CENG 223 Take Home Exam 2

Student Information

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Q. 1

$$(A \cup B)\backslash(A \cap B) \text{ Part1}$$
$$(x \in A \cup B) \wedge (x \notin A \cap B) \longrightarrow \text{definition of difference}$$
$$(x \in A \cup B) \wedge \neg(x \in A \cap B) \longrightarrow \text{definition of } \in$$
$$(x \in A \vee x \in B) \wedge \neg(x \in A \cap B) \longrightarrow \text{definition of union}$$
$$(x \in A \vee x \in B) \wedge \neg(x \in A \wedge x \in B) \longrightarrow \text{definition of intersection}$$
$$(x \in A \vee x \in B) \wedge (\neg(x \in A) \vee \neg(x \in B)) \longrightarrow \text{De Morgan's law for logical operations}$$
$$(x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B) \longrightarrow \text{x2 definition of } \notin$$
$$((x \in A \vee x \in B) \wedge x \notin A) \vee ((x \in A \vee x \in B) \wedge x \notin B) \longrightarrow \text{distribution law}$$
$$((x \in A \wedge x \notin A) \vee (x \in B \wedge x \notin A)) \vee ((x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin B)) \longrightarrow \text{x2 distribution law}$$
$$(\emptyset \vee (x \in B \wedge x \notin A)) \vee ((x \in Ax \notin B) \vee \emptyset) \longrightarrow \text{x2 complement law}$$
$$(x \in B \wedge x \notin A) \vee (x \in A \wedge x \notin B) \longrightarrow \text{x2 identity law}$$
$$(x \in B\backslash A) \vee (x \in A\backslash B) \longrightarrow \text{x2 definition of difference}$$
$$(x \in A\backslash B) \vee (x \in B\backslash A) \longrightarrow \text{commutative law}$$
$$x \in [(A\backslash B) \cup (B\backslash A)] \longrightarrow \text{definition of union}$$
$$(A \cup B)\backslash(A \cap B) \subseteq (A\backslash B) \cup (B\backslash A)$$

$$(A\backslash B) \cup (B\backslash A) \text{ Part2}$$
$$(x \in (A\backslash B)) \vee (x \in (B\backslash A)) \longrightarrow \text{definition of union}$$
$$(x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A) \longrightarrow \text{x2 definition of difference}$$
$$((x \in A \wedge x \notin B) \vee x \in B) \wedge ((x \in A \wedge x \notin B) \vee x \notin A) \longrightarrow \text{x2 distribution law}$$
$$((x \in A \vee x \in B) \wedge (x \notin B \vee x \in B)) \wedge ((x \in A \vee x \notin A) \wedge (x \notin B \vee x \notin A)) \longrightarrow \text{x2 distribution law}$$
$$((x \in A \vee x \in B) \wedge U) \vee (U \wedge (x \notin B \wedge x \notin A)) \longrightarrow \text{x2 complement law}$$
$$(x \in A \vee x \in B) \wedge (x \notin B \vee x \notin A) \longrightarrow \text{x2 identity law}$$
$$(x \in A \vee x \in B) \wedge (\neg(x \in B) \vee \neg(x \in A)) \longrightarrow \text{definition of } \notin$$
$$(x \in A \vee x \in B) \wedge \neg(x \in B \wedge x \in A) \longrightarrow \text{De Morgan's Law for logical operators}$$
$$x \in (A \cup B) \wedge x \notin (A \cap B) \longrightarrow \text{definition of intersection,union with commutative law}$$
$$x \in [(A \cup B)\backslash(A \cap B)] \longrightarrow \text{definition of difference}$$
$$(A\backslash B) \cup (B\backslash A) \subseteq ((A \cup B)\backslash(A \cap B))$$

Since;
$(A\backslash B) \cup (B\backslash A) \subseteq ((A \cup B)\backslash(A \cap B))$ and,
$(A \cup B)\backslash(A \cap B) \subseteq (A\backslash B) \cup (B\backslash A)$
Given sets are equivalent.

Q. 2

———First part———

Assume $f \mid f \subseteq N \times \{0,1\}$ is countable set

$N \times \{0,1\}$:

(0,0), (0,1)

(1,0), (1,1)

(2,0), (2,1)

(3,0), (3,1)

... ...

First attempt for enumeration:

$f_0 = \{(0,0),(0,1)\}$

$f_1 = \{(1,0),(1,1)\}$

$f_2 = \{(2,0),(2,1)\}$

$f_3 = \{(3,0),(3,1)\}$

....

By design f $\in \{(0,0),(0,1),(1,0),(1,1)\}$ and it is missed by enumeration.

Second attempt for enumeration:

$f_0 = \{(0,0),(0,1),(1,0),(1,1),(2,0),(2,1)...\}$

$f_1 = \{(1,0),(1,1),(3,0),(3,1),(5,0),(5,1)...\}$

$f_2 = \{(2,0),(2,1),(4,0),(4,1),(6,0),(6,1)...\}$

$f_3 = \{(3,0),(3,1),(6,0),(6,1),(9,0),(9,1)...\}$

....

By design f $\in \{(0,0),(0,1)\}$ and it is missed by enumeration.

Third attempt for enumeration:

$f_0 = \{(0,0),(3,0)\}$

$f_1 = \{(0,0),(5,1),(67,0),(3,1),(78,1)\}$

$f_2 = \{(2,0),(6,0),(6,1)\}$

$f_3 = \{(6,0),(6,1),(9,0),(9,1)...\}$

....

f : $= \{i \in N \mid i \notin f_i\}$

f : $= \{(2,1),(58,0),(7,1),(11,0)\}$

$(2,1) \notin f_0, \quad (58,0) \notin f_1, \quad (7,1) \notin f_2, \quad (11,0) \notin f_3,$

f is missed by enumeration. Indeed f can not put in bijection with $f \mid f \subseteq N \times \{0,1\}$

$\perp$ We reached contradiction.

$f \mid f \subseteq N \times \{0,1\}$ is uncountable. And we call it A for next proofs.

———Second Part———

$f \mid f : N\{0,1\} \longrightarrow N$

f(a) = b

	f(0) = 0	f(0) = 1	f(0) = 2	f(0) = 3 ....
	f(1) = 0	f(1) = 1	f(1) = 2	f(1) = 3 ....
f(0) = 0	f(0) = 1, f(1) = 0	f(0) = 2, f(1) = 1	f(1) = 2, f(0) = 3	
a+b=0	a+b = 1	a+b = 2	a+b = 3	

Countbly infinite by Cantor's diagonalization method.

We can call it B for later proofs.

Assume A $\backslash B$ is countable

We found that B is countable on Part2

So we can say that $(A \backslash B) \cup B$ is countable.(Finite union of countable sets is clearly countable)

So, $A \subseteq (A \backslash B) \cup B$ A contained in countable set,

$\perp$ We reached a contradiction, We found A uncountable on PART 1

So A $\backslash$ B is uncountable. $f \mid f \subseteq N \times \{0,1\} \backslash f \mid f : N\{0,1\} \longrightarrow N$ is uncountable.

Q. 3

$$\text{Assume } 4^n + 5n^2 * log(n) \text{ is } O(2^n)$$
Then there exists C,k such that:
$$4^n + 5n^2 * log(n) \leq C * 2^n \text{ for } x > k$$
$$2^n * 2^n + 5n^2 * log(n) \leq C * 2^n$$
$$2^n + \frac{5n^2 * log(n)}{2^n} \leq C$$
$$\perp$$
We reached a contradiction since we can not find $2^n + \frac{5n^2 * log(n)}{2^n} \leq C$ It can not hold for all n

Our assumption is false, $4^n + 5n^2 * log(n)$ is not $O(2^n)$

Q. 4

$$(2x-1)^n - x^2 \equiv -x - 1 \qquad mod(x-1)$$
We need to get rid of $(2x-1)^n$. Manipulating equations with adding modulo x-1
$$\big((2x-1) - 2(x-1)\big)^n - (x - (x-1))^2 \equiv -x - 1 + 2(x-1) \qquad mod(x-1)$$
$$1^n - 1^2 \equiv x - 3 \qquad mod(x-1)$$
$$x \equiv 3 \qquad mod(x-1)$$