Limits to Buyer Power: Input Prices and Pass-Through in Horizontal Merger Policy

Rebekah Dix[†] Todd Lensman[‡]

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Abstract

We re-examine the "buyer power" defense to horizontal mergers using models of price competition in which input prices are set before goods prices. We derive a measure of unilateral incentives to adjust input prices after a downstream merger, Input Pricing Pressure, and we use it to show that mergers often incentivize *higher* input prices. Consumer surplus-maximizing antitrust policy is often too lax when input prices are assumed fixed, and it should be biased against buyer power claims. In an empirical application to local retail beer markets, endogenizing input prices substantially raises the consumer harm from mergers of retailers.

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[†]MIT, rdix@mit.edu

[‡]MIT, tlensman@mit.edu

1 Introduction

The standard framework used by competition authorities to evaluate horizontal mergers emphasizes a trade-off between market power and cost efficiencies. As Williamson (1968) articulates, a merger between competitors reduces competition in the goods market and incentivizes the merged firm to raise prices, while reductions in marginal costs due to realized economies of scale or scope incentivize lower prices. If these *efficiencies* are large enough, the merger may lead to lower prices and higher consumer surplus. More recently, in several merger cases the defendants have argued for an alternative source of "efficiencies:" improved contracting terms vis-á-vis upstream firms.[1] As the defense goes, the merger between the downstream firms will improve their bargaining position and allow them to negotiate lower quality-adjusted input prices. The savings will be passed through to their customers, so the merger should be permitted on consumer surplus grounds.

In this paper, we evaluate the "buyer power" defense in models of price competition in which input prices are set before goods prices. If firms anticipate changes in goods prices when bargaining over input prices, we show that input prices often *increase* after a downstream merger, contrary to the buyer power defense. For settings in which such "upstream-leading" conduct is plausible, our results reveal limits to the buyer power defense and suggest that merger policy should be biased against claims that increased bargaining leverage will reduce input prices. We also show that input prices may rise so significantly after a merger that goods prices increase regardless of real efficiencies: A merger may be inherently anticompetitive once endogenous input prices are taken into account.

To illustrate the mechanism behind our results, and to demonstrate that it does not depend on bargaining per se, we focus primarily on the canonical model of vertical contracting in which upstream firms unilaterally set input prices (Spengler, 1950). That is, we suppose that downstream firms compete in goods prices holding input prices fixed, while upstream firms compete in input prices, anticipating the resulting changes in goods prices. We then consider a merger between two downstream firms and study an upstream firm's incentive to adjust its price after the merger. We find that this incentive depends crucially on how the merger alters downstream pass-through rates and the merged firm's production technology.

The intuition is as follows: With upstream-leading conduct, downstream pass-through rates are key determinants of input prices because they affect how changes in input prices are transmitted to downstream output and hence demand for inputs. After a downstream merger between single-product firms, pass-through rates adjust such that the output of either down-

[1] The most prominent example is the proposed merger between Anthem and Cigna, two of the four largest health insurers in the United States. See the district court's opinion (*United States v. Anthem, Inc.*, 2017) for a discussion of the defendants' argument.

stream good becomes more sensitive to its own marginal cost, but less sensitive to the marginal cost of the other good. If an upstream firm's input is used to produce only one good, these adjustments provide an incentive for the firm to lower its price. This holds because the upstream firm effectively observes more elastic demand for its input after the merger. If instead the input is used to produce both goods, the upstream firm often has an incentive to raise its price. In either case, if the merged firm becomes more efficient in its use of the input, downstream output always becomes less sensitive to the input price, and the upstream firm has an additional incentive to increase its price. Altogether our results indicate that with upstream-leading conduct, downstream mergers are more likely to raise goods prices and must yield additional real efficiencies to leave consumers unharmed.

For simplicity, we prove these results in a stylized model with a merger to monopoly downstream, linear consumer demand, and Leontief production technologies. However, we provide additional theoretical results and simulations that show our conclusions generalize with (i) arbitrary production technologies, (ii) competition from non-merging downstream firms, and (iii) nonlinear demand. We also show that the key economic mechanism behind our results is still present when we allow (upstream-leading) bargaining over input prices. In this case, simulations show that our pass-through and efficiency effects often dominate the standard bargaining leverage effect studied in the literature reviewed below, so that input prices rise after a downstream merger.

We contribute to two lines of research in industrial organization and antitrust law related to buyer power. The first assesses the basic argument that horizontal mergers can improve the merging firms' bargaining leverage and reduce input prices, and it belongs to a larger literature that studies the effects of horizontal market structure in settings with vertical contracting. Studies in this larger literature generally estimate structural models of price competition and bargaining in which downstream firms simultaneously choose goods prices and bargain with upstream firms over input prices (Crawford and Yurukoglu, 2012; Gowrisankaran, Nevo, and Town, 2015; Ho and Lee, 2017; Crawford, Lee, Whinston, and Yurukoglu, 2018; Ho and Lee, 2019; see Lee, Whinston, and Yurukoglu, 2021 for a review). Sheu and Taragin (2021) explicitly address the buyer power defense, and broadly they show that a downstream merger can indeed raise the merging firms' bargaining leverage relative to an upstream firm by altering each party's outside option. Given this positive finding, the second line of research asks if, under existing legal frameworks for antitrust, the resulting decline in input prices should provide a legitimate defense ("cognizable efficiency") in merger enforcement cases (Carlton and Israel, 2011; Hemphill and Rose, 2017). We depart from these studies by assuming that input prices are set before goods prices, the traditional timing assumption in the vertical contracting literature. In this case, we characterize new effects of pass-through rates and efficiencies on input prices, and we demonstrate that input prices often rise after downstream mergers. Our results rebut the argument that input prices necessarily fall due to changes in bargaining leverage, and they support additional skepticism about buyer power claims.

We also contribute to a smaller theoretical literature that considers the effects of downstream market structure on input prices. These studies share our focus on upstream-leading conduct, but we differ from them in two key respects. First, to maintain tractability these papers generally assume that downstream firms are symmetric (Dobson and Waterson, 1997; Iozzi and Valletti, 2014; Gaudin, 2018). This assumption obscures the mechanisms by which downstream market structure affects input pricing incentives, and by dispensing with it we can describe how input prices are expected to change under a more general set of conditions. Second, these papers generally do not draw implications for consumer surplus-maximizing merger policy.[2] In our main results, we focus on mergers that are neutral with respect to consumer surplus if input prices are held fixed, and we describe conditions under which such mergers will raise or lower goods prices *because of endogenous input prices*. We argue that consumer surplus-maximizing merger policy assuming fixed input prices is often too lax when input prices are actually endogenous.

To organize our analysis, in Section 2 we extend the first-order approach to merger analysis of Farrell and Shapiro (2010) and Jaffe and Weyl (2013) to incorporate endogenous input prices. When price-setting conduct is upstream-leading, we provide an approximation formula for post-merger price changes that strictly generalizes that of Jaffe and Weyl (2013) and is expressed in sufficient statistics. We define a new term, Input Pricing Pressure (IPP), that is analogous to the Generalized Pricing Pressure (GePP) of Jaffe and Weyl (2013) and precisely measures the incentives for input prices to adjust after the merger. In Section 3, we study IPP in a simple model to demonstrate how changes in pass-through rates and production technologies may incentivize input price changes after a downstream merger. We also consider more general models and show that the mechanism we emphasize is robust to a number of assumptions.

In Section 4, we provide an empirical application of our theory to local retail beer markets. We calibrate logit demand systems for retail beer products using Nielsen scanner data, and we impute a simple distribution network by which upstream distributors sell beer to retailers at wholesale prices. We follow Villas-Boas (2007) to calculate distributor marginal costs, and we simulate mergers between beer retailers both with and without endogenous wholesale prices. Even when the mergers generate no efficiencies, retail prices increase more post-merger when we allow wholesale prices to adjust endogenously, complementing our theoretical findings. Section 5 concludes.

[2]For example, Lommerud, Straume, and Sørgard (2005) focus primarily on determining how endogenous input prices affect the profitability of downstream mergers.

2 First-Order Approach and Input Pricing Pressure

In this section, we introduce our general model and show how the first-order approach to merger analysis extends to settings with endogenous input prices. We initially place only weak conditions on price-setting conduct, and we clarify the assumptions needed to obtain approximation formulas for post-merger price changes in sufficient statistics (Proposition 2). In Section 3, we use one of these statistics, Input Pricing Pressure, to study how downstream mergers can incentivize input price changes in a class of simple models.

2.1 Setup

The economy exists in partial equilibrium and has two sets of products. Downstream products, or *goods*, are indexed by $g \in \mathcal{G}$ and sold to consumers. The set of downstream firms is $\mathcal{F}_{\mathcal{G}}$, where each firm $G \in \mathcal{F}_{\mathcal{G}}$ is identified with the set of goods that it produces, $G \subseteq \mathcal{G}$. Upstream products, or *inputs*, are indexed by $i \in \mathcal{I}$ and sold to downstream firms. The set of upstream firms is $\mathcal{F}_{\mathcal{I}}$, where each firm $I \in \mathcal{F}_{\mathcal{I}}$ is identified with the set of inputs it produces, $I \subseteq \mathcal{I}$.

All transactions are facilitated by linear prices, and we let $p := (p_g)_{g \in \mathcal{G}}$ and $w := (w_i)_{i \in \mathcal{I}}$ denote the price vectors for goods and inputs. The production technology for each downstream firm G is described by the cost function $C_G(y_G, w)$, and similarly each upstream firm I has cost function $C_I(z_I)$. Here $y_G := (y_g)_{g \in \mathcal{G}}$ and $z_I := (z_i)_{i \in I}$ denote the output vectors for firms G and I. Consumer demand for good g is $y_g(p)$, and the demand for input i from downstream firm G is given by Shephard's Lemma, $\partial C_G(y_G, w)/\partial w_i$. Total demand for input i is equal to the sum of the input demands from downstream firms:

$$z_i(y, w) := \sum_{G \in \mathcal{F}_{\mathcal{G}}} \frac{\partial C_G(y_G, w)}{\partial w_i}.$$

To illustrate the generality of the first-order approach, and to demonstrate how different conduct assumptions affect our analysis, we initially make only mild assumptions about equilibrium price setting. We suppose that equilibrium prices (p^*, w^*) are given by the solution to a system of equations:

$$0 = f(p^*, w^*) := \begin{pmatrix} f_{\mathcal{G}}(p^*, w^*) \\ f_{\mathcal{I}}(p^*, w^*) \end{pmatrix}. \tag{1}$$

Here $f_{\mathcal{G}}(p, w)$ is the $\mathbb{R}^{\mathcal{G}}$-valued *first-order condition (FOC) function* for goods prices. In the examples of conduct described below, $f_{\mathcal{G}}$ corresponds precisely to the first-order conditions of maximization problems whose solutions determine the goods prices p^*. Similarly, $f_{\mathcal{I}}(p, w)$

is the FOC function for input prices. We assume that the "stacked" FOC function f is twice continuously differentiable and that the system (1) has a unique solution.

2.2 Conduct Examples

Several common modes of conduct can be represented by a system of the form (1). Here we describe a few examples that we analyze in detail in Section 3, and we give additional examples in Appendix C.[3]

Our benchmark mode of conduct for downstream firms is Bertrand-Nash competition, whereby each firm $G \in \mathcal{F}_{\mathcal{G}}$ chooses its prices $p_G := \left(p_g\right)_{g \in G}$ to maximize its profits, holding all remaining prices fixed:

$$\max_{p_G \geq 0} p_G \cdot y_G(p) - C_G(y_G(p), w). \tag{2}$$

Suppressing the dependence of y on p, the FOC function for good $g \in G$ is then

$$f_g(p, w) = y_g + \sum_{g' \in G} \left(p_{g'} - c_{g'}(y_G, w)\right) \frac{\partial y_{g'}}{\partial p_g}.$$

Here $c_{g'}(y_G, w) := \partial C_G(y_G, w) / \partial y_{g'}$ denotes the marginal cost of good g'. Our framework also accommodates a number of other downstream equilibrium concepts using conjectural variations; see Jaffe and Weyl (2013) for details.

More importantly, our analysis allows for many different conduct assumptions for upstream firms. For example, upstream firms may also compete à la Bertrand-Nash, holding goods prices and remaining input prices fixed when maximizing profits. Firm $I \in \mathcal{F}_{\mathcal{I}}$ then solves

$$\max_{w_I \geq 0} w_I \cdot z_I(y(p), w) - C_I(z_I(y(p), w)). \tag{3}$$

The corresponding FOC function for input $i \in I$ is

$$f_i(p, w) = z_i(y, w) + \sum_{i' \in I} (w_{i'} - c_{i'}(z_I(y, w))) \frac{\partial z_{i'}(y, w)}{\partial w_i}. \tag{4}$$

Here $c_{i'}(z_I(y, w))$ denotes the marginal cost of input i'.

Alternatively, in the spirit of the successive monopoly model common in the literature on vertical contracting, we could suppose that upstream firms anticipate changes in goods prices

[3] Throughout, we assume that the equilibrium in each price-setting game is unique and characterized by interior first-order conditions.

when choosing input prices. Let $p^*(w)$ denote the equilibrium goods prices given input prices w, obtained as the solution to the system of equations

$$0 = f_{\mathcal{G}}(p^*(w), w).$$

Then the demand function for input i observed by firm I is

$$z_i(w) := z_i(y(p^*(w)), w).$$

Note that firm I now expects downstream firms to adjust to input price changes along two margins: (i) substitution between inputs in production and (ii) changes in goods prices. Only the first margin operates when upstream firms engage in Bertrand-Nash competition, because they hold goods prices constant when choosing input prices. In this "upstream-leading" price-setting game, each upstream firm I now solves

$$\max_{w_I \geq 0} w_I \cdot z_I(w) - C_I(z_I(w)). \tag{5}$$

This problem is identical to (3), but with the demand function $z_I(w)$ in place of $z_I(y, w)$. The corresponding FOC function for input $i \in I$ is

$$f_i(w) = z_i(w) + \sum_{i' \in I} (w_{i'} - c_{i'}(z_I(w))) \frac{dz_{i'}(w)}{dw_i}. \tag{6}$$

Note two key differences from the Bertrand-Nash upstream FOC function (3): First, the total derivative $dz_{i'}(w)/dw_i$ appears in the second term, indicating that downstream price responses as well as input substitution are taken into account by firm I.[4] Second, the FOC function f_i depends only on input prices w. This property is common to all upstream-leading modes of conduct, and we will see that it substantially simplifies the first-order approach to merger analysis with endogenous input prices.

Following the seminal contribution of Horn and Wolinsky (1988), a large literature has explored the implications of bilateral bargaining in input markets for classic questions about the effects of horizontal and vertical mergers, price discrimination, and exclusive dealing (see Lee et al., 2021 for a review). Our framework can also accommodate standard versions of the "Nash-in-Nash bargaining" solution concept of Horn and Wolinsky (1988) that appears in many recent analyses of vertical contracting environments.[5] For brevity, we leave details about

[4] Here $\frac{dz(w)}{dw} := \frac{\partial z(y(p^*(w)),w)}{\partial y} \frac{\partial y(p^*(w))}{\partial p} \frac{\partial p^*(w)}{\partial w} + \frac{\partial z(y(p^*(w)),w)}{\partial w}$.

[5] Examples include Draganska, Klapper, and Villas-Boas (2010); Crawford and Yurukoglu (2012); Grennan (2013); Gowrisankaran et al. (2015); Ho and Lee (2017); Crawford et al. (2018); and Dubois and Sæthre (2020).

these solution concepts and their FOC functions in Appendix C.

These examples indicate that our setting can accommodate many conduct assumptions found in the literature on vertical contracting. However, it is not completely general. We make the strong assumption that all transactions are mediated by linear prices, ruling out nonlinear pricing schedules as well as quantity-based bargaining games. This assumption limits the generality of our results, but it provides a reasonable and tractable starting point for understanding the effects of vertical relationships on horizontal mergers.

2.3 Downstream Merger

Our counterfactual of interest is a merger between two downstream firms. Let $G_M, G_{M'} \in \mathcal{F}_{\mathcal{G}}$ denote the merging firms, let $M := G_M \cup G_{M'}$ denote the merged firm, and let $\hat{\mathcal{F}}_{\mathcal{G}}$ denote the set of downstream firms following the merger.[6]

The merger may have two effects on the economic environment. First, the merged firm may obtain a new production technology by reallocating production across plants, attaining economies of scale or scope, engaging in cost-reducing investments after the merger, or from a number of other unmodeled sources. We capture these *efficiencies* in a reduced-form way by supposing that merged firm attains a new cost function $C_M(y_M, w)$. As a by-product of this technological change, the upstream firms may now observe a different demand function for each input i:

$$\hat{z}_i(y, w) := \sum_{G \in \hat{\mathcal{F}}_{\mathcal{G}} \setminus \{M\}} \frac{\partial C_G(y_G, w)}{\partial w_i} + \frac{\partial C_M(y_M, w)}{\partial w_i}.$$

Second, the merger may alter price-setting conduct. For example, if downstream firms engage in Bertrand-Nash competition, the merger allows the merged firm to jointly maximize profits over the prices $p_M = (p_{G_M}, p_{G_{M'}})$. In the absence of marginal cost-reducing efficiencies, this conduct change generally implies higher equilibrium goods prices for fixed input prices when goods are substitutes in consumption (Deneckere and Davidson, 1985). As we discuss at length in Section 3, changes in downstream conduct may also indirectly change upstream conduct. For example, the downstream equilibrium price function $p^*(w)$ is a key input for any of the upstream-leading modes of conduct described in Section 2.2 or Appendix C. If downstream conduct changes following the merger, the equilibrium price function $\hat{p}^*(w)$ will also change, and input prices will adjust accordingly. To capture these conduct changes while remaining agnostic about their details, we suppose that post-merger equilibrium prices $(\hat{p}^*, \hat{w}^*)$ are given

[6] We denote post-merger objects using the decoration "^".

by the solution to the system of equations

$$0 = \hat{f}\left(\hat{p}^*, \hat{w}^*\right) := \begin{pmatrix} \hat{f}_{\mathcal{G}}\left(\hat{p}^*, \hat{w}^*\right) \\ \hat{f}_{\mathcal{I}}\left(\hat{p}^*, \hat{w}^*\right) \end{pmatrix}. \tag{7}$$

Here $\hat{f}_{\mathcal{G}}$ and $\hat{f}_{\mathcal{I}}$ are the post-merger FOC functions for goods prices and input prices. Just as before the merger, we assume that the stacked FOC function $\hat{f}$ is twice continuously differentiable and that the system (7) has a unique solution.

2.4 First-Order Approach: Generalized and Input Pricing Pressure

The first-order approach to merger analysis hews closely to the first-order approach to general comparative statics initiated by Samuelson (1947). In both cases, we attempt to understand how equilibrium objects adjust to changes in the economic environment by studying the resulting changes in the first-order conditions that characterize equilibrium. The key difference is that the first-order approach to comparative statics provides *exact* changes in equilibrium objects in response to *marginal* changes in the environment, whereas the first-order approach to merger analysis provides *approximate* changes in equilibrium prices in response to *non-marginal* changes in conduct and production technologies. In this section, we demonstrate how the first-order approach to merger analysis pioneered by Farrell and Shapiro (2010) and Jaffe and Weyl (2013) generalizes to economies with endogenous input prices.

The task to be solved by the first-order approach is as follows: Given a pre-merger equilibrium (p^*, w^*) and a post-merger FOC function $\hat{f}$, how can we approximate the post-merger equilibrium $(\hat{p}^*, \hat{w}^*)$, and what properties of price-setting conduct influence the direction and magnitude of price changes? We assume that equilibria are determined by FOC systems (1, 7), so we can make progress on these questions by comparing the pre- and post-merger FOC functions. This is the basis for the main approximation result of the first-order approach:

Proposition 1. Suppose $\hat{f}$ is invertible. Then the post-merger price changes are

$$\left(\hat{p}^* - p^*, \hat{w}^* - w^*\right) = -\left[D\hat{f}\left(p^*, w^*\right)\right]^{-1}\left(\hat{f}\left(p^*, w^*\right) - f\left(p^*, w^*\right)\right) + R, \tag{8}$$

where the error vector R satisfies

$$R_k = \frac{1}{2}\sum_{l,m=1}^{|\mathcal{G}|+|\mathcal{I}|} \frac{\partial^2 \hat{f}_k^{-1}(r)}{\partial r_l \partial r_m}\left(\hat{f}_l\left(p^*, w^*\right) - f_l\left(p^*, w^*\right)\right)\left(\hat{f}_m\left(p^*, w^*\right) - f_m\left(p^*, w^*\right)\right) \tag{9}$$

for some vector $r \in \left[0, \hat{f}\left(p^*, w^*\right)\right]$.

Like Theorem 1 in Jaffe and Weyl (2013), the result follows from using the post-merger FOC system (7) to write

$$(\hat{p}^* - p^*, \hat{w}^* - w^*) = \hat{f}^{-1}(0) - \hat{f}^{-1}\left(\hat{f}(p^*, w^*)\right),$$

and then linearizing $\hat{f}^{-1}$ around $\hat{f}(p^*, w^*)$.[7] The error formula (9) indicates that the approximation is accurate when the post-merger FOC function $\hat{f}$ has little curvature or when the pre-merger prices (p^*, w^*) nearly satisfy the post-merger FOC system (7); it is exact when $\hat{f}$ is linear or in the trivial case when the pre- and post-merger equilibria coincide. These conditions are unsurprising given that, as Jaffe and Weyl (2013) recognize, the approximation is the first step in the Newton-Raphson algorithm when finding the root of $\hat{f}$ beginning at (p^*, w^*).

Provided that the error is small, the calculation (8) demonstrates that post-merger equilibrium prices can be approximated with only knowledge of post-merger price-setting conduct $\hat{f}$ around the pre-merger equilibrium (p^*, w^*). More importantly, it also provides some insight as to the factors that determine the direction and magnitude of price changes after the merger. In particular, the prices changes are jointly determined by (i) the change in the FOC function at the pre-merger equilibrium and (ii) the sensitivity of the post-merger FOC function to prices. The former measures the unilateral incentives for firms to adjust prices, while the latter incorporates price responses between products and measures the size of price changes needed to reach the post-merger equilibrium.

To convey this intuition more clearly and to connect our first-order approach to that of Farrell and Shapiro (2010) and Jaffe and Weyl (2013), we can express the basic approximation formula in terms of interpretable sufficient statistics: pass-through rates, the Generalized Pricing Pressure (GePP) vector of Jaffe and Weyl (2013), and the Input Pricing Pressure (IPP) vector that we introduce below. To do this, we make two additional assumptions:

Assumption 1. FOC functions for input prices do not depend directly on goods prices p, and we reparametrize them by

$$f_{\mathcal{I}}(w, c_{\mathcal{I}}(z_{\mathcal{I}}(w))) \quad \text{and} \quad \hat{f}_{\mathcal{I}}(w, c_{\mathcal{I}}(\hat{z}_{\mathcal{I}}(w))).$$

Assumption 2. Input prices w only affect the FOC functions for goods prices through marginal costs, and we reparametrize them by

$$f_{\mathcal{G}}\left(p, c_{\mathcal{G}}\left(y_{\mathcal{G}}(p), w\right)\right) \quad \text{and} \quad \hat{f}_{\mathcal{G}}\left(p, \hat{c}_{\mathcal{G}}\left(y_{\mathcal{G}}(p), w\right)\right).$$

[7] The assumption that $\hat{f}$ is invertible is the "non-marginal" analogue of the local invertibility assumption in marginal comparative statics.

As we discussed in Section 2.2, Assumption 1 holds for upstream-leading modes of conduct. Assumption 2 is fairly innocuous; it is satisfied, for example, when downstream firms engage in Bertrand-Nash competition (or one of its conjectural versions). We maintain these assumptions in the remainder of the paper.

We can now define the sufficient statistics that appear in the approximation formulas below. The post-merger pass-through rate functions for downstream and upstream firms are, respectively,

$$\begin{aligned}\hat{\rho}_{\mathcal{G}}(p,w) &:= -\left[\frac{d\hat{f}_{\mathcal{G}}\left(p, c_{\mathcal{G}}\left(y_{\mathcal{G}}(p), w\right)\right)}{dp}\right]^{-1} \frac{\partial \hat{f}_{\mathcal{G}}\left(p, c_{\mathcal{G}}\left(y_{\mathcal{G}}(p), w\right)\right)}{\partial c_{\mathcal{G}}}, \\ \hat{\rho}_{\mathcal{I}}(w) &:= -\left[\frac{d\hat{f}_{\mathcal{I}}(w, c_{\mathcal{I}}(z_{\mathcal{I}}(w)))}{dw}\right]^{-1} \frac{\partial \hat{f}_{\mathcal{I}}(w, c_{\mathcal{I}}(z_{\mathcal{I}}(w)))}{\partial c_{\mathcal{I}}}.\end{aligned} \tag{10}$$

Evaluated at the post-merger equilibrium, $\hat{\rho}_{\mathcal{G}}(\hat{p}^*, \hat{w}^*) \times t_{\mathcal{G}}$ describes the change in post-merger goods prices after introducing a vector $t_{\mathcal{G}}$ of small specific taxes on goods, holding input prices fixed. Similarly, $\hat{\rho}_{\mathcal{I}}(\hat{w}^*) \times t_{\mathcal{I}}$ describes the change in post-merger input prices in response to a vector $t_{\mathcal{I}}$ of small specific taxes on inputs, but allowing goods prices to vary.[8] The key "merger-specific" statistics are the Generalized Pricing Pressure and Input Pricing Pressure vectors:

$$\mathrm{GePP}(w) := \left[\frac{\partial \hat{f}_{\mathcal{G}}}{\partial c_{\mathcal{G}}}\right]^{-1} \left(\hat{f}_{\mathcal{G}} - f_{\mathcal{G}}\right), \tag{11}$$

where all functions are evaluated at prices $(p^*(w), w)$, and

$$\mathrm{IPP} := \left[\frac{\partial \hat{f}_{\mathcal{I}}}{\partial c_{\mathcal{I}}}\right]^{-1} \left(\hat{f}_{\mathcal{I}} - f_{\mathcal{I}}\right), \tag{12}$$

where all functions are evaluated at input prices w^*. $\mathrm{GePP}(w)$, introduced by Jaffe and Weyl (2013) to generalize the Upward Pricing Pressure heuristic of Farrell and Shapiro (2010), is essentially the change in the FOC function for goods prices after the merger, but renormalized in "units" of downstream marginal costs. The change in the FOC function measures the incentives for firms to adjust prices after the merger; the normalization allows us to conceptualize these incentives as a simple change in marginal costs. IPP similarly measures the incentives for upstream firms to adjust input prices after the merger, in units of upstream marginal costs.

The next proposition gives our approximation result, a refinement of Proposition 1.

[8] The pre-merger pass-through rate functions $\rho_{\mathcal{G}}(p,w)$ and $\rho_{\mathcal{I}}(w)$ are defined similarly using the pre-merger FOC functions $f_{\mathcal{G}}$ and $f_{\mathcal{I}}$.

Proposition 2. Suppose $\hat{f}_{\mathcal{G}}\left(\cdot, c_{\mathcal{G}}\left(y_{\mathcal{G}}(\cdot), w^*\right)\right)$ and $\hat{f}_{\mathcal{I}}\left(\cdot, \hat{c}_{\mathcal{I}}\left(\hat{z}_{\mathcal{I}}(\cdot)\right)\right)$ are invertible. Then the post-merger price changes are

$$\hat{p}^* - p^* \quad = \quad \hat{\rho}_{\mathcal{G}} \times \text{GePP} + \hat{\rho}_{\mathcal{G}} \times \frac{\partial \hat{c}_{\mathcal{G}}}{\partial w} \times \hat{\rho}_{\mathcal{I}} \times \text{IPP} + R_{\mathcal{G}}, \tag{13}$$

$$\hat{w}^* - w^* \quad = \quad \hat{\rho}_{\mathcal{I}} \times \text{IPP} + R_{\mathcal{I}}, \tag{14}$$

where the error vector $R = \left(R_{\mathcal{G}}, R_{\mathcal{I}}\right)$ coincides with (9) and all functions are evaluated at the pre-merger equilibrium (p^*, w^*).

The proof is found in Appendix A and makes use of the observation that the post-merger FOC system (7) is block-diagonal when $\hat{f}_{\mathcal{I}}$ does not depend directly on goods prices (Assumption 1).[9] To interpret the formulas in Proposition 2, we begin with the simpler equation (14). This equation implies that the change in input prices after the merger can be approximated by the IPP vector multiplied by the post-merger pass-through rate function, evaluated at pre-merger prices. This is the basis of our claim that IPP quantifies the incentives for input price changes in units of upstream marginal costs. Put differently, $\text{GePP}(w^*)$ and IPP are exactly the vectors of subsidies that must be applied after the merger for all prices stay fixed. The first-order effect of the merger on input prices can be calculated by removing the fictitious IPP subsidies and applying the pass-through rates $\hat{\rho}_{\mathcal{I}}(w^*)$, which are exact for marginal changes around the subsidized equilibrium. Error arises because removing the IPP subsidies is a non-marginal change in the environment, so the "locally exact" pass-through rates $\hat{\rho}_{\mathcal{I}}(w^*)$ may not perfectly describe the effect on input prices.[10]

The first two terms in (13) have similar interpretations. These are most easily understood by analogy to the following exact decomposition of the post-merger change in goods prices:

$$\hat{p}^*(\hat{w}^*) - p^*(w^*) \quad = \quad \underbrace{\hat{p}^*(w^*) - p^*(w^*)}_{\substack{\text{merger with fixed} \\ \text{input prices}}} \quad + \quad \underbrace{\hat{p}^*(\hat{w}^*) - \hat{p}^*(w^*)}_{\substack{\text{effect of endogenous} \\ \text{input prices}}}. \tag{15}$$

The first term in this decomposition is the post-merger change in goods prices when input prices are held fixed. Standard analyses of horizontal mergers describe this object, which is crucial for understanding the effect of mergers on consumer surplus. Theorem 1 of Jaffe and

[9] This assumption eliminates a feedback loop between the changes in goods prices and the changes in input prices after the merger. This feedback loop significantly complicates the basic approximation (8) and does not permit a general approximation formula in easily interpretable sufficient statistics.

[10] By redefining $\text{GePP}(w)$ and IPP, we can use other pass-through rates in place of $\hat{\rho}_{\mathcal{G}}(p^*, w^*)$ and $\hat{\rho}_{\mathcal{I}}(w^*)$ in (13, 14). We use these pass-through rates and the associated definitions of $\text{GePP}(w^*)$ and IPP so that (i) no knowledge of post-merger equilibrium prices is needed to compute the approximations (13, 14) and (ii) we can interpret $\text{GePP}(w^*)$ and IPP as the compensating subsidies needed to keep all prices fixed post-merger.

Weyl (2013) establishes that the first term in (13) provides a first-order approximation:

$$\hat{p}^*(w^*) - p^*(w^*) \quad \approx \quad \hat{\rho}_{\mathcal{G}}(p^*, w^*) \times \text{GePP}(w^*).$$

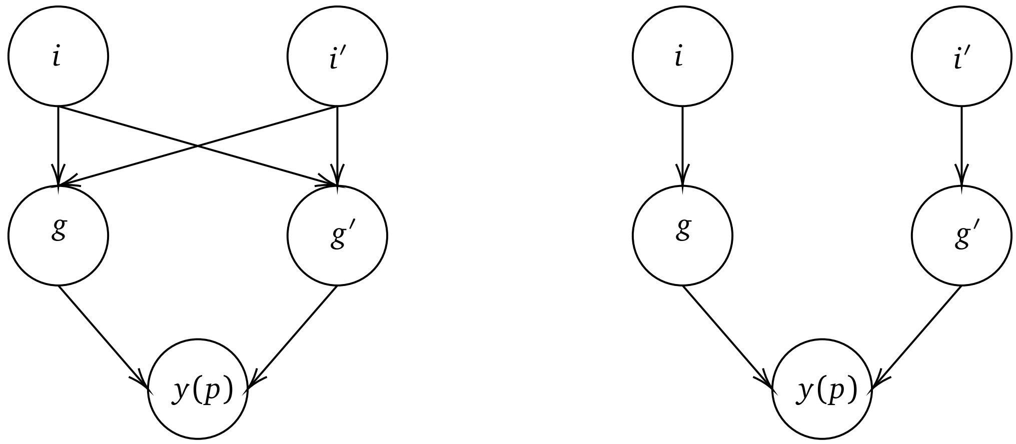
This result is formally a special case of Proposition 2 with $\hat{f}_{\mathcal{I}}(w) = w - w^*$. Our generalization shows that a similar approximation applies to the second term in (15), which describes the additional change in goods prices resulting from endogenous changes in input prices after the merger. The corresponding approximation is exactly the second term in (13):

$$\hat{p}^*(\hat{w}^*) - \hat{p}^*(w^*) \quad \approx \quad \hat{\rho}_{\mathcal{G}}(p^*, w^*) \times \frac{\partial \hat{c}_{\mathcal{G}}(y(p^*), w^*)}{\partial w} \times \hat{\rho}_{\mathcal{I}}(w^*) \times \text{IPP}.$$

We can then determine the effect of endogenous input prices by calculating the (approximate) changes in input prices $\hat{\rho}_{\mathcal{I}} \times \text{IPP}$, converting them into shifts in downstream marginal costs using the derivative $\partial \hat{c}_{\mathcal{G}} / \partial w$, and applying the downstream pass-through rates $\hat{\rho}_{\mathcal{G}}$.

We find the approximation formulas of Proposition 2 useful because they cleanly describe how post-merger changes in the economic environment map into price changes. For example, pass-through rates are the subject of a substantial theoretical and empirical literature, so we may feel comfortable conjecturing values for $\hat{\rho}_{\mathcal{G}}(p^*, w^*)$ and $\hat{\rho}_{\mathcal{I}}(w^*)$ for a merger of interest. All that remains to implement the approximations (13, 14) is to determine (or guess) how sensitive marginal costs are to input prices post-merger and how the merger affects pricing incentives in marginal cost-equivalent terms. We admit that this is more information-intensive than implementing the approximations of Farrell and Shapiro (2010) and Jaffe and Weyl (2013) with fixed input prices, but even simple observations provide information about the likely direction of input price changes. For example, under many conduct assumptions the own-price pass-through rates $\hat{\rho}_{ii}(w^*)$ are the largest entries in $\hat{\rho}_{\mathcal{I}}(w^*)$, so just examining IPP_i can provide a good indication of the likely change in the price of input i. This is the premise of our theoretical analysis in Section 3.

Nonetheless, we view the primary contribution of Proposition 2 as conceptual: With linear pricing and upstream-leading conduct, the economics of horizontal mergers are not fundamentally different with endogenous input prices. Rather, we can simply adjust our standard estimate for the consumer price effect of mergers by an additive term that captures the effect of input price changes. We study this adjustment theoretically in simple economies in Section 3, and we provide a quantitative analysis using a calibrated demand system and a realistic input-output structure in Section 4.



(a) general (b) independent input suppliers

Figure 1: Input-output configurations in the simple model.

3 Merger Policy and Input Prices

In this section, we apply Proposition 2 in a special case of our model to study how input prices change following a downstream merger. We find that upstream firms have incentives to raise prices when downstream output expands or becomes less sensitive to input prices, and that these incentives are often stronger when the merger yields higher efficiencies. This provides a channel whereby merger efficiencies can place upward pressure on goods prices, in contrast to their standard price-reducing effect. Considering the implications for policy, we find that the consumer surplus-maximizing merger policy assuming fixed input prices is likely too lenient when input prices are endogenous. This finding is robust to a number of modeling assumptions, and simulations indicate that the effect of endogenous input prices on goods prices is quantitatively significant (and often has a similar magnitude to the standard price effect under fixed input prices). We also provide an example indicating that a merger may be inherently anticompetitive because of endogenous input prices: "Offsetting efficiencies" that lead to lower goods prices may not exist.

3.1 Simple Model

Consider the following special case of the economy of Section 2: There are two goods, g and g', and two inputs, i and i'. Before the merger, each product is owned by a different firm. Each input i has constant marginal cost c_i, and each good g has constant marginal cost

$$c_g(w) = v_g + x_{gi}w_i + x_{gi'}w_{i'}.$$

Here v_g denotes the component of marginal cost due to use of the numeraire (e.g., labor). This marginal cost function implies that good g is potentially produced using both inputs i and i', but in fixed proportions (Figure 1a). A useful special case that we discuss below is one in which good g is produced using only input i ($x_{gi'} = 0$) and good g' is produced using only input i' ($x_{g'i} = 0$). We say that such an economy features "independent input suppliers" (Figure 1b).

The consumer demand system $y(p)$ is linear, Slutsky symmetric, and features gross substitutes, and we let

$$D_{gg'} := -\frac{\partial y_{g'}/\partial p_g}{\partial y_g/\partial p_g} \tag{16}$$

denote the *diversion ratio* from g to g'. This quantity gives the fraction of demand diverted from g to g' after an increase in the price of g, providing a (directed) measure of the substitutability of goods g and g'.[11] Demand for input i satisfies $z_i(y,w) = \sum_g y_g x_{gi}$.

Before the merger, the downstream firms engage in Bertrand-Nash competition as in (2), and the upstream firms engage in upstream-leading price competition as in (5). We assume that the upstream firms cannot price discriminate across the downstream firms. After downstream firms g and g' merge, the merged firm M realizes a new marginal cost function for each good:

$$\hat{c}_g(w) = \left(1 - E_{gv}\right) v_g + (1 - E_i)\, x_{gi} w_i + (1 - E_{i'})\, x_{gi'} w_{i'}.$$

Here E_{gv}, E_i, and $E_{i'}$ are the *efficiencies* attained by the merged firm on each component of the marginal cost of good g. For simplicity, we assume that the input-specific efficiencies E_i and $E_{i'}$ are symmetric across downstream goods. The post-merger demand for input i is then $\hat{z}_i(y,w) = (1 - E_i)\sum_g y_g x_{gi}$. The merged firm sets goods prices as a multiproduct monopolist, while the upstream firms continue to engage in upstream-leading price competition.

We remark that a number of other features of the model – two downstream firms, linear demand, constant returns to scale with linear marginal costs – are highly stylized. These assumptions are made largely to simplify the exposition of our main results in Section 3.3. We provide additional theoretical results in Section 3.4 to indicate how they generalize, and we give numerical results from merger simulations in Section 4 and Appendix B to address cases not covered by our propositions.

[11] $D_{gg'}$ is non-negative since g and g' are gross substitutes in consumption, and we further assume that it is strictly smaller than 1.

3.2 An Example

Before studying IPP in the simple model, we provide an example to show that endogenous input prices may reverse basic intuitions about horizontal mergers and merger policy. In particular, downstream mergers may be inherently anticompetitive once endogenous input prices are taken into account.

Claim 1. Goods prices may increase post-merger for all efficiencies because of endogenous input prices.

To see this, consider a special case of the simple model with a single upstream wholesaler and two downstream retailers. Goods g and g' are produced using only input i, with marginal costs $c_g(w) = c_{g'}(w) = w_i$. The goods are symmetrically differentiated, with consumer demand

$$y_g(p) = V - p_g + Dp_{g'}.$$

Here $V > 0$ controls the own-price elasticity of demand. The downstream merger yields efficiencies E: $\hat{c}_g(w) = \hat{c}_{g'}(w) = (1-E)w_i$.

The pre- and post-merger equilibria can be computed by hand, and we do so in Appendix A. Most importantly, we observe that the input price always rises after the merger:

$$\hat{w}_i^* = \frac{V + (1-D)(1-E)c_i}{2(1-D)(1-E)} \geq \frac{V + (1-D)c_i}{2(1-D)} = w_i^*,$$

where the inequality is strict when $E > 0$. These expressions show that post-merger downstream marginal costs $(1-E)\hat{w}_i^*$ are decreasing in efficiencies E, but more slowly than when the input price is fixed at its pre-merger value. As a result, efficiencies are less effective at constraining downstream prices when the input price is endogenous. If the merger is to leave goods prices unchanged, it must then achieve greater efficiencies when the input price is endogenous than when it is fixed at its pre-merger value.

More formally, let E^{W} denote the efficiencies that keep goods prices unchanged at a fixed input price, $\hat{p}^*(w^*) = p^*(w^*)$. We refer to these efficiencies as *Werden efficiencies* given the general characterization by Werden (1996). Let E^{T} denote the "true" offsetting efficiencies that keep goods prices unchanged when the input price varies endogenously, $\hat{p}^*(\hat{w}^*) = p^*(w^*)$. Direct calculation yields

$$E^{\mathrm{T}} = \frac{D}{2-D} \frac{\frac{V}{c} - (1-D)}{1-D} > \frac{D}{2-D} \frac{\frac{V}{c} - (1-D)}{\frac{V}{c} + 1 - D} = E^{\mathrm{W}}.$$

True offsetting efficiencies are always higher than Werden efficiencies, and when demand is sufficiently inelastic they are above one. In this case, goods prices increase after the merger regardless of any efficiencies realized by the merged firm. This holds precisely because the input price is endogenous since the Werden efficiencies E^{W} are bounded below one. By implication, a consumer surplus-maximizing antitrust authority should always enjoin the merger between firms g and g', but it may fail to do so if it does not recognize that the input price adjusts after the merger.

In this example, Williamson's (1968) trade-off between market power and cost efficiencies works as expected when the input price is held fixed: Goods prices are unchanged if marginal costs fall sufficiently. However, when the input price changes endogenously after the merger, the trade-off resolves decisively against consumers, and goods prices always rise. This stark conclusion does not apply generally to mergers with endogenous input prices, but it indicates that an understanding of upstream pricing incentives may be crucial for accurate merger analysis. In the next section, we use IPP to study these incentives in the simple model.

3.3 IPP and Merger Policy

Motivated by Proposition 2 and our subsequent discussion, we say that firm i has an incentive to raise its price after the merger if $\mathrm{IPP}_i > 0$.[12] To determine when this is the case, we can specialize the upstream-leading FOC function (6) to our setting and directly calculate

$$\mathrm{IPP}_i \propto \sum_g x_{gi} \left\{ \underbrace{y_g\left(\hat{p}^*(w^*)\right) - y_g\left(p^*(w^*)\right)}_{\Delta \text{ downstream output}} + \left(w_i^* - c_i\right) \underbrace{\left(\frac{d y_g\left(\hat{p}^*(w^*)\right)}{d w_i} - \frac{d y_g\left(p^*(w^*)\right)}{d w_i} \right)}_{\Delta \text{ sensitivity of output to input price}} \right\}. \tag{17}$$

Here $\propto$ indicates that IPP_i is proportional to the expression on the right side. Firm i has an incentive to increase its price when the merged firm expands downstream output or when downstream output becomes less sensitive to the input price w_i after the merger. Both effects tend to reduce the elasticity of demand for input i, incentivizing firm i to raise its input price and extract greater profits from inframarginal purchases.

As we show directly in Appendix A, the sign of each term in (17) depends crucially on the efficiencies generated by the merger. For example, if no efficiencies are realized, the merged firm will find it profitable to raise goods prices and lower output when input prices are held

[12] As (14) indicates, a more precise estimate of the price change would weigh all entries of IPP using the pass-through rates $\hat{\rho}_{\mathcal{I}}$. In Lemma A.1 in Appendix A, we show that the own-price pass-through rate $\hat{\rho}_{ii}$ is generally much larger than the cross-good pass-through rate $\hat{\rho}_{ii'}$, so IPP_i is likely to dominate in determining the sign of the price change $\hat{w}_i^* - w_i^*$. Our focus on IPP_i is supported by our numerical results and is consistent with the use of Upward Pricing Pressure to diagnose unilateral pricing incentives for merging firms (Farrell and Shapiro, 2010).

fixed. The first term in (17) is then negative, suggesting that the upstream firm i may want to lower its price because of lower inframarginal purchases of input i. This effect is generally mitigated as efficiencies rise. Similarly, the second term in (17) increases as the merged firm becomes more efficient in its use of input i (as E_i rises): The marginal cost of each good becomes less sensitive to the input price w_i, so goods prices and outputs also become less sensitive to w_i.

To assess the implications of endogenous input prices for merger policy, we consider the following thought experiment: Suppose the merger attains Werden efficiencies, so that goods prices remain unchanged post-merger when input prices are held fixed. Will upstream firms then have an incentive to raise input prices? If so, goods prices will also rise, and consumers will be harmed precisely because input prices are endogenous. We can then claim that consumer surplus-maximizing merger policy assuming fixed input prices is too lax. The next proposition gives conditions under which this holds:

Proposition 3. Suppose the merger is Werden-efficient, $\hat{p}^*(w^*) = p^*(w^*)$. Then $\mathrm{IPP}_i > 0$ if and only if

$$E_i > \bar{E}_i, \quad \text{where} \quad \bar{E}_i \leq \frac{D_{gg'}D_{g'g}}{4 - D_{gg'}D_{g'g}}. \tag{18}$$

The bound for $\bar{E}_i$ is attained with independent input suppliers.

This result demonstrates that, for a Werden-efficient merger, upstream firm i has an incentive to increase its price if the merged firm attains sufficient i-specific efficiencies. For intuition, first consider the case with independent input suppliers. Werden efficiencies imply that the first term in the IPP expression (17) is zero, while the decomposition for the second term is

$$\frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i} = \frac{\partial y_g}{\partial p_g}\left(\hat{\rho}_{gg}(1-E_i) - \rho_{gg}\right)x_{gi} + \frac{\partial y_g}{\partial p_{g'}}\left(\hat{\rho}_{g'g}(1-E_i) - \rho_{g'g}\right)x_{gi}. \tag{19}$$

The key comparative static is then the change in downstream pass-through rates $\hat{\rho}_{\mathcal{G}} - \rho_{\mathcal{G}}$ after the merger. By direct calculation, we find that all pass-through rates fall, $\hat{\rho}_{\mathcal{G}} \ll \rho_{\mathcal{G}}$. To see why, suppose an exogenous increase in the marginal cost c_g of good g. Before the merger, only the price of good g increases directly in response, but all goods prices increase in equilibrium through complementary price responses ($\rho_{gg} > \rho_{g'g} > 0$). After the merger, the merged firm has an incentive to *decrease* the price of good g' directly in response to the increase in c_g, diverting demand away from the lower margin good g and increasing profits. This is the "Edgeworth-Salinger effect" coined by Luco and Marshall (2020), and it reduces all pass-through rates following the merger. In fact, it reduces the cross-good pass-through rate $\hat{\rho}_{g'g}$ much more than the own-good pass-through rate $\hat{\rho}_{gg}$, so that (19) is negative when E_i is sufficiently low.

However, as E_i increases and the merged firm becomes more efficient in its use of input i, direct calculation shows that the whole expression (19) increases. This illustrates a key trade-off with independent input suppliers: The post-merger decline in pass-through rates tends to raise the sensitivity of downstream output y_g to the input price w_i, while input-specific efficiencies tend to reduce it. For a Werden-efficient merger, the resolution of this trade-off determines whether firm i will have an incentive to raise or lower its price after the merger. Proposition 3 quantifies this trade-off and indicates that, to compensate for a larger fall in cross-good pass-through rates post-merger, greater input-specific efficiencies are needed for more substitutable goods.

Suppose now that input i is used to produce good g' as well as good g. Then the proposition implies that the efficiencies threshold $\bar{E}_i$ is *weakly smaller*, so firm i has an incentive to raise prices more often.[13] Intuition again follows from the Edgeworth-Salinger effect: In response to an increase in the marginal cost of good g, the merged firm raises the price of good g' by a smaller amount than before the merger in order to divert demand from g to g'. When input i is used to produce only good g, this tends to increase the sensitivity of demand for input i to its price w_i. However, when input i is also used to produce good g', firm i internalizes some of the "input demand" diverted from g to g'. This mitigating effect tends to reduce the sensitivity of total demand for input i to its price w_i, thereby reducing firm i's incentive to lower its price. In some cases, this effect is strong enough that the threshold efficiencies $\bar{E}_i$ are negative.

Claim 2. Suppose both goods require equal quantities of input i, $x_{gi} = x_{g'i}$, and suppose the diversion ratios are equal, $D := D_{gg'} = D_{g'g}$. Then $\bar{E}_i = -\frac{D/2}{1-D/2} < 0$.

With $\bar{E}_i < 0$, firm i has an incentive to raise its price after a Werden-efficient merger regardless of i-specific efficiencies. The example in Section 3.2 shows that in further special cases, we can even dispense with the assumption of Werden efficiencies, and firm i will always have an incentive to raise its price after the merger.

Proposition 3 has direct implications for merger policy. When input prices are fixed, Werden-efficient mergers lie precisely on the boundary between mergers that are allowed and mergers that are blocked by a consumer surplus-maximizing antitrust authority. Proposition 3 demonstrates that this boundary must generally shift when input prices are endogenous. In particular, when input-specific efficiencies are sufficiently large, the antitrust authority should block Werden-efficient mergers (and, by continuity, a subset of mergers that would have yielded

[13]This inequality is strict provided that input i is used to produce both goods and the goods are strict gross substitutes in consumption.

consumer surplus gains with fixed input prices). The opposite holds when input-specific efficiencies are sufficiently small.

We can use inequality (18) to assess the relative likelihood of these scenarios. To start, note that a Werden-efficient merger must yield specific reductions in the marginal cost of each good:

$$\frac{c_g - \hat{c}_g}{c_g} = E_g^{\mathrm{W}} := \frac{D_{gg'}}{1 - D_{gg'} D_{g'g}} \left(\frac{c_{g'}}{c_g} \mu_{g'}^* + D_{g'g} \mu_g^* \right) \quad g \in \mathcal{G}, \tag{20}$$

where all functions are evaluated at pre-merger input prices w^* and $\mu_g^* := p_g^* / c_g(w^*) - 1$ denotes the pre-merger *markup* for good g.[14] These reductions in marginal cost can only come from more efficient use of the numeraire or upstream inputs. In percentage terms, the total efficiencies must be a weighted average of the efficiencies from each source:

$$\frac{c_g - \hat{c}_g}{c_g} = \frac{v_g}{c_g} E_{gv} + \frac{x_{gi} w_i^*}{c_g} E_i + \frac{x_{gi'} w_{i'}^*}{c_g} E_{i'}.$$

A reasonable benchmark for the input-specific efficiencies E_i is then the Werden efficiencies themselves (20).[15] The next set of results describes sufficient conditions for inequality (18) to hold in this benchmark case.

Proposition 4. Suppose the merger is Werden-efficient, and suppose $E_i \geq \max\left\{E_g^{\mathrm{W}}, E_{g'}^{\mathrm{W}}\right\}$. Then $\mathrm{IPP}_i > 0$ if either of the following hold:

(i) the pre-merger markup μ_g^* is greater than 0.25;

(ii) demand $y(p)$ is symmetric, pre-merger marginal costs for both goods are equal $c_g(w^*) = c_{g'}(w^*)$, and the pre-merger markup μ_g^* is greater than 0.07.

This proposition follows readily by comparing the Werden efficiencies (20) to the right side of inequality (18). We view the sufficient conditions of Proposition 4 as fairly mild; they are the basis of our argument that optimal merger policy under fixed input prices is likely too lax when input prices vary endogenously.[16] However, we emphasize that these sufficient conditions require that the merged firm achieve *input-specific efficiencies*, which may not always be a reasonable assumption. As Claim 2 indicates, this requirement may be relaxed if input i is used to produce both downstream goods.

[14] This expression can be derived following Werden (1996) or by noting that $c_g - \hat{c}_g = \mathrm{GePP}_g(w^*)$.

[15] For example, $E_i = \frac{c_g - \hat{c}_g}{c_g}$ if all merger-induced changes to production technologies are Hicks-neutral.

[16] For example, Döpper, MacKay, Miller, and Stiebale (2021) estimate markups for over 100 categories of consumer products in the United States and find that the median markups have increased from .63 in 2006 to 1 in 2019, well above the threshold established in Proposition 4. Note however that Döpper et al. (2021) assume a random coefficients logit demand system, whereas the demand system assumed in Proposition 4 is linear.

The results above indicate when we should expect input prices to rise post-merger: when downstream output expands or becomes less sensitive to input prices, which are more likely when the merged firm achieves input-specific efficiencies. However, Propositions 3 and 4 do not indicate when the corresponding effects on goods prices are likely to be large as a function of pre-merger observables (e.g., markups and diversion ratios). These comparative statics would be helpful for screening anticompetitive mergers, but they are difficult to determine analytically. In Appendix B we provide simulation results that suggest for a Werden-efficient merger, the effects on goods prices are larger with higher pre-merger markups and higher diversion ratios. Though less helpful for merger screening, with independent input suppliers we can determine comparative statics with respect to efficiencies:

Proposition 5. With independent input suppliers, the change in the price of good g due to input prices $\hat{p}_g^*(\hat{w}^*) - \hat{p}_g^*(w^*)$ is

(i) increasing in good g efficiencies $\left(E_{gv}, E_i\right)$, holding pre-merger objects fixed;

(ii) decreasing in good g' efficiencies $\left(E_{g'v}, E_{i'}\right)$, holding pre-merger objects fixed.

The proof follows by noting that approximation (13) is exact in the simple model, and that the second term is precisely equal to the change in goods prices due to changing input prices. These comparative statics are consistent with our discussion of the IPP_i expression (17), but they hold for goods prices instead of input prices.[17]

We emphasize that the comparative statics in part (i) of Proposition 5 run counter to the standard price effects of efficiencies. With fixed input prices, higher efficiencies generally imply lower equilibrium goods prices. This is the essence of Williamson's (1968) trade-off, but Proposition 5 suggests a qualification: Larger efficiencies may also place upward pressure on goods prices through higher input prices. As we saw in Section 3.2, this countervailing effect may be so strong that true "offsetting efficiencies" do not exist, and goods prices may necessarily increase after the merger.

3.4 Extensions

Below we describe how our results extend beyond the simple model of Section 3.1. We focus on generalizing Propositions 3 and 4, which we view as our most policy-relevant results.

[17] Part (i) holds despite a countervailing force: As input-specific efficiencies for g increase, the price of g becomes less sensitive to the price of input i.

Production Technology and Many Input Suppliers

The simple model severely restricts the downstream production technologies and the input-output structure: Downstream marginal costs are constant in output and linear in input prices, and there are only two inputs. To relax these assumptions, we start with the simple model and now suppose that there are an arbitrary number $|\mathcal{I}| \geq 1$ of upstream firms. Each upstream firm i produces one input and sets a uniform price w_i that applies to both downstream firms. Downstream production technologies are fully general: The pre-merger cost function for good g is $C_g\left(y_g, w\right)$, and the post-merger cost function for the merged firm is $C_M\left(y, w\right)$.[18] We keep all remaining aspects of the simple model unchanged.

To determine when upstream firm i has an incentive to raise its price after the merger, we again consider IPP_i. In Appendix A,[19] we show that for a Werden-efficient merger

$$\begin{aligned} \text{IPP}_i \quad \propto \quad & \underbrace{\sum_{g\in\mathcal{G}} \frac{\partial \log z_i}{\partial y_g}\left(\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i} - \frac{dy_g\left(p^*\left(w^*\right)\right)}{dw_i}\right)}_{\Delta \text{ sensitivity of output to input price}} \\ & + \underbrace{\sum_{g\in\mathcal{G}}\left(\frac{\partial \log \hat{z}_i}{\partial y_g} - \frac{\partial \log z_i}{\partial y_g}\right)\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}}_{\Delta \text{ elasticity of input demand to output}} + \underbrace{\frac{\partial \log \hat{z}_i}{\partial w_i} - \frac{\partial \log z_i}{\partial w_i}}_{\Delta \text{ elasticity of input demand to input price}}, \end{aligned} \tag{21}$$

where all total demand functions for input i are evaluated at pre-merger outputs $y\left(p^*\right)$ and input prices w^*. This decomposition differs in two key ways from the analogue (17) in the simple model. First, (17) holds for an arbitrary merger while (21) holds for a Werden-efficient merger, so (21) does not have a term reflecting a change in downstream outputs. Second, the last two terms in (21) do not appear in (17), because they account for effects that do not arise under the production technologies of the simple model. The second term in (21) captures any change in the elasticity of total input demand with respect to downstream outputs. It is positive when total input demand becomes less elastic to outputs, implying that firm i observes a smaller decline in input demand for a given increase in price. The third term in (21) captures any change in the elasticity of total input demand to the input price, holding fixed downstream outputs. It is similarly positive when the "partial elasticity" of input demand to input price falls in magnitude. These new terms may be quantitatively important for firm i's pricing incentives after the merger, but they depend exclusively on the change in downstream production technologies, which is exogenous in our model. As a result, for the remainder of

[18] Our results require the weak assumption that the marginal cost for each good g is strictly increasing in the price of each input i that firm g purchases before the merger. This holds, for example, when pre-merger production technologies are homothetic.

[19] See the proof of Proposition $3'$.

this section we restrict our focus to the more familiar first term in (21).

Assumption 3. The partial elasticities of total input demand to outputs y and the input price w_i are unchanged after the merger:

$$\frac{\partial \log \hat{z}_i}{\partial \log y_g} = \frac{\partial \log z_i}{\partial \log y_g} \quad g \in \mathcal{G} \quad \text{and} \quad \frac{\partial \log \hat{z}_i}{\partial \log w_i} = \frac{\partial \log z_i}{\partial \log w_i},$$

where all functions are evaluated at pre-merger outputs and input prices $(y(p^*), w^*)$.

To state our generalization of Proposition 3, let

$$E_{gi} := 1 - \left(\frac{\partial}{\partial y_g} \frac{\partial C_M(y(p^*), w^*)}{\partial w_i}\right)\left(\frac{\partial}{\partial y_g} \frac{\partial C_g\left(y_g(p^*), w^*\right)}{\partial w_i}\right)^{-1}$$

denote the *i-specific efficiencies* attained by the merged firm in the production of good g. This quantity is the reduction in the component of marginal cost of good g due to expenditure on input i, naturally extending the concept of input-specific efficiencies from the simple model. We again assume that input-specific efficiencies are symmetric across downstream goods, $E_{gi} = E_{g'i} = E_i$. With this condition and Assumption 3, we obtain the following strict generalization of Proposition 3:

Proposition 3′. Suppose the merger is Werden-efficient. Then $\text{IPP}_i > 0$ if and only if

$$E_i > \bar{E}_i, \quad \text{where} \quad \bar{E}_i \leq \frac{D_{gg'}D_{g'g}}{4 - D_{gg'}D_{g'g}}.$$

This result demonstrates that Proposition 3 holds for general downstream production technologies and input-output structures. This immediately implies that the sufficient conditions of Proposition 4 also apply to the more general model of this section, provided the assumptions of Proposition 3′ are satisfied.

We remark that we have assumed symmetric input-specific efficiencies to shut down composition effects: If good g attains greater i-specific efficiencies than good g', then g' will tend to comprise a greater share of the total demand for i after the merger than before the merger. If the post-merger input demand from g' is less elastic than that from g at pre-merger input prices, then this places additional upward pressure on the price of i. This composition effect is absent when i-specific efficiencies are equal or when the elasticities of post-merger demand for i from g and g' are equal. When efficiencies and input demand elasticities are heterogeneous, this composition effect must be weighed against the pass-through effects we have highlighted (and the effects assumed away by Assumption 3) to determine the net incentive for a change in the input price w_i.

Downstream "Outsiders"

The analysis of Section 3 has assumed that initially only two firms compete in the market for consumer goods. Though consistent with other studies of unilateral pricing incentives after mergers (e.g., Farrell and Shapiro, 2010; Schmalensee, 2009), this assumption likely yields biased estimates of merger price effects when the merging firms face competition from non-merging rivals, or *outsiders*, that produce substitutable products. Most relevant for our results, under Bertrand-Nash conduct outsiders' prices are strategic complements to the merging firms' prices, so outsiders generally increase their prices after an increase in the merging firms' marginal costs. This response (i) raises pre- and post-merger pass-through rates and (ii) creates an additional channel through which downstream output y_g is affected by the input price w_i. The net effect on Input Pricing Pressure is not obvious. Below we give a result indicating that this effect is likely small and often strengthens our main conclusions.

To incorporate an outsider into the simple model, we now suppose that there is a third good $o \in \mathcal{G}$ owned by a non-merging firm. For simplicity, we assume that this good is produced at constant marginal cost c_o using only the numeraire. The outsider o competes à la Bertrand-Nash with firms g and g' before the merger and with firm $M = \{g, g'\}$ after the merger. For tractability, we also assume independent input suppliers, so that input i is used to produce only good g. We keep all remaining aspects of the simple model unchanged.

To see how Proposition 3 extends, suppose the merger between firms g and g' is Werden-efficient. Then IPP_i is again proportional to the change in the sensitivity of output y_g to the input price w_i, which we can decompose as follows:

$$\frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i} = \frac{\partial y_g}{\partial p_g}\left(\hat{\rho}_{gg}(1-E_i) - \rho_{gg}\right)x_g + \frac{\partial y_g}{\partial p_{g'}}\left(\hat{\rho}_{g'g}(1-E_i) - \rho_{g'g}\right)x_g$$
$$+ \frac{\partial y_g}{\partial p_o}\left(\hat{\rho}_{og}(1-E_i) - \rho_{og}\right)x_g. \tag{22}$$

As discussed above, this decomposition differs from the analogue (19) in the simple model because the pass-through rates in the first two terms are generally higher, and a new third term captures the direct change in downstream output because of an adjustment in the outsider's price p_o. Input-specific efficiencies E_i again enter linearly, so there exists a threshold $\bar{E}_i^{\text{out}}$ such that $\text{IPP}_i > 0$ if and only if $E_i > \bar{E}_i^{\text{out}}$. This threshold is a complicated nonlinear function of all diversion ratios, so we simplify by assuming that all diversion ratios are symmetric. Proposition 3 then extends as follows:

Proposition 3′′. Suppose the diversion ratios between all pairs of goods are equal to $D \in [0, 0.5]$. If the merger is Werden-efficient, then $\mathrm{IPP}_i > 0$ if and only if

$$E_i > \bar{E}_i^{\mathrm{out}}, \quad \text{where} \quad \bar{E}_i^{\mathrm{out}} \leq \frac{D^2}{4 - D^2}.$$

This result demonstrates that Proposition 3 continues to hold in the presence of an outsider when the diversion ratios are equal, and moreover that the efficiencies threshold $\bar{E}_i^{\mathrm{out}}$ is weakly smaller than in the simple model. As a corollary, we find that the sufficient conditions of Proposition 4 also apply with an outsider under equal diversion ratios. Our numerical results in Appendix B show that Proposition 3′′ continues to hold when the diversion ratio between the outsider's good and the merging firms' goods is not equal to the diversion ratio between the merging firms' goods. In fact, the threshold $\bar{E}_{gi}^{\mathrm{out}}$ becomes *smaller* as the diversion ratio between the outsider's good and the merging firms' goods increases.

Nonlinear Demand

In the simple model, we make the strong assumption that the consumer demand system $y(p)$ is linear. We do this for analytical tractability: The incentive for an upstream firm to raise its price after a downstream merger depends crucially on the change in downstream pass-through rates, $\hat{\rho}_{\mathcal{G}} - \rho_{\mathcal{G}}$. With linear demand, this change is easy to compute because pass-through rates are constant and determined exclusively by diversion ratios. This allows us to characterize the efficiencies threshold in Proposition 3 as well as the bounds in Propositions 3′ and 3′′ in terms of model primitives. With nonlinear demand, pass-through rates depend additionally on the equilibrium margins and the local curvature of demand, so they are more difficult to manipulate analytically.[20] This precludes a simple generalization of Proposition 3, but it does not fundamentally change the economics of mergers with endogenous input prices. Departing from the simple model only in allowing nonlinear consumer demand, we again find that an upstream firm i has an incentive to raise its price after a Werden-efficient merger when the merged firm attains sufficient i-specific efficiencies:

Proposition 3′′′. Suppose the merger is Werden-efficient. Then there exists a threshold $\bar{E}_i^{\mathrm{NL}}$ such that $\mathrm{IPP}_i > 0$ if and only if $E_i > \bar{E}_i^{\mathrm{NL}}$.

We derive a formula for the efficiencies threshold $\bar{E}_i^{\mathrm{NL}}$ in the proof of the proposition. It is difficult to analytically determine the difference between the thresholds of Propositions 3 and 3′′′, but our numerical results in Appendix B suggest that it is small with logit demand and

[20] For example, see Weyl and Fabinger (2013).

does not materially affect the policy implications of our model: Optimal merger policy under fixed input prices is often too lax when input prices are endogenous.

Simultaneous Pricing and Bargaining

In the simple model and all generalizations above, we maintain two assumptions on upstream conduct: Upstream firms unilaterally set input prices, and they anticipate changes in goods prices when doing so. In these ways, we depart from the recent literature on horizontal market structure with vertical contracting, in which it is commonly assumed that input prices are set through Nash-in-Nash bargaining and simultaneously with goods prices.[21] Our upstream-leading conduct assumption is responsible for the substantive differences between our analysis and this literature; the unilateral price-setting assumption is expositionally convenient, but as we discuss shortly it is not crucial for our results.

We focus on upstream-leading conduct to study how changes in downstream pass-through rates and merger efficiencies may affect input pricing incentives after downstream mergers. These effects are relatively understudied, and they are absent when input prices are set simultaneously with goods prices. When upstream firms set input prices while taking goods prices as given, they anticipate only substitution between inputs in response to price changes. Input pricing incentives are then determined by (i) the elasticity of demand by each downstream firm for each input *conditional on downstream outputs*, (ii) the share of each downstream firm in total demand for each input, and (iii) upstream marginal costs – see (4). With Werden efficiencies, changes in the elasticities of conditional input demands after the merger are determined exclusively by changes in the merged firm's production technology, which are exogenous in our model. The same holds for changes in the shares of downstream firms in total demand for each input, and in many cases such changes in demand composition do not affect input prices at all.[22] Because of these observations, we find the version of our model in which upstream firms set prices simultaneously with downstream firms uninformative for merger analysis.

Many recent analyses maintain the assumption of simultaneous pricing, but with Nash-in-Nash bargaining between downstream and upstream firms over input prices. These studies focus on the effect of downstream market structure on bargaining leverage, often to assess how market concentration affects negotiated input prices. The consensus from the literature appears to be that a downstream merger can improve the bargaining position of the merged firm and lead to lower input prices, because controlling additional goods raises the merged firm's outside option or reduces the upstream firm's outside option in negotiations over an

[21]As Sheu and Taragin (2021) note, the simultaneous pricing assumption is often made for computational tractability.

[22]For example, this holds when all downstream firms share a common homothetic production technology.

input price. However, this bargaining leverage effect alone is generally insufficient to prevent goods prices from rising when downstream prices are set in Bertrand-Nash equilibrium (Sheu and Taragin, 2021).

Our results suggest that, with upstream-leading conduct, post-merger input price changes may actually be biased in the opposite direction, leading to higher prices for inputs and goods. In Appendix C, we argue that this is even the case when (upstream-leading) Nash-in-Nash bargaining is incorporated into the simple model of Section 3.1. We show that IPP_i is the sum of three terms: The first term is proportional to Input Pricing Pressure in the unilateral price-setting case and is exactly equal to the right side of (17). The results presented above apply immediately to this term. The second term captures the standard effect of the merger on bargaining leverage, and it is negative when the downstream firm's gains from trade relative to the upstream firm's gains from trade are smaller after the merger. Per the discussion above, we generally expect that it is negative as the merged firm improves its bargaining position relative to upstream firms. The third term captures a change in the sensitivity of downstream profits to the input price w_i, and it is negative for a Werden-efficient merger because the merged firm internalizes demand diverted between consumer goods.

The IPP_i decomposition under Nash-in-Nash bargaining then suggests that, for a Werden-efficient merger, incentives for input price increases are muted relative to the unilateral price-setting benchmark. This is consistent with the naive intuition that input prices must be fairly close to upstream marginal costs when downstream firms have substantial bargaining power, so they cannot increase much (or may decrease) following the merger. In any case, the simulations described in Appendix C demonstrate that the first term in IPP_i often still dominates for a variety of bargaining parameters, so that input prices may still rise after a Werden-efficient downstream merger. In other words, the pass-through and efficiencies effects we analyze are still quantitatively important under Nash-in-Nash bargaining, and they may dominate the standard bargaining effects discussed in the literature.

4 Quantitative Analysis of Beer Retailer Mergers

We now turn to a quantitative analysis of mergers with endogenous input prices. We consider local consumer retail markets for beer, and using a calibrated demand system and cost structure, we simulate mergers between beer retailers. Even for mergers that generate no efficiencies, we find that goods prices increase more post-merger when we allow input prices to adjust endogenously. This stylized exercise suggests that input price adjustments and their effect on consumer prices are quantitatively significant, and it also provides a test of our theory in environments that are difficult to study analytically.

4.1 Setting and Data

Our setting is the retail beer market in Ohio, and we use Retail Scanner Data from Kilts Nielsen for the sales and prices of retail beer products at the UPC-store-week level in 2010. We consider a collection of non-overlapping geographic markets with generic index m, where each market is identified with a designated market area (DMA) defined by Nielsen. The set of goods in market m is $\mathcal{G}_m$, where each good is a brand- and retailer-specific beer product. The set of downstream firms in market m is $\mathcal{F}_{\mathcal{G}_m}$, where we identify a firm with a retail chain. We aggregate sales and price data from the store level to the retail chain level, and we include food, mass merchandise, convenience, and liquor stores when doing so. Within each retail chain, we similarly aggregate data from the UPC level to the brand level, weighting by the volume of each product of a given brand. We also aggregate weekly data to the month level. For computational tractability, we keep only the 15 highest-selling brands by volume sold.

Retail chains purchase beer at wholesale prices from distributors, which constitute the upstream firms $\mathcal{F}_{\mathcal{I}_m}$ in this setting.[23] We do not observe the beer distribution networks in each market m, so we make the simplifying assumption that each brand of beer is distributed by a unique distributor in each market. We assume that in each market m, retail chains engage in Bertrand-Nash competition while distributors engage in upstream-leading price competition. The former assumption is standard, while a number of regulations on beer distribution suggest that the latter is reasonable. For example, federal regulations prohibit slotting allowances in the sale of alcoholic beverages in retails establishments (Gundlach and Bloom, 1998). State regulations in Ohio additionally prohibit distributors from offering volume discounts or retail credit, and they place no restrictions on distributor markups.[24] Altogether these regulations suggest that upstream-leading linear price competition is a reasonable approximation to market conduct by distributors.

4.2 Demand and Supply Calibration Procedure

For simplicity, we suppose that consumer demand in each market is given by a logit demand system. The indirect utility experienced by consumer l in market m from purchasing good g is

$$u_{lgm} = -\beta_m p_{gm} + \xi_{gm} + \epsilon_{lgm},$$

[23]A complete model of beer distribution would also include a third tier: beer manufacturers. We do not model price-setting by beer manufacturers for simplicity and to ensure that the quantitative exercise stays within the confines of our general model.

[24]See the 2018 report on wholesale pricing restrictions from the Substance Abuse and Mental Health Services Administration <link>.

where p_{gm} is the per-ounce price of good g in market m, ξ_{gm} is a brand-retail chain fixed effect, and ϵ_{lgm} is an unobserved logit error term. We normalize the utility of the outside option to zero.

We calibrate the parameters β_m and $\xi_m := \left(\xi_{gm}\right)_{g\in\mathcal{G}_m}$ to match (i) the median market elasticity of demand estimated in a similar setting by Miller and Weinberg (2017) and (ii) the market shares for each good observed in the data:[25]

$$\hat{\mathcal{E}}_m = \sum_{g\in\mathcal{G}_m}\sum_{g'\in\mathcal{G}_m} \frac{\partial \log y_{gm}(p_m;\beta_m,\xi_m)}{\partial \log p_{g'm}},$$
$$\hat{y}_{gm} = y_{gm}(p_m;\beta_m,\xi_m) \qquad\qquad g\in\mathcal{G}_m,$$

where $\hat{\mathcal{E}}_m = -0.7$ is the market elasticity of demand, $\hat{y}_{gm}$ is the market share of good g in market m observed in the data, and $y_{gm}(p;\beta_{pm},\xi_{gm})$ is the market share of good g in market m implied by the logit demand system.

To calculate market shares $\hat{y}_{gm}$, for each month we compute the share of each good g in total volume sold in market m in our data, and we average these shares across all months. To scale these "inside" shares, we construct the share of the outside good as follows: Using the Consumer Panel Data from Kilts Nielsen, we determine the number of panelists in market m who purchased beer at some point during the year. We then calculate the proportion of beer-purchasing panelists who did not purchase beer in market m in a given month, and we use this as the share of the outside good for that month. We average across months to obtain a final estimate of the outside good's share, and we scale all inside shares to compute our final market shares $\hat{y}_{gm}$.

The final parameters needed to conduct merger simulations are distributor marginal costs, which we impute following Villas-Boas (2007). Let $\Omega^{\mathcal{G}_m}$ denote the ownership matrix for goods in market m, where $\Omega^{\mathcal{G}_m}_{gg'} = 1$ if goods g and g' are owned by the same retail chain and $\Omega^{\mathcal{G}_m}_{gg'} = 0$ otherwise. Define $\Omega^{\mathcal{I}_m}$ similarly as the ownership matrix for brands (inputs); in our baseline specification of the distribution network, $\Omega^{\mathcal{I}_m}$ is the identity. Inverting the downstream Bertrand-Nash FOCs, we can compute retail margins:

$$p - c_{\mathcal{G}_m} = -\left[\Omega^{\mathcal{G}_m} \odot \frac{dy}{dp}^T\right]^{-1} y.$$

Here $\odot$ denotes the Hadamard (pointwise) product. Given the calibrated demand system and observed goods prices, we solve this equation to determine retailer marginal costs $c_{\mathcal{G}_m}$. For sim-

[25] In ongoing work, we are estimating a random coefficients nested logit model for consumer demand. The calibrated logit demand system serves a placeholder until the full estimation has been completed.

plicity, we assume that all marginal costs incurred by a retail chain are attributable to wholesale prices, so we can compute implied wholesale prices: $w_i = c_{gm}$ if good g is a retail product for brand i.[26] We can similarly invert the upstream price-setting FOCs to compute distributor margins:

$$w - c_{\mathcal{I}_m} = -\left[\Omega^{\mathcal{I}_m} \odot \frac{dz}{dw}^T\right]^{-1} z.$$

Given the calibrated demand system and imputed wholesale prices, we solve this equation to determine distributor marginal costs $c_{\mathcal{I}_m}$.[27]

4.3 Merger Simulations

Using our calibrated demand system and cost structure, we simulate mergers between retailers, both holding wholesale prices fixed and allowing wholesale prices to vary endogenously. We observe 8 markets in our data, with 3-9 retail chains in each market.[28] We simulate pairwise mergers between all chains in a given market, assuming that the mergers yield no efficiencies to the merging firms.

Table 1 presents results from these simulations. For each market, we calculate the average change in retail prices across all goods and all mergers, weighting price changes by pre-merger shares. In every market, retail prices increase more on average when wholesale prices are endogenous than when they are exogenous, with a typical difference of 4-6 percentage points. Similarly, the average decline in consumer surplus across all mergers in a market is always larger with endogenous wholesale prices: The retail mergers have almost no effect on consumer surplus with exogenous wholesale prices, because we observe relatively small market shares for each retail chain in our data. As a result, the mergers have only a small effect on market concentration as measured by the projected change in HHI. However, allowing wholesale prices to adjust endogenously lowers consumer surplus by nearly 1% on average in each market, despite the small changes in concentration.

Figure 2 plots the distributions of retail price changes after the mergers, where price changes are weighted by pre-merger market shares. When wholesale prices are exogenous, retail price changes are heavily concentrated near zero, reflecting the small changes to market concen-

[26] This inversion gives us a $|\mathcal{G}_m|$ vector of wholesale prices. We assume there is no price discrimination, so we take a mean by brand to calculate the final wholesale price per brand. However, we observe little variation in the wholesale prices across different retailers if we allow for price discrimination.

[27] The resulting distributions of pre-merger wholesale prices and distributor marginal costs are given in Appendix D.

[28] There are 12 DMAs defined by Nielsen that include parts of Ohio, but 4 of these also overlap with other states. We exclude DMAs spanning more than one state.

Table 1: Simulation Summary

	1	2	3	4	5	6	7	8
Num Chains	8	6	7	7	6	8	3	4
Chain HHI	479.06	638.54	548.83	660.71	779.06	446.57	863.96	767.57
Mean Δ HHI	8.93	2.15	16.87	9.15	5.52	5.9	2.74	42.35
β_p	-54.81	-59.61	-50.62	-52.37	-53.78	-59.87	-49.28	-49.13
Median Own Elasticity	-4.45	-4.21	-4.38	-4.05	-4.14	-4.31	-3.52	-3.95
Median Cross Elasticity	0.01	0.02	0.02	0.03	0.02	0.01	0.06	0.03
Mean Δp, Exogenous	0.12	0.03	0.21	0.12	0.08	0.08	0.21	0.6
Mean Δp, Endogenous	3.72	6.54	6.34	6.09	7.14	4.45	4.68	5.39
Mean Δw, Endogenous	-0.02	-0.18	0.29	0.14	0.42	-0.1	3.56	2.53
Mean ΔCS, Exogenous	-0.03	-0.01	-0.06	-0.03	-0.02	-0.02	-0.14	-0.15
Mean ΔCS, Endogenous	-0.87	-1.41	-1.58	-1.48	-1.74	-0.96	-1.36	-1.36

Notes: Each column corresponds to a local retail beer market (DMA) in Ohio. The last 5 rows are expressed in percent changes. For the rows showing the percent change in consumer prices for the exogenous and endogenous input price counterfactuals, we construct these means weighting by pre-merger product market shares. HHI and Δ HHI are calculated at the retail chain level, and Δ HHI is defined as the projected change in HHI using pre-merger shares.

tration associated with the mergers. When wholesale prices are endogenous, the distribution of retail price changes displays greater dispersion and is shifted to the right, with a mean of 5.47 %. Figure 3 plots the distribution of wholesale price changes for mergers with endogenous wholesale prices; the price changes are again weighted by pre-merger market shares.[29] Wholesale price changes are concentrated near zero, but the distribution is again right-skewed, and many wholesale prices rise in excess of 1%.

These quantitative results broadly support our argument that downstream mergers incentivize higher input prices and that endogenous input prices exacerbate the consumer harm from downstream mergers. Notably, we find increases in input (wholesale) prices even though we simulate mergers with no input-specific efficiencies. As we discussed in Section 3.4, this is more likely when a single upstream firm sells to both merging firms and to outsiders, as we have assumed in our description of the beer distribution network. In addition, we find that endogenous changes in input prices can have a quantitatively large effect on goods prices even when the goods market is fairly unconcentrated. In any case, the quantitative results reported here do not follow directly from our propositions, so we view this exercise as a reassuring test of the external validity of our theory.

[29] The market share for a brand is the sum of the shares of the retail goods that belong to the brand.

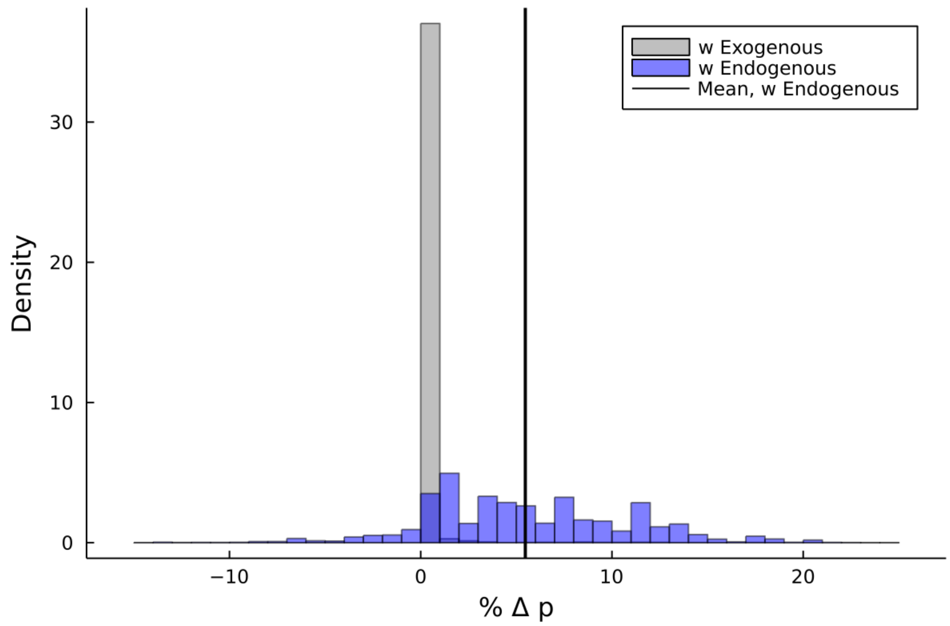


Figure 2: Histogram of percent change in retail prices after mergers with and without endogenous wholesale prices. Price changes are weighted by pre-merger market shares.

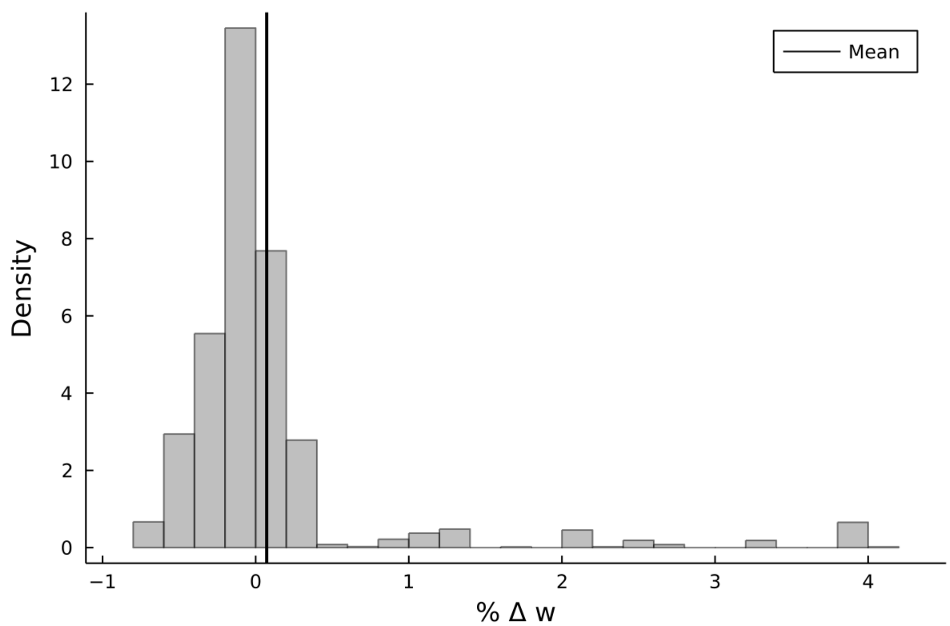


Figure 3: Histogram of percent change in wholesale prices after mergers with endogenous wholesale prices. Price changes are weighted by pre-merger market shares.

5 Conclusion

Recent merger cases have sparked debate about the buyer power defense: Can a merger between competitors raise their bargaining leverage relative to upstream firms, leading to lower prices for inputs and consumer goods? We examined this question using models of price competition in which input prices are set (or bargained) in anticipation of their effect on goods prices. To do so, we generalized the first-order approach of Farrell and Shapiro (2010) and Jaffe and Weyl (2013) and derived a measure of the incentives for input prices to adjust after a downstream merger, Input Pricing Pressure. We used this measure to show that, for mergers that have no effect on goods prices at fixed input prices, upstream firms often have incentives to *raise* input prices post-merger. These incentives hinge on changes in downstream pass-through rates and cost efficiencies generated by the merger, and they are absent under the common assumption that input prices and goods prices are set simultaneously. By implication, we argued that consumer surplus-maximizing antitrust policy is often too lax when input prices are assumed fixed, and it should be biased against claims of buyer power.

In an empirical application to local retail beer markets in Ohio, we found that allowing for endogenous input prices when simulating mergers between retailers raises average changes in goods prices as well as average consumer harm. We think of this exercise as an encouraging "out of sample test" of the pass-through effects we have highlighted: We allowed no merger efficiencies and considered a demand system and downstream market structure that do not satisfy the assumptions of our propositions. Nonetheless, we found that endogenous changes in input prices were still generally positive and had a substantial positive effect on goods prices, in line with the conclusions from the theory.

Our analysis suggests several productive areas for future work. First, we currently lack empirical evidence about changes in input prices after consummated downstream mergers. A merger retrospective study that accounts for endogenous input prices would shed additional light on the empirical relevance of our theory. Similarly, it would be helpful to determine if modeling endogenous input prices can significantly improve the performance of merger simulations, which may be inaccurate (Weinberg and Hosken, 2013; Björnerstedt and Verboven, 2016). Finally, we maintain the crucial assumption that all transactions between upstream and downstream firms are mediated by linear prices. In many contexts firms may transact using more efficient contracts (e.g., nonlinear price schedules or quantity-based bargaining), and it is not clear how our results would generalize to these settings. Loertscher and Marx (2021) have made progress on this point by microfounding inefficient contracting through incomplete information, but they have not drawn conclusions about the effects of downstream mergers on consumers.

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Appendices

A Proofs

Proof of Proposition 2. Recall the basic first-order approximation (8), and note that under the assumptions of Proposition 2, the matrix to be inverted takes the block form

$$D\hat{f}\left(p^*, w^*\right) = \begin{bmatrix} \frac{d\hat{f}_{\mathcal{G}}}{dp} & \frac{d\hat{f}_{\mathcal{G}}}{dw} \\ 0 & \frac{d\hat{f}_{\mathcal{I}}}{dw} \end{bmatrix}.$$

Here we suppress arguments for notational simplicity. Our assumptions imply that this matrix is invertible, with block inverse

$$\begin{aligned} \left[D\hat{f}\left(p^*, w^*\right)\right]^{-1} &= \begin{bmatrix} \left[\frac{d\hat{f}_{\mathcal{G}}}{dp}\right]^{-1} & -\left[\frac{d\hat{f}_{\mathcal{G}}}{dp}\right]^{-1} \frac{d\hat{f}_{\mathcal{G}}}{dw} \left[\frac{d\hat{f}_{\mathcal{I}}}{dw}\right]^{-1} \\ 0 & \left[\frac{d\hat{f}_{\mathcal{I}}}{dw}\right]^{-1} \end{bmatrix} \\ &= \begin{bmatrix} -\hat{\rho}_{\mathcal{G}} \left[\frac{\partial \hat{f}_{\mathcal{G}}}{\partial c_{\mathcal{G}}}\right]^{-1} & -\hat{\rho}_{\mathcal{G}} \frac{\partial \hat{c}_{\mathcal{G}}}{\partial w} \hat{\rho}_{\mathcal{I}} \left[\frac{\partial \hat{f}_{\mathcal{I}}}{\partial c_{\mathcal{I}}}\right]^{-1} \\ 0 & -\hat{\rho}_{\mathcal{I}} \left[\frac{\partial \hat{f}_{\mathcal{I}}}{\partial c_{\mathcal{I}}}\right]^{-1} \end{bmatrix}. \end{aligned}$$

The sufficient statistics approximations (13, 14) then follow immediately from the basic approximation (8) and the definitions of GePP and IPP (11, 12). ■

Calculations for Claim 1. Pre-merger equilibrium prices are

$$\begin{aligned} p_g^*(w) &= \frac{V + w}{2 - D} & g \in \mathcal{G}, \\ w^* &= \frac{V + (1-D)c}{2(1-D)}, \\ p_g^*(w^*) &= \frac{(3-2D)V + (1-D)c}{2(2-D)(1-D)} & g \in \mathcal{G}. \end{aligned}$$

Post-merger equilibrium prices are

$$\begin{aligned} \hat{p}_g^*(w) &= \frac{V + (1-D)(1-E)w}{2(1-D)} & g \in \mathcal{G}, \\ \hat{w}^* &= \frac{V + (1-D)(1-E)c}{2(1-D)(1-E)}, \\ \hat{p}_g^*(\hat{w}^*) &= \frac{3V + (1-D)(1-E)c}{4(1-D)} & g \in \mathcal{G}. \end{aligned}$$

We calculate E^{W} by solving $\hat{p}_g^*(w^*) = p_g^*(w^*)$, and we calculate E^{T} by solving $\hat{p}_g^*(\hat{w}^*) = p_g^*(w^*)$. ■

Throughout the remainder of Appendix A, we often write $\hat{x}_{gi} := (1-E_i)\,x_{gi}$ to simplify notation. The next two lemmas provide formulas used to prove the results in Section 3.

Lemma A.1. The following hold in the simple model (Section 3.1):

$$\rho_{\mathcal{G}} = \frac{1/2}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\begin{bmatrix} 1 & \frac{D_{gg'}}{2} \\ \frac{D_{g'g}}{2} & 1 \end{bmatrix} \tag{23}$$

$$\hat{\rho}_{\mathcal{G}} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{24}$$

$$\hat{\rho}_{\mathcal{I}} = \frac{1/2}{1-\frac{\hat{D}_{ii'}}{2}\frac{\hat{D}_{i'i}}{2}}\begin{bmatrix} 1 & \frac{\hat{D}_{ii'}}{2} \\ \frac{\hat{D}_{i'i}}{2} & 1 \end{bmatrix}, \tag{25}$$

where

$$\hat{D}_{ii'} := -\frac{d\hat{z}_{i'}/dw_i}{d\hat{z}_i/dw_i} = -\frac{\sum_g \hat{x}_{gi'}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right)}{\sum_g \hat{x}_{gi}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right)}. \tag{26}$$

Proof of Lemma A.1. The downstream pass-through matrices (23, 24) are readily calculated from the definition (10), noting that the pre- and post-merger FOC functions for goods prices are

$$f_{\mathcal{G}}\left(p, c_{\mathcal{G}}\right) = y\,(p) + \operatorname{diag}\left[\frac{\partial y}{\partial p}\right]\left(p - c_{\mathcal{G}}\right), \tag{27}$$

$$\hat{f}_{\mathcal{G}}\left(p, \hat{c}_{\mathcal{G}}\right) = y\,(p) + \left[\frac{\partial y}{\partial p}\right]^{\top}\left(p - \hat{c}_{\mathcal{G}}\right). \tag{28}$$

To calculate the upstream post-merger pass-through matrix (25), we note that the upstream demand system $\hat{z}\,(w)$ is linear, so we can apply the downstream pre-merger formula (23) using the appropriate diversion ratios. We calculate these as follows:

$$\frac{d\hat{z}_i}{dw_i} = \sum_g \hat{x}_{gi} \left\{ \frac{\partial y_g}{\partial p_g} \left(\hat{\rho}_{gg} \hat{x}_{gi} + \hat{\rho}_{gg'} \hat{x}_{g'i} \right) + \frac{\partial y_g}{\partial p_{g'}} \left(\hat{\rho}_{g'g} \hat{x}_{gi} + \hat{\rho}_{g'g'} \hat{x}_{g'i} \right) \right\} \tag{29}$$

$$= \frac{1}{2} \sum_g \hat{x}_{gi} \frac{\partial y_g}{\partial p_g} \left(\hat{x}_{gi} - D_{gg'} \hat{x}_{g'i} \right), \tag{30}$$

$$\frac{d\hat{z}_{i'}}{dw_i} = \sum_g \hat{x}_{gi'} \left\{ \frac{\partial y_g}{\partial p_g} \left(\hat{\rho}_{gg} \hat{x}_{gi} + \hat{\rho}_{gg'} \hat{x}_{g'i} \right) + \frac{\partial y_g}{\partial p_{g'}} \left(\hat{\rho}_{g'g} \hat{x}_{gi} + \hat{\rho}_{g'g'} \hat{x}_{g'i} \right) \right\} \tag{31}$$

$$= \frac{1}{2} \sum_g \hat{x}_{gi'} \frac{\partial y_g}{\partial p_g} \left(\hat{x}_{gi} - D_{gg'} \hat{x}_{g'i} \right), \tag{32}$$

$$\hat{D}_{ii'} := -\frac{d\hat{z}_{i'}/dw_i}{d\hat{z}_i/dw_i} = -\frac{\sum_g \hat{x}_{gi'} \frac{\partial y_g}{\partial p_g} \left(\hat{x}_{gi} - D_{gg'} \hat{x}_{g'i} \right)}{\sum_g \hat{x}_{gi} \frac{\partial y_g}{\partial p_g} \left(\hat{x}_{gi} - D_{gg'} \hat{x}_{g'i} \right)}. \tag{33}$$

This verifies (26), and (25) follows by substituting $\hat{D}_{ii'}$ for $D_{gg'}$ in (23). ■

Lemma A.2. In the simple model of Section 3.1,

$$\mathrm{IPP}_i \propto \sum_g x_{gi} \left\{ y_g(\hat{p}^*(w)) - y_g(p^*(w)) + \left(w_i^* - c_i \right) \left(\frac{dy_g(\hat{p}^*(w))}{dw_i} - \frac{dy_g(p^*(w))}{dw_i} \right) \right\} \tag{34}$$

and

$$\begin{aligned} &(1 - E_i) \left(\sum_g x_{gi} \left(-\frac{\partial y_g}{\partial p_g} \right) \left(x_{gi} - D_{gg'} x_{g'i} \right) \right) \mathrm{IPP}_i \\ &= \sum_g x_{gi} \left(-\frac{\partial y_g}{\partial p_g} \right) \left\{ c_g(w^*) - \hat{c}_g(w^*) - D_{gg'} \left(p_{g'}^* - \hat{c}_{g'}(w^*) \right) \right\} \\ &\quad + \left(w_i^* - c_i \right) \sum_g x_{gi} \left(-\frac{\partial y_g}{\partial p_g} \right) \left\{ x_{gi} \left(E_i - \frac{\frac{D_{gg'}}{2} \frac{D_{g'g}}{2}}{1 - \frac{D_{gg'}}{2} \frac{D_{g'g}}{2}} \right) - D_{gg'} x_{g'i} \left(E_i - \frac{1}{2} \frac{1 - D_{gg'} \frac{D_{g'g}}{2}}{1 - \frac{D_{gg'}}{2} \frac{D_{g'g}}{2}} \right) \right\}. \end{aligned} \tag{35}$$

Proof of Lemma A.2. To derive relations (34, 35), note that the pre- and post-merger FOC functions for input i are

$$f_i(w, c_i) = \sum_g x_{gi} \left\{ y_g(p^*(w)) + (w_i - c_i) \frac{dy_g(p^*(w))}{dw_i} \right\},$$

$$\hat{f}_i(w, c_i) = \sum_g \hat{x}_{gi} \left\{ y_g(\hat{p}^*(w)) + (w_i - c_i) \frac{dy_g(\hat{p}^*(w))}{dw_i} \right\}.$$

IPP_i is then

$$
\begin{aligned}
\mathrm{IPP}_i &= -\left(\sum_g \hat{x}_{gi} \frac{dy_g(\hat{p}^*(w))}{dw_i}\right)^{-1} \left\{\hat{f}_i(w^*, c_i) - f_i(w^*, c_i)\right\} \\
&= -\left(\sum_g \hat{x}_{gi} \frac{dy_g(\hat{p}^*(w))}{dw_i}\right)^{-1} \left\{\hat{f}_i(w^*, c_i) - (1-E_i) f_i(w^*, c_i)\right\} \\
&= -\left(\frac{1}{2}\sum_g \hat{x}_{gi} \frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right)\right)^{-1} \qquad (36) \\
&\quad \times (1-E_i)\sum_g x_{gi}\left\{y_g(\hat{p}^*(w)) - y_g(p^*(w)) + \left(w_i^* - c_i\right)\left(\frac{dy_g(\hat{p}^*(w))}{dw_i} - \frac{dy_g(p^*(w))}{dw_i}\right)\right\}
\end{aligned}
$$

The second line makes use of the pre-merger FOC for i. This calculation verifies (34).

To derive the exact IPP_i formula (35), we observe that linear consumer demand, constant returns to scale everywhere, and linear marginal costs downstream jointly imply that the FOC functions f and $\hat{f}$ are linear, so the approximation formulas of Proposition 2 are exact. Beginning with the first term in brackets in (36), we can write

$$
y_g(\hat{p}^*(w^*)) - y_g(p^*(w^*)) = \frac{\partial y}{\partial p}(\hat{p}^*(w^*) - p^*(w^*)).
$$

To calculate the change in goods prices at fixed input prices, we can use the first term in approximation (13). The pre- and post-merger FOC functions for goods prices are

$$
\begin{aligned}
f_{\mathcal{G}}\left(p, c_{\mathcal{G}}\right) &= y(p) + \mathrm{diag}\left[\frac{\partial y}{\partial p}\right]\left(p - c_{\mathcal{G}}\right), \\
\hat{f}_{\mathcal{G}}\left(p, \hat{c}_{\mathcal{G}}\right) &= y(p) + \left[\frac{\partial y}{\partial p}\right]^{\top}\left(p - \hat{c}_{\mathcal{G}}\right).
\end{aligned}
$$

Making use of Slutsky symmetry, we can then calculate

$$
\begin{aligned}
\frac{\partial y}{\partial p}(\hat{p}^*(w^*) - p^*(w^*)) &= -\frac{\partial y}{\partial p}\left[\frac{d\hat{f}_{\mathcal{G}}}{dp}\right]^{-1} \left\{\hat{f}_{\mathcal{G}}\left(p^*, \hat{c}_{\mathcal{G}}(w^*)\right) - f_{\mathcal{G}}\left(p^*, c_{\mathcal{G}}(w^*)\right)\right\} \\
&= -\frac{1}{2}\left\{\left(\frac{\partial y}{\partial p} - \mathrm{diag}\left[\frac{\partial y}{\partial p}\right]\right)\left(p^* - c_{\mathcal{G}}(w^*)\right) + \frac{\partial y}{\partial p}\left(c_{\mathcal{G}}(w^*) - \hat{c}_{\mathcal{G}}(w^*)\right)\right\} \\
&= -\frac{1}{2}\,\mathrm{diag}\left[\frac{\partial y}{\partial p}\right]\begin{pmatrix} c_g(w^*) - \hat{c}_g(w^*) - D_{gg'}\left(p_{g'}^* - \hat{c}_{g'}(w^*)\right) \\ c_{g'}(w^*) - \hat{c}_{g'}(w^*) - D_{g'g}\left(p_g^* - \hat{c}_g(w^*)\right) \end{pmatrix}.
\end{aligned}
$$

This equation expresses the key trade-off of Williamson (1968): increased market power (internalization of diverted sales) tends to lower output, while cost efficiencies tend to raise output.

Considering now the second term of (36), we can write

$$\frac{dy\left(\hat{p}^*(w^*)\right)}{dw}-\frac{dy\left(p^*(w^*)\right)}{dw}=\frac{\partial y}{\partial p}\left(\hat{\rho}_{\mathcal{G}}\frac{\partial \hat{c}_{\mathcal{G}}}{\partial w}-\rho_{\mathcal{G}}\frac{\partial c_{\mathcal{G}}}{\partial w}\right).$$

Using the pass-through formulas (23, 24), the right side can be written

$$\frac{\partial y}{\partial p}\left(\hat{\rho}_{\mathcal{G}}\frac{\partial \hat{c}_{\mathcal{G}}}{\partial w}-\rho_{\mathcal{G}}\frac{\partial c_{\mathcal{G}}}{\partial w}\right)=\frac{1}{2}\frac{\partial y}{\partial p}\begin{bmatrix}\hat{x}_{gi}-\frac{x_{gi}+\frac{D_{gg'}}{2}x_{g'i}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}} & \hat{x}_{gi'}-\frac{x_{gi'}+\frac{D_{gg'}}{2}x_{g'i'}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\\ \hat{x}_{g'i}-\frac{\frac{D_{g'g}}{2}x_{gi}+x_{g'i}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}} & \hat{x}_{g'i'}-\frac{\frac{D_{g'g}}{2}x_{gi'}+x_{g'i'}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\end{bmatrix}.$$

Direct calculation yields

$$\begin{aligned}&\frac{dy_g\left(\hat{p}^*(w^*)\right)}{dw_i}-\frac{dy_g\left(p^*(w^*)\right)}{dw_i}\\&=-\frac{1}{2}\frac{\partial y_g}{\partial p_g}\left\{x_{gi}\left(\frac{x_{gi}-\hat{x}_{gi}}{x_{gi}}-\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)-D_{gg'}x_{g'i}\left(\frac{x_{g'i}-\hat{x}_{g'i}}{x_{g'i}}-\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\right\}\end{aligned}$$

We can substitute into (36) to verify (35):

$$\begin{aligned}&(1-E_i)\left(\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left(x_{gi}-D_{gg'}x_{g'i}\right)\right)\mathrm{IPP}_i\\&=\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left\{c_g(w^*)-\hat{c}_g(w^*)-D_{gg'}\left(p^*_{g'}-\hat{c}_{g'}(w^*)\right)\right\}\\&\quad+\left(w^*_i-c_i\right)\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left\{x_{gi}\left(E_i-\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)-D_{gg'}x_{g'i}\left(E_i-\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\right\}.\end{aligned}$$

■

Proof of Proposition 3. When the merger yields Werden efficiencies, Lemma A.2 implies

$$\frac{1-E_i}{w^*_i-c_i}\mathrm{IPP}_i=\frac{\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left\{x_{gi}\left(E_i-\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)-D_{gg'}x_{g'i}\left(E_i-\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\right\}}{\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left(x_{gi}-D_{gg'}x_{g'i}\right)}.$$

Note that the expression on the right is increasing in E_i, and with $D_{gg'}, D_{g'g} \in [0,1]$ we have

$$\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}} \geq \frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}.$$

Setting

$$E_i = \frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}},$$

we then find $\mathrm{IPP}_i \geq 0$, where the inequality is strict if $D_{gg'}D_{g'g} \in (0,1)$ and $x_{gi}, x_{g'i} > 0$. ■

Proof of Claim 2. Following the proof of Proposition 3 and using the assumptions $x_{gi} = x_{g'i}$ and $D_{gg'} = D_{g'g} = D$, we can calculate

$$\bar{E}_i = \frac{\sum_g x_{gi}^2\left(-\frac{\partial y_g}{\partial p_g}\right)\left(\frac{\frac{D^2}{4}}{1-\frac{D^2}{4}} - \frac{D}{2}\frac{1-\frac{D^2}{2}}{1-\frac{D^2}{4}}\right)}{\sum_g x_{gi}^2\left(-\frac{\partial y_g}{\partial p_g}\right)(1-D)} = -\frac{D}{2}\frac{1-\frac{D}{2}-\frac{D^2}{2}}{(1-D)\left(1-\frac{D^2}{4}\right)} = -\frac{\frac{D}{2}}{1-\frac{D}{2}}.$$

■

Proof of Proposition 4. (i) Equation (20) yields the bound

$$E_g^{\mathrm{W}} \geq \frac{D_{gg'}D_{g'g}}{1-D_{gg'}D_{g'g}}\mu_g^*.$$

Comparing this lower bound to the right side of inequality (18),

$$\frac{D_{gg'}D_{g'g}}{1-D_{gg'}D_{g'g}}\mu_g^* \geq \frac{D_{gg'}D_{g'g}}{4-D_{gg'}D_{g'g}} \iff \mu_g^* \geq \frac{1-D_{gg'}D_{g'g}}{4-D_{gg'}D_{g'g}}.$$

The result follows from Proposition 3, noting that the right side of the final inequality is bounded above by 0.25 since $D_{gg'}, D_{g'g} \in [0,1)$.

(ii) The symmetry assumptions imply $p_g^* = p_{g'}^*$ and $D_{gg'} = D_{g'g} =: D$, so equation (20) becomes

$$E_g^{\mathrm{W}} = \frac{D}{1-D}\mu_g^*.$$

Comparing this value to the right side of inequality (18),

$$\frac{D}{1-D}\mu_g^* \geq \frac{D^2}{4-D^2} \iff \mu_g^* \geq \frac{D(1-D)}{4-D^2}.$$

The result follows from Proposition 3, noting that the right side of the final inequality is bounded above by $\frac{2-\sqrt{3}}{4} \approx 0.06699$. ■

Proof of Proposition 5. Making use of the second term in approximation (13), the change in goods prices due to endogenous input prices is

$$\hat{p}^*(\hat{w}^*) - \hat{p}^*(w^*) = \hat{\rho}_{\mathcal{G}} \frac{\partial \hat{c}_{\mathcal{G}}}{\partial w} \hat{\rho}_{\mathcal{I}} \text{IPP}.$$

Applying Lemma A.1 with $x_{g'i} = x_{gi'} = 0$, we have

$$\hat{p}^*(\hat{w}^*) - \hat{p}^*(w^*) = \frac{(1/2)^2}{1 - \frac{D_{gg'}}{2}\frac{D_{g'g}}{2}} \begin{bmatrix} \hat{x}_{gi} & \hat{x}_{g'i'}\frac{D_{gg'}}{2} \\ \hat{x}_{gi}\frac{D_{g'g}}{2} & \hat{x}_{g'i'} \end{bmatrix} \text{IPP}.$$

Hence

$$\begin{aligned}
&\hat{p}_g^*(\hat{w}^*) - \hat{p}_g^*(w^*) \\
&\quad = \frac{(1/2)^2}{1 - \frac{D_{gg'}}{2}\frac{D_{g'g}}{2}} \\
&\quad\quad \times \left\{ c_g(w^*) - \hat{c}_g(w^*) - D_{gg'}\left(p_{g'}^* - \hat{c}_{g'}(w^*)\right) + x_g\left(E_i - \frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1 - \frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\left(w_i^* - c_i\right) \right. \\
&\quad\quad\quad \left. + \frac{D_{gg'}}{2}\left[c_{g'}(w^*) - \hat{c}_{g'}(w^*) - D_{g'g}\left(p_g^* - \hat{c}_g(w^*)\right) + x_{g'}\left(E_{i'} - \frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1 - \frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\left(w_{i'}^* - c_{i'}\right)\right]\right\}.
\end{aligned}$$

We observe the following comparative statics:

$$\frac{\partial\left(\hat{p}_g^*(\hat{w}^*)-\hat{p}_g^*(w^*)\right)}{\partial E_i} \propto x_g\left(w_i^*-c_i\right)+\left(1-D_{g'g}\frac{D_{gg'}}{2}\right)x_{gi}w_i^*,$$
$$\frac{\partial\left(\hat{p}_g^*(\hat{w}^*)-\hat{p}_g^*(w^*)\right)}{\partial E_{gv}} \propto \left(1-D_{g'g}\frac{D_{gg'}}{2}\right)v_g,$$
$$\frac{\partial\left(\hat{p}_g^*(\hat{w}^*)-\hat{p}_g^*(w^*)\right)}{\partial E_{i'}} \propto -\frac{D_{gg'}}{2}x_{g'i'}c_{i'},$$
$$\frac{\partial\left(\hat{p}_g^*(\hat{w}^*)-\hat{p}_g^*(w^*)\right)}{\partial E_{g'v}} \propto -\frac{D_{gg'}}{2}v_{g'}.$$

■

Proof of Proposition 3′. We denote the pre- and post-merger input demand functions by

$$x_{gi}\left(y_g,w\right):=\frac{\partial C_g\left(y_g,w\right)}{\partial w_i} \quad \text{and} \quad \hat{x}_i(y,w):=\frac{\partial C_M(y,w)}{\partial w_i}.$$

The pre- and post-merger FOC functions for input i are

$$\begin{aligned}
f_i(w,c_i) &= \sum_{g\in\mathcal{G}} x_{gi}\left(y_g(p^*(w)),w\right) \\
&\quad +(w_i-c_i)\sum_{g\in\mathcal{G}}\left(\frac{\partial x_{gi}\left(y_g(p^*(w)),w\right)}{\partial y_g}\frac{dy_g(p^*(w))}{dw_i}+\frac{\partial x_{gi}\left(y_g(p^*(w)),w\right)}{\partial w_i}\right),\\
\hat{f}_i(w,c_i) &= \hat{x}_i(y(\hat{p}^*(w)),w) \\
&\quad +(w_i-c_i)\left(\sum_{g\in\mathcal{G}}\frac{\partial \hat{x}_i(y(\hat{p}^*(w)),w)}{\partial y_g}\frac{dy_g(\hat{p}^*(w))}{dw_i}+\frac{\partial \hat{x}_i(y(\hat{p}^*(w)),w)}{\partial w_i}\right).
\end{aligned}$$

Where there is no risk of confusion, we suppress the arguments of all functions on the right sides of these equations. Define the reduction in total demand for input i after the merger, holding fixed pre-merger outputs and input prices:

$$1-\chi_i:=\frac{\hat{x}_i(y(p^*),w^*)}{\sum_{g\in\mathcal{G}}x_{gi}\left(y_g(p^*),w^*\right)}.$$

Making use of the pre-merger FOC for i, IPP_i then satisfies

$$\begin{aligned}
\mathrm{IPP}_i &\propto \hat{f}_i(w^*, c_i) - (1-\chi_i) f_i(w^*, c_i) \\
&= \hat{x}_i - (1-\chi_i)\sum_{g\in\mathcal{G}} x_{gi} \\
&\quad + \left(w_i^* - c_i\right)\sum_{g\in\mathcal{G}} (1-\chi_i)\frac{\partial x_{gi}}{\partial y_g}\left(\frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i}\right) \\
&\quad + \left(w_i^* - c_i\right)\sum_{g\in\mathcal{G}}\left(\frac{\partial \hat{x}_i}{\partial y_g} - (1-\chi_i)\frac{\partial x_{gi}}{\partial y_g}\right)\frac{dy_g(\hat{p}^*(w^*))}{dw_i} \\
&\quad + \left(w_i^* - c_i\right)\left(\frac{\partial \hat{x}_i}{\partial w_i} - (1-\chi_i)\sum_{g\in\mathcal{G}}\frac{\partial x_{gi}}{\partial w_i}\right).
\end{aligned}$$

Since the merger is Werden-efficient, the first line equals zero, and this relation becomes

$$\begin{aligned}
\mathrm{IPP}_i \propto &\sum_{g\in\mathcal{G}} (1-\chi_i)\frac{\partial x_{gi}}{\partial y_g}\left(\frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i}\right) \\
&+ \sum_{g\in\mathcal{G}}\left(\frac{\partial \hat{x}_i}{\partial y_g} - (1-\chi_i)\frac{\partial x_{gi}}{\partial y_g}\right)\frac{dy_g(\hat{p}^*(w^*))}{dw_i} + \frac{\partial \hat{x}_i}{\partial w_i} - (1-\chi_i)\sum_{g\in\mathcal{G}}\frac{\partial x_{gi}}{\partial w_i}.
\end{aligned}$$

Dividing through by $\hat{x}_i$ yields

$$\begin{aligned}
\mathrm{IPP}_i \propto &\sum_{g\in\mathcal{G}} \frac{1}{\sum_{g\in\mathcal{G}} x_{gi}}\frac{\partial x_{gi}}{\partial y_g}\left(\frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i}\right) \\
&+ \sum_{g\in\mathcal{G}}\left(\frac{1}{\hat{x}_i}\frac{\partial \hat{x}_i}{\partial y_g} - \frac{1}{\sum_{g\in\mathcal{G}} x_{gi}}\frac{\partial x_{gi}}{\partial y_g}\right)\frac{dy_g(\hat{p}^*(w^*))}{dw_i} + \frac{1}{\hat{x}_i}\frac{\partial \hat{x}_i}{\partial w_i} - \frac{1}{\sum_{g\in\mathcal{G}} x_{gi}}\sum_{g\in\mathcal{G}}\frac{\partial x_{gi}}{\partial w_i}.
\end{aligned}$$

Under the assumptions of the proposition, the second line is precisely zero, so we have

$$\mathrm{IPP}_i \propto \sum_{g\in\mathcal{G}} \frac{\partial x_{gi}}{\partial y_g}\left(\frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i}\right).$$

The remainder of the proof follows exactly the derivation of (35) in Lemma A.2 and the subsequent proof of Proposition 3. (Exchange the notation x_{gi} and $\hat{x}_{gi}$ in those proofs for $\frac{\partial c_g}{\partial w_i} = \frac{\partial x_{gi}}{\partial y_g}$ and $\frac{\partial \hat{c}_g}{\partial w_i}$ here, respectively.) ■

Proof of Proposition 3′′. With Werden efficiencies, the same calculation as in the proof of

Lemma A.2 yields

$$\text{IPP}_i \propto \frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i}.$$

The output sensitivity decomposition (22) then implies

$$\text{IPP}_i > 0 \iff 1 - E_i \leq \frac{\rho_{gg} - D_{gg'}\rho_{g'g} - D_{go}\rho_{og}}{\hat{\rho}_{gg} - D_{gg'}\hat{\rho}_{g'g} - D_{go}\hat{\rho}_{og}}.$$

The threshold efficiencies $\bar{E}_i^{\text{out}}$ are such that this inequality binds. To calculate $\bar{E}_i^{\text{out}}$, we must then determine the downstream pass-through rates $\rho_{\mathcal{G}}$ and $\hat{\rho}_{\mathcal{G}}$. To do this, note that the pre- and post-merger downstream FOC functions are

$$\begin{aligned}
f_g(p, c_g) &= y_g(p) + \frac{\partial y_g}{\partial p_g}(p_g - c_g) && g \in \mathcal{G} \setminus \{o\}, \\
f_o(p, c_o) &= y_o(p) + \frac{\partial y_o}{\partial p_o}(p_o - c_o), \\
\hat{f}_g(p, c_g) &= y_g(p) + \frac{\partial y_g}{\partial p_g}(p_g - c_g) + \frac{\partial y_{g'}}{\partial p_g}(p_{g'} - c_{g'}) && g \in \mathcal{G} \setminus \{o\}, \\
\hat{f}_o(p, c_o) &= f_o(p, c_o).
\end{aligned}$$

Using the definition (10), the downstream pass-through rates satisfy

$$\rho_{\mathcal{G}} = \frac{1}{2}(\mathbf{I} - B)^{-1} \quad \text{and} \quad \hat{\rho}_{\mathcal{G}} = \frac{1}{2}\left(\mathbf{I} - \hat{B}\right)^{-1}, \quad \text{where} \quad B = \begin{bmatrix} 0 & \frac{D}{2} & \frac{D}{2} \\ \frac{D}{2} & 0 & \frac{D}{2} \\ \frac{D}{2} & \frac{D}{2} & 0 \end{bmatrix} \quad \text{and} \quad \hat{B} = \begin{bmatrix} 0 & D & \frac{D}{2} \\ D & 0 & \frac{D}{2} \\ \frac{D}{2} & \frac{D}{2} & 0 \end{bmatrix}.$$

Inverting yields

$$\begin{aligned}
\rho_{\mathcal{G}} &= \frac{\frac{1}{2}}{(1-D)\left(1+\frac{D}{2}\right)} \begin{bmatrix} 1-\frac{D}{2} & \frac{D}{2} & \frac{D}{2} \\ \frac{D}{2} & 1-\frac{D}{2} & \frac{D}{2} \\ \frac{D}{2} & \frac{D}{2} & 1-\frac{D}{2} \end{bmatrix}, \\
\hat{\rho}_{\mathcal{G}} &= \frac{\frac{1}{2}}{\left(1-D-\frac{D^2}{2}\right)(1+D)} \begin{bmatrix} 1-\frac{D^2}{4} & D\left(1+\frac{D}{4}\right) & \frac{D}{2}(1+D) \\ D\left(1+\frac{D}{4}\right) & 1-\frac{D^2}{4} & \frac{D}{2}(1+D) \\ \frac{D}{2}(1+D) & \frac{D}{2}(1+D) & 1-D^2 \end{bmatrix}.
\end{aligned}$$

We can substitute these expressions into the definition of the threshold $\bar{E}_i^{\text{out}}$ to find

$$
\begin{aligned}
1-\bar{E}_i^{\text{out}} &= \frac{\left(1-D-\frac{D^2}{2}\right)(1+D)}{(1-D)\left(1+\frac{D}{2}\right)} \frac{1-\frac{D}{2}-D^2}{1-\frac{D^2}{4}-D^2\left(1+\frac{D}{4}\right)-\frac{D^2}{2}(1+D)} \\
&= \frac{1-D-\frac{D^2}{2}}{(1-D)\left(1+\frac{D}{2}\right)} \frac{1-\frac{D}{2}-D^2}{\left(1-\frac{3}{2}D\right)\left(1+\frac{D}{2}\right)}.
\end{aligned}
$$

Then

$$
\bar{E}_i^{\text{out}} \leq \frac{D^2}{4-D^2} \iff \frac{1-D-\frac{D^2}{2}}{1-D} \frac{1-\frac{D}{2}-D^2}{1-\frac{3}{2}D} \geq \frac{1-\frac{D^2}{2}}{1-\frac{D}{2}}.
$$

It is easy to verify that this inequality holds for $D \in [0, 0.5]$, and strictly for $D \neq 0$. ■

Proof of Proposition 3$'''$. By the same calculation as in the proof of Lemma A.2,

$$
\begin{aligned}
\text{IPP}_i \propto & \sum_g x_{gi} \left\{ \frac{d y_g(\hat{p}^*(w^*))}{d w_i} - \frac{d y_g(p^*(w^*))}{d w_i} \right\} \\
= & \sum_g x_{gi} \left\{ \left(\frac{\partial y_g}{\partial p_g} \hat{\rho}_{gg} + \frac{\partial y_g}{\partial p_{g'}} \hat{\rho}_{g'g} \right) (1-E_i) x_{gi} + \left(\frac{\partial y_g}{\partial p_g} \hat{\rho}_{gg'} + \frac{\partial y_g}{\partial p_{g'}} \hat{\rho}_{g'g'} \right) (1-E_i) x_{g'i} \right\} \\
& - \sum_g x_{gi} \left\{ \left(\frac{\partial y_g}{\partial p_g} \rho_{gg} + \frac{\partial y_g}{\partial p_{g'}} \rho_{g'g} \right) x_{gi} + \left(\frac{\partial y_g}{\partial p_g} \rho_{gg'} + \frac{\partial y_g}{\partial p_{g'}} \rho_{g'g'} \right) x_{g'i} \right\}.
\end{aligned}
$$

where all demand derivatives and pass-through rate functions are evaluated at prices (p^*, w^*). Hence $\text{IPP}_i > 0$ if and only if $E_i > \bar{E}_i^{\text{NL}}$, where

$$
\bar{E}_i^{\text{NL}} = 1 - \frac{\sum_g x_{gi} \frac{\partial y_g}{\partial p_g} \left\{ \left(\rho_{gg} - D_{gg'} \rho_{g'g}\right) x_{gi} + \left(\rho_{gg'} - D_{gg'} \rho_{g'g'}\right) x_{g'i} \right\}}{\sum_g x_{gi} \frac{\partial y_g}{\partial p_g} \left\{ \left(\hat{\rho}_{gg} - D_{gg'} \hat{\rho}_{g'g}\right) x_{gi} + \left(\hat{\rho}_{gg'} - D_{gg'} \hat{\rho}_{g'g'}\right) x_{g'i} \right\}}.
$$

■

B Simulations

B.1 Price Changes in the Simple Model

In this section, we compute post-merger changes in goods and input prices in the simple model of Section 3.1. We assume independent input suppliers, unit input requirements $x_{gi} = x_{g'i'} = 1$ and $v_g = v_{g'} = 0$, and upstream marginal costs $c_i = c_{i'} = 1$. We also assume a symmetric linear demand system

$$y_g(p) = V - p_g + Dp_{g'} \quad g \in \mathcal{G}.$$

We calibrate (V, D) to match the specified diversion ratios and pre-merger markups.

Figures 4, 5, 6, 7 show the percent change in the goods and input prices after the downstream merger. Figures 4 and 5 show these price changes for a merger with Hicks-neutral Werden efficiencies. We see that goods and input prices increase for all diversions and pre-merger markups displayed. Figures 6 and 7 show these price changes for a merger with zero efficiencies. Input prices decline for all diversion ratios and pre-merger markups displayed, but goods prices always increase.

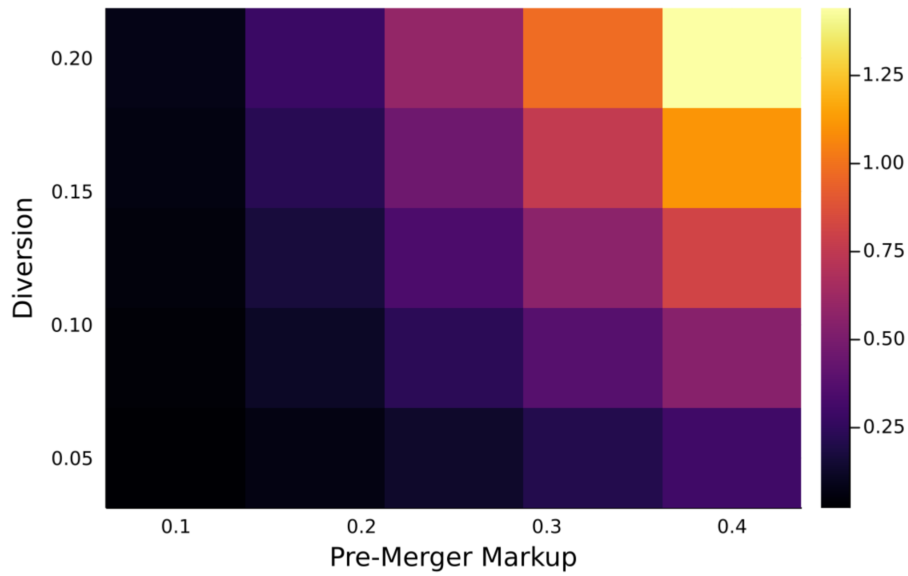

Figure 4: Percent change in consumer prices with endogenous input prices at Werden efficiencies.

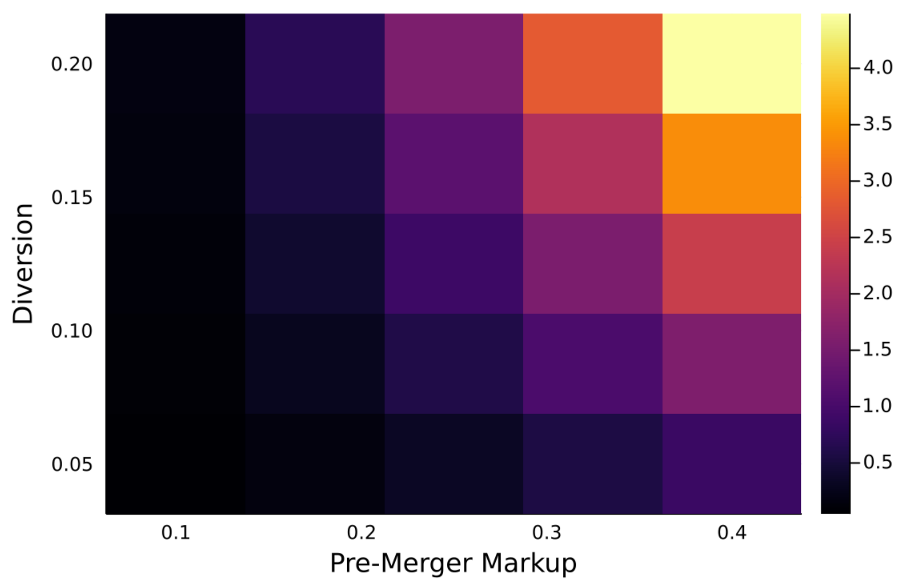

Figure 5: Percent change in input prices with endogenous input prices at Werden efficiencies.

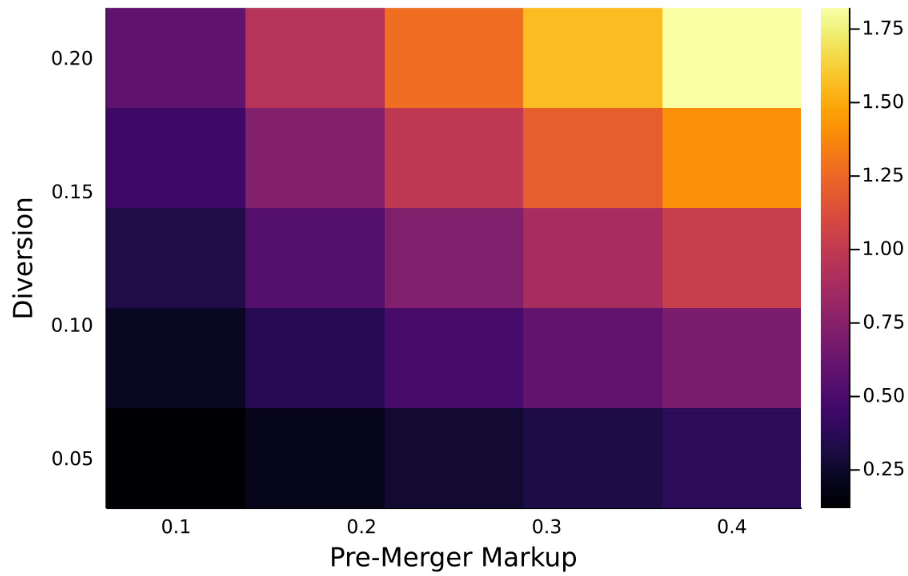

Figure 6: Percent change in consumer prices with endogenous input prices at zero efficiencies.

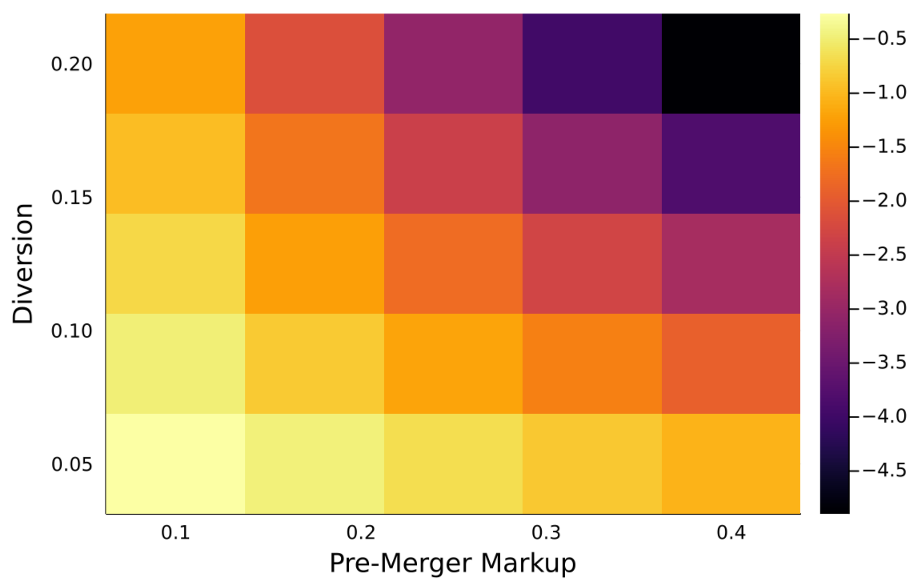

Figure 7: Percent change in input prices with endogenous input prices at zero efficiencies.

B.2 Outsiders

Here we explore how Proposition 3″ extends when the symmetric diversion ratio between the merging firms' goods $D_M := D_{gg'} = D_{g'g}$ is different from the symmetric diversion ratio between the outsider's good and the merging firms' goods $D_o := D_{go} = D_{og} = D_{g'o} = D_{og'}$. Figure 8 plots $\bar{E}_i^{\text{out}} - \frac{D_M^2}{4-D_M^2}$ as a function of D_M and D_o. This figure shows that the threshold $\bar{E}_{gi}^{\text{out}}$ becomes *smaller* as the diversion ratio between the outsider's good and the merging firms' goods increases. Note that we have imposed the constraints that the *aggregate diversion ratios* $D_M + D_o$ and $2D_o$ are below 1.

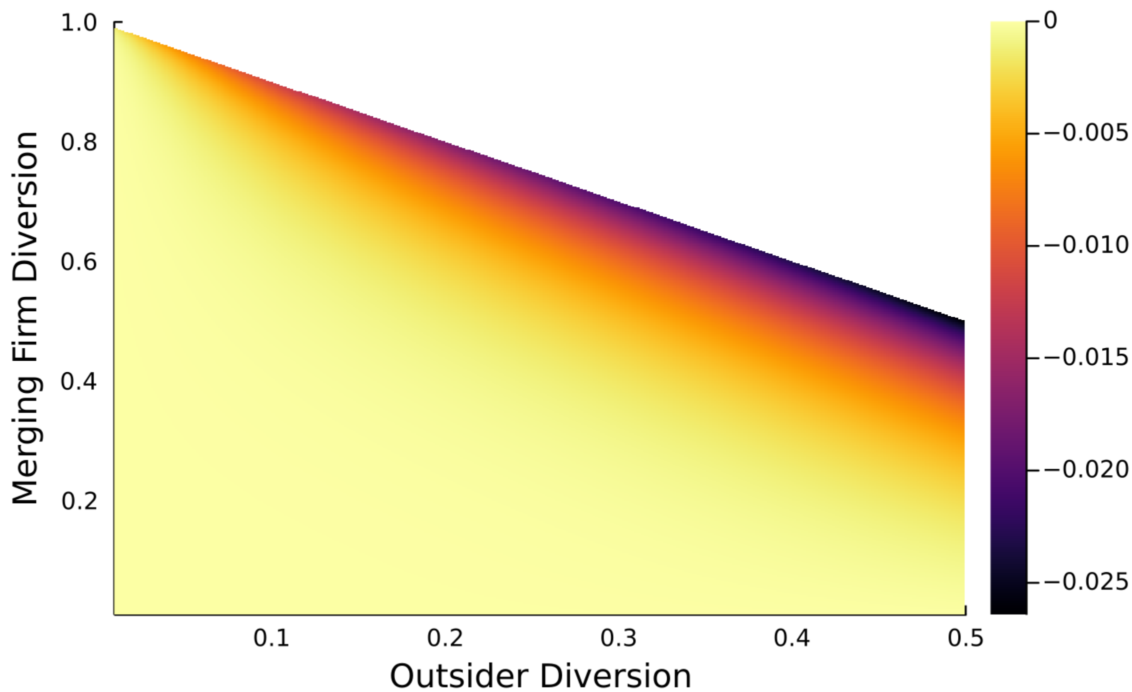

Figure 8: Comparison of efficiencies threshold with and without an outsider.

B.3 Nonlinear Demand and Approximation Error

The results in Section 3.3 partially characterize IPP but do not directly speak to equilibrium consumer and input price changes. Here we consider a version of the simple model with nonlinear demand, and we calculate equilibrium price changes numerically. We confirm that the approximation formulas given in Proposition 2 correctly predict the sign of the equilibrium changes in input and goods prices after the merger, and moreover that the approximation for goods prices is also close in magnitude to the equilibrium change.

Input Prices

Consider the simple model of Section 3.1 with independent input suppliers, unit input requirements $x_{gi} = x_{g'i'} = 1$ and $v_g = v_{g'} = 0$, and upstream marginal costs $c_i = c_{i'} = 1$. Suppose consumer demand $y(p)$ is described by a symmetric logit demand system, so that the indirect utility of good g for an atomistic consumer l takes the form

$$u_{lg} = \alpha - \beta p_g + \epsilon_{lg},$$

where ϵ_{lg} is a logit error term. In Figures 9 and 10, we calibrate the parameters of the logit demand system (α, β) to give the specified (symmetric) diversion ratio and pre-merger markup. The plot marker denotes the diversion ratio and the color denotes the pre-merger markup. We plot the (symmetric) percent change in input prices in equilibrium versus the approximate percent change corresponding to equation (14) in Proposition 2. Figure 9 shows these price changes after a merger with Hicks-neutral Werden efficiencies, and Figure 10 with zero efficiencies. We observe in both cases that the sign of the input price change predicted by IPP is the same as the sign of the input price change in equilibrium; the input price change due to IPP underestimates the magnitude of the equilibrium change. The approximation is better for lower diversion ratios and lower pre-merger markups. In both cases, higher diversion ratios and higher pre-merger markups are associated with larger post-merger input price changes.

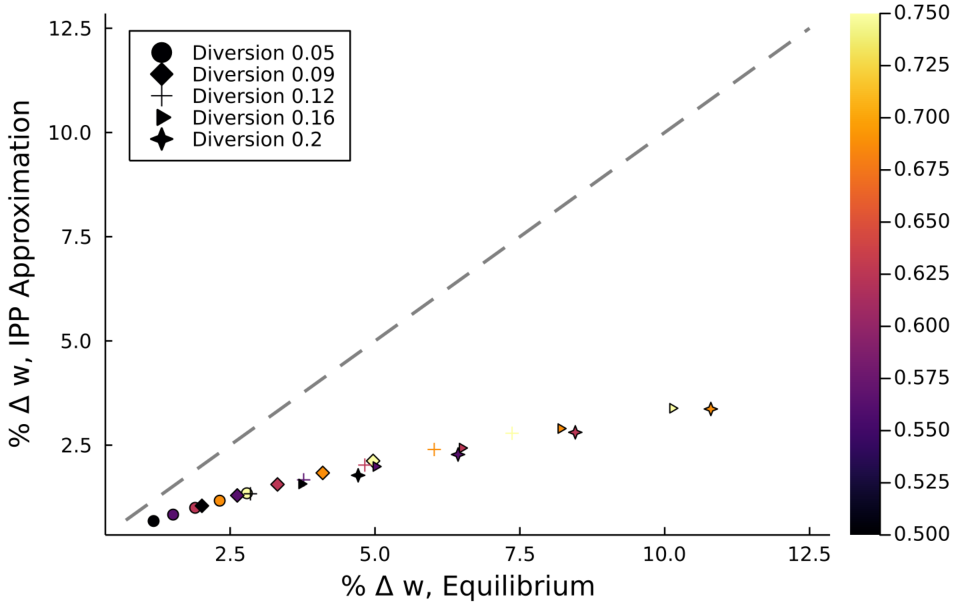

Figure 9: Equilibrium versus approximate input price changes at Werden efficiencies.

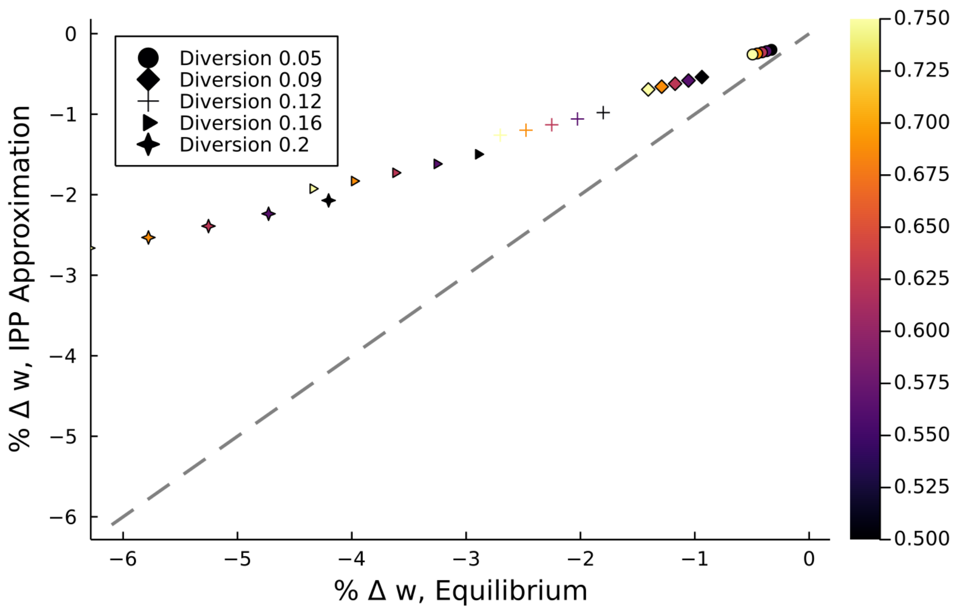

Figure 10: Equilibrium versus approximate input price changes at zero efficiencies.

Goods Prices

Using the same model as above, we now consider changes in goods prices. Figure 11 shows exact and approximate changes in goods prices after a merger with Hicks-neutral Werden efficiencies, and Figure 12 with zero efficiencies. We plot the (symmetric) percent change in goods prices in equilibrium versus the approximate percent change corresponding to equation (13) in Proposition 2. We again observe that the approximate price change has the same sign as the equilibrium goods price change and that the approximation understates the magnitude of the price change. The approximation is better for lower diversions and pre-merger markups. In both cases, higher diversion ratios and higher pre-merger markups are associated with higher post-merger goods price changes.

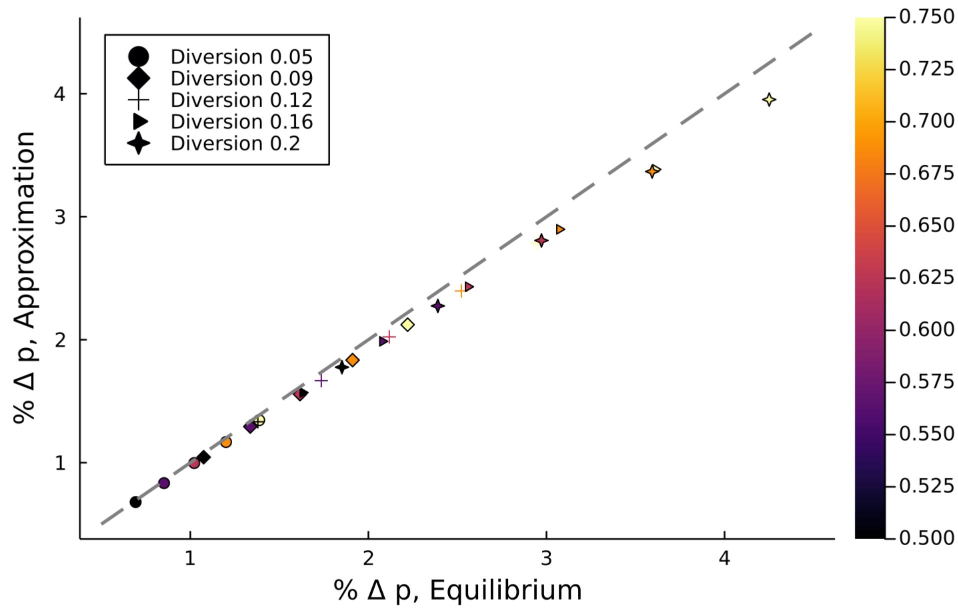

Figure 11: Equilibrium versus approximate goods price changes at Werden efficiencies.

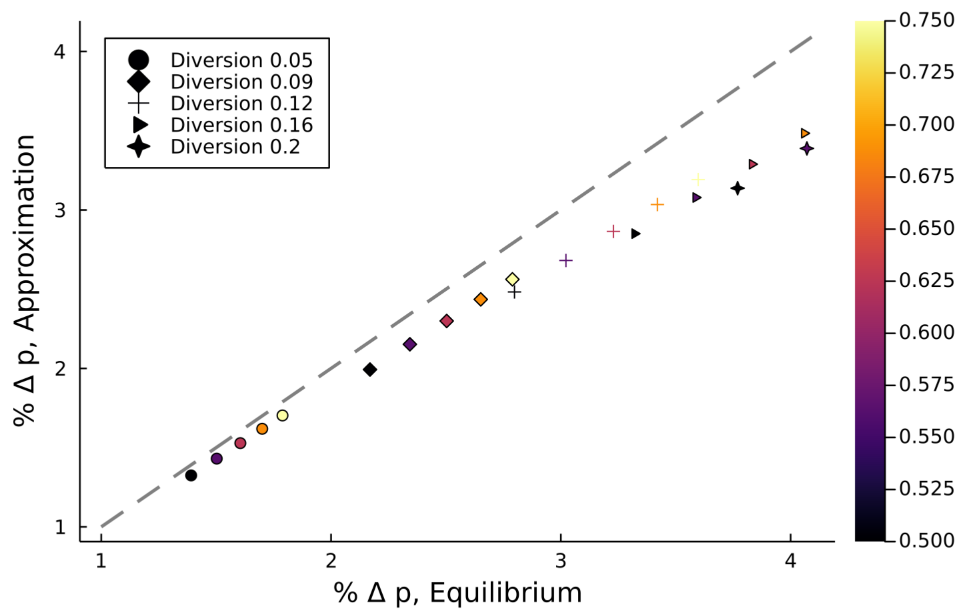

Figure 12: Equilibrium versus approximate goods price changes at zero efficiencies.

C Bargaining

In this section, we describe how standard versions of the Nash-in-Nash bargaining model of Horn and Wolinsky (1988) can be incorporated into the analysis of Sections 2 and 3.

C.1 Conduct Examples

As with modes of conduct in which upstream firms unilaterally set input prices, we consider two timing assumptions for the bargaining model, both of which have been employed in recent structural empirical work.[30] First, suppose that a subset of inputs $I_G \in I$ produced by firm I is only used by downstream firm G.[31] Then in "simultaneous" Nash-in-Nash bargaining, the price vector w_{I_G} for inputs I_G is chosen to maximize the weighted Nash product between firms G and I, holding goods prices and remaining input prices fixed:

$$\max_{w_{I_G} \geq 0} \Big[\underbrace{\pi_G(p,w) - \pi_G(p,(\varnothing, w_{-I_G}))}_{\mathrm{GFT}_{G,I_G}(p,w)} \Big]^{1-\beta_{I_G}} \times \Big[\underbrace{\pi_I(p,w) - \pi_I(p,(\varnothing, w_{-I_G}))}_{\mathrm{GFT}_{I,I_G}(p,w)} \Big]^{\beta_{I_G}}.$$

Here $\beta_{I_G} \in [0,1]$ is the Nash bargaining weight on firm I,

$$\pi_G(p,w) := p_G \cdot y_G(p) - C_G(y_G(p), w),$$
$$\pi_I(p,w) := w_I \cdot z_I(y(p), w) - C_I(z_I(y(p), w))$$

denote the profits earned by firms G and I after agreement on input prices w_{I_G}, and

$$\pi_G(p,(\varnothing, w_{-I_G})) := \max\left\{p_G \cdot y_G(p) - C_G\left(y_G(p), \left(\infty, w_{-I_G}\right)\right), 0\right\},$$
$$\pi_I(p,(\varnothing, w_{-I_G})) := \max\left\{w_{I\setminus I_G} \cdot z_{I\setminus I_G}(y(p), w) - C_I\left(0, z_{I\setminus I_G}(y(p), w)\right), 0\right\}.$$

denote the profits earned by firms G and I after a bargaining breakdown. The functions $\mathrm{GFT}_{G,I_G}(p,w)$ and $\mathrm{GFT}_{I,I_G}(p,w)$ describe the "gains-from-trade" realized by G and I if the bargained input prices are w_{I_G} and the remaining prices are (p, w_{-I_G}). The FOC function for input

[30] For examples of simultaneous Nash-in-Nash bargaining, see Draganska et al. (2010); Crawford and Yurukoglu (2012); Grennan (2013); Gowrisankaran et al. (2015); Ho and Lee (2017); Crawford et al. (2018); and Dubois and Sæthre (2020).

[31] Alternatively, assume that each downstream firm-upstream firm pair can negotiate different linear prices over inputs that are commonly used by many downstream firms.

price $i \in I_G$ is then

$$f_i(p,w) = z_i(y,w) + \sum_{i' \in I} (w_{i'} - c_{i'}(z_I(y,w))) \frac{\partial z_{i'}(y,w)}{\partial w_i} - \frac{1-\beta_{I_G}}{\mathrm{GFT}_{G,I_G}(p,w)} \frac{\mathrm{GFT}_{I,I_G}(p,w)}{\beta_{I_G}} z_i(y,w). \tag{37}$$

This function is exactly the Bertrand-Nash FOC function (3) less a term that reflects the profits lost by firm G after a marginal increase in the price of input i.

We could instead assume that all firms anticipate changes in goods prices when negotiating input prices. In "upstream-leading" Nash-in-Nash bargaining, the price vector w_{I_G} solves

$$\max_{w_{I_G} \geq 0} \mathrm{GFT}_{G,I_G}(p^*(w),w)^{1-\beta_{I_G}} \times \mathrm{GFT}_{I,I_G}(p^*(w),w)^{\beta_{I_G}}.$$

The resulting FOC function for input price $i \in I_G$ is

$$\begin{aligned} f_i(w) = {} & z_i(w) + \sum_{i' \in I} (w_{i'} - c_{i'}(z_I(w))) \frac{dz_{i'}(w)}{dw_i} \\ & + \frac{1-\beta_{I_G}}{\mathrm{GFT}_{G,I_G}(p^*(w),w)} \frac{\mathrm{GFT}_{I,I_G}(p^*(w),w)}{\beta_{I_G}} \\ & \times \left[-z_i(w) + \sum_{g \in G} \left(p_g - c_g(y_G(p^*(w)),w) \right) \sum_{g' \in \mathcal{G} \setminus G} \frac{dy_g(p^*(w))}{dp_{g'}} \frac{\partial p^*_{g'}(w)}{\partial w_i} \right]. \end{aligned} \tag{38}$$

This FOC function differs from the simultaneous Nash-in-Nash FOC function (37) in two ways: First, the demand function $z(w)$ replaces $z(y,w)$ since changes in goods prices in response to changes in input prices are now taken into account. Second, the firms now internalize how a change in the price of input $i \in I_G$ will incentivize price changes by firm G's competitors, which have a first-order impact on firm G's profits by shifting the consumer demand for firm G's goods. This second effect is captured by the second term in the brackets. Finally, note that just as with unilateral price setting, the "upstream-leading" assumption implies that the FOC function $f_{\mathcal{I}}$ depends only on input prices w.

C.2 IPP and Merger Policy

We now discuss how the main results of Section 3 generalize when input prices are set through Nash-in-Nash bargaining instead of upstream profit maximization. Consider the simple model of Section 3.1, and assume the economy features independent input suppliers. We now suppose that before the merger, the input price w_i is set by upstream-leading Nash bargaining between firms g and i. Specializing (38) to the simple model, the resulting pre-merger FOC function is

$$f_i(w,c_i) = x_{gi} y_g(p^*(w)) + (w_i - c_i) x_{gi} \frac{dy_g(p^*(w))}{dw_i} + \frac{1-\beta_i}{\mathrm{GFT}_{g,i}(p^*(w),w)} \frac{\mathrm{GFT}_{i,i}(p^*(w),w)}{\beta_i} \left[\left(p_g - c_g(w)\right) \frac{dy_g}{dp_{g'}} \frac{\partial p^*_{g'}(w)}{\partial w_i} - x_{gi} y_g(p^*(w)) \right],$$

where $\beta_i \in (0,1]$ denotes the bargaining weight on firm i and the gains-from-trade terms are equal to on-path profits:

$$\mathrm{GFT}_{i,g}(p^*(w),w) = \pi_g(p^*(w),w) \quad \text{and} \quad \mathrm{GFT}_{i,i}(p^*(w),w) = \pi_i(p^*(w),w).$$

After the merger, we assume that the merged firm bargains independently with each upstream firm before setting goods prices. The input price w_i is then chosen to solve

$$\max_{w_i \geq 0} \Big[\underbrace{\pi_M(\hat{p}^*(w),w) - \pi_M(\hat{p}^*(\varnothing, w_{i'}),(\varnothing, w_{i'}))}_{\hat{\mathrm{GFT}}_{M,i}(\hat{p}^*(w),w)} \Big]^{1-\beta_i} \times \Big[\underbrace{\pi_i(\hat{p}^*(w),w)}_{\hat{\mathrm{GFT}}_{i,i}(\hat{p}^*(w),w)} \Big]^{\beta_i}.$$

For simplicity, we assume that the exogenous bargaining weight β_i remains unchanged after the merger. The resulting post-merger FOC function is

$$\hat{f}_i(w,c_i) = \hat{x}_{gi} y_g(\hat{p}^*(w)) + (w_i - c_i)\hat{x}_{gi} \frac{dy_g(\hat{p}^*(w))}{dw_i} - \frac{1-\beta_i}{\hat{\mathrm{GFT}}_{M,i}(\hat{p}^*(w),w)} \frac{\hat{\mathrm{GFT}}_{i,i}(\hat{p}^*(w),w)}{\beta_i} \hat{x}_{gi} y_g(\hat{p}^*(w)),$$

where we have written $\hat{x}_{gi} := (1-E_i)x_{gi}$ for simplicity.

To determine the incentives for an increase in the input price w_i after the merger, we examine IPP_i:

$$\begin{aligned}\text{IPP}_i &\propto \hat{f}_i(w, c_i) - \frac{\hat{x}_{gi}}{x_{gi}} f_i(w, c_i) \\ &\propto \underbrace{y_g(\hat{p}^*(w)) - y_g(p^*(w)) + (w_i - c_i)\left(\frac{dy_g(\hat{p}^*(w))}{dw_i} - \frac{dy_g(p^*(w))}{dw_i}\right)}_{\propto\ \text{IPP}_i \text{ with upstream price setting}} \\ &\quad + \underbrace{\frac{1-\beta_i}{\beta_i}\left(\frac{\text{GFT}_{i,i}(p^*(w), w)}{\text{GFT}_{g,i}(p^*(w), w)} - \frac{\hat{\text{GFT}}_{i,i}(\hat{p}^*(w), w)}{\hat{\text{GFT}}_{M,i}(\hat{p}^*(w), w)}\right) y_g(\hat{p}^*(w))}_{\Delta \text{ downstream bargaining leverage}} \\ &\quad + \underbrace{\frac{1-\beta_i}{\beta_i}\frac{\text{GFT}_{i,i}(p^*(w), w)}{\text{GFT}_{g,i}(p^*(w), w)}\left[y_g(p^*(w)) - y_g(\hat{p}^*(w)) - \left(p_g - c_g(w)\right)\frac{\partial y_g}{\partial p_{g'}}\rho_{g'g}\hat{x}_{gi}\right]}_{\Delta \text{ sensitivity of downstream profits to input price}}.\end{aligned} \tag{39}$$

The first term in this decomposition is proportional to the Input Pricing Pressure on i when input prices are set unilaterally by upstream firms – see (17). The remaining terms arise because the Nash bargaining solution weighs the effect of a higher input price w_i on downstream profits as well as upstream profits. In particular, the second term reflects the change in bargaining leverage after the merger: When the merged firm's gains from trade increase relative to firm i's gains from trade after the merger, this term is negative and incentivizes a decrease in the input price w_i. With independent input suppliers, the change in relative bargaining leverage can be written

$$\begin{aligned}&\frac{\text{GFT}_{i,i}(p^*(w), w)}{\text{GFT}_{g,i}(p^*(w), w)} - \frac{\hat{\text{GFT}}_{i,i}(\hat{p}^*(w), w)}{\hat{\text{GFT}}_{M,i}(\hat{p}^*(w), w)} \\ &\quad = \frac{\pi_i(p^*(w), w)}{\pi_g(p^*(w), w)} - \frac{\pi_i(\hat{p}^*(w), w)}{\pi_M(\hat{p}^*(w), w) - \pi_M(\hat{p}^*(\varnothing, w_{i'}), (\varnothing, w_{i'}))}.\end{aligned} \tag{40}$$

The sign of this term is theoretically ambiguous. Firm i's profits may rise or fall post-merger, depending on the change in downstream output and the extent of i-specific efficiencies attained by the merged firm. Moreover, the change in the gains-from-trade that accrue downstream is also indeterminate, since the merged firm both earns higher on-path profits and has a higher threat point in its negotiations with i because it now controls the price of good g'.

The last term in the decomposition (39) reflects the change in the sensitivity of downstream profits to the input price w_i. Downstream profits naturally become more sensitive to the input price after the merger because the merged firm internalizes the value of demand diverted between consumer goods. This diversion is not internalized by the competing downstream firms before the merger, so it softens the impact of a higher input price on firm g's profits.

Internalization of diverted demand thus tends to lower the third term in (39). Downstream profits may also become more sensitive to the input price if the merged firm raises the output of good g, requiring additional expenditure on input i.

This discussion indicates that the two new terms in the IPP decomposition (39) have ambiguous signs, but we generally expect them to be negative for Werden-efficient mergers. This is clearly true for the third term, because downstream outputs remain unchanged after a Werden-efficient merger. The second term is the primary object of interest in the recent literature examining the effects of horizontal market structure in economies with vertical contracting. The consensus from this literature appears to be that, for mergers without marginal cost efficiencies, this term is negative as the merged firm improves its bargaining position vis-à-vis the upstream firms (Barrette, Gowrisankaran, and Town, 2020; Craig, Grennan, and Swanson, 2021; Sheu and Taragin, 2021). However, these studies show that the resulting decline in input prices is not sufficient to prevent goods prices from rising when downstream firms engage in Bertrand-Nash competition. Our simulations suggest that the second term in the IPP decomposition (39) is generally negative for Werden-efficient mergers. As a result, IPP_i is generally smaller under Nash-in-Nash bargaining than under the upstream price-setting benchmark, but the simulation results presented below indicate that for a range of bargaining parameters IPP_i remains positive for a merger with Hicks-neutral Werden efficiencies.

C.3 Simulations

Below we compute the IPP decomposition (39) for the simple model under the following assumptions: independent input suppliers, unit input requirements $x_{gi} = x_{g'i'} = 1$ and $v_g = v_{g'} = 0$, upstream marginal costs $c_i = c_{i'} = 1$, and symmetric linear demand

$$y_g(p) = V_\beta - p_g + D_\beta p_{g'} \quad g \in \mathcal{G}.$$

We calibrate (V_β, D_β) for each bargaining weight β on upstream firms to match the diversion ratio 0.063 and the pre-merger markup 0.404 .

Figures 13 and 14 plot the terms in the decomposition as a function of the bargaining weight β for mergers with Hicks-neutral Werden efficiencies and zero efficiencies, respectively.[32] Figure 13 shows that the term proportional to IPP with upstream price setting is weakly positive for all bargaining weights, while the term showing the change in downstream bargaining leverage is negative for most bargaining weights.

[32]We normalize each term by the scaling factor such that relation (39) hold with equality: $\left(\frac{\partial \hat{f}_i}{\partial c_i}\right)^{-1}$, where $\frac{\partial \hat{f}_i}{\partial c_i} = -\hat{x}_{gi} \frac{d y_g(\hat{p}^*(w))}{d w_i} + \frac{1-\beta_i}{\hat{\mathrm{GFT}}_{M,i}(\hat{p}^*(w), w)} \frac{\left(\hat{x}_{gi} y_g(\hat{p}^*(w))\right)^2}{\beta_i}$.

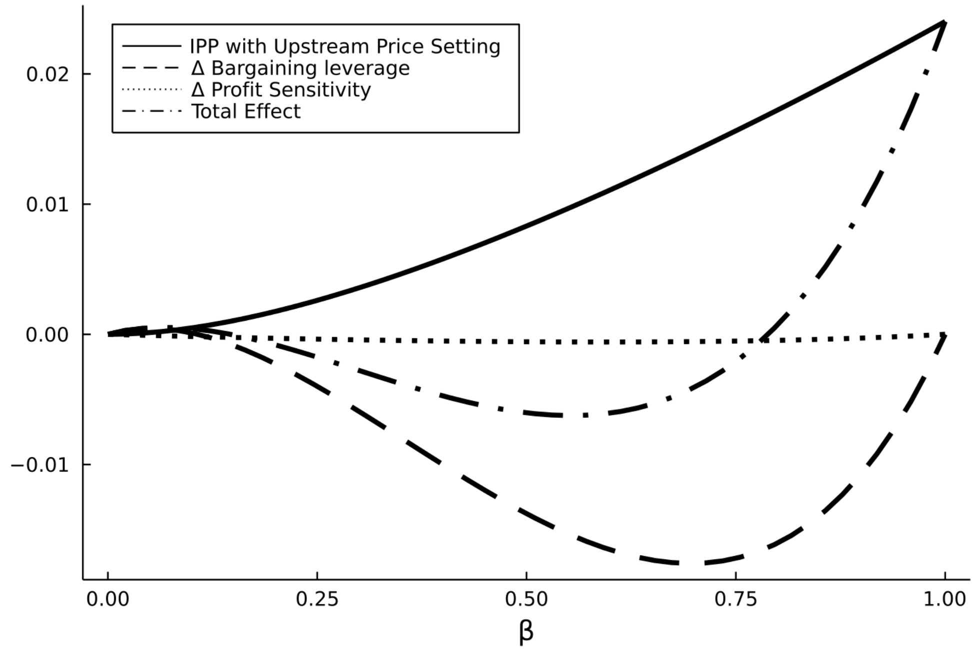

Figure 13: Bargaining IPP decomposition at Werden efficiencies.

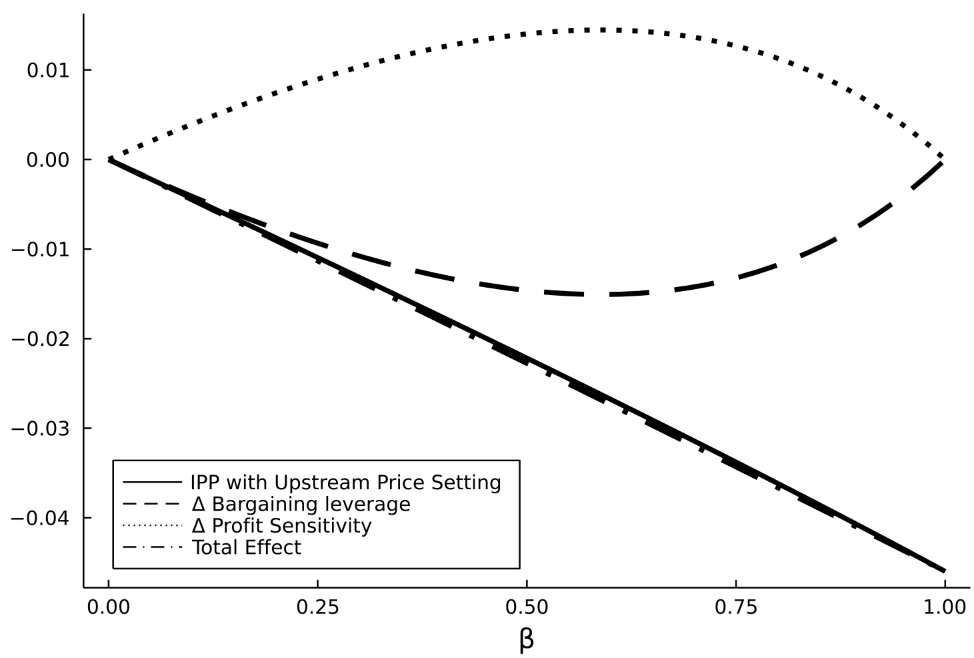

Figure 14: Bargaining IPP decomposition at zero efficiencies.

D Additional Figures from Merger Simulations

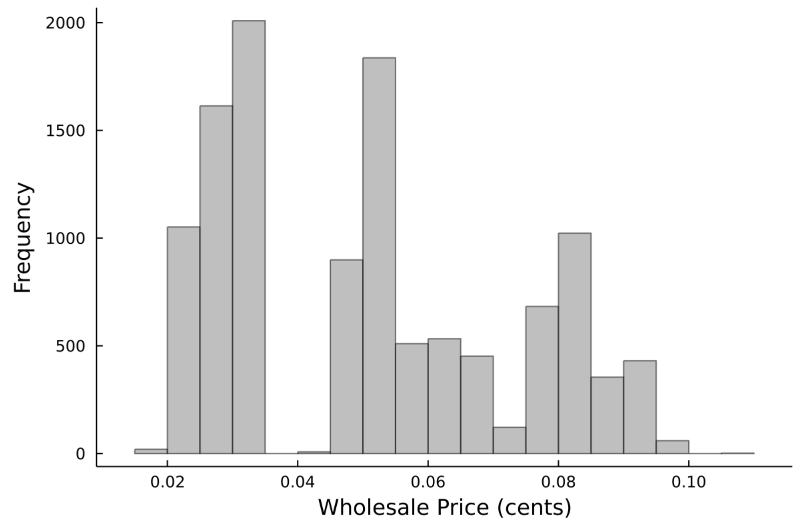

Figure 15: Distribution of pre-merger wholesale prices per ounce, in cents.

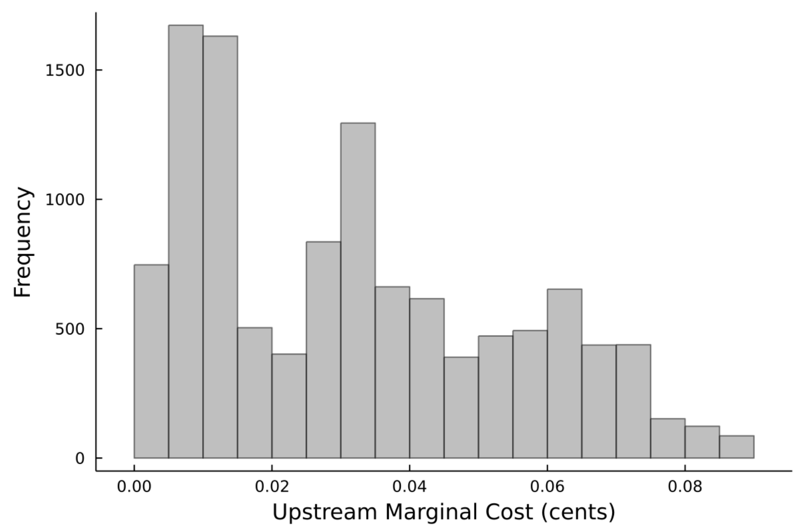

Figure 16: Distribution of upstream marginal costs per ounce, in cents.

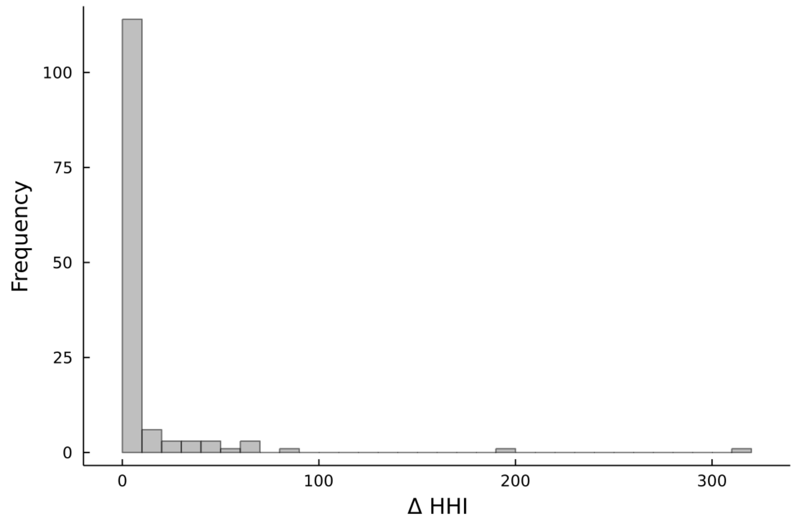

Figure 17: Distribution of Δ HHI for the simulated mergers.