

- 1) The runtime of the algorithm is given by

$$T(n) = \sum_{i=1}^n \sum_{j=1}^n 1 = n^2$$

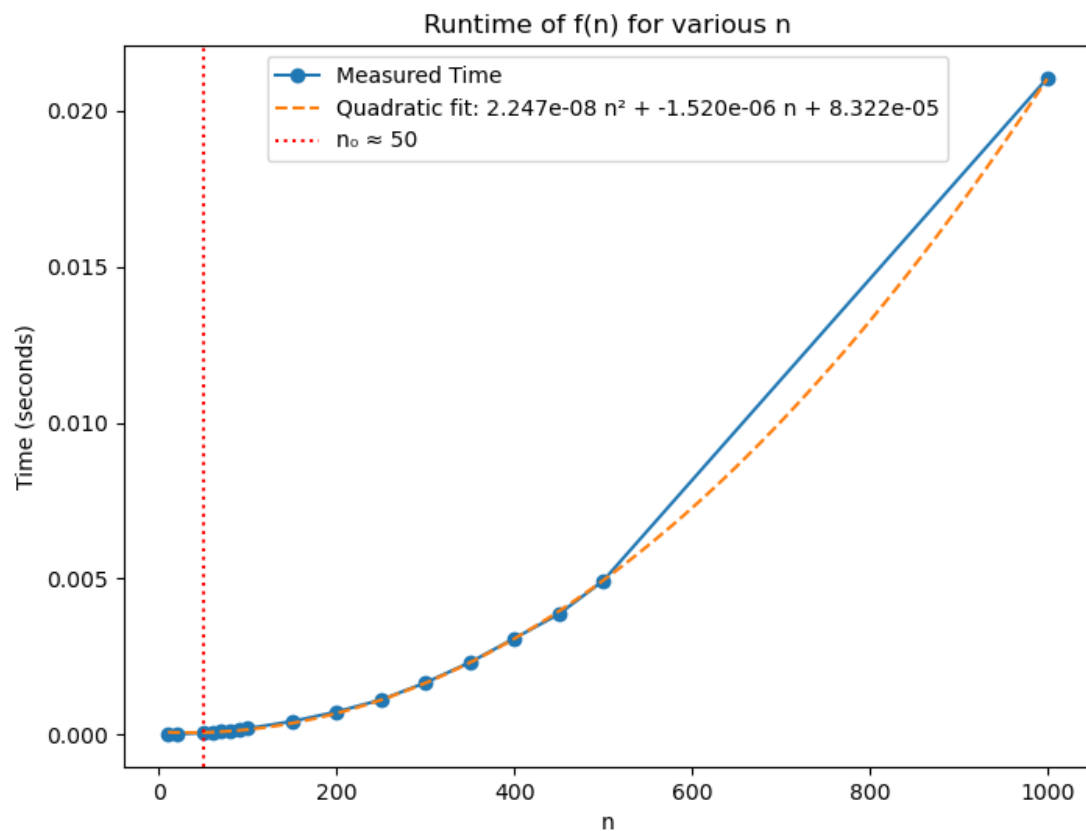
Hence

Big Theta: $\Theta(n^2)$

Big Omega: $\Omega(n^2)$

Big O: $O(n^2)$

- 2) The polynomial graph is provided below:



- 3) Since the n^2 term dominates for large n , we can state that there exist positive constants c_1 and c_2 such that, for all $n \geq n_0$ (for some threshold n_0):

$$c_1 n^2 \leq T(n) \leq c_2 n^2$$

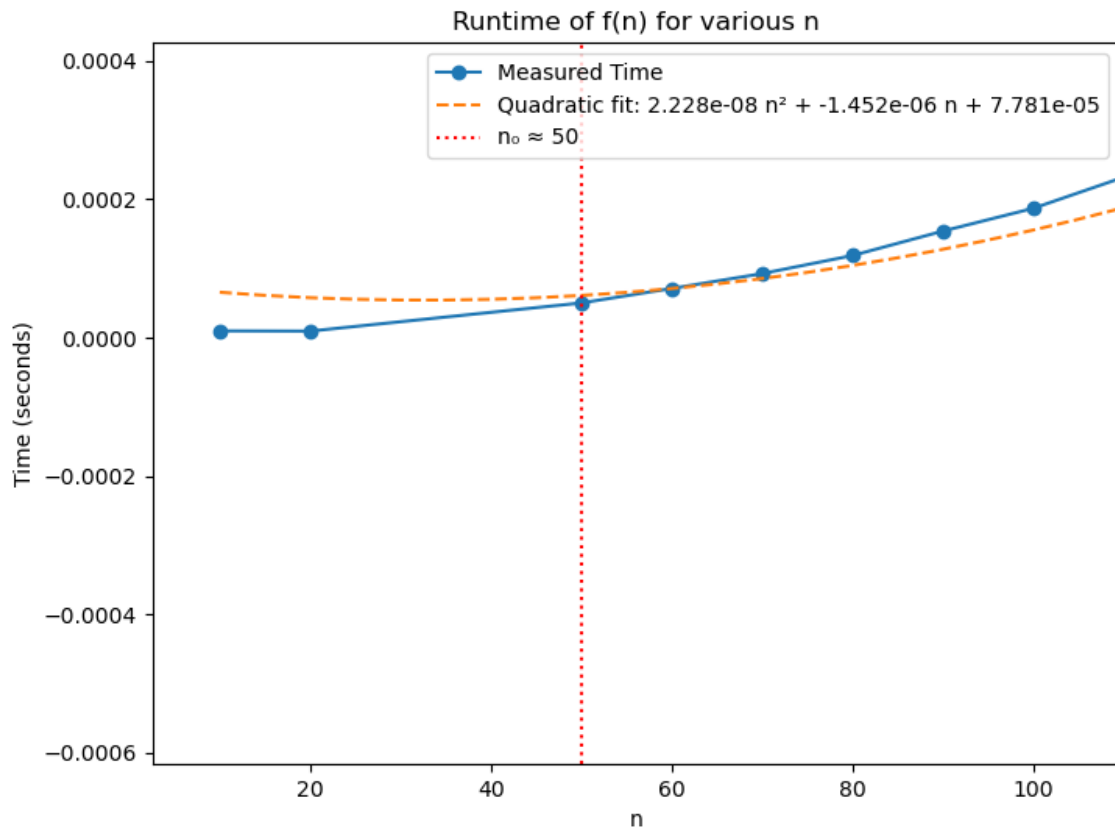
Hence

Big-O : $O(n^2)$ (upper bound)

Big Omega: $\Omega(n^2)$ (lower bound)

Big Theta: $\Theta(n^2)$ (tight bound)

- 4) For $n < 50$, the timings deviate noticeably from the quadratic curve For $n \geq 50$, the data aligns well with the quadratic polynomial, indicating that the n^2 term dominates and the overhead is negligible.



- 4) In the modified function, the inner loop now performs **two** constant-time operations (one for updating x and one for updating y) instead of just one. This effectively increases the constant factor of the runtime.
- 5) It will not affect the asymptotic analysis from Part #1. Even though the actual runtime increases by a constant factor, the asymptotic behavior remains the same.