The impact of hyperons in the Neutron Star core equation of state

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Contents

Current Section

Introduction to Neutron Stars (NS)

- Remnants of a gravitational (' core-collapse') supernova when the mass of the progenitor star is $8M_{\odot} \lesssim M \lesssim (20-30)M_{\odot}$
- First discovered in late 60's via pulsars : very fast rotating NS (rotation period from around 1s-1ms) that emits an EM pulsed signal
- Very compact object : typical mass range of $\mathbf{M} \sim \mathbf{1.4M_{\odot}}$, but radius $\mathbf{R} \sim \mathbf{10km}$

Introduction to Neutron Stars (NS)

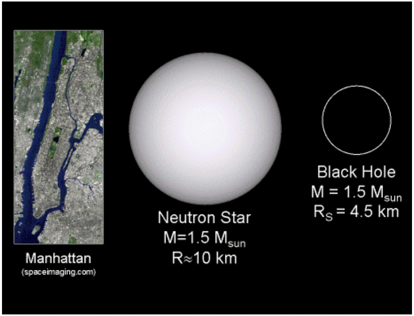


FIGURE 1 – Comparison of two compact objects to the size of Manhattan

This allows us to introduce the compacity parameter $C \equiv \frac{M}{R}$, the relevant parameter to judge how relevant GR is to study a stellar object (~ 0.15 for NS, 0.5 for Schw BH, 10^{-9} for Earth)

Current Section

General Relativity reminders

- General form of a static spherically symmetric metric

$$ds^2 = -e^{2\Phi(r)/c^2} c^2 dt^2 + (1 - \frac{2Gm(r)}{rc^2}) dr^2 + r^2 d\Omega^2$$

- + Stress-energy tensor $T_{\mu\nu}$ of the NS approximated by a perfect fluid with ρ, P energy density and pressure

$$T_{\alpha\beta} = (\rho + P) u_\alpha u_\beta + P g_{\alpha\beta}$$

- All that in Einstein's equation and conservation of stress energy tensor

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{and} \quad \nabla^\mu T_{\mu\nu} = 0$$

TOV Equations

We derive the Tolman-Oppenheimer-Volkoff (TOV) equations

TOV Equations

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$
$$\frac{d\Phi}{dr} = (1 - \frac{2Gm(r)}{rc^2})^{-1}(\frac{Gm(r)}{r^2} + \frac{4\pi G}{c^2} rP(r))$$
$$\frac{dP}{dr} = -(\rho(r) + P(r))\frac{d\Phi}{dr}$$

Another equation is needed to close the system of equations : The Equation of State (EoS) $\boxed{P(\rho, T) = P(\rho)}$ here because we consider a cold neutron star.

Integrating the TOV equations

- In practice, we compute the energy and pressure as a function of the baryonic number density and make a correspondance between $P(n), \rho(n)$
- TOV system = First-order differential equations, can be integrated starting from $m(r = 0) = 0$, and a given density at the center (around 3-6 times nuclear saturation density $n_{sat} \sim 0.16 fm^{-3}$)
- Then we integrate until the pressure reaches $P(R) = 0$. Then the metric reduces to the Schwarschild one with mass $M = m(R)$
- In short, given a certain EoS,

$$\text{one } P_{center} \Leftrightarrow \text{one } (M, R) \text{ couple}$$

Mass-Radius diagram and maximal mass

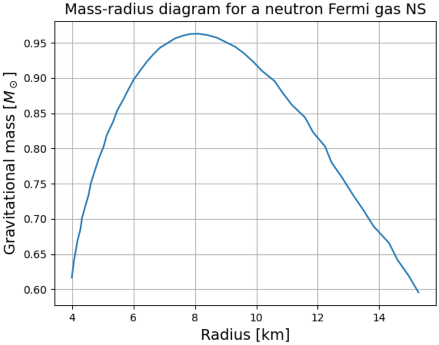


FIGURE 2 – Mass-Radius Diagram for a free Fermi gas of neutrons, **no nuclear interactions**!

Basically two branches, one unstable where M grows with R, the other stable where M decreases when R grows.stability criterion is $\frac{\partial M}{\partial P_{CENTER}} > 0$. between the two there is a maximum mass.

Tidal effects in GR for NS in binary systems 1/2

- Other than the mass and the radius of a NS, one of the other important observables is the tidal deformability : in the case of binary mergers involving NS (so either NS-NS or NS-BH), the dynamics and the gravitationnal waves emitted depend on the tidal deformability Λ.
- We consider a NS in a binary system, subject to an outside displacement quadripolar field $\mathcal{E}_{ij}$. Then the deformability coefficient is defined as $Q_{ij} = \lambda \mathcal{E}_{ij}$, then $\Lambda = \lambda \times G \left(\frac{c^2}{GM} \right)^5$.

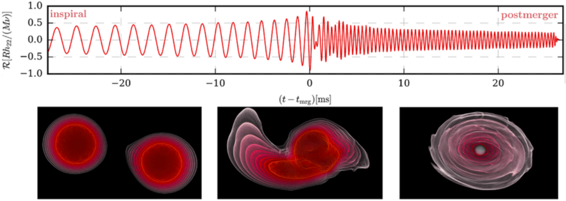


FIGURE 3 – Gravitationnal wave emitted by a BNS merger

Tidal effects in GR for NS in binary systems 2/2

- The tidal deformability is computed perturbatively, we write $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta}$, then $|h| \propto H(R)$, H satisfies $H'' + H'[\frac{2}{r} + e^{2\alpha}(\frac{2m(r)}{r^2} + 4\pi r(p-\rho))] + H[-\frac{6e^{2\alpha}}{r^2} + 4\pi e^{2\alpha}(5\rho + 9P + \frac{\rho+P}{(dP/d\rho)}) - (2\frac{d\nu}{dr})^2] = 0$
  (new equation to integrate in parallel to the TOV)
- and the tidal deformability depends on M,R,H'(R),H(R).
  All these observables are deeply related to the **EoS** of NS matter !
- For now we'll focus on the core, because it's both the region we know the least and the more impactful on the observables M, R, Λ

NS structure

- Atmosphere composed of plasma
- Outer crust : atomic nuclei, until the density reaches $\rho_{ND} \approx 4 \times 10^{11} g.cm^{-3}$
- Inner crust, mixture of nuclei and neutrons (can be superfluid)
- Outer core, mixture of n, p, e, μ
- Inner core, highest densities, hyperons can appear, condensates, possible transition to quark matter... least known

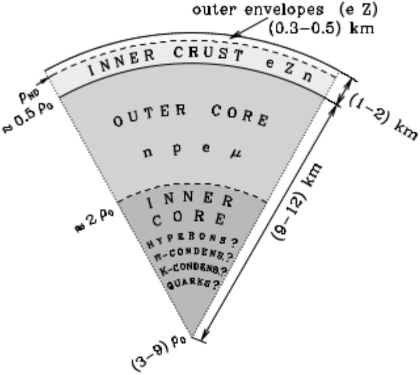


FIGURE 4 – NS layers

Current Section

Lagrangian of nuclear matter

- Baryons interact by exchanging effective bosons, here we take 3 types of mesons :
- scalar isoscalar σ meson : attractive force
- vector isoscalar ω meson : short-range repulsion
- vector isovector ρ meson : repulsion different for neutrons and protons
- Then, we have, for a Dirac spinor ψ representing neutrons or protons :

$$\mathcal{L} = \bar{\psi}\left[g^{\mu}(i\partial_{\mu} - g_{\omega}A_{\mu}^{(\omega)} - g_{\rho}\vec{\tau}\cdot\vec{A}_{\mu}^{(\rho)}) - (m - g_{\sigma}\sigma)\right]\psi + \mathcal{L}_{\sigma} + \mathcal{L}_{\omega} + \mathcal{L}_{\rho}$$

$+$ Mean field approach

Methodology

Self coherent eqation on σ,and other conditions on chemical potentials have to be fulfilled : beta equilibrium sets $\boxed{\mu_n = \mu_p + \mu_l}$ with $\mu_l = \mu_e = \mu_\mu$ and charge neutrality $\boxed{n_p = n_e + n_\mu}$

Algorithm 1 Process to find the EoS

Input: Couplings meson-baryons

Output: consistant values of mean fields and densities of each species n_i

1: **for** $n_B \in [0.5 n_{sat}, 8 n_{sat}]$ **do**

2: find root of vector functions of conditions :
$$\begin{cases} \text{f1}(\sigma, n_i) = \sigma - \sigma(n_i(\sigma)) \text{ ensures } \sigma \text{ is a fixed point of } \sigma(n_i(\sigma)) \\ \text{f2}(\sigma, n_i) = \mu_n - (\mu_p + \mu_l) \text{ ensures beta equilibrium} \\ \text{f3}(\sigma, n_i) = n_p - (n_e + n_\mu) \text{ ensures charge neutrality} \end{cases}$$

3: Compute thermodynamic values (Energy, Pressure,enthalpy...)

4: **end for**

Methodology 2/2

- Then we can compute the energy ε, And the pressure can be derived from thermodynamic consistency $\boxed{P = \sum_i \mu_i n_i - \varepsilon}$

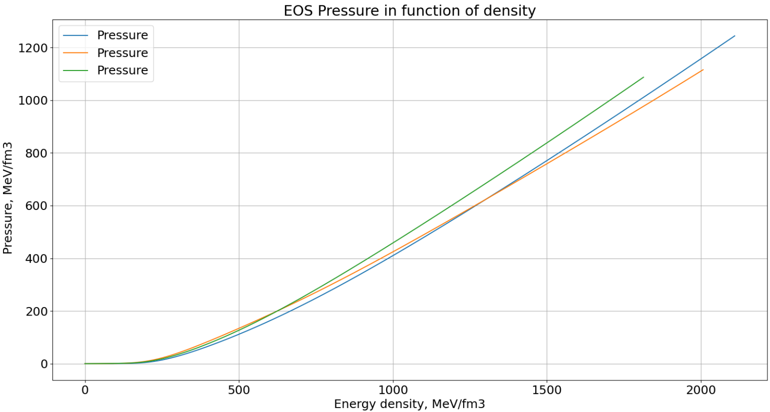
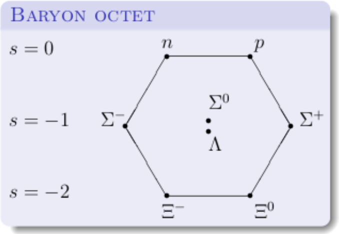


FIGURE 5 – Nuclear matter EoS, Pressure and energy, 3 models

Current Section

Hyperon phenomenology



- We consider the neutral and of strangeness -1 Λ baryon, of mass 1115 MeV.
- In NS cores, hyperons can appear when chemical potential of neutron μ_B becomes larger than the effective mass of the hyperon. We expect it to happen at $\sim 2 - 3n_{sat}$

We expect hyperons to soften the eos and get a smaller max mass because we add more degrees of freedom to the system $\rightarrow$ Pressure lowered

Hyperonic terms in the Lagrangian

- Same form, except that since Λ has isospin 0, the coupling to the ρ meson is 0. We add a meson that only couples strange mesons, we call it σ^*

$$\mathcal{L}_\Lambda = \bar{\psi}_\Lambda \left[\gamma^\mu (i\partial_\mu - g_{\Lambda\omega} A_\mu^{(\omega)}) - (m_\Lambda - g_{\Lambda\sigma}\sigma - g_{\Lambda\sigma^*}\sigma^*) \right] \psi_\Lambda$$

- Appears the chemical potential of the lambda hyperon $\mu_\Lambda = \mu_B + \mu_s$ with the chemical potential of strangeness μ_s. At weak equilibrium a new condition appears $\boxed{\mu_\Lambda = \mu_B}$ (or $\mu_s = 0$)

Modified algorithm

So we add two new functions to find the root of : we will now apply a damped Newton-Raphson algorithm to the following function :

$$\overrightarrow{f} \begin{cases} \text{f1}(\sigma,\sigma^*,n_i) = \sigma - \sigma(n_i,\sigma,\sigma^*) \text{ ensures } \sigma \text{ is a fixed point of } \sigma(n_i,\sigma,\sigma^*) \\ \text{f2}(\sigma,\sigma^*,n_i) = \mu_n - (\mu_p + \mu_l) \text{ ensures beta equilibrium} \\ \text{f3}(\sigma,\sigma^*,n_i) = n_p - (n_e + n_\mu) \text{ ensures charge neutrality} \\ \text{f4}(\sigma,\sigma^*,n_i) = \sigma^* - \sigma^*(n_i,\sigma,\sigma^*) \text{ ensures } \sigma^* \text{ is a fixed point of } \sigma^*(n_i,\sigma, \\ \text{f5}(\sigma,\sigma^*,n_i) = \mu_n - \mu_\Lambda \text{ ensures strange equilibrium} \end{cases}$$

($n_i \in \{n_n, n_p, n_\Lambda\}$)

What do we expect ? Depends on the strength of the couplings.

Relevant quantities

- We vary couplings to mesons of hyperons compared to nuclear matter :

$$R_{\sigma\Lambda} = \frac{g_{\sigma\Lambda}}{g_{\sigma N}}; R_{\omega\Lambda} = \frac{g_{\omega\Lambda}}{g_{\omega N}}; R_{\sigma^*\Lambda} = \frac{g_{\sigma^*\Lambda}}{g_{\sigma N}}$$

- One crucial value is the density at which hyperons appear. It should be superior to n_{sat}, decrease with $R_{\omega\Lambda}$ decreasing (less repulsion so it can appear sooner) and increase with both $R_{\sigma\Lambda}$ and $R_{\sigma^*\Lambda}$ decreasing (less attraction so it can appear later)
- One quantity constrained by experiment is the potential of Λ in **symmetric nuclear matter** $U_{\Lambda N}$
  It should have a value around -30MeV around n_{sat} (value computed with hypernuclei experiments)

Crust-core matching

- We now have a core eos. To have M, R, Λ, we need an eos for the crust as well. We use an expansion of the eos around n_{sat} to match what we know from crust and what we know from core, imposing continuity. Lingering problem of jump in pressure
- For each nuclear model we vary the hyperonic parameter, but not the crust associated ! because the crust is low to very low density, no hyperons.

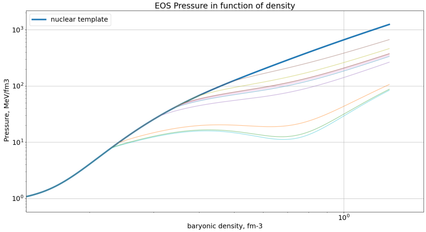


FIGURE 6 – Different eos with hyperons and the nuclear-only model associated

Current Section

Results

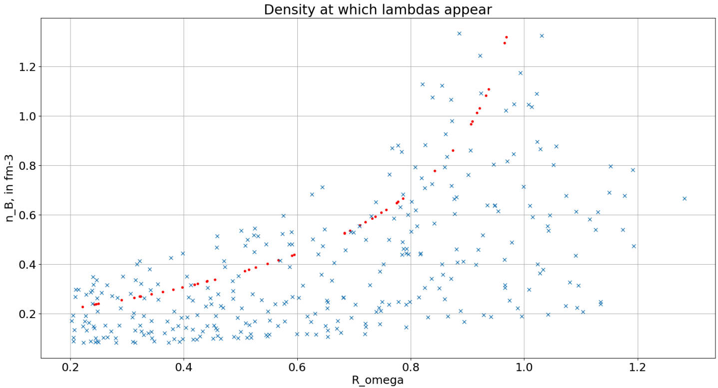


FIGURE 7 – Density at which hyperons appear, random sampling of size 1000 of the 3 ratios. For $R_\omega > 1$, apparition density is higher than the one we compute the eos to. Red points are when we vary R_ω with $R_\sigma, R_{\sigma*}$ fixed.

Results

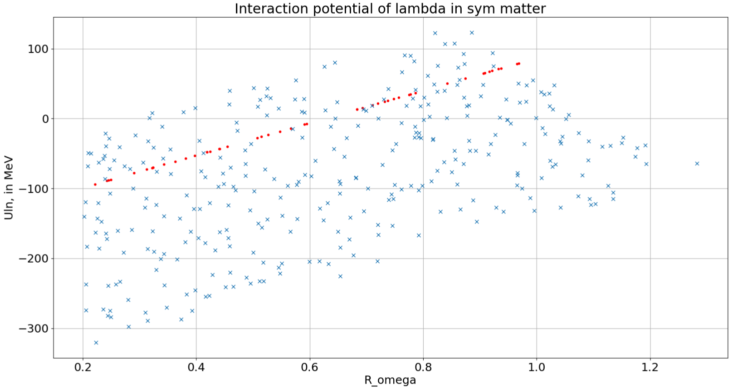


FIGURE 8 – Interaction potential of hyperons in nuclear matter, we should stay around 0.5

Results

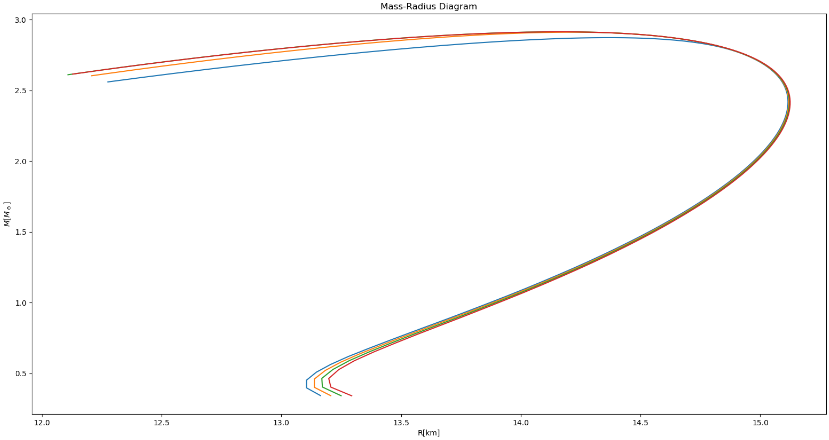


FIGURE 9 – Mass-Radius relation for different R_{σ^*}

Outlook/perspectives

- The couplings are density-dependant, we model that using 4 parameters. In the internship, I only modified the ratio compared to nuclear matter : the next step would be to vary each of the 4 parameters
- A more robust crust-core matching procedure that doesn't cause jumps.
- Study of non-monotony of pressure, appearance of phase and phase transitions, we can see that better in a $P(\mu_B)$ plot

Take home messages

- Need microscopic EoS to integrate GR equations and get macroscopic observables.
- The NS core is very very dense, too much for LQCD and not enough for perturbative QCD $\rightarrow$ Phenomenological models or models solving many-body problem on the basis of a few-body baryonic interaction
- Is very useful for extracting NS radius and dense matter properties from measured deformabilities via GWs : because deformability affects the waveform. (O4 running as we speak)

Mean Field approach

We consider the matter to be static and homogenous, and we replace the meson fields by their expectation values given by the Euler-Lagrange equations

$$m_\sigma^2 \sigma = g_\sigma \langle \bar{\psi}\psi \rangle$$
$$m_\omega^2 \omega_0 = g_\omega \langle \bar{\psi}\gamma_0\psi \rangle$$
$$m_\rho^2 \rho_3^0 = \frac{1}{2} g_\rho \langle \bar{\psi}\gamma_0\tau_3\psi \rangle$$

$\langle \bar{\psi}\psi \rangle = n^s$ is the scalar density and $\langle \bar{\psi}\gamma_0\psi \rangle = n = (3\pi^2 p_F)^{1/3}$ the more usual number density.
This causes the effective mass of baryons to shift $m \to m^* = m - g_\sigma \sigma$.
But then the fermi momentum $p_F = \sqrt{\epsilon_F - m^2}$ changes as well, so the densities and scalar densities as well $\to$ new value of mean fields and so on

GDFM Model and rearrangement term

- We consider density dependant couplings for each meson i -baryon j coupling, of the form

$$g_{ij}(x) = a_{ij} + (b_{ij} + d_{ij}x^3)e^{-c_{ij}x}$$

  with $x = n/n_0$ with n the number density and n_0 a scaling parameter that we choose.

- This induces a new term in the chemical potential $\mu = \frac{\partial \varepsilon}{\partial n} = (\mu_{old} \propto g_i) + \Sigma^R)$ with Σ^R the rearrangement term which takes the value

$$\Sigma^R = \sum_{j \in B} \frac{\partial g_{\omega B}}{\partial n_j} \omega_0 n_j + t_{3j} \frac{\partial g_{\rho B}}{\partial n_j} \rho_3^0 n_j - \frac{\partial g_{\sigma B}}{\partial n_j} \sigma n_j^s$$

Current Section

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