11. Determine λ cut set from the given fuzzy set

SU-U4
SC-U3

$S_1 = \{x \mid \mu_s(x) \geq 1\}$

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Q.11) $S_1 = \left\{ \frac{0}{0} + \frac{0.5}{20} + \frac{0.65}{40} + \frac{0.85}{60} + \frac{1}{80} + \frac{1.0}{100} \right\}$

$S_2 = \left\{ \frac{0}{0} + \frac{0.45}{20} + \frac{0.6}{40} + \frac{0.86}{60} + \frac{0.95}{80} + \frac{1.0}{100} \right\}$

① $S_1 \cap S_2 \rightarrow$ min of their membership values for Each Element

$$\mu_{S_1 \cap S_2}(x) = \min(\mu_{S_1}(x), \mu_{S_2}(x))$$

$\Rightarrow$ for $x=0$: $\mu_{S_1 \cap S_2}(0) = \min(0, 0) = 0$
for $x=20$: $\mu_{S_1 \cap S_2}(20) = \min(0.5, 0.45) = 0.45$
for $x=40$: $\mu_{S_1 \cap S_2}(40) = \min(0.65, 0.6) = 0.6$
for $x=60$: $\mu_{S_1 \cap S_2}(60) = \min(0.85, 0.86) = 0.85$
for $x=80$: $\mu_{S_1 \cap S_2}(80) = \min(1, 0.95) = 0.95$
for $x=100$: $\mu_{S_1 \cap S_2}(100) = \min(1, 1) = 1$

Result $\Rightarrow S_1 \cap S_2 = \left\{ \frac{0}{0} + \frac{0.45}{20} + \frac{0.6}{40} + \frac{0.85}{60} + \frac{0.95}{80} + \frac{1}{100} \right\}$

$\Rightarrow (S_1 \cap S_2)_\lambda = \{40, 60, 80, 100\}$

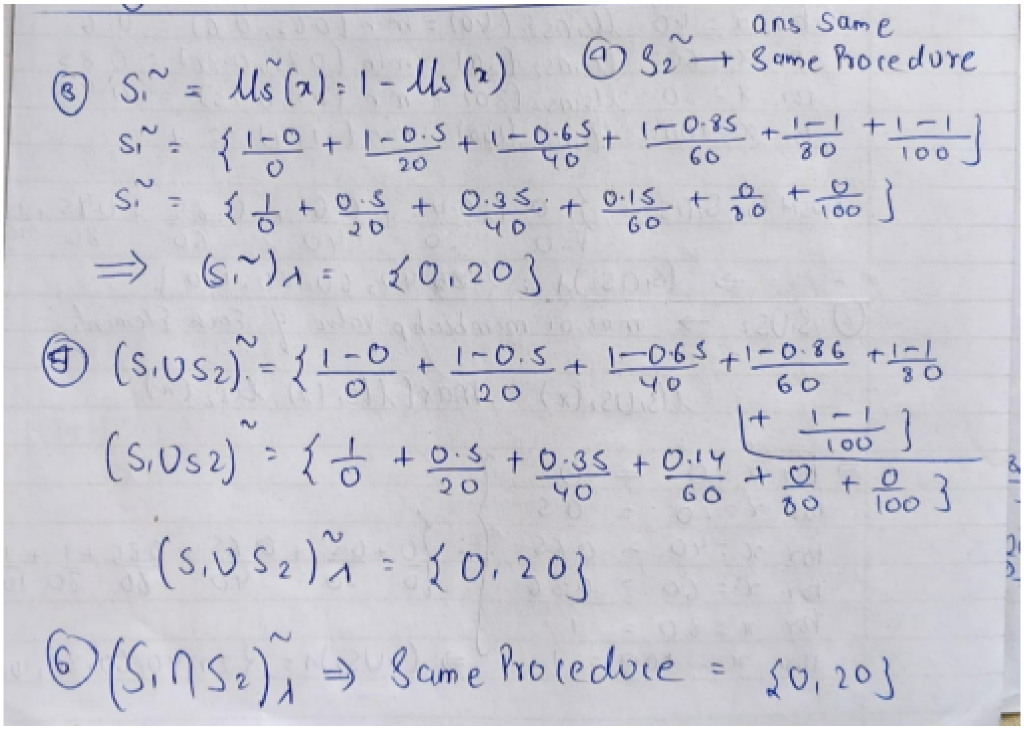
② $S_1 \cup S_2 \rightarrow$ max of membership value of Each Element

$$\mu_{S_1 \cup S_2}(x) = \max(\mu_{S_1}(x), \mu_{S_2}(x))$$

$\Rightarrow$ for $x = 0$ = 0
for $x = 20$ = 0.5
for $x = 40$ = 0.65
for $x = 60$ = 0.86
for $x = 80$ = 1
for $x = 100$ = 1

$= \left\{ \frac{0}{0} + \frac{0.5}{20} + \frac{0.65}{40} + \frac{0.86}{60} + \frac{1}{80} + \frac{1}{100} \right\}$

$\Rightarrow (S_1 \cup S_2)_\lambda = \{20, 40, 60, 80, 100\}$



ans same

(3) $\tilde{S_1} = \tilde{\mu_s}(x) = 1 - \mu_s(x)$ (4) $\tilde{S_2} \rightarrow$ Same Procedure

$\tilde{S_1} = \left\{ \frac{1-0}{0} + \frac{1-0.5}{20} + \frac{1-0.65}{40} + \frac{1-0.85}{60} + \frac{1-1}{80} + \frac{1-1}{100} \right\}$

$\tilde{S_1} = \left\{ \frac{1}{0} + \frac{0.5}{20} + \frac{0.35}{40} + \frac{0.15}{60} + \frac{0}{80} + \frac{0}{100} \right\}$

$\Rightarrow (\tilde{S_1})_\lambda = \{0, 20\}$

(5) $\widetilde{(S_1 \cup S_2)} = \left\{ \frac{1-0}{0} + \frac{1-0.5}{20} + \frac{1-0.65}{40} + \frac{1-0.86}{60} + \frac{1-1}{80} + \frac{1-1}{100} \right\}$

$\widetilde{(S_1 \cup S_2)} = \left\{ \frac{1}{0} + \frac{0.5}{20} + \frac{0.35}{40} + \frac{0.14}{60} + \frac{0}{80} + \frac{0}{100} \right\}$

$\widetilde{(S_1 \cup S_2)}_\lambda = \{0, 20\}$

(6) $\widetilde{(S_1 \cap S_2)}_\lambda \Rightarrow$ Same Procedure $= \{0, 20\}$

---

12. Define fuzzification and defuzzification. List the different methods of membership value of assignment. in detail

- Fuzzification and defuzzification are two fundamental processes in fuzzy logic systems.
- These processes help in converting crisp values to fuzzy values and vice versa, enabling the implementation of fuzzy inference systems in real-world applications.

1. Fuzzification

Definition:

- Fuzzification is the process of converting a crisp (precise) numerical value into a fuzzy value, represented by membership functions in a fuzzy set.
- It allows the system to handle uncertainty and vagueness in real-world problems.

Steps in Fuzzification:

1. **Identify Input Variables:** Determine the crisp input values from sensors or user inputs.
2. **Define Universe of Discourse:** Establish the range of values for the input variable.
3. **Select Membership Functions:** Assign appropriate membership functions (triangular, trapezoidal, Gaussian, etc.).
4. **Compute Membership Degree:** Determine the degree of belonging of the input value to different fuzzy sets.

Example of Fuzzification:

Consider a temperature control system where the input temperature is **30°C**. The fuzzy sets can be defined as:

- **Cold:** $\mu(30) = 0.2$
- **Warm:** $\mu(30) = 0.7$
- **Hot:** $\mu(30) = 0.1$

Here, the crisp value (30°C) is converted into fuzzy values using membership functions.

2. Defuzzification

Definition:

- Defuzzification is the process of converting fuzzy values back into a crisp value to obtain a final output in a decision-making system.
- This is necessary to make practical, real-world decisions.

Steps in Defuzzification:

1. **Obtain Fuzzy Output:** Apply fuzzy inference rules to get the fuzzy output.
2. **Apply a Defuzzification Method:** Use an appropriate defuzzification technique to convert the fuzzy output into a crisp value.

Example of Defuzzification:

In a fuzzy temperature control system, if the fuzzy output suggests:

- **Low Fan Speed:** $\mu = 0.3$
- **Medium Fan Speed:** $\mu = 0.6$
- **High Fan Speed:** $\mu = 0.1$

A defuzzification method, such as the **Centroid Method**, is applied to calculate the final crisp fan speed.

3. Different Methods of Membership Value Assignment

Membership values in a fuzzy system define how much an element belongs to a fuzzy set. There are various ways to assign these values:

(i) Intuition-Based Assignment

- Membership values are assigned based on expert knowledge and human intuition.
- Example: In an air conditioning system, a temperature of **30°C** can intuitively belong **70% to warm** and **30% to hot**.

(ii) Statistical Methods

- Data-driven methods are used to determine membership values from historical data.
- Example: If a survey reveals that most people feel comfortable at 25°C, the membership function for “Comfortable” can be statistically adjusted.

(iii) Learning-Based Methods

- Machine learning techniques like neural networks or clustering (e.g., Fuzzy C-Means) are used to learn membership values from data.
- Example: A fuzzy classifier trained on customer purchase data can assign membership values for "High Spending" and "Low Spending" customers.

(iv) Expert Systems

- Experts define the membership values based on domain knowledge and experience.

- Example: In a medical diagnosis system, doctors define fuzzy sets like "High Blood Pressure" based on patient records.

(v) Histogram-Based Methods

- Membership functions are constructed based on frequency distributions of collected data.
- Example: If a dataset of student grades shows that most students score between **40-60**, the fuzzy set "Average Score" can be defined accordingly.

---

13. Methods of Defuzzification

- Defuzzification is the process of converting a fuzzy output into a single crisp value in a fuzzy logic system.
- There are several methods to achieve this, depending on the application and required accuracy.

Below are the most commonly used defuzzification methods:

1. Centroid Method (Center of Gravity - COG)

- **Definition:** It finds the center of area under the fuzzy set curve.
- **Formula: a**re the output values andμi\mu_iμiare their corresponding membership degrees.
- **Example:**
  If a fuzzy set has values:
  - (10,0.3)(10, 0.3)(10,0.3)
    (20,0.6)(20, 0.6)(20,0.6)
    (30,0.9)(30, 0.9)(30,0.9)
    then the centroid method calculates a weighted average of these values.
- **Pros:** Provides a smooth and balanced result.
- **Cons:** Computationally expensive.

---

2. Mean of Maximum (MOM)

- **Definition:** It takes the average of all values that have the highest membership degree.
- **Formula:**

- **Example:**

  If the highest membership degree is **0.9**, and the corresponding values are **20 and 30**, then:

  $Z* = 20 + 30/2 = 25$

- **Pros:** Simple and easy to compute.
- **Cons:** Can give multiple outputs if there are multiple max values.

---

3. Bisector of Area (BOA)

- **Definition:** It finds the vertical line that divides the fuzzy set into two equal areas.
- **Example:**

  If a fuzzy set has two peaks at **20 and 30**, the bisector might be at **25**.
- **Pros:** Useful when a balanced decision is needed.
- **Cons:** Computationally complex.

14,15 . Features of Membership Function

Definition

- A **membership function (MF)** in fuzzy logic defines the degree to which a given input belongs to a fuzzy set.
- It is a mathematical representation that maps each input value to a membership value between **0 and 1**.

Characteristics of Membership Functions

A membership function must satisfy the following conditions:

1. **Range:** The membership value should always be between **0 and 1**.
2. **Boundary Values:** At least one element should have a membership value of **1** (fully belonging to the set).
3. **Continuity:** The function can be continuous or discrete, depending on the application.

Key Features of Membership Functions

1. Shape of the Membership Function

- Different shapes of membership functions can be used, such as:
  - **Triangular**
  - **Trapezoidal**
  - **Gaussian**
  - **Sigmoidal**
  - **Bell-shaped**
- The choice of shape depends on the problem requirements.

2. Membership Value (Degree of Membership)

- The function assigns a value between **0 and 1** to represent the degree of belonging.
- Example: If **Temperature = 30°C**, the membership values might be:
  - **Cold:** 0.2
  - **Warm:** 0.7
  - **Hot:** 0.1

3. Universality (Support and Core)

- **Support:** The range where the membership function is nonzero.
- **Core:** The region where the membership function has a value of **1**.

4. Normalization

- A membership function is **normalized** if at least one element has a membership value of **1**.

5. Fuzziness

- It helps in handling **uncertainty** by allowing gradual transition between membership levels.

Types of Membership Functions

Below are common types of membership functions with their mathematical representations:

1. Triangular Membership Function

- Defined by three points (a, b, c).
- Formula:

$$\mu(x) = \begin{cases} 0, & x \leq a \text{ or } x \geq c \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \end{cases}$$

- **Example Diagram:**

```css
      /\
     /  \
    /    \
---a------b------c---
```

2. Trapezoidal Membership Function

- Defined by four points (a, b, c, d).
- Formula:

$$\mu(x) = \begin{cases} 0, & x \leq a \text{ or } x \geq d \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \end{cases}$$

- **Example Diagram:**

```css
     ______
    /      \
   /        \
--a--b---c---d--
```

3. Gaussian Membership Function

- Defined by a mean **m** and standard deviation **σ**.
- Formula:

$$\mu(x) = e^{-\frac{(x-m)^2}{2\sigma^2}}$$

- **Example Diagram:**

```css
      ___
     /   \
    /     \
-----m---------
```

4. Sigmoidal Membership Function

- Defined by a steepness parameter **α** and midpoint **m**.
- Formula:

$$\mu(x) = \frac{1}{1 + e^{-\alpha(x-m)}}$$

- **Example Diagram:**

```css
     ______
    /
   /
  /
--m---
```

Applications of Membership Functions

- **Fuzzy Control Systems:** Air conditioning, washing machines.
- **Medical Diagnosis:** Assigning disease risk levels.
- **Robotics:** Navigation and obstacle avoidance.
- **Artificial Intelligence:** Decision-making in expert systems.

---

16. Using your own intuition and definition of universe of discourse, plot fuzzy membership functions for “weights of people.” in detail

Fuzzy Membership Functions for "Weights of People"

1. Understanding Universe of Discourse

The **universe of discourse** is the complete range of values a variable can take. For "weights of people," we define the universe of discourse as:

U=[30,150] kgU = [30, 150] \text{ kg}

This range represents a reasonable spread of human weights, from **very light (30 kg)** to **very heavy (150 kg)**.

---

2. Defining Fuzzy Sets

We divide weight into five fuzzy categories:

- **Very Light (VL)**
- **Light (L)**
- **Medium (M)**
- **Heavy (H)**
- **Very Heavy (VH)**

Each category is defined by a **membership function** that assigns a weight value a degree of belonging.

---

3. Membership Functions and Equations

We use **trapezoidal and triangular** membership functions for simplicity.

a) Very light (VL)

→ People weight 30-45 Kg belong to this category

→ Trapezoidal function is used

$$\mu_{VL}(x) = \begin{cases} 1 & x \leq 35 \\ \frac{45-x}{10} & 35 \leq x \leq 45 \\ 0 & x \geq 45 \end{cases}$$

b) Light (L)

→ People b/w 40-60 Kg

→ Triangular function

$$\mu_{L}(x) = \begin{cases} \frac{x-40}{10}, & 40 \leq x \leq 50 \\ \frac{60-x}{10}, & 50 \leq x \leq 60 \\ 0, & \text{otherwise} \end{cases}$$

c) Medium (M)

→ people b/w 55 - 80kg

→ Triangular funct^n used

$$\mu_{M}(x) = \begin{cases} \frac{x-55}{10}, & 55 \leq x \leq 65 \\ \frac{80-x}{15}, & 65 \leq x \leq 80 \\ 0, & \text{otherwise} \end{cases}$$

d) Heavy (H)

→ People b/w 75-110 kg

→ Triangular function

$$\mu_{H}(x) = \begin{cases} \frac{x-75}{10}, & 75 \leq x \leq 85 \\ \frac{110-x}{25}, & 85 \leq x \leq 110 \\ 0, & \text{otherwise} \end{cases}$$

e) Very Heavy → above 100

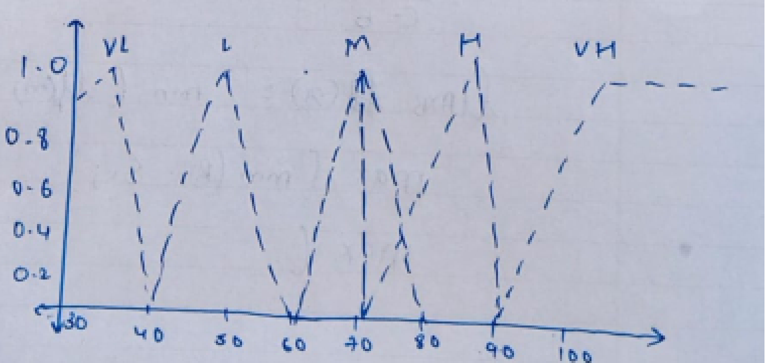
→ Trapezoidal funct^n

$$\mu_{VH}(x) = \begin{cases} \frac{x-100}{20}, & 100 \leq x \leq 120 \\ 1, & x \geq 120 \end{cases}$$

*) Graphical Rep

x → Weight

y → M. Value (0 to 1)



5. Interpretation

- If a person weighs **42 kg**, they belong **mostly to "Light" but slightly to "Very Light".**
- If someone weighs **90 kg**, they are **mostly "Heavy" but slightly "Very Heavy".**
- **Overlapping regions** ensure smooth transitions between weight categories.

17. Illustrate Max membership principle and centroid method of defuzzification with suitable diagram.

Defuzzification is the process of converting a fuzzy set into a **crisp (precise) value**. Two important methods used for defuzzification are:

1. **Max Membership Principle (MM)**
2. **Centroid Method (COG - Center of Gravity)**

Let's explore both methods with examples and diagrams.

---

1. Max Membership Principle (MM)

Definition

- Also known as **Maximum Criterion** or **Height Method**.
- The **crisp output** is the value that has the **highest degree of membership** in the fuzzy set.
- If multiple values have the same maximum membership, **any of them can be chosen arbitrarily** or the average can be taken.

Formula

$x*=\arg\max \mu A(x)$

where **$x*$** is the defuzzified output, and **$\mu A(x)$** is the membership function.

Example

Suppose a fuzzy output set represents the temperature of a system:

Temperature (°C)	20	30	40	50	60
Membership (μ)	0.2	0.5	**0.9**	0.7	0.3

- The maximum membership value is **0.9**, occurring at **40°C**.

- So, **defuzzified output = 40°C**.

Result: 40°C (Max Membership)

Advantages

✔ Simple and fast

✔ Works well if there is a single maximum point

Disadvantages

❌ Not suitable for complex distributions

❌ Ignores the shape of the membership function

```
1.0 |
    |            /\
0.9 |------------/  \
0.8 |
0.7 |----------/      \
0.6 |
0.5 |------/            \
0.4 |
0.3 |---/                  \
0.2 |-/
0.1 |
0.0 --------------------------------
     20    30    40    50    60
```

Result: 40°C (Max Membership)

---

2. Centroid Method (COG - Center of Gravity)

Definition

- The most **widely used** and **accurate** defuzzification method.

- Computes the **center of mass** of the fuzzy region.
- It considers **all membership values** and balances them like a **seesaw**.

Formula

$x* = \sum xi \cdot \mu A(xi) / \sum \mu A(xi)$

where:

- $x*$= Defuzzified value
- xi= Discrete values of the universe
- $\mu A(xi)$= Corresponding membership values

Example

Consider the same fuzzy set:

Temperature (°C)	20	30	40	50	60
Membership (μ)	0.2	0.5	0.9	0.7	0.3

Result: 41.54°C (Center of Gravity)

Advantages

✔ Uses all available data

✔ More **accurate** than Max Membership

Disadvantages

❌ **Computationally complex**

❌ More **time-consuming**

---

18. Center of Largest Area (CoLA) and First of Maxima (FoM) Defuzzification Methods

Defuzzification is the process of obtaining a **crisp** value from a fuzzy set. Two popular methods for defuzzification are:

1. **Center of Largest Area (CoLA)**
2. **First of Maxima (FoM)**

1. Center of Largest Area (CoLA) Method

Definition

- In **CoLA**, the fuzzy set is divided into multiple **sub-areas**.
- The **largest** area among the fuzzy subsets is **identified**.
- The defuzzified output is the **centroid** of that largest area.

Formula

If the fuzzy set is divided into regions **A1,A2,A3,…**, we calculate:

where:

- x^*= Defuzzified output (centroid of the largest area)
- x_i= Discrete values of the universe of discourse
- $\mu A(x_i)$= Membership values of correspondingxix_ixi

Example

Consider a fuzzy set representing **room temperature preferences** with the following values:

Temperature (°C)	20	30	40	50	60
Membership (μ)	0.2	0.6	0.9	0.9	0.5

- The **largest area** is between **30°C to 50°C**.
- The centroid of this region is calculated.

x∗=(30×0.6)+(40×0.9)+(50×0.9) / 0.6+0.9+0.9

x∗=18+36+452.4=992.4=41.25°Cx^* = \frac{18 + 36 + 45}{2.4} = \frac{99}{2.4} = 41.25°Cx∗=2.418+36+45 =2.499=41.25°C

20. llustration of Intuition and Inference of Fuzzification

1. Introduction to Fuzzification

Fuzzification is the process of transforming **crisp input values** into **fuzzy values** by mapping them into **fuzzy sets** using membership functions. It helps in dealing with uncertainty and imprecise data in real-world applications like **control systems, decision-making, and artificial intelligence**.

Example of Fuzzification

Suppose we want to classify **temperature** into three categories:

- **Cold**
- **Warm**
- **Hot**

A crisp temperature value (e.g., **30°C**) does not explicitly fall into any one of these categories—it could be slightly warm or moderately hot. **Fuzzification assigns degrees of membership** to each category.

---

2. Intuition of Fuzzification

In **classical (crisp) logic**, a value either **belongs to a set or does not** (binary: 0 or 1). However, in fuzzy logic, a value **partially belongs** to multiple sets.

Real-World Example: Weather Perception

Imagine asking **three different people** if 30°C is hot:

1. **Person A** (from a cold country) says, "30°C is **very hot**."
2. **Person B** (from a temperate region) says, "30°C is **moderate**."
3. **Person C** (from a desert) says, "30°C is **cool**."

This **subjective perception** of temperature motivates **fuzzification**, which assigns **partial degrees of truth** to different sets.

---

3. Membership Function for Fuzzification

A **membership function (MF)** assigns a degree of membership μ\muμ(ranging from**0 to 1**) to each value.

For **temperature classification**, a **triangular membership function** can be defined as:

Membership Functions for Temperature

Cold(T)={1,T≤1020−T10,10<T<200,T≥20\text{Cold}(T) = \begin{cases} 1, & T \leq 10 \\\frac{20 - T}{10}, & 10 < T < 20 \\0, & T \geq 20\end{cases}
At **T = 30°C**:

- **Cold** membership μ=0\mu = 0μ=0
- **Warm** membership μ=0.5\mu = 0.5μ=0.5
- **Hot** membership μ=0.5\mu = 0.5μ=0.5

This means **30°C is equally warm and hot** in fuzzy logic.

---

4. Inference of Fuzzification

Once data is **fuzzified**, we use it in a **fuzzy inference system (FIS)** to make decisions. The two major inference methods are:

a) Mamdani Fuzzy Inference Method

- Uses fuzzy **IF-THEN rules** to infer results.
- Example rule:
  - **IF** temperature is hot **THEN** fan speed is high.
  - **IF** temperature is warm **THEN** fan speed is medium.

Example Rule Application for T = 30°C:

- Warm: 0.5 → Fan speed = Medium (50%)
- Hot: 0.5 → Fan speed = High (75%)
- **Final speed** = weighted average of both.

b) Sugeno Fuzzy Inference Method

- Instead of outputting a fuzzy value, **it generates a crisp number**.
- Example:

- **IF** temperature is hot, **THEN fan speed = 1.2 × T + 10**
- **IF** temperature is warm, **THEN fan speed = 0.8 × T + 5**

For **T = 30°C**, the inferred speed would be:

(0.5×(1.2×30+10))+(0.5×(0.8×30+5))0.5+0.5\frac{(0.5 \times (1.2 \times 30 + 10)) + (0.5 \times (0.8 \times 30 + 5))}{0.5 + 0.5}0.5+0.5(0.5×(1.2×30+10))+(0.5×(0.8×30+5))=(0.5×46)+(0.5×29)1=23+14.51=37.5= \frac{(0.5 \times 46) + (0.5 \times 29)}{1} = \frac{23 + 14.5}{1} = 37.5=1(0.5×46)+(0.5×29)=123+14.5=37.5

So, **Fan speed = 37.5%**.