

# Deep Learning for ASL Recognition:

## Model Theory and Concepts

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# 1 Introduction

American Sign Language (ASL) is a visual language used by the Deaf and Hard-of-Hearing communities, primarily in the United States and parts of Canada. Unlike spoken languages, ASL relies heavily on hand gestures, facial expressions, and body movements to convey meaning. These non-verbal cues form the building blocks of communication within these communities. ASL is a rich, dynamic language with its own grammar, syntax, and nuances, and it is integral to the daily lives of millions of people. For those outside the Deaf and Hard-of-Hearing communities, however, interpreting ASL can be challenging, as it requires both familiarity with the specific signs and the broader context in which they are used.

ASL recognition — the process of automatically interpreting these signs and converting them into text or speech — plays a crucial role in bridging the communication gap between sign language users and non-users. By enabling real-time translation of ASL, this technology can make communication more accessible, inclusive, and effective in a variety of contexts, from educational settings to social interactions and professional environments.

With the rapid advancement of deep learning, ASL recognition systems have become increasingly sophisticated. Deep learning, especially through the use of neural networks, has revolutionized many fields by enabling machines to learn from large datasets and recognize intricate patterns within data. This approach has proven especially effective in ASL recognition, where models can automatically detect and interpret complex hand shapes, positions, orientations, and other key features required for accurate translation.

In this project, the focus is on the real-time recognition of static ASL alphabet signs captured through a live video feed. Each frame of the video is processed in real-time, detecting the hand shapes corresponding to individual letters of the ASL alphabet. The ability to interpret these signs accurately and instantly is essential for creating practical tools that can be used by sign language users in everyday settings.

At the core of this system is the use of Convolutional Neural Networks (CNNs), a class of deep learning models that excel in tasks involving visual data. CNNs are particularly well-suited for ASL recognition because they are designed to automatically detect spatial hierarchies and patterns within images, such as the unique features of hand shapes and movements. By applying CNNs to the live video feed, the system can effectively extract and interpret features that correspond to static ASL signs, making real-time ASL recognition a reality. The ultimate goal of this project is to advance the development of real-time ASL interpretation systems, contributing to the seamless communication between sign language users and others, enhancing accessibility and fostering inclusion.

## 2 Convolutional Neural Networks (CNNs)

Convolutional Neural Networks (CNNs) are a class of deep learning algorithms that have been extremely successful in various computer vision tasks. CNNs are designed to automatically and adaptively learn spatial hierarchies of features from input images. Unlike fully connected networks, CNNs use convolutional layers to capture spatial relationships in data by applying local receptive fields and weight sharing.

The key building blocks of CNNs are:

- **Convolutional Layers:** Apply a set of learnable filters (kernels) to the input image, extracting essential features.
- **Activation Functions:** Apply a non-linear operation to the output of the convolutional layer (e.g., ReLU) to introduce non-linearity.
- **Pooling Layers:** Downsample the feature maps to reduce spatial dimensions, improving computational efficiency and providing spatial invariance.
- **Dropout Layers:** Randomly deactivate neurons during training to prevent overfitting and help the model generalize better.
- **Batch Normalization:** Normalize activations across the mini-batch to stabilize training, improving convergence speed.
- **Global Pooling:** Reduce the feature map to a single value, summarizing the most important features and avoiding overfitting.
- **Fully Connected Layers:** Make predictions based on the learned features by connecting each neuron to every neuron in the previous layer.
- **Softmax Activation:** Convert the raw output of the model into a probability distribution, especially useful in multi-class classification problems.

## 2.1 Key Layers and Concepts

### 2.1.1 Convolutional Layer

Convolutional layers are the core building blocks of CNNs. A convolution operation applies a filter (also called a kernel) to the input image. This filter is a small matrix (often of size  $3 \times 3$ ,  $5 \times 5$ , etc.) that slides over the input image and computes a weighted sum at each spatial location. The purpose of the convolution is to extract features such as edges, textures, or other patterns in the image.

**Kernels (Filters)** are learnable parameters that are optimized during training. Each filter is designed to detect a specific feature in the input image. As the filter slides (or convolves) across the image, it produces a feature map that captures the presence of the detected feature in different parts of the image. By stacking multiple convolutional layers with different kernels, CNNs can learn increasingly abstract and hierarchical features from the data.

The size and number of filters (kernels) in each convolutional layer affect the model's ability to capture different types of features. Larger kernels capture more global information, while smaller kernels capture fine-grained local details.

**Why Convolution Works:** To understand why convolution filters are effective in feature extraction, consider the underlying operation. Convolution applies a filter (kernel) to the input image via matrix multiplication. This process enables the model to capture spatial relationships and patterns within the image.

A key aspect of convolution is that the filter (or kernel) identifies local spatial patterns by scanning small regions of the input image at each step. These regions often correspond to fundamental features, such as edges, textures, and corners, that recur throughout the image. Once these patterns are detected across the image, the network can learn more abstract representations from these localized details.

Mathematically, the convolution operation is described as a sliding window over the input image  $X$  with the kernel  $K$ . At each location, the kernel performs element-wise multiplication between the filter  $K$  and the corresponding region of the image  $X$ . The resulting products are then summed to produce a single value for the output feature map.

The convolutional operation for an input image  $X$  with kernel  $K$  is defined as:

$$Y = X * K = \sum_{i,j} X(i,j) \cdot K(i,j)$$

Where:

- $Y$  is the output feature map,
- $X(i,j)$  is the pixel value of the image at position  $(i,j)$ ,
- $K(i,j)$  is the value of the filter at position  $(i,j)$ ,
- $*$  denotes the convolution operation.

**Theoretical Rationale:** The filter matrix  $K$  is designed to highlight specific features in the image. When the filter slides over the image, it computes a weighted sum of pixel values at each position, effectively "measuring" the local structure of the image. This local measurement enables the network to learn patterns, such as edges or textures, in different spatial regions of the image. By learning multiple filters, the network can detect multiple types of features in different parts of the image simultaneously.

This ability to detect and capture hierarchical features, starting from simple patterns like edges in early layers to more complex shapes and textures in deeper layers, is what makes convolution so powerful in feature extraction. The fact that the same filter is applied across the whole image (shared weights) means the model is not sensitive to the location of features, enabling translation invariance, a key property for tasks like object recognition.

Additionally, by stacking multiple convolutional layers, CNNs can learn increasingly abstract and complex features, as each subsequent layer combines the simpler features detected in previous layers into more meaningful patterns.

**Example:** Consider the following input image  $X$  and kernel  $K$ :

$$X = \begin{bmatrix} 105 & 102 & 100 \\ 103 & 99 & 103 \\ 101 & 98 & 104 \end{bmatrix}, \quad K = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

To compute the convolution of  $X$  and  $K$ , we perform element-wise multiplication and sum the results:

$$Y = (105 \cdot 0 + 102 \cdot (-1) + 100 \cdot 0) + (103 \cdot (-1) + 99 \cdot 5 + 103 \cdot (-1)) + (101 \cdot 0 + 98 \cdot (-1) + 104 \cdot 0)$$

$$Y = (0 - 102 + 0) + (-103 + 495 - 103) + (0 - 98 + 0)$$

$$Y = -102 + 289 - 98 = 89$$

Thus, the output feature map value at the position of the kernel is  $Y = 89$ .

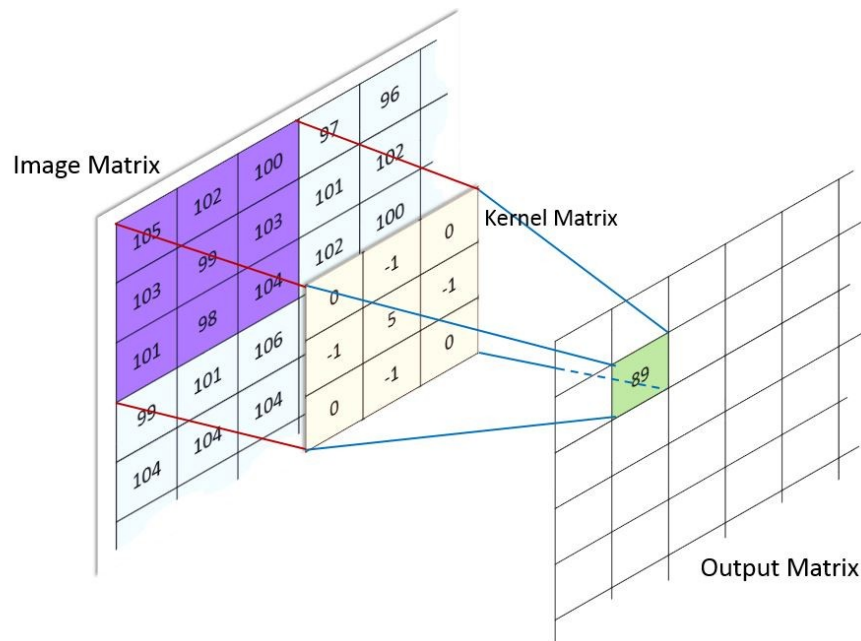


Figure 1: Convolutional Filter

### 2.1.2 Stride and Padding

Two important concepts in convolutional layers are **stride** and **padding**, which control how the convolution operation processes the input data.

**Stride:** The stride determines the step size of the convolutional filter as it moves across the input. A larger stride skips more pixels, effectively reducing the spatial dimensions of the output feature map. The stride is denoted as  $s$ , and the output size is given by:

$$\text{Output size} = \frac{\text{Input size} - \text{Filter size}}{s} + 1$$

**Example:** If we apply a  $3 \times 3$  filter to a  $5 \times 5$  input with:

- Stride  $s = 1$ : The filter slides one pixel at a time, producing a  $3 \times 3$  output.
- Stride  $s = 2$ : The filter skips one pixel at each step, producing a  $2 \times 2$  output.

**Padding:** Padding refers to adding extra border pixels (often zeros) around the input. This helps control the output size and ensures that the filter can process the edges of the input. There are two common types of padding:

- **Valid Padding:** No padding is applied, and the output size shrinks based on the filter size.
- **Same Padding:** Padding is added so that the output size matches the input size.

The output size with padding  $p$  is calculated as:

$$\text{Output size} = \frac{\text{Input size} + 2p - \text{Filter size}}{s} + 1$$

**Example:** If a  $3 \times 3$  filter is applied to a  $5 \times 5$  input:

- Without padding ( $p = 0$ ): The output size is  $(5 - 3 + 1) = 3 \times 3$ .
- With  $p = 1$  (same padding): The input size becomes  $7 \times 7$ , and the output size remains  $5 \times 5$ .

**Visualization:**

- Without padding (*valid padding*):

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- With padding (*same padding*):

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



By adjusting stride and padding, convolutional layers can control the size of the output feature map and manage the computational cost while preserving critical spatial information in the input.

### 2.1.3 ReLU Activation Function

The Rectified Linear Unit (ReLU) is one of the most commonly used activation functions in CNNs. It introduces non-linearity to the network, allowing it to learn complex patterns. The ReLU activation function is defined as:

$$\text{ReLU}(x) = \max\{0, x\}$$

This operation outputs  $x$  if  $x$  is positive and 0 otherwise. ReLU helps the network converge faster during training by reducing the likelihood of the vanishing gradient problem, where gradients become too small to propagate through the network during backpropagation.

**Example:** Given the input vector:

$$x = [-3, -1, 0, 2, 4]$$

The output after applying the ReLU function is:

$$\text{ReLU}(x) = [0, 0, 0, 2, 4]$$

### 2.1.4 Pooling Layer

Pooling layers are used to reduce the spatial dimensions of the feature maps while retaining important information. Two common types of pooling are:

- **Max Pooling:** Selects the maximum value from each region of the feature map.
- **Average Pooling:** Computes the average value of each region.

Max pooling is more common, and it is typically used to downsample the feature map, reducing the computational load and making the model invariant to small translations of the input.

The operation for max pooling with a kernel size of 2x2 is defined as:

$$Y(i, j) = \max\{X(2i, 2j), X(2i + 1, 2j), X(2i, 2j + 1), X(2i + 1, 2j + 1)\}$$

Where:

- $Y$  is the pooled feature map,
- $X$  is the input feature map.

**Example:** given a  $4 \times 4$  feature map as input:

$$X = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 4 & 6 & 5 & 3 \\ 7 & 8 & 9 & 6 \\ 2 & 4 & 3 & 8 \end{bmatrix}$$

After applying max pooling with a  $2 \times 2$  kernel:

$$Y = \begin{bmatrix} 6 & 5 \\ 8 & 9 \end{bmatrix}$$

The pooling operation selects the maximum value from each  $2 \times 2$  region, reducing the spatial dimensions from  $4 \times 4$  to  $2 \times 2$ .

This dimensionality reduction not only speeds up the computation but also helps to prevent overfitting by discarding non-critical details.

### 2.1.5 Dropout Layer

Dropout is a regularization technique used to prevent overfitting in deep neural networks. During training, dropout randomly disables a fraction of the neurons in the network. This encourages the network to learn more robust and redundant representations of the data, preventing reliance on any one specific neuron. The fraction of neurons to be dropped is controlled by the *dropout rate*, a hyperparameter.

Mathematically, the output  $y$  of a neuron with dropout is:

$$y = \frac{z}{p}$$

Where:

- $z$  is the output of the neuron before applying dropout,
- $p$  is the probability of retaining the neuron (i.e.,  $1 - \text{dropout rate}$ ).

During training:

- Each neuron is retained with probability  $p$ ,
- If a neuron is dropped, its output is set to zero.

During inference:

- Dropout is not applied, but the weights of all neurons are scaled by  $p$ . This ensures that the expected output remains consistent between training and inference.

### Why Dropout Works:

- **Reduces Overfitting:** By randomly dropping neurons during training, dropout forces the network to rely on a distributed representation of the data, improving generalization.
- **Implicit Ensembling:** Dropout can be interpreted as training an ensemble of sub-networks. During training, each forward pass operates on a different subset of neurons, which helps the model learn diverse features. In inference, all neurons are used, but their contributions are scaled, effectively averaging the predictions of the ensemble.

**Example:** consider a fully connected layer with 4 neurons:

$$\text{Input} \rightarrow [h_1, h_2, h_3, h_4] \rightarrow \text{Dropout}(p = 0.5) \rightarrow \text{Output}$$

During training, dropout might randomly drop  $h_2$  and  $h_4$ , resulting in:

$$\text{Active neurons: } [h_1, h_3]$$

The outputs of  $h_1$  and  $h_3$  are scaled by  $\frac{1}{0.5}$  to maintain the expected sum of activations.

This simple yet effective technique makes deep neural networks less prone to overfitting and more capable of generalizing to unseen data.

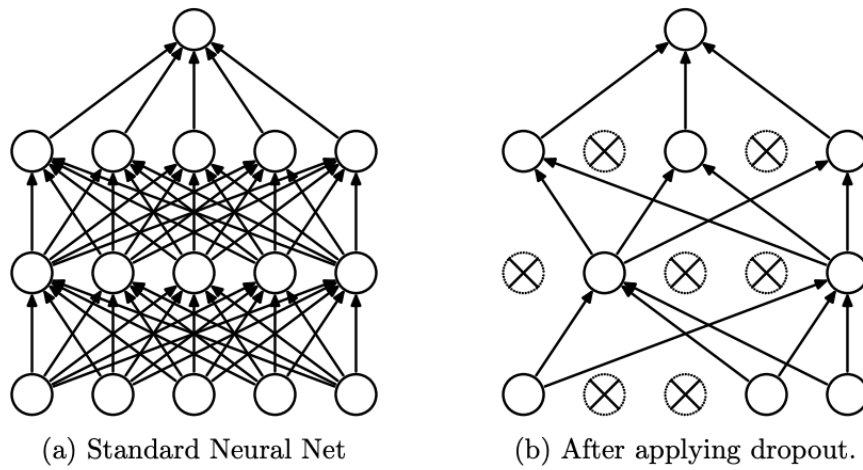


Figure 2: Dropout Layer

### 2.1.6 Mini-Batches

A mini-batch is a small subset of the training dataset used in one iteration of training a machine learning model. The dataset is divided into these smaller groups to optimize computational efficiency and model performance.

In **mini-batch gradient descent**, instead of using the entire dataset (batch gradient descent) or a single example (stochastic gradient descent), gradients are computed and weights are updated for each mini-batch.

Key advantages of using mini-batches include:

- **Faster Training:** Mini-batches allow for more frequent updates compared to batch gradient descent.
- **Memory Efficiency:** Only a mini-batch needs to be loaded into memory, which is beneficial when working with large datasets.
- **Stable Convergence:** Compared to stochastic gradient descent, mini-batch updates are less noisy and provide smoother convergence.

For example, if a dataset contains 10,000 samples and the mini-batch size is 100, the model processes 100 samples per iteration, resulting in 100 iterations per epoch (i.e., one pass over the dataset).

### 2.1.7 Batch Normalization

Batch Normalization is a technique used to normalize the activations of neurons across a mini-batch. It reduces the internal covariate shift, improving both the training speed and stability. Batch normalization is typically applied after the linear or convolutional layer and before the activation function.

The output of the batch normalization layer is defined as:

$$y = \frac{x - \mu}{\sigma} \cdot \gamma + \beta$$

Where:

- $x$  is the input to the batch normalization layer,
- $\mu$  is the mean of the mini-batch,
- $\sigma$  is the standard deviation of the mini-batch,
- $\gamma$  and  $\beta$  are learnable parameters that allow scaling and shifting of the normalized output.

#### Example:

Consider a mini-batch of 3 examples with a single feature for simplicity:

$$X = [2, 4, 6]$$

The mean  $\mu$  and standard deviation  $\sigma$  of the mini-batch are calculated as:

$$\mu = \frac{2 + 4 + 6}{3} = 4, \quad \sigma = \sqrt{\frac{(2 - 4)^2 + (4 - 4)^2 + (6 - 4)^2}{3}} = 1.632$$

Batch normalization then normalizes each value in the mini-batch:

$$y_1 = \frac{2 - 4}{1.632} = -1.225, \quad y_2 = \frac{4 - 4}{1.632} = 0, \quad y_3 = \frac{6 - 4}{1.632} = 1.225$$

Next, scaling and shifting are applied using  $\gamma$  and  $\beta$ , which are learnable parameters (e.g.,  $\gamma = 1$ ,  $\beta = 0$ ):

$$y_1 = (-1.225 \cdot 1) + 0 = -1.225, \quad y_2 = (0 \cdot 1) + 0 = 0, \quad y_3 = (1.225 \cdot 1) + 0 = 1.225$$

This process ensures that the outputs of the batch normalization layer are normalized, making the network more stable and faster during training.

### 2.1.8 Global Pooling Layer

Global pooling is used to reduce the dimensions of a feature map to a single value, summarizing the most important feature. Global average pooling computes the average value of the entire feature map:

$$Y = \frac{1}{H \times W} \sum_{i=1}^H \sum_{j=1}^W X(i, j)$$

Where:

- $H$  and  $W$  are the height and width of the feature map,
- $X(i, j)$  is the value at the position  $(i, j)$  in the feature map.

Global max pooling works similarly, but instead of averaging, it takes the maximum value across the entire feature map.

**Example:** Given a  $4 \times 4$  feature map  $X$ :

$$X = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

Global average pooling computes the average of all values:

$$Y = \frac{1}{4 \times 4} \sum_{i=1}^4 \sum_{j=1}^4 X(i, j) = \frac{1}{16} \times 136 = 8.5$$

### 2.1.9 Fully Connected Layer

The fully connected (dense) layer connects each neuron in the current layer to every neuron in the previous layer. It is used to make predictions based on the features learned by previous layers. The output of the fully connected layer is given by:

$$y = W \cdot x + b$$

Where:

- $W$  is the weight matrix,
- $x$  is the input vector,
- $b$  is the bias vector.

**Example:** Consider a fully connected layer with 3 input neurons, 2 output neurons, and the following weights and biases:

$$W = \begin{bmatrix} 0.2 & 0.4 & 0.6 \\ 0.5 & 0.1 & 0.3 \end{bmatrix}, \quad b = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

And an input vector  $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ .

The output of the fully connected layer is:

$$y_1 = (0.2 \times 1) + (0.4 \times 2) + (0.6 \times 3) + 0.1 = 2.6$$

$$y_2 = (0.5 \times 1) + (0.1 \times 2) + (0.3 \times 3) + 0.2 = 1.8$$

Thus, the output of the fully connected layer is  $y = \begin{bmatrix} 2.6 \\ 1.8 \end{bmatrix}$ .

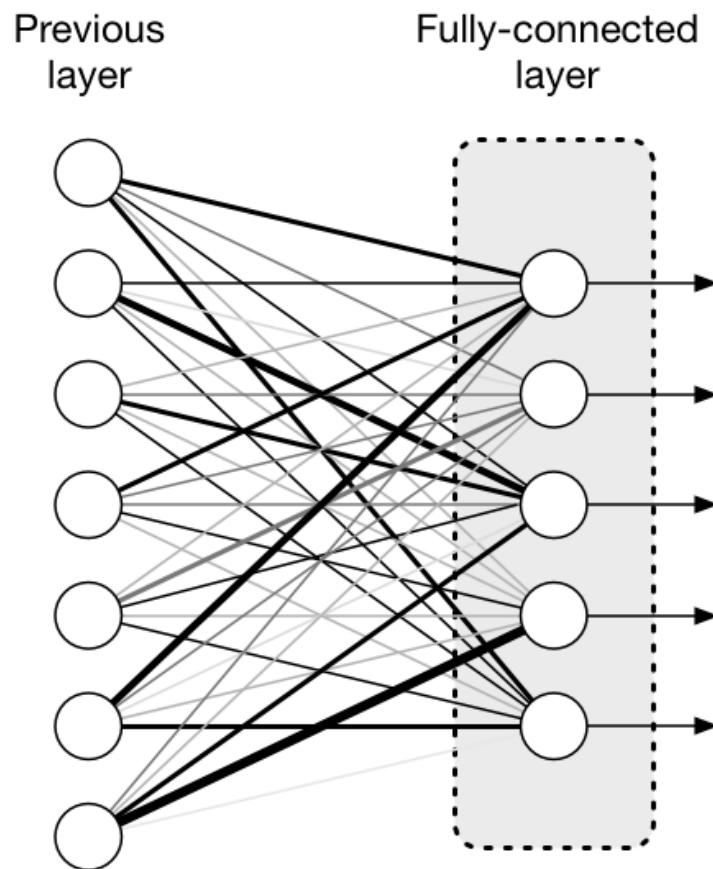


Figure 3: Fully Connected Layer

### 2.1.10 Softmax Activation Function

Softmax is commonly used in the output layer of neural networks, especially for classification tasks, to convert the raw output values (logits) into probabilities. This transformation ensures that the network's outputs are interpretable as probabilities, as they sum to 1 and lie within the range  $[0, 1]$ . This is important because, in classification tasks, we want to assign a probability to each class, which can then be used for decision-making.

The Softmax function is defined as:

$$\text{Softmax}(z_i) = \frac{e^{z_i}}{\sum_j e^{z_j}}$$

Where:

- $z_i$  is the logit for the  $i$ -th class (raw output of the model),
- $\sum_j e^{z_j}$  is the sum of the exponentials of all logits, serving as a normalization factor to ensure the output values sum to 1.

#### Properties of Softmax:

- *Probability Distribution*: Converts logits into a valid probability distribution, with all outputs summing to 1.
- *Amplification of Differences*: The exponential function emphasizes larger logits, making predictions more confident while reducing the impact of smaller logits.
- *Differentiability*: The function is smooth and differentiable, allowing effective optimization during backpropagation.
- *Interpretability*: Outputs represent class probabilities, making predictions straightforward to interpret.

**Example:** Consider a vector of raw outputs (logits)  $z = \begin{bmatrix} 2.0 \\ 1.0 \\ 0.1 \end{bmatrix}$ .

First, compute the exponentials of the logits:

$$e^{z_1} = e^{2.0} \approx 7.389, \quad e^{z_2} = e^{1.0} \approx 2.718, \quad e^{z_3} = e^{0.1} \approx 1.105$$

Next, compute the sum of exponentials:

$$\sum_j e^{z_j} = 7.389 + 2.718 + 1.105 = 11.212$$

Finally, apply the softmax function to each logit:

$$\text{Softmax}(z_1) = \frac{7.389}{11.212} \approx 0.659, \quad \text{Softmax}(z_2) = \frac{2.718}{11.212} \approx 0.242, \quad \text{Softmax}(z_3) = \frac{1.105}{11.212} \approx 0.099$$

The output of the softmax function is  $\begin{bmatrix} 0.659 \\ 0.242 \\ 0.099 \end{bmatrix}$ , which represents the probabilities for each class.

### 2.1.11 Sigmoid Activation Function

The Sigmoid activation function is widely used in neural networks, particularly in binary classification tasks or as an activation in intermediate layers. It maps input values to a range of  $(0, 1)$ , which can be interpreted as probabilities in binary classification problems.

The Sigmoid function is defined as:

$$\text{Sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

Where:

- $x$  is the input to the function, which could be the weighted sum of inputs in a neural network layer,
- $e^{-x}$  is the exponential term that ensures smooth, non-linear mapping.

#### Properties of Sigmoid:

- *Range*: The output of the Sigmoid function lies in the range  $(0, 1)$ , making it suitable for representing probabilities.
- *Non-linearity*: It introduces non-linearity to the model, allowing it to capture complex relationships between inputs and outputs.
- *Smooth Gradient*: The function is differentiable, and its gradient is smooth, which facilitates optimization during backpropagation.
- *Vanishing Gradient*: For very large or very small input values, the gradient approaches zero, which can slow down learning in deep networks.

**Example:** Consider an input  $x = 2.0$ :

First, compute the exponential term:

$$e^{-x} = e^{-2.0} \approx 0.135$$

Next, apply the Sigmoid function:

$$\text{Sigmoid}(x) = \frac{1}{1 + 0.135} \approx 0.881$$

Thus, the Sigmoid activation outputs 0.881 for an input of 2.0.



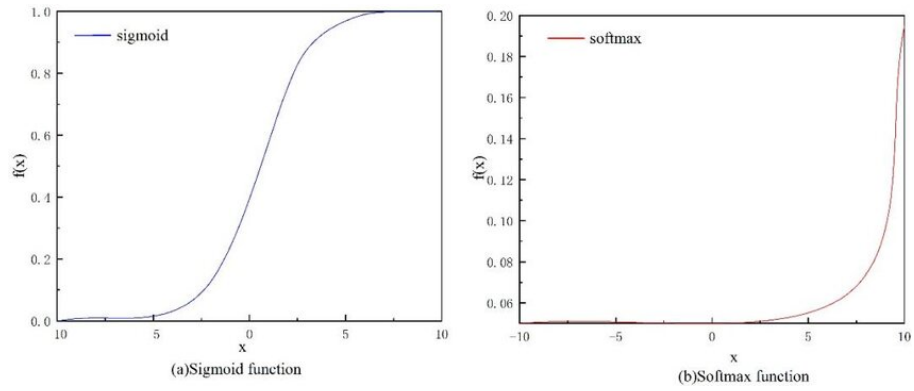


Figure 4: Sigmoid vs. Softmax Comparison

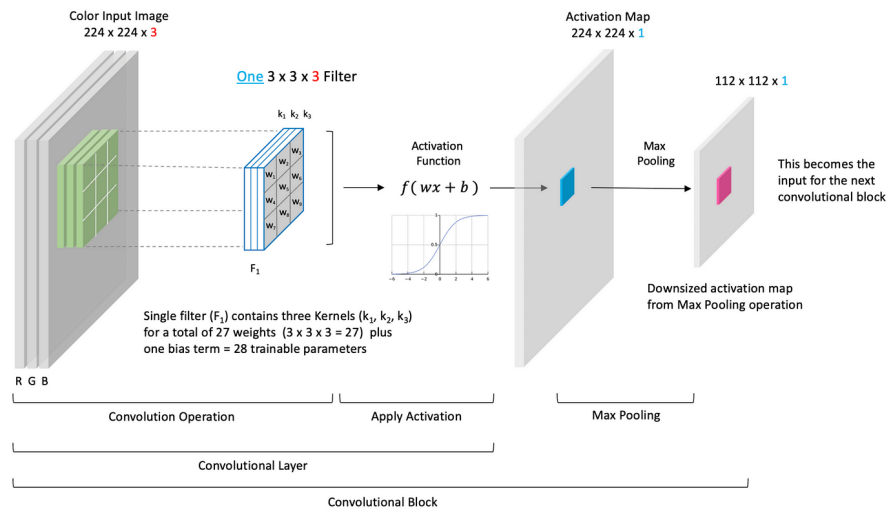


Figure 5: Convolutional Block

## 2.2 Advanced Convolutional Models

This section discusses two prominent neural network architectures, **MobileNetV2** and **ResNet18**, both designed for efficient and effective deep learning applications.

### 2.2.1 MobileNetV2

MobileNetV2 is a lightweight neural network architecture optimized for mobile and embedded devices. It utilizes **depthwise separable convolutions**, which split the convolution operation into two parts:

1. **Depthwise Convolutions:** Applies a separate filter to each input channel, effectively reducing the computation by processing each channel independently.
2. **Pointwise Convolutions:** Applies a  $1 \times 1$  convolution across all input channels to combine them into the desired number of output channels.

This approach significantly reduces computational costs compared to standard convolutions, enhancing efficiency. The mathematical representation of these convolutions is:

$$\text{Depthwise Output} = X * K_{\text{depthwise}}$$

$$\text{Pointwise Output} = \text{Depthwise Output} * K_{\text{pointwise}}$$

Additionally, MobileNetV2 introduces a **linear bottleneck layer**, which compresses the feature space by reducing the number of intermediate channels before restoring them with a final  $1 \times 1$  convolution. This design minimizes computational costs while retaining essential information.

Unlike traditional layers that apply non-linear activations after every operation, the bottleneck layer avoids using non-linearity after the final  $1 \times 1$  convolution. This ensures that no critical information is lost during the compression step, enabling the network to better preserve and utilize features in subsequent layers.

**Example:** Consider an input tensor of size  $128 \times 128 \times 32$  (spatial dimensions  $128 \times 128$ , with 32 channels). The operation of the linear bottleneck layer can be broken down as follows:

1. **Expansion (Pointwise Convolution):** A  $1 \times 1$  convolution is applied to expand the number of channels from 32 to 128. This increases the representation capacity of the feature map. The output tensor is now of size  $128 \times 128 \times 128$ .
2. **Depthwise Convolution:** A  $3 \times 3$  depthwise convolution is applied to the expanded tensor, processing each of the 128 channels independently. This operation reduces the spatial dimensions slightly (depending on stride and padding), resulting in an output tensor of size  $126 \times 126 \times 128$  (assuming a stride of 1 and no padding).
3. **Compression (Pointwise Convolution):** A second  $1 \times 1$  convolution is applied to reduce the number of channels from 128 back down to 64. This compression step retains the most important features while minimizing the computational cost, resulting in a final tensor of size  $126 \times 126 \times 64$ .

The linear bottleneck design helps to maintain the critical feature information while reducing the computational burden, making MobileNetV2 efficient for mobile and embedded applications. The operations are optimized to balance model performance and computational efficiency, ensuring that the model can run effectively on devices with limited resources.

### 2.2.2 ResNet18

ResNet18, a variation of the ResNet (Residual Network) architecture, addresses the vanishing gradient problem by introducing **residual connections**. These connections provide a direct path for gradient flow, bypassing intermediate layers and simplifying the training of deeper networks.

A residual block in ResNet18 can be defined mathematically as:

$$y = F(x, \{W_i\}) + x$$

Where:

- $x$  is the input to the block,
- $F(x, \{W_i\})$  represents the operations applied to  $x$ , such as convolutions, batch normalization, and activation functions,
- $y$  is the output of the block, combining  $F(x, \{W_i\})$  with the original input  $x$ .

**Example:** A typical residual block includes:

- A  $3 \times 3$  convolution layer that applies  $F(x)$ ,
- Batch normalization and activation (e.g., ReLU),
- A shortcut connection that directly adds  $x$  to  $F(x)$ .

If the input tensor  $x$  has dimensions  $64 \times 64 \times 64$  (spatial dimensions  $64 \times 64$ , with 64 channels), the residual block performs:

- Convolution: A  $3 \times 3$  convolution changes spatial dimensions (depending on stride and padding).
- Addition: The shortcut connection ensures the input dimensions match the output dimensions (padding or  $1 \times 1$  convolution might be applied if needed).

The output tensor maintains the same dimensions,  $64 \times 64 \times 64$ , facilitating smooth gradient flow.

## 2.3 Attention Mechanisms

Attention mechanisms allow the model to focus on the most relevant parts of the input. In the case of CNNs, attention can be applied to channels or spatial locations.

### 2.3.1 Channel Attention

The channel attention mechanism aims to assign a weight to each channel of the input tensor  $X$  based on its importance in contributing to the model's output. This can be calculated as:

$$\mathbf{z} = \sigma(\text{FC}_2(\text{ReLU}(\text{FC}_1(\text{Global Average Pooling}(X))))))$$

Where:

- $\sigma$  is the sigmoid activation function,
- $\text{FC}_1, \text{FC}_2$  are fully connected layers,
- $X$  is the input tensor.

The final output of the channel attention mechanism is obtained by element-wise multiplication of the input tensor  $X$  with the attention weights  $\mathbf{z}$ :

$$\text{Attended Output} = X \times \mathbf{z}$$

This means that each channel in the input tensor is scaled according to its importance, allowing the model to focus on the most informative channels while suppressing irrelevant ones.

**Why It Works:** The channel attention mechanism works because it helps the network focus on the most relevant features by assigning higher importance to certain channels based on the input data. In a convolutional neural network (CNN), each channel captures different features, but not all of them contribute equally to the task at hand. By dynamically adjusting the importance of channels, the network can emphasize the most informative features and suppress irrelevant ones.

This is particularly beneficial in tasks like image classification, where certain patterns (e.g., edges, textures) are more important than others. By focusing on these key channels, the model can learn more discriminative features, improving its performance. Additionally, by leveraging global context through techniques like global average pooling, the model can capture larger, more complex patterns that may span across different spatial regions of the image, leading to better overall feature representation. This makes the model more effective at recognizing meaningful structures in the data.

### 3 Optimization and Loss Functions

In machine learning, a **loss function** (also called a **cost function**) is a mathematical function that measures the difference between the predicted output of a model and the true target output. The goal during training is to minimize this loss, which indicates that the model's predictions are becoming more accurate. The loss function is crucial in guiding the optimization process, helping the model adjust its parameters (such as weights and biases in neural networks) to improve performance. In most cases, the loss function is minimized using optimization techniques like **gradient descent**.

The choice of loss function depends on the type of task being performed. For regression tasks, common loss functions include Mean Squared Error (MSE), while for classification tasks, Cross-Entropy Loss is typically used.

#### 3.1 Cross-Entropy Loss

In classification tasks, the **cross-entropy loss** is used to measure the difference between the predicted and actual labels. It is particularly effective for tasks where the goal is to output probabilities, such as in multi-class classification. The loss function quantifies how far the predicted probability distribution ( $\hat{y}$ ) is from the true distribution ( $y$ ).

For a multi-class classification problem, the cross-entropy loss is defined as:

$$\mathcal{L} = - \sum_{i=1}^C y_i \log(\hat{y}_i)$$

Where:

- $C$  is the number of classes,
- $y_i$  is the true label (one-hot encoded),
- $\hat{y}_i$  is the predicted probability of class  $i$ .

This loss function works by penalizing the model more heavily when the predicted probability for the correct class ( $\hat{y}_i$ ) is low, and less when it is high. The use of the logarithm (log) magnifies the impact of incorrect predictions, particularly when the model is confident but wrong.

The logarithmic term serves to:

- Amplify the penalty for predictions far from the true label, as the log function grows negative rapidly when  $\hat{y}_i$  is close to 0.
- Penalize confident but incorrect predictions heavily, making it clear to the model that it needs to avoid high confidence in the wrong class.
- Provide a smooth gradient for optimization, ensuring stable learning during training.

Intuitively, the closer the predicted probability for the correct class is to 1, the smaller the loss becomes. Conversely, if the model assigns a low probability to the correct class (i.e., a value close to 0), the loss increases significantly, pushing the model to correct its prediction in the next iteration. This behavior helps the model improve its predictions over time, gradually becoming more confident in the correct classes.

## 3.2 Gradient Descent

Gradient descent is an optimization algorithm used to minimize the loss function by iteratively adjusting the model parameters in the direction that reduces the error or loss. This method works by computing the gradient (or derivative) of the loss function with respect to the model parameters and updating them in the opposite direction of the gradient, which leads to a decrease in the loss over time.

The core idea behind gradient descent is to move towards the minimum of the loss function, which corresponds to the optimal parameters for the model. In practice, the update rule for the parameters is:

$$\theta = \theta - \eta \cdot \frac{\partial \mathcal{L}}{\partial \theta}$$

Where:

- $\theta$  represents the model parameters, which can include weights and biases in machine learning models.
- $\eta$  is the learning rate, a hyperparameter that controls the step size during each update. A smaller  $\eta$  makes the model update more slowly, while a larger  $\eta$  makes the updates more aggressive, which can sometimes lead to overshooting the optimal solution.
- $\frac{\partial \mathcal{L}}{\partial \theta}$  is the gradient of the loss function  $\mathcal{L}$  with respect to the model parameters  $\theta$ . The gradient indicates the direction and rate of the fastest increase in the loss function. By subtracting this value, we move in the direction that minimizes the loss.

The gradient descent algorithm is applied iteratively, and at each step, the model parameters are updated based on the computed gradient. This continues until the algorithm converges, meaning that the updates become very small, or it reaches a predefined number of iterations.

**Why it works:** The reason gradient descent works is that the gradient points in the direction of the steepest ascent of the loss function. Since we are interested in minimizing the loss, we take steps in the opposite direction of the gradient, which gradually reduces the loss function. By adjusting the parameters in small increments (controlled by the learning rate), the model converges to the local (or global, in convex problems) minimum.

**Convergence Issues:** While gradient descent is effective, choosing the right learning rate is crucial. If  $\eta$  is too large, the algorithm may not converge or may oscillate around the minimum. If  $\eta$  is too small, the convergence can be very slow. Techniques like learning rate scheduling or adaptive methods (e.g., Adam, RMSprop) can help mitigate such issues.

**Example:** Suppose we are optimizing a simple linear regression model with one parameter  $\theta$ , and the loss function is the mean squared error (MSE):

$$\mathcal{L}(\theta) = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

Where:

- $y_i$  is the true value,

- $\hat{y}_i = \theta x_i$  is the predicted value, and
- $N$  is the number of data points.

To update the parameter  $\theta$ , we calculate the gradient of the loss function with respect to  $\theta$ :

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{-2}{N} \sum_{i=1}^N x_i (y_i - \theta x_i)$$

Using this gradient, the update rule becomes:

$$\theta = \theta - \eta \cdot \frac{-2}{N} \sum_{i=1}^N x_i (y_i - \theta x_i)$$

At each iteration,  $\theta$  is updated in the direction opposite to the gradient. This process continues until  $\theta$  converges to the optimal value that minimizes the MSE.

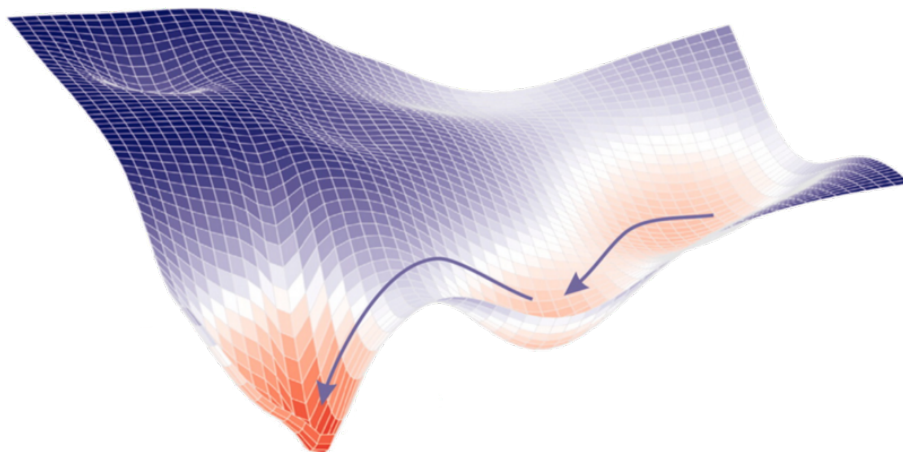


Figure 6: Gradient Descent

### 3.3 Adam Optimizer

The Adam (Adaptive Moment Estimation) optimizer is an extension of the standard gradient descent algorithm that adapts the learning rate for each parameter by taking into account both the first and second moments of the gradients. This adaptation allows Adam to handle sparse gradients and noisy problems more effectively, making it a very popular optimization algorithm in deep learning.

Adam combines two key concepts:

- **Momentum:** It uses the first moment (mean) of the gradients to give more weight to recent gradients, thus accelerating convergence.
- **RMSprop (Root Mean Square Propagation):** It uses the second moment (variance) of the gradients to normalize the gradient, addressing the issue of varying gradient magnitudes.

**Update Rules:** Adam maintains two moment estimates for each parameter:

- The **first moment**  $m_t$  (mean of gradients), which helps to smooth out the gradients and prevent oscillations.
- The **second moment**  $v_t$  (variance of gradients), which stabilizes the updates by scaling with the variance of the gradients.

The update rules for the first and second moment estimates are as follows:

$$\begin{aligned}m_t &= \beta_1 m_{t-1} + (1 - \beta_1) \nabla_{\theta} \mathcal{L} \\v_t &= \beta_2 v_{t-1} + (1 - \beta_2) \nabla_{\theta}^2 \mathcal{L}\end{aligned}$$

Where:

- $m_t$  is the first moment estimate (mean of gradients), which helps to capture the moving average of the gradients.
- $v_t$  is the second moment estimate (variance of gradients), which captures the moving average of the squared gradients.
- $\beta_1$  and  $\beta_2$  are decay rates (typically set to values like 0.9 and 0.999, respectively), which control the exponential decay of the moving averages.
- $\nabla_{\theta} \mathcal{L}$  is the gradient of the loss function with respect to the model parameters  $\theta$ .

**Parameter Update:** The final parameter update rule in Adam is given by:

$$\theta = \theta - \frac{\eta \cdot m_t}{\sqrt{v_t} + \epsilon}$$

Where:

- $\eta$  is the learning rate, controlling the step size.
- $\epsilon$  is a small constant (typically  $10^{-8}$ ) added to avoid division by zero.
- $m_t$  and  $v_t$  are the first and second moment estimates that are used to adjust the learning rate for each parameter dynamically.



**Why Adam Works:** Adam is special because it adapts the learning rate for each parameter individually. This means:

- **Adaptive Learning Rate:** The learning rate is adjusted based on the gradient's history for each parameter, which helps handle problems with sparse gradients or noisy datasets.
- **Bias Correction:** Since the moment estimates are initialized to zero, they are biased during the early training iterations. Adam corrects this bias by applying bias correction terms:

$$\hat{m}_t = \frac{m_t}{1 - \beta_1^t}, \quad \hat{v}_t = \frac{v_t}{1 - \beta_2^t}$$

This correction ensures that the moment estimates are unbiased and the learning rate update is more stable early in training.

**What Makes Adam Special:** Adam combines the benefits of both momentum (via  $m_t$ ) and adaptive learning rates (via  $v_t$ ), making it robust to noisy data and sparse gradients. It is computationally efficient, has little memory overhead, and performs well in practice for a wide range of machine learning tasks, particularly in training deep neural networks.

Additionally, Adam tends to converge faster than traditional gradient descent methods, which is particularly helpful in large-scale datasets and complex models where convergence speed is critical.

In practice, Adam is often the default optimizer in many deep learning frameworks due to its effectiveness and ease of use. Its ability to handle varying learning rates for different parameters and adjust those rates over time makes it a highly reliable choice for optimizing complex neural networks.

## 4 Application to ASL Recognition

### 4.1 Dataset

For ASL recognition, the dataset typically consists of images of hands performing different signs. These images are preprocessed to resize them to a consistent size and normalized. In order to enhance model generalization and prevent overfitting, data augmentation techniques are applied to artificially expand the dataset. These techniques include random rotation, flipping, scaling, and other transformations.

### 4.2 Data Augmentation

Data augmentation helps to create a more robust model by introducing slight variations of the images during training, which makes the model more invariant to changes in the input. The following data augmentation techniques are employed:

- **Flipping:** Horizontal flipping is applied with a 50% probability to mimic mirror-image variations of signs.
- **Scaling:** The image is resized randomly within a predefined scale factor, which helps the model adapt to signs performed at different distances from the camera.
- **Translation:** Random shifts in the horizontal and vertical directions help to simulate different positions of the hand within the frame.

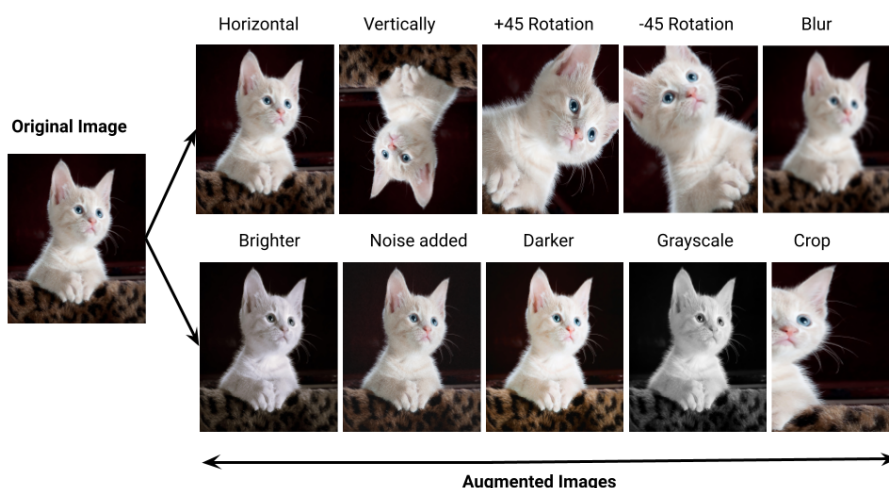


Figure 7: Data Augmentation

### 4.3 Model Architecture for ASL Recognition

The models used for ASL recognition in this project include:

#### 4.3.1 Attention Mechanism - Channel Attention Module

The **Channel Attention Module** is an essential component used to enhance the performance of the models. The Channel Attention mechanism helps the model focus on the most informative channels by adaptively recalibrating channel-wise feature responses. This is achieved through the following steps:

1. The input feature map is first passed through an adaptive average pooling layer to aggregate global information.
2. The pooled features are then passed through a two-layer fully connected network (implemented as convolutions with kernel size 1) to capture channel dependencies.
3. A sigmoid activation function is applied to the output of the fully connected layers to produce a set of channel-wise attention weights.
4. These attention weights are multiplied element-wise with the original input feature map, re-weighting the channels based on their importance.

This attention mechanism enables the model to emphasize relevant features and suppress less informative ones, which can lead to improved recognition accuracy, especially in the case of complex and subtle variations in ASL signs.

#### 4.3.2 CustomCNN

The **CustomCNN** model is a convolutional neural network designed specifically for ASL recognition. The architecture of CustomCNN consists of several convolutional layers followed by fully connected layers for classification. The convolutional layers are equipped with batch normalization and attention mechanisms to enhance feature learning and spatial hierarchies. After each convolutional layer, a corresponding *Channel Attention Module* is applied, which allows the network to focus on the most informative channels, thus improving the network's ability to identify subtle patterns in the ASL signs. The model incorporates dropout regularization to mitigate overfitting and is particularly effective in recognizing spatial patterns in images of ASL signs. The final output is a classification layer that maps the extracted features to the ASL classes.

The **CustomCNN** architecture includes:

- Four convolutional layers, each followed by batch normalization, activation (ReLU), and channel attention.
- A max-pooling layer after each convolution to reduce spatial dimensions.
- Fully connected layers with dropout to further prevent overfitting.
- The output layer produces predictions for the number of ASL classes.

### 4.3.3 CustomMobileNetV2

The **CustomMobileNetV2** model utilizes MobileNetV2, an efficient deep learning architecture known for its reduced computational cost and model size, making it well-suited for mobile and embedded applications. MobileNetV2 uses depthwise separable convolutions, which break down the standard convolution operation into two simpler steps—depthwise convolutions (which apply a filter to each input channel independently) and pointwise convolutions (which combine the results from depthwise convolutions). This approach reduces the number of parameters and computations, maintaining performance while improving efficiency.

In this custom implementation, attention mechanisms are applied to specific layers within the MobileNetV2 architecture to enhance the network’s ability to focus on the most informative features at each stage. The attention layers are added after key blocks in the network, corresponding to the channel dimensions at different stages. The final classification layer has been updated to output the correct number of ASL classes.

The **CustomMobileNetV2** architecture includes:

- Pre-trained MobileNetV2 layers with added channel attention after specific blocks.
- A custom classifier layer with dropout regularization to prevent overfitting.
- Global average pooling to reduce the spatial dimensions before the final classification.

### 4.3.4 CustomResNet18

The **CustomResNet18** model is based on the ResNet18 architecture, a deep convolutional network that utilizes residual blocks to mitigate the vanishing gradient problem, enabling the training of deeper networks. The residual connections allow gradients to flow more easily through the network, improving the model’s training stability and performance. In the context of ASL recognition, the ResNet18 model benefits from these residual blocks, enabling it to learn complex representations of ASL signs.

Channel attention mechanisms are applied after specific residual layers to improve the model’s focus on informative features. The model is further enhanced by modifying the fully connected layer to output the appropriate number of ASL classes.

The **CustomResNet18** architecture includes:

- Residual blocks with attention applied after selected layers.
- A modified classifier that produces output for the ASL classes.
- Final average pooling and flattening before classification.

## 4.4 Training the Model and Optimizations

The training process for the ASL recognition models involves the following steps:

#### 4.4.1 Data Preprocessing and Augmentation

Data preprocessing is essential to ensure the images are in a suitable format for training. The dataset is first loaded from a ‘.npz’ file, which contains images and their corresponding labels. The images are resized to a consistent shape (224x224 pixels) to match the input size expected by the model. Additionally, they are normalized using standard mean and standard deviation values, which help the model converge faster and improve performance by ensuring the input features are within a similar scale.

A series of data augmentations, including resizing, normalization, and transformation into tensors, are applied during training to increase the diversity of the training data and improve generalization. This can also help the model become more robust to slight variations in input data, such as different lighting conditions or background noise in ASL images.

#### 4.4.2 Training Setup

The training process is based on the following hyperparameters:

- **Batch size:** 100 images per batch. This size strikes a balance between memory usage and efficient model updates.
- **Learning rate:** 0.001. The learning rate is carefully chosen to ensure the model converges while avoiding overshooting the optimal solution. The learning rate can be adjusted during training using learning rate scheduling or adaptive optimization techniques.
- **Epochs:** 10 epochs. The number of epochs is selected based on the dataset’s size and complexity. For large datasets, fewer epochs may be sufficient.
- **Loss function:** Cross-entropy loss, suitable for multi-class classification tasks.
- **Optimizer:** Adam optimizer, which combines the advantages of both the AdaGrad and RMSProp algorithms. Adam adapts the learning rate for each parameter, making it effective for training deep networks.

#### 4.4.3 Model Initialization and Device Setup

The model is initialized based on the `CustomMobileNetV2` architecture, with the number of output classes set to match the unique labels in the dataset. The model is moved to the GPU (if available) to accelerate training.

#### 4.4.4 Model Training

During training, the model’s parameters are optimized using the Adam optimizer. For each batch of images, the following steps are executed:

- **Forward pass:** The input image is passed through the model to generate predictions.
- **Loss calculation:** The predictions are compared to the true labels, and the loss is calculated using the cross-entropy loss function.

- **Backpropagation:** The gradients of the loss with respect to the model parameters are computed.
- **Parameter update:** The optimizer adjusts the model parameters using the gradients to minimize the loss.

The training process is monitored, and validation is performed at the end of each epoch to track the model's performance on unseen data. Early stopping can be used if the validation accuracy plateaus or starts to degrade, preventing overfitting.

#### 4.4.5 Evaluating the Model

After training, the model is evaluated on the validation set to assess its performance. The evaluation step involves calculating metrics such as accuracy, precision, recall, and F1 score. This gives an indication of how well the model generalizes to new, unseen data. The evaluation process includes:

- Running the model on the validation dataset.
- Calculating the loss and accuracy for the validation set.
- Generating confusion matrices and performance reports to further analyze the model's strengths and weaknesses.

#### 4.4.6 Hyperparameter Tuning and Optimizations

During training, several optimization strategies were employed to improve model performance:

- **Early Stopping:** Training was stopped early when the validation accuracy did not improve after a certain number of epochs, preventing overfitting.
- **Data Augmentation:** Data augmentation techniques, such as random cropping, rotation, and flipping, were applied during training to improve the model's robustness.
- **Dropout:** Dropout layers were used during training to randomly set a fraction of input units to zero, helping to prevent the model from overfitting.

#### 4.4.7 Transfer Learning and Fine-Tuning

Transfer learning was applied to improve model performance, especially with limited datasets. This involved leveraging pre-trained models and adapting them for the specific task.

- **Pre-Trained Models:** Pre-trained models such as ResNet, and MobileNet, trained on large datasets like ImageNet, were used as feature extractors. The initial layers of these models captured general features like edges and textures, which were useful for the task.
- **Fine-Tuning:** After using pre-trained models, the top layers were fine-tuned on the specific dataset to help the model adjust to task-specific features.

- **Channel Attention:** Channel Attention mechanisms were incorporated to focus on more informative features in the input data. This technique helped the model to dynamically adjust the attention weights to the most relevant channels, improving performance.

## 4.5 Sign Language Detection Pipeline

The pipeline for the sign language recognition system follows these steps:

### 4.5.1 Input Capture

The system continuously captures frames from the webcam using OpenCV's `cv2.VideoCapture(0)`. Each captured frame is then processed by the system to detect hand signs.

### 4.5.2 Hand Detection with MediaPipe

MediaPipe is used to detect hand landmarks within the captured frames. The `mp.solutions.hands.Hand` model processes each frame and extracts the 3D coordinates (x, y, z) of hand landmarks.

### 4.5.3 Feature Extraction and Masking

The detected hand landmarks are drawn onto a blank black image (mask) to isolate the hand from the background. The mask is mirrored to account for any orientation variation, and both masks are passed through transformations to ensure consistent input for the model.

### 4.5.4 Data Transformation

The hand feature masks are transformed by:

- Resizing the image to 224x224 pixels.
- Converting the image to a tensor.
- Normalizing the image with pre-defined mean and standard deviation values for color channels.

### 4.5.5 Model Prediction

The transformed features are passed to the **CustomMobileNetV2** model to make predictions. Both the original and mirrored feature masks are processed to accommodate hand orientation variations. The model output is filtered using a confidence threshold (`CONFIDENCE_THRESHOLD`) to reject low-confidence predictions.

### 4.5.6 Prediction Smoothing

To improve prediction stability, a sliding window approach is used. The last `PREDICTION_WINDOW` predictions are stored in a queue, and the most frequent prediction is selected as the final prediction.

#### **4.5.7 Output and Visualization**

The system visualizes the predicted sign by drawing a bounding box around the detected hand with the predicted sign label. This is displayed on the webcam feed in real-time.

#### **4.5.8 Real-Time Processing Loop**

The system continuously processes each frame, makes predictions, and updates the GUI. The processed frames are displayed with the predicted sign label, allowing the user to see the results immediately.

#### **4.5.9 Thresholding and Filtering**

The system applies a confidence threshold to filter out predictions with low confidence, ensuring that only valid predictions are used for the final output.