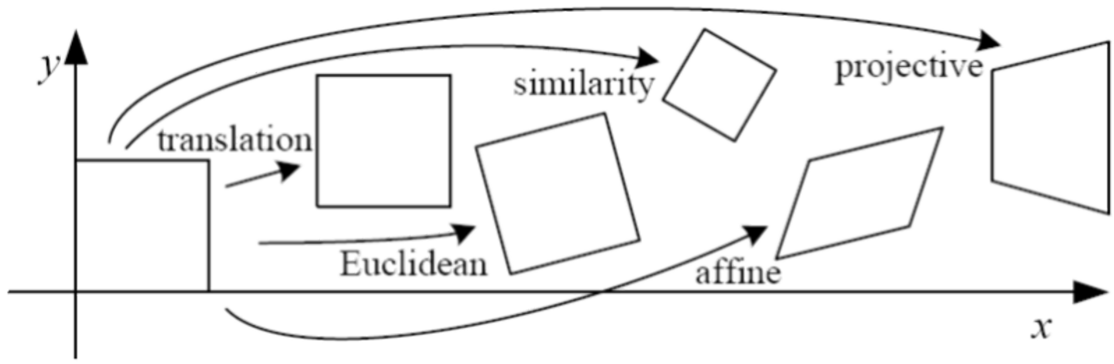
EE 046746 - Technion - Computer Vision

Tutorial 09 - Homography, Alignment & Panoramas



Agenda

- Matching Local Features
- Parametric Transformations
- Computing Parametric Transformations
  - Affine
  - Projective
  - Excercise 1 (Spring 2020 Q1A))
- RANSAC
  - Excercise 2 (Spring 2020 Q1D) - Ransac---Ransac)
- Panorama
  - Warping
    - Excercise 3 (Spring 2020 Q1B))
  - Image Blending (Feathering)
- Kornia & Transformations in Deep Learning
- Recommended Videos
- Credits

The largest panorama in the world (2014): Mont Blanc

In2WHITE Video (https://youtu.be/Dwyx0h9h8zg)

In2WHITE Full Image (http://www.in2white.com/)

Homographic usage examples:

[http://blog.flickr.net/en/2010/01/27/a-look-into-the-past/ (http://blog.flickr.net/en/2010/01/27/a-look-into-the-past/)](http://blog.flickr.net/en/2010/01/27/a-look-into-the-past/)

[https://www.instagram.com/albumplusart/ (https://www.instagram.com/albumplusart/)](https://www.instagram.com/albumplusart/)

In [1]:

```
%%html
<iframe src="http://www.in2white.com/" width="700" height="600"></iframe>
```

Filippo B. (http://www.filippoblengini.com) Sponsored by:

In [2]:

```
# imports for the tutorial
import numpy as np,sys
import matplotlib.pyplot as plt
from PIL import Image
import cv2
from scipy import signal
%matplotlib inline
```

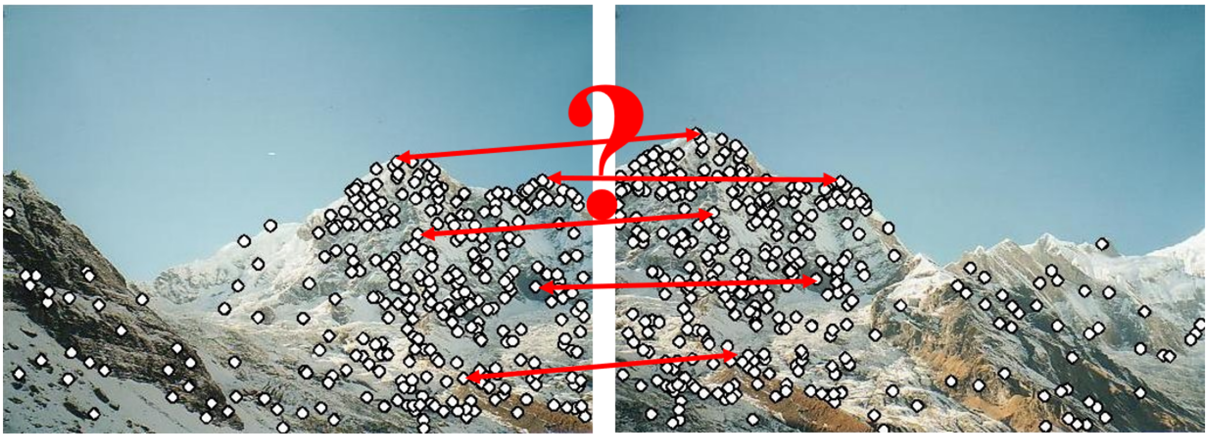
In [3]:

```
# plot images function
def plot_images(image_list, title_list, subplot_shape=(1,1), axis='off', fontsize=30, f
igsize=(4,4), cmap=['gray']):
    plt.figure(figsize=figsize)
    for ii, im in enumerate(image_list):
        c_title = title_list[ii]
        if len(cmap) > 1:
            c_cmap = cmap[ii]
        else:
            c_cmap = cmap[0]
        plt.subplot(subplot_shape[0], subplot_shape[1],ii+1)
        plt.imshow(im, cmap=c_cmap)
        plt.title(c_title, fontsize=fontsize)
        plt.axis(axis)
```

Matching Local Features

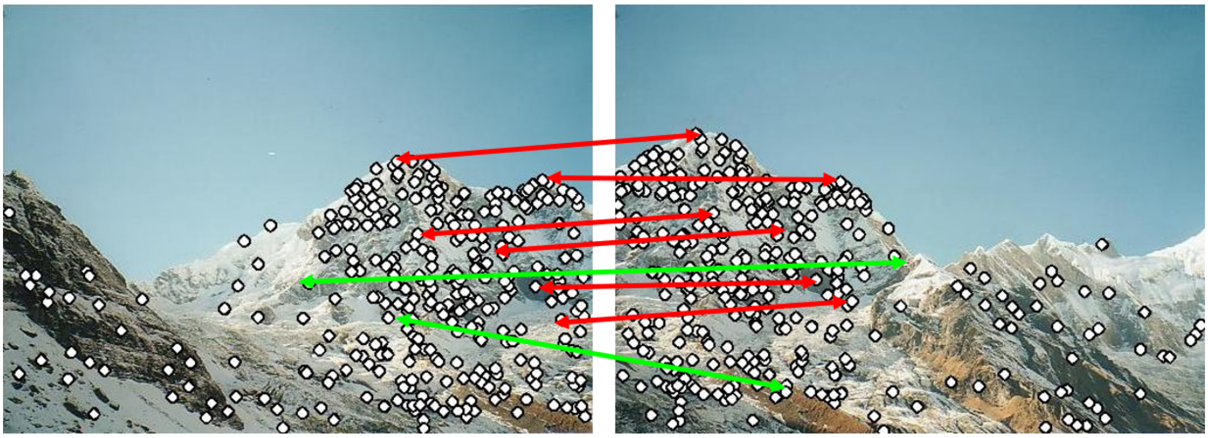
Feature matching

- We know how to detect and describe good points
- Next question: How to match them?



Typical feature matching results

- Some matches are correct
- Some matches are incorrect

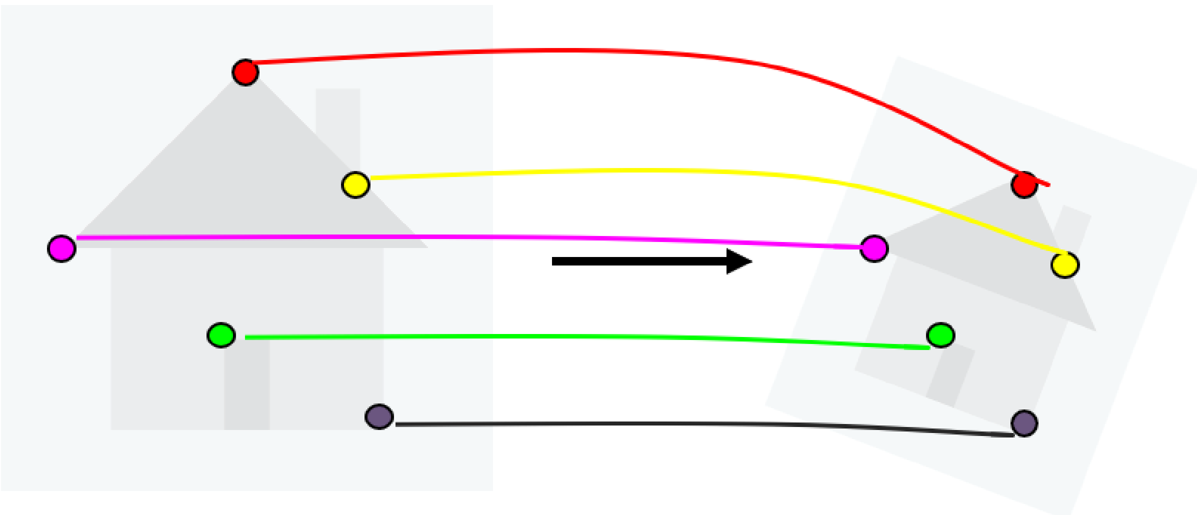


- Solution: search for a set of geometrically consistent matches

Parametric Transformations

Image Alignment

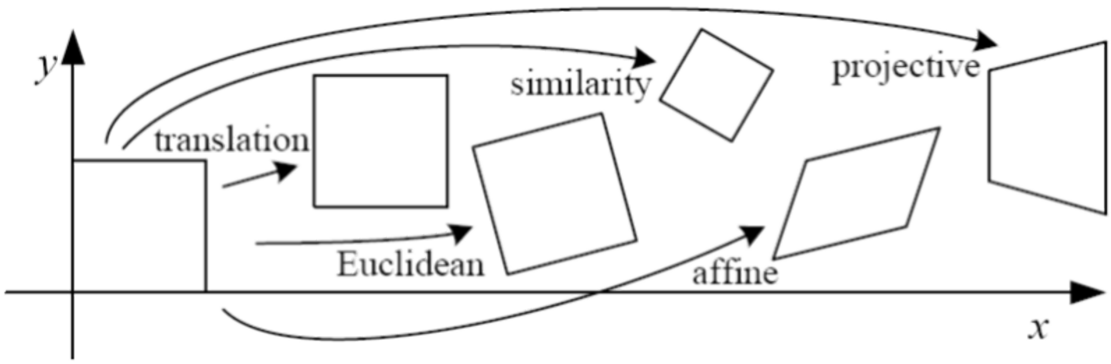
Given a set of matches, what parametric model describes a geometrically consistent transformation?



Basic 2D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} R \mid t \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{2\times 3}$	4	angles	
affine	$\begin{bmatrix} A \end{bmatrix}_{2\times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3\times 3}$	8	straight lines	

(*x* is in *homogeneous* coordinates)



Parametric (Global) Warping

Examples of parametric warps:

In [4]:

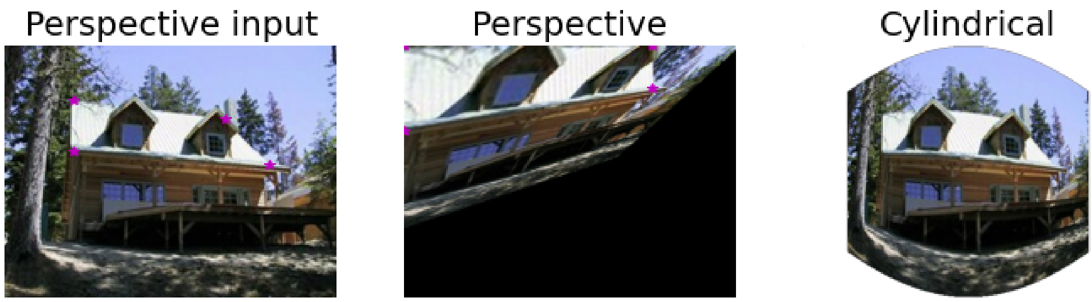
```
im = cv2.imread('./assets/tut_8_exm.jpg')
im = cv2.cvtColor(im,cv2.COLOR_BGR2RGB)
rows,cols,d = im.shape
print(im.shape)

# Translation:
tx,ty = [10,20]
h_T = np.float32([[1,0,tx],[0,1,ty],[0,0,1]])

# Rotation
theta = np.deg2rad(20)
h_R = np.float32([[np.cos(theta),-np.sin(theta),0],[np.sin(theta),np.cos(theta),0],[0,0
,1]])

# Affine
h_AFF = np.array([[  1.26666667,-0.5,-60],[ -0.33333333,1,66.66666667],[0,0,1]])

H = [h_T,h_R,h_AFF]

img = [im]
for h in H:
    img.append(cv2.warpPerspective(im, h,(cols,rows)))
Titles = ['Original','Translation','Rotation','Affine']


# Perspective
pts1 = np.float32([[100,77],[320,105],[100,150],[385,170]])
pts2 = np.float32([[0,0],[300,0],[0,100],[300,50]])
M = cv2.getPerspectiveTransform(pts1,pts2)
im_perspective = cv2.warpPerspective(im,M,(400,300))

# Cylindrical (non linear)
def cylindricalWarp(img, K):
    """This function returns the cylindrical warp for a given image and intrinsics matr
ix K"""
    h_,w_ = img.shape[:2]
    # pixel coordinates
    y_i, x_i = np.indices((h_,w_))
    X = np.stack([x_i,y_i,np.ones_like(x_i)],axis=-1).reshape(h_*w_,3) # to homog
    Kinv = np.linalg.inv(K)
    X = Kinv.dot(X.T).T # normalized coords
    # calculate cylindrical coords (sin\theta, h, cos\theta)
    A = np.stack([np.sin(X[:,0]),X[:,1],np.cos(X[:,0])],axis=-1).reshape(w_*h_,3)
    B = K.dot(A.T).T # project back to image-pixels plane
    # back from homog coords
    B = B[:,:-1] / B[:,[-1]]
    # make sure warp coords only within image bounds
    B[(B[:,0] < 0) | (B[:,0] >= w_) | (B[:,1] < 0) | (B[:,1] >= h_)] = -1
    B = B.reshape(h_,w_,-1)

    img_rgba = cv2.cvtColor(img,cv2.COLOR_RGB2RGBA) #BGR2BGRA for transparent border
s...
    # warp the image according to cylindrical coords
    return cv2.remap(img_rgba, B[:,:,0].astype(np.float32), B[:,:,1].astype(np.float32
),
                      cv2.INTER_AREA, borderMode=cv2.BORDER_TRANSPARENT)
# Camera parameters:
```

```
K = np.array([[200,0,cols/2],[0,400,rows/2],[0,0,1]]) # mock intrinsics
img_cyl = cylindricalWarp(im, K)
```

```
(362, 484, 3)
```

In [5]:

```
plot_images(img,Titles,(1,4),figsize=(16,4),fontsize=20)

plt.figure(figsize=(12,4))
plt.subplot(131)
plt.imshow(im),plt.title('Perspective input',fontsize=20)
plt.plot(pts1[:,0],pts1[:,1],'m*')
plt.axis('off')
plt.subplot(132),plt.imshow(im_perspective),plt.title('Perspective',fontsize=20)
plt.plot(pts2[:,0],pts2[:,1],'m*')
plt.axis('off')
plt.subplot(133),plt.imshow(img_cyl)
plt.title('Cylindrical',fontsize=20)
_ = plt.axis('off')
```



- [Code source (https://opencv-python-tutroals.readthedocs.io/en/latest/py_tutorials/py_imgproc/py_geometric_transformations/py_geometric_tra](https://opencv-python-tutroals.readthedocs.io/en/latest/py_tutorials/py_imgproc/py_geometric_transformations/py_geometric_tra) - OpenCV
- [Cylinder code source (https://www.morethantechnical.com/2018/10/30/cylindrical-image-warping-for-panorama-stitching/)](https://www.morethantechnical.com/2018/10/30/cylindrical-image-warping-for-panorama-stitching/) - More Than Technical
- [Cylinder coordinates (https://en.wikipedia.org/wiki/Cylindrical_coordinate_system)](https://en.wikipedia.org/wiki/Cylindrical_coordinate_system)

Basic 2D Transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective
(Homography)

When can we use a homography matrix to describe the transformation of the scene?

Homography maps between:

- Points that lay on a plane in the real world and their positions in an image.
- Points in two different images that lay on the **same** plane.
- Two images of a 3D object where the camera has been rotated but has not been translated.

Homographies are used to approximate the transformations of far away objects:

- Works fine for small viewpoint changes.

Excercise 1 (Spring 2020 Q1A)

Write down the homography H ($3x3$), that maps from image-1 to image-2 in the following cases:

1. Image-2 is a translated image 1 by 100 pixels to the left.
2. Image-2 is a $3\times$ zoom of image 1, so that the pixel indexed $(0,0)$ stays in its original position.

Answer:

1. Image-2 is a translated image 1 by 100 pixels to the left.

We know that for a translation the homography matrix becomes:

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

The translation of the image is to the left, which means

$$x_2 = x_1 - 100$$

and the position in the y axis stays the same:

$$y_2 = y_1$$

Therefore, the homography that describes the given translation is given by:

$$\begin{bmatrix} 1 & 0 & -100 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sanity check:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -100 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 - 100 \\ y_1 \\ 1 \end{bmatrix}$$

In [6]:

```
im = cv2.imread('./assets/tut_8_exm.jpg')
im = cv2.cvtColor(im,cv2.COLOR_BGR2RGB)
rows,cols,d = im.shape
h_T = np.float32([[1, 0, -100],
                  [0, 1, 0],
                  [0, 0, 1]])
im_T = cv2.warpPerspective(im, h_T,(cols,rows))
Titles = ['Original','Translation']

plot_images([im,im_T],Titles,(1,2),figsize=(16,4),fontsize=20)
```

Original

Translation

1. Image-2 is a $3\times$ zoom of image 1, so that the pixel indexed $(0,0)$ stays in its original position.

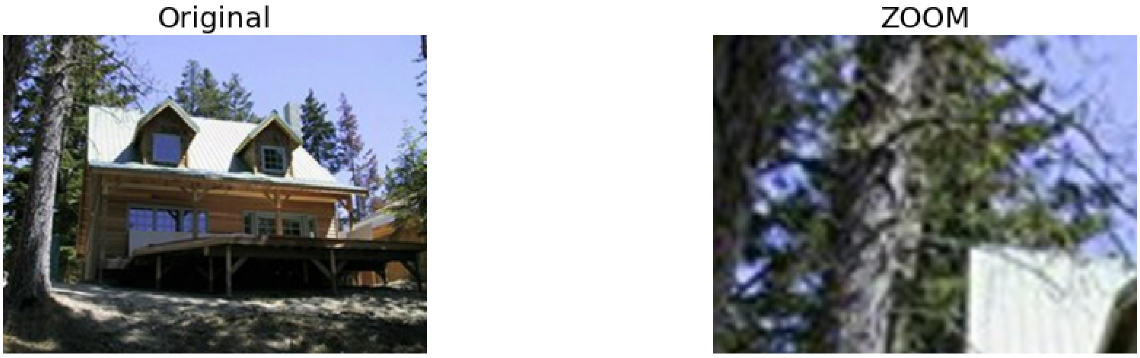
Answer:

The scaled image is different by 1 DOF, since we scale the X-axis and the Y-axis the same amount (same zoom). Going back to the homography matrix, this means only the s parameter changes:

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In [7]:

```
h_S = np.float32([[3,0,0],[0,3,0],[0,0,1]])
im_S = cv2.warpPerspective(im, h_S,(cols,rows))
Titles = ['Original','ZOOM']

plot_images([im,im_S],Titles,(1,2),figsize=(16,4),fontsize=20)
```



Computing Parametric Transformations

- Affine
- Projective

Computing Affine Transformation

- Assuming we know correspondences, how do we get the transformation?

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix} = \begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i & y_i & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{bmatrix}$$

$$b = Ah$$

- Solve with Least-squares $||Ah - b||^2$

$$h = (A^T A)^{-1} A^T b$$

  In Python:

```
A_inv = pinv(A)

h = np.linalg.pinv(A)@b
```

- How many matches (correspondence pairs) do we need to solve?
- Once we have solved for the parameters, how do we compute the coordinates of the corresponding point for any pixel (x_{new}, y_{new})?

$$H = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x'_{new} = H x_{new}$$

Computing Projective Transformation

- Recall working with homogenous coordinates

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$
$$x'_i = u_i / w_i$$
$$y'_i = v_i / w_i$$

- We get the following non-linear equation:

$$x'_i = \frac{h_1 x_i + h_2 y_i + h_3}{h_7 x_i + h_8 y_i + h_9}$$
$$y'_i = \frac{h_4 x_i + h_5 y_i + h_6}{h_7 x_i + h_8 y_i + h_9}$$

- We can re-arrange the equation

$$\begin{bmatrix} & & & & & \dots & & & \\ x_i & y_i & 1 & 0 & 0 & 0 & -x_i x'_i & -y_i x'_i & -x'_i \\ 0 & 0 & 0 & x_i & y_i & 1 & -x_i y'_i & -y_i y'_i & -y'_i \\ & & & & & \dots & & & \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} \dots \\ 0 \\ 0 \\ \dots \end{bmatrix}$$

- We want to find a vector h satisfying

$$Ah = 0$$

where A is full rank. We are obviously not interested in the trivial solution $h = 0$ hence we add the constraint
$$||h|| = 1$$

- Thus, we get the homogeneous Least square equation:

$$arg \min_h ||Ah||_2^2, s.t ||h||_2^2 = 1$$

Compute Projective transformation using SVD:
$$arg \min_h ||Ah||_2^2, s.t ||h||_2^2 = 1$$

- Let decompose A using SVD: $A = UDV^T$, where U and V are orthonomal matrix, and D is a diagonal matrix.
  - Need a reminder on SVD? [Click Here (https://nbviewer.jupyter.org/github/taldatech/cs236756-intro-to-ml/blob/master/cs236756_tutorial_03_linear_algebra.ipynb)](https://nbviewer.jupyter.org/github/taldatech/cs236756-intro-to-ml/blob/master/cs236756_tutorial_03_linear_algebra.ipynb)

- From orthonormality of U and V:

$$||UDV^Th|| = ||DV^Th||$$
$$||V^Th|| = ||h||$$

Hence, we get the following minimization problem:

$$arg\min_h ||DV^Th|| \text{ s.t. } ||V^Th|| = 1$$

- Subsitute $y = V^Th$:

$$arg\min_h ||Dy|| \text{ s.t. } ||y|| = 1$$

- D is a diagonal matrix with decreasing values. Then, it is clear that $y = [0, 0, \dots, 1]^T$.

- Therefore, choosing h to be the last column in V will minimize the equation.

In Python:

```
(U,D,Vh) = np.linalg.svd(A,False)

h = Vh.T[:,-1]
```

Some more options to find h:

- Lagrange multipliers - [Least–squares Solution of Homogeneous Equations (http://cmp.felk.cvut.cz/cmp/courses/XE33PVR/WS20072008/Lectures/Supporting/constrained_lsq.pdf)](http://cmp.felk.cvut.cz/cmp/courses/XE33PVR/WS20072008/Lectures/Supporting/constrained_lsq.pdf)
- Using EVD (eigenvalue decomposition) on A^TA.
- If we know our transformation is nearly Affine we can get an approximate solution using linear least squares

- As we said, some of the points may be a good match, and some won't.
- If we use a wrong point in the algorithm, we will get the wrong transformation.
- How can we solve that major problem?

- We can use our prior knowledge on the type of the trasformation, and use points that have strong evidence that are correct.
- For example, we know that homographys preserve lines, so we can try to look for keypoints that lay in a line, and use them to find the transformation.
- How would we find keypoints that lay on a line?

RANSAC

The RANSAC algorithm is extremly simple, **but** it often:

- Does not produce correct model with user-defined probability
- Outputs an inaccurate model
- Does not handle degeneracies
- Can be sped up (by orders of magnitude)
- Does not gurantee minimum running time
- Needs information about scale of the noise
- Does not handle multiple models efficiently

Many improved algorithms:

- PROSAC (https://dspace.cvut.cz/bitstream/handle/10467/9496/2005-Matching-with-PROSAC-progressive-sample-consensus.pdf;jsessionid=AB7FDDCCABE0CA249D253E7E8F9D1DC7?sequence=1)
    - Key idea is to assume that the similarity measure predicts correctness of a match
- Randomized RANSAC (http://www.bmva.org/bmvc/2002/papers/50/full_50.pdf)
    - Each step take a random subset of the query points and perform RANSAC
- KALMANSAC (https://ieeexplore.ieee.org/abstract/document/1541313)
- and more... (https://en.wikipedia.org/wiki/Random_sample_consensus#Development_and_improvements)
- Estimating homogrpahy with RANSAC in OpenCV: `cv2.findHomography(src_pts, dst_pts, cv2.RANSAC)`

Excercise 2 (Spring 2020 Q1D) - Ransac

We'd like to fit an ellipse to a given set of 200 points, $X_i = (x_i, y_i), i = 1, \dots, 200$.
As you know, an ellipse can be described by the implicit equation $a^T(x^2, xy, y^2, x, y, 1) = 0$ where $a = (a_1, a_2, a_3, a_4, a_5, a_6)$ is a coefficients vector. The data points might be noisy, such that some of the points may not reside on the perimeter of the ellipse (outliers).

- Describe the RANSAC algorithm needed to fit the ellipse (and find a).

Answer:

- to define an ellipse, we need at least 5 points ([Why? (http://www.lee-mac.com/5pointellipse.html#:~:text=Five%20points%20are%20required%20to%20define%20a%20unique](http://www.lee-mac.com/5pointellipse.html#:~:text=Five%20points%20are%20required%20to%20define%20a%20unique)
  This means a is defined up to a scale factor.

Step 1: Sample 5 points from X_i randomly. **Step 2**: Fit an ellipse to the points with Least Squares.

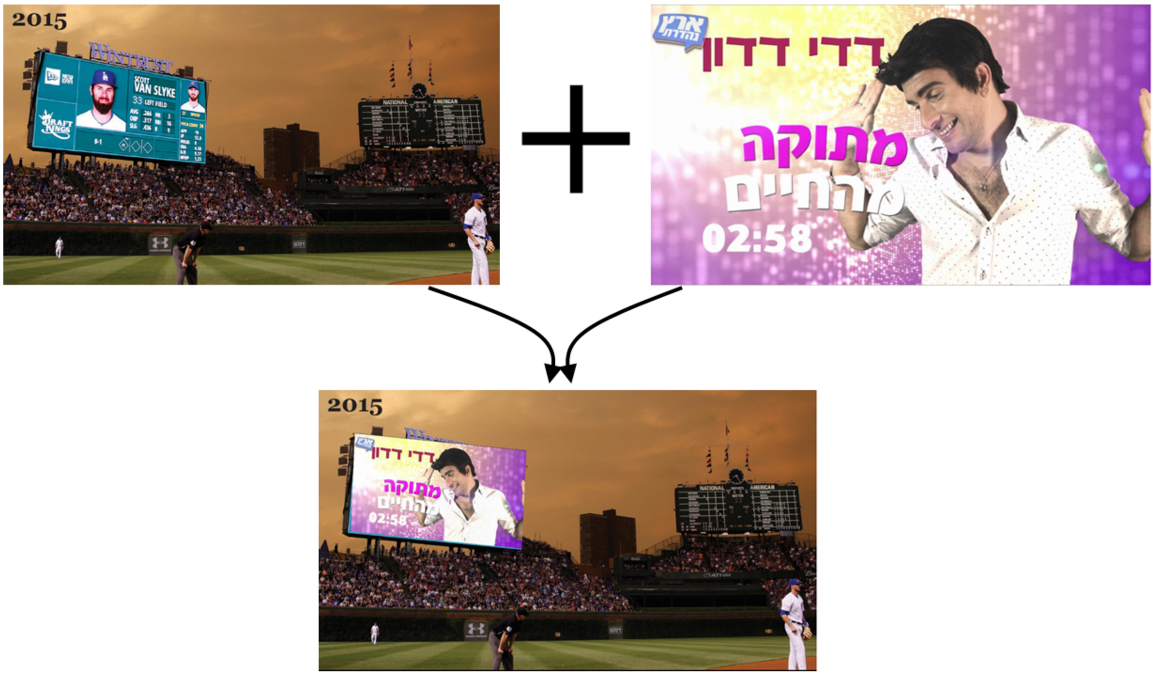
- $$\begin{bmatrix} & & \cdots & & & \\ x_i^2 & x_i y_i & y_i^2 & x_i & y_i & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = 0$$
- This can be denoted as $Sa = 0$, and solved via SVD under the constraint $||a|| = 1$

Step 3: Find the points that fit to the model (inliers), i.e. points with a fit error below a threshold chosen in advanced.
Step 4: Refine our model according to all the inliers, and count them. The number of inliers is the "score" of the current model (from this iteration).
Step 5: Repeat 1-4, until the stop criteria has been met.

When do we stop?

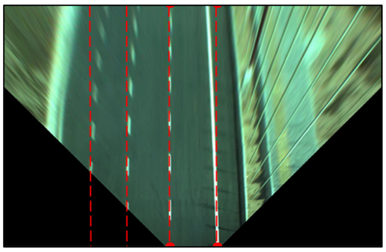
- The percentage of inliers out of all the points is above a certain percentage.
- The amount of inliers is above a certain amount.
- An error threshold (for the LS) is reached.
- A predefined amount of iterations is reached.

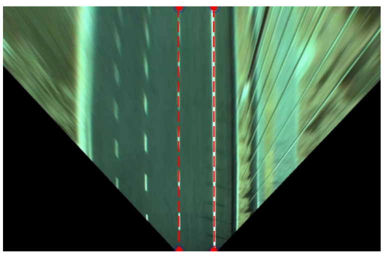
application 1: Planting images into other images



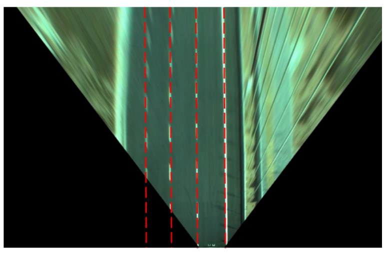
application 2: BirdEye

Original Road Image





After Iterations



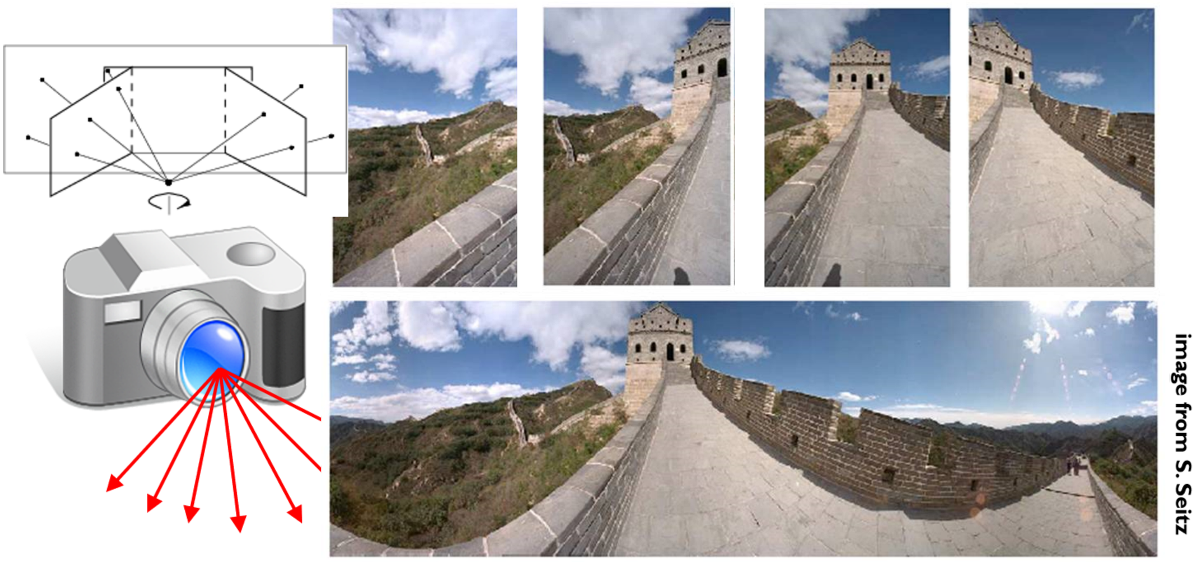
Multi-Lanes Merge

- [Image Source (https://csyhhu.github.io/2015/07/09/IPM/)](https://csyhhu.github.io/2015/07/09/IPM/)

application 3: Looking into the past

- [Image Source from Flickr (http://blog.flickr.net/en/2010/01/27/a-look-into-the-past/)](http://blog.flickr.net/en/2010/01/27/a-look-into-the-past/)

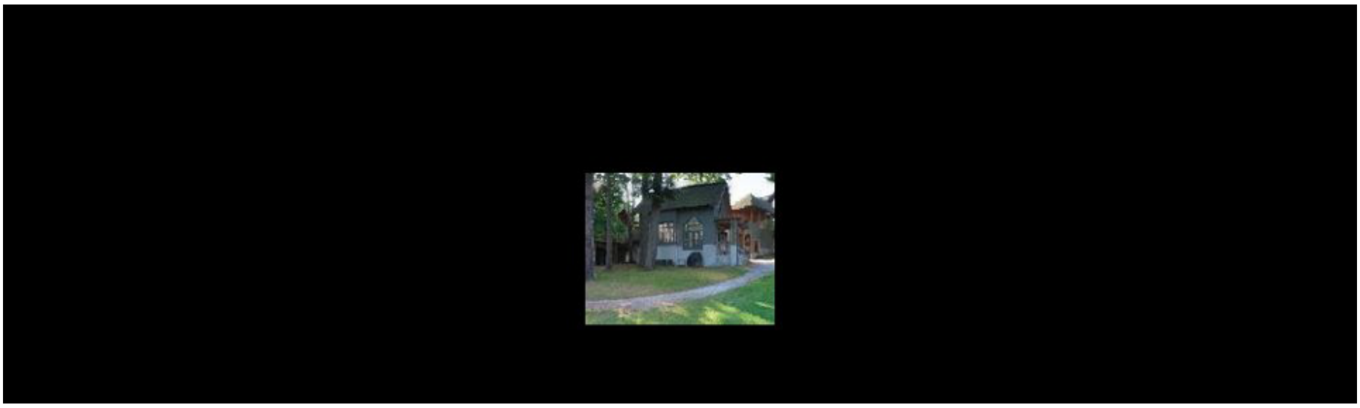
application 4: Panorama



Obtain a wider angle view by combining multiple images

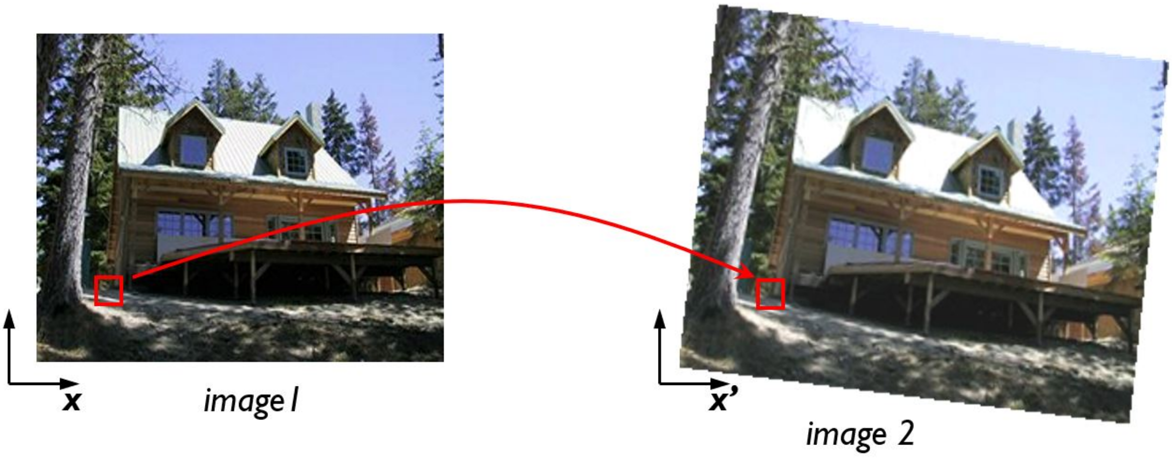
Panorama

- [Warping]()
- [Image Blending (Feathering)]()



Warp - What we need to solve?

- Given source and target images, and the transformation between them, how do we align them?
- Send each pixel x in image1 to its corresponding location x' in image 2



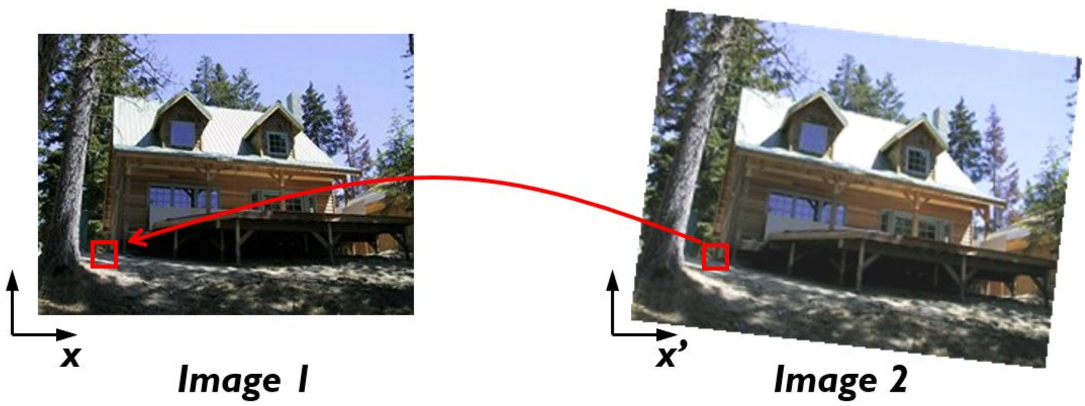
Forward Warping

- What if pixel lands “between” two pixels?
- Answer: add “contribution” to several pixels and normalize (splatting)

- Limitation: Holes (some pixels are never visited)

Inverse Warping

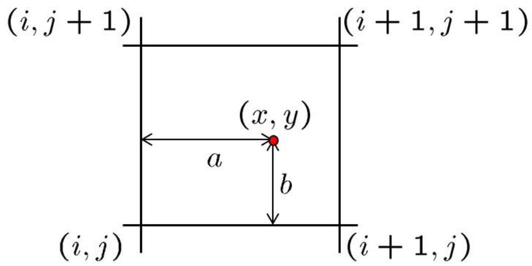
- For each pixel x’ in image 2 find its origin x in image 1
- Problem: What if pixel comes from “between” two pixels?



- Answer: interpolate color value from neighbors

Bilinear Interpolation

Sampling at $f(x, y)$:



$$\begin{aligned} f(x, y) = &(1-a)(1-b) f[i, j] \\ &+ a(1-b) f[i+1, j] \\ &+ ab f[i+1, j+1] \\ &+ (1-a) b f[i, j+1] \end{aligned}$$

Python:

- `interp2d()` - [https://docs.scipy.org/doc/scipy/reference/generated/scipy.interpolate.interp2d.html (https://docs.scipy.org/doc/scipy/reference/generated/scipy.interpolate.interp2d.html)](https://docs.scipy.org/doc/scipy/reference/generated/scipy.interpolate.interp2d.html)
- Inverse warping in OpenCV: `cv2.warpPerspective(im,*,*,cv2.WARP_INVERSE_MAP)`

Excercise 3 (Spring 2020 Q1B)

1. Describe forward and backward wraping.
2. For which out of the two cased before will there be a difference between using inverse or forward warping?
3. For the case which a difference exists, what type of warping should be used and why?

reminder:

- Image-2 is a translated image 1 by 100 pixels to the left
- Image-2 is a 3x zoom of image 1, so the index (0,0) stay in place

Answer

1. Read the tutorial :)
2. For the translation there will be no difference between forward/inverse warping, while for the zooming there will be a difference - we'll get holes.

To understand why, let's denote the pixel of image k at the indices (i, j) as x_{ij}^k.
In the first case, each pixel is moved to the left by a whole number, therefore for forward warping we follow the warp defined by:

$$x^2_{(i-100)j} = x^1_{ij}$$

And for inverse warping we use:

$$x^2_{ij} = x^1_{(i+100)j}$$

There's no need to interpolate anything here - because the pixel moved by an integer, and thus there is no difference.
In the second case we get for forward warping:

$$x^2_{(3i)(3j)} = x^1_{ij}$$

And for inverse warping we get an interpolation dependant on α, β:

$$x^2_{ij} = (1-\alpha)(1-\beta)x^1_{\left\lfloor \frac{i}{3} \right\rfloor \left\lfloor \frac{j}{3} \right\rfloor} + \alpha(1-\beta)x^1_{\left(\left\lfloor \frac{i}{3} \right\rfloor+1\right)\left\lfloor \frac{j}{3} \right\rfloor} + \alpha\beta x^1_{\left\lfloor \frac{i}{3} \right\rfloor\left(\left\lfloor \frac{j}{3} \right\rfloor+1\right)} + (1-\alpha)\beta x^1_{\left(\left\lfloor \frac{i}{3} \right\rfloor+1\right)\left(\left\lfloor \frac{j}{3} \right\rfloor+\right.}$$

We can see that for the forward case we get "holes" in image 2, for example $x^2_{1,1}$ has no origin in image 1.

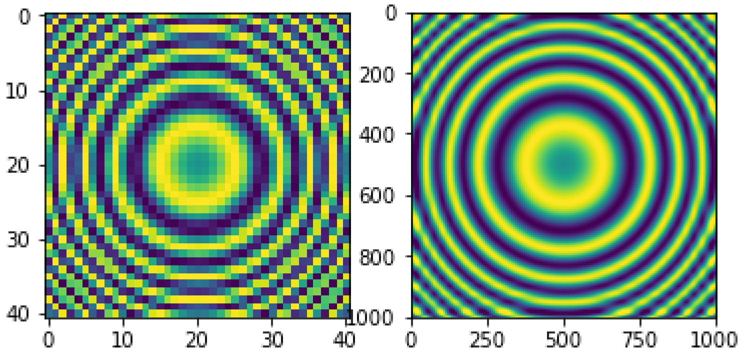
1. The backward wraping will be of course better, since as we saw no holes will be created.

In [8]:

```
import numpy as np
from scipy import interpolate
import matplotlib.pyplot as plt

# original samples
x = np.arange(-5.01, 5.01, 0.25)
y = np.arange(-5.01, 5.01, 0.25)
xx, yy = np.meshgrid(x, y)
z = np.sin(xx**2+yy**2)

# interpolated samples
xnew = np.arange(-5.01, 5.01, 1e-2)
ynew = np.arange(-5.01, 5.01, 1e-2)
```

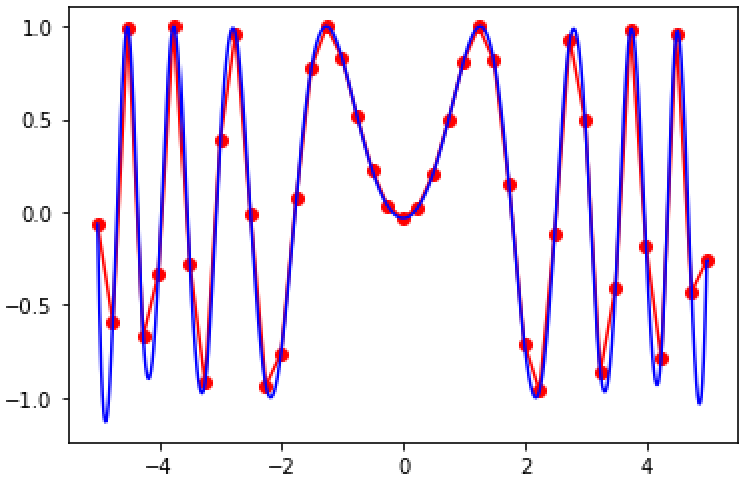
In [9]:

```
## Cubic
f = interpolate.interp2d(x, y, z, kind='cubic')
znew = f(xnew, ynew)

plt.figure()
plt.subplot(121)
plt.imshow(z,vmin=-1,vmax=1)
plt.subplot(122)
plt.imshow(znew,vmin=-1,vmax=1)
plt.show()
```



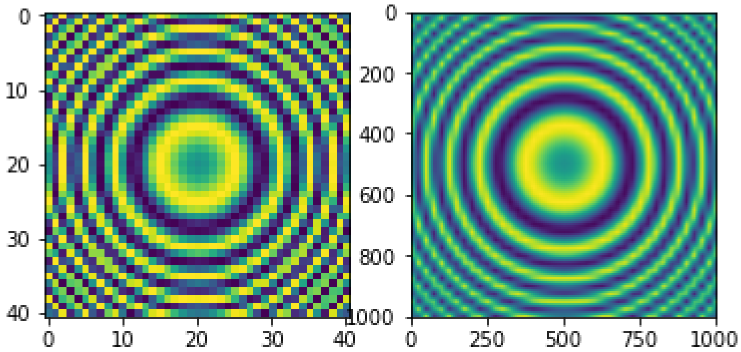
In [10]:

```
plt.figure()
plt.plot(x, z[0, :], 'ro-', xnew, znew[0, :], 'b-')
plt.show()
```



In [11]:

```
##linear
f2 = interpolate.interp2d(x, y, z, kind='linear')
znew2 = f2(xnew, ynew)

plt.figure()
plt.subplot(121)
plt.imshow(z,vmin=-1,vmax=1)
plt.subplot(122)
plt.imshow(znew2,vmin=-1,vmax=1)
plt.show()
```



In [12]:

```
plt.figure()
plt.plot(x, z[0, :], 'ro-', xnew, znew2[0, :], 'b-')
plt.show()
```

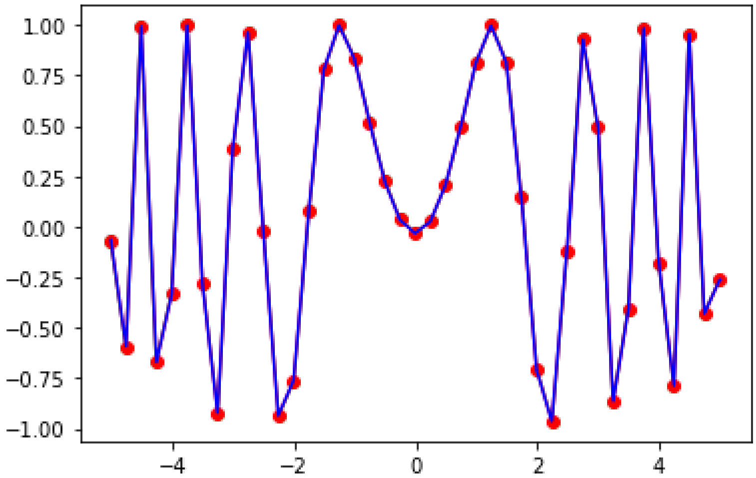
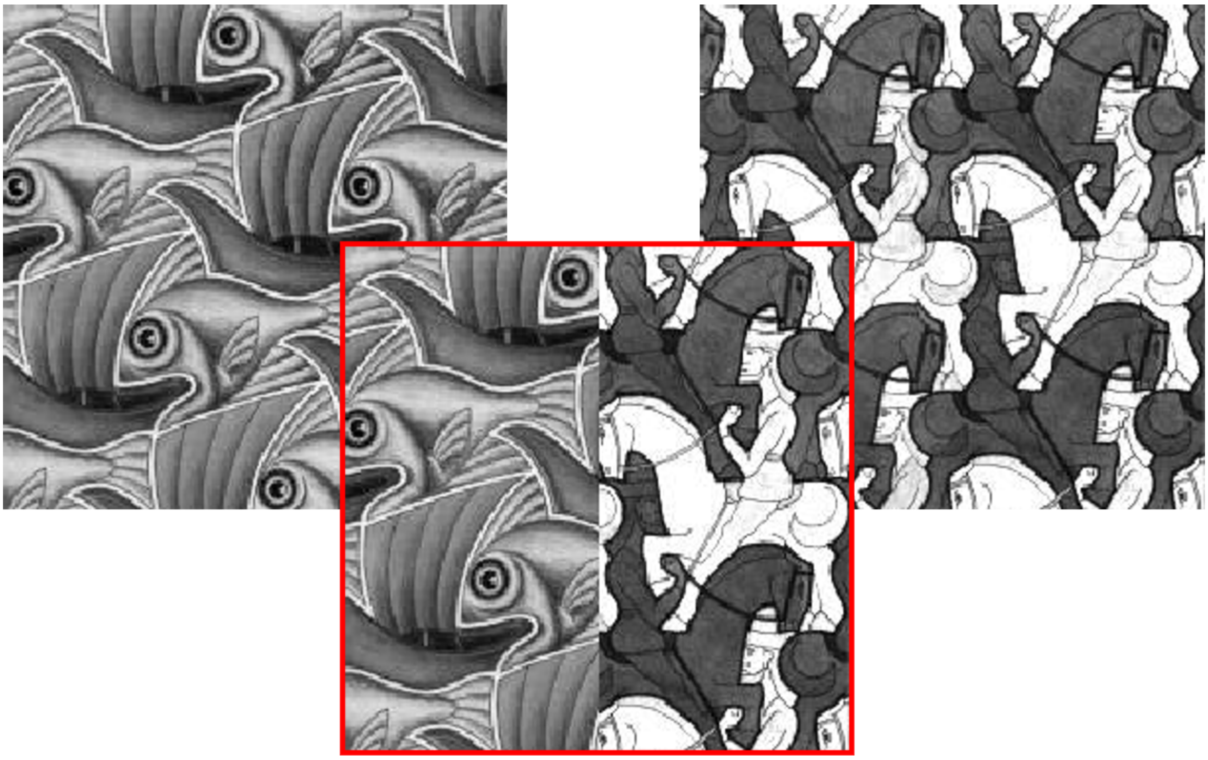
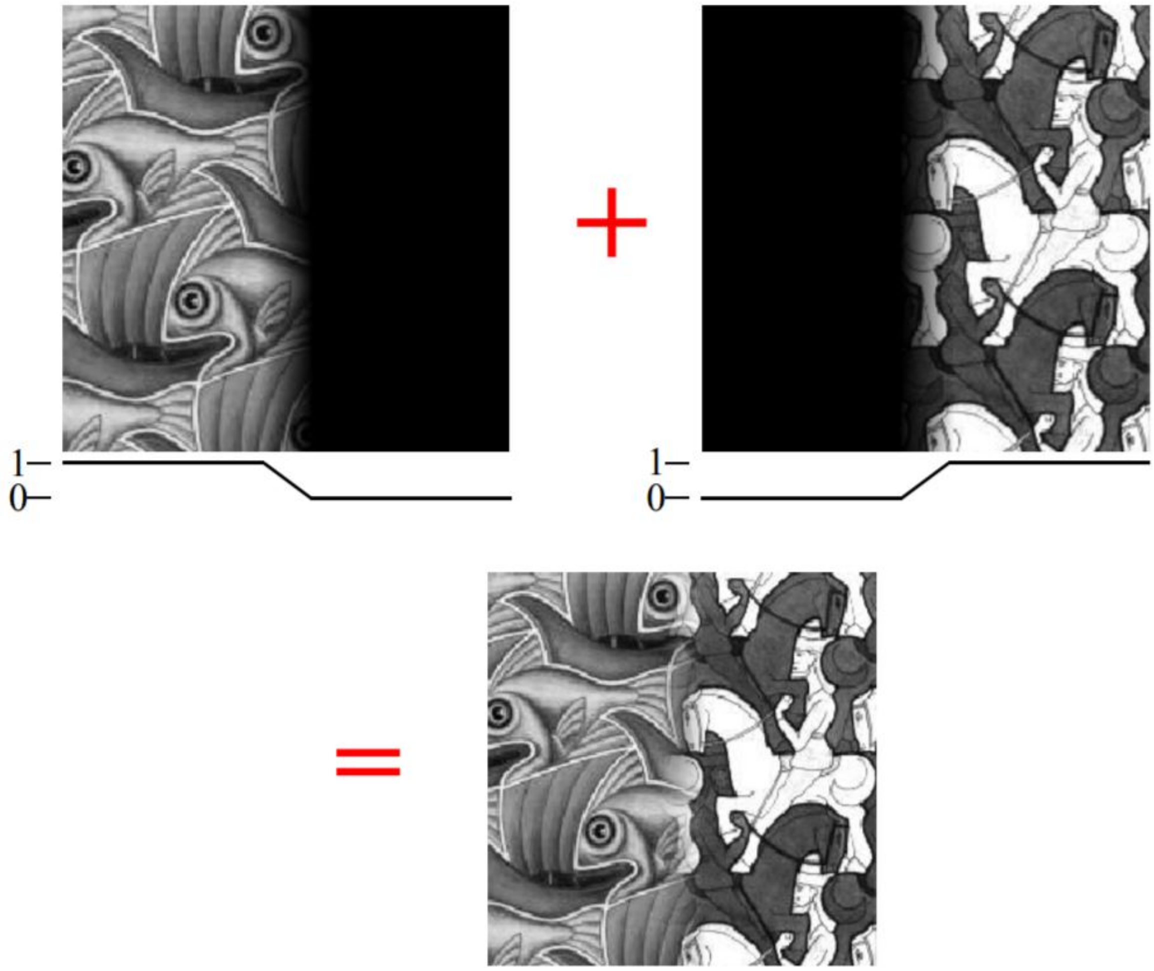


Image Blending



- Alpha blending
- Pyramid blending

Alpha Blending



Pyramid Blending:

1. Build a Gaussian pyramid for each image
2. Build the Laplacian pyramid for each image
3. Decide/find the blending border (in the example: left half belongs to image 1, and right half to image 2 -> the blending border is `cols/2`)
   - Split by index, or
   - Split using a 2 masks (can be weighted masks)
4. Constract a new mixed pyramid - mix each level seperatly acording to (3)
5. Reconstract a blend image from the mixed pyramid

In [13]:

```
def create_pyrs(A,B):
    # generate Gaussian pyramid for A
    G = A.copy()
    gpA = [G]
    for i in range(6):
        G = cv2.pyrDown(G)
        gpA.append(G)
    # generate Gaussian pyramid for B
    G = B.copy()
    gpB = [G]
    for i in range(6):
        G = cv2.pyrDown(G)
        gpB.append(G)
    # generate Laplacian Pyramid for A
    lpA = [gpA[5]]
    for i in range(5,0,-1):
        GE = cv2.pyrUp(gpA[i])
        L = cv2.subtract(gpA[i-1],GE)
        lpA.append(L)
    # generate Laplacian Pyramid for B
    lpB = [gpB[5]]
    for i in range(5,0,-1):
        GE = cv2.pyrUp(gpB[i])
        L = cv2.subtract(gpB[i-1],GE)
        lpB.append(L)
    return lpA,lpB
```

In [14]:

```
def blend_images(A,B):
    lpA,lpB = create_pyrs(A,B)
    # Now add left and right halves of images in each level
    LS = []
    for la,lb in zip(lpA,lpB):
        rows,cols,dpt = la.shape
        ls = np.hstack((la[:,0:int(cols/2)], lb[:,int(cols/2):])) #mixing can also be d
one with a mask
        LS.append(ls)
    # now reconstruct
    ls_ = LS[0]
    for i in range(1,6):
        ls_ = cv2.pyrUp(ls_)
        ls_ = cv2.add(ls_, LS[i])
    # image with direct connecting each half
    real = np.hstack((A[:,:int(cols/2)],B[:,int(cols/2):]))
    return real, ls_
```

In [15]:

```
def switch_texture(A,B):
    lpA,lpB = create_pyrs(A,B)
    # Now add left and right halves of images in each level
    LS = []
#     for la,lb in zip(lpA,lpB):
#         rows,cols,dpt = la.shape
#         ls = np.hstack((la[:,0:int(cols/2)], lb[:,int(cols/2):])) #mixing can also be
done with a mask
#         LS.append(ls)
    # now reconstruct
    ls_ = lpA[0]
    for i in range(1,6):
        ls_ = cv2.pyrUp(ls_)
        ls_ = cv2.add(ls_, lpB[i])
    # image with direct connecting each half
    real = np.hstack((A[:,:int(cols/2)],B[:,int(cols/2):]))
    return real, ls_
```

In [16]:

```
def alpha_blend(A,B,MASK):
    A = np.float32(A)
    B = np.float32(B)
    return np.uint8(A*MASK), np.uint8(A*MASK+B*(1-MASK))
```

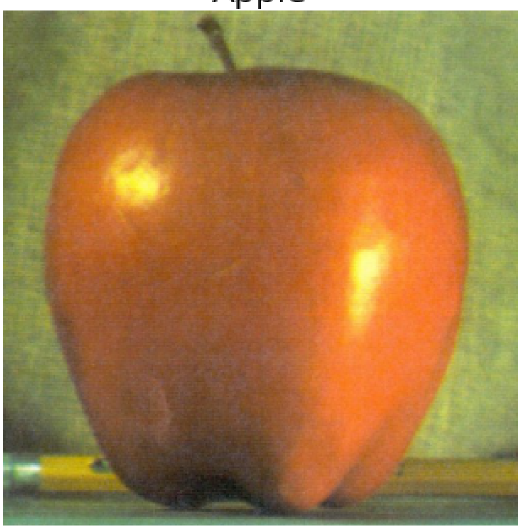
In [17]:

```
A = cv2.imread('./assets/apple.jpg')
B = cv2.imread('./assets/orange.jpg')
real,ls_ = blend_images(A,B)

A_ = cv2.cvtColor(A,cv2.COLOR_BGR2RGB)
B_ = cv2.cvtColor(B,cv2.COLOR_BGR2RGB)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

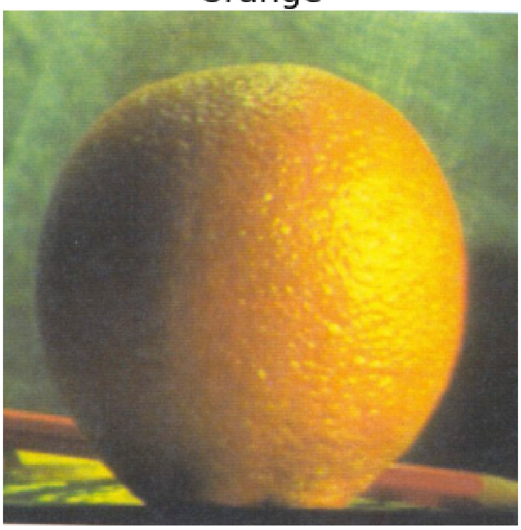
In [18]:

```
plot_images([A_,B_],['Apple','Orange'],(1,2),figsize=(12,12),fontsize=20)
```

Apple

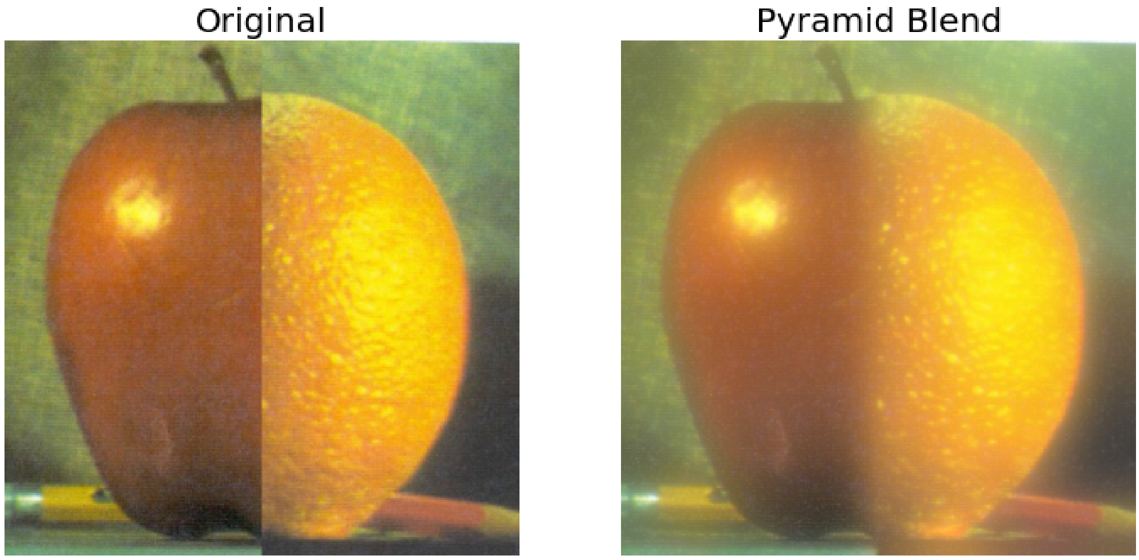


Orange



In [19]:

```
plot_images([real,ls_],['Original','Pyramid Blend'],(1,2),figsize=(12,12),fontsize=20)
```



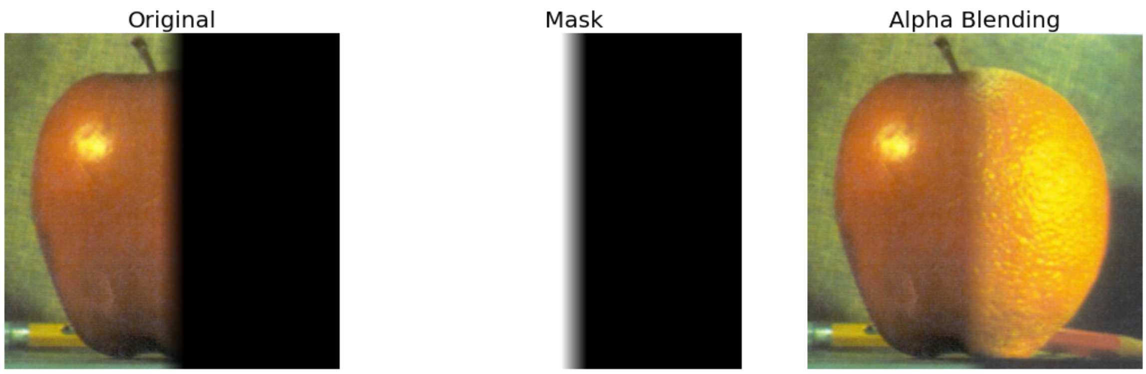
- Alpha blending example:

In [20]:

```
MASK = np.ones_like(A,np.float32)
rows,cols,dpt = MASK.shape
w=20
v_dec = np.linspace(1,0,2*w)
MASK[:,int(cols/2):]=0
MASK[:,(int(cols/2)-w):(int(cols/2)+w)]=np.tile(np.reshape(v_dec,[1,-1,1]),[rows,1,3])
real,ls_ = alpha_blend(A,B,MASK)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

In [21]:

```
plot_images([real,MASK,ls_],['Original','Mask','Alpha Blending'],(1,3),figsize=(18,12),
fontsize=20)
```



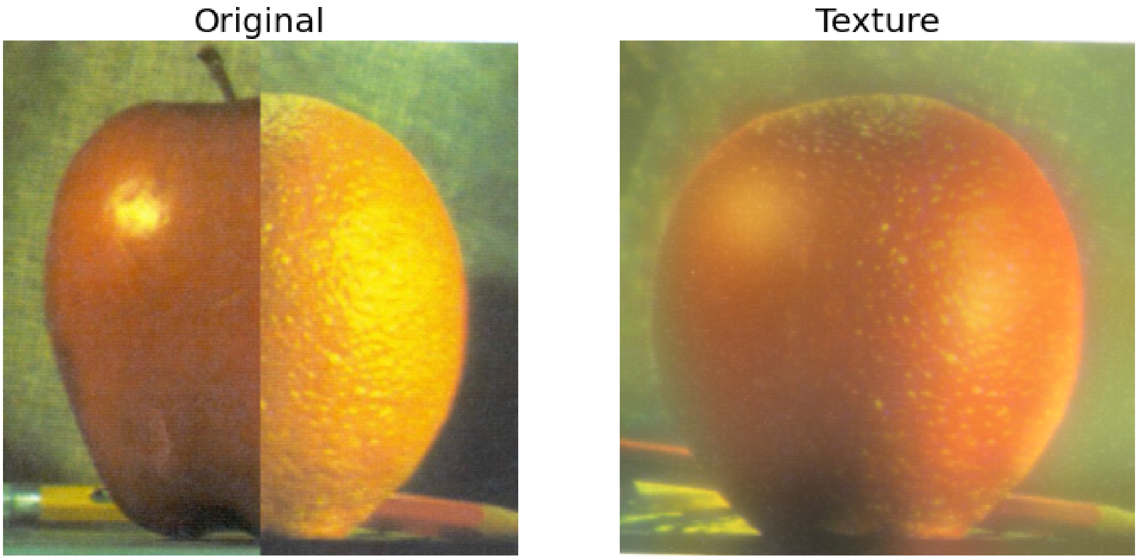
- Stylize image using pyramids:

In [22]:

```
real,ls_ = switch_texture(A,B)
ls_ = cv2.cvtColor(ls_,cv2.COLOR_BGR2RGB)
real = cv2.cvtColor(real,cv2.COLOR_BGR2RGB)
```

In [23]:

```
plot_images([real,ls_],['Original','Texture'],(1,2),figsize=(12,12),fontsize=20)
```



Blending Improvments

- Many algorithms have different variations of combining alpha and pyramid blending (different masks for different frequencies)
- Find the boundaries using **segmentation**

Panorama - Summary

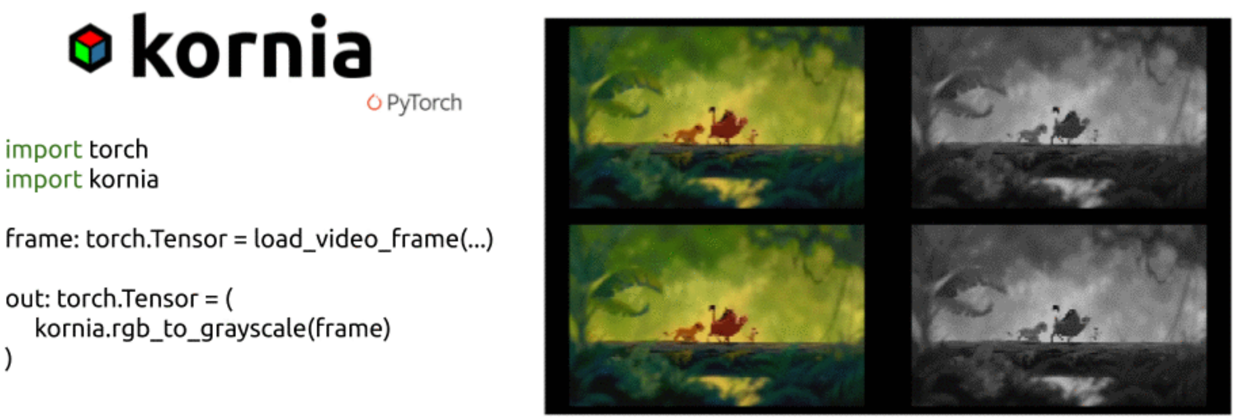
- Detect features
- Compute transformations between pairs of frames
- Can Refine transformations using RANSAC
- Warp all images onto a single coordinate system
- Find mixing borders (e.g. using segmentation)
- Blend

Transformations in Deep Learning

- Can we incorporate transformations in the pipeline of a deep learning algorithm?
    - Moreover, can we accelerate these transformations by performing them on a GPU?
- **YES!**

Kornia - Computer Vision Library for PyTorch

- [Kornia is a differentiable computer vision library for PyTorch (https://github.com/kornia/kornia)](https://github.com/kornia/kornia)
    - That means you can have gradients for the transformations!
- Inspired by OpenCV, this library is composed by a subset of packages containing operators that can be inserted within neural networks to train models to perform image transformations, epipolar geometry, depth estimation, and low-level image processing such as filtering and edge detection that operate directly on tensors.
- Check out `kornia.geometry` - [https://kornia.readthedocs.io/en/latest/geometry.html (https://kornia.readthedocs.io/en/latest/geometry.html)](https://kornia.readthedocs.io/en/latest/geometry.html)
- [Warp image using perspective transform (https://kornia.readthedocs.io/en/latest/tutorials/warp_perspective.html)](https://kornia.readthedocs.io/en/latest/tutorials/warp_perspective.html)



Recommended Videos

Warning!

- These videos do not replace the lectures and tutorials.
- Please use these to get a better understanding of the material, and not as an alternative to the written material.

Video By Subject

- Homography
  - Image geometry and planar homography - ENB339 lecture 9: Image geometry and planar homography (https://www.youtube.com/watch?v=fVJeJMWZcq8)
  - Homography - Homography in computer vision explained (https://www.youtube.com/watch?v=MlaIWymLCD8)
- Transformations - Lect. 5(1) - Linear and affine transformations (https://www.youtube.com/watch?v=4I2S5Xhf24o)
- Matching Local Features
  - SIFT - CSCI 512 - Lecture 12-1 SIFT (https://www.youtube.com/watch?v=U0wqePj4Mx0)

Credits

- EE 046746 Spring 2022 - Moshe Kimhi (https://www.linkedin.com/in/moshekimhi/), Hila Manor (https://github.com/HilaManor)
- EE 046746 Spring 2020 - Dahlia Urbach (https://il.linkedin.com/in/dahlia-urbach-97a816123)
- Slides - Elad Osherov (Technion), Simon Lucey (CMU)
- Multiple View Geometry in Computer Vision - Hartley and Zisserman - Section 2
- Least–squares Solution of Homogeneous Equations (cmp.felk.cvut.cz/cmp/courses/XE33PVR/WS20072008/Lectures/Supporting/constrained_lsq.pdf) - Center for Machine Perception - Tomas Svoboda
- Computer vision: models, learning and inference (http://www.computervisionmodels.com/) , Simon J.D. Prince - Section 15.1
- Computer Vision: Algorithms and Applications (https://www.springer.com/gp/book/9781848829343) - Richard Szeliski - Sections 2,4,6, 9 (Free for Technion students via remote library (https://piazza.com/class/k81fypit1f75kx?cid=122))
- Icons from Icon8.com (https://icons8.com/) - https://icons8.com (https://icons8.com)