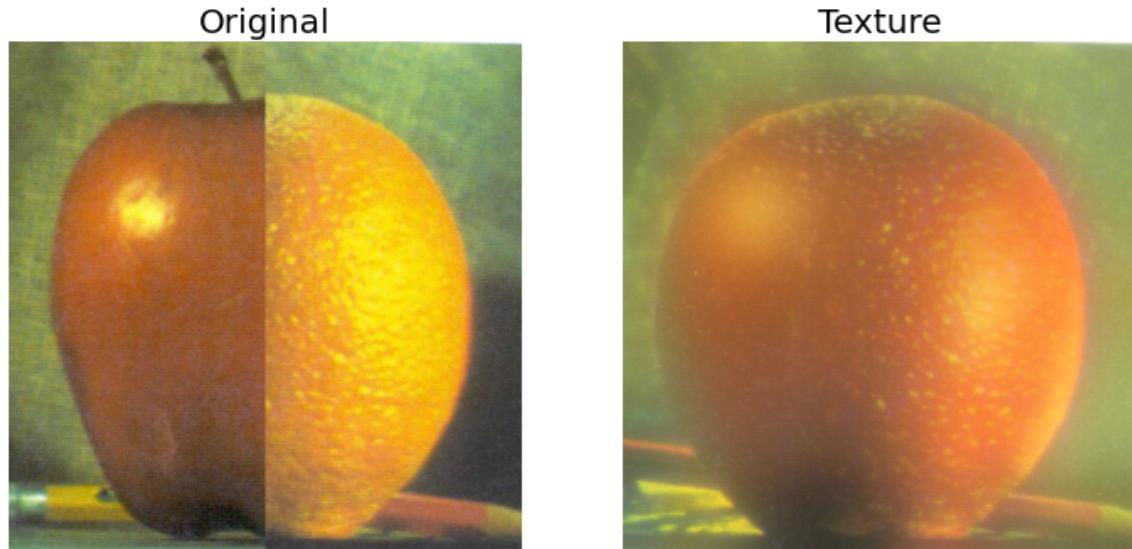
- Stylize image using pyramids:

In [29]:

```
real,ls_ = switch_texture(A,B)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

In [30]:

```
plot_images([real,ls_],[ 'Original','Texture'],(1,2),figsize=(12,12),fontsize=20)
```



### Blending Improvements

- Many algorithms have different variations of combining alpha and pyramid blending (different masks for different frequencies)
- Find the boundaries using **segmentation**



### Panorama - Summary

- Detect features
- Compute transformations between pairs of frames
- Can Refine transformations using RANSAC
- Warp all images onto a single coordinate system
- Find mixing borders (e.g. using segmentation)

- Blend



## Transformations in Deep Learning

- Can we incorporate transformations in the pipeline of a deep learning algorithm?
  - Moreover, can we accelerate these transformations by performing them on a GPU?
- YES!

### Kornia - Computer Vision Library for PyTorch



- Kornia is a differentiable computer vision library for PyTorch
  - That means you can have gradients for the transformations!
- Inspired by OpenCV, this library is composed by a subset of packages containing operators that can be inserted within neural networks to train models to perform image transformations, epipolar geometry, depth estimation, and low-level image processing such as filtering and edge detection that operate directly on tensors.
- Check out `kornia.geometry` - <https://kornia.readthedocs.io/en/latest/geometry.html>
- Warp image using perspective transform



PyTorch

```
import torch
import kornia

frame: torch.Tensor = load_video_frame(...)

out: torch.Tensor = (
    kornia.rgb_to_grayscale(frame)
)
```



### Recommended Videos



#### Warning!

- These videos do not replace the lectures and tutorials.
- Please use these to get a better understanding of the material, and not as an alternative to the written material.

### Video By Subject

- Homography
  - Image geometry and planar homography - [ENB339 lecture 9: Image geometry and planar homography](#)

- Homography - [Homography in computer vision explained](#)
- Transformations - [Lect. 5\(1\) - Linear and affine transformations](#)
- Matching Local Features
  - SIFT - [CSCI 512 - Lecture 12-1 SIFT](#)
- RANSAC (see [Tutorial 2](#))



## Credits

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- EE 046746 Spring 2020 - [Dahlia Urbach](#)
- Slides - Elad Osherov (Technion), Simon Lucey (CMU)
- Multiple View Geometry in Computer Vision - Hartley and Zisserman - Section 2
- [Least-squares Solution of Homogeneous Equations](#) - Center for Machine Perception - Tomas Svoboda
- [Computer vision: models, learning and inference](#), Simon J.D. Prince - Section 15.1
- [Computer Vision: Algorithms and Applications](#) - Richard Szeliski - Sections 2,4,6, 9 ([Free for Technion students via remote library](#))
- Icons from [Icon8.com](#) - <https://icon8.com>