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Q. 1

1. $\neg(p \wedge q) \leftrightarrow (\neg q \rightarrow p)$
2. $\equiv (\neg p \vee \neg q) \leftrightarrow (\neg q \rightarrow p) \longrightarrow$ Using De Morgans' second law for left hand side proposition
3. $\equiv (\neg p \vee \neg q) \leftrightarrow (p \vee q) \longrightarrow$ Using definition of implication and Commutative law
4. $\equiv ((\neg p \vee \neg q) \rightarrow (p \vee q)) \wedge ((p \vee q) \rightarrow (\neg p \vee \neg q) \longrightarrow$ Using definition of double implication
5. $\equiv ((p \wedge q) \vee (p \vee q)) \wedge ((\neg p \wedge \neg q) \vee (\neg p \vee \neg q) \rightarrow$ Using def. of implication and De Morgan's law.
6. $\equiv \big[((p \wedge q) \vee p) \vee ((p \wedge q) \vee q)\big] \wedge \big[((\neg p \wedge \neg q) \vee \neg p) \vee ((\neg p \wedge \neg q) \vee \neg q)\big] \longrightarrow$ Using Distributive law
7. $\equiv (p \vee q) \wedge (\neg p \vee \neg q) \longrightarrow$ Using Absorption law for all 4 compound proposition
8. As can be seen from statement 7, the first proposition given is equal to the second.

Q. 2

2.a) $\forall a, b, z, y (a \neq b)\big[\big(I(a,y) \wedge I(b,y)\big) \rightarrow \big(E(a,z) \wedge \neg E(b,z)\big)\big]$

2.b) $\exists x \forall y \forall z (z \neq x)\big(I(x,y) \wedge S(x,x) \wedge \neg S(x,z)\big)$

2.c) $\forall k, t$(t=medicine)$\neg\exists(x,y,z)\big[I(x,t) \wedge I(y,t) \wedge I(z,t) \wedge A(x, J(k,t)) \wedge A(y, J(k,t)) \wedge A(z, J(k,t) \wedge x \neq y \wedge y \neq z \wedge x \neq z\big]$

Q. 3

Proof1 for first question: $\neg p, p \vee q \vdash q$

1. $\neg p$	premise
2. $p \vee q$	premise
3. p	assumption
4. $\perp$	$\neg e$ 1,3
5. q	$\perp e$ 4
6. q	assumption
7. q	$\vee e$ 2, 3-5, 6-6

Proof2 for first question: $\neg p, p \vee \neg q \vdash \neg q$

1. $\neg p$	premise
2. $p \vee \neg q$	premise
3. p	assumption
4. $\perp$	$\neg e$ 1,3
5. $\neg q$	$\perp e$ 4
6. $\neg q$	assumption
7. $\neg q$	$\vee e$ 2, 3-5, 6-6

1.

$$p \lor \neg q, p \lor r \vdash (r \to q) \to p$$

1. $p \lor \neg q$	premise
2. $p \lor r$	premise
3. $\neg p$	assumption
4. r	proof1 2,3
5. $\neg q$	proof2 1,3
6. $(r \land \neg q)$	$\land i$ 4,5
7. $\neg p \to (r \land \neg q)$	$\to i$ 3-6
8. $r \to q$	assumption
9. $\neg p$	assumption
10. $r \land \neg q$	$\to e$ 7,9
11. r	$\land e$ 10
12. q	$\to e$ 8,11
13. $\neg q$	$\land e$ 10
14. $\perp$	$\neg e$ 12,13
15. p	$\neg i$ 9-14
16. $(r \to q) \to p$	$\to i$ 8-15

2.

$$\vdash ((q \to p) \to q) \to q$$

1.	$(q \to p) \to q$	assumption
2.	$q \vee \neg q$	lemma
3.	q	assumption
4.	q	copy 3
5.	$\neg q$	assumption
6.	q	assumption
7.	$\perp$	$\neg e$ 5,6
8.	p	$\perp e$ 7
9.	$q \to p$	$\to i$ 6-8
10.	q	$\to e$ 1,9
11.	q	$\vee e$ 2, 3-4, 5-10
12.	$((q \to p) \to q) \to q$	$\to i$ 1-11

Q. 4

1.

$$\forall x(P(x) \to Q(x)) \vdash \exists x(P(x) \land \neg Q(x))$$

1. $\neg\forall x(P(x) \to Q(x))$	premise
2. $\neg\exists x(P(x) \land \neg Q(x))$	assumption
3. x_0	fresh name
4. $\neg(P(x_0) \land \neg Q(x_0))$	assumption
5. $P(x_0)$	assumption
6. $\neg Q(x_0)$	assumption
7. $P(x_0) \land \neg Q(x_0)$	$\land i$ 4,5
8. $\perp$	$\neg e$ 3,6
9. $\neg\neg Q(x_0)$	$\neg i$ 5-7
10. $Q(x_0)$	$\neg\neg e$ 9
11. $P(x_0) \to Q(x_0)$	$\to i$ 5-10
12. $P(x_0) \to Q(x_0)$	copy 11
13. $\forall x(P(x) \to Q(x))$	$\forall i$ 3-12
14. $\perp$	$\neg e$ 1,13
15. $\neg\neg\exists x(P(x) \land \neg Q(x))$	$\neg i$ 2-14
16. $\exists x(P(x) \land \neg Q(x))$	$\neg\neg e$ 15