Appendix

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A. Probability of misdetection for $N_T > 2$ agents

We employ a similar model to 2 agent case:

$$I_{i,t} = I_{i,t,\text{true}} + n_{i,t}, \tag{1}$$

where $n_{i,t} \sim N(0, \sigma_u^2)$ if the state is U, and $n_{i,t} \sim N(0, \sigma_c^2)$ if the state is C, and $\sigma_u > \sigma_c$, where $I_{i,t}$ is the noisy ARSS measurement of agent i due to all other agents. The joint state is C if there is at least one agent i such that $I_{i,t} > I_{thr}$. We define the probability of error as follows:

$$\begin{aligned} P_{\text{err}} &= P(\text{detect uncoor} \mid \text{actual coor})P(\text{actual coor}) + P(\text{detect coor} \mid \text{actual uncoor})P(\text{actual uncoor}). && (2)\\ &= P(\text{detect uncoor} \mid \text{actual coor})P(\text{actual coor}) + (1 - P(\text{detect uncoor} \mid \text{actual uncoor}))P(\text{actual uncoor}), && (3) \end{aligned}$$

where $P(\text{actual coor}) = (1-\alpha) = \frac{|\mathcal{S}_c|}{|\mathcal{S}|}$ and $P(\text{actual uncoor}) = \alpha = \frac{|\mathcal{S}_u|}{|\mathcal{S}|}$.

We first want to find a lower bound on P_{err}. To do so, we first find a lower bound on $P(\text{detect uncoor} \mid \text{actual coor})$:

$$\begin{aligned} P(\text{detect uncoor} \mid \text{actual coor}) &= P(\max_i\{I_{i,t}\} < I_{thr} \mid \text{actual coor}). && (4)\\ &\geq 1 - \sum_{i=1}^{N_T} P(I_{i,t} > I_{thr} \mid \text{actual coor}). && (5)\\ &= 1 - \sum_{i=1}^{N_T} Q\Big(\frac{I_{thr} - I_{i,t,true}}{\sigma_c}\Big) && (6) \end{aligned}$$

where (4) follows from the criteria for detecting coordination, (5) follows from Boole's inequality and (6) follows from (1). We next find an upper bound on $P(\text{detect uncoor} \mid \text{actual uncoor})$:

$$\begin{aligned} P(\text{detect uncoor} \mid \text{actual uncoor}) &= P(\max_i\{I_{i,t}\} < I_{thr} \mid \text{actual uncoor}). && (7)\\ &= P(I_{i,1} < I_{thr}, I_{i,2} < I_{thr}, ... \mid \text{actual uncoor}). && (8)\\ &\leq \min_i P(I_{i,i} < I_{thr} \mid \text{actual uncoor}). && (9)\\ &= 1 - \min_i Q\Big(\frac{I_{thr} - I_{i,t,true}}{\sigma_u}\Big). && (10)\\ &= 1 - Q\Big(\frac{I_{thr} - I_{i',t,true}}{\sigma_u}\Big), && (11) \end{aligned}$$

where (7) follows from the criteria for detecting coordination, (8) and (9) follow from the properties of probability, (10) follows from (1), (11) follows from the notation that $I_{i',t,true} = \min_{i=1,\ldots,N} I_{i,t,true}$. By combining (6) and (11), we obtain:

$$P_{err} = f(I_{thr}) \geq \Big(1 - \sum_{i=1}^{N_T} Q\Big(\frac{I_{thr} - I_{i,t,true}}{\sigma_c}\Big)\Big)(1-\alpha) + Q\Big(\frac{I_{thr} - I_{i',t,true}}{\sigma_u}\Big)\alpha, \tag{12}$$

We use the second-order Taylor expansion of the Q-function around x = 0:

$$Q(x) = \frac{1}{2} - \frac{x}{\sqrt{2\pi}} - \frac{x^3}{6\sqrt{2\pi}}. \tag{13}$$

Combining (13) with 12, we obtain:

$$f(I_{\text{thr}}) \geq \left(1 - \sum_{i=1}^{N_T} \left(\frac{1}{2} - \frac{I_{\text{thr}} - I_{i,t,\text{true}}}{\sqrt{2\pi}\sigma_c} - \frac{(I_{\text{thr}} - I_{i,t,\text{true}})^3}{6\sqrt{2\pi}\sigma_c^3}\right)\right)(1-\alpha) + \left(\frac{1}{2} - \frac{I_{\text{thr}} - I_{i',t,true}}{\sqrt{2\pi}\sigma_u} - \frac{(I_{\text{thr}} - I_{i',t,true})^3}{6\sqrt{2\pi}\sigma_u^3}\right)\alpha. \tag{14}$$

Lets call $I_{i',t,true} = c$. To find the value of I_{thr} that minimizes this term, we take the gradient of RHS of (14) with respect to I_{thr}:

$$\frac{d}{dI_{\text{thr}}} f(I_{\text{thr}}) = (1-\alpha)\sum_{i=1}^{N_T}\left(\frac{1}{\sqrt{2\pi}\sigma_c} + \frac{(I_{\text{thr}} - I_{i,t,\text{true}})^2}{2\sqrt{2\pi}\sigma_c^3}\right) + \alpha\left(-\frac{1}{\sqrt{2\pi}\sigma_u} - \frac{(I_{\text{thr}} - c)^2}{2\sqrt{2\pi}\sigma_u^3}\right). \tag{15}$$

$$= \frac{2(1-\alpha)N}{\sigma_c} + \frac{(1-\alpha)}{\sigma_c^3}\sum_{i=1}^{N_T}(I_{\text{thr}} - I_{i,t,\text{true}})^2 - \frac{2\alpha}{\sigma_u} - \frac{\alpha}{\sigma_u^3}(I_{\text{thr}} - c)^2. \tag{16}$$

$$= \frac{2(1-\alpha)N}{\sigma_c} - \frac{2\alpha}{\sigma_u} + \frac{(1-\alpha)N}{\sigma_c^3} I_{\text{thr}}^2 - \frac{2(1-\alpha)}{\sigma_c^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}} I_{\text{thr}} + \frac{(1-\alpha)}{\sigma_c^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}}^2 - \frac{\alpha}{\sigma_u^3} I_{\text{thr}}^2 + \frac{2\alpha}{\sigma_u^3} I_{\text{thr}} c - \frac{\alpha}{\sigma_u^3} c^2. \tag{17}$$

$$= A I_{\text{thr}}^2 + B I_{\text{thr}} + C, \tag{18}$$

where

$$A = \frac{(1-\alpha)N}{\sigma_c^3} - \frac{\alpha}{\sigma_u^3}. \tag{19}$$

$$B = -\frac{2(1-\alpha)}{\sigma_c^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}} + \frac{2\alpha}{\sigma_u^3} c. \tag{20}$$

$$C = \frac{2(1-\alpha)N}{\sigma_c} - \frac{2\alpha}{\sigma_u} + \frac{(1-\alpha)}{\sigma_c^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}}^2 - \frac{\alpha}{\sigma_u^3} c^2. \tag{21}$$

Herein, the critical points are $I_{thr} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$. One can show that $I^*_{thr} = \frac{-B - \sqrt{B^2-4AC}}{2A}$ is a global minimizer as $\frac{d^2}{dI_{\text{thr}}^2}|_{I_{thr} = I^*_{thr}} > 0$, which follows from $|\mathcal{S}_C|\sigma_u > |\mathcal{S}_U|\sigma_c$, $c < \sum_{i=1}^{N_T} I_{i,t,\text{true}}^2$ and $N > 1$. Thus, the lower bound on P_{err} is given by:

$$P_{err} \geq \Big(1 - \sum_{i=1}^{N_T} Q\Big(\frac{I^*_{thr} - I_{i,t,true}}{\sigma_c}\Big)\Big)\frac{|\mathcal{S}_c|}{|\mathcal{S}|} + Q\Big(\frac{I^*_{thr} - I_{i',t,true}}{\sigma_u}\Big)\frac{|\mathcal{S}_u|}{|\mathcal{S}|}. \tag{22}$$

1) Upper bound: We take a very similar approach to the lower bound. We first find a lower bound on $P(\text{detect uncoor} \mid \text{actual uncoor})$:

$$P(\text{detect uncoor} \mid \text{actual uncoor}) \geq 1 - \sum_{i=1}^{N_T} Q\Big(\frac{I_{thr} - I_{i,t,true}}{\sigma_u}\Big), \tag{23}$$

which follows from (6). We next find the upper bound on $P(\text{detect uncoor} \mid \text{actual coor})$:

$$P(\text{detect uncoor} \mid \text{actual coor}) \leq 1 - Q\Big(\frac{I_{thr} - I_{i',t,true}}{\sigma_c}\Big), \tag{24}$$

which follows from (11). Combining (23) with (24), we obtain:

$$P_{err} = f(I_{thr}) \leq \sum_{i=1}^{N_T} Q\Big(\frac{I_{thr} - I_{i,t,true}}{\sigma_u}\Big)\alpha + \Big(1 - Q\Big(\frac{I_{thr} - I_{i',t,true}}{\sigma_c}\Big)\Big)(1-\alpha). \tag{25}$$

If we use (13) in (25), and take the gradient with respect to I_{thr}, we obtain:

$$\frac{d}{dI_{\text{thr}}} f(I_{\text{thr}}) = \tilde{A} I_{\text{thr}}^2 + \tilde{B} I_{\text{thr}} + \tilde{C}, \tag{26}$$

where

$$\tilde{A} = \frac{-\alpha N}{\sigma_u^3} + \frac{(1-\alpha)}{\sigma_c^3} \tag{27}$$

$$\tilde{B} = \frac{2\alpha}{\sigma_u^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}} - \frac{2(1-\alpha)}{\sigma_c^3} c \tag{28}$$

$$\tilde{C} = \frac{-2\alpha N}{\sigma_u} + \frac{2(1-\alpha)}{\sigma_c} - \frac{\alpha}{\sigma_u^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}}^2 + \frac{(1-\alpha)}{\sigma_c^3} c^2 \tag{29}$$

One can show that $I_{thr}^{**} = \frac{-\tilde{B}+\sqrt{\tilde{B}^2-4\tilde{A}\tilde{C}}}{2\tilde{A}}$ is a global maximizer as $\frac{d^2}{dI_{\text{thr}}^2}|_{I_{thr}=I_{thr}^{**}} < 0$. Thus, the upper bound on P_{err} is given by:

$$P_{err} \leq \sum_{i=1}^{N_T} Q\Big(\frac{I_{thr}^{**} - I_{i,t,true}}{\sigma_u}\Big)\frac{|\mathcal{S}_u|}{|\mathcal{S}|} + \Big(1 - Q\Big(\frac{I_{thr}^{**} - I_{i',t,true}}{\sigma_c}\Big)\Big)\frac{|\mathcal{S}_c|}{|\mathcal{S}|}. \tag{30}$$

Combining (22) and (30), we can bound P_{err} for arbitrary N agents:

$$P_{err} \in \Bigg[\Big(1 - \sum_{i=1}^{N_T} Q\Big(\frac{I_{thr}^{*} - I_{i,t,true}}{\sigma_c}\Big)\Big)\frac{|\mathcal{S}_c|}{|\mathcal{S}|} + Q\Big(\frac{I_{thr}^{*} - I_{i',t,true}}{\sigma_u}\Big)\frac{|\mathcal{S}_u|}{|\mathcal{S}|}, \tag{31}$$

$$\sum_{i=1}^{N_T} Q\Big(\frac{I_{thr}^{**} - I_{i,t,true}}{\sigma_u}\Big)\frac{|\mathcal{S}_u|}{|\mathcal{S}|} + \Big(1 - Q\Big(\frac{I_{thr}^{**} - I_{i',t,true}}{\sigma_c}\Big)\Big)\frac{|\mathcal{S}_c|}{|\mathcal{S}|}\Bigg], \tag{32}$$

where

$$I_{thr}^{*} = \frac{-B - \sqrt{B^2 - 4AC}}{2A}. \tag{33}$$

$$I_{thr}^{**} = \frac{-\tilde{B} + \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}}{2\tilde{A}}. \tag{34}$$

$$A = \frac{(1-\alpha)N}{\sigma_c^3} - \frac{\alpha}{\sigma_u^3}. \tag{35}$$

$$B = -\frac{2(1-\alpha)}{\sigma_c^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}} + \frac{2\alpha}{\sigma_u^3} I_{i',t,true}. \tag{36}$$

$$C = \frac{2(1-\alpha)N}{\sigma_c} - \frac{2\alpha}{\sigma_u} + \frac{(1-\alpha)}{\sigma_c^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}}^2 - \frac{\alpha}{\sigma_u^3} I_{i',t,true}^2. \tag{37}$$

$$\tilde{A} = \frac{-\alpha N}{\sigma_u^3} + \frac{(1-\alpha)}{\sigma_c^3}. \tag{38}$$

$$\tilde{B} = \frac{2\alpha}{\sigma_u^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}} - \frac{2(1-\alpha)}{\sigma_c^3} I_{i',t,true}. \tag{39}$$

$$\tilde{C} = \frac{-2\alpha N}{\sigma_u} + \frac{2(1-\alpha)}{\sigma_c} - \frac{\alpha}{\sigma_u^3}\sum_{i=1}^{N_T} I_{i,t,\text{true}}^2 + \frac{(1-\alpha)}{\sigma_c^3} I_{i',t,true}^2. \tag{40}$$

$$I_{i',t,true} = \min_{i=1,\ldots,N} I_{i,t,true}. \tag{41}$$