DIAVIK DIAMOND MINES

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1. Introduction

Diavik Diamond Mines employs over 700 in the process of recovering diamonds. The diamond business in the Northwest Territories has become an important part of the regional economy. Diamonds mined by Diavik have been prized for their distinctiveness and value as they guarantee an issue-free origin. Their mining practices operate in one of the most difficult environments while still maintaining sustainable and minimal impacts on the land and wildlife. The rock formation of the Northwest Territories, specifically the North Slave Region has led Canada to be a world-leader in diamond mining and production. Geologically, diamonds are found in deep cylindrical pipes of kimberlite surrounded by granite. Diavik Diamond Mines has located one of these diamond-rich kimberlite pipes and they want to build a mine in order to recover the diamonds. This type of open-pit mine is dug in a series of circular levels called benches. The problem that the company faces is that the kimberlite pipe is covered by water in the form of a lake. They can overcome this by building a dike around the pipe so the rock becomes accessible. This raises the question of how big to build such a dike. The size of the dike depends on how deep the mine will be. These parameters must be determined before construction can start. Now, obviously the deeper the mine is the more diamonds can be recovered causing an increase in revenue. A deep mine also means a larger dike, and a larger volume of rock being mined, therefore increasing costs. A mining plan that provides a maximal amount of profit needs to be established.

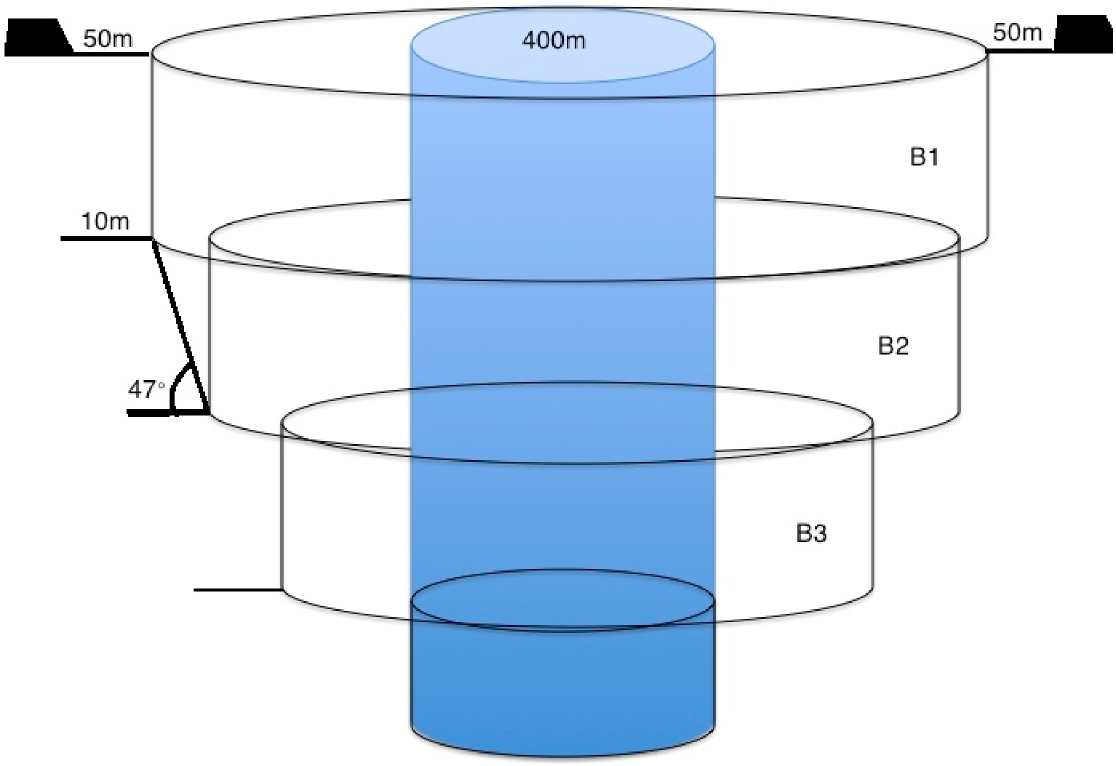
2. Iteration 1

2.1. **Assumptions.** Diavik Diamond Mines provided us with the following physical and financial data which allows us to make certain assumptions about the problem. They are illustrated in the table below:

Physical Constraints		
Parameter	Units	Values
Maximum pit slope angle	Degrees	47
Minimum distance from dike to pit	Meters	50
Diameter of kimberlite pipe	Meters	400
Density of kimberlite	Tonnes/cubic meter	1.5
Density of granite	Tonnes/cubic meter	2.0
Kimberlite grade	Carats of diamonds/tonne kimberlite	2.0
Maximum kimberlite processing rate	Tonnes/year	4.35 million
Dike construction rate	Kilometers/year	2.0

Financials (in 2014 dollars)		
Parameter	Units	Values
Dike construction cost	$ per 10 linear meters	1,000,000
Kimberlite processing cost	$ per tonne kimberlite	18.0
Mining cost 0-100m depth	$ per tonne	4.50
100-200m depth	$ per tonne	5.50
200-300m depth	$ per tonne	6.50
300-400m depth	$ per tonne	8.00
400-500m depth	$ per tonne	12.00
>500m depth	$ per tonne	18.00
Discount rate	% per year	8.0
Inflation rate	% per year	3.0
Diamond value	$ per carat	74

In order to model the problem, we have made a number of reasonable assumptions. We can assume a fixed bench hight of 10 meters. We also assume that the kimberlite pipe is a perfect cylinder with a 200 m radius. This is depicted in the following diagram.



We assume the mine has no maximum depth as we expect costs will prevent a mine of infinite depth. There is no geological way of determining the spread of diamond within a pipe so we assume a constant spread. Our final assumption is that the mine will be dug at a constant rate. This is reasonable as the mining speed is limited by the rate at which the kimberlite can be processed. When a bench is dug, the granite is moved to a till pile while the kimberlite has to be processed in order to extract the diamonds. This processing can only be done so fast and there is no sense is mining faster than the extracting process. Since the volume of kimberlite in any bench will be the same, we assume each bench will be dug in the same amount of time, we say 6 months.

2.2. **Decision Variables and Objective Function.** The objective of the problem is to determine how many benches should be dug, and thus how large the dike needs to be. All in order to maximize the profits. The profit function is defined as follows:

$$\begin{aligned}\text{Profit} &= \text{Diamond Revenue - Processing Costs} \\ &\text{- Mining Costs - Dike Construction Costs.}\end{aligned} \tag{1}$$

Where each of the revenue and costs functions are defined in the code (see Diavik.mod). In order to model this, we have chosen to define our decision variables as a Boolean value indicating whether or not a bench has been dug in a certain dig period (where each dig period is 6 months). This allows us to introduce an expression that sums the value of all the decision variables giving the optimal number of benches to dig.

2.2.1. *Diamond Revenue Definiton.* We now consider defining the Diamond Value function. We have assumed the pipe is a perfect cylinder so the volume of the kimberlite mined per bench is a constant value, given by:

$$\Pi \times r^2 \times h = \Pi \times 200^2 \times 10m^3/\text{bench} \tag{2}$$

The value of the diamonds is measured in dollars per carat, so we want to know how many carats are being recovered from each bench. Since we have also assumed a constant spread of diamonds, this parameter will also be a constant value. It is calculated using the estimated value of diamonds with their worth being \$74/carat, kimberlite having 2 carats per tonne and the density of kimberlite being 1.5 tonnes per metres cubed. This yields the following equation:

$$\Pi \times 200^2 \times 10m^3/\text{bench} \times 1.5\text{tonnes}/m^3 \tag{3}$$

This gives us an estimated diamond revenue of \$278,973,427.64 per bench. We model this by introducing a parameter, diamondValue and setting the value to \$278,973,427.64 in our data file. The profit equation will then simply be diamondValue multiplied by the number of benches dug. In our program, we define an expression, nbBenches, to be the number of benches dug:

(4) nbBenches = sum (b in bench, d in digPeriod)(miningPlan[b][d]);

Thus, our objective function will begin with:

$$\text{diamondValue} \times \ \text{nbBenches} \tag{5}$$

Which gives the total profit from all diamonds recovered.

2.2.2. *Processing Cost Definition.* The processing costs must also be considered so we introduce a processing costs function. It is similar because it is a constant parameter across all bench depths. When a bench is dug, the granite is deposited in the till area while the kimberlite is moved to a processing plant so the diamonds can be extracted. We have assumed the volume of kimberlite pulled from each bench is the same, so we also assume a constant cost of processing. The plant used by Diavik has estimated a cost of $18 per tonne of kimberlite. The cost to process the kimberlite from one bench is given by:

$$\text{tonnes of kimberlite per bench} \times \$18 \text{ per tonne} \tag{6}$$

In our profit equation, Processing Costs are represented by $33,929,200.66 multiplied by the number of benches dug. This is easily modelled by defining a parameter, processingCost and putting the following expression into the objective function:

$$\text{processingCost} \times \ \text{nbBenches}; \tag{7}$$

2.2.3. *Dike Construction Definition.* The costs of building the dike is dependent on the radius of the top bench, which depends on the number of benches dug. We know that the maximum pit slope is 47 degrees. Using a smaller angle would incase the size of the dike required, thus we can assume a constant slope of 47 degrees. There are calculations that allow us to work from the top down where each bench will decrease in radius by 9.33 metres. We can now define the radius of the top bench in terms of the number of benches dug as followed:

$$\text{Radius} = (200m + 9.33m \ (\text{number of benches} - 1)) \tag{8}$$

In order to have trucks and machinery outside the top bench there must be at least 50 metres between the edge of the mine and the edge of the dike. The circumference of the dike is given by:

$$\text{Circ} = 2 \times \Pi \times (\text{radius} + 50m) \tag{9}$$

Using data from previous mines, Diavik has estimated a construction cost of $100,000 per linear meter, so the cost of building the dike is represented as:

$$\text{Dike Construction Cost} = \text{Circ} \times \$100,000 \tag{10}$$

This function is what is used in the profit equation.

2.2.4. *Mining Cost Definition.* Modelling the cost of mining is more difficult. Diavik has estimated a mining cost in dollars per tonne of material dug. It is more expensive to dig the deeper you go. The estimated costs Diavik has provided are outlined in the above table. We model these costs by introducing six parameters, miningCost10 through miningCost60 which have the values of 4.5 through 18 respectively (from the table). The cost to dig a specific bench will be calculated as the appropriate miningCost multiplied by the sum of the tonnes of granite and kimberlite. We number the benches from the top down so the top bench is described as b1. Now we can define the radius of a given bench, b_n as:

$$\text{Radius} = 200 + 9.33 \text{ (number of benches - n)} \tag{11}$$

We know that the number of tonnes of kimberlite dug is the same in every bench (1884955.59 tonnes). We also know the volume of kimberlite per bench, 1256637.06 metres cubed. We can write the weight of granite mined from b_n:

$$\text{Tonnes of Granite} = 2 \text{ tonnes}/m^3 \times (\Pi \times 10 \times radius_n^2 - 1256637.06). \tag{12}$$

Which gives us an equation for the cost of mining bench n:

$$\text{Cost} = \text{miningCost} \times (2 \times (\Pi \times 10 \times radius_n^2 - 1256637.06) + 1884955.59) \tag{13}$$

Notice here that the radius has been squared, and so the equation is no longer linear. We have tried a few different ways of dealing with non-linearity. Initially we tried to break down the numbers and see if a pattern could be represented by a linear equation. We have found that this sort of pattern would exist but would be complicated to understand. We also tried running the program using a non-linear cost equation but ran into various programming issues. As we are not computer scientists nor IBM Cplex Optimization Studio experts, we sought external and more expert advice. The advice led us to consider a small change to the interpretation of the problem. We had first defined bench b1 to be the top of the mine, which forced our equation for the tonnes of granite dug from a given bench to be in terms of how many benches are dug. Instead, if we consider b_1 to be the bottom bench, we make the problem easier to define. First we must develop a new equation for the radius of a given bench in metres:

$$Radius_n = 200 + 9.33 \times (n - 1) \tag{14}$$

The tonnes of granite dug from each bench can be defined without knowing how deep the mine will eventually be. Thus it can be defined as:

$$\begin{aligned}
&\text{Tonnes of granite from bench n} \\
&= \text{Density of granite} \times (\text{volume of granite}) \\
&= \text{Density of granite} \\
&\times (\text{volume of bench - volume of kimberlite}) \\
&= \text{Density of granite} \times (\Pi \times r^2 \times h \\
&- \times(\Pi \times \text{pipe radius}^2 \times h) \\
&= 2 \times (\Pi \times r^2 \times 10 - \Pi \times 200^2 \times 10)
\end{aligned} \tag{15}$$

As our equation for the radius of any particular bench is well defined we can easily calculate the tonnes of granite dug from any particular bench. This allows us to introduce a variable, tnsGranite[bench] to our model, and fill in the data using the above equation.

Our expression in the objective function for the mining cost is going to be a series of expressions,

$$\begin{aligned}
&\text{Sum (b in benchMax-9..benchMax, d in digPeriod)(miningCost10)} \\
&\times (\text{tnsGranite[b]} + \text{tnsKimberlite}) \times \text{miningPlan[b][d]} \\
&\text{Sum (b in benchMax-19..benchMax, d in digPeriod)} \\
&\text{miningCost20*(tnsGranite[b]+tnsKimberlite)(miningPlan[b][d])} \\
&\ldots \\
&\text{Sum (b in benchMax-59..benchMax-50, d in digPeriod)} \\
&\text{(miningCost60(tnsGranite[b]+tnsKimberlite)(miningPlan[b][d])}
\end{aligned} \tag{16}$$

The first expression will give the total mining cost of digging the bottom 10 benches. The second expression will give the total mining cost of digging the following 10 benches up and the pattern follows. We now have a value for each piece of the objective function.

2.3. **Constraints.** Our main concern is to look at modelling the constraints. The client had very few specifications about restrictions to the problem but there are still many that need to be considered from a modelling perspective. The one restriction that was given to us is the maximum rate at which the kimberlite can be processed. Since we have defined our decision variables by if a bench is dug or not (in a 6 month period), the processing constraint is simple to define. All that is required is that we restrict the number of benches dug in a specific dig period to be less than one. We introduce the following to define this:

$$\text{forall(d in digPeriod) sum(b in bench) miningPlan[b][d]} \leq 1; \tag{17}$$

This ensures that the rate of mining does not exceed the rate of processing. We need another restriction that a bench can only be dug once. We also need to ensure that if a certain bench is not dug, then the bench below it is certainly not dug either. Which will force the solutions to have benches dug in a descending order.

Now recall that the kimberlite pipe is located underwater, so the first step in the mining process is to the build the dike. This means that mining cannot begin until the dike has been fully constructed so the rock below it has been exposed. We want to introduce a constraint of the following form to represent this

$$\begin{aligned}&\text{forall(b in bench, d in digPeriod : d} \leq \text{constructionTime)}\\&\text{miningPlan[b][d] == 0;}\end{aligned} \tag{18}$$

This ensures that no bench is dug in a dig period preceding the completion of the dike, represented by constructionTime. There are some complications here, how do we define the value of the constructionTime parameter? We first need to estimate how long it will take to construct a dike. We have an equation defining the circumference of the dike in meters so it would be desirable to have a construction rate given in meters per time period. Diavik has provided an estimated construction rate of 2 kilometres per year. Which is easily converted to 1 kilometre per dig period. The number of dig periods required to complete the dike will be defined as the circumference divided by 1000 and rounded up to the nearest integer. This concludes the modelling information and data collection, now we have enough information to obtain a solution.

2.4. **Solution.** When we run the program we find that the profit from the recovered diamonds of a bench is far exceeding the costs of mining an additional bench. Alas, our solution will always advise digging to the maximum number of benches (in such case 60 benches). However, in our initial meetings with the client they had said that a mine would only very rarely be deeper than 500 metres (50 benches). In a brief discussion with the client, they advised that we could reasonably assume an inflation rate of 3% per year on both the mining cost and the diamond value parameters. This was easily incorporated into our model (see Diavik1.mod), but was not sufficient to limit the depth of the mine and was thus disregarded. Before meeting with the client again, we considered playing with the maximum time value. Previously, we had been allowing enough time so that every bench could be dug if that was the optimal solution. If we reduce the allowed amount of dig periods, the model is forced to dig less than the maximum number of benches as there is not enough time to do so. What we find is that the benches the solutions choose to not dig are the bottom two. This is not what we would expect, as there is no value in omitting the two smallest benches. Doing this, would mean you are digging more granite and building a larger dike than is required to recover the same amount of diamonds. This should cause an increase in costs. We thus expect that if a solution is forced to omit two benches, it should omit the top two (bench 59 and 60). Clearly there is an aspect in our model that needs adjusting.

3. Iteration 2

After reexamining our code, we found an issue with the constraints that forced the mining plan to dig benches in the appropriate order, from the top down over time. We modified the constraints to be the following:

(19)
$$\begin{aligned}
&\text{ctDigOrder1:}\\
&\text{forall(b in 2..benchMax, d in 1..timeMax-1)}\\
&\text{miningPlan[b][d]} \leq \text{miningPlan[b-1][d+1]}\\
&\text{ctDigOrder2:}\\
&\text{forall(b in 2..benchMax}\\
&\text{sum (d in digPeriod)miningPlan[b][d]} \leq\\
&\text{sum(d in digPeriod)miningPlan[b-1][d]}
\end{aligned}$$

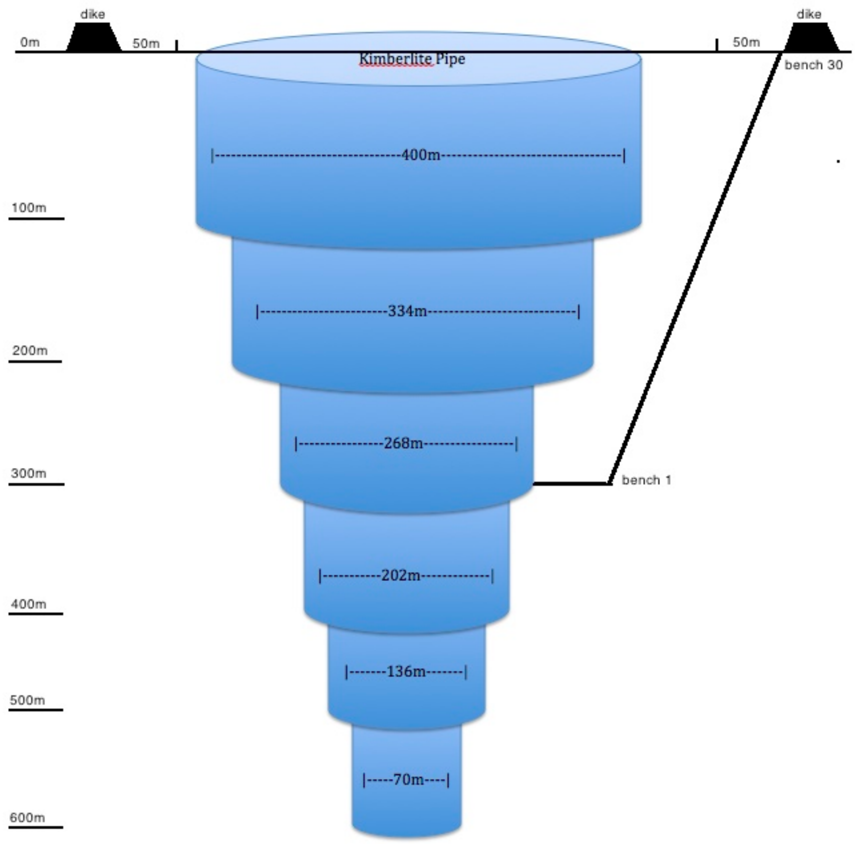
This however, was not the overall problem.

3.1. **Assumptions.**

We scheduled a meeting with our client and provided them with a report of our findings to this point (see appendix). After discussing the results, they advised that the assumption of a kimberlite pipe with uniform diameter is unreasonable. Diavik provided us with new information that depicts a pipe with a decreasing diameter. For the sake of simplicity, we chose to have the pipe decrease in diameter by 33 every 100 meters. Our model now requires some changes as the volume of kimberlite is no longer constant. In our new model (see Diavik2.mod), we maintain the data parameters and decision variables outlined in our first model. Our first change is to redefine the radius of a given bench. We define:

(20)
$$\text{radiusSquared[b]} = \text{pow(200+9.33} \times \text{(b-1),2);}$$

We then proceed to define the amount of kimberlite mined from a given bench. We have the new assumption of a kimberlite pipe in six decreasing segments, so the volume of kimberlite mined from a bench will be same within each 100 meter segment. These values are calculated and then included in the model as data parameters volKimberlite1 through volKimberlite6. A diagram of the decreasing diameter kimberlite pipe is described below:



3.2. **Decision Variables and Objective Function.** The decision variables and objective function of our model have not changed going into the second iteration. Only the diamond revenue was changed and everything else stayed the same.

3.2.1. *Diamond Revenue Definition.* To define the diamond profit we must calculate the profit from each 100 meter section of the kimberlite pipe and then sum them up. We introduce an expression,

$$\begin{aligned}&\text{diamondProfit1} = \text{sum}(\text{b in } 1..10 : \text{b} \leq \text{benchMax},\\ &(\text{d in digPeriod})(\text{diamondValue} \times \text{tnsKimberlite1} \times \text{miningPlan[b][d]})\end{aligned} \tag{21}$$

for each section. This expression provides the total value of diamonds recovered from the first 10 benches, or fewer, if less than 10 are dug. The expression diamondProfit2 will give the profit from any diamonds recovered from the next 10 benches, and so on. We then let totalDiamondProfit be the sum of these six expression and the parameter we use in our objective function.

3.2.2. *Processing Cost Definition.* Processing costs are modelled in the same way, but with a processing cost instead of a diamond value multiplied by the tonnes of kimberlite.

3.2.3. *Construction Cost Definition.* There are no changes to our model in terms of the dike construction cost.

3.2.4. *Mining Cost Definition.* We now examine the definition of mining costs. We want expressions similar to those for the diamond profit and processing cost, where the total mining cost is a sum of the costs per section. We define, for 1 through 6

$$\text{tMiningCost1} = \text{miningCost} \times \text{tnsMateral1}; \tag{22}$$

Now we outline what value tnsMaterial1 has. We want it to represent the total weight of the material (kimberlite and granite) dug from a particular section. This is tricky because the way we have defined the radius and the labelling of the benches. We represent the total weight by using an execute loop, seen in the Diavik2.mod file. This loop ensures that the radius of the top bench is minimized, meaning the minimal amount of material is dug. We now have a value for tnsMaterial1 through tnsMaterial6, and thus a total mining cost.

3.3. **Constraints.** We have also added two constraints to our new model. They are trivial in the sense that they do not impact the objective value, but they ensure a reasonable mining plan. The first new constraint, ctmustDig, forces the solution to dig at least one bench. This is included as it is a useful forcing constraint during our modelling and working through errors in the code. The second constraint, ctForcedTime, forces the mining plan to begin digging at the earliest possible time. Which will ensure that if we have set timeMax exceeding the time required to complete the mine, the plan will not wait a years before digging. We are now able to run the model and obtain a solution!

3.4. **Solution.** After extensive analysis, we established a reasonable solution and sent a final report of our second iteration (see Second Iteration Report.pdf) to the company. Our final report advised Diavik Diamond Mines to construct a dike with radius 520.57 meters. This will allow for a mine that is 300 meters in depth without exceeding the maximum pit slope angle of 47 degrees. If this plan is followed, it will take approximately 17 years to develop which includes a two-year span for dike-construction. Revenue from the diamonds will exceed \$5.987 billion, the total cost of kimberlite will exceed \$728 million and mining costs will exceed \$1.296 billion. With an additional \$327 million of cost of building the dike, Diavik can expect an estimated profit of \$3.635 billion, before taxes. After a final meeting with our client to discuss this solution, they are satisfied with the mining plan we have outlined above.

4. Conclusion

Having located a diamond-rich area in the Northwest Territories, Diavik Diamond Mines is interested in constructing an open bit bench mine. We have worked with them to develop an outline of what an optimal mining plan could look like for them. This plan details how deep their mine should be, and thus how large of a dike they are required to construct in order to maximize profits. We have cautioned the company that the values and plan we have provided from our model are based on a number of assumptions and are thus only estimates and not exact values. If the solution and mining plan we have made for the company were not suitable, we would require more data and information to make adjustments to better fit their needs. Further investigation on our part could be done to evaluate inflation costs as well as a time to money value constraint if a 17-year develop is too lenghtily. We hope that Diavik Diamond Mines are satisfied with our model and solution and we have appreciated the opportunity to work with the company.

Appendix A

(1) Diavik.mod
  - The very first model made for this problem, that was not used

(2) Diavik1.mod
  - The model for the first iteration

(3) Diavik1.dat
  - The data file for the first iteration

(4) First Iteration Report.pdf
  - The report sent to the company with the solution of the first iteration

(5) Diavik2.mod
  - The model for the second iteration, which contains the final solution

(6) Diavik2.dat
  - The data file for the second iteration

(7) Second Iteration Report.pdf
  - The report sent to the company with the solution of the second iteration