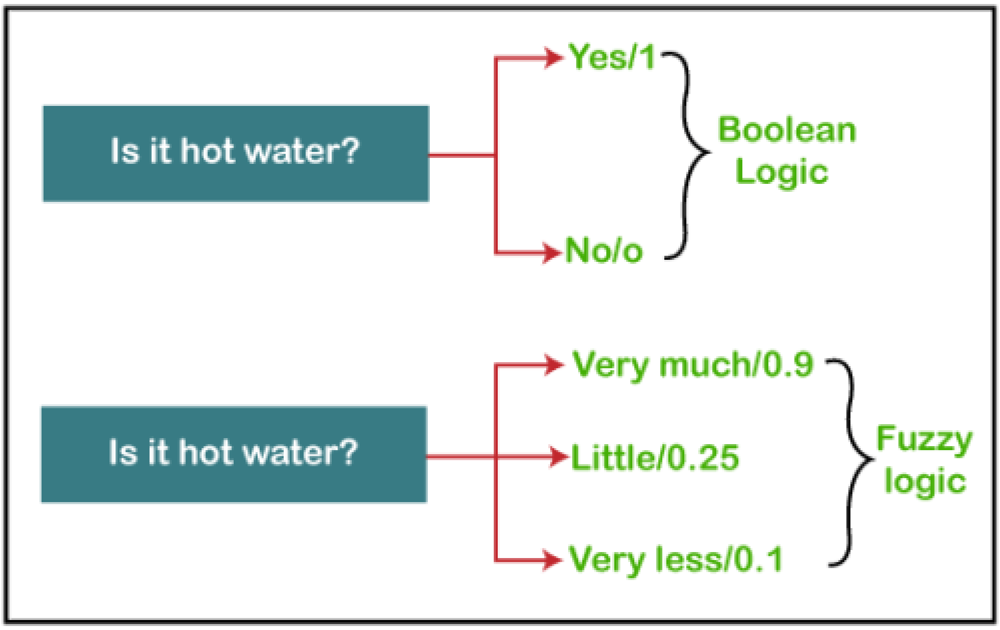
SC U3

1. Explain Fuzzy logic with a suitable diagram and example.

- Fuzzy logic is a form of multi-valued logic that deals with reasoning that is approximate rather than fixed and exact.
- Unlike classical binary logic (which has only True/False values), fuzzy logic allows for a range of truth values between 0 and 1.

Key Characteristics of Fuzzy Logic:

- **It allows for partial truth values.**
- **It is useful in situations where human reasoning is involved.**
- **It is widely used in control systems, decision-making, and AI.**



Key Concepts in Fuzzy Logic

1. Fuzzy Sets: In classical set theory, an element either belongs to a set or does not. In fuzzy logic, an element can belong to a set to a certain degree, represented by a membership value between 0 and 1.

2. Membership Function: This function defines how each point in the input space is mapped to a membership value between 0 and 1. Common shapes for membership functions include triangular, trapezoidal, and Gaussian.
3. Linguistic Variables: These are variables whose values are words or sentences in a natural or artificial language. For example, "temperature" can be a linguistic variable with values like "cold," "warm," and "hot."
4. Fuzzy Rules: These are if-then rules that describe how the input variables are transformed into output variables. For example, "If the temperature is cold, then the heater should be on high."

Architecture of a Fuzzy Logic System

In the architecture of the Fuzzy Logic system, each component plays an important role. The architecture consists of the different four components which are given below.

1. Rule Base
2. Fuzzyification
3. Inference Engine
4. Defuzzification

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2. Find the power set and cardinality of the given set, X = {2,4,6,8,10,12}. Also, find the cardinality of the power set.

2) → i) Power Set of Set X is the Set of all possible Subset of X, including Empty set & X itself

ii) The Cardinality of a set is a Number of Element in the Set.

Eg) $X = \{2, 4, 6, 8, 10, 12\}$

Step 1) Find Power Set of X.

→ The power of Set X denoted as P(x) is set of all Subset of X, for a set with n Elements, the power set has 2n Elements

→ Since X has 6 Elements Pow (x) = $2^{(6)} = 64$ Subset

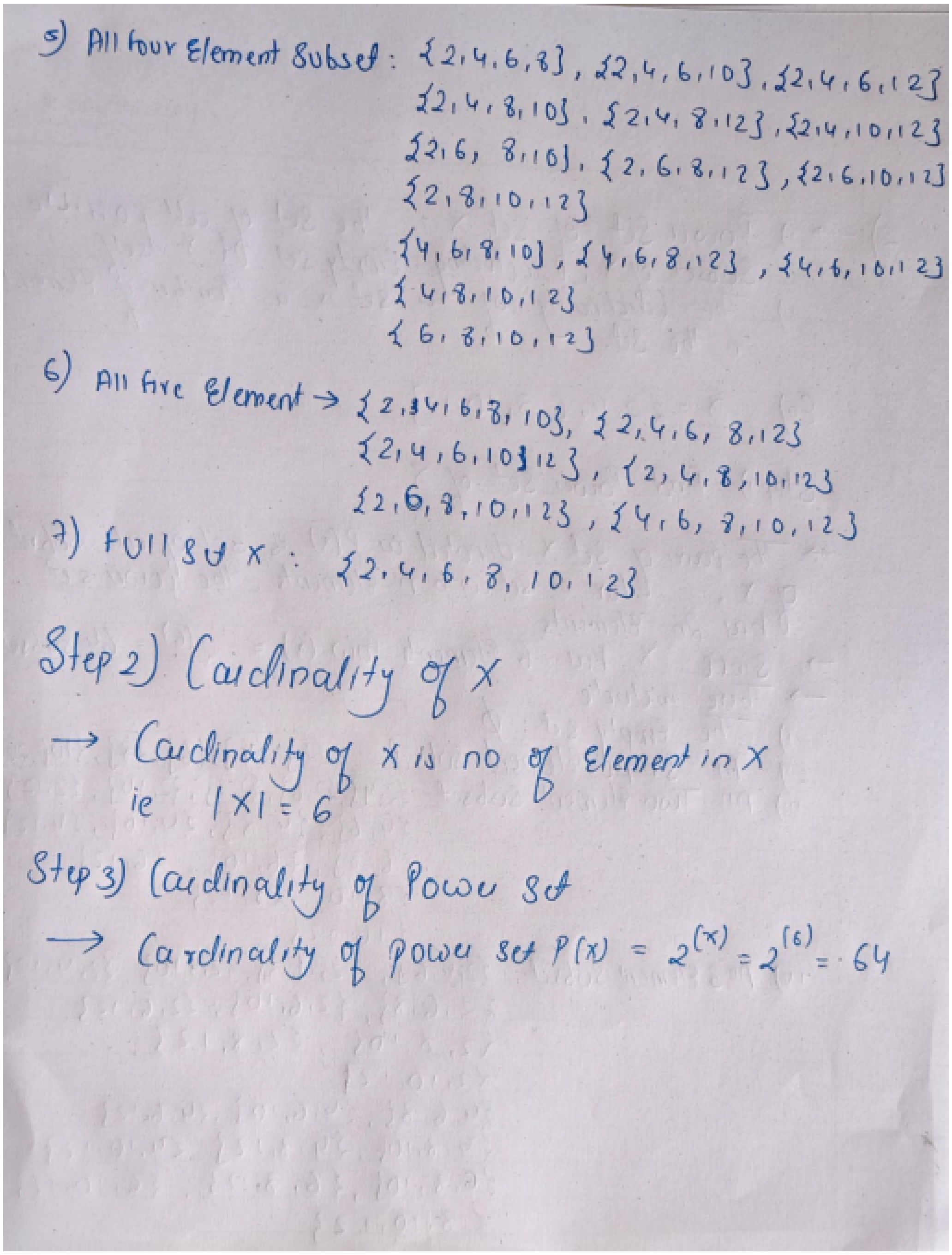
→ These include

i) The Empty set : $\emptyset$

ii) All Single Element Subset : {2}, {4}, {6}, {8}, {10}, {12}

iii) All Two Element Subset : {2,4}, {2,6}, {2,8}, {2,10}, {2,12}
{4,6}, {4,8}, {4,10}, {4,12}
{6,8}, {6,10}, {6,12}
{8,10}, {8,12}
{10,12}

iv) All 3 Element Subset : {2,4,6}, {2,4,8}, {2,4,10}, {2,4,12}
{2,6,8}, {2,6,10}, {2,6,12}
{2,8,10}, {2,8,12}
{2,10,12}
{4,6,8}, {4,6,10}, {4,6,12}
{4,8,10}, {4,8,12} {4,10,12}
{6,8,10}, {6,8,12}, {6,10,12}
{8,10,12}



5) All four Element Subset : {2,4,6,8}, {2,4,6,10}, {2,4,6,12}
{2,4,8,10}, {2,4,8,12}, {2,4,10,12}
{2,6,8,10}, {2,6,8,12}, {2,6,10,12}
{2,8,10,12}
{4,6,8,10}, {4,6,8,12}, {4,6,10,12}
{4,8,10,12}
{6,8,10,12}

6) All five Element → {2,4,6,8,10}, {2,4,6,8,12}
{2,4,6,10,12}, {2,4,8,10,12}
{2,6,8,10,12}, {4,6,8,10,12}

7) Full set X : {2,4,6,8,10,12}

Step 2) Cardinality of X

→ Cardinality of X is no of Element in X
ie $|X| = 6$

Step 3) Cardinality of Power set

→ Cardinality of power set $P(X) = 2^{(x)} = 2^{(6)} = 64$

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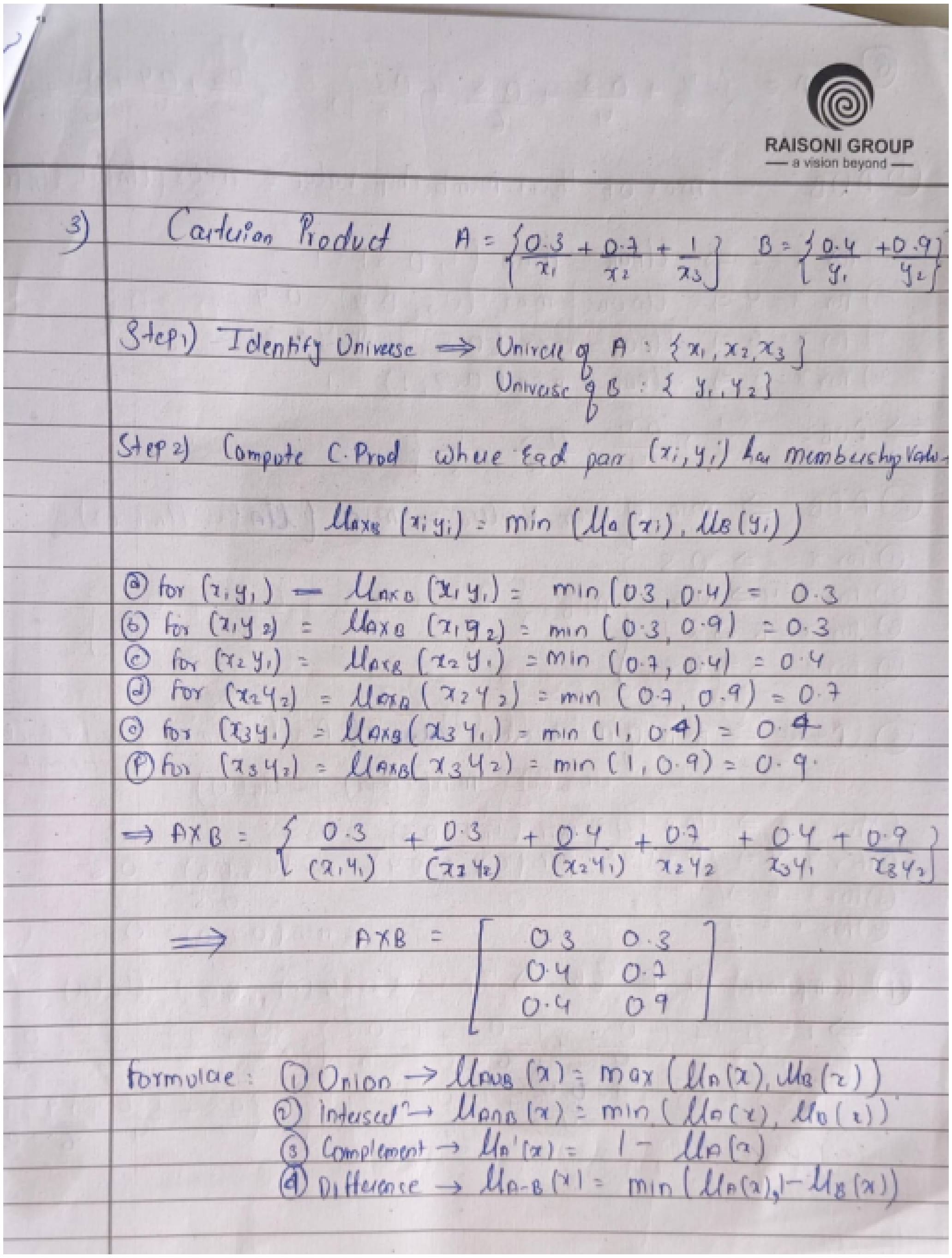
3. Consider the given fuzzy set, A = {0.3/x1 + 0.7/x2 + 1/x3} & B={0.4/y1 + 0.9/y2 } Perform Cartesian product over these given fuzzy set.



3) Cartesian Product $A = \left\{\frac{0.3}{x_1} + \frac{0.7}{x_2} + \frac{1}{x_3}\right\}$ $B = \left\{\frac{0.4}{y_1} + \frac{0.9}{y_2}\right\}$

Step1) Identify Universe ⇒ Universe of A : $\{x_1, x_2, x_3\}$
Universe of B : $\{y_1, y_2\}$

Step 2) Compute C.Prod where Each pair (x_i, y_i) has membership Value-

$$\mu_{A \times B}(x_i\, y_i) = \min(\mu_A(x_i), \mu_B(y_i))$$

ⓐ for $(x_1 y_1)$ — $\mu_{A \times B}(x_1 y_1) = \min(0.3, 0.4) = 0.3$
ⓑ for $(x_1 y_2)$ = $\mu_{A \times B}(x_1 y_2) = \min(0.3, 0.9) = 0.3$
ⓒ for $(x_2 y_1)$ = $\mu_{A \times B}(x_2 y_1) = \min(0.7, 0.4) = 0.4$
ⓓ for $(x_2 y_2)$ = $\mu_{A \times B}(x_2 y_2) = \min(0.7, 0.9) = 0.7$
ⓔ for $(x_3 y_1)$ = $\mu_{A \times B}(x_3 y_1) = \min(1, 0.4) = 0.4$
ⓕ for $(x_3 y_2)$ = $\mu_{A \times B}(x_3 y_2) = \min(1, 0.9) = 0.9$.

$$\Rightarrow A \times B = \left\{\frac{0.3}{(x_1 y_1)} + \frac{0.3}{(x_1 y_2)} + \frac{0.4}{(x_2 y_1)} + \frac{0.7}{x_2 y_2} + \frac{0.4}{x_3 y_1} + \frac{0.9}{x_3 y_2}\right\}$$

$$\Longrightarrow \quad A \times B = \begin{bmatrix} 0.3 & 0.3 \\ 0.4 & 0.7 \\ 0.4 & 0.9 \end{bmatrix}$$

formulae : ① Union → $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$
② Intersect^n → $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$
③ Complement → $\mu_{A'}(x) = 1 - \mu_A(x)$
④ Difference → $\mu_{A-B}(x) = \min(\mu_A(x), 1 - \mu_B(x))$

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4. Explain classical set with different operations performed on classical set with Venn diagram.

Classical Set:

- A classical set is a well-defined collection of distinct objects where each element either belongs to the set or does not.
- There is no ambiguity in membership.
- Classical sets are fundamental in mathematics and form the basis for more advanced concepts like fuzzy logic.

Operations on Classical Sets with Venn Diagrams

Venn diagrams are graphical representations of sets and their relationships. Below are the operations on classical sets with corresponding Venn diagrams.

1. Union (A∪B)

The union of two sets A and B includes all elements that are in A, in B, or in both.

Venn Diagram:

- Two overlapping circles represent sets A and B.
- The shaded area covers both circles.

2. Intersection (A∩B)

The intersection of two sets A and B includes only the elements that are in both A and B.

Venn Diagram:

- Two overlapping circles represent sets A and B.
- The shaded area is the overlapping region.

3. Complement (A′)

The complement of a set A includes all elements in the universe U that are not in A.

Venn Diagram:

- A rectangle represents the universe U.
- A circle inside the rectangle represents set A.
- The shaded area is everything outside the circle.

4. Difference (A−B)

The difference of two sets A and B includes all elements that are in A but not in B.

Venn Diagram:

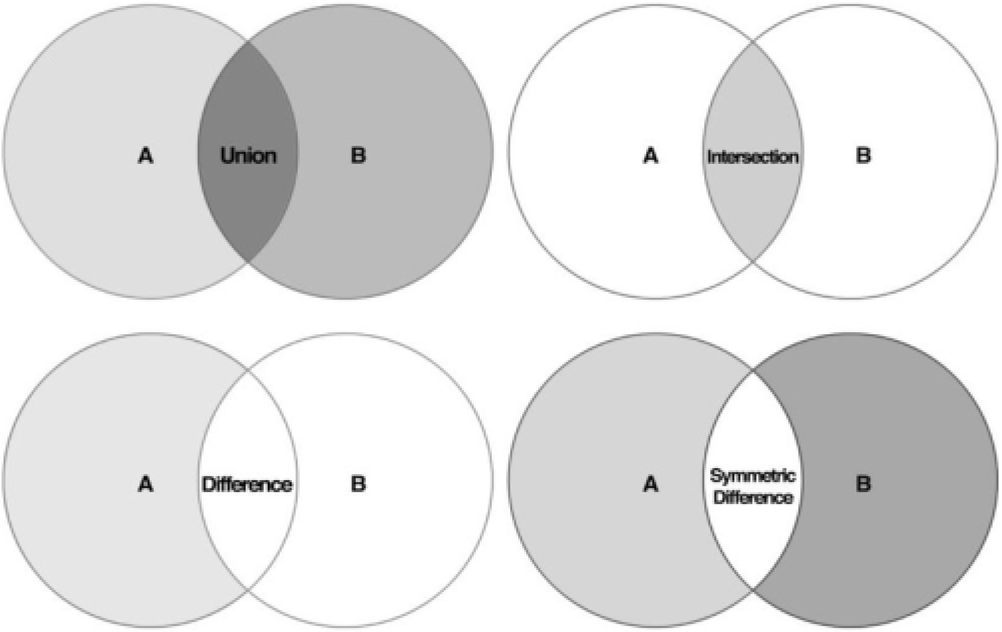
- Two overlapping circles represent sets A and B.
- The shaded area is the part of A that does not overlap with B.

5. Symmetric Difference (AΔB)

The symmetric difference of two sets A and B includes all elements that are in A or B but not in both.

Venn Diagram:

- Two overlapping circles represent sets A and B.
- The shaded area is the non-overlapping parts of both circles.



5. Explain fuzzy set with different operations performed on fuzzy set with a suitable diagram.

- Fuzzy sets are an extension of classical sets where elements can have partial membership.
- In classical sets, an element either belongs to a set or does not, but in fuzzy sets, an element can belong to a set to a certain degree, represented by a membership value between 0 and 1.
- This makes fuzzy sets particularly useful for handling imprecise or subjective information.

Key Concepts in Fuzzy Sets

1. **Membership Function**: A function that defines how each element in the universe of discourse is mapped to a membership value between 0 and 1.
2. **Linguistic Variables**: Variables whose values are words or sentences in a natural or artificial language. For example, "temperature" can be a linguistic variable with values like "cold," "warm," and "hot."
3. **Fuzzy Set Representation**: A fuzzy set A is represented as:
   $A=\{\mu A(x1)/x1 + \mu A(x2)/x2 + \cdots + \mu A(xn)/xn\}$
   where $\mu A(xi)$ is the membership value of xi in A.

Operations on Fuzzy Sets

The operations on fuzzy sets are analogous to those on classical sets but are defined in terms of membership functions.

1. Union ($A \cup BA \cup B$)

The union of two fuzzy sets A and B is a fuzzy set whose membership function is the maximum of the membership functions of A and B:
$\mu A \cup B(x)=\max(\mu A(x),\mu B(x))$
Example:
Let
$A=\{0.3/x1 + 0.7/x2\}$ and $B=\{0.6/x1 + 0.4/x2\}$

Then:

$A \cup B = \{\max(0.3,0.6)/x1 + \max(0.7,0.4)/x2\} = \{0.6x1+0.7x2\}$

2. Intersection (A∩B)

The intersection of two fuzzy sets A and B is a fuzzy set whose membership function is the minimum of the membership functions of A and B:

$\mu A \cap B(x) = \min(\mu A(x), \mu B(x))$

Example:

Using the same

$A \cap B = \{\min(0.3,0.6)/x1 + \min(0.7,0.4)/x2\} = \{0.3/x1+0.4/x2\}$

3. Complement (A′)

The complement of a fuzzy set A is a fuzzy set whose membership function is:

$\mu A'(x) = 1 - \mu A(x)$

Example:

For

$A = \{0.3/x1 + 0.7/x2\}$

$A' = \{1-0.3/x1 + 1-0.7/x2\} = \{0.7/x1 + 0.3/x2\}$

4. Difference (A−B)

The difference of two fuzzy sets A and B is a fuzzy set whose membership function is:

$\mu A - B(x) = \min(\mu A(x), 1 - \mu B(x))$

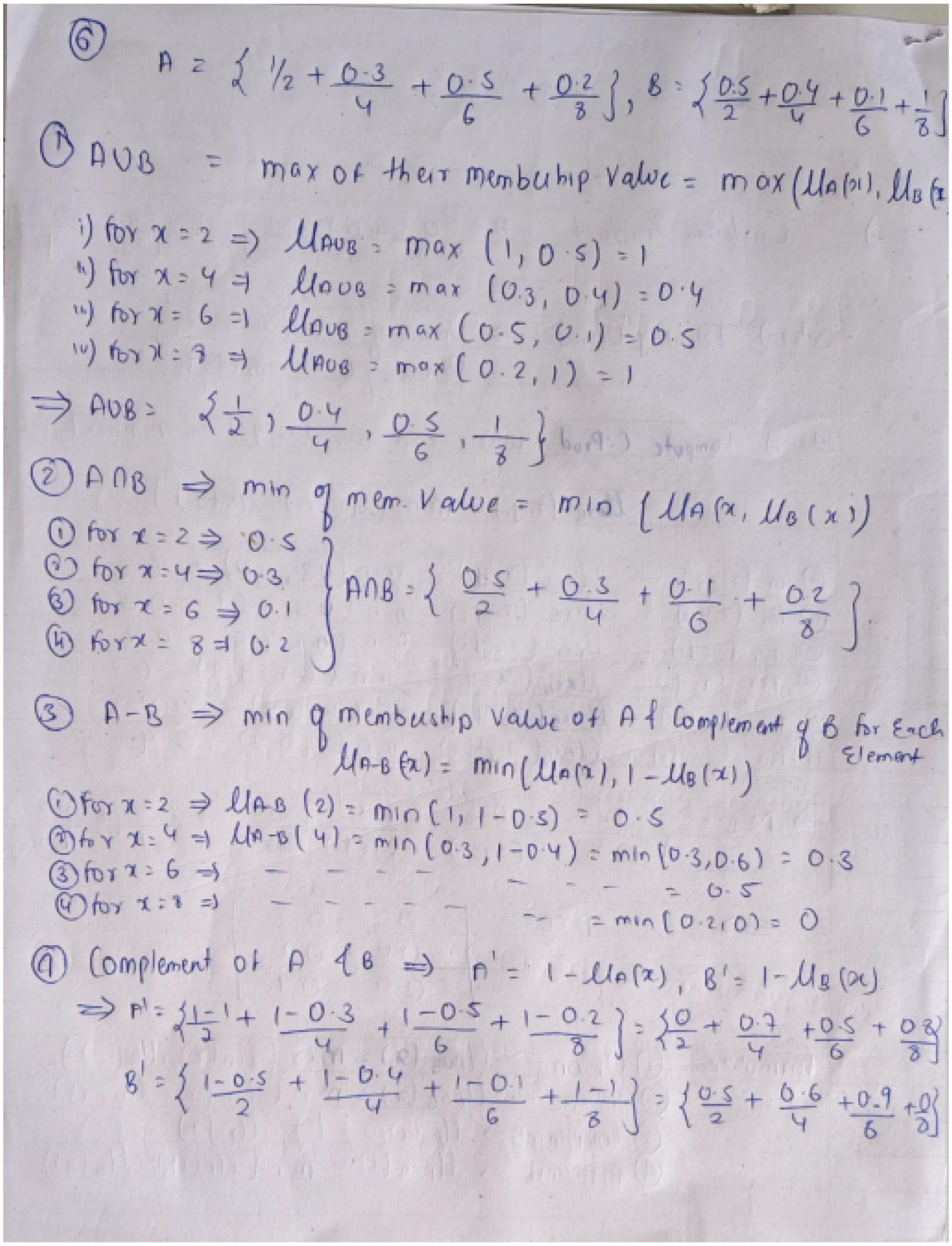
Example:

Using the same

as above:

$A - B = \{\min(0.3,1-0.6)/x1 + \min(0.7,1-0.4)/x2\} = \{0.3x1+0.6x2\}$

6. Perform Union, Intersection, Difference, and Complement for given fuzzy sets



⑥ $A = \left\{ 1/2 + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8} \right\}, B = \left\{ \frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8} \right\}$

① $A \cup B$ = max of their membuhip value = $\max(\mu_A(x), \mu_B($

i) for $x = 2 \Rightarrow \mu_{A \cup B} = \max(1, 0.5) = 1$

ii) for $x = 4 \Rightarrow \mu_{A \cup B} = \max(0.3, 0.4) = 0.4$

iii) for $x = 6 \Rightarrow \mu_{A \cup B} = \max(0.5, 0.1) = 0.5$

iv) for $x = 8 \Rightarrow \mu_{A \cup B} = \max(0.2, 1) = 1$

$\Rightarrow A \cup B = \left\{ \frac{1}{2}, \frac{0.4}{4}, \frac{0.5}{6}, \frac{1}{8} \right\}$

② $A \cap B \Rightarrow$ min of mem. value = $\min(\mu_A(x, \mu_B(x))$

① for $x = 2 \Rightarrow 0.5$

② for $x = 4 \Rightarrow 0.3$

③ for $x = 6 \Rightarrow 0.1$

④ for $x = 8 \Rightarrow 0.2$

$A \cap B = \left\{ \frac{0.5}{2} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.2}{8} \right\}$

③ $A - B \Rightarrow$ min of membuship value of A & Complement of B for Each Element

$\mu_{A-B}(x) = \min(\mu_A(x), 1 - \mu_B(x))$

① for $x = 2 \Rightarrow \mu_{AB}(2) = \min(1, 1 - 0.5) = 0.5$

② for $x = 4 \Rightarrow \mu_{A-B}(4) = \min(0.3, 1 - 0.4) = \min(0.3, 0.6) = 0.3$

③ for $x = 6 \Rightarrow$ — — — — — — — $= 0.5$

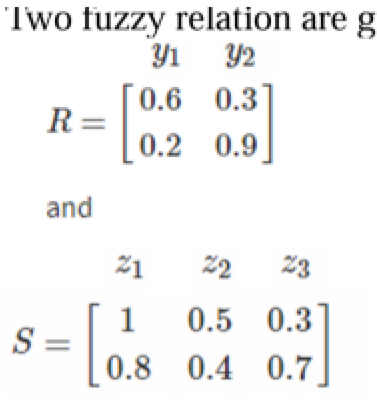
④ for $x = 8 \Rightarrow$ — — — — — — $= \min(0.2, 0) = 0$

④ Complement of A & B $\Rightarrow A' = 1 - \mu_A(x), B' = 1 - \mu_B(x)$

$\Rightarrow A' = \left\{ \frac{1-1}{2} + \frac{1-0.3}{4} + \frac{1-0.5}{6} + \frac{1-0.2}{8} \right\} = \left\{ \frac{0}{2} + \frac{0.7}{4} + \frac{0.5}{6} + \frac{0.8}{8} \right\}$

$B' = \left\{ \frac{1-0.5}{2} + \frac{1-0.4}{4} + \frac{1-0.1}{6} + \frac{1-1}{8} \right\} = \left\{ \frac{0.5}{2} + \frac{0.6}{4} + \frac{0.9}{6} + \frac{0}{8} \right\}$

7,8 . Two fuzzy relations are given by
Obtain fuzzy relation T as composition between the fuzzy relations using
1.max-min product composition.
2.max product composition

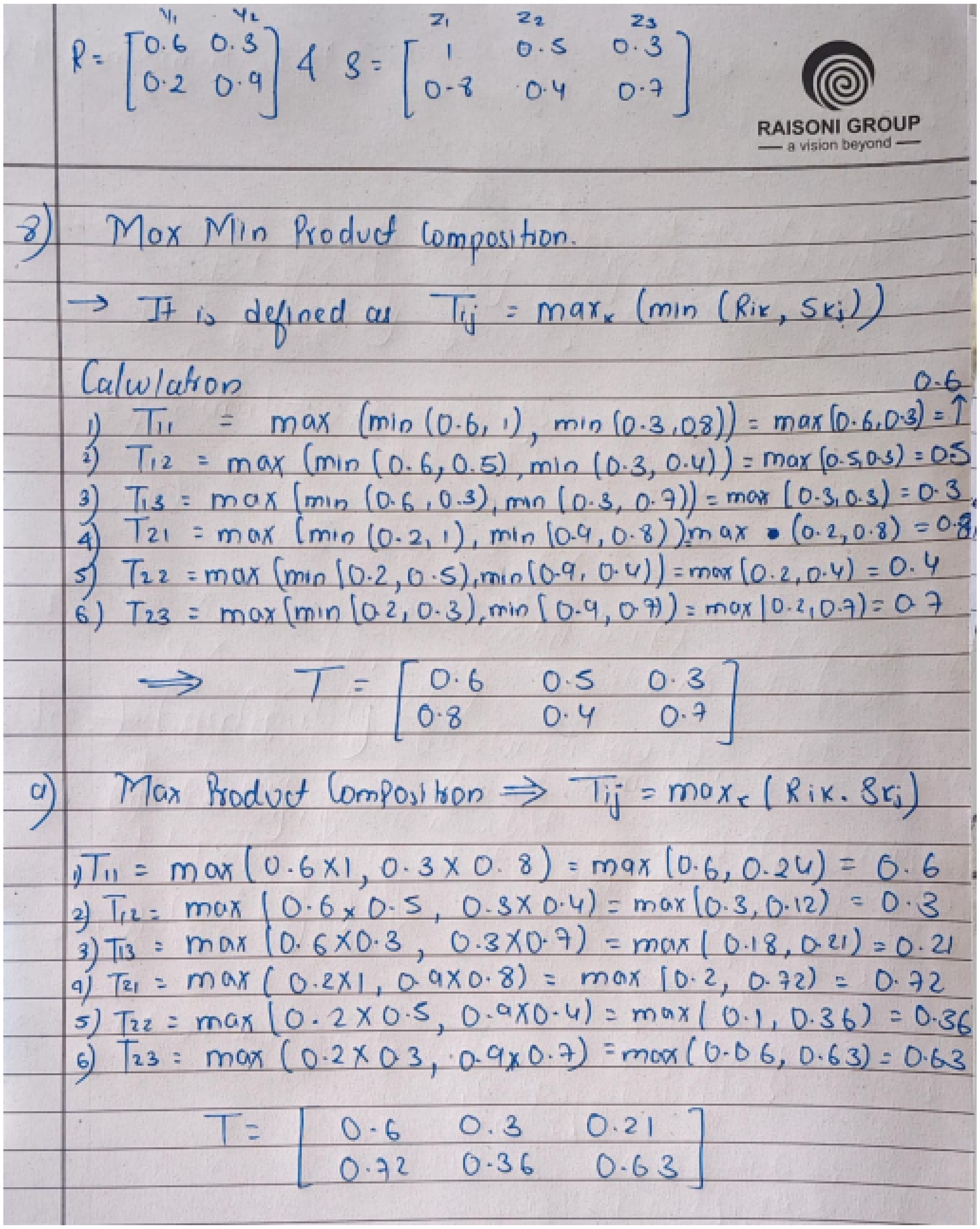


Two fuzzy relation are given by

$$R = \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

and

$$S = \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$



$$R = \begin{array}{cc} y_1 & y_2 \\ \left[\begin{array}{cc} 0.6 & 0.3 \\ 0.2 & 0.9 \end{array}\right] \end{array} \quad \& \quad S = \begin{array}{ccc} z_1 & z_2 & z_3 \\ \left[\begin{array}{ccc} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{array}\right] \end{array}$$

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8) Max Min Product Composition.

→ It is defined as $T_{ij} = \max_k (\min (R_{ik}, S_{kj}))$

Calculation

1) $T_{11} = \max (\min (0.6, 1), \min (0.3, 08)) = \max (0.6, 0.3) = \overset{0.6}{\uparrow}$

2) $T_{12} = \max (\min (0.6, 0.5), \min (0.3, 0.4)) = \max (0.5, 0.3) = 0.5$

3) $T_{13} = \max (\min (0.6, 0.3), \min (0.3, 0.7)) = \max (0.3, 0.3) = 0.3$

4) $T_{21} = \max (\min (0.2, 1), \min (0.9, 0.8)) = \max (0.2, 0.8) = 0.8$

5) $T_{22} = \max (\min (0.2, 0.5), \min (0.9, 0.4)) = \max (0.2, 0.4) = 0.4$

6) $T_{23} = \max (\min (0.2, 0.3), \min (0.9, 0.7)) = \max (0.2, 0.7) = 0.7$

$$\Longrightarrow \quad T = \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix}$$

9) Max Product Composition $\Rightarrow T_{ij} = \max_k (R_{ik} \cdot S_{kj})$

1) $T_{11} = \max (0.6 \times 1, 0.3 \times 0.8) = \max (0.6, 0.24) = 0.6$

2) $T_{12} = \max (0.6 \times 0.5, 0.3 \times 0.4) = \max (0.3, 0.12) = 0.3$

3) $T_{13} = \max (0.6 \times 0.3, 0.3 \times 0.7) = \max (0.18, 0.21) = 0.21$

4) $T_{21} = \max (0.2 \times 1, 0.9 \times 0.8) = \max (0.2, 0.72) = 0.72$

5) $T_{22} = \max (0.2 \times 0.5, 0.9 \times 0.4) = \max (0.1, 0.36) = 0.36$

6) $T_{23} = \max (0.2 \times 0.3, 0.9 \times 0.7) = \max (0.06, 0.63) = 0.63$

$$T = \begin{bmatrix} 0.6 & 0.3 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix}$$

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9. Illustrate composition with suitable example

Composition of fuzzy relations is a way to combine two fuzzy relations to form a new fuzzy relation. There are two common types of composition:

1. **Max-Min Composition -- T=max(min(Rik,Skj))**
2. **Max-Product Composition -- t = max(Rik . Skj)**

give example of question 8,9

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10. Binary Relation and Relation Matrix - Detailed Explanation

1. What is a Binary Relation?

A **binary relation** R between two setsA and Bis a subset of their Cartesian productA×B. It defines a relationship between elements of these two sets.

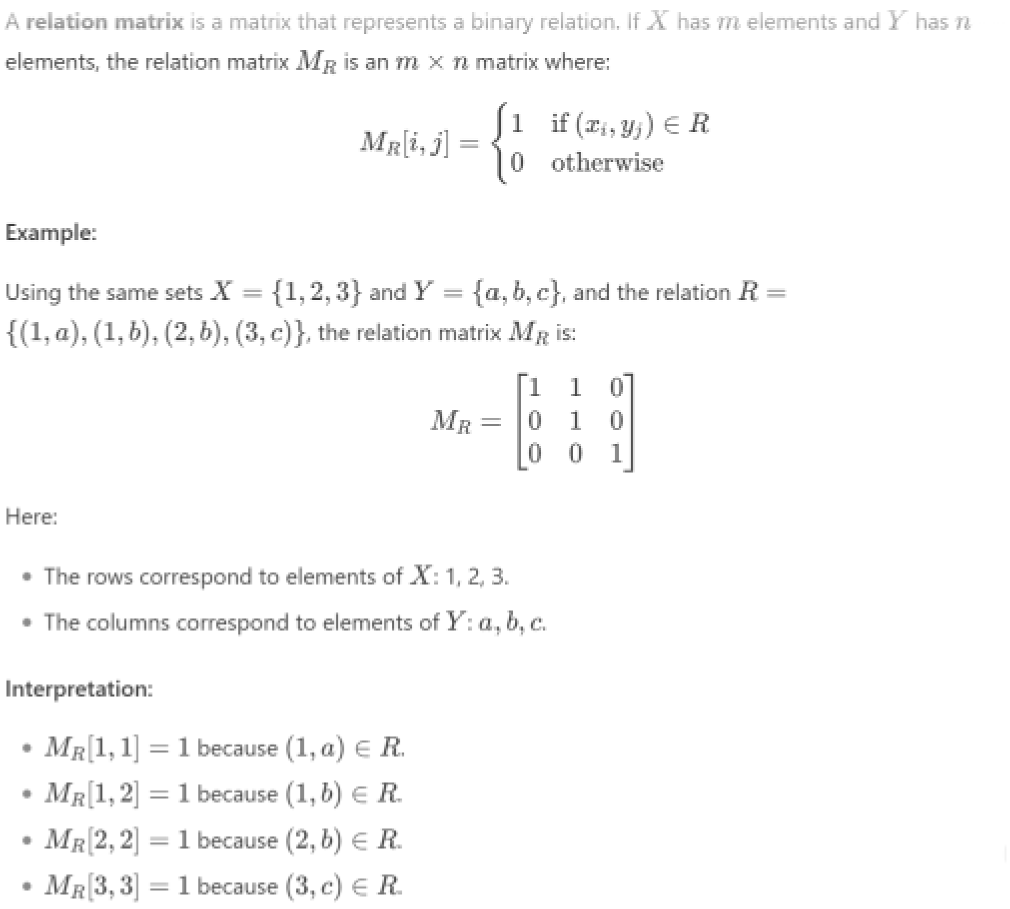
Example of a Binary Relation:

Let
A={1,2,3} and B={a,b,c}

A binary relation
R can be defined as:
R={(1,a),(2,b),(3,c),(1,c)}
This means:

- Element **1** in Ais related to**a** and **c** in BBB.
- Element **2** in Ais related to**b** in BBB.
- Element **3** in Ais related to**c** in BBB.

Relation matrix



A **relation matrix** is a matrix that represents a binary relation. If X has m elements and Y has n elements, the relation matrix M_R is an $m \times n$ matrix where:

$$M_R[i,j] = \begin{cases} 1 & \text{if } (x_i, y_j) \in R \\ 0 & \text{otherwise} \end{cases}$$

Example:

Using the same sets $X = \{1,2,3\}$ and $Y = \{a,b,c\}$, and the relation $R = \{(1,a),(1,b),(2,b),(3,c)\}$, the relation matrix M_R is:

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

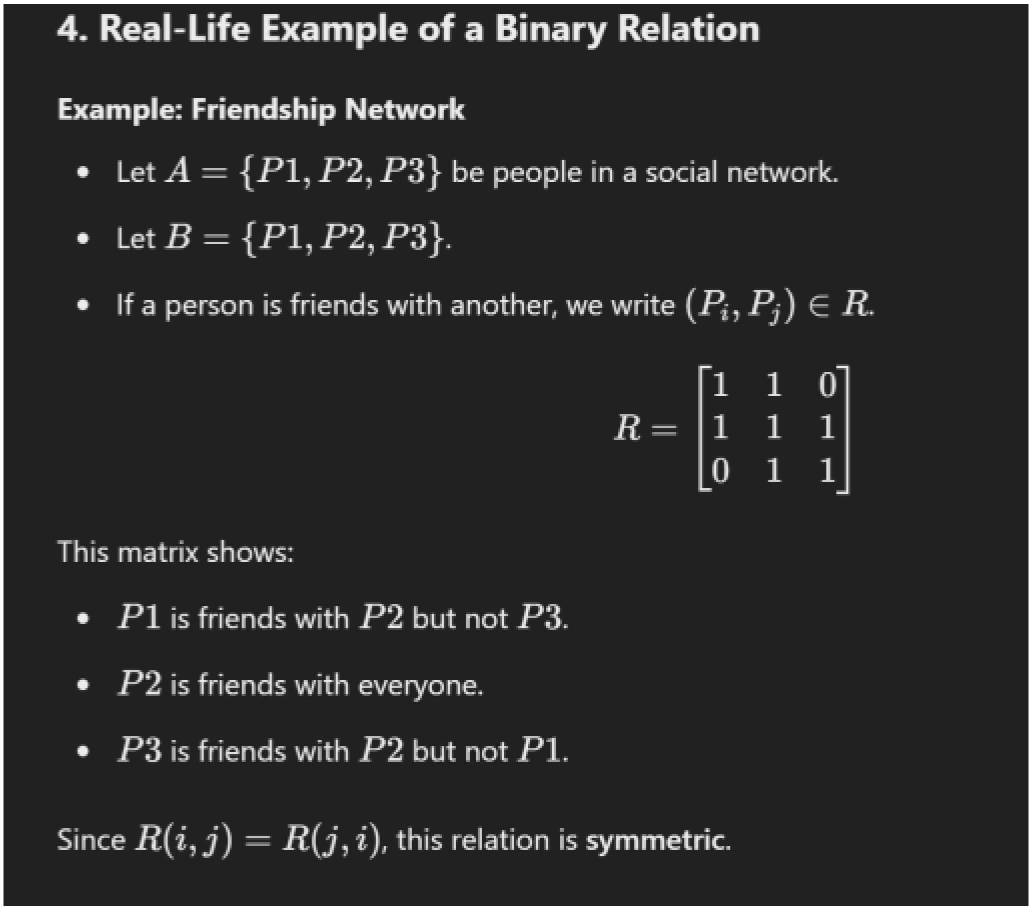
Here:

- The rows correspond to elements of X: 1, 2, 3.
- The columns correspond to elements of Y: a, b, c.

Interpretation:

- $M_R[1,1] = 1$ because $(1,a) \in R$.
- $M_R[1,2] = 1$ because $(1,b) \in R$.
- $M_R[2,2] = 1$ because $(2,b) \in R$.
- $M_R[3,3] = 1$ because $(3,c) \in R$.

Real-Life Example of a Binary Relation



4. Real-Life Example of a Binary Relation

Example: Friendship Network

- Let $A = \{P1, P2, P3\}$ be people in a social network.
- Let $B = \{P1, P2, P3\}$.
- If a person is friends with another, we write $(P_i, P_j) \in R$.

$$R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

This matrix shows:

- $P1$ is friends with $P2$ but not $P3$.
- $P2$ is friends with everyone.
- $P3$ is friends with $P2$ but not $P1$.

Since $R(i, j) = R(j, i)$, this relation is **symmetric**.