

$$1.) X = \{0.3, 0.6, 0.8, 0.9\}$$

$$MOM: f(x) = \begin{cases} \frac{2 \cdot \theta^2}{x^3}, & \theta \leq x < \infty \\ 0, & \text{otherwise} \end{cases}$$

$$M_1 = E[x] = \int_{\theta}^{\infty} x \cdot f(x) dx = \int_{\theta}^{\infty} \frac{2\theta^2}{x^2} dx = 2\theta^2 \int_{\theta}^{\infty} \frac{1}{x^2} dx = 0 - (-2\theta) = 2\theta$$

$$m_1 = \bar{x} = \frac{0.3 + 0.6 + 0.8 + 0.9}{4} = 0.65$$

$$m_1 = M_1 \Rightarrow 0.65 = 2\theta$$

$$\hat{\theta} = 0.325$$

$$MLE: \prod_{i=1}^4 \frac{2\theta^2}{x_i^3}$$

$$\ln f(x) = \sum_{i=1}^4 \ln\left(\frac{2\theta^2}{x_i^3}\right) = \sum_{i=1}^4 (\ln 2\theta^2 - \ln x_i^3)$$

$$= 4 \ln(2\theta^2) - 3 \cdot (\ln x_1 + \ln x_2 + \ln x_3 + \ln x_4)$$

$$= 4 \ln(2\theta^2) - 3 \cdot (\ln x_1 \cdot x_2 \cdot x_3 \cdot x_4)$$

$$\frac{d \ln f(x)}{d \theta} = \frac{8}{\theta}$$

When x becomes zero, it reaches its maximum value. 0.3 chosen because 0.3 is closer to 0 than 0.9.

$$2.) f(x) = \begin{cases} \frac{2 \cdot \theta^2}{x^3}, & \theta \leq x < \infty \\ 0, & \text{otherwise} \end{cases}$$

we need $F(x)$.

$$F(x) = \int_{\theta}^x f(x) dx = \int_{\theta}^x \frac{2 \cdot \theta^2}{x^3} dx \Rightarrow -\frac{\theta^2}{x^2} + 1$$

$$F(x) = -\frac{\theta^2}{x^2} + 1$$

$$F(x)^{-1} = \frac{\theta}{\sqrt{1-x}}$$

$$= \boxed{\frac{2 \cdot 4}{\sqrt{1-x}}}$$

If θ is negative, it comes before $-$, but it is known that θ is 2.4.

3.) I create 2 arrays, one inside and one outside. In the loop, we generate 100 000 samples from random numbers. We add them to the array of uniform function we produced. Then we find their std and mean.

- As N increased, the values of the MoM function increased, and the values of MLE function decreased.
- I prefer the MLE function. Because maximum likelihood estimators have a higher probability of being close to the quantities to be estimated.