UNIT II : Physical Layer

The Theoretical basis for data communication

Fourier Analysis

- Fourier analysis is a fundamental branch of mathematics and signal processing named after the French mathematician and physicist **Joseph Fourier**.
- It deals with the study of **periodic functions and their representation as a sum of simple sine and cosine functions (**simple trigonometric functions**)**.
- The main idea behind Fourier analysis is that any complex and periodic function can be expressed as a combination of simple **harmonic functions** with different frequencies and amplitudes.
- These harmonic functions are the sine and cosine waves, which are fundamental building blocks in signal processing and understanding the behaviour of periodic phenomena.
- The representation of a function in terms of sine and cosine waves is known as the **Fourier series**. It allows us to break down a complex function into its constituent frequency components, which can be analysed separately.
- This is particularly **useful in fields like** engineering, physics, and signal processing, where many real-world phenomena can be approximated or understood better by analyzing their frequency content.

Fourier analysis is widely **used in various applications**, including:

1. **Signal processing**: In digital signal processing, Fourier analysis is used to transform signals between the time domain and the frequency domain. This enables tasks such as filtering, compression, and spectral analysis.
2. **Communications**: Fourier analysis is employed in various communication systems to understand and analyze signal propagation, noise, and channel characteristics.
3. **Image processing**: In image processing, Fourier analysis is used to analyze and manipulate image data in the frequency domain. It is the basis for techniques like image filtering and compression.
4. **Quantum mechanics**: In quantum mechanics, Fourier analysis plays a crucial role in understanding the wave nature of particles and representing wavefunctions.

The most common Fourier transforms are the continuous Fourier transform and the **discrete Fourier transform (DFT**). The latter is widely used in digital signal processing and is the foundation for the popular **Fast Fourier Transform (FFT)**

algorithm, which efficiently computes the DFT. These transforms are essential tools in many fields of science and engineering, allowing researchers to gain insights into complex phenomena by analyzing their frequency components

The Fourier series decomposes a periodic function as a sum of sine and cosine components as expressed below:

$$g(t) = \frac{c}{2} + \sum_{n=1}^{\infty} a_n \sin(2\pi n f t) + \sum_{n=1}^{\infty} b_n \cos(2\pi n f t)$$

where, $g(t)$ is the periodic function

T is the time period

f is the fundamental frequency expressed as 1/T

a_n is the sine amplitude of the n^{th} harmonic

b_n is the cosine amplitude of the n^{th} harmonic

c is a constant

The values of a_n, b_n and c are computed by the following expressions:

$$a_n = \frac{2}{T} \int_0^T g(t) \sin(2\pi n f t)\, dt$$

$$b_n = \frac{2}{T} \int_0^T g(t) \cos(2\pi n f t)\, dt$$

$$c = \frac{2}{T} \int_0^T g(t) dt$$

Bandwidth limited Signals

Bandwidth

The range of frequencies that are used for transmitting a signal without being substantially attenuated is called the bandwidth. It is calculated as the difference between the highest and the lowest frequencies. It is expressed in Hertz (Hz).

For example, if the minimum frequency is 100 Hz and the maximum frequency is 1000 Hz, the bandwidth will be 900 Hz.

The bandwidth of a transmission medium is the frequency width of the medium and is dependent upon its physical characteristics like thickness, material, length etc. For example, the bandwidth of a coaxial cable is 750 MHz

Baseband

Baseband transmissions are those requiring low – pass channels, i.e. the frequency range starts from 0. The bandwidth of a baseband channel is simply its maximum frequency.

Bandpass and Passband

Bandpass is an electronic filter that allows frequencies within a particular range to pass through it, while screening out other frequencies. The output of a bandpass filter is a passband signal.

Bandwidth – Limited Signal

A signal is called bandwidth – limited or simply band-limited when the amplitude of the spectrum goes to zero whenever its frequency crosses the allowable limits. Thus, its Fourier transform is non-zero only for a finite frequency interval. A band-limited signal is represented by a finite number of harmonics.

In most applications, an analog signal is sampled, converted to digital form on which operations are performed, which is finally reconstructed to analog form.

For data communications, a signal, which is to be transmitted, has an infinite number of terms in its Fourier transform. However, when this signal needs to be transmitted through a channel of fixed bandwidth, band-limiting is required.

It can be observed that among the infinite Fourier components, only the first few terms (harmonics) suffice to reconstruct the signal. So, if the bandwidth of the channel permits these harmonics to be transmitted, then the original signal can be reconstructed with sufficient accuracy.

Limiting the bandwidth of a signal will limit the data rate, even if the channel is perfect with very less noise. A solution is to use coding schemes with different voltage levels.

The Maximum Data Rate of a Channel

Data rate refers to the speed of data transfer through a channel. It is generally computed in bits per second (bps). Higher data rates are expressed as Kbps ("Kilo" bits per second, i.e.1000 bps), Mbps ("Mega" bits per second, i.e.1000 Kbps), Gbps ("Giga" bits per second, i.e. 1000 Mbps) and Tbps ("Tera" bits per second, i.e. 1000 Gbps).

One of the main objectives of data communications is to increase the data rate. There are three factors that determine the data rate of a channel:

- Bandwidth of the channel
- Number of levels of signals that are used
- Noise present in the channel

Data rate can be calculated using two theoretical formulae:

- Nyquist Bit Rate – for noiseless channel
- Shannon’s Capacity – for noisy channel

Nyquist Bit Rate

Nyquist bit rate was developed by Henry Nyquist who proved that the transmission capacity of even a perfect channel with no noise has a maximum limit.

The theoretical formula for the maximum bit rate is:

maximum bit rate = $2 \times$ *Bandwidth* $\times$ log_2V

Here, *maximum bit rate* is calculated in bps

Bandwidth is the bandwidth of the channel

V is the number of discrete levels in the signal

For example, if there is a noiseless channel with a bandwidth of 4 KHz that is transmitting a signal with 4 discrete levels, then the maximum bit rate will be computed as, *maximum bit rate* = $2 \times 4000 \times log_2 4 = 16{,}000$ *bps* = 16 *kbps*

Shannon's Capacity

Claude Shannon extended Nyquist's work for actual channels that are subject to noise. Noise can be of various types like thermal noise, impulse noise, cross-talks etc. Among all the noise types, thermal noise is unavoidable. The random movement of electrons in the channel creates an extraneous signal not present in the original signal, called the thermal noise. The amount of thermal noise is calculated as the ratio of the signal power to noise power, *SNR*.

Signal-to-Noise Ratio,SNR = Average Signal Power/Average Noise Power

Since *SNR* is the ratio of two powers that varies over a very large range, it is often expressed in decibels, called SNR_{db} and calculated as: $SNR_{db} = 10log_{10}SNR$.

Shannon's Capacity gives the theoretical maximum data rate or capacity of a noisy channel. It is expressed as:

Capacity = *Bandwidth* $\times$ $log_2(1+SNR)$

Here, *Capacity* is the maximum data rate of the channel in bps

Bandwidth is the bandwidth of the channel

SNR is the signal – to – noise ratio

For example, if the bandwidth of a noisy channel is 4 KHz, and the signal to noise ratio is 100, then the maximum bit rate can be computed as:

Capacity = $4000 \times log_2(1+100) = 26{,}633$ *bps* = 26.63 *kbp*