

**CENG 222**  
**Probability and Statistics**  
**HOMEWORK 4&5**  
**Method of Moments and Maximum Likelihood Estimation**

**DUE DATE: 21.05.2022 - 23:55**

You are expected to implement the following experiments in Python and answer the corresponding questions in a report.

**a) MoM and MLE Estimation**

- In your **report**, estimate the parameter  $\theta$  using Method of Moments (MoM) and Maximum Likelihood Estimation (MLE) for the distribution with the following density:

$$f(x) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Apply your findings to  $X = \{0.3, 0.6, 0.8, 0.9\}$  and calculate the two estimates for this sample set.

- In your **code**, implement the two functions which take a sample set  $X$  and return the calculated estimate of the parameter  $\theta$  using MoM and MLE. Call these two functions to calculate the estimates for the same  $X$  given above and print the results.

**b) Population Generation**

- In your **report**, do the necessary calculations to use the Inverse Transform Method to generate random samples of the given distribution.
- In your **code**, create a population  $P$  of size 10 million which has the same distribution as in (a), using the Inverse Transform Method. Set the parameter  $\theta$  as 2.4.

**c) Experiment Simulation**

- In your **code**, implement a function that takes the population ( $P$ ) and sample size ( $N$ ) as its input, uses 100000 samples of size  $N$  from the population  $P$  to calculate MoM and MLE estimates of the parameter  $\theta$  from. Use `np.random.random_integers` to create indices for random sampling. After plotting the histograms of both estimators in a single figure (use 100 bins and  $\alpha=0.5$ ), it returns the mean and the variance of the two estimators.

Call the implemented function for  $N = [1, 2, 3, 4, 5, 10, 50, 100, 500, 1000]$ . Print the estimator mean and variances for each  $N$ .

- In your **report**, comment on the histograms and the findings. How does the numbers change with respect to  $N$ ? Which estimator would you prefer? Why?

**Submission Rules:**

1. You should submit your assignments through TEAMS until due date.
2. You have to submit one zipped file including one file for your code (**py**) and one file for your report (**pdf**).
3. Your homework should be named as **CENG222\_HW4-5\_studentID.zip**
4. Write your **student ID** both in your code and report files.