Measurement of Muon life-time with STEREO

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Abstract

We measure the mean life time of muon, using the delay of time between the Cherenkov radiation emitted by cosmic rays muons and the other one emitted by an electron muon's daughter in a water tank. We find the mean life time to be $\tau = 2.201 \pm 0.023 \pm 0.045 \mu s$. Our results seems in agreement with the value from PDG $\tau = 2.1969811 \pm 0.0000022 \mu s$ [1].

1 Introduction

Muons μ^- and antimuons μ^+ are created in the atmosphere after the interaction of the highly energetic cosmic rays with the molecules of the earth's atmosphere leading to hadronic showers. These muons are the daughters of the pions ($\pi^+ \to \mu^+ \nu_\mu$ or $\pi^- \to \mu^- \bar{\nu}_\mu$) whereas pions are products of cosmic ray protons.

When a charged particle ($\mu^\pm, e^\pm, p$...) is travelling with a velocity higher than the velocity of light in a dielectric medium, the molecules of the medium will be polarised thus, after a certain delay of time (order of ns) they will return to the ground state emitting a radiation known as Cherenkov Radiation.

When a muon enters in a liquid medium it will lose energy by dE/dx (among the lost energy a very small fraction comes from Cherenkov effect so we can neglect it when we consider the trajectory of the muon) approximately given by the Bethe-Bloch formula, thus after a while, if the energy of the muon can hold for a path inside the medium smaller than the length of the tank, it will stop in the tank; in other word if the energy of the muon is not high enough to cross the tank, the charged particle will stop and then it will decay as following $\mu^+ \to e^+ \nu_e \bar{\nu_\mu}$ or $\mu^- \to e^- \bar{\nu_e} \nu_\mu$. The produced electron, in turn, will emit Cherenkov radiation that can be detected.

So measuring the delay of time between the two Cherenkov radiations will lead to find the lifetime of the muon.

The flux of the muon at sea level is proportional to the square of the cosine of the angle between the vertical and the direction of propagation of the beam [2]:

$$I(P,\theta)(m^2 s^{-1} sr^{-1}) = f(P) \times cos^2(\theta) \tag{1}$$

where P is the momentum of the μ. Integrating all over the 4 sides and the roof of the tank we estimate:

$$\Phi_{tot} \approx 246 \text{ muons}/s \tag{2}$$

The order of magnitude is compatible with our measurement (see section 2.2). This is the estimated flux of all muons at sea level entering the tank. In fact, in our case we are interested in muons with energy lower than $500MeV$, more precisely $159 < E_\mu < 450MeV$. The lower limit is the minimal energy required for the muon so that the Cherenkov effect can occur, while the upper limit is calculated by requiring the muon to decay inside the tank considering that the maximal path length is $\approx 1m$ (water tank length). The muon with energy higher than the upper limit will most likely cross the tank without decaying contributing to the background, and even if they decay, the second pulse would be in the dead time of the detector. These low energy muons consist around 15% of the total flux of muons.

Measuring the life-time of muon serves the calculation of Fermi coupling constant G_F which is an universal constant defining the strength of all weak interactions. the relation is as following :

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1 + \Delta q) \tag{3}$$

Where Δq represents the higher order QED and QCD corrections calculated in the Fermi theory [4]

2 Experimental Setup

2.1 Apparatus

Our experimental apparatus to measure the muon lifetime is described in figure 1. A tank of volume $V \approx 0.5 \times 0.5 \times 1m^3$ is placed at room temperature. This tank is made of a stainless steel vessel to avoid any light leak and any corrosion. From the top a filling tube is injected to add water if needed, and oil is added to protect the electronics inside the tank. Mirrors are fixed on the sides of the tank in order to reflect the light produced by Cherenkov effect to the detectors.

On top of the tank are mounted two PMTs facing down, in order to detect the photons emitted inside the tank : when a photon is caught by the photocathode of the PMT an electron is emitted by photoelectric effect. Then a focusing electrode is used to guide this electron toward the series of dynode where an avalanche is produced. Each PMT is connected to a front end board to convert the analog signal into a numeric signal, where a triggering system is applied to eliminate the unwanted events. The numeric signal amplitude has arbitrary units, that we estimated around $1mV \simeq 200u.a$ using a pulse generator with fixed amplitude checked with an oscilloscope as well. The board is then connected to a computer where this data acquisition can be saved.

A high voltage power supply is used to provide the PMTs with a sufficient voltage to reach a gain large enough to have a detectable signal of the photons emitted by the Cherenkov effect of both the muons and the electrons. In order to find the optimal voltage and threshold's amplitude, we started by measuring the variation of the event's number per second as a function of ΔV for each PMT apart. To take the data we fixed the trigger threshold at 100 u.a, where the threshold is the minimal amplitude of the pulse for it to be saved. When $\Delta V < 700V$ roughly no event was detected, then the number starts to increase linearly. When the applied voltage exceeds 1100V the number of events diverges which means that the the amplitude of the noise events starts to exceed 100u.a. We measured also the the variation of the amplitude median with different applied voltages, to study the gain. Consistent with previous results, we found that the median was too low below 700V then it starts to increase, until 1100V where it decrease drastically. So we fixed the applied voltage at 1000V to be in the safe zone, in other word minimise the noise as much as possible and ensure having a gain large enough (so that most Cherenkov radiations are saved) at the same time. After that, we connected both PMTs to the board and we kept only the coincident events to reduce the noise, and measured the number of events per second for different DAQ threshold, we decided to fix it at 150u.a, since above this threshold $\frac{dN}{d(thres)}$ starts to decrease (saturation).

The choice of taking data when both PMTs are in coincidence was especially important because of **Afterpulses** : With no coincidence, we obtained a gaussian centered around $7\mu s$ in the time distribution. This gaussian is due to electrons ionizing atoms in the PMT because the vacuum is not perfect. This ionized atom traveled in the dynode series as well, but because its mass is around 2000 times the electron's mass, the time of flight is lower, comparable to the lifetime of a muon, so without coincidence a small bump appears around $7\mu s$. To avoid this type of events, we select only coincidence events; the probability of ionising two atoms in both PMTs at the same time is negligible in this case.

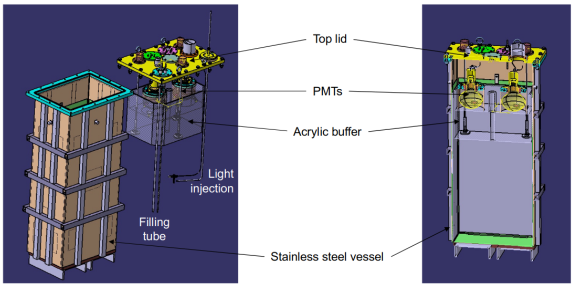

Figure 1: The Water Tank.

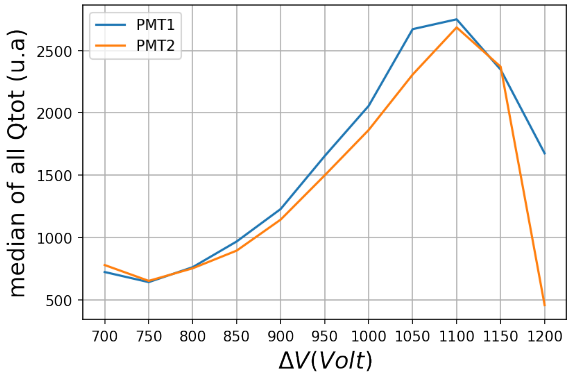

Figure 2: Evolution of the median of Q_{tot} in function of the applied voltage.

2.2 Data Analysis

The DAQ card provides us with the timing of each event, thus we can calculate Δt between two consecutive events. The sources of the events presented in our data are: the muons in the tank emitting Cherenkov light, the electrons (muon's daughter) in the tank emitting Cherenkov light, unstable radiative sources in the tank, spontaneous thermic photoelectron emission in the photocathode and ionized atoms in the PMTs. Reducing the background (unwanted events) was done by choosing carefully the threshold, the voltage, the coincidence as explained in section 2.1.

For now, we cannot differentiate between high energy muons; $E_\mu > 450MeV$; that most probably will not decay in the tank, and low energy muons; $450 > E_\mu > 159MeV$. Thus, the high energy muons will contribute to the background, and these events are characterised by Poisson distribution, in fact we consider the event "a muon enters the tank and emits Cherenkov light" as a process with no memory and a specific occurrence rate.

A priori, we know that τ_μ is equal to few μs, equivalent to a decay rate of the order of $5.10^5 s^{-1}$ thus, we expect the Δt distribution to be a negative exponential that will go to ≈ 0 around 10 to $20\mu s$ (assuming there is no background). The muonic background is also characterised by a negative exponential but with a different characteristic time ($>>$ few μs), the theoretical estimation we have calculated is 246 muons/s, while we measured 90 to 100 events/s. The measured rate depends on the threshold and the voltage chosen, as well as the thickness of the tank, material and many other factors. Still, the order of magnitude is coherent with our calculation. Once this background rate is fixed, the analysis will be performed in a range of $30\mu s$ as we will explain in the section 3.

To complete our model we have to take into consideration the radiative negative muon capture. μ^- capture is the capture of a negative muon by, most probably, an oxygen nucleus of a water molecule H_2O in our case, resulting in the disappearance of the muon and production of a neutron and a neutrino. Since the fraction of μ^+/μ^- in the cosmic shower is estimated to be between 1.2 and 1.6 because it depends on angle and energy of cosmic ray [5], we will define two new parameters r and Γ where r is the ratio μ^+/μ^- and Γ is the rate of μ^- capture. we consider that the muonic capture rate in water is the same as the capture rate of a muon in an oxygen atom Γ_O estimated around $10^5 s^{-1}$ [3]. This hypothesis is equivalent to say that all captured muons in the tank gets caught near an oxygen nucleus, neglecting the two hydrogen atoms effect (Z=1 vs Z=8). Finally our Final model is given by:

$$\frac{dN}{dt} = Ae^{-\Delta t/\tau}(r + e^{-\Gamma\Delta t}) + Ce^{-100\Delta t} \tag{4}$$

Where A is the Amplitude of the signal distribution and C the Amplitude of the background distribution are parameters of the fit while r and Γ are estimated to be 1.4 ± 0.2 and $1.02 \times 10^5 \pm 0.10 \times 10^5 s^{-1}$ respectively.

In the results section we will calculate τ and discuss the systematic errors coming from our approximations of r and Γ.

3 Results

Our data sample consists of around 14×10^6 events selected after triggering. In order to plot Δt distribution we should find the best binning and range to minimize the deviation between our model and the data. We applied the following procedure:

We start the fitting from the value of the second bin, because width of the first one could intercept with the deadtime of the detector (around 250 ns), then we fit the distribution by varying the binning from 40 to 200 bins and the upper range between 10 and $50\mu s$, we fixed also τ to PDG value, Γ at $1.02 \times 10^5 s^{-1}$ and ratio at 1.4.

Then for each fit we calculated $\frac{\chi^2}{\text{d.o.f}} = \frac{1}{d.o.f} \sum \frac{(y_{data} - y_{model})^2}{\sigma^2}$ (should be close to 1) where d.o.f means the degree of freedom = nb of points of data - nb of parameters, and the Cumulative distribution function (cdf) of $\frac{\chi^2}{\text{d.o.f}}$ (should be close to 0,5) using the function built in "scipy". When varying the number of bins, the χ^2 information is not enough, and so we need to specify the value of the cdf. However, once the binning is fixed, χ^2/dof is enough to compare the compatibility between the model and the data for different fits. Finally, we settled for : number of bins=100 and upper range= $30\mu s$

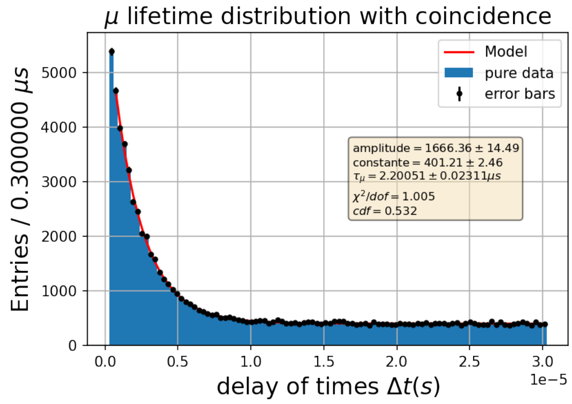

Figure 3: Fit of the life time distribution. We notice that the first bin is not fitted.

Figure 3 shows the performed fit on the Δt distribution, after choosing the considered binning, range and also fixing Γ to $1.02 \times 10^5 s^{-1}$ and ratio to 1.4. So we obtained $\tau = 2.201 \pm 0.023\mu s$. The statistical error is calculated by "scipy.optimize.curve_fit".

Systematic errors will be added to this result by studying the variation of τ in function of Γ and ratio, in fact fixing these parameters using previous studies values and some approximations, was necessary to obtain the best possible values of τ. Otherwise, we might have lot's of triplet $\{\tau, \Gamma, r\}$ that can fit the data since these three variables are correlated and their phase space can be non trivial and have a lot of local minimas. So fixing two of them will reduce the phase space to the point where $\tau = \tau_{nominal}$ projection of this point on the τ axis.

So we have studied the variation of τ, in function of these two parameters. Figure 4 shows that τ decreases when ratio increases. To interpret this observation we know that when ratio increases the fraction of μ^+ in the shower increases and thus the the number of captured muons decreases, so for the same data sample the τ fitted will decrease.

On the other hand, Figure 5 shows that for a fixed ratio, increasing Γ will lead to increase τ also, because the term $e^{-\Gamma t}$ decreases, so to compensate the τ has to increase.

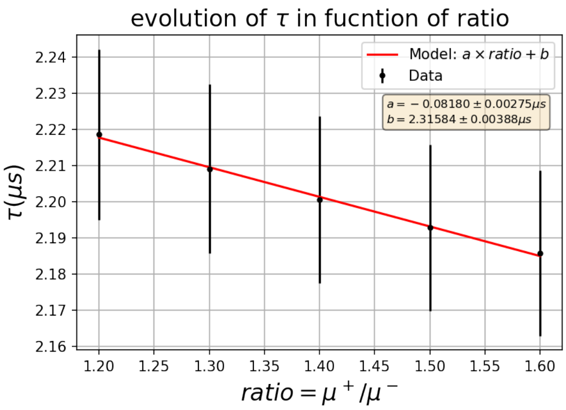

Figure 4: Variation of τ in function of ratio $\frac{\mu^+}{\mu^-}$.

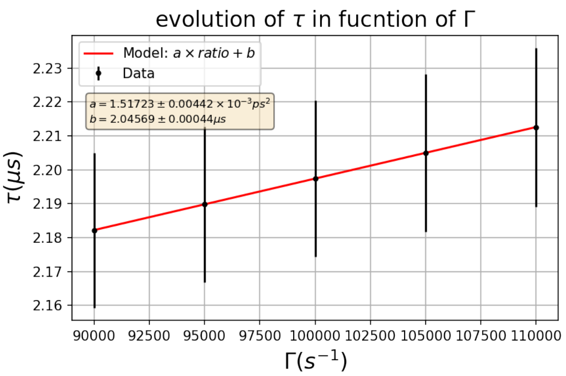

Figure 5: Variation of τ in function of Γ.

Our final result is then:

$$\tau = \tau_{nominal} \pm \Delta\tau_{statistical} \pm \Delta\tau_{systematic} \tag{5}$$

with

$$\Delta\tau_{systematic} = \sqrt{(\frac{\partial\tau}{\partial\Gamma}\Delta\Gamma)^2 + (\frac{\partial\tau}{\partial r}\Delta r)^2} \tag{6}$$

Then:

$$\tau = 2.201 \pm 0.023 \pm 0.045\mu s \tag{7}$$

4 Discussion

In this work, we search the muon life time using the Cherenkov effect in water tank. We were able through this study to discriminate between the positive muon life time and the negative muon lifetime. In reality the fact that μ^- can be captured by the medium molecules, contrary to μ^+, pushes us to define the life time of a particle within the medium itself, in our case water (H_2O). We can therefore assume that the calculated life time belong only to μ^+, while the life time of μ^- in this medium is calculated by inverting the sum of the radiative capture rate with the "normal" decay rate $\tau_{\mu^-} \approx 1/(10^5 + 1/2.10^{-6}) \approx 1.66\mu s$.

Our assumption that the rate of μ^- capture in water is equal to the same rate in O, can be seen as a good approximation but still needs more investigation. In fact this assumption can be convincing if one take into account that the interaction cross section is proportional to the magnitude of the nucleus wave function, thus since the wave function of oxygen is more spread out in the space comparing with the H's one, thus a muon is more likely to interact with the O rather than H. Assuming that all the other parameters of the model are well fitted, one can conclude from the variation of τ in function of Γ, that the exact value of the capture rate is not so far from the one assumed in our study, since the PDG value of τ, can be obtained by decreasing slightly Γ (if we want to reverse-engineer our parameters *a posteriori*).

Another source of the deviation between our values of τ and the PDG's one come from the ambiguity on the ratio μ^+/μ^-, in fact this value depends mainly on the energy scale of the muons so having a precise value is not achievable in the sens that not all the muons have the same energy. We can imagine a procedure to separate both muons as the following: Instead of using only one water tank we can use two tanks well isolated (so that only muons coming perpendicularly to the ground are caught, then by applying a horizontal electric field in the middle one can separate negative charged particles from the positive charged particles and thus each tank can serve then the measurement of even the (+) or (-) muons. Then with the same procedure one can estimate the ratio by measuring the number of events coming from each tank.

References

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