

Projet Techniques d'Optimisation de la Parallélisation (TOP)

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## Seismic core - 3D stencil optimization

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# 1 Introduction

The project focuses on optimizing a seismic core algorithm that performs a 3D stencil computation. The algorithm involves a 16th-order, 49-point stencil operation, which is crucial for seismic data processing in various scientific and engineering applications.

The primary objective of the optimization effort is to enhance the computational efficiency of the algorithm while maintaining its accuracy and reliability. The optimization process aims to reduce the overall computational time required for the stencil computation, thereby enabling faster seismic data analysis and processing.

Key challenges in optimizing the algorithm include :

Memory Access Patterns : Efficient management of memory access patterns is essential to minimize data transfer overhead and maximize cache utilization during stencil computation. Parallelization : Leveraging parallel processing techniques such as multithreading or vectorization to distribute the computational workload across multiple processing units can significantly accelerate the stencil computation. Algorithmic Optimization : Exploring alternative algorithmic approaches or optimization strategies to minimize redundant computations and improve arithmetic intensity can lead to further performance gains. By addressing these challenges and implementing effective optimization techniques, the project aims to achieve substantial improvements in the performance and scalability of the seismic core algorithm, thereby facilitating faster and more efficient seismic data analysis and interpretation.

## 2 Environnement

Tous les tests ont été exécutés dans cet environnement :

- OS : Arch Linux x86 64 Kernel 6.5.7-arch1-1
- CPU : Intel(R) Core(TM) i5-6300U CPU @ 2.40GHz
- RAM : 16GB
- Compilateur : gcc version 11.4.0 Ubuntu clang version 14.0.0-1
- profilers : perf strace gprof flamegraphs maqao

## 3 Optimization Steps

### 3.1 Initial Profiling and Analysis

Initially, the program encountered compilation errors. The CMake configuration needed correction to properly detect and compile MPI. Additionally, we identified issues with error management in the mesh initialization module. After addressing these issues, we achieved a functioning code that compiles successfully.

Our first step was to run the program with a 100x100x100 configuration file for 10 iterations to obtain an initial speed reading.

Subsequently, we employed the 'perf' tool to profile the code, recording metrics such as cycles, instructions, cache-references, and cache-misses. Additionally, we utilized 'gprof' to generate a call graph of the program. Our initial optimization involved compiler settings. For the initial profiling, we did not utilize compiler optimization flags. However, subsequent runs were conducted with 'O3' and 'Ofast' optimization flags enabled.

Performance counter stats for './top-stencil ../config.txt':

```
13407600584      cycles
10991068847      instructions          #    0,82  insn per cycle
1957075053       cache-references
726958179        cache-misses          #   37,145 % of all cache refs
27,965110103 seconds time elapsed
4,563284000 seconds user
0,060095000 seconds sys
```

Result\_no\_flag initial AVERAGE = 4028.01

Result flag O3 AVERAGE=3614.29

Result flag ofast Average = 2447.98

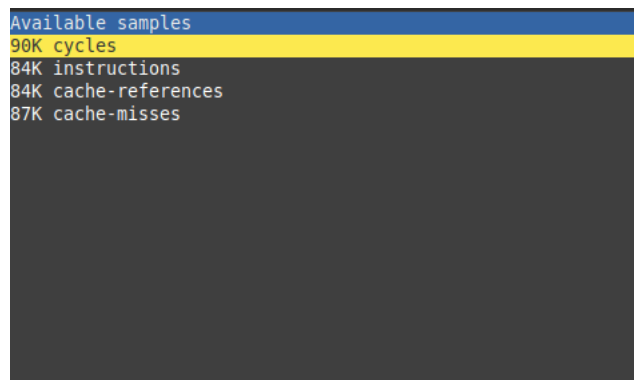


FIGURE 1 – perf screenshots

Reference is 60.32% faster than result.

Samples: 84K of event 'instructions', Event count (approx.): 100498696128					
Children	Self	Command	Shared Object	Symbol	
99,20%	0,00%	top-stencil	libc.so.6	[.]	libc_start_call_main
99,20%	0,00%	top-stencil	top-stencil	[.]	main
64,82%	64,82%	top-stencil	libm.so.6	[.]	ieee754_pow_fma
26,47%	25,80%	top-stencil	libstencil.so	[.]	solve_jacobi
5,58%	5,58%	top-stencil	libm.so.6	[.]	pow@GLIBC 2.29
0,66%	0,66%	top-stencil	libstencil.so	[.]	mesh_copy_core
0,60%	0,60%	top-stencil	libstencil.so	[.]	0x00000000000011f0
0,60%	0,00%	top-stencil	libstencil.so	[.]	0x00007f24e60881f0
0,51%	0,00%	top-stencil	libstencil.so	[.]	init_meshes
0,48%	0,48%	top-stencil	libstencil.so	[.]	0x00000000000011f4
0,48%	0,00%	top-stencil	libstencil.so	[.]	0x00007f24e60881f4
0,44%	0,44%	top-stencil	libm.so.6	[.]	0x000000000000e300
0,44%	0,00%	top-stencil	libm.so.6	[.]	0x00007f24e5c0e300
0,38%	0,38%	top-stencil	libm.so.6	[.]	_cos_fma
0,28%	0,28%	top-stencil	libm.so.6	[.]	0x000000000000e304
0,28%	0,00%	top-stencil	libm.so.6	[.]	0x00007f24e5c0e304
0,25%	0,14%	top-stencil	libstencil.so	[.]	setup_mesh_cell_values
0,23%	0,14%	top-stencil	libstencil.so	[.]	setup_mesh_cell_kinds
0,16%	0,16%	top-stencil	libm.so.6	[.]	_sin_fma
0,10%	0,10%	top-stencil	libstencil.so	[.]	mesh_set_cell_kind
0,06%	0,06%	top-stencil	libstencil.so	[.]	compute_core_pressure
0,06%	0,00%	top-stencil	[kernel.kallsyms]	[k]	asm_exc_page_fault
0,06%	0,00%	top-stencil	[kernel.kallsyms]	[k]	entry_SYSCALL_64_after_hwframe
0,06%	0,00%	top-stencil	[kernel.kallsyms]	[k]	exc_page_fault
0,06%	0,00%	top-stencil	[kernel.kallsyms]	[k]	do_syscall_64

FIGURE 2 – perf screenshots

Samples: 90K of event 'cycles', Event count (approx.): 63185610320					
Children	Self	Command	Shared Object	Symbol	
98,85%	0,00%	top-stencil	libc.so.6	[.]	_libc_start_call_main
98,85%	0,00%	top-stencil	top-stencil	[.]	main
50,14%	47,55%	top-stencil	libstencil.so	[.]	solve_jacobi
43,26%	40,69%	top-stencil	libm.so.6	[.]	ieee754_pow_fma
5,85%	5,79%	top-stencil	libm.so.6	[.]	pow@GLIBC 2.29
2,14%	2,12%	top-stencil	libstencil.so	[.]	mesh_copy_core
0,56%	0,56%	top-stencil	[kernel.kallsyms]	[k]	irqentry_text_end
0,41%	0,00%	top-stencil	libm.so.6	[.]	0x00007f24e5c0e300
0,41%	0,00%	top-stencil	libstencil.so	[.]	0x00007f24e60881f4
0,40%	0,40%	top-stencil	libm.so.6	[.]	0x000000000000e300
0,40%	0,40%	top-stencil	libstencil.so	[.]	0x00000000000011f4
0,37%	0,00%	top-stencil	libstencil.so	[.]	init_meshes
0,29%	0,01%	top-stencil	[kernel.kallsyms]	[k]	asm_exc_nmi
0,29%	0,29%	top-stencil	[kernel.kallsyms]	[k]	exc_nmi
0,28%	0,00%	top-stencil	libstencil.so	[.]	0x00007f24e60881f0
0,27%	0,27%	top-stencil	libstencil.so	[.]	0x00000000000011f0
0,26%	0,01%	top-stencil	[kernel.kallsyms]	[k]	asm_sysvec_apic_timer_interrupt
0,25%	0,00%	top-stencil	[kernel.kallsyms]	[k]	sysvec_apic_timer_interrupt
0,25%	0,25%	top-stencil	libm.so.6	[.]	_cos_fma
0,21%	0,00%	top-stencil	[kernel.kallsyms]	[k]	_sysvec_apic_timer_interrupt
0,20%	0,01%	top-stencil	[kernel.kallsyms]	[k]	hrtimer_interrupt
0,20%	0,20%	top-stencil	libm.so.6	[.]	_sin_fma
0,20%	0,10%	top-stencil	libstencil.so	[.]	setup_mesh_cell_values
0,20%	0,01%	top-stencil	libc.so.6	[.]	int_malloc
0,19%	0,01%	top-stencil	[kernel.kallsyms]	[k]	_hrtimer_run_queues

FIGURE 3 – perf screenshots

Samples: 87K of event 'cache-misses', Event count (approx.): 101435457

	Children	Self	Command	Shared Object	Symbol
+	99,41%	0,00%	top-stencil	libc.so.6	[.] __
+	99,41%	0,00%	top-stencil	top-stencil	[.] ma
+	47,98%	42,10%	top-stencil	libstencil.so	[.] so
+	44,85%	44,65%	top-stencil	libm.so.6	[.] __
+	5,66%	5,64%	top-stencil	libstencil.so	[.] me
+	5,12%	5,09%	top-stencil	libm.so.6	[.] po
	0,40%	0,00%	top-stencil	[kernel.kallsyms]	[k] as
	0,39%	0,00%	top-stencil	[kernel.kallsyms]	[k] sy
	0,35%	0,00%	top-stencil	libstencil.so	[.] in
	0,35%	0,00%	top-stencil	[kernel.kallsyms]	[k] __
	0,34%	0,00%	top-stencil	[kernel.kallsyms]	[k] hr
	0,34%	0,34%	top-stencil	libstencil.so	[.] 0x
	0,34%	0,00%	top-stencil	libstencil.so	[.] 0x
	0,33%	0,01%	top-stencil	[kernel.kallsyms]	[k] __
	0,30%	0,00%	top-stencil	libstencil.so	[.] 0x
	0,30%	0,30%	top-stencil	libstencil.so	[.] 0x
	0,29%	0,00%	top-stencil	[kernel.kallsyms]	[k] ti
	0,27%	0,00%	top-stencil	libm.so.6	[.] 0x
	0,26%	0,26%	top-stencil	libm.so.6	[.] 0x
	0,20%	0,20%	top-stencil	libm.so.6	[.] 0x
	0,20%	0,00%	top-stencil	libm.so.6	[.] 0x
	0,19%	0,00%	top-stencil	[kernel.kallsyms]	[k] ti
	0,19%	0,01%	top-stencil	[kernel.kallsyms]	[k] up
	0,17%	0,11%	top-stencil	libstencil.so	[.] se
	0,17%	0,00%	top-stencil	[kernel.kallsyms]	[k] as

FIGURE 4 – perf screenshots

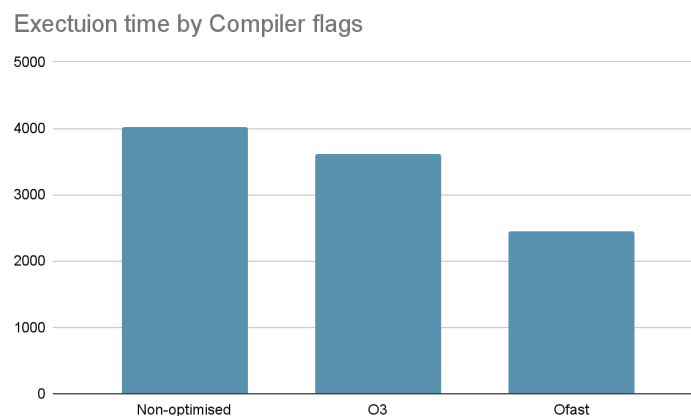


FIGURE 5 – compiler flags comparison

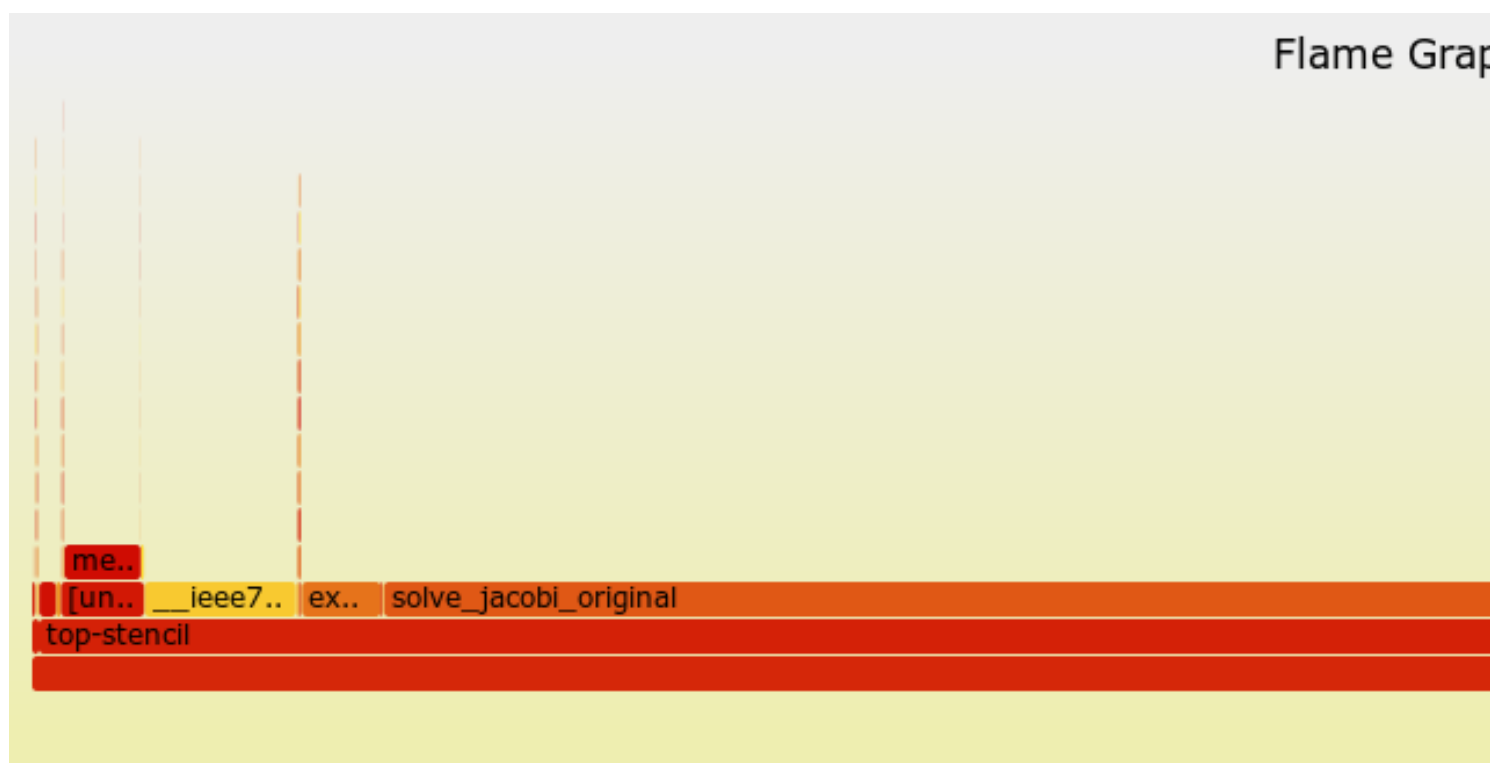


FIGURE 6 – flamegraph visualisation

### 3.1.1 Optimization Attempt 2 : using Strace to remove obsfuction of malicious code

so for our second attempt at optimizing the code, we corrected some bugs we didn't see in the first phase, such a bug that didn't allow other ranks to access the all reduce mp function in the main. This allowed the field in our results to be corrected.

We stuck with the Ofast flag for the rest of the project. We ran strace on the program and found a small little system call that called sleep(), which obviously was contributing massively to our performance drag.

Result R2 ofast = 362.9053

Performance counter stats for './top-stencil ../config.txt':

12088115629	cycles		
10987954787	instructions	#	0,91 insn per cycle
1949147064	cache-references		
681035780	cache-misses	#	34,940 % of all cache refs
4,391701852	seconds time elapsed		
4,028489000	seconds user		
0,048149000	seconds sys		

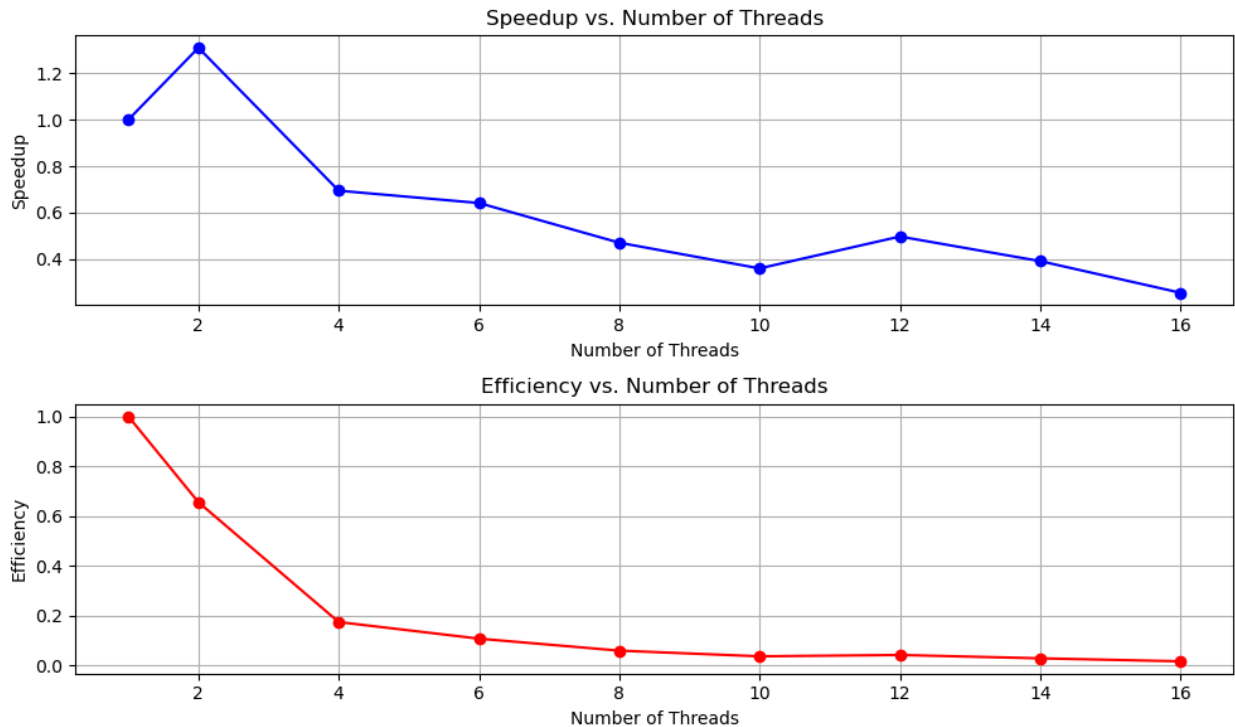


FIGURE 7 – speed-up/by number of threadq

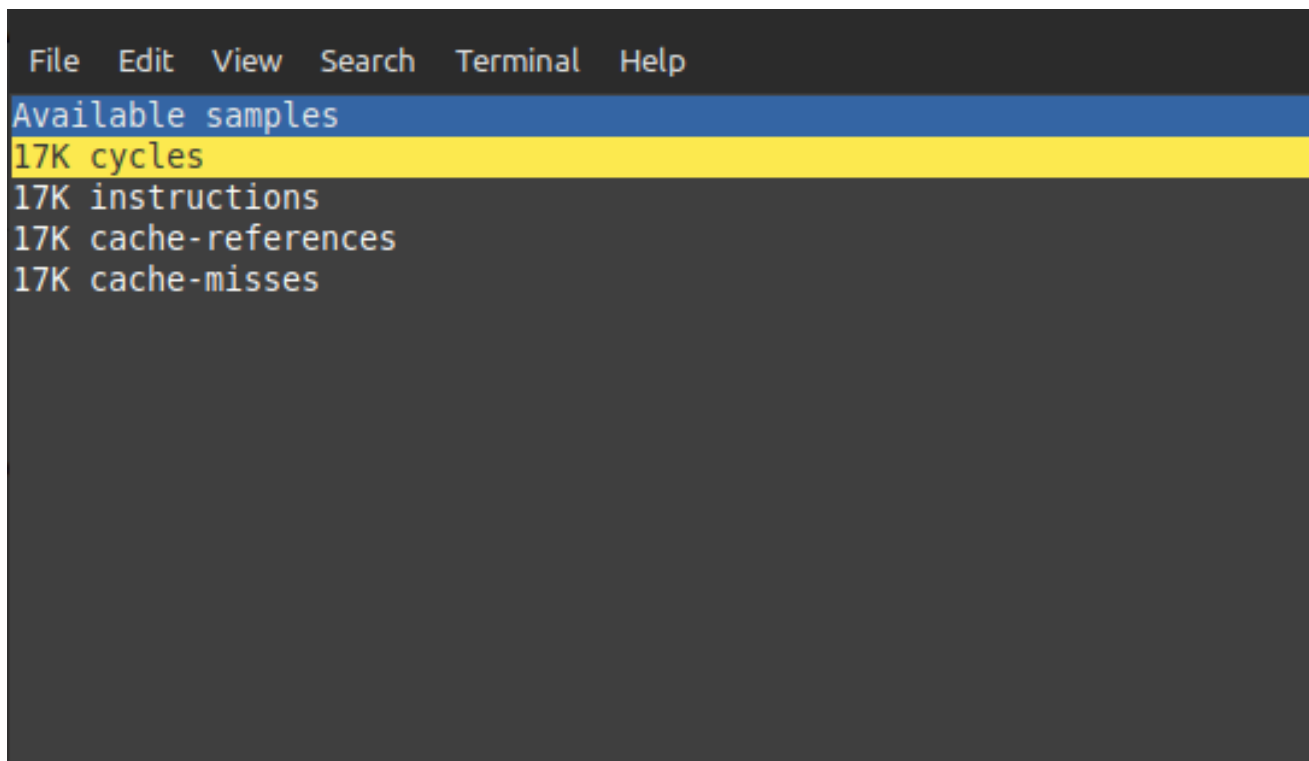


FIGURE 8 – perf screenshots

Samples: 17K of event cycles, Event count (approx.): 12641380078

Children	Self	Command	Shared Object	Symbol
+ 94,59%	0,00%	top-stencil	libc.so.6	[.] __libc_start_call_main
+ 94,59%	0,00%	top-stencil	top-stencil	[.] main
+ 79,22%	78,36%	top-stencil	libstencil.so	[.] solve_jacobi
+ 5,40%	5,36%	top-stencil	libm.so.6	[.] _ieee754_exp_fma
+ 3,67%	3,65%	top-stencil	libstencil.so	[.] mesh_copy_core
+ 3,41%	3,36%	top-stencil	libm.so.6	[.] exp@@GLIBC_2.29
+ 2,26%	0,18%	top-stencil	libstencil.so	[.] init_meshes
+ 1,15%	1,15%	top-stencil	libc.so.6	[.] __mcount_internal
+ 1,00%	1,00%	top-stencil	libm.so.6	[.] __sin_fma
+ 1,00%	0,11%	top-stencil	libc.so.6	[.] __int_malloc
+ 0,98%	0,97%	top-stencil	libm.so.6	[.] __cos_fma
+ 0,66%	0,09%	top-stencil	[kernel.kallsyms]	[k] asm_exc_page_fault
+ 0,58%	0,03%	top-stencil	[kernel.kallsyms]	[k] entry_SYSCALL_64_after_hwframe
+ 0,58%	0,58%	top-stencil	[kernel.kallsyms]	[k] __irqentry_text_end
+ 0,57%	0,00%	top-stencil	[kernel.kallsyms]	[k] exc_page_fault
+ 0,55%	0,00%	top-stencil	[kernel.kallsyms]	[k] do_syscall_64
+ 0,47%	0,01%	top-stencil	[kernel.kallsyms]	[k] do_user_addr_fault
+ 0,45%	0,45%	top-stencil	libstencil.so	[.] setup_mesh_cell_values
+ 0,45%	0,02%	top-stencil	[kernel.kallsyms]	[k] handle_mm_fault
+ 0,41%	0,04%	top-stencil	[kernel.kallsyms]	[k] __handle_mm_fault
+ 0,38%	0,02%	top-stencil	[kernel.kallsyms]	[k] handle_pte_fault
+ 0,37%	0,05%	orted	[kernel.kallsyms]	[k] entry_SYSCALL_64_after_hwframe
+ 0,31%	0,00%	orted	[kernel.kallsyms]	[k] do_syscall_64
+ 0,31%	0,00%	top-stencil	[kernel.kallsyms]	[k] do_anonymous_page
+ 0,30%	0,02%	top-stencil	[kernel.kallsyms]	[k] asm_sysvec_apic_timer_interrupt

Cannot load tips.txt file, please install perf!

FIGURE 9 – perf screenshots



Samples: 17K of event 'cache-misses', Event count (approx.): 694745482

Children	Self	Command	Shared Object	Symbol
+ 99,22%	0,00%	top-stencil	libc.so.6	[.] __libc_start_call_main
+ 99,22%	0,00%	top-stencil	top-stencil	[.] main
+ 83,45%	83,20%	top-stencil	libstencil.so	[.] solve_jacobi
+ 7,33%	7,31%	top-stencil	libm.so.6	[.] __ieee754_exp_fma
+ 3,81%	3,78%	top-stencil	libm.so.6	[.] exp@@GLIBC_2.29
+ 3,71%	3,69%	top-stencil	libstencil.so	[.] mesh_copy_core
0,39%	0,04%	top-stencil	libstencil.so	[.] init_meshes
0,31%	0,00%	top-stencil	[kernel.kallsyms]	[k] asm_sysvec_apic_timer_interrupt
0,31%	0,00%	top-stencil	[kernel.kallsyms]	[k] sysvec_apic_timer_interrupt
0,23%	0,00%	top-stencil	[kernel.kallsyms]	[k] asm_exc_page_fault
0,23%	0,00%	top-stencil	[kernel.kallsyms]	[k] exc_page_fault
0,23%	0,00%	top-stencil	[kernel.kallsyms]	[k] do_user_addr_fault
0,23%	0,00%	top-stencil	[kernel.kallsyms]	[k] handle_mm_fault
0,23%	0,00%	top-stencil	[kernel.kallsyms]	[k] __handle_mm_fault
0,23%	0,00%	top-stencil	[kernel.kallsyms]	[k] handle_pte_fault
0,21%	0,01%	top-stencil	libc.so.6	[.] __int_malloc
0,21%	0,00%	top-stencil	[kernel.kallsyms]	[k] do_anonymous_page
0,21%	0,21%	top-stencil	libc.so.6	[.] __mcount_internal
0,19%	0,19%	top-stencil	libstencil.so	[.] setup_mesh_cell_values
0,17%	0,01%	top-stencil	[kernel.kallsyms]	[k] __sysvec_apic_timer_interrupt
0,17%	0,00%	top-stencil	[kernel.kallsyms]	[k] hrtimer_interrupt
0,16%	0,00%	top-stencil	[kernel.kallsyms]	[k] __hrtimer_run_queues
0,16%	0,16%	top-stencil	libm.so.6	[.] 0x0000000000000e380
0,16%	0,00%	top-stencil	libm.so.6	[.] 0x00007f7ae8f37380
0,15%	0,00%	top-stencil	[kernel.kallsyms]	[k] entry_SYSCALL_64_after_hwframe

FIGURE 10 – perf screenshots

```

percent  B->cells[i][j - o][k].value / pow(17.0, (f64)o);
0,53    mov     -0x48(%r8,%r12,8),%r9
0,62    mov     %r13,%r8
        for (usz o = 1; o <= STENCIL_ORDER; ++o) {
0,12    add     $0x1,%r13
        shl     $0x4,%r8
        B->cells[i - o][j][k].value / pow(17.0, (f64)o);
0,28    mulsd   %xmm2,%xmm3
        C->cells[i][j][k].value += A->cells[i - o][j][k].value *
0,66    addsd   %xmm3,%xmm7
0,78    movsd   %xmm7, (%r14)
        C->cells[i][j][k].value += A->cells[i][j + o][k].value *
0,55    movsd   (%r11,%rbx,1),%xmm4
11,73    mulsd   (%rax,%rbx,1),%xmm4
10,03    mov     %r12,%r11
0,38    shl     $0x4,%r11
        B->cells[i][j + o][k].value / pow(17.0, (f64)o);
0,13    mulsd   %xmm2,%xmm4
        C->cells[i][j][k].value += A->cells[i][j + o][k].value *
1,60    addsd   %xmm7,%xmm4
2,28    movsd   %xmm4, (%r14)
        C->cells[i][j][k].value += A->cells[i][j - o][k].value *
0,68    movsd   (%r10,%rbx,1),%xmm1
3,57    mulsd   (%r9,%rbx,1),%xmm1
        B->cells[i][j - o][k].value / pow(17.0, (f64)o);

```

FIGURE 11 – perf screenshots

plotting cache misses compared to the first version .  
cache misses for opt1:17K  
cache misses for opt2:87K  
plotting cycles compared to the first version :  
execution cycles for opt1 : 17K  
execution cycles for opt2 : 90K

cache misses plotted for initial version and optimised version

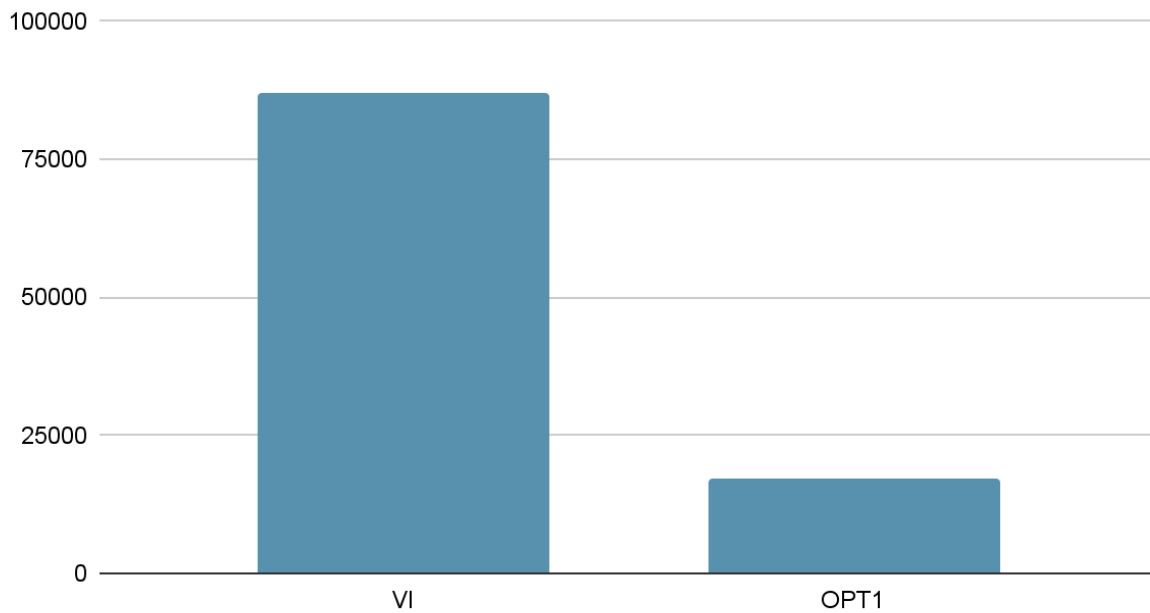


FIGURE 12 – cache misses plotted

### cycles per version

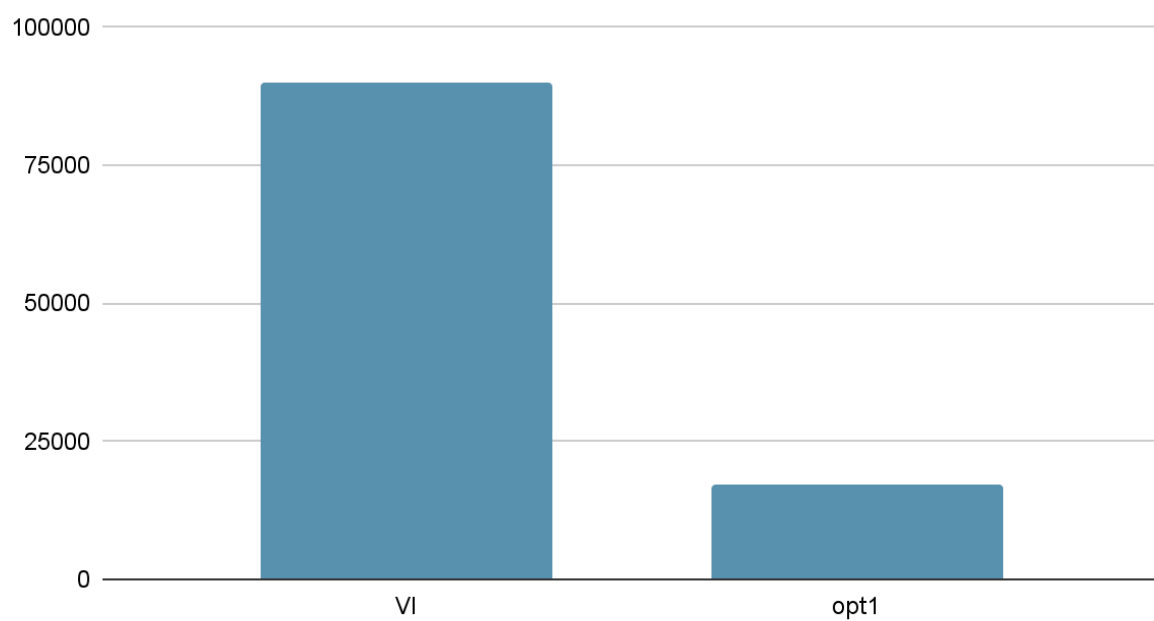


FIGURE 13 – cycles by optim

Result is 140.31% faster than Reference

### 3.1.2 Optimization Attempt 3 : Precomputing Pows

in our third optimization we wanted to reduce the cost of pow operations , so we precomputed the pows in a loop table (array) and just accessed hem along the mesh access

we recorded a small gain in performance.

R3 Ofast : Average = 336.3882

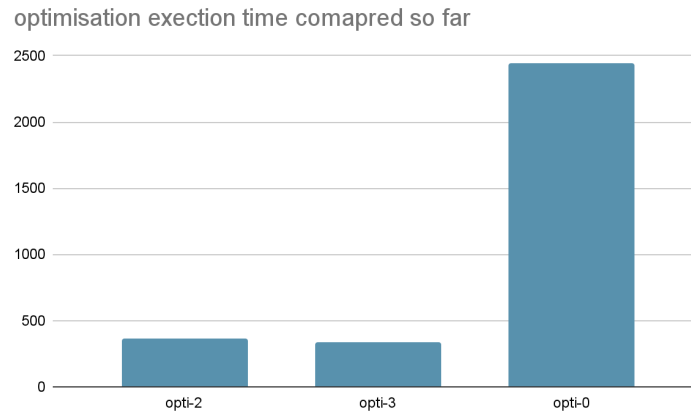


FIGURE 14 – Optimization comparison

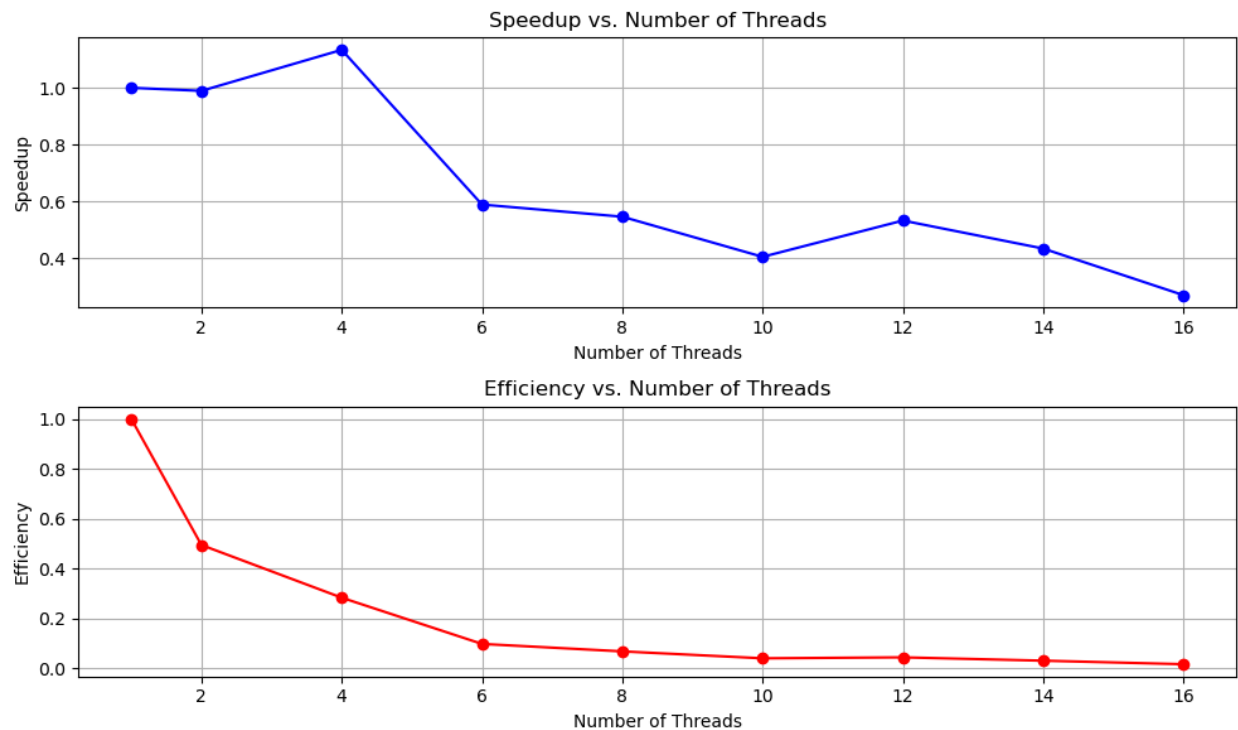


FIGURE 15 – scaling plot

Result is 199.43% faster than reference

### 3.1.3 Optimization Attempt 4 : Specific Pow Implementation

We took on from earlier, and we decided to optimize the pow function to see if there was a performance gain to accompany the pre-computation of the values. We did also gain a small chunk of performance from the specific pow implementation.

R4 Ofast Average = 298.5749

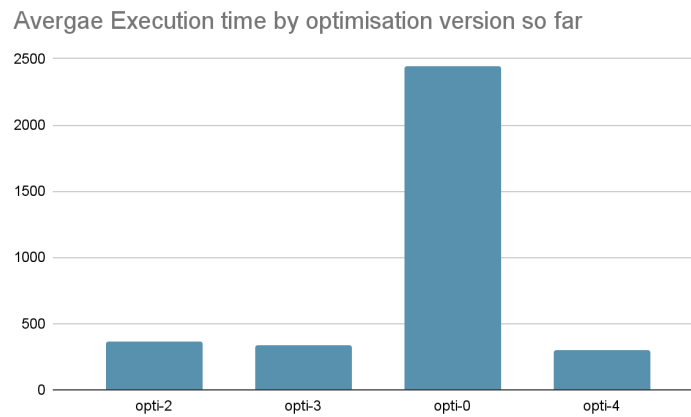


FIGURE 16 – scaling plot

Result is 253.37% faster than reference

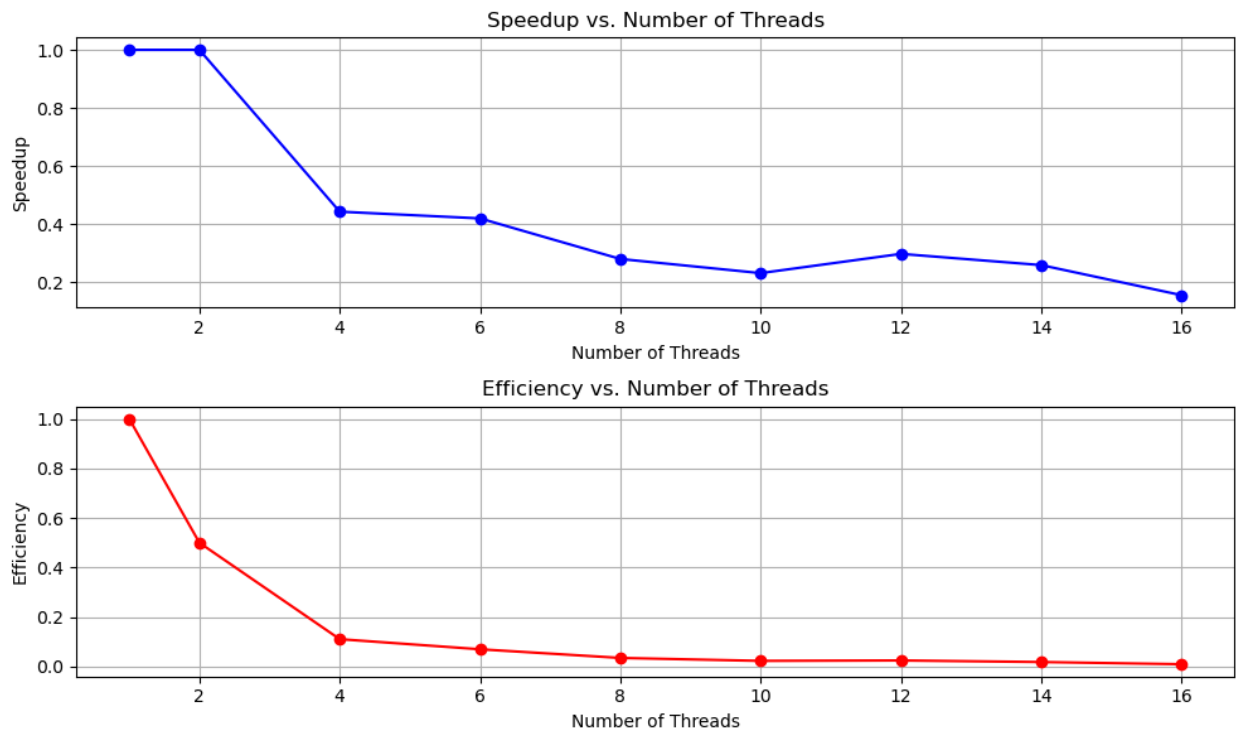


FIGURE 17 – scaling plot

### 3.1.4 Optimization Attempt 5 : implementing Cache blocking

Cache blocking is a fundamental optimization technique crucial for enhancing memory access patterns and reducing cache misses. By breaking down data into smaller blocks that fit efficiently into cache levels, cache blocking maximizes cache utilization and minimizes cache miss rates.

In our optimization endeavors, we integrated cache blocking techniques into our 5th attempt. Specifically, we adopted blocks of size 64 to mitigate cache misses and improve memory locality. This strategic optimization led to a discernible enhancement in performance, streamlining memory access patterns and reducing cache misses.

We varied the cache size from and measured the execution time. We found with size 16 we had the best average execution time and reduction in cache misses , the blocks were similar in all direction (x,y,z) , although we tried to experiment with a different access pattern such as 8 blocks in the X direction and 4 in both Y and Z directions and also an 8/1/1 split but it was much less effective for our problem size, it might be interesting in larger problems .

R5 Ofast Average with Cache size 64 = 218.8748 .

with cache size 8 we were able to obtain Average = 173.7580.

with cache size 4 the Average = 189.44

with cache size 16 the average is 169.1500

with cache size 32 the average is 200.8483

with cache size split of 8/4/4 the average is 193.3652

with cache size split of 8/1/1 the Average is 295.3421

We ran perf on this version and recorded these measures :

Performance counter stats for './top-stencil ../config.txt':

5662365742	cycles		
5031504709	instructions	#	0,89 insn per cycle
403648349	cache-references		
175646578	cache-misses	#	43,515 % of all cache refs
2,252407360	seconds time elapsed		
1,846213000	seconds user		
0,068229000	seconds sys		

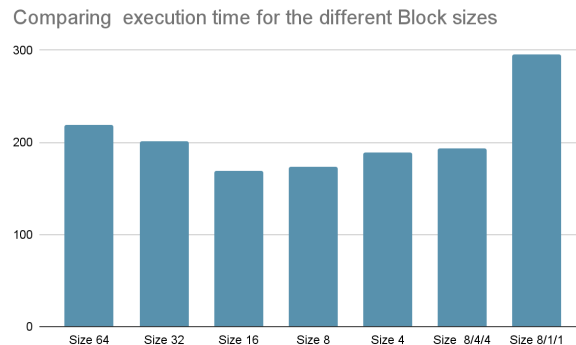


FIGURE 18 – block size comparison for cache-blocking

### 3.1.5 Optimization Attempt 6 : Rearranging Mesh Data structure

in this optimization attempt, we restructured the mesh data from a Structure of Arrays (SoA) to an Array of Structures (AoS). This change in data structure organization aimed to enhance memory locality and improve cache utilization.

After implementing the new AoS data structure, we observed a notable boost in performance. Specifically, when running with the 'Ofast' compiler optimization flag (referred to as 'R6 ofast'), we achieved an average execution time of 63.9168.

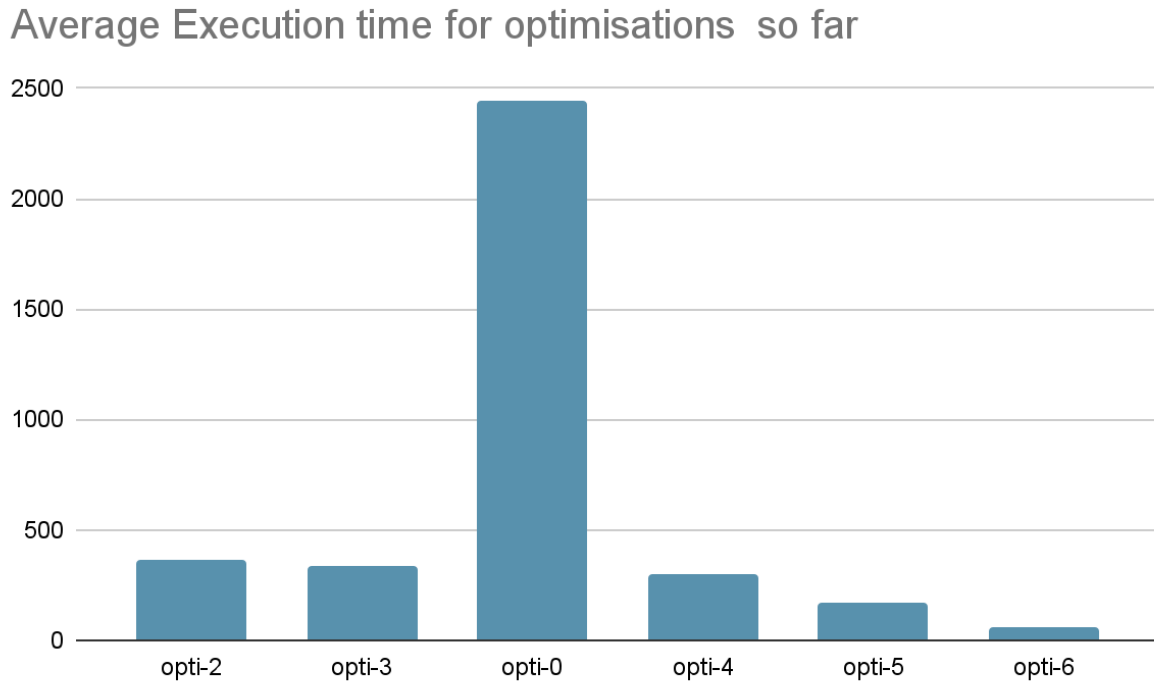


FIGURE 19 – execution time plots

### 3.1.6 Optimization Attempt 7 : using openmp

We used openmp in the solve jacobi to distribute the main loop. using the openmp collapse clause, which Specifies how many loops in a nested loop should be collapsed into one large iteration space and divided according to the schedule clause

execution time per number of thread.

```
2 threads: 54.9468
4 threads: 39.7711
6 threads: 45.7734
8 threads: 43.7071
10 threads: 45.0123
12 threads: 45.0981
14 threads: 43.747
16 threads: 44.783
```

Result is 2820.83% faster than reference



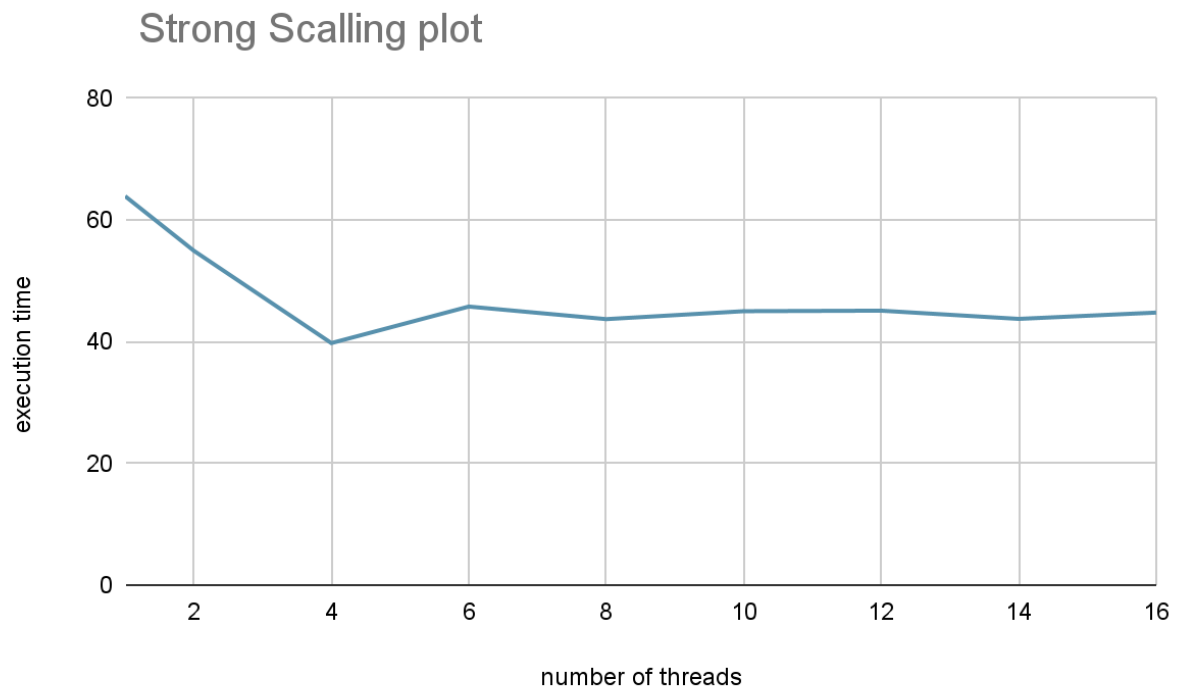


FIGURE 20 – scaling plot

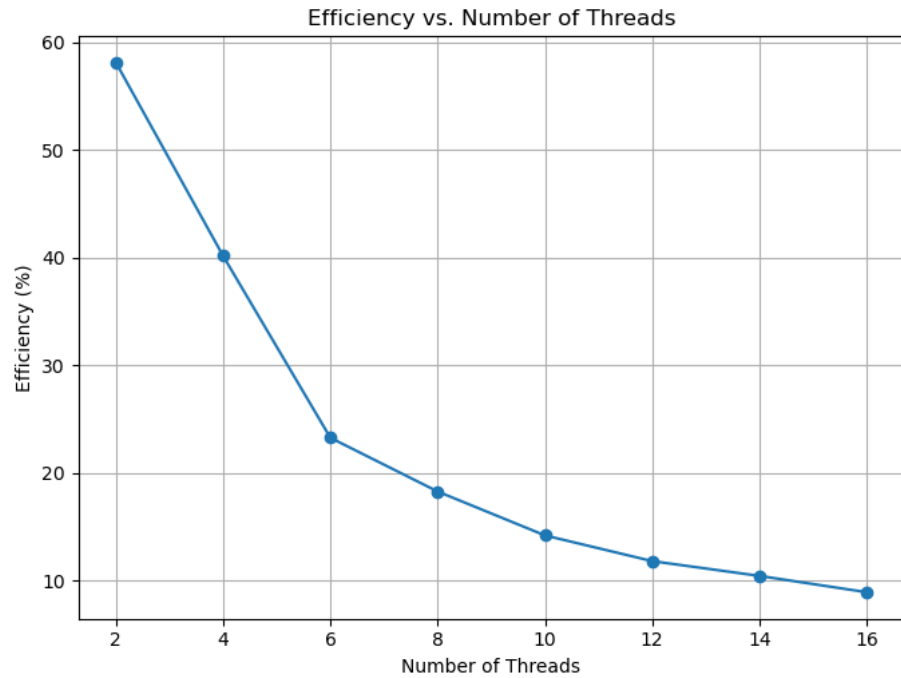


FIGURE 21 – efficiency/speed-up plot

## 4 Conclusion

In this report, we embarked on a journey to optimize the performance of our computational program through systematic strategies and targeted optimizations. Our efforts began by addressing compilation errors and optimizing compiler settings, laying the groundwork for subsequent optimization attempts.

Profiling tools such as 'perf' and 'gprof' provided invaluable insights into the program's performance characteristics, guiding our optimization endeavors. Through careful analysis of performance metrics, we identified key areas for improvement and devised targeted optimization strategies.

One notable optimization attempt involved restructuring the mesh data from a Structure of Arrays (SoA) to an Array of Structures (AoS). This data structure rearrangement aimed to enhance memory access patterns and cache utilization, resulting in a significant boost in performance. Leveraging compiler optimization flags, further contributed to the efficiency gains, culminating in a speedup of 2820.83

Our optimization journey underscores the importance of systematic optimization strategies in improving program performance. By addressing compilation errors, optimizing compiler settings, and leveraging profiling tools, we successfully enhanced the efficiency and speed of the program without radically changing the initial code. By changing the strategies we use to access and manipulate data we were able to see a huge jump in performance, and by going through the multiple techniques from removing redundancies and utilizing our cache line more effectively, we learned that the optimizations that net the biggest gain aren't always the most extravagant and complicated, but are simply the ones that show a fundamental understanding of what we were trying to achieve with our code whether its simulating natural phenomena or visualizing observations, It's important to leverage the understanding of your environment and its parameters. Due to time constraints, we left a chunk of performance on the table, especially with vectorizing our code to leverage 128 and 256 bit registers. These optimizations demonstrate the effectiveness of considering various factors such as memory access patterns, cache efficiency, and parallel scaling in achieving performance improvements in computational tasks.

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