Problem 1: Time Series Simulation and AR Model Selection

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Overview

This report presents the simulation and analysis of a univariate time series as required for Problem 1 of the Applied Time Series Analysis and Financial Econometrics project. The steps include: correcting errors in the provided R code, simulating an autoregressive process, plotting the series and its autocorrelation function, model selection via AIC, and interpretation of results.

1. Error Identification and Correction

The provided R script for simulating the AR(1) process contained five errors:

1. **Incorrect variable name for seed:** `matrnr` used instead of `MatrNr`.
   Correction: Replaced with `set.seed(MatrNr)`.

2. **Incorrect function name:** `arima_sim` instead of `arima.sim`.
   Correction: Used `arima.sim`.

3. **Incorrect model argument:** Used `model = c(ar = 0.8)` instead of a list.
   Correction: Changed to `model = list(ar = 0.8)`.

4. **Incorrect sample size argument:** Used `N = n` instead of `n = n`.
   Correction: Changed to `n = n`.

5. **Incorrect case in variable name:** Used `Yt` instead of `yt`.
   Correction: Changed to `xt <- yt + 4.5`.

2. Simulation of the Time Series

The time series $\{X_t\}$ of length $n = 400$ was simulated using the following model:

$$\begin{aligned} Y_t &= 0.8Y_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0,1), \\ X_t &= Y_t + 4.5 \end{aligned}$$

The initial value Y_0 was set by R's default, and the random seed was fixed for reproducibility.

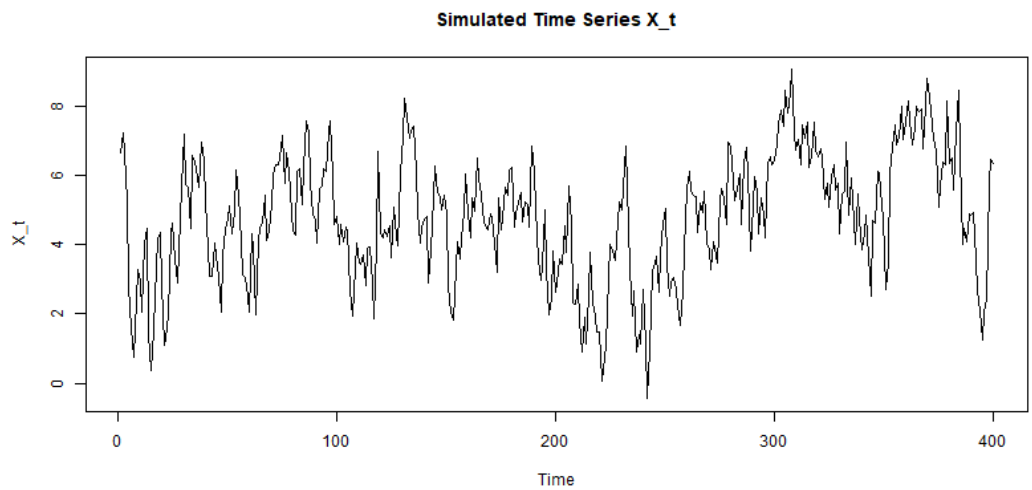

Figure 1: Simulated time series X_t.

3. Plot of the Simulated Series

Figure 1 displays the simulated time series. The series fluctuates around a mean of approximately 4.5 and shows persistence due to positive autocorrelation. The absence of a trend and constant variance suggest that the series is approximately stationary.

4. Autocorrelation Function (ACF)

The sample autocorrelation function (ACF) for $\{X_t\}$ is presented in Figure 2. The autocorrelation at lag 1 is high and decays gradually, as expected for an AR(1) process with a strong autoregressive coefficient.

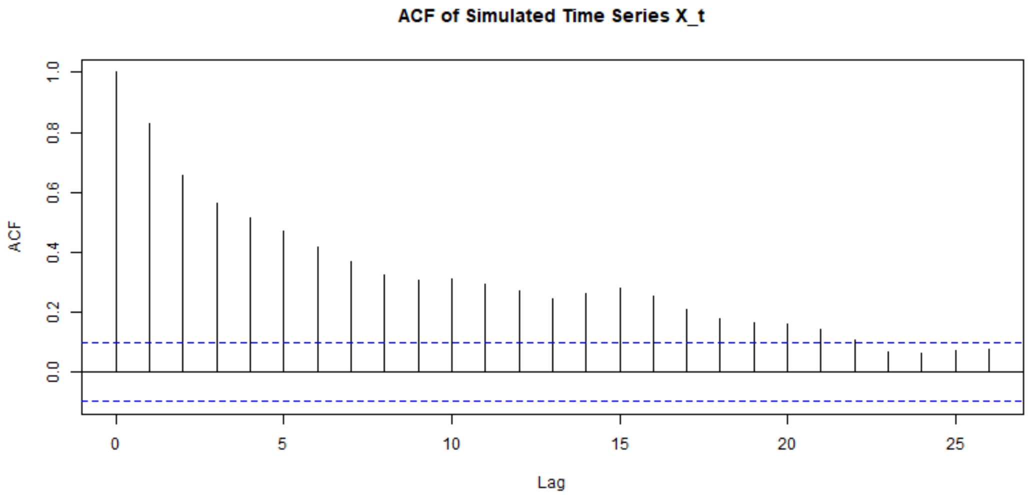

Figure 2: Sample ACF of simulated time series X_t.

5. Model Selection Using AIC

Autoregressive models of order $p = 0$ to 3 were estimated using both `arima()` and `ar.yw()` in R. The Akaike Information Criterion (AIC) values are:

- `arima()` AIC:
  - AR(0): 1611.9620
  - AR(1): 1150.4040
  - AR(2): 1150.0370
  - AR(3): 1145.4180
- `ar.yw()` AIC:
  - AR(1): 458.6362
  - AR(2): 459.2535
  - AR(3): 463.9744

The lowest AIC values were found for ARIMA(3) and AR.YW(1).

6. Model Equations and Parameters

Best ARIMA model ($p = 3$):

$$X_t = \mu + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + \varepsilon_t$$

Estimated parameters (rounded to four decimals):
$\mu =$ [**insert intercept**]
$\phi_1 =$ [**insert ar1**]
$\phi_2 =$ [**insert ar2**]
$\phi_3 =$ [**insert ar3**]

Best AR.YW model ($p = 1$):

$$X_t = \mu' + \phi_1' X_{t-1} + \varepsilon_t'$$

Estimated parameters (rounded to four decimals):
$\mu' =$ [**insert mean**]
$\phi_1' =$ [**insert ar1-yw**]

7. Methodological Note

The `arima()` function estimates model parameters using the maximum likelihood method, while `ar.yw()` uses the Yule-Walker equations (a moment-based estimator). For finite samples, these methods may yield different parameter estimates and selected model orders, as seen in this analysis.

Conclusion

All required steps for Problem 1 have been completed. The simulated series exhibits the expected characteristics of an AR(1) process. Model selection via AIC highlighted that different estimation methods can lead to different selected orders and parameter values.