CENG 223

Discrete Computational Structures

Fall 2021-2022

Take Home Exam 1

Due date: Nov 14 2021, Sunday, 23:55

Question 1 (10 pts)

Prove that the compound proposition

$$\neg(p \wedge q) \leftrightarrow (\neg q \rightarrow p)$$

is logically equivalent to

$$(p \vee q) \wedge (\neg p \vee \neg q)$$

using logical identities and algebraic manipulation techniques. Write down the identity used in each step.

Question 2 (30 pts)

Translate the following English sentences into compound predicate logic propositions using the predicates below.

$I(x, y)$: x is an intern in faculty y.
$E(x, y)$: x has employee id number y.
$S(x, y)$: x is supervised by y.
$A(x, y)$: x is admitted to job position y.
$J(x, y)$: x is a job position in faculty y.

Besides the indicated predicates, you are **only** allowed to use additional variables, existential and universal quantifiers along with logical connectives, and equals ($=$) and not equals ($\neq$) relations if necessary. Use of any other notation within your statements will cause the corresponding answers to be evaluated as 0.

a. Two different interns in the same faculty cannot have the same employee id number.

b. There are some interns in all faculties who are supervised by no one but themselves.

c. At most two interns can be admitted to each job position in the medicine faculty.

Question 3 (30 pts)

Using natural deduction rules for propositional logic, prove the following statements.

a. $p \lor \neg q,\ p \lor r \vdash (r \to q) \to p$.

b. $\vdash ((q \to p) \to q) \to q$.

Question 4 (30 pts)

Using natural deduction rules for propositional and predicate logic, prove the following statements.

a. $\neg \forall x (P(x) \to Q(x)) \vdash \exists x (P(x) \land \neg Q(x))$.

b. $\forall x \forall y (P(x, y) \to \neg P(y, x)),\ \forall x \exists y P(x, y) \vdash \neg \exists v \forall z P(z, v)$.

Regulations

1. Your submission should be a single vector-based PDF document with the name "the1.pdf".

2. **Late Submission:** Not allowed.

3. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.

4. **Updates & Announces:** You must follow the odtuclass for discussions and possible updates. You can ask your questions freely in the Student Forum on the course page in odtuclass.

5. **Evaluation:**Your .pdf file will be checked for plagiarism automatically using "black-box" technique and manually by assistants.

Submission

Submission will be done via odtuclass. For those who prefer to use LaTeX to generate the vector-based pdf file, a template answer file "the1.tex" will be provided in odtuclass. You need to compile the filled template yourselves and submit the generated .pdf file only.