Input-Price Responses to Horizontal Mergers and the Bargaining-Leverage Defense

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Abstract

We study the implications of endogenous input prices for horizontal merger policy when input prices are set before goods prices. Generalizing the first-order approach to merger analysis, we derive a measure of unilateral incentives to adjust input prices after a downstream merger, Input Pricing Pressure. We use this measure to show that mergers often incentivize *higher* input prices, and that these incentives hinge on changes in downstream pass-through rates, marginal cost efficiencies generated by the merger, and input-output linkages. By implication, consumer surplus-maximizing antitrust policy may be too lax when input prices are assumed fixed, and it should be biased against claims that input prices will fall after a downstream merger. In an empirical application to local retail beer markets, endogenizing input prices substantially raises the consumer harm from mergers of retailers.

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1 Introduction

Competition authorities evaluate horizontal mergers based on a trade-off between market power and cost efficiencies. As Williamson (1968) articulates, a merger between competitors reduces competition in the goods market and incentivizes the merged firm to raise prices, while reductions in marginal costs due to realized economies of scale or scope incentivize lower prices. If such *efficiencies* are large enough, the merger may lead to lower prices and higher consumer surplus. This analysis generally holds the input prices faced by the merging firms fixed, but in several recent merger cases the defendants have argued that lower input prices may constitute an alternative source of "efficiencies." For example, in *United States v. Anthem, Inc.*, health insurers Anthem and Cigna argued that merging would have allowed them to obtain more favorable quality-adjusted reimbursement rates from healthcare providers. They further argued that almost all of the resulting savings would have been passed through to their customers, so the merger should have been permitted on consumer surplus grounds (*United States v. Anthem, Inc.*, D.D.C. 2017).

A recent literature in industrial organization integrates similar vertical contracting concerns into the analysis of horizontal market structure. These studies generally estimate structural models of price competition and bargaining in which downstream firms simultaneously choose goods prices and bargain with upstream firms over input prices (Gowrisankaran, Nevo, and Town, 2015; Ho and Lee, 2017; Ho and Lee, 2019; see Lee, Whinston, and Yurukoglu, 2021 for a review). In these settings, a downstream merger affects input prices by altering firms' outside options in negotiations. For example, the merger may raise the merged firm's outside option, improving its *bargaining leverage* vis-á-vis upstream firms and lowering input prices in equilibrium (Sheu and Taragin, 2021).[1]

In this paper, we instead study the effects of downstream mergers when input prices are set before goods prices. If firms anticipate changes in goods prices when bargaining over input prices, we show that input prices may *increase* after a downstream merger. For settings in which such *successive* conduct is plausible, our results suggest that merger policy should be biased against claims that increased bargaining leverage will reduce input prices. We also show that input prices may rise so significantly after a merger that goods prices increase regardless of real efficiencies: A merger may be inherently anticompetitive once endogenous input prices are taken into account.

To illustrate the mechanism behind our results, and to demonstrate that it does not depend

[1]Notably, in their opinions issued in *United States v. Anthem, Inc.*, the district and circuit courts recognized that the firms' defense was implicitly predicated on the assumption that the merged firm would have greater bargaining leverage vis-á-vis healthcare providers, allowing it to negotiate more favorable rates (*United States v. Anthem, Inc.*, D.D.C. 2017; *United States v. Anthem, Inc.*, D.C. Cir. 2017).

on bargaining per se, we focus primarily on the canonical model of vertical contracting in which upstream firms unilaterally set input prices (Spengler, 1950). That is, we suppose that downstream firms compete in goods prices holding input prices fixed, while upstream firms compete in input prices, anticipating the resulting changes in goods prices. We then consider a merger between two downstream firms and study an upstream firm's incentive to adjust its price after the merger. We find that this incentive depends crucially on how the merger alters downstream pass-through rates and the merged firm's production technology.

The intuition for our main results is as follows: With successive conduct, downstream pass-through rates are key determinants of input prices because they affect how changes in input prices are transmitted to downstream output and hence demand for inputs. After a downstream merger between single-product firms, pass-through rates adjust such that the output of either downstream good becomes more sensitive to its own marginal cost, but less sensitive to the marginal cost of the other good. If an upstream firm's input is used to produce only one good, these adjustments provide an incentive for the firm to lower its price. This holds because the upstream firm effectively observes more elastic demand for its input after the merger. If instead the input is used to produce both goods, the upstream firm often has an incentive to raise its price. In either case, if the merged firm becomes more efficient in its use of the input, downstream output always becomes less sensitive to the input price, and the upstream firm has an additional incentive to increase its price. Altogether our results indicate that with successive conduct, downstream mergers often raise goods prices and must yield additional real efficiencies to leave consumers unharmed.

For simplicity, we prove these results in a stylized model with a merger to monopoly downstream, linear consumer demand, and Leontief production technologies. However, we provide additional theoretical results and simulations that show our conclusions generalize with (i) arbitrary production technologies, (ii) competition from non-merging downstream firms, and (iii) nonlinear demand. We also show that the key economic mechanism behind our results is still present when we allow successive bargaining over input prices. In this case, simulations show that our pass-through and efficiency effects often dominate the standard bargaining leverage effect studied in the literature, so that input prices rise after a downstream merger.

To organize our analysis, in Section 2 we extend the first-order approach to merger analysis of Farrell and Shapiro (2010) and Jaffe and Weyl (2013) to incorporate endogenous input prices. When price-setting conduct is successive, we provide an approximation formula for post-merger price changes that strictly generalizes that of Jaffe and Weyl (2013) and is expressed in sufficient statistics. We define a new term, Input Pricing Pressure (IPP), that is analogous to the Generalized Pricing Pressure (GePP) of Jaffe and Weyl (2013) and precisely measures the incentives for input prices to adjust after the merger. In Section 3, we study IPP in a simple model with successive price competition to demonstrate how changes in pass-through rates and production technologies may incentivize input price changes after a downstream merger. We also consider more general models and show that the mechanism we emphasize is robust to a number of assumptions. In Section 4, we show that the effects and policy implications of endogenous input prices described in Section 3 also arise when input prices are set through successive Nash-in-Nash bargaining.

In Section 5, we provide an empirical application of our theory to local retail beer markets. We calibrate logit demand systems for retail beer products using Nielsen scanner data, and we impute a simple distribution network by which upstream distributors sell beer to retailers at wholesale prices. We follow Villas-Boas (2007) to calculate distributor marginal costs, and we simulate mergers between beer retailers both with and without endogenous wholesale prices. Even when the mergers generate no efficiencies, retail prices increase more post-merger when we allow wholesale prices to adjust endogenously, complementing our theoretical findings.

This paper contributes primarily to the literature mentioned above on the effects of horizontal market structure in settings with vertical contracting. We identify conditions under which vertical relationships are important for the normative analysis of horizontal mergers, and our results show that the timing of price setting can crucially affect policy counterfactuals. We also contribute to a smaller theoretical literature that considers the effects of downstream market structure on input prices under successive conduct (Dobson and Waterson, 1997; Lommerud, Straume, and Sørgard, 2005; Iozzi and Valletti, 2014; Gaudin, 2018). We depart from these studies by (i) relaxing the assumption that the downstream firms are symmetric, which obscures the mechanisms by which downstream market structure affects input pricing incentives; and (ii) focusing on the normative implications of endogenous input prices. Finally, our concern for merger policy also relates to a small literature in antitrust law that asks if, under existing legal frameworks for antitrust, post-merger declines in input prices should provide a legitimate defense ("cognizable efficiency") in merger enforcement cases (Carlton and Israel, 2011; Hemphill and Rose, 2017). Our results suggest that these declines in input prices may fail to materialize under successive conduct, supporting additional skepticism about this defense from a purely economic perspective.

2 First-Order Approach and Input Pricing Pressure

In this section, we introduce our general model and show how the first-order approach to merger analysis extends to settings with endogenous input prices. We initially place only weak conditions on price-setting conduct, and we clarify the assumptions needed to obtain approximation formulas for post-merger price changes in sufficient statistics (Proposition 2).

In Sections 3 and 4, we use one of these statistics, Input Pricing Pressure, to study how downstream mergers can lead to input price changes in a class of simple models.

2.1 Setup

The economy exists in partial equilibrium and has two sets of products. Downstream products, or *goods*, are indexed by $g \in \mathcal{G}$ and sold to consumers. The set of downstream firms is $\mathcal{F}_{\mathcal{G}}$, where each firm $G \in \mathcal{F}_{\mathcal{G}}$ is identified with the set of goods that it produces, $G \subseteq \mathcal{G}$. Upstream products, or *inputs*, are indexed by $i \in \mathcal{I}$ and sold to downstream firms. The set of upstream firms is $\mathcal{F}_{\mathcal{I}}$, where each firm $I \in \mathcal{F}_{\mathcal{I}}$ is identified with the set of inputs it produces, $I \subseteq \mathcal{I}$.

All transactions are facilitated by linear prices, and we let $p := (p_g)_{g \in \mathcal{G}}$ and $w := (w_i)_{i \in \mathcal{I}}$ denote the price vectors for goods and inputs.[2] The production technology for each downstream firm G is described by the cost function $C_G(y_G, w)$, and similarly each upstream firm I has cost function $C_I(z_I)$, where $y_G := (y_g)_{g \in G}$ and $z_I := (z_i)_{i \in I}$ denote the output vectors for firms G and I. Consumer demand for good g is $y_g(p)$, and the demand for input i from downstream firm G given outputs y_G and input prices w is $\partial C_G(y_G, w) / \partial w_i$. Total demand for input i equals the sum of the input demands from downstream firms:

$$z_i(y, w) := \sum_{G \in \mathcal{F}_{\mathcal{G}}} \frac{\partial C_G(y_G, w)}{\partial w_i}.$$

To illustrate the generality of the first-order approach, and to demonstrate how different conduct assumptions affect our analysis, we initially make only mild assumptions about equilibrium price setting. We suppose that equilibrium prices (p^*, w^*) are given by the solution to a system of equations:

$$0 = f(p^*, w^*) := \begin{pmatrix} f_{\mathcal{G}}(p^*, w^*) \\ f_{\mathcal{I}}(p^*, w^*) \end{pmatrix}. \tag{1}$$

Here $f_{\mathcal{G}}(p, w)$ is the $\mathbb{R}^{\mathcal{G}}$-valued *first-order condition (FOC) function* for goods prices. In the examples of conduct described below, $f_{\mathcal{G}}$ corresponds precisely to the first-order conditions of maximization problems whose solutions determine the goods prices p^*. Similarly, $f_{\mathcal{I}}(p, w)$ is the FOC function for input prices. We assume that the "stacked" FOC function f is twice continuously differentiable and that the system (1) has a unique solution.

[2] Price discrimination can be accommodated by introducing distinct inputs for different downstream firms.

2.2 Conduct Examples

Several common modes of conduct can be represented by a system of the form (1). Here we describe a few examples based on price competition that we analyze in detail in Section 3, and we discuss an additional example based on bargaining in Section 4.[3]

Our benchmark mode of conduct for downstream firms is *price competition*, whereby each firm $G \in \mathcal{F}_{\mathcal{G}}$ chooses its prices $p_G := \left(p_g\right)_{g \in G}$ to maximize its profits, holding all other prices fixed:

$$\max_{p_G \geq 0} p_G \cdot y_G(p) - C_G(y_G(p), w). \tag{2}$$

Suppressing the dependence of y on p, the FOC function for good $g \in G$ is then

$$f_g(p,w) = y_g + \sum_{g' \in G} \left(p_{g'} - c_{g'}(y_G, w)\right) \frac{\partial y_{g'}}{\partial p_g}.$$

Here $c_{g'}(y_G, w) := \partial C_G(y_G, w) / \partial y_{g'}$ denotes the marginal cost of good g'. Our framework also accommodates a number of other downstream equilibrium concepts using conjectural variations; see Jaffe and Weyl (2013) for details.

More importantly, our analysis allows for many different conduct assumptions for upstream firms. In Section 3 we focus primarily on the case in which upstream firms also compete in prices, holding remaining input prices fixed but anticipating changes in goods prices. To describe such *successive* price competition, let $p^*(w)$ denote the equilibrium goods prices given input prices w, obtained as the solution to the system of equations

$$0 = f_{\mathcal{G}}(p^*(w), w).$$

Then the demand function for input i observed by firm I is

$$z_i(w) := z_i(y(p^*(w)), w).$$

Note that firm I expects downstream firms to adjust to input price changes along two margins: (i) substitution between inputs in production and (ii) changes in goods prices. Only the first margin would operate if upstream firms instead engaged in *simultaneous* price competition, holding goods prices constant when choosing input prices. With successive price competition,

[3] Throughout, we assume that the equilibrium in each price-setting game is unique and characterized by interior first-order conditions.

each upstream firm I solves

$$\max_{w_I \geq 0} w_I \cdot z_I(w) - C_I(z_I(w)). \tag{3}$$

Suppressing the dependence of z_I on w, the corresponding FOC function for input $i \in I$ is

$$f_i(w) = z_i + \sum_{i' \in I} (w_{i'} - c_{i'}(z_I)) \frac{dz_{i'}}{dw_i}. \tag{4}$$

Here $c_{i'}(z_I)$ denotes the marginal cost of input $i' \in I$. Note that the *total* derivative $dz_{i'}/dw_i$ appears in the second term, indicating that downstream price responses as well as input substitution are taken into account by firm I.[4] As a result, the FOC function f_i depends only on input prices w. This property is common to all successive modes of conduct in which input prices are set before goods prices, and we will see that it substantially simplifies the first-order approach to merger analysis with endogenous input prices.

Following the seminal contribution of Horn and Wolinsky (1988), a large literature has explored the implications of bilateral bargaining in input markets for classic questions about the effects of horizontal and vertical mergers, price discrimination, and exclusive dealing (see Lee, Whinston, and Yurukoglu, 2021 for a review). Our framework can also accommodate standard versions of the Nash-in-Nash bargaining solution concept of Horn and Wolinsky (1988) that appears in many recent analyses of vertical contracting environments.[5] We analyze this possibility in Section 4. The conduct examples discussed above and in Section 4 indicate that our setting can accommodate many conduct assumptions found in the literature on vertical contracting, but it is not completely general. We make the strong assumption that all transactions are mediated by linear prices, ruling out nonlinear pricing schedules as well as quantity-based bargaining games. This assumption limits the generality of our results, but it provides a reasonable and tractable starting point for understanding the effects of vertical relationships on horizontal mergers. It is also assumed in all of the empirical literature cited above.

2.3 Downstream Merger

Our counterfactual of interest is a merger between two downstream firms $G_M, G_{M'} \in \mathcal{F}_\mathcal{G}$. Let $M := G_M \cup G_{M'}$ denote the merged firm, and let $\hat{\mathcal{F}}_\mathcal{G}$ denote the set of downstream firms following the merger. Throughout, we denote post-merger objects using the decoration $\hat{}$.

[4]Here $\frac{dz(w)}{dw} := \frac{\partial z(y(p^*(w)),w)}{\partial y} \frac{\partial y(p^*(w))}{\partial p} \frac{\partial p^*(w)}{\partial w} + \frac{\partial z(y(p^*(w)),w)}{\partial w}$.

[5]Examples include Draganska, Klapper, and Villas-Boas (2010); Crawford and Yurukoglu (2012); Grennan (2013); Gowrisankaran, Nevo, and Town (2015); Ho and Lee (2017); Crawford, Lee, Whinston, and Yurukoglu (2018); and Dubois and Sæthre (2020).

The merger may have two effects on the economic environment. First, the merged firm may obtain a new production technology by reallocating production across plants, attaining economies of scale or scope, or from a number of other unmodeled sources. We capture these *efficiencies* in a reduced-form way by supposing that merged firm attains a new cost function $C_M(y_M, w)$. As a by-product of this technological change, the upstream firms may now observe a different demand function for each input i:

$$\hat{z}_i(y,w) := \frac{\partial C_M(y_M,w)}{\partial w_i} + \sum_{G \in \hat{\mathcal{F}}_{\mathcal{G}} \setminus \{M\}} \frac{\partial C_G(y_G,w)}{\partial w_i}.$$

Second, the merger may alter conduct. For example, if downstream firms engage in price competition, the merger allows the merged firm to jointly maximize profits over the prices $p_M = (p_{G_M}, p_{G_{M'}})$. In the absence of marginal cost-reducing efficiencies, this conduct change generally implies higher equilibrium goods prices for fixed input prices when goods are gross substitutes (Deneckere and Davidson, 1985). As we discuss at length in Section 3, changes in downstream conduct may also indirectly change upstream conduct. For example, the downstream equilibrium price function $p^*(w)$ is a key input for any of the successive modes of conduct described in Section 2.2 or Section 4. If downstream conduct changes following the merger, the equilibrium price function $\hat{p}^*(w)$ will also change, and input prices will adjust accordingly. To capture these conduct changes while remaining agnostic (for now) about their details, we suppose that post-merger equilibrium prices $(\hat{p}^*, \hat{w}^*)$ are given by the solution to the system of equations

$$0 = \hat{f}(\hat{p}^*, \hat{w}^*) := \begin{pmatrix} \hat{f}_{\mathcal{G}}(\hat{p}^*, \hat{w}^*) \\ \hat{f}_{\mathcal{I}}(\hat{p}^*, \hat{w}^*) \end{pmatrix}. \tag{5}$$

Here $\hat{f}_{\mathcal{G}}$ and $\hat{f}_{\mathcal{I}}$ are the post-merger FOC functions for goods prices and input prices. Just as before the merger, we assume that the stacked FOC function $\hat{f}$ is twice continuously differentiable and that the system (5) has a unique solution.

2.4 First-Order Approach: Generalized and Input Pricing Pressure

The first-order approach to merger analysis hews closely to the first-order approach to general comparative statics initiated by Samuelson (1947). In both cases, we attempt to understand how equilibrium objects adjust to changes in the economic environment by studying the resulting changes in the first-order conditions that characterize equilibrium. The key difference is that the first-order approach to comparative statics provides *exact* changes in equilibrium objects in response to *marginal* changes in the environment, whereas the first-order approach to merger analysis provides *approximate* changes in equilibrium prices in response to *non-marginal* changes in conduct and production technologies. In this section, we demonstrate how the first-order approach to merger analysis pioneered by Farrell and Shapiro (2010) and Jaffe and Weyl (2013) generalizes when we allow for endogenous input price responses. Our aim in doing so is to provide a framework for investigating the factors that determine input price responses to downstream horizontal mergers, an investigation that we pursue in Sections 3-5.

The task to be solved by the first-order approach is as follows: Given a pre-merger equilibrium (p^*, w^*) and a post-merger FOC function $\hat{f}$, how can we approximate the post-merger equilibrium $(\hat{p}^*, \hat{w}^*)$, and what properties of price-setting conduct influence the direction and magnitude of price changes? We assume that the pre- and post-merger equilibria are determined by FOC systems (1, 5), so we can make progress on these questions by comparing the pre- and post-merger FOC functions. This is the basis for the main approximation result of the first-order approach:

Proposition 1. *Suppose $\hat{f}$ is invertible. Then the post-merger price changes are*

$$(\hat{p}^* - p^*, \hat{w}^* - w^*) = -\left[D\hat{f}\,(p^*, w^*)\right]^{-1}\left(\hat{f}\,(p^*, w^*) - f\,(p^*, w^*)\right) + R, \tag{6}$$

where the error vector R satisfies

$$R_k = \frac{1}{2}\sum_{l,m=1}^{|\mathcal{G}|+|\mathcal{I}|}\frac{\partial^2 \hat{f}_k^{-1}(r)}{\partial r_l \partial r_m}\hat{f}_l\,(p^*, w^*)\,\hat{f}_m\,(p^*, w^*) \tag{7}$$

for some vector $r \in \left[0, \hat{f}\,(p^*, w^*)\right]$.

Like Theorem 1 in Jaffe and Weyl (2013), the result follows by using the post-merger FOC system (5) to write

$$(\hat{p}^* - p^*, \hat{w}^* - w^*) = \hat{f}^{-1}(0) - \hat{f}^{-1}\left(\hat{f}\,(p^*, w^*)\right),$$

and then linearizing $\hat{f}^{-1}$ around $\hat{f}\,(p^*, w^*)$.[6] The error formula (7) indicates that the approximation is accurate when the post-merger FOC function $\hat{f}$ has little curvature or when the pre-merger prices (p^*, w^*) nearly satisfy the post-merger FOC system (5). These conditions are unsurprising since the approximation is just the first step in the Newton-Raphson algorithm when finding the root of $\hat{f}$ beginning at (p^*, w^*).

[6]The assumption that $\hat{f}$ is invertible is the "non-marginal" analogue of the local invertibility assumption in marginal comparative statics.

If the error is small, expression (6) demonstrates that post-merger equilibrium prices can be approximated with only knowledge of post-merger price-setting conduct $\hat{f}$ around the pre-merger equilibrium (p^*, w^*). More importantly, it also provides some insight as to the factors that determine the direction and magnitude of price changes after the merger. In particular, the prices changes are jointly determined by (i) the change in the FOC function at the pre-merger equilibrium and (ii) the sensitivity of the post-merger FOC function to prices. The former measures the unilateral incentives for firms to adjust prices, while the latter incorporates price responses between products and measures the size of price changes needed to reach the post-merger equilibrium.

To convey this intuition more clearly, we can express the basic approximation formula in terms of sufficient statistics: pass-through rates, the Generalized Pricing Pressure (GePP) vector of Jaffe and Weyl (2013), and the Input Pricing Pressure (IPP) vector that we introduce below. To do this, we make two additional assumptions:

Assumption 1. *FOC functions for input prices do not depend directly on goods prices p, and we reparametrize them by*

$$f_{\mathcal{I}}(w, c_{\mathcal{I}}(z_{\mathcal{I}}(w))) \quad \textit{and} \quad \hat{f}_{\mathcal{I}}(w, c_{\mathcal{I}}(\hat{z}_{\mathcal{I}}(w)))\,.$$

Assumption 2. *FOC functions for goods prices depend on input prices only through downstream marginal costs, and we reparametrize them by*

$$f_{\mathcal{G}}\left(p, c_{\mathcal{G}}\left(y_{\mathcal{G}}(p), w\right)\right) \quad \textit{and} \quad \hat{f}_{\mathcal{G}}\left(p, \hat{c}_{\mathcal{G}}\left(y_{\mathcal{G}}(p), w\right)\right).$$

As we discussed in Section 2.2, Assumption 1 holds for successive modes of conduct. Assumption 2 is fairly innocuous; it is satisfied, for example, when downstream firms engage in price competition or one of its conjectural versions. We maintain Assumptions 1 and 2 in the remainder of the paper.

We can now define the sufficient statistics that appear in the approximation formulas below. The post-merger pass-through rate functions for downstream and upstream firms are, respectively,

$$\hat{\rho}_{\mathcal{G}}(p, w) := -\left[\frac{d\hat{f}_{\mathcal{G}}\left(p, c_{\mathcal{G}}\left(y_{\mathcal{G}}(p), w\right)\right)}{dp}\right]^{-1} \frac{\partial \hat{f}_{\mathcal{G}}\left(p, c_{\mathcal{G}}\left(y_{\mathcal{G}}(p), w\right)\right)}{\partial c_{\mathcal{G}}}, \tag{8}$$
$$\hat{\rho}_{\mathcal{I}}(w) := -\left[\frac{d\hat{f}_{\mathcal{I}}(w, c_{\mathcal{I}}(z_{\mathcal{I}}(w)))}{dw}\right]^{-1} \frac{\partial \hat{f}_{\mathcal{I}}(w, c_{\mathcal{I}}(z_{\mathcal{I}}(w)))}{\partial c_{\mathcal{I}}}.$$

Evaluated at the post-merger equilibrium, $\hat{\rho}_{\mathcal{G}}(\hat{p}^*, \hat{w}^*) \times t_{\mathcal{G}}$ describes the change in post-merger goods prices after introducing a vector $t_{\mathcal{G}}$ of small specific taxes on goods, holding input prices fixed. Similarly, $\hat{\rho}_{\mathcal{I}}(\hat{w}^*) \times t_{\mathcal{I}}$ describes the change in post-merger input prices in response to a vector $t_{\mathcal{I}}$ of small specific taxes on inputs, but allowing goods prices to vary in response to input price changes.[7] The key merger-specific statistics are the Generalized Pricing Pressure and Input Pricing Pressure vectors:

$$\text{GePP} := \left[\frac{\partial \hat{f}_{\mathcal{G}}}{\partial c_{\mathcal{G}}}\right]^{-1} \left(\hat{f}_{\mathcal{G}} - f_{\mathcal{G}}\right), \tag{9}$$

$$\text{IPP} := \left[\frac{\partial \hat{f}_{\mathcal{I}}}{\partial c_{\mathcal{I}}}\right]^{-1} \left(\hat{f}_{\mathcal{I}} - f_{\mathcal{I}}\right), \tag{10}$$

where all functions are evaluated at pre-merger prices (p^*, w^*). GePP, introduced by Jaffe and Weyl (2013) to generalize the Upward Pricing Pressure heuristic of Farrell and Shapiro (2010), is essentially the change in the FOC function for goods prices after the merger, normalized in "units" of downstream marginal costs. The change in the FOC function measures the incentives for downstream firms to adjust prices after the merger; the normalization allows us to conceptualize these incentives as a simple change in marginal costs. Similarly, IPP is our new measure of the incentives for upstream firms to adjust prices after the merger, normalized in units of upstream marginal costs. When the FOC functions $\hat{f}_{\mathcal{G}}$ and $\hat{f}_{\mathcal{I}}$ are linear in the respective marginal costs $c_{\mathcal{G}}$ and $c_{\mathcal{I}}$, as they are for all modes of conduct described in this paper, the normalizations imply that we can also interpret GePP and IPP as the vectors of subsidies that would leave all prices unchanged after the merger.

The next proposition gives our approximation result, a refinement of Proposition 1.

Proposition 2. *Suppose $\hat{f}_{\mathcal{G}}\left(\cdot, c_{\mathcal{G}}\left(y_{\mathcal{G}}(\cdot), w^*\right)\right)$ and $\hat{f}_{\mathcal{I}}(\cdot, \hat{c}_{\mathcal{I}}(\hat{z}_{\mathcal{I}}(\cdot)))$ are invertible. Then the post-merger price changes are*

$$\hat{p}^* - p^* \quad = \quad \left(\hat{\rho}_{\mathcal{G}} \times \text{GePP}\right) + \left(\hat{\rho}_{\mathcal{G}} \times \frac{\partial \hat{c}_{\mathcal{G}}}{\partial w} \times \hat{\rho}_{\mathcal{I}} \times \text{IPP}\right) + R_{\mathcal{G}}, \tag{11}$$

$$\hat{w}^* - w^* \quad = \quad \left(\hat{\rho}_{\mathcal{I}} \times \text{IPP}\right) + R_{\mathcal{I}}, \tag{12}$$

where the error vector $R = \left(R_{\mathcal{G}}, R_{\mathcal{I}}\right)$ coincides with (7) and all functions are evaluated at the pre-merger equilibrium (p^*, w^*).

The proof is found in Appendix A and makes use of the observation that the post-merger FOC system (5) is block-diagonal when $\hat{f}_{\mathcal{I}}$ does not depend directly on goods prices (Assump-

[7] The pre-merger pass-through rate functions $\rho_{\mathcal{G}}(p, w)$ and $\rho_{\mathcal{I}}(w)$ are defined similarly using the pre-merger FOC functions $f_{\mathcal{G}}$ and $f_{\mathcal{I}}$.

tion 1).[8] To interpret the formulas in Proposition 2, we begin with the simpler equation (12). This equation implies that the change in input prices after the merger can be approximated by the IPP vector multiplied by the post-merger pass-through rate function, evaluated at pre-merger prices. This is the basis of our claim that IPP quantifies the incentives for input price changes in units of upstream marginal costs. Put differently, GePP and IPP are the vectors of subsidies that must be applied after the merger for all prices stay fixed. The first-order effect of the merger on input prices can be calculated by removing the fictitious IPP subsidies and applying the pass-through rates $\hat{\rho}_{\mathcal{I}}(w^*)$, which are exact for marginal changes around the subsidized equilibrium. Error arises because removing the IPP subsidies is a non-marginal change in the environment, so the "locally exact" pass-through rates $\hat{\rho}_{\mathcal{I}}(w^*)$ may not perfectly describe the effect on input prices.[9]

The first two terms in (11) have similar interpretations. These are most easily understood by analogy to the following exact decomposition of the post-merger change in goods prices:

$$\hat{p}^*(\hat{w}^*) - p^*(w^*) \quad = \quad \underbrace{\hat{p}^*(w^*) - p^*(w^*)}_{\substack{\text{merger with fixed} \\ \text{input prices}}} \quad + \quad \underbrace{\hat{p}^*(\hat{w}^*) - \hat{p}^*(w^*)}_{\substack{\text{effect of endogenous} \\ \text{input prices}}}. \tag{13}$$

The first term in this decomposition is the post-merger change in goods prices when input prices are held fixed. Standard analyses of horizontal mergers describe this object, which is crucial for understanding the effect of mergers on consumer surplus. Theorem 1 of Jaffe and Weyl (2013) establishes that the first term in (11) provides a first-order approximation:[10]

$$\hat{p}^*(w^*) - p^*(w^*) \quad \approx \quad \hat{\rho}_{\mathcal{G}}(p^*, w^*) \times \text{GePP}.$$

Our generalization shows that a similar approximation applies to the second term in (13), which describes the additional change in goods prices resulting from endogenous changes in input prices. This approximation is exactly the second term in (11):

$$\hat{p}^*(\hat{w}^*) - \hat{p}^*(w^*) \quad \approx \quad \hat{\rho}_{\mathcal{G}}(p^*, w^*) \times \frac{\partial \hat{c}_{\mathcal{G}}(y(p^*), w^*)}{\partial w} \times \hat{\rho}_{\mathcal{I}}(w^*) \times \text{IPP}.$$

In words, we can determine the effect of endogenous input prices on goods prices by calculating

[8] This assumption eliminates a feedback loop between the changes in goods prices and the changes in input prices after the merger. This feedback loop significantly complicates the basic approximation (6) and does not permit a general approximation formula in easily interpretable sufficient statistics.

[9] By redefining GePP and IPP, we could use other pass-through rates in place of $\hat{\rho}_{\mathcal{G}}(p^*, w^*)$ and $\hat{\rho}_{\mathcal{I}}(w^*)$ in (11, 12). We use these pass-through rates and the associated definitions of GePP and IPP so that (i) no knowledge of post-merger equilibrium prices is needed to compute the approximations (11, 12) and (ii) we can interpret GePP and IPP as the compensating subsidies needed to keep all prices fixed post-merger.

[10] This result is formally a special case of Proposition 2 with $\hat{f}_{\mathcal{I}}(w) = w - w^*$.

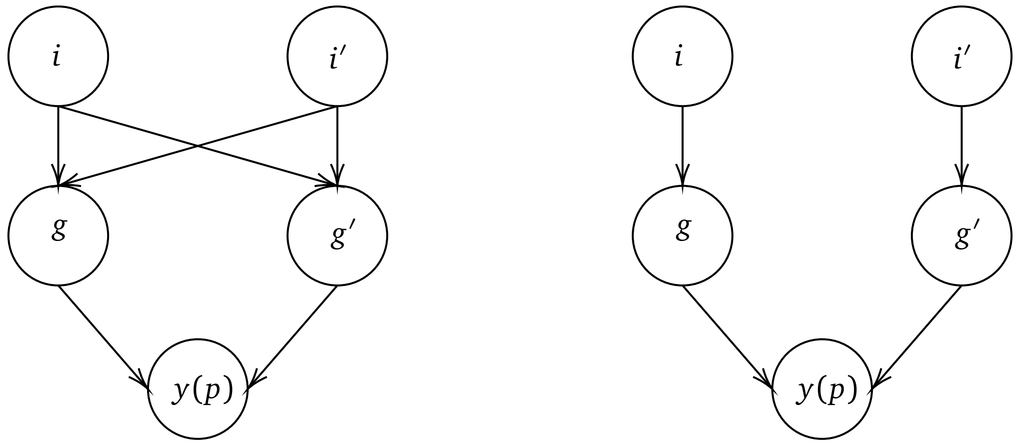
the (approximate) changes in input prices $\hat{\rho}_{\mathcal{I}} \times \text{IPP}$, converting them into shifts in downstream marginal costs using the derivative $\partial \hat{c}_{\mathcal{G}} / \partial w$, and converting these shifts into changes in goods prices by applying the downstream pass-through rates $\hat{\rho}_{\mathcal{G}}$.

We find the approximation formulas of Proposition 2 useful because they cleanly describe how post-merger changes in the economic environment map into price changes. For example, pass-through rates are the subject of a substantial theoretical and empirical literature, so we may feel comfortable conjecturing values for $\hat{\rho}_{\mathcal{G}}(p^*, w^*)$ and $\hat{\rho}_{\mathcal{I}}(w^*)$ for a merger of interest. All that remains to implement the approximations (11, 12) is to determine how sensitive marginal costs are to input prices post-merger and how the merger affects pricing incentives in marginal cost-equivalent terms. We admit that this is more information-intensive than implementing the approximations of Farrell and Shapiro (2010) and Jaffe and Weyl (2013) with fixed input prices, but even simple observations provide information about the likely direction of input price changes. For example, under many conduct assumptions the own-price pass-through rates $\hat{\rho}_{ii}(w^*)$ are the largest entries in $\hat{\rho}_{\mathcal{I}}(w^*)$, so just examining IPP_i can provide a good indication of the likely change in the price of input i. This is the premise of our theoretical analysis in Sections 3 and 4.

Nonetheless, we view the primary contribution of Proposition 2 as conceptual: With linear pricing and successive conduct, we can simply adjust our standard estimate for the consumer price effect of mergers by an additive term that captures the effect of input price changes. We study this adjustment theoretically in simple settings in Sections 3 and 4, and we provide a quantitative analysis using an estimated demand system and a realistic input-output structure in Section 5.

3 Merger Policy and Input Prices

In this section, we apply Proposition 2 in a special case of our model with successive price competition to study how input prices change following a downstream merger. We find that upstream firms have incentives to raise prices when downstream output expands or becomes less sensitive to input prices, and that these incentives are often stronger when the merger yields higher efficiencies. As a result, efficiencies are less effective at mitigating merger-induced increases in goods prices than when input prices are held fixed. Considering the implications for policy, we find that the consumer surplus-maximizing merger policy assuming fixed input prices may be too lenient when input prices are endogenous. This finding is robust to a number of modeling assumptions, and our simulations in Section 5 and Appendix C indicate that the effect of endogenous input prices on goods prices is quantitatively significant and often larger than the standard price effect of the merger under fixed input prices. We also provide an



(a) general

(b) specialized input suppliers

Figure 1: Input-output configurations in the simple model.

example indicating that a merger may be *inherently anticompetitive* because of endogenous input prices: "Offsetting efficiencies" that lead to lower goods prices may not exist.

3.1 A Simple Model

Consider a special case of the economy of Section 2: There are two goods, g and g', and two inputs, i and i'. Before the merger, each product is owned by a different firm. Each input i has constant marginal cost c_i, and each good g has constant marginal cost

$$c_g(w) = v_g + x_{gi}w_i + x_{gi'}w_{i'}.$$

Here v_g denotes the component of marginal cost due to use of the numeraire (e.g., labor). This marginal cost function implies that good g is produced using both inputs i and i' in fixed proportions (Figure 1a). A useful special case that we discuss below is one in which good g is produced using only input i ($x_{gi'} = 0$) and good g' is produced using only input i' ($x_{g'i} = 0$). We say that such a setting features "specialized input suppliers" (Figure 1b).

The consumer demand system $y(p)$ is linear, so that all price derivatives $\partial y_g / \partial p_g$ and $\partial y_g / \partial p_{g'}$ are independent of the price vector p. The demand system is also Slutsky symmetric and features gross substitutes, so that the cross-price derivatives are equal and weakly positive:

$$\frac{\partial y_g}{\partial p_{g'}} = \frac{\partial y_{g'}}{\partial p_g} \geq 0.$$

We denote the *diversion ratio* from g to g' by

$$D_{gg'} := -\frac{\partial y_{g'}/\partial p_g}{\partial y_g/\partial p_g}. \tag{14}$$

This quantity gives the fraction of lost demand diverted from g to g' after an increase in the price of g, providing a measure of the substitutability of goods g and g'. The diversion ratio $D_{gg'}$ is non-negative since g and g' are gross substitutes, and we further assume that it is strictly smaller than 1. Demand for input i satisfies $z_i(y,w) = \sum_g y_g x_{gi}$.

Before the merger, the downstream firms engage in price competition as in (2), and the upstream firms engage in successive price competition as in (3). We assume that the upstream firms cannot price discriminate across the downstream firms. After downstream firms g and g' merge, the merged firm M realizes a new marginal cost function for each good:

$$\hat{c}_g(w) = \left(1 - E_{gv}\right) v_g + (1 - E_i) x_{gi} w_i + (1 - E_{i'}) x_{gi'} w_{i'}.$$

Here E_{gv}, E_i, and $E_{i'}$ are the *efficiencies* attained by the merged firm on each component of the marginal cost of good g. For simplicity, we assume that the input-specific efficiencies E_i and $E_{i'}$ are symmetric across downstream goods. The post-merger demand for input i is then $\hat{z}_i(y,w) = (1 - E_i)\sum_g y_g x_{gi}$. The merged firm sets goods prices as a multiproduct monopolist, while the upstream firms continue to engage in successive price competition.

Many features of this simple model – two downstream firms, linear demand, Leontief production – are highly stylized. These assumptions are made largely to simplify the exposition of our main results in Section 3.3. We provide additional theoretical results in Section 3.5 to indicate how they generalize, and we give numerical results from merger simulations in Section 5 and Appendix C to address cases not covered by our propositions.

3.2 Example: Inherently Anticompetitive Mergers

Before studying the simple model in detail, we provide an example to show how endogenous input prices may reverse basic intuitions about horizontal mergers and merger policy. This example shows that downstream mergers may be *inherently* anticompetitive once endogenous input prices are taken into account: Goods prices may increase post-merger *regardless* of efficiencies because input prices are endogenous. To see this, consider a special case of the simple model with a single upstream wholesaler and two downstream retailers. Goods g and g' are produced using only input i, with marginal costs $c_g(w) = c_{g'}(w) = w_i$. The goods are symmetrically differentiated, with consumer demand

$$y_g(p) = V - p_g + Dp_{g'}.$$

Here $V > 0$ controls the own-price elasticity of demand and D is the (symmetric) diversion ratio. The downstream merger yields efficiencies E: $\hat{c}_g(w) = \hat{c}_{g'}(w) = (1-E)w_i$.

We calculate the pre- and post-merger input prices in Appendix A and find that the input price always rises after the merger:

$$w_i^* = \frac{V + (1-D)c_i}{2(1-D)} \leq \frac{V + (1-D)(1-E)c_i}{2(1-D)(1-E)} = \hat{w}_i^*,$$

where the inequality is strict when $E > 0$. These expressions show that post-merger downstream marginal costs $(1-E)\hat{w}_i^*$ are decreasing in efficiencies E, but more slowly than when the input price is fixed at its pre-merger value. As a result, efficiencies are less effective at constraining goods prices when the input price is endogenous. Thus, for the merger to leave goods prices unchanged, it must achieve greater efficiencies when the input price is endogenous than when the input price is fixed at its pre-merger value.

More formally, let E^{W} denote the efficiencies that keep goods prices unchanged at a fixed input price, $\hat{p}^*(w^*) = p^*(w^*)$. We refer to these efficiencies as *Werden efficiencies* given the general characterization by Werden (1996). Let E^{T} denote the "true" offsetting efficiencies that keep goods prices unchanged when the input price changes endogenously from w_i^* to $\hat{w}_i^*$, $\hat{p}^*(\hat{w}^*) = p^*(w^*)$. Direct calculation yields

$$E^{\mathrm{T}} = \frac{D}{2-D}\frac{\frac{V}{c_i} - (1-D)}{1-D} > \frac{D}{2-D}\frac{\frac{V}{c_i} - (1-D)}{\frac{V}{c_i} + 1 - D} = E^{\mathrm{W}}.$$

True offsetting efficiencies E^{T} are always higher than Werden efficiencies, and they are *above one* when demand is sufficiently inelastic (i.e., when V is sufficiently large).[11] In this case, goods prices increase after the merger *regardless of efficiencies realized by the merged firm*. By implication, a consumer surplus-maximizing antitrust authority should always block the merger between firms g and g', but it may fail to do so if it does not recognize that the input price adjusts after the merger.

In this example, Williamson's (1968) trade-off between market power and cost efficiencies works as expected when the input price is held fixed: Goods prices are unchanged if the merger generates large enough efficiencies. However, when the input price changes endogenously after the merger, the trade-off may resolve decisively against consumers so that goods prices

[11] In contrast, Werden efficiencies E^{W} are always less than one.

always rise. This stark conclusion does not apply generally to mergers with endogenous input prices, but it indicates that an understanding of upstream pricing incentives may be crucial for accurate merger analysis. In the next two sections, we use IPP to study these incentives in the simple model.

3.3 IPP and Merger Policy in the Simple Model

Returning to the simple model, we seek to determine how input prices adjust after the merger and to draw implications for merger policy. Instead of solving directly for equilibrium prices as in the example, a more tractable approach is to apply the input price approximation in Proposition 2, which is exact in the simple model because all FOC functions are linear:[12]

$$\hat{w}_i^* - w_i^* \quad = \quad \hat{\rho}_{ii} \times \text{IPP}_i \quad + \quad \hat{\rho}_{ii'} \times \text{IPP}_{i'}. \tag{15}$$

The price of input i may change because firm i has a unilateral incentive to adjust its price ($\text{IPP}_i \neq 0$) or firm i' has a unilateral incentive to adjust its price ($\text{IPP}_{i'} \neq 0$), to which firm i responds in equilibrium. In this section, we focus on understanding these incentives and their determinants. This focus is consistent with the use of Upward Pricing Pressure or GePP to diagnose unilateral pricing incentives for merging firms (Farrell and Shapiro, 2010; Jaffe and Weyl, 2013), and in the next section we study the overall price change $\hat{w}_i^* - w_i^*$ and argue that IPP_i is often dispositive of its sign.

Turning to an examination of IPP_i, we can specialize the FOC function for successive price competition (4) to our setting and directly calculate

$$\text{IPP}_i \propto \sum_g x_{gi} \left\{ \underbrace{y_g\left(\hat{p}^*(w^*)\right) - y_g\left(p^*(w^*)\right)}_{\Delta \text{ downstream output}} + \left(w_i^* - c_i\right) \underbrace{\left(\frac{dy_g\left(\hat{p}^*(w^*)\right)}{dw_i} - \frac{dy_g\left(p^*(w^*)\right)}{dw_i} \right)}_{\Delta \text{ sensitivity of output to input price}} \right\}. \tag{16}$$

Here $\propto$ indicates that IPP_i is proportional to the expression on the right side. Firm i has an incentive to increase its price when the merged firm expands downstream output or when downstream output becomes less sensitive to the input price w_i after the merger. Both effects tend to reduce the elasticity of demand for input i, incentivizing firm i to raise its input price and extract greater profits from inframarginal purchases.

The sign of each term in (16) depends crucially on the efficiencies generated by the merger.[13]

[12] We characterize the pass-through rates $\hat{\rho}_{ii}$ and $\hat{\rho}_{ii'}$ in Appendix A and note that they do not depend on input prices because the demand system observed by upstream firms $\hat{z}(w)$ is linear.

[13] See Lemma A.2 in Appendix A.

For example, if no efficiencies are realized, the merged firm will find it profitable to raise goods prices and lower output when input prices are held fixed. The first term in (16) is then negative, suggesting that the upstream firm i may want to lower its price because of lower inframarginal purchases of input i. This effect is generally mitigated as efficiencies rise. The second term in (16) similarly increases as the merged firm becomes more efficient in its use of input i (as E_i rises): The marginal cost of each good is less sensitive to the input price w_i with higher i-specific efficiencies, so goods prices and outputs are also less sensitive to w_i.

To assess the implications of endogenous input prices for merger policy, we consider the following thought experiment: Suppose the merger attains Werden efficiencies, so that goods prices remain unchanged post-merger when input prices are held fixed. Will upstream firms then have an incentive to raise input prices? If so, goods prices will rise, consumers will be harmed, and consumer surplus-maximizing merger policy assuming fixed input prices will be too lax. Proposition 3 below gives conditions under which this result holds. An upstream firm i has an incentive to increase its price after the merger if i-specific efficiencies E_i are sufficiently high, provided that *overall* efficiencies would keep goods prices unchanged at fixed input prices:

Proposition 3. *Suppose the merger is Werden-efficient, $\hat{p}^*(w^*) = p^*(w^*)$. Then*

$$\mathrm{IPP}_i > 0 \quad \textit{if and only if} \quad E_i > \bar{E}_i, \quad \textit{where} \quad \bar{E}_i \leq \frac{D_{gg'}D_{g'g}}{4 - D_{gg'}D_{g'g}}. \tag{17}$$

The $\bar{E}_i$ bound is attained for independent goods ($D_{gg'} = D_{g'g} = 0$) or specialized input suppliers.

For intuition, first consider the case with specialized input suppliers. Werden efficiencies imply that the output term in the IPP expression (16) is zero, and we can decompose the output sensitivity term as

$$\frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i} = \frac{\partial y_g}{\partial p_g}\left(\hat{\rho}_{gg}(1-E_i) - \rho_{gg}\right)x_{gi} + \frac{\partial y_g}{\partial p_{g'}}\left(\hat{\rho}_{g'g}(1-E_i) - \rho_{g'g}\right)x_{gi}. \tag{18}$$

After the merger, output y_g becomes more or less sensitive to the input price w_i only if goods prices p become more or less sensitive to w_i. In turn, goods prices p become less sensitive to w_i if pass-through rates decline or the merger yields i-specific efficiencies. For example, the first term in the decomposition captures the effect of a change in the sensitivity of p_g to w_i on the sensitivity of y_g to w_i: A given increase in w_i generates a smaller increase in p_g if the pass-through rate $\hat{\rho}_{gg}$ falls or i-specific efficiencies E_i rise. A smaller increase in p_g then yields a smaller decline in output y_g. The second term similarly shows that as $p_{g'}$ becomes less sensitive to w_i, a given increase in w_i generates a smaller increase in output y_g.

A key comparative static is then the change in downstream pass-through rates $\hat{\rho}_{\mathcal{G}} - \rho_{\mathcal{G}}$ after the merger. By direct calculation (Lemma A.1), all pass-through rates fall:

$$\hat{\rho}_{\mathcal{G}} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ll \frac{1/2}{1 - \frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\begin{bmatrix} 1 & \frac{D_{gg'}}{2} \\ \frac{D_{g'g}}{2} & 1 \end{bmatrix} = \rho_{\mathcal{G}}.$$

To see why, consider an exogenous increase in the marginal cost c_g of good g. Before the merger, only the price of good g increases directly in response, but all goods prices increase in equilibrium through complementary price responses. After the merger, the merged firm has an incentive to *decrease* the price of good g' directly in response to the increase in c_g, diverting demand away from the lower margin good g and increasing profits. This is the "Edgeworth-Salinger effect" coined by Luco and Marshall (2020), and it reduces all pass-through rates following the merger. In fact, it reduces the cross-good pass-through rate $\hat{\rho}_{g'g}$ much more than the own-good pass-through rate $\hat{\rho}_{gg}$, so that output y_g becomes more sensitive to the input price w_i when E_i is sufficiently low. In this case, (18) and hence IPP_i are negative: We can expect input prices to fall after the merger if overall efficiencies equal Werden efficiencies but input-specific efficiencies are low.

However, as E_i increases and the merged firm becomes more efficient in its use of input i, (18) increases. This holds because higher i-specific efficiencies E_i reduce the sensitivity of downstream marginal costs to the input price w_i. For given downstream pass-through rates $\hat{\rho}_{\mathcal{G}}$, this implies that goods prices p become less sensitive to the input price w_i. This effect is much larger for p_g than for $p_{g'}$, because the cross-good pass-through rate $\hat{\rho}_{g'g}$ falls much more than the own-good pass-through rate $\hat{\rho}_{gg}$ after the merger. As a result, output y_g also becomes less sensitive to the input price w_i as E_i increases, raising (18). We then observe a key trade-off with specialized input suppliers: The post-merger decline in pass-through rates tends to raise the sensitivity of downstream output y_g to the input price w_i, while input-specific efficiencies tend to reduce it. For a Werden-efficient merger, the resolution of this trade-off determines whether firm i will have an incentive to raise or lower its price after the merger. Proposition 3 quantifies this trade-off and indicates that, to compensate for a larger Edgeworth-Salinger effect, greater input-specific efficiencies are needed for $\text{IPP}_i > 0$ when goods are more substitutable.

Suppose now that input i is used to produce good g' as well as good g. Then the proposition implies that the efficiencies threshold $\bar{E}_i$ is *weakly smaller*, so firm i has an incentive to raise prices more often.[14] Intuition again follows from the Edgeworth-Salinger effect: In response to an increase in the marginal cost of good g, the merged firm raises the price of good g' by a smaller amount than before the merger in order to divert demand from g to g'. When input i

[14]This inequality is strict provided that input i is used to produce both goods and the goods are strict gross substitutes.

is used to produce only good g, this tends to increase the sensitivity of demand for input i to its price w_i. However, when input i is also used to produce good g', firm i internalizes some of the "input demand" diverted from g to g'. This mitigating effect tends to reduce the sensitivity of total demand for input i to its price w_i, thereby reducing firm i's incentive to lower its price. In some cases, this effect is strong enough that the threshold efficiencies $\bar{E}_i$ are negative, so we can expect input prices to rise for any Werden-efficient merger.

Result 1. *Suppose both goods require equal quantities of input i, $x_{gi} = x_{g'i}$, and suppose the diversion ratios are equal, $D := D_{gg'} = D_{g'g}$. Then $\bar{E}_i = -\frac{D/2}{1-D/2} < 0$.*

With $\bar{E}_i < 0$, firm i has an incentive to raise its price after a Werden-efficient merger regardless of i-specific efficiencies. The example in Section 3.2 shows that in further special cases, we can even dispense with the assumption of Werden efficiencies, and firm i will always have an incentive to raise its price after the merger.

Proposition 3 has direct implications for merger policy. When input prices are fixed, Werden-efficient mergers lie precisely on the boundary between mergers that are allowed and mergers that are blocked by a consumer surplus-maximizing antitrust authority. The proposition demonstrates that this boundary must generally shift when input prices are endogenous. In particular, when input-specific efficiencies are sufficiently large, the antitrust authority should block Werden-efficient mergers (and, by continuity, a subset of mergers that would yield consumer surplus gains with fixed input prices). The opposite holds when input-specific efficiencies are sufficiently small.

We can use inequality (17) to assess the likelihood of each scenario by choosing a benchmark for input-specific efficiencies. To start, note that a Werden-efficient merger must yield specific reductions in the marginal cost of each good:

$$\frac{c_g - \hat{c}_g}{c_g} = E_g^{\mathrm{W}} := \frac{D_{gg'}}{1 - D_{gg'}D_{g'g}} \left(\frac{c_{g'}}{c_g}\mu_{g'}^* + D_{g'g}\mu_g^* \right) \quad g \in \mathcal{G}, \tag{19}$$

where all functions are evaluated at pre-merger input prices w^* and $\mu_g^* := p_g^*/c_g(w^*) - 1$ denotes the pre-merger *markup* for good g.[15] We argue that the Werden efficiencies E_g^{W} and $E_{g'}^{\mathrm{W}}$ are reasonable benchmarks for the i-specific efficiencies E_i. For example, if the production technology for good g changes in a Hicks-neutral fashion after a Werden-efficient merger, then the i-specific efficiencies must be equal to the Werden efficiencies for good g, $E_i = E_g^{\mathrm{W}}$. More generally, if i-specific efficiencies exceed the Werden efficiencies for some good g, Proposition 4 establishes conditions on the pre-merger markup μ_g^* under which firm i has an incentive to increase its price after the merger:

[15]This expression can be derived following Werden (1996) or by noting that $c_g - \hat{c}_g = \mathrm{GePP}_g$.

Proposition 4. *Suppose the merger is Werden-efficient, and suppose $E_i \geq E_g^{\mathrm{W}}$. Then $\mathrm{IPP}_i > 0$ if either of the following hold:*

(i) the pre-merger markup μ_g^* is greater than 0.25;

(ii) demand $y(p)$ is symmetric, pre-merger marginal costs for both goods are equal $c_g(w^*) = c_{g'}(w^*)$, and the pre-merger markup μ_g^* is greater than 0.07.

This proposition follows readily by comparing the Werden efficiencies (19) to the right side of inequality (17). We view the sufficient conditions of Proposition 4 as fairly mild, indicating that input prices, and hence goods prices, may often rise after Werden-efficient downstream mergers. However, we emphasize that the proposition requires *input-specific efficiencies*, which may not always be a reasonable assumption. As Result 1 indicates, this requirement can be relaxed if the downstream goods have similar unit input requirements for input i.

3.4 Overall Price Effects

In the previous section, we described conditions under which a downstream merger generates positive Input Pricing Pressure on an upstream firm i, $\mathrm{IPP}_i > 0$. However, recalling the decomposition (15) for the change in the price of input i, we observe that IPP_i is generally insufficient to predict the sign or the magnitude of the overall price change $\hat{w}_i^* - w_i^*$. This is because IPP_i measures only the *unilateral incentive* for firm i to adjust its price. The price of input i also responds to changes in the other input price $w_{i'}$ in the post-merger equilibrium, and IPP_i itself must be converted into a price change using the pass-through rate $\hat{\rho}_{ii}$.

Despite these complications, we argue that IPP_i alone is informative of input i's price change, and that it is often dispositive of the *sign* of the price change. For example, if the merger symmetrically affects pricing incentives across both upstream firms so that $\mathrm{IPP}_i = \mathrm{IPP}_{i'}$, then IPP_i necessarily determines the sign of input i's price change:[16]

$$\hat{w}_i^* - w_i^* \quad = \quad (\hat{\rho}_{ii} + \hat{\rho}_{ii'}) \times \mathrm{IPP}_i.$$

Alternatively, under successive price competition the price of input i often responds much more strongly to changes in its own marginal cost than the marginal cost of input i', $\hat{\rho}_{ii} > |\hat{\rho}_{ii'}|$.[17] If the merger generates IPP of similar magnitudes for each input, then the first term in the decomposition dominates, and we can sign the post-merger price change $\hat{w}_i^* - w_i^*$ by ignoring

[16] The sum of pass-through rates in parentheses is positive for many specifications of the simple model – see Lemma A.1 in Appendix A for a closed-form expression for $\hat{\rho}_{\mathcal{I}}$. In fact, every pass-through rate is positive when the inputs are gross substitutes under the demand system $\hat{z}$, $d\hat{z}_i/dw_{i'} > 0$ (e.g., with specialized input suppliers).

[17] Lemma A.1 in Appendix A shows that this holds provided that inputs i and i' are not used in substantially different proportions in downstream production.

$\text{IPP}_{i'}$:

$$\text{sgn}\left(\hat{w}_i^* - w_i^*\right) \quad = \quad \text{sgn}\left(\hat{\rho}_{ii} \times \text{IPP}_i\right).$$

Finally, our numerical results for the simple model and for our empirical application in Section 5 confirm that IPP_i accurately predicts the sign of the post-merger price change for input i. Based on these arguments, we have focused primarily on characterizing IPP_i in our theoretical analysis of endogenous input price changes.

Given these arguments, we interpret the results of Section 3.3 as indicating when we should expect input prices to rise post-merger: when downstream output expands or becomes less sensitive to input prices, which are more likely when the merged firm achieves input-specific efficiencies. However, Propositions 3 and 4 do not indicate when the corresponding effects on input and goods prices are likely to be large as a function of pre-merger observables (e.g., markups and diversion ratios). These comparative statics would be helpful for screening anti-competitive mergers, but they are difficult to determine analytically. In Appendix C we provide simulation results that suggest for a Werden-efficient merger, endogenous changes in input prices and the resulting changes in goods prices are larger with higher pre-merger markups $\left(\mu_g^*, \mu_{g'}^*\right)$ and higher diversion ratios $\left(D_{gg'}, D_{g'g}\right)$. These comparative statics hold for both linear and logit consumer demand systems.

Higher pre-merger markups and diversion ratios also predict greater consumer harm from a merger under fixed input prices, and standard intuition based on Williamson's (1968) trade-off suggests that such a merger must yield greater efficiencies to prevent consumer harm.[18] Do higher efficiencies also mitigate the downstream effects of endogenous input prices? Proposition 5 below shows that the answer is generally "no," and that higher input-specific efficiencies can *exacerbate* the effect of endogenous input prices on downstream marginal costs and goods prices. Before stating the result, we note that good g's marginal cost can change after the merger both because of the direct effect of efficiencies and because of changes in input prices:

$$\hat{c}_g\left(\hat{w}^*\right) - c_g\left(w^*\right) \quad = \quad \underbrace{-E_{gv} v_g - \sum_i E_i x_{gi} w_i^*}_{\text{efficiencies}} + \underbrace{\sum_i \left(1 - E_i\right) x_{gi}\left(\hat{w}_i^* - w_i^*\right)}_{\Delta \text{ input prices}}. \tag{20}$$

Here $\left(1 - E_i\right) x_{gi}\left(\hat{w}_i^* - w_i^*\right)$ is the change in the marginal cost of good g that results from the change in the price of input i. The next proposition describes how $\left(1 - E_i\right)\left(\hat{w}_i^* - w_i^*\right)$ and the change in the price of good g due to endogenous input prices, $\hat{p}_g^*\left(\hat{w}^*\right) - \hat{p}_g^*\left(w^*\right)$, vary with

[18] This follows from (19), which shows that Werden efficiencies are increasing in pre-merger markups and diversion ratios.

efficiencies:

Proposition 5. *The following comparative statics hold in the simple model:*

(i) $(1-E_i)\left(\hat{w}_i^* - w_i^*\right)$ *is increasing in* E_i.

(ii) $(1-E_i)\left(\hat{w}_i^* - w_i^*\right)$ *is increasing in* $E_{i'}$ *if and only if* $d\hat{z}_{i'}/dw_i < 0$.

(iii) $\hat{p}_g^*(\hat{w}^*) - \hat{p}_g^*(w^*)$ *is increasing in* E_i *if and only if* $x_{gi} > B_i x_{gi'}$, *where* B_i *is independent of* g *and is negative if* $d\hat{z}_{i'}/dw_i < 0$.

The proof follows by noting that the approximations in Proposition 2 are exact in the simple model. Part (i) of the proposition indicates that the increase in the marginal cost of each good g due to the change in the price of input i is increasing in i-specific efficiencies. This result is consistent with our discussion of the IPP_i expression (16): Higher i-specific efficiencies tend to raise downstream outputs at fixed input prices and make downstream outputs less sensitive to the input price w_i. These two mechanisms incentivize an increase (or a smaller decrease) in the price of input i as i-specific efficiencies rise. These mechanisms are strong enough so that the effect of the endogenous input price w_i on downstream marginal costs is increasing in E_i.

Part (ii) similarly indicates that the increase in the marginal cost of each good g due to the change in the price of input i is increasing in i'-specific efficiencies, assuming the inputs are gross complements under the demand system $\hat{z}$, $d\hat{z}_{i'}/dw_i < 0$. This holds because, holding the price of input i' fixed at $w_{i'}^*$, raising i'-specific efficiencies lowers the effective price of input i' $(1-E_{i'})w_{i'}^*$ observed by the merged firm. When inputs i and i' are gross complements under the demand system $\hat{z}$, this raises demand for input i and incentivizes firm i to raise its price and extract revenues. This effect dominates any increase in $\hat{w}_{i'}^*$ that arises in equilibrium, which would reduce the demand for input i under gross complements.

Part (iii) aggregates the marginal cost effects from parts (i) and (ii) and characterizes how the change in the price of good g due to endogenous input prices $\hat{p}_g^*(\hat{w}^*) - \hat{p}_g^*(w^*)$ varies with input-specific efficiencies. In particular, if inputs i and i' are gross complements or the production of good g is sufficiently more intensive in its use of i than i', then $\hat{p}_g^*(\hat{w}^*) - \hat{p}_g^*(w^*)$ is increasing in i-specific efficiencies. For intuition, first consider the case in which inputs i and i' are gross complements. Parts (i) and (ii) of the proposition then imply that the effect of the change in any input price on the marginal cost of good g is increasing in E_i, so that the total effect of endogenous input prices on the marginal cost of good g is increasing in E_i. When instead inputs i and i' are gross substitutes, raising E_i increases the effect of the change in w_i on the marginal cost of good g but lowers the effect of the change in $w_{i'}$. The former dominates when the production of good g is sufficiently i-intensive, so that the net effect on the marginal cost and the price of good g are increasing in E_i.

As the discussion above emphasizes, the comparative statics in parts (ii) and (iii) of Proposition 5 turn on whether the inputs are gross complements or substitutes in the demand system $\hat{z}$. Since the inputs are perfect complements in downstream production, the extent to which the inputs are gross complements in total input demand is driven by the structure of the input-output network (i.e, the unit input requirements $x_{gi}, x_{gi'}, x_{g'i}, x_{g'i'}$), the consumer demand system y, and the successive pricing assumption. For example, the inputs are gross substitutes when the economy features specialized input suppliers: An increase in the price w_i of input i raises only the marginal cost of good g, and the merged firm responds by raising only the price of good g. This price change diverts consumer demand toward good g' and raises the demand for input i', so that the total effect of the increase in the price of input i on demand for input i' is positive, $d\hat{z}_{i'}/dw_i > 0$. Alternatively, the inputs are gross complements when both goods require the same quantity of each input: An increase in w_i raises marginal costs equally for both downstream goods, so that total downstream output and hence total demand for input i' declines, $d\hat{z}_{i'}/dw_i < 0$.

We stress that many of the comparative statics in Proposition 5 run counter to the standard price effect of efficiencies. With fixed input prices, higher efficiencies generally imply lower equilibrium goods prices. This is the essence of Williamson's (1968) trade-off, but Proposition 5 suggests a qualification: Larger efficiencies may also place upward pressure on goods prices through higher input prices. This countervailing effect is generally weaker than the standard price effect of efficiencies, so that goods prices are decreasing in efficiencies overall. However, as we saw in Section 3.2, it may be strong enough that true "offsetting efficiencies" do not exist, and goods prices necessarily increase after the merger.

3.5 Extensions

While it provides useful insight into how input prices respond to downstream horizontal mergers, the simple model is very special. In Section 5, we empirically explore input price responses in a much richer setting: mergers between beer retailers in local geographic markets. Here we discuss theoretically the effects of relaxing some of the restrictive features of the simple model. We focus on generalizing Propositions 3 and 4, which we view as our most policy-relevant results. We briefly summarize our extensions in this section, and we leave proposition statements, proofs, and more detailed discussions in Appendix B.

Production Technology and Many Input Suppliers

The simple model severely restricts the downstream production technologies and the input-output structure: Downstream marginal costs are constant in output and linear in input prices, and there are only two inputs. In Appendix B, we show that Propositions 3 and 4 apply essentially unchanged with arbitrary downstream cost functions and an arbitrary number of single-product upstream firms. We only require the additional assumption that any merger-induced changes to downstream cost functions do not alter returns to scale or conditional input demand elasticities at pre-merger input prices. Any such changes can be taken into account separately from the pass-through and efficiencies effects that we have identified to determine the net change in input pricing incentives.

Downstream "Outsiders"

The analysis of Section 3 assumes that initially only two firms compete in the market for consumer goods. Though consistent with other studies of unilateral pricing incentives after mergers (e.g., Schmalensee, 2009; Farrell and Shapiro, 2010), this assumption likely yields biased estimates of merger price effects when the merging firms face competition from non-merging rivals, or *outsiders*, that produce substitutable products. Most relevant for our results, when downstream firms compete in prices, outsiders' prices are strategic complements to the merging firms' prices, so outsiders generally increase their prices after an increase in the merging firms' marginal costs. This response (i) raises pre- and post-merger pass-through rates and (ii) creates an additional channel through which downstream output y_g is affected by the input price w_i. The net effect on IPP is not obvious.

In Appendix B, we give a result indicating that this effect is likely small and may even strengthen our main conclusions. We incorporate a third good owned by a non-merging downstream firm into the simple model. To maintain tractability, we assume that this good is produced at constant marginal cost using only the numeraire and that the diversion ratios between all pairs of goods are equal. In this case, we find that Propositions 3 and 4 continue to hold. Our numerical results in Appendix C indicate that this result generalizes to settings with asymmetric diversion ratios, in which our model is not analytically tractable.

Nonlinear Demand

In the simple model, we make the strong assumption that the consumer demand system $y(p)$ is linear. We do this for analytical tractability: The incentive for an upstream firm to raise its price after a downstream merger depends crucially on the change in downstream pass-through rates, $\hat{\rho}_{\mathcal{G}} - \rho_{\mathcal{G}}$. With linear demand, this change is easy to compute because pass-through rates are constant and determined exclusively by diversion ratios. This allows us to characterize the efficiencies threshold in Proposition 3 as well as the bounds in Propositions 3′ and 3″ in terms of model primitives. With nonlinear demand, pass-through rates depend additionally on the equilibrium margins and the local curvature of demand, so they are more difficult to manipulate analytically.[19] This precludes a simple generalization of Proposition 3, but it does not fundamentally change the economics of mergers with endogenous input prices. Departing from the simple model only in allowing nonlinear consumer demand, in Appendix B we again find that an upstream firm i has an incentive to raise its price after a Werden-efficient merger when the merged firm attains sufficient i-specific efficiencies $\bar{E}_i^{\mathrm{NL}}$. It is difficult to analytically determine the difference between $\bar{E}_i^{\mathrm{NL}}$ and the threshold $\bar{E}_i$ of Proposition 3, but our numerical results in Appendix C suggest that it is small with logit demand and does not materially affect the policy implications of our model: Optimal merger policy under fixed input prices may be too lax when input prices are endogenous.

Asymmetric Input-Specific Efficiencies

Throughout our analysis, we assume that the merged firm achieves symmetric input-specific efficiencies E_i for each good g. More generally, we could suppose that different input-specific efficiencies are realized for each good after the merger:

$$\hat{c}_g(w) = \left(1 - E_{gv}\right) v_g + \left(1 - E_{gi}\right) x_{gi} w_i + \left(1 - E_{gi'}\right) x_{gi'} w_{i'},$$

where potentially $E_{gi} \neq E_{g'i}$. We assume symmetric input-specific efficiencies $E_i = E_{gi} = E_{g'i}$ to shut down composition effects: If good g attains greater i-specific efficiencies than good g', then g' will tend to comprise a greater share of the total demand for i after the merger than before the merger. If the post-merger input demand from g' is less elastic than that from g at pre-merger input prices, this places additional upward pressure on the price of i. This composition effect is absent when i-specific efficiencies are equal or when the elasticities of post-merger demand for i from g and g' are equal. When efficiencies and input demand elasticities are heterogeneous, this composition effect must be weighed against the pass-through and efficiencies effects we have highlighted to determine the net change in input pricing incentives.

4 Bargaining

Throughout Section 3, we assumed that input prices are set through successive price competition: Upstream firms unilaterally set input prices and anticipate changes in goods prices when doing so. We departed in these two ways from the recent literature on horizontal market structure with vertical contracting, in which it is commonly assumed that input prices are set

[19] See Weyl and Fabinger (2013).

through Nash-in-Nash bargaining and simultaneously with goods prices. In this section we explain how the effects and policy implications we have highlighted persist when input prices are set through *successive* Nash-in-Nash bargaining. The pass-through and efficiency effects described in Section 3 again push toward higher post-merger input prices for Werden-efficient mergers, mitigating the price-reducing effects of changes in bargaining leverage.

4.1 Conduct

Consider the simple model, and to avoid complications from price discrimination suppose the model features specialized input suppliers. Before the merger, firms g and i Nash bargain over the input price w_i, holding the input price $w_{i'}$ fixed but allowing goods prices to vary:

$$\max_{w_i \geq 0} \Big[\underbrace{\pi_g(w) - \pi_g(\infty, w_{i'})}_{\mathrm{GFT}_g(w)} \Big]^{1-\beta_i} \times \Big[\underbrace{\pi_i(w) - \pi_i(\infty, w_{i'})}_{\mathrm{GFT}_i(w)} \Big]^{\beta_i}. \tag{21}$$

Here $\beta_i \in [0,1]$ is the Nash bargaining weight for firm i, and we define the profits earned by firms g and i at input prices w by

$$\begin{aligned} \pi_g(w) &:= \left(p_g^*(w) - c_g(w)\right) y_g(p^*(w)), \\ \pi_i(w) &:= (w_i - c_i) z_i(w). \end{aligned}$$

The *gains from trade* functions $\mathrm{GFT}_g(w)$ and $\mathrm{GFT}_i(w)$ describe the additional profits firms g and i earn if they agree to price w_i for input i, relative to the profits they earn if negotiations break down and they do not trade (so that effectively $w_i = \infty$). With specialized input suppliers, these gains from trade are exactly equal to the profits earned by each firm after agreement:

$$\mathrm{GFT}_g(w) = \pi_g(w) \quad \text{and} \quad \mathrm{GFT}_i(w) = \pi_i(w).$$

This holds because input i is essential to the production of good g and because firm i sells only to firm g, so both firms make zero profits after a bargaining breakdown. Note that this specification of Nash bargaining is *successive* because firms anticipate changes in equilibrium goods prices $p^*(w)$ when bargaining over input prices.

After the merger, the merged firm M bargains separately with firms i and i' to determine input prices. The input price w_i is then chosen to maximize the post-merger Nash product betwen firms M and i, holding the input price $w_{i'}$ fixed:

$$\max_{w_i \geq 0} \Big[\underbrace{\hat{\pi}_M(w) - \hat{\pi}_M(\infty, w_{i'})}_{\hat{\mathrm{GFT}}_{M,i}(w)} \Big]^{1-\beta_i} \times \Big[\underbrace{\hat{\pi}_i(w) - \hat{\pi}_i(\infty, w_{i'})}_{\hat{\mathrm{GFT}}_i(w)} \Big]^{\beta_i}. \tag{22}$$

Here we assume that the exogenous bargaining weight β_i remains unchanged after the merger, and we define the post-merger profits earned by firms M and i at prices w by

$$\hat{\pi}_M(w) := \sum_{g \in \mathcal{G}} \left(\hat{p}^*_g(w) - \hat{c}_g(w) \right) y_g(\hat{p}^*(w)),$$
$$\hat{\pi}_i(w) := (w_i - c_i)\, \hat{z}_i(w).$$

With specialized input suppliers, firm i's gains from trade are again equal to the profits it earns after agreement: $\hat{\mathrm{GFT}}_i(w) = \hat{\pi}_i(w)$. However, the merged firm's gains from trade $\hat{\mathrm{GFT}}_{M,i}$ are generally *less* than its profits after agreement, because it may still earn positive profits from the sale of good g' after a bargaining breakdown with firm i: $\hat{\mathrm{GFT}}_{M,i}(w) \leq \hat{\pi}_M(w)$, where the inequality is strict provided that $y_{g'}(\hat{p}^*(\infty, w_{i'})) > 0$. For simplicity, we assume that the merged firm cannot renegotiate the price for input i' after a breakdown with firm i.

4.2 IPP and Merger Policy under Bargaining

How does the downstream merger affect input prices when input prices are determined through bargaining? We again study IPP to uncover the determinants of input price changes. In Appendix A, we derive the FOC functions corresponding to successive Nash-in-Nash bargaining (21, 22), and we find that IPP_i can be decomposed into three terms:

$$\mathrm{IPP}_i \quad \propto \quad \mathrm{IPP}_i^{\text{price}} \quad + \quad \Delta_i^{\text{profit sensitivity}} \quad + \quad \Delta_i^{\text{leverage}}. \tag{23}$$

The first term $\mathrm{IPP}_i^{\text{price}}$ is precisely equal to IPP for firm i under successive price competition; the analysis of Section 3.3 applies unchanged to this term. The second term $\Delta_i^{\text{profit sensitivity}}$ captures the change in the sensitivity of downstream profits to the input price w_i after the merger:

$$\Delta_i^{\text{profit sensitivity}} \quad \propto \quad \frac{1-\beta_i}{\beta_i} \frac{\mathrm{GFT}_i(w^*)}{\mathrm{GFT}_g(w^*)} \left[y_g(p^*(w^*)) - y_g(\hat{p}^*(w^*)) - \left(p_g - c_g(w^*)\right) \frac{\partial y_g}{\partial p_{g'}} \rho_{g'g} \hat{x}_{gi} \right]$$

This term is negative when the merged firm's profits $\hat{\pi}_M$ are more sensitive to the input price w_i after the merger than were the profits π_g of independent firm g before the merger.[20] When this

[20] I.e., when $\frac{\partial \hat{\pi}_M(w^*)}{\partial w_i} < \frac{\partial \pi_g(w^*)}{\partial w_i}$.

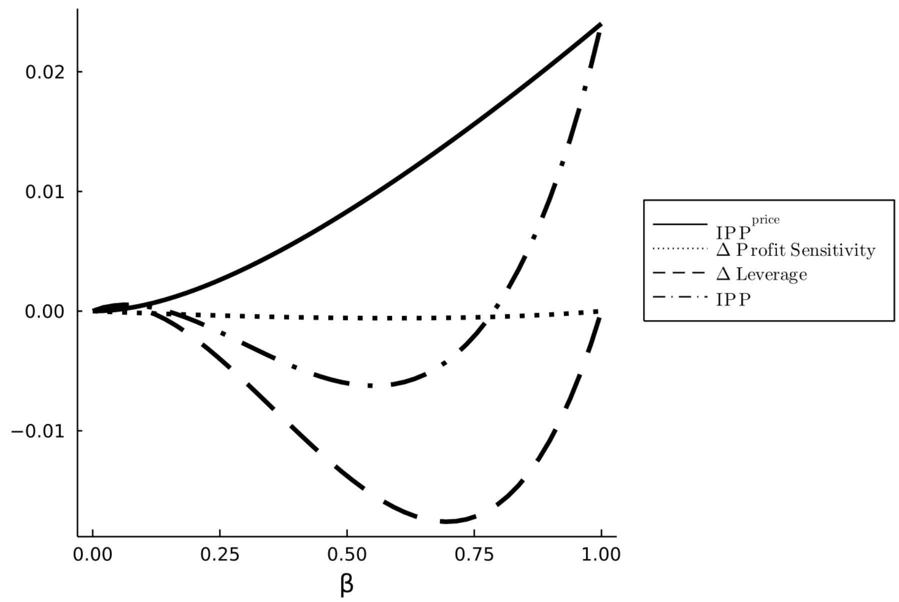


Figure 2: Bargaining IPP decomposition with Werden efficiencies. We parametrize the simple model with specialized input suppliers by $x_{gi} = x_{g'i'} = 1$, $v_g = v_{g'} = 0$, and $c_i = c_{i'} = 1$. The demand system takes the symmetric form $y_g(p) = V_\beta - p_g + D_\beta p_{g'}$, where for each bargaining weight $\beta \in [0,1]$, we calibrate (V_β, D_β) to match a diversion ratio of 6.3% and a pre-merger markup of 40.4% for both downstream goods.

holds, the merged firm is harmed more by a small increase in the input price w_i, incentivizing an decrease in w_i to raise the merged firm's profits and the post-merger Nash product (22). Considering the expression in brackets, we observe that this may hold for two reasons. First, the merged firm's profits are more sensitive to the input price w_i if the merged firm raises the output of good g, requiring additional expenditure on input i. This effect is precisely zero for a Werden-efficient merger since downstream prices and hence outputs are unchanged at fixed input prices. Second, the merged firm's profits are naturally more sensitive to the input price because the merged firm internalizes the value of demand diverted between consumer goods. Before the merger, an increase in w_i raises firm g's marginal cost, and firm g' responds in equilibrium by raising its price. The increase in the price of good g' raises demand for good g and has a first-order effect on firm g's profits, softening the impact of the higher input price. The merged firm jointly optimizes over downstream prices after the merger, so this effect is absent. To summarize, we observe that the sign of $\Delta_i^{\text{profit sensitivity}}$ is generally indeterminate but negative for Werden-efficient mergers, providing a force for lower post-merger input prices. However, our simulations suggest that this term is often quantitatively small (Figure 2).

The third term $\Delta_i^{\text{leverage}}$ reflects the change in bargaining leverage after the merger:

$$\Delta_i^{\text{leverage}} \quad \propto \quad \frac{1-\beta_i}{\beta_i}\left(\frac{\text{GFT}_i(w^*)}{\text{GFT}_g(w^*)} - \frac{\hat{\text{GFT}}_i(w^*)}{\hat{\text{GFT}}_{M,i}(w^*)}\right) y_g\left(\hat{p}^*(w^*)\right).$$

When the merged firm's gains from trade fall relative to firm i's gains from trade after the merger, this term is negative and incentivizes a decrease in the input price w_i. The intuition is standard: The merged firm benefits when the cost of a bargaining breakdown becomes relatively higher for firm i. The sign of $\Delta_i^{\text{leverage}}$ is also theoretically ambiguous, but this term is the primary object of interest in the recent literature examining the effects of horizontal market structure in economies with vertical contracting. The consensus from this literature appears to be that, for mergers without marginal cost efficiencies, this term is negative as the merged firm improves its bargaining position vis-à-vis the upstream firms.[21] Our simulations indicate that $\Delta_i^{\text{leverage}}$ is also often negative for Werden-efficient mergers (Figure 2).

To summarize, the IPP_i decomposition (23) clarifies that the mechanism studied in Section 3.3 is also present when input prices are determined through successive Nash-in-Nash bargaining: Merger-induced changes in downstream pass-through rates and efficiencies can generate endogenous changes in input prices ($\text{IPP}_i^{\text{price}}$). Our Propositions 3 and 4 provide conditions under which this mechanism counteracts the standard effect of the merger on bargaining leverage, which tends to reduce input prices ($\Delta_i^{\text{leverage}}$). On the basis of these observations and our simulations, we argue that downstream mergers may incentivize higher input prices when conduct is successive, in contrast to predictions of lower input prices based solely on changes in bargaining leverage. Our analysis supports additional skepticism about bargaining leverage claims when conduct is successive, particularly when the merger may yield input-specific efficiencies.

5 Quantitative Analysis of Beer Retailer Mergers

We now turn to a quantitative analysis of mergers with endogenous input prices. We consider local consumer retail markets for beer, and using an estimated demand system and cost structure, we simulate mergers between beer retailers. We make the exercise stylized to be similar to a preliminary analysis an antitrust authority might conduct. Even for mergers that generate no efficiencies, we find that goods prices increase more post-merger when we allow input prices to adjust endogenously. This stylized exercise suggests that input price adjustments and their effect on consumer prices are quantitatively significant, and it also provides a test of our theory in environments that are difficult to study analytically.

[21] See Barrette, Gowrisankaran, and Town (2020); Craig, Grennan, and Swanson (2021); and Sheu and Taragin (2021). These studies also show that the resulting decline in input prices is not sufficient to prevent goods prices from rising when downstream firms engage in price competition.

5.1 Setting and Data

Our setting is the retail beer market in Ohio, and we use Retail Scanner Data from Kilts Nielsen for the sales and prices of retail beer products at the UPC-store-week level in 2010. We consider a collection of non-overlapping geographic markets with generic index m, where each market is identified with a designated market area (DMA) defined by Nielsen. The set of goods in market m is $\mathcal{G}_m$, where each good is a brand- and retailer-specific beer product. The set of downstream firms in market m is $\mathcal{F}_{\mathcal{G}_m}$, where we identify a firm with a retail chain. We aggregate sales and price data from the store level to the retail chain level, and we include food, mass merchandise, convenience, and liquor stores when doing so. Within each retail chain, we similarly aggregate data from the UPC level to the brand level, weighting by the volume of each product of a given brand. We also aggregate weekly data to the month level. For computational tractability, we keep only the 15 highest-selling brands by volume sold.

Retail chains purchase beer at wholesale prices from distributors, which constitute the upstream firms $\mathcal{F}_{\mathcal{I}_m}$ in this setting.[22] We do not observe the beer distribution networks in each market m, so we make the simplifying assumption that each brand of beer is distributed by a unique distributor in each market. We assume that in each market m, retail chains engage in Bertrand-Nash competition while distributors engage in successive price competition. The former assumption is standard, while a number of regulations on beer distribution suggest that the latter is reasonable. For example, federal regulations prohibit slotting allowances in the sale of alcoholic beverages in retails establishments (Gundlach and Bloom, 1998). State regulations in Ohio additionally prohibit distributors from offering volume discounts or retail credit, and they place no restrictions on distributor markups.[23] Altogether these regulations suggest that successive price competition is a reasonable approximation to market conduct by distributors.

5.2 Demand and Supply Estimation Procedure

For simplicity, we suppose that consumer demand in each market is given by a logit demand system. The indirect utility experienced by consumer l in market m from purchasing good g is

$$u_{lgm} = -\gamma_m p_{gm} + \xi_{gm} + \epsilon_{lgm},$$

[22] A complete model of beer distribution would also include a third tier: beer manufacturers. We do not model price-setting by beer manufacturers for simplicity and to ensure that the quantitative exercise stays within the confines of our general model.

[23] See the 2018 report on wholesale pricing restrictions from the Substance Abuse and Mental Health Services Administration <link>.

where p_{gm} is the per-ounce price of good g in market m, γ_m is the price sensitivity parameter of market m, ξ_{gm} is a brand-retail chain fixed effect, and ϵ_{lgm} is an unobserved logit error term. We normalize the utility of the outside option to zero.

We calibrate the parameters γ_m and $\xi_m := \left(\xi_{gm}\right)_{g\in\mathcal{G}_m}$ to match (i) the median market elasticity of demand estimated in a similar setting by Miller and Weinberg (2017) and (ii) the market shares for each good observed in the data:

$$\begin{aligned}
\hat{\mathcal{E}}_m &= \sum_{g\in\mathcal{G}_m}\sum_{g'\in\mathcal{G}_m} \frac{\partial \log y_{gm}(p_m;\gamma_m,\xi_m)}{\partial \log p_{g'm}}, \\
\hat{y}_{gm} &= y_{gm}(p_m;\gamma_m,\xi_m) \qquad\qquad g\in\mathcal{G}_m,
\end{aligned}$$

where $\hat{\mathcal{E}}_m = -0.7$ is the market elasticity of demand, $\hat{y}_{gm}$ is the market share of good g in market m observed in the data, and $y_{gm}(p;\gamma_{pm},\xi_{gm})$ is the market share of good g in market m implied by the logit demand system.

To calculate market shares $\hat{y}_{gm}$, for each month we compute the share of each good g in total volume sold in market m in our data, and we average these shares across all months. To scale these "inside" shares, we construct the share of the outside good as follows: Using the Consumer Panel Data from Kilts Nielsen, we determine the number of panelists in market m who purchased beer at some point during the year. We then calculate the proportion of beer-purchasing panelists who did not purchase beer in market m in a given month, and we use this as the share of the outside good for that month. We average across months to obtain a final estimate of the outside good's share, and we scale all inside shares to compute our final market shares $\hat{y}_{gm}$.

The final parameters needed to conduct merger simulations are distributor marginal costs, which we impute following Villas-Boas (2007). Let $\Omega^{\mathcal{G}_m}$ denote the ownership matrix for goods in market m, where $\Omega^{\mathcal{G}_m}_{gg'} = 1$ if goods g and g' are owned by the same retail chain and $\Omega^{\mathcal{G}_m}_{gg'} = 0$ otherwise. Define $\Omega^{\mathcal{I}_m}$ similarly as the ownership matrix for brands (inputs); in our baseline specification of the distribution network, $\Omega^{\mathcal{I}_m}$ is the identity. Inverting the downstream Bertrand-Nash FOCs, we can compute retail margins:

$$p - c_{\mathcal{G}_m} = -\left[\Omega^{\mathcal{G}_m} \odot \frac{dy}{dp}^{T}\right]^{-1} y.$$

Here $\odot$ denotes the Hadamard (pointwise) product. Given the calibrated demand system and observed goods prices, we solve this equation to determine retailer marginal costs $c_{\mathcal{G}_m}$. For simplicity, we assume that all marginal costs incurred by a retail chain are attributable to wholesale prices, so we can compute implied wholesale prices: $w_i = c_{gm}$ if good g is a retail product for brand i.[24] We can similarly invert the upstream price-setting FOCs to compute distributor margins:

$$w - c_{\mathcal{I}_m} = -\left[\Omega^{\mathcal{I}_m} \odot \frac{dz}{dw}^T \right]^{-1} z.$$

Given the calibrated demand system and imputed wholesale prices, we solve this equation to determine distributor marginal costs $c_{\mathcal{I}_m}$.[25]

5.3 Merger Simulations

Using our calibrated demand system and cost structure, we simulate mergers between retailers, both holding wholesale prices fixed and allowing wholesale prices to vary endogenously. We observe 8 markets in our data, with 3-9 retail chains in each market.[26] We simulate pairwise mergers between all chains in a given market, assuming that the mergers yield no efficiencies to the merging firms.

Table 1 presents results from these simulations. For each market, we calculate the average change in retail prices across all goods and all mergers, weighting price changes by pre-merger shares. In every market, retail prices increase more on average when wholesale prices are endogenous than when they are exogenous, with a typical difference of 4-6 percentage points. Similarly, the average decline in consumer surplus across all mergers in a market is always larger with endogenous wholesale prices: The retail mergers have almost no effect on consumer surplus with exogenous wholesale prices, because we observe relatively small market shares for each retail chain in our data. As a result, the mergers have only a small effect on market concentration as measured by the projected change in HHI. However, allowing wholesale prices to adjust endogenously lowers consumer surplus by nearly 1% on average in each market, despite the small changes in concentration.

Figure 3 plots the distributions of retail price changes after the mergers, where price changes are weighted by pre-merger market shares. When wholesale prices are exogenous, retail price changes are heavily concentrated near zero, reflecting the small changes to market concentration associated with the mergers. When wholesale prices are endogenous, the distribution of retail price changes displays greater dispersion and is shifted to the right, with a mean of

[24] This inversion gives us a $|\mathcal{G}_m|$ vector of wholesale prices. We assume there is no price discrimination, so we take a mean by brand to calculate the final wholesale price per brand. However, we observe little variation in the wholesale prices across different retailers if we allow for price discrimination.

[25] The resulting distributions of pre-merger wholesale prices and distributor marginal costs are given in Appendix D.

[26] There are 12 DMAs defined by Nielsen that include parts of Ohio, but 4 of these also overlap with other states. We exclude DMAs spanning more than one state.

Table 1: Simulation Summary

	1	2	3	4	5	6	7	8
# Chains	8	6	7	7	6	8	3	4
Chain HHI	479.06	638.54	548.83	660.71	779.06	446.57	863.96	767.57
Mean Δ HHI	8.93	2.15	16.87	9.15	5.52	5.9	2.74	42.35
γ_m	-54.81	-59.61	-50.62	-52.37	-53.78	-59.87	-49.28	-49.13
Median Own Elasticity	-4.45	-4.21	-4.38	-4.05	-4.14	-4.31	-3.52	-3.95
Median Cross Elasticity	0.01	0.02	0.02	0.03	0.02	0.01	0.06	0.03
Mean Δp, Exogenous	0.12	0.03	0.21	0.12	0.08	0.08	0.21	0.6
Mean Δp, Endogenous	3.72	6.54	6.34	6.09	7.14	4.45	4.68	5.39
Mean Δw, Endogenous	-0.02	-0.18	0.29	0.14	0.42	-0.1	3.56	2.53
Mean ΔCS, Exogenous	-0.03	-0.01	-0.06	-0.03	-0.02	-0.02	-0.14	-0.15
Mean ΔCS, Endogenous	-0.87	-1.41	-1.58	-1.48	-1.74	-0.96	-1.36	-1.36

Notes: Each column corresponds to a local retail beer market (DMA) in Ohio. The last 5 rows are expressed in percent changes. For the rows showing the percent change in consumer prices for the exogenous and endogenous input price counterfactuals, we construct these means weighting by pre-merger product market shares. HHI and Δ HHI are calculated at the retail chain level, and Δ HHI is defined as the projected change in HHI using pre-merger shares.

5.47 %. Figure 4 plots the distribution of wholesale price changes for mergers with endogenous wholesale prices; the price changes are again weighted by pre-merger market shares.[27] Wholesale price changes are concentrated near zero, but the distribution is again right-skewed, and many wholesale prices rise in excess of 1%.

We find that after mergers with endogenous input prices, retail price changes are generally larger than wholesale price changes. In a logit demand system, the pass-through rate of idiosyncratic marginal cost shocks to own prices $\frac{\partial p_g}{\partial c_g}$ is bounded below one.[28] However, the pass-through rate of changes in wholesale prices to retail prices $\frac{dp_i}{dw_j}$ can be above one in our environment since multiple retailers may use a particular input. When multiple retailers demand the same input, the pass-through rate is amplified relative to an idiosyncratic marginal cost shock.

These quantitative results broadly support our argument that downstream mergers incentivize higher input prices and that endogenous input prices exacerbate the consumer harm from downstream mergers. Notably, we find increases in input (wholesale) prices even though we simulate mergers with no input-specific efficiencies. As we discussed in Section 3.5, this is more likely when a single upstream firm sells to both merging firms and to outsiders, as we have assumed in our description of the beer distribution network. In addition, we find that endogenous changes in input prices can have a quantitatively large effect on goods prices even when the goods market is fairly unconcentrated. In any case, the quantitative results reported here do not follow directly from our propositions, so we view this exercise as a reassuring test

[27] The market share for a brand is the sum of the shares of the retail goods that belong to the brand.

[28] See, for example, Verboven and van Dijk (2009).

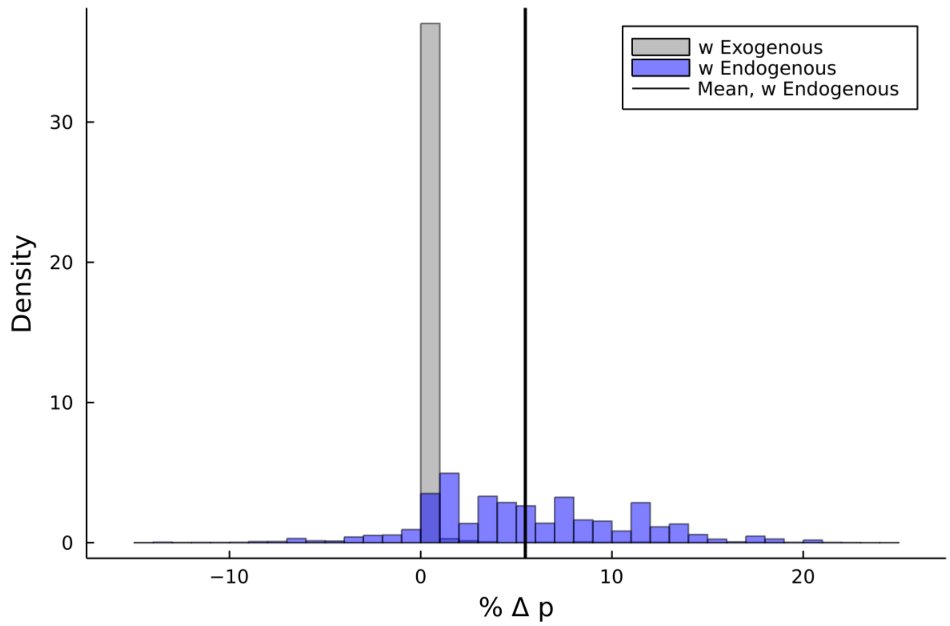


Figure 3: Histogram of percent change in retail prices after mergers with and without endogenous wholesale prices. Price changes are weighted by pre-merger market shares.

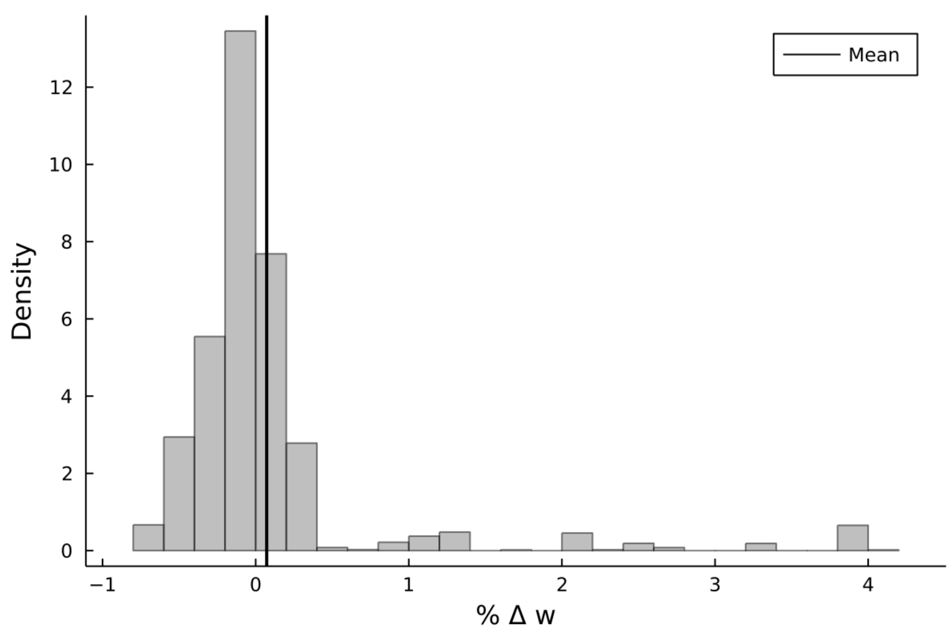


Figure 4: Histogram of percent change in wholesale prices after mergers with endogenous wholesale prices. Price changes are weighted by pre-merger market shares.

of the external validity of our theory.

6 Conclusion

Recent merger cases have sparked debate about the buyer power defense: Can a merger between competitors raise their bargaining leverage relative to upstream firms, leading to lower prices for inputs and consumer goods? We examined this question using models of imperfect competition in which input prices are set in anticipation of their effect on goods prices. To do so, we generalized the first-order approach of Farrell and Shapiro (2010) and Jaffe and Weyl (2013) and derived a measure of the incentives for input prices to adjust after a downstream merger, Input Pricing Pressure. We used this measure to show that, for mergers that have no effect on goods prices at fixed input prices, upstream firms often have incentives to *raise* input prices post-merger. These incentives hinge on changes in downstream pass-through rates and cost efficiencies generated by the merger, and they are absent under the common assumption that input prices and goods prices are set simultaneously. By implication, we argued that consumer surplus-maximizing antitrust policy may be too lax when input prices are assumed fixed, and it should be biased against claims of buyer power.

In an empirical application to local retail beer markets in Ohio, we found that allowing for endogenous input prices when simulating mergers between retailers raises average changes in goods prices as well as average consumer harm. We think of this exercise as an encouraging "out of sample test" of the pass-through effects we have highlighted: We allowed no merger efficiencies and considered a demand system and downstream market structure that do not satisfy the assumptions of our propositions. Nonetheless, we found that endogenous changes in input prices were still generally positive and had a substantial positive effect on goods prices, in line with the conclusions from the theory.

Our analysis suggests several productive areasq for future work. First, we currently lack empirical evidence about changes in input prices after consummated downstream mergers. A merger retrospective study that accounts for endogenous input prices would shed additional light on the empirical relevance of our theory. Similarly, it would be helpful to determine if modeling endogenous input prices can significantly improve the performance of merger simulations, which may be inaccurate (Weinberg and Hosken, 2013; Björnerstedt and Verboven, 2016). Finally, we maintain the crucial assumption that all transactions between upstream and downstream firms are mediated by linear prices. In many contexts firms may transact using more efficient contracts (e.g., nonlinear price schedules or quantity-based bargaining), and it is not clear how our results would generalize to these settings. Loertscher and Marx (2021) have made progress on this point by microfounding inefficient contracting through incomplete information, but they have not drawn conclusions about the effects of downstream mergers on consumers.

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Appendices

A Proofs for Results in the Main Text

A.1 Section 2

Proof of Proposition 2. Recall the basic first-order approximation (6), and note that under the assumptions of Proposition 2, the matrix to be inverted takes the block form

$$D\hat{f}\left(p^*, w^*\right) = \begin{bmatrix} \frac{d\hat{f}_{\mathcal{G}}}{dp} & \frac{d\hat{f}_{\mathcal{G}}}{dw} \\ 0 & \frac{d\hat{f}_{\mathcal{I}}}{dw} \end{bmatrix}.$$

Here we suppress arguments for notational simplicity. Our assumptions imply that this matrix is invertible, with block inverse

$$\begin{aligned}
\left[D\hat{f}\left(p^*, w^*\right)\right]^{-1} &= \begin{bmatrix} \left[\frac{d\hat{f}_{\mathcal{G}}}{dp}\right]^{-1} & -\left[\frac{d\hat{f}_{\mathcal{G}}}{dp}\right]^{-1} \frac{d\hat{f}_{\mathcal{G}}}{dw}\left[\frac{d\hat{f}_{\mathcal{I}}}{dw}\right]^{-1} \\ 0 & \left[\frac{d\hat{f}_{\mathcal{I}}}{dw}\right]^{-1} \end{bmatrix} \\
&= \begin{bmatrix} -\hat{\rho}_{\mathcal{G}}\left[\frac{\partial\hat{f}_{\mathcal{G}}}{\partial c_{\mathcal{G}}}\right]^{-1} & -\hat{\rho}_{\mathcal{G}}\frac{\partial\hat{c}_{\mathcal{G}}}{\partial w}\hat{\rho}_{\mathcal{I}}\left[\frac{\partial\hat{f}_{\mathcal{I}}}{\partial c_{\mathcal{I}}}\right]^{-1} \\ 0 & -\hat{\rho}_{\mathcal{I}}\left[\frac{\partial\hat{f}_{\mathcal{I}}}{\partial c_{\mathcal{I}}}\right]^{-1} \end{bmatrix}.
\end{aligned}$$

The sufficient statistics approximations (11, 12) then follow immediately from the basic approximation (6) and the definitions of GePP and IPP (9, 10). ■

A.2 Calculations for Example in Section 3.2

Pre-merger equilibrium prices are

$$\begin{aligned}
p_g^*\left(w_i\right) &= \frac{V + w_i}{2 - D} & g \in \mathcal{G}, \\
w_i^* &= \frac{V + (1-D)c_i}{2(1-D)}, & \\
p_g^*\left(w_i^*\right) &= \frac{(3-2D)V + (1-D)c_i}{2(2-D)(1-D)} & g \in \mathcal{G}.
\end{aligned}$$

Post-merger equilibrium prices are

$$\hat{p}_g^*(w_i) = \frac{V + (1-D)(1-E)w_i}{2(1-D)} \quad g \in \mathcal{G},$$
$$\hat{w}_i^* = \frac{V + (1-D)(1-E)c_i}{2(1-D)(1-E)},$$
$$\hat{p}_g^*(\hat{w}_i^*) = \frac{3V + (1-D)(1-E)c_i}{4(1-D)} \quad g \in \mathcal{G}.$$

We calculate E^{W} by solving $\hat{p}_g^*(w_i^*) = p_g^*(w_i^*)$, and we calculate E^{T} by solving $\hat{p}_g^*(\hat{w}_i^*) = p_g^*(w_i^*)$.

A.3 Helpful Lemmas

Throughout the remainder of Appendix A, we often write $\hat{x}_{gi} := (1-E_i)x_{gi}$ to simplify notation. The next two lemmas provide formulas used to prove the results in Section 3.

Lemma A.1. *The following hold in the simple model (Section 3.1):*

$$\rho_{\mathcal{G}} = \frac{1/2}{1 - \frac{D_{gg'}}{2}\frac{D_{g'g}}{2}} \begin{bmatrix} 1 & \frac{D_{gg'}}{2} \\ \frac{D_{g'g}}{2} & 1 \end{bmatrix} \tag{24}$$

$$\hat{\rho}_{\mathcal{G}} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{25}$$

$$\hat{\rho}_{\mathcal{I}} = \frac{1/2}{1 - \frac{\hat{D}_{ii'}}{2}\frac{\hat{D}_{i'i}}{2}} \begin{bmatrix} 1 & \frac{\hat{D}_{ii'}}{2} \\ \frac{\hat{D}_{i'i}}{2} & 1 \end{bmatrix}, \tag{26}$$

where

$$\hat{D}_{ii'} := -\frac{d\hat{z}_{i'}/dw_i}{d\hat{z}_i/dw_i} = -\frac{\sum_g \hat{x}_{gi'}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right)}{\sum_g \hat{x}_{gi}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right)}. \tag{27}$$

Moreover,

$$\sum_g \hat{x}_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right) > 0 \quad \textit{for} \quad \left(\hat{x}_{gi}, \hat{x}_{g'i}\right) \neq (0,0). \tag{28}$$

Proof of Lemma A.1. The downstream pass-through matrices (24, 25) are readily calculated from the definition (8), noting that the pre- and post-merger FOC functions for goods prices are

$$f_{\mathcal{G}}\left(p, c_{\mathcal{G}}\right) = y\left(p\right) + \operatorname{diag}\left[\frac{\partial y}{\partial p}\right]\left(p - c_{\mathcal{G}}\right),$$
$$\hat{f}_{\mathcal{G}}\left(p, \hat{c}_{\mathcal{G}}\right) = y\left(p\right) + \left[\frac{\partial y}{\partial p}\right]^{\top}\left(p - \hat{c}_{\mathcal{G}}\right).$$

To calculate the upstream post-merger pass-through matrix (26), we note that the upstream demand system $\hat{z}\left(w\right)$ is linear: Post-merger downstream marginal costs $\hat{c}_{\mathcal{G}}\left(w\right)$ are linear in input prices w, and the post-merger FOC function $\hat{f}_{\mathcal{G}}$ for goods prices is linear in goods prices p and marginal costs $\hat{c}_{\mathcal{G}}$. As a result, the solution $\hat{p}^*\left(w\right)$ to the post-merger FOC system for goods prices must be linear in w. But total demand for each input i $\hat{z}_i\left(w\right) = \sum_g y_g\left(\hat{p}^*\left(w\right)\right)\hat{x}_{gi}$ must then be linear in w, because the consumer demand function $y\left(p\right)$ is assumed linear. As a result, we can apply the downstream pre-merger formula for pass-through rates (24) using the appropriate diversion ratios. We calculate these as follows:

$$\begin{aligned}
\frac{d\hat{z}_i}{dw_i} &= \sum_g \hat{x}_{gi}\left\{\frac{\partial y_g}{\partial p_g}\left(\hat{\rho}_{gg}\hat{x}_{gi} + \hat{\rho}_{gg'}\hat{x}_{g'i}\right) + \frac{\partial y_g}{\partial p_{g'}}\left(\hat{\rho}_{g'g}\hat{x}_{gi} + \hat{\rho}_{g'g'}\hat{x}_{g'i}\right)\right\} \\
&= \frac{1}{2}\sum_g \hat{x}_{gi}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right), \\
\frac{d\hat{z}_{i'}}{dw_i} &= \sum_g \hat{x}_{gi'}\left\{\frac{\partial y_g}{\partial p_g}\left(\hat{\rho}_{gg}\hat{x}_{gi} + \hat{\rho}_{gg'}\hat{x}_{g'i}\right) + \frac{\partial y_g}{\partial p_{g'}}\left(\hat{\rho}_{g'g}\hat{x}_{gi} + \hat{\rho}_{g'g'}\hat{x}_{g'i}\right)\right\} \\
&= \frac{1}{2}\sum_g \hat{x}_{gi'}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right), \\
\hat{D}_{ii'} &:= -\frac{d\hat{z}_{i'}/dw_i}{d\hat{z}_i/dw_i} = -\frac{\sum_g \hat{x}_{gi'}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right)}{\sum_g \hat{x}_{gi}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi} - D_{gg'}\hat{x}_{g'i}\right)}.
\end{aligned}$$

This verifies (27), and (26) follows by substituting $\hat{D}_{ii'}$ for $D_{gg'}$ in (24). Finally, (28) follows by noting that the left side of the inequality equals

$$\begin{pmatrix}\hat{x}_{gi} & \hat{x}_{g'i}\end{pmatrix}\left[-\frac{\partial y}{\partial p}\right]\begin{pmatrix}\hat{x}_{gi} \\ \hat{x}_{g'i}\end{pmatrix},$$

where the matrix $-\partial y/\partial p$ is positive definite since we assume $D_{gg'}, D_{g'g} < 1$. ■

Lemma A.2. *In the simple model of Section 3.1,*

$$\mathrm{IPP}_i \propto \sum_g x_{gi}\left\{y_g\left(\hat{p}^*(w)\right)-y_g\left(p^*(w)\right)+\left(w_i^*-c_i\right)\left(\frac{dy_g\left(\hat{p}^*(w)\right)}{dw_i}-\frac{dy_g\left(p^*(w)\right)}{dw_i}\right)\right\} \tag{29}$$

and

$$\begin{aligned}(1-E_i)\,\mathrm{IPP}_i =& \frac{\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left\{c_g(w^*)-\hat{c}_g(w^*)-D_{gg'}\left(p^*_{g'}-\hat{c}_{g'}(w^*)\right)\right\}}{\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left(x_{gi}-D_{gg'}x_{g'i}\right)} \\ &+\left(w_i^*-c_i\right)\frac{\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left\{x_{gi}\left(E_i-\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)-D_{gg'}x_{g'i}\left(E_i-\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\right\}}{\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left(x_{gi}-D_{gg'}x_{g'i}\right)}.\end{aligned} \tag{30}$$

Proof of Lemma A.2. To derive relations (29, 30), note that the pre- and post-merger FOC functions for input i are

$$\begin{aligned} f_i(w,c_i) &= \sum_g x_{gi}\left\{y_g\left(p^*(w)\right)+(w_i-c_i)\frac{dy_g\left(p^*(w)\right)}{dw_i}\right\}, \\ \hat{f}_i(w,c_i) &= \sum_g \hat{x}_{gi}\left\{y_g\left(\hat{p}^*(w)\right)+(w_i-c_i)\frac{dy_g\left(\hat{p}^*(w)\right)}{dw_i}\right\}. \end{aligned}$$

IPP_i is then

$$\begin{aligned}\mathrm{IPP}_i &= -\left(\sum_g \hat{x}_{gi}\frac{dy_g\left(\hat{p}^*(w)\right)}{dw_i}\right)^{-1}\left\{\hat{f}_i(w^*,c_i)-f_i(w^*,c_i)\right\} \\ &= -\left(\sum_g \hat{x}_{gi}\frac{dy_g\left(\hat{p}^*(w)\right)}{dw_i}\right)^{-1}\left\{\hat{f}_i(w^*,c_i)-(1-E_i)f_i(w^*,c_i)\right\} \\ &= -\left(\frac{1}{2}\sum_g \hat{x}_{gi}\frac{\partial y_g}{\partial p_g}\left(\hat{x}_{gi}-D_{gg'}\hat{x}_{g'i}\right)\right)^{-1} \\ &\quad\times(1-E_i)\sum_g x_{gi}\left\{y_g\left(\hat{p}^*(w)\right)-y_g\left(p^*(w)\right)+\left(w_i^*-c_i\right)\left(\frac{dy_g\left(\hat{p}^*(w)\right)}{dw_i}-\frac{dy_g\left(p^*(w)\right)}{dw_i}\right)\right\}\end{aligned} \tag{31}$$

The second line makes use of the pre-merger FOC for i. This calculation verifies (29).

To derive the exact IPP_i formula (30), we observe that linear consumer demand, constant returns to scale everywhere, and linear marginal costs downstream jointly imply that the FOC functions f and $\hat{f}$ are linear, so the approximation formulas of Proposition 2 are exact. Beginning with the first term in brackets in (31), we can write

$$y_g\left(\hat{p}^*\left(w^*\right)\right)-y_g\left(p^*\left(w^*\right)\right)=\frac{\partial y}{\partial p}\left(\hat{p}^*\left(w^*\right)-p^*\left(w^*\right)\right).$$

To calculate the change in goods prices at fixed input prices, we can use the first term in approximation (11). The pre- and post-merger FOC functions for goods prices are

$$\begin{aligned}
f_{\mathcal{G}}\left(p,c_{\mathcal{G}}\right)&=y\left(p\right)+\operatorname{diag}\left[\frac{\partial y}{\partial p}\right]\left(p-c_{\mathcal{G}}\right),\\
\hat{f}_{\mathcal{G}}\left(p,\hat{c}_{\mathcal{G}}\right)&=y\left(p\right)+\left[\frac{\partial y}{\partial p}\right]^{\top}\left(p-\hat{c}_{\mathcal{G}}\right).
\end{aligned}$$

Making use of Slutsky symmetry, we can then calculate

$$\begin{aligned}
\frac{\partial y}{\partial p}\left(\hat{p}^*\left(w^*\right)-p^*\left(w^*\right)\right)&=-\frac{\partial y}{\partial p}\left[\frac{d\hat{f}_{\mathcal{G}}}{dp}\right]^{-1}\left\{\hat{f}_{\mathcal{G}}\left(p^*,\hat{c}_{\mathcal{G}}\left(w^*\right)\right)-f_{\mathcal{G}}\left(p^*,c_{\mathcal{G}}\left(w^*\right)\right)\right\}\\
&=-\frac{1}{2}\left\{\left(\frac{\partial y}{\partial p}-\operatorname{diag}\left[\frac{\partial y}{\partial p}\right]\right)\left(p^*-c_{\mathcal{G}}\left(w^*\right)\right)+\frac{\partial y}{\partial p}\left(c_{\mathcal{G}}\left(w^*\right)-\hat{c}_{\mathcal{G}}\left(w^*\right)\right)\right\}\\
&=-\frac{1}{2}\operatorname{diag}\left[\frac{\partial y}{\partial p}\right]\begin{pmatrix}c_g\left(w^*\right)-\hat{c}_g\left(w^*\right)-D_{gg'}\left(p^*_{g'}-\hat{c}_{g'}\left(w^*\right)\right)\\ c_{g'}\left(w^*\right)-\hat{c}_{g'}\left(w^*\right)-D_{g'g}\left(p^*_{g}-\hat{c}_{g}\left(w^*\right)\right)\end{pmatrix}.
\end{aligned}$$

This equation expresses the key trade-off of Williamson (1968): increased market power (internalization of diverted sales) tends to lower output, while cost efficiencies tend to raise output.

Considering now the second term of (31), we can write

$$\frac{dy\left(\hat{p}^*\left(w^*\right)\right)}{dw}-\frac{dy\left(p^*\left(w^*\right)\right)}{dw}=\frac{\partial y}{\partial p}\left(\hat{\rho}_{\mathcal{G}}\frac{\partial\hat{c}_{\mathcal{G}}}{\partial w}-\rho_{\mathcal{G}}\frac{\partial c_{\mathcal{G}}}{\partial w}\right).$$

Using the pass-through formulas (24, 25), the right side can be written

$$\frac{\partial y}{\partial p}\left(\hat{\rho}_{\mathcal{G}}\frac{\partial\hat{c}_{\mathcal{G}}}{\partial w}-\rho_{\mathcal{G}}\frac{\partial c_{\mathcal{G}}}{\partial w}\right)=\frac{1}{2}\frac{\partial y}{\partial p}\begin{bmatrix}\hat{x}_{gi}-\frac{x_{gi}+\frac{D_{gg'}}{2}x_{g'i}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}} & \hat{x}_{gi'}-\frac{x_{gi'}+\frac{D_{gg'}}{2}x_{g'i'}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\\ \hat{x}_{g'i}-\frac{\frac{D_{g'g}}{2}x_{gi}+x_{g'i}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}} & \hat{x}_{g'i'}-\frac{\frac{D_{g'g}}{2}x_{gi'}+x_{g'i'}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\end{bmatrix}.$$

Direct calculation yields

$$\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}-\frac{dy_g\left(p^*\left(w^*\right)\right)}{dw_i}$$
$$=-\frac{1}{2}\frac{\partial y_g}{\partial p_g}\left\{x_{gi}\left(\frac{x_{gi}-\hat{x}_{gi}}{x_{gi}}-\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)-D_{gg'}x_{g'i}\left(\frac{x_{g'i}-\hat{x}_{g'i}}{x_{g'i}}-\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\right\}$$

We can substitute into (31) to verify (30):

$$\left(1-E_i\right)\left(\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left(x_{gi}-D_{gg'}x_{g'i}\right)\right)\mathrm{IPP}_i$$
$$=\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left\{c_g\left(w^*\right)-\hat{c}_g\left(w^*\right)-D_{gg'}\left(p^*_{g'}-\hat{c}_{g'}\left(w^*\right)\right)\right\}$$
$$+\left(w^*_i-c_i\right)\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left\{x_{gi}\left(E_i-\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)-D_{gg'}x_{g'i}\left(E_i-\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\right\}.$$

■

A.4 Section 3

Proof of Proposition 3. When the merger yields Werden efficiencies, the first term on the right side of (30) equals zero. Lemma A.2 then implies

$$\frac{1-E_i}{w^*_i-c_i}\mathrm{IPP}_i=\frac{\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left\{x_{gi}\left(E_i-\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)-D_{gg'}x_{g'i}\left(E_i-\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\right)\right\}}{\sum_g x_{gi}\left(-\frac{\partial y_g}{\partial p_g}\right)\left(x_{gi}-D_{gg'}x_{g'i}\right)}.$$

The expression on the right is increasing in E_i, and Lemma A.2 implies that the denominator is strictly positive. With $D_{gg'}, D_{g'g}\in[0,1]$ we have

$$\frac{1}{2}\frac{1-D_{gg'}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}\geq\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}.$$

Setting

$$E_i=\frac{\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}}{1-\frac{D_{gg'}}{2}\frac{D_{g'g}}{2}},$$

we then find $\mathrm{IPP}_i \geq 0$, where the inequality is strict if and only if $D_{gg'}D_{g'g} \in (0,1)$ and $x_{gi}, x_{g'i} > 0$. ■

Proof of Result 1. Following the proof of Proposition 3 and using the assumptions $x_{gi} = x_{g'i}$ and $D_{gg'} = D_{g'g} = D$, we can calculate

$$\bar{E}_i = \frac{\sum_g x_{gi}^2 \left(-\frac{\partial y_g}{\partial p_g}\right)\left(\frac{\frac{D^2}{4}}{1-\frac{D^2}{4}} - \frac{D}{2}\frac{1-\frac{D^2}{2}}{1-\frac{D^2}{4}}\right)}{\sum_g x_{gi}^2 \left(-\frac{\partial y_g}{\partial p_g}\right)(1-D)} = -\frac{D}{2}\frac{1-\frac{D}{2}-\frac{D^2}{2}}{(1-D)\left(1-\frac{D^2}{4}\right)} = -\frac{\frac{D}{2}}{1-\frac{D}{2}}.$$

■

Proof of Proposition 4.

(i) Equation (19) yields the bound

$$E_g^{\mathrm{W}} \geq \frac{D_{gg'}D_{g'g}}{1-D_{gg'}D_{g'g}}\mu_g^*.$$

Comparing this lower bound to the right side of inequality (17),

$$\frac{D_{gg'}D_{g'g}}{1-D_{gg'}D_{g'g}}\mu_g^* \geq \frac{D_{gg'}D_{g'g}}{4-D_{gg'}D_{g'g}} \iff \mu_g^* \geq \frac{1-D_{gg'}D_{g'g}}{4-D_{gg'}D_{g'g}}.$$

The result follows from Proposition 3, noting that the right side of the final inequality is bounded above by 0.25 since $D_{gg'}, D_{g'g} \in [0,1)$.

(ii) The symmetry assumptions imply $p_g^* = p_{g'}^*$ and $D_{gg'} = D_{g'g} =: D$, so equation (19) becomes

$$E_g^{\mathrm{W}} = \frac{D}{1-D}\mu_g^*.$$

Comparing this value to the right side of inequality (17),

$$\frac{D}{1-D}\mu_g^* \geq \frac{D^2}{4-D^2} \iff \mu_g^* \geq \frac{D(1-D)}{4-D^2}.$$

The result follows from Proposition 3, noting that the right side of the final inequality is bounded above by $\frac{2-\sqrt{3}}{4} \approx 0.06699$. ■

Proof of Proposition 5.

(i) Approximation (12) is exact in the simple model, so the post-merger change in input prices is

$$\hat{w}^* - w^* = \hat{\rho}_{\mathcal{I}}\text{IPP} = \frac{\frac{1}{2}}{1 - \frac{\hat{D}_{ii'}}{2}\frac{\hat{D}_{i'i}}{2}} \begin{pmatrix} \text{IPP}_i + \frac{\hat{D}_{ii'}}{2}\text{IPP}_{i'} \\ \frac{\hat{D}_{i'i}}{2}\text{IPP}_i + \text{IPP}_{i'} \end{pmatrix}.$$

The second equality follows from Lemma A.1. We then have

$$(1 - E_i)\left(\hat{w}_i^* - w_i^*\right) = \frac{\frac{1}{2}}{1 - \frac{\hat{D}_{ii'}}{2}\frac{\hat{D}_{i'i}}{2}} \left\{(1 - E_i)\,\text{IPP}_i + \frac{\hat{D}_{ii'}}{2}(1 - E_i)\,\text{IPP}_{i'}\right\}. \tag{32}$$

First note that the factor before the braces is strictly positive: Again by Lemma A.1,

$$\hat{D}_{ii'}\hat{D}_{i'i} = \frac{d\hat{z}_{i'}/dw_i}{d\hat{z}_i/dw_i}\frac{d\hat{z}_i/dw_{i'}}{d\hat{z}_{i'}/dw_{i'}} = \frac{(d\hat{z}_{i'}/dw_i)^2}{(d\hat{z}_i/dw_i)(d\hat{z}_{i'}/dw_{i'})}.$$

The second equality holds because the post-merger input demand system $\hat{z}$ is Slutsky symmetric. Finally, letting $\langle\cdot,\cdot\rangle$ denote the inner product on $\mathbb{R}^2$ induced by the positive definite matrix $-\partial y/\partial p$, the Cauchy-Schwarz inequality implies

$$\begin{aligned} \left(\frac{d\hat{z}_{i'}}{dw_i}\right)^2 &= \left\langle \begin{pmatrix} \hat{x}_{gi} \\ \hat{x}_{g'i} \end{pmatrix}, \begin{pmatrix} \hat{x}_{gi'} \\ \hat{x}_{g'i'} \end{pmatrix} \right\rangle^2 \\ &\leq \left\langle \begin{pmatrix} \hat{x}_{gi} \\ \hat{x}_{g'i} \end{pmatrix}, \begin{pmatrix} \hat{x}_{gi} \\ \hat{x}_{g'i} \end{pmatrix} \right\rangle \left\langle \begin{pmatrix} \hat{x}_{gi'} \\ \hat{x}_{g'i'} \end{pmatrix}, \begin{pmatrix} \hat{x}_{gi'} \\ \hat{x}_{g'i'} \end{pmatrix} \right\rangle \\ &= \frac{d\hat{z}_i}{dw_i}\frac{d\hat{z}_{i'}}{dw_{i'}}. \end{aligned}$$

We then find $\hat{D}_{ii'}\hat{D}_{i'i} \leq 1$, so that the factor before the braces in (32) is strictly positive. Differentiating (32) and noting that $\hat{D}_{ii'}\hat{D}_{i'i}$ does not depend on $(E_i, E_{i'})$, we have

$$\frac{\partial\,(1 - E_i)\left(\hat{w}_i^* - w_i^*\right)}{\partial E_i} \propto \frac{\partial\,(1 - E_i)\,\text{IPP}_i}{\partial E_i} + \frac{\partial}{\partial E_i}\left(\frac{\hat{D}_{ii'}}{2}(1 - E_i)\,\text{IPP}_{i'}\right).$$

Using Lemma A.2, we can calculate the first term on the right side:

$$\frac{\partial\,(1 - E_i)\,\text{IPP}_i}{\partial E_i} = 2w_i^* - c_i.$$

Similarly, the second term is

$$\begin{aligned}
&\frac{\partial}{\partial E_i}\left(\frac{\hat{D}_{ii'}}{2}(1-E_i)\,\mathrm{IPP}_{i'}\right)\\
&\quad=-\frac{1}{2}\frac{\sum_g x_{gi'}\frac{\partial y_g}{\partial p_g}\left(x_{gi}-D_{gg'}x_{g'i}\right)}{\sum_g x_{gi}\frac{\partial y_g}{\partial p_g}\left(x_{gi}-D_{gg'}x_{g'i}\right)}\frac{\partial\,(1-E_{i'})\,\mathrm{IPP}_{i'}}{\partial E_i}\\
&\quad=-\frac{w_i^*}{2}\frac{\sum_g x_{gi'}\frac{\partial y_g}{\partial p_g}\left(x_{gi}-D_{gg'}x_{g'i}\right)}{\sum_g x_{gi}\frac{\partial y_g}{\partial p_g}\left(x_{gi}-D_{gg'}x_{g'i}\right)}\frac{\sum_g x_{gi'}\frac{\partial y_g}{\partial p_g}\left(x_{gi}-D_{gg'}x_{g'i}\right)}{\sum_g x_{gi'}\frac{\partial y_g}{\partial p_g}\left(x_{gi'}-D_{gg'}x_{g'i'}\right)}\\
&=-\frac{w_i^*}{2}\hat{D}_{ii'}\hat{D}_{i'i}.
\end{aligned}$$

We then find

$$\frac{\partial\,(1-E_i)\left(\hat{w}_i^*-w_i^*\right)}{\partial E_i}=\frac{\frac{1}{2}}{1-\frac{\hat{D}_{ii'}}{2}\frac{\hat{D}_{i'i}}{2}}\left\{w_i^*-c_i+w_i^*\left(1-\frac{\hat{D}_{ii'}\hat{D}_{i'i}}{2}\right)\right\}>0.$$

The final strict inequality holds because $\hat{D}_{ii'}\hat{D}_{i'i}\leq 1$.

(ii) Using the calculations above,

$$\begin{aligned}
\frac{\partial\,(1-E_i)\left(\hat{w}_i^*-w_i^*\right)}{\partial E_{i'}}&\propto\frac{\partial\,(1-E_i)\,\mathrm{IPP}_i}{\partial E_{i'}}+\frac{\partial}{\partial E_{i'}}\left(\frac{\hat{D}_{ii'}}{2}(1-E_i)\,\mathrm{IPP}_{i'}\right)\\
&=-w_{i'}^*\frac{1-E_i}{1-E_{i'}}\hat{D}_{ii'}+\left(2w_{i'}^*-c_{i'}\right)\frac{1-E_i}{1-E_{i'}}\frac{\hat{D}_{ii'}}{2}\\
&=-\frac{c_{i'}}{2}\frac{1-E_i}{1-E_{i'}}\hat{D}_{ii'}.
\end{aligned}$$

This expression is positive if and only if $\hat{D}_{ii'}<0\iff d\hat{z}_{i'}/dw_i<0$.

(iii) Approximation (11) is exact in the simple model, so together with Lemma (A.1) we have

$$\hat{p}_g^*(\hat{w}^*)-\hat{p}_g^*(w^*)=\frac{1}{2}\left\{x_{gi}(1-E_i)\left(\hat{w}_i^*-w_i^*\right)+x_{gi'}(1-E_{i'})\left(\hat{w}_{i'}^*-w_{i'}^*\right)\right\}.$$

Differentiating with respect to E_i and using the calculations above,

$$\begin{aligned}\frac{\partial\left(\hat{p}_g^*(\hat{w}^*)-\hat{p}_g^*(w^*)\right)}{\partial E_i} &= \frac{1}{2}\left\{x_{gi}\frac{\partial(1-E_i)\left(\hat{w}_i^*-w_i^*\right)}{\partial E_i}+x_{gi'}\frac{\partial(1-E_{i'})\left(\hat{w}_{i'}^*-w_{i'}^*\right)}{\partial E_i}\right\}\\ &\propto x_{gi}\left\{w_i^*-c_i+w_i^*\left(1-\frac{\hat{D}_{ii'}\hat{D}_{i'i}}{2}\right)\right\}-x_{gi'}\frac{1-E_{i'}}{1-E_i}\hat{D}_{i'i}\frac{c_i}{2}.\end{aligned}$$

This expression is positive if and only if $x_{gi} > B_i x_{gi'}$, where

$$B_i := \frac{1}{2}\frac{1-E_{i'}}{1-E_i}\frac{\hat{D}_{i'i}}{\frac{w_i^*}{c_i}\left(2-\frac{\hat{D}_{ii'}\hat{D}_{i'i}}{2}\right)-1}.$$

Clearly $B_i < 0 \iff \hat{D}_{i'i} < 0 \iff d\hat{z}_{i'}/dw_i < 0$. ∎

A.5 Section 4

Proposition A.1. *The FOC functions corresponding to the successive Nash-in-Nash bargaining problems (21, 22) are*

$$f_i(w,c_i) = x_{gi}y_g(p^*(w))+(w_i-c_i)x_{gi}\frac{dy_g(p^*(w))}{dw_i} + \frac{1-\beta_i}{\mathrm{GFT}_g(w)}\frac{\mathrm{GFT}_i(w)}{\beta_i}\left[\left(p_g-c_g(w)\right)\frac{dy_g}{dp_{g'}}\frac{\partial p_{g'}^*(w)}{\partial w_i}-x_{gi}y_g(p^*(w))\right], \tag{32}$$

$$\hat{f}_i(w,c_i) = \hat{x}_{gi}y_g(\hat{p}^*(w))+(w_i-c_i)\hat{x}_{gi}\frac{dy_g(\hat{p}^*(w))}{dw_i} - \frac{1-\beta_i}{\hat{\mathrm{GFT}}_{M,i}(w)}\frac{\hat{\mathrm{GFT}}_i(w)}{\beta_i}\hat{x}_{gi}y_g(\hat{p}^*(w)). \tag{33}$$

IPP_i *then satisfies*

$$\mathrm{IPP}_i \propto \underbrace{y_g\left(\hat{p}^*\left(w^*\right)\right) - y_g\left(p^*\left(w^*\right)\right) + \left(w_i^* - c_i\right)\left(\frac{d y_g\left(\hat{p}^*\left(w^*\right)\right)}{d w_i} - \frac{d y_g\left(p^*\left(w^*\right)\right)}{d w_i}\right)}_{\propto \mathrm{IPP}_i^{\text{price}}} \tag{34}$$
$$+ \underbrace{\frac{1-\beta_i}{\beta_i}\frac{\mathrm{GFT}_i\left(w^*\right)}{\mathrm{GFT}_g\left(w^*\right)}\left[y_g\left(p^*\left(w^*\right)\right) - y_g\left(\hat{p}^*\left(w^*\right)\right) - \left(p_g - c_g\left(w^*\right)\right)\frac{\partial y_g}{\partial p_{g'}}\rho_{g'g}\hat{x}_{gi}\right]}_{\propto \Delta_i^{\text{profit sensitivity}}}$$
$$+ \underbrace{\frac{1-\beta_i}{\beta_i}\left(\frac{\mathrm{GFT}_i\left(w^*\right)}{\mathrm{GFT}_g\left(w^*\right)} - \frac{\hat{\mathrm{GFT}}_i\left(w^*\right)}{\hat{\mathrm{GFT}}_{M,i}\left(w^*\right)}\right) y_g\left(\hat{p}^*\left(w^*\right)\right),}_{\propto \Delta_i^{\text{leverage}}}$$

where the proportionality factor is $\hat{x}_{gi}\left(\frac{\partial \hat{f}_i}{\partial c_i}\right)^{-1}$*, where*

$$\frac{\partial \hat{f}_i}{\partial c_i} = -\hat{x}_{gi}\frac{d y_g\left(\hat{p}^*\left(w^*\right)\right)}{d w_i} + \frac{1-\beta_i}{\hat{\mathrm{GFT}}_{M,i}\left(w^*\right)}\frac{\left(\hat{x}_{gi} y_g\left(\hat{p}^*\left(w^*\right)\right)\right)^2}{\beta_i}. \tag{35}$$

Proof of Proposition A.1. The FOC functions (32, 33) follow by differentiating the successive Nash-in-Nash objectives (21, 22). The IPP_i relation then follows by observing

$$\mathrm{IPP}_i \propto \frac{1}{\hat{x}_{gi}}\hat{f}_i\left(w^*, c_i\right) - \frac{1}{x_{gi}} f_i\left(w^*, c_i\right).$$

■

B Extensions

B.1 Production Technology and Many Input Suppliers

Start with the simple model, and now suppose that there are an arbitrary number $|\mathcal{I}| \geq 1$ of upstream firms. Each upstream firm i produces one input and sets a uniform price w_i that applies to both downstream firms. Downstream production technologies are fully general: The pre-merger cost function for good g is $C_g\left(y_g, w\right)$, and the post-merger cost function for the merged firm is $C_M(y, w)$.[29] We keep all remaining aspects of the simple model unchanged.

To determine when upstream firm i has an incentive to raise its price after the merger, we again consider IPP_i. In the proof of Proposition 3′ below, we show that for a Werden-efficient merger

$$\begin{aligned}
\mathrm{IPP}_i \quad \propto \quad & \underbrace{\sum_{g \in \mathcal{G}} \frac{\partial \log z_i}{\partial y_g}\left(\frac{d y_g\left(\hat{p}^*\left(w^*\right)\right)}{d w_i}-\frac{d y_g\left(p^*\left(w^*\right)\right)}{d w_i}\right)}_{\Delta \text{ sensitivity of output to input price}} \qquad (36)\\
& +\underbrace{\sum_{g \in \mathcal{G}}\left(\frac{\partial \log \hat{z}_i}{\partial y_g}-\frac{\partial \log z_i}{\partial y_g}\right) \frac{d y_g\left(\hat{p}^*\left(w^*\right)\right)}{d w_i}}_{\Delta \text{ elasticity of input demand to output}} \quad + \underbrace{\frac{\partial \log \hat{z}_i}{\partial w_i}-\frac{\partial \log z_i}{\partial w_i}}_{\Delta \text{ elasticity of input demand to input price}},
\end{aligned}$$

where all total demand functions for input i are evaluated at pre-merger outputs $y\left(p^*\right)$ and input prices w^*. This decomposition differs in two key ways from the analogue (16) in the simple model. First, (16) holds for an arbitrary merger while (36) holds for a Werden-efficient merger, so (36) does not have a term reflecting a change in downstream outputs. Second, the last two terms in (36) do not appear in (16), because they account for effects that do not arise under the production technologies of the simple model. The second term in (36) captures any change in the elasticity of total input demand with respect to downstream outputs. It is positive when total input demand becomes less elastic to outputs, implying that firm i observes a smaller decline in input demand for a given increase in price. The third term in (36) captures any change in the elasticity of total input demand to the input price, holding fixed downstream outputs. It is similarly positive when the "partial elasticity" of input demand to input price falls in magnitude. These new terms may be quantitatively important for firm i's pricing incentives after the merger, but they depend exclusively on the change in downstream production technologies, which is exogenous in our model. As a result, for the remainder of this section we restrict our focus to the more familiar first term in (36).

[29] Our results require the weak assumption that the marginal cost for each good g is strictly increasing in the price of each input i that firm g purchases before the merger. This holds, for example, when pre-merger production technologies are homothetic.

Assumption 3. *The partial elasticities of total input demand to outputs y and the input price w_i are unchanged after the merger:*

$$\frac{\partial \log \hat{z}_i}{\partial \log y_g} = \frac{\partial \log z_i}{\partial \log y_g} \quad g \in \mathcal{G} \quad \textit{and} \quad \frac{\partial \log \hat{z}_i}{\partial \log w_i} = \frac{\partial \log z_i}{\partial \log w_i},$$

where all functions are evaluated at pre-merger outputs and input prices $(y(p^*), w^*)$.

To state our generalization of Proposition 3, let

$$E_{gi} := 1 - \left(\frac{\partial}{\partial y_g} \frac{\partial C_M (y(p^*), w^*)}{\partial w_i} \right) \left(\frac{\partial}{\partial y_g} \frac{\partial C_g \left(y_g (p^*), w^* \right)}{\partial w_i} \right)^{-1}$$

denote the *i-specific efficiencies* attained by the merged firm in the production of good g. This quantity is the reduction in the component of marginal cost of good g due to expenditure on input i, naturally extending the concept of input-specific efficiencies from the simple model. We again assume that input-specific efficiencies are symmetric across downstream goods, $E_{gi} = E_{g'i} = E_i$. With this condition and Assumption 3, we obtain the following strict generalization of Proposition 3:

Proposition 3′. *Suppose the merger is Werden-efficient. Then* $\text{IPP}_i > 0$ *if and only if*

$$\text{IPP}_i > 0 \quad \textit{if and only if} \quad E_i > \bar{E}_i, \quad \textit{where} \quad \bar{E}_i \leq \frac{D_{gg'} D_{g'g}}{4 - D_{gg'} D_{g'g}}.$$

The $\bar{E}_i$ bound is attained for independent goods ($D_{gg'} = D_{g'g} = 0$) or specialized input suppliers.

Proof of Proposition 3′. We denote the pre- and post-merger input demand functions by

$$x_{gi} \left(y_g, w \right) := \frac{\partial C_g \left(y_g, w \right)}{\partial w_i} \quad \text{and} \quad \hat{x}_i (y, w) := \frac{\partial C_M (y, w)}{\partial w_i}.$$

The pre- and post-merger FOC functions for input i are

$$\begin{aligned} f_i(w, c_i) &= \sum_{g \in \mathcal{G}} x_{gi} \left(y_g (p^*(w)), w \right) \\ &\quad + (w_i - c_i) \sum_{g \in \mathcal{G}} \left(\frac{\partial x_{gi} \left(y_g (p^*(w)), w \right)}{\partial y_g} \frac{d y_g (p^*(w))}{d w_i} + \frac{\partial x_{gi} \left(y_g (p^*(w)), w \right)}{\partial w_i} \right), \\ \hat{f}_i(w, c_i) &= \hat{x}_i (y(\hat{p}^*(w)), w) \\ &\quad + (w_i - c_i) \left(\sum_{g \in \mathcal{G}} \frac{\partial \hat{x}_i (y(\hat{p}^*(w)), w)}{\partial y_g} \frac{d y_g (\hat{p}^*(w))}{d w_i} + \frac{\partial \hat{x}_i (y(\hat{p}^*(w)), w)}{\partial w_i} \right). \end{aligned}$$

Where there is no risk of confusion, we suppress the arguments of all functions on the right sides of these equations. Define the reduction in total demand for input i after the merger, holding fixed pre-merger outputs and input prices:

$$1-\chi_i := \frac{\hat{x}_i\left(y\left(p^*\right),w^*\right)}{\sum_{g\in\mathcal{G}} x_{gi}\left(y_g\left(p^*\right),w^*\right)}.$$

Making use of the pre-merger FOC for i, IPP_i then satisfies

$$\begin{aligned}
\mathrm{IPP}_i &\propto \hat{f}_i\left(w^*,c_i\right)-\left(1-\chi_i\right)f_i\left(w^*,c_i\right)\\
&= \hat{x}_i-\left(1-\chi_i\right)\sum_{g\in\mathcal{G}} x_{gi}\\
&\quad+\left(w_i^*-c_i\right)\sum_{g\in\mathcal{G}}\left(1-\chi_i\right)\frac{\partial x_{gi}}{\partial y_g}\left(\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}-\frac{dy_g\left(p^*\left(w^*\right)\right)}{dw_i}\right)\\
&\quad+\left(w_i^*-c_i\right)\sum_{g\in\mathcal{G}}\left(\frac{\partial \hat{x}_i}{\partial y_g}-\left(1-\chi_i\right)\frac{\partial x_{gi}}{\partial y_g}\right)\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}\\
&\quad+\left(w_i^*-c_i\right)\left(\frac{\partial \hat{x}_i}{\partial w_i}-\left(1-\chi_i\right)\sum_{g\in\mathcal{G}}\frac{\partial x_{gi}}{\partial w_i}\right).
\end{aligned}$$

Since the merger is Werden-efficient, the first line equals zero, and this relation becomes

$$\begin{aligned}
\mathrm{IPP}_i \propto &\sum_{g\in\mathcal{G}}\left(1-\chi_i\right)\frac{\partial x_{gi}}{\partial y_g}\left(\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}-\frac{dy_g\left(p^*\left(w^*\right)\right)}{dw_i}\right)\\
&+\sum_{g\in\mathcal{G}}\left(\frac{\partial \hat{x}_i}{\partial y_g}-\left(1-\chi_i\right)\frac{\partial x_{gi}}{\partial y_g}\right)\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}+\frac{\partial \hat{x}_i}{\partial w_i}-\left(1-\chi_i\right)\sum_{g\in\mathcal{G}}\frac{\partial x_{gi}}{\partial w_i}.
\end{aligned}$$

Dividing through by $\hat{x}_i$ yields

$$\begin{aligned}
\mathrm{IPP}_i \propto &\sum_{g\in\mathcal{G}}\frac{1}{\sum_{g\in\mathcal{G}} x_{gi}}\frac{\partial x_{gi}}{\partial y_g}\left(\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}-\frac{dy_g\left(p^*\left(w^*\right)\right)}{dw_i}\right)\\
&+\sum_{g\in\mathcal{G}}\left(\frac{1}{\hat{x}_i}\frac{\partial \hat{x}_i}{\partial y_g}-\frac{1}{\sum_{g\in\mathcal{G}} x_{gi}}\frac{\partial x_{gi}}{\partial y_g}\right)\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}+\frac{1}{\hat{x}_i}\frac{\partial \hat{x}_i}{\partial w_i}-\frac{1}{\sum_{g\in\mathcal{G}} x_{gi}}\sum_{g\in\mathcal{G}}\frac{\partial x_{gi}}{\partial w_i}.
\end{aligned}$$

Under the assumptions of the proposition, the second line is precisely zero, so we have

$$\mathrm{IPP}_i \propto \sum_{g\in\mathcal{G}}\frac{\partial x_{gi}}{\partial y_g}\left(\frac{dy_g\left(\hat{p}^*\left(w^*\right)\right)}{dw_i}-\frac{dy_g\left(p^*\left(w^*\right)\right)}{dw_i}\right).$$

The remainder of the proof follows exactly the derivation of (30) in Lemma A.2 and the subsequent proof of Proposition 3. (Exchange the notation x_{gi} and $\hat{x}_{gi}$ in those proofs for $\frac{\partial c_g}{\partial w_i} = \frac{\partial x_{gi}}{\partial y_g}$ and $\frac{\partial \hat{c}_g}{\partial w_i}$ here, respectively.) ■

This result demonstrates that Proposition 3 holds for general downstream production technologies and input-output structures. This immediately implies that the sufficient conditions of Proposition 4 also apply to the more general model of this section, provided the assumptions of Proposition 3′ are satisfied.

B.2 Downstream "Outsiders"

The analysis of Section 3 has assumed that initially only two firms compete in the market for consumer goods. Though consistent with other studies of unilateral pricing incentives after mergers (e.g., Farrell and Shapiro, 2010; Schmalensee, 2009), this assumption likely yields biased estimates of merger price effects when the merging firms face competition from non-merging rivals, or *outsiders*, that produce substitutable products. Most relevant for our results, under Bertrand-Nash conduct outsiders' prices are strategic complements to the merging firms' prices, so outsiders generally increase their prices after an increase in the merging firms' marginal costs. This response (i) raises pre- and post-merger pass-through rates and (ii) creates an additional channel through which downstream output y_g is affected by the input price w_i. The net effect on Input Pricing Pressure is not obvious. Below we give a result indicating that this effect is likely small and often strengthens our main conclusions.

To incorporate an outsider into the simple model, we now suppose that there is a third good $o \in \mathcal{G}$ owned by a non-merging firm. For simplicity, we assume that this good is produced at constant marginal cost c_o using only the numeraire. The outsider o competes à la Bertrand-Nash with firms g and g' before the merger and with firm $M = \{g, g'\}$ after the merger. For tractability, we also assume specialized input suppliers, so that input i is used to produce only good g. We keep all remaining aspects of the simple model unchanged.

To see how Proposition 3 extends, suppose the merger between firms g and g' is Werden-efficient. Then IPP_i is again proportional to the change in the sensitivity of output y_g to the input price w_i, which we can decompose as follows:

$$\frac{d y_g(\hat{p}^*(w^*))}{dw_i} - \frac{d y_g(p^*(w^*))}{dw_i} = \frac{\partial y_g}{\partial p_g}\left(\hat{\rho}_{gg}(1-E_i) - \rho_{gg}\right)x_g + \frac{\partial y_g}{\partial p_{g'}}\left(\hat{\rho}_{g'g}(1-E_i) - \rho_{g'g}\right)x_g$$
$$+ \frac{\partial y_g}{\partial p_o}\left(\hat{\rho}_{og}(1-E_i) - \rho_{og}\right)x_g. \tag{37}$$

As discussed above, this decomposition differs from the analogue (18) in the simple model because the pass-through rates in the first two terms are generally higher, and a new third term captures the direct change in downstream output because of an adjustment in the outsider's price p_o. Input-specific efficiencies E_i again enter linearly, so there exists a threshold $\bar{E}_i^{\text{out}}$ such that $\text{IPP}_i > 0$ if and only if $E_i > \bar{E}_i^{\text{out}}$. This threshold is a complicated nonlinear function of all diversion ratios, so we simplify by assuming that all diversion ratios are symmetric. Proposition 3 then extends as follows:

Proposition 3″. *Suppose the diversion ratios between all pairs of goods are equal to $D \in [0, 0.5]$. If the merger is Werden-efficient, then*

$$\text{IPP}_i > 0 \quad \textit{if and only if} \quad E_i > \bar{E}_i^{\text{out}}, \quad \textit{where} \quad \bar{E}_i^{\text{out}} \leq \frac{D^2}{4 - D^2}.$$

Proof of Proposition 3″. With Werden efficiencies, the same calculation as in the proof of Lemma A.2 yields

$$\text{IPP}_i \propto \frac{d y_g\left(\hat{p}^*\left(w^*\right)\right)}{d w_i} - \frac{d y_g\left(p^*\left(w^*\right)\right)}{d w_i}.$$

The output sensitivity decomposition (37) then implies

$$\text{IPP}_i > 0 \iff 1 - E_i \leq \frac{\rho_{gg} - D_{gg'}\rho_{g'g} - D_{go}\rho_{og}}{\hat{\rho}_{gg} - D_{gg'}\hat{\rho}_{g'g} - D_{go}\hat{\rho}_{og}}.$$

The threshold efficiencies $\bar{E}_i^{\text{out}}$ are such that this inequality binds. To calculate $\bar{E}_i^{\text{out}}$, we must then determine the downstream pass-through rates $\rho_{\mathcal{G}}$ and $\hat{\rho}_{\mathcal{G}}$. To do this, note that the pre- and post-merger downstream FOC functions are

$$\begin{aligned}
f_g\left(p, c_g\right) &= y_g(p) + \frac{\partial y_g}{\partial p_g}\left(p_g - c_g\right) && g \in \mathcal{G} \setminus \{o\}, \\
f_o\left(p, c_o\right) &= y_o(p) + \frac{\partial y_o}{\partial p_o}\left(p_o - c_o\right), && \\
\hat{f}_g\left(p, c_g\right) &= y_g(p) + \frac{\partial y_g}{\partial p_g}\left(p_g - c_g\right) + \frac{\partial y_{g'}}{\partial p_g}\left(p_{g'} - c_{g'}\right) && g \in \mathcal{G} \setminus \{o\}, \\
\hat{f}_o\left(p, c_o\right) &= f_o\left(p, c_o\right). &&
\end{aligned}$$

Using the definition (8), the downstream pass-through rates satisfy

$$\rho_{\mathcal{G}} = \frac{1}{2}(\mathbf{I} - B)^{-1} \quad \text{and} \quad \hat{\rho}_{\mathcal{G}} = \frac{1}{2}\left(\mathbf{I} - \hat{B}\right)^{-1}, \quad \text{where} \quad B = \begin{bmatrix} 0 & \frac{D}{2} & \frac{D}{2} \\ \frac{D}{2} & 0 & \frac{D}{2} \\ \frac{D}{2} & \frac{D}{2} & 0 \end{bmatrix} \quad \text{and} \quad \hat{B} = \begin{bmatrix} 0 & D & \frac{D}{2} \\ D & 0 & \frac{D}{2} \\ \frac{D}{2} & \frac{D}{2} & 0 \end{bmatrix}.$$

Inverting yields

$$\rho_{\mathcal{G}} = \frac{\frac{1}{2}}{(1-D)\left(1+\frac{D}{2}\right)} \begin{bmatrix} 1-\frac{D}{2} & \frac{D}{2} & \frac{D}{2} \\ \frac{D}{2} & 1-\frac{D}{2} & \frac{D}{2} \\ \frac{D}{2} & \frac{D}{2} & 1-\frac{D}{2} \end{bmatrix},$$

$$\hat{\rho}_{\mathcal{G}} = \frac{\frac{1}{2}}{\left(1-D-\frac{D^2}{2}\right)(1+D)} \begin{bmatrix} 1-\frac{D^2}{4} & D\left(1+\frac{D}{4}\right) & \frac{D}{2}(1+D) \\ D\left(1+\frac{D}{4}\right) & 1-\frac{D^2}{4} & \frac{D}{2}(1+D) \\ \frac{D}{2}(1+D) & \frac{D}{2}(1+D) & 1-D^2 \end{bmatrix}.$$

We can substitute these expressions into the definition of the threshold $\bar{E}_i^{\text{out}}$ to find

$$\begin{aligned} 1-\bar{E}_i^{\text{out}} &= \frac{\left(1-D-\frac{D^2}{2}\right)(1+D)}{(1-D)\left(1+\frac{D}{2}\right)} \frac{1-\frac{D}{2}-D^2}{1-\frac{D^2}{4}-D^2\left(1+\frac{D}{4}\right)-\frac{D^2}{2}(1+D)} \\ &= \frac{1-D-\frac{D^2}{2}}{(1-D)\left(1+\frac{D}{2}\right)} \frac{1-\frac{D}{2}-D^2}{\left(1-\frac{3}{2}D\right)\left(1+\frac{D}{2}\right)}. \end{aligned}$$

Then

$$\bar{E}_i^{\text{out}} \leq \frac{D^2}{4-D^2} \iff \frac{1-D-\frac{D^2}{2}}{1-D} \frac{1-\frac{D}{2}-D^2}{1-\frac{3}{2}D} \geq \frac{1-\frac{D^2}{2}}{1-\frac{D}{2}}.$$

It is easy to verify that this inequality holds for $D \in [0, 0.5]$, and strictly for $D \neq 0$. ■

This result demonstrates that Proposition 3 continues to hold in the presence of an outsider when the diversion ratios are equal, and moreover that the efficiencies threshold $\bar{E}_i^{\text{out}}$ is weakly smaller than in the simple model. As a corollary, we find that the sufficient conditions of Proposition 4 also apply with an outsider under equal diversion ratios. Our numerical results in Appendix C show that Proposition 3″ continues to hold when the diversion ratio between the outsider's good and the merging firms' goods is not equal to the diversion ratio between the merging firms' goods. In fact, the threshold $\bar{E}_{gi}^{\text{out}}$ becomes *smaller* as the diversion ratio between the outsider's good and the merging firms' goods increases.

B.3 Nonlinear Demand

Departing from the simple model only in allowing nonlinear consumer demand, we again find that an upstream firm i has an incentive to raise its price after a Werden-efficient merger when the merged firm attains sufficient i-specific efficiencies:

Proposition 3‴. *Suppose the merger is Werden-efficient. Then there exists a threshold $\bar{E}_i^{\mathrm{NL}}$ such that* $\mathrm{IPP}_i > 0$ *if and only if* $E_i > \bar{E}_i^{\mathrm{NL}}$.

Proof of Proposition 3‴. By the same calculation as in the proof of Lemma A.2,

$$\begin{aligned}
\mathrm{IPP}_i &\propto \sum_g x_{gi}\left\{\frac{dy_g(\hat{p}^*(w^*))}{dw_i} - \frac{dy_g(p^*(w^*))}{dw_i}\right\} \\
&= \sum_g x_{gi}\left\{\left(\frac{\partial y_g}{\partial p_g}\hat{\rho}_{gg} + \frac{\partial y_g}{\partial p_{g'}}\hat{\rho}_{g'g}\right)(1-E_i)\,x_{gi} + \left(\frac{\partial y_g}{\partial p_g}\hat{\rho}_{gg'} + \frac{\partial y_g}{\partial p_{g'}}\hat{\rho}_{g'g'}\right)(1-E_i)\,x_{g'i}\right\} \\
&\quad - \sum_g x_{gi}\left\{\left(\frac{\partial y_g}{\partial p_g}\rho_{gg} + \frac{\partial y_g}{\partial p_{g'}}\rho_{g'g}\right)x_{gi} + \left(\frac{\partial y_g}{\partial p_g}\rho_{gg'} + \frac{\partial y_g}{\partial p_{g'}}\rho_{g'g'}\right)x_{g'i}\right\}.
\end{aligned}$$

where all demand derivatives and pass-through rate functions are evaluated at prices (p^*, w^*). Hence $\mathrm{IPP}_i > 0$ if and only if $E_i > \bar{E}_i^{\mathrm{NL}}$, where

$$\bar{E}_i^{\mathrm{NL}} = 1 - \frac{\sum_g x_{gi}\frac{\partial y_g}{\partial p_g}\left\{\left(\rho_{gg} - D_{gg'}\rho_{g'g}\right)x_{gi} + \left(\rho_{gg'} - D_{gg'}\rho_{g'g'}\right)x_{g'i}\right\}}{\sum_g x_{gi}\frac{\partial y_g}{\partial p_g}\left\{\left(\hat{\rho}_{gg} - D_{gg'}\hat{\rho}_{g'g}\right)x_{gi} + \left(\hat{\rho}_{gg'} - D_{gg'}\hat{\rho}_{g'g'}\right)x_{g'i}\right\}}.$$

■

C Simulations

C.1 Price Changes in the Simple Model

In this section, we compute post-merger changes in goods and input prices in the simple model of Section 3.1. We assume specialized input suppliers, unit input requirements $x_{gi} = x_{g'i'} = 1$ and $v_g = v_{g'} = 0$, and upstream marginal costs $c_i = c_{i'} = 1$. We also assume a symmetric linear demand system

$$y_g(p) = V - p_g + D p_{g'} \quad g \in \mathcal{G}.$$

We calibrate (V, D) to match the specified diversion ratios and pre-merger markups.

Figures 5, 6, 7, 8 show the percent change in the goods and input prices after the downstream merger. Figures 5 and 6 show these price changes for a merger with Hicks-neutral Werden efficiencies. We see that goods and input prices increase for all diversions and pre-merger markups displayed. Figures 7 and 8 show these price changes for a merger with zero efficiencies. Input prices decline for all diversion ratios and pre-merger markups displayed, but goods prices always increase.

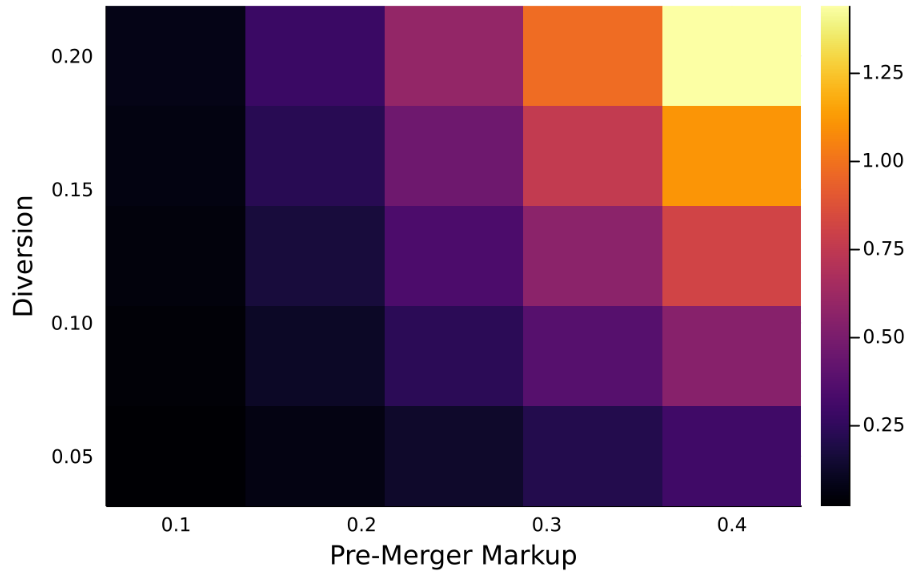

Figure 5: Percent change in consumer prices with endogenous input prices at Werden efficiencies.

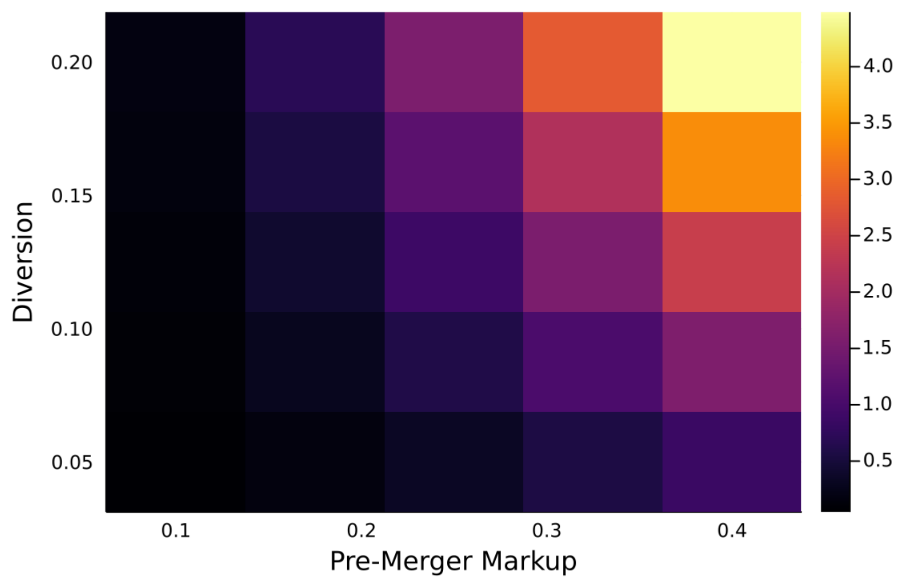

Figure 6: Percent change in input prices with endogenous input prices at Werden efficiencies.

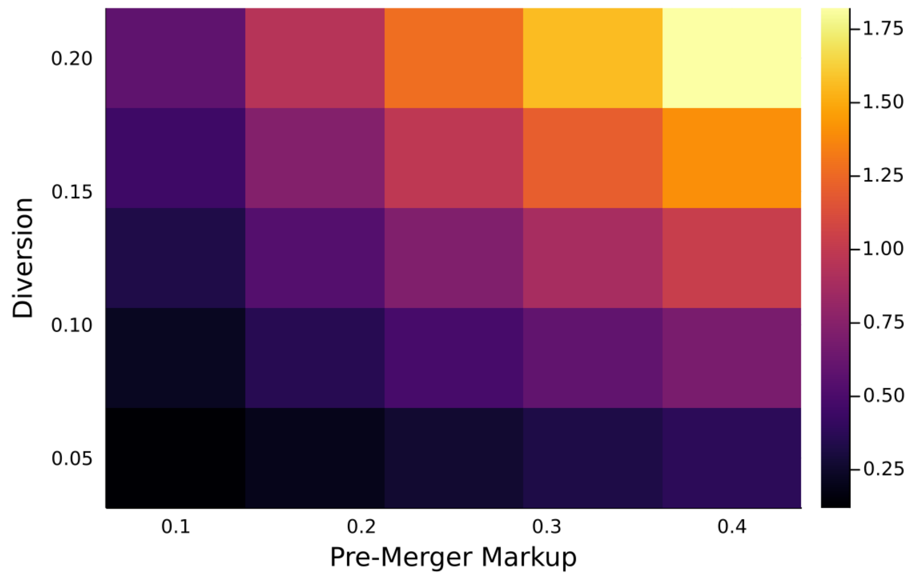

Figure 7: Percent change in consumer prices with endogenous input prices at zero efficiencies.

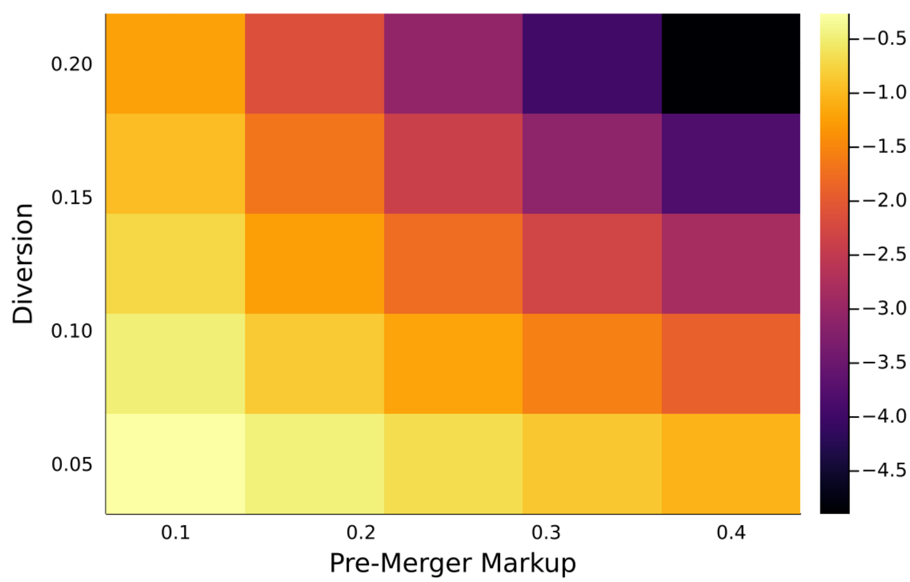

Figure 8: Percent change in input prices with endogenous input prices at zero efficiencies.

C.2 Outsiders

Here we explore how Proposition 3′′ extends when the symmetric diversion ratio between the merging firms' goods $D_M := D_{gg'} = D_{g'g}$ is different from the symmetric diversion ratio between the outsider's good and the merging firms' goods $D_o := D_{go} = D_{og} = D_{g'o} = D_{og'}$. Figure 9 plots $\bar{E}_i^{\text{out}} - \frac{D_M^2}{4-D_M^2}$ as a function of D_M and D_o. This figure shows that the threshold $\bar{E}_{gi}^{\text{out}}$ becomes *smaller* as the diversion ratio between the outsider's good and the merging firms' goods increases. Note that we have imposed the constraints that the *aggregate diversion ratios* $D_M + D_o$ and $2D_o$ are below 1.

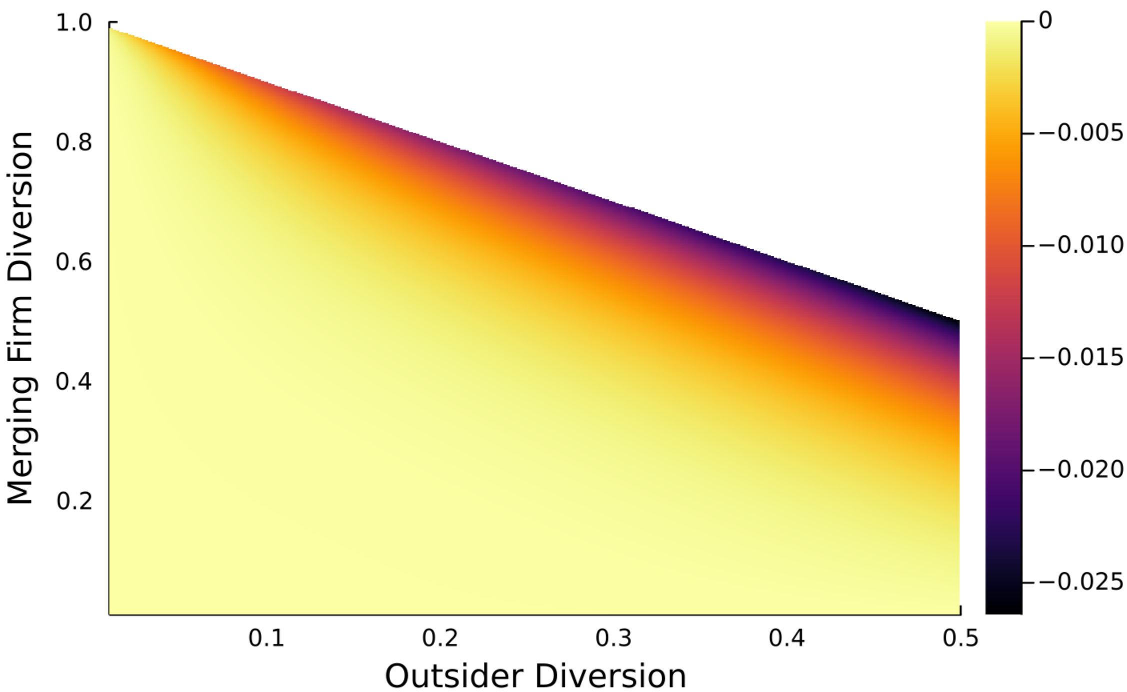

Figure 9: Comparison of efficiencies threshold with and without an outsider.

C.3 Nonlinear Demand and Approximation Error

The results in Section 3.3 partially characterize IPP but do not directly speak to equilibrium consumer and input price changes. Here we consider a version of the simple model with nonlinear demand, and we calculate equilibrium price changes numerically. We confirm that the approximation formulas given in Proposition 2 correctly predict the sign of the equilibrium changes in input and goods prices after the merger, and moreover that the approximation for goods prices is also close in magnitude to the equilibrium change.

Input Prices

Consider the simple model of Section 3.1 with specialized input suppliers, unit input requirements $x_{gi} = x_{g'i'} = 1$ and $v_g = v_{g'} = 0$, and upstream marginal costs $c_i = c_{i'} = 1$. Suppose consumer demand $y(p)$ is described by a symmetric logit demand system, so that the indirect utility of good g for an atomistic consumer l takes the form

$$u_{lg} = \alpha - \beta p_g + \epsilon_{lg},$$

where ϵ_{lg} is a logit error term. In Figures 10 and 11, we calibrate the parameters of the logit demand system (α, β) to give the specified (symmetric) diversion ratio and pre-merger markup. The plot marker denotes the diversion ratio and the color denotes the pre-merger markup. We plot the (symmetric) percent change in input prices in equilibrium versus the approximate percent change corresponding to equation (12) in Proposition 2. Figure 10 shows these price changes after a merger with Hicks-neutral Werden efficiencies, and Figure 11 with zero efficiencies. We observe in both cases that the sign of the input price change predicted by IPP is the same as the sign of the input price change in equilibrium; the input price change due to IPP underestimates the magnitude of the equilibrium change. The approximation is better for lower diversion ratios and lower pre-merger markups. In both cases, higher diversion ratios and higher pre-merger markups are associated with larger post-merger input price changes.

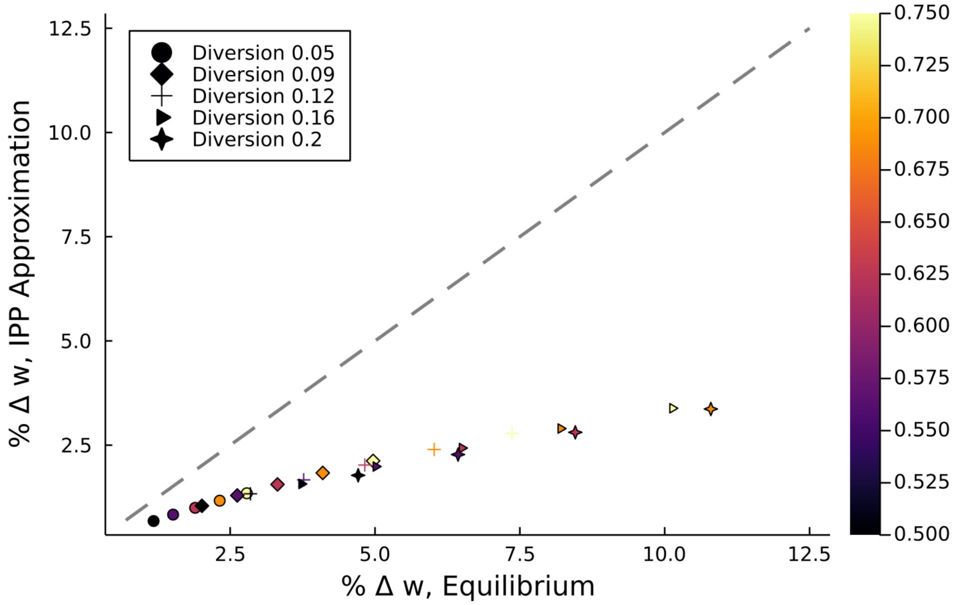

Figure 10: Equilibrium versus approximate input price changes at Werden efficiencies.

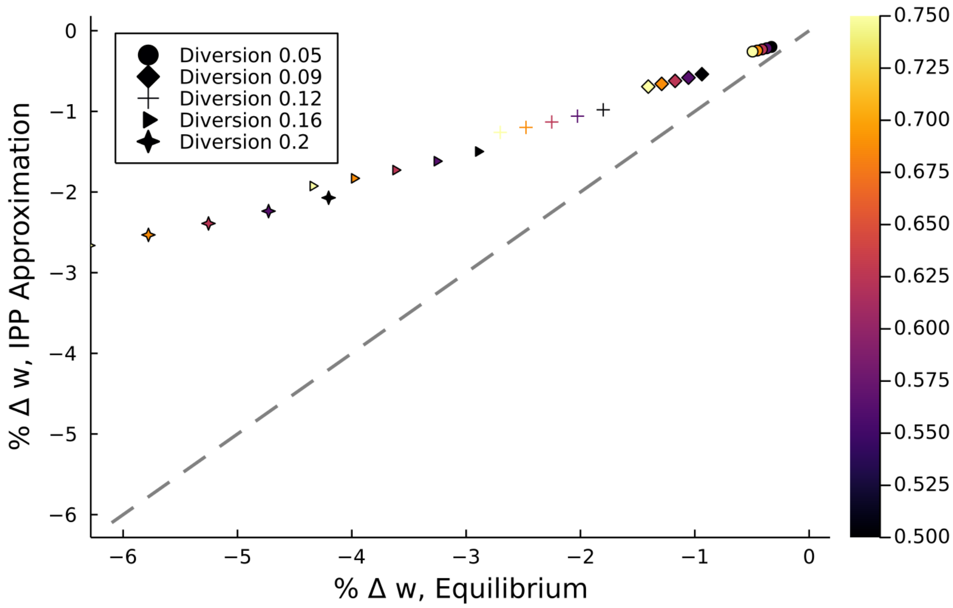

Figure 11: Equilibrium versus approximate input price changes at zero efficiencies.

Goods Prices

Using the same model as above, we now consider changes in goods prices. Figure 12 shows exact and approximate changes in goods prices after a merger with Hicks-neutral Werden efficiencies, and Figure 13 with zero efficiencies. We plot the (symmetric) percent change in goods prices in equilibrium versus the approximate percent change corresponding to equation (11) in Proposition 2. We again observe that the approximate price change has the same sign as the equilibrium goods price change and that the approximation understates the magnitude of the price change. The approximation is better for lower diversions and pre-merger markups. In both cases, higher diversion ratios and higher pre-merger markups are associated with higher post-merger goods price changes.

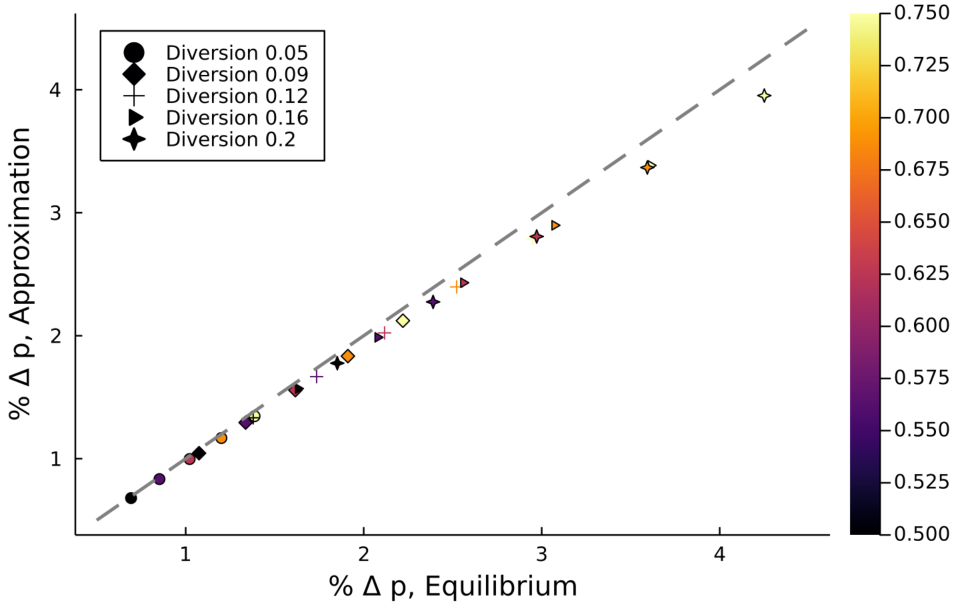

Figure 12: Equilibrium versus approximate goods price changes at Werden efficiencies.

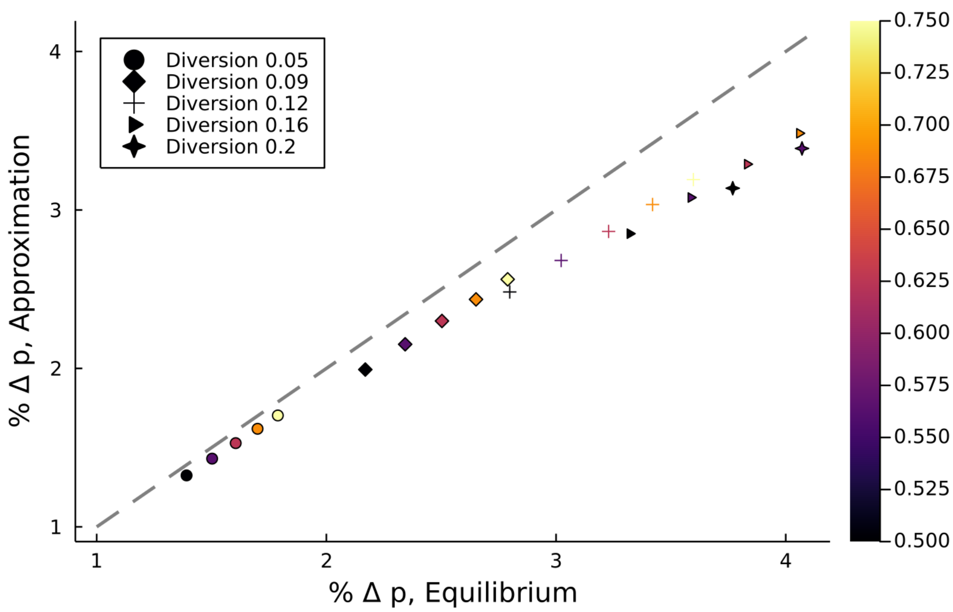

Figure 13: Equilibrium versus approximate goods price changes at zero efficiencies.

D Additional Figures from Merger Simulations

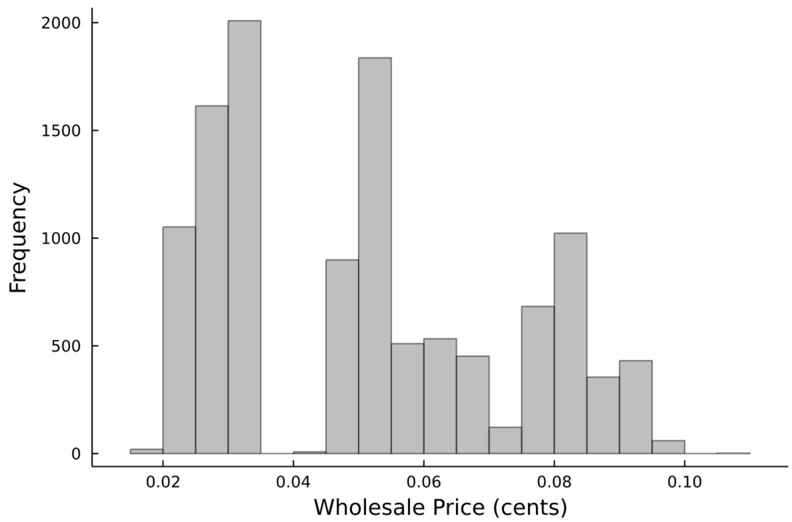

Figure 14: Distribution of pre-merger wholesale prices per ounce, in cents.

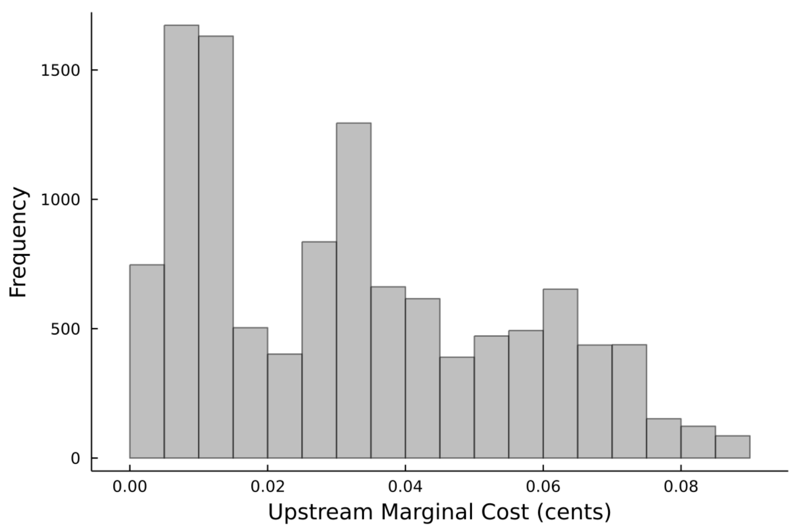

Figure 15: Distribution of upstream marginal costs per ounce, in cents.

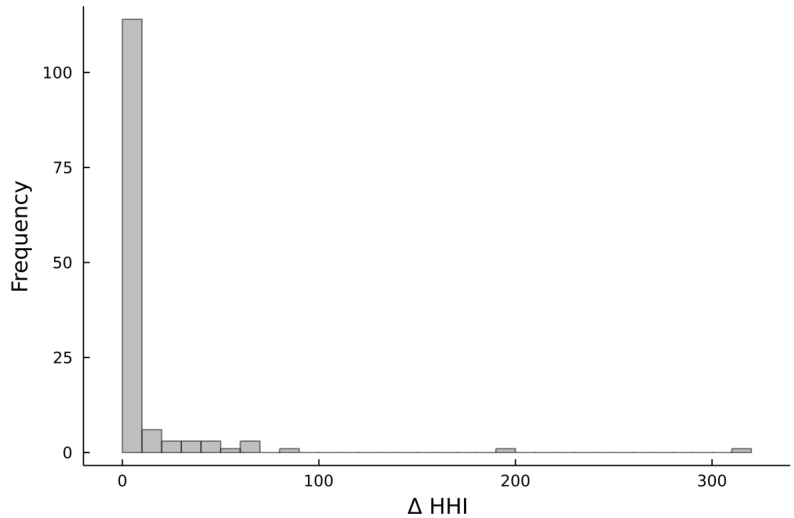

Figure 16: Distribution of Δ HHI for the simulated mergers.